Communications

A Comment on "This Age of Leontief . . . and Who?"*

Levine claims that Sraffa has been misunderstood on issues concerning, "(1) [the] use of production coefficients and the question of returns to scale, (2) the centerpiece of the price-determining apparatus, (3) numeraire problems, (4) consumer demand, and (5) Sraffa's place in the classical tradition" [4, 1974, p. 877]. Unfortunately, Levine's discussion of (1), (2), and (3) is incomplete, and below I will attempt to set the record straight on these important issues; I refrain from comment on (4) and (5) because of limited space.

Returns to Scale

Levine quotes Quandt's statement, "One notable difference between Leontief's system and Sraffa's is that the latter nowhere defines the equivalent of input coefficients" [6, 1961, p. 500], and he then states that, "One wonders about the relevance of this remark, unless there is in it an implication that Sraffa's use of total rather than unit coefficients is in some way mystifying. If so, such an implication would be incorrect: after all, the unit coefficients of Leontief's notation would be of no use to Sraffa's joint-production model" [4, 1974, pp. 877–78].

The latter statement may be misleading since a Leontief model is a special case of a (generalized) von Neumann model having joint production and constant returns to scale. In such a model the j^{th} production activity, operated at the unit intensity level, requires an input vector $(a_{0j}, a_{1j}, \ldots, a_{nj})$ to produce the output vector $(b_{1j}, \ldots, b_{nj})$. In this notation a_{0j} designates the input of labor to the j^{th} activity, a_{ij} designates the input of the i^{th} commodity to the j^{th} activity, and b_{ij} designates the output of the i^{th} commodity from the j^{th} production activity. Constant returns implies that if $\{(a_{0j}, a_{1j}, \ldots, a_{nj}), (b_{1j}, \ldots, b_{nj})\}$ is a feasible input-output vector, then so is $\{(\lambda a_{0j}, \lambda a_{1j}, \ldots, \lambda a_{nj}), (\lambda b_{1j}, \ldots, \lambda b_{nj})\}$ for any $\lambda > 0$. Without joint production, as in Leontief models or in the first part of Sraffa (1960), the j^{th} activity may be associated uniquely with the production of the j^{th} commodity; all the output vectors are then of the special form $(0, \ldots 0, b_{jj} = 1, 0, \ldots, 0)$, $j = 1, \ldots, n$.

Of course, one may deny constant returns to scale, but it is crucial then to determine what economically meaningful propositions remain valid. For example, Sraffa derives the relationship between the profit rate, r, labor's share of net product in terms of his Standard Commodity, w, and the maximum profit rate, $R = r^*$, namely:

$$r = R(1 - w) \qquad [1]$$

or

$$w = 1 - \frac{r}{r^*}.^{1} \qquad [2]$$

For simplicity, assume $n = 2$, that there is "equal organic composition of capital," and that only labor plus commodity i is used to produce commodity i, $i = 1, 2$. The Sraffa pricing conditions then are

$$P_i X_i = W L_i + (1 + r) P_i X_{ii}, \quad i = 1, 2, \qquad [3]$$

where X_i is the output of commodity i, L_i is the labor input, paid a nominal wage W, and X_{ii} is

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[1] See Sraffa [7, 1960, p. 22] and Burmeister [1, 1968, p. 86].

the amount of commodity i used to produce commodity i. From [3] we derive:

$$\frac{W}{P_i} = \frac{1 - \frac{(1+r)X_{ii}}{X_i}}{\frac{L_i}{X_i}}, \quad i = 1, 2. \qquad [4]$$

