Individual Incentives within Team Competitions

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Contents

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Abstract

This paper develops a theoretical model to show the effects of simultaneously having both a competition between groups and one among the individuals of a tournament. The players are divided up among teams and compete for a flat bonus for winning the team competition. At the same time, their efforts also determine if they will win another flat bonus from winning the individual game among all the tournament's players. With the use of a specific team production function demonstrating substitution among the player's efforts, the individual award seems to garner more overall group output than the group award. Under a specific production function demonstrating complementary efforts, players seem to be indifferent between the group and individual award. Lastly, a general production function that incorporates a variable measuring the degree of substitution was analyzed. The analysis showed that an individual award was beneficial to increasing overall team effort. The results imply that the payment scheme should be structured in a way that allocates larger individual rewards when the team efforts are more substitutable.

1 Introduction

Team sports have been one of the major forms of entertainment since the earliest of human civilizations. They successfully combine two of the most basic interactions in human society, teamwork and competition, into one event. Today, sports have become one of the largest and most lucrative businesses in the market, with major sports leagues comprising a multi-billion dollar industry. However, the driving factor behind sports today is still the same as it was in the past: competition. Generally speaking, the most exciting matches are those that have teams playing all out. Therefore, when establishing the rules of the game, the savvy businessmen behind these sports league aim to create incentives for winning to motivate their players to play to the fullest extent of their abilities.

Perhaps the most interesting of these incentives is the Most Valuable Player, or MVP, award. As the name implies, this award is given to the player who contributed the most to the team cause. A version of this prolific award can be found in nearly every team sport ranging from college basketball (NCAA Player of the Year) to World Cup Football (World Cup MVP). In professional sports today, winning this award generally give the athlete a financial bonus; however, the major bonus is the windfall of recognition that the player receives from the media, audience, and fellow athletes alike. The player can then use this instant recognition as a bargaining chip for future endorsement deals, better contracts and all around superstar status.

The designation of an MVP is ironic given the collaborative nature of team sports because it essentially hands out individual recognition for group efforts. One would think that in team sports the team aspect should be emphasized along the lines of the popular idiom, "There is no I in team". The question is raised of whether this added individual incentive will translate to improved team play. If there are benefits, how should the organizer set up the payments for winning the MVP award versus winning the team competition?

Since the MVP award is ubiquitous in most major sports leagues, perhaps it has a positive effect on team competition. However, having the MVP award may change the dynamics of a team sport. The players may no longer focus solely on winning the group event; they might also take into consideration the individual competition that they are facing from their own teammates and their competitors. This may cause the player to focus his efforts on improving his individual odds for the MVP at the expense of his team's performance. For example, in the sport of basketball a player motivated to win the MVP may hog the ball and take ineffective shots himself instead of passing it off to a teammate.

This paper will try to determine if awards for individual performances in a group setting lead to better team play. A theoretical model will be constructed to understand how an organizer of these competitions can create awards that motivate the players to improve the competition among the groups.

As an initial hypothesis, the MVP award probably does improve overall team play since it is present in so many sports; thus organizers should include an individual award to motivate the players to do better as a team. This paper will attempt to provide insight as to whether there is a benefit to the team effort from the inclusion of an individual award and if so, how the organizer should structure payments to take advantage of it. First off, this paper will introduce some of the previous studies related to the topic of team interactions. The following section will introduce the basic framework for the original model while the section afterward will expand the analysis of the model with specific examples. Lastly, the paper will conclude with some answers to the proposed questions.

2 Literature Review

When individuals are working together in a group to produce an output, there is an introduction of group interaction factors that are not present when the individuals are working by themselves. Although some types of group interactions such positive synergy are good, others actually hinder the overall output. One of these factors is the classic economic problem of the free-rider effect in which individuals within a group under-contribute to a group's public output.

Due to its wide variety of applications, the study of group interactions has a rather elaborate array of research behind it. Most of the current literature focuses on the type of reward scheme (and thus the interaction) that will induce the largest amount of total effort from players working in a group environment.

The idea of the winner takes all reward scheme in sports falls under the study of tournaments. Under tournament compensation schemes, individuals or groups are rewarded based on their relative performance to other individuals or groups.

A major paper in this field of study was published by Edward Lazear and

Sherwin Rosen in 1981. In their research they showed that compensation under rank-order tournaments can be optimal over traditional compensation based on individual output levels (Lazear, Rosen, 1981). Their paper has led to much other research within this topic as to how to structure multiple players into tournaments. Specific organization schemes that have been the focus of much of the research are intra-team competition and inter-team competition.

There have been a number of experimental studies on the effects of intra-team and inter-team competition. In 2002, Bornstein, Gneezy and Nagel devised an empirical experiment that introduced competition between groups. Their results showed that when groups were producing under the minimum effort game (in which total effort of the group is equal to the minimum individual contribution), the inter-team competition helped encourage more team member coordination than the just the regular team production (Bornstein, Gneezy, Nagel, 2002). Along the same lines of thought, Fatas, Neugebauer, and Perote, implemented an experiment that introduced competition among fellow team members producing under the minimum effort game. However, this time, the team member whose effort was the minimum of the group earned a lower payoff than the other members. Their results showed that this intra-team competition significantly increased the overall effort of the group (Fatas, Neugebauer, Perote, 2006). Fatas and Neugebauer actually conducted another experiment that contrasted the difference in output induced by inter-team competition versus intra-team competition. Interestingly, it showed that intra-team incentives elicit more output than its inter-team counterparts (Fatas, Neugebauer, 2007).

