

# Asymmetric Correlations in Financial Markets

by

Sati Mehmet Ozsoy

Department of Economics  
Duke University

Date: \_\_\_\_\_

Approved:

---

Craig Burnside, Co-Supervisor

---

Cosmin Ilut, Co-Supervisor

---

Francesco Bianchi

---

Juan Rubio-Ramirez

Dissertation submitted in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy in the Department of Economics  
in the Graduate School of Duke University  
2013

ABSTRACT

Asymmetric Correlations in Financial Markets

by

Sati Mehmet Ozsoy

Department of Economics  
Duke University

Date: \_\_\_\_\_

Approved:

---

Craig Burnside, Co-Supervisor

---

Cosmin Ilut, Co-Supervisor

---

Francesco Bianchi

---

Juan Rubio-Ramirez

An abstract of a dissertation submitted in partial fulfillment of the requirements for  
the degree of Doctor of Philosophy in the Department of Economics  
in the Graduate School of Duke University  
2013

Copyright © 2013 by Sati Mehmet Ozsoy  
All rights reserved except the rights granted by the  
Creative Commons Attribution-Noncommercial Licence

# Abstract

This dissertation consists of three essays on asymmetric correlations in financial markets. In the first essay, I have two main contributions. First, I show that dividend growth rates have symmetric correlations. Second, I show that asymmetric correlations are different than correlations being counter-cyclical. The correlation asymmetry I study in this dissertation should not be confused with correlations being counter-cyclical, i.e. being higher during recessions than during booms. I show that while counter-cyclical correlations can simply be explained by counter-cyclical aggregate market volatility, the correlation asymmetry with respect to joint upside and downside movements of returns are not just due to the heightened market volatility during those times.

In the second essay I present a model in order to explain the correlation asymmetry observed in the data. This is the first paper to offer an explanation for observed correlation asymmetry. I formalize the explanation using an equilibrium model. The model is useful to understand both the cross-section and time-series of correlation asymmetry. By the means of my model, we can answer questions about why some stocks have higher correlation asymmetry, and why the correlation asymmetry was higher during 1990s? In the model asset prices respond the realization of dividends and news about the future. However, price responses to news are asymmetric and this asymmetry is endogenous. Price responses are endogenously stronger conditional on bad news than conditional on good news. This asymmetry also generates

the observed correlation asymmetry. The price responses are asymmetric due to the ambiguity about the news quality. Information about the quality of the signal is incomplete in the sense that the exact precision of the signal is unknown; it is only known to be in an interval, which makes the representative agent treat news as ambiguous. To model ambiguity aversion, I use Gilboa and Schmeidler (1989)'s max-min expected utility representation. The agent has a set of beliefs about the quality of signals, and the ambiguity-averse agent behaves as if she maximizes expected utility under a worst-case scenario. This incomplete information about the news quality, together with ambiguity-averse agents, generates an asymmetric response to news. Endogenous worst-case scenarios differ depending on the realization of news. When observing “bad” news, the worst-case scenario is that the news is reliable and the prices of trees decrease strongly. On the other hand, when “good news” is observed, under the worst-case scenario the news is evaluated as less reliable, and thus the price increases are mild. Therefore, price responses are stronger conditional on a negative signal and this asymmetry creates a higher correlation conditional on a negative signal than conditional on a positive signal. I also show that the results are robust to the smooth ambiguity aversion representation.

Motivated by the model, I uncover a new empirical regularity that is unknown in the literature. I show that correlation asymmetry is related to idiosyncratic volatility: the higher the idiosyncratic volatility, the higher the correlation asymmetry. This novel empirical finding is also useful to understand the time-series and cross-sectional variation in correlation asymmetry. Stocks with smaller market capitalizations have greater correlation asymmetry compared to stocks with higher market capitalization. However, an explanation for this finding has been lacking. According to the explanation offered in this paper, smaller size stocks have greater correlation asymmetry compared to bigger size stocks because small size stocks tend to have higher idiosyncratic volatilities compared to bigger size stocks. In the time-series, correlation

asymmetry shows quite significant variation as well. The average correlation asymmetry is especially high for the 1990s and decreases significantly at the beginning of the 2000s. This pattern in times-series can also be explained in terms of the time-series behavior of idiosyncratic volatilities. Several papers including Brandt et al. (2010), document higher idiosyncratic volatilities during 1990s while the aggregate volatility stays fairly stable. Basically, the high idiosyncratic volatilities during the 1990s also caused greater correlation asymmetry.

In the third essay, I study the correlation of returns in government bond markets. Similar to the findings in equity markets, I show that there is some evidence for asymmetric correlations in government bond markets. First, I show that the maturity structure matters for correlation asymmetry in bonds markets: Unlike long-maturity bonds, shorter-maturity bonds tend to have asymmetric correlations. Second, I show that the correlation asymmetry observed in European bond markets disappears with the formation of a common currency area. Lastly, I study the correlation between equity and bond returns in different countries. For long-maturity bonds, correlations with the domestic equity returns are asymmetric for half of the countries in the sample, including the U.S. These findings show that results on asymmetric correlations from equity markets can generalize, at least to some extent, to other financial markets.

# Contents

<b>Abstract</b>	<b>iv</b>
<b>List of Tables</b>	<b>ix</b>
<b>List of Figures</b>	<b>xi</b>
<b>Acknowledgements</b>	<b>xii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Literature Review . . . . .	2
1.2 Correlation Asymmetry in Returns and Dividends . . . . .	4
1.2.1 Data and methodology . . . . .	4
1.2.2 Testing for correlation asymmetry . . . . .	6
1.3 Correlation Asymmetry vs. Counter-cyclical Correlations: Two Different Phenomena . . . . .	10
1.4 Conclusion . . . . .	14
1.5 Tables and Figures . . . . .	14
<b>2 Ambiguity and Asymmetric Correlations</b>	<b>21</b>
2.1 Introduction . . . . .	21
2.2 Literature Review . . . . .	25
2.3 The Model . . . . .	29
2.3.1 An Asset Market with Ambiguous News . . . . .	32
2.3.2 News . . . . .	32
2.3.3 Asset Pricing Properties . . . . .	35

2.3.4	Conditional Correlations . . . . .	37
2.4	Testing the model's prediction: New empirical relationship . . . . .	41
2.5	Data . . . . .	41
2.6	Idiosyncratic volatility measurement . . . . .	41
2.7	Does higher idiosyncratic volatility imply greater correlation asymmetry? . . . . .	43
2.8	Conclusion . . . . .	50
2.9	Tables and Figures . . . . .	52
<b>3</b>	<b>Correlation Asymmetry in Government Bond Markets</b>	<b>61</b>
3.1	Introduction . . . . .	61
3.2	Data and Methodology . . . . .	63
3.3	International Bond Return Correlations . . . . .	65
3.3.1	Correlation Asymmetry: The Case of Europe . . . . .	66
3.4	Bond-Equity Return Correlations . . . . .	68
3.5	Conclusion . . . . .	68
3.6	Tables and Figures . . . . .	69
<b>A</b>	<b>Appendix to Chapter 2</b>	<b>90</b>
A.1	Model Solution . . . . .	90
A.1.1	Prices in Stationary Equilibria . . . . .	90
A.1.2	Conditional Correlations . . . . .	92
A.1.3	Proof of Proposition 1 . . . . .	94
A.1.4	Proof of Proposition 2 . . . . .	94
A.2	The Case of Smooth Ambiguity Aversion . . . . .	100
	<b>Bibliography</b>	<b>104</b>
	<b>Biography</b>	<b>108</b>



# List of Tables

1.1	Correlation Asymmetry Tests: Role of Dividends . . . . .	15
1.2	Correlation Asymmetry Test for Dividend Growth Rates . . . . .	16
1.3	Correlation Asymmetry: Effect of Market Volatility . . . . .	17
1.4	Higher Correlations During Downside Movements: Effect of Market Volatility . . . . .	18
1.5	Correlation Asymmetry over the Business Cycle: Effect of Market Volatility . . . . .	19
1.6	Correlation Asymmetry over the Business Cycle: Effect of Market Volatility . . . . .	20
2.1	Determinants of Correlation Asymmetry for Portfolio Returns: Panel Regressions . . . . .	53
2.2	Testing Correlation Asymmetry with Portfolios Formed on Size and Idiosyncratic Volatility . . . . .	54
2.3	Testing Correlation Asymmetry with Portfolios Formed on Size and Idiosyncratic Volatility, <i>NYSE Breakpoints</i> . . . . .	55
2.4	Determinants of Correlation Asymmetry: Idiosyncratic Volatility and Ambiguity . . . . .	56
2.5	Determinants of Correlation Asymmetry: Liquidity . . . . .	57
3.1	Correlation Asymmetry for Bonds with 1 to 3 Years of Maturity . . . . .	70
3.2	Correlation Asymmetry for Bonds with 7 to 10 Years of Maturity . . . . .	71
3.3	Correlation Asymmetry for Bonds with more than 10 Years of Maturity . . . . .	72
3.4	Correlation Asymmetry for Bonds with all Years of Maturity . . . . .	73

3.5	Correlation Asymmetry for Bonds with 1 to 3 Years of Maturity, France	74
3.6	Correlation Asymmetry for Bonds with 7 to 10 Years of Maturity, France	75
3.7	Correlation Asymmetry for Bonds with more 10 Years of Maturity, France . . . . .	76
3.8	Correlation Asymmetry for Bonds with All Years Maturity, France . .	77
3.9	Correlation Asymmetry Europe, pre-Euro, France . . . . .	78
3.10	Correlation Asymmetry Europe, post-Euro, France . . . . .	79
3.11	Correlation Asymmetry Europe, pre-Euro, Germany . . . . .	80
3.12	Correlation Asymmetry Europe, post-Euro, Germany . . . . .	81
3.13	Correlation Asymmetry Europe, pre-Euro, France . . . . .	82
3.14	Correlation Asymmetry Europe, post-Euro, France . . . . .	83
3.15	Correlation Asymmetry Europe, pre-Euro, Germany . . . . .	84
3.16	Correlation Asymmetry Europe, post-Euro, Germany . . . . .	85
3.17	Equity Bond Correlations 1 to 3 Years Maturity . . . . .	86
3.18	Equity Bond Correlations 10 Years and Longer Maturity . . . . .	87

# List of Figures

2.1	Decomposing Correlation . . . . .	58
2.2	Idiosyncratic Volatility Sorted Portfolios . . . . .	59
2.3	Time Variation in Correlation Asymmetry . . . . .	60
3.1	Correlation Asymmetry: Austria, Belgium, Italy . . . . .	88
3.2	Correlation Asymmetry: Portugal, Spain, Sweden . . . . .	89
3.3	Correlation Asymmetry: Canada, Germany, United Kingdom . . . . .	89

# Acknowledgements

I am grateful to my advisors Professors Craig Burnside and Cosmin Ilut for the advice and encouragement. I am also grateful to the members of my committee Professors Francesco Bianchi and Juan Rubio-Ramirez for all their help. I have benefited from discussions with Nir Jaimovich and Andrew Patton. I thank seminar participants of Duke's Macroeconomics Lunch Groups for their helpful comments and suggestions. I owe special thanks to my friends and family for their constant support and encouragement.

# 1

## Introduction

Many empirical studies document that the correlations in the financial markets are higher during joint downside movements than during joint upside movements. For instance, Longin and Solnik (2001) show that the correlation of returns to the U.S. aggregate stock market and the U.K. aggregate stock market is more correlated during joint downside movements – when both of the returns are below their average levels – than during joint upside movements – when both of the returns are above their average levels.<sup>1</sup> This asymmetry has important implications for portfolio allocation, risk diversification and, potentially, asset pricing. For example, correlation asymmetry implies that diversification benefits offered by a group of financial instruments will be extremely limited during market downturns due to the higher correlations between these instruments. Moreover, these limited diversification benefits may be unfavorable for investors so that the latter may require premia to hold assets with correlation asymmetry.

The correlation asymmetry I study in this paper should not be confused with

---

<sup>1</sup> Using monthly data from January 1959 to December 1996, Longin and Solnik (2001) calculate the correlation between the U.S. equity index return and the U.K. equity index return to be 0.53 during the joint downside movements while it is only 0.41 during the joint upside movements.

correlations being counter-cyclical, i.e. being higher during recessions than during booms. While counter-cyclical correlations can simply be explained by counter-cyclical aggregate market volatility, the correlation asymmetry with respect to joint upside and downside movements of returns are not just due to the heightened market volatility during those times. This distinction is crucial and I discuss it more in Section 1.3. In the present paper I offer an explanation for the reason why the correlations of stock returns are higher during joint downside movements than during joint upside movements. To the best of my knowledge, I am the first one to offer an explanation for the relationship between the realized returns and the correlations of returns.

I also show that dividend growth rates – unlike returns – have symmetric correlations. Ribeiro and Veronesi (2002) and Aydemir (2008) also run similar studies and find similar results for correlations of cross country industrial production and GDP growth rates over the business cycle. I differ from those papers in two dimensions. First, I define market upturns and downturns similar to the empirical literature – with respect to level of returns, not with respect to the business cycle. Second, I look at the portfolio of stocks within U.S. stock markets, and study the correlation of dividend growth rates, rather than the growth rates of industrial production or GDP. This finding puts some restrictions on my model. In the light of this result I assume that correlation of dividends is constant.

## 1.1 Literature Review

The papers in this literature mainly try to show the correlation asymmetry in the financial markets and explain the potential roots of the asymmetry. Unlike the model I describe in section 2, most of the explanations in the literature are statistical.<sup>2</sup>

---

<sup>2</sup> Exceptions are Ribeiro and Veronesi (2002), Aydemir (2008), Ehling and Heyerdahl-Larsen (2011), Mueller et al. (2012).

Longin and Solnik (2001) are among the first ones to show the existence of asymmetry after controlling for the bias coming from the conditioning. They show that the return correlation between U.S. stock market and some developed economies' stock markets is higher during the times of stock markets fall.

Ang and Chen (2002) make a similar observation for the U.S. stock markets. Looking at the correlations between U.S. stocks and the aggregate U.S. stock market conditional on downside and upside moves, their test results reject the null hypothesis of multivariate normal distributions at daily, weekly and monthly frequencies. They also show that asymmetric conditional correlations are fundamentally different from other measures of asymmetries, such as skewness and co-skewness.

Similar patterns have been discovered for exchange rate markets. Patton (2006) finds evidence that the mark-dollar and yen-dollar exchange rates are more correlated when they are depreciating against the dollar than when they are appreciating. Observing very similar patterns in different markets can be a signal of common source like investor behavior. Before moving to the discussion about possible sources of asymmetry, let us briefly examine the consequences of conditional correlation asymmetry.

The findings regarding conditional correlation asymmetry are important for several reasons. Considering an extreme scenario, if the stocks prices fall all together for instance, the diversification strategies which are developed ignoring conditional correlation asymmetry and using unconditional correlations will not be optimal. Ang and Chen (2002) show that if correlations increase on the downside relative to a bivariate normal distribution, the potential utility losses are economically significant. Hong et al. (2007) and Buraschi et al. (2010) also find that incorporating the asymmetry in portfolio choice decision bring significant gains.

What can be the source of this asymmetry? Ang and Bekaert (2002) show that a general asymmetric GARCH model cannot reproduce the documented asymmetric

correlations. Ang and Chen (2002) compare GARCH models with Poisson Jump models and several regime-switching models. According to their conclusions, although the regime-switching models are better in generating the asymmetric conditional correlations compared to the GARCH models, their ability to explain the empirical facts is still limited.

My main theoretical contribution is to offer a potential explanation for the source of this widespread asymmetry in the financial markets.

## 1.2 Correlation Asymmetry in Returns and Dividends

In this section I provide an overview of the role of fundamentals for the correlation asymmetry. Throughout the paper I study correlation with the aggregate market, unless otherwise noted. I concentrate on the assets with the highest correlation asymmetry in returns. In addition to replicating one of the most robust findings in the literature I question the role of dividends in these findings. The data and the methodology are explained in the next subsection. The results and discussion follow in Section 1.2.2.

### 1.2.1 *Data and methodology*

I use monthly data for the publicly traded US stocks. I obtain data on stock returns, stock prices, shares outstanding, and exchange listings for the universe of stocks available from the Center for Research on Security Prices (CRSP). I also obtain monthly risk-free rates from the data library of Kenneth French.<sup>3</sup> The data spans the period between July 1965 and December 2011.<sup>4</sup>

Similarly to most of the studies in this literature I concentrate on portfolios of

---

<sup>3</sup> The data library is available at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

<sup>4</sup> Sample period extends the data used in Hong et al. (2007). The results are not sensitive to sample chosen.



stocks rather than on individual stocks. This is due to the following reasons. First of all, forming portfolios reduces the residual variance inherent in the individual stock returns. Second, for portfolios – unlike individual stocks – we can back out relatively smooth cash flow data. This is a priori consistent with the model I propose, which focuses explicitly on infinitely lived assets. Thus, I choose to perform the empirical exercises on portfolios of stocks that are by definition generating cash flows for long periods of time.

In principle there are countless ways of forming portfolios of stocks. I concentrate on portfolio formations known to generate very high correlation asymmetry. Thus, I follow Ang and Chen (2002) and Hong et al. (2007), among others, and form portfolios according to the market capitalization (size). To the best of my knowledge, neither these papers nor any others in the literature have any theoretical ground to form portfolios in a specific way. The model I propose in Section 2 provides a specific way to sort portfolios, which is not studied in the literature. Moreover, the model helps to understand the underlying reason as to why sorting portfolios by market capitalization generates high correlation asymmetry.

To test for correlation asymmetry I apply the standard methods in the literature. I use the exceedance (threshold) correlations to facilitate comparison with other papers.<sup>5</sup> In the simplest version of this approach, two separate correlations are calculated for two subsamples, and these two correlation estimates are tested for statistically significant difference. The name exceedance (or threshold) refers to the criteria to choose subsamples: Observations across subsamples are sorted so that levels of the returns are above or below some threshold level. More specifically,  $\rho^-$  represents the correlation during downturns, when both of the excess returns are

---

<sup>5</sup> Ang and Chen (2002), Hong et al. (2007) and Longin and Solnik (2001), among others, use exceedance correlations although their statistical tests which compare the exceedance correlations differ. Andersen et al. (2001) provide a regression based way to test for a correlation asymmetry, which I utilize in Section 2.7.

below their means  $\rho^- = \rho(r_i, r_m | r_i < 0, r_m < 0)$  and  $\rho^+$  represent the correlation during market upturns  $\rho^+ = \rho(r_i, r_m | r_i > 0, r_m > 0)$ .  $r_i$  and  $r_m$  are the excess returns to portfolio  $i$  and to the aggregate market respectively, and they are both standardized, as is common in the literature. The excess return is derived by subtracting the one-month Treasury bill rate from the monthly return.

Given the two correlation estimates, we want to test to see whether the estimates are statistically different. The null hypothesis of symmetric correlation is

$$H_0 : \rho^+ = \rho^-$$

That is, if we fail to reject the null hypothesis, it means that the correlation estimates are equal across times of joint upward moves and joint downward moves. The alternative hypothesis is

$$H_A : \rho^+ \neq \rho^-$$

Hong et al. (2007) develops the asymptotic distribution of the test statistics under the null hypothesis of symmetry. The test is similar to the Wald test (Hansen (1982)) in generalized method of moments (GMM) framework but utilizes conditional moment conditions rather than unconditional ones.<sup>6</sup> In the next subsection I test for a correlation asymmetry between the size sorted portfolios using the test statistic developed by Hong et al. (2007).

### 1.2.2 *Testing for correlation asymmetry*

I start by testing the correlations between the returns of size sorted portfolios and the aggregate market return. I follow Hong et al. (2007) and form portfolios according to the market capitalization of individual stocks. The smallest size portfolio

---

<sup>6</sup> The test of Hong et al. (2007) is more general and allows other threshold levels than zero. However, there is no theoretical guidance about how to choose the threshold levels, so I only consider zero as threshold level, which is also studied, by Ang and Chen (2002) and Hong et al. (2007).

consists of stocks with market value in the lowest decile and the largest size portfolio consists of stocks with market valuation in the highest decile. Once the constituents of each portfolio are determined, I take a value weighted average of the returns of the constituent stocks in order to calculate the returns of the portfolios. Then the exceedance correlations between the returns of the size-sorted portfolios and the market return are calculated. The left panel of Table 1.1 presents the results. These results replicates the results of Hong et al. (2007) in an extended sample. Furthermore, they are in line with the findings of Ang and Chen (2002). Correlations are higher conditional on joint downward moves than conditional on joint upward moves. The second column has the p-values for the correlation asymmetry test. The p-values that are less than 5 percent suggest that the correlations are asymmetric. For most of the portfolios the difference in correlations is statistically significant and the correlation asymmetry is monotonically decreasing in portfolio size. Interestingly, the correlation asymmetry for smaller size portfolios is substantial. For the smallest size portfolio, the correlation with the aggregate market during market downturns is almost four times as big as the correlation with the aggregate market during market upturns.

To understand the role of dividends in correlation asymmetry, I perform the same statistical analysis with one essential difference: Rather than using the total returns (the sum of capital gains and dividend yield), only the capital gains are used. The purpose of doing that is to understand whether the realizations of dividend payments play any role in the correlation asymmetry. The right panel of Table 1.1 presents the results. As can be seen from the table, the conditional correlation estimates do not differ much from the original estimates in the left panel. Correlation estimates in the two panels are difficult to distinguish. And again the same 6 out of 10 portfolios have p-values smaller than 5 percent. This gives support to the idea that the correlation asymmetry is not caused by the correlation asymmetry in fundamentals only.

As a more direct check, I test for the correlation asymmetry in the dividend growth rates for the same portfolios. Here I study the correlation between dividend growth rates of the different portfolios with the aggregate dividend growth rate. Before commenting on the results, I will discuss the procedure to derive the dividend growth rates. I follow the methodology used in Bansal et al. (2005) and Hansen et al. (2008). The two return series available in the data set are denoted by  $r_{t+1}$  and  $rx_{t+1}$  respectively, where the former includes cash flows and the latter excludes them:

$$r_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t} \quad (1.1) \quad rx_{t+1} = \frac{p_{t+1}}{p_t} \quad (1.2)$$

Using equations 1.1 and 1.2, we can back out the cash flow series. The exact procedure is as follows:

$$\frac{d_{t+1}}{p_t} = r_{t+1} - rx_{t+1} \quad (1.3)$$

$$\frac{p_t}{p_0} = \prod_{j=1}^t rx_j \quad (1.4)$$

$$\frac{d_{t+1}}{d_0} = \frac{d_{t+1}}{p_t} \frac{p_t}{p_0} \quad (1.5)$$

Starting from  $p_0$ , we can get the whole price series using the capital gains  $rx_{t+1} = \frac{p_{t+1}}{p_t}$ . Once we have the price series, using the returns including cash flows, we can back out the dividends. Hence we have got the dividends and the price series up to an arbitrary scale factor. This scale factor  $p_0$  is not essential for our purposes since the object of interest for our analysis is the correlation, which is scale free.<sup>7</sup>

I employ the strategy discussed above to derive the dividends for the aggregate market and for the size-sorted portfolios. Having the data on dividends, for each

<sup>7</sup> Bansal et al. (2005) normalize  $p_0$  to 1. Hansen et al. (2008) choose  $p_0$  such that for each portfolio quarterly dividends in 1947Q1 is same as personal consumption of nondurable and services. I could follow a similar procedure as they did, however it does not affect the results.

portfolio I calculate the correlation with the aggregate dividend growth rate, conditional on market upturns and downturns. Importantly, I define market upturns and downturns with respect to the level of excess returns in order to identify the same subsamples as used before. Thus, conditional correlations of dividends are defined as follows:  $\rho^- = \rho(\Delta d_i, \Delta d_m | r_i < 0, r_m < 0)$  and  $\rho^+ = \rho(\Delta d_i, \Delta d_m | r_i > 0, r_m > 0)$ , where  $\Delta d_i$  is the dividend growth rate for portfolio  $i$  and  $\Delta d_m$  is the dividend growth rate for the aggregate market.

The results are collected in Table 1.2. From the last two columns of the table we can see that the dividend correlations, unlike return correlations, are very similar across downturns and upturns. As a result, p-values are very high except for two cases. In one of these two cases the correlation is higher during market upturns, which is the opposite case to the correlation asymmetry for returns. Thus, both of the analyses discussed in this subsection suggest that there is no correlation asymmetry for dividend growth rates. In the light of this result, I model the correlation of dividends as constant.

