Coordination Mechanism Design for Sustainable Global Supply Networks

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Business Administration
in the Graduate School of Duke University
2011

Abstract

(Operations Management)

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Abstract

This dissertation studies coordination mechanism design for sustainable supply networks in a globalized environment, with the goal of achieving long-term profitability, environmental friendliness and social responsibility. We examine three different types of supply networks in detail.

The first network consists of one supplier and multiple retailers. The main issue is how to efficiently share a scarce resource, such as capacities for green technology, among all members with private information under dynamically changing environment. We design a shared surplus supply agreement among the members which can lead to both efficient private investments and efficient capacity allocation under unpredictable and unverifiable market conditions. The second network is a serial supply chain. The source node provides critical raw material (like coffee cherries) for the entire chain and is typically located in an underdeveloped economy, the end node is a retailer serving consumer at a developed economy (like Starbucks Co.). We construct a dynamic supply agreement that takes into account the changing market and production conditions to ensure fair compensations so that the partners have the right incentives to work together to develop sustainable quality supply.

The third network is a stylized global production network of a multinational company consisting of a home plant and a foreign branch. The branch serves the foreign market but receives a key component from the home plant. The distinctive feature is that both facilities belong to the same company, governed by the headquarters, yet

they each also have their own autonomies. We analyze the role of the headquarters in designing coordination mechanism to improve efficiency. We show the headquarters can delegate the coordination effort to the home plant, as long as it keeps veto power.

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List of Abbreviations and Symbols

Symbols

Notations for Chapter 2:

$i = \mathcal{S}, \mathcal{R}$

Δ^t_{r-s}	the number of developing stage to developed stage after period t.
δ	discount factor.
ϵ^t_i	the random part of $i's$ condition.
θ^t_s	$\mathcal{S}$'s production condition in period t.
θ^t_r	$\mathcal{R}$'s market condition in period t.
θ^t	(θ^t_s, θ^t_s).
θ^t_{-i}	member different from i's type in period t.
$\tilde{\theta}^t_i$	i's report of θ^t_i in period t.
$\tilde{\theta}^t_{-i}$	member different from i's reports of type in period t.
θ^t_{iw}	i's worst off type in period t.
λ	increment rate of θ^t_s and θ^t_r given θ^{t-1}_s and θ^{t-1}_r when θ^t_s and θ^t_r are identical.
ν^t_i	increment rate of θ^t_i given θ^{t-1}_i.
$\Pi^t(\theta^t)$	the supply chain profit in period t given θ^t.
$\Pi^t_i(x, \theta^t_i)$	i's profit in period t given θ^t_i and fix x and τ.
ρ^t_i	i's control right in period t.

ρ^t	(ρ^t_s, ρ^t_r).
ρ^{t*}_i	i's control right in period t that maximizes supply chain's total participation fee.
ρ^{t*}	$(\rho^{t*}_s, \rho^{t*}_r)$.
τ^t_i	i's transfer payment received in period t.
τ^t	(τ^t_s, τ^t_r).
$\tilde{\tau}^t_i$	i's AVG transfer payment received in period t.
$\tilde{\tau}^t$	$(\tilde{\tau}^t_s, \tilde{\tau}^t_r)$.
$B^t(\rho^{t*})$	the difference between the supply chain profit and the total profit each member must receive under the SQSA mechanism.
b^t_i	i's bid in period t.
$c(x)$	$\mathcal{S}$'s effort to provide quality x.
$C(x^t, \theta^t_s)$	$\mathcal{S}$'s production cost given quality x^t and θ^t_s.
$F^t_i(\theta^t_i \vert \theta^{t-1} \vert i)$	the c.d.f. of θ^t_i in period t given θ^{t-1}_i.
$\tilde{H}^t$	set of possible period t reported histories.
h^t_i	i's history upto period t.
$\tilde{h}^t_i$	i's reported history upto period t.
I_i	i's investment.
I	(I_s, I_r).
I^*_i	i's optimal investment under SQSA.
I^*	(I^*_s, I^*_r).
$K_i(\theta^{t-1})$	$E_{\theta^t_{-i}} V^t(\theta^t) - W^t_i(\theta^{t-1}, \theta^t_i)$.
$M \left\langle x^t, \tau^t, r^{t+1} \right\rangle_{t=1}^{t=T}$	the mechanism.
m_i	i's payment received under the all pay auction.
$N_h(t)$	number of developed stage after period t.
$N_l(t)$	number of developing stage after period t.

$O_i^t(\rho^t)$ — i's outside option payoff in period t given ρ^t.

$\tilde{p}_i^t$ — i's net payment in period t.

$R(x^t, \theta_r^t)$ — $\mathcal{R}$'s revenue given quality x^t and θ_r^t.

$\mathcal{R}$ — retailer.

$\mathcal{S}$ — supplier.

T — number of period.

V_i^t — i's expected profit in period t given the quality x and transfer payment τ are fixed from period t onwards.

V^{t*} — expected supply chain maximum profit in period t.

W_i^t — i's maximum expected profit in period t.

w_i^t — i's willingness to pay in period t.

x — quality of the product.

x^t — quality of the product in period t.

x_i^o — i's choice of quality if the contract terminates.

Notations for Chapter 3:

δ — discount factor.

$\bar{\theta}$ — the upper bound of θ_S^t.

$\bar{\theta}_c$ — the upper bound of θ_B^t.

θ_i^t — i's type in period t.

θ_{-i}^t — member different from i's type in period t.

θ^t — $(\theta_1^t, ..., \theta_N^t)$.

$\tilde{\theta}_i^t$ — i's report of θ_i^t in period t.

$\tilde{\theta}_{-i}^t$ — member different from i's reports of type in period t.

θ_{iw}^t — i's worst off type in period t.

λ — the probability that the supply chain is compatible.

$\Pi^t(\theta^t, x^t)$	the supply chain profit in period t given θ^t and x^t.
$\pi_i^t(\theta_i^t, x_i^t)$	i's profit in period t given θ_i^t and x_i^t.
$\Pi^*(\theta^t)$	the supply chain maximum profit in period t.
τ_i^t	i's transfer payment received in period t.
τ^t	$(\tau_1^t, ..., \tau_N^t)$.
$\tilde{\tau}_i^t$	i's AVG transfer payment received in period t.
$\tilde{\tau}^t$	$(\tilde{\tau}_1^t, ..., \tilde{\tau}_N^t)$.
$B^t(\rho^{t*})$	the difference between the supply chain profit and the total profit each member must receive under the SSS mechanism.
B	buyer.
$b_i^t(\theta_i^t)$	i's bid function in period t.
$\tilde{b}_i^t$	i's bid in period t.
c	compatibility.
$\mathcal{F}_{\rangle}^{\sqcup}$	i's entry fee in period t.
$F_i^t(\theta_i^t \vert \theta_i^{t-1})$	the c.d.f. of θ_i^t in period t given θ_i^{t-1}.
$f_i^t(\theta_i^t \vert \theta_i^{t-1})$	the p.d.f. of θ_i^t in period t given θ_i^{t-1}.
$\tilde{H}^t$	set of possible period t reported histories.
h_i^t	i's history upto period t.
$\tilde{h}_i^t$	i's reported history upto period t.
I_i	i's investment.
I	$(I_1, ..., I_N)$.
I_i^*	i's optimal investment under SSS.
I^*	$(I_1^*, ..., I_N^*)$.
$K_i(\theta^{t-1})$	$E_{\theta_{-i}^t} V^t(\theta^t) - W_i^t(\theta^{t-1}, \theta_i^t)$.
$M \langle x^t, \tau^t, r^{t+1} \rangle_{t=1}^{t=T}$	the mechanism.
m_i	i's payment received under the all pay auction.

N	number of members in the supply chain.
$\tilde{p}_i^t$	i's net payment in period t.
r_i^t	i's control right in period t.
r^t	$(\rho_1^t, ..., \rho_N^t)$.
r_i^{t*}	i's control right in period t that maximizes supply chain's total participation fee.
r^{t*}	$(\rho_1^{t*}, ..., \rho_N^{t*})$.
S	supplier.
T	number of period.
$U(I)$	expected channel surplus.
$U^{H,1}$	expected channel surplus of a T-period SSS agreement for a H supply chain.
$U^{L,1}$	expected channel surplus of a T-period SSS agreement for a L supply chain.
V_i^t	i's expected profit in period t given the quality x and transfer payment τ are fixed from period t onwards.
V^t	expected supply chain maximum profit in period t.
W_i^t	i's maximum expected profit in period t.
$\tilde{w}_i^t(\theta^t)$	i's expected participation fee in period t.
$w_i^t(\theta^t)$	i's willingness to pay in period t.
$\bar{w}_i^t(r^t, \theta^t)$	i's participation fee in period t before maximizing over r^t.
$\bar{x}$	total capacity.
x_i^t	i's capacity in period t.
$x_i^*(\theta^t)$	i's capacity in period t when the supply chain surplus is maximized given θ^t.
x^t	$(x_1^t, ..., x_N^t)$.

Notations for Chapter 4:

α^*	$\mathcal{C}$'s optimal α.
α	$\mathcal{H}$'s share of k.
α^h	$\mathcal{H}$'s share of k that minimizes $\mathcal{H}$'s cost.
$\beta(k)$	$\sqrt{\omega(k)/k}$.
λ	deterministic demand rate.
γ	$\lambda hb/(2(h+b))$.
μ	production capacity.
ν	shelf capacity.
η	$H(t(k))/B^u(k)$.
θ	$Pr(D(t) > 0)/t$.
ΔH	cost savings of $\mathcal{H}$ under the optimal cost sharing contract.
ΔB	cost savings of $\mathcal{B}$ under the optimal cost sharing contract.
ΔC	cost savings of $\mathcal{C}$ under the optimal cost sharing contract.
ξ_H	$\Delta H/\Delta C$.
ξ_B	$\Delta B/\Delta C$.
$\varphi(x)$	$F(x)/f(x)$.
$\omega(x)$	$x + \varphi(x)$.
$A(x)$	compensation to $\mathcal{B}$ given x.
$\mathcal{B}$	foreign branch.
$\widehat{B}(k, T(k))$	$\mathcal{B}$'s minimum cost given k and $T(k)$.
$B(s, t\|k)$	$\mathcal{B}$'s long run average cost.
$B^u(k)$	$B(s(k), t(k)\|k)$.
$\mathcal{C}$	supply chain.
c_e	unit expediting cost.
c	unit production cost.
$\widehat{C}(s, t\|k)$	total supply chain cost.

