

Essays in Macroeconomics

by

Christopher James Roark

Department of Economics
Duke University

Date: _____

Approved:

A. Craig Burnside, Supervisor

Cosmin Ilut

Emma Rasiel

Michelle Connolly

Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
2017

ABSTRACT

Essays in Macroeconomics

by

Christopher James Roark

Department of Economics
Duke University

Date: _____

Approved:

A. Craig Burnside, Supervisor

Cosmin Ilut

Emma Rasiel

Michelle Connolly

An abstract of a dissertation submitted in partial fulfillment of the requirements for
the degree of Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
2017

Copyright © 2017 by Christopher James Roark
All rights reserved except the rights granted by the
Creative Commons Attribution-Noncommercial Licence

Abstract

This dissertation will address the dynamic interrelationship between multiple macro-economic fundamentals. The second chapter details a solution to the excess volatility puzzle described in the macro-economic labor literature in which properly calibrated search and matching models are unable to accurately match both the volatility of vacancies and productivity. The model described utilizes agents who abstract from rational expectations via knightian uncertainty in order to break the tight mechanism that links both productivity and vacancies together. The third chapter documents previously unknown long and short run dynamic interrelationships between the real exchange rate and various macro-economic fundamentals. It refutes the Backus-Smith model's assertion that relative consumption growth and the real exchange rate should be tightly correlated and documents a novel relationship between relative investment and output growth. More specifically, it notes that higher relative investment today should signal future appreciations of the exchange rate. The fourth chapter takes the novel result from the third chapter and proposes a model in order to define a mechanism which may explain the interesting interrelationship between investment and the real exchange rate at medium and long horizons. In particular, the model utilizes bonds as a form of collateral in the production of the investment good in order to more tightly link the two fundamentals.

I dedicate this thesis to my dog Hestia, whose unwavering loyalty and affection has sustained me through this endeavor

Contents

Abstract	iv
List of Tables	ix
List of Figures	x
List of Abbreviations and Symbols	xiii
Acknowledgements	xiv
1 Introduction	1
2 Ambiguity and the Shimer Puzzle	5
2.1 Introduction	5
2.2 The Baseline Model	9
2.2.1 Social Planner’s Problem	9
2.2.2 Competitive Market Setup	15
2.3 Introducing Ambiguity to the Model	18
2.3.1 Multiple Priors	19
2.3.2 The Social Planner’s Problem with Ambiguous Agents	20
2.3.3 Solution method	22
2.4 Empirical Implications of the model	24
2.4.1 Calibrations	24
2.4.2 Simulation Method and Results	27
2.5 Conclusion	30

3	Real Exchange Rates: Exploring Fundamentals of the Cross Section	41
3.1	Introduction	41
3.2	Dynamic Correlations	47
3.2.1	Data Creation	48
3.2.2	Correlation Results	49
3.2.3	Panel Regressions	57
3.3	Vector Autoregressions	68
3.3.1	VAR Environment	70
3.3.2	Impulse Response Functions and Variance Decomposition . . .	72
3.3.3	Simulation Results	76
3.3.4	Country Specific Robustness	81
3.4	Conclusion	82
4	Investment Driven Exchange Rate Dynamics	84
4.1	Introduction	84
4.2	Empirical Motivation	86
4.2.1	The International Risk Sharing condition	86
4.2.2	Summary Data	88
4.3	A Small Open Economy Model	90
4.3.1	Model Environment	91
4.3.1.1	Firms	91
4.3.1.1.1	Final Goods	91
4.3.1.1.2	Intermediate Goods	92
4.3.1.2	Households	96
4.3.1.3	Government	99
4.3.2	Equilibrium of the Model	101

4.4 Conclusion	104
5 Conclusion	106
A Data Sources and Construction	108
B Log-Linearization and Solution for Ambiguity and the Shimer Puzzle	111
C Expenditure Minimization Problem, Final Good Producer's Problem and the Intermediate Goods Producer's Problem for Investment Driven Exchange Rate Dynamics	115
D Log-Linearization for Investment Driven Exchange Rate Dynamics	118
Bibliography	123
Biography	128

List of Tables

2.1	Parameter Values for Calibration	24
2.2	Simulated Moments $\gamma = 1$	38
2.3	Counterfactual Simulated Moments $\gamma = 1$	39
2.4	Simulated Moments $\gamma = 0.4$	40
3.1	Contemporaneous Correlations	50
3.2	Variance Decomposition	76
3.3	AIC Comparison values between Benchmark and Country Specific . .	82
4.1	Model Calibrations	104

List of Figures

2.1	Impulse Responses to a one time shock to productivity in baseline model $\gamma = 1$	32
2.2	Impulse Responses to a one time shock to productivity $\gamma = 1$	33
2.3	Impulse Responses to a one time shock to ambiguity, $\gamma = 1$	34
2.4	Impulse Responses to a one time shock to productivity in baseline model $\gamma = 0.4$	35
2.5	Impulse Responses to a one time shock to productivity $\gamma = 0.4$	36
2.6	Impulse Responses to a one time shock to ambiguity $\gamma = 0.4$	37
3.1	Sample of Country Specific Dynamic Cross Correlations $\rho_{\Delta c, \Delta reer}$	51
3.2	Sample of Country Specific Dynamic Cross Correlations $\rho_{\Delta y, \Delta reer}$	51
3.3	Sample of Country Specific Dynamic Cross Correlations $\rho_{\Delta i, \Delta reer}$	52
3.4	Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 1-3 years: $\rho_{c, reer}$	53
3.5	Dynamic Cross Correlation with Output for Individual Countries with periodicities from 1-3 years: $\rho_{y, reer}$	54
3.6	Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 1-3 years: $\rho_{i, reer}$	55
3.7	Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 3-5 years: $\rho_{c, reer}$	56
3.8	Dynamic Cross Correlation with Output for Individual Countries with periodicities from 3-5 years: $\rho_{y, reer}$	57
3.9	Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 3-5 years: $\rho_{i, reer}$	58

3.10	Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 5-10 years: $\rho_{c, reer}$	59
3.11	Dynamic Cross Correlation with Output for Individual Countries with periodicities from 5-10 years: $\rho_{y, reer}$	60
3.12	Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 5-10 years: $\rho_{i, reer}$	61
3.13	Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 1+ years: $\rho_{c, reer}$	62
3.14	Dynamic Cross Correlation with Output for Individual Countries with periodicities from 1+ years: $\rho_{i, reer}$	63
3.15	Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 1+ years: $\rho_{i, reer}$	64
3.16	Estimated β for $\Delta c_{t+k}, \Delta reer_t$	64
3.17	Estimated β for $c_{t+k}, reer_t$ for periodicities from 1 to 3 years	65
3.18	Estimated β for $c_{t+k}, reer_t$ for periodicities from 3 to 5 years	65
3.19	Estimated β for $c_{t+k}, reer_t$ for periodicities from 5 to 10 years	66
3.20	Estimated β for $c_{t+k}, reer_t$ for periodicities greater than 1 year	66
3.21	Estimated β for $\Delta y_{t+k}, \Delta reer_t$	67
3.22	Estimated β for $y_{t+k}, reer_t$ for periodicities from 1 to 3 years	67
3.23	Estimated β for $y_{t+k}, reer_t$ for periodicities from 3 to 5 years	68
3.24	Estimated β for $y_{t+k}, reer_t$ for periodicities from 5 to 10 years	68
3.25	Estimated β for $y_{t+k}, reer_t$ for periodicities greater than 1 year	69
3.26	Estimated β for $\Delta i_{t+k}, \Delta reer_t$	69
3.27	Estimated β for $i_{t+k}, reer_t$ for periodicities from 1 to 3 years	70
3.28	Estimated β for $i_{t+k}, reer_t$ for periodicities from 3 to 5 years	70
3.29	Estimated β for $i_{t+k}, reer_t$ for periodicities from 5 to 10 years	71
3.30	Estimated β for $i_{t+k}, reer_t$ for periodicities greater than 1 year	71
3.31	Impulse Responses to a 1 Standard Deviation Monetary Policy Shock	72

3.32	Impulse Responses to a 1 Standard Deviation Orthogonalized Output Shock	74
3.33	Impulse Responses to a 1 Standard Deviation Orthogonalized Consumption Shock	74
3.34	Impulse Responses to a 1 Standard Deviation Investment Shock	75
3.35	Impulse Responses to a 1 Standard Deviation Orthogonalized Real Exchange Rate Shock	75
3.36	Dynamic Cross Correlations for Raw Simulated Data	76
3.37	Dynamic Cross Correlations for Simulated band-pass 3-5 year Data	77
3.38	Median Simulated Cross Correlations with Y shock only	78
3.39	Median Simulated Cross Correlations with C shock only	79
3.40	Median Simulated Cross Correlations with I shock only	79
3.41	Median Simulated Cross Correlations with MP shock only	80
3.42	Median Simulated Cross Correlations with REER shock only	81
4.1	Spectrograms for Japan, US, UK and Norway for REER, REC, REI and REY	89
4.2	Standardized HP filtered data with smoothing parameter $\lambda = 100000$ for US, UK and Japan	89
4.3	Standardized HP filtered data with smoothing parameter $\lambda = 100000$ for Norway	90

List of Abbreviations and Symbols

Abbreviations

IMRS	Intertemporal Marginal Rate of Substitution .
VAR	Vector Autoregression
REER	Real Effective Exchange Rate
RER	Real Exchange Rate
CPI	Consumer Price Index
HP Filter	Hoderick-Prescott Filter
OECD	Organization for Economic Cooperation and Development
REC	Relative Real Effective Consumption
REY	Relative Real Effective Output
REI	Relative Real Effective Investment
CES	Constant Elasticity of Substitution

Acknowledgements

I first would like to thank my family, namely my mother, father and Aunt Catherine for their unending support. Without their constant encouragement I never would have made this journey. I would next like to thank my friends in particular; Dr. Nick Ingwersen, Dr. Dale Payne, Dr. Kalina Staub, John Herlin, and Stephanie Rogers esq., without whose constant support and encouragement through graduate school I would never have been able to complete this paper. I would next like to thank my fellow graduate students, Dr. Rosen Valchev, Adam Bergeron and Vasco de Castro Bothelho, who have provided me with many hours of capable listening ears as I developed the ideas in this paper. I would also like to thank the members of my committee, Dr. Emma Rasiel, Dr. Cosmin Ilut and Dr. Michelle Connolly, for being on my committee and giving me valuable feedback and input throughout the process. Finally I would like to thank my advisor, Dr. Craig Burnside. If he had not continued to support and believe in me even in my weakest times I would have long ago lost myself along the way.

1

Introduction

A key feature of macroeconomics is its focus on dynamic relationships between various macro-economic fundamentals. As macroeconomists we hope to glean a smaller understanding of complex economic systems by defining puzzles in the time series and at the same time building structural models in order to explain those puzzles. The use of structural models allows us to perform simple policy experiments that have large and immediate ramifications for policy makers in the public arena. Therefore it behooves us to make sure when designing our models that we try to truly encapsulate and study the various dynamic interrelationships that are present. This dissertation attempts to approach a small part of that problem, looking at both failings of conventional labor market models and international models. Through the course of this work I propose solutions to existing problems and document a few future empirical issues that the literature has so far largely ignored.

The second chapter of this dissertation focuses on cyclical employment fundamentals. In particular the volatility puzzle described by Shimer (2005), also referred to as the Shimer puzzle. The Shimer puzzle itself is described as the inability of a properly calibrated search and matching model to simultaneously match the volatil-

ity of vacancies and productivity in the data. The problem itself lies in the inability of the central mechanism to generate enough movement in the labor market while still matching the weak procyclical productivity. Any sort of solution to this problem must simultaneously generate a significant increase in the action taken by agents in the labor market while still keeping productivity, and therefore the fundamentals of the economy relatively unaffected. I propose an interesting solution to this puzzle in the form of endowing agents with Knightian uncertainty. Similar to the mechanism proposed by Ilut and Schneider (2012). When agents face Knightian uncertainty, this allows for a divergence from a typical rational expectations solution. Instead of having complete knowledge of the exogenous parameters of the model, agents instead form beliefs over a set of priors about these parameter values. This in turn causes agents to instead act with regard to what they believe may be the worst case value of the parameter in question. In the model, this departure from rational expectations is focused on the mean of the exogenous productivity process. Since beliefs themselves are now relatively removed from the realizations of the fundamental technology process agents will engage in search activity under their worst case beliefs about productivity. This in turn generates increased volatility in model implied vacancies relative to the volatility of output.

In the third chapter I explore the cross section of the fundamentals in international macro-economics. Specifically, hoping to better define linkages between fundamental macroeconomic variables such as consumption, output and investment and movements in the real exchange rate. The most famous result of which comes from Backus and Smith (1995), in which their defined model shows a close linkage between the relative consumption growth across countries and the growth of the real exchange rate. Regardless of how you define the model most models come to the conclusion that the change in the real exchange rate is the ratio of stochastic discount factors (SDF) across countries. The finance literature has largely focused

on finding ways to define the SDF and testing the international risk sharing condition through proxies. The international macroeconomic literature also attempts to redefine this risk sharing condition whether that be through long run-risk models¹, recursive preferences over durable and non-durable consumption², or habit formation³. Most of these are using modeling tricks which make it difficult to measure the relevant macroeconomic aggregate in the data and thus can satisfy their redefined risk sharing condition since it is empirically poorly defined. In this chapter I take a departure from this literature instead of testing the risk sharing condition directly I look at the dynamic interrelationship between the macro fundamentals and the real exchange rate. What is unique about this analysis is that I look at a large panel of countries and analyze these relationships where each country is compared against the rest of the panel rather than bilaterally. In addition I explore not only the normal raw data but also look at longer periodicities of the data in order to understand the relationship between long and medium run components of the real exchange rate and the macro-economic fundamentals. I find in my analysis a confirmation of the Backus and Smith conjecture, that consumption growth is not correlated or slightly negatively correlated with real exchange rate growth at all periodicities of the data. I also find a previously unknown relationship between the medium and long run components of relative investment growth and the real exchange rate that has so far been undocumented in the literature.

Finally in the fourth chapter I propose a small open economy model with the intent to propose a possible mechanism for the interesting dynamic linkages described in chapter 3. The proposed model unlike other models in the literature actively includes the investment sector as a key component of the model, in order to reflect

¹ For examples see Bansal and Shaliastovich (2013) or Colacito and Croce (2011)

² See Lustig and Verdelhen(2010)

³ See Verdelhan (2010).

and model the interesting investment dynamics seen in chapter 3. The model itself uses a similar mechanism to that used by Valchev (2016) who constructed a two country model that utilize a convenience yield in a government supplied bond to accurately model the dynamics of the uncovered interest parity condition as shown by both Valchev and Engel (2015). The model, however lacks an investment sector and when a simple investment sector is put into place is unable to accurately model the interrelationship documented in chapter 3. Following the collateral constraint literature, whose best example is the model detailed by Kiyotaki and Moore (1997), I instead endow the bonds with a role as collateral in the production of the investment good in the model. This in turn creates a wedge in the euler equation governing the international risk sharing condition, linking investment and changes in the real exchange rate more tightly to each other.

Ambiguity and the Shimer Puzzle

2.1 Introduction

A common theme in the search and matching literature is the ability of a business cycle model with a matching friction to be able to generate statistics similar to the US economy. In particular it fails to simultaneously match the volatility of output and vacancy postings. Diamond (1982), Mortensen (1982) and Pissardes (1985), are the first to create models with search frictions. The search model itself therefore bears their initials and is referred to as the DMP model. The model puts a friction within the labor market which prevents the market from instantly clearing. This search friction allows for the model to have a well defined unemployed sector of the economy that is otherwise missing from other macro models. Following this work, both Andolfatto (1996) and Merz (1995) incorporate the DMP model into a tractable general equilibrium format in an attempt to analyze the business cycle moments. That is the relative volatility of productivity, output, and unemployment. Shimer (2005a) notes the inability of this model to be able to generate enough volatility of unemployment relative to the volatility of labor productivity. In particular, Shimer

notes that a technology shock is unable to simultaneously match both the volatilities of unemployment and labor productivity in a reasonably calibrated search and matching model. This phenomenon, also called the Shimer Puzzle has, for the most part, dominated the recent literature. Mortensen and Nagypál (2007) point out that any hope of solving this puzzle must also contend with the fact that the data also suggests that unemployment and productivity are only slightly negatively correlated unlike what the standard model seems to imply, which is that, productivity and unemployment should be strongly negatively correlated. That is to say, that when productivity is high we should see low unemployment as firms rush to hire workers to take advantage of the high returns to labor.

A strand of the literature attempts to answer this question using many different methods. Hall (2005) proposes a model with wage stickiness, and is able to ameliorate the volatility puzzle. Hall and Milgram (2008) look at the bargaining process and create a model in which firms and workers have a cost to delaying bargaining, as they do not see the outside threat of leaving the bargaining table as credible. This framework also makes wages less responsive to unemployment and helps solve the excess volatility puzzle. Costain and Reiter (2008) show that with certain calibrations it is possible to match the volatility of unemployment in the data or the response of unemployment to a change in the unemployment benefit. However, the model requires either sticky wages or embodied technological progress in order to match both simultaneously. Gervais et al (2011), look at a model with shocks to the learning rate of new technologies and are able to explain both the excess volatility puzzle and the correlation puzzle posed by Mortensen and Nagypál.¹

Ambiguity aversion may be another interesting lens through which to look at the search and matching literature. I propose to abstract from the typical rational ex-

¹ This is only a small list of papers that explore the mechanisms of the matching function while trying to satisfy the Shimer critique. For a more comprehensive look at the mechanism I direct the reader to Hornstein, Krusell and Violante (2005).

pecations assumption and instead view agents as ambiguity averse. Ellsberg (1961) notes that rational individuals tend to prefer known probabilities over unknown probabilities. Gilboa and Schmeidler(1989), extend upon this by proposing a min-max utility representation where agents will minimize over their set of multiple priors about the stochastic process and act to maximize the worst case. There are a few examples in the literature where individuals look at models with ambiguity averse agents. Epstein and Wang(1994) look at asset pricing with agents who are ambiguity averse, and Cagetti et al(2002), look at a model where agents act out of concern for model misspecification in a stochastic growth model. Hansen et al (1999) also look at the effect on quantities and prices implied by a dynamic stochastic general equilibrium model in which consumers have a concern for robustness. More importantly, for my purposes, Ilut and Schneider(2012) propose a tractable way to incorporate ambiguity into a real business cycle framework.

In this chapter I attempt to build upon the literature and look at a model with ambiguity averse agents, hoping to discover if ambiguity has any role in explaining the Shimer puzzle within the context of a business cycle model. I consider a model proposed by Merz (1995), and deviate from the assumption of rational expectations by making agents ambiguity averse. Ilut and Schneider (2012) provide a method of introducing ambiguity aversion in such a manner as to have first order effects on the implied dynamics. I follow their approach in which agents, in the model, possess a set of multiple priors about their beliefs about future fundamentals. In the context of the model I propose this is the belief over the mean of the technology shock for the next period. I can then introduce shocks to these agents beliefs, causing their set of priors to increase and decrease in size. Since agents are ambiguity averse, they operate as if the parameter will take on the value which will lead them to the worst outcome. This in turn means that as their belief set widens or narrows so too does their worst case scenario. The intuition is that agents will respond to these shocks

to their belief set like they would a news shock about future fundamentals, but when the next period comes that shock is not realized. This in turn will create volatility in the choice variables but keep fundamentals relatively unaffected. There is evidence in the literature as to the merits of news shocks in models with a search and matching friction. Den Haan and Kaltenbrunner (2009), look at a search and matching model with wage stickiness and a large outside option for workers, and are able to generate procyclical movement in employment with news shocks. Krusell and McKay(2010) look at the ability of news shocks to generate comovement in models. In particular they look at a search and matching model with news shocks and demonstrate how the model can show comovement, though they also set the reservation wage extremely high. Despite this my results indicate the shocks themselves are unable to generate any significant additional volatility and are largely unable to solve the Shimer puzzle. However, the result in and of itself is a qualitative improvement over the benchmark, suggesting that further research in this area may produce a significant quantitative result.

This chapter is organized in the following manner. In section 2, I outline the basic search and matching model and solve both the social planner's problem and a competitive market problem. In section 3, I explain how I intend to introduce ambiguity to the model. In section 4, I outline the calibrations used in my model, the impulse response functions implied by the dynamics of the model and I simulate the model to generate business cycle statistics implied by the model. I show that ambiguity, in the setup I have described, is largely unable to create a quantitative result that matches the business cycle moments of output and vacancies in the data. I believe this may be attributed to the constraints placed upon the volatility of the ambiguity shock and the simplistic framework. If other frictions are added it may amplify the qualitative result. Finally in section 5, I conclude.

2.2 The Baseline Model

I first describe the details of the economic model. The model itself is taken from Merz (1995). Where I differ is in the fact that all unemployed workers search for work with a constant search intensity at zero cost. This is in order to maintain the empirically identified negative relationship between vacancies and unemployment, within the model. The economy is populated by a continuum of infinitely lived households on the unit interval. Likewise there is a continuum of competitive firms which produce an identical product with an identical production technology. Each household, consists of a continuum of individuals. These individuals in each household perfectly insure each other against changes in income caused by changes in employment or unemployment. In addition households own the capital stock. Households will buy consumption goods from firms who will in turn rent the factors of production from the households. Capital and the consumption good will be traded in perfectly competitive markets. Labor factors on the other hand will be subject to a search externality. The ability of a firm to find a worker and the ability of an unemployed worker to find employment will be determined by the amount of unemployed workers relative to the number of firms posting vacancies in the economy. The mechanism will allow for individuals to have increased gains to finding a job or filling a vacancy if there are more individuals on the opposite side of the market also searching. To post a vacancy a firm must incur a cost for each vacancy it posts, as posting help wanted ads and engaging in search activities typically costs either time or resources. All matches are also assumed to dissolve at an exogenous rate.

2.2.1 Social Planner's Problem

The following is the social planner's problem to the above economy. I will then later show a competitive market framework and how the competitive outcome can satisfy

the Hosios conditions and therefore also achieve the social planner's maximum.

Households wish to maximize the discounted stream of utility given by their consumption and leisure. This is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (2.1)$$

Where β is the household's discount factor, C_t is the household's per period consumption and N_t is its per period labor supply. I normalize the total supply of labor services in the economy to 1. For now I assume that expectations are rational. That is to say that, households know the true data generating process and make decisions based off of their expectations of that process. I will later abstract from this in introducing ambiguity but for the baseline model I will assume that the expectations above are not distorted. $U(C_t, N_t)$ is given by the following equation:

$$U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1-\frac{1}{\nu}}}{(1-\frac{1}{\nu})}$$

Where γ is simultaneously the coefficient for the household's relative risk aversion and also controls their intertemporal elasticity of substitution. The limiting case where $\gamma = 1$ is where the above reduces to $\log(C_t)$. Likewise ν is the negative of the household's Frisch elasticity of labor supply. Output can either be consumed by households, spent on investment into capital or spent on search activities. The economy wide resource constraint is given by:

$$C_t + I_t + \exp\{\Lambda t\}gV_t \leq Y_t \quad (2.2)$$

Where Y_t is per period output in terms of the consumption good. V_t is the per period number of vacancies posted by firms, g is the constant cost per vacancy posted in terms of the consumption good. This can be thought of as either the cost of advertising or other resources expended in finding potential workers. I_t is per

period investment. Λ represents the rate at which all variables in the economy grow, except for vacancies and employment which I will assume to be stationary. Output is produced according to the technology:

$$Y_t = \exp[(1 - \alpha)(\Lambda t + z_t)] K_t^\alpha N_t^{1-\alpha} \quad (2.3)$$

Where, K_t is per period capital stock. $0 \leq \alpha \leq 1$ represents the capital share of production. In addition, $\exp[(1 - \alpha)(\Lambda t + z_t)]$ represents a labor augmenting technology, which grows at the rate of growth of the economy but is also governed by the exogenous variable z_t . I will assume z_t follows an AR(1) process given by:

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{z,t+1} + \mu_t \quad (2.4)$$

Here, ρ_z is the autoregressive coefficient, ε_{z_t} is assumed to be i.i.d. normally distributed white noise, with mean 0 and variance 1. σ_z governs the variance of the process. Finally μ_t represents the average of the AR process. For the baseline model this is assumed to be zero. Capital evolves according to the following process:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (2.5)$$

Where, capital is assumed to depreciate at the rate $0 \leq \delta \leq 1$. Employment is constrained by a search friction so employment evolves according to the following law of motion.

$$N_{t+1} = (1 - \Psi)N_t + M_t \quad (2.6)$$

Where M_t is the total number of matches in time t , and $0 \leq \Psi \leq 1$ represents the exogenous separation rate. The matches will be resolved according to a matching function given by the following technology:

$$M_t = V_t^{1-\lambda}(1 - N_t)^\lambda \quad (2.7)$$

The above Matching Technology implies that the per period hazard rate of any vacancy finding an employee ξ_t and the per period hazard rate of any unemployed

worker finding a job ζ_t are functions of the labor tightness ratio or $\theta_t = \frac{(1-N_t)}{V_t}$. Specifically:

$$\xi_t = \frac{M_t}{V_t} = V_t^{-\lambda}(1 - N_t)^\lambda = \theta_t^\lambda$$

$$\zeta_t = \frac{M_t}{(1 - N_t)} = V_t^{1-\lambda}(1 - N_t)^{\lambda-1} = \theta_t^{\lambda-1}$$

This implies that as unemployment increases the probability of being matched with a vacancy decreases. Likewise the higher the number of vacancies the lower the probability of each vacancy being filled.

The Social Planner's maximization problem is to then choose sequences of values at time 0 of K_t, C_t, V_t, N_t for all possible states and times, in order to maximize the household's objective function (2.1) subject to the constraints given by (2.2),(2.3) (2.4), (2.5), (2.6) and (2.7), and given initial levels of capital and labor. In each period the social planner is given all information available at that time. It is worth noting that the social planner cannot affect the level of output in period t, but can instead only affect the investment and capital and the posting of vacancies in order to affect future output levels, as new matches and new capital do not become productive until the next period.

I solve the model using linear methods and so will detrend the model. A detrended variable is denoted by a tilde, and in particular these detrended values are given by the following:

$$\tilde{K}_{t+t} = \frac{K_{t+1}}{\exp(\Lambda t)}, \tilde{C}_t = \frac{C_t}{\exp(\Lambda t)}$$

I omit the final detrended variable I_t though it follows the same rule as C_t . In addition, as part of the detrending I will make use of the following notation:

$$\phi = -\alpha\mu, \delta' = 1 - 1(1 - \delta) \exp(-\Lambda)$$

The Social Planner's problem can now be written as a recursive Bellman equation given by:

$$V(z_t, \tilde{K}_t, N_t) = \max_{\{\tilde{I}_t, V_t\}} U(\tilde{C}_t, N_t) + \beta E_t[V(z_{t+1}, \tilde{K}_{t+1}, N_{t+1})] \quad (2.8)$$

subject to

$$\tilde{C}_t = \exp\{(1 - \alpha)z_t\} \phi \tilde{K}_t^\alpha N_t^{1-\alpha} \tilde{I}_t - gV_t \quad (2.9)$$

$$\tilde{K}_{t+1} = (1 - \delta')\tilde{K}_t + \tilde{I}_t \quad (2.10)$$

$$N_{t+1} = (1 - \Psi)N_t + V_t^{1-\lambda}(1 - N_t)^\lambda \quad (2.11)$$

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{z,t+1} + \mu_t \quad (2.12)$$

The solution to this problem will be a set of policy functions $\tilde{K}_{t+1} = G(s_t)$ and $N_{t+1} = O(s_t)$ for each set of states s_t such that the above problem is satisfied. I will solve the problem using the linear methods detailed by Sims(2002). I choose this method rather than value function iteration for computational ease. However I acknowledge that the model has many non-linearities and it may be interesting to look into solving the model via value function iteration.

