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## The evolution of airplanes

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The prevailing view is that we cannot witness biological evolution because it occurred on a time scale immensely greater than our lifetime. Here, we show that we can witness evolution in our lifetime by watching the evolution of the flying human-and-machine species: the airplane. We document this evolution, and we also predict it based on a physics principle: the constructal law. We show that the airplanes must obey theoretical allometric rules that unite them with the birds and other animals. For example, the larger airplanes are faster, more efficient as vehicles, and have greater range. The engine mass is proportional to the body size: this scaling is analogous to animal design, where the mass of the motive organs (muscle, heart, lung) is proportional to the body size. Large or small, airplanes exhibit a proportionality between wing span and fuselage length, and between fuel load and body size. The animal-design counterparts of these features are evident. The view that emerges is that the evolution phenomenon is broader than biological evolution. The evolution of technology, river basins, and animal design is one phenomenon, and it belongs in physics. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4886855>]

### I. AIRPLANES ARE GETTING BIGGER

The things we see and touch are changing. If not from day to day, or from year to year, then from decade to decade. Just look at the airplanes that carry more and more people all over the globe. Look at airport gates, and in the sky.

Much simpler is to look at Fig. 1. The data represent the sizes of the new airplane models and the years when they were put in service. Each new model was presumably more economical (efficient) than its predecessors of the same size, because otherwise it would not have been successful to be adopted. This trend (toward greater efficiency) is not visible in Fig. 1. Visible is another trend: although the new models come in all sizes, the biggest airplanes of one decade are joined by even bigger models in the next decade.

Why is this, and why should we care?

In this article, we answer these questions, but we do it in reverse order. Yes, we should care because bird's-eye-views such as Fig. 1 open everybody's eyes to the natural phenomenon called "evolution." Evolution means a flow organization (design) that changes over time. In biology, evolution is largely a mental construct built on imagination, because the time scale of animal evolution is immense relative to the time available to us for observations. We cannot witness animal evolution, and this places the biology argument for evolution at a disadvantage. It would be useful to have access to the evolution of one species in real time.

Looking at Fig. 1 satisfies precisely this need. The species to watch is the human-and-machine species. New airplane models do not happen by themselves. They are extensions, enclosures of the humans who come together to ride on them. Airplanes are flying buildings. Every model is a new human-and-machine design for moving our bodies,

groups, and belongings over the entire globe. This design is changing, and what evolves with it is the movement of humans on the globe. This spreading flow gets better, faster, more efficient, and farther reaching, in accord with the constructal law.

This is just like the evolution of animal fliers, Fig. 2. The bigger fly faster, but this is well established,<sup>1–7</sup> and it is not the reason for showing Fig. 2 here. The reason is that the invisible evolution of animal fliers has led numerous forms of animal mass flow on the globe to converge on the same design—the same scaling rules—as the evolution of human fliers.

Equally important is the observation that over time the cloud of fliers has been expanding to the right in Fig. 2. In the beginning were the insects, later the birds and the insects, and even later the airplanes, the birds, and the insects. The animal mass that sweeps the globe today is a weave of few large and many small. The new are the few and large. The old are the many and small.

The airplanes evolved in the same way. In the beginning was the DC-3 and many smaller airplanes; then, the DC-3 was joined by the DC-8 and the B737; next the B747 joined the smaller and older models still in use. In this evolutionary direction, the size record is broken every time. This trend unites human fliers (Fig. 1) with animal fliers (Fig. 2) and leads back to the first question: Why is this?

### II. TECHNOLOGY EVOLUTION

Think of a vehicle that consumes fuel and moves on the world map, and ask how large one of the organs of this vehicle should be, for example, a duct with fluid flowing through it, or the heat exchanger surface of the environmental control system. Because the size of the organ is finite, the vehicle is penalized (in fuel terms) by the component in two ways.