Moreover, suppose that the scale and original units of measurement were such that the identities

$$a_{0i} \equiv \frac{L_i}{X_i}$$

and

$$a_{ii} \equiv \frac{X_{ii}}{X_i}$$

satisfy

$$a_{0i} + a_{ii} = 1,$$
$$0 < a_{0i} = \alpha < 1, \ i = 1, 2.$$

Then Lemma (1) in Burmeister [1, 1968, p. 85] applies, and we may conclude

$$\frac{W}{P_i} = 1 - \frac{r}{r^*}, \qquad [5]$$

implying that

$$\frac{1 - \frac{(1+r)X_{ii}}{X_i}}{\frac{L_i}{X_i}} = 1 - \frac{r}{r^*}.$$

In this case of "equal organic composition of capital" either commodity one or two (or any linear combination of them) can serve as a Standard Commodity; suppose we choose commodity one. Sraffa's relationship [2] becomes:

$$w = \frac{W}{P_1} = 1 - \frac{r}{r^*}, \qquad [7]$$

and from [6] we see that

$$\frac{1 - \frac{(1+r)X_{11}}{X_1}}{\frac{L_1}{X_1}} = 1 - \frac{r}{r^*} = \frac{W}{P_1} = w. \qquad [8]$$

Assume an initial equilibrium position for fixed X_1, X_{11}, and L_1, and let $r = ½\ r^*$ be given (so that w is fixed at ½). Suppose X_1 changes, thereby changing one (or both) of the ratios X_{11}/X_1 and L_1/X_1 because constant returns to scale do not prevail. Will [7] correctly predict the new equilibrium level of w? Answer: *No.*

Conclusion: The Sraffa relationship $w = 1 - \frac{r}{r^*}$ and the determination of relative prices as functions of the profit rate both either (i) are valid only where all quantities are held fixed, or (ii) involve division by X_i's and hence the implicit assumption of constant returns to scale. The above argument elaborates my intention in 1968 when I wrote, "Unless it is assumed that the economy exhibits constant returns to scale with the matrix of input coefficients $\left[\frac{a_0}{a}\right]$ *fixed,* then the above analysis is meaningless if even a single quantity X_j changes" [1, 1968, p. 87]. That is, if constant returns to scale do not exist, and if we compare two situations in which quantities are allowed to vary, then Sraffa's relationship $w = 1 - \frac{r}{r^*}$ is *not* valid.

Price Determination

Levine asserts that "There does not seem to be sufficient grasp of the essentials here, although Quandt did perhaps show that he caught at least a glimpse of the truth when he wrote, in the following sentence, that 'It is not surprising . . . that, to use Debreu's terms, a price system is inherent in a production' " [4, 1974, p. 878].

What is an essential feature of Sraffa's price system? One answer which Levine overlooked deserves mention: The Nonsubstitution Theorem or the Factor-Price Frontier. In a Sraffa-type model without joint-production, a steady-state equilibrium point must lie *on* the economy's factor-price frontier, thereby determining the real wage rate in terms of any (every) commodity as numeraire; see Burmeister

[2, 1974] for one elementary exposition of this issue.[2]

Numeraire

There is no doubt that economics would be an easier subject if God had imposed the restriction "equal organic composition of capital" on the world, a *technological* restriction of nature as immutable as the laws of physics. In reality, however, we have more freedom and price ratios

$$\frac{P_i/W}{P_j/W} = \frac{P_i}{P_j}$$

do vary with the profit rate r (except in cases of "equal composition"). When price ratios vary with r, the manner in which the real wage rate changes as the rate of profit changes depends on the commodity selected as numeraire, *i.e.*,

$$\frac{W}{P_i} = f_i(r), \quad i = 1, \ldots, n, \qquad [9]$$

with $f_i(r) \neq f_j(r)$ except in special cases.

When there is "equal organic composition of capital," $f_i(r) = f_j(r) = f(r)$ for all commodities and [9] is replaced by:

$$\frac{W}{P_i} = 1 - \frac{r}{r^*} = f(r), \qquad 0 \leqq r < r^*, \; i = 1, \ldots, n.^3 \qquad [10]$$

Sraffa's Standard Commodity has the property that the basket weights are selected in a manner which preserves the right-hand side of [10], namely Sraffa finds positive weights $C_1^* > 0, \ldots, C_n^* > 0$ such that

$$\frac{W}{\sum_{i=1}^{n} P_i C_i^*} = w = 1 - \frac{r}{r^*}, \qquad 0 \leqq r < r^*. \qquad [11]$$

The C_i^* weights and hence w do not depend in any way on consumer preferences, nor are they derived from the maximizing behavior of any individual or any economic planner. Thus while the use of Sraffa's Standard Commodity as numeraire yields a mathematical theorem, namely [11], it is not an appropriate measure of value with properties similar to an ordinary price or cost-of-living index. For example, Sraffa's Standard Commodity might require relatively high weights for commodities that are never consumed (such as pig iron) and relatively low weights for commodities which are used almost entirely as final consumption goods (such as pies). Accordingly, higher values of Sraffa's measure w given by [11] do not, in general, imply anything about economic welfare.