The above problem gives the following first order conditions for the model:

$$\tilde{I}_t : \tilde{C}_t^{-\gamma} = \beta E_t[V_K(z_{t+1}, \tilde{K}_{t+1}, N_{t+1})] \quad (2.13)$$

$$V_t : g\tilde{C}_t^{-\gamma} = \beta(1 - \lambda)\theta_t^\lambda E_t[V_N(z_{t+1}, \tilde{K}_{t+1}, N_{t+1})] \quad (2.14)$$

Here the first order condition with respect to \tilde{I}_t , (2.13), relates the marginal utility of consumption today with the discounted future marginal value of capital, as in the typical model. The first order condition with respect to V_t , (2.14), equates the cost of a vacancy in marginal utility terms, to the discounted future value of the match. However, the match will make it less likely for further vacancies to be filled so it will

be multiplied times the marginal change in the probability of filling future vacancies. The implied envelope conditions of the model are given by:

$$V_K(z_t, \tilde{K}_t, N_t) = \alpha \tilde{C}_t^{-\gamma} e^{(1-\alpha)z_t} \phi \tilde{K}_t^{\alpha-1} N_t^{1-\alpha} + \beta E_t[V_K(z_{t+1}, \tilde{K}_{t+1}, N_t + 1)](1 - \delta') \quad (2.15)$$

$$V_N(z_t, \tilde{K}_t, N_t) = (1 - \alpha) \tilde{C}_t^{-\gamma} e^{(1-\alpha)z_t} \phi \tilde{K}_t^\alpha N_t^{-\alpha} - N_t^{-\frac{1}{\nu}} + \beta E_t[V_N(z_{t+1}, \tilde{K}_{t+1}, N_t + 1)][(1 - \Psi) - \lambda \theta_t^{\lambda-1}] \quad (2.16)$$

Here the envelope condition with respect to capital, (2.15), has the usual implication. That is that the marginal value of capital in the model is equal to the marginal productivity gained by the additional unit of capital, put into marginal utility terms, plus the discounted future value of capital times the amount of the capital that is undepreciated. The envelope condition for labor, (2.16), in the model gives similar intuition. This is because labor must adjust slowly in the model and its payoff is not realized until future periods. The envelope condition states that the marginal value of labor to the social planner is equal to the marginal product of labor, times the marginal utility of consumption, gained from the extra output minus the marginal disutility of labor, plus the discounted future value of the match times the probability that the match isn't destroyed and the marginal effect the additional employment causes to the probability of an unemployed worker finding a match. Solving and substituting the Envelope conditions and putting them into the first order conditions will give me the 2 Euler equations. Combined with the resource constraint as well as the law of motions for capital and labor it will give me 5 equations in 5 unknowns which will allow me to fully characterize the social planner's optimum. These equations are given below:

$$\tilde{C}_t^{-\gamma} = \beta E_t[\alpha \tilde{C}_{t+1}^{-\gamma} [e^{(1-\alpha)z_{t+1}} \phi \tilde{K}_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1 - \delta)']] \quad (2.17)$$

$$\frac{g}{(1-\lambda)\theta_t^\lambda \tilde{C}_t^\gamma} = \beta E_t \left[\frac{g}{(1-\lambda)\theta_{t+1}^\lambda \tilde{C}_{t+1}^\gamma} [(1-\Psi) - \lambda\theta_{t+1}^{\lambda-1}] - N_{t+1}^{-\frac{1}{\nu}} + \frac{1-\alpha}{\tilde{C}_{t+1}^\gamma} [e^{(1-\alpha)z_t} \phi \tilde{K}_{t+1}^\alpha N_{t+1}^{-\alpha}] \right] \quad (2.18)$$

$$N_{t+1} = (1-\Psi)N_t + V_t^{1-\lambda}(1-N_t)^\lambda \quad (2.19)$$

$$\tilde{K}_{t+1} = (1-\delta')\tilde{K}_t + \tilde{I}_t \quad (2.20)$$

$$\tilde{C}_t = \exp\{(1-\alpha)z_t\} \phi \tilde{K}_t^\alpha N_t^{1-\alpha} - \tilde{I}_t - gV_t \quad (2.21)$$

2.2.2 Competitive Market Setup

To outline the competitive market setup I first note that the technology process, the production function, matching technology and stochastic elements remain the same. For this section I will assume, that agents have rational expectations just to outline the model. I will later relax this assumption. In addition I will assume the households own the firms. This is so that when I relax the assumption that households have rational expectations their distorted beliefs will translate to the firms via their stochastic discount factor. Households consume consumption goods, own the capital stock which they will rent at rental rate r_t , and provide labor services, for which they will receive a wage w_t . In addition households will be able to purchase one period risk free bonds which cost price q_t and will pay 1 unit of consumption good one period in the future. Since the households own the firms they will receive profits each period in the form of dividends, Π_t . In addition both firms and households take the probability that they are able to find a match as given. The household's problem can then be written as the following recursive value function:

$$\max_{B_{t+1}, k_{t+1}} V(\tilde{K}_t, N_t, z_t, B_t) = U(\tilde{C}_t, N_t) + \beta E_t[V(\tilde{K}_{t+1}, N_{t+1}, z_{t+1}, B_{t+1})]$$

subject to

$$\tilde{C}_t + q_t B_{t+1} + \tilde{K}_{t+1} - (1 - \delta')\tilde{K}_t \leq w_t N_t + B_t + r_t \tilde{K}_t + \Pi_t$$

The household's first order necessary conditions are then given by:

$$\tilde{K}_{t+1} : \tilde{C}_t^{-\gamma} = \beta E_t[V_K(\tilde{K}_{t+1}, N_{t+1}, z_{t+1}, B_{t+1})] \quad (2.22)$$

$$B_{t+1} : q_t \tilde{C}_t^{-\gamma} = \beta E_t[V_B(\tilde{K}_{t+1}, N_{t+1}, z_{t+1}, B_{t+1})] \quad (2.23)$$

The envelope conditions are then given by:

$$V_K(\tilde{K}_t, N_t, z_t, B_t) = \tilde{C}_t^{-\gamma}[r_t + (1 - \delta')] \quad (2.24)$$

$$V_B(\tilde{K}_t, N_t, z_t, B_t) = \tilde{C}_t^{-\gamma} \quad (2.25)$$

Combining (2.22), with (2.24) gives us household's conditions for capital accumulation or that

$$\tilde{C}_t^{-\gamma} = \beta E_t \left[\tilde{C}_{t+1}^{-\gamma} [r_{t+1} + (1 - \delta')] \right] \quad (2.26)$$

That is that the marginal value of consumption today must be equal to the discounted future expected rent of the additional capital, plus the amount of the capital that will not depreciate next period times its value in marginal utility terms tomorrow. The conditions for bonds gives us a standard stochastic discount factor or that the price of a bond today which pays 1 unit of consumption tomorrow, must be equal to the discounted relative value of the marginal utility of consumption tomorrow divided by the marginal utility of consumption today.

$$q_t = \beta E_t \left[\frac{\tilde{C}_t^{-\gamma}}{\tilde{C}_{t+1}^{-\gamma}} \right] \quad (2.27)$$

Workers, though they do not choose the amount of labor contributed to the economy, do have an incentive compatibility constraint. In the sense that the marginal worker

would be better off not working should the wage be less than their reservation wage. This can be expressed in the following condition:

$$w_t \geq N_t^{-\frac{1}{\nu}} C_t^\gamma \quad (2.28)$$

The risk neutral firm wishes to maximize the discounted stream of future profits. Since the households own the firms they will discount all future values by the stochastic discount factor q_t . They will choose the quantity of capital services to rent from the household and will also hire a fraction of the household via labor services. The firm however will be constrained by the search friction and will take the rate at which a vacancy is filled, ξ_t as given. Thus the firm's recursive problem is given by:

$$\max_{\tilde{K}_t^d, V_t} \Pi(\tilde{K}_t, N_t, z_t, B_t) = Y_t - w_t N_t - \tilde{K}_t^d r_t - g V_t + q_t \Pi(\tilde{K}_{t+1}, N_{t+1}, z_{t+1}, B_{t+1})$$

subject to

$$N_{t+1} = (1 - \Psi) N_t + \xi_t V_t$$

$$Y_t = \phi e^{(1-\alpha)z_t} (\tilde{K}_t^d)^\alpha N_t^{1-\alpha}$$

The Firm's First order conditions are then:

$$V_t : q_t \Pi_N(\tilde{K}_{t+1}, N_{t+1}, z_{t+1}, B_{t+1}) \xi_t = g \quad (2.29)$$

$$\tilde{K}_t^d : \alpha e^{(1-\alpha)z_t} \tilde{K}_t^{\alpha-1} N_t^{1-\alpha} = r_t \quad (2.30)$$

Its envelope condition is given by:

$$\Pi_N(\tilde{K}_t, N_t, z_t, B_t) = (1 - \alpha) e^{(1-\alpha)z_t} \tilde{K}_t^\alpha N_t^{-\alpha} - w_t + q_t \Pi_N(\tilde{K}_{t+1}, N_{t+1}, z_{t+1}, B_{t+1}) (1 - \Psi) \quad (2.31)$$

I define the recursive competitive equilibrium as a set of functions for prices $\{q_t, r_t, w_t\}$ and policy functions for allocations $\{\tilde{K}_t, \tilde{C}_t, N_t, B_t\}$, such that all markets clear, profits are equal to zero, capital supplied is equal to capital demanded, and labor supplied

is equal to labor demanded. From the firm's equation it is possible to pin down r_t from (2.30), combining this with (2.26) will give the intertemporal euler equation from the social planner's problem, (2.17). It is common in search models, since the wage is not explicitly pinned down to have firms and households enter into Nash Bargaining over the wage. Ideally households would like to be paid the marginal value of their labor minus the additional cost the firm expended in creating the match per unemployed worker, while firms would want to pay the reservation wages of the workers. Therefore any set of wages within the set given by:

$$w_t = \eta \left[(1 - \alpha)e^{(1-\alpha)z_t} \tilde{K}_t^\alpha N_t^{-\alpha} + \frac{gV_t}{(1 - N_t)} \right] + (1 - \eta)[N_t^{-\frac{1}{\nu}} C_t^\gamma] \quad (2.32)$$

will be acceptable to both parties. Where $0 \leq \eta \leq 1$ represents the bargaining power of the household, and $(1 - \eta)$ represents the bargaining power of the firm. Any wage selected in the set will be Pareto optimal. Thus if $\eta = \lambda$, the wage bargaining solution will be acceptable by both the household and firm. In addition by combining (2.32) with (2.31) and solving and substituting into the first order condition of the firm for vacancies (2.29), I achieve the social planner's euler equation given by (2.18). This choice also will satisfy the conditions outlined by Hosios(1990), in order to achieve the social planner's optimum.

2.3 Introducing Ambiguity to the Model

The following section explains one method of implementing agents who are averse to Knightian uncertainty. I abstract from the assumption on rational expectations and instead endow the agents with no more knowledge than econometricians. Agents have the best estimate of the technology process but are concerned that the true value may be a slight deviation from the estimated value. The Ellsberg paradox shows that agents prefer the lotteries with known probabilities to lotteries with unknown

probabilities. In this sense they act taking concern as to what they deem will be the worst case. I therefore endow these agents with a set of multiple priors as to the true value of the mean process, as defined by Gilboa and Schmeidler(1989). Agents do not know the true mean of the technology process and instead will act out of concern for the worst case. I first set up the process governing the set of priors that the agents posses, following the guidelines of Ilut and Schneider (2012). By having ambiguity about the mean of the process, the model will have first order effects from the ambiguity process. I then rewrite the social planner’s problem and derive the equilibrium conditions, as well as discuss the solution of the model.

2.3.1 Multiple Priors

Agents in the model are aware of the best estimate of the mean for the technology process. However also believe that the true mean may be within a set of bounds set by the agents confidence in this estimate. Agents know that the in the long run μ_t will be equal to zero, however they are unsure about what it means for the conditional mean of μ_t for the next period. In particular they are unsure if the true time t conditional mean of z_{t+1} is really $\rho_z z_t$. They are concerned that they cannot actually estimate the value of μ_t in real time. They wish to forecast z_{t+1} but have a range of beliefs, representing their doubt, in what μ_t is, and so will act out of concern for those beliefs. Recalling (2.4), I center agents beliefs about μ_t around zero. In particular I define the set of multiple priors by the following:

$$\mu_t \in [-a_t, -a_t + 2|a_t|]$$

Where a_t represents the agents confidence in their beliefs about the one step ahead value μ_t . I allow agents beliefs to evolve according to an exogenous AR(1) process given by:

$$a_{t+1} = (1 - \rho_a)\bar{a} + \rho_a a_t + \sigma_a \varepsilon_{a,t+1}$$

Where \bar{a} represents the average bound of the process. Large values of a_t represent a decrease in confidence whereas small values represent an increase in confidence. However, I would want to restrict these belief sets to within reasonable bounds. The three parameters \bar{a} , ρ_a and σ_a , control the agent's spread of beliefs. I want to bound \bar{a} to be constrained by the volatility given by the z_t process in order for agents' set of priors to have reasonable bounds. Ilut and Schneider propose bounding this by the following formula:

$$\bar{a} = n\sigma_z$$

Where $n \in [0, 1]$, to represent the amount of volatility of z that agents attribute to the ambiguous process. The other restriction is that I do not want the value of a_t to become negative. This is because I would like the confidence bounds of the agents to stay centered around the long run mean of μ of zero. Thus I will require \bar{a} , the mean of the process, to be greater than 3 standard deviations away from zero. or that

$$\bar{a} \geq 3 \frac{\sigma_a}{\sqrt{1 - \rho_a^2}} \quad (2.33)$$

This means that a_t will have only a .15% chance of becoming negative and is consistent with Ilut and Schneider(2012). Thus choosing \bar{a} and ρ_a will give me σ_a . I will later take the calibrations for ρ_a and σ_a from Ilut and Schneider(2012), due to a lack of any other priors for these values, as I do not estimate the parameter values myself in this chapter.

2.3.2 *The Social Planner's Problem with Ambiguous Agents*

Now that I have defined the process for agents beliefs I can now rewrite the social planner's problem. The only assumption I change is the rational expectations assumption, that agents, and therefore the social planner will be unsure of the true process for μ_t and will act out of concern for the worst case scenario. That is to

say they will maximize the value function taking into account the minimum value of their future expected value with respect to their beliefs about a_t . More explicitly this is written as:

$$V(z_t, \tilde{K}_t, N_t, a_t) = \max_{\{\tilde{I}_t, V_t\}} U(\tilde{C}_t, N_t) + \beta \min_{\mu_t \in [-a_t, -a_t + 2|a_t|]} E_{\mu_t} [V(z_{t+1}, \tilde{K}_{t+1}, N_{t+1}, a_{t+1})] \quad (2.34)$$

subject to

$$\tilde{C}_t = \exp \{ (1 - \alpha) z_t \} \phi \tilde{K}_t^\alpha N_t^{1-\alpha} \tilde{I}_t - gV_t \quad (2.35)$$

$$\tilde{K}_{t+1} = (1 - \delta') \tilde{K}_t + \tilde{I}_t \quad (2.36)$$

$$N_{t+1} = (1 - \Psi) N_t + V_t^{1-\lambda} (1 - N_t)^\lambda \quad (2.37)$$

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{z,t+1} + \mu_t \quad (2.38)$$

$$a_{t+1} = (1 - \rho_a) \bar{a} + \rho_a a_t + \sigma_a \varepsilon_{a,t+1} \quad (2.39)$$

Where my notation indicates they take expectations with respect to the value of μ_t that minimizes their future value function given their choices this period and their belief set concerning μ_t given by a_t . I posit, without formal proof, that the value that will minimize the future expected value to the social planner is the value $-a_t$. This is because the expected future value of output is exactly governed by the technology process. A larger value for z_t will enable the agents productivity of their labor to increase, raising their return to labor and therefore the amount of the consumption good that is available for consumption, investment and vacancies. Therefore the value that minimizes the future process z_t will be the value of μ_t that corresponds with making the z_t process as small as possible or $-a_t$. Looking at the competitive market setup, I posit that the value is the same as in the social planner's outcome. This is because for firms, their profits rely on the productive value of their inputs. The value which minimizes this in the future will be the value of μ_t that minimizes

the technology process z_t . In this case that is once again $-a_t$. This also holds for households, as their wage w_t is a function of their labor productivity. Labor productivity is increasing in z_t and therefore the value that minimizes z_t will once again be $-a_t$. Thus the minimizing value for all versions of the model and all states will be $-a_t$. Given this we can now solve for the social planner's maximum, which will be given by the following equilibrium conditions:

$$\tilde{C}_t^{-\gamma} = \beta E_{\mu_t = -a_t} [\alpha \tilde{C}_{t+1}^{-\gamma} [e^{(1-\alpha)z_{t+1}} \phi \tilde{K}_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1 - \delta')]] \quad (2.40)$$

$$\begin{aligned} \frac{g}{(1-\lambda)\theta_t^\lambda \tilde{C}_t^\gamma} = \\ \beta E_{\mu_t = -a_t} \left[\frac{g}{(1-\lambda)\theta_{t+1}^\lambda \tilde{C}_{t+1}^\gamma} [(1-\Psi) - \lambda \theta_{t+1}^{\lambda-1}] - N_{t+1}^{-\frac{1}{\nu}} + \frac{1-\alpha}{\tilde{C}_{t+1}^\gamma} [e^{(1-\alpha)z_t} \phi \tilde{K}_{t+1}^\alpha N_{t+1}^{-\alpha}] \right] \end{aligned} \quad (2.41)$$

$$N_{t+1} = (1-\Psi)N_t + V_t^{1-\lambda}(1-N_t)^\lambda \quad (2.42)$$

$$\tilde{K}_{t+1} = (1-\delta')\tilde{K}_t + \tilde{I}_t \quad (2.43)$$

$$\tilde{C}_t = \exp\{(1-\alpha)z_t\} \phi \tilde{K}_t^\alpha N_t^{1-\alpha} - \tilde{I}_t - gV_t \quad (2.44)$$

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{z,t+1} + -a_t \quad (2.45)$$

$$a_{t+1} = (1-\rho_a)\bar{a} + \rho_a a_t + \sigma_a \varepsilon_{a,t+1} \quad (2.46)$$

Where the only difference is in the fact that expectations are taken with respect to the value of $\mu_t = -a_t$.

2.3.3 Solution method

Ilut and Schneider(2012), posit that the above equilibrium conditions can be linearized and then solved via any normal linearization methods. I will first log linearize the model around the distorted steady state given by the ambiguous agents beliefs about future fundamentals. I will refer to this state as the ambiguous steady state.

The model can then be written in a state space form where the states are given by the vector $s_t = [E[v_{t+1}], c_t, E[c_{t+1}], v_t, k_{t+1}, n_{t+1}, i_t, z_{t+1}, a_{t+1}]'$, where lower case letters represent log deviations from the steady state. The state space form is given by:

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + Q \epsilon_t + J \eta_t$$

Where ϵ_t is the vector of i.i.d. normal shocks to the model $\varepsilon_{a,t+1}, \varepsilon_{z,t+1}$ and η_t is a vector of expectational errors. I relegate the log-linearization and the explicit state space matrices to the appendix. Sims(2002), gives us a method of solving such that the system can be reduced to

$$s_t = G_1 s_{t-1} + P \epsilon_t$$

Now however, since the agents are acting out of concern for what they perceive as the worst case, to the econometrician estimating this model this then appears that the average technology shock is different from zero. Where the econometrician has estimated the mean of the process to be zero. This implies that instead I want to solve for what Ilut and Schneider(2012) refer to as the zero-risk steady state² The zero risk steady state being the fact that agents perceive the fact that they are at this steady state but believe that under the worst case the dynamics will converge back to the ambiguous steady state. Another way of looking at this is absent technology shocks agents will perceive the fact that μ_t is not in fact $-a_t$ and will act as if they are experiencing small positive shocks to technology. Agents will react to this and it will change the dynamics slightly. In particular the new dynamics around this zero risk steady state are actually given by:

$$s_t = G_1 s_{t-1} + P \epsilon_t + P \begin{bmatrix} a_t \\ \sigma_z \\ 0 \end{bmatrix}$$

² Rather than go into detail on this specifically I direct the reader to Ilut and Schneider(2012).

Where the average shock to technology is now equal to $\frac{a_t}{\sigma_z}$. With this in hand it is now possible to create implied statistics from the model as well as impulse response functions given a calibration for the parameter values.

2.4 Empirical Implications of the model

Now that the model can be implemented and solved I can now simulate the economy in order to gain statistics. In this section I will discuss calibration and discuss my empirical findings as well as why I believe the model does not create the volatility, that I first thought it may.

2.4.1 Calibrations

Table 2.1: Parameter Values for Calibration

α	0.36	0.36
β	0.99	0.99
δ	0.022	0.022
Λ	0.004	0.004
ν	-1.25	-1.25
λ	0.4	0.4
Ψ	0.0623	0.0623
g	0.05	0.05
\bar{a}	0.00435	0.00435
ρ_z	0.95	0.95
ρ_a	0.96	0.96
γ	1	0.4
σ_a	0.00044	0.00044
σ_z	0.0045	0.0045

Table 2.1 gives the parameter values I used in my calibration. For the most part I have tried to stay consistent with the literature. α is set to be one minus the labor share of income, consistent with Merz (1995). Likewise I set my $\Lambda = 0.004$ which is consistent with an annual growth rate of 1.6% per year. $\beta = 0.99$, which is equivalent

to a 4.01% annual interest rate. $\delta = 0.022$ will mean that $\delta' = 0.0259$ which implies that the depreciation rate per year will be 9.96%. Following Shimer (2005b), I calibrate the matching elasticity of unemployment $\lambda = 0.4$. I follow Merz(1995), in calculating the exogenous separation parameter $\Psi = 0.0623$ where I calculate it as the average unemployment rate divided by the average number employed using data from Quarter 1 1953 to Quarter 2 2011. I follow Merz (1995) for the calibration of the cost $g = 0.05$. I choose ρ_a and \bar{a} to be consistent with the estimates gained from Ilut and Schneider (2012). This implies a maximum volatility of $\sigma_a = 0.0044$. I will look at the results for different values of the parameter γ . The first where $\gamma = 1$ or the case of log utility and the second where $\gamma = 0.4$ as the model has far different results for the two. Finally I choose ρ_z and σ_z to be consistent with Ilut and Schneider(2012).

4.2 Model Implied Impulse Response Functions

I look at two models in order to see if ambiguity aversion can improve upon the relative volatility of unemployment to productivity, as well as several other moments in the data. The first model, which I will call the baseline model, is a model without ambiguity averse agents, it's the exact model described in section 2 of this chapter. The second model is the model in which agents are ambiguity averse as described in section 3. I generate impulse response functions to gain some understanding of the model. Figure 2.1 shows the impulse response to a single standard deviation shock to productivity in the baseline model when we keep $\gamma = 1$. Comparing it to Figure 2.2, which is a single standard deviation shock to productivity in the ambiguity model, we see that the two predominantly act the same. Consumption drops by a lot on impact. Vacancies, employment and output all also drop on impact with largely the same reaction. Investment on the other hand seems to increase when the shock hits and then decreases. This is most likely due to the relatively small intertemporal

elasticity of substitution. Agents have a wealth effect due to the shock, and save more in the form of capital initially. Agents also perceive a substitution effect in the sense that capital is now less productive now than it was in the past, and so would have an incentive to dissave. However, it appears that initially this wealth effect dominates and agents will later reduce their investment in the period after the shock as the substitution effect begins to dominate again. Figure 2.3 shows an impulse response function to a one standard deviation shock to their bound on their belief set. The shock itself will act like an unrealized news shock. Like news shocks the wealth effect and substitution effect play a huge role. As agents feel they will be poorer in the future, they initially invest more and increase the number of vacancies in order to get more output initially. They also drop their consumption on impact and allow it to slowly recover. Because they increase their output and capital in the future, output in the next period jumps as the news shock is unrealized. However agents still feel uncertain about the future and though they readjust their level of vacancies after the initial increase, vacancies still remain high relative to the steady state. Investment slowly reduces to the steady state level as their belief set shrinks back to the steady state level.

Seeing how the wealth effect seems to dominate the reactions of agents to ambiguity shocks. I then decide to compare the two models when $\gamma = 0.4$ in order to greatly increase the agents intertemporal elasticity of substitution. Figure 2.4 shows the impulse response function for the baseline model to a one standard deviation shock to productivity. For comparison Figure 2.5 shows the impulse response function to a one standard deviation shock to productivity to the model with ambiguity. On impact both vacancies, investment and output drop. One period later in response to this capital and labor also drops. On impact consumption increases in both models as agents try and consume as much today as possible. This is because since their IES is so high the consumption smoothing motive decreases greatly. Agents no

longer feel as bad about the future drop in consumption. They adjust their investment and vacancies, by more than they would in the log model in order to take into account the fact that now both of those factors are relatively unproductive, while using the rest of their income to consume today. They then slowly drop their consumption below steady state, as output decreases before slowly bringing it back to the steady state level. Figure 2.6 shows the impulse response function to a one time standard deviation shock to the interval of beliefs. Here the substitution effect now dominates as both investment and vacancies drop on impact. Y drops this time as both capital and employment in the future also drop. Though agents would most likely increase their consumption as they would in the case of a productivity shock, they also must increase it by an additional amount as output is fixed in the current period, and investment and vacancies both drop. I refrain from saying anything in this section about the relative size of each shock's effect on the model as the shocks are not standard in size. However, I will explore this somewhat more in the following section.

2.4.2 Simulation Method and Results

I now simulate the models in order to generate relative business cycle moments of the data. I create 5000 time series of 900 observations each in order to look at the statistical properties of the model and compare it to US data. The US data is taken in logs and then HP filtered with smoothing parameter equal to 1600. I relegate further discussion on my data to the Appendix. The subscript "prod" denotes the natural log of labor productivity and the subscript "m" denotes the natural log of the job finding rate. u is the natural log of unemployment. The subscript theta denotes the natural log of the labor tightness ratio which I have defined earlier in this chapter. Table 2.2 gives the simulated moments from the baseline model and the model with ambiguity when $\gamma = 1$. Below each of the simulated moments I include the standard

deviation in parentheses. Both the benchmark and the ambiguity model are able to match the variance of output and the variance of production quite well each barely outside of one standard deviation from the value given by the US data. Similarly both models match the relative volatility of consumption to output. Unfortunately this is where it's ability to match ends, both models are only able to generate about half as much volatility in investment relative to output, similarly the volatility of employment relative to output in both models is a little more than one third the volatility implied by the data. The benchmark model is only capable of explaining 30% of the relative volatility of the job finding rate to productivity and similarly the model with ambiguity is only able to explain 33% of it. Likewise neither comes close to matching the volatility of vacancies relative to output, accounting for only 38% in the benchmark and 41% in the ambiguity model. The volatility of unemployment to productivity is only 32% in the baseline, and 36% in the model with ambiguity, of the volatility implied in the data. It is apparent neither model works very well. Both models however perform surprisingly well in matching the correlations of consumption, investment and employment to output, that is present in the data. However the ambiguity model only improves slightly over the baseline on the correlations of unemployment to productivity and the correlation of the job finding rate to productivity. The three main facts which I had hoped ambiguity to help ameliorate (relative volatility of unemployment to productivity, relative volatility of the job finding rate to productivity, and the correlation between unemployment and productivity), it is largely unable to explain. Despite this it does appear that the model with ambiguity however does slightly better, in explaining the statistics in the labor market. However, I hesitate to place any weight on this as all the differences are within one standard deviation of the mean found in the baseline model, and so their differences do not appear to be significant in the least.

Discouraged by this finding I then decide to see the effect on the various moments

of the ambiguity shock if the productivity shock is shut down. I therefore perform a counterfactual where I simulate moments from the data with the productivity shocks removed. Though the exercise allows me to gain intuition to how the ambiguity shock is affecting the model, it remains largely unrealistic and the results gained here must be taken with a degree of skepticism. Table 2.3 presents these findings. For obvious reasons the shock is unable to generate near enough volatility in output and productivity, just by the fact that the ambiguity shock's variance must be low and restricted. However it is able to greatly enhance the relative volatility of investment, employment and vacancies to output. Similarly it is able to increase the volatility of matches, the tightness ratio and unemployment relative to productivity. This most likely happens as agents react to their future beliefs as if it will be news about future productivity. However, the change in productivity never actually occurs. This means investment, the job finding rate, the labor tightness ratio and unemployment all vary significantly more than output as the labor augmenting technology process does not change as it would in the case of a technology or realized news shock. Looking at the correlations, by itself the ambiguity shock appears to break the strong negative correlation of unemployment to productivity as is normally implied by the model. This is most likely due to the reason stated above. Both investment and vacancies move in the same direction, however the shock itself isn't realized. Therefore, though labor productivity will increase due to the increased capital stock, the increase in employment coupled with no change in the labor augmenting technology factor will subdue the increase in productivity. The one correlation that does significantly worse is the correlation of consumption with output. This can largely be attributed to the fact that consumption does not comove with employment or capital, implying that it should also not comove with output, as the technology process no longer varies.

Finally, wanting to see if subduing the wealth effect will enhance the effect ambiguity has on the model, I simulate moments from the benchmark and the ambiguity

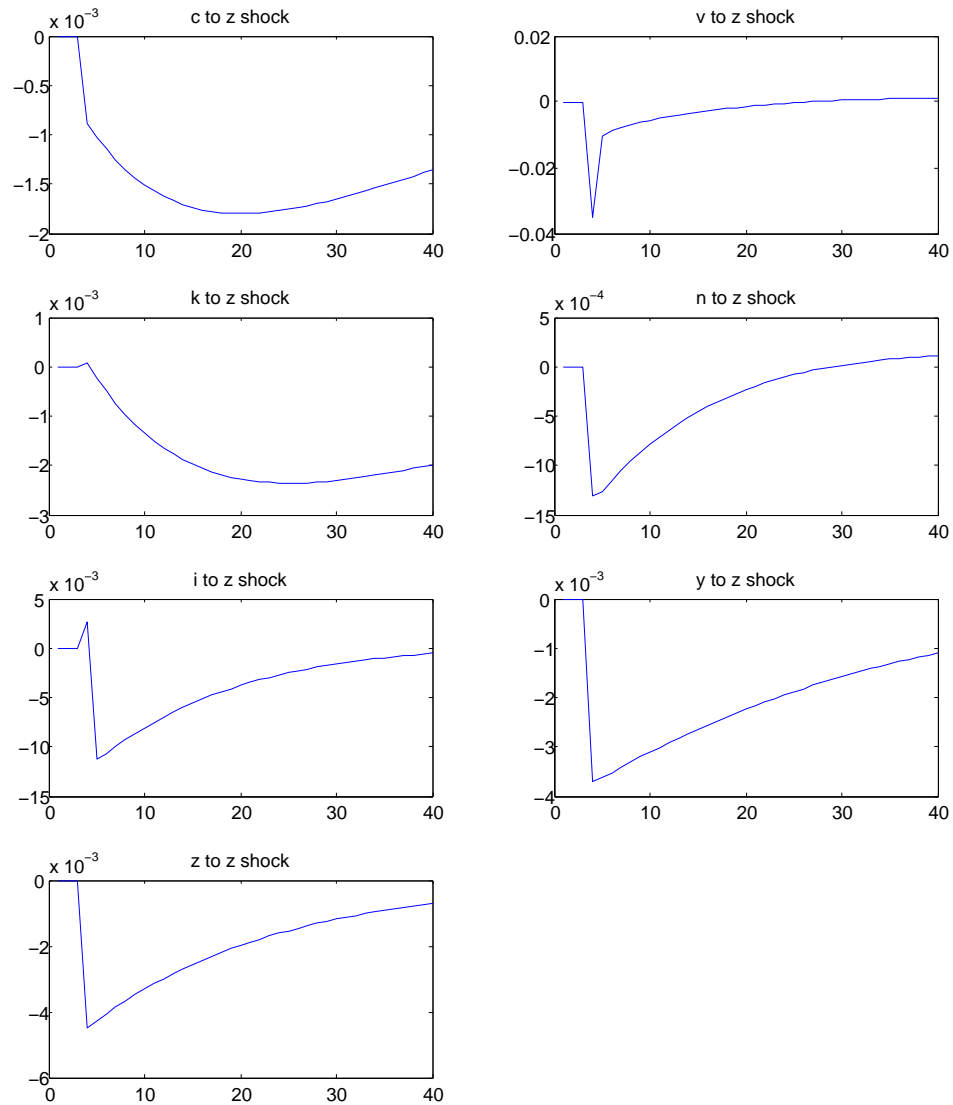
model with $\gamma = 0.4$. Table 2.4 gives my results. Both models once again seem to act the exact same way. Both the volatility of output and productivity are high relative to the data. However, with the increase in the IES, the rest of the simulated data becomes far more volatile. It appears that greatly increasing the intertemporal elasticity of substitution helps the ambiguity model, but also helps the benchmark just as well if not more. There is no real statistical difference between any of the simulated moments of the data. Though volatility improves for almost every variable, the correlation between both unemployment and productivity and matching and productivity become much closer to one for both models. This is counter to what is implied in the data. Together with the fact that the benchmark model also improves greatly seems to imply that most of the action for all the variables is coming through the technology shock. When total labor factor productivity declines, so too do vacancies, investment and consumption, as implied by the impulse responses. Whereas this is not normally the case with regards to the ambiguity shock since total factor productivity does not change when an ambiguity shock hits, creating a decreased effect on productivity. Given this result and the previous results I tentatively conclude that though ambiguity seems to have the desired effect, because the calibration of the variance of the parameter must be so low, for mechanical as well as some theoretical reasons, its effect is almost entirely drowned out by the technology process. This drowning out effect is magnified as the IES increases, as agents react more to changes in productivity.

2.5 Conclusion

In this chapter I have looked at two models with an imbedded search friction in the hopes of explaining several facts about the US labor market that exist in the data. These are the excess volatility puzzle, as pointed out by Shimer(2005a), and the lack of correlation between unemployment and labor productivity, as per Mortensen and

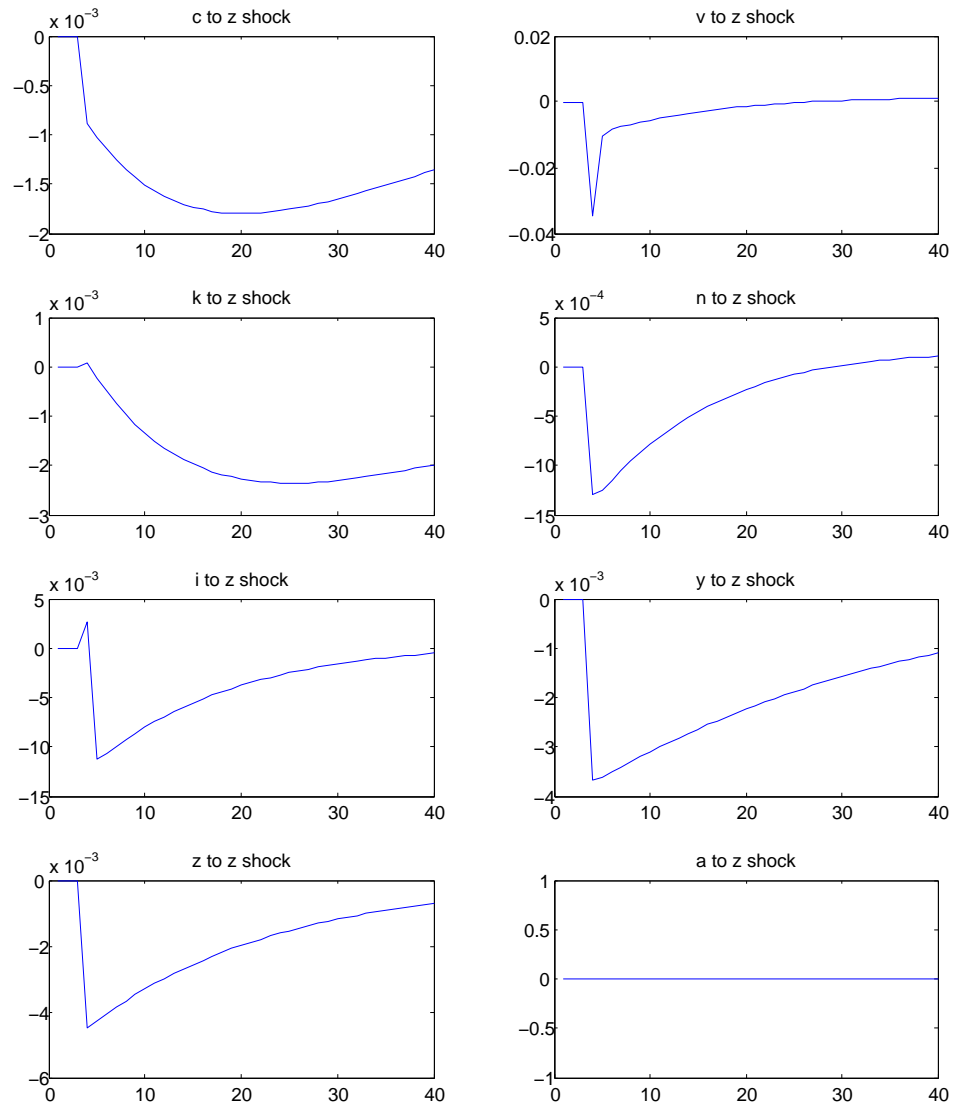
Nagyál (2007). I first look at a benchmark model which has been shown in the past as unable to solve either of these and compare it to a similar model in which agents are ambiguity averse. I find that the model with ambiguity averse agents does not show any real difference as compared to the benchmark in helping to explain these moments. However, both models are able to explain the excess volatility puzzle when the agents intertemporal elasticity of substitution greatly increases, but at the cost of creating an almost perfect negative correlation between unemployment and labor productivity, which we do not see in the data.