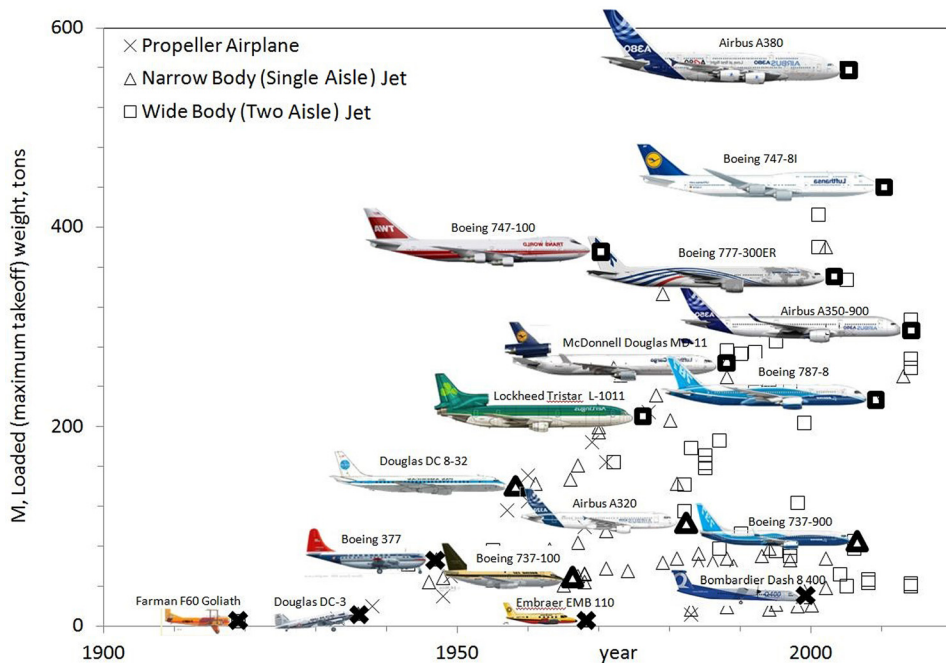


FIG. 1. The evolution of the major airplane models during the 100-yr history of aviation (Source: Table 1 in Supplementary material).

First, the organ is alive with currents that flow by overcoming resistances, obstacles, and all kinds of “friction.” In thermodynamics, this universal phenomenon is called irreversibility, or destruction of useful energy (exergy), or entropy generation.<sup>8</sup> This fuel penalty is smaller when the organ is larger, because larger means wider ducts and larger heat transfer surfaces. In this limit, larger is better, because the organ poses less resistance to the flow of fluid, heat, mass, and stresses.

Second, the vehicle must burn fuel in order to transport the organ. The fuel penalty for carrying the organ is proportional to the weight of the organ. This second penalty

commands that the smaller is better, and it is in conflict with the first. From this conflict emerges the discovery that the organ should have a characteristic size that is finite, not too large, not too small, but just right for that particular vehicle.<sup>9</sup>

The organ size recommended by this trade off is such that large organs (pipes, heat exchangers, pumps, compressors, turbines) belong on large vehicles, and small organs belong on small vehicles. This prediction is evident in Fig. 3, which shows that during the evolution of airplanes (Fig. 1) an approximate proportionality has emerged between the mass of the heat engine ( $M_e$ ) and the mass of the whole