In this sense, then, I stand with my 1968 statement, quoted by Levine, that, "It is dubious what *economic significance* can be attached to Mr. Sraffa's proposed composite commodity" [1, 1968, p. 85].

[2] It is important to realize that Sraffa's formulation implicitly assumes a steady-state equilibrium in which prices are constant with

$$P_i(t+1) = P_i(t) = P_i \text{ for all } i = 1, \ldots, n.$$

This fact is evident from the price equations which in our notation become

Value of Output = Cost of Production

or

$$P_i X_i = R_1 X_{1i} + \ldots + R_n X_{ni}, \; i = 1, \ldots, n, \qquad [1']$$

where R_i is the gross rental rate for the i^{th} factor of production.

The conclusion is immediate once it is observed that Sraffa sets

$$R_i = (1 + r) P_i, \quad i = 1, \ldots, n. \qquad [2']$$

In general, prices are not constant out of steady-state equilibrium and the appropriate rental rates in [1′] are not given by [2′], but instead they would be

$$R_i(t) = (1 + r) P_i(t) - [P_i(t+1) - P_i(t)], \quad i = 1, \ldots, n. \qquad [3']$$

A detailed discussion of this issue, including the timing of the production process and of factor payments, is provided in Burmeister and Dobell [3, 1970, pp. 228–34].

[3] See Burmeister [1, 1968, p. 85].

Concluding Remarks

I anticipate that Levine's criticism of Samuelson also applies to me—the shoe does fit, since I frequently have referred to "Leontief-Sraffa models" as, for example, in my recent survey of capital theory (see Burmeister [2, 1974]). My reason for this usage is quite simple, namely the equations of a Leontief model with a positive rate of profit, written in extensive form, are *identical* to Sraffa's equations when joint production is absent. Since many of Sraffa's theorems are meaningful only when constant returns to scale exist, Leontief's input-output notation is the most familiar and convenient way to express Sraffa's model in intensive form; moreover, Sraffa's somewhat cumbersome verbal arguments can be replaced by the straightforward application of theorems in linear algebra, and this, in turn, greatly simplifies the task of resolving questions such as, "Under what conditions is Sraffa's Standard Commodity unique?"[4]

In conclusion, I share Levine's feeling that Sraffa has been misunderstood by many economists and that *Productions of commodities by means of commodities* is not fully appreciated. The greatest tragedy is that its publication was delayed for perhaps as long as 25 years; had the book borne a 1935 publication date, it would have been acclaimed for giving birth to much of modern linear economics.

[4] See Miyao [5] who has found the definitive answer to this question.

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"This Age of Leontief . . . and Who?" A Reply

In his Comment [5, 1975] on my "This Age of Leontief . . . and Who? An Interpretation" [10, 1974], Edwin Burmeister reports that my discussion of production coefficients, returns to scale, price determination and the numeraire in the Sraffa model [16, 1960] is "incomplete." Burmeister proposes to try to complete it—to "attempt to set the record straight," as he puts it—by a retrospective glance at his theorem [4, 1968] on the linear wage-profit relationship in Sraffa's standard system, which glance is part of a larger disquisition on the need for a constant-returns-to-scale assumption both within and without the ambit of that theorem; a reminder about the relevance of Samuelson's Nonsubstitution Theorem [13, 1951; 14, 1961] and the factor-price frontier [15, 1962] to "an essential feature of Sraffa's price system" [5, Burmeister, 1975, p. 455]; and some comments on the weights of that special commodity basket that serves as Sraffa's standard commodity.

On the question of the need for a constant-returns-to-scale assumption, I accept Sraffa's position [16, 1960, p. v]; Burmeister does not. In the matter of Samuelson's Nonsubstitution Theorem and the factor-price frontier, Bur-