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Chaos in percepts?

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Abstract. Multistability in perceptual tasks has suggested that the mechanisms underlying our percepts might be modeled as nonlinear, deterministic systems that exhibit chaotic behavior. We present evidence supporting this view, obtaining an estimate of 3.5 for the dimensionality of such a system. A surprising result is that this estimate applies for a rather diverse range of perceptual tasks.

1 Introduction

The multistability of impoverished visual displays, such as the Necker cube or the crater illusion presented in Fig. 1, is well known. Less appreciated is the fact that even complex scenes, although rich in visual information, also may have many interpretations – indeed, perhaps even an infinity, depending on the level of detail desired to "explain" the scene. Different patterns of eye movements are often associated with these alternate interpretations, as shown by Yarbus (1967). Hence, despite our impressions to the contrary, the chosen percept typically entails the selection of one of many possible interpretations of the input, even if the context remains unchanged (Jepson and Richards 1993). Here we present evidence that processes searching for the appropriate percept have some characteristics associated with low-dimensional, chaotic dynamical systems. Our results confirm a proposal by Poston and Stewart (1978) and conclusions reached by Ta'eed et al. (1988) that multistable percepts can be modeled as nonlinear dynamical systems.

2 Method

A dynamical system can be characterized by the geometry of the space of its output. These outputs are typically a sequence of state changes, either temporal or spatial. Hence, possible measures that we can use to evaluate a perceptual dynamical system are (1) the sequence of time intervals between perceptual states, such as the duration of successive flips of a Necker cube, or (2) the sequence of spatial vectors, such as when the eye moves from one (x, y) position to another. In either case the data will be a sequence of values, $\{v_i\}$ for i typically greater than 200. To determine whether an observed pattern exhibits chaotic behavior characteristic of a dynamical system, we chose a correlation technique outlined elsewhere (Bergé et al. 1984; Grassberger and Procaccia 1983). This technique has been especially successful in using time-series data to analyze nonlinear physical systems, such as turbulent flow (Essex et al. 1987; Malraison et al. 1983) or semiconductor resonators (Van Buskirk and Jeffries 1985; Theiler 1990). Briefly, the method first computes the number of vectors of length

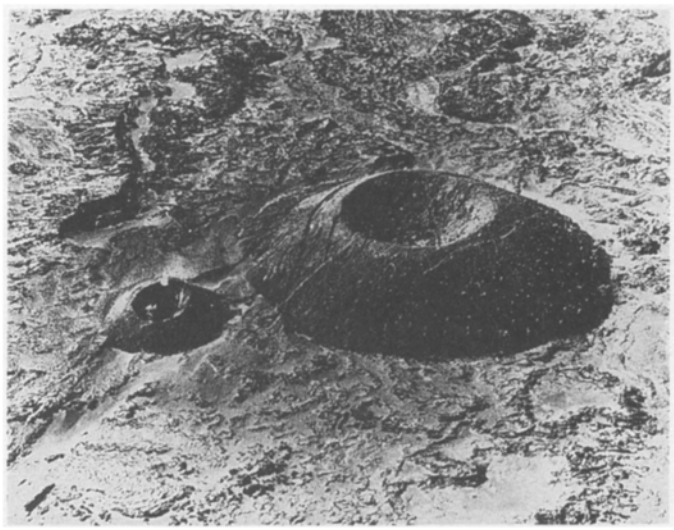

Fig. 1. Although the photograph is actually of two cones in the desolate Ka'u Desert lava field in Hawaii, occasionally the alternate interpretation of two craters is preferred. This preference can be reinforced by turning the picture upside down. (Courtesy of Wide World of Photos, Nov. 1972)

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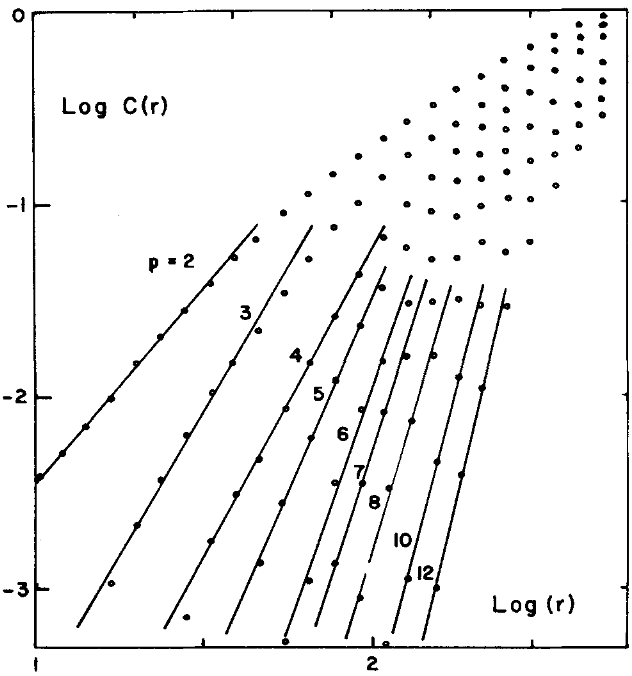