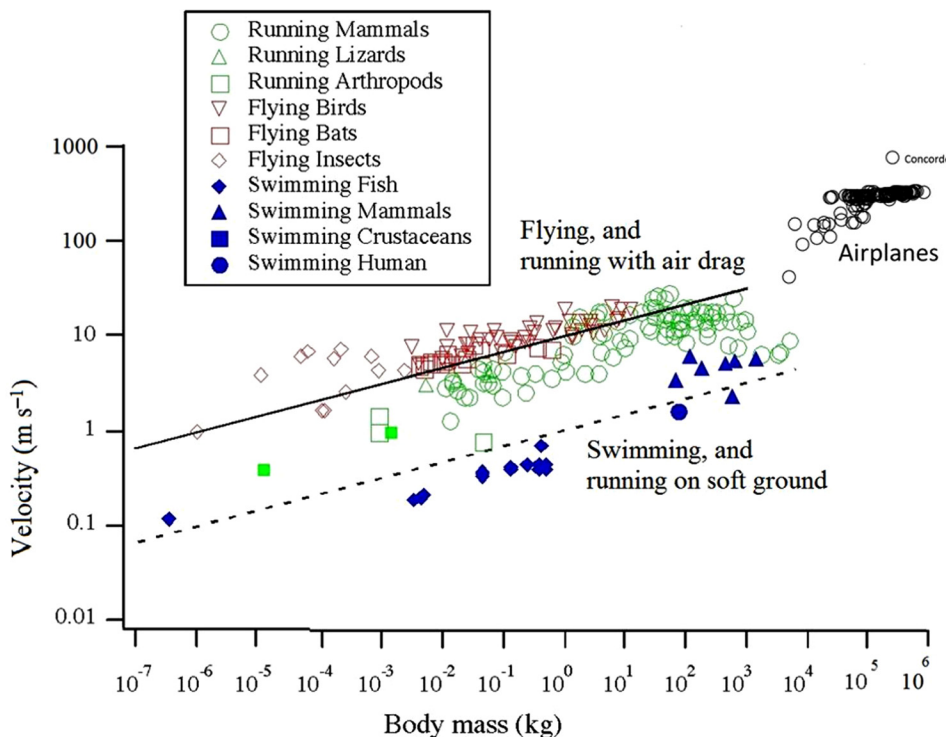


FIG. 2. The characteristic speeds of all the bodies that fly, run, and swim (insects, birds, and mammals). The sources of the animal locomotion data are indicated in Ref. 1.

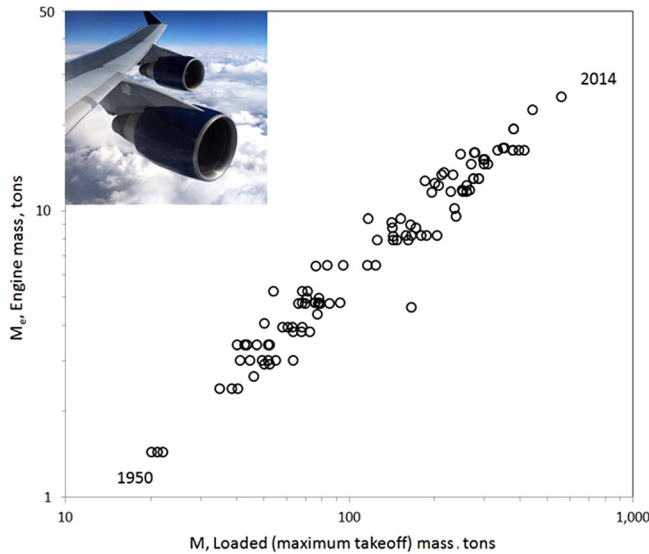


FIG. 3. During the evolution of airplanes (Fig. 1), the engine sizes have increased almost proportionally with the airplane sizes (Source: Table 1; the data refer only to turbine (jet) engine airplanes).

aircraft (M). The data are correlated in a statistically meaningful way<sup>10,11</sup> as

$$M_e = 0.13 M^{0.83}, \tag{1}$$

with  $R^2 = 0.95$ , where both M and  $M_e$  are expressed in tons (the correlation is statistically meaningful because its P-value is 0.0001, and it is less than 0.05 so that the null hypothesis can be rejected).

Noteworthy is also the time arrow indicated by the cloud of data in Fig. 3: the sizes of engines and airplanes increased by factors of order 20 from 1950 to 2014. This time arrow is oriented in the same direction as in Fig. 2, toward the large and few.