In the end it appears that the effect of the ambiguity shock is completely drowned out by the productivity shock in simulations of the model. I don't necessarily believe that this is the final note on ambiguity in search and matching models and more work can certainly be done. Certainly it appears that the intuition behind how the ambiguity shock behaves seems correct based off the counterfactual simulation and this may have some promise for the future. It may be worth looking at ambiguity in models outside of a real business cycle setup like appears in Merz, and instead remaining consistent with the models used by Andolfatto(1996), which many other papers in the literature adopt. However, I at this point leave that to future work.



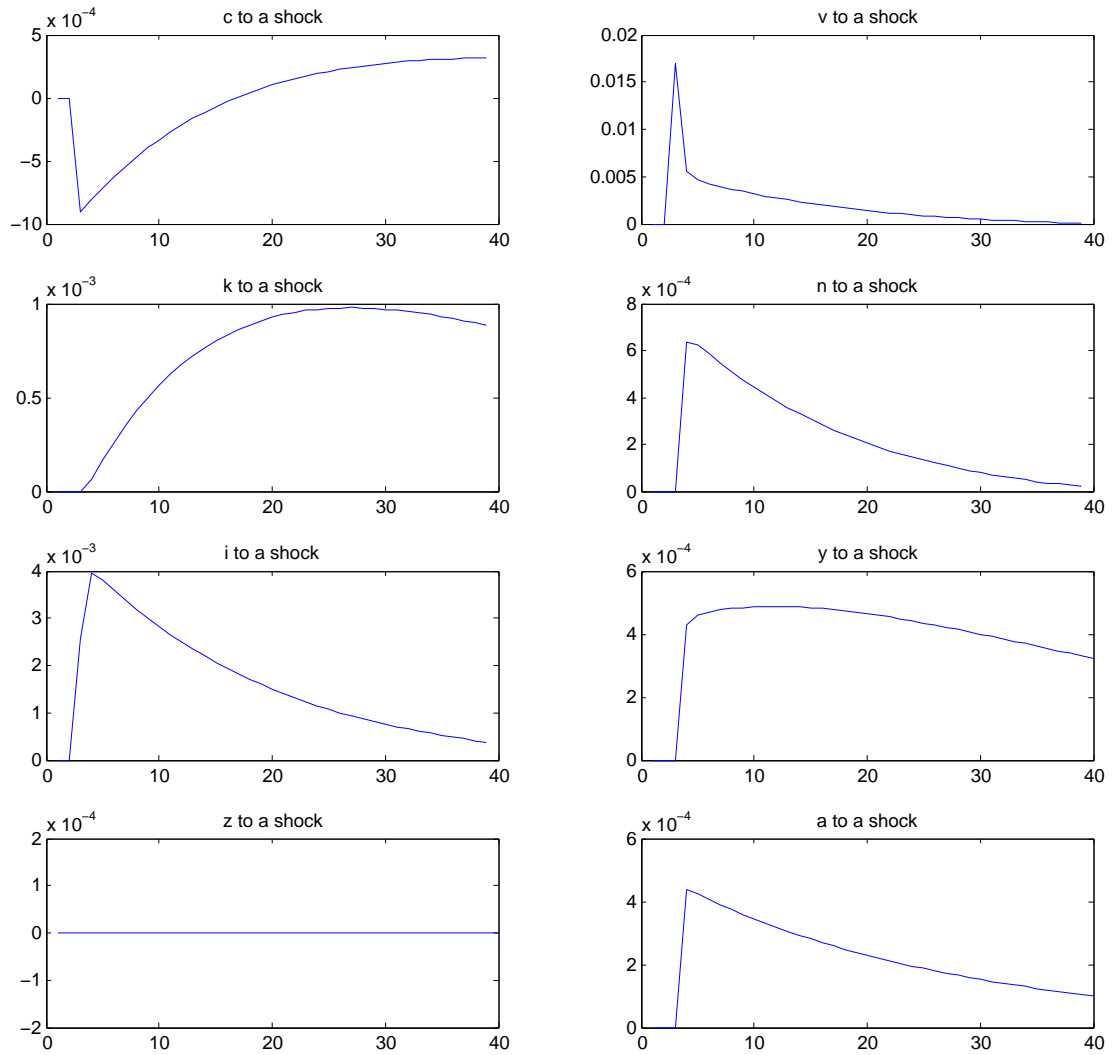
Student Version of MATLAB

FIGURE 2.1: Impulse Responses to a one time shock to productivity in baseline model $\gamma = 1$



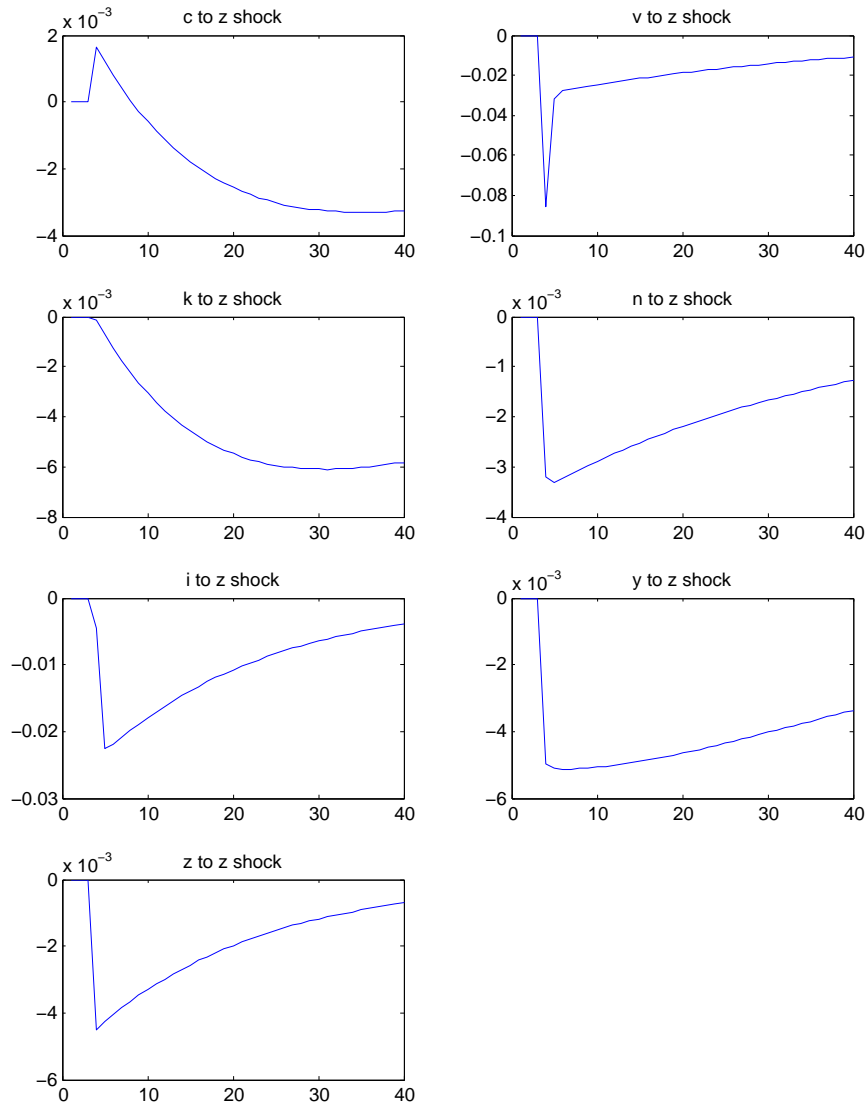
Student Version of MATLAB

FIGURE 2.2: Impulse Responses to a one time shock to productivity $\gamma = 1$



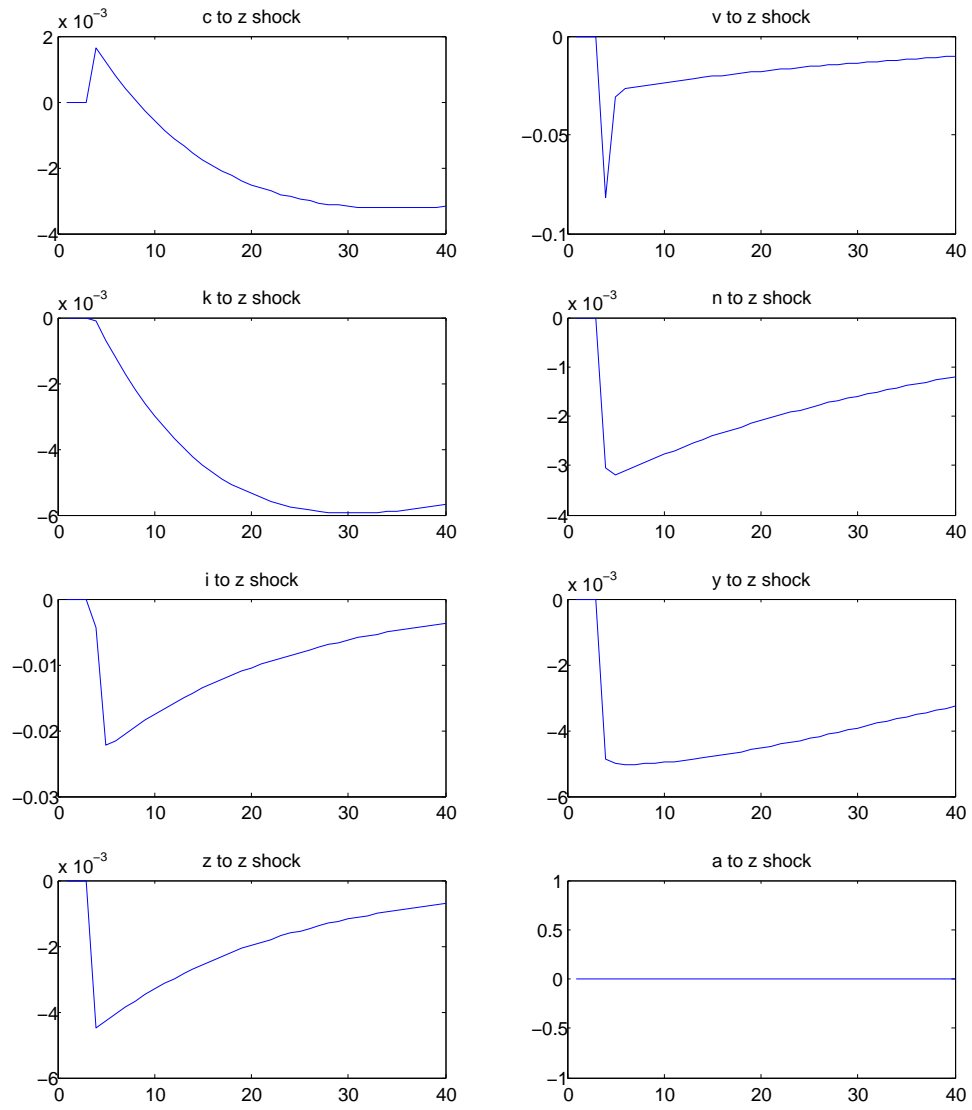
Student Version of MATLAB

FIGURE 2.3: Impulse Responses to a one time shock to ambiguity, $\gamma = 1$



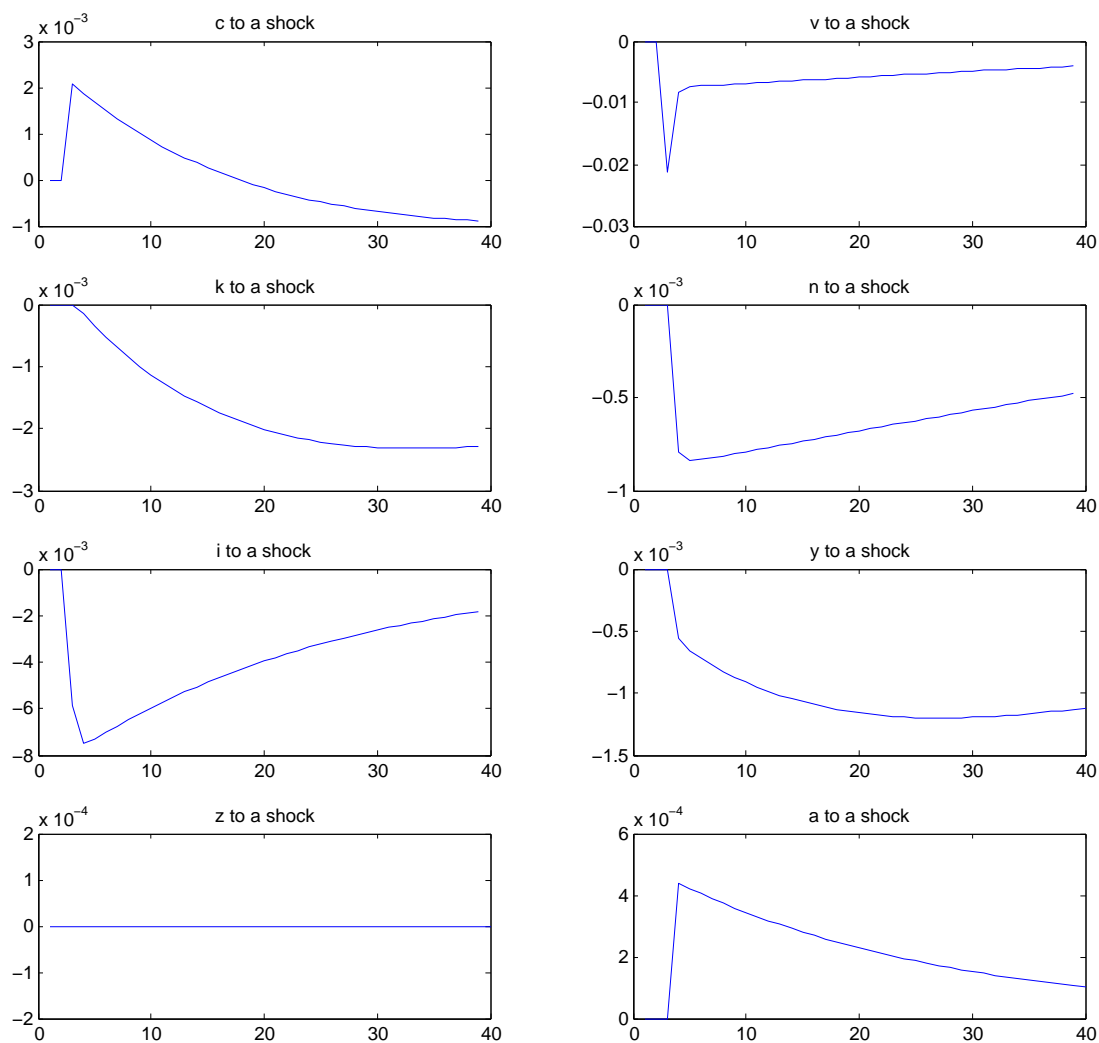
Student Version of MATLAB

FIGURE 2.4: Impulse Responses to a one time shock to productivity in baseline model $\gamma = 0.4$



Student Version of MATLAB

FIGURE 2.5: Impulse Responses to a one time shock to productivity $\gamma = 0.4$



Student Version of MATLAB

FIGURE 2.6: Impulse Responses to a one time shock to ambiguity $\gamma = 0.4$

Table 2.2: Simulated Moments $\gamma = 1$

	US Data	Benchmark Model	Ambiguity Model
σ_y	0.0161	0.0142 (0.0018)	0.0145 (0.0018)
σ_{prod}	0.0135	0.0120 (0.0017)	0.0122 (0.0017)
$\frac{\sigma_c}{\sigma_y}$	0.7925	0.7364 (0.0381)	0.7474 (0.0396)
$\frac{\sigma_i}{\sigma_y}$	4.6926	2.1700 (0.1345)	2.2838 (0.1550)
$\frac{\sigma_n}{\sigma_y}$	0.6492	0.2350 (0.0206)	0.2612 (0.0249)
$\frac{\sigma_v}{\sigma_y}$	7.8652	2.9829 (0.297)	3.2610 (0.3379)
$\frac{\sigma_m}{\sigma_{prod}}$	8.4567	2.5511 (0.2963)	2.8198 (0.3462)
$\frac{\sigma_\theta}{\sigma_{prod}}$	18.6109	6.377 (0.7408)	7.0495 (0.8655)
$\frac{\sigma_u}{\sigma_{prod}}$	9.6877	3.1074 (0.3503)	3.4527 (0.4181)
$\rho_{c,y}$	0.8885	0.8865 (0.0209)	0.8469 (0.0340)
$\rho_{i,y}$	0.9109	0.8527 (0.0133)	0.8365 (0.0193)
$\rho_{n,y}$	0.7986	0.7374 (0.0336)	0.7001 (0.0430)
$\rho_{v,u}$	-0.9003	-0.8311 (0.0133)	-0.8350 (0.0133)
$\rho_{m,prod}$	0.2509	0.5328 (0.0297)	0.4663 (0.0468)
$\rho_{u,prod}$	-0.2735	-0.5979 (0.0331)	-0.5250 (0.0521)

US data is HP filtered data with smoothing parameter $\lambda = 1600$, taken from Q1:1953 to Q2:2011. m represents the log of the number of matches. Matches were created following the method outlined in Shimer(2005b). Simulated moments from the models is the mean from simulating the model 5000 times for 1000 periods and throwing out the first 100 periods.

Table 2.3: Counterfactual Simulated Moments $\gamma = 1$

	US Data	Ambiguity Model
σ_y	0.0161	0.0028 (0.0004)
σ_{prod}	0.0135	0.002 (0.0003)
$\frac{\sigma_c}{\sigma_y}$	0.7925	0.9803 (0.092)
$\frac{\sigma_i}{\sigma_y}$	4.6926	4.2832 (0.3927)
$\frac{\sigma_n}{\sigma_y}$	0.6492	0.6504 (0.0716)
$\frac{\sigma_v}{\sigma_y}$	7.8652	7.8072 (0.9927)
$\frac{\sigma_m}{\sigma_{prod}}$	8.4567	8.061 (1.0294)
$\frac{\sigma_\theta}{\sigma_{prod}}$	18.6109	20.1527 (2.5736)
$\frac{\sigma_u}{\sigma_{prod}}$	9.6877	10.0354 (1.224)
$\rho_{c,y}$	0.8885	0.1307 (0.0144)
$\rho_{i,y}$	0.9109	0.8091 (0.0120)
$\rho_{n,y}$	0.7986	0.7031 (0.0151)
$\rho_{v,u}$	-0.9003	-0.8505 (0.0143)
$\rho_{m,prod}$	0.2509	0.0144 (0.0976)
$\rho_{u,prod}$	-0.2735	-0.0731 (0.0984)

US data is HP filtered data with smoothing parameter $\lambda = 1600$, taken from Q1:1953 to Q2:2011. m represents the log of the number of matches. Simulated moments from the models is the mean from simulating the model 5000 times for 1000 periods and throwing out the first 100 periods.

Table 2.4: Simulated Moments $\gamma = 0.4$

	US Data	Benchmark Model	Ambiguity Model
σ_y	0.0161	0.0294 (0.0049)	0.0298 (0.005)
σ_{prod}	0.0135	0.0160 (0.0030)	0.0165 (0.003)
$\frac{\sigma_c}{\sigma_y}$	0.7925	0.7452 (0.0246)	0.7652 (0.0266)
$\frac{\sigma_i}{\sigma_y}$	4.6926	2.4890 (0.2123)	2.65 (0.2257)
$\frac{\sigma_n}{\sigma_y}$	0.6492	0.4790 (0.0168)	0.4735 (0.0174)
$\frac{\sigma_v}{\sigma_y}$	7.8652	4.9933 (0.3275)	4.9076 (0.3259)
$\frac{\sigma_m}{\sigma_{prod}}$	8.4567	7.4475 (0.5154)	7.203 (0.5032)
$\frac{\sigma_\theta}{\sigma_{prod}}$	18.6109	18.6188 (1.2885)	18.0075 (1.258)
$\frac{\sigma_u}{\sigma_{prod}}$	9.6877	9.7996 (0.5798)	9.4801 (0.5740)
$\rho_{c,y}$	0.8885	0.7657 (0.0701)	0.7358 (0.0781)
$\rho_{i,y}$	0.9109	0.8661 (0.0107)	0.8540 (0.0163)
$\rho_{n,y}$	0.7986	0.9760 (0.0035)	0.9714 (0.0051)
$\rho_{v,u}$	-0.9003	-0.9180 (0.0.173)	-0.9188 (0.0174)
$\rho_{m,prod}$	0.2509	0.8753 (0.0240)	0.8615 (0.0287)
$\rho_{u,prod}$	-0.2735	-0.9155 (0.0162)	-0.9015 (0.0213)

US data is HP filtered data with smoothing parameter $\lambda = 1600$, taken from Q1:1953 to Q2:2011. m represents the log of the number of matches. Matches were created following the method outlined in Shimer(2005b). Simulated moments from the models is the mean from simulating the model 5000 times for 1000 periods and throwing out the first 100 periods so as not to have the initial conditions affect the result.

Real Exchange Rates: Exploring Fundamentals of the Cross Section

3.1 Introduction

The asset market view of exchange rates has been the dominant paradigm in the recent international finance literature on exchange rates.¹ According to this view, the gross change in an exchange rate between two numeraire, over time, is equal to the ratio of stochastic discount factors (SDFs) expressed in the respective currency units. In and of itself, this statement is tautological, as it is always possible to derive an SDF in one set of currency units in terms of an SDF in another set of currency units using the change of units itself.² The relationship moves beyond pure tautology, however, in structural exchange rate models. Typically, in these models, the gross change in the real exchange rate between the two economies, over time, is equal to the ratio of the intertemporal marginal rates of substitution (IMRSs) defined over the consumption baskets of representative agents in each economy.

The real exchange rate between two economies (domestic and foreign), e , mea-

¹ For a summary discussion, see Lustig and Verdelhan (2012).

² See Burnside and Graveline (2014).

asures the cost of the foreign agent’s consumption basket relative to the cost of the domestic agent’s consumption basket when these are measured in a common numeraire. Empirically $e = SP^*/P$ where S is the nominal exchange rate expressed as domestic currency units per foreign currency unit, P is the consumer price index (CPI) in the domestic economy, and P^* is the CPI in the foreign economy.³ Defining the IMRS of the domestic and foreign agents over aggregate consumption, respectively, as m and m^* , the asset market view is that

$$\frac{m_{t+1}^*}{m_{t+1}} = \frac{e_{t+1}}{e_t}. \quad (3.1)$$

As discussed in Burnside and Graveline (2014), this condition holds in any structural model in which asset markets are complete.⁴

In structural models, m and m^* are determined in equilibrium, and are functions of underlying fluctuations in exogenous economic fundamentals. An early example in the literature is the seminal work of Backus and Smith (1993). They propose a model in which the representative agent in each economy has standard preferences over a consumption aggregator, which is, itself, a homothetic function of the agent’s consumption of traded and nontraded goods. They present an example where the agent in the domestic economy has the instantaneous utility function

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \quad (3.2)$$

where

$$c = c_T^\alpha c_N^{1-\alpha}, \quad (3.3)$$

³ When e falls this is referred to as a real depreciation of the foreign currency in the same way that if S falls, this is a nominal depreciation of the foreign currency (given that it buys less domestic currency units).

⁴ A standard way of ensuring that equation (3.1) holds in a structural model is to assume that there exists a complete set of state contingent claims denominated in a frictionlessly tradable numeraire, and that all agents have the same costless access to these securities.

c_T is the agent's consumption of the traded good, c_N is the agent's consumption of the nontraded good, $0 < \alpha < 1$ and $\gamma > 0$. Assuming that the agent discounts future utility by a factor $0 < \beta < 1$ relative to currency utility, this means that

$$m_{t+1} = \beta (c_{t+1}/c_t)^{-\gamma} . \quad (3.4)$$

If the foreign agent has identical preferences and has consumption denoted c^* we also have

$$m_{t+1}^* = \beta (c_{t+1}^*/c_t^*)^{-\gamma} . \quad (3.5)$$

Equation (3.1), then, implies that the log change in the real exchange rate between the two economies is

$$\ln(e_{t+1}/e_t) = \gamma[\ln(c_{t+1}/c_t) - \ln(c_{t+1}^*/c_t^*)] . \quad (3.6)$$

Clearly, equation (3.6) illustrates a close connection between the real exchange rate and economic fundamentals. In the Backus and Smith model the underlying exogenous processes are the endowments of the nontraded goods in the two economies. Because the model assumes complete asset markets, any risk associated with the endowments of the traded good are shared and traded consumption growth in both countries always equals the growth rate of the global endowment of the traded good. Consequently

$$\ln(e_{t+1}/e_t) = (1 - \alpha)\gamma[\ln(y_{Nt+1}/y_{Nt}) - \gamma \ln(y_{Nt+1}^*/y_{Nt}^*)] , \quad (3.7)$$

where y_N and y_N^* represent the endowments of the nontraded good in the domestic and foreign economies, respectively.

Whether one focuses on equation (3.6) or equation (3.7), a common message coming from the Backus and Smith model is that when times are relatively good, or, more precisely, improve more in the foreign economy than in the domestic economy,

its real exchange rate depreciates.⁵ Backus and Smith (1993) use bilateral real exchange rate and consumption data to test equation (3.6). They find little evidence in favor of it. Rather than finding the a tight positive correlation between the variables on the left and right-hand sides of equation (3.6), they find almost no correlation, or perhaps a small negative correlation. Thus, with respect to the proposed model there is a disconnect between fundamentals and the exchange rate.

In the face of Backus and Smith’s evidence, the literature has gone in a number of directions to try to explain exchange rate behavior. As mentioned above, a significant strand of the literature has pursued models in which asset markets are still assumed to be complete. Therefore, equation (3.1) continues to hold. However, the mapping from fundamentals to m and m^* changes. Lustig and Verdelhan (2007), for example, explore a model in which agents have recursive preferences over nondurable and durable consumption goods. In this model, m and m^* cannot be represented as simple functions of aggregate consumption growth. Nonetheless, there is an implied connection between macroeconomic fundamentals and the real exchange rate. Similarly, Verdelhan (2010) considers a model in which agents have preferences over the consumption aggregate that display habit formation. In this model, m and m^* are functions of a distributed lag of consumption growth in the two economies. Colacito and Croce (2011) and Bansal and Shaliastovich (2013) consider different models with recursive preferences over the consumption aggregate where agents face long run risk. Here, apart from the recursive utility formulation, the complication is that the law of motion of consumption growth is modified by the long-run risk feature. One difficulty in testing these models is that it is difficult to measure the needed

⁵ Equation (3.6) also emerges from variants of the Backus-Smith setup with different structures underlying the aggregate consumption construct. For example, one can also write down a version of the model with two frictionlessly traded goods, but in which the agents have different weights on these goods in their consumption aggregators. Ready, Roussanov and Ward (2013) develop a model with shipping costs, which also implies that equation (3.6) holds for the consumption aggregate. Their model relies on the notion that one country produces final goods while the other is a commodity producer.

macroeconomic aggregates in order to calculate empirical counterparts of m and m^* . Therefore, none of these models appear to have been tested in the same direct way as Backus and Smith's (1993) model, through a direct test of equation (3.1).

In this chapter, I revisit the relationship between macroeconomic aggregates and real exchange rates. My analysis is not, per se, a test of equation (3.1). Instead, I explore the dynamic interrelationship between consumption, output, investment and the real exchange rate. At one level, this analysis could be viewed as a generalization of Backus and Smith (1993). At another, it could be viewed as developing a set of empirical facts that any exchange rate model ought to be consistent with. While Backus and Smith (1993) shows a disconnect between one macroeconomic fundamental (consumption growth) and the real exchange rate, my question is whether this disconnect is broader and holds dynamically, as well as for other macroeconomic aggregates.

The disconnect between exchange rates and fundamentals is echoed in the asset pricing literature by a disconnect between asset returns and macroeconomic fundamentals. Related to my exploration of the exchange rate disconnect, here, Albuquerque et al. (2014), offer an exploration of the disconnect between asset returns and macro fundamentals. Instead of focusing on the contemporaneous relationships between time series, they, instead, consider longer periodicities of the data and their correlations. I take a similar approach in this chapter to real exchange rates. Rather than looking only at high frequency fluctuations (say quarterly changes), I consider medium to long-term components of real exchange rates and macro fundamentals.

A novel feature of my analysis is that, rather than looking only at bilateral exchange rate movements with the US, I instead look at a panel of 19 OECD countries. For each country in the data set, I create a series of dynamic trade weights, which I use to create a rest-of-the-world basket of currencies with which to construct a real effective exchange rate. Similarly, I construct indices of relative economic per-

formance based on aggregate measures of consumption, output, and investment. I construct these using the same set of trade weights, so that these indices measures, in a sense, when times are relatively “good” or “bad” in each country compared to the rest-of-the-world.

When exploring the dynamic relationship between real exchange rates and economic performance, I consider statistics at the country level, as well as exploiting the large cross section to draw sharper conclusions. Similar to the literature, my results do not seem to align with prevailing theories of exchange rate growth determination. I find, first, that contemporaneously and dynamically there is very little consistency across countries and very little of the movement in the real exchange rate can be “explained” by links to macroeconomic variables. When I impose cross-sectional restrictions in the full panel I confirm Backus and Smith type results for the relationship between output, consumption and the real exchange rate. In particular, the correlation between consumption growth and real exchange rate changes is, if anything, opposite to what theory suggests.

As previously mentioned, similar to Albuquerque et al. (2014) I look at these relationships using different periodicities of the data, using a band-pass filter. I find little correlation between consumption and the real exchange rate at low frequencies. However, I identify interesting dynamic patterns in the correlations of the real exhcng rate with investment and output at low frequencies. . These findings are novel and suggest a variety of benchmarks which new exchange rate models could be tested against. Most interesting, among my findings, is that an increase in the relative investment of the home country signals a subsequent real appreciation of the home currency at low frequencies. That is, when times are relatively “good” today, in terms of investment, in the home country, we tend to see a later real appreciation of its exchange rate.

To more fully explore the data, I take a similar approach to analyzing the cross

section as Eichenbaum and Evans (1995) do using US bilateral data. That is, I create a vector autoregression (VAR) using the entire panel of countries, in order to see if orthogonalized monetary policy shocks, or other orthogonalized shocks, can explain these long run dynamics. Simulating the estimated VAR while, alternatively shutting down different sets of shocks, I find that no single shock appears to be able to explain the dynamic cross correlations between the fundamentals and the real exchange rate. I do, however, find that individual country results can be explained somewhat through individual specific shocks despite the restriction that all countries must have the same parameters in the VAR.

This chapter consists of three more sections. In section 2, I present empirical findings relating to dynamic correlations in the cross-section including results for different periodicities of the data. In section 3 I present the panel VAR and some simulation results. Section 4 concludes.