Comparing Tables 1.1 and 1.2, we can see that the correlation levels of dividends are much lower than the ones of stock returns. More interestingly, this discrepancy seems to be higher during market downturns. Thus, dividends seem to have a lesser role in the comovements of returns during market downturns than during market upturns. In other words, market downturns seem to be responsible for a relatively larger portion of excess comovements in returns.<sup>8</sup>

---

<sup>8</sup> Pindyck and Rotemberg (1993) and Shiller (1989) show that the comovements of returns are too high to be explained by the comovements of dividends. In a much simpler way I make a similar observation. However, my observation is conditional on market upturns and downturns while theirs are unconditional.

### 1.3 Correlation Asymmetry vs. Counter-cyclical Correlations: Two Different Phenomena

Unfortunately, there has been a bit of a confusion surrounding the concept of asymmetric correlations. The aim of this subsection is to eliminate this confusion. Looking at the extensive literature on correlation asymmetry, one would see that the term *asymmetric correlations* refers to two different types of time variation in correlations. The first one, which is the subject matter of this paper, relates the asymmetric correlations to the realized returns: when the realized returns are relatively low, correlations are relatively high. The second type, however, pertains to the correlations over the business cycles: correlations during recessions are higher relative to correlations during booms. Erb et al. (1994), for example, refer to both types of time-variation in correlations when they study the pairwise correlations of international equity returns. They segment the data according to ex-post returns with respect to joint downside movements – when both of the returns are below their average levels (considered to capture bear markets) and joint upside movements – when both of the returns are above their average levels (capturing bull market). With this in mind, they show that the international equity correlations are higher during joint downside movements (bear market) compared to joint upside movements (bull market). They also study the correlations of international equity returns over the business cycles and show that correlations are counter-cyclical, meaning that the latter are higher during recessions than during booms.

In this paper I study the correlation asymmetry of stock returns with respect to joint upside and downside movements of the latter with the aggregate stock market. In this subsection I compare the correlation asymmetry in that sense with the counter-cyclical correlations. I stress that these two phenomena are different in nature and should not be confused. I show that, unlike the asymmetric correlations,

the counter-cyclical correlations are driven by the counter-cyclical market volatility. This result is implied by some other findings in the literature, which I discuss below. However, to the best of my knowledge, it has not been shown explicitly. The aim of this subsection is to clarify the distinction between the asymmetric correlations and the counter-cyclical correlations without leaving any room for confusion.

As in the previous subsection, I work with the size-sorted portfolios and study the correlation of their excess returns with the aggregate market excess return. However, here I limit the number of portfolios to five rather than ten for expositional purposes. Unlike the previous subsection, I follow a regression based analysis which allows explicit control for time variation in the aggregate market volatility.

As it is known in the literature, high correlations can be a byproduct of high volatility.<sup>9</sup> Even if the unconditional correlations are constant, conditioning on high volatility time periods can create spuriously high correlations. For instance, a simple model of asset returns, such as the bivariate normal distribution with a constant correlation, would generate relatively high correlations for periods of high volatility. Boyer et al. (1997), among others, derive this result in a closed form for the case of the bivariate normal distribution. Therefore, one needs to be careful while comparing the correlations estimates from different subsamples of data if those are generated according to the ex post realizations of a series. In our case, we will have higher correlations during periods of high volatility than periods of low volatility by construction. Namely, splitting the sample into subsamples induces a conditioning bias in the correlation estimates.

To illustrate the effects of the time-varying volatility on correlations, I apply a statistical analysis similar to Andersen et al. (2001). I run panel regressions of the following form:

---

<sup>9</sup> For a detailed discussion, please see Boyer et al. (1997), Stambaugh (1995), Corsetti et al. (2005), Ronn et al. (2009), and Forbes and Rigobon (2002).

$$corr_{i,t} = \delta_0 + \delta_1 \mathbb{I}(R_{i,t} * R_{m,t} > 0) + \delta_2 \mathbb{I}(R_{i,t} < 0, R_{m,t} < 0) + \beta_0 \sigma_{m,t} + \beta_1 corr_{i,t-1} \quad (1.6)$$

where  $corr_{i,t}$  is the correlation between the excess returns of portfolio  $i$  and of the aggregate market in month  $t$ .  $R_{i,t}$  and  $R_{m,t}$  are the monthly excess returns of portfolio  $i$  and of the aggregate market, respectively. As is common in the literature both excess returns are standardized.<sup>10</sup>  $\mathbb{I}(\cdot)$  is the indicator function which takes the value of one when the condition in parentheses is satisfied and zero otherwise.  $\mathbb{I}(R_{i,t} * R_{m,t} > 0)$  captures the effect of joint upside and downside movements in returns while  $\mathbb{I}(R_{i,t} < 0, R_{m,t} < 0)$  controls only for the joint downside movements. Therefore, the impact of upside movements on correlations is  $\delta_1$  and the impact of downside movements is  $\delta_1 + \delta_2$ . Thus,  $\delta_2$  captures the additional effect of the downside movements and a statistically significant positive  $\delta_2$  implies that correlations are asymmetric, being higher during joint downside movements.

In Table 1.3, the first column reports the result of the panel regression, excluding the market volatility as a regressand. The coefficient  $\delta_2$  of the dummy variable  $\mathbb{I}(R_{i,t} < 0, R_{m,t} < 0)$  controlling for the downside movements is positive and statistically significant. In the second column we have the results of the previous regression with the addition of the market volatility as a regressand. The coefficient we are interested in, namely  $\delta_2$ , is still positive and statistically significant. First, this result confirms the findings of the previous subsection and of the empirical literature: correlations are higher conditional on joint downside movements. More importantly, this is true even after controlling for the effect of the aggregate market volatility. In other words, the higher correlations observed during downside movements are not just due to the heightened market volatility during those times. Below I show that the same statement does not hold for the counter-cyclical correlations.

---

<sup>10</sup> See Ang and Chen (2002), Hong et al. (2007).



Next I run the same analysis for the counter-cyclical correlations over the business cycle. The panel regression is specified as follows

$$corr_{i,t} = \delta_0 + \gamma_1 Recession_t + \beta_0 \sigma_{m,t} + \beta_1 corr_{i,t-1} \quad (1.7)$$

The results are shown in the third and the fourth columns of Table 1.3.  $Recession_t$  is a dummy variable which takes the value of one for the months during which the economy is in recession according to the NBER. The coefficient of the Recession dummy is positive and statistically significant. That is consistent with correlations being counter-cyclical. However, when the effect of the market volatility is incorporated, the coefficient of the Recession dummy changes sign and loses its statistical significance.

The panel regression can be restrictive given the heterogeneity in the level of correlation asymmetry across size-sorted portfolios. To circumvent this potential problem, I run two time-series regressions (the ones given by equations 1.6 and 1.7) for each size-sorted portfolio separately. The results are reported in Tables 1.4 and 1.5. The same conclusion holds: unlike the correlation asymmetry with respect to the joint upside and downside return movements, the counter-cyclical correlations are driven by the heightened aggregate market volatility. Table 1.4 reports the results for correlation asymmetry with respect to the joint upside and downside return movements. The coefficient of  $\mathbb{I}(R_{i,t} < 0, R_{m,t} < 0)$  is positive and statistically significant, even after controlling for the effect of the aggregate market volatility. However, as we can see in Table 1.5, the coefficient of the Recession dummy loses its statistical significance once the aggregate market volatility is introduced into the regressions.

For robustness purposes, I run one more set of regressions. The NBER's Business Cycle Dating Committee announces recession dates for quarters. Therefore the

dummy variable used in the above regressions is constant, being either zero or one during each quarter. That may be a problem for identifying the effects of Recessions on return correlations. To address this issue, I replace the recession dummy with a continuous variable: the growth rate of the real industrial production in the U.S. The results of this set of time-series regressions are reported in Table 1.6. The main conclusion does not change. When the industrial production growth is negative, i.e. during recessionary periods, correlations are higher. However, this is purely driven by the higher aggregate market volatility.

## 1.4 Conclusion

In this chapter I have two main contributions. First, I show that dividend growth rates – unlike returns – have symmetric correlations. This is important because the observed asymmetry in correlations of returns could simply follow from the asymmetric correlations of dividends. Therefore by showing that the correlations of dividends are symmetric we eliminate one potential explanation.

Second, I show that asymmetric correlations are different than correlations being counter-cyclical. The correlation asymmetry I study in this paper should not be confused with correlations being counter-cyclical, i.e. being higher during recessions than during booms. I show that while counter-cyclical correlations can simply be explained by counter-cyclical aggregate market volatility, the correlation asymmetry with respect to joint upside and downside movements of returns are not just due to the heightened market volatility during those times.

## 1.5 Tables and Figures

Table 1.1: Correlation Asymmetry Tests: Role of Dividends

Portfolio	Returns including Dividends			Returns excluding Dividends		
	P-values	$\rho^-$	$\rho^+$	P-values	$\rho^-$	$\rho^+$
Smallest ME	0.00	0.77	0.20	0.00	0.77	0.23
Size 2	0.01	0.80	0.30	0.05	0.80	0.34
Size 3	0.09	0.82	0.38	0.22	0.83	0.42
Size 4	0.53	0.82	0.46	1.01	0.82	0.49
Size 5	1.05	0.85	0.50	1.82	0.85	0.54
Size 6	2.87	0.87	0.59	3.96	0.87	0.61
Size 7	9.23	0.90	0.69	11.87	0.90	0.70
Size 8	30.98	0.90	0.77	33.76	0.90	0.78
Size 9	61.28	0.93	0.86	61.76	0.93	0.87
Biggest ME	94.34	0.99	0.98	94.84	0.99	0.98

The table collects the result of the correlation asymmetry tests between the market excess return and the excess return on one of the size sorted portfolios. Monthly data spans the period from Jan, 1965 to December, 2011 (564 observations). In columns 2 through 4 the return definition includes the dividends, therefore replicates the results of Hong et al. (2007) in an extended sample. In columns 5 through 7, the return definition excludes the dividends, therefore equals to capital gains. Columns 2 and 5 report the P-values, in percentage points, of the correlation asymmetry test for 2 different return definitions. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_m | r_i < 0, r_m < 0)$  and  $\rho^+ = \rho(r_i, r_m | r_i > 0, r_m > 0)$ .  $r_i$  and  $r_m$  are the excess return to portfolio  $i$  and to the aggregate market, respectively, and they are both standardized, as it is common in literature. The excess return is derived by subtracting by the one-month Treasury bill rate.

Table 1.2: Correlation Asymmetry Test for Dividend Growth Rates

Portfolio	P-values	$\rho^- - \rho^+$	$\rho^-$	$\rho^+$
Smallest ME	0.00	-0.09	-0.03	0.05
Size 2	4.81	0.05	0.18	0.13
Size 3	50.07	-0.04	0.37	0.41
Size 4	81.78	-0.02	0.49	0.51
Size 5	80.54	0.02	0.63	0.61
Size 6	37.90	-0.08	0.67	0.75
Size 7	45.46	-0.09	0.71	0.80
Size 8	73.14	-0.03	0.78	0.80
Size 9	70.58	-0.03	0.86	0.88
Biggest ME	99.35	0.00	0.99	0.99

The table collects the result of the correlation asymmetry tests between the dividend growth rate of aggregate market and that of one of the size sorted portfolios. Monthly data spans the period from Jan, 1965 to December, 2011 (564 observations). The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ , and the conditioning to calculate correlation is subsamples is consistent with Table 1.1,  $\rho^- = \rho(\Delta d_i, \Delta d_m | r_i < 0, r_m < 0)$  and  $\rho^+ = \rho(\Delta d_i, \Delta d_m | r_i > 0, r_m > 0)$ . The conditioning variables,  $r_i$  and  $r_m$  are the excess return to portfolio  $i$  and to the aggregate market, respectively, and they are both standardized, as it is common in literature. The excess return is derived by subtracting by the one-month Treasury bill rate. The P-values are reported in column 2, again in percentage points.

Table 1.3: Correlation Asymmetry: Effect of Market Volatility

Dependent variable: $\text{corr}_{i,t}$	I	II	III	IV
$I(r_{i,t} * r_{m,t} > 0)$	-0.00 (-0.58)	-0.00 (-0.26)		
$I(r_{i,t} < 0 \& r_{m,t} < 0)$	0.03 (2.97)	0.02 (2.91)		
$\text{Log}(\sigma_{m,t})$		0.02 (3.10)		0.02 (3.22)
$\text{corr}_{i,t-1}$	0.62 (8.57)	0.60 (7.39)	0.62 (8.30)	0.61 (7.33)
Recession			0.01 (1.87)	-0.01 (-2.64)
Intercept	0.34 (5.49)	0.51 (4.63)	0.34 (5.46)	0.54 (4.66)
Observations	2,905	2,905	2,905	2,905
R-squared	0.42	0.44	0.40	0.43

The table reports estimates from panel regressions, including coefficient estimates and t-statistics (in parenthesis). The dependent variable is the monthly correlation with the aggregate market excess return, for five portfolios sorted according to the market capitalization. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for five portfolios in cross section. The regressors are as follows: First and second independent variables are dummy variables, the second one identifying the market downturns. If the condition in the parenthesis is satisfied the dummy variable takes the value of one and otherwise zero. Other regressors are logarithm of aggregate market volatility, a dummy variable which takes the value of one for the months within the NBER determined recession periods, and the lagged correlation ( the first lag of the regressand). Estimates from four different specifications are reported, where the standard errors are clustered for time and cross sectional dependence, with the method proposed by Petersen(2009). Return variables are in excess of risk free rate which is approximated by the one-month Treasury bill rate. As is in the literature, the return variables have been standardized so that each variable has a mean zero and a standard deviation of one.

Table 1.4: Higher Correlations During Downside Movements: Effect of Market Volatility

Dep var: $\text{corr}_{i,t}$	I	II	III	IV	V	I	II	III	IV	V
$I(r_{i,t} * r_{m,t} > 0)$	-0.02 (-1.49)	-0.01 (-1.17)	-0.01 (-1.70)	-0.01 (-1.79)	0.00 (0.81)	-0.02 (-1.31)	-0.01 (-0.68)	-0.01 (-1.51)	-0.01 (-1.35)	0.00 (0.87)
$I(r_{i,t} < 0 \& r_{m,t} < 0)$	0.05 (5.58)	0.04 (6.74)	0.03 (6.00)	0.02 (6.47)	0.00 (1.59)	0.04 (4.34)	0.03 (5.32)	0.02 (4.32)	0.01 (4.19)	0.00 (0.22)
$\text{Log}(\sigma_{m,t})$						0.03 (6.13)	0.02 (7.56)	0.02 (8.24)	0.02 (7.01)	0.00 (5.69)
$\text{corr}_{i,t-1}$	0.41 (9.16)	0.40 (9.28)	0.39 (8.13)	0.37 (5.02)	0.46 (7.64)	0.38 (8.48)	0.35 (8.05)	0.30 (6.48)	0.27 (4.04)	0.42 (7.14)
Intercept	0.49 (12.45)	0.52 (13.16)	0.56 (12.51)	0.61 (8.56)	0.53 (9.01)	0.78 (12.37)	0.81 (14.63)	0.86 (15.57)	0.85 (11.38)	0.60 (10.38)
Observations	581	581	581	581	581	581	581	581	581	581
R-squared	0.21	0.22	0.20	0.18	0.22	0.26	0.30	0.32	0.30	0.28

The table reports estimates from time-series regressions, including coefficient estimates and t-statistics (in parenthesis). The dependent variable is the monthly correlation with the aggregate market excess return, for five portfolios sorted according to the market capitalization. Column numbers from I to V indicates the corresponding size portfolio, I being the smallest and V being the largest. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for five portfolios in cross section. The regressors are as follows: First and second independent variables are dummy variables, the second one identifying the market downturns. If the condition in the parenthesis is satisfied the dummy variable takes the value of one and otherwise zero. Other regressors are logarithm of aggregate market volatility, a dummy variable which takes the value of one for the months within the NBER determined recession periods, and the lagged correlation ( the first lag of the regressand). Estimates from four different specifications are reported, where the standard errors are clustered for time and cross sectional dependence, with the method proposed by Petersen(2009). Return variables are in excess of risk free rate which is approximated by the one-month Treasury bill rate. As is in the literature, the return variables have been standardized so that each variable has a mean zero and a standard deviation of one.

Table 1.5: Correlation Asymmetry over the Business Cycle: Effect of Market Volatility

Dep var: $\text{corr}_{i,t}$	I	II	III	IV	V	I	II	III	IV	V
Recession	0.03 (2.61)	0.02 (3.77)	0.02 (4.29)	0.01 (4.95)	0.00 (3.51)	-0.01 (-0.73)	-0.01 (-0.95)	-0.01 (-1.55)	-0.00 (-1.50)	0.00 (0.20)
$\text{Log}(\sigma_{m,t})$						0.03 (6.53)	0.03 (7.95)	0.02 (8.45)	0.02 (7.41)	0.00 (5.27)
$\text{corr}_{i,t-1}$	0.40 (9.01)	0.39 (8.70)	0.37 (7.30)	0.34 (4.48)	0.45 (7.51)	0.38 (8.44)	0.35 (7.91)	0.30 (6.20)	0.26 (3.95)	0.42 (7.17)
Intercept	0.49 (12.76)	0.53 (13.23)	0.57 (12.14)	0.62 (8.46)	0.55 (9.31)	0.83 (12.80)	0.85 (15.03)	0.89 (15.43)	0.88 (11.88)	0.60 (10.40)
Observations	581	581	581	581	581	581	581	581	581	581
R-squared	0.18	0.17	0.16	0.14	0.22	0.24	0.27	0.30	0.28	0.28

The table reports estimates from time-series regressions, including coefficient estimates and t-statistics (in paranthess). The dependent variable is the monthly correlation with the aggregate market excess return, for five portfolios sorted according to the market capitalization. Column numbers from I to V indicates the corresponding size portfolio, I being the smallest and V being the largest. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for five portfolios in cross section. The regressors are as follows: the logarithm of aggregate market volatility, a dummy variable which takes the value of one for the months within the NBER determined recession periods, and the lagged correlation ( the first lag of the regressand). Return variables are in excess of risk free rate which is approximated by the one-month Treasury bill rate. As is in the literature, the return variables have been standardized so that each variable has a mean zero and a standard deviation of one.

Table 1.6: Correlation Asymmetry over the Business Cycle: Effect of Market Volatility

Dep var: $\text{corr}_{i,t}$	I	II	III	IV	V	I	II	III	IV	V
Ind. Prod.	-0.01 (-1.24)	-0.01 (-1.55)	-0.00 (-1.65)	-0.00 (-1.78)	-0.00 (-1.95)	0.00 (0.92)	0.00 (1.06)	0.00 (1.10)	0.00 (1.54)	0.00 (0.18)
$\text{Log}(\sigma_{m,t})$						0.03 (6.87)	0.03 (8.17)	0.02 (8.53)	0.02 (7.96)	0.00 (5.52)
$\text{corr}_{i,t-1}$	0.41 (9.21)	0.40 (8.94)	0.38 (7.51)	0.35 (4.56)	0.46 (7.69)	0.38 (8.45)	0.35 (7.93)	0.30 (6.23)	0.26 (3.94)	0.42 (7.24)
Intercept	0.49 (12.63)	0.53 (13.05)	0.57 (12.00)	0.62 (8.28)	0.54 (9.13)	0.83 (13.11)	0.85 (15.13)	0.89 (15.48)	0.87 (11.94)	0.60 (10.43)
Observations	581	581	581	581	581	581	581	581	581	581
R-squared	0.17	0.17	0.15	0.13	0.22	0.24	0.27	0.30	0.28	0.28

The table reports estimates from time-series regressions, including coefficient estimates and t-statistics (in paranthesss). The dependent variable is the monthly correlation with the aggregate market excess return, for five portfolios sorted according to the market capitalization. Column numbers from I to V indicates the corresponding size portfolio, I being the smallest and V being the largest. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for five portfolios in cross section. The regressors are as follows: the logarithm of aggregate market volatility, change in real industrial production, and the lagged correlation ( the first lag of the regressand). Return variables are in excess of risk free rate which is approximated by the one-month Treasury bill rate. As is in the literature, the return variables have been standardized so that each variable has a mean zero and a standard deviation of one.



## Ambiguity and Asymmetric Correlations

### 2.1 Introduction

Despite the extensive empirical literature studying asymmetric correlations, there is little theoretical research investigating the underlying reasons for this phenomenon.<sup>1</sup> In this paper, I propose an equilibrium model that explains this phenomenon and has interesting implications for empirical studies of correlation asymmetry.

The present paper has several contributions to the literature on asymmetric correlations in financial markets. First and foremost, I introduce a Lucas (1978) tree model with two trees in order to explain the correlation asymmetry observed in the data. The trees pay stochastic dividends each of which have two components—an aggregate component that is common to both dividend streams and an idiosyncratic component that is specific to each tree. Agents receive news in the form of a noisy signal about future dividends. Although this noisy signal is informative about next period's aggregate innovation to dividends, the informativeness of the signal diminishes with its noise. Therefore, the representative agent solves a signal extraction

---

<sup>1</sup> Notable papers are Ribeiro and Veronesi (2002), Aydemir (2008), Ehling and Heyerdahl-Larsen (2011), Mueller et al. (2012).

problem, and the strength of her response to the signal depends on the signal's quality, measured by the signal-to-noise ratio. For instance, a very noisy signal would be considered a low quality one, and vice versa.

Furthermore, I assume that it is difficult to determine the reliability of news. I model this by assuming incomplete information about the quality of the signal. Information about the quality of the signal is incomplete in the sense that the exact precision of the signal is unknown; it is only known to be in an interval, which makes the representative agent treat news as ambiguous. The agent has a set of beliefs about the quality of signals, and the ambiguity-averse agent behaves as if she maximizes expected utility under a worst-case scenario. This incomplete information about the news quality, together with ambiguity-averse agents, generates an asymmetric response to news. Endogenous worst-case scenarios differ depending on the realization of news. When observing "bad" news, the worst-case scenario is that the news is reliable and the prices of trees decrease strongly. On the other hand, when "good news" is observed, under the worst-case scenario the news is evaluated as less reliable, and thus the price increases are mild. Therefore, price responses are stronger conditional on a negative signal and this asymmetry creates a higher correlation conditional on a negative signal than conditional on a positive signal.

The main distinction of this paper from the few theoretical models studying correlation asymmetries is how I relate the time-varying correlations to the prices and returns. In these models the relation between time-varying correlations and realized returns is missing. Usually, these models incorporate a state variable that identifies the evolution of business cycles and the correlations are higher during recessions or when the state variable is low. In these models, it is not necessarily true that the returns are below their means or prices are decreasing during recessionary periods. In other words, this set of models generates higher correlations for recessionary periods, but not necessarily below the mean excess returns for stocks. Thus the link between

correlations and the level of returns is missing.

Second, I discuss the implications of correlation asymmetry. In the model, agents have no desire for risk diversification due to a risk-neutrality assumption. Therefore, there is neither a premium for correlation asymmetry, nor any gain to investors from accounting for correlation asymmetry in portfolio allocation decisions. However, the correlation asymmetry may look as if it is priced due to the ambiguity premium. Similar to Epstein and Schneider (2008), asymmetric response of prices to news causes investors to demand a premium which is called an ambiguity premium.

Greater ambiguity requires a higher ambiguity premium, even in a case with only one stock or tree in which there are no correlations to consider, let alone the correlation asymmetry. Therefore, the ambiguity premium is not a premium for correlation asymmetry. However, greater ambiguity also leads to a more pronounced correlation asymmetry. Thus, higher correlation asymmetry is simultaneously observed with a higher premium on returns, due to a third common factor: ambiguity. That is why the observation of higher correlation asymmetry together with higher returns does not imply the compensation for correlation asymmetry. An econometrician trying to estimate the premium for correlation asymmetry will also take the ambiguity premium as a part of the premium for correlation asymmetry. This can be potentially important for the interpretation of empirical studies of premium for correlation asymmetry.<sup>2</sup>

Third, I uncover a new empirical regularity that is, to the best of my knowledge, unknown in the literature. Motivated by the model, I show that correlation asymmetry is related to idiosyncratic volatility: the higher the idiosyncratic volatility, the higher the correlation asymmetry. To see this, consider the two sources of correlation in the model: Correlation between the dividends, and correlation due to the common signal. The correlation due to the common signal is equal to 1, which

---

<sup>2</sup> Ang et al. (2006b).

is the absolute maximum for correlation. Dividends, on the other hand, are imperfectly correlated. The correlation of dividends is positive but less than 1. Thus, the overall correlation is a weighted average of the correlations due to the signal and due to the dividends. The weight of the signal in overall correlation increases as we move from market upturns to downturns. During market downturns, stronger price responses to signals translate into signals being responsible for a higher share of the movements (volatility) in returns. Therefore, during downturns the signals drive more of the total volatility in returns; hence, the weight of the signals in the overall correlation increases in downturns. As a result, correlations are higher during downturns because of the higher share of signals in the overall correlation.