### III. BODY SIZE SCALING

The size effect uncovered in Fig. 3 can be predicted analytically. Consider a vehicle that travels the distance L while consuming the amount of fuel  $M_f$ . The vehicle mass M has two main components, the fuel mass  $M_f$ , and the engine mass  $M_e$ . The burning of  $M_f$  delivers the heat input  $Q = M_f H$  to the motor, where H is a factor representing the heating value of the fuel. The work produced from Q is destroyed during the L travel, namely  $W = \mu M g L$ , where  $\mu$  is a dimensionless factor, and Mg is the weight of the loaded vehicle.<sup>7</sup> This W formula holds (with slightly different  $\mu$  values) for all modes of transportation and animal locomotion: land, sea, and air.

Larger vehicles and animals are more efficient movers of mass on the world map. The energy conversion efficiency of a moving body ( $\eta = W/Q$ ) exhibits a size effect known as economies of scale. This effect is present in all power generators and power users: larger machines are more efficient than smaller machines because they operate with less friction (with wider passages for fluid flow) and less heat transfer irreversibility (with larger surface for heat transfer).<sup>12</sup> The data on the efficiency of the largest motors in use, gas

turbines<sup>13</sup> and steam turbines,<sup>14</sup> show that the size effect is captured by a relationship of type  $\eta = C_1 M_e^\alpha$ , where  $C_1$  and  $\alpha$  are two constants, and where  $\alpha$  is of order 1 but smaller than 1 because the curve  $\eta(M_e)$  must be concave as it tends toward its Carnot ceiling. Combining the Q, W, and  $\eta$  expressions, we find that the total movement of mass on the landscape (ML) scales as

$$ML \sim (C_1 H / \mu g) M_e^\alpha M_f. \tag{2}$$

Because of the total mass constraint  $M = M_e + M_f$ , the product ML (or the product  $M_e^\alpha M_f$ ) is maximal when the ratio  $M_f / M_e$  is a constant of order 1.

In conclusion, there must be a proportionality between the size of the engine, the size of the fuel used, and the mass of the whole vehicle. Both  $M_e$  and  $M_f$  are represented by the scale of their sum, which is M. The airplanes have all evolved such that larger motors and fuel loads belong on larger vehicles. This prediction is supported by the data in Figs. 3 and 4, and by all the vehicles that have evolved, man-made or animal.

### IV. RANGE

Larger vehicles also travel farther. From the ML formula above, we conclude that when  $M_e$  and  $M_f$  scale with M, the distance traveled (the range) is  $L \sim (C_1 H / \mu g) f(\alpha) M^\alpha$ , where the group  $f(\alpha) = \alpha^\alpha / (1 + \alpha)^{1 + \alpha}$  is a constant of order 1. The range L is predicted to vary in proportion with  $M^\alpha$ : larger vehicles cover greater territories. This is confirmed by the L vs M data for airplane evolution (Fig. 5), which are correlated as

$$L = 323.91 M^{0.64}, \tag{3}$$

with L [km] and M [tons], and statistically significant with  $P = 0.0001$ ,  $R^2 = 0.80$ . Note the M exponent  $\alpha = 0.64$ , which conforms to the analysis above.

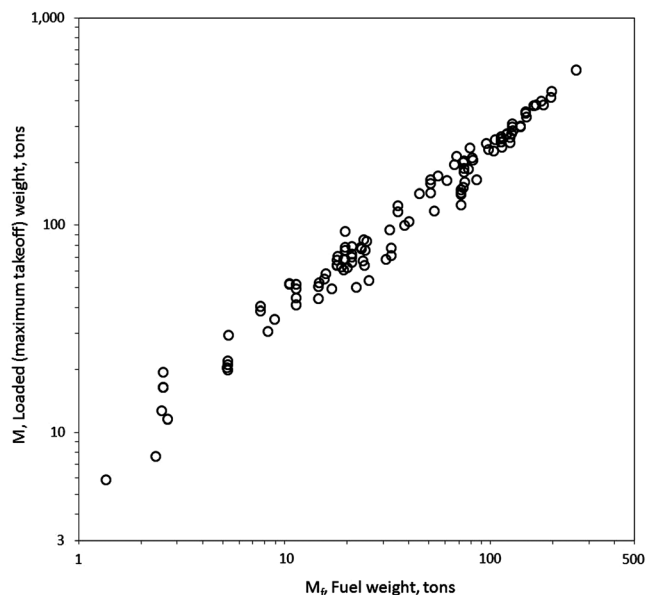


FIG. 4. The proportionality between fuel mass and airplane mass (Source: Table 1).