Fig. 2. First stage in the analysis of the time sequence of state changes for the crater illusion of Fig. 1. Abscissa is the radius r of the hypersphere of dimension p. Each point shows the value for that $C_p(r)$. The slopes on this log-log plot yield the values for e_p

$(\mathbf{x}_i - \mathbf{x}_j)$ falling within a p-dimensional hypersphere of radius r, where $\{\mathbf{x}\}$ is the set of m sample points (e.g., time intervals) collected:

$$C_p(r) = \frac{1}{m^2} \sum_{i,j=1 \text{ to } m} H[r - |\mathbf{x}_i - \mathbf{x}_j|]$$

$C_p(r)$ is the average number of vectors or the "correlation coefficient" for the chosen radius, p is the "embedding" dimension, m is the total number of values in the sequence, and H is the Heaviside function, which is evaluated to 1 if the distance between the pair i, j is less than r, otherwise zero. In other words, the correlation function counts the number of pairs with distance $|\mathbf{x}_i - \mathbf{x}_j|$ smaller than r, where r is the radius of a p-dimensional hypersphere. The method thus provides an assessment of the geometry of the space from which the samples are taken. For example, if the samples are uniformly distributed on a plane, then increasing r will increase the correlation count as the square of the radius. For each embedding dimension p, an exponent $e_p(r) = \log[C(r)]/\log[r]$ is calculated over r. The maximum value of this exponent e_p is then determined and plotted against p. If a random time series is evaluated by this method, this exponent rises with the embedding dimension (i.e., $e_p = p$, or more correctly, $e_p \doteq p$ for stochastic series). If a deterministic chaotic series is evaluated, such as that generated by a Henon attractor, then the maximal values of e_p are asymptotic at some $p_{\max}$ for all $p \geqslant p_{\max}$. For a typical predator-prey dynamical system, such as May's logistic function, $p_{\max} \doteq 1.2$. Figure 2 illustrates this method. For each p, $\log C_p(r)$ was plotted vs $\log r$, and the steepest

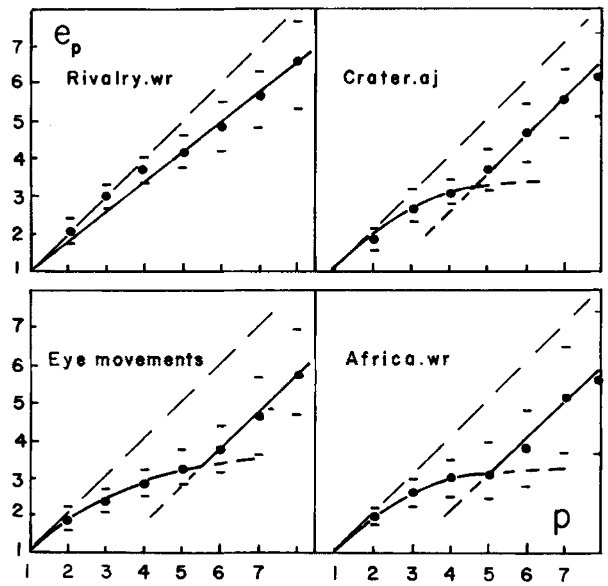

Fig. 3. Plots of e_p vs the embedding dimension p obtained from data like that illustrated in Fig. 2. *Upper left*, binocular rivalry; *upper right*, the crater illusion of Fig. 1; *lower left*, monkey saccadic eye movements; *lower right*, perceptual "eras" in "Out of Africa"

slope e_p was computed using six points for r separated by factors of 1.2 (i.e., $2^{\frac{1}{4}}$). Note that as p increases, so does e_p. These slopes were then plotted vs p, as shown in Fig. 3. The error bars show the range of slopes, based on adjacent points, within the six-point average plotted as a filled circle. The weakness of the method is also illustrated here: for large embedding dimensions, $p > 5$, the uncertainty in e_p increases substantially, especially when the number of data points is small (i.e, $N \doteq 200$). Hence, in our study estimates of e_p are restricted to $\leqslant 8$.

3 Experiments

We obtained data for four sets of perceptual tasks: binocular rivalry, fixation patterns during search, simple multistable percepts, and perceptual segments or "eras" in several movies. The panels in Fig. 4, plus Fig. 1, show examples of the multistable stimuli used. Table 1 gives the number of reversals recorded and the mean reversal time.