3.2 Dynamic Correlations

In this section I lay out the basic empirical work relating to the dynamic cross correlations of real exchange rates and macroeconomic fundamentals. I begin by discussing the construction of my data set. Then I describe results for individual countries, which appear to display few common patterns that could readily be explained with prevalent theoretic models. I also introduce my panel analysis. My panel regressions, which identify pseudo-correlations between the macro variables and real exchange rates, return values which are very similar to the median point estimates from the country regressions. Whether the restrictions embodied in my panel estimates are indicative of a common set of relationships pertaining to all countries is an open question. Nonetheless, these dynamic correlation patterns with respect to the cross section are novel findings relative to the literature.

3.2.1 Data Creation

My sample consists of 20 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK and the US.⁶ I treat each country in the sample as a “home” country in the creation of the data. References to the “home” country are for ease of explanation. However the process is repeated for every country in the sample. The real exchange rate of the home economy and its relative economic performance are measured against the “rest-of-the-world”, represented by the other countries in the data set.

First I create a measure of the Real Effective Exchange Rate (REER). In order to do this I create a series of dynamic weights based on bilateral trade flows between countries taken from the United Nations Comtrade Database for each year from 1970–2014. Each country in the rest-of-the world is weighted by the proportion of their trade with the “home” country versus the “home” country’s total self-reported exports and imports with the rest-of-the-world. I then use these weights to create a weighted basket of nominal currency performance relative to the rest-of-the-world which I will refer to as the Nominal Effective Exchange Rate (NEER). This measure is based on quarterly nominal exchange rate data taken from the International Financial Statistics (IFS) from 1970–2014.⁷ I then use the reported quarterly consumer price index (CPI) for each country, from the OECD database, and the same weights, to create a REER index. The log change of the REER of country j ’s currency versus the rest of the world is given by:

$$\Delta \ln REER_{j,t} = \sum_{i \neq j}^n [w_{j,i,t} \Delta \ln e_{i,j,t}] , \quad (3.8)$$

⁶ I relegate some discussion of my data selection and creation to the appendix.

⁷ I repeat this procedure using annual data, but do not report the results in this chapter as they are virtually identical to the results obtained with quarterly data.

where $e_{i,j,t}$ is the real exchange rate between country j and country i in terms of country j 's currency, and $w_{j,i,t}$ is the weight of country i in country j 's trade at time t . An index of the level of the REER can be created by accumulating the period-by-period growth rates.

I next consider measures of real consumption, real investment and real output, also obtained from the OECD database. I measure the relative consumption, investment and output growths of each country in the sample relative to the home country. I then use the aforementioned trade weights to create a series of indices which I refer to as Real Effective Consumption (REC), Real Effective Investment (REI) and Real Effective Output (REO). REC growth for country j at time t is constructed as

$$\Delta \ln REC_{j,t} = \Delta \ln c_{j,t} - \sum_{i \neq j}^n [w_{j,i,t} \Delta \ln c_{i,t}] , \quad (3.9)$$

where $c_{i,t}$ is the real consumption of country i at time t .

There are two simple motivations for constructing these measures. One obvious motivation is that the simplest theory, in Backus and Smith (1993), suggests that there is a close theoretical relationship between the log change of the real exchange rate and relative consumption growth. At some level, my work extends this logic to other variables of interest, investment and output. The reason to use multilateral REERs instead of bilateral REERs for pairs of countries is simply that the theoretical relationships, if they hold for two countries, must also hold for weighted averages of countries.

3.2.2 Correlation Results

Using my created data series, I compute basic correlations for each country between the per period growth rates of the macroeconomics variables and the per period growth rate of the REER.. Table (3.1) shows the contemporaneous correlations

Table 3.1: Contemporaneous Correlations

	$\rho_{c, reer}$	$\rho_{y, reer}$	$\rho_{i, reer}$		$\rho_{c, reer}$	$\rho_{y, reer}$	$\rho_{i, reer}$
Australia	-0.02	-0.05	-0.07	Japan	0.01	0.04	0.01
Austria	-0.03	0.04	-0.00	Netherlands	-0.05	0.13	0.09
Belgium	-0.14	0.02	-0.04	New Zealand	0.06	0.02	-0.02
Canada	-0.07	-0.02	-0.02	Norway	0.08	0.05	-0.02
Denmark	0.17	0.01	0.18	Portugal	0.12	-0.00	-0.08
Finland	0.02	-0.09	-0.06	Spain	-0.17	-0.13	-0.18
France	0.01	0.00	-0.05	Sweden	-0.10	-0.06	-0.06
Germany	0.00	0.02	0.02	Switzerland	-0.13	0.02	0.03
Ireland	0.02	-0.10	0.09	UK	-0.06	-0.21	0.02
Italy	-0.12	-0.07	-0.06	USA	0.14	0.11	0.01

between the REER and consumption, output and investment. The main takeaway from the table is that there is very little consistency across the different country results. For consumption, estimated correlations range from -0.17 to 0.12. For output they range from -0.21 to 0.13 and for investment they range from -0.18 to 0.18. Countries for which the correlation is positive for consumption do not necessarily have positive correlations for investment and output. The other main takeaway is the fact that most of these results are not statistically different from zero confirming previous results, that at least contemporaneously, the real exchange rate is very poorly linked to fundamentals.

Although there appears to be little in the way of contemporaneous relationships between REERs and macro fundamentals, an open question is whether there are more interesting and informative relationships with regard to the dynamics of these

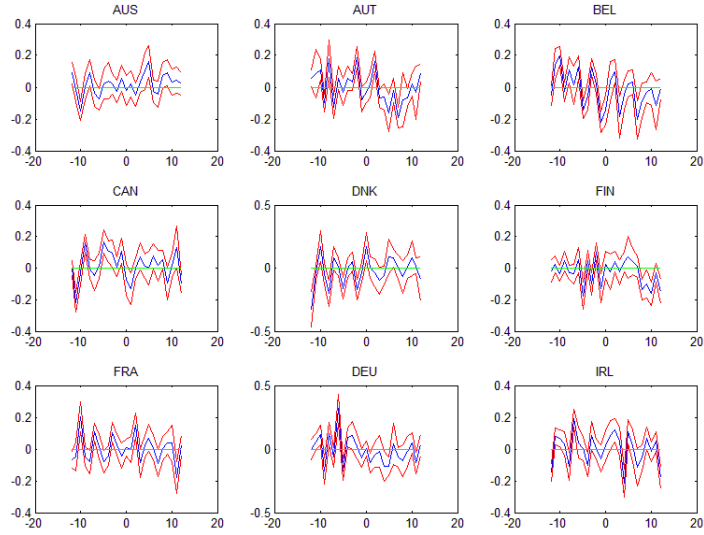


FIGURE 3.1: Sample of Country Specific Dynamic Cross Correlations $\rho_{\Delta c, \Delta reer}$

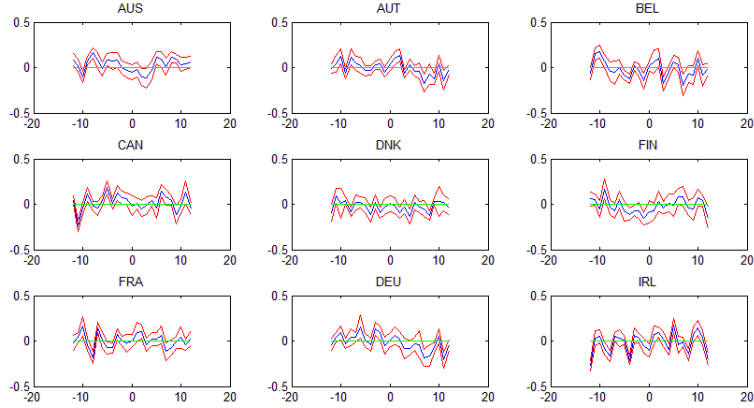


FIGURE 3.2: Sample of Country Specific Dynamic Cross Correlations $\rho_{\Delta y, \Delta reer}$

variables. Therefore, I measure the dynamic cross correlations for each of these variables. Figures 3.1, 3.2, 3.3 display graphically the dynamic cross correlations between the REER and consumption, output and investment respectively.

. For country j , and consumption, for example, I look at the correlations

$$\text{corr}(\Delta \ln REC_{j,t+k}, \Delta \ln REER_{j,t}), \quad (3.10)$$

for $k = -12, -11, \dots, 11, 12$. I only display results for a representative subset of the

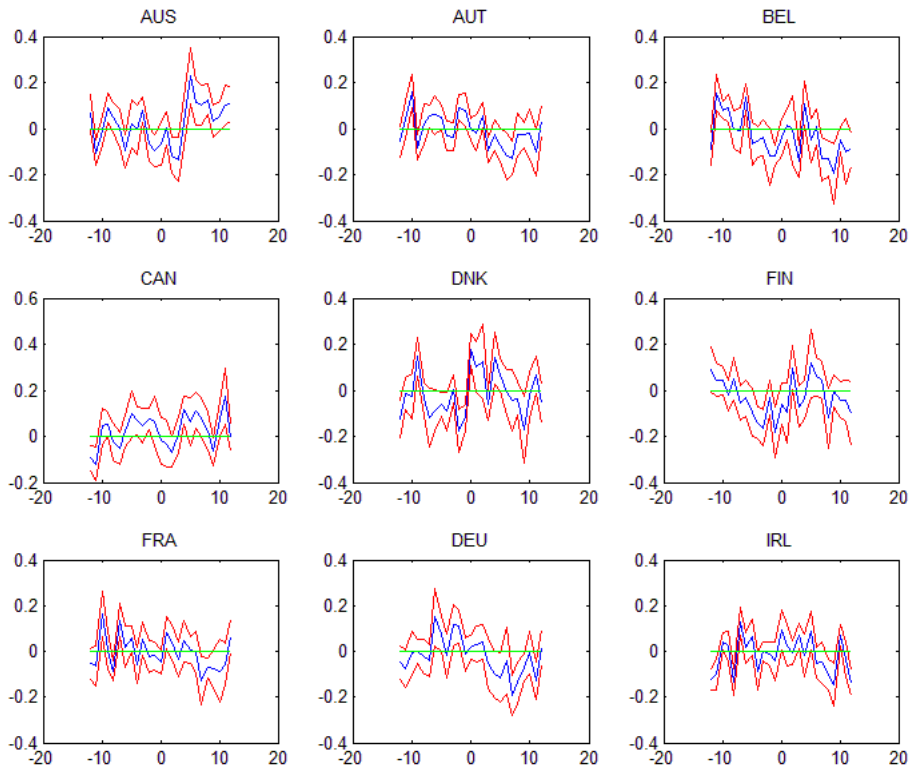


FIGURE 3.3: Sample of Country Specific Dynamic Cross Correlations $\rho_{\Delta i, \Delta reer}$

individual countries . The first point learned is that none of the dynamic patterns are very similar across countries. The second is that very few of them are statistically different from zero.

Although there doesn't seem to be any sort of interesting dynamic correlation pattern between the growth rates of consumption, output and investment and the REER, that is somewhat consistent across countries, I also passed data for the individual countries through a band-pass filter. I do this primarily to gauge if the REER may be more linked to fundamentals in the long and medium run than it is at high frequencies To be more specific, I specifically look at data passed through an optimum truncated band-pass filter as outlined in Baxter and King (1999), with periodicities of 1-3 years, 3-5 years and 5-10 years. In addition, I also look at the

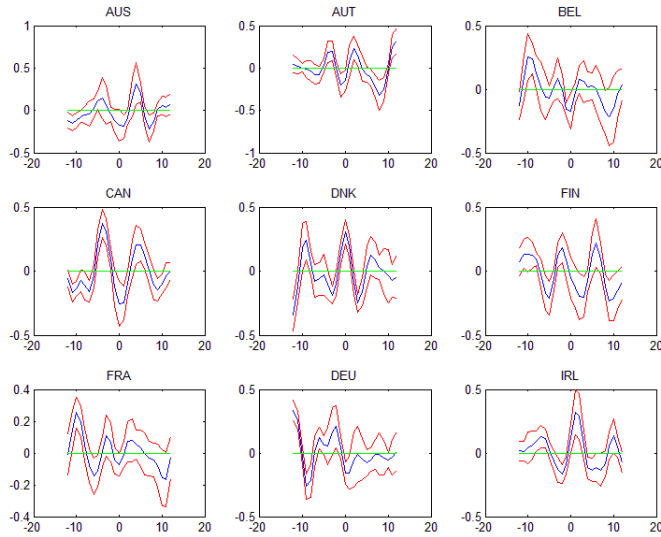


FIGURE 3.4: Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 1-3 years: $\rho_{c, reer}$

data through a low-pass filter with frequencies lower than 1 year.

Figures 3.4, 3.5, and 3.6 show the dynamic cross correlation for a subset of countries for the 1–3 year band-pass filter. As before, I do not show the full set of countries in order to save space. Once again, there is little consistency across countries, but there are some results which are interesting and statistically significant. Rather than just the sporadic hovering around zero present in the growth rate correlation patterns, we see several spikes which are significantly different than zero. Despite this there is very little crosssectional consistency in the signs and magnitudes of the correlations. For example, in figure 3.4, we can see that Denmark has a positive contemporaneous correlation between consumption and the REER at this periodicity, whereas France and Canada seems to have significantly strong negative correlations.

Repeating this exercise, figures 3.7, 3.8, and 3.9 show the cross correlation between REER, and consumption, output and investment, respectively, for periodicities between 3 and 5 years. The main take-away is how much smoother and consistent

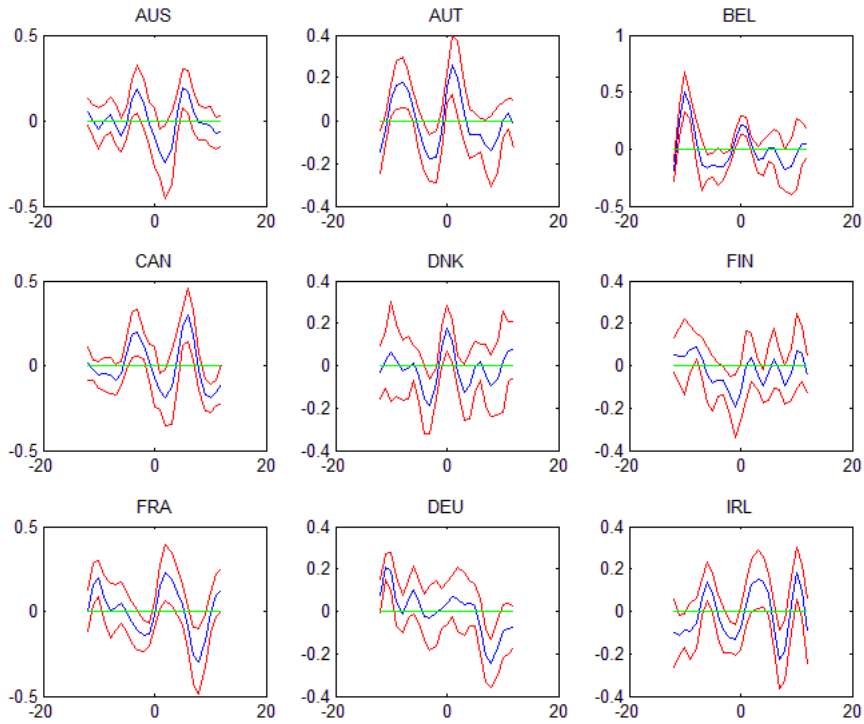


FIGURE 3.5: Dynamic Cross Correlation with Output for Individual Countries with periodicities from 1-3 years: $\rho_{y, reer}$

the results are. The smoothness isn't terribly surprising considering the filtering process however there are a few important things to note. The first, as can be seen in figure 3.7, is that the dynamic correlations between consumption and the REER are noticeably and consistently positive. Given how the data are constructed, this implies that when home consumption is relatively high we see an appreciation of the domestic currency, in general, across most of the dynamics. If this held consistently across all countries it would be a strict rejection of the risk sharing condition in the long run that is present in the Backus and Smith (1993) model. We revisit this finding in the next section.

Similarly, output and investment also display interesting dynamic relationships with REERs that appear more sinusoidal than the patterns in the consumption

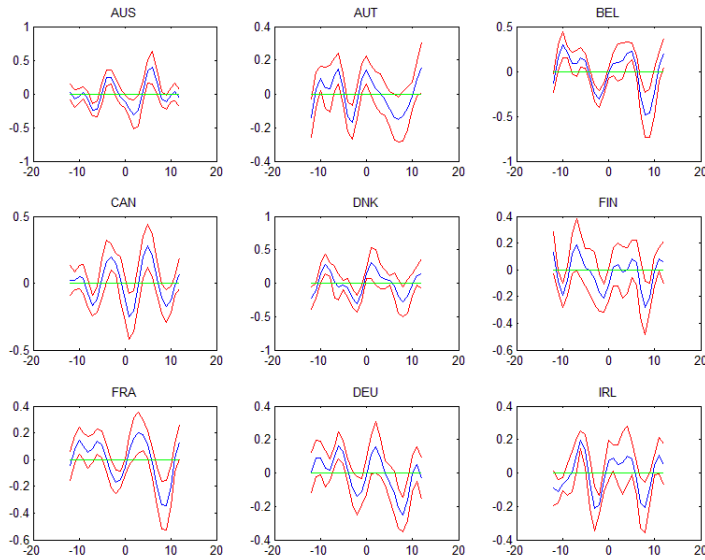


FIGURE 3.6: Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 1-3 years: $\rho_{i, reer}$

correlations. Figures 3.8 and 3.9 attest to this fact. Similarities also seemingly emerge across countries. In the figures presented we see that though the point estimates differ, the behavior of the cross correlations are incredibly similar for example in the case of Finland, Australia, Ireland, France, and Germany. Obviously there are exceptions but the shapes of the dynamic cross correlations seem to be relatively more consistent across countries with respect to these longer term periodicities.

Finally I present the results for the long run band-pass filter for 5-10 years and the low-pass filters in figures 3.10, 3.11, and 3.12. Many of the countries maintain the relationships they displayed for the medium run band-pass filter. However, many of the individual point estimates lose their statistical significance. This may be due to the fact that the sample itself is only 40 years long so there may be insufficient data to accurately identify the longer run correlations. It may also be the case that fundamentals are not as tied to the real exchange rate as we look at longer and longer run periodicities. Regardless it is nonetheless encouraging that the sinusoidal

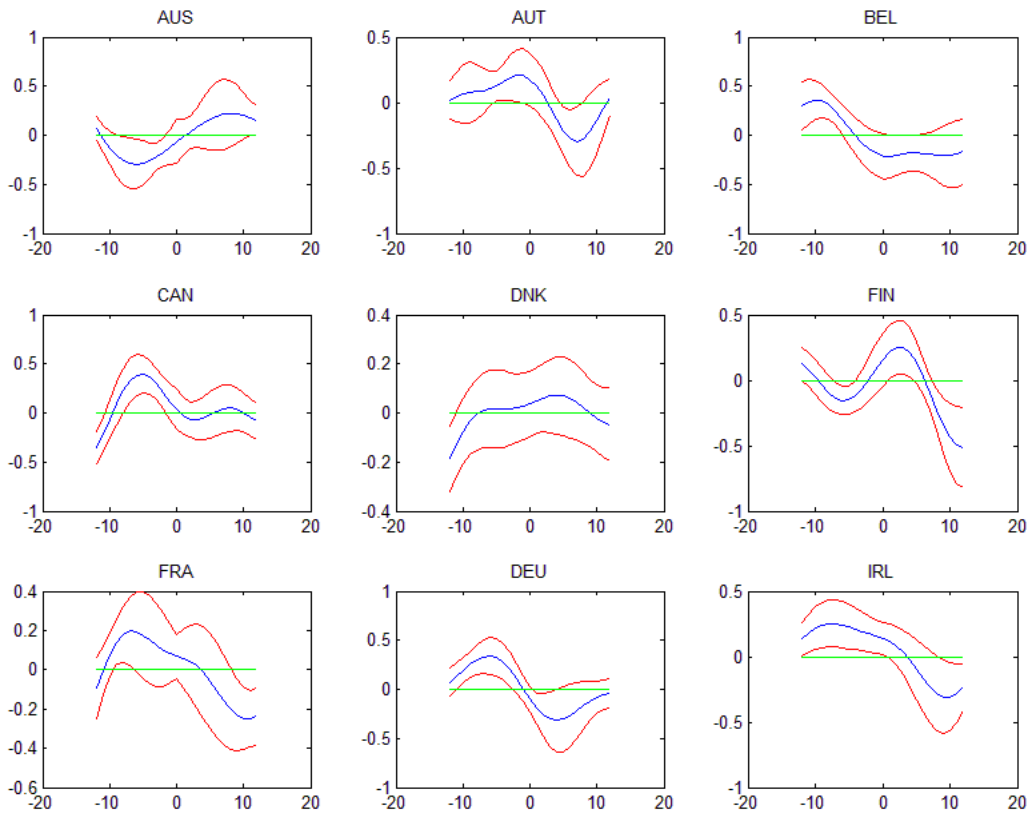


FIGURE 3.7: Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 3-5 years: $\rho_{c, reer}$

relationship still seems to exist with regards to output and investment.

Figures 3.13, 3.14, and 3.15 display the dynamic cross correlations for the same subset of individual countries, this time using a low-pass filter with periodicities longer than 4 quarters. The lowpass consumption results are largely inconsistent across countries. In addition they are, with a few exceptions, not significantly different than zero. The low-pass output and investment results, on the other hand, also lack significance for large portions of the results, however their relative shape seems consistent with what was seen in the band-pass results. This most likely is because the higher frequency components dampen the medium and longer frequency

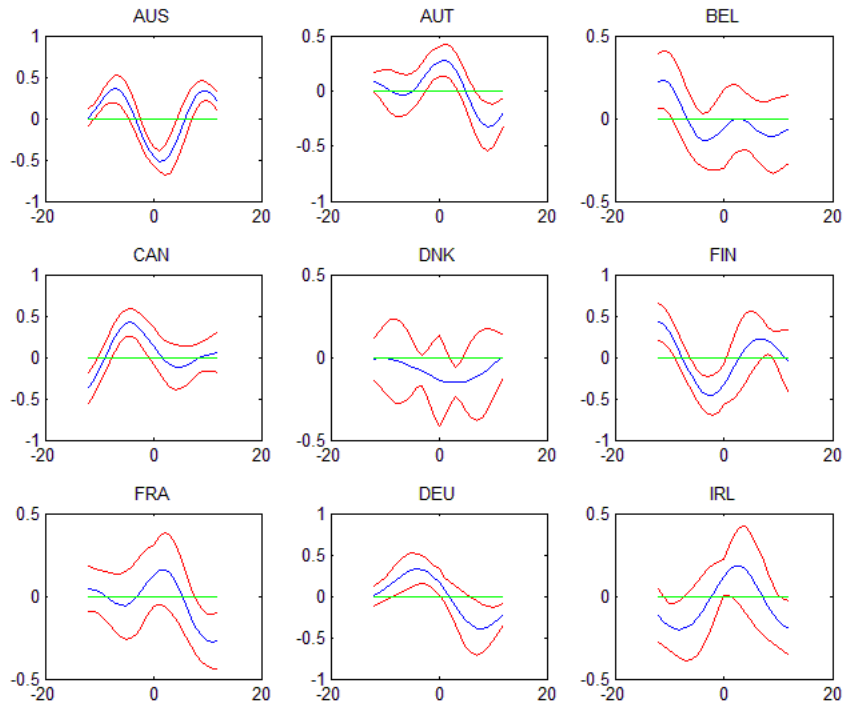


FIGURE 3.8: Dynamic Cross Correlation with Output for Individual Countries with periodicities from 3-5 years: $\rho_{y, reer}$

relationship.

I also note that I have repeated the same exercise with respect to the NEER. I suppress the results only because they are consistent with the results given with regards to the cross correlations of the fundamental variables and the real exchange rate. This is unsurprising given the fact that the real and nominal exchange rates are so tightly correlated in the data, with a correlation coefficient exceeding 0.9, in most cases.

3.2.3 Panel Regressions

I now add more structure to the empirical exercise. I make the assumption that, despite the large cross sectional variation with regards to the high frequency components, there exist similar relationships across countries with respect to the lower

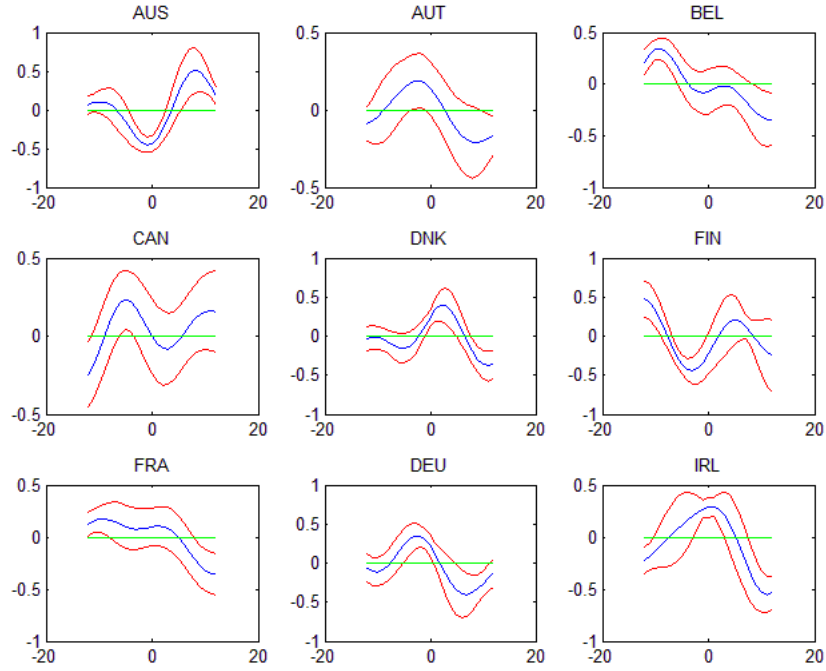


FIGURE 3.9: Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 3-5 years: $\rho_{i, reer}$

frequency components of the data series. This is not too bold an assumption given the similarities across countries, especially when we look at the individual country dynamic correlation coefficients for periodicities between 3 and 5 years. In order to do this, I run panel regressions, in which I estimate the following model for consumption:

$$\Delta \ln REER_{i,t} = \alpha + \beta_k \Delta \ln REC_{i,t+k} + \varepsilon_{i,t} \quad k \in [-12, 12] \quad (3.11)$$

Likewise in order to be robust to the effect of fixed country effects I also estimate:

$$\Delta \ln REER_{i,t} = \alpha_i + \beta_{k, FE} \Delta \ln REC_{i,t+k} + \varepsilon_{i,t} \quad k \in [-12, 12] \quad (3.12)$$

If equation (3.6) is correct, one would expect both β and β_{FE} to be less than 0, at least contemporaneously, implying that when times are relatively bad in the home country that we should expect an appreciation in real terms of the home currency. Since the

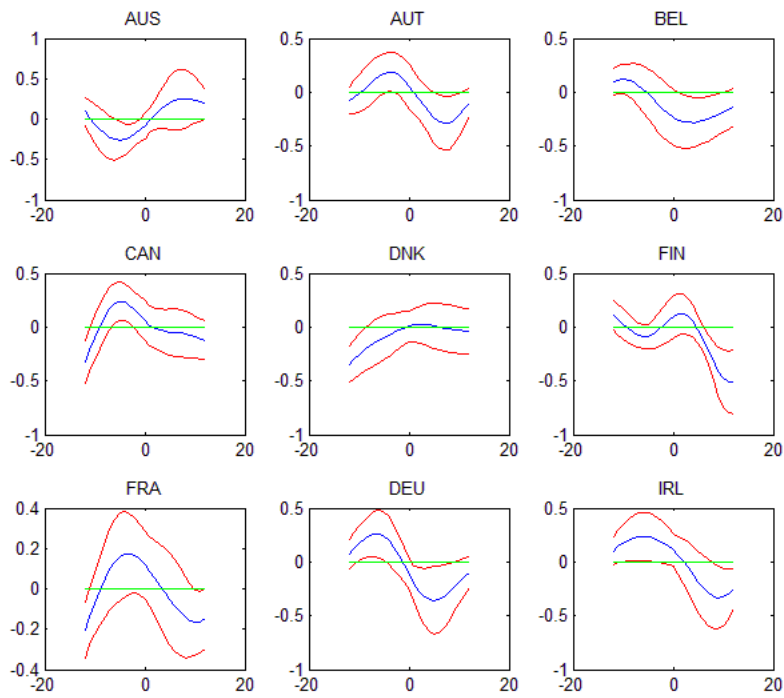


FIGURE 3.10: Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 5-10 years: $\rho_{c, reer}$

results are similar between the fixed effects and the non-fixed effects regressions I report only the fixed effects regressions. For expediency, I also note that I run the above regression in terms of consumption, for each of the series that has gone through the band-pass and low-pass filters. Figure (3.16) shows the results for the panel regression for the first differenced data. Unsurprisingly, much like the country specific regressions, we see no clear pattern, as well as little to no significance in terms of the results. Some of the point estimates do show significance but due to the scattered nature of the betas it is difficult to see any sort of pattern.

Meanwhile figures (3.17), (3.18), (3.19) show the results for the panel regression on the band-pass filtered data for periodicities between 1 to 3, 3 to 5 and 5 to 10 years respectively. Given the power gained from the panel we see a clear and highly signif-

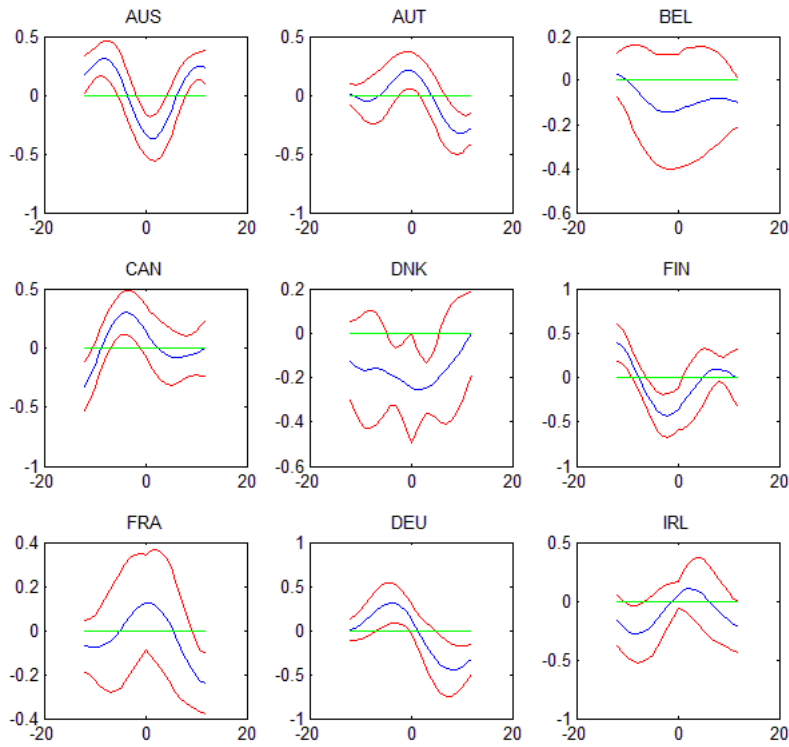


FIGURE 3.11: Dynamic Cross Correlation with Output for Individual Countries with periodicities from 5-10 years: $\rho_{y, reer}$

icant pattern in the betas in each of the graphs. In the higher frequency component we do see a slight negative correlation contemporaneously between the REER and the relative consumption measure. However, we see at both lead and lag that this relationship becomes positive signaling that current high relative consumption implies that the component with periodicity between 1 and 3 years of the real effective exchange rate should experience an appreciation in the future. Similarly figure (3.18) displays a similar however far less significant pattern as the longer run components of relative consumption and the REER seem to be far less related. Finally in figure (3.19), we see, similar to the 3 to 5 year periodicity, that there is a flattening out of the curve. If anything it goes slightly negative which implies that the risk sharing

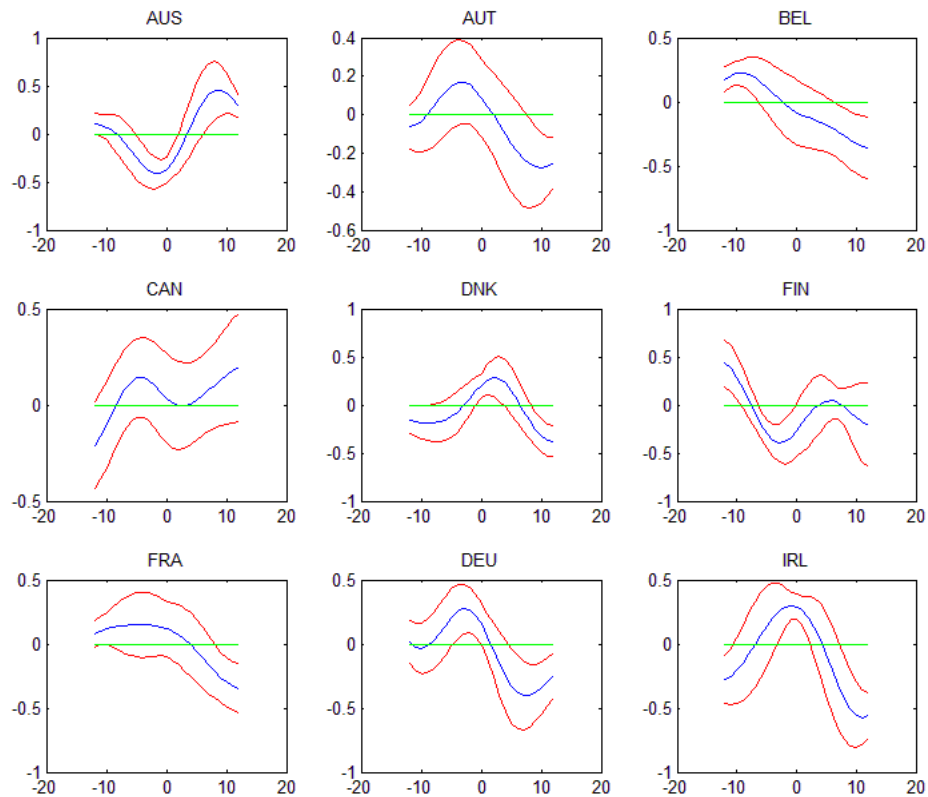


FIGURE 3.12: Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 5-10 years: $\rho_{i, reer}$

condition implied by equation (3.6) is stronger in terms of longer run periodicities.