The mechanism described above also explains why higher idiosyncratic volatility is associated with higher correlation asymmetry. On the one hand, stocks with higher idiosyncratic volatilities have a lower correlation due to dividends. Therefore their overall correlation will be a weighted average of the lower correlation due to dividends and correlation due to the signals. On the other hand, stocks with low idiosyncratic volatility have a strong anchor due to dividend correlation and, as the weight of the signal changes, the change in correlation will be less for stocks with low idiosyncratic volatility.

The model provides a new vantage point to explore asymmetric correlations in the data. To test this prediction, each month I sort U.S. stocks into 100 portfolios according to their idiosyncratic volatilities and calculate the correlation asymmetry for each portfolio. There is a clear relationship, observed in Figure 2.2, between the idiosyncratic volatilities and the correlation asymmetry. I also run a panel regression to show that the relationship is statistically significant. Further details are described in Section 3.

## 2.2 Literature Review

In this subsection, I discuss the explanations in the literature for correlation asymmetry over the business cycle: correlations are higher during recession than during booms. It is important to note that the empirical evidence goes beyond correlation asymmetry over the business cycle. As shown by, Longin and Solnik (2001) , Ang and Chen (2002) and Hong et al. (2007), among others, there is a strong relationship between the correlation of returns and the realized returns. Even within recessionary and expansionary periods correlations vary significantly and this demands an explanation that accounts for more than just the effects of the business cycles alone. The nature of the correlation asymmetry over the business cycle and correlation asymmetry with respect to joint upside and downside market movements is different and I discuss this in Section 1.3. More precisely, correlation asymmetry over the business cycle could be explained by changes in the variance of returns while the correlation asymmetry with respect to downside and upside market movements cannot be.

Before discussing the papers one by one, I highlight one common feature of these papers. All of the explanations offered work through the business cycle and explain the higher correlations during recessions. In other words, the link between time-varying correlations and the realized returns is missing. Establishing this link is the main theoretical contribution of this paper.

Ribeiro and Veronesi (2002) are the first ones to offer an equilibrium model to explain higher correlations during recessions. Their model is based on Bayesian learning where dividends have an unobservable and common business cycle component that follows a two-state regime-switching Markov process. Agents form subjective beliefs about the current state by observing the realizations of dividends. Returns are affected by changes in dividends and by changes in agents' beliefs. Because beliefs are formed with regards to a common business cycle component, they generate co-

movement in returns. Therefore, when beliefs become more volatile, the correlation of returns increases and vice versa.

As a result of Bayesian learning, beliefs change as agents observe dividend realizations. Intuitively, when they observe a high realization of dividends, agents update their beliefs towards the high growth regime. Therefore, beliefs are sensitive to the dividend realization. Importantly, beliefs are the most responsive to dividend realizations when both regimes are equally likely. When agents have strong beliefs about the current state, i.e. the probability of the high growth state is close to zero or one, the beliefs are not very responsive to dividend realizations. However, when agents assign approximately the same probabilities to the two regimes, any information coming from the dividend realizations becomes important and beliefs are adjusted accordingly. Returns are very volatile when agents are not sure about the state of the economy. So the relationship is non-monotonic. When agents are fairly confident that the economy is in the low growth or the high growth regime, correlations are low. But when agents are not sure which regime is in effect, beliefs become more volatile and correlations go up. Hence, to the extent that recessions are relatively more uncertain times, during those times beliefs are going to be more volatile and correlations are going to be higher.

Aydemir (2008) proposes a model with time-varying risk aversion that is due to the external habit formation a-la Campbell and Cochrane (1999). In a two-country one-good setup, he studies the correlation of returns for equities paying each country's outputs as their dividends. Each country is inhabited by a representative agent that has external habit formation preferences. In low consumption states risk aversion is more volatile, generating more volatile discount rates. Therefore, in low consumption states the discount rate volatility drives most of the movements in returns. To the extent that discount rates are more correlated across countries than outputs are, in low consumption states correlations are going to be higher than in high consumption

states.

In that paper, the return volatilities are due to changes in discount rates and changes in dividends. In low consumption states the volatilities of the discount rates are higher than they are in high consumption states and therefore during those periods the discount rates generate relatively more movements in returns. Thus discount rates generate bigger proportion of the return correlations in low consumption states as compared to high consumption states. However, the effect on return correlations depends on the correlation of discount rates across countries. If the risk sharing among countries is strong enough the correlation of discount rates will be larger than the correlation of outputs. Therefore, as discount rates become more volatile, the correlation of returns increases.

Ehling and Heyerdahl-Larsen (2011) also offer an explanation based on time-varying risk aversion, similar to Aydemir (2008). However, the mechanism that generates time-varying risk aversion is different and it is also able to explain the level of correlations for the different industries. Equity returns respond to changes in aggregate risk aversion as well as to changes in cash flows. Aggregate risk aversion is a common component and it is more volatile during low consumption states than during high consumption states. Hence the correlation of equity returns is higher during recessions.

In their setup, endogenous aggregate risk aversion is due to heterogeneous agents. There are two types of agents with high and low risk aversion respectively. As the agents' relative consumption shares change the aggregate risk aversion changes as well. More specifically, as consumption decreases more risk-averse agents get a bigger share of the aggregate consumption. This is due to the inverse relationship between the coefficient of risk aversion and the intertemporal elasticity of substitution. That is why the aggregate risk aversion is sensitive to the changes in consumption shares and the consumption sharing rule between the two types of agents is steeper at lower

consumption levels.<sup>3</sup> The crucial point is that when the sharing rule is steeper, small changes in consumption lead to relatively larger changes in consumption shares, so that the aggregate risk-aversion becomes very volatile. Therefore, in low consumption states changes in aggregate risk-aversion drive most of the movements in returns. Hence, because the aggregate risk aversion is a common factor in returns, in low consumption states the correlations are going to be higher than in high consumption states.

Mueller et al. (2012) study the correlation of exchange rates in a multi-country multi-good model with home bias and external habit formation preferences. Under the complete markets assumption, there exists perfect risk sharing between countries but home bias impedes the perfect consumption pooling. However, habit formation allows for time-varying risk aversion, which in turn generates more international risk sharing during low consumption states. The interaction of the higher international risk sharing and the greater home bias in the domestic country generates higher correlations during recessions.

According to the aforementioned paper, exchange rates respond both to the consumption risks of the domestic and the foreign country and to the risk aversion of the representative agents in each country. When the representative agents are symmetric in terms of their risk aversion, the exchange rate is only going to be a function of domestic and foreign consumption risks. In this setting, two exchange rates, i.e. pound/dollar and euro/dollar, are correlated due to the domestic consumption risk for the U.S., which is a common factor. Time-varying risk aversion affects the incentives for international risk sharing, which in turn affects the consumption risks. In low consumption states, high risk aversion leads to higher international risk sharing, decreasing the consumption risk in each country. Therefore, in low consumption states, consumption risks are smaller. However, due to higher home bias in the do-

---

<sup>3</sup> See Dumas (1989).



mestic country, the domestic consumption risk decreases less relative to the foreign countries' consumption risks. For example, if the domestic country has maximum home bias and only consumes its domestic output, then the domestic consumption risk is going to be constant while foreign countries' consumption risks are going to decrease with increasing risk aversion. Therefore, in low consumption states, the common component of exchange rates is relatively more volatile, or the idiosyncratic components in exchange rates are relatively smaller, which leads to higher correlation of the exchange rates. Mueller et al. (2012) also empirically show that time-variation in correlations is priced in the cross-section of the exchange rate risk premia. High interest rate currencies provide lower returns when correlations are higher, while low interest rate currencies are safe heavens in providing a hedge for higher correlations.

### 2.3 The Model

In this section I describe the model and derive the implications for conditional correlations. The main feature of the model is the signal extraction problem when the knowledge about the quality of the signal is incomplete, i.e. the market participants do not know how reliable the information they receive is.

This incomplete nature of information is modeled by ambiguity. The investors do not know the exact variance of the noise term in the signal but they have a range for it. Therefore, they do not have a unique likelihood to update when they receive the signal, but rather they have a family of likelihoods. For example, assuming  $\theta$  is the parameter that we want to learn, and the signal is

$$s = \theta + \epsilon, \text{ where } \epsilon \sim \mathbb{N}(0, \sigma_s^2) \text{ and } \sigma_s^2 \in [\underline{\sigma_s^2}, \overline{\sigma_s^2}]$$

Here  $\epsilon$  would be the noise or measurement error term and the variance of it determines the reliability of the signal. With a unique  $\sigma_s^2$ , value the maximum likelihood

approach would give the best prediction upon observing the signal  $s$ . However, we will not have a unique likelihood to update when  $\sigma_s^2$  value is not unique.

Agents' behavior in this environment is described by ambiguity aversion. To model ambiguity aversion, we use Epstein and Schneider (2003)'s recursive multiple-priors utility, which is the extension of Gilboa and Schmeidler (1989)'s model to an intertemporal setting. In that setting agents behave as if they maximize the expected utility every period but under the worst-case belief. In our case the beliefs are about the variance of the noise term, and the objective function of the agent would be:

$$\max_{\{c_t\}} \min_{\sigma_s^2 \in [\underline{\sigma}_s^2, \overline{\sigma}_s^2]} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(c_t) \quad (2.1)$$

In the case of no ambiguity, i.e. when  $\sigma_s^2$  is unique, the objective function of the agent would be the same as in the standard expected utility maximization problem.

The representative agent is assumed to be risk neutral but ambiguity averse. To see the asset pricing implications under this setup, let us start with a simple case: the agent is risk neutral and there is no ambiguity. Risk neutrality assumption will make the stochastic discount factor independent of consumption.<sup>4</sup> Assuming no ambiguity will shrink the multiple priors set  $[\underline{\sigma}_s^2, \overline{\sigma}_s^2]$  to a singleton, therefore the min operator will be dropped.

Thus, under those assumptions a Lucas tree model of asset pricing with multiple trees would be:

$$\max_{\{c_t, x_{t+1}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(c_t) \quad \text{such that} \quad c_t + q_t x_{t+1} \leq (q_t + d_t) x_t \quad \text{for every } t \quad (2.2)$$

where  $c_t$  is the consumption,  $q_t$  is the price vector for the shares of assets,  $x_t$  is the

<sup>4</sup> Risk neutrality assumption is very important for tractability. Even without ambiguity the multiple tree Lucas model becomes quite complicated without risk neutrality assumption. See Cochrane et al. (2008) and Martin (2013) for further discussion.

asset shares held by the agent and  $d_t$  is the vector representing the dividend of each asset. The agent maximizes his lifetime utility by choosing how much to consume and how many shares of assets to buy. In that setup the price vector for shares of assets is:

$$q_t = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (q_{t+1} + d_{t+1}) \right\} \quad (2.3)$$

where  $\lambda_t$  is the stochastic discount factor and equal to  $\beta^t \frac{U'(c_{t+1})}{U'(c_t)}$ . However, the risk neutrality assumption implies that  $\lambda_t = \beta^t$ . Therefore, the asset prices under risk neutrality and without ambiguity would be:

$$q_t = \beta \mathbb{E}_t \{ q_{t+1} + d_{t+1} \} \quad (2.4)$$

When we have ambiguity, the pricing function is:

$$q_t = \min_{\sigma_s^2 \in [\underline{\sigma}_s^2, \overline{\sigma}_s^2]} \beta \mathbb{E}_t \{ q_{t+1} + d_{t+1} \} \quad (2.5)$$

I will be using that formula to get the asset prices throughout the paper.

The model I present here is closely related to Epstein and Schneider (2008). They show that agents respond more strongly to bad news than to good news and use that feature to explain the equity premium, the excess volatility of prices and the skewness of returns.

In the next subsection I will define the asset markets and information structure more specifically.

### 2.3.1 An Asset Market with Ambiguous News

There are three dates, labeled 0, 1, and 2. To get correlation properties, I define two assets: asset  $i$  and  $j$ . There is an equal number of shares outstanding for each asset, where each share is a claim to a dividend stream

$$d^i = m + \varepsilon^a + \varepsilon^i \quad (2.6)$$

$$d^j = m + \varepsilon^a + \varepsilon^j \quad (2.7)$$

where  $m$  is the mean dividend,  $\varepsilon^a$  is an aggregate shock that affects both of the assets, and  $\varepsilon^i$  and  $\varepsilon^j$  are idiosyncratic shocks that affects only asset  $i$  and  $j$ , respectively. We can consider  $\varepsilon^a + \varepsilon^i$  as a dividend innovation for asset  $i$ , and similarly  $\varepsilon^a + \varepsilon^j$  for asset  $j$ . In what follows, all shocks are assumed to be mutually independent and normally distributed with mean zero.

### 2.3.2 News

Dividends are revealed at date 2. At date 1, one random news is realized in the form of a signal, only about the aggregate component.<sup>5</sup> In other words, the following signal is observed at date 1 before the realization of dividends:

$$s = \varepsilon^a + \varepsilon^s$$

Since  $\varepsilon^a$  is common in both dividend innovations, the signal carries information relevant for both of the assets. Thus, upon observing the signal, both of the prices are updated. The properties of  $\varepsilon^s$  are important for our study. The variance of the shock  $\varepsilon^s$  is known only to lie in some range,  $[\underline{\sigma}_s^2, \overline{\sigma}_s^2]$ . This captures the agent's lack of confidence in the signal's precision.

---

<sup>5</sup> We can generalize the structure of signal and allow it to carry information about the idiosyncratic innovations as well. Results are provided in the online appendix.

The set of one-step-ahead beliefs about  $s^i$  at date 0 consists of normals with mean zero and variance  $\sigma_a^2 + \sigma_s^2$ , for  $\sigma_s^2 \in [\underline{\sigma_s^2}, \overline{\sigma_s^2}]$ . After observing the signal, at date 1, the posteriors are formed according to the standard updating rules. We can think of this as a regression since a linear regression would coincide with the conditional expectation function in the case of normal errors, which is the case here.<sup>6</sup> Therefore, after observing the signal  $s$ , the posterior for  $\varepsilon^a + \varepsilon^i$  would have mean  $\eta(\sigma_{s,i}^2)s^i$ , where  $\eta(\sigma_{s,i}^2)$  is the regression coefficient from regressing the dividend on the signal. The posterior mean for the other dividend after observing the signal is the same since the signal has the same information content for both assets. The posterior mean for  $d^j$  and  $d^i$  will be  $\mathbb{E}[d^j|s] = \mathbb{E}[d^i|s] = m + \eta(\sigma_s^2)s$ .

$$\eta(\sigma_s^2) = \frac{\text{cov}(d^i, s)}{\text{var}(s)} = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_s^2} \quad (2.8)$$

where  $\sigma_s^2$  ranges over  $[\underline{\sigma_s^2}, \overline{\sigma_s^2}]$ . Hence the regression coefficient also varies, tracing out a family of posteriors. In case of a single prior, the forecast would be a singleton. However, with ambiguity the forecast spans an interval as  $\sigma_s^2$  ranges over  $[\underline{\sigma_s^2}, \overline{\sigma_s^2}]$ . In other words, the ambiguous news  $s$  introduces ambiguity into beliefs about fundamentals.

I will now calculate the prices. Recall that the agent is risk neutral but ambiguity averse. As discussed in the previous subsection, with recursive multiple-priors utility, actions are evaluated under the worst-case conditional probability. We also know that the representative agent must hold all assets in equilibrium. It follows that, as we discussed at the beginning of this section, the worst-case conditional probability

---

<sup>6</sup> See Goldberger (1991) chapter 16 for a general discussion

minimizes conditional mean payoffs. Therefore, the price of asset  $i$  at date 1 is: <sup>7</sup>

$$q_1^i(s) = \min_{(\sigma_s^2) \in [\underline{\sigma}_s^2, \overline{\sigma}_s^2]} \mathbb{E}[d^i | s] = \begin{cases} m + \underline{\eta} s & \text{if } s \geq 0, \\ m + \overline{\eta} s & \text{if } s < 0, \end{cases} \quad (2.9)$$

A crucial property of ambiguous news is that the worst-case likelihood used to interpret the signal depends on the value of the signal itself. Here the agent interprets bad news ( $s < 0$ ) as very informative, whereas good news is viewed as imprecise.

At date 0, the agent knows that an ambiguous signal will arrive at date 1. His one-step-ahead conditional beliefs about the signal are normals with mean zero and variances  $\sigma_a^2 + \sigma_s^2$ . Again, the worst-case probability is used to evaluate payoffs. Since the date 1 price is concave in the signal  $s$ , the date 0 conditional mean return is minimized by selecting the highest possible variance  $\overline{\sigma}_s^2$ . Thus, we have

$$\begin{aligned} q_0^i &= \min_{\sigma_s^2 \in [\underline{\sigma}_s^2, \overline{\sigma}_s^2]} \mathbb{E}[q_1^i] = m + \min_{\sigma_s^2} \{ (\overline{\eta} - \underline{\eta}) \mathbb{E}[s | s < 0] \} \\ &= m - \left\{ (\overline{\eta} - \underline{\eta}) \frac{1}{\sqrt{2\pi}} \sqrt{\sigma_a^2 + \overline{\sigma}_s^2} \right\} \end{aligned}$$

Without the ambiguity aversion, the asset prices would be equal to mean dividend  $m$  under a risk neutral valuation. However, here the prices have discount terms, date 0 prices include a premium for ambiguity. The amount of the premium is directly related to the amount of the ambiguity,  $(\overline{\eta} - \underline{\eta})$ . As it is clear in the prices, the premium for ambiguity is increasing in the volatility of fundamentals.

---

I only derive the price of asset  $i$ , however the price of asset  $j$  is identical.<sup>8</sup>

<sup>7</sup> Since date 2 is the terminal date, price of the tree will be zero,  $q_2^i = q_2^j = 0$ . Therefore, the prices at date 1,  $q_1^i$  and  $q_1^j$  will just be a function of expected dividends.

<sup>8</sup> When we move to infinite horizon with autoregressive dividends, the realization of idiosyncratic shocks are going to make the prices different, however, unconditionally the assets are identical.

### 2.3.3 Asset Pricing Properties

To compare the predictions of the model to data, I embed the above three date model of news release into an infinite-horizon asset pricing model. Specifically, in every period there are going to be dividend realizations as well as a signal about the future dividends. Agents observe one signal about the next innovation in dividends before that innovation is revealed and the next learning episode starts. That is, in each period agents observe dividends and signal about the future dividends.

The level of dividends for the assets is given by a mean-reverting process,

$$d_t = \kappa \bar{d} + (1 - \kappa)d_{t-1} + u_t, \quad (2.10)$$

where  $\kappa \in (0, 1)$  is the mean reversion parameter for dividends. For  $d_t^i$ ,  $u_t^i$  is equal to  $\varepsilon_t^a + \varepsilon_t^i$  and for  $d_t^j$ ,  $u_t^j$  is equal to  $\varepsilon_t^a + \varepsilon_t^j$ . Hence, the assets are identical except for the realization of idiosyncratic shocks.

In each period, agents observe an ambiguous signal about the aggregate component of dividend innovations. The assets pay dividends each period:

$$s_t = \varepsilon_{t+1}^a + \varepsilon_t^s \quad (2.11)$$

$$d_t^i = \kappa \bar{d} + (1 - \kappa)d_{t-1}^i + \varepsilon_t^a + \varepsilon_t^i \quad (2.12)$$

$$d_t^j = \kappa \bar{d} + (1 - \kappa)d_{t-1}^j + \varepsilon_t^a + \varepsilon_t^j \quad (2.13)$$

The goal is to derive asset pricing properties that would be observed by an econometrician who studies the above asset market. Thus, I assume that there is a true variance of noise  $\sigma_s^{*2} \in [\underline{\sigma}_s^2, \overline{\sigma}_s^2]$ . I further assume that the true distributions of the fundamentals are known to the agent. Therefore, the ambiguity does not stem from

the fundamentals but from the difficulty of forecasting fundamentals. The point is that market participants typically have access to ambiguous information, other than past dividends, that is not observed by the econometrician.

Due to the risk-neutrality assumption, the price of a risk-free bond is constant, which implies a constant interest rate  $r$  in terms of the exogenous time discount factor:  $\beta = \frac{1}{1+r}$ . However, the stock prices vary as they respond to the dividend realizations and ambiguous signals. Let  $q_t^i$  and  $q_t^j$  denote the stock prices. In equilibrium, the price at time- $t$  must be the worst-case conditional expectation of the price plus dividend in period  $t+1$ :

$$q_t^i = \min_{(\sigma_{s,t}^2, \sigma_{s,t+1}^2)} \beta \mathbb{E}[q_{t+1}^i + d_{t+1}^i] \quad (2.14)$$

$$q_t^j = \min_{(\sigma_{s,t}^2, \sigma_{s,t+1}^2)} \beta \mathbb{E}[q_{t+1}^j + d_{t+1}^j] \quad (2.15)$$

I focus on stationary equilibria. The prices<sup>9</sup> are given by

$$q_t^i = \frac{\bar{d}}{r} + \frac{1-\kappa}{r+\kappa}(d_t^i - \bar{d}) + \frac{1}{r+\kappa}\eta_t s_t + \bar{Q}^i \quad (2.16)$$

$$q_t^j = \frac{\bar{d}}{r} + \frac{1-\kappa}{r+\kappa}(d_t^j - \bar{d}) + \frac{1}{r+\kappa}\eta_t s_t + \bar{Q}^j \quad (2.17)$$

where

$$\eta_t = \begin{cases} \underline{\eta} & \text{if } s_t \geq 0 \\ \bar{\eta} & \text{if } s_t < 0 \end{cases}$$

$$\text{and } \bar{Q}^i = \bar{Q}^j = -\frac{1}{r(r+\kappa)} \frac{1}{2} \frac{1}{\sqrt{2\pi}} \left[ (\bar{\eta} - \underline{\eta}) \sqrt{\sigma_a^2 + \sigma_s^2} \right].$$

---

<sup>9</sup> Conjecture a time-invariant price function of the type  $q_t = \bar{Q} + Q_d \hat{d}_t + Q_s \mu_t s_t$ . Inserting the guess into equations (2.14)-(2.15) and matching undetermined coefficients delivers (2.16)-(2.17). The calculations are left to the appendix.



The first two terms in prices reflect the present discounted value of dividends without news, where prices are determined only by the interest rate and the current dividend level. The third term captures the response to the current ambiguous signal. As in equation (A.15), this response is asymmetric: The distribution of  $\eta_t$  implies that bad news is incorporated into prices more strongly. In addition, the strength of the reaction now depends on the persistence of dividends: If  $\kappa$  is smaller, then the effect of news on prices is stronger since the information matters more for payoffs beyond just the next period. The fourth term captures anticipation of future ambiguous news; it is the present discounted value of the premium in  $q_0^i$  and  $q_0^j$ .

#### 2.3.4 Conditional Correlations

Since we are interested in the conditional correlation properties of excess returns, I define the excess returns first. Per share excess returns can be defined as:

$$R_{t+1} = q_{t+1} + d_{t+1} - (1 + r)q_t \quad (2.18)$$

Using equation (2.18) for the two assets we have, we get the following representations for excess returns:

$$R_{t+1}^i = \frac{1+r}{r+\kappa} (u_{t+1}^i - \eta_t s_t) + \frac{1}{r+\kappa} \eta_{t+1} s_{t+1} + \tilde{Q} \quad (2.19)$$

$$R_{t+1}^j = \frac{1+r}{r+\kappa} (u_{t+1}^j - \eta_t s_t) + \frac{1}{r+\kappa} \eta_{t+1} s_{t+1} + \tilde{Q} \quad (2.20)$$

where  $\tilde{Q} = \frac{1}{r+\kappa} \frac{1}{\sqrt{2\pi}} \left[ (\bar{\eta} - \underline{\eta}) \frac{\sqrt{\sigma_a^2}}{\sqrt{\underline{\eta}}} \right]$ .

The first term in parentheses captures the surprise component of dividend realizations. The second term incorporates the information into the prices through the

signal.