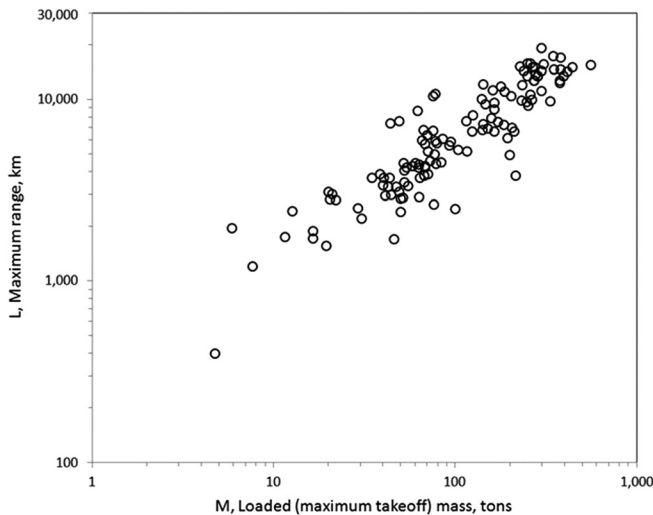


FIG. 5. The effect of the size of the aircraft and its range (Source: Table 1).

**V. EFFICIENCY**

The evolution of aircraft shows that commercial air travel is becoming more efficient, less costly. Reported in Table 1 in the supplementary material<sup>18</sup> is the unit cost (*f*) expressed as liters of fuel spent for one seat and 100 km flown. The trend is visibly downward: the *f* values have decreased by one order of magnitude during the past half century. On an average, every year there has been a 1.2% decrease in fuel burn per seat.<sup>15</sup> The evolution toward greater efficiency goes hand in hand with the trend toward larger flying bodies, noted earlier in Figs. 1 and 2.

In sum, technology evolution is about the evolving design of the human movement on the Earth’s surface: people, goods, material, construction, mining, etc. As the whole vehicle or animal evolves, its architecture to become better at moving mass on the landscape, the organs remain imperfect, because each has a finite size, not an infinite size. The whole (the vehicle) is a construct of organs that are “imperfect” only when examined in isolation. The vehicle design evolves over time and becomes a better construct for moving the vehicle mass on the world map.

Everything that we can say about vehicles in relation to Figs. 3–5 applies unchanged to animal organs and the whole animal. Every organ must have a certain characteristic size, which is larger when the animal is larger. Every organ is imperfect because of its finite size. If the animal is the analog of the human-made vehicle (e.g., the airplane) then the organs that constitute the motor system of the animal (muscles, heart, lung) are the counterparts of the engine of the vehicle. In biology, it is well known that the muscle mass, the heart mass, and the lung volume are empirically proportional to the animal body mass.<sup>2–6</sup> The animal organ scaling is the same as the engine mass vs vehicle mass proportionality revealed by Fig. 3, and this means that the theory that predicted Fig. 3 also holds for predicting the organ-size allometric relations recognized empirically in biology.

**VI. WHY AIRPLANES LOOK ALIKE**

Small or large, airplanes are evolving such that they look more and more like airplanes, not like birds. They do not flap wings, hover, or glide. They have engines that provide steady power for cruising speed and constant altitude. Unlike in birds, in airplanes the motor and lift functions are performed by two distinct organs, the engine and the wings. Yet, airplanes obey allometric rules that unite them with birds and other animals. Their engines scale with their body sizes and with their fuel loads. The larger airplanes are more efficient vehicles of mass, and travel farther, just like the larger animals.