3.1 Binocular rivalry

When two patterns having significantly different spatial structure are presented, each to separate eyes, typically one sees an alternation of one pattern with the other. For example, if one eye is presented with vertical red and black bars, whereas the other sees horizontal green and black bars, the percept is dynamic alternation between each. The site of this competition or "rivalry" is known to be early in the visual pathway, namely, near the initial cortical area of processing (Julesz 1971; Richards 1970).

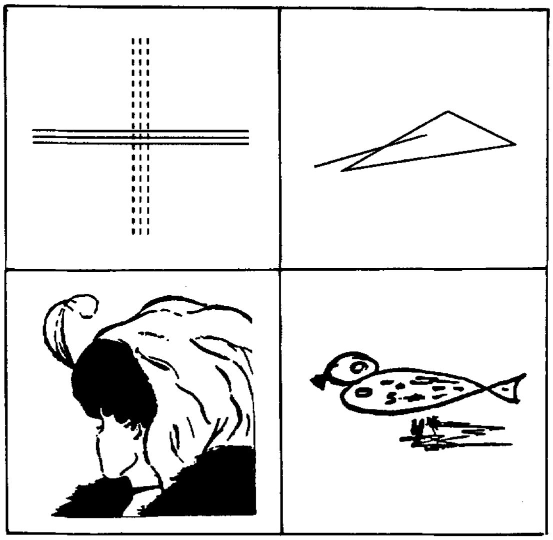

Fig. 4. Some of the stimuli used that have multiple-stable states. *Upper left*, a rivalrous pattern (*dashed lines* indicate the set presented to one eye, the *solid lines* indicate the set presented to the other); *upper right*, triangle plus stick (stick is commonly seen as sitting on the edge of the triangle, or alternatively as angled in space above the triangle with its right end touching the interior plane); *lower right*, a single duck vs two fish reversal; *lower left*, Boring's ambiguous-mother-in-law: an old woman vs a young girl

Table 1. Number of samples N and estimates of dimension underlying dynamical system, p_{max}

Stimulus	N	Mean duration (s)	Dimension, p_{max}
Binocular rivalry	258	2.7	–
Crater illusion (observer 1)	200	3.2	3.4
Crater illusion (observer 2)	200	3.0	3.6
Triangle and stick	400	3.2	3.5
Duck-fish	400	4.3	4.2
Hag-girl	200	2.2	4.8
Eye movements	200	0.2	3.5
"Out of Africa"	320	25.2	3.3
"ET"	350	16.8	3.7
"Chariots of Fire"	350	19.3	3.5
"Star Wars"	400	15.2	3.6
		AVG.	3.7

The eye movement analysis was performed on the spatial, not the temporal patterns of fixation; the eye movement duration is the mean intersaccadic interval

We measured the time series for 200 alternations for two subjects and computed the correlation coefficients and exponents e_p for embedding dimensions $p = 1$ to 8. As shown in Fig. 3, a plot of e_p vs p is roughly a line of unit slope, specifically 0.85 for this subject. This result, regardless of the underlying cause, is in agreement with Levelt (1965) and is typical of a stochastic process such as the pattern of rainfall. Hence, we conclude that binocular rivalry is not a deterministic chaotic process typical of a predator-prey competition (Kadanov 1986; May 1974, 1976).

3.2 Cognitive rivalry

Unlike "passive" binocular rivalry, the flip-flops of the crater picture in Fig. 1 reflect processes that are evaluating the image as a depiction of a world state or "scene". The competition, if you will, is between alternate models that "explain" the image. It is easy to imagine a process, such as the fluctuations in predator-prey populations, where one explanation dominates for a while, and then another takes over. We studied several simple displays such as the illusion of Fig. 1, a picture of a triangle with a stick lying across one edge, as well as some cognitive rivalry tasks invented by Manfred Fahle (Fahle and Palm 1990). All of these examples revealed hints of chaos. One of the clearer results was obtained from the time series of flip-flops for the crater illusion. This display is particularly well suited for study because the perceptual biases can be manipulated by rotating the pattern, placing the illuminant above-below or left-right, the latter being chosen because it yields a more equal competition between percepts.

The data for observer 1 are presented in Fig. 3, upper right. (The results for observer 2 were similar; see Table 1). Note that, in contrast to binocular rivalry, we do not see e_p increasing linearly with p, but rather rising and leveling off near $e_p = 3$ for $p \doteq 4$, then rising again. This "notch" in the curve is significantly different from the linear extrapolation of the points for $p > 5$. We propose that this "notch" results from a process exhibiting deterministic chaos of dimensionality, roughly 3.5, corrupted by a noisy stochastic process that "takes over" as the embedding dimension increases.