Since we have a closed form solution for returns, we can get the correlation function in closed form as well in terms of the model parameters. The following two propositions constitute the main results of this paper.

**Proposition 1.** *Correlation asymmetry exists. More specifically, correlation conditional on bad news is higher than correlation conditional on good news:  $\rho^- = \text{corr}(R_{t+1}^i, R_{t+1}^j | s_{t+1} < 0) > \rho^+ = \text{corr}(R_{t+1}^i, R_{t+1}^j | s_{t+1} > 0)$  and the asymmetry is larger for larger ambiguity.*

The proof of the proposition can be found in the appendix. The intuition is as follows: We can decompose the correlation of returns into two components. The signal generates common movements in returns and thus it is a source of correlation. Returns can also comove due to the comovement in dividends. Therefore, we can represent the correlation of returns as a weighted average of correlations due to signals and due to the dividends.

$$\text{corr}(R_t^1, R_t^2) = \underbrace{\text{corr}(s_t, s_t)}_{=1} \omega_t^s + \text{corr}(d_t^1, d_t^2) \omega_t^d$$

where  $\omega_t^s + \omega_t^d = 1$

$$\omega_t^s \propto \frac{\eta_t \text{var}(s_t)}{\sigma_1 \sigma_2}$$

The weight of signals is increasing in  $\eta_t$ . This is intuitive because  $\eta_t$  represents how strong prices respond to the signals; hence, if price response to signals is stronger, the share of the movements in returns (volatility) attributed to signals is higher. Thus, the share of signals in the overall correlation increases with  $\eta_t$ .

Given this decomposition, it is evident that the overall correlation changes as the weight of signal changes. The correlation due to the common signal is equal to 1, which is the absolute maximum for correlation. Dividends, on the other hand, are imperfectly correlated. Hence, the overall correlation is higher conditional on a negative signal, which generates larger movements in returns.

To summarize, prices respond more strongly to negative signals compared to positive signals. The weight of signals,  $\omega_t^s$ , in generating correlation is higher when the signals are negative. Thereby the correlations are higher conditional on negative signals.

The following proposition states the relationship between the idiosyncratic volatility and the correlation asymmetry. This relationship is useful in understanding the time-series and cross-sectional variation in correlation asymmetry.

**Proposition 2.** *Defining  $\xi = \frac{\sigma_i^2}{\sigma_a^2}$  as the ratio of idiosyncratic-to-aggregate volatility, correlation asymmetry is greater for higher  $\xi$ .*

The proof is in the appendix. For higher idiosyncratic-to-aggregate volatility ratios, the correlation asymmetry is greater in percentage terms. For example, if the correlation asymmetry is 10 percent for a low idiosyncratic volatility asset, the asymmetry for an asset with higher idiosyncratic volatility is more than 10 percent. More interestingly, the asymmetry in absolute terms is also increasing in  $\xi$  for a large empirically relevant region. As the idiosyncratic-to-aggregate volatility ratio rises, the correlation asymmetry increases in absolute terms. For example, if the correlation asymmetry is 10 percentage points, i.e.  $\rho^- - \rho^+ = 0.10$ , for a low idiosyncratic volatility asset, the asymmetry for an asset with higher idiosyncratic volatility is more than 10 percentage points. However, this relationship is non-monotonic. For very high idiosyncratic values the asymmetry is high in percentage terms, but due to the low levels of correlation, the asymmetry is low in percentage points. The

asymmetry in absolute terms is equal to asymmetry in percentage terms multiplied by the level of correlations. Thus, at the very high values of idiosyncratic volatilities the level of correlations is low, which decreases the asymmetry in absolute terms. The domain of the correlation asymmetry in absolute terms can be split in two regions: In one region, correlation asymmetry is increasing in  $\xi$ , while in the other one it is decreasing in  $\xi$ .

I provide the characterization of the regions in the appendix. However, in the data the idiosyncratic volatilities do not seem to be high enough to observe the second region, at least at a portfolio level. In the next section, I empirically study the correlation asymmetry where the asymmetry is defined in absolute terms.

Before moving to the next section, I provide the intuition of the relationship between the idiosyncratic volatility and the correlation asymmetry. As we discovered when discussing Proposition 1, the correlation of returns can be decomposed into two components, one component due to the common signals and one component due to the dividends. The overall correlation is a weighted average of these two correlations. Although the correlation due to the signals is independent of idiosyncratic volatilities, the correlation of dividends is decreasing in the idiosyncratic volatilities. The correlation of dividends is lower for high idiosyncratic volatility stocks. Therefore, as the weight of signals changes, the change in overall correlation is larger for high idiosyncratic volatility stocks. To better illustrate this point, I have a representative relationship in Figure 2.1. The vertical axis represents the level of overall correlation, and the horizontal axis represents the weight of signals. As discussed earlier, the higher the weight of signals, the higher the correlation. To observe the difference across different idiosyncratic volatility levels, we have two separate lines for two hypothetical stocks with low and high idiosyncratic volatility. The dashed blue line represents the correlation when the idiosyncratic volatility is high. The line representing the correlation is steeper for higher idiosyncratic volatility, which

means as the weight of the signals changes, the change in overall correlation will be stronger for high idiosyncratic volatility stocks. On the other hand, the correlation of dividends is high for low idiosyncratic volatility stocks, which acts like an anchor to stabilize the overall correlation.

## 2.4 Testing the model's prediction: New empirical relationship

In this section I analyze the relationship between the idiosyncratic volatility and the correlation asymmetry for which my model has a clear prediction. As I run the analysis I refer to the literature that deals with the properties of idiosyncratic volatilities. By the means of my model I discuss some implications of the findings in that literature. I further borrow the methodology used to estimate idiosyncratic volatilities. In the next two subsections, I explain the data and the aforementioned methodology. In subsection 2.7, I provide the statistical analyses that shows the relationship between idiosyncratic volatilities and asymmetric correlations.

## 2.5 Data

I obtain the daily stock returns, stock prices, shares outstanding, and exchange listings for the universe of stocks available from the Center for Research on Security Prices (CRSP). I also obtain daily Fama-French factor returns and daily risk-free rates from Kenneth French's data library.<sup>10</sup> The sample period ranges from July 1, 1963 through December 30, 2011.

## 2.6 Idiosyncratic volatility measurement

To measure idiosyncratic volatilities, I follow an approach that is common in the literature. Similar to Ang et al. (2006a), the idiosyncratic volatility is measured

---

<sup>10</sup> The data library is available at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

by the Fama and French (1993) three-factor model. The excess stock returns are regressed on risk factors and the volatility of the residuals from the regression is estimated to be the idiosyncratic volatility. In order to capture the effect of time varying betas the regressions are run for every month, using daily observations.<sup>11</sup>

For each stock, I estimate the following regression every month using the daily returns in order to measure idiosyncratic volatilities. To be more precise, this is simply the standard Fama and French (1993) model run monthly.

$$r^i = \alpha^i + \beta^i MKT + s^i SMB + h^i HML + \varepsilon^i$$

where  $r^i$  is the daily return of stock  $i$ , in excess of the one-month U.S. T-bill rate. The market factor  $MKT$  is computed as the value-weighted average of excess returns of all stocks. The returns on zero-cost portfolios  $SMB$  and  $HML$ , measure size and value premiums, respectively. The  $SMB$  factor is the return of the smallest one-third of stocks less the return on the firms in the top one third ranked by market capitalization. The value factor  $HML$  is the return of the portfolio that goes long on the top one third of stocks with the highest book-to-market ratios and shorts the bottom one third of stocks with the lowest book-to-market ratios. This regression is run for every month for each stock and the idiosyncratic volatility for stock  $i$  at month  $t$ ,  $\sigma_{i,t}$  is measured as the standard deviation of the residuals  $\varepsilon$  obtained from this regression. To measure the idiosyncratic volatilities of portfolios I follow the same procedure, by replacing the stock returns with portfolio returns.

---

<sup>11</sup> This method does not impose any constraint on the betas, thus called as “a direct approach” by Xu and Malkiel (2003), as opposed the approach in Campbell et al. (2001) which assumes that the systematic risks are captured by the industry return and that firms have unit betas with respect to the industry to which they belong. Unfortunately the second method is not suitable for my analysis, because it requires the estimation of idiosyncratic volatilities for portfolios as well as stocks. However, the estimates of these two methods are known to be highly correlated.

## 2.7 Does higher idiosyncratic volatility imply greater correlation asymmetry?

In this subsection I study the relationship between idiosyncratic volatility and correlation asymmetry. As discussed in Proposition 2 the model predicts a positive relationship between idiosyncratic volatility and correlation asymmetry. In principle this relation should be observed at stock level. However, in the model correlations are always positive by construction although in the data that it is not necessarily the case. This discrepancy is especially prevalent at stock level while it is less of an issue at portfolio level. As long as a portfolio includes sizable amount of stocks, that portfolio's returns are usually positively correlated with the aggregate market. There is also an advantage of using portfolios in the estimation of idiosyncratic volatilities. Since portfolios have smoother return series compared to stocks the idiosyncratic volatility estimates are more precise. Therefore, I focus on portfolios rather than on stocks although the results are similar at stock level for stocks with non-negative correlations with the aggregate market.<sup>12</sup>

One needs to be careful when forming portfolios for this analysis since the idiosyncratic volatility is a property of individual stock returns. Therefore, when we form portfolios that consist of random stocks, returns of portfolios may have little idiosyncratic volatility. Although that is less of an issue for non-random portfolios, where we form portfolios according to some criteria, different portfolios may end up having similar levels of idiosyncratic volatilities. To address these potential issues, I form portfolios according to the level of the idiosyncratic stock volatilities and estimate the idiosyncratic volatility of the resulting portfolios. In the next paragraph I discuss the details of the portfolio formation and the estimation of the idiosyncratic volatilities. Before doing that, I discuss the choice of criterion for portfolio formation. In

---

<sup>12</sup> It is also difficult to interpret the correlation asymmetry when the correlation of a stock turns from negative to positive.

principle any set of portfolios with non-negligible idiosyncratic risk provides a room to test the prediction of the model. However, if there is not enough dispersion in the idiosyncratic volatility of different portfolios, identifying the relationship will be much more difficult and the statistical methods will be less powerful. Thus I choose to sort stocks into portfolios according to their idiosyncratic volatilities with the goal that the resulting portfolios will have dispersion in their idiosyncratic volatilities.<sup>13</sup> Below I show that this strategy does indeed create dispersion in the idiosyncratic volatilities of the resulting portfolios.

Details of portfolio formation are as follows. First, I estimate the idiosyncratic volatilities of individual stocks and use those estimates to sort stocks into 100 portfolios on a monthly basis. The portfolios are updated each month.<sup>14</sup> Second, I take a value weighted average of the daily returns of their constituent stocks in order to calculate returns of portfolios. Once I have the daily portfolio returns, I can estimate their idiosyncratic volatilities and correlations with the aggregate market. Using the daily returns over a month and following the procedure described in the previous subsection, I estimate the idiosyncratic volatilities relative to the three-factor model of Fama and French (1993). For each month the correlation with the aggregate market is calculated using the daily returns in excess of the risk free rate.

Before discussing the main statistical test, I provide a graphical analysis first. In Figure 2.2 I plot the main relationship I am interested in the cross-section: the relationship between idiosyncratic volatility and correlation asymmetry. For each portfolio, first the upward and downward correlations are calculated and the difference is then referred to as correlation asymmetry. Similar to the literature, for

---

<sup>13</sup> This idea utilizes two findings in the literature, idiosyncratic risk of stocks seems to be cross-correlated and somewhat persistent. Fu (2009) estimates the average first-order autocorrelation of idiosyncratic volatility to be around 0.33.

<sup>14</sup> Less frequent adjustment of portfolios may not create dispersion in the idiosyncratic volatilities since the idiosyncratic volatilities are not very persistent at stock level.



each portfolio, the upward correlation is calculated over the months where both the excess return of portfolio and the market is above their sample means. Downward correlations are measured in a similar fashion. We can represent those, using the standardized excess returns, as  $\rho_i^- = \text{corr}(r_{i,t}, r_{m,t} | r_{i,t} < 0, r_{m,t} < 0)$ . Similarly, upward correlations are  $\rho_i^+ = \text{corr}(r_{i,t}, r_{m,t} | r_{i,t} > 0, r_{m,t} > 0)$ . For each portfolio correlation asymmetry is estimated as  $\rho_i^- - \rho_i^+$  and those estimates are plotted across portfolios. As it can be seen in Figure 2.2, there is a clear relationship between the idiosyncratic volatility and the correlation asymmetry – the higher the idiosyncratic volatility, the higher the correlation asymmetry.

Although the graphical analysis shows a clear pattern, it cannot be conclusive for a couple of reasons. First, the slope of the relationship in Figure 2.2 needs to be statistically significant. It is possible that high and low correlation asymmetry is not statistically different. Second, even if the relation is statistically significant, i.e. the slope in Figure 2.2 is significantly positive in statistical sense, it may be due to a third factor which is correlated with the idiosyncratic volatility. To address these concerns I run a statistical analysis similar to Andersen et al. (2001). I run panel regressions in the following form:

$$\text{corr}_{i,t} = \delta_0 + \delta_1 \mathbb{I}(R_{i,t} * R_{m,t} > 0) + \delta_2 \mathbb{I}(R_{i,t} < 0, R_{m,t} < 0) + \beta X_{i,t} \quad (2.21)$$

where  $\text{corr}_{i,t}$  is the correlation between portfolio  $i$  and aggregate market in month  $t$ ,  $R_{i,t}$  and  $R_{m,t}$  are monthly excess returns to portfolio  $i$  and aggregate market, respectively. As common in the literature both excess returns are standardized.<sup>15</sup>  $X_{i,t}$  includes some variables I am interested in as well as some control variables which are to be discussed shortly.  $\mathbb{I}(\cdot)$  is the indicator function and  $\mathbb{I}(R_{i,t} * R_{m,t} > 0)$  captures the joint effect of market upturns and downturns while  $\mathbb{I}(R_{i,t} < 0, R_{m,t} < 0)$

---

<sup>15</sup> See Ang and Chen (2002), Hong et al. (2007)

indicates only the market downturns. Therefore, the impact of market upturns on correlations is  $\delta_1$  and the impact of market downturns is  $\delta_1 + \delta_2$ . Thus,  $\delta_2$  captures the additional influence of market downturns and a statistically significant positive  $\delta_2$  implies that correlations are asymmetric, with higher correlations during market downturns.

The first column in Table 2.1 presents the results of this initial panel regression. As it can be inferred from the t-statistics the effect of market downturns is statistically significant, which confirms the findings in the literature. Before moving to the next step of analysis I would like to explain how the t-statistics are calculated.

It is well known that OLS standard errors are biased when the residuals are correlated. In panel data, such as the one I consider here, residuals may be correlated across time, or for a particular time period the residuals may be correlated across the cross-section. In my case, for instance, monthly correlations with the aggregate market may be high due to unobserved reasons. If this is due to a variable accounted for in the regression, i.e. high market volatility, this will not be a problem. To the extent that this is not accounted for by the right hand side variables, the residuals will be correlated across portfolios. Moreover a portfolio with a high correlation with the aggregate market may tend to have a high correlation with the market over the next period as well. Again, if the right hand side variables capture the reason for that persistence, residuals may be uncorrelated. That said, I claim by no means that the relationship is good enough to capture the movements in correlations. Moreover, the correlation in residuals may also arise due to unobservable factors. Therefore, these potential biases in standard errors should be addressed. To do so, I follow Cameron et al. (2011) and Petersen (2009) and report the results from pooled ordinary least squares (OLS) regressions after adjusting the standard errors for heteroskedasticity, serial-, and cross-sectional correlation using a two-dimensional cluster at the portfolio

and at the month level.<sup>16</sup>

The second column of Table 2.1 presents the results of the same panel regression including an additional explanatory variable: market volatility. As it is known in the literature, high correlations can be a byproduct of high volatility.<sup>17</sup> Thus, I include the variance of the aggregate market to control for the effect of volatility. The market volatility is calculated by taking the standard deviation of the daily market returns over each month, where the market return is the value-weighted average of the constituent stock returns. As we compare the first and second columns of Table 2.1 we see that asymmetric correlations are not driven by volatility. Although the effect of market volatility is present, the effect of market downturns  $\delta_2$  is positive and statistically significant, meaning that correlations are higher during market downturns.

To test the main prediction of the model, I incorporate into the regression 2.21 three new variables: the estimates of the idiosyncratic volatilities as well as interaction variables between the old dummies and the idiosyncratic volatilities. The coefficient on  $\mathbb{I}(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{Log}(\sigma_{i,t})$  captures the effect of idiosyncratic volatility on correlation asymmetry. If the prediction of the model is true in the data, we expect that coefficient to be statistically significant and positive since the model predicts that higher idiosyncratic volatility implies higher correlation asymmetry.

It is known that the correlation asymmetry is higher for small size (market capitalization) stocks. Moreover, as shown by Malkiel and Xu (1997) small size stocks tend to have larger idiosyncratic risk. Therefore it is possible that the regression

---

<sup>16</sup> The code is kindly provided by Mitchell A. Petersen at <http://www.kellogg.northwestern.edu/faculty/petersen>

<sup>17</sup> Even if the unconditional correlations are constant, conditioning on high volatile time periods can create spuriously high correlations. For a detailed discussion, please see Boyer et al. (1997) and Forbes and Rigobon (2002).

results are picking up the effect of firm size rather than the effect of idiosyncratic volatilities. Unless the effect of idiosyncratic volatilities is separated from the effect of market capitalization, the regression results cannot be interpreted to validate the model since the model is silent about the size.<sup>18</sup> Therefore, I incorporate *Size* variable into the regression and interact it with the dummy variables, similar to the procedure for idiosyncratic volatilities. The results are shown in the fourth column of Table 2.1. The coefficient of the interaction term of size variable for downturns is negative and statistically significant. This result is consistent with Ang and Chen (2002) and Hong et al. (2007) and shows that the smaller market capitalization is related to bigger correlation asymmetry. More importantly, the conclusion on the effect of idiosyncratic volatilities does not change. The interaction variable for market downturns is still statistically significant and positive for idiosyncratic volatilities. Hence, we can conclude that the prediction of the model is supported by the data.

We can also analyze the relationship between idiosyncratic volatilities and asymmetric correlations in time-series. The panel regressions already exploit the variations in cross-section and time-series. Here, similar to Figure 2.2, I provide a graphical analysis. While Figure 2.2 shows the relationship in the cross-section, now I explore the relationship in time-series. To observe the time-series behavior of correlation asymmetry, I calculate the correlation asymmetry over a moving window. For each portfolio, the correlation asymmetry,  $\rho_i^- - \rho_i^+$ , is calculated using the approach to construct Figure 2.2. However, this time the asymmetry is calculated using a subset of the data, rather than the whole sample. The subsample of the data consists of a window of 100 monthly observations, approximately 8 years of data. Once the correlation asymmetry is calculated for each portfolio, the average is taken over 100 portfolios. By way of moving the window over time we can observe the time-series

<sup>18</sup> However, when the signals are modeled as to have idiosyncratic components – similar to Epstein and Schneider (2008) – idiosyncratic risk is priced and higher idiosyncratic risk implies lower market capitalization. Results for this case is provided in online appendix.

behavior of the average correlation asymmetry.

In Figure 2.3 the average correlation asymmetry is plotted on the vertical axis while the midpoint of the data window is on the horizontal axis. Figure 2.3 shows quite significant time-variation in the correlation asymmetry. The correlation asymmetry seems to be especially high for the 1990s and decreases significantly at the beginning of the 2000s. We can think about this in terms of the time-series behavior of idiosyncratic volatilities. Several papers including Campbell et al. (2001) and Brandt et al. (2010) document the significant changes in idiosyncratic volatilities over time, while the aggregate volatility stays fairly stable.<sup>19</sup> Brandt et al. (2010) note higher idiosyncratic volatilities during the 1990s and their empirical finding can explain the high levels of correlation asymmetry during the 1990s when considered in conjunction with Proposition 2 of this paper. So the relationship between the idiosyncratic volatilities and the asymmetric correlations reflects itself in the time-series as well. Basically, the high idiosyncratic volatilities during the 1990s caused greater correlation asymmetry. Therefore, after analyzing Figure 2.2 and Figure 2.3, together with Table 2.1, we can conclude that the prediction of the model is useful to understand both the cross-section and the time-series variation in correlation asymmetry.

---

<sup>19</sup> Papers in this literature are mainly concerned about the time-series behavior of idiosyncratic volatility and the reasons behind this time variation. Campbell et al. (2001) show that the idiosyncratic volatilities have been increasing although the aggregate volatilities stayed stationary. Brandt et al. (2010) convincingly show that the increase in idiosyncratic volatilities was temporary and the trend is reversed after 2000. The proposed explanations include firm fundamentals becoming more volatile (Wei and Zhang (2006)), increased institutional ownership (Bennett, Sias, and Starks 2003, Xu and Malkiel (2003)), tradings of retail investors (Brandt et al. (2010)), newly listed firms becoming increasingly younger (Fink et al. 2009) and riskier (Brown and Kapadia 2007), and product markets becoming more competitive (Irvine and Pontiff 2009).

## 2.8 Conclusion

In this chapter, I offer an explanation for the correlation asymmetry observed in the data. Empirical studies document a robust relationship between the realized returns and realized correlations in financial markets: correlations are higher when realized returns are relatively lower or prices of financial assets decrease. The explanation offered in this paper is formalized by an equilibrium model, which is based on ambiguity aversion. Ambiguity averse agents receive an aggregate signal with ambiguous precision. When observing ambiguous news, investors maximize their expected utility under the endogenous worst-case scenarios. When observing bad news, the worst case is that the news is very precise. On the other hand, good news under the worst case scenario is perceived as noisy or less precise compared to bad news. As a result of this endogenous mechanism, bad news is treated as a stronger signal than good news. Therefore, price decreases are sharper conditional on a bad news and this asymmetry creates a higher correlation conditional on a bad news than conditional on good news. Similar to Epstein and Schneider (2008), this mechanism also generates large equity premium, excess volatility of prices and skewness in returns.

The model provides a unified explanation for the time-series and cross-sectional variation in correlation asymmetry. The mechanism described above also predicts a relationship between the idiosyncratic volatility and the correlation asymmetry. I empirically analyze this prediction and show that it holds in the data. This novel empirical finding is also useful to understand the time-series and cross-sectional variation in correlation asymmetry. In the empirical literature it is well documented that stocks with smaller market capitalizations have greater correlation asymmetry compared to stocks with higher market capitalization.<sup>20</sup> However an explanation for this finding has been lacking. According to the explanation offered in this paper,

---

<sup>20</sup> Please see Ang and Chen (2002) and Hong, Tu and Zhou (2007).

smaller size stocks have greater correlation asymmetry compared to bigger size stocks because small size stocks tend to have higher idiosyncratic volatility. In the time-series, correlation asymmetry shows quite significant variation as well. In Figure 2.3 we can see that the average correlation asymmetry is especially high for the 1990s and decreases significantly at the beginning of the 2000s. That pattern in time-series can also be explained in terms of the time-series behavior of idiosyncratic volatilities. Several papers including Brandt et. al. (2010), document higher idiosyncratic volatilities during 1990s while the aggregate volatility stays fairly stable. Basically, the high idiosyncratic volatilities during the 1990s caused greater correlation asymmetry. Therefore, the prediction of the model is useful to understand both the cross-section and the time-series variation in correlation asymmetry.

The explanation proposed in this paper has also interesting asset pricing implication. Ambiguity acts like a third common factor in the model, driving both the ambiguity premium and the correlation asymmetry simultaneously. Thus, higher correlation asymmetry is simultaneously observed with a higher premium on returns. However, the ambiguity premium is not a premium for correlation asymmetry. An econometrician trying to calculate the premium for correlation asymmetry will also take the ambiguity premium as a part of the premium for correlation asymmetry. This can be potentially important for the interpretation of empirical studies of premium for correlation asymmetry.