$D(\tau)$	cumulative demand of the key component at $\mathcal{B}$ within $[0, \tau)$.
$F(k)$	c.d.f. of k.
$f(k)$	p.d.f. k.
$\mathcal{H}$	home plant.
$H(t)$	$\mathcal{H}$'s long run average cost.
$\tilde{H}(k, \alpha, t)$	$H(t) + (1 - \alpha)k\theta(t)$.
h	unit hold cost.
k	fixed order cost.
$\bar{k}$	upper bound of k.
$\underline{k}$	lower bound of k.
$\mathcal{M}$	the mechanism.
R^+	$(0, +\infty)$.
$r = (r_H, r_B)$	the control rights of $\mathcal{H}$ and $\mathcal{B}$.
$\mathcal{S}_\mathcal{B}$	$\mathcal{B}$'s set of strategies.
s	base stock level.
$s^*(k)$	$\mathcal{C}$'s optimal base stock level.
$s(t\|k)$	$arg\min_s B(s, t\|k)$.
$s(k)$	$arg\min_s B(s, t(k)\|k)$.
$T(x)$	menu of replenishment interval.
t	replenishment interval.
$t^*(k)$	$\mathcal{C}$'s optimal replenishment interval.
$t(s\|k)$	$arg\min_t B(s, t\|k)$.
$t(k)$	$arg\min_t B(s(k), t\|k)$.
X_n	$\mathcal{H}$'s demand in the nth period.

Abbreviations

BB	budget balance.
BIC	Bayesian incentive compatible.
EE	ex post efficient.
IIR	interim individual rationality.
SQSA	sustainable quality supply agreement.
AVG	d'Aspremont and Gérard-Varet.
SSS	shared surplus supply.
EOQ	efficient order quantity.
IC	incentive compatible.
IR	individual rationality.
HA	headquarter's agreement.

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1

Introduction

This dissertation studies coordination mechanism design for sustainable supply networks in a globalized environment. As companies seek global resources and markets, the supply networks are becoming more and more geographically dispersed across different economies and cultures, involving long distance communication and transportation. As a result, companies are facing increasingly pressing sustainability issues, such as long term profitability, environmental friendliness and social responsibility. To meet these challenges, it requires the members in the supply networks to build long-term relationships with mutual trust and economic accountability. The purpose of this dissertation research is to design coordination mechanisms to enable companies to achieve these requirements. We examine three different types of supply networks in detail.

Chapter 2 studies a network consisting of one supplier and multiple retailers. The main issue is how to efficiently share a scarce resource, such as capacities for green technology, among all members with private information under dynamically changing environment. This setting is applicable to many situations such as an organic produce farmer and a set of supermarkets, a hybrid car manufacturer and a set of dealers, or a contract manufacturer and a set of OEMs. The goal is to induce optimal investments in both the production and applications of green technology and to allocate the limited production resource to the mostly needed. We design a shared surplus supply agreement among the members which can lead to both efficient private investments and efficient capacity allocation under unpredictable and unverifiable market conditions.

Chapter 3 studies a serial supply chain. The source node provides critical raw material (like coffee cherries) for the entire chain and is typically located in an underdeveloped economy, the end node is a retailer serving consumer at a developed economy (like Starbucks Co.). With information asymmetry and unevenly distributed bargaining powers along the global supply, the supplier at the source is often unfairly compensated, leaving it difficult to make ends meet, not to mention having the resources and ability to invest in the production process to maintain the quality of the critical material, which endangers the sustainability of the entire chain. We construct a dynamic supply agreement that takes into account the changing market and production conditions to ensure fair compensations so that the partners have the right incentives to work together to develop sustainable quality supply.

Chapter 4 studies a stylized global production network of a multinational company consisting of a home plant and a foreign branch. The branch serves the foreign market but receives a key component from the home plant. The distinctive feature is that both facilities belong to the same company, governed by the headquarters, yet they each also have their own autonomies. The existing production and logistics cost allocation schemes result in great inefficiency in the cross-continental supply chain with complex international logistics, contributing adversely to the environmental and economic outcomes. We analyze the role of the headquarters in designing coordination mechanism to improve efficiency. We show the headquarters can delegate the coordination effort to the home plant, as long as it keeps veto power.

The Appendix contains all the proofs.

2

Shared Supply Surplus Mechanism

2.1 Introduction

A manufacturer and $N-1$ independent retailers come together to form a simple supply chain. Examples of the supply chain we envision include an organic-produce farmer and a set of supermarkets, a hybrid-car manufacturer and a set of dealers, or a contract manufacturer and a set of OEMs. The retailers are seeking a reliable supply of goods to sell to their customers. The manufacturer wants a reliable source of retailers to buy his stock of goods. Each supply chain member benefits by establishing a coordinated supply channel that provides for an ample supply and efficiently allocates capacity to the highest demand retailers. With a supply chain in place, the members profit by making investments to create capacity (in the case of the manufacturer) and to increasing demand for their merchandise (in the case of retailers). Moreover these investments are complementary; the greater the supply, the more profitable it is for the retailer to invest to increase his demand, and the greater the demand, the more profitable it is for the manufacturer to increase his capacity.

The members' mutual gains from a coordinated supply channel are great, but so are the problems in attaining coordination. As Cachon and Lariviere (1999) demonstrate, coordination among buyers and sellers of capacity is hard to achieve because the retailers' demand and the manufacturer's costs fluctuate across periods in ways that are difficult to predict and impossible to verify. Whereas a simple revenue sharing arrangement is sufficient for coordination when demand and supply conditions can be observed and contracted on, such agreements are unenforceable when the revenues and costs of the channel members are private knowledge. Consider for instance, the producer who agrees to provide retailers with specified levels of supply each period. What is he to do when retail orders exceed his capacity or when retail orders fall short of his supply? Whereas a cost or revenue sharing agreement between the supplier and retailers could in principle resolve this problem, it's generally difficult, if not impossible to measure the supplier's cost or the retailer's revenues upon which the reallocation is based. Other processes for reallocation, such as proportional rationing of original orders that the supplier may commit to are contentious and are susceptible to strategic manipulation by the retailers.

Whenever there is imperfect allocation of supply capacity, either because stocking demands are unobservable or noncontractible, Cachon and Lariviere (1999) and Plambeck and Taylor (2006) demonstrate the incentives for members to make relationship specific investments to increase supply chain value are distorted as well. In our context the less reliable supply becomes, the less the retailer will invest to increase demand for his merchandise. In turn, the less retail demand there is, the less the manufacturer will invest in capacity. Generally the failure to coordinate one downstream activity like reallocating supplies to meet excess demand, can lead to "holdup" problems (e.g. Rogerson (1992), and Plambeck and Taylor (2006)) that adversely impacts upstream investment in manufacturing capacity and product innovation.

In this paper we view supply chain coordination as an exercise in mechanism design directed to address three fundamental questions:

- *What supply capacity allocations can be implemented?*
- *What initial investments in capacity and demand can be induced?*
- *What agreements for coordination are self enforcing and sustainable?*

The first question is formally addressed by designing a mechanism that allocates supplies among the members based on the supplier's report of cost and the retailers' reports of their demand. The mechanism characterizes the set of allocations that can be implemented by payments of transfers between the channel members based on their reports. The mechanism requires, without any loss of generality, that the transfers be designed to induce the members to report truthfully.

Interestingly enough, the mechanism reveals that the ex post efficient supply allocation is possible. Moreover it can be implemented with transfer payments that are similar to the revenue sharing agreements employed to coordinate channel supply with verifiable costs and revenues (see Cachon and Lariviere (2005) and Bernstein and Federgruen (2003)). The transfers which we call *Shared Supply Surplus* or SSS extends the revenue sharing concept to settings where costs and revenues aren't observed and allocations are repeatedly renegotiated. Under SSS the manufacturer receives compensation equal to the *reported* revenues generated by the retailers. In turn each retailer pays the producer's *reported* costs and receives the *reported* revenues of all other retailers. Since costs and revenues are not publicly observed, the sharing of surplus is entirely based on the self reported cost and revenues of the supply chain members.

The effect of SSS is to make all members the owners (or residual claimants) of the expected supply channel surplus. As a result, the incentives of the supplier and retailers are perfectly aligned to allocate supplies efficiently to maximize supply chain surplus. To illustrate, under SSS each retailer pays the manufacturing cost and receives the revenues generated by the other retailers. Therefore he has an incentive to gather and report information on market demand to maximize his and other members' net revenues. Likewise, the manufacturer has incentives to gather and report information on his cost to allocate supply in order to maximize his and other members' channel surplus. In each instance, SSS induces members to coordinate decisions to produce efficiently.

The question of investment is also efficiently resolved by SSS agreements. We demonstrate that SSS induces members to invest efficiently in the supply chain, even for instance, when an investment by one member may hurt or harm another member. The SSS agreement induces members to internalize spillovers when investing by making each a residual claimant to the total chain surplus.

Finally, the last question of sustainability is addressed by analyzing what SSS agreements are sufficient to sustain each member's voluntary participation in the channel. Supply agreements can not be enforced against members who refuse to participate. When an agreement is breached members may exercise their residual control rights to determine what claims they have to supply capacity in the future. Default rights can vary. One allocation of rights allows the supplier to buy out his obligation to serve the retailers. Another allocation allows one or more retailers to buy out the supplier to obtain control of his capacity. We find agreements that allocate rights to capacity in *reverse order* of how much the member would pay to control supply, are sustainable. So if retailers value the supply more than the manufacturer, then the manufacturer is allocated exclusive rights of control. The agreement is sustained by giving members who are most likely to breach the contract the least rights of control.

Our findings are related to a number of ideas that have appeared in the supply chain management literature and in the incentives literature in economics. With regards to supply chain management our analysis is most closely related to the papers by Cachon and Lariviere (1999, 2005), Bernstein and Federgruen (2003) and Plambeck and Taylor (2005, 2006, 2007) [1]. This literature emphasizes the role of contracts partnerships and joint ventures in coordinating supply and facilitating investment. The extent to which channel coordination is achieved depends on what coordinating devices are employed, what variables, like revenues and costs, can be observed and what activities, including supply and investment, are contractible. This literature considers a class of contracts restricted to known allocation mechanisms, and concludes that complete coordination can be achieved if and only if market conditions are verifiable and actions are contractible. Our analysis differs from this literature in two respects. First, we consider the larger unrestricted class of mechanisms for implementing coordinated supply. And second, as a result, we find that coordination is generally possible even when market conditions are unverifiable, provided that channel members can contract over default control rights.