Note that data for the reversals of a Necker cube do not appear in Table 1. As was found for binocular rivalry, only the hint of a "notch" was observed in the plot of e_p vs p. This result suggests that the dominant process causing the reversals of the Necker cube may be formally similar to the process that leads to the competition observed during binocular rivalry.

3.3 Eye movements

Again using the "notch" as our indicator, evidence for chaos was also observed in the spatial pattern of 200 eye movements obtained from a monkey, who was searching a $17 \times 17°$ field for a target [see Sommer (1993) for details]. Note that in this case the analysis was not performed on the temporal intervals, but rather on the magnitude of the saccade[1]. The location of the notch in the curve of e_p vs p again suggests an underlying dynamical system of dimension 3.5 which becomes masked by a stochastic process at the higher embedding dimensions. We have attempted to model these data using a combination of deterministic and stochastic processes, but to date have not succeeded. [Although the unit slope observed for $p > 5$ is suggestive of additive "random" noise, as yet we do not have a model that adequately explains all

[1] The data we report were taken from the vertical component of the saccade

our $C(r)$ vs (r) observations. The data are not typical of the expected effects of additive noise observed in physical systems (Ben-Mizrachi et al. 1984; Theiler 1990).]

3.4 Movies

The most complex perceptual activity we explored was the time sequence of segments that appear in films. Typically the camera will be directed to a person or event for a duration, then the scene flips to another person or event as the story unfolds. Think then of the camera as the eye of the film editor, who unfolds the story, attempting to communicate the salient points. Certainly this process entails a very high level of perceptual understanding. However, the lengths of the camera segments are rather easy to measure without excessive subjective bias. For "Out of Africa", "Chariots of Fire", "Star Wars", and "ET", 200 to 400 segments were recorded. These films were chosen to represent a spectrum of fast- to slow-paced films. The "Out of Africa" analysis is presented in the fourth panel of Fig. 3. As summarized in Table 1, it can be seen that the other three films were similar, again suggesting an underlying dynamical system of dimension of about 3 to 4, corrupted by stochastic noise as the embedding dimension exceeded 5. [Of interest is that "Out of Africa" also yielded an embedding dimension of less than 3 for the upper component of the $C(r)$ vs (r) plot[2].]

4 Discussion

If one views the brain as a society of competing neurons or neural processes, then their characterization as a dynamical system is not surprising. Indeed, Poston and Stewart (1978) proposed such a model for multistable percepts, and such behavior has previously been reported for adaptive systems and neural activity with dimensionality estimates ranging from 3 to 7 (Altman 1991; Priesol et al. 1991; Skarda and Freeman 1987; Xu and Li 1986). It is reassuring that the observed dimensionality of their conscious counterpart – our percepts – falls in this range. What is surprising is that our estimate lies at the lower end, with little variance. A dimension of less than 4 for any dynamical system offers a reasonable possibility for model development and study.

A second surprise is that regardless of the complexity of the perceptual task, when chaotic behavior is plausible, the inferred dimensionality remains roughly the same. It is difficult to believe that the processes underlying the reversals in the crater and the temporal sequence of segments in the movies share the same neural machinery to any great degree. Yet they both exhibit the same dimensionality! A type of process that would elicit such a result is one that is organized in a hierarchical manner, having a fractal composition with roughly the same number of components collected together in each level of the hierarchy. In other words, in such a scheme, neural subsystems would compete for the attention of their "parental" systems etc. The fractal dimension common to all tasks would then indicate that each parent has roughly the same number of "offspring". Here, we have simply adopted a proposal made elsewhere for social choice, where each state of decision is not a single rational choice, but rather an aggregation of many hierarchical choice problems (Richards 1991). In this domain the fractal dimension also is surprisingly low.

Thirdly, with respect to the eye movement data and results, which are spatial and not temporal patterns, again the dimensionality appears to be about 3.5. This suggests that perceptual mechanisms may be directing the spatial pattern of eye movements during the search for a target's appearance. Note that this "chaotic" search occurs even though the intersaccadic duration intervals are short, around 200 ms (Sommer 1993). The scanning strategy, then, appears controlled by a nonrandom, dynamical spatial representation, but not a temporal one. Again, a hierarchical search pattern, where small and large step sizes are nested as in a tree or branching structure, would be one strategy that could exhibit such deterministic chaotic behavior.

Finally, although our percepts appear to exhibit some properties consistent with deterministic chaos, this should not be construed to imply that the percepts themselves are chaotic! Indeed, the percepts can be quite rational and stable. However, the state changes underlying the development of these percepts seem characteristic of a special kind of dynamical system that is well suited for searching rapidly through a large data base of variable resolution, where many states are to be evaluated quickly, probably in parallel.

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[2] The other film data also suggested a lower-dimensional component for values of $C(r) > 0.03$, but this observation remains tentative

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