## 2.9 Tables and Figures



Table 2.1: Determinants of Correlation Asymmetry for Portfolio Returns: Panel Regressions

Dependent variable: $\text{corr}_{i,t}$	I	II	III	IV
$I(r_{i,t} * r_{m,t} > 0)$	0.01 (1.78)	0.02 (3.76)	-0.07 (-1.88)	-0.11 (-2.04)
$I(r_{i,t} < 0 \& r_{m,t} < 0)$	0.08 (9.55)	0.05 (7.99)	0.23 (6.55)	0.34 (6.11)
$\text{Log}(\sigma_{m,t})$		0.07 (14.77)	0.10 (14.92)	0.09 (10.82)
$\text{Log}(\sigma_{i,t})$			-0.15 (-6.78)	-0.14 (-5.96)
$I(r_{i,t} * r_{m,t} > 0) \times \text{log}(\sigma_{i,t})$			-0.02 (-2.14)	-0.01 (-1.16)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{log}(\sigma_{i,t})$			0.04 (5.88)	0.03 (4.06)
$\text{Log}(\text{Size}_{i,t})$				0.01 (1.82)
$I(r_{i,t} * r_{m,t} > 0) \times \text{Log}(\text{Size}_{i,t})$				0.00 (1.05)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{Log}(\text{Size}_{i,t})$				-0.01 (-2.94)
$\text{corr}_{i,t-1}$	0.53 (21.69)	0.46 (14.91)	0.25 (6.81)	0.23 (8.11)
Intercept	0.27 (12.44)	0.96 (15.36)	0.71 (15.13)	0.52 (5.56)
Observations	57,560	57,560	57,560	57,560
R-squared	0.32	0.38	0.54	0.54

The table reports estimates from panel regressions, including coefficient estimates and t-statistics (in parenthesis). The dependent variable is the monthly correlation with the aggregate market excess return, for 100 portfolios sorted according to idiosyncratic volatility. Each month stocks are sorted into 100 portfolios according to their idiosyncratic volatility and value-weighted returns are calculated for these 100 portfolios. Then the idiosyncratic volatility of those 100 portfolios are estimated relative to 3 factor-model of Fama and French (1993), using daily data over a month. Again using the daily value-weighted returns of 100 portfolios, correlation with the aggregate market excess return is calculated over the month. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for 100 portfolios in cross section. The regressors are as follows: First and second independent variables are dummy variables, the second one identifying the market downturns. If the condition in the parenthesis is satisfied the dummy variable takes the value of one and otherwise zero. Other regressors are logarithm of aggregate market volatility, logarithm of idiosyncratic volatility for portfolio  $i$ , logarithm of market capitalization(size), and the lagged correlation (the first lag of the regressand). There are also two interaction terms, representing the interaction of the dummy variable identifying the market downturns with the size and the idiosyncratic volatility variable. Estimates from four different specifications are reported, where the standard errors are clustered for time and cross sectional dependence, with the method proposed by Petersen (2009). Return variables are in excess of risk free rate which is approximated by the one-month Treasury bill rate. As is in the literature, the return variables have been standardized so that each variable has a mean zero and a standard deviation of one.

Table 2.2: Testing Correlation Asymmetry with Portfolios Formed on Size and Idiosyncratic Volatility

Panel A: P-values					
	Small ME	ME-2	ME-3	ME-4	Large ME
Low $\sigma_i$	6.22	8.71	19.51	26.63	83.32
$\sigma_i$ 2	13.51	13.78	19.53	21.49	90.82
$\sigma_i$ 3	0.84	3.73	3.91	17.51	49.28
$\sigma_i$ 4	0.59	0.34	0.47	7.01	10.98
High $\sigma_i$	0.00	0.01	0.01	0.22	0.06

Panel B: Correlation Asymmetry $\rho^- - \rho^+$					
	Small ME	ME-2	ME-3	ME-4	Large ME
Low $\sigma_i$	0.27	0.24	0.19	0.17	0.03
$\sigma_i$ 2	0.23	0.21	0.18	0.16	0.01
$\sigma_i$ 3	0.37	0.27	0.27	0.16	0.08
$\sigma_i$ 4	0.37	0.35	0.33	0.21	0.17
High $\sigma_i$	0.53	0.46	0.44	0.32	0.31

Panel C: Average Number of Stocks in Each Portfolio					
	Small ME	ME-2	ME-3	ME-4	Large ME
Low $\sigma_i$	154.11	173.06	208.64	251.40	378.70
$\sigma_i$ 2	103.33	157.30	208.03	298.72	398.15
$\sigma_i$ 3	139.65	213.10	270.26	300.64	241.83
$\sigma_i$ 4	232.48	300.95	295.60	223.22	113.28
High $\sigma_i$	536.36	321.10	182.95	91.55	33.49

Table report the result of the correlation asymmetry test between the market excess return and the excess return on one of the double sorted portfolios. P-values are based on the test of Hong et al. (2007). Monthly data spans the period from July, 1963 to December, 2011 (582 observations). In Panel A, the P-values of the asymmetry test are reported, in percentage points. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_m | r_i < 0, r_m < 0)$  and  $\rho^+ = \rho(r_i, r_m | r_i > 0, r_m > 0)$ .  $r_i$  and  $r_m$  are the excess return to portfolio  $i$  and to the aggregate market, respectively, and they are both standardized, as it is common in literature. The excess return is derived by subtracting by the one-month Treasury bill rate. The estimated correlation asymmetry is reported in Panel B while the number of stocks in each portfolio is in Panel C. The portfolio returns are calculated as follows. Each month stocks are sorted into 5 portfolios according to their market capitalization (price times shares outstanding) and idiosyncratic volatility. Portfolios are re-formed monthly given that idiosyncratic volatilities are not very persistent. Idiosyncratic volatilities are computed relative to 3 factor-model of Fama and French (1993), using daily data over a month. The interaction of these two sorts yields 25 double sorted portfolios. Value-weighted returns are calculated for these 25 portfolios.

Table 2.3: Testing Correlation Asymmetry with Portfolios Formed on Size and Idiosyncratic Volatility, *NYSE Breakpoints*

Panel A: P-values					
	Small ME	ME-2	ME-3	ME-4	Large ME
Low $\sigma_i$	0.60	37.19	47.82	41.22	66.77
$\sigma_i$ 2	33.12	29.56	42.27	54.59	86.76
$\sigma_i$ 3	25.17	32.49	34.94	62.58	99.59
$\sigma_i$ 4	11.26	15.88	18.30	47.11	42.93
High $\sigma_i$	1.16	4.79	5.42	8.79	0.07

Panel B: Correlation Asymmetry $\rho^- - \rho^+$					
	Small ME	ME-2	ME-3	ME-4	Large ME
Low $\sigma_i$	0.27	0.15	0.11	0.11	0.06
$\sigma_i$ 2	0.14	0.15	0.11	0.08	0.02
$\sigma_i$ 3	0.16	0.13	0.12	0.06	0.00
$\sigma_i$ 4	0.21	0.18	0.17	0.09	0.09
High $\sigma_i$	0.33	0.26	0.24	0.21	0.39

Panel C: Average Number of Stocks in Each Portfolio					
	Small ME	ME-2	ME-3	ME-4	Large ME
Low $\sigma_i$	3690.80	84.10	79.20	86.80	121.90
$\sigma_i$ 2	52.70	69.90	85.00	104.50	129.10
$\sigma_i$ 3	58.90	80.30	96.30	105.60	100.20
$\sigma_i$ 4	84.50	100.70	102.00	91.20	62.90
High $\sigma_i$	176.00	106.20	78.70	53.10	27.00

Table report the result of the correlation asymmetry test between the market excess return and the excess return on one of the double sorted portfolios. P-values are based on the test of Hong et al. (2007). Monthly data spans the period from July, 1963 to December, 2011 (582 observations). In Panel A, the P-values of the asymmetry test are reported, in percentage points. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_m | r_i < 0, r_m < 0)$  and  $\rho^+ = \rho(r_i, r_m | r_i > 0, r_m > 0)$ .  $r_i$  and  $r_m$  are the excess return to portfolio  $i$  and to the aggregate market, respectively, and they are both standardized, as it is common in literature. The excess return is derived by subtracting by the one-month Treasury bill rate. The estimated correlation asymmetry is reported in Panel B while the number of stocks in each portfolio is in Panel C. The portfolio returns are calculated as follows. Each month stocks are sorted into 5 portfolios according to their market capitalization (price times shares outstanding) and idiosyncratic volatility. Portfolios are re-formed monthly given that idiosyncratic volatilities are not very persistent. Idiosyncratic volatilities are computed relative to 3 factor-model of Fama and French (1993), using daily data over a month. The interaction of these two sorts yields 25 double sorted portfolios. Value-weighted returns are calculated for these 25 portfolios. NYSE quintile breakpoints are calculated each month using only the NYSE stocks.

Table 2.4: Determinants of Correlation Asymmetry: Idiosyncratic Volatility and Ambiguity

Dependent variable: $\text{corr}_{i,t}$	I	II	III	IV	V
$I(r_{i,t} * r_{m,t} > 0)$	-0.01 (-1.40)	-0.00 (-0.75)	-0.11 (-2.48)	-0.19 (-3.44)	-0.18 (-3.41)
$I(r_{i,t} < 0 \& r_{m,t} < 0)$	0.06 (7.49)	0.04 (6.19)	0.20 (6.32)	0.36 (6.28)	0.35 (6.24)
$\text{Log}(\sigma_{m,t})$		0.06 (13.79)	0.10 (14.39)	0.10 (13.60)	0.10 (13.57)
$\text{Log}(\sigma_{i,t})$			-0.16 (-6.62)	-0.17 (-6.27)	-0.17 (-5.99)
$I(r_{i,t} * r_{m,t} > 0) \times \text{log}(\sigma_{i,t})$			-0.02 (-2.52)	-0.02 (-1.77)	-0.02 (-1.49)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{log}(\sigma_{i,t})$			0.04 (5.84)	0.03 (4.78)	0.03 (4.43)
$\text{Log}(\text{Size}_{i,t})$				0.00 (0.08)	0.00 (0.16)
$I(r_{i,t} * r_{m,t} > 0) \times \text{Log}(\text{Size}_{i,t})$				0.01 (1.66)	0.01 (1.63)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{Log}(\text{Size}_{i,t})$				-0.01 (-3.75)	-0.01 (-3.65)
$\text{Log}(\text{Disp}_{i,t})$					0.00 (0.80)
$I(r_{i,t} * r_{m,t} > 0) \times \text{Log}(\text{Disp}_{i,t})$					-0.00 (-0.87)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{Log}(\text{Disp}_{i,t})$					0.01 (1.87)
$\text{corr}_{i,t-1}$	0.49 (17.06)	0.44 (13.26)	0.20 (5.00)	0.19 (5.08)	0.20 (5.16)
Intercept	0.34 (13.44)	0.95 (16.20)	0.75 (13.72)	0.75 (12.16)	0.75 (12.25)
Observations	43,100	43,100	43,100	43,100	43,100
R-squared	0.26	0.32	0.53	0.53	0.53

The table reports estimates from panel regressions, including coefficient estimates and t-statistics (in parentheses). The dependent variable is the monthly correlation with the aggregate market excess return, for 100 portfolios sorted according to idiosyncratic volatility. Each month stocks are sorted into 100 portfolios according to their idiosyncratic volatility and value-weighted returns are calculated for these 100 portfolios. Then the idiosyncratic volatility of those 100 portfolios are estimated relative to 3 factor-model of Fama and French (1993), using daily data over a month. Again using the daily value-weighted returns of 100 portfolios, correlation with the aggregate market excess return is calculated over the month. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for 100 portfolios in cross section. The regressors are as follows: First and second independent variables are dummy variables, the second one identifying the market downturns. If the condition in the parenthesis is satisfied the dummy variable takes the value of one and otherwise zero. Other regressors are logarithm of aggregate market volatility, logarithm of idiosyncratic volatility for portfolio  $i$ , logarithm of market capitalization(size), and the lagged correlation ( the first lag of the regressand). There are also two interaction terms, representing the interaction of the dummy variable identifying the market downturns with the size and the idiosyncratic volatility variable. Estimates from four different specifications are reported, where the standard errors are clustered for time and cross sectional dependence, with the method proposed by Petersen (2009). Return variables are in excess of risk free rate which is approximated by the one-month Treasury bill rate. As is in the literature, the return variables have been standardized so that each variable has a mean zero and a standard deviation of one.

Table 2.5: Determinants of Correlation Asymmetry: Liquidity

Dependent variable: $\text{corr}_{i,t}$	I	II	III	IV	V
$I(r_{i,t} * r_{m,t} > 0)$	0.02 (2.29)	0.02 (3.77)	0.02 (3.16)	-0.08 (-2.10)	-0.12 (-2.23)
$I(r_{i,t} < 0 \& r_{m,t} < 0)$	0.07 (8.47)	0.05 (7.97)	0.05 (7.60)	0.23 (6.63)	0.35 (6.15)
$\text{Log}(\sigma_{m,t})$		0.06 (13.47)	0.06 (13.50)	0.10 (14.80)	0.09 (9.79)
$\text{Log}(\sigma_{i,t})$				-0.15 (-6.78)	-0.14 (-5.93)
$I(r_{i,t} * r_{m,t} > 0) \times \text{log}(\sigma_{i,t})$				-0.02 (-2.26)	-0.01 (-1.25)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{log}(\sigma_{i,t})$				0.04 (5.98)	0.03 (4.14)
$\text{Log}(\text{Size}_{i,t})$					0.01 (1.93)
$I(r_{i,t} * r_{m,t} > 0) \times \text{Log}(\text{Size}_{i,t})$					0.00 (1.09)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times \text{Log}(\text{Size}_{i,t})$					-0.01 (-2.96)
$LIQ_t$	-0.40 (-7.63)	-0.02 (-0.30)	-0.01 (-0.06)	0.05 (0.53)	-0.01 (-0.05)
$I(r_{i,t} * r_{m,t} > 0) \times LIQ_t$			-0.08 (-0.78)	-0.21 (-2.02)	-0.21 (-2.01)
$I(r_{i,t} < 0 \& r_{m,t} < 0) \times LIQ_t$			0.11 (1.06)	0.16 (1.95)	0.16 (1.84)
$\text{corr}_{i,t-1}$	0.52 (20.77)	0.46 (14.90)	0.46 (14.86)	0.25 (6.80)	0.23 (8.14)
Intercept	0.27 (12.05)	0.96 (14.32)	0.96 (14.35)	0.70 (14.49)	0.48 (4.69)
Observations	57,560	57,560	57,560	57,560	57,560
R-squared	0.33	0.38	0.38	0.54	0.54

The table reports estimates from panel regressions, including coefficient estimates and t-statistics (in parenthesis). The dependent variable is the monthly correlation with the aggregate market excess return, for 100 portfolios sorted according to idiosyncratic volatility. Each month stocks are sorted into 100 portfolios according to their idiosyncratic volatility and value-weighted returns are calculated for these 100 portfolios. Then the idiosyncratic volatility of those 100 portfolios are estimated relative to 3 factor-model of Fama and French (1993), using daily data over a month. Again using the daily value-weighted returns of 100 portfolios, correlation with the aggregate market excess return is calculated over the month. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for 100 portfolios in cross section. The regressors are as follows: First and second independent variables are dummy variables, the second one identifying the market downturns. If the condition in the parenthesis is satisfied the dummy variable takes the value of one and otherwise zero. Other regressors are logarithm of aggregate market volatility, logarithm of idiosyncratic volatility for portfolio i, logarithm of market capitalization(size), and the lagged correlation ( the first lag of the regressand). There are also two interaction terms, representing the interaction of the dummy variable identifying the market downturns with the size and the idiosyncratic volatility variable.  $LIQ_t$  is the market-wide liquidity factor of Pástor and Stambaugh (2003).<sup>21</sup> Estimates from four different specifications are reported, where the standard errors are clustered for time and cross sectional dependence, with the method proposed by Petersen (2009). Return variables are in excess of risk free rate which is approximated by the one-month Treasury bill rate. As is in the literature, the return variables have been standardized so that each variable has a mean zero and a standard deviation of one.

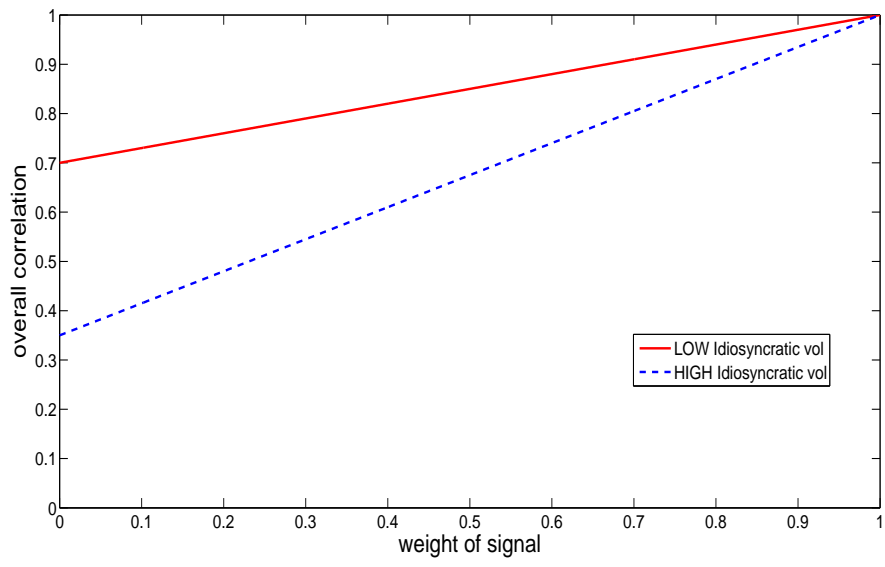


FIGURE 2.1: Decomposing Correlation

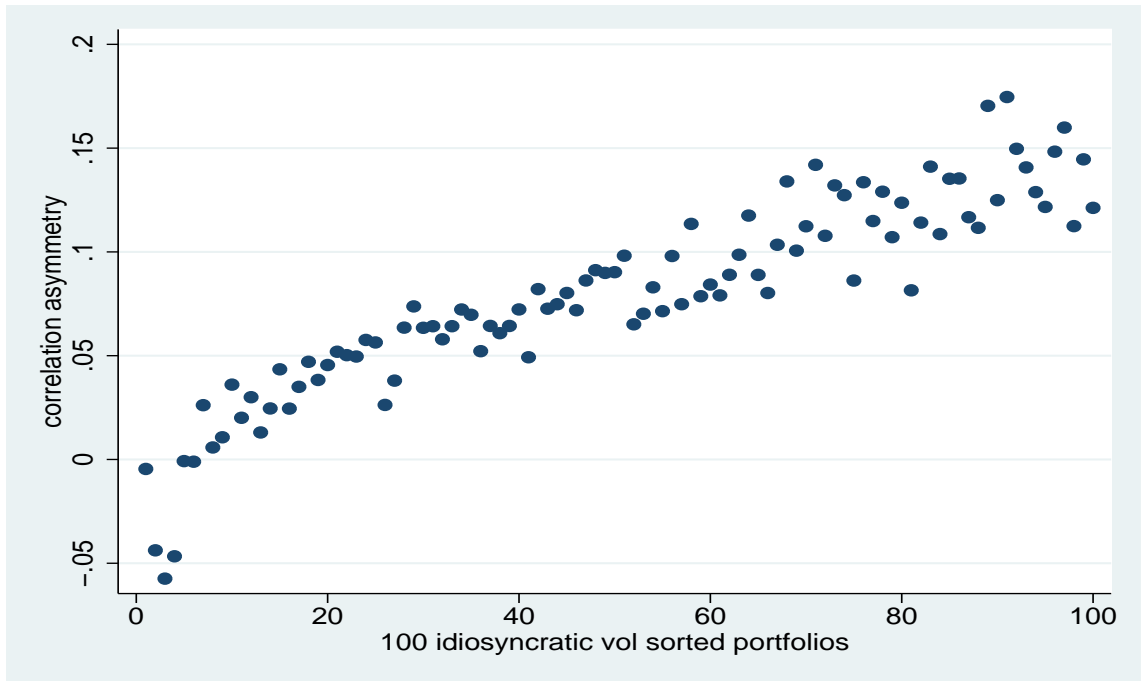


FIGURE 2.2: Idiosyncratic Volatility Sorted Portfolios: Each month stocks are sorted into 100 portfolios according to their idiosyncratic volatility and value-weighted returns are calculated for these 100 portfolios. The idiosyncratic volatilities are estimated relative to 3 factor-model of Fama and French (1993), using daily data over a month. Using the daily value-weighted returns of 100 portfolios, correlation with the aggregate market excess return is calculated over the month. Then for each portfolio, correlations are averaged over market downturns ( $r_i < 0, r_m < 0$ ) and over market upturns ( $r_i > 0, r_m > 0$ ). The difference is plotted as the correlation asymmetry of portfolio. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for 100 portfolios in cross section.

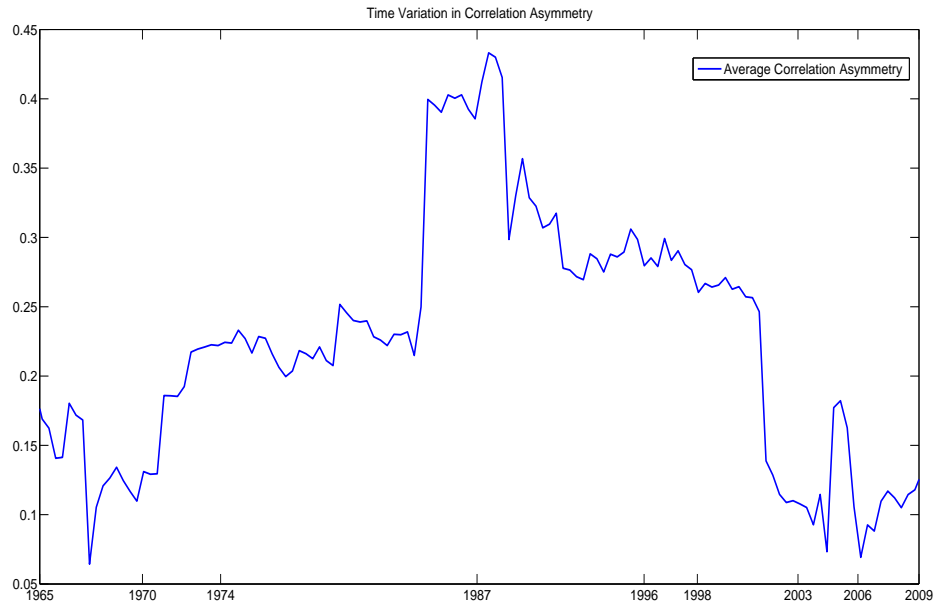


FIGURE 2.3: Time Variation in Correlation Asymmetry: Each month stocks are sorted into 100 portfolios according to their idiosyncratic volatility and value-weighted returns are calculated for these 100 portfolios. The idiosyncratic volatilities are estimated relative to 3 factor-model of Fama and French (1993), using daily data over a month. Using the daily value-weighted returns of 100 portfolios, correlation with the aggregate market excess return is calculated over the month. Then for each portfolio, correlations are averaged over market downturns ( $r_i < 0, r_m < 0$ ) and over market upturns ( $r_i > 0, r_m > 0$ ). The difference is plotted as the correlation asymmetry of portfolio. Monthly data spans the period from July, 1963 to December, 2011 (582 observations), for 100 portfolios in cross section.



# Correlation Asymmetry in Government Bond Markets

## 3.1 Introduction

In this chapter I study the correlation of returns in government bond markets. Correlation of stock returns has been widely studied and they have been shown to be asymmetric: the correlations of stock markets are higher during the downside movements than during the upside movements.<sup>1</sup> For instance, Longin and Solnik (2001) show that the correlation of returns to the U.S. aggregate stock market and the U.K. aggregate stock market is more correlated during joint downside movements – when both of the returns are below their average levels – than during joint upside movements – when both of the returns are above their average levels. Ang and Chen (2002), among others, make a similar observation for the U.S. stock markets. Looking at the correlations between U.S. stocks and the aggregate U.S. stock market conditional on downside and upside moves, they provide evidence against the symmetry of correlations at daily, weekly and monthly frequencies. They also show that

---

<sup>1</sup> See Lin et al. (1994), Longin and Solnik (1995), Karolyi and Stulz (1996), and Das and Uppal (2005).

asymmetric conditional correlations are fundamentally different from other measures of asymmetries, such as skewness and co-skewness. Correlation asymmetry is also observed in other financial markets. For example, Patton (2006) shows that the mark-dollar and yen-dollar exchange rates are more correlated when they are depreciating against the dollar than when they are appreciating. One may question how widespread the asymmetric correlations across financial markets or whether they are specific to certain financial markets.