Given these results we can look at the low-pass filter results and see that, unsurprisingly the betas seem to be a combination of the three different periodicity results, leaving the conclusion that consumption has almost a zero correlation dynamically with the real effective exchange rate. This is a far cry from the expected strong negative relationship that equation (3.6) suggests.

As with consumption I do a similar exercise with output growth. I run a panel regression in which I estimate the following model

$$\Delta \ln REER_{i,t} = \alpha_i + \beta_{k, FE} \Delta \ln REY_{i,t+k} + \varepsilon_{i,t} \quad k \in [-12, 12] \quad (3.13)$$

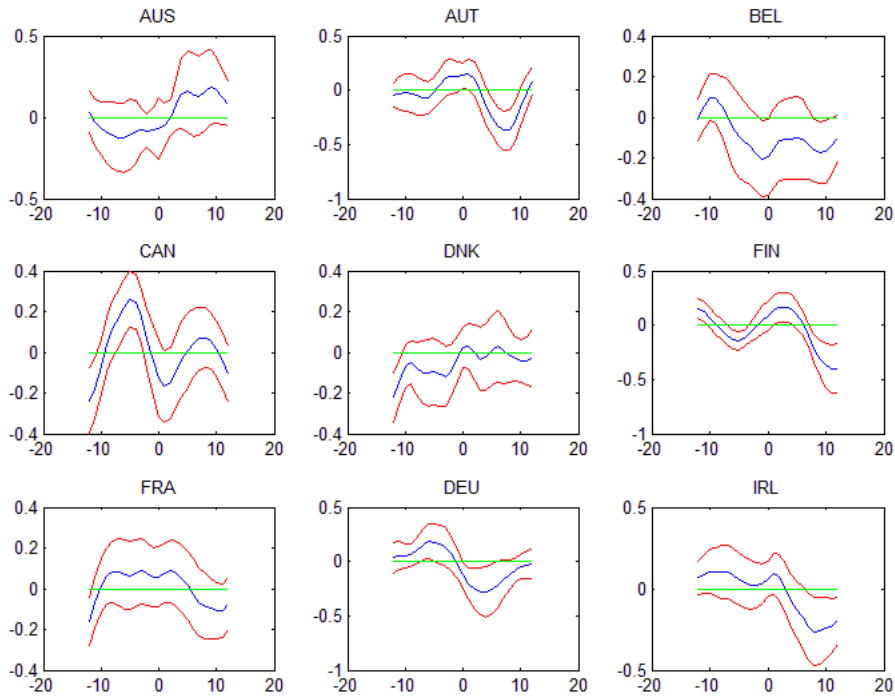


FIGURE 3.13: Dynamic Cross Correlation with Consumption for Individual Countries with periodicities from 1+ years: $\rho_{c, reer}$

Once again we might expect the estimated β 's to be less than 0, if the story from the theoretical model is correct, assuming we use relative output as a substitute for consumption in measuring the stochastic discount factor. In truth whether we thought this would be a good substitute or not one would expect similar results to the consumption panel since both consumption and output, and therefore relative consumption and output are tightly linked in the data. Figure (3.21) graphs the panel results from the regression of the growth rates of the REER and the measure of relative output. Similar to consumption we see that relative output and the real effective exchange rate have no discernible dynamic correlation pattern. Many of the results are close to zero and the few significant results are inconsistent across the different lags. We see similar results with the regression on the band-pass filtered

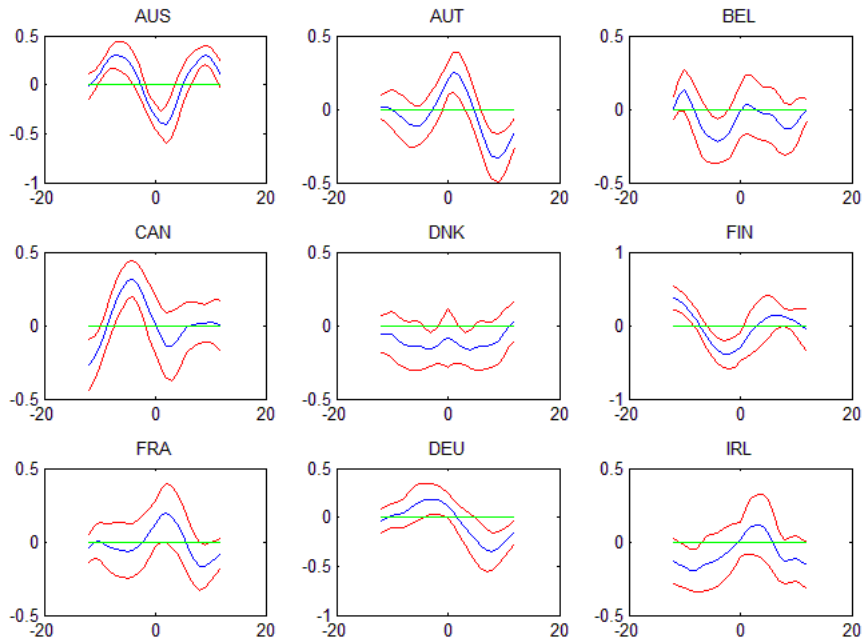


FIGURE 3.14: Dynamic Cross Correlation with Output for Individual Countries with periodicities from 1+ years: $\rho_{i, reer}$

data with periodicities from 1 to 3 years as displayed in figure (3.22).

However when we look at the regression run on the data with periodicities between 3 and 5 and 5 and 10 years we see a clear and distinct pattern from the pattern shown via the consumption regressions. Figure (3.23) shows this best. We see that good times in terms of output today signal a real appreciation of the exchange rate tomorrow. However, contemporaneously, we see a strong negative coefficient. Unlike consumption this pattern does not disappear as evidenced by figure (3.25) where we see that the sinusoidal nature and the strong negative contemporaneous correlation and the positive correlations suggesting future appreciations of the exchange rate given high relative output is still present in the dynamics.

Finally, I run the same regression on the measure of relative investment. Where

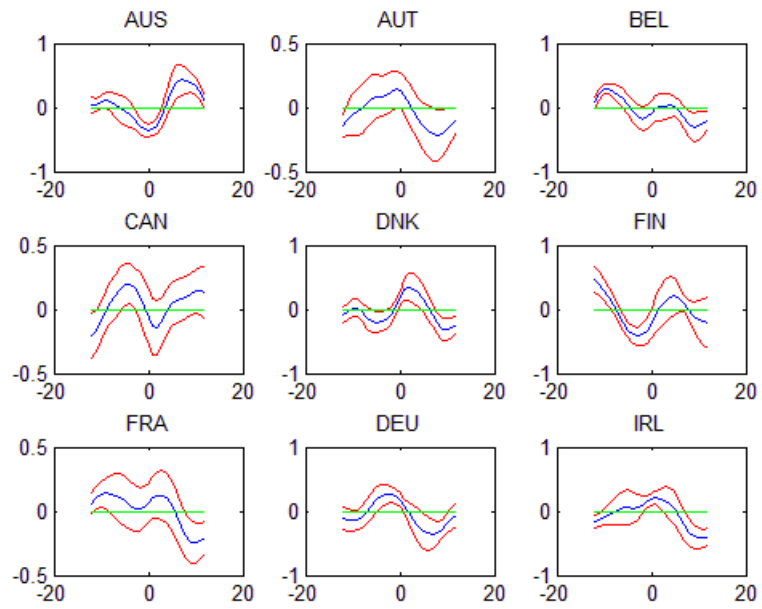


FIGURE 3.15: Dynamic Cross Correlation with Investment for Individual Countries with periodicities from 1+ years: $\rho_{i, reer}$

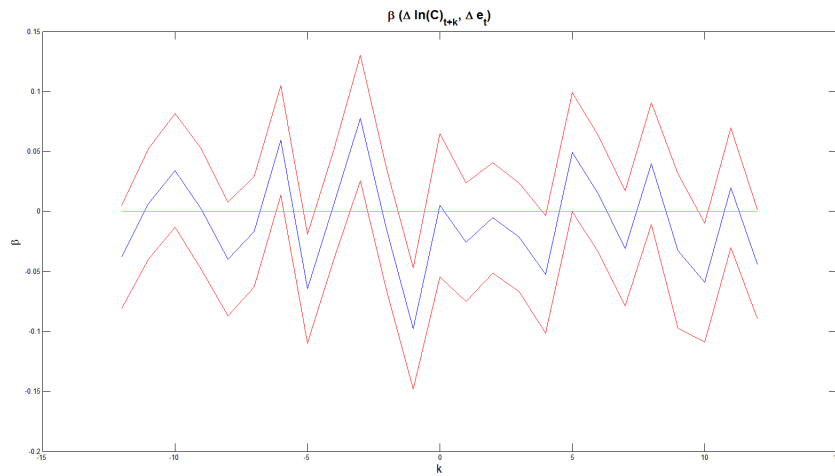


FIGURE 3.16: Estimated β for Δc_{t+k} , $\Delta reer_t$

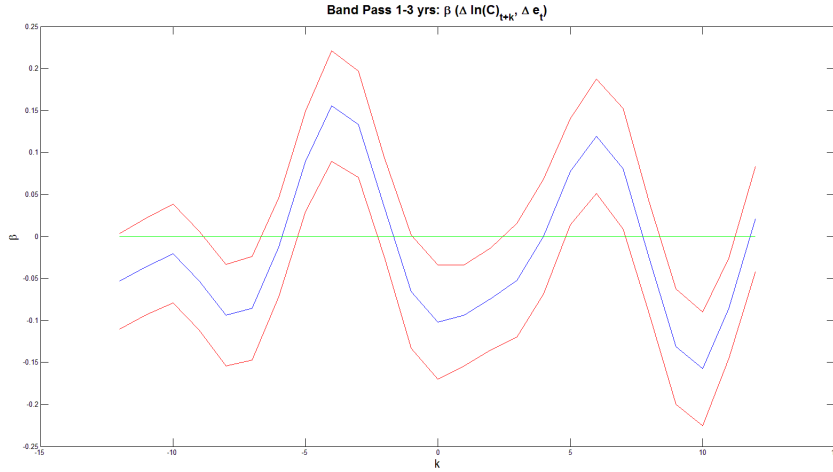


FIGURE 3.17: Estimated β for c_{t+k} , $reer_t$ for periodicities from 1 to 3 years

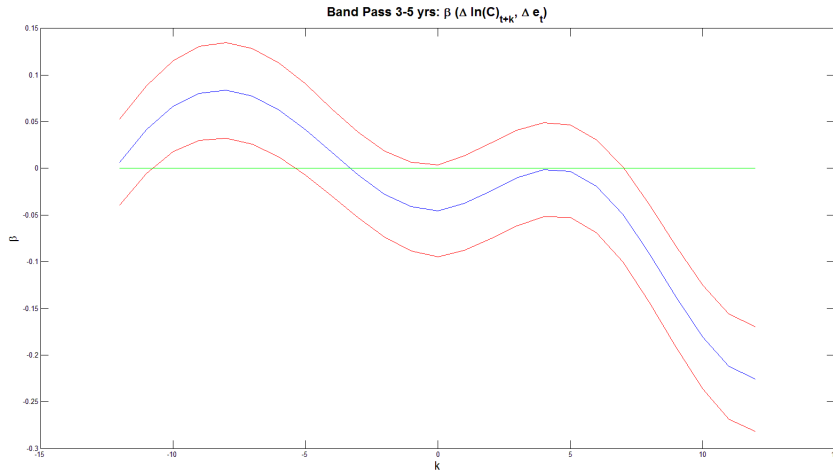


FIGURE 3.18: Estimated β for c_{t+k} , $reer_t$ for periodicities from 3 to 5 years

the model is given by:

$$\Delta \ln REER_{i,t} = \alpha_i + \beta_{k,FE} \Delta \ln REI_{i,t+k} + \varepsilon_{i,t} \quad k \in [-12, 12] \quad (3.14)$$

Figure (3.26) provides the results for these panel regressions. Unsurprisingly the graph is similar to both the results from the consumption and output-based regressions. Figure (3.27) shows the results from the similar regression using data with periodicities between 1 and 3 years. It is important to notice that unlike for output,

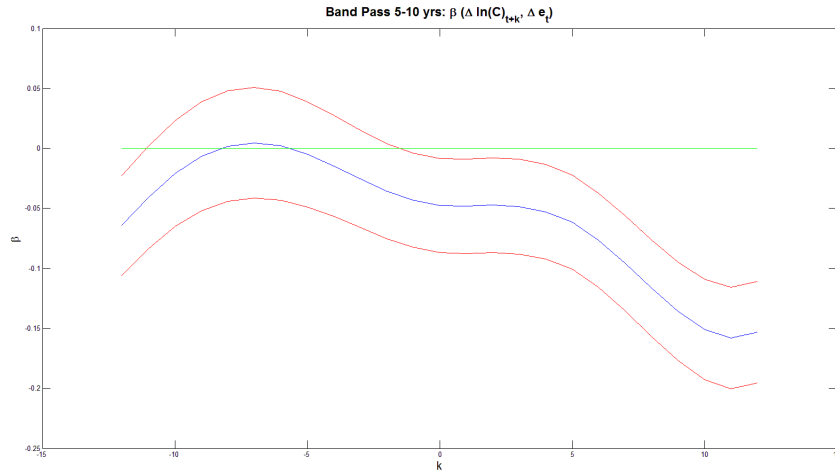


FIGURE 3.19: Estimated β for c_{t+k} , $reer_t$ for periodicities from 5 to 10 years

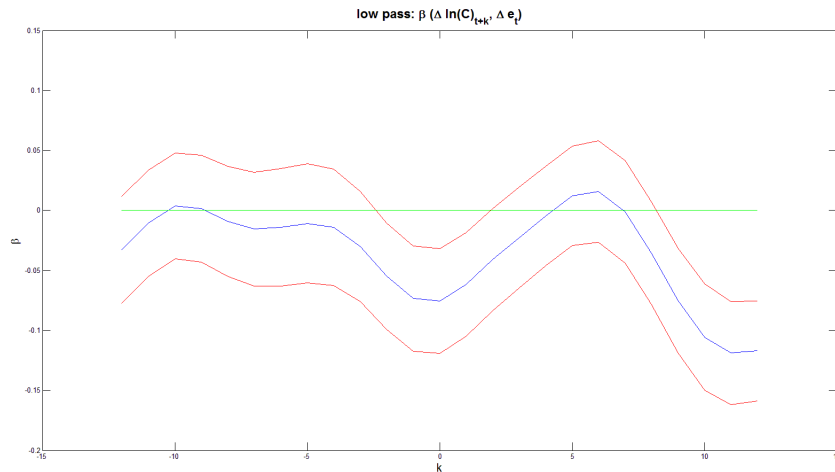


FIGURE 3.20: Estimated β for c_{t+k} , $reer_t$ for periodicities greater than 1 year

a clear pattern emerges in the relatively higher frequency components of the data. This is even more evident when we compare it to the results from the regression in figure (3.28). However the pattern is virtually identical to that expressed in the output betas. That is a strong negative contemporaneous correlation. However, we also see that an increase in relative investment today prompts a future appreciation of the exchange rate.

Before moving on, I note the following: All of these estimated betas, though sta-

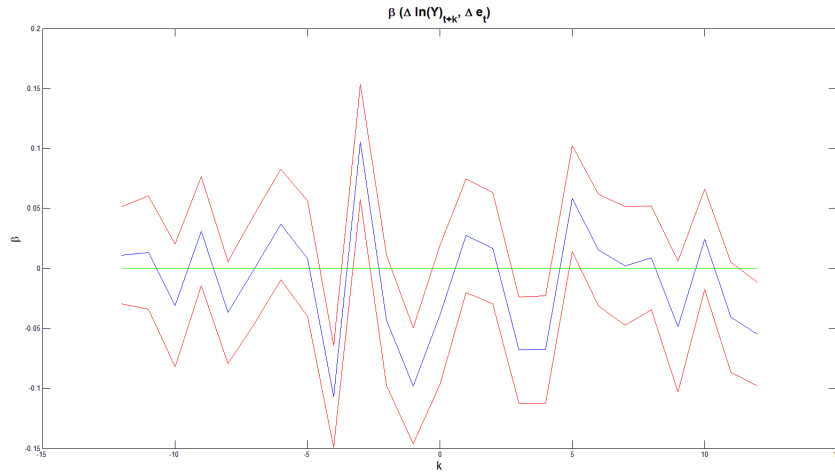


FIGURE 3.21: Estimated β for $\Delta y_{t+k}, \Delta reer_t$

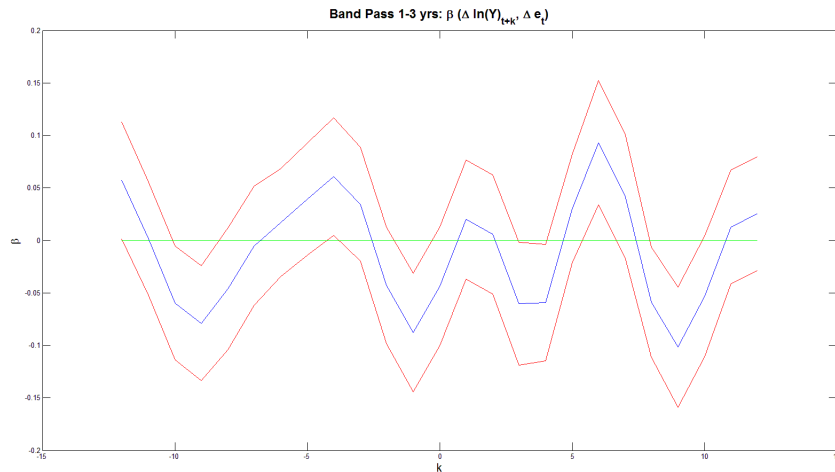


FIGURE 3.22: Estimated β for $y_{t+k}, reer_t$ for periodicities from 1 to 3 years

tistically significant, are still relatively small. This means that the fundamentals are still not tightly linked to the REER. Despite this it is encouraging though surprising to see different correlations in regards to the medium to long run periodicities of the data for consumption versus those for investment. It is further surprising that investment seems to have a very distinct dynamic pattern with regards to the REER when most modeling attempts have largely ignored investment dynamics. Regardless, the long run dynamic correlations imply an interesting and complicated modeling chal-

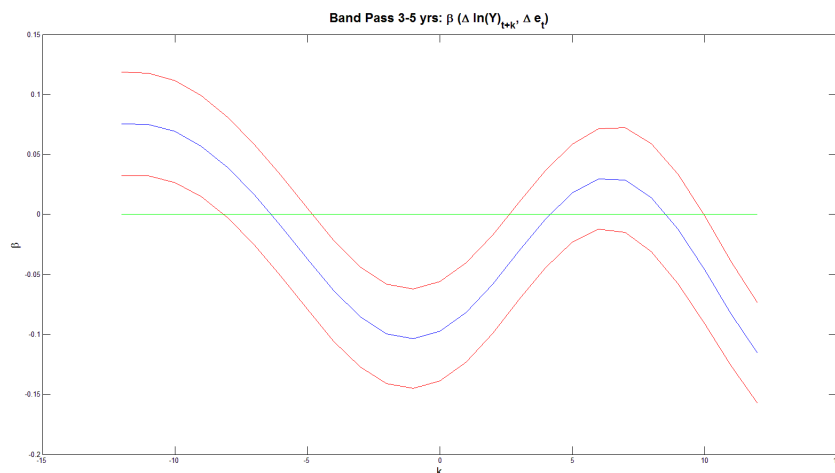


FIGURE 3.23: Estimated β for y_{t+k} , $reer_t$ for periodicities from 3 to 5 years

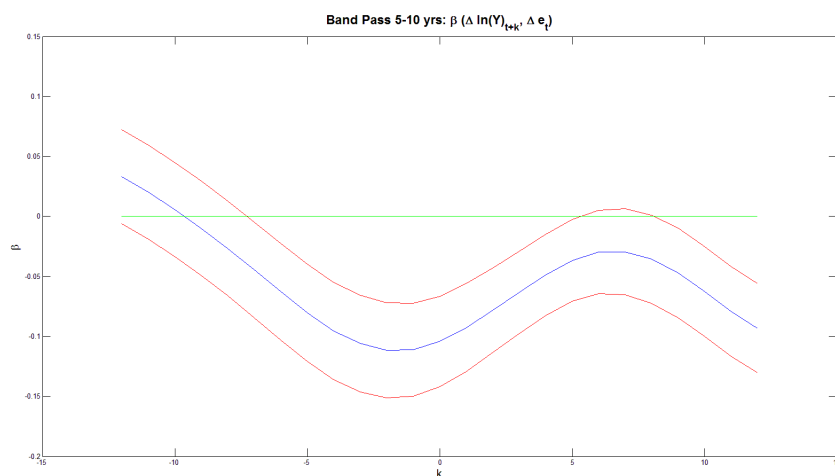


FIGURE 3.24: Estimated β for y_{t+k} , $reer_t$ for periodicities from 5 to 10 years

length that has so far been unaddressed in the literature.

3.3 Vector Autoregressions

The results from the previous section do provide us an interesting opportunity to look further at the empirical dynamics. In order to do this, I estimate a VAR using the panel, with the hope that I can identify a shocks that lead to the dynamics in question. This is in a similar vein to the work of Eichenbaum and Evans (1995),

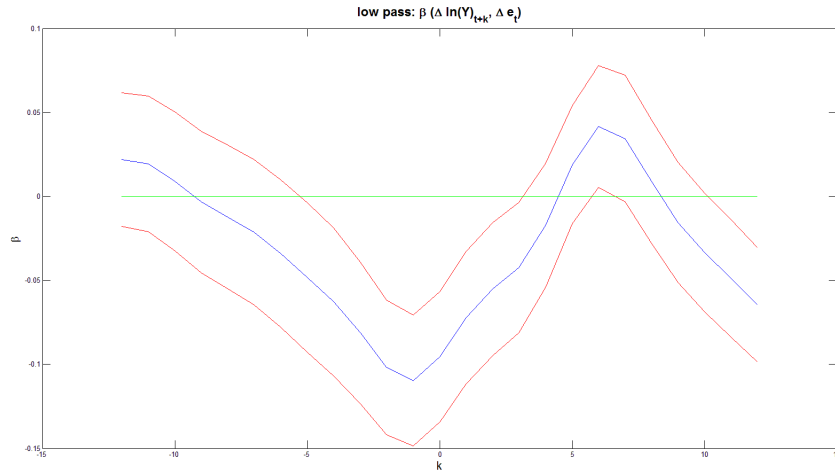


FIGURE 3.25: Estimated β for y_{t+k} , $reer_t$ for periodicities greater than 1 year

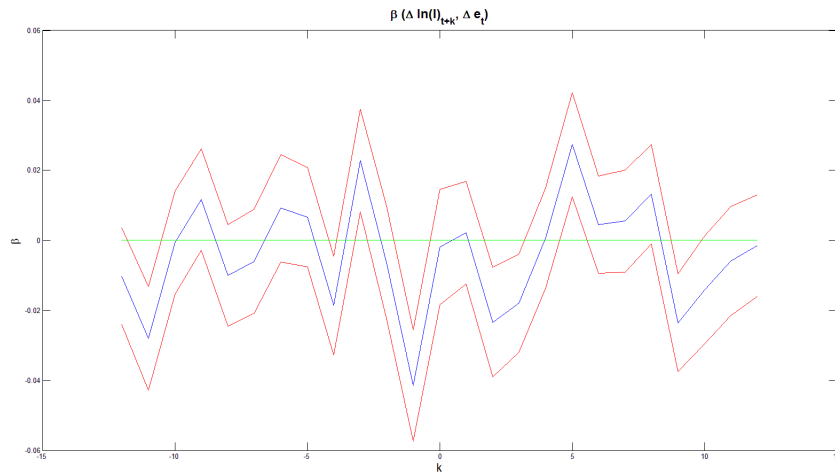


FIGURE 3.26: Estimated β for Δi_{t+k} , $\Delta reer_t$

except that I look at the data in a large panel rather than just bilaterally between the US and a small sample of countries. I'm also interested in discovering whether a VAR can explain the cross sectional variation in the dynamics that we observed. I would also like to test the assumption that it is reasonable to assume that each country has a similar structure in regards to the relationship between the fundamentals and the real exchange rate.

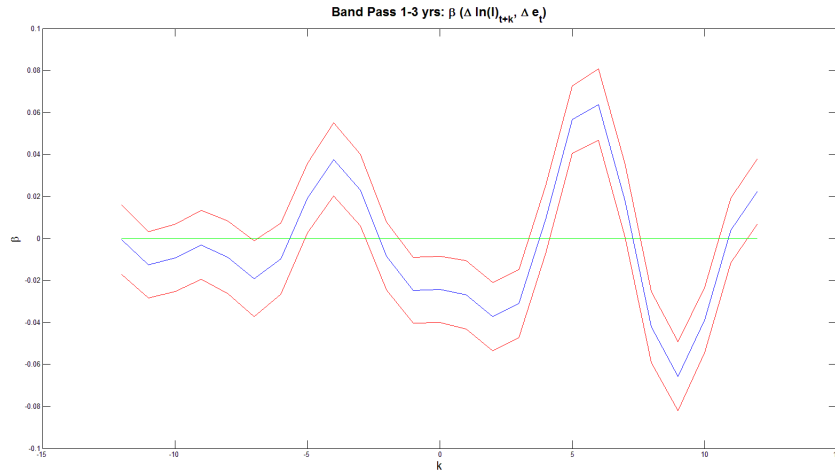


FIGURE 3.27: Estimated β for i_{t+k} , $reer_t$ for periodicities from 1 to 3 years

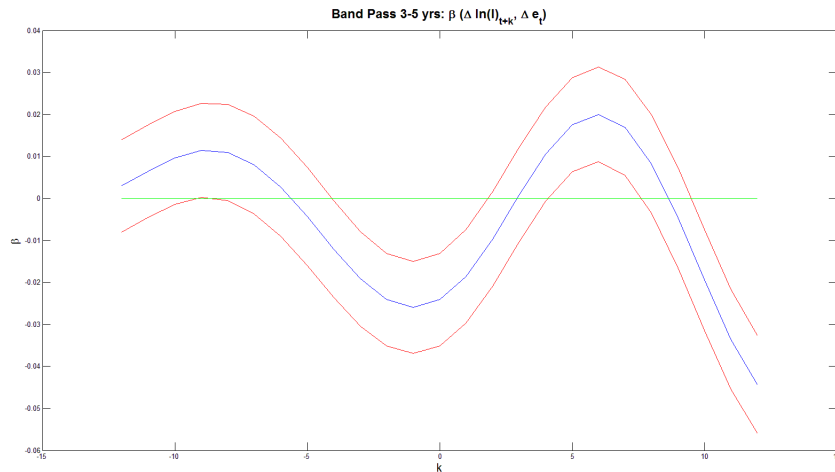


FIGURE 3.28: Estimated β for i_{t+k} , $reer_t$ for periodicities from 3 to 5 years

3.3.1 VAR Environment

In order to estimate the VAR and identify the orthogonalized shocks it's important that I establish a relatively reasonable Wold ordering. I assume the following Wold ordering: $[\Delta REY_t, \Delta REC_t, \Delta REI_t, rdifft_t, \Delta REER_t]$. The new variable $rdifft_t$ is the interest rate differential between the home and foreign countries. More specifically I construct it using the weighted differential on annualized 10 year treasury

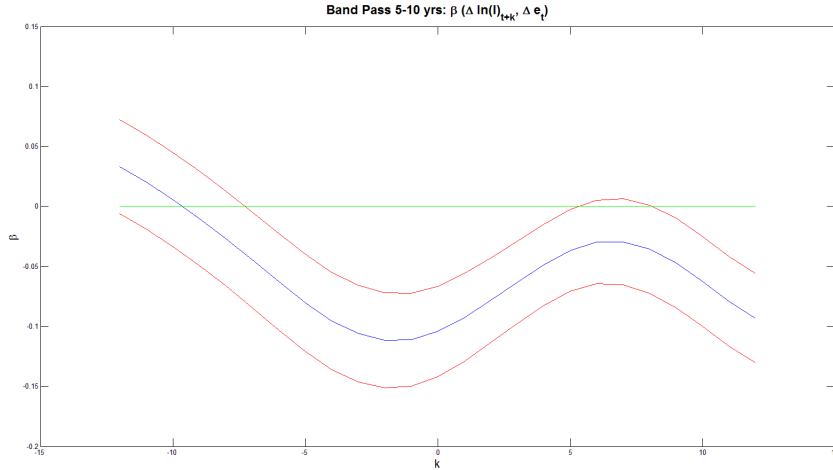


FIGURE 3.29: Estimated β for i_{t+k} , $reer_t$ for periodicities from 5 to 10 years

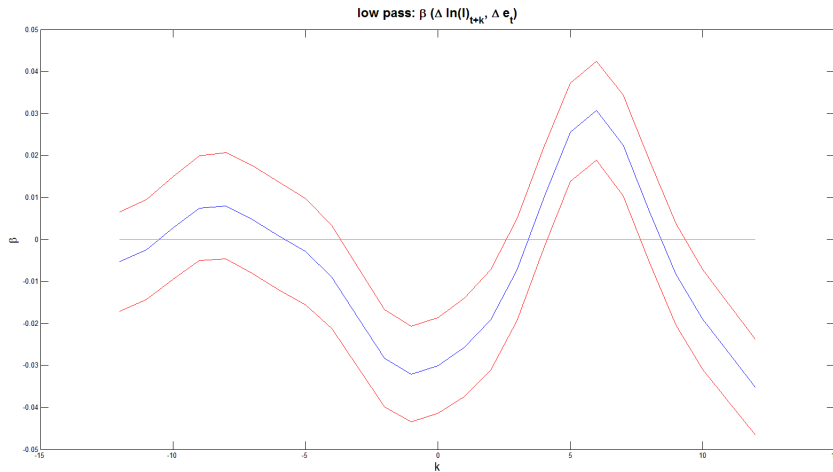


FIGURE 3.30: Estimated β for i_{t+k} , $reer_t$ for periodicities greater than 1 year

returns for each country. Specifically it can be written as:

$$rdiff_{j,t} = r_{j,t} - \sum_{i \neq j} w_{j,i} r_{i,t} \quad (3.15)$$

I add the interest rate differential despite not having used it previously in the analysis for the purpose of identifying monetary policy shocks. Similar to Eichenbaum and Evans (1995), I wish to be able to say something about the impact as well as the contribution of a structurally identified shock to the dynamics witnessed in the data.

