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The Stahl–Alexeev Paradox: A Note[1]

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We offer a simple proof of the paradox discovered by Stahl and Alexeev that the introduction of black markets in a fixed price economy with a rationing mechanism consisting of waiting line queues is not necessarily a Pareto improvement. We show that the paradoxical result arises from the fact that queue length is endogenous and illustrates the general principle that permitting an additional market may be undesirable in an economy which has other distortions. *Journal of Economic Literature* Classification Number: 027.

Recently, in this journal, Stahl and Alexeev [1] considered a centrally planned economy with some fixed prices and a rationing mechanism consisting of waiting line queues. They introduce a "black market" in which individuals can trade commodities that they have acquired through the official economy. They discover, quite contrary to intuition, that the introduction of black markets is not necessarily a Pareto improvement. Here, we offer a simpler proof of their proposition, which involves only two types of agent, no compensation payments, and easier arithmetic.

Consider an economy made up of 100 Giants, 100 Midgets, some geese, and 101 golden eggs to be rationed by queue, with a maximum of one egg to be given away per queuer. Giants and Midgets both have additive utility

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functions in leisure, golden eggs and geese with constant marginal utilities of geese. They value leisure at 4 geese/hr and 1 goose/hr, respectively. Giants value golden eggs at 4 geese/egg. Midgets value their first egg at 2 geese, but golden eggs being so heavy to carry and geese being able to walk or fly, they value additional eggs at zero.

If eggs cannot be resold, equilibrium will occur where the queue is one hour long with all 100 Midgets standing in it alongside one Giant. Each Midget will reap 2 geese worth of benefit as a consumer and sacrifice one goose worth of leisure for a net gain of one goose of consumer–queuer surplus and the lone Giant will sacrifice 4 geese worth of leisure in exchange for 4 geese worth of egg for a zero net gain. Thus, the net social gain from the egg give-away is 100 geese.

Opening up a black market will result in Giants hiring Midgets to stand in queues for them. Since there are 101 eggs and only 100 Midgets, we need to postulate two different give-aways, one in the morning and one in the evening with the number of eggs to be given away at each preannounced. Each queue will end up being four hours long. Giants will reap a consumer benefit of 404 geese, all of which will be paid out to the Midgets who queue for them and in the process forgo leisure valued at 404 geese. The social surplus is zero. Thus the introduction of the black market immizerizes Midgets without benefiting Giants.

If, on the other hand, we had postulated that Giants place a zero value on eggs beyond the first one while Midgets value each egg at 2 geese, without the black market there would have been a two-hour queue full of Midgets and no social surplus. With the black market, the length of queues would have stayed at two hours with the Giants reaping

$$[(4 \text{ geese/egg}) - (1 \text{ goose/Midget-hr})(2 \text{ Midget-hr/egg})]\ 100 \text{ eggs} = 200 \text{ geese}$$

of surplus and the Midgets reaping zero. Thus, in this case introducing the black market benefits Giants while leaving Midget welfare unchanged. In the first case eliminating the black market is Pareto improving even without compensation and in the second case creating a black market is Pareto improving, again even without compensation.

Suppose for each individual there is at least one clone with identical preferences and endowments. If geese and eggs are perfect substitutes for anyone who ends up consuming eggs, and geese and leisure are perfect substitutes for anyone who ends up queuing, then the price of eggs in goose numeraire will settle at the highest price anyone is willing to pay for the first unit. Similarly, the price of leisure in goose numeraire will settle at the lowest reservation wage at which anyone is willing to supply the first unit. Then the existence of the egg give-away will make no one better off. When utility is additive, the only way for anyone to benefit from the eggs is for at least one of those who actually queue to have an upward sloping supply of leisure or for one of those who actually consume to have a downward sloping demand for eggs, where all prices are in goose numeraire. In our case 1 the condition for a gain for anyone is met only without the black market and in case 2 only with it.

In both cases introducing the black market shifts consumption of the queued-for item toward those who value it the most and puts or keeps lowest reservation-wage people in the queue. However, in the first case, these beneficial effects are more than offset by the lengthening of the queue which results from the existence of the black market. Thus the paradoxical result of case 1 arises from the fact that queue length is endogenous. Finally, the paradox serves to illustrate the general principle that permitting an additional market to flourish may be undesirable in an economy which is suffering from other distortions.

Reference

1. D. O. Stahl II and M. Alexeev, The influence of black markets on a queue-rationed centrally planned economy, *J. Econ. Theory* **35** (1985), 234–250.