Studying correlations in government bond markets is important for several reasons. Before discussing the reasons behind that, I discuss the importance of correlation in financial markets in general. An investor who cares about risk should potentially care about correlation of financial instruments in her portfolio. Because the amount of dependency between the financial instruments determines how much diversification benefit is possible. Therefore an increase in correlation implies diminished diversification benefits.

Why should we study correlation of government bond returns? First, relative to stock or exchange rates, markets for government bonds are more relevant for many investors. Legal restrictions, informational or other types of frictions form barriers for investors to enter the stock and exchange rate markets. For instance, some investment funds, retirement and saving funds tend to have restrictions on what kind of investment they can make, how much risk they can take. For those institutions government bonds are the main interest. Government bonds are more attractive from the risk and liquidity perspective. Therefore government bonds are held by many investors, and correlations properties of bonds matter for those investors.

Studying the correlation asymmetry in government bond markets has further merits on its own as well. It is important to understand whether the correlation asymmetry observed in other financial markets extend to the bond markets as well. Observing correlation asymmetry in bonds markets along with stock and exchange

rate markets would point to a common factor causing asymmetric correlations in different financial markets. On the other hand if the bond markets differ in terms of correlation asymmetry from stock markets and exchange rate markets that would put more restriction to potential explanations.

The rest of the chapter is organized as follows: The next section includes the description of the data and the methodology.

## 3.2 Data and Methodology

I use daily data on international bond returns provided by Datastream. The data on daily equity returns also comes from Datastream. Similar to Ang and Chen (2002) I use normalized, continuously compounded returns data, and in the case of equity returns the data is in excess of risk free return. The data spans the period from January 1976 to March 2010. For each country the data on government bonds is provided in different categories depending on the maturity of bonds. The first category includes the bonds with maturity from 1 to 3 years. The bonds with maturity of more than 10 years form the last category. For the maturities in between, Datastream provides data in three categories: 3 to 5 years of maturity, 5 to 7 and 7 to 10 years of maturity. Each category is formed by weighting different bonds according to their volume in the market. For example a country may have two different bonds being traded in the market, with maturities of three and a half years and five years. Both of these bonds will be in the second category and their returns are weighted according to their volumes. Datastream also provides an aggregate measure for each country which is calculated by weighting all outstanding bonds of different maturities and forming a single price index.

To test for correlation asymmetry I apply the standard methods in the literature. I use the exceedance (threshold) correlations to facilitate comparison with

other papers.<sup>2</sup> In the simplest version of this approach, two separate correlations are calculated for two subsamples, and these two correlation estimates are tested for statistically significant difference. The name exceedance (or threshold) refers to the criteria to choose subsamples: Observations across subsamples are sorted so that levels of the returns are above or below some threshold level. More specifically,  $\rho^-$  represents the correlation during downturns, when both of the excess returns are below their means  $\rho^- = \rho(r_i, r_m | r_i < 0, r_m < 0)$  and  $\rho^+$  represent the correlation during market upturns  $\rho^+ = \rho(r_i, r_m | r_i > 0, r_m > 0)$ .  $r_i$  and  $r_m$  are the excess returns to portfolio  $i$  and to the aggregate market respectively, and they are both standardized, as is common in the literature. The excess return is derived by subtracting the one-month Treasury bill rate from the monthly return.

Given the two correlation estimates, we want to test to see whether the estimates are statistically different. The null hypothesis of symmetric correlation is

$$H_0 : \rho^+ = \rho^-$$

That is, if we fail to reject the null hypothesis, it means that the correlation estimates are equal across times of joint upward moves and joint downward moves. The alternative hypothesis is

$$H_A : \rho^+ \neq \rho^-$$

Hong et al. (2007) develops the asymptotic distribution of the test statistics under the null hypothesis of symmetry. The test is similar to the Wald test (Hansen (1982)) in generalized method of moments (GMM) framework but utilizes conditional moment conditions rather than unconditional ones.<sup>3</sup> In the next subsection I test

<sup>2</sup> Ang and Chen (2002), Hong et al. (2007) and Longin and Solnik (2001), among others, use exceedance correlations although their statistical tests which compare the exceedance correlations differ. Andersen et al. (2001) provide a regression based way to test for a correlation asymmetry, which I utilize as well.

<sup>3</sup> The test of Hong et al. (2007) is more general and allows other threshold levels than zero.

for a correlation asymmetry between the size sorted portfolios using the test statistic developed by Hong et al. (2007).

### 3.3 International Bond Return Correlations

I start with looking at the correlation between the returns to the US government bonds and to other countries' bonds. Governments tend to issue bonds at different maturities and bonds of different maturities may be exposed to different factors or may be exposed to the same factors but with different sensitivities. For example long-maturity bonds are relatively more sensitive to discount rates compared to short-maturity bonds. Thus I analyze them separately. Similar to the literature on the asymmetric correlations of equities, I consider the US to be the base country and study the correlation of different countries' bond returns with the US bond returns. However, in the following sections other countries are also going to be considered as the base country. For instance, in Section 3.3.1 as we study the correlation in European government bond markets correlations with the French and German government bonds are analyzed.

The results for the shortest-maturity bonds are reported in Table 3.1. In the data that I use, the shortest maturity corresponds to maturities from 1 to 3 years. Using the statistical test of Hong et al. (2007), which is described in the previous section, I try to understand whether the international bond returns become more correlated with the US bond returns during the downside movements. In the first column of Table 3.1 I have the names of countries. The correlation of bond returns in bear markets ( $\rho_N$ , both returns are negative) and the correlations conditional on bull markets ( $\rho_P$ ) are shown in the second and the third columns respectively. From the table we can see that for half of the countries in the sample correlations are

---

However, there is no theoretical guidance about how to choose the threshold levels, so I only consider zero as threshold level, which is also studied, by Ang and Chen (2002) and Hong et al. (2007).

asymmetric, i.e.  $\rho_N > \rho_P$ . Correlation of Italian government bonds, for instance, is 8 basis point during upside moves while it rises to 22 basis points for downside movements. To visualize the correlation asymmetry I plot the figure in the spirit of Ang and Chen (2002). Figure 3.1 plots the correlation asymmetry for Austria, Belgium and Italy. Tables 3.2 and 3.3 reports the results for longer-maturity bonds, and as it is clear from the tables the correlation asymmetry disappears. In Table 3.4 results are reported for the composite return index which is constructed from traded government bonds of all maturities. As it can be seen from the table, the longer maturities dominates and the asymmetry observed in the correlation of short-maturity bonds disappear when aggregated with longer-maturity bonds. Proposing an explanation for why the correlations of long-maturity bonds are different than the short-maturity bonds in term of the studied asymmetry is beyond the scope of the current paper and therefore is left for future research.

### *3.3.1 Correlation Asymmetry: The Case of Europe*

In this subsection I study the correlation asymmetry of bond returns for countries in Europe. Focusing on the European bond markets has important advantages. First, in terms of “flight to quality phenomenon”, where investors move capital from equities to safer assets during financial turmoils, the U.S. government bonds may be peculiar. Since we studied the correlation of bond returns with the U.S. bond returns in the previous section, the results can be specific to the U.S. bonds. In that respect, by focusing the European bond markets, we study countries similar in terms of how they are affected from the “flight to quality phenomenon”.

Table 3.5 depicts the results of correlation asymmetry test for European government bonds with maturities from 1 to 3 years. The base country is France, so the table reports the correlation of different European countries’ bonds with the French bonds. As is in the previous section for each country two correlation level are esti-

mated, one for the downside movements,  $\rho_N$  and one for the upside movements,  $\rho_P$ . The results are consistent with the previous section: For half of the countries studied correlations are asymmetric in the short-maturity bonds. Government bonds of Austria, Belgium, Portugal and Sweden have asymmetric correlation with French bonds as they had with the U.S. bonds. In Tables 3.6 and 3.7 the correlation asymmetry with the French bond returns are studied using the longer-maturity bonds. Consistent with the previous section, correlations seem to more be symmetric for long-maturity bonds compared to short-maturity bonds.

European bond markets is also interesting for studying correlations due the formation of the monetary union. As noted in the literature, i.e. Cappiello et al. (2006), formation of the common currency area caused structural breaks in correlation levels. How should we think about the launch of Euro as a common currency in terms of the consequences for correlations in European bond markets? In some way we can consider the adoption of common currency as elimination of some idiosyncratic risk since bonds denominated in different currencies were exposed to risks associated with the currencies. For instance investors may have different expected depreciation rates for different currencies and this leads to idiosyncratic risks associated with different currencies. With the launch of Euro this channel of idiosyncracies is eliminated and we actually see that the correlation of bond returns increased with the common currency area.

To understand the effects of a common currency I apply subsample analysis. Tables 3.9 and 3.10 collect the results for correlation asymmetry test for pre and post-Euro area respectively. As it can be seen from Table 3.9 almost all of the countries studied have asymmetric correlations with the French bond returns. From Table 3.10 it is clear that the asymmetry disappears in the post-Euro sample. Tables 3.11 and 3.12 show that the same result hold when the base country is Germany rather than France.

### 3.4 Bond-Equity Return Correlations

The correlation between the returns of bonds and stocks plays a key role in studying the asset allocation and investment strategies. As shown in Engle (2002) the correlation between the US bond returns and the US stock returns is time varying and very volatile. One possible cause for that can be the asymmetry in conditional correlations. If conditional correlation asymmetry exists we would expect correlations to increase as we move from bull markets to bear markets. To explore that channel I look at the conditional correlation of bond and stock returns in different countries.

Results for the correlation between bond and stock returns are collected in Tables 3.17 and 3.18. For each country downside and upside correlations between bonds and equity returns are calculated and tested for the correlation asymmetry. In table 3.17 the shorter-maturity bonds are used while in Table 3.18 returns to longer-maturity bonds and equity returns are studied. The correlation between equity returns and shorter-maturity bond returns seem to be symmetric. However, with longer-maturity bond returns for half of the countries in the sample correlations are found to be asymmetric. For instance, the correlation between the U.S. equity returns and the long-maturity U.S. bond returns increases from 0.21 to 0.29, which is statistically significant at 10 percent level. These results suggest that the change in conditional correlations can account to some extent for the time variation in bond-equity correlations documented by Engle (2002). However, the conditional correlations between bond and equity returns are always non-negative. Therefore, there must be reasons beyond conditional correlations to account for the negative equity-bond correlation observed during certain times and documented by Engle (2002).

### 3.5 Conclusion

In this chapter I study the correlation of returns in government bond markets. Similar



to the findings in equity markets, I show that there is some evidence for asymmetric correlations in government bond markets. First, I show that the maturity structure matters for correlation asymmetry in bonds markets: Unlike long-maturity bonds, shorter-maturity bonds tend to have asymmetric correlations. Second, I show that the correlation asymmetry observed in European bond markets disappears with the formation of a common currency area. Lastly, I study the correlation between equity and bond returns in different countries. For long-maturity bonds, correlations with the domestic equity returns are asymmetric for half of the countries in the sample, including the U.S. These findings show that results on asymmetric correlations from equity markets can generalize, at least to some extent, to other financial markets.

### 3.6 Tables and Figures

Table 3.1: Correlation Asymmetry for Bonds with 1 to 3 Years of Maturity

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.25	0.14	0.04
Belgium	0.29	0.18	0.02
Canada	0.46	0.41	0.26
Denmark	0.06	-0.02	0.00
France	0.23	0.30	0.42
Germany	0.27	0.26	0.94
Greece	0.11	0.06	0.06
Ireland	0.17	0.20	0.55
Italy	0.22	0.08	0.00
Japan	0.02	0.06	0.05
Netherlands	0.31	0.26	0.29
Portugal	0.27	0.13	0.01
Spain	0.30	0.21	0.11
Sweden	0.12	0.05	0.01
United Kingdom	0.22	0.18	0.29

Table report the result of the correlation asymmetry test between the US government bond return and the bond return of the corresponding countries. Bonds with 1 to 3 years of maturity remaining are used for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{us} | r_i < 0, r_{us} < 0)$  and  $\rho^+ = \rho(r_i, r_{us} | r_i > 0, r_{us} > 0)$ .  $r_i$  and  $r_{us}$  are the return to country  $i$  and to the US government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.2: Correlation Asymmetry for Bonds with 7 to 10 Years of Maturity

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.29	0.30	0.90
Belgium	0.30	0.29	0.96
Canada	0.54	0.50	0.28
Denmark	0.14	0.10	0.22
France	0.26	0.27	0.68
Germany	0.26	0.33	0.12
Greece	0.11	0.12	0.86
Ireland	0.23	0.26	0.49
Italy	0.20	0.18	0.62
Japan	0.06	0.06	0.96
Netherlands	0.33	0.38	0.39
Portugal	0.26	0.27	0.79
Spain	0.26	0.21	0.12
Sweden	0.19	0.21	0.53
United Kingdom	0.28	0.30	0.66

Table report the result of the correlation asymmetry test between the US government bond return and the bond return of the corresponding countries. Bonds with 7 to 10 years of maturity remaining are used for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{us} | r_i < 0, r_{us} < 0)$  and  $\rho^+ = \rho(r_i, r_{us} | r_i > 0, r_{us} > 0)$ .  $r_i$  and  $r_{us}$  are the return to country i and to the US government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.3: Correlation Asymmetry for Bonds with more than 10 Years of Maturity

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.31	0.30	0.81
Belgium	0.32	0.31	0.77
Canada	0.53	0.54	0.93
Denmark	0.16	0.19	0.40
France	0.25	0.28	0.57
Germany	0.25	0.30	0.27
Greece	0.14	0.19	0.17
Ireland	0.22	0.24	0.59
Italy	0.18	0.12	0.07
Japan	0.12	0.07	0.17
Netherlands	0.34	0.35	0.84
Portugal	0.33	0.15	0.00
Spain	0.23	0.27	0.30
Sweden	0.21	0.22	0.82
United Kingdom	0.31	0.34	0.49

Table report the result of the correlation asymmetry test between the US government bond return and the bond return of the corresponding countries. Bonds with more than 10 years of maturity remaining are used for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{us} | r_i < 0, r_{us} < 0)$  and  $\rho^+ = \rho(r_i, r_{us} | r_i > 0, r_{us} > 0)$ .  $r_i$  and  $r_{us}$  are the return to country i and to the US government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.4: Correlation Asymmetry for Bonds with all Years of Maturity

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.29	0.31	0.61
Belgium	0.31	0.28	0.48
Canada	0.52	0.51	0.62
Denmark	0.15	0.10	0.06
France	0.26	0.29	0.51
Germany	0.27	0.34	0.22
Greece	0.10	0.12	0.41
Ireland	0.26	0.26	0.91
Italy	0.23	0.22	0.77
Japan	0.06	0.07	0.82
Netherlands	0.34	0.36	0.77
Portugal	0.30	0.27	0.55
Spain	0.29	0.27	0.54
Sweden	0.17	0.19	0.42
United Kingdom	0.30	0.32	0.59

Table report the result of the correlation asymmetry test between the US government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{us} | r_i < 0, r_{us} < 0)$  and  $\rho^+ = \rho(r_i, r_{us} | r_i > 0, r_{us} > 0)$ .  $r_i$  and  $r_{us}$  are the return to country i and to the US government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.5: Correlation Asymmetry for Bonds with 1 to 3 Years of Maturity, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.50	0.42	0.06
Belgium	0.53	0.42	0.01
Denmark	0.24	0.24	0.97
Germany	0.45	0.54	0.14
Greece	0.27	0.35	0.06
Ireland	0.24	0.23	0.74
Italy	0.46	0.49	0.64
Netherlands	0.56	0.49	0.09
Portugal	0.52	0.37	0.00
Spain	0.59	0.49	0.05
Sweden	0.34	0.24	0.02
United Kingdom	0.24	0.32	0.12

Table report the result of the correlation asymmetry test between the France government bond return and the bond return of the corresponding countries. Bonds with 1 to 3 years of maturity remaining are used for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country i and to the France government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.6: Correlation Asymmetry for Bonds with 7 to 10 Years of Maturity, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.58	0.50	0.03
Belgium	0.67	0.54	0.00
Denmark	0.40	0.37	0.60
Germany	0.65	0.59	0.18
Greece	0.26	0.36	0.01
Ireland	0.51	0.45	0.08
Italy	0.59	0.50	0.01
Netherlands	0.73	0.65	0.05
Portugal	0.63	0.53	0.01
Spain	0.59	0.49	0.01
Sweden	0.47	0.34	0.01
United Kingdom	0.51	0.44	0.09

Table report the result of the correlation asymmetry test between the France government bond return and the bond return of the corresponding countries. Bonds with 7 to 10 years of maturity remaining are used for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country i and to the France government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.7: Correlation Asymmetry for Bonds with more than 10 Years of Maturity, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.85	0.81	0.50
Belgium	0.80	0.74	0.26
Denmark	0.52	0.44	0.13
Germany	0.72	0.67	0.29
Greece	0.32	0.47	0.00
Ireland	0.52	0.48	0.27
Italy	0.64	0.53	0.02
Netherlands	0.81	0.76	0.27
Portugal	0.63	0.54	0.30
Spain	0.72	0.65	0.15
Sweden	0.50	0.43	0.10
United Kingdom	0.54	0.41	0.00

Table report the result of the correlation asymmetry test between the France government bond return and the bond return of the corresponding countries. Bonds with more than 10 years of maturity remaining are used for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country  $i$  and to the France government bond, respectively, and they are both standardized, as it is common in literature.



Table 3.8: Correlation Asymmetry for Bonds with All Years Maturity, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.62	0.55	0.08
Belgium	0.66	0.56	0.00
Denmark	0.42	0.38	0.31
Germany	0.69	0.66	0.46
Greece	0.26	0.42	0.00
Ireland	0.57	0.51	0.08
Italy	0.64	0.55	0.04
Netherlands	0.72	0.63	0.02
Portugal	0.68	0.57	0.01
Spain	0.69	0.60	0.03
Sweden	0.40	0.34	0.05
United Kingdom	0.52	0.45	0.11

Table report the result of the correlation asymmetry test between the France government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country i and to the France government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.9: Correlation Asymmetry Europe, pre-Euro, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.62	0.55	0.08
Belgium	0.66	0.56	0.00
Denmark	0.42	0.38	0.31
Germany	0.69	0.66	0.46
Greece	0.26	0.42	0.00
Ireland	0.57	0.51	0.08
Italy	0.64	0.55	0.04
Netherlands	0.72	0.63	0.02
Portugal	0.68	0.57	0.01
Spain	0.69	0.60	0.03
Sweden	0.40	0.34	0.05
United Kingdom	0.52	0.45	0.11

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *before* the launch of Euro, from January, 1st 1985 to January, 1st 2002. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country i and to the French government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.10: Correlation Asymmetry Europe, post-Euro, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.96	0.96	0.97
Belgium	0.96	0.94	0.68
Denmark	0.48	0.43	0.31
Germany	0.97	0.96	0.90
Greece	0.24	0.43	0.00
Ireland	0.74	0.82	0.12
Italy	0.84	0.81	0.58
Netherlands	0.98	0.98	0.99
Portugal	0.73	0.74	0.81
Spain	0.92	0.92	0.96
Sweden	0.62	0.61	0.81
United Kingdom	0.69	0.64	0.32

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *after* the launch of Euro, from January, 1st 2002 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country i and to the French government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.11: Correlation Asymmetry Europe, pre-Euro, Germany

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.74	0.63	0.09
Belgium	0.66	0.52	0.01
Denmark	0.55	0.33	0.00
France	0.52	0.49	0.58
Greece	0.51	0.47	0.64
Ireland	0.34	0.27	0.10
Italy	0.52	0.37	0.03
Netherlands	0.65	0.58	0.24
Portugal	0.59	0.36	0.00
Spain	0.55	0.38	0.00
Sweden	0.37	0.28	0.07
United Kingdom	0.33	0.31	0.80

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *before* the launch of Euro, from January, 1st 1985 to January, 1st 2002. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{gr} | r_i < 0, r_{gr} < 0)$  and  $\rho^+ = \rho(r_i, r_{gr} | r_i > 0, r_{gr} > 0)$ .  $r_i$  and  $r_{gr}$  are the return to country i and to the German government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.12: Correlation Asymmetry Europe, post-Euro, Germany

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.94	0.94	0.97
Belgium	0.93	0.91	0.66
Denmark	0.51	0.44	0.21
France	0.97	0.96	0.90
Greece	0.35	0.42	0.16
Ireland	0.77	0.81	0.45
Italy	0.81	0.80	0.93
Netherlands	0.97	0.97	0.94
Portugal	0.77	0.74	0.50
Spain	0.92	0.90	0.72
Sweden	0.65	0.61	0.57
United Kingdom	0.70	0.65	0.41

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *after* the launch of Euro, from January, 1st 2002 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{gr} | r_i < 0, r_{gr} < 0)$  and  $\rho^+ = \rho(r_i, r_{gr} | r_i > 0, r_{gr} > 0)$ .  $r_i$  and  $r_{gr}$  are the return to country i and to the German government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.13: Correlation Asymmetry Europe, pre-Euro, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.62	0.55	0.08
Belgium	0.66	0.56	0.00
Denmark	0.42	0.38	0.31
Germany	0.69	0.66	0.46
Greece	0.26	0.42	0.00
Ireland	0.57	0.51	0.08
Italy	0.64	0.55	0.04
Netherlands	0.72	0.63	0.02
Portugal	0.68	0.57	0.01
Spain	0.69	0.60	0.03
Sweden	0.40	0.34	0.05
United Kingdom	0.52	0.45	0.11

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *before* the launch of Euro, from January, 1st 1985 to January, 1st 1999. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country i and to the French government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.14: Correlation Asymmetry Europe, post-Euro, France

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.91	0.92	0.90
Belgium	0.94	0.91	0.58
Denmark	0.51	0.46	0.20
Germany	0.93	0.93	0.99
Greece	0.26	0.42	0.00
Ireland	0.73	0.80	0.13
Italy	0.80	0.78	0.73
Netherlands	0.94	0.96	0.76
Portugal	0.72	0.72	0.98
Spain	0.89	0.89	0.88
Sweden	0.62	0.60	0.72
United Kingdom	0.66	0.61	0.33

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *after* the launch of Euro, from January, 1st 1999 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{fr} | r_i < 0, r_{fr} < 0)$  and  $\rho^+ = \rho(r_i, r_{fr} | r_i > 0, r_{fr} > 0)$ .  $r_i$  and  $r_{fr}$  are the return to country i and to the French government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.15: Correlation Asymmetry Europe, pre-Euro, Germany

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.68	0.54	0.09
Belgium	0.62	0.46	0.01
Denmark	0.51	0.31	0.00
France	0.48	0.47	0.87
Ireland	0.45	0.32	0.11
Italy	0.66	0.55	0.17
Netherlands	0.52	0.27	0.01
Portugal	0.41	0.26	0.04
Spain	0.34	0.27	0.23
Sweden	0.31	0.33	0.82
United Kingdom	0.65	0.60	0.31

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *before* the launch of Euro, from January, 1st 1985 to January, 1st 1999. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{gr} | r_i < 0, r_{gr} < 0)$  and  $\rho^+ = \rho(r_i, r_{gr} | r_i > 0, r_{gr} > 0)$ .  $r_i$  and  $r_{gr}$  are the return to country i and to the German government bond, respectively, and they are both standardized, as it is common in literature.



Table 3.16: Correlation Asymmetry Europe, post-Euro, Germany

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.93	0.93	0.98
Belgium	0.89	0.87	0.66
Denmark	0.58	0.48	0.03
France	0.93	0.93	0.99
Greece	0.38	0.43	0.28
Ireland	0.73	0.75	0.64
Italy	0.79	0.78	0.85
Netherlands	0.90	0.91	0.76
Portugal	0.75	0.71	0.38
Spain	0.91	0.88	0.62
Sweden	0.63	0.58	0.37
United Kingdom	0.65	0.60	0.31

Table report the result of the correlation asymmetry test between the German government bond return and the bond return of the corresponding countries. Bonds of different maturity are weighted by their volume to derive a single return for each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period *after* the launch of Euro, from January, 1st 1999 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_i, r_{gr} | r_i < 0, r_{gr} < 0)$  and  $\rho^+ = \rho(r_i, r_{gr} | r_i > 0, r_{gr} > 0)$ .  $r_i$  and  $r_{gr}$  are the return to country i and to the German government bond, respectively, and they are both standardized, as it is common in literature.