There is also a literature on contracting in economics that our paper builds on. Our analyses extends previous work on efficient exchange mechanisms by Cramton, Gibbons and Klemperer (1987), Edlin and Reichelstein (1996), Krahmer and Strausz (2006), Rogerson (1992), Schweizer (2006) and Segal and Whinston (2009) to a dynamic setting. The analyses closest to ours are recent papers by Bergemann and Valimaki (2006, 2010) and Athey and Segal (2007a,b). These papers construct contracts implementing efficient exchange and investment, but fail (in Athey and Segal) to always be individually rational and (in Bergemann and Valimaki (2006, 2010)) to balance the budget. Our paper extends these analyses by constructing agreements

[1] See also the excellent surveys by Cachon (2003) and Chen (2003) that describe earlier work on contracting in supply chain coordination.

that are always ex post efficient, budget balancing and individually rational.[2] The enabling mechanisms for achieving budget balance and voluntary participation is to allow the members to exchange control rights upon the arrival of new information.

The rest of the paper is organized as follows: The next section outlines our model of channel coordination. Section 3 constructs SSS agreements that allocate supplies efficiently, are budget balancing and self enforcing. Section 4 demonstrates how channel coordination and investment may be implemented in theory and in practice.

2.2 Shared Surplus Supply Chain

2.2.1 Model of the Supply Chain

We consider the multi-period version of the capacity choice and allocation model analyzed in Cachon and Lariviere (1999). A supply chain consisting of N risk neutral members is formed. Member $i = 1$, is the designated supplier who sells to the retail buyers $j = 2, .., N$, although we make no real distinction between a supplier and buyer in our analysis.[3] There is a fixed total capacity $\bar{x}$ (normalized to 1) that is allocated among the members in each period, $t = 1, .., T$ in the amount of x_i^t, where $x_i^t \geq 0$ and $\Sigma_i x_i^t = 1$. We are implicitly assuming that supply is fixed and is perishable so that it must be used by the end of the period, or it is lost. Alternatively one can imagine that the supplier is able to sell his unused supply to businesses outside of the supply chain at a price determined on the spot market.

Before each period, each i privately observes his demand type represented by $\theta_i^t \in [0, 1]$ for that period. Member $1's$ demand represents the opportunity cost of selling his supply to retailers, which he could otherwise use for self production or for

[2] Kuribko and Lewis (2010) derive an agreement with the same properties for the governance of a joint development partnership.

[3] The restriction to a single supplier can easily be modified to allow for multiple suppliers without any substantive change to our results.

sale outside of the supply chain. The retailers' demands are for the products they distribute to their customers. Member $i's$ single period profit, $\pi_i(\theta_i^t, x_i^t)$, is a function of his demand θ_i^t and supply, x_i^t, and is twice continuously differentiable with,

$$\frac{\partial \pi_i(\theta_i^t, x_i^t)}{\partial x_i^t} > 0, \ \frac{\partial \pi_i(\theta_i^t, x_i^t)}{\partial \theta_i^t} > 0, \ \frac{\partial^2 \pi_i(\theta_i^t, x_i^t)}{\partial \left(x_i^t\right)^2} < 0, \ \frac{\partial^2 \pi_i(\theta_i^t, x_i^t)}{\partial x_i^t \partial \theta_i^t} > 0 \text{ for } \theta_i^t \in (0, 1]$$

Profits are increasing in supply at a decreasing rate and total and marginal profits are increasing in demand.

The demand valuation is a privately observed finite random variable, $\theta_i^t \in [0, 1]$, which is serially correlated with time, but independently distributed across members. The demands evolve according to a first order Markov process.[4] Member $i's$ demand θ_i^t is drawn form a *c.d.f.* ,$F_i^t(\theta_i^t|\theta_i^{t-1})$,with a bounded, strictly positive and continuous density, $f_i^t\left(\theta_i^t|\theta_i^{t-1}\right) > 0$. For now we assume the initial value distribution $F_i^1(\theta_i^1|I)$ depends on an initial investment $I \equiv (I_1, .., I_N)$ the members make in period $t = 0$ prior to formation of the supply channel, (and is described below in section 4). The demands in different periods are assumed to be stochastically ordered by,

$$\frac{\partial F_i^t\left(\theta_i^t|z\right)}{\partial \left(z\right)} \leq 0, \tag{FOSD}$$

that is an increase in previous period demand shifts the present period distribution towards higher demands. This allows for demands to be positively correlated or to be independently distributed over time. When there is strictly positive correlation, a further technical assumption that is convenient to impose in some instances is,

$$\frac{\partial^2 F_i^t\left(\theta_i^t|z\right)}{\partial \left(z\right)^2} = 0. \tag{LFOSD}$$

[4] The restriction to Markov processes is for convenience in characterizing the efficient supply agreement, and can be extended to more general settings with some additional complications that we discuss below.

The *linear FOSD* condition ($LFOSD$) indicates that there is a constant shift in the distribution function for any increase in previous period costs. This is a regularity condition that ensures that the expected surplus functions are convex and therefore there is a unique demand realization where the members' participation constraint binds.[5]

2.2.2 The Optimal Coordinated Chain

The fully coordinated supply chain achieves the maximum surplus. This requires the members to coordinate on a capacity allocation each period, $x^t \equiv \{x_i^t\}_{i=1}^N$, that maximizes the channel value. Given $\theta^t \equiv (\theta_i^t)_{i=1}^N$ and x^t the period t aggregate supply surplus is

$$\Pi(\theta^t, x^t) \equiv \sum_i \pi_i(\theta_i^t, x_i^t).$$

Previous assumptions on $\pi_i(\cdot)$ and $F_i^t(\cdot)$ ensure there is a unique surplus maximizing capacity allocation, $x^*(\theta^t)$, such that

$$\Pi^*(\theta^t) \equiv \Pi(\theta^t, x^*\left(\theta^t\right)) = \sum_i \pi_i(\theta_i^t, x_i^*\left(\theta^t\right))$$

is the maximized period t channel surplus and

$$V^t\left(\theta^t\right) \equiv \Sigma_{k=t}^T \delta^{k-t} E\Pi^*(\theta^k \mid \theta^t)$$

is the total expected discounted value of the supply channel starting in period t with demand, θ^t, given $\delta \in (0,1]$ is the common discount factor and $E\Pi^*(\theta^k \mid \theta^t)$ is the expected chain surplus in period k, given θ^t.

[5] In static models, the convexity condition is automatically satisfied. Additional structure on the demand distributions are required to ensure this holds for the multi-period case when demands are positively correlated over time.

2.2.3 *Shared Surplus Supply Agreement*

This section proposes a direct mechanism to determine the set of capacity allocations and investments that the supply chain can implement. The mechanism can indicate the conditions under which full coordination of the supply channel is possible. Once we determine this we specify the agreement required for complete coordination. This approach does not presume a particular supply policy is optimal, but instead it determines what allocations are possible given the constraints that the members operate under.

Formally, a mechanism is an agreement specifying the capacity allocation and transfers each member receives in each period. One constructs the mechanism by having the members report their current period capacity demands, θ_i^t. Based on the reports the capacity allocation is assigned and transfers between members are designated. This construction is without loss of generality as the *Revelation Principle* ensures any feasible allocation can be implemented through a direct mechanism that allocates capacity based on the members' reports.[6]

The interaction between the members proceeds as follows. Initially the members agree to a multi period direct mechanism, denoted by[7]

$$M\left\langle x^t, \tau^t, r^{t+1}\right\rangle_{t=1}^{t=T} \tag{2.1}$$

In each period t, M stipulates a vector of supplies, transfers and control rights, denoted respectively by $\left\{x^t\left(\tilde{h}^t\right), \tau^t\left(\tilde{h}^t\right), r^{t+1}\left(\tilde{h}^t\right)\right\}$ which is based on the history, $\tilde{h}^t = \left(\tilde{h}_1^t, .., \tilde{h}_N^t\right)$ of reported capacity demands through period t. Each $i's$ reported history $\tilde{h}_i^t = \left(\tilde{\theta}_i^1, .., \tilde{\theta}_i^t\right)$ consists of the disclosed capacity demands through period t,

[6] The Revelation Principle is due to Myerson(1986). It asserts any feasible allocation of resources to privately informed agents can also be implemented by committing to an allocation based on the agents' truthful disclosure of their information.

[7] Note in period T, the supply channel is dissolved so that r^{T+1} is undefined.

which can't be verified and may differ from $i's$ actual history of demands denoted by $h_i^t = (\theta_i^1, .., \theta_i^t)$.

Let $\tilde{H}^t$ be the set of all reported demands. Then the capacity allocation under M is determined by the mapping,

$$x^t : \tilde{H}^t \to X, \tag{2.2}$$

where x^t is the t period allocation and X is set of all feasible allocation such that $x_i^t \geq 0$ and $\Sigma_i x_i^t = 1$ for all i, t. After determining the allocation, each i makes (or receives) a transfer τ_i^t, determined by the mapping,

$$\tau^t : \tilde{H}^t \to \mathbb{R}^N. \tag{2.3}$$

The last and most noteworthy provision of M is the assignment of *control rights.* The rights for period t are allocated by the mapping,

$$r^t \ : \tilde{H}^{t-1} \to [0,1]^N \ \ s.t. \sum\nolimits_i r_i^t = 1. \tag{2.4}$$

Control of the supply capacity is awarded to member i with probability, r_i^t, if the *SSS* is dissolved in period t.[8] In practice, most contracts provide for the enforcement of default rights of control when the parties fail to agree. These rights stipulate who controls the asset(s), which is the supply capacity in our case. For instance, the manufacturer may receive exclusive rights to his supply or the supply rights may be awarded to one or more retailers to use. In our setting we allow for *probabilistic rights* so that partial control can be distributed to several members in contrast to vesting complete control with one member.[9]

[8] It is reasonable to assume that the rights are allocated independent of who initiates the breakup of the supply chain agreement. Practically, courts are only able to observe the dissolution of an agreement , but not the member(s) who precipitated the breakup. This is similar to a "contracting at will" standard where courts only enforce agreements that are contingent on the level of trade, but not on who precipitates the trade disruption.

[9] It is also possible to give members control of fixed portions of the capacity. However, this would be hard to enforce without a formal agreement.

In our application rights are allocated to enable members to dissolve the agreement if they desire. We assume each period any party may dissolve the agreement after observing their current period demand. In that case member i is assigned control right r_i^t, the probability that he is allowed to employ the capacity for his own use. With this right i may thereby earn expected continuation profits equal to,

$$r_i^t V_i^t \left(\theta_i^t, 1\right) = r_i^t \Sigma_{k=t}^T \delta^{k-t} E\left[\pi_i \left(\theta_i^k, 1\right) \mid \theta_i^t\right],$$

where $V_i^t\left(\theta_i^t, 1\right)$ is the expected profit for member i with current demand θ_i^t who controls the supply chain capacity starting in period t.