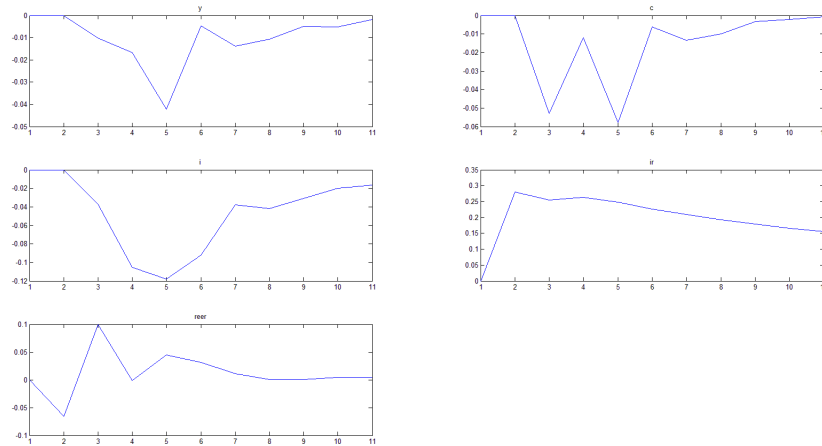


FIGURE 3.31: Impulse Responses to a 1 Standard Deviation Monetary Policy Shock

Therefore I use the proposed Wold decomposition to identify orthogonalized shocks to the interest differential as structural monetary policy shocks. There are several other ways to do this, however since the international data do not lend themselves to allowing us to see changes in non-borrowed reserves for each country's central bank, and there is no clear cut narrative series like that given by Romer and Romer (19XX), this will have to suffice as our method of identifying monetary policy shocks. We will see that the other orthogonalized shocks have a role to play however their structural interpretations are not clear. Finally, to be specific, I use five lags in the VAR.

3.3.2 Impulse Response Functions and Variance Decomposition

In order to try and understand the effect of each orthogonalized shock I present the impulse response functions for the estimated panel VAR. Figure 3.31 shows the response to a one time, one standard deviation, contractionary monetary policy shock. A huge difference between this result and the results shown in Eichenbaum and Evans is that we see an initial depreciation of the home currency on impact and then an appreciation one period later. After that we see a very similar hump

shaped response to the shock. The initial depreciation is unique and tells us that it seems initially a rise in the interest rate differential implies an initial depreciation, as expected through theory, but then a persistent appreciation, which is similar to results suggested by Valchev (2015) and Engel (2014).

Figures 3.32, 3.33, and 3.34 show the impulse response functions for a one standard deviation expansionary orthogonalized shock to output, consumption and investment, respectively. Despite having interesting dynamics for the REER, the sizes of the responses are incredibly small. Only the orthogonalized output and consumption shocks gets anywhere close to the same magnitude of response for the REER. An expansionary output shock has a similar effect on the REER as the contractionary monetary policy shock. The consumption shock itself has very little persistence in the case of consumption however it has a prolonged and delayed effect on the REER. Consumption also has an interesting reaction causing an initial slight depreciation but a much later appreciation, despite it not having much of a persistence in terms of consumption. It appears the appreciation may come through the persistence exhibited by the effect of the shock on the interest rate differential, investment and through output. Given this it becomes a little more apparent as to why investment and output seem to exhibit more interesting dynamic correlations with respect to the REER.

Finally figure 3.35 displays the response of the REER to a one time, one positive standard deviation, REER shock. Despite what seems to be a large response by the fundamental variables the magnitude is at least an order smaller in terms of response to the other shocks. The exchange rate on the other hand has a much greater response to its own orthogonalized shock. This seems to suggest that the exchange rate is largely affected by its own shock with some slight crossover from the other shocks. Since the exchange rate has so little response to the other shocks and the exchange rate shock has so little effect on the fundamentals it is not surprising

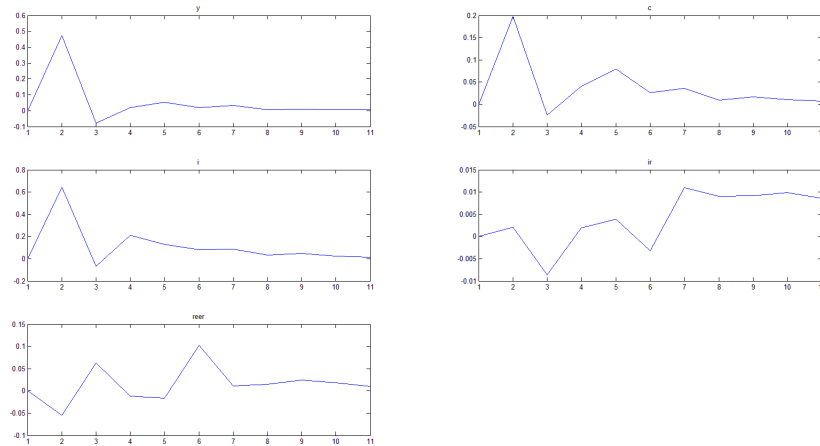


FIGURE 3.32: Impulse Responses to a 1 Standard Deviation Orthogonalized Output Shock

about how disconnected the two are at least, contemporaneously.

Table 3.2 shows the variance decomposition of the VAR. The table itself plays out what we suspected from the impulse response functions. That is that most of the variation of the REER comes from its own orthogonalized shock. However both the monetary policy shock and the output shock have some mild effect on the REER.

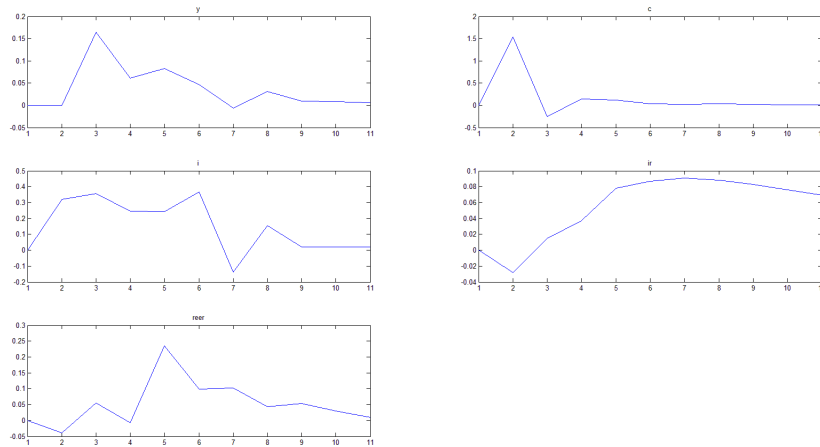


FIGURE 3.33: Impulse Responses to a 1 Standard Deviation Orthogonalized Consumption Shock

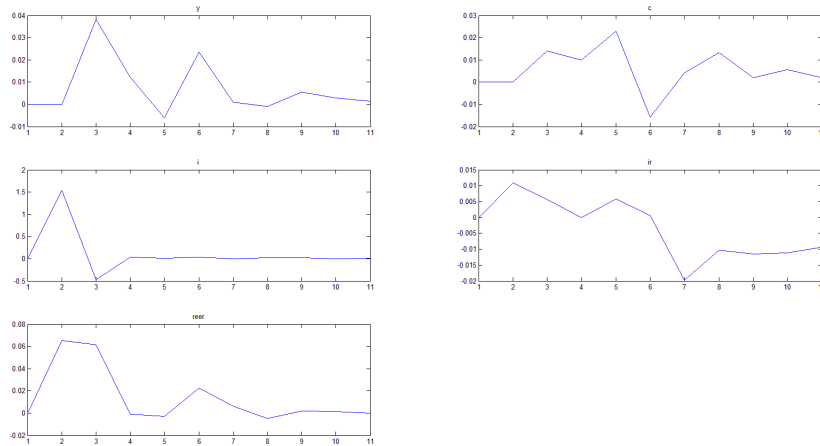


FIGURE 3.34: Impulse Responses to a 1 Standard Deviation Investment Shock

Given the countervailing effect the two different shocks have on the real exchange rate it begins to explain why consumption's dynamic cross correlations are so close to zero, especially considering the lack of persistence given by any individual shock to consumption. Investment itself is largely moved by it's own orthogonalized shock and the next largest contributor to its variance is the orthogonalized output shock.

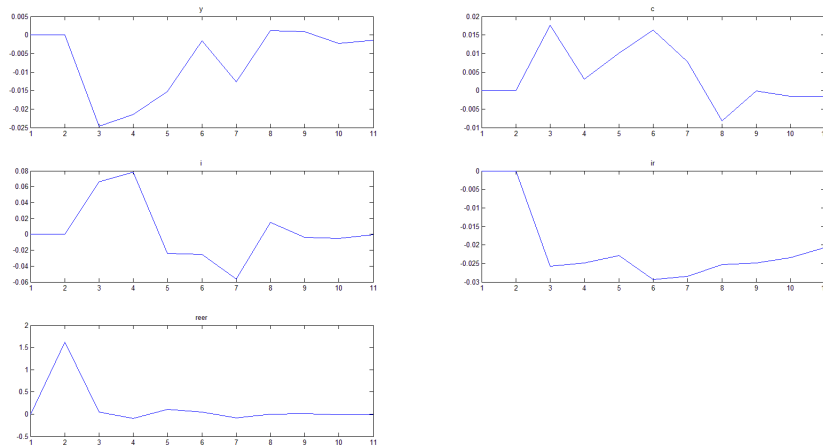


FIGURE 3.35: Impulse Responses to a 1 Standard Deviation Orthogonalized Real Exchange Rate Shock

Table 3.2: Variance Decomposition

	ε_y	ε_c	ε_i	ε_r	ε_{reer}
y	0.961	0.013	0.009	0.010	0.006
c	0.198	0.768	0.005	0.026	0.003
i	0.157	0.013	0.813	0.013	0.005
r	0.002	0.010	0.002	0.973	0.013
$reer$	0.007	0.003	0.003	0.007	0.981

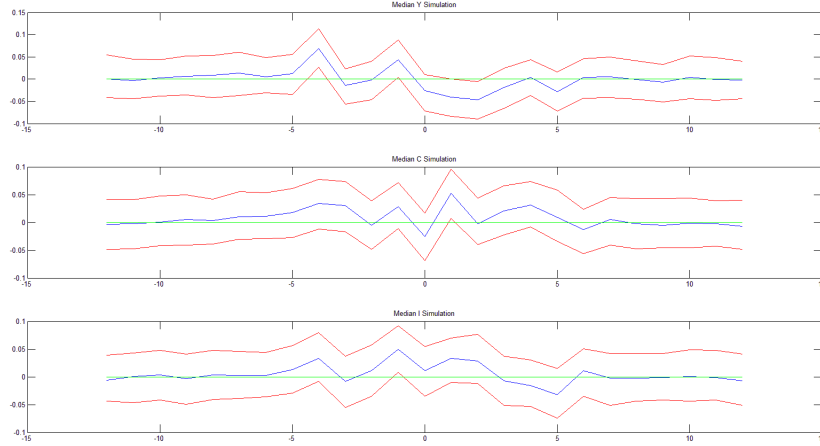


FIGURE 3.36: Dynamic Cross Correlations for Raw Simulated Data

3.3.3 Simulation Results

In order to get a better idea of the VAR’s ability to explain the dynamic cross correlations created by the panel, and within the cross-section, I simulate the VAR. I allow for a 50 period burn in, then simulate 1000 samples of the same sample size as an individual country in the empirical VAR. I then take the data and pass it through a band-pass filter with periodicities from 3-5 years in order to be able to compare it to the results from the previous section. I record the median and distributional confidence bands for the simulated dynamic cross correlations between the listed fundamental variable and the REER. Figure 3.36 shows the cross correlations created via the simulations. Just as in the actual data the results seem to be both messy, and hover around zero with very few instances of significance.

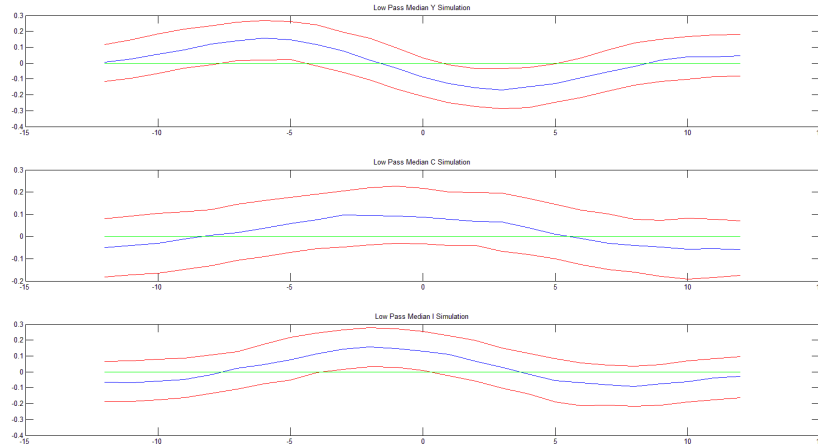


FIGURE 3.37: Dynamic Cross Correlations for Simulated band-pass 3-5 year Data

Figure 3.37 shows the dynamic cross correlations for the simulated data after reconstructing the index and passing it through a band-pass filter. Though the results do not look significant there are at least a few points to talk about. First of all, the error bands are much wider than they would be for a typical panel since the size of the simulations are restricted to the same number of time periods as a single country. With a larger simulated sample size these error bands should constrict around the median. Despite this, it's important to note that consumption still maintains a zero correlation dynamically throughout, which is similar to our results in the previous section. More importantly to note is the sinusoidal dynamic cross correlation pattern exhibited by both investment and output. It's stronger with regards to output but nonetheless, still there. As before, increases in output, today, signal a future appreciation of the REER. Likewise, a rise in investment today signals an appreciation in the future REER. Consumption however largely stays removed from the exchange rate dynamics, which is consistent with our earlier results and results previously in the literature.

In order to try and better understand what is causing these dynamic relationships and perhaps understand the variation we see in the cross section, I resimulate

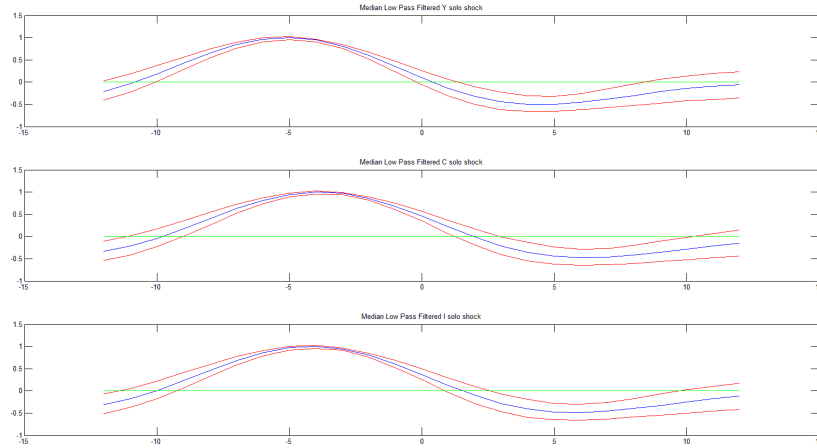


FIGURE 3.38: Median Simulated Cross Correlations with Y shock only

the VAR. This time, however, I shut down subsets of the shocks so that only one shock is varying at any given time. I then pass that new simulated series through the 3-4 year optimum band-pass filter, in order to understand the dynamic cross correlational component due to each individual orthogonalized shock. Figure 3.38 shows the dynamic cross correlation in band-pass filtered simulated data. In terms of the positive and initial component of the dynamic cross correlation both the 3-5 year periodicities of the REER and output, consumption and investment are almost perfectly correlated such that a rise in output attributed to the output orthogonalized shock will certainly mean an appreciation of the 3-5 year component of the real exchange rate 5 quarters later. The result is similar for both consumption and investment relative to the REER. Similarly, for the orthogonalized consumption shock Figure 3.39 shows the results. Like the output shock we see that there is a close to perfect correlation between output, consumption, investment and the REER. Figure 3.40 reports the results for the dynamic cross correlations in relation to the investment shock. Here it is very telling in terms of the Wold ordering as to how the shock effects the various dynamic correlations. Like the previous two, a rise in investment caused by an investment orthogonalized shock, indicates a future appreciation of the

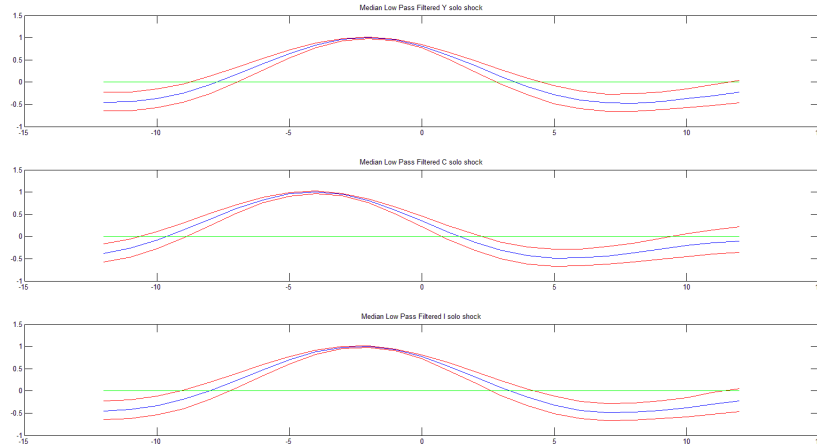


FIGURE 3.39: Median Simulated Cross Correlations with C shock only

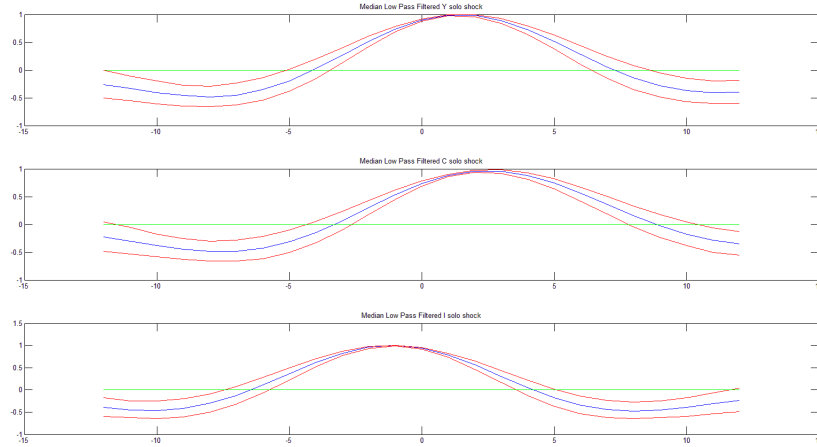


FIGURE 3.40: Median Simulated Cross Correlations with I shock only

real exchange rate. Output and consumption, however, respond differently, dynamically, as as they lag changes in the real exchange rate. Given that all three have relatively the same effect on the dynamic correlations it cannot be the case that either of these two can explain the differentiation we see between consumption and investment and output.

Figure 3.41 shows the cross correlation components caused by monetary policy shocks. We see that, as opposed to the other results, we have a close to perfect

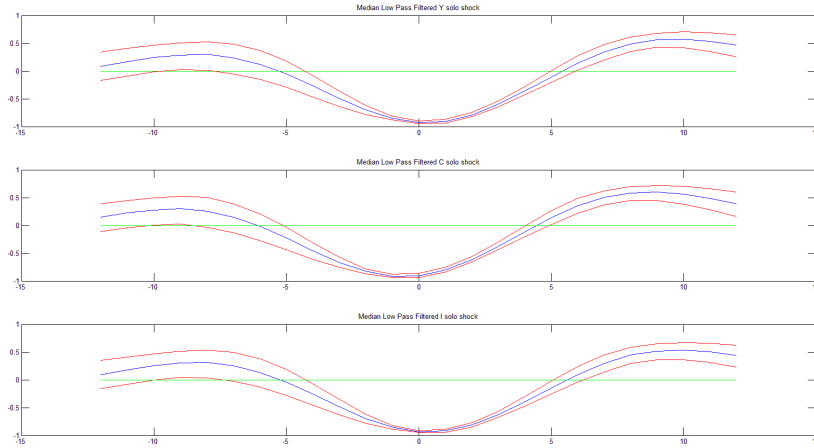


FIGURE 3.41: Median Simulated Cross Correlations with MP shock only

negative correlation between the fundamentals and the REER contemporaneously. If it was just interest rate shocks we should see the results implied by equation 3.6. That is, that contemporaneously “bad times” in terms of the fundamentals would suggest an appreciation of the REER. The result itself seems to suggest not that equation 3.6 is incorrect, rather that models are actually missing important components of the relationship between the REER and fundamentals. This, of course, is once again only looking at the component of the data with periodicities between 3 and 5 years. It’s important to remember that the relationships that are being displayed are only long to medium run relationships.

Finally, we see, in figure 3.42, the dynamic cross correlation relationship implied with just the orthogonalized shock to the REER. Finally we see a stark difference between the response of output and consumption and investment. The cross correlation of output exhibits the sinusoidal relationship that we saw earlier. That is, a current rise in the 3-5 year component of output implies a future appreciation of the REER, and similarly a current appreciation of the REER implies a future drop in output. Consumption, on the other hand, has a much different response. Contemporaneously it appears consumption and the REER, at least the 3-5 year component, have an

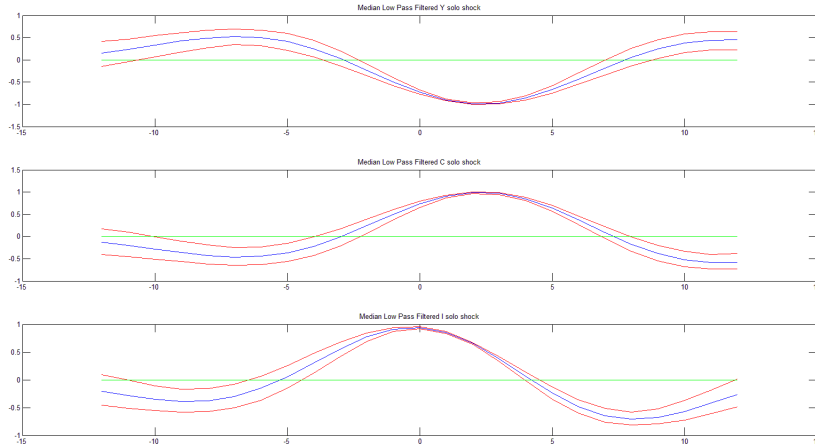


FIGURE 3.42: Median Simulated Cross Correlations with REER shock only

opposite relationship to what the simplest models imply. That is that “good times” in terms of consumption signal an appreciation of the real exchange rate. Finally, investment has a similar sinusoidal structure to other two however it is marginally shifted to the left. Considering the amount of importance this shock has on the REER, we may be able to extrapolate that this shock has a large responsibility to explain the differences in the cross correlations between output, consumption and to a lesser extent investment.

3.3.4 Country Specific Robustness

The above results and the VAR depend exclusively on the assumption that the parameter estimates of the VAR are not unreasonable estimates for defining the relationship of the REER, the macroeconomic variables and the interest rates for each individual country. That is that we are gaining power from utilizing the panel at the expense of country specific parameter estimates that may be better at defining the relationship in each individual country. In order to test the implicit assumptions in the panel, I estimate individual VARs for each country. I then compare the Akaike information criterion (henceforth AIC) for each country specific regression and com-

Table 3.3: AIC Comparison values between Benchmark and Country Specific

	$\frac{AIC_{ind}}{AIC_{bm}}$		$\frac{AIC_{ind}}{AIC_{bm}}$
Australia	0.96	Netherlands	0.95
Austria	0.95	New Zealand	0.98
Belgium	0.94	Norway	0.97
Canada	0.95	Portugal	0.96
Denmark	0.96	Spain	0.95
France	0.94	Sweden	0.96
Germany	0.95	Switzerland	0.95
Ireland	0.97	UK	0.96
Italy	0.95	USA	0.95
Japan	0.96		

pare to the AIC given by the benchmark panel VAR. I do this primarily as a rough measure of how good the initial VAR is at explaining the variation in each country relative to the VAR identified with only country specific data. Table 3.3 shows the ratio of the individual AIC divided by the AIC created by the benchmark model. Obviously some countries are better explained by the VAR than others however, almost universally the difference in the AIC is less than 10 percent. Therefore we can conclude at least roughly that the panel is not too much of a stretch in terms of explaining some of the variation in the data. That cross-sectional differences may arise from a few differences in parameterization or how the shocks themselves are realized in each individual country.

3.4 Conclusion

To conclude, I have presented many empirical regularities in the dynamic relationship between the REER and macroeconomic fundamentals within a large cross section of OECD countries. I record that at least the long-term and medium-term components of the REER and output and investment have sharply different interrelationships than we see for consumption. These relationships appear to be undocumented in other bilateral empirical studies. In addition, I have reported a large degree of cross-

sectional dispersion in the cross correlations among the developed countries in my sample.

I also estimate a VAR in order to explain and identify the cross correlational relationship with respect to orthogonalized structural shocks, similar to the exercise in Eichenbaum and Evans (1995). I find that the differentiation between the relationship of the REER with consumption and output and that with investment must come from the exchange rate shock. I also offer an argument that these variations in the response to each orthogonalized shock may help explain some of the cross sectional variation in the relationship between the REER and the fundamentals displayed by each individual country.

This chapter underlines the need to explore the dynamic correlations rather than the just the contemporaneous correlations the literature has focused on. This chapter also suggests the need for future research to explain the dynamics identified here. These dynamics have largely been unexplored as investment when building international models. If investment and output have such different relationships with the REER, it is important that our models should be able to explain this. Finally, I stress the need for future research to structurally explain the cross sectional variation present in the data.

Investment Driven Exchange Rate Dynamics

4.1 Introduction

The previous chapter highlighted important relationships in the dynamic interrelationship between macro-fundamentals and the real exchange rate. Of particular note is the interesting dynamic relationship between relative investment and the real exchange rate. Previous structural models have taken several tacts to address previously noted problems. Some have focused on creating frictions that drive a wedge between the relative IMRS of the two countries and the growth of the real exchange rate. Examples of these include Ready et al (2013), who describe a model in which iceberg costs create a friction in the model in order to explain excess returns in the carry trade, and Valchev (2016), who describes a two country model in which bonds having convenience yields help describe the empirical violations of the uncovered interest parity condition found in the data. Other models have focused on redefining the SDF, either through things such as habit formation and recursive preferences over different kinds of goods¹, or models in which agents are concerned about the

¹ See Lustig and Verdelhan (2007), and Verdlehan (2010)

long run risk component of consumption ². All of these models either define the SDF in a way that masks the underlying macroeconomic fundamentals making their empirical counterpart hard to define or still maintain a strong link to between the real exchange rate relative consumption growth similar to Backus and Smith (1993). Papers such as Chari et al (2002), Clarida et al (2008), put these models into a new-keynesian framework in order to answer important questions about monetary policy.

Regardless each of these models typically ignores investment in the model design or it is added as an extra sector and an afterthought. The problem with misdefining these interrelationships in model design could have large effects concerning monetary and fiscal policy analysis, and asset pricing implications. Additionally since investment is such a strong procyclical component of GDP, understanding its relationship in the context of the underlying economic mechanisms should be a promising avenue for future development in the area of international exchange rate research. In this chapter I attempt to propose a model to stimulate future conversation about relative investment and it's role in exchange rate determination across countries.

Using the data set constructed from chapter 3, I motivate with further empirical work, the importance of looking at the interest rate and trying to align a model in which we capture these dynamic relationships. In particular I stress the need to focus on medium to long run moments of the data and motivate that at least in terms of US data, that this is an important issue. I then propose a unique structural model that is built around the idea that investment demand is a key component for exchange rate determination.

Similar to Valchev (2016), who uses a bond with a non-pecuniary liquidity value in order to address the Uncovered Interest Parity puzzle, I propose a model in which zero risk bonds exchanged between agents have both a finite supply and are used

² See Colacito and Croce (2011) and Bansal and Shaliastovich (2013)

as collateral in investment decisions. This enables investment demand to drive a wedge between the essential risk sharing condition that binds exchange rate growth to the relative changes in the IMRS. I take an abstraction from the setup given in Kiyotaki and Moore (1997), primarily for tractability purposes, however try to stay true to the spirit of their analysis. That is that a subset of agents may be credit constrained and must hold some sort of asset as collateral, in this case government issued bonds, in order to engage in investment activity. The idea being that bonds are valued not only for their ability to engage in risk sharing but in their ability to help in the production of the capital good. Using this model I attempt to better define the role investment plays in exchange rate dynamics and be able to match the empirical dynamic relationship that I find which seems to exist between the two time series.

The paper is organized in the following manner. In section 2, I detail the empirical motivation behind the paper. In section 3, I develop a small open economy model which uses sticky prices as well as the bond collateral mechanism in order to address the empirical link between investment and the real exchange rate. Finally in section 4, I conclude.

4.2 Empirical Motivation

4.2.1 *The International Risk Sharing condition*

For completeness I start by defining the risk sharing condition derived from the Backus and Smith(1993) model, to better motivate why previous studies have focused on the relative consumption growths of countries. The condition itself is given by the following Euler equation³:

$$\frac{\tilde{m}}{m} = \frac{e'}{e} \tag{4.1}$$

³ For a more detailed description of the model environment that gives rise to this condition I direct the reader to Backus and Smith(1993) or Burnside and Graveline(2013).

where e is defined as the exchange rate or the price of the consumption basket in the foreign country over the price of the home consumption basket, and a prime denotes the expected exchange rate tomorrow. m is typically defined as $\frac{\beta U_e(c')}{U_c(c)}$ or the marginal utility growth of the home country and the variable with a tilde represents the same but for the foreign country. This follows from several assumptions, including complete markets, no frictions in the trade of the tradable good, and identical preferences across countries with respect to the tradable and non tradable good. Moreover, with constant relative risk aversion utility functions it directly implies a perfect correlation between the exchange rate and consumption growth. Though the above example holds in a two country setting it can be shown relatively trivially that this will hold for all bilateral pairs of countries. Previous studies have focused on this condition and empirically have found zero or slightly negative correlation between consumption growth and the real exchange rate. Solutions for this empirical anomaly exist. Benigno and Thoenissen(2006) as well as Burnside and Graveline(2013), and Corsetti, Dedola and Leduc(2007) point out that this condition is broken once agents are forced to operate under incomplete markets. In fact subtracting the complete markets assumption can account for most of the correlation puzzle that we find in the data. Another popular way in which to break the risk sharing condition is by breaking the law of one price by having exporters price to market. The law of one price fails to hold as these prices are assumed to be sticky and therefore unable to adjust to changing economic conditions between countries. Studies of pricing to market include Kollmann(1995), Chari, Kehoe and McGratten(1998). Most of these studies however have very little to say about investment dynamics in general. Either the economies themselves are endowment economies or their investment sector does not enter into the risk sharing condition. I would like to point out that I bring up the risk sharing condition not to say this paper directly addresses the Backus Smith puzzle, but more in order to motivate the point out the previous research in this area

and the way in which this paper differs.

4.2.2 Summary Data

Despite evidence in the previous chapter the question remains why do we care about these medium and long run frequency relationships? The fact is that most of the variation in the real exchange rate and the other macro-economic variables occurs in the long and medium run components of the data. This fact is supported by Figure 4.1 where the spectrograms for the US, Japan, UK and Norway are shown. Though the countries shown are only a subset of the countries, they are first of all chosen somewhat ad hoc but also for their regional distribution in order to paint a relatively clear picture. In addition the variables themselves (Real Effective Exchange Rate, Relative Real Effective Consumption, Relative Real Effective Output and Relative Real Effective Investment), still include the other countries not shown as they are constructed using the same methodology described in Chapter 3. Here we see that most of the variation especially in the REER occurs at longer run periodicities, no matter which country we observe. Since investment is also quite volatile in the long run if we combine this fact with the documented facts from the previous chapter it becomes a natural candidate to help explain this long run volatility in exchange rates.