Likewise, the theoretical field has taken the time to analyze the effects of intra-team and inter-team competition. Ramakrishnan and Thakor developed a theoretical model which differentiated between the organizers benefits from inducing cooperation versus competition among individual teams. Their results indicated that the optimal team contract from the principal's perspective depended on the correlation between the individual team members outputs. For highly correlated outputs, competitive contracts were preferred whereas cooperative contracts favored outputs with low correlation (Ramakrishnan and Thakor, 1991). Similarly, Hideshi Itoh researched a model to further understand the effects of team cooperation in hierarchical organizations. His main question asked whether cooperation among certain members in an organization could be beneficial to the organization as a whole. It dealt with two types of cooperation: induced and delegated. From his model he concluded that cooperation could be beneficial to the organization. Specifically, he stated that induced cooperation was preferred by individuals who had high productivity interaction. Likewise, induced cooperation was preferred by individuals who are not risk-adverse (Itoh, 1992).

Along similar lines, McAfee and McMillan developed a theoretical model to represent an optimal team contract from the principals perspective. However, theirs differs from Ramakrishnan and Thakor in the fact that they tailor their model to handle the moral hazard that arises from incomplete information between the principal and the team about each members ability. They concluded that the moral hazard problem can be minimized if the principal allowed the team members to choose among contracts with different compensation schemes. In choosing a contract, the members effectively reveal their abilities to the principal and thus minimize the incomplete information (McAfee and McMillan, 1991). One of the best know papers dealing with team interactions comes from Bengt Holmstrom's research on the moral hazard among teams. He showed that competition was an optimal way to extract previously unknown information (Holmstrom, 1982).

Further research has gone into detail about how the payoffs for team members should be structured efficiently. Eyal Winter constructed a payoff structure for teams that produce their output under a perfect complements production function; it suggested that due to externalities, even with symmetric team members, no two members should be paid the same (2004). Dickinson and Isaac also focused on intra-team payoff structures; however, a notable difference is that they assume that players have different initial abilities. They compared giving absolute rewards to high contributing members versus relative rewards based on the ratio of the member's initial ability and his contribution. Their experiment showed that the highest team production and the lowest dispersion of individual contributions came from high relative rewards as players of lower initial ability were not discouraged from competing (Dickinson, Isaac, 1998).

Another notable collection of empirical literature focuses on the development of hybrid compensation structures that include both a team component and an individual component. Irlenbusch and Ruchala conducted an experiment that compared the use of a hybrid compensation structure versus a pure team payoff. Here, the pure team payoff gave each member the same compensation amount which was proportional to the team's output. In the hybrid structure, the team member that contributed the most to the group event was also given a flat bonus. Their empirical results showed that a pure team-based compensation motivates voluntary cooperation among team members. Furthermore, the introduction of a high individual award caused individual efforts to increase; however, it crowded out voluntary contributions within the team (Irlenbusch, Ruchala, 2006). Similar results were found by Wageman's experimental study which showed that both pure team-based compensations and pure individual-based compensations fared better than hybrid compensation structures (1995).

This paper develops an original theoretical model to analyze two main points. First, it aims to analyze the effects on total team production caused by inducing a tournament among individuals already situated in a group tournament setting. Second it aims to develop the optimal payment structure under this situation.

3 Model

The model used for this game must incorporate the game's various participants and stages. The underlying assumption within this model is that all the participants are rational persons who make their decisions in an attempt to maximize their individual payoff.

Let's begin by laying out the setup of the game that the participants will be playing in. Here, the players cooperate in teams to compete against another team. In conjunction with this team competition, there is also a separate individual award that the players are eligible for.

3.1 The Basic Model

To start off, let's begin with the simplest form of the model. There are two types of participants in this game: the organizer and the player. The organizer is the individual who has set up the tournament rules and the payments for the winners of the individual competition (MVP) and the team competition. For this basic model, there is one organizer. In order to maintain consistency in the notations, let

$$r = \text{payment to winner of individual competition}$$
$$w = \text{payment to each member of the winning team}$$

The second type of participants, the players, are those who are competing in the actual tournament set up by the organizer. Each player must be on a team. This is a group that has a total of at least two players. This team will be competing with all the other teams in the tournament. For this particular model, let's assume that there are two teams, X and Y. Furthermore, let's assume that each team consists of a total of two players, $i = \{1, 2\}$. In regards to the individual players, each player is able to exert one type of effort, e_i, in an attempt to win both the group and individual contest. At the same time, the player also incurs a cost, $C(e_i)$, which is a function that is increasing in terms of e_i. Thus in this simple model, let

$$x_i = e_i = \text{effort of player } i \text{ in team X}$$
$$y_i = e_i = \text{effort of player } i \text{ in team Y}$$

This is a sequential two-stage game in which the organizer moves in the first stage and the players move in the second stage after observing the organizer's moves. As such, in order to find the optimal equilibrium moves for all the participants in this game, we will use backwards induction.

The first step in this process is to find the equilibrium moves for the second stage of this game. During this stage, the objective of each player involved in the actual tournament is to maximize their total individual payoff. They increase their overall payoff whenever they win the the individual or team competition. In contrast, their payoff decreases as they expend more effort at a cost. The strategy of each player is his individual choice of how much effort to exert. Each player's choice of effort level is unknown to the other players since they all choose their efforts simultaneously.

First off, let's construct the team competition's payoff function. Each team's production function is given by $Q^T = Q(e_1, e_2)$. There are certain assumptions associated with this production function.

(i) Positive Marginal Product

$$\frac{\partial}{\partial e_i} Q^T > 0$$

This means that each additional unit of effort, e_i, will increase the total output of the group.

(ii) Diminishing Marginal Return

$$\frac{\partial^2}{\partial e_i^2} Q^T < 0$$

Although each additional unit of effort increases the overall output of the group, the amount by which the group output increases actually shrinks as the amount of total effort goes up. For instance, if the third unit of effort increases output by 100, the 50th unit of effort may only increase output by 5.

Furthermore, there are a couple of definitions that are essential to the model. For the second order derivatives of the production function, if $\frac{\partial^2}{\partial e_1 \partial e_2} Q^T > 0$, then the players' efforts are by definition complementary. Notice that $\frac{\partial^2}{\partial e_1 \partial e_2} Q^T = \frac{\partial}{\partial e_1}\left(\frac{\partial}{\partial e_2} Q^T\right) > 0$. This means that for complements, the marginal product of player 1 actually increases the as the marginal product of player 2 increasses. A particular example of a complementary function that will be used in the next section is $Q^T = \sqrt{e_1 e_2}$. On the same note, if $\frac{\partial^2}{\partial e_1 \partial e_2} Q^T < 0$, the players' efforts are substitutes. Once rewritten as $\frac{\partial}{\partial e_1}\left(\frac{\partial}{\partial e_2} Q^T\right) < 0$, it can be seen that the marginal product of player 1 decreases as the marginal product of player 2 increases and vice-versa. An example of a substitution function is $Q^T = \sqrt{e_1 + e_2}$.

Using this information, the probability of team T winning the team competition is

$$\Pr(\text{team } T \text{ wins}) = \frac{Q^T}{Q^X + Q^Y} \tag{1}$$

The construction of the individual contest's payoff is very similar to that of the group competition. Since each player's efforts are contributed to the group output, we need a method of measuring an individual's value to the team. Let's define the value of the individual, V_i^T as

$$\begin{aligned} V_1^T &= Q^T(e_1, e_2) - Q^T(0, e_2) \\ V_2^T &= Q^T(e_1, e_2) - Q^T(e_1, 0) \end{aligned} \tag{2}$$

Based on this definition, the probability that a player wins the individual competition is

$$\Pr(\text{player } i \text{ of team } T \text{ wins}) = \frac{V_i^T}{V_1^X + V_2^X + V_1^Y + V_2^Y} \tag{3}$$

Finally, with the cumulation of the previous information, the payoff function, π_{player}, for each individual player i in team T can be written as

$$\pi_{player} = \Pr(\text{team } T \text{ wins})w + \Pr(\text{player } i \text{ is MVP})r - C(e_i) \tag{4}$$

$$= \frac{Q^T}{Q^X + Q^Y}w + \frac{V_i^T}{V_1^X + V_2^X + V_1^Y + V_2^Y}r - C(e_i) \tag{5}$$

Each player's payoff is a function of the group award w, the individual award r, the other players' effort levels, and of course, his own effort level. As the players are rational persons, the equilibrium effort levels are found from solving

$$\max_{e_i} \pi_{player} = \frac{Q^T}{Q^X + Q^Y}w + \frac{V_i^T}{V_1^X + V_2^X + V_1^Y + V_2^Y}r - C(e_i) \tag{6}$$

Once the equilibrium effort levels are found from the second stage, the first stage equilibrium follows. Here, the organizer's objective is to maximize the total efforts of the players subject to his budget constraint. He is the first mover in this game and is given no prior information on the movements of the players. His payoff function, $\pi_{organizer}$, in this basic model is

$$\pi_{organizer} = x_1^* + x_2^* + y_1^* + y_2^* \tag{7}$$

Since the cost to the organizer is his overall income, which is a constant, it is not included in the payoff function. However, it does play into consideration when the organizer is determining his equilibrium strategy. Recall that the organizer has to pay w to each member of the winning team and r to the MVP winner. From this, his budget constraint is

$$2w + r = M \tag{8}$$

where M is his overall income. Following this logic, the organizer uses his strategy of choosing w and r to solve

$$\begin{aligned} \max_{w,r} \pi_{organizer} &= x_1^* + x_2^* + y_1^* + y_2^* \\ s.t. \quad 2w + r &= M \end{aligned} \tag{9}$$

This is a constrained optimization problem involving two variables. Recall that the equilibrium efforts, e^*, that were found from the previous stage are functions of w and r. For this particular model, there are non-negativity constraints on both the individual and group awards.

$$\begin{aligned} w &\geq 0 \\ r &\geq 0 \end{aligned} \tag{10}$$

These constraints imply that there is no explicit punishment for losing either of the contests. Likewise, they also impose a maximum value for both rewards in which

$$\begin{aligned} w &\leq \frac{M}{2} \\ r &\leq M \end{aligned} \tag{11}$$

4 Analysis

Now that the model for the tournament has been established, it yields some results that can further our understanding of this game beyond our basic intuition. Since the definition of the production function Q^T is so pivotal to the model, I have broken down the analysis of the basic model into four sections. The first introduces a general production function case to the model. The latter three sections will extend the ideas of the general case with three specific examples.

4.1 The General Case

In this case, the production function has not been explicitly stated and is left in the general form of $Q^T = Q(e_1, e_2)$. The individual's problem (in terms of team X's first player) is

$$\max_{x_1} \pi_{x_1} = \frac{Q(x_1, x_2)}{Q(x_1, x_2) + Q(y_1, y_2)} w + \frac{Q(x_1, x_2) - Q(0, x_2)}{2Q(x_1, x_2) + 2Q(y_1, y_2) - Q(0, x_2) - Q(x_1, 0) - Q(0, y_2) - Q(y_1, 0)} r - C(x_1) \quad (12)$$

A notable assumption that I made when solving for the equilibrium effort levels of the players is that all players are symmetric. This means that they all face the same production and cost functions. As such, their equilibrium efforts are all be the same.

$$x_1^* = x_2^* = y_1^* = y_2^* \quad (13)$$

Using this assumption, the first order condition, $\frac{\partial}{\partial x_1}$, of the payoff function, π_{x_1}, is

$$\frac{\partial}{\partial x_1}\pi_{x_1} \mid_e = \frac{Q_1(e,e)}{4Q(e,e)}w + \frac{2Q_1(e,e)+Q_1(e,0)}{16[Q(e,e)-Q(e,0)]}r - C'(e) = 0 \tag{14}$$

where $Q_1(e,e)$ is the partial of $Q(x_1, x_2)$ taken with respect to x_1 and then evaluated at $x_1 = e$

This general result will work for all production and cost functions that are differentiable. From the properties of production functions, the coefficients of equation (12) can be narrowed down. Let's first look at the coefficient in front of the team wage, w. It will always be positive due to the fact that the first order derivative of a production function is always positive. By definition, $Q(e,e) > Q(e,0)$; therefore, the coefficient in front of the individual reward r is always positive as well. Lastly, as cost is increasing in terms of e, the first order derivative of cost will always be positive.

4.2 A Substitution Example

The insights found from the general case can be further explored through the analysis of specific examples of production functions. One extreme example that can be explored is the substitution production function. Recall that the production function of the team is defined as a substitute if, $\frac{\partial^2}{\partial e_1 \partial e_2}Q^T < 0$. Here is the specific function that we shall be analyzing.

$$Q^T = \sqrt{e_1 + e_2} \tag{15}$$

$$V_1^T = \sqrt{e_1 + e_2} - \sqrt{e_2} \tag{16}$$

This type of production function can be used to describe team projects in which the efforts of each individual team member are interchangeable with the others. Going back to the sports example of an MVP, this production function is more applicable when describing sports such as basketball, where the team members' roles in the game tend to overlap.

Finally, the last piece of information needed to obtain a concrete solution to this problem is a cost function which will be defined as

$$C(e_i) = \frac{1}{2}e_i^2 \tag{17}$$

This is a commonly used example of a cost function that is increasing with respect to effort.

From the model developed in the previous section, the first step is to solve for the players' equilibrium efforts in the second stage of the game. Each player's objective is to solve the following maximization problem.

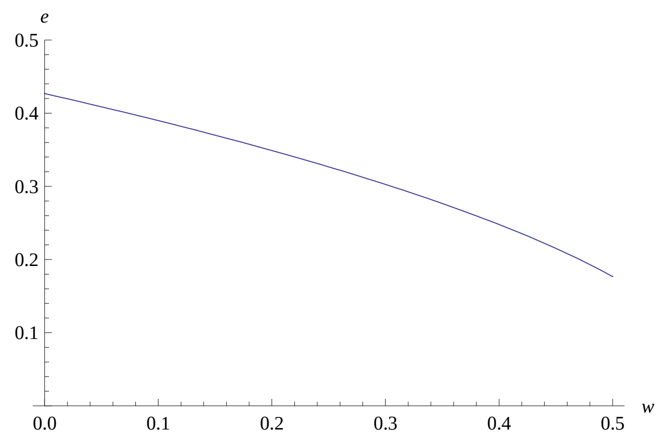
$$\begin{aligned} \max_{e_i} \pi_{x_1} = & \frac{\sqrt{x_1 + x_2}}{\sqrt{x_1 + x_2} + \sqrt{y_1 + y_2}} w \\ & + \frac{\sqrt{x_1 + x_2} - \sqrt{x_2}}{2\sqrt{x_1 + x_2} - \sqrt{x_1} - \sqrt{x_2} + 2\sqrt{y_1 + y_2} - \sqrt{y_1} - \sqrt{y_2}} r - \frac{1}{2}x_1^2 \end{aligned} \tag{18}$$

By differentiating each of the player's payoff function with respect to his effort level, four best response functions can be found from the FOC. Once again, in order to avoid solving this system of linear equations, we will be using assumption of symmetry (11) to simplify the problem for now. In the end, the individual equilibrium effort levels for all of the players can be written as a function of w and r

$$e^* = \frac{1}{4}\sqrt{w + \frac{r}{6 - 4\sqrt{2}}} \tag{19}$$

Notice the coefficients of w and r in this particular example. Effort is increasing both in terms of w and r. This is expected because the higher the reward for succeeding, the more likely an individual or a group will feel motivated to try harder. Not only do the signs of the coefficient matter, the magnitudes of the coefficients are important as well. Notice that r coefficient is larger than w. This means that a unit increase in r has a greater positive change in e^* than the same unit increase in w.

Graphically, Equation(19) is shown below.



Following the theoretical model, this result can be used to solve the organizer's problem in the first stage of the game.

$$\begin{aligned} w^* &= 0 \\ r^* &= M \end{aligned} \tag{20}$$

This result implies that the players are more motivated by individual rewards than team rewards. This observation falls in line with the substitutability of the team efforts. By awarding a high individual award, the free-rider problem that is common with interchangeable efforts within a group is minimized. In order to increase his odds of winning the individual award, the individual must increase his effort. Whereas, in the group contest, the individual has the incentive to slack off and leave the work to the his teammate because only the sum of their efforts matter to winning. As long as his teammate increases his effort, the player feels the same benefit as if he himself has worked harder. Therefore, increasing the group reward w is not the most effective way to encourage the player to commit more effort since part of the increase is lost to the free-rider inefficiencies. Therefore, in this particular example, the organizer should allocate all of his budget to the individual reward and disregard the team reward in this particular case.

4.3 A Complementary Example

On the other extreme end from the substitution case is the complementary case. Here, each member of the team has a distinct role that must be completed for the overall team effort to be successful. In terms of sports, this is perhaps a good model for baseball. Here, each team member is assigned a very distinct role while on the field; for example, the pitcher generally does not stray far from the mound and the first baseman does not go near third base.

Recall that for the efforts of a production function to be complementary, $\frac{\partial^2}{\partial e_1 \partial e_2} Q^T > 0$. The specific production function that will be the focus of this analysis is

$$Q^T = \sqrt{e_1 e_2} \tag{21}$$

$$V_1^T = \sqrt{e_1 e_2} \tag{22}$$

Likewise, the cost function will be the same as the one defined in equation (15).

Following the theoretical model, the individual players once again face the problem of maximizing their payoff function.

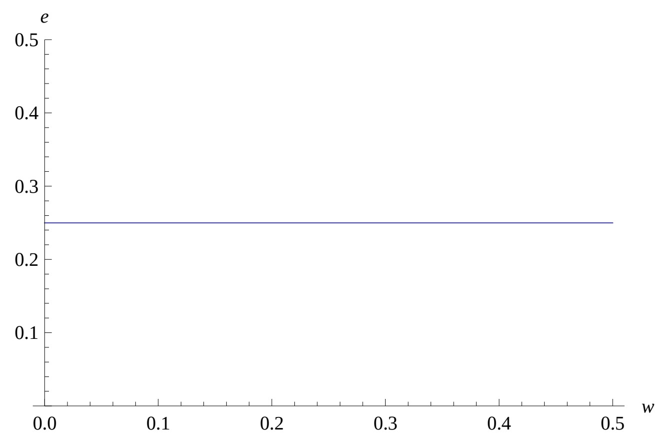
$$\max_{x_1} \pi_{x_1} = \frac{\sqrt{x_1 x_2}}{\sqrt{x_1 x_2} + \sqrt{y_1 y_2}} w + \frac{\sqrt{x_1 x_2}}{2\sqrt{x_1 x_2} + 2\sqrt{y_1 y_2}} r - \frac{1}{2} x_1^2 \tag{23}$$

For simplicity, once again the assumption of symmetry (11) is used to reduce the system of equations to a single equation. Each individual's equilibrium efforts can be written as a function of w and r

$$e^* = \frac{1}{4}\sqrt{2w + r} \tag{24}$$

The effort is increasing with respect to w and r; just like the substitution case, it follows the intuition that higher rewards offer individuals higher incentives to work harder. On the other hand, unlike the substitute case, the magnitude of w's coefficient is twice as large as that of r. This means that increasing the group reward by a unit will have twice the positive effect on the individual's effort than the same unit increase of the individual award.

Graphically, Equation(24) is shown below.



With this knowledge of the players' equilibrium, the organizer wants to maximize the total output of all the individuals subject to his limited and finite income, M.

$$\max_{w,r} \pi_{organizer} = 4\sqrt{\frac{w}{8} + \frac{r}{16}} \qquad s.t. \quad 2w + r = M \tag{25}$$

This is yet another constrained maximization problem. The result of this organizer's problem is

$$w^* = \left[0,\ \frac{M}{2}\right] \qquad r^* = M - 2w^* \tag{26}$$

The result from this particular example is rather interesting as it suggests that the individual is indifferent between the individual and group awards. Given the complementary nature of the production function, this result is reasonable because a player's effort is equally valuable in both the team and individual competition when considering the structure of the payoffs. Unlike the substitution case, the player's effort does not suffer from the inefficiencies of the free-rider problem. In order to raise his probability of winning the group award, the player must increase his own efforts when all other things are held equal.

The specific results from the substitution and complementary cases highlight some interesting points about this type of contest. The first thing to note is the production function's relation to the equilibrium points. In the substitution case, an equilibrium for the organizer states that there should only be an individual award in order to maximize efforts. At least with the substitution case of the basic model, the MVP award is vital to the success of organizer. Without the MVP award ($r = 0$) the utility of the organizer will be lower; therefore, he would be worst off.

In contrast, the results of the basic model's complementary case imply that the players are indifferent to the actual size of each reward relative to the other. This brings up an interesting question as to the purpose of the MVP award in the first place when teams exhibit complementary efforts. The MVP award does not add any extra value above the group award's value to the organizer or the players. Ironically, in this complementary case, the MVP award is actually serving as a substitute for the group award. These results only apply to these specific examples; nevertheless, they provide some insight to the problems faced by employers when trying to developed an optimal incentive plans for their employees. Sports leagues keen on increasing audience interests in their teams by making the competitions more competitive can use these results as a springboard to develop a payoff structure that is tailored to their own needs.

4.4 A General Example

Ultimately we would like to understand whether there are situations in which a team competition would benefit from an inclusion of an individual award. The resulting outcomes of the two examples offers some insights on the effects of a game's substitutability on the optimal reward structure. To further understand the effects of substitution on the optimal reward structure, we can utilize a production function which includes an added variable that measures the degree of substitution in a particular contest. Let this production function be defined as

$$Q^T = \alpha\sqrt{(e_1 + e_2)} + (1 - \alpha)\sqrt{e_1 e_2} \tag{27}$$

$$V_1^T = \alpha(\sqrt{e_1 + e_2} - \sqrt{e_2}) + (1 - \alpha)\sqrt{e_1 e_2} \tag{28}$$

This production function includes a new variable, α, which represents the degree of substitution among the team dynamics. For the purposes of this model, α will be constrained to

$$0 \leq \alpha \leq 1 \tag{29}$$

As α increases, the production function of a team's efforts become more substitutable. Notice that this new production function encompasses both of our previous examples. When $\alpha = 0$, the production function is equivalent to our complimentary case. Similarly, this new production function is the same as our substitution case when $\alpha = 1$.

This general production function can be used to describe a variety of team interactions with different levels of substitution.

As with the previous two examples, the cost function is defined as

$$C(e_i) = \frac{1}{2}e_i^2 \tag{30}$$

Once again, each individual player aims to optimize his own payoff function. For player 1 on team X, his optimization problem is

$$\max_{x_1} \pi_{x_1} = \left(\frac{\alpha\sqrt{(x_1+x_2)} + (1-\alpha)\sqrt{x_1x_2}}{\alpha\sqrt{(x_1+x_2)} + (1-\alpha)\sqrt{x_1x_2} + \alpha\sqrt{(y_1+y_2)} + (1-\alpha)\sqrt{y_1y_2}}\right)w$$
$$+\left(\frac{\alpha(\sqrt{x_1+x_2}-\sqrt{x_2}) + (1-\alpha)\sqrt{x_1x_2})}{\alpha(\sqrt{x_1+x_2}-\sqrt{x_2}) + (1-\alpha)\sqrt{x_1x_2}) + \alpha(\sqrt{y_1+y_2}-\sqrt{y_2}) + (1-\alpha)\sqrt{y_1y_2})}\right)r$$
$$-\frac{1}{2}x_1^2 \tag{31}$$

All the players are solving a symmetric version of player 1's optimization problem. Thus the optimal effort level in equilibrium, e, for all of the players is the one that satisfies the following equation

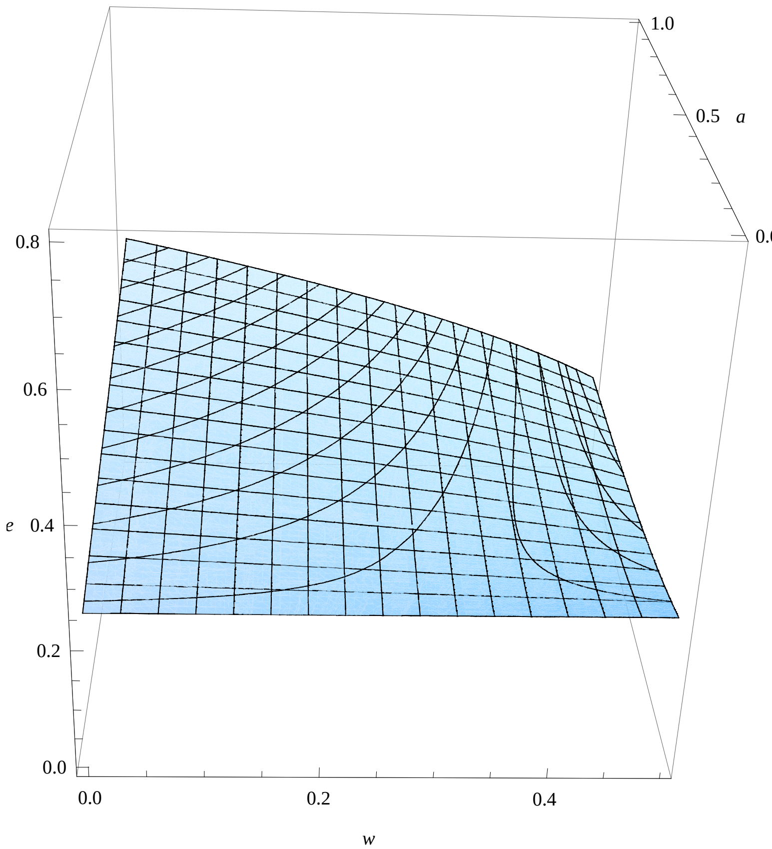
$$\frac{\partial}{\partial x_1}\pi_{x_1}\mid_e = \frac{1}{16}\frac{\left(\frac{\alpha\sqrt{2}}{\sqrt{e}} + 2(1-\alpha)\right)w}{\alpha\sqrt{2e} + (1-\alpha)e} + \frac{1}{16}\left[\frac{\left(\frac{1}{2}\frac{\alpha\sqrt{2}}{\sqrt{e}} + (1-\alpha) + \frac{1}{2}\frac{\alpha}{\sqrt{e}}\right)r}{\alpha\sqrt{2e} + (1-\alpha)e - \alpha\sqrt{e}}\right] - e = 0 \tag{32}$$

