Web-based Supporting Materials for “Bayesian Hierarchical Joint Modeling Using Skew-Normal/Independent Distributions” by Geng Chen and Sheng Luo

Table 1: Setting I: simulation results from models RM_N, RM_T and RM_{ST} when there were skewness and 5% outliers in the continuous outcome.

		RM_N				RM_T				RM_{ST}			
	True	Bias	SD	SE	CP	Bias	SD	SE	CP	Bias	SD	SE	CP
a_1	25.000	**4.390**	0.774	0.713	**0.000**	**5.954**	0.618	0.584	**0.000**	0.421	1.055	1.074	0.941
b_1	10.000	**0.345**	0.801	0.657	**0.843**	**0.587**	0.461	0.458	**0.734**	−0.147	0.525	0.509	0.946
σ_1	5.000	**7.447**	1.332	0.376	**0.000**	**−1.227**	0.254	0.267	**0.013**	**−1.369**	0.247	0.257	**0.008**
a_{21}	−2.700	0.099	0.205	0.183	0.858	0.187	0.173	0.162	0.760	−0.045	0.197	0.187	0.925
a_{22}	−0.600	0.018	0.149	0.140	0.925	0.048	0.137	0.129	0.911	−0.029	0.148	0.142	0.936
a_{23}	2.000	−0.060	0.158	0.160	0.923	−0.101	0.153	0.147	0.883	−0.001	0.167	0.163	0.941
a_{24}	2.800	−0.069	0.187	0.183	0.915	−0.133	0.175	0.168	0.875	0.018	0.192	0.187	0.938
a_{25}	5.000	−0.074	0.345	0.316	0.918	−0.185	0.328	0.293	0.846	0.086	0.353	0.321	0.928
a_{26}	6.000	0.076	0.512	0.467	0.948	−0.056	0.487	0.445	0.932	0.256	0.518	0.473	0.922
b_2	2.000	−0.103	0.152	0.145	0.871	−0.211	0.124	0.119	0.594	−0.005	0.148	0.144	0.936
a_{31}	−0.100	0.009	0.074	0.076	0.954	0.015	0.073	0.074	0.940	0.004	0.074	0.075	0.949
a_{32}	1.000	0.012	0.082	0.083	0.954	0.016	0.081	0.082	0.945	0.010	0.083	0.082	0.952
a_{33}	1.800	0.012	0.096	0.103	0.961	0.016	0.096	0.102	0.958	0.009	0.096	0.102	0.971
a_{34}	2.600	0.032	0.142	0.140	0.951	0.036	0.141	0.140	0.953	0.031	0.142	0.140	0.968
a_{35}	3.300	0.069	0.196	0.193	0.938	0.074	0.198	0.193	0.940	0.068	0.197	0.193	0.941
a_{36}	4.000	0.149	0.285	0.278	0.928	0.154	0.291	0.279	0.924	0.149	0.289	0.279	0.928
b_3	0.400	0.004	0.060	0.060	0.954	−0.010	0.056	0.056	0.958	−0.007	0.057	0.056	0.946
β_{10}	0.400	**−1.459**	0.191	0.188	**0.000**	**−1.246**	0.163	0.160	**0.000**	**−1.278**	0.173	0.173	**0.000**
β_{11}	−0.500	**0.464**	0.214	0.207	**0.374**	**0.395**	0.182	0.184	**0.440**	**0.388**	0.194	0.199	**0.504**
ρ	0.400	−0.223	0.147	0.164	0.714	-0.217	0.105	0.109	0.438	−0.212	0.107	0.111	0.477
σ_u	1.300	−0.526	0.124	0.118	0.034	-0.417	0.090	0.090	0.018	−0.334	0.101	0.101	0.147
γ	−1.000	**0.304**	0.158	0.109	**0.273**	**0.303**	0.162	0.109	**0.286**	**0.305**	0.157	0.109	**0.273**
η_1	0.400	—	—	—	—	—	—	—	—	—	—	—	—
η_2	1.000	—	—	—	—	—	—	—	—	—	—	—	—

Note: Large bias and poor CR are highlighted in bold.