Small or large, airplanes are evolving such that they look the same. The airplane body has two main parts, a fuselage that carries passengers and freight, and wings that lift the fuselage. This two-part structure is shown schematically in Fig. 6. Every aspect ratio (shape) of this structure is predictable from the constructal law<sup>1,16</sup> that predicted the evolutionary trends discussed until now. Here is how.

The primary objective of commercial airplanes is to carry as many people as possible to a specified distance while using as little fuel as possible. The fuel consumed is proportional to the work delivered by the engine over the distance, and the work is equal to the total drag force experienced by the airplane times the traveled distance. In sum, to reduce the fuel requirement of an airplane of specified size is to reduce the total drag force subject to constraints that are described next.

Assume that the fuselage has the cross sectional area *A* and length *L*, and the wings have the span *S*, thickness *t*, and swept length *L<sub>w</sub>*. The following analysis is based on the method of scale analysis.<sup>17</sup> The total drag force (*F*) on the airplane has two main components, fuselage and wing. Each component has two subcomponents, drag due to the stagnation of the approaching air of speed *V* and density  $\rho_a$ , and tangential drag due to skin friction,

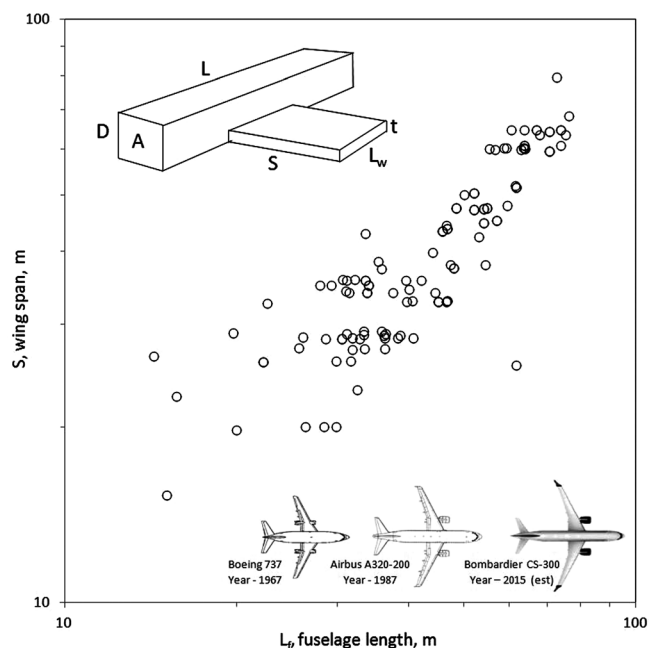


FIG. 6. During the evolution of airplanes (Fig. 1), the fuselage length has increased proportionally with the wing span (Source: Table 1).

$$\frac{F}{\frac{1}{2}\rho_a V^2} \sim (A + C_f p L)_{fuselage} + (tS + C_f S L_w)_{wing}. \quad (4)$$

Here,  $p$  is the perimeter of the fuselage cross section, and  $C_f$  is the skin friction coefficient, which is nearly constant (independent of  $V$ ) and of order  $10^{-2}$ . Of all the shapes of the  $A$  cross-section, the shape that offers minimum drag is the round cross section.<sup>9</sup> Consequently, we write  $D$  for the diameter of the fuselage, and in Eq. (2) we substitute  $A \sim D^2$  and  $p \sim D$ .

In Sec. III and Fig. 5, we showed that the evolution toward less fuel consumption during travel (animal or vehicle) leads the design to a rough proportionality between the amount of fuel (or food, for animals) and the rest of the mass of the moving body. On a modern commercial airplane, the fuel is loaded in the wings. The size of the airplane is fixed, and is represented by its mass  $M = M_{fuselage} + M_{wing}$ ; however, because  $M \sim M_{wing}$ , the total mass scales with either  $M_{fuselage}$  or  $M_{wing}$ , therefore the body mass scaling is

$$M \sim \rho D^2 L, \text{ fixed}, \quad (5)$$

where  $\rho$  is the average density of the fuselage and the wing.