Table 3.17: Equity Bond Correlations 1 to 3 Years Maturity

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.06	0.12	0.06
Belgium	0.13	0.09	0.26
Canada	0.18	0.11	0.38
Denmark	0.24	0.20	0.50
France	0.28	0.22	0.58
Germany	0.16	0.08	0.02
Greece	0.34	0.29	0.76
Ireland	0.23	0.25	0.82
Italy	0.26	0.29	0.64
Japan	0.08	0.16	0.10
Netherlands	0.08	0.11	0.18
Portugal	0.29	0.39	0.68
South Africa	0.20	0.09	0.05
Spain	0.18	0.22	0.78
Sweden	0.20	0.28	0.28
Switzerland	0.09	0.05	0.15
United Kingdom	0.26	0.26	0.95
United States	0.19	0.18	0.76

Table report the result of the correlation asymmetry test between the equity market return and the bond return in each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_e, r_b | r_e < 0, r_b < 0)$  and  $\rho^+ = \rho(r_e, r_b | r_e > 0, r_b > 0)$ .  $r_e$  and  $r_b$  are the return on the aggregate equity market and the bond return for each country, and both returns are standardized, as it is common in literature.

Table 3.18: Equity Bond Correlations 10 Years and Longer Maturity

Country	$\rho_N$	$\rho_P$	p-values
Austria	0.17	0.07	0.05
Belgium	0.23	0.14	0.07
Canada	0.22	0.12	0.00
Denmark	0.25	0.10	0.00
France	0.31	0.26	0.27
Germany	0.25	0.14	0.00
Greece	0.19	0.31	0.36
Ireland	0.17	0.21	0.59
Italy	0.25	0.25	1.00
Japan	0.07	0.13	0.13
Netherlands	0.21	0.15	0.26
Portugal	0.05	0.00	0.68
South Africa	0.30	0.13	0.00
Spain	0.21	0.12	0.27
Sweden	0.26	0.26	0.94
Switzerland	0.17	0.09	0.01
United Kingdom	0.32	0.23	0.11
United States	0.29	0.21	0.08

Table report the result of the correlation asymmetry test between the equity market return and the bond return in each country. p-values are based on the test of Hong et al. (2007). Daily data spans the period from January, 1st 1985 to May, 11th 2010. The exceedance correlations are estimated with respect to the exceedance level  $c = 0$ ,  $\rho^- = \rho(r_e, r_b | r_e < 0, r_b < 0)$  and  $\rho^+ = \rho(r_e, r_b | r_e > 0, r_b > 0)$ .  $r_e$  and  $r_b$  are the return on the aggregate equity market and the bond return for each country, and both returns are standardized, as it is common in literature.

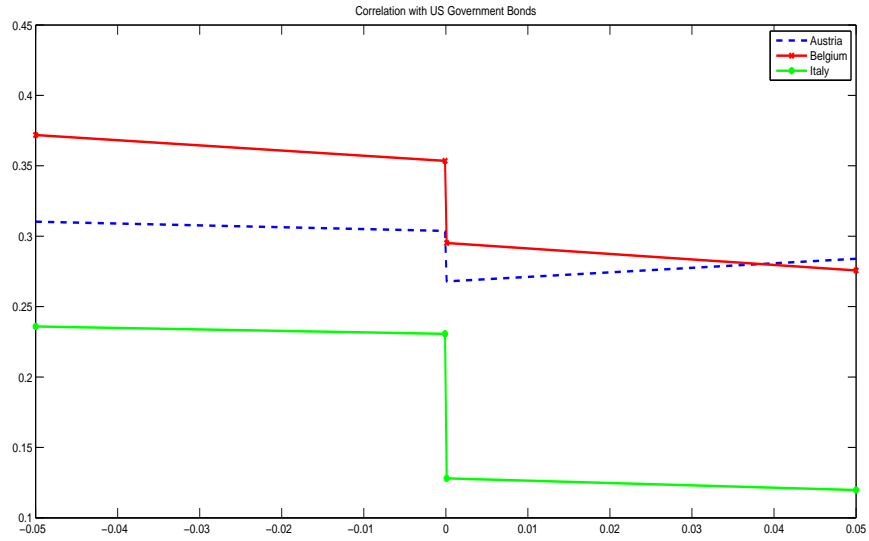


FIGURE 3.1: Correlation Asymmetry in Government Bond Markets

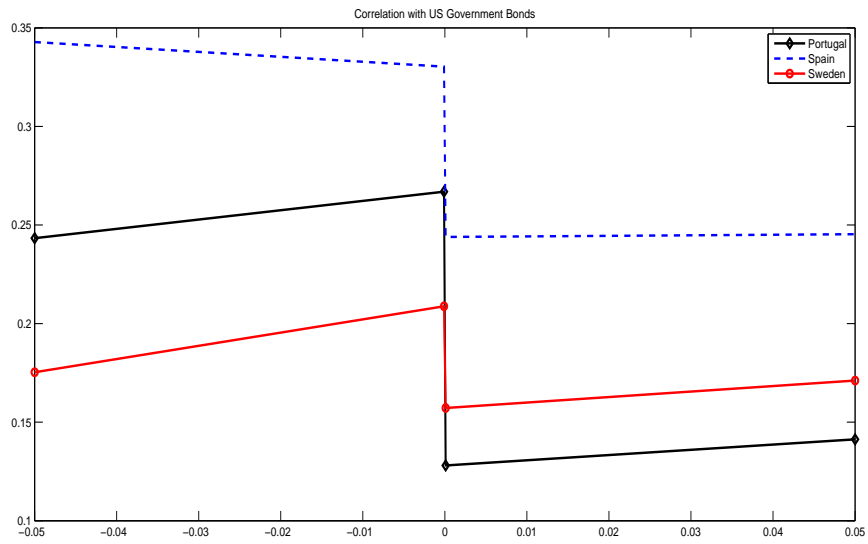


FIGURE 3.2: Correlation Asymmetry in Government Bond Markets

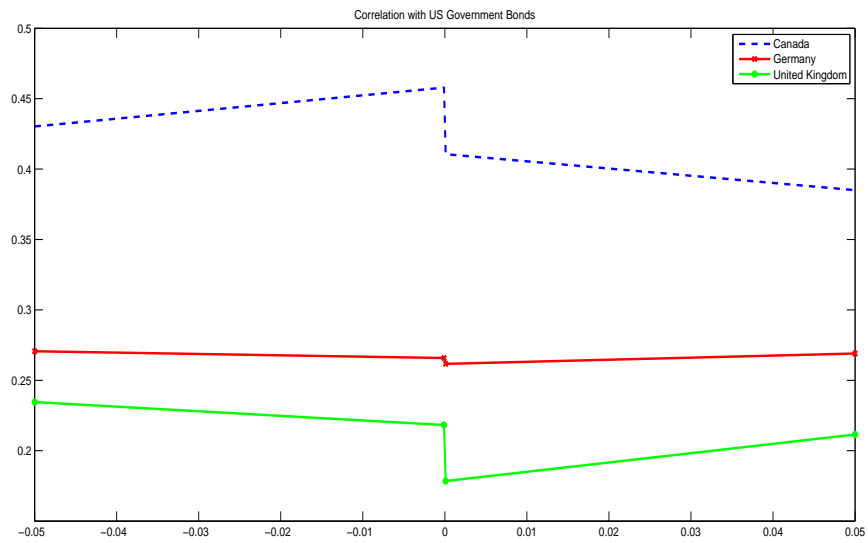


FIGURE 3.3: Correlation Asymmetry in Government Bond Markets

# Appendix A

## Appendix to Chapter 2

### A.1 Model Solution

#### A.1.1 Prices in Stationary Equilibria

Here we derive the prices in stationary equilibrium. We conjecture a time-invariant price function of the type  $q_t = \bar{Q} + Q_d \hat{d}_t + Q_s \eta_t s_t$ . Inserting the guess into equations (2.14)-(2.15) and matching undetermined coefficients will yield the solution. The equations defining the equilibrium prices (equations (2.14)-(2.15) in the text)

$$q_t^i = \min_{(\sigma_{s,t}^2, \sigma_{s,t+1}^2)} \beta \mathbb{E}[q_{t+1}^i + d_{t+1}^i]$$

$$q_t^j = \min_{(\sigma_{s,t}^2, \sigma_{s,t+1}^2)} \beta \mathbb{E}[q_{t+1}^j + d_{t+1}^j]$$

$$q_t^i = \min_{(\sigma_{s,t}^2, \sigma_{s,t+1}^2)} \frac{1}{1+r} \mathbb{E}_t \left[ \underbrace{\bar{Q}^i + Q_d(d_{t+1}^i - \bar{d}) + Q_s \eta_{t+1} s_{t+1}}_{q_{t+1}^i} + d_{t+1}^i \right] \quad (\text{A.1})$$

$$= \min_{(\sigma_{s,t}^2, \sigma_{s,t+1}^2)} \frac{1}{1+r} \mathbb{E}_t \left[ \bar{Q}^i + \underbrace{(Q_d + 1)(d_{t+1}^i - \bar{d})}_{\text{part II}} + \bar{d} + \underbrace{Q_s \eta_{t+1} s_{t+1}}_{\text{part III}} \right] \quad (\text{A.2})$$

For *Part III*;

$$\mathbb{E}[Q_s \eta_{t+1} s_{t+1}] = Q_s \left[ \int_{-\infty}^0 \bar{\eta} s_{t+1} \frac{1}{\sqrt{2\pi \text{var}(s)}} e^{-\frac{1}{2} \frac{s_{t+1}^2}{\text{var}(s)}} ds + \int_0^{\infty} \underline{\eta} s_{t+1} \frac{1}{\sqrt{2\pi \text{var}(s)}} e^{-\frac{1}{2} \frac{s_{t+1}^2}{\text{var}(s)}} ds \right]$$

$$\mathbb{E}[Q_s \eta_{t+1} s_{t+1}] = -Q_s \left[ (\bar{\eta} - \underline{\eta}) \frac{\sqrt{\text{var}(s)}}{\sqrt{2\pi}} \right]$$

For *Part II*;

$$\begin{aligned} (Q_d + 1) [\mathbb{E}_t d_{t+1} - \bar{d}] &= (Q_d + 1) \left[ \kappa \bar{d} + (1 - \kappa) d_t^i + \underbrace{\mathbb{E}_t(u_{t+1} | s_t)}_{\eta_t s_t} - \bar{d} \right] \\ &= (Q_d + 1) [(1 - \kappa)(d_t^i - \bar{d}) + \eta_t s_t] \end{aligned}$$

In order to minimize the right hand side the *Part III* will be evaluated at the upper bound of  $\text{var}(s_t)$ , which is  $\sigma_a^2 + \bar{\sigma}_s^2$ . Using  $\underline{\eta} = \frac{\text{cov}(s_t, u_{t+1})}{\text{var}(s)}$ , we get the following;

$$\begin{aligned} q_t^i &= \bar{Q}^i + Q_d(d_t^i - \bar{d}) + Q_s \eta_t s_t \\ &= \frac{\bar{Q}^i + \bar{d}}{1+r} - \frac{Q_s}{1+r} (\bar{\eta} - \underline{\eta}) \frac{\sigma_a^2}{\sqrt{2\pi \underline{\eta}}} + \underbrace{\frac{1}{1+r} (Q_d + 1) [(1 - \kappa)(d_t^i - \bar{d}) + \eta_t s_t]}_{\text{Part II}} \end{aligned}$$

Now all we need to do is to solve for coefficients;

$$\text{For } Q_d; \quad Q_d = \frac{(Q_d+1)(1-\kappa)}{1+r} \Rightarrow Q_d = \frac{1-\kappa}{r+\kappa}$$

$$\text{For } Q_s; \quad Q_s = \frac{(Q_s+1)}{1+r} \Rightarrow Q_s = \frac{1}{r+\kappa}$$

$$\text{For } \bar{Q}^i; \quad \bar{Q}^i = \frac{\bar{Q}^i + \bar{d}}{1+r} - \frac{Q_s}{1+r} \left[ (\bar{\eta} - \underline{\eta}) \frac{\sigma_a^2}{\sqrt{2\pi\underline{\eta}}} \right]$$

$$\Rightarrow \bar{Q}^i = \frac{\bar{d}}{r} - \frac{1}{r(r+\kappa)} \frac{1}{\sqrt{2\pi}} \left[ (\bar{\eta} - \underline{\eta}) \frac{\sqrt{\sigma_a^2}}{\sqrt{\underline{\eta}}} \right]$$

Thus we have the solution for  $q_t^i$  and the solution for  $q_t^j$  is the same in this case.

### A.1.2 Conditional Correlations

As in the main body, we denote the response coefficient as  $\eta(\sigma_s^2) = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_s^2}$ . Given  $q_t^i$  and  $q_t^j$ , we define the *excess return per share* as  $R_{t+1}^i = q_{t+1}^i + d_{t+1}^i - (1+r)q_t^i$

$$R_{t+1}^i = \frac{1+r}{r+\kappa} (u_{t+1}^i - \eta_t s_t) + \frac{1}{r+\kappa} \eta_{t+1} s_{t+1} + \tilde{Q} \quad (\text{A.3})$$

$$R_{t+1}^j = \frac{1+r}{r+\kappa} (u_{t+1}^j - \eta_t s_t) + \frac{1}{r+\kappa} \eta_{t+1} s_{t+1} + \tilde{Q} \quad (\text{A.4})$$

$$\text{where } \tilde{Q} = \frac{1}{r+\kappa} \frac{1}{\sqrt{2\pi}} \left[ (\bar{\eta} - \underline{\eta}) \frac{\sqrt{\sigma_a^2}}{\sqrt{\underline{\eta}}} \right]$$

$$\text{cov}(R_{t+1}^i, R_{t+1}^j | s_{t+1} \geq 0) = \eta_{t+1}^2 \tau^2 \text{var}(s_{t+1} | s_{t+1} \geq 0) + \tau^2 \Omega_{ij} \quad (\text{A.5})$$

where  $\Omega_{ij}$  is independent of  $s_{t+1}$  but is a function of  $s_t$ . And where  $\tau = \frac{1}{r+\kappa}$ .

$$\text{var}(R_{t+1}^i | s_{t+1} \geq 0) = \eta^2 \tau^2 \text{var}(s_{t+1} | s_{t+1} \geq 0) + \tau^2 \Omega_i \quad (\text{A.6})$$

$$\text{var}(R_{t+1}^j | s_{t+1} \geq 0) = \eta^2 \tau^2 \text{var}(s_{t+1} | s_{t+1} \geq 0) + \tau^2 \Omega_j \quad (\text{A.7})$$



$$\begin{aligned}
\Omega_{ij} &= (1+r)^2 \text{cov}(u_{t+1}^i - \eta_t s_t, u_{t+1}^j - \eta_t s_t) \\
&= (1+r)^2 [\text{cov}(u_{t+1}^i, u_{t+1}^j) - \eta_t \text{cov}(u_{t+1}^i, s_t) - \eta_t \text{cov}(u_{t+1}^j, s_t) + \eta_t^2 \text{var}(s_t)] \\
&= (1+r)^2 [\sigma_a^2 - 2\eta_t \sigma_a^2 + \eta_t^2 \text{var}(s_t)] \\
&= (1+r)^2 (1 - \eta_t) \sigma_a^2
\end{aligned}$$

$$\begin{aligned}
\Omega_i &= (1+r)^2 \text{var}(u_{t+1}^i - \eta_t s_t) \\
&= (1+r)^2 [\text{var}(u_{t+1}^i) - 2\eta_t \text{cov}(u_{t+1}^i, s_t) + \eta_t^2 \text{var}(s_t)] \\
&= (1+r)^2 [\sigma_a^2 + \sigma_i^2 - 2\eta_t \sigma_a^2 + \eta_t^2 \text{var}(s_t)] \\
&= (1+r)^2 [(1 - \eta_t) \sigma_a^2 + \sigma_i^2]
\end{aligned}$$

$$\begin{aligned}
\Omega_j &= (1+r)^2 \text{var}(u_{t+1}^j - \eta_t s_t) \\
&= (1+r)^2 [\text{var}(u_{t+1}^j) - 2\eta_t \text{cov}(u_{t+1}^j, s_t) + \eta_t^2 \text{var}(s_t)] \\
&= (1+r)^2 [\sigma_a^2 + \sigma_j^2 - 2\eta_t \sigma_a^2 + \eta_t^2 \text{var}(s_t)] \\
&= (1+r)^2 [(1 - \eta_t) \sigma_a^2 + \sigma_j^2]
\end{aligned}$$

Defining

$$\Omega_{ij} = (1+r)^2 (1 - \eta_t) \sigma_a^2 \tag{A.8}$$

$$\Omega_i = (1+r)^2 [(1 - \eta_t) \sigma_a^2 + \sigma_i^2] \tag{A.9}$$

$$\Omega_j = (1+r)^2 [(1 - \eta_t) \sigma_a^2 + \sigma_j^2] \tag{A.10}$$

**Lemma 3.**  $\Omega_i > \Omega_{ij}$  and  $\Omega_j > \Omega_{ij}$ .

*Proof:* Obvious from comparing equations (A.8), (A.9) and (A.10) as long as  $\sigma_i^2 > 0$  and  $\sigma_j^2 > 0$ .

### A.1.3 Proof of Proposition 1

Here we describe the strategy and provide the proof for the symmetric case, i.e.  $\sigma_i = \sigma_j$ . The nonsymmetric case is deferred to next subsection.

In the symmetric case, i.e.  $\sigma_i = \sigma_j$ ,  $\Omega_i = \Omega_j$ .

Looking at the complete symmetric case and using the variance and covariance terms from equations (A.5),(A.6) and (A.7)

$$\text{corr}(R_{t+1}^i, R_{t+1}^j | s_{t+1}) = \text{corr}(\eta(\sigma_{s,t+1}^2)) = \frac{\eta^2 \text{varc} + \Omega_{ij}}{\eta^2 \text{varc} + \Omega_i}$$

where  $\text{varc} = \text{var}(s_{t+1} | s_{t+1} \geq 0)$ . The derivative of the correlation function with respect to  $\eta(\sigma_{s,t+1}^2)$  is<sup>1</sup>

$$\frac{2\eta(\sigma_{s,t+1}^2) \text{varc}[\Omega_i - \Omega_{ij}]}{(\eta(\sigma_{s,t+1}^2)^2 \text{varc} + \Omega_i)^2} \quad (\text{A.11})$$

Lemma 3 proves that  $\Omega_i > \Omega_{ij}$ , which implies that the correlation increases with the response coefficient  $\eta$ . Thus we have shown that stronger responses lead to higher conditional correlation.

### A.1.4 Proof of Proposition 2

We can define the correlation asymmetry in percentage terms,  $\log \text{corr}(\bar{\eta}) - \log \text{corr}(\underline{\eta})$ . We need to prove that the asymmetry is greater for assets with high idiosyncratic volatilities. Defining  $\xi = \frac{\sigma_i^2}{\sigma_a^2}$ , we need to show that the asymmetry is increasing in  $\xi$ .

To that end, all we need to prove is  $\frac{\partial^2 \log \text{corr}(\eta)}{\partial \eta \partial \xi} > 0$ . Because,

---

<sup>1</sup> Note that we are taking derivative with respect to  $\eta(\sigma_{s,t+1}^2)$ , not  $\eta(\sigma_{s,t}^2)$ .

$$\frac{\partial}{\partial \xi} [\log \text{corr}(\bar{\eta}) - \log \text{corr}(\underline{\eta})] = \frac{\partial}{\partial \xi} \log \text{corr}(\bar{\eta}) - \frac{\partial}{\partial \xi} \log \text{corr}(\underline{\eta})$$

$$\frac{\partial^2 \log \text{corr}(\eta)}{\partial \eta \partial \xi} \text{ being positive} \implies \frac{\partial}{\partial \xi} \log \text{corr}(\bar{\eta}) > \frac{\partial}{\partial \xi} \log \text{corr}(\underline{\eta})$$

We provide the proof for two cases. In the first case, the assets are assumed to be symmetric. In other words, their idiosyncratic volatilities are the same. This makes the assets identical up to the realization of shocks. The second case is more general and algebraically a little bit more involved. It also completes the proof of Proposition 1 for nonsymmetric case.

**Symmetric case:**  $\sigma_i^2 = \sigma_j^2$

Looking at the complete symmetric case,

$$\text{corr}(R_{t+1}^i, R_{t+1}^j | s_{t+1}) = \text{corr}(\eta(\sigma_{s,t+1}^2)) = \frac{\eta^2 \text{varc} + \Omega_{ij}}{\eta^2 \text{varc} + \Omega_i}$$

$$\log \text{corr}(\eta(\sigma_{s,t+1}^2)) = \log(\eta^2 \text{varc} + \Omega_{ij}) - \log(\eta^2 \text{varc} + \Omega_i)$$

The derivative of the log correlation function with respect to  $\eta(\sigma_{t+1}^2)$  is

$$\frac{\sigma_a^2 [\Omega_i - \Omega_{ij}]}{[\eta(\sigma_{s,t+1}^2)^2 \text{varc} + \Omega_{ij}][\eta(\sigma_{s,t+1}^2)^2 \text{varc} + \Omega_i]} \quad (\text{A.12})$$

Again Lemma 3 proves that  $\Omega_i > \Omega_{ij}$ , which implies that the correlation increases with the response coefficient  $\eta$ . Thus we have shown that stronger responses lead to higher conditional correlation.