Wanting a more concrete picture than this. Figure 4.2 shows the detrended time series for the US, Japan and the UK of the REER compared to the REC, REI and REY, and figure 4.3 shows the same for Norway. I use a very large smoothing parameter, $\lambda = 100000$, only to take out the extreme long term trends in each of the time series. In addition I normalize each series by its own standard deviation so that I can compare each series in terms of standard deviations to each other. Though not a strict statistical test, the pattern in the comovements is striking. Though we know from before that there is little to no contemporaneous correlation between the

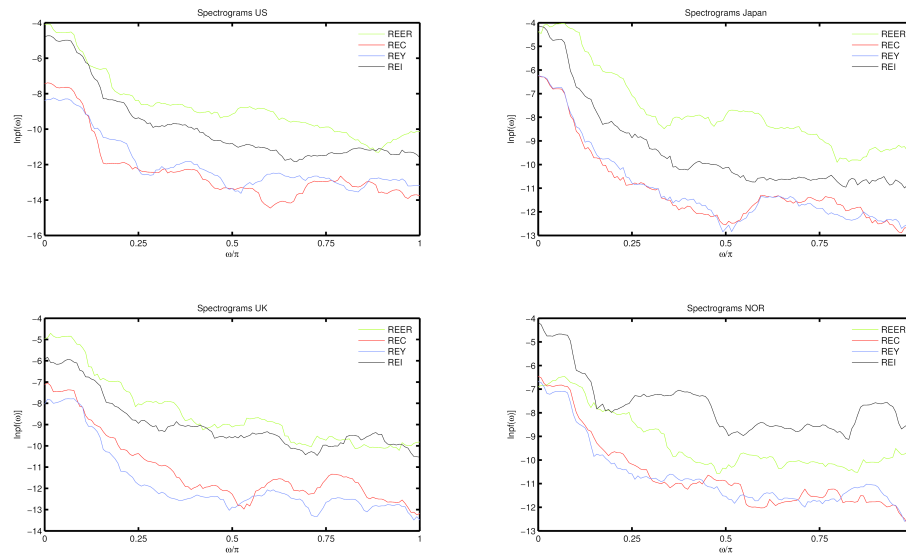


FIGURE 4.1: Spectrograms for Japan, US, UK and Norway for REER, REC, REI and REY



FIGURE 4.2: Standardized HP filtered data with smoothing parameter $\lambda = 100000$ for US, UK and Japan

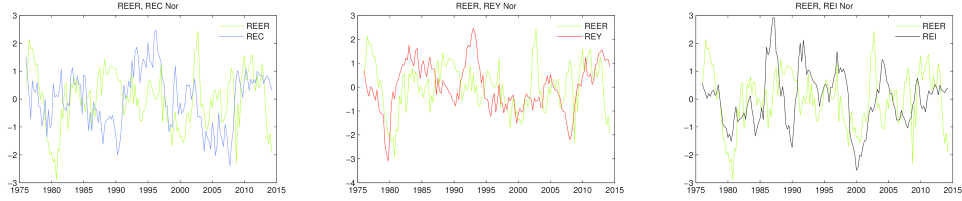


FIGURE 4.3: Standardized HP filtered data with smoothing parameter $\lambda = 100000$ for Norway

real effective exchange rate and real effective investment, or any of the other variables for that matter it appears that there is at the very least something interesting going on dynamically between the two variables. This is particularly poignant for the US and to a lesser extent Japan and Norway in the figures. The US detrended REI series appears to be a time shifted version of the investment series. Therefore, if we're interested in explaining US exchange rate movements and how they relate to macro-fundamentals then we cannot ignore these comovements.

4.3 A Small Open Economy Model

In this section I build a small open economy model with the intent to address the investment dynamics explored in the previous chapter. For the most part the model is similar to a small open economy version of Corsetti, Dedola and Luduc (2008), in which finished goods in each country are made using a CES production technology buying intermediate goods from monopolistically competitive intermediate firms. Where I differ is in the inclusion of capital and in the properties of the risk free bond issued by the government. Similar to Valchev (2015) my bond has value for the individual agents beyond its basic interest bearing return. Valchev's model uses the bond as a less liquid form of money thus giving the bond a convenience yield which has time varying value that matches the dynamic properties of the Uncovered Interest Parity puzzle explored by both Valchev (2015) and Engel (2012). Unlike Valchev however the model endows the bond with a collateral value that is used

in the production of investment goods. Inspiration for this comes from the work of Kiyotaki and Moore(1997) and Iacoviello (2005), who create models of credit constrained heterogenous agents with need of physical capital in order to borrow from a decentralized lender.

I utilize a small open economy model, primarily for ease of analysis. Since there is no other country in the model any positive change in investment will be a relative increase in investment for the home country. In addition due to the setup of the model changes in the price level of goods will be a direct change in the exchange rate since the foreign economy is assumed to be stable.

4.3.1 Model Environment

4.3.1.1 Firms

I begin by first defining the environment of the firms for the home economy. The economy consists of a final goods producer which takes in a basket of intermediate goods from monopolistically competitive intermediate good firms. The intermediate good firms are constrained by a Calvo pricing friction in which only a randomly chosen subset of them can change their prices in each period.

4.3.1.1.1 *Final Goods* A representative firm in the home country uses a CES technology over a continuum of intermediate goods to produce Y units of the home final good:

$$y = \left(\int_0^1 y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1.$$

Demand for intermediate good i is given by:

$$y_i = \left(\frac{P_i}{P_h} \right)^{-\varepsilon} y$$

Given that the final goods firm minimizes cost, and that the market for final goods is perfectly competitive, the price index for the final good is:

$$P_h = \left(\int_0^1 P_i^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

See the appendix for details. Given that the final goods producer is competitive and the technology is CRTS, profits are zero.

4.3.1.1.2 Intermediate Goods The intermediate inputs are produced domestically. Intermediate goods producer i uses the production technology

$$y_i = A n_i^{\mu(1-\alpha)} k_i^{\mu\alpha} \quad (4.2)$$

where n_i and k_i are its inputs of labor and capital, respectively. $0 < \mu < 1$ so that the firm experiences decreasing returns to scale and thus intermediate firms will have a positive markup value. The firm's cost minimization problem is

$$\min_{n_i, k_i} W n_i + Q k_i$$

subject to $y_i = A n_i^{\mu(1-\alpha)} k_i^{\mu\alpha}$, so that total cost is

$$\Psi(y_i, W, Q) = \frac{W^{1-\alpha} Q^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \left(\frac{y_i}{A} \right)^{\frac{1}{\mu}} \quad (4.3)$$

Marginal cost is

$$\Psi_y(y_i, W, Q) = \frac{1}{\mu} \frac{W^{1-\alpha} Q^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{y_i^{(1-\mu)/\mu}}{A^{1/\mu}} = \frac{\Psi}{\mu y_i} \quad (4.4)$$

The firm faces a Calvo pricing friction and has probability $1 - \theta$ of being able to adjust its price in any period. A firm that is randomly given the opportunity to change price in period t will solve the problem

$$\max_{P_{ht}^*} E_t \sum_{k=0}^{\infty} \theta^k M_{t,t+k} (P_{ht}^* y_{t+k|t} - \Psi_{t+k|t})$$

where

- $y_{t+k|t}(P_{ht}^*) = (P_{ht}^*/P_{ht+k})^{-\varepsilon} y_{t+k}$ is the demand faced by the firm in period $t+k$ if it has not changed its price since period t , with P_{ht}^* representing the price it sets at that time, P_{ht+k} representing the $t+k$ aggregate price index for home goods, and y_{t+k} being the aggregate demand for home goods at $t+k$
- $\Psi_{t+k}(P_{ht}^*) = \frac{W_{t+k}^{1-\alpha} Q_{t+k}^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \left(\frac{(P_{ht}^*/P_{ht+k})^{-\varepsilon} y_{t+k}}{A_t} \right)^{\frac{1}{\mu}}$ is total cost at $t+k$ if the firm has not changed its price since period t ,
- $M_{t,t+k} = \beta^k (c_{t+k}/c_t)^{-\sigma} (P_t/P_{t+k})$ is the discount factor used by the household for evaluating date $t+k$ nominal payoffs at date t

It is important to note the distinction between P_t , which is the price index for consumption, and P_{ht} , which is the price index for home-produced goods. The term θ^k is the probability of arriving in period $t+k$ without being able to change price since period t (so that P_{ht}^* still matters for the firm's profits). The firm's first order condition (dropping some notation) is

$$E_t \sum_{k=0}^{\infty} \theta^k M_{t,t+k} (y_{t+k|t} + P_{ht}^* y'_{t+k|t} - \Psi'_{t+k|t} y'_{t+k|t}) = 0 \quad (4.5)$$

where

$$\Psi'_{t+k|t} = \Psi_y [y_{t+k|t}, W_{t+k}, Q_{t+k}]$$

and

$$y'_{t+k|t} = -\varepsilon y_{t+k|t} / P_{ht}^*$$

so we have

$$E_t \sum_{k=0}^{\infty} \theta^k M_{t,t+k} y_{t+k|t} (1 - \varepsilon + \varepsilon \Psi'_{t+k|t} / P_{ht}^*) = 0$$

or, multiplying through by $P_{ht}^*/(1 - \varepsilon)$:

$$E_t \sum_{k=0}^{\infty} \theta^k M_{t,t+k} y_{t+k|t} (P_{ht}^* - \nu \Psi'_{t+k|t}) = 0. \quad (4.6)$$

Next we divide (4.6) through by P_{ht-1} to get

$$E_t \sum_{k=0}^{\infty} \theta^k M_{t,t+k} y_{t+k|t} \left(\frac{P_{ht}^*}{P_{ht-1}} - \nu \frac{\Psi'_{t+k|t} P_{ht+k}}{P_{ht+k} P_{ht-1}} \right) = 0. \quad (4.7)$$

If we assume a steady state in which home goods and consumption goods inflation are both zero then, in that steady state, $M_{t,t+k} = \beta^k$, $y_{t+k|t} = y$ and the term in parentheses is zero. The latter result comes from the fact that the price ratio term is 1, while the markup P_h/Ψ' is equal to ν in the steady state. Hence, if we linearize (4.7) in the neighborhood of that steady state we have

$$(p_{ht}^* - p_{ht-1}) \sum_{k=0}^{\infty} (\beta\theta)^k \approx E_t \sum_{k=0}^{\infty} (\beta\theta)^k (mc_{t+k|t} + \ln \nu + p_{ht+k} - p_{ht-1}). \quad (4.8)$$

where small letters denote logs and $mc_{t+k|t} = \ln(\Psi'_{t+k|t}/P_{ht+k})$.

Wanting to rewrite this expression in terms of home inflation. We now define two variables to help with ease of analysis.

$$\Theta = \frac{\mu}{\mu + \varepsilon(1 - \mu)} \leq \mu$$

$$\widehat{mc}_{t+k} = mc_{t+k} - \ln \nu^{-1}.$$

So \widehat{mc}_{t+k} is the log deviation of the typical real marginal cost of a firm from the steady state real marginal cost. (It's the negative of the log deviation of the typical markup.) After some significant algebra we can rewrite the home price as a difference equation:

$$p_{ht}^* - p_{ht-1} = \beta\theta E_t (p_{ht+1}^* - p_{ht}) + (1 - \beta\theta)\Theta \widehat{mc}_t + \pi_{ht}. \quad (4.9)$$

To proceed further I derive an expression for π_{ht} in terms of $p_{ht}^* - p_{ht-1}$. Notice that the aggregate price index is

$$P_{ht} = \left(\int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

Any firm that is setting its price has price P_{ht}^* . Denoting the set of firms who are not changing price as \mathcal{S}_t , we can write

$$P_{ht} = \left[(1 - \theta) (P_{ht}^*)^{1-\varepsilon} + \int_{\mathcal{S}_t} P_{it-1}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}.$$

Since the ability to change price is independent of any characteristics of the firm, or any past ability to change price, the firms in the set \mathcal{S}_t are spread across the distribution of P_{it-1} identically to how the firms in $[0, 1]$ are spread across the distribution of P_{it-1} . Hence $\int_{\mathcal{S}_t} P_{it-1}^{1-\varepsilon} di = \theta \int_0^1 P_{it-1}^{1-\varepsilon} di = \theta P_{ht-1}^{1-\varepsilon}$ and

$$P_{ht} = \left[(1 - \theta) (P_{ht}^*)^{1-\varepsilon} + \theta P_{ht-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (4.10)$$

So we have

$$\Pi_{ht} \equiv \frac{P_{ht}}{P_{ht-1}} = \left[(1 - \theta) \left(\frac{P_{ht}^*}{P_{ht-1}} \right)^{1-\varepsilon} + \theta \right]^{\frac{1}{1-\varepsilon}}.$$

To a first order approximation around the zero-inflation steady state this is

$$\pi_{ht} = (1 - \theta) (p_{ht}^* - p_{ht-1}).$$

Using this to substitute out $p_{ht}^* - p_{ht-1}$ in (4.9) we get

$$\pi_{ht} = \beta E_t \pi_{ht+1} + \frac{(1 - \theta)(1 - \beta\theta)\Theta}{\theta} \widehat{m}c_t = \beta E_t \pi_{ht+1} + \tilde{\Theta} \widehat{m}c_t. \quad (4.11)$$

4.3.1.2 Households

The representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\phi}}{1+\phi} \right)$$

where c_t is consumption, n_t is labor, $\sigma > 0$ and $\phi > 0$. Consumption is a CES aggregate of home (H) and foreign (F) final goods,

$$c_t = \left(a_h^{\frac{1}{\eta}} c_{ht}^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} c_{ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (4.12)$$

where c_{ht} and c_{ft} are the amounts of the home final good and the foreign good that the household consumes, η is the elasticity of substitution between the two goods and the weights a_h and a_f , normalized to sum to 1, determine the degree of home bias in consumption.

As shown in the appendix, if P_{ht} and P_{ft} are the prices of the home and foreign good in domestic currency, the household sets

$$c_{ht} = a_h (P_{ht}/P_t)^{-\eta} c_t \quad (4.13)$$

$$c_{ft} = a_f (P_{ft}/P_t)^{-\eta} c_t. \quad (4.14)$$

where P_t is the consumption price index:

$$P_t = (a_h P_{ht}^{1-\eta} + a_f P_{ft}^{1-\eta})^{\frac{1}{1-\eta}}. \quad (4.15)$$

The price of the foreign good in the foreign currency is equal to one and with the further assumption of purchasing power parity, $P_{ft} = S_t$, where S_t is the nominal exchange rate expressed as domestic currency units per foreign currency unit. Let the relative price of the two goods be denoted $\Xi_t = P_{ft}/P_{ht}$. The optimality conditions stated above can be rewritten as

$$c_{ht} = a_h (a_h + a_f \Xi_t^{1-\eta})^{\frac{\eta}{1-\eta}} c_t \quad (4.16)$$

$$c_{ft} = a_f \left(a_h \Xi_t^{\eta-1} + a_f \right)^{\frac{\eta}{1-\eta}} c_t. \quad (4.17)$$

Households own the capital stock and can invest in new capital by purchasing home and foreign goods. Investment is the same CES aggregate of these goods as is consumption:

$$i_t = \left(a_h^{\frac{1}{\eta}} i_{ht}^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} i_{ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (4.18)$$

This implies that the household sets

$$i_{ht} = a_h \left(a_h + a_f \Xi_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} i_t \quad (4.19)$$

$$i_{ft} = a_f \left(a_h \Xi_t^{\eta-1} + a_f \right)^{\frac{\eta}{1-\eta}} i_t. \quad (4.20)$$

and that the same price index applies to investment as to consumption.

The overall inflation rate is

$$\Pi_t = \frac{P_t}{P_{t-1}} = \left(\frac{a_h P_{ht}^{1-\eta} + a_f P_{ft}^{1-\eta}}{a_h P_{ht-1}^{1-\eta} + a_f P_{ft-1}^{1-\eta}} \right)^{\frac{1}{1-\eta}}$$

which can be written as

$$\Pi_t = \left(\frac{a_h + a_f \Xi_t^{1-\eta}}{a_h + a_f \Xi_{t-1}^{1-\eta}} \Pi_{ht}^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

So around a zero inflation steady state and assuming $a_f = 1 - a_h$ and $\Xi = 1$ we have

$$\pi_t = \pi_{ht} + a_f (\xi_t - \xi_{t-1}) \quad (4.21)$$

where the lower case letters represent logarithms (or decimal percent deviations from steady state value). We also will define:

$$s_t \equiv \frac{S_t}{P_t} = \frac{P_{ft}}{P_t} = \frac{P_{ft}}{\left(a_h P_{ht}^{1-\eta} + a_f P_{ft}^{1-\eta} \right)^{\frac{1}{1-\eta}}} = \left(a_h \Xi_t^{\eta-1} + a_f \right)^{\frac{1}{\eta-1}}. \quad (4.22)$$

The budget constraint of the household denominated in domestic currency units is

$$P_{ht}(c_{ht}+i_{ht})+P_{ft}(c_{ft}+i_{ft})+T_t+B_{h,t+1}^c+S_tB_{f,t+1}^c = W_tn_t+Q_tk_t+D_t+B_{h,t}^cR_{t-1}+S_tB_{f,t}^cR_{t-1}^*,$$

or

$$P_t(c_t + i_t) + T_t + B_{h,t+1}^c + S_tB_{f,t+1}^c = W_tn_t + Q_tk_t + D_t + B_{h,t}^cR_{t-1} + S_tB_{f,t}^cR_{t-1}^*,$$

where T_t is lump sum taxes paid to the government, $B_{h,t}^c$ is domestic bonds purchased by the household at time $t-1$ and maturing at time t , $B_{f,t}^c$ is foreign bonds purchased by the household at time $t-1$ and maturing at time t , W_t is the nominal wage rate, Q_t is the nominal rental rate of capital, D_t is any profits distributions from firms owned by the household, R_t is the gross time t domestic nominal interest rate and R_t^* is the gross time t foreign nominal interest rate.

The law of motion for the capital stock is

$$k_{t+1} = (1 - \delta)k_t + \chi i_t^\gamma z_t^{1-\gamma} \quad (4.23)$$

where

$$z_t = \frac{B_{ht}^c + \xi S_t B_{ft}^c}{P_t}. \quad (4.24)$$

This is first of all an abstraction from the Kiyotaki and Moore framework. I take this shortcut primarily for tractability. Credit constrained heterogenous agents, would add another layer of complication and provide zero clarity. Instead I assume and calibrate the model such that a certain proportion of investment is backed by collateral in the form of foreign and domestic bonds, abstracting from a set distribution and instead assuming a proportion of consumers are credit constrained. Due to this the model should be isomorphic to one in which a fixed number of individuals need to hold either foreign or home bonds in order to invest. Observationally

this should be equivalent to bonds appearing to be an input into capital production. In addition it's important that the two bonds not be perfect substitutes, otherwise home bonds and foreign bonds will have the same value to the agents preventing the wedge from being non-trivial. Letting Λ_t be the lagrange multiplier associated with the budget constraint and Ω_t be the lagrange multiplier associated with equation (4.23), the optimality conditions for the household's problem are

$$\begin{aligned}
n_t^\phi &= w_t c_t^{-\sigma} \\
\omega_t &= \frac{1}{\gamma \chi} c_t^{-\sigma} \left(\frac{i_t}{z_t} \right)^{1-\gamma} \\
c_t^{-\sigma} &= \beta c_{t+1}^{-\sigma} \left[R_t + \left(\frac{1-\gamma}{\gamma} \right) \frac{i_{t+1}}{z_{t+1}} \right] \frac{1}{\Pi_{t+1}} \\
c_t^{-\sigma} &= \beta c_{t+1}^{-\sigma} \left[R_t^* + \xi \left(\frac{1-\gamma}{\gamma} \right) \frac{i_{t+1}}{z_{t+1}} \right] \frac{\Delta_{t+1}}{\Pi_{t+1}} \\
\omega_t &= (1-\delta)\beta\omega_{t+1} + \beta c_{t+1}^{-\sigma} q_{t+1}
\end{aligned}$$

Of Particular note are the 3rd and 4th equation above. Those two give us the risk sharing condition. the imperfect substitutes allow the wedge to be trivially non-zero and instead be the ratio of the relative value of the foreign and domestic bond in investment. Thus the desired amount of investment normalized by the supply of effective collateral in the form of bonds directly enters the risk sharing condition.

4.3.1.3 Government

The monetary authority holds any unsold government bonds and sends any revenue gained by the holding of those bonds back to the fiscal authority. They therefore satisfy the following budget constraint:

$$T_t^m = R_{t-1} B_{h,t}^m - B_{h,t+1}^m$$

The fiscal authority collects taxes and satisfies the budget constraint:

$$B_{ht+1}^g + T_t + T_t^m = B_{h,t}^g R_{t-1} + P_{ht} g_t$$

where we have assumed that the government only purchases home goods.

In addition we assume the government follows a tax rule:

$$\frac{T_t}{P_{ht} Y_t} = \rho \frac{T_{t-1}}{P_{ht-1} Y_{t-1}} + (1 - \rho) \kappa_b \frac{B_{h,t}^g}{P_{ht-1} Y_{t-1}}$$

All home bonds are willingly held so that:

$$B_{ht}^g = B_{ht}^c + B_{h,t}^m$$

If you consolidate the government and the monetary authority's budget constraints (and use this last condition) you get

$$B_{ht+1}^c + T_t = B_{h,t}^c R_{t-1} + P_{ht} g_t$$

and you can rewrite the government tax rule as

$$\frac{T_t}{P_{ht} Y_t} = \rho \frac{T_{t-1}}{P_{ht-1} Y_{t-1}} + (1 - \rho) \kappa_b \frac{B_{h,t}^c + B_{h,t}^m}{P_{ht-1} Y_{t-1}}$$

The two equations can be restated for simplicity as

$$B_{ht+1}^c + \tau_t P_{ht} Y_t = B_{h,t}^c R_{t-1} + P_{ht} g_t$$

$$\tau_t = \rho \tau_{t-1} + (1 - \rho) \kappa_b \frac{B_{h,t}^c}{P_{ht-1} Y_{t-1}} + u_t$$

where $u_t = (1 - \rho) \kappa_b \frac{B_{h,t}^m}{P_{ht-1} Y_{t-1}}$. The market clearing condition for home goods is

$$c_{ht} + i_{ht} + g_t + x_t = y_t \tag{4.25}$$

where y_t is the production of home goods and x_t is some exogenous foreign demand for the home good. This allows the household budget constraint in equilibrium to be rewritten as

$$P_{ft}(c_{ft} + i_{ft}) + T_t + B_{h,t+1}^c + S_t B_{f,t+1}^c = P_{ht}(g_t + x_t) + B_{h,t}^c R_{t-1} + S_t B_{f,t}^c R_{t-1}^*.$$

When this is combined with the government budget constraint you get

$$P_{ft}(c_{ft} + i_{ft}) + S_t B_{f,t+1}^c = P_{ht}x_t + S_t B_{f,t}^c R_{t-1}^*.$$

Given that $P_{ft} = S_t$ this simplifies to

$$c_{ft} + i_{ft} + B_{f,t+1}^c = \frac{x_t}{\Xi_t} + B_{f,t}^c R_{t-1}^*. \quad (4.26)$$

We also assume the monetary authority sets the nominal interest rate according to the following simple Taylor Rule

$$\ln R_t - \ln R = \rho_r (\ln R_{t-1} - \ln R) + (1 - \rho_r)\phi_\pi \pi_t + v_t$$

The definition of z_t , the government budget constraint and tax rule can be restated in “real” terms using $b_{ht}^c = B_{ht}^c/P_{ht-1}$ as

$$\begin{aligned} b_{ht+1}^c + \tau_t y_t &= b_{h,t}^c \frac{R_{t-1}}{\Pi_t} + g_t \\ \tau_t &= \rho \tau_{t-1} + (1 - \rho) \kappa_b \frac{b_{h,t}^c}{y_{t-1}} + u_t \\ z_t &= \frac{b_{ht}^c}{\Pi_t} + \xi s_t B_{ft}^c. \end{aligned} \quad (4.27)$$

4.3.2 Equilibrium of the Model

We can therefore collect everything from the previous sections and define the equilibrium in the model as a set of prices, and allocations, for firms and households such that it satisfies the following equations:

$$c_t = \left(a_h^\frac{1}{\eta} c_{ht}^\frac{\eta-1}{\eta} + a_f^\frac{1}{\eta} c_{ft}^\frac{\eta-1}{\eta} \right)^\frac{\eta}{\eta-1}$$

$$i_t = \left(a_h^\frac{1}{\eta} i_{ht}^\frac{\eta-1}{\eta} + a_f^\frac{1}{\eta} i_{ft}^\frac{\eta-1}{\eta} \right)^\frac{\eta}{\eta-1}$$

$$y_t = A_t n_t^{\mu(1-\alpha)} k_t^{\mu\alpha}$$

$$z_t = \frac{b_{ht}^c}{\Pi_t} + \Xi s_t B_{ft}^c$$

$$s_t = (a_h \Xi_t^{\eta-1} + a_f)^\frac{1}{\eta-1}$$

$$\Pi_t = \left(\frac{a_h + a_f \Xi_t^{1-\eta}}{a_h + a_f \Xi_{t-1}^{1-\eta}} \Pi_{ht}^{1-\eta} \right)^\frac{1}{1-\eta}$$

$$\Delta_{t+1} = \frac{s_{t+1}}{s_t} \Pi_{t+1}$$

$$k_{t+1} = (1 - \delta)k_t + \chi i_t^\gamma z_t^{1-\gamma}$$

$$b_{ht+1}^c + \tau_t y_t = b_{h,t}^c \frac{R_{t-1}}{\Pi_t} + g_t$$

$$\tau_t = \rho \tau_{t-1} + (1 - \rho) \kappa_b \frac{b_{h,t}^c}{Y_{t-1}} + u_t$$

$$\ln R_t - \ln R = \rho_r (\ln R_{t-1} - \ln R) + (1 - \rho_r) \phi_\pi \pi_t + v_t$$

$$MC_t = \frac{1}{\mu} \frac{w_t^{1-\alpha} q_t^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{y_t^{(1-\mu)/\mu}}{A_t^{1/\mu}} (a_h + a_f \Xi_t^{1-\eta})^\frac{1}{1-\eta}$$

$$\frac{n_t}{k_t} = \frac{1 - \alpha}{\alpha} \frac{q_t}{w_t}$$

$$c_{ht} = a_h (a_h + a_f \Xi_t^{1-\eta})^{\frac{\eta}{1-\eta}} c_t \quad (4.28)$$

$$c_{ft} = a_f (a_h \Xi_t^{\eta-1} + a_f)^{\frac{\eta}{1-\eta}} c_t. \quad (4.29)$$

$$i_{ht} = a_h (a_h + a_f \Xi_t^{1-\eta})^{\frac{\eta}{1-\eta}} i_t \quad (4.30)$$

$$i_{ft} = a_f (a_h \Xi_t^{\eta-1} + a_f)^{\frac{\eta}{1-\eta}} i_t. \quad (4.31)$$

$$n_t^\phi = w_t c_t^{-\sigma}$$

$$\omega_t = \frac{1}{\gamma \chi} c_t^{-\sigma} \left(\frac{i_t}{z_t} \right)^{1-\gamma}$$

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[R_t + \left(\frac{1-\gamma}{\gamma} \right) \frac{i_{t+1}}{z_{t+1}} \right] \frac{1}{\Pi_{t+1}}$$

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[R_t^* + \xi \left(\frac{1-\gamma}{\gamma} \right) \frac{i_{t+1}}{z_{t+1}} \right] \frac{\Delta_{t+1}}{\Pi_{t+1}}$$

$$\omega_t = (1 - \delta) \beta \omega_{t+1} + \beta c_{t+1}^{-\sigma} q_{t+1}$$

$$\pi_{ht} = \beta E_t \pi_{ht+1} + \frac{(1 - \theta)(1 - \beta\theta)\Theta}{\theta} \widehat{m}_t^c = \beta E_t \pi_{ht+1} + \tilde{\Theta} \widehat{m}_t^c. \quad (4.32)$$

$$c_{ht} + i_{ht} + g_t + x_t = y_t$$

$$c_{ft} + i_{ft} + B_{f,t+1}^c = \frac{x_t}{\Xi_t} + B_{f,t}^c R_{t-1}^*.$$

Where the above are the solutions for the firm's and household's problems, given prices, and equations such that all markets will clear. If this is the case then we can

Table 4.1: Model Calibrations

Parameter	Value	Source
n	0.375	American Time Use Survey: Hours worked/24
bc	0.7	Fraction of Public Debt to GDP
y	1	Normalization
c	0.63	NIPA quarterly average of C/GDP
i	0.17	NIPA quarterly average of I/GDP
g	0.2	So $c+i+g=y$
x	0.08	NIPA quarterly average of exports/GDP
δ	0.022	Standard depreciation rate
σ	2	
β	0.97	
ξ	0.8	
ν	22	So that $R^* > R > 1$
ϕ	1.5	
α	0.36	Capital share of income
μ	0.9	

linearize the model around the zero inflation steady state in order to acquire a quantitative solution using a method such as that used by King and Watson (1998). In order to solve the model I propose the following set of calibrations for the parameter values. Table 4.1 displays the proposed calibrations for the model as well as their sources, where applicable. Some parameters are free parameters and are calibrated in order to maintain certain ratios in the steady state that are fairly consistent in the data. Utilizing both this calibration and the above equilibrium conditions it is possible to solve the model.

4.4 Conclusion

In conclusion, building from the results of chapter 3, we see that at least for the US relative investment dynamics may have a compelling relationship with the real exchange rate. At least in terms of long run episodes of appreciation and depreciation it appears that relative investment series may have some explanatory power in defining exchange rate fluctuations. Indeed, it appears that out of the fundamentals

investment may provide the best series with which to tie the real exchange rate to macro-fundamentals. I believe more work can be done on this as it's entirely believable that some unobserved fundamental is linking both these time series. I have also provided a modest model with which to help explain a possible linkage mechanism via the utilization of bonds as collateral in investment decisions. The hope is to open discussion in the role investment may play at least in terms of the long term dynamics of the exchange rate. If anything this paper stresses the need that we start looking at classes of international models that are not silent on the role that relative investment may play in international economics. Given the investment dynamics presented here, and the fact that they differ so sharply from consumption dynamics in regards to the real exchange rate it seems that we must make our models able to not only explain the consumption risk sharing puzzle but at the very least be able to explain the relative linkage that seems to occur between investment and the real exchange rate.

Conclusion

The focus of this dissertation has been primarily on the interrelationships between various macroeconomic variables. Through defining new empirical facts and then attempting to create mechanisms to explain the various interrelationships, this dissertation hopes to better document empirical facts present across countries (in the case of the international risk sharing puzzle) or create mechanisms with which to explain these new or existing puzzles in the data. This dissertation if nothing else hopes to stress the need to pay attention to these interrelationships as we as macroeconomists strive to create models and explain mechanisms with the ultimate goal of better policy proposals. If nothing else the new facts documented here as well as the novel mechanical innovations to existing models present a great avenue for future macroeconomic research.

Chapter 2's focus on the search and matching mechanism suggests that abstracting from rational expectations in the form of Knightian uncertainty may provide a possible means of ameliorating the current search and matching framework to the relevant data on productivity and vacancies. It suggests that merely with a simple multiplying friction such as sticky prices we could see a possible increase in the

responsiveness of agents within the model to a small change in productivity. If nothing else the chapter suggests a possible new application for models with ambiguity aversion.