For vertical equilibrium, the wing must provide a lift force that balances the weight of the airplane ( $Mg$ ), therefore

$$\frac{1}{2}\rho_a V^2 C_l S L_w \sim Mg. \quad (6)$$

Here,  $C_l$  is the lift coefficient (assumed constant and of order 1) and  $S L_w$  is the wing surface that provides lift.

The wing must be strong in bending, to support the bending moment exerted by the body weight on the wing ( $MgS$ ). If  $\sigma$  is the allowable stress level in the wing as a beam in pure bending,<sup>9</sup> then the bending moment in a vertical longitudinal section through the wing is of order ( $\sigma t L_w$ ),

$$\sigma t L_w \sim \rho D^2 L g S. \quad (7)$$

The objective is to discover the shape of the flying body such that  $F$  is minimum subject to Eqs. (6) and (7). First, we note that the contribution that the fuselage makes to  $F$  in Eq. (4) is proportional to  $A + C_f p L \sim D^2 + C_f D L$ . This quantity varies as  $D$  and  $L$  vary subject to  $D^2 L \sim M/\rho$ , constant, cf. Eq. (5). It reaches its minimum value when

$$\frac{D}{L} \sim \frac{C_f}{2} < 1, \quad (8)$$

which means that the fuselage must continue to evolve toward a slender body of revolution. This result also means that in Eq. (4) the  $A$  and  $pL$  terms are of the same order of magnitude, and Eq. (4) reduces to

$$\frac{F}{\frac{1}{2}\rho_a V^2} \sim D^2 + tS + C_f S L_w. \quad (9)$$

It also means that [in view of Eq. (5)] the scales of  $D$  and  $L$  are fixed,

$$D \sim \left(\frac{C_f M}{2\rho}\right)^{1/3}, \quad L \sim \left(\frac{2}{C_f}\right)^{2/3} \left(\frac{M}{\rho}\right)^{1/3}. \quad (10)$$

Likewise, the shape of the wing profile ( $t/L_w$ ) can be selected by minimizing the wing contribution to  $F$ , Eq. (4), subject to fixed profile area  $tL_w$ . The result is

$$t \sim C_f L_w, \quad (11)$$

which shows that the last two terms in Eq. (9) are of the same order, and Eq. (9) reduces to

$$\frac{F}{\frac{1}{2}\rho_a V^2} \sim D^2 + C_f S L_w. \quad (12)$$

With  $D$  known from Eq. (10), the dimensions that are left to be determined are  $S$  and  $L_w$ . Combining Eqs. (6), (7), and (11) we find

$$S \sim a^{-1/4} b^{3/4} C_f^{1/2}, \quad (13)$$

$$L_w \sim a^{1/4} b^{1/4} C_f^{1/2}, \quad (14)$$

where

$$a = \frac{g}{\sigma} M, \quad b = \frac{Mg}{\frac{1}{2}\rho_a V^2 C_l}. \quad (15)$$

The key result of this analysis is that the ratio between the wing span and the fuselage length should scale as

$$\frac{S}{L} \sim M^{1/6} g^{1/2} \rho^{1/3} \sigma^{1/4} (\rho_a V^2 C_l)^{-3/4} 2^{1/4} C_f^{7/6}. \quad (16)$$

Because the scaling of speed with body mass is broadly a power law of type  $V \sim M^{1/6}$ , cf. Fig. 2, the conclusion is that the ratio  $S/L$  should vary as  $M^{-1/12}$ , which indicates a negligible effect of body size on  $S/L$ . In the  $M$  range covered by commercial aircraft (Figs. 1 and 2), the predicted ratio  $S/L$  should be constant. This prediction is confirmed by the data assembled in Fig. 6.