The rights are allocated to ensure each member's participation in the SSS is voluntary. The allocation distributes *all* of the capacity *only* to the members so that, $\Sigma_i r_i^t = 1$. This prevents the destruction or diversion of assets to a third party who serves as a budget breaker (e.g. Plambeck and Taylor (2006)) to prevent disagreement. This provision reduces the private cost of dissolving the agreement by providing some assets to the members if the channel agreement is terminated. Therefore it makes it more difficult to sustain the agreement because members can leave at lower cost. Nonetheless we retain this assumption, because in practice the confiscation of members' assets is illegal and therefore difficult to implement. Moreover, we are only able to assess the limits of self sustaining agreements, by assuming members are free to quit.

In summary, the provisions of the agreement are specified by the dynamic mechanism, M that is implemented each period in two stages:

Stage 1: Decision to Participate: *Each i privately observes θ_i^t and decides to remain in or quit the supply chain. If a member(s) quit, the chain is dissolved and i receives the rights to control supply with probability r_i^t. Otherwise the agreement moves to the next stage.*

Stage 2: Reporting: *Each i reports $\tilde{\theta}_i^t \in [0,1]$ and the capacity allocation–*

transfers–rights provisions, $\left\langle x\left(\tilde{h}^t\right), \tau_i^t\left(\tilde{h}^t\right), r_i^{t+1}\left(\tilde{h}^t\right)\right\rangle$ *are implemented. If* $t = T$, *the agreement ends, otherwise if* $t < T$ *the agreement moves to period* $t+1$.

2.2.4 Equilibrium

The mechanism M induces a sequential simultaneous move game, wherein the members choose a sequence of disclosure rules in each period to maximize their expected surplus. (Section 4 extends this game to include initial investment.) Member $i's$ expected payoff from reporting a sequence of capacity demands $\tilde{\theta}_i = \left\{\tilde{\theta}_i^t\right\}$, given members different from i report $\tilde{\theta}_{-i} = \left\{\tilde{\theta}_{-i}^t\right\}$, is

$$E\sum_{t=1}^{T}\left[\delta^{t-1}\left(\tau_i^t\left(\tilde{h}^{t-1}, \tilde{\theta}_i^t, \tilde{\theta}_{-i}^t\right) + \pi_i\left(\theta_i^t, x_i^t\left(\tilde{h}^{t-1}, \tilde{\theta}_i^t, \tilde{\theta}_{-i}^t\right)\right)\right)\right].$$

Member i selects a disclosure, that maps from his history of reports and period t demand,

$$\tilde{\theta}_i^t : \left[\tilde{h}^{t-1}, \theta_i^t\right] \to [0,1]$$

to solve the following sequential maximization problem. Let $W_i^t\left[\tilde{h}^{t-1}, \theta_i^t\right]$ be $i's$ maximized surplus in period t, given his history of reports and his period t demand. Then $W_i^t\left[\tilde{h}^{t-1}, \theta_i^t\right]$ is determined as the solution to this dynamic program,

$$\begin{aligned} W_i^t\left[\tilde{h}^{t-1}, \theta_i^t\right] &= \max_{\tilde{\theta}_i^t} E_{\tilde{\theta}_{-i}^t}\left\{\tau_i^t\left(\tilde{h}^{t-1}, \tilde{\theta}_i^t, \tilde{\theta}_{-i}^t\right) + \pi_i\left(\theta_i^t, x_i^t\left(\tilde{h}^{t-1}, \tilde{\theta}_i^t, \tilde{\theta}_{-i}^t\right)\right)\right. \\ &\quad \left. + \delta E W_i^{t+1}\left[\tilde{h}^t, \theta_i^{t+1} \mid \theta_i^t\right]\right\}, \qquad \text{(BIC)} \end{aligned}$$

where $EW_i^t\left[\tilde{h}^t, \theta_i^{t+1} \mid \theta_i^t\right]$ is $i's$ expected continuation surplus in period $t+1$ given θ_i^t.

The reporting strategies of members $i = 1, ..., N$ comprise a *Bayesian incentive compatible* equilibrium, (BIC), provided that truth telling is a mutual best response for all i.[10] Truth telling is a best response for member i, provided that if all members different from i are truthfully disclosing then the solution to the following dynamic program

$$W_i^t\left[\tilde{h}^{t-1},\theta_i^t\right] = \max_{\tilde{\theta}_i^t} E_{\tilde{\theta}_{-i}^t}\left\{\tau_i^t\left(\tilde{h}^{t-1},\tilde{\theta}_i^t,\theta_{-i}^t\right)+\pi_i\left(\theta_i^t,x^t\left(\tilde{h}^{t-1},\tilde{\theta}_i^t,\theta_{-i}^t\right)\right)\right.$$
$$\left.+\,\delta E W_i^{t+1}\left[\tilde{h}^t,\theta_i^{t+1}\mid\theta_i^t\right]\right\} \qquad \text{(BIC)}$$

is $\tilde{\theta}_i^t = \theta_i^t$ for all t. Expressed in words, (BIC) says that a necessary and sufficient condition for truth telling, is that member i does not wish to deviate once from truth telling, provided all other agents are truthfully reporting.[11]

The equilibrium is *ex post efficient,* (EE), provided

$$x^t\left(\theta^t\right) = x^*\left(\theta^t\right), \qquad \text{(EE)}$$

so the efficient allocation $x^*\left(\theta^t\right)$ is employed for all θ^t.

Given M, the equilibrium is *interim individual rationality,* (IIR), provided for all i, t and θ_i^t,

$$W_i^t\left[\tilde{h}^{t-1},\theta_i^t\right] \geq r_i^t V_i^t\left(\theta_i^t,1\right), \qquad \text{(IIR)}$$

Each $i's$ expected payoff $W_i^t[\cdot]$ exceeds his expected outside option from quitting. The equilibrium satisfies *ex post budget balance*, (BB), if for all i, t and $\tilde{h}^t$

$$\sum_i \tau_i^t\left(\tilde{h}^t\right) = 0. \qquad \text{(BB)}$$

It is important to note the compromises in equilibrium selection we have made to preserve some desired features of our agreement. The SSS is voluntary or IIR,

[10] See Fudenberg and Tirole (1991) Chapter 7 for an elaboration of Bayesian Incentive Compatible equilibrium in mechanism design.

[11] This condition is known as the single stage deviation principle. It asserts that it is necessary and sufficient that member i does not wish to deviate once from truth telling, in order for truth telling to be a best reply to truthful reporting by other agents. The single stage deviation principle applies in all settings, including our model, where the member's private information follows a first order stochastic Markov process. See Pavan et al (2009)

to ensure members participate. This is required of any long term arrangement that members initially agree to. Moreover the agreement is ex post budget balanced. This ensures the channel is self supporting without a third party–budget breaker to subsidize or to divert resources. This is required of any self sufficient private agreement. These requirements are satisfied at a cost, however. By requiring voluntary participation and budget balance, it is not possible to implement ex post efficient exchange as an *ex post* incentive compatible equilibrium, where truth telling is each member's best response independent of the distribution of other members' types.[12] Instead, we must adopt a short cut by assuming the SSS agreement can be supported by a Bayesian incentive equilibrium.

2.2.5 Remarks and Observations

We conclude this section with a few remarks about our modeling approach which may be unfamiliar to some analysts of supply chain management. Most studies of supply chain coordination, exemplified by Cachon and Lariviere (1999), focus on the performance of some known mechanism such as, linear and individually responsive allocations. The degree of supply coordination achieved is determined then by the comparative performance of allocations in the consideration set. The advantage this approach offers is that it evaluates known mechanisms, used in practice, that have desirable properties. The analysis reveals the strengths and weaknesses of these mechanisms and typically leads to some increase in coordination. The limitation of this approach is that it is tethered to existing practices and therefore less likely to yield breakthrough discoveries of new methods for coordination.

In contrast to this we model coordination as a mechanism design problem. We propose a mechanism that identifies what allocations are possible when members

[12] Each member prefers to report truthfully, independent of the types (private preferences) of the other players in an ex post incentive compatible equilibrium. This is a stronger, more robust equilibrium concept that is preferable to Bayesian incentive compatibility.

are privately informed about their demands, their exchanges of supply capacity are contractible while their investments are not. In comparison to previous analyses, we do not restrict attention to a specific class of mechanism.[13] Only after we characterize the set of possible allocations, do we enquire which specific mechanism exists to implement the desired coordination. This approach permits us to show whether a mechanism exists that allocates supply efficiently, induces members to make efficient investments, balances the budget and is voluntary. The advantage of this approach is that it identifies what allocations are and are not possible and therefore reveals the implicit cost of requiring allocations to have desired properties.[14] The limitation of mechanism design is that it does not address how the desirable allocations can be implemented. This is left to the analyst to do. In section 4 we demonstrate how the SSS allocation may be implemented by a conventional supply auction similar to business to business auctions we observe in practice.

2.3 Constructing the Shared Surplus Supply Agreement

The SSS agreement is constructed by making each member the residual claimant to (or the owner of) the expected supply chain surplus in equilibrium. An agreement with this property is known as an $AGV-Arrow$ mechanism.[15] AGV mechanisms ensure each member is incented to truthfully disclose its private capacity demand to maximize its surplus. Once we construct this mechanism it remains for us to show it is also individually rational and ex post budget balancing to complete our

[13] We do restrict attention to mechanisms that are incentive compatible. However this restriction is without loss of generality by the revelation principle, e.g., Myerson (1986).

[14] Mechanism design is also useful in explaining, in hindsight, why some market mechanisms, like second price auctions, are so useful in practice.

[15] The *AGV-Arrow* mechanism, independently derived by d'Aspremont and Varet (1979) and Arrow (1979), pays each member the *expected* surplus created by the other members. This mechanism is closely related to the better known VCG mechanisms, developed by Vickrey (1961), Clark (1971) and Groves (1973) in which each member is paid the actual (as opposed to expected) surplus created by the other members. The AGV mechanism has the advantage that it provides for a balanced budget, whereas the VCG mechanism generally doesn't.

construction.

The construction of the *SSS* agreements proceeds in three steps [16]

1. First, we propose an AGV mechanism that endows each member as a residual claimant of the expected surplus.

2. Second, we identify the conditions on M sufficient for individual rationality and budget balancing.

3. Third, we construct transfers satisfying these properties and confirm the AGV mechanism implements the ex post efficient capacity allocations as a Bayesian Incentive Compatible equilibrium.

2.3.1 Constructing the AGV Mechanism

Consider an AGV mechanism, M, where each i is a residual claimant to the expected channel surplus. For any AGV mechanism $i's$ continuation surplus at time t with demand θ_i^t can be written as

$$W_i^t \left[\theta^{t-1}, \theta_i^t\right] = E_{\theta_{-i}^t} V^t \left(\theta^t\right) - K_i \left(\theta^{t-1}\right), \tag{2.5}$$

where $K_i\left(\theta^{t-1}\right)$ is a surplus, independent of $\tilde{\theta}_i^t$, $i's$ reported demand, that depends only on the prior period demand conditions.[17] Hence, member i is promised the expected channel surplus (minus a term, independent of its report) in equilibrium.