JAGS code for fitting model $\mathrm{JM}_{\mathrm{ST}}$

```
model
{
  for (i in 1:obs)  # obs: number of total observations
  {
    w1[i] ~ dgamma(nu,nu)
    Y.conti[i] ~ dnorm(mu.conti[i], taudy[i]) # continuous outcome
    taudy[i]<-w1[i]*tau.conti
    # two ordinal outcomes
    Y.ordi.1[i] ~ dcat(prob.y.1[i, 1:n.1])
    Y.ordi.2[i] ~ dcat(prob.y.2[i, 1:n.2])
  }

  # construct ST distribution on the mean of the continuous outcome
  for (i in 1:obs)
{
  mu.conti[i] <- a.conti + b.conti * theta[i] + delta.sn*(w.sn[subject[i]])
}

    # Construct the probability vector for the ordinal variables
    for (i in 1:obs)
{
for (l in 1:(n.1-1)) { logit(psi.1[i, l]) <- a.ordi.1[l] - b.ordi.1*theta[i] }
prob.y.1[i, 1] <- psi.1[i, 1]
for (l in 2:(n.1-1)) { prob.y.1[i, l] <- psi.1[i, l] - psi.1[i, l-1] }
prob.y.1[i, n.1] <- 1 - psi.1[i,(n.1-1)]

for (l in 1:(n.2-1)) { logit(psi.2[i, l]) <- a.ordi.2[l] - b.ordi.2*theta[i] }
prob.y.2[i, 1] <- psi.2[i, 1]
for (l in 2:(n.2-1)) { prob.y.2[i, l] <- psi.2[i, l] - psi.2[i, l-1] }
prob.y.2[i, n.2] <- 1 - psi.2[i,(n.2-1)]
}


  for (i in 1:N)
    {
    # construct random effects
    u[i, 1:2] ~ dmnorm(zero[], precision[,])
    # construct variable for the skewness parameter
    w.sn[i]~dnorm(0,1)I(0,)
}

    # construct the variance-covariance matrix for random effects
    precision[1:2,1:2]<-inverse(sigma[,])
    sigma[1,1]<-1
    sigma[1,2]<-rho*sig1
    sigma[2,1]<-sigma[1,2]
    sigma[2,2]<-sig1*sig1

  # construct theta, the latent variable of subject i at time j
  for (i in 1:obs)
  {
     theta[i] <- u[subject[i], 1] + (beta[1] + beta[2]*treat[i]
                 + u[subject[i], 2])*t[i]
  }

  # construct survival part
  for (i in 1:N)
  {
```

```
    # use zero-trick to specify the likelihood
    phi[i] <- -lL[i]
    zeros[i] ~ dpois(phi[i])

    # k is the number of time interval for baseline step function
    for (k in 1:3) {
      h0[i,k] <- inprod(g[k],I0[i,k])
      gt[i,k] <- inprod(g[k],dt1[i,k])
    }

## take log of the survival function
    lh[i] <- gam*treat.pts[i] + omega2*u[i, 1] + omega3*u[i,2] + log(sum(h0[i,]))
    lS[i] <- -(exp(gam*treat.pts[i] + omega2*u[i, 1] + omega3*u[i,2])*sum(gt[i,]))
    # event=1 for event; 0 for censored
    lL[i] <- event[i]*lh[i] + lS[i] -log(1.0E+08)
  }

  # prior for g
  for (k in 1:3)
  {
    g[k] ~dunif(0,20)
  }
  # prior for parameters gam, omega2, and omega3
  gam ~ dnorm(0, 0.01)
  omega2 ~ dnorm(0, 0.01)
  omega3 ~ dnorm(0, 0.01)

  # prior for regression coefficients
  for (i in 1:2)
  {
      beta[i] ~ dnorm(0, 0.01)
  }

  # specify prior distributions
  rho ~ dunif(-1, 1)
  sig1 ~  dgamma(0.01, 0.01)

  # prior for continuous variable's parameters
  b.conti ~ dgamma(0.001,0.001)
  a.conti ~ dnorm(0, 0.0005)
  tau.conti ~ dgamma(0.001,0.001)
  sd.conti <- 1/sqrt(tau.conti)
  b.ordi.1 ~ dgamma(0.001,0.001)
  b.ordi.2 ~ dgamma(0.001,0.001)

  a.ordi.1[1] ~ dnorm(0,0.001)
  for (l in 2:(n.1-1)) { a.ordi.1[l] <- a.ordi.1[l-1] + delta.1[l-1] }
  for (i in 1:(n.1-2)) {delta.1[i] ~ dnorm(0,0.01)I(0,) }
  a.ordi.2[1] ~ dnorm(0,0.001)
  for (l in 2:(n.2-1)) { a.ordi.2[l] <- a.ordi.2[l-1] + delta.2[l-1] }
  for (i in 1:(n.2-2)) {delta.2[i] ~ dnorm(0,0.01)I(0,) }

  # prior distribution for df and skewness parameters
  nu~dgamma(0.001,0.001)
  delta.sn~dnorm(0,0.001)I(0,)
}
```