The results in Chapter 3, show an interesting dynamic interrelationship between the real exchange rate and macroeconomic fundamentals. In fact it suggests that at least dynamically and in the long run that the consumption risk sharing puzzle does not hold. In particular that when times are "good" in the home country we should expect to see a future appreciation of the exchange rate. The VAR exercise suggests a need to find a structural explanation for the orthogonalized exchange rate shock as this seems to be the chief cause of the divergence in the relationship between relative output and relative consumption and the real exchange rate. The chapter itself stresses the need for future modelers to take these dynamic fundamental relationships into account when forming their models. In addition it documents the large heterogeneity in interrelationships across countries, suggesting a need for a future structural explanation for this large variety in the cross section.

Chapter 4 proposes a novel small open economy model that takes into account the possible linkage between the real exchange rate and investment that is documented in chapter 3. The main mechanism involved is the use of government issued bonds as collateral in the production of the capital good. The model itself also utilizes a fiscal and monetary authority which allows the model to, in theory, better match the interesting long run dynamic fundamentals documented in Chapter 3. This chapter hopes to allow for a tractable model that future researchers can use as a base that will contain a mechanism which will explain the relative investment dynamics documented in the prior chapter. It also begins a point of comparison for future modelers as they try to understand the underlying mechanism involved that creates the dynamic correlation pattern present in the data.

Appendix A

Data Sources and Construction

I construct the data for Chapter 2, from a variety of sources. All data prior to constructing statistics are logged and then HP filtered with a smoothing parameter $\lambda = 1600$ for quarterly data. All data is from Q1:1953 to Q2:2012. I choose these dates based on the availability of data. For output, I use Real GDP in billions of chained 2005 dollars from the NIPA series. Consumption is taken from real personal consumption expenditures, also in billions of chained 2005 dollars, from the NIPA series. Investment is from Real Gross Private Domestic Investment in billions of chained 2005 dollar from the NIPA series. For unemployment I use the quarterly seasonally adjusted Civilian Unemployment Rate from the Bureau of Labor Statistics. Productivity is taken from Output per Person in the Nonfarm Business Sector as constructed by the BLS. Vacancies come from the quarterly average of the monthly data series created by Barnichon(2010), which is a combination of the old Conference Board Help Wanted Advertising Index and the newer Conference Board Online Help Wanted Index. I construct the labor tightness ratio as the unemployment rate divided by the number of vacancies. Employment is taken from the Household Sur-

vey of the Civilian Employment-Population Ratio constructed by the BLS. Finally I construct the job finding rate following the method outlined in Shimer(2005a), where I use the seasonally adjusted monthly unemployment level from the Current Population Survey created by the BLS, and the seasonally adjusted monthly series of Number Unemployed for Less than 5 Weeks as the short term unemployment rate. I then use the following formula to calculate the time t job finding rate m_t .

$$m_t = \frac{u_{t+1} - u_{t+1}^s}{u_t}$$

Where u_t is the time t unemployment level and u_t^s is the time t short term unemployment level.

For Chapter 3 I built a data set for 21 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. Nominal exchange rate data were taken from the IFS E-Library as the end of period exchange rate given in US Dollars. Countries which became part of the euro system, have the euro converted back into their respective currency using the irrevocable exchange rate. CPI data is also taken from the IFS E-Library for each country. Real consumption, output and investment data are taken from the OECD statistics website at stats.OECD.org. in their national currency. In order to construct a series of dynamic trade weights I use data on bilateral imports and exports from the UN comtrade database using the SITC Revision 2 data series. In particular the trade weight for a particular country against another is the proportion of self reported imports and exports relative to the total amount of imports and exports in all 21 countries in the sample for a given year. In order to then create a measure for the NEER, I construct the following:

$$\Delta \ln NEER_{j,t} = \sum_{i \neq j}^N [w_{j,i,t} \Delta \ln ne_{i,t}] - \Delta \ln ne_{j,t}$$

Where $ne_{i,t}$ is the nominal exchange rate of country i in terms of dollars. I then convert this into the REER using a similar operation using the CPI. Similarly I perform the same operation using the constructed weights to construct measures for real effective consumption, investment and output.

Appendix B

Log-Linearization and Solution for Ambiguity and the Shimer Puzzle

I first solve for the steady state values under ambiguity. I note first that the steady state value of z under the ambiguous steady state is different from zero in particular it is equal to

$$z = \frac{-\bar{a}}{1 - \rho_z}$$

With this solved the rest of the steady states under ambiguity are the solution to the following system of equations, where capital letters represent steady state values

of the choice variables:

$$\begin{aligned}
1 &= \beta [\alpha e^{(1-\alpha)z} \phi K^{\alpha-1} N^{1-\alpha} + (1 - \delta')] \\
\frac{g}{(1-\lambda)\theta^\lambda C^\gamma} &= \beta \left[\frac{g}{(1-\lambda)\theta^\lambda C^\gamma} [(1-\Psi) - \lambda\theta^{\lambda-1}] - N^{-\frac{1}{\nu}} + \frac{1-\alpha}{C^\gamma} [e^{(1-\alpha)z} \phi K^\alpha N^{-\alpha}] \right] \\
N &= (1-\Psi)N + V^{1-\lambda}(1-N)^\lambda \\
I &= \delta'K \\
C &= \exp \{ (1-\alpha)z \} \phi K^\alpha N^{1-\alpha} - I - gV \\
Y &= \exp \{ (1-\alpha)z \} \phi K^\alpha N^{1-\alpha} \\
\theta &= \frac{(1-N)}{V}
\end{aligned}$$

We can now log linearize the model around the steady state. Log linearization of the equilibrium conditions yields the following results, where hatted lower case letters are to represent log differences from the steady state and large letters represent steady state values.

$$\begin{aligned}
& - \frac{g}{(1-\lambda)\theta^\lambda C} \hat{c}_t + \frac{\lambda g}{(1-\lambda)\theta^\lambda C} \hat{v}_t + \frac{\lambda g V^\lambda N}{(1-\lambda)(1-N)^{\lambda-1} C} \hat{n}_t = \\
\beta E_{\mu_t=-at} & \left[\left(\frac{-g}{(1-\lambda)\theta^\lambda C} (1-\Psi - \lambda\theta^{\lambda-1}) - \frac{(1-\alpha)Y}{CN} \right) \hat{c}_{t+1} + \left(\frac{\alpha(1-\alpha)Y}{CN} \right) \hat{k}_{t+1} \right. \\
& + \left[\frac{\lambda g V^\lambda}{(1-\lambda)(1-N)^{\lambda-1} C} (1-\Psi) - \frac{\lambda g V N}{(1-\lambda)(1-N)^2 C} + \frac{1}{\nu} N^{-\frac{1}{\nu}} - \frac{\alpha(1-\alpha)Y}{CN} \right] \hat{n}_{t+1} \\
& \left. + \left(\frac{\lambda g}{(1-\lambda)\theta^\lambda C} (1-\Psi) - \frac{\lambda g V}{(1-\lambda)(1-N)C} \right) \hat{v}_{t+1} + \left(\frac{(1-\alpha)^2 Y}{CN} \right) \hat{z}_{t+1} \right]
\end{aligned}$$

$$\begin{aligned}
-\gamma C^{-\gamma} \hat{c}_t &= -\beta E_{\mu_t = -a_t} \left[-\gamma C^{-\gamma} \left(\alpha \frac{Y}{K} + 1 - \delta' \right) \hat{c}_{t+1} + (1 - \alpha) \alpha C^{-\gamma} \frac{Y}{K} \hat{z}_{t+1} \right. \\
&\quad \left. + (\alpha - 1) \alpha C^{-\gamma} \frac{Y}{K} \hat{k}_{t+1} + (1 - \alpha) \alpha C^{-\gamma} \frac{Y}{K} \hat{n}_{t+1} \right] \\
N \hat{n}_{t+1} &= [(1 - \Psi) - \lambda V^{1-\lambda} (1 - N)^{\lambda-1}] N \hat{n}_t + (1 - \lambda) V^{1-\lambda} (1 - N)^{\lambda} \hat{v}_t \\
K \hat{k}_{t+1} &= (1 - \delta') K \hat{k}_t + I \hat{i}_t \\
C \hat{c}_t &= (1 - \alpha) Y \hat{z}_t + \alpha Y \hat{k}_t + (1 - \alpha) Y \hat{n}_t - I \hat{i}_t - g V \hat{v}_t \\
\hat{z}_{t+1} &= \rho_z \hat{z}_t - \hat{a}_t + \sigma_z \varepsilon_{z,t+1} \\
\hat{a}_{t+1} &= \rho_a \hat{a}_t + \sigma_a \varepsilon_{a,t+1}
\end{aligned}$$

Finally now this can be rewritten in a state space representation and solved using the methods outline in Sims(2002). This state space representation has the following format:

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + Q \epsilon_t + J \eta_t$$

Where:

$$s_t = \begin{bmatrix} E_t [v_{t+1}] \\ c_t \\ E_t [c_{t+1}] \\ v_t \\ k_{t+1} \\ n_{t+1} \\ i_t \\ z_{t+1} \\ a_{t+1} \end{bmatrix}$$

$$\epsilon_t = \begin{bmatrix} \varepsilon_{z,t+1} \\ \varepsilon_{a,t+1} \end{bmatrix}$$

$$\eta_t = \begin{bmatrix} E_{t-1}[v_t] - v_t \\ E_{t-1}[c_t] - c_t \end{bmatrix}$$

The solution will yield a system in which the states evolve according to a VAR(1) with the following formula:

$$s_t = G_1 s_{t-1} + P \epsilon_t$$

However this gives log deviations from the ambiguous steady state. What we are more interested in is the log deviations from the "zero-risk" steady state. This however, will evolve according to the same relative process except each period the agents will receive corrections to their beliefs in the sense that they will on average receive positive shocks to the technology process equal to a_t . In this case the solution now becomes:

$$s_t = G_1 s_{t-1} + P \epsilon_t + P \begin{bmatrix} \frac{a_t}{\sigma_z} \\ 0 \end{bmatrix}$$

Appendix C

Expenditure Minimization Problem, Final Good Producer's Problem and the Intermediate Goods Producer's Problem for Investment Driven Exchange Rate Dynamics

The Expenditure Minimization Problem

To solve the expenditure minimization problem we have

$$\min_{c_h, c_f} P_h C_h + P_f C_f$$

subject to $c = \left(a_h^{\frac{1}{\eta}} c_h^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} c_f^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$. The first order conditions are

$$P_h = \lambda \frac{\eta}{\eta-1} \left(a_h^{\frac{1}{\eta}} c_h^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} c_f^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}-1} a_h^{\frac{1}{\eta}} \frac{\eta-1}{\eta} c_h^{\frac{\eta-1}{\eta}-1}$$

$$P_f = \lambda \frac{\eta}{\eta-1} \left(a_h^{\frac{1}{\eta}} c_h^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} c_f^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}-1} a_f^{\frac{1}{\eta}} \frac{\eta-1}{\eta} c_f^{\frac{\eta-1}{\eta}-1}$$

which simplify to

$$P_h = \lambda c^{\frac{1}{\eta}} a_h^{\frac{1}{\eta}} c_h^{-\frac{1}{\eta}}$$

$$P_f = \lambda c^{\frac{1}{\eta}} a_f^{\frac{1}{\eta}} c_f^{-\frac{1}{\eta}}$$

Hence we have

$$\frac{P_h}{P_f} = \frac{a_h^{\frac{1}{\eta}} c_h^{-\frac{1}{\eta}}}{a_f^{\frac{1}{\eta}} c_f^{-\frac{1}{\eta}}}$$

or

$$c_f = \frac{a_f}{a_h} \left(\frac{P_f}{P_h} \right)^{-\eta} c_h.$$

Plugging this result into the definition of c we have

$$c_H = \frac{a_h P_h^{-\eta}}{(a_h P_h^{1-\eta} + a_f P_f^{1-\eta})^{\frac{\eta}{\eta-1}}} c$$

so

$$c_F = \frac{a_f P_f^{-\eta}}{(a_h P_h^{1-\eta} + a_f P_f^{1-\eta})^{\frac{\eta}{\eta-1}}} c.$$

So total expenditure is

$$P_h c_h + P_f c_f = [a_h P_h^{1-\eta} + a_f P_f^{1-\eta}]^{\frac{1}{1-\eta}} c,$$

hence

$$P = [a_h P_h^{1-\eta} + a_f P_f^{1-\eta}]^{\frac{1}{1-\eta}},$$

since total expenditure must equal Pc .

The Final Good producer's problem and the Intermediate Goods producer's problem

The cost minimization problem of the final goods producer is have

$$\min_{\{Y_i\}_{i \in [0,1]}} \int_0^1 P_i Y_i di$$

subject to

$$Y = \left(\int_0^1 Y_i^{\frac{\lambda-1}{\lambda}} di \right)^{\frac{\lambda}{\lambda-1}}.$$

The first order conditions are

$$P_i = \frac{\lambda}{\lambda-1} \left(\int_0^1 Y_i^{\frac{\lambda-1}{\lambda}} di \right)^{\frac{\lambda}{\lambda-1}-1} \frac{\lambda-1}{\lambda} Y_i^{\frac{\lambda-1}{\lambda}-1}$$

which simplify to

$$P_i = Y^{\frac{1}{\lambda}} Y_i^{-\frac{1}{\lambda}}$$

Hence we have

$$\frac{P_i}{P_j} = \frac{Y_i^{-\frac{1}{\lambda}}}{Y_j^{-\frac{1}{\lambda}}}$$

for any pair of intermediate goods, i and j . Hence

$$Y_i = \left(\frac{P_i}{P_j} \right)^{-\lambda} Y_j.$$

Plugging this result into the definition of Y we have

$$Y_j = \frac{P_j^{-\lambda}}{\left(\int_0^1 P_i^{1-\lambda} di \right)^{\frac{\lambda}{\lambda-1}}} Y.$$

So total expenditure is

$$\int_0^1 P_i Y_i di = \left(\int_0^1 P_i^{1-\lambda} di \right)^{\frac{1}{1-\lambda}} Y =,$$

hence

$$P_h = \left(\int_0^1 P_i^{1-\lambda} di \right)^{\frac{1}{1-\lambda}},$$

since total cost must equal $P_h Y$.

Appendix D

Log-Linearization for Investment Driven Exchange Rate Dynamics

Assume that $B_f^c = 0$. Assume we are in a zero inflation (of all prices) steady state. Assume the units in which things are measured are such that $\Xi = P_f/P_h = 1$. With $\Xi = 1$, the fifth equation can be used to show that

$$s = 1$$

Normalize n to be some number. Normalize output to $y = 1$. It follows from the assumptions so far that

$$z = b_h^c.$$

The value of b_h^c can be set equal to some reasonable fraction of GDP. Also let c , i , g and x correspond to shares of GDP from the data. We also have simple results such as

$$c_h = a_h c$$

$$c_f = a_f c = (1 - a_h) c$$

In the steady state

$$\delta k = \chi i^\gamma z^{1-\gamma}$$

but I also want $\delta k = i$ so this means set $\chi = (i/z)^{1-\gamma}$ (but I haven't calibrated γ yet).

And it also means that $k = i/\delta$ for some sensible choice of δ . Given the production function this means that A is determined as

$$A = \frac{y}{n^{\mu(1-\alpha)} k^{\mu\alpha-1}}$$

We get the real wage from the foc for labor

$$w = n^\phi c^\sigma$$

and the rental rate of capital from the firm's optimal choices of capital and labor

$$q = \frac{\alpha}{1-\alpha} w \frac{n}{k}$$

From the Euler equation for capital I have

$$\omega = \frac{\beta c^{-\sigma} q}{1 - (1-\delta)\beta}$$

and then I also have

$$\omega = \frac{1}{\gamma \chi} c^{-\sigma} \left(\frac{i}{z} \right)^{1-\gamma}$$

With the stipulated assumption that $\chi = (i/z)^{1-\gamma}$ the latter equation means

$$\gamma = \frac{1}{\omega} c^{-\sigma} = \frac{1 - (1-\delta)\beta}{\beta q}.$$

The home bond Euler equation implies

$$R = \beta^{-1} - \left(\frac{1-\gamma}{\gamma} \right) \frac{i}{z}$$

$$R^* = R + (1 - \xi) \left(\frac{1 - \gamma}{\gamma} \right) \frac{i}{z}$$

Government budget constraint means

$$\tau = b_h^c (R - 1) + g$$

And the tax evolution equation means

$$\tau = \kappa_b b_h^c$$

which determines κ_b .

Then we have

$$MC = \frac{1}{\mu} \frac{w^{1-\alpha} q^\alpha}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \frac{y^{(1-\mu)/\mu}}{A^{1/\mu}} = \frac{1}{\mu} \frac{w^{1-\alpha} q^\alpha}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \frac{1}{A^{1/\mu}}$$

$$c_f + i_f = x.$$

Hatted variables are all of the form $\hat{x} = dx/x$ except where indicated. Also note that all gross inflation rates are 1 in the steady state.

$$\hat{c}_t = \kappa_{ch} \hat{c}_{ht} + (1 - \kappa_{ch}) \hat{c}_{ft} \quad \kappa_{ch} = \frac{a_h^{\frac{1}{\eta}} c_{ht}^{\frac{\eta-1}{\eta}}}{a_h^{\frac{1}{\eta}} c_{ht}^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} c_{ft}^{\frac{\eta-1}{\eta}}}$$

$$\hat{i}_t = \kappa_{ch} \hat{i}_{ht} + (1 - \kappa_{ch}) \hat{i}_{ft}$$

$$\hat{y}_t = \hat{A}_t + \mu(1 - \alpha) \hat{n}_t + \mu\alpha \hat{k}_t$$

$$\hat{z}_t = \kappa_{zh} \hat{b}_{ht}^c - \kappa_{zh} \pi_t + (1 - \kappa_{zh}) \hat{s}_t + (1 - \kappa_{zh}) \hat{B}_{ft}^c \quad \kappa_{zh} = \frac{b_h^c}{b_h^c + \xi_s B_f^c}$$

$$\hat{s}_t = \kappa_{sh} \xi_t \quad \kappa_{sh} = \frac{a_h \Xi^{\eta-1}}{a_h \Xi^{\eta-1} + a_f}$$

$$\pi_t = \pi_{ht} + a_f (\xi_t - \xi_{t-1})$$

$$\delta_{t+1} = s_{t+1} - s_t + \pi_{t+1}$$

$$\hat{k}_{t+1} = (1 - \delta) \hat{k}_t + \gamma \delta \hat{i}_t + (1 - \gamma) \delta \hat{z}_t$$

$$b_{ht+1}^c + \tau_t y_t = b_{h,t}^c \frac{R_{t-1}}{\Pi_t} + g_t$$

$$\tau_t = \rho \tau_{t-1} + (1 - \rho) \kappa_b \frac{b_{h,t}^c}{Y_{t-1}} + u_t$$

$$\ln R_t - \ln R = \rho_r (\ln R_{t-1} - \ln R) + (1 - \rho_r) \phi_\pi \pi_t + v_t$$

Optimality Conditions

$$MC_t = \frac{1}{\mu} \frac{w_t^{1-\alpha} q_t^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{y_t^{(1-\mu)/\mu}}{A_t^{1/\mu}} (a_h + a_f \Xi_t^{1-\eta})^{\frac{1}{1-\eta}}$$

$$\frac{n_t}{k_t} = \frac{1 - \alpha}{\alpha} \frac{q_t}{w_t}$$

$$c_{ht} = a_h (a_h + a_f \Xi_t^{1-\eta})^{\frac{\eta}{1-\eta}} c_t \tag{D.1}$$

$$c_{ft} = a_f (a_h \Xi_t^{\eta-1} + a_f)^{\frac{\eta}{1-\eta}} c_t. \tag{D.2}$$

$$i_{ht} = a_h (a_h + a_f \Xi_t^{1-\eta})^{\frac{\eta}{1-\eta}} i_t \tag{D.3}$$

$$i_{ft} = a_f (a_h \Xi_t^{\eta-1} + a_f)^{\frac{\eta}{1-\eta}} i_t. \tag{D.4}$$

$$n_t^\phi = w_t c_t^{-\sigma}$$

$$\omega_t = \frac{1}{\gamma\chi} c_t^{-\sigma} \left(\frac{i_t}{z_t} \right)^{1-\gamma}$$

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[R_t + \left(\frac{1-\gamma}{\gamma} \right) \frac{i_{t+1}}{z_{t+1}} \right] \frac{1}{\Pi_{t+1}}$$

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[R_t^* + \xi \left(\frac{1-\gamma}{\gamma} \right) \frac{i_{t+1}}{z_{t+1}} \right] \frac{\Delta_{t+1}}{\Pi_{t+1}}$$

$$\omega_t = (1-\delta)\beta\omega_{t+1} + \beta c_{t+1}^{-\sigma} q_{t+1}$$

$$\pi_{ht} = \beta E_t \pi_{ht+1} + \frac{(1-\theta)(1-\beta\theta)\Theta}{\theta} \widehat{m}c_t = \beta E_t \pi_{ht+1} + \tilde{\Theta} \widehat{m}c_t. \quad (\text{D.5})$$

Market Clearing Conditions

$$\frac{c_h}{y} \hat{c}_{ht} + \frac{i_h}{y} \hat{i}_{ht} + \frac{g}{y} \hat{g}_t + \frac{x}{y} \hat{x}_t = \hat{y}_t$$

$$\frac{\Xi c_f}{y} \hat{c}_{ft} + \frac{\Xi i_f}{y} \hat{i}_{ft} + \hat{B}_{f,t+1}^c = \frac{x}{y} \hat{x}_t - \frac{x}{y} \xi_t + R^* \hat{B}_{f,t}^c + \frac{\Xi}{y} B_f^c dR_{t-1}^*$$

where $\hat{B}_{ft}^c = \Xi dB_{ft}^c/y$. Why define it this way? Because I'd like to be able to assume that $B_f^c = 0$ in the initial steady state so that trade is balanced. I.e. I want to have

$$c_h + i_h + g + x = y$$

$$c_f + i_f = \frac{x}{\Xi}.$$

If $B_f^c = 0$ then the last equation becomes

$$\frac{\Xi c_f}{y} \hat{c}_{ft} + \frac{\Xi i_f}{y} \hat{i}_{ft} + \hat{B}_{f,t+1}^c = \frac{x}{y} \hat{x}_t - \frac{x}{y} \xi_t + R^* \hat{B}_{f,t}^c$$

Bibliography

- Albuquerque, R., Eichenbaum, M., Papanikolaou, D., and Rebelo, S. (2015), “Long-run bulls and bears,” *Journal of Monetary Economics*, 76, S21–S36.
- Anderson, E. (2005), “The dynamics of risk-sensitive allocations,” *Journal of Economic Theory*, 125, 2005.
- Anderson, T. and Hsiao, C. (1982), “Formulation and estimation of dynamic models using panel data,” *Journal of Econometrics*, 18, 47–82.
- Andolfatto, D. (1996), “Business cycles and labor-market search,” *American Economic Review*, 86, 112–132.
- Backus, D. and Smith, G. (1993), “Consumption and real exchange rates in dynamic economies with non-traded goods,” *Journal of International Economics*, 35, 297–316.
- Bansal, R. and Shaliastovich, I. (2013), “A long-run risks explanation of predictability puzzles in bond and currency markets,” *The Review of Financial Studies*, 26, 1–33.
- Barnichon, R. (2010), “Building a composite help-wanted index,” *Economics Letter*, 109, 175–178.
- Baxter, M. and King, R. (1999), “Measuring business cycles: approximate band-pass filters for economic time series,” *The Review of Economics and Statistics*, 81, 575–593.
- Benigno, G. and Thoenissen, C. (2005), “Consumption and real exchange rates with incomplete markets and non-traded goods,” *Journal of International Money and Finance*, 27, 926–948.
- Brandt, M., Cochrane, J., and Santa-Clara, P. (2006), “International risk sharing is better than you think, or exchange rates are too smooth,” *Journal of Monetary Economics*, 53, 671–698.
- Burnside, A. and Graveline, J. (2014), “Exchange rate determination, risk sharing and the asset market view,” NBER working paper no. 18646.

- Burstein, A., Eichenbaum, M., and Rebelo, S. (2006), “The importance of nontradable goods’ prices in cyclical real exchange rate fluctuations,” *Japan and the World Economy*, 18, 247–253.
- Burstein, A., Kurz, C., and Tesar, L. (2008), “Trade, production sharing and the international transmission of business cycles,” *Journal of Monetary Economics*, 55, 775–795.
- Cagetti, M., Hansen, L., Sargent, T., and Williams, N. (2002), “Robustness and pricing with uncertain growth,” *Review of Financial Studies*, 15, 363–404.
- Calvo, G. (1983), “Staggered prices in a utility-maximizing framework,” *Journal of Monetary Economics*, 12, 383–398.
- Chari, V., Kehoe, P., and McGrattan, E. (2002), “Can sticky price models generate volatile and persistent real exchange rates,” *Review of Economic Studies*, 69, 533–563.
- Chéron, A. and Langot, F. (2004), “Labor market search and real business cycles: reconciling Nash bargaining with the real wage dynamics,” *Review of Economic Dynamics*, 7, 476–493.
- Chinn, M. (2006), “A primer on real effective exchange rates: determinants, overvaluation, trade flows and competitive devaluation,” *Open Economies Review*, 17, 115–143.
- Christiano, L. and Fitzgerald, T. (2003), “The band pass filter,” *International Economic Review*, 44, 435–465.
- Christiano, L., Ilut, C., Motto, R., and Rostagno, M. (2008), “Monetary policy and stock market boom-bust cycles,” ECB Working Papers 955.
- Clarida, R., Gali, J., and Gertler, M. (2002), “A simple framework for international monetary policy analysis,” *Journal of Monetary Economics*, 49, 879–904.
- Colacito, R. and Croce, M. (2011), “Risks for the long run and the real exchange rate,” *Journal of the Political Economy*, 119, 153–181.
- Corsetti, G., Dedola, L., and Leduc, S. (2008), “International risk sharing and the transmission of productivity shocks,” *The Review of Economic Studies*, 75, 443–479.
- Costain, J. and Reiter, M. (2008), “Business cycles, unemployment insurance, and the calibration of matching models.” *Journal of Economic Dynamics and Control*, 32, 1360–1385.

- den Haan, W. and Kaltenbrunner, G. (2009), “Anticipated growth and business cycles in matching models,” *Journal of Monetary Economics*, 56, 309–327.
- Diamond, P. (1982), “Aggregate demand management in search equilibrium,” *Journal of Political Economy*, 90, 881–894.
- Dominguez, K. and Tesar, L. (2006), “Exchange rate exposure,” *Journal of International Economics*, 68, 188–218.
- Eichenbaum, M. and Evans, C. (1995), “Some empirical evidence on the effects of shocks to monetary policy on exchange rates,” *The Quarterly Journal of Economics*, 110, 975–1009.
- Ellsberg, D. (1961), “Risk Ambiguity and the savage axioms,” *The Quarterly Journal of Economics*, 75, 643–669.
- Engel, C. (1999), “Accounting for U.S. real exchange rate changes,” *Journal of Political Economy*, 107, 507–538.
- Engel, C. (2012), “The real exchange rate, real interest rates, and the risk premium,” Tech. rep., National Bureau of Economic Research.
- Epstein, L. and Wang, T. (1994), “Intertemporal asset pricing under knightian uncertainty,” *Econometrica*, 62, 283–322.
- Gilboa, I. and Schmeidler, D. (1989), “Maxmin expected utility with non-unique prior,” *Journal of Mathematical Economics*, 18, 141–153.
- Hall, R. (2005), “Employment fluctuations with equilibrium wage stickiness,” *American Economic Review*, 95, 50–65.
- Hall, R. and Milgrom, P. (2008), “The limited influence of unemployment on the wage bargain,” *American Economic Review*, 98, 1653–1674.
- Hansen, L., Sargent, T., and Tallarini, T. (1999), “Robust Permanent Income and Pricing,” *Review of Economic Studies*, 66, 873–907.
- Hodrick, R. and Prescott, E. (1997), “Postwar U.S. business cycles: an empirical investigation,” *Journal of Money, Credit and Banking*, 29, 1–16.
- Holtz-Eakin, D., Newey, W., and Rosen, H. (1988), “Estimating vector autoregressions with panel data,” *Econometrica*, 56, 1371–1395.
- Hornstein, A., Krusell, P., and Violante, G. (2005), “Unemployment and vacancy fluctuations in the matching model: Inspecting the mechanism,” *Federal Reserve Bank of Richmond Economic Quarterly*, 91, 19–51.

- Hosios, A. (1990), “On the efficiency of matching and related models of search and unemployment,” *Review of Economic Studies*, 57, 279–298.
- Iacoviello, M. (2005), “House prices, borrowing constraints, and monetary policy in the business cycle,” *The American Economic Review*, 95, 739–764.
- Ilut, C. and Schneider, M. (2012), “Ambiguous Business Cycles,” ERID Working Paper No. 123.
- King, R. and Watson, M. (1998), “The solution of singular linear difference systems under rational expectations,” *International Economic Review*, 39, 1015–1026.
- Kiyotaki, N. and Moore, J. (1997), “Credit Cycles,” *Journal of Political Economy*, 105, 211–248.
- Kollmann, R. (1995), “Consumption, real exchange rates and the structure of international asset markets,” *Journal of International Money and Finance*, 14, 191–211.
- Kollmann, R. (2005), “Macroeconomic effects of nominal exchange rate regimes: new insights into the role of price dynamics,” *Journal of International Money and Finance*, 24, 275–292.
- Krusell, P. and McKay, A. (2010), “News shocks and the business cycle,” *Economic Quarterly*, 96, 272–397.
- Lafrance, R., Osakwe, P., St-Amant, P., et al. (1998), *Evaluating alternative measures of the real effective exchange rates*, Bank of Canada.
- Lustig, H. and Verdelhan, A. (2007), “The cross section of foreign currency risk premia and consumption growth risk,” *The American Economic Review*, 97, 89–117.
- Merz, M. (1995), “Search in the labor market and the real business cycle,” *Journal of Monetary Economics*, 36, 269–300.
- Mortensen, D. (1982), “Property rights and efficiency in mating, racing and related games,” *American Economic Review*, 72, 968–979.
- Mortensen, D. and Nagypál, E. (2007), “More on unemployment and vancancy fluctuations,” *Review of Economic Dynamics*, 10, 327–347.
- Pissarides, C. (1985), “Short-run equilibrium dynamics of unemployment, vancancies, and real wages,” *American Economic Review*, 75, 1339–1369.
- Ravn, M. (2001), “Consumption dynamics and real exchange rates,” unpublished.

- Ravn, M., Schmitt-Grohé, S., and Uribe, M. (2012), “Consumption, government spending and the real exchange rate,” *Journal of Monetary Economics*, 59, 215–234.
- Ready, R., Roussanov, N., and Ward, C. (2013), “Commodity trade and the carry trade: a tale of two countries,” Tech. rep., National Bureau of Economic Research.
- Shimer, R. (2005a), “The cyclical behavior of equilibrium unemployment and vacancies,” *American Economic Review*, 95, 25–49.
- Shimer, R. (2005b), “The cyclical behavior of hires, separations, and job-to-job transitions.” *Federal Reserve Bank of St. Louis Review*, 87, 493–507.
- Sims, C. (2002), “Solving Linear Rational Expectations Models,” *Computational Economics*, 20, 1–2.
- Stock, J. and Watson, M. (2008), “Heteroskedasticity-robust standard errors for fixed effects panel data regression,” *Econometrica*, 76, 155–174.
- Stockman, A. and Tesar, L. (1995), “Tastes and technology in a two-country model of the business cycle: explaining international comovements,” *The American Economic Review*, 85, 168–185.
- Valchev, R. (2016), “Exchange Rates and UIP violations at short and long horizons,” Unpublished.
- Verdelhan, A. (2010), “A habit-based explanation of the exchange rate risk premium,” *The Journal of Finance*, 65, 123–146.

Biography

1. Christopher James Roark
2. Born: September 19th 1986 Houston Texas
3. BA French and Economics, The University of Texas at Austin.
4. MA Economics, Duke University
5. Ph.D Economics, Duke University.
6. Graduate School Summer Fellowship Receptient 2013
7. Martin Bronfenbrenner Fellowship, Duke University 2014
8. "The evolution of U.S. spectrum values over time" - Joint with Michelle Connolly, Nelson Sa and Azeem Zaman.
9. Will be continuing as a Lecturer at The University of Chicago