[16] Segal and Whinston (2009) employ a similar process in constructing one period efficient mechanisms.

[17] Note the history of reported demands is $\tilde{h}^{t-1} = h^{t-1}$ on the equilibrium path since all members truthfully report their costs each period. h^{t-1} is completely summarized by θ^{t-1} since demand conditions are first order Markov. Consequently we can represent $i's$ period t continuation surplus as, $W_i^t \left[\tilde{h}^{t-1}, \theta_i^t\right] = W_i^t \left[\theta^{t-1}, \theta_i^t\right]$.

2.3.2 *Ensuring Individual Rationality and a Budget Surplus*

In order for M to satisfy IIR we require for all i , t and θ_i^t,

$$W_i^t \left[\theta^{t-1}, \theta_i^t\right] - r_i^t V_i^t \left(\theta_i^t, 1\right) = E_{\theta_{-i}^t} V^t \left(\theta_i^t, \theta_{-i}^t \right) - K_i \left(\theta^{t-1}\right) - r_i^t V_i^t \left(\theta_i^t, 1\right) \geq 0 \quad \text{(IIR)}$$

Define

$$\bar{w}_i^t \left(r^t, \theta_i^t\right) = E_{\theta_{-i}^t} V^t \left(\theta_i^t, \theta_{-i}^t\right) - r_i^t V_i^t \left(\theta_i^t, 1\right)$$

as the differential surplus for member i of participating in the channel and receiving $E_{\theta_{-i}^t} V^t \left(\theta_i^t, \theta_{-i}^t\right)$ or dissolving the channel and receiving $r_i^t V_i^t \left(\theta_i^t, 1\right)$. Given our assumption, $\bar{w}_i^t \left(r^t, \theta_i^t\right)$ is minimized at some critical demand, $\theta_{iw}^t \in [0, 1]$ that we refer to as the "worst off type ". In the analysis to follow we assume there is a unique minimizer $\theta_{iw}^t \left(r^t\right)$ which is a differentiable function of r^t. This requires the differential surplus function $\bar{w}_i^t \left(r^t, \theta_i^t\right)$ is sufficiently well behaved so that IIR binds at a unique point. A sufficient condition for this is that the distribution functions, $F_i \left(\theta_i^t \mid \theta_i^{t-1}\right)$ satisfy $(LFOSD)$ which ensures that the $\bar{w}_i^t \left(r^t, \theta_i^t\right)$ function is strictly convex in θ_i^t. [18]

The following Lemma gives conditions on the independent surplus term, $K_i \left(\theta^{t-1}\right)$, required for (IIR):

Lemma 1. *M is IIR provided*

$$K_i \left(\theta^{t-1}\right) \leq w_i^t \left(r^t\right) \equiv \min_{\theta_i^t} \left[\bar{w}_i^t \left(r^t, \theta_i^t\right)\right]$$

To balance the budget the total expected value the members receive must be less than the channel surplus; otherwise members receive more surplus than they create for some demand realizations. This requires,

$$E\left[V^t \left(\theta^t\right) \mid \theta^{t-1}\right] - \Sigma_i E W_i^t \left[\theta^{t-1}, \theta_i^t\right] \geq 0. \tag{2.6}$$

[18] $(FOSD)$ ensures period t's profit is strictly convex in θ_i^twhile $(LFOSD)$ensures the expected profit staring from period $t+1$is strictly convex in θ_i^t.

Since $K_i\left(\theta^{t-1}\right) \leq w_i\left(r^t\left(\theta^{t-1}\right)\right)$, (2.6) implies

$$B^t\left(r^t\right) \equiv \Sigma_i w_i^t\left(r^t\left(\theta^{t-1}\right)\right) - (N-1)E\left[V^t\left(\theta^t\right) \mid \theta^{t-1}\right] \geq 0. \tag{2.7}$$

$B^2\left(r^t\right)$ measures the sum of participation payments in excess of the members' net surplus. Participation payments are a function of the control rights $r^t\left(\theta^{t-1}\right)$. The budget surplus is most likely to be positive when the control rights, r^{*t} are chosen to maximize the aggregate willingness to pay. The next Lemmas characterize these rights, r^{*t}, and show they produce a strictly positive budget surplus.

Lemma 2. *Assume $\theta_{iw}^t\left(r^t\right)$ is continuously differentiable. Then the control rights, r^{*t}, that maximize total willingness to pay, $\sum_i w_i^t\left(r^t\right)$, minimize the aggregate outside option value, $\sum_i r_i^t V_i^t\left(\theta_{iw}^t\left(r^t\right), 1\right)$.*

Lemma 2 shows that the optimal control rights r^{*t} that maximize the sum of the member's willingness to pay, do so by minimizing the sum of the member's outside options, evaluated at their worst off types.[19] It seems intuitive that surplus maximizing rights r^{*t} should minimize the members' aggregate outside option, $\sum_i r_i^t V_i^t\left(\theta_{iw}^t\left(r^t\right), 1\right)$. In a loose sense, members' willingness to participate is greatest when their incentives to leave the channel to pursue their outside options is minimized. In section 4 we describe what this implies for the determination of control rights for different channel settings.

Lemma 3. $B^t\left(r^{*t}\right) > 0$.

The proof proceeds by demonstrating the willingness to pay exceeds the transfer payments required for making each i a residual claimant. Lemma 3 indicates the rights, r^{*t}, maximizing the total willingness to pay, provide a strictly positive budget surplus. For convenience, we will use the r^{*t} rights in constructing the mechanism, noting there may be other control right allocations that would work as well.

[19] Schweizer (2006) employs a similar method to identify an initial allocation rule that permits budget balancing in his analysis of static efficient budget balancing mechanisms.

2.3.3 Transfers that Ensure IC and a BB AGV Mechanism

To complete the construction we need to confirm there are transfers that ensure IIR, BB and implement the efficient allocation as a Bayesian incentive compatible equilibrium. Denote $\tilde{p}_i^t\left(\tilde{h}^t\right)$ as the the net payment to i in period t where,

$$\tilde{p}_i^t\left(\tilde{h}^t\right) = \tilde{\tau}_i^t\left(\tilde{h}^t\right) - \tilde{w}_i^t\left(r^{*t}\left(\tilde{\theta}^{t-1}\right)\right) + \delta E_{\tilde{\theta}_i^t}\tilde{w}_i^{t+1}\left(r^{*(t+1)}\left(\tilde{\theta}^t\right)\right) \tag{2.8}$$

such that

$$\tilde{\tau}_i^t\left(\tilde{h}^t\right) = \underbrace{E_{\tilde{\theta}_{-i}^t}\sum_{j\neq i}\pi_j\left(\tilde{\theta}_j^t, x^*\left(\tilde{\theta}^t\right)\right)}_{i\text{'s transfer}},$$

$$\tilde{w}_i^t\left(r^{*t}\left(\tilde{\theta}^{t-1}\right)\right) = \underbrace{\left(w_i^t\left(r^{*t}\left(\tilde{\theta}^{t-1}\right)\right) - \frac{B^t\left(r^{*t}\left(\tilde{\theta}^{t-1}\right)\right)}{N}\right)}_{\text{i's expected participation payment for } t}$$

$$\delta E_{\tilde{\theta}_{-i}^t}\tilde{w}_i^{t+1}\left(r^{*(t+1)}\left(\tilde{\theta}^t\right)\right) = \underbrace{\delta E_{\tilde{\theta}_{-i}^t}\left(w_i^{t+1}\left(r^{*(t+1)}\left(\tilde{\theta}^t\right)\right) - \frac{B^{t+1}\left(r^{*(t+1)}\left(\tilde{\theta}^t\right)\right)}{N}\right)}_{\text{i's expected participation payment for } t+1}$$

Member $i's$ net payment consists of three transfers. The first transfer, $\tilde{\tau}_i^t\left(\tilde{h}^t\right)$, is the expected surplus created by the other channel members. By transferring $\tilde{\tau}_i^t\left(\tilde{h}^t\right)$ to i, he becomes the residual claimant of all the expected channel surplus. The second transfer, $\tilde{w}_i^t\left(r^{*t}\left(\tilde{\theta}^{t-1}\right)\right)$, is $i's$ fee to participate in period t. The third transfer, $\delta E\tilde{w}_i^{t+1}\left(r^{*(t+1)}\left(\tilde{\theta}^t\right)\right)$, equals $i's$ expected participation fee in period $t+1$. This transfer ensures i doesn't benefit from misreporting current market conditions to reduce future participation costs.

To balance the budget the net payments of all members must sum to zero. This requires each i receive adjusted net transfers, $\tau_i^t\left(\tilde{h}^t\right)$, equal to his net payments minus $i's$ share of net payments to other members $j \neq i$.

$$\tau_i^t\left(\tilde{h}^t\right) = \tilde{p}_i^t\left(\tilde{h}^t\right) - \frac{1}{N-1}\left(\Sigma_{j\neq i}\left(\tilde{p}_j^t\left(\tilde{h}^t\right) - E_\theta \tilde{p}_j^t\left(\tilde{h}^t\right)\right)\right) \tag{2.9}$$

We are now ready to prove our main result,

Proposition 4. *The dynamic mechanism,* $M\left\langle x^*\left(\tilde{\theta}^t\right), \tau^t\left(\tilde{h}^t\right), r^{*(t+1)}\left(\tilde{\theta}^t\right)\right\rangle_{t=1}^T$, *implements the efficient allocation x^* as a Bayesian incentive compatible, individually rational and balanced budget equilibrium.*

The proof of Proposition 4 provides an algorithm for implementing efficient exchange. The process requires each member receives an AGV transfer, $\tilde{\tau}_i^t$. This makes the member a residual claimant and therefore induces him to truthfully disclose his demand condition in all periods to maximize social surplus. Members are assessed a participation fee, $\tilde{w}_i^t\left(r^{*t}\right)$, to fund these transfers. The fee maximizes collections subject to ensuring individually rational participation. To construct the control rights that support these fees, requires knowledge of the members' future outside options. This information is common knowledge in a one period mechanism. However with multiple periods, the information must be solicited from the members. To eliminate incentives for misreporting, each member receives a payment equal to his discounted expected future participation fee.

2.4 Applications and Examples

In this section, we illustrate the model and the mechanism through three application settings.

2.4.1 A Single Period Problem

What should the supplier do when retailer orders exceed her capacity? Cachon and Lariviere (1999) studies the capacity allocation problem under many practical supply chain settings: allocation of nicotine patches based on sale histories and location, turn and earn used by car companies, explicit preferences for Frito-Lay to have access to P&G and the allocation based on demand priority used widely in toy industry. Due to the heterogeneity of the retailers' demand distribution, the retailers may game the system when deciding their order quantities and hence cause supply chain inefficiency. In particular, they try to answer the following three questions:

1) Which allocation mechanisms are manipulable and which are truth-inducing? A larger class of mechanisms is manipulable

2) Does truth telling helps the supply chain?