Unlike the previous examples, the explicit function for e in terms of r, w, and α cannot be simply computed. Thus it is not obviously clear what effects the degree of substitution, α, and the rewards, w and r, have on e.

However, we can still gleam some information from this equation by utilizing numerical analysis. For simplicity, let's normalize the value of the organizer's budget, M, to

$$M = 1 \tag{33}$$

so that the individual prize, r, can be rewritten in terms of the team prize, w. Now the function in (32) is reduced to a three variable function. To guide the analysis, the following graphical representation of this production function is helpful.



The illustrated surface shows the different combinations of w, α, and e that causes the function (32) to equal zero and thus making it optimal. As a reference, the horizontal axis indicates the values of w. The axis on the top indicates the α values while the remaining axis are the various e values. In this particular graph, the domains are as follows

$$
\begin{aligned}
0 \le w \le 0.5 \\
0 \le \alpha \le 1 \\
0 \le e \le 0.8
\end{aligned}
\tag{34}
$$

In order to start the numerical analysis, the equilibrium effort levels at various combinations of w and α were calculated (Table 1). The maximum effort level occurs when $w = 0$ and $\alpha = 1$ while the minimum effort level occurs when $w = 0.5$ and $\alpha = 1$. Notice that when $\alpha = 0$ the effort stays constant at $e = 0.25$ regardless of the w value. This falls in line with our complementary example in which all possible combinations of the team and individual rewards were optimal.

As with the other examples, it will be helpful to see how the equilibrium effort levels change with respect to changes in the other variables. Looking at the equilibrium efforts at the various game structures, it seems that generally speaking, effort is increasing with respect to α. Likewise, when α is held constant, the effort is decreasing in w for all $\alpha > 0$. In particular, we would like to see how w's effects on effort differ depending on the α levels. By holding α constant, the percentage change of the effort levels when the team prize value changes was found (Table 1).

The more interesting results come to play when taking a look at the percentage changes of effort. A sample of the results is shown here.

Table: Sample Numerical Results from General Example

α	w	e	Change in Effort
0.2	0	0.286	N/A
0.2	0.1	0.275	-0.038
0.2	0.2	0.264	-0.043
0.2	0.3	0.251	-0.048
0.2	0.4	0.237	-0.055
0.2	0.5	0.222	-0.065

α	w	e	Change in Effort
0.8	0	0.387	N/A
0.8	0.1	0.357	-0.076
0.8	0.2	0.323	-0.093
0.8	0.3	0.286	-0.117
0.8	0.4	0.241	-0.157
0.8	0.5	0.183	-0.240

First off, when $\alpha > 0$ and is constant, the magnitude of the effort's percentage change increases when the team reward increases. Let's remember that as the team reward increases, the individual reward simultaneously decreases. This results hints at a couple of observations. This implies that when there is a substitution element within a team's production function, the team is faced with a free rider problem which requires a component of an individual reward to counter it.

The most noteworthy result is that the magnitude of the percentage change of effort for the same changes in w increases as alpha increases as shown in the table above. This implies that as the team efforts become more substitutable, the free rider effect becomes more prominent in an individual's effort decision. The individual team reward value becomes more significant to the motivation of an individual player in order to limit the negative effects of the free rider problem. With smaller α the size of the individual reward is not as influential to the overall effort production as it is when α is higher.

Overall, the numerical analysis of this general example implies that the equilibrium efforts are inversely related to the team reward when the game has a positive degree of substitution. Likewise the inverse relationship between the efforts and team rewards become higher in magnitude as the degree of substitution increases. Based on this result, organizers of team competitions should include an individual reward in their compensation structure when the game includes a positive degree of substitution. Furthermore, they should really focus on increasing the size of the individual reward for games with higher degrees of substitution.

5 Conclusion

The main result of this study is that including a reward for an individual within a team competition can be beneficial to the organizer. Furthermore, based on the analysis of the general example, it seems that this benefit is correlated with the degree of substitution within an individual teams production function. Teams whose efforts exhibit higher levels of substitution are more motivated by the inclusion of an individual reward than those who have more complementary efforts.

These results offer some interesting suggestions to organizers who are trying increase the overall effort within a competition. For instance, the organizer of a baseball game, where efforts are complementary, should offer a lower individual prize than an organizer of the more substitutable basketball game in order to induce the optimal amount of total effort. These results can be extended beyond sports to topics such as management in the workforce. For instance, an award found in the workforce that is similar to the MVP award is the Employee of the Month. Here, even if the employee was working with other team members to succeed in their division, they are also eligible to gain additional individual recognition. Managers can use these results as a way to effective structure their employees' compensation scheme. In fact, team interactions play a role in many industries ranging from the entertainment to the political world. It goes to show that although there is no "I" in team there is certainly a "me".

An interesting caveat arises when the general example is applied to sports. Recall that the highest equilibrium effort level came when $\alpha = 1$ and $w = 0$. If the total effort is representative with the overall entertainment value of a sport, then this result implies that sports with higher degrees of substitution are more entertaining.

Further research can extend these results to more realistic scenarios. For instance, the current model only deals with symmetric players. However, in reality, most players in a team competition are not symmetric. Most sports teams generally have a superstar player among its regular players. Future research can examine the superstar's impact on the competition payment structure. Likewise, future research can extend the model to an n-player case to see whether the addition of players will change the optimal environments.

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Appendix

Substitution Example

FOC for Player 1 in Team X

$$\frac{\partial\pi}{\partial x_1}=\frac{\frac{\sqrt{y_1+y_2}}{2\sqrt{x_1+x_2}}}{(\sqrt{x_1+x_2}+\sqrt{y_1+y_2})^2}w$$
$$+\left[\frac{\frac{\sqrt{y_1+y_2}}{\sqrt{x_1+x_2}}-\frac{\sqrt{x_1}+\sqrt{x_2}-\sqrt{y_1}-\sqrt{y_2}}{2\sqrt{x_1+x_2}}+\frac{\sqrt{x_1+x_2}-\sqrt{x_2}}{2\sqrt{x_1}}}{\left(2\sqrt{x_1+x_2}+2\sqrt{y_1+y_2}-\sqrt{x_1}-\sqrt{x_2}-\sqrt{y_1}-\sqrt{y_2}\right)^2}\right]r-x_1=0 \tag{35}$$

Complementary Example

FOC for Player 1 in Team X

$$\frac{\partial\pi}{\partial x_1}=\frac{\frac{x_2\sqrt{y_1y_2}}{2\sqrt{x_1x_2}}}{(\sqrt{x_1x_2}+\sqrt{y_1y_2})^2}w+\frac{\frac{x_2\sqrt{y_1y_2}}{\sqrt{x_1x_2}}}{(2\sqrt{x_1x_2}+2\sqrt{y_1y_2})^2}r-x_1=0$$

General Example

Equilibrium Effort

$$\frac{\partial}{\partial x_1}\pi_{x_1}\mid_e=\frac{1}{16}\frac{\left(\frac{\alpha\sqrt{2}}{\sqrt{e}}+2(1-\alpha)\right)w}{\alpha\sqrt{2e}+(1-\alpha)e}+\tfrac{1}{16}\left[\frac{\left(\frac{1}{2}\frac{\alpha\sqrt{2}}{\sqrt{e}}+(1-\alpha)+\frac{1}{2}\frac{\alpha}{\sqrt{e}}\right)r}{\alpha\sqrt{2e}+(1-\alpha)e-\alpha\sqrt{e}}\right]-e=0$$

Table 1: General Example Numerical Results

α	w	e	Change in Effort
0	0	0.25	N/A
0	0.1	0.25	0
0	0.2	0.25	0
0	0.3	0.25	0
0	0.4	0.25	0
0	0.5	0.25	0
0.2	0	0.286	N/A
0.2	0.1	0.275	-0.038
0.2	0.2	0.264	-0.043
0.2	0.3	0.251	-0.048
0.2	0.4	0.237	-0.055
0.2	0.5	0.222	-0.065
0.4	0	0.319	N/A
0.4	0.1	0.301	-0.057
0.4	0.2	0.281	-0.066
0.4	0.3	0.259	-0.078
0.4	0.4	0.234	-0.098
0.4	0.5	0.203	-0.130

α	w	e	Change in Effort
0.6	0	0.352	N/A
0.6	0.1	0.328	-0.068
0.6	0.2	0.301	-0.081
0.6	0.3	0.271	-0.100
0.6	0.4	0.236	-0.130
0.6	0.5	0.191	-0.189
0.8	0	0.387	N/A
0.8	0.1	0.357	-0.076
0.8	0.2	0.323	-0.093
0.8	0.3	0.286	-0.117
0.8	0.4	0.241	-0.157
0.8	0.5	0.183	-0.240
1	0	0.427	N/A
1	0.1	0.39	-0.087
1	0.2	0.349	-0.105
1	0.3	0.303	-0.133
1	0.4	0.248	-0.181
1	0.5	0.177	-0.287