Now we show that that slope is steeper for assets with high idiosyncratic variance.

$$\begin{aligned}
\frac{\partial \log \text{corr}(\eta)}{\partial \eta} &= \frac{\sigma_a^2 [\Omega_i - \Omega_{ij}]}{[\eta(\sigma_{s,t+1}^2 \text{varc} + \Omega_{ij})][\eta(\sigma_{s,t+1}^2 \text{varc} + \Omega_i)]} \\
&= \frac{\sigma_a^2 \sigma_i^2 (1+r)^2}{[\eta \sigma_a^2 + (1+r)^2 (\sigma_a^2 - \eta \sigma_a^2)][\eta \sigma_a^2 + (1+r)^2 (\sigma_a^2 + \sigma_i^2 - \eta \sigma_a^2)]} \\
&= \frac{\sigma_a^2 \sigma_i^2 (1+r)^2}{\sigma_a^2 [\eta + (1+r)^2 (1-\eta)][\eta \sigma_a^2 + (1+r)^2 (\sigma_a^2 + \xi \sigma_a^2 - \eta \sigma_a^2)]} \\
&= \frac{\sigma_a^2 \sigma_i^2 (1+r)^2}{\sigma_a^2 \sigma_a^2 [\eta + (1+r)^2 (1-\eta)][\eta + (1+r)^2 (1+\xi - \eta)]} \\
&= \frac{\xi (1+r)^2}{[\eta + (1+r)^2 (1-\eta)][\eta + (1+r)^2 (1+\xi - \eta)]} \\
&= \frac{(1+r)^2}{[\eta + (1+r)^2 (1-\eta)] \left[ \frac{\eta + (1+r)^2 (1-\eta)}{\xi} + (1+r)^2 \right]}
\end{aligned}$$

where  $\xi = \frac{\sigma_i^2}{\sigma_a^2}$  and it is clear from the last equation that  $\frac{\partial \log \text{corr}(\eta)}{\partial \eta}$  is increasing in  $\xi$ . That means the asymmetry (in percentage terms) in conditional correlations is higher for assets with more idiosyncratic volatilities.<sup>2</sup>

**Nonsymmetric case:**  $\sigma_i^2 \neq \sigma_j^2$

$$\text{corr}(\eta(\sigma_{s,t+1}^2)) = \frac{\eta^2 \text{varc} + \Omega_{ij}}{\sqrt{(\eta^2 \text{varc} + \Omega_i)(\eta^2 \text{varc} + \Omega_j)}}$$

$$\implies \log \text{corr}(\eta(\sigma_{s,t+1}^2)) = \log(\eta^2 \text{varc} + \Omega_{ij}) - \frac{1}{2} \log(\eta^2 \text{varc} + \Omega_i) - \frac{1}{2} \log(\eta^2 \text{varc} + \Omega_j)$$

---

<sup>2</sup> To get simple expressions I take the derivate around  $\eta(\sigma_s^{*2})$ , which implies  $\eta \text{varc} = \sigma_a^2$

$$\frac{\partial \log \text{corr}(\eta)}{\partial \eta} = \frac{2\eta \text{ varc} + \Omega_{ij}}{\eta^2 \text{ varc} + \Omega_{ij}} - \frac{1}{2} \frac{2\eta \text{ varc} + \Omega_{ij}}{\eta^2 \text{ varc} + \Omega_i} - \frac{1}{2} \frac{2\eta \text{ varc} + \Omega_{ij}}{\eta^2 \text{ varc} + \Omega_j} \quad (\text{A.13})$$

which is equal to

$$\begin{aligned} &= \eta \text{ varc} \frac{2(\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j)}{(\eta^2 \text{ varc} + \Omega_{ij})(\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j)} \\ &\quad - \eta \text{ varc} \frac{(\eta^2 \text{ varc} + \Omega_{ij})(\eta^2 \text{ varc} + \Omega_j)(\eta^2 \text{ varc} + \Omega_{ij})(\eta^2 \text{ varc} + \Omega_i)}{(\eta^2 \text{ varc} + \Omega_{ij})(\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j)} \\ &= \eta \text{ varc} \frac{(\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j - \eta^2 \text{ varc} - \Omega_{ij})}{(\eta^2 \text{ varc} + \Omega_{ij})(\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j)} \\ &\quad + \eta \text{ varc} \frac{(\eta^2 \text{ varc} + \Omega_j)(\eta^2 \text{ varc} + \Omega_i - \eta^2 \text{ varc} - \Omega_{ij})}{(\eta^2 \text{ varc} + \Omega_{ij})(\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j)} \\ &= \eta \text{ varc} \frac{(\eta^2 \text{ varc} + \Omega_i)(1+r)^2\sigma_j^2 + (\eta^2 \text{ varc} + \Omega_j)(1+r)^2\sigma_i^2}{(\eta^2 \text{ varc} + \Omega_{ij})(\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j)} \end{aligned}$$

Note that the previous line shows that  $\frac{\partial \log \text{corr}(\eta)}{\partial \eta} > 0$ , which completes the proof of Proposition 1 for non symmetric case. Picking up from the last line, we continue to prove Proposition 2.

$$\begin{aligned} \frac{\partial \log \text{corr}(\eta)}{\partial \eta} &= \sigma_a^2 \frac{(\eta^2 \text{ varc} + \Omega_i)(1+r)^2\sigma_j^2 + (\eta^2 \text{ varc} + \Omega_j)(1+r)^2\sigma_i^2}{[\eta + (1+r)^2(1-\eta)](\eta^2 \text{ varc} + \Omega_i)(\eta^2 \text{ varc} + \Omega_j)} \\ &= \frac{(1+r)^2}{[\eta + (1+r)^2(1-\eta)](\eta^2 \text{ varc} + \Omega_j)} \frac{(\eta^2 \text{ varc} + \Omega_i)\sigma_j^2 + (\eta\sigma_a^2 + \Omega_j)\xi\sigma_a^2}{\eta^2 \text{ varc} + \Omega_i} \\ &= \frac{(1+r)^2}{[\eta + (1+r)^2(1-\eta)](\eta^2 \text{ varc} + \Omega_j)} \frac{X(\xi)\sigma_j^2 + (\eta\sigma_a^2 + \Omega_j)\xi\sigma_a^2}{X(\xi)} \end{aligned}$$

where  $X(\xi) = (\eta^2 \text{ varc} + \Omega_i) = \sigma_a^2[\eta + (1+r)^2(1+\xi-2\eta+\eta)]$

Remember that we are trying to show that  $\frac{\partial \log \text{corr}(\eta)}{\partial \eta}$  is increasing in  $\xi$ . The first

ratio in the last line is independent of  $\xi$ , so we analyze the second ratio, which is

$$\frac{X(\xi)\sigma_j^2 + (\eta\sigma_a^2 + \Omega_j)\xi\sigma_a^2}{X(\xi)} = \sigma_j^2 + (\eta\sigma_a^2 + \Omega_j)\sigma_a^2 \frac{\xi}{X(\xi)}$$

In Lemma 4 we show that  $\frac{\xi}{X(\xi)}$  is increasing in  $\xi$ , which completes the proof.

**Lemma 4.**  $\frac{\xi}{X(\xi)}$  is increasing in  $\xi$ .

*Proof:*

$$\text{Derivative of } \frac{\xi}{X(\xi)} \text{ with respect to } \xi: \frac{X(\xi) - X'(\xi)\xi}{X(\xi)^2} = \frac{X(\xi) - X'(\xi)\xi}{X(\xi)^2}$$

$$\text{where } X'(\xi) = \sigma_a^2(1+r)^2$$

$$\begin{aligned} X(\xi) - X'(\xi)\xi &= \sigma_a^2[\eta + (1+r)^2(1+\xi-\eta)] - \xi\sigma_a^2(1+r)^2 \\ &= \sigma_a^2[\eta + (1+r)^2(1-\eta)] > 0 \end{aligned}$$

which implies that  $\frac{\xi}{X(\xi)}$  is increasing in  $\xi$ .

### Characterizing the regions

Here we consider  $R^j$  as the market return, therefore  $\sigma_j^2 = 0$  and the correlation represents the correlation with the market. Due to the assumption  $\sigma_j^2 = 0$ , now  $\Omega_j = \Omega_{ij}$  and  $\Omega_i = \Omega_j + (1+r)^2\sigma_i^2$

$$\begin{aligned} \text{corr}(R_{t+1}^i, R_{t+1}^j | s_{t+1}) &= \text{corr}(\eta(\sigma_{s,t+1}^2)) = \frac{\eta^2 \text{varc} + \Omega_j}{\sqrt{(\eta^2 \text{varc} + \Omega_j)(\eta^2 \text{varc} + \Omega_j + (1+r)^2\sigma_i^2)}} \\ &= \frac{\eta^2 \text{varc} + \Omega_j}{\sqrt{(\eta^2 \text{varc} + \Omega_j)^2 + (\eta^2 \text{varc} + \Omega_j)(1+r)^2\sigma_i^2}} \end{aligned}$$

$$\frac{\partial \text{corr}(\eta)}{\partial \eta} = \frac{2\eta \text{varc} \sqrt{\Theta} - (\eta^2 \text{varc} + \Omega_j) \frac{1}{2} \frac{2(\eta^2 \text{varc} + \Omega_j) 2\eta \text{varc} + (1+r)^2 \sigma_i^2 2\eta \text{varc}}{\sqrt{\Theta}}}{\Theta}$$

where  $\Theta = (\eta^2 \text{varc} + \Omega_j)^2 + (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2$ .

$$\begin{aligned} &= \frac{2\eta \text{varc} \sqrt{\Theta}}{\Theta} - (\eta^2 \text{varc} + \Omega_j) \frac{1}{2} \frac{2(\eta^2 \text{varc} + \Omega_j) 2\eta \text{varc} + (1+r)^2 \sigma_i^2 2\eta \text{varc}}{\Theta \sqrt{\Theta}} \\ &= \frac{2\eta \text{varc} \Theta - (\eta^2 \text{varc} + \Omega_j) [( \eta^2 \text{varc} + \Omega_j) 2\eta \text{varc} + (1+r)^2 \sigma_i^2 \eta \text{varc}]}{\Theta \sqrt{\Theta}} \end{aligned}$$

plugging for  $\Theta$  only in the numerator

$$\begin{aligned} &= \frac{2\eta \text{varc} [(\eta^2 \text{varc} + \Omega_j)^2 + (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2]}{\sqrt{\Theta}} \\ &\quad - \frac{\eta \text{varc} [2(\eta^2 \text{varc} + \Omega_j)^2 + (1+r)^2 \sigma_i^2 (\eta^2 \text{varc} + \Omega_j)]}{\Theta \sqrt{\Theta}} \\ &= \frac{2\eta \text{varc} (1+r)^2 \sigma_i^2 (\eta^2 \text{varc} + \Omega_j) - (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2 \eta \text{varc}}{\sqrt{\Theta}} \\ &= \frac{\eta \text{varc} (1+r)^2 \sigma_i^2 (\eta^2 \text{varc} + \Omega_j)}{\Theta \sqrt{\Theta}} > 0 \end{aligned}$$

The previous line shows that the slope of correlation with respect to  $\eta_{t+1}$  is positive, thus proves the existence of the correlation asymmetry. Now I take derivative with respect to  $\sigma_i^2$  to show the region where the slope is steeper for higher idiosyncratic volatilities.

$$\frac{\eta \text{varc} (\eta^2 \text{varc} + \Omega_j) (1+r)^2 \sigma_i^2}{\Theta^{3/2}} = \frac{\eta \text{varc} (\eta^2 \text{varc} + \Omega_j) (1+r)^2 \sigma_i^2}{[(\eta^2 \text{varc} + \Omega_j)^2 + (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2]^{3/2}}$$

with respect to  $\sigma_i^2$

$$\begin{aligned}
&= \frac{\eta \text{varc}(\eta^2 \text{varc} + \Omega_j)(1+r)^2 \Theta^{3/2}}{\Theta^3} \\
&\quad - \frac{\frac{3}{2} \Theta^{1/2} (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \eta \text{varc}(\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2}{\Theta^3} \\
&= \frac{\eta \text{varc}(\eta^2 \text{varc} + \Omega_j)(1+r)^2 \Theta^{1/2} \left\{ \Theta - \frac{3}{2} (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2 \right\}}{[(\eta^2 \text{varc} + \Omega_j)^2 + (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2]^3} \\
&= \frac{\eta \text{varc}(\eta^2 \text{varc} + \Omega_j)(1+r)^2 \Theta^{1/2} \left\{ (\eta^2 \text{varc} + \Omega_j)^2 - \frac{1}{2} (\eta^2 \text{varc} + \Omega_j)(1+r)^2 \sigma_i^2 \right\}}{\Theta^3} \\
&= \frac{\eta \sigma_a^2 + (1+r)^2 (1-\eta) \sigma_a^2 - \frac{1}{2} (1+r)^2 \sigma_i^2}{\Theta^3}
\end{aligned}$$

$$\frac{2\sigma_a^2[\eta + (1+r)^2(1-\eta)]}{(1+r)^2} > \sigma_i^2$$

The previous line defines an upper bound for the idiosyncratic volatility. If the idiosyncratic volatility is above the upper bound the correlation asymmetry decreases in absolute terms as the idiosyncratic volatility increases.

## A.2 The Case of Smooth Ambiguity Aversion

The purpose in this section is to show that the main result of the paper is not specific to Gilboa and Schmeidler's (1989) max-min expected utility representation of ambiguity aversion and it holds under the Smooth Ambiguity Aversion representation of Klibanoff, Marinacci, and Mukerji (2005). Here I do not intend to discuss the decision theoretic foundations that lead to these two different representations. However, the interested reader can refer to Klibanoff, Marinacci, and Mukerji (2005), Epstein (2010) and Klibanoff, Marinacci, and Mukerji (2005).

To make the setting even simpler I assume that the variance of noise term can take only two values:  $\overline{\sigma}_s$  or  $\underline{\sigma}_s$ . In the main text, the ambiguity region consists of an



interval rather than of two scalars. However, this simplification is not crucial and the results shown below can be extended to to the case of an interval as well. The two different values for the variance of noise correspond to two different signal-to-noise ratios, or response coefficients  $\eta(\sigma_s)$ . Based on the formula:

$$\eta(\sigma_s^2) = \frac{\text{cov}(d^i, s)}{\text{var}(s)} = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_s^2} \quad (\text{A.14})$$

$\underline{\eta} = \frac{\sigma_a^2}{\sigma_a^2 + \underline{\sigma}_s^2}$  and  $\bar{\eta} = \frac{\sigma_a^2}{\sigma_a^2 + \bar{\sigma}_s^2}$ . Since there is one-to-one relationship between  $\sigma_s$  and  $\eta$ , I proceed by using  $\eta$  –the response to news coefficient.

I start with Gilboa and Schmeidler’s (1989) max-min expected utility representation for the sake of completeness in this simplified case. Then I proceed with Klibanoff, Marinacci, and Mukerji’s (2005) Smooth Ambiguity Aversion representation.

In max-min expected utility representation there is no distributional assumption over the ambiguity set. More specifically there is no assumption on the probability of  $\sigma_s$  being equal to  $\underline{\sigma}_s$  and the agent behaves as if the realization  $\sigma_s$  will be the worst for her utility. In that setup, similar to the main text, the price responses will be less strong conditional on good news compared to responses conditional on bad news. Because conditional on good news,  $\bar{\sigma}_s$  implies lower expected utility compared to the expected utility implied by  $\underline{\sigma}_s$ . Equation (A.15) in the main text:

$$q_1^i(s) = \min_{\sigma_s^2 \in \{\bar{\sigma}_s, \underline{\sigma}_s\}} \mathbb{E}[d^i | s] = \min_{\sigma_s^2 \in \{\bar{\sigma}_s, \underline{\sigma}_s\}} m + \eta(\sigma_s^2)s = \begin{cases} m + \underline{\eta} s & \text{if } s \geq 0, \\ m + \bar{\eta} s & \text{if } s < 0, \end{cases} \quad (\text{A.15})$$

Next I show that the asymmetric response to news is also observed in the Smooth Ambiguity Aversion representation. In this representation, agents have a subjective probability distribution over the possible set of variances,  $\{\bar{\sigma}_s, \underline{\sigma}_s\}$ . For simplicity I

assume that both values are equally likely for  $\sigma_s$ . To capture the ambiguity aversion there is also an additional concave function,  $\phi(x)$  representing the preferences over the probability distribution. The functional form of  $\phi(x)$  used below allows for tractability and is borrowed from Klibanoff, Marinacci, and Mukerji (2005).

$$q_1^i(s) = \mathbb{E}^\mu [\phi(\mathbb{E}[d^i|s])] \quad (\text{A.16})$$

where  $\phi(x) = \frac{1-e^{-\alpha x}}{1-e^{-\alpha}}$ . Thus, equation (A.16) is a specific case of smooth ambiguity aversion under the risk neutrality assumption. The inner expectation is due to the risk preferences. Risk neutral agents only care about the mean dividend level, thus we have  $\mathbb{E}[d^i|s]$ . However, this expectation depends on the variance of noise term or distribution of it. The outer expectation integrates over the different values of  $\sigma_s$ , and the concavity of  $\phi(x)$  represent the ambiguity aversion.

$$\mathbb{E}[d^i|s] = m + \eta s = \begin{cases} m + \underline{\eta} s & \text{with prob 0.5} \\ m + \bar{\eta} s & \text{with prob 0.5} \end{cases} \quad (\text{A.17})$$

$$q_1^i(s) = \mathbb{E}^\mu [\phi(\mathbb{E}[d^i|s])] = \frac{0.5}{1-e^{-\alpha}} \{1 - e^{-\alpha(m+\bar{\eta}s)} + 1 - e^{-\alpha(m+\underline{\eta}s)}\} \quad (\text{A.18})$$

Now, given the closed form solution for price I show that price is increasing in the signal  $s$ . Here I want to note the importance of risk neutrality assumption. Under the standard Constant Relative Risk Aversion utility specifications the same parameter identifies both the risk aversion and elasticity of intertemporal substitution (EIS). In that setup higher risk aversion implies smaller EIS. For example risk aversion being larger than 1 implies that the EIS is less than 1. Getting a good news generates two opposite effects, a substitution and an income effect. When the EIS is less than 1 the agent prefers to smooth her consumption or in other words the income effect dominates. So the consumption goes up and the saving goes down, resulting in less

demand for saving instruments, which leads to lower prices for saving instruments, i.e. stocks. Therefore, by assuming risk neutrality we avoid this counter intuitive result. In order to incorporate risk aversion, we can refer to Epstein and Zin (1989) type utility functions which allow for separation of risk aversion from the EIS. Doing this is beyond the scope of the current paper and is left for future work.

$$\frac{\partial q^i}{\partial s} = \frac{0.5}{1 - e^{-\alpha}} \{ \alpha \bar{\eta} e^{-\alpha(m + \bar{\eta}s)} + \alpha \underline{\eta} e^{-\alpha(m + \underline{\eta}s)} \} > 0 \quad (A.19)$$

$$\frac{\partial^2 q^i}{\partial s^2} = \frac{0.5}{1 - e^{-\alpha}} \{ -\alpha^2 \bar{\eta}^2 e^{-\alpha(m + \bar{\eta}s)} - \alpha^2 \underline{\eta}^2 e^{-\alpha(m + \underline{\eta}s)} \} < 0 \quad (A.20)$$

Equation (A.19) shows that prices are increasing in the signal  $s$ . Thus, prices increase when the agent observes a positive signal and decrease when she observes a negative one. However, as can be seen in equation (A.20) the greater the signal, the lower is the response. Therefore in this more general setup we still observe asymmetric response to signals, but in a more continuous fashion. The worse the signal is the stronger is the response. In the multiple priors setup I studied in the paper, there are only two different slope coefficients, depending on whether the signal is positive or negative. Here the slope changes continuously and it is steeper for worse signals.

Given the asymmetric response to news, the proof of Proposition 1 guarantees the asymmetric correlations in this case as well.

# Bibliography

- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Ebens, H. (2001), “The distribution of realized stock return volatility,” *Journal of Financial Economics*, 61, 43–76.
- Ang, A. and Bekaert, G. (2002), “International asset allocation with regime shifts,” *Review of Financial Studies*, 15, 1137–1187.
- Ang, A. and Chen, J. (2002), “Asymmetric correlations of equity portfolios,” *Journal of Financial Economics*, 63, 443–494.
- Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006a), “The cross-section of volatility and expected returns,” *The Journal of Finance*, 61, 259–299.
- Ang, A., Chen, J., and Xing, Y. (2006b), “Downside Risk,” *Review of Financial Studies*, 19, 1191–1239.
- Aydemir, C. (2008), “Risk sharing and counter-cyclical variation in market correlations,” *Journal of Economic Dynamics and Control*, 32, 3084–3112.
- Bansal, R., Dittmar, R. F., and Lundblad, C. T. (2005), “Consumption, dividends, and the cross section of equity returns,” *The Journal of Finance*, 60, 1639–1672.
- Boyer, B. H., Gibson, M. S., and Loretan, M. (1997), “Pitfalls in tests for changes in correlations,” *Board of Governors of the Federal Reserve System (US) International Finance Discussion Papers*.
- Brandt, M. W., Brav, A., Graham, J. R., and Kumar, A. (2010), “The idiosyncratic volatility puzzle: Time trend or speculative episodes?” *Review of Financial Studies*, 23, 863–899.
- Buraschi, A., Porchia, P., and Trojani, F. (2010), “Correlation risk and optimal portfolio choice,” *The Journal of Finance*, 65, 393–420.
- Cameron, A. C., Gelbach, J. B., and Miller, D. L. (2011), “Robust inference with multiway clustering,” *Journal of Business & Economic Statistics*, 29, 238–249.

- Campbell, J. Y. and Cochrane, J. H. (1999), “By force of habit: A consumption-based explanation of aggregate stock market behavior,” *The Journal of Political Economy*, 107, 205–251.
- Campbell, J. Y., Lettau, M., Malkiel, B. G., and Xu, Y. (2001), “Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk,” *The Journal of Finance*, 56, 1–43.
- Cappiello, L., Engle, R. F., and Sheppard, K. (2006), “Asymmetric dynamics in the correlations of global equity and bond returns,” *Journal of Financial econometrics*, 4, 537–572.
- Cochrane, J. H., Longstaff, F. A., and Santa-Clara, P. (2008), “Two trees,” *Review of Financial Studies*, 21, 347–385.
- Corsetti, G., Pericoli, M., and Sbracia, M. (2005), “Some contagion, some interdependence: More pitfalls in tests of financial contagion,” *Journal of International Money and Finance*, 24, 1177–1199.
- Das, S. R. and Uppal, R. (2005), “Systemic risk and international portfolio choice,” *The Journal of Finance*, 59, 2809–2834.
- Dumas, B. (1989), “Two-person dynamic equilibrium in the capital market,” *Review of Financial Studies*, 2, 157–188.
- Ehling, P. and Heyerdahl-Larsen, C. (2011), “Correlations,” in *AFA 2011 Denver Meetings Paper*.
- Engle, R. (2002), “Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models,” *Journal of Business & Economic Statistics*, 20, 339–350.
- Epstein, L. G. and Schneider, M. (2003), “Recursive multiple-priors,” *Journal of Economic Theory*, 113, 1–31.
- Epstein, L. G. and Schneider, M. (2008), “Ambiguity, information quality, and asset pricing,” *The Journal of Finance*, 63, 197–228.
- Erb, C. B., Harvey, C. R., and Viskanta, T. E. (1994), “Forecasting international equity correlations,” *Financial Analysts Journal*, pp. 32–45.
- Fama, E. F. and French, K. R. (1993), “Common risk factors in the returns on stocks and bonds,” *Journal of financial economics*, 33, 3–56.
- Forbes, K. J. and Rigobon, R. (2002), “No contagion, only interdependence: measuring stock market comovements,” *The Journal of Finance*, 57, 2223–2261.

- Gilboa, I. and Schmeidler, D. (1989), “Maxmin expected utility with non-unique prior,” *Journal of mathematical economics*, 18, 141–153.
- Goldberger, A. S. (1991), *A course in econometrics*, Harvard University Press.
- Hansen, L. P. (1982), “Large sample properties of generalized method of moments estimators,” *Econometrica: Journal of the Econometric Society*, pp. 1029–1054.
- Hansen, L. P., Heaton, J. C., and Li, N. (2008), “Consumption strikes back? Measuring long-run risk,” *Journal of Political Economy*, 116, 260–302.
- Hong, Y., Tu, J., and Zhou, G. (2007), “Asymmetries in stock returns: Statistical tests and economic evaluation,” *Review of Financial Studies*, 20, 1547–1581.
- Karolyi, G. A. and Stulz, R. M. (1996), “Why do markets move together? An investigation of US-Japan stock return comovements,” *Journal of Finance*, pp. 951–986.
- Lin, W.-L., Engle, R. F., and Ito, T. (1994), “Do bulls and bears move across borders? International transmission of stock returns and volatility,” *Review of Financial Studies*, 7, 507–538.
- Longin, F. and Solnik, B. (1995), “Is the correlation in international equity returns constant: 1960–1990?” *Journal of international money and finance*, 14, 3–26.
- Longin, F. and Solnik, B. (2001), “Extreme correlation of international equity markets,” *The Journal of Finance*, 56, 649–676.
- Lucas, R. E. (1978), “Asset prices in an exchange economy,” *Econometrica: Journal of the Econometric Society*, pp. 1429–1445.
- Malkiel, B. G. and Xu, Y. (1997), “Risk and return revisited,” *The Journal of Portfolio Management*, 23, 9–14.
- Martin, I. (2013), “The Lucas orchard,” *Econometrica*, 81, 55–111.
- Mueller, P., Vedolin, A., and Stathopoulos, A. (2012), “International correlation risk,” *Available at SSRN 2049809*.
- Pástor, L. and Stambaugh, R. F. (2003), “Liquidity risk and expected stock returns,” *Journal of Political Economy*, 111, 642–685.
- Patton, A. J. (2006), “Modelling asymmetric exchange rate dependence,” *International Economic Review*, 47, 527–556.
- Petersen, M. A. (2009), “Estimating standard errors in finance panel data sets: Comparing approaches,” *Review of Financial Studies*, 22, 435–480.

- Pindyck, R. S. and Rotemberg, J. J. (1993), “The comovement of stock prices,” *The Quarterly Journal of Economics*, 108, 1073–1104.
- Ribeiro, R. and Veronesi, P. (2002), “The excess comovement of international stock markets in bad times: A rational expectations equilibrium model,” .
- Ronn, E., Sayrak, A., and Tompaidis, S. (2009), “The impact of large changes in asset prices on inter- and intra-market correlations in the stock and bond markets,” *Financial Review*, 44, 405–436.
- Shiller, R. J. (1989), “Comovements in stock prices and comovements in dividends,” *Journal of Finance*, pp. 719–729.
- Stambaugh, R. (1995), “Discussion of Why do markets move together? An investigation of US–Japan stock return comovements” by A. Karolyi, and R. Stulz presented at the NBER Conference on Financial Risk Management,” .
- Wei, S. X. and Zhang, C. (2006), “Why did individual stocks become more volatile?” *The Journal of Business*, 79, 259–292.
- Xu, Y. and Malkiel, B. G. (2003), “Investigating the behavior of idiosyncratic volatility,” *The Journal of Business*, 76, 613–645.

# Biography

I am Sati Mehmet Ozsoy, and I was born in Istanbul, Turkey on January 26th, 1983. I earned my B.A. in Economics, with High Honors, from Bogazici University in 2006, and my M.A. in Economics from Koc University, Istanbul in 2008. I wrote my masters thesis under the supervision of Professor Sumru Altug. Since 2008 I have been studying at Duke University. I am planning to graduate from Duke University with a doctorate degree in Economics in the spring of 2013.