3) How the chosen allocation mechanism influences the supplier's capacity choice?

They find that a large class of allocation mechanisms applied in practise is manipulatable. Moreover, compared with the truth telling mechanisms, the manipulable mechanisms result in greater supply chain efficiency. On the supplier side, early investment in capacity is restricted to intensify competition between retailers (retailers will order a larger quantity).

We compare our mechanism with Cachon and Lariviere (1999) by restricting $\mathcal{M}$ to a single period allocation mechanism. Before the period begins, the supplier has chosen an investment to improve his capacity value, simultaneously the retailers have chosen investments respectively to improve the product's market value. Each member's investment affects the distribution of their private information and it cannot be verified or credibly announced.

During the period, the following sequence of events occurs: 1) the supplier and the retailers simultaneously learn their private information and decided to participate or not; 2) the supplier and the retailers simultaneously submit their private information to a third party; 3) the third party enforces the allocation and transfer payments based on the mechanism; 4) the supplier and the retailers consume capacity.

The mechanism is an agreement specifying the capacity allocation and transfer payments the supplier and the retailers receive. It is denoted by two terms:

$$M\langle x,\tau\rangle. \tag{2.10}$$

The first term $x=(x_1,x_2,...,x_N)$ characterizes the allocation mechanism. $x_i(\tilde{\theta})$ is the proportion of capacity allocating to member i given all member's reports $\tilde{\theta}$. The second term $\tau=(\tau_1,\tau_2,...,\tau_N)$ characterizes the transfer payments. $\tau_i(\tilde{\theta})$ is the amount of payment member i receives given all member's reports $\tilde{\theta}$.

The capacity is jointly owned by all members. The ownership is characterized by a probability distribution called the control rights $r=(r_1,r_2,...,r_N)$. For any given mechanism M, if any member decides not to participate in the mechanism, the partnership dissolves and member i receives the entire capacity with probability r_i.

We would like to construct a mechanism that has the following desired properties under Bayesian Equilibrium:

1. Incentive Compatibility.

   Each member reports truthfully his private information (capacity value or market value)

   $$\tilde{\theta}=\theta.$$

2. Supply chain optimality.

   The supply chain optimal allocation is selected based on $\tilde{\theta}$.

   $$x(\tilde{\theta})=x^*(\tilde{\theta}).$$

3. Voluntary participation.

   Each member's expected profit is maximized when he chooses to participate in the partnership.

   $$E_{\tilde{\theta}_{-i}}[\pi_i(\theta_i,x_i(\tilde{\theta}))+\tau_i(\tilde{\theta})]\geq r_i\pi_i(\theta_i,1)+\sum_{j\neq i}r_j\pi_i(\theta_i,0).$$

4. The transfer payments are balanced.

$$\sum_i \tau_i(\tilde{\theta}) = 0.$$

Proposition 5. *For any given control rights r, construct mechanism M as follows:*

$$x(\tilde{\theta}) = x^*(\tilde{\theta}), \qquad \tau_i(\tilde{\theta}) = \tilde{p}_i(\tilde{\theta}) - \frac{1}{N-1}\sum_{j\neq i}\left(\tilde{p}_j(\tilde{\theta}) - E_{\tilde{\theta}}\tilde{p}_j(\tilde{\theta})\right) \tag{2.11}$$

where

$$\begin{aligned}
\tilde{p}_i(\tilde{\theta}) &= \tilde{\tau}_i(\tilde{\theta}) - \tilde{w}_i(r), \\
\tilde{\tau}_i(\tilde{\theta}) &= E_{\tilde{\theta}_{-i}}\sum_{j\neq i}\pi_j\left(\tilde{\theta}_j, x^*(\tilde{\theta})\right), \\
\tilde{w}_i(r) &= w_i(r) - \frac{B(r)}{N}, \\
w_i(r) &= E_{\theta_{-i}}[\Pi^*(\theta)] - r_i\pi_i(\theta_i, 1), \\
B(r) &= \sum_i w_i(r) - (N-1)E[\Pi^*(\theta)].
\end{aligned}$$

1) Mechanism M satisfies the four requirements if $B(r) \geq 0$.

2) There exists a control right allocation r^* such that $B(r^*) \geq 0$. Especially, $B(r^*) > 0$ when

$$r^* = \arg\max\left\{\sum_i w_i(r) | \sum r_i = 1,\ r_i \geq 0,\ i = 1, 2, ..., N\right\}.$$

3) $\mathcal{R} = \{r | B(r) \geq 0,\ \sum r_i = 1,\ r_i \geq 0,\ i = 1, 2, ..., N\}$ is a non empty closed convex set.

Cachon and Lariviere (1999) defined an allocation mechanism to be *individually responsive (IR)* if a retailer is receiving a positive allocation and orders more, the retailer gets more unless he has already been allocated all of the capacity. Example of individually responsive allocations are proportional allocation, linear allocation and Pareto allocation. They show that all members truthfully reporting their optimal stocking level is not an IR Bayesian equilibrium when the capacity is allocated based on a whole sale price scheme. By relaxing the pricing scheme and introduce probabilistic control rights, Proposition 5 illustrates an allocation that is Pareto, and hence individually responsive, and induces truth telling under Bayesian equilibrium. Moreover, contrary to Cachon and Lariviere (1999)'s finding that truth telling mechanisms are supply chain wise less efficient, the above mechanism guarantees supply chain optimality.

However, we can still observe the gap between our mechanism and the allocation schemes used in practise. In the next section we focus on the implementation of our mechanism to reduce this gap. In particular, our mechanism can be implemented by an all pay auction and possesses similar properties to a turn and earn policy: a higher θ (better selling history) leads to a higher potential to receive more capacity in the future.

2.4.2 IFRC

International Federation of Red Cross and Red Crescent (IFRC) is an non-profit organization that carries out and manages disaster reliefs in different regions around the world. In particular, one of IFRC's responsibility is to obtain and allocate resources to each region according to the disaster conditions. This includes coordinating the donors (National Societies (NS)), distributing basic resources as well as sending emergency response units to the most needed regions.

On the one hand, IFRC is uncertain about the price to obtain the total basic supplies. This is due to two reasons. First, IFRC is uncertain about how much money or basic supply it can receive from the donors. Second, IFRC needs to negotiate the purchase price of the basic supplies before disaster happens. The final purchase price changes as the world economy changes.

On the other hand, IFRC needs to optimize the allocation process. First, IFRC needs to keep a base line of supply every period to anticipate disasters. Second, to prevent the arrival of unsolicited goods from "over-enthusiastic" NSs, IFRC needs to integrate the donors and gives incentives to each region to share their private information on the types of supplies and the quantity needed.

We propose an allocation process for a single good that takes into account the uncertainty on the supply side and also guarantees efficient allocation.

Assume IFRC needs to distribute a single type of good (i.e. rice, flour etc.) to $N-1$ regions for T periods. IFRC keeps a basic supply of 1 unit good every period. The capacity in the previous period cannot carry over to the next period because they will expire. In period t, let $\theta_1^t \in [0,1]$ be the percentage of goods donated by the donors. We assume $\{\theta_1^t,\ t = 1, ..., T\}$ is private information of IFRC and it is i.i.d. with c.d.f. $F(\cdot)$. With out loss of generality, assume the purchase price for the good is c_1 and it is publicly known(this is reasonable if we take into account the uncertainty of θ_1^t). IFRC's percentage of good allocated is $x_1^t \in [0,1]$. Then the total cost of IFRC is $c_1[1 - x_1^t - \theta_1^t]$. Normalize the cost into profit, we have $\pi_1(\theta_1^t, x_1^t) = c_1[x_1^t + \theta_1^t]$.

Let $\theta_i^t,\ i = 2, ..., N$ be region i's disaster condition. It can take value 0 (disaster does not happen/not severe) or 1 (disaster happens/severe). Assume $\{\theta_i^t,\ t = 1, ..., T\}$ is private information of region i and it is i.i.d. and $Pr(\theta_t^i = 1) = p_i$. Let c_i be the cost per unit incurred if there is a disaster in region i. k_i is the maximum percentage of good needed if a disaster happened in region i. Both c_i and k_i are publicly known. x_i^t be the percentage of allocated goods to region i. The cost in region i is $c_i\theta_i^t(1-x_i^t)$. Normalize the cost into profit, we have $\pi_i(\theta_i^t, x_i^t) = c_i\theta_i^t(x_i^t \wedge k_i)$. Without loss of generality, assume that $0 < c_1 < c_i$ for all $i = 2, ..., N$ and $\sum_{i=2}^N k_i \leq 1$. All the participants share a common discount factor δ. Let $[x]^+ = \max\{0,\ x\}$.

In period t, given θ^t, the efficient allocation solves the following maximization problem

$$\max_{x_i^t} \quad \left\{ \sum_{i=2}^{N} \theta_i^t c_i (x_i^t \wedge k_i) + c_1[\theta_1^t + x_1^t] \right\}$$

$$s.t. \sum_{i=1}^{N} x_i^t = 1,\ x_i^t \geq 0.$$

Solve the above problem we have $x_i^{*t} = \theta_i^t k_i,\ i = 2, ..., N$ and $x_1^{*t} = 1 - \sum_{i=2}^{N} x_i^{*t}$. The supply chain profit under the efficient allocation is

$$\Pi^*(\theta^t) = \sum_{i=2}^{N} \theta_i^t (c_i - c_1) k_i + c_1(1 + \theta_1^t).$$

The expected supply chain profit under optimal allocation is

$$V^t(\theta^t) = \sum_{n=t}^{T} \delta^{n-t} E[\Pi^*(\theta^n|\theta^t)].$$

Given r_i^t, θ_{iw}^t is the minimizer of the problem below:

$$\min_{\theta_i^t \in \{0,1\}} \Delta_i^t(\theta_i^t) = \min_{\theta_i^t \in \{0,1\}} \left\{ E_{\theta_{-i}^t}[V^t(\theta^t)] - r_i^t \sum_{n=t}^{T} \delta^{n-t} E[\pi_i(\theta_i^n, 1)|\theta_i^t] \right\}.$$

When $i = 2, ...N$,

$$E_{\theta_{-i}^t}[V^t(\theta^t)] = \sum_{j=2, j\neq i}^{N} p_j k_j (c_j - c_1) + \theta_i^t k_i (c_i - c_1)$$
$$+ c_1 E(1 + \theta_1^t) + \delta E_{\theta_{-i}^t}[V^{t+1}(\theta^{t+1})],$$

$$r_i^t \sum_{n=t}^{T} \delta^{n-t} E[\pi_i(\theta_i^n, 1)|\theta_i^t] = r_i^t \theta_i^t c_i k_i + r_i^t \sum_{n=t+1}^{T} \delta^{n-t} p_i c_i k_i.$$

It is equivalent to solving

$$\min\left\{0,\ k_i(c_i - c_1) - r_i^t c_i k_i\right\}.$$

Note that when $r_i^t = 0$, $\theta_{iw}^t = 0$; when $r_i^t = 1$, $\theta_{iw}^t = 1$. Moreover, θ_{iw}^t is increasing in r_i^t, there exists a $r_i^{0t} = (c_i - c_1)/c_i$ such that θ_{iw}^t switches from 0 to 1. When $i = 1$,

$$\begin{aligned}\Delta_1^t(\theta_1^t) &= \sum_{j=2}^{N} p_j k_j (c_j - c_1) + c_1(1 + \theta_1^t) \\ &\quad + \delta E_{\theta_{-1}^t}[V^{t+1}(\theta^{t+1})] - r_1^t c_1 (1 + \theta_1^t) - r_1^t \sum_{n=t+1}^{T} \delta^{n-t} c_1 (1 + E[\theta_1])\end{aligned}$$

is increasing in θ_1^t. Therefore, $\theta_{1w}^t = 0$.