An additional result is obtained by substituting the scales of  $S$  and  $L_w$  in Eq. (12),

$$\frac{F}{(1/2)\rho_a V^2} \sim D^2 + C_f^2 b. \quad (17)$$

On the right side, both terms are proportional to  $M^{2/3}$  (note that  $b$  varies as  $M/V^2$ , with  $V \sim M^{1/6}$ ). The right side of Eq. (17) is proportional to  $M^{2/3}$ , and consequently  $F$  is proportional to  $M^{2/3} V^2 \sim M$ , because  $V \sim M^{1/6}$ .

The conclusion that the necessary force during travel is proportional to the body mass is in accord with the known scaling of animal locomotion and vehicle movement. This is why the progress toward greater fuel efficiency is monitored by calculating the ratio  $F/M$ , because the fuel spent on a specified distance is proportional to  $F$ . The ratio  $F/M$  decreases over time because of evolutionary improvement in the configuration of the flow systems involved, the engine, the shaping of the body and wing, the miniaturization of avionics, etc.

## VII. CONCLUSION

In summary, the fuselage and the wing must have similarly slender profiles ( $D < L$ ;  $t < L_w$ ), the fuselage cross

section A must be roundish (shown as a square in Fig. 6), and the wing span (S) must be proportional to the fuselage length (L). The predicted proportionality  $S \sim L$  is supported statistically by the measurements displayed in Fig. 6, which cover the wide range of body sizes of the airplane evolution viewed in this paper.

Looking at the graphs of this paper, we see that there is an outlier, the Concorde, which was perhaps the most radical departure from the traditional swept wing commercial airplane. The Concorde's primary goal was to fly fast. In chasing an "off the charts" speed rating the Concorde deviated from the evolutionary path traced by successful airplanes that preceded it. It was small, had limited passenger capacity, long fuselage, short wingspan, massive engines, and poor fuel economy relative to the airplanes that preceded it. Even when it was in service, the Concorde did not sell, and only 20 units were ever produced (whereas successful Boeing and Airbus models were produced by the thousands). Eventually, due to lack of demand and safety concerns, the Concorde was retired in 2003.

The carbon fiber revolution will mark a dramatic shift in commercial aviation. Until recently, all commercial airplane structures were manufactured from the same material—aluminum. Four years ago Boeing introduced the 787, which is the first commercial airplane made primarily from carbon fiber reinforced polymer. This advanced material is substantially lighter than aluminum and is likely to be utilized on every future commercial airplane as its weight efficiency results in improved fuel economy and more streamlined aerodynamics.

Airbus has built and is currently testing their answer to the 787—the A350. This airplane is also made primarily from carbon fiber and is slightly larger than the 787. Boeing is currently developing a still larger airplane to replace their popular 777—dubbed the 777X. This airplane will also utilize carbon fiber and will feature the largest carbon fiber wing in commercial aviation history. The size record continues to be broken in this new material era.

Technology evolution is about us, about the evolutionary design of all the flows and movements that facilitate human flow (life) on the Earth's surface (people, goods, etc.). The evolution of airplanes illustrates this convincingly.

What works is kept. Flow architectures that offer greater access persist, and are joined by even better ones. Together,

the vascular tapestry of old and new carries the global human flow easier and farther than the old alone. Air mass transport with new and old airplane models mixes the global sphere more effectively than in the absence of new models.

Flow architectures are evolving right now, throughout nature and in our technologies, in accord with the constructal law.<sup>7,16</sup> The legacy of all flow systems (animate and inanimate) is this: they have moved mass (they have "mixed" the Earth's crust) more because of design evolution than in the absence of design evolution.

## ACKNOWLEDGMENTS

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- <sup>18</sup>See supplementary material at <http://dx.doi.org/10.1063/1.4886855> for the sources of the data displayed in Figs. 1–6.