The optimal control rights r_i^{*t} solves the maximization problem below:

$$\max_{r_i^t} \sum_{i=1}^{N} \Delta_i(\theta_{iw}^t)$$

$$s.t. \sum_{i=1}^{N} r_i^t = 1,\ r_i^t \geq 0.$$

This is equivalent to

$$\max_{r_i^t} \left\{ \sum_{i=2}^{N} \left\{\theta_{iw}^t k_i (c_i - c_1) - r_i^t \theta_{iw}^t c_i k_i - r_i^t p_i c_i k_i \xi(t)\right\} - r_1^t c_1 \left(1 + \xi(t)\left(1 + E[\theta_1]\right)\right) \right\}$$

$$s.t \quad \sum_{i=1}^{N} r_i^t = 1,\ r_i^t \geq 0.$$

where $\xi(t) = \sum_{n=t+1}^{T} \delta^{n-t}$. Rank $\{p_i c_i k_i \xi(t),\ (1+p_i\xi(t))c_i k_i,\ c_1(1+\xi(t)(1+E[\theta_1])|\ i = 2, ..., N\}$ from the least to the most. Let

$$i_0 = \arg\min_i \{(1 + p_i \xi(t)) c_i k_i|_{i=2,\ldots,N}, c_i(1 + \xi(t)(1 + E[\theta_i])|_{i=1}\},$$

$$C(i_0) = \min_i \{(1 + p_i \xi(t)) c_i k_i|_{i=2,\ldots,N}, c_i(1 + \xi(t)(1 + E[\theta_i])|_{i=1}\},$$

and

$$S(i_0) = \{i, p_i c_i k_i \xi(t) \leq C(i_0)\}.$$

$S(i_0)$ is non empty because $i_0 \in S(i_0)$. Rank $S(i_0)$ in ascending order of $p_i c_i k_i$:

$$p_{i(1)} c_{i(1)} k_{i(1)} < p_{i(2)} c_{i(2)} k_{i(2)} < \ldots < p_{i(m)} c_{i(m)} k_{i(m)},$$

where $i(l) \in S(i_0)$ and $m = |S(i_0)|$ is the number of elements in $S(i_0)$. Let $r_1^{0t} = 1$. The optimal control rights allocation can be characterized as follows:

$$\begin{aligned} r_{i(l)}^{*t} &= [\min\{r_{i(l)}^{0t},\ 1 - \sum_{j=1}^{l-1} r_{i(j)}^{0t}\}]^+, \quad i(l) \neq i_0, i(l) \in S(i_0), \\ r_{i_0}^{*t} &= 1 - \sum_{j=1, j \neq i_0}^{m} r_{i(j)}^{*t}, \\ r_i^{*t} &= 0, \quad i \notin S(i_0). \end{aligned}$$

From the above expression we know that there is at most one $\theta_{iw}^t = 1$.

Next we discuss the properties of r^{*t} as t changes. First, $\xi(t)$ is decreasing in t. Second, the rate of change for member i in $\xi(t)$ is $p_i c_i k_i$ for $i = 2, ..., N$ and $c_1(1 + E[\theta_1])$ for $i = 1$. The member with a larger rate of change receives less control right at the beginning of the horizon and more control right towards the end of the horizon. This implies a more volatile control right change in time. Third, $c_i k_i$ for $i = 2, ..., N$ and c_1 characterizes the control right allocation towards the end of the horizon. member i receives control right towards the end of the horizon if $c_i k_i$ for $i = 2, ..., N$ or c_1 for $i = 1$ is small. Finally, the magnitude of the control right is measured by c_i. Member i receives greater control rights if c_i is large.

This analysis gives suggestions of how to allocate control rights. For example, a region with low probability of having a disaster (low p_i) but high disaster cost (high $c_i k_i$) will should receive larger control rights at the beginning of the partnership but less control rights towards the end. That is because at the beginning of the partnership region i has the incentive to stay for the large cost of the future disasters but towards the end, his incentive drops because of the low occurring probability. Our control allocation scheme ensures that region i is staying in the partnership by assigning him less control right when his incentive for quitting is high.

Finally we verify that $B(r^{*t}) \geq 0$.

$$\begin{aligned} B(r^{*t}) &= \sum_{i=1}^{N} [E_{\theta_{-i}^t} V^t(\theta^t|\theta^{t-1})|_{\theta_i^t=\theta_{iw}^t} - r_i^{*t} V_i^t(\theta_{iw}^t|\theta_i^{t-1})] - (N-1)E[V(\theta^t|\theta^{t-1})] \\ &= \delta E[V^{t+1}(\theta^{t+1})] - \left[\sum_{i=2}^{N} r_i^{*t}\xi(t)p_i c_i k_i + r_1^{*t}\xi(t)c_1(1+E[\theta_1])\right] \\ &\quad + \sum_{i=2}^{N} \theta_{iw}^t k_i(c_i - c_1) - \sum_{i=2}^{N} r_i^{*t}\theta_{iw}^t k_i c_i + c_1(1+\theta_{1w}^t) - r_1^{*t} c_1(1+\theta_{1w}^t). \end{aligned}$$

Note that $E[V^{t+1}(\theta^{t+1})] - \left[\sum_{i=2}^{N} r_i^{*t}\xi(t)p_i c_i k_i + r_1^{*t}\xi(t)c_1(1+E[\theta_1])\right] \geq 0$ because the expected supply chain profit under optimal allocation is always greater than the weighted sum of the expected profit under non optimal allocation. If there is no $\theta_{iw}^t = 1$, then $B(r^{*t}) \geq 0$. If there is one $\theta_{iw}^t = 1$ and $i \neq 1$, then $r_1^{*t} = 0$ and

$$\begin{aligned} &\sum_{i=2}^{N} \theta_{iw}^t k_i(c_i - c_1) - \sum_{i=2}^{N} r_i^{*t}\theta_{iw}^t k_i c_i + c_1(1+\theta_{1w}^t) - r_1^{*t} c_1(1+\theta_{1w}^t) \\ &= k_i(c_i - c_1) - r_i^{*t}\theta_{iw}^t k_i c_i + c_1 > 0; \end{aligned}$$

if $\theta_{1w}^t = 1$, then

$$\sum_{i=2}^{N} \theta_{iw}^t k_i(c_i - c_1) - \sum_{i=2}^{N} r_i^{*t}\theta_{iw}^t k_i c_i + c_1(1+\theta_{1w}^t) - r_1^{*t} c_1(1+\theta_{1w}^t) = 2c_1 - 2r_1^{*t} c_1 > 0.$$

This guarantees the existence of the mechanism.

2.4.3 Capacity Planning in Pharmaceutical Industry

Consider a single manufacture provides production capacity for $N-1$ drug dealers for T periods to innovate in new medicine. The capacity needs to be allocated to the drug dealers during the process. However, once the capacity is allocated to a certain member, it cannot be taken back. At the end of the T periods, the success of innovation is observed and profit is obtained. At the beginning of the horizon, the manufacturer has a total capacity of 1. The problem is to allocate the capacity through time so that the expected supply chain profit is maximized.

In period t, the manufacturer incurs a maintenance fee for the capacity that has not been allocated. The maintenance fee has a maximum rate of c per unit, and θ_1^t is the percentage paid by the manufacturer. $\{\theta_1^t\}$ is i.i.d. with c.d.f. $F(\cdot)$. The capacity allocated to the manufacturer is $x_1^t \in [0,1]$. It is the capacity that has not yet been allocated to the drug dealers. Thus, the manufacture needs to pay $c_1\theta_1^t x_1^t$ in period t. And the manufacturer's corresponding profit in period t is $\pi_1(\theta_1^t, x_1^t) = -c_1\theta_1^t x_1^t$.

$\theta_i^t \in [0,1]$ is the drug dealers' estimate in period t of the probability of success at the end of the horizon. It follows a Markov Process that $\theta_i^t = \alpha_i\theta_i^{t-1} + \beta_i(1-\theta_i^{t-1})$. α_i and β_i are random variables with c.d.f. $G_i(\cdot)$ and $H_i(\cdot)$. Each period, $\alpha_i \in [0,1]$ and $\beta_i \in [0,1]$ are drown independently from $G_i(\cdot)$ and $H_i(\cdot)$. α_i is the percentage of signals indicating success in the previous period that stay positive this period while β_i is the percentage of signals indicating failure in the previous period that turn positive this period. Assume $E[\alpha_i] > E[\beta_i]$ so that the $\{\theta_i^t\}_{t=1}^T$ is stable. x_i^t is the total capacity allocated to drug dealer i in period t. The profit in period t for the drug dealer is zero. The expected profit of member i at the end of the horizon is $E[p_i\theta_i^T x_i^T | \theta_i^t, x_i^t]$, where p_i is the profit rate if drug dealer i succeed at the end of the horizon.

The optimal allocation in period t, x^{*t}, is obtained by solving the following dynamic program:

$$
\begin{aligned}
V^t(\theta^t,\ x^{t-1}) &= \max\left\{-c\theta_1^t x_1^t + E_{\theta^{t+1}} V^{t+1}(\theta^{t+1},\ x^t)\right\}, \\
& \quad s.t.\ x_i^t \geq x_i^{t-1},\ i = 2, ..., N, \\
& \quad x_1^t = 1 - \sum_{i=2}^{N} x_i^t.
\end{aligned}
$$

and

$$
\begin{aligned}
V^{t+1}(\theta^{T+1}, x^T) &= \sum_{i=2}^{N} p_i \theta_i^{T+1} x_i^T - c x_1^T \theta_1^{T+1} \\
\theta^{T+1} &= 0, \quad or \quad 1.
\end{aligned}
$$

Substitute $x_1^t = 1 - \sum_{i=2}^{N} x_i^t$ into the dynamic program and reduce one dimension of the problem, we have:

$$
\begin{aligned}
V^t(\theta^t,\ \tilde{x}^{t-1}) &= \max\left\{-c\theta_1^t \left(1 - \sum_{i=2}^{N} x_i^t\right) + E_{\theta^{t+1}} V^{t+1}(\theta^{t+1},\ \tilde{x}^t)\right\}, \\
& \quad s.t.\ \tilde{x}_i^t \geq \tilde{x}_i^{t-1},\ i = 2, ..., N.
\end{aligned}
$$

and

$$
V^{t+1}(\theta^{T+1}, \tilde{x}^T) = \sum_{i=2}^{N} (p_i \theta_i^{T+1} - c\theta_1^{T+1}) \tilde{x}_i^T - c\theta_1^{T+1},
$$

where $\tilde{x}^t = (x_2^t, ..., x_N^t)$.

Let

$$
J^t(\theta^t,\ \tilde{x}^t) = c_1 \theta_1^t (1 - \sum_{j=2}^{N} x_j^t) + E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)],
$$

$$
\Delta_{\tilde{x}_i^t} J^t = c_1 \theta_1^t + \partial E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)] / \partial x_j^t
$$

and

$$\Delta^t(\theta^t) = c_1\theta_1^t + \max_j\{\partial E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)]/\partial x_j^t\}.$$

The following properties solves the optimal allocation problem.

Proposition 6. *1) $V^t(\theta^t,\ \tilde{x}^{t-1})$ is coordinatewise linear in $\tilde{x}^{t-1}$ and hence $E_{\theta^t}[V^t(\theta^t,\ \tilde{x}^{t-1})]$ is coordinatewise linear in $\tilde{x}^{t-1}$. Thus, $\Delta_{\tilde{x}_i^t}J^t$ and $\Delta^t(\theta^t)$ are independent with $\tilde{x}^t$.*
2) The optimal allocation is: if $\Delta^t(\theta^t) \geq 0$, then $\tilde{x}_j^{*t} = 1 - \sum_{i\neq j}\tilde{x}_i^t$ and $\tilde{x}_i^{*t} = \tilde{x}_i^{t-1}$ for all $i \neq j$, where $j = \arg\max_j\{\partial E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)]/\partial x_j^t\}$. If $\Delta^t(\theta^t) < 0$, then $\tilde{x}_i^{*t} = \tilde{x}_i^{t-1}$.
3) The supply chain optimal cost, $V^t(\theta^t,\ \tilde{x}^{t-1})$, given θ^t is convex in θ^t. Moreover, it is also concave in θ^t implying that $V^t(\theta^t,\ \tilde{x}^{t-1})$ is a hyperplane in θ^t.

Assume that at the beginning of the horizon, the manufacturer holds all the capacity. Then the optimal policy is to allocate all the capacity to drug dealer i in period t if $\Delta^t(\theta^t) > 0$ for the first time and $i = \arg\max_j\{\partial E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)]/\partial \tilde{x}_j^t\}$. Then the process terminates. Thus, at the beginning of period t, the allocation of capacity is fully characterized by $\Delta^t(\theta^t)$, which is a function of θ^t. This guarantees that the mechanism can be constructed similarly to the ones we discussed in the previous sections.

After we characterized the optimal allocation scheme, we calculate θ_{iw}^t. For simplicity, we use the original cost function with allocation x^t instead of $\tilde{x}^t$.

$$\theta_{iw}^t = \arg\min_{\theta_i^t}\{E_{\theta_{-i}^t}V^t(\theta^t, x^*(\theta^t)) - r_i^t V_i^t(\theta_i^t, 1)\}$$

Since $V^t(\theta^t, x^*(\theta^t))$ is a hyperplane, there exists r_i^{0t} such that

$$\frac{\partial E_{\theta_{-i}^t}V^t(\theta^t, x^*(\theta^t))}{\partial \theta_i^t} = r_i^{0t}\frac{\partial V_i^t(\theta_i^t, 1)}{\partial \theta_i^t}$$

and

$$\theta_{iw}^t = \begin{cases} 0 & r_i^t \leq r_i^{0t}, \\ 1 & r_i^t > r_i^{0t}. \end{cases} \qquad i = 2, ..., N,$$

$$\theta_{1w}^t = \begin{cases} 0 & r_1^t \geq r_1^{0t}, \\ 1 & r_1^t < r_1^{0t}. \end{cases} .$$

Now we calculate r^{*t}

$$r^{*t} = \arg\max_r \left\{ \sum_{i=1}^N E_{\theta_{-i}^t} V^t(\theta^t, x^*(\theta^t))|_{\theta_i^t = \theta_{iw}^t} - \sum_{i=1}^N r_i^t V_i^t(\theta_{iw}^t, 1) \right\}$$

$$s.t. \sum_{i=1}^N r_i^t = 1, \quad r_i^t \geq 0.$$

Since $\sum_{i=1}^N E_{\theta_{-i}^t} V^t(\theta^t, x^*(\theta^t))|_{\theta_i^t=\theta_{iw}^t} - \sum_{i=1}^N r_i^t V_i^t(\theta_{iw}^t, 1)$ is increasing in r_1^t and decreasing in r_i^t for $i = 2, ..., N$.

$$r_1^{*t} = 1, \quad r_i^t = 0, \quad i = 2, ..., N.$$

The manufacturer always has full control right over the capacity before they are allocated. This is because the manufacturer has the least incentive to control the capacity that he is the only one who pays if the capacity belongs to him.

Finally we verify that $B^t(r^{*t}) \geq 0$:

$$\begin{aligned} B^t(r^{*t}) &= \sum_{i=1}^N E_{\theta_{-i}^t} V^t(\theta^t, x^*(\theta^t))|_{\theta_i^t=0} - V_1^t(0,1) - (N-1)E[V^t(\theta^t|\theta^{t-1})] \\ &\geq E_{\theta^t} V^t(\theta_w^t, x^*(\theta^t)) - V_1^t(0,1) \geq 0. \end{aligned}$$

The second inequality follows because allocating the capacity randomly results in more profit than not allocating the capacity.

2.5 Implementation and Investment in SSS Agreements

This section extends our results to address the implementation of desired allocations and ex ante investment.

2.5.1 Implementing the SSS Agreement

Given the rather abstract nature of the SSS agreement, it is important to know if it can be implemented by conventional means such as an auction for example. Following Cramton et al (1987) we construct an all pay auction that proceeds as follows. At the beginning of each period, before observing their demands, θ_i^t, each member i pays an entry fee,

$$F_i^t\left(r^{*t}\right) = \tilde{w}_i^t\left(r^{*t}\right)$$

that is equal to the participation payment provided for in the direct mechanism, M. These payments ensure that the all pay auction breaks even in each period. Following this, each member i observes his current θ_i^t and decides whether to dissolve the SSS agreement and receive his outside option value $r_i^{*t}V_i^t\left(\theta_i^t, 1\right)$, or to remain a member and submit a bid, $b_i^t\left(\theta_i^t\right) = \tilde{b}_i^t$.[20] (In equilibrium, of course, all members submit bids.)

The auction induces a bidding game. We conjecture the equilibrium to this game is supported by bid functions, $b_i\left(\theta_i^t\right)$, that are strictly increasing in θ_i^t. These functions can be inverted to determine each member's demand. The capacity allocation for period t is given by $x^*\left(\left(b_1^t\right)^{-1}\left(\tilde{b}_1^t\right), .., \left(b_N^t\right)^{-1}\left(\tilde{b}_N^t\right)\right)$, where $\left(b_i^t\right)^{-1}\left(\tilde{b}_i^t\right)$ is the inverse of the bid function. Since this is an all pay auction, all bidders i receive a payment, whether they win or lose. The payment,

$$\begin{aligned} m_i^t\left(\tilde{b}_i^t, \tilde{b}_{-i}^t\right) &= \tilde{b}_i^t - \frac{\sum_{j\neq i}\left(\tilde{b}_j^t - E_{\theta^t}\tilde{b}_j^t\right)}{N-1} \\ &\quad + \delta\tilde{w}_i^{t+1}\left(r^{*(t+1)}\left(\left(b_1^t\right)^{-1}\left(\tilde{b}_1^t\right), ..., \left(b_N^t\right)^{-1}\left(\tilde{b}_N^t\right)\right)\right) \end{aligned} \quad (2.12)$$

[20] Note that $b_i^t\left(\theta_i^t\right)$ is the bid function and $\tilde{b}_i^t$ is the amount of the bid.

is equal to the weighted sum of bids plus a credit for the next period entry fee.

Proposition 7. *A sequential all-pay auction with entry fees and bid payments* $\left\{F_i^t\left(r^{*t}\right), m_i^t\left(\tilde{b}_i^t, \tilde{b}_{-i}^t\right)\right\}_{i=1}^N$ *in each period* $t = 1, ..., T$ *implements the SSS agreement. In equilibrium each member i bids,*

$$b_i^t\left(\theta_i^t\right) = E_{\theta_{-i}^t}\left(\Sigma_{j\neq i}\pi_j\left(\theta_j^t, x_j^*\left(\theta^t\right)\right)\right) \tag{2.13}$$

The derivation of the sequential auction as summarized in Proposition 2 allows us to clarify how the supply channel's allocation of capacity and control rights can be decentralized. The auction that implements the SSS agreement differs from the well known first price or sealed bid auctions in several respects. First, as we emphasize above, this is an all pay auction in which all bidders make or receive some payment whether or not their bid is the winning one. An all pay auction is needed to implement the transfer payments between the members that balance the budget and make each member a residual claimant. Second, the primary purpose of the auction is to allocate capacity efficiently and to distribute control rights that sustain the SSS agreement. In equilibrium members reveal their demands by the amount they bid. A higher bid signals the member has a higher demand. However, since the members typically draw their demands from different distributions, the highest bidder is not necessarily the member with the highest demand. Hence the auction does not necessarily allocate greater current capacity to the highest bidder, but instead allocates capacity to members with the highest demand *as revealed* by their bid. Finally, the auction assigns next period control rights, $r^{*(t+1)}$, and the corresponding entry fees $\tilde{w}_i^{t+1}\left(r^{*(t+1)}\right)$ for the next period according to the demands that are revealed by each member's bid.

2.5.2 Illustration

To illustrate how the SSS agreement is implemented in practice we consider an example of supply chains where supply compatibility between members may differ.

Example 8. *A supply channel consists of a supplier S with one unit of capacity and a buyer, B. The supplier'svaluation for capacity in period t , θ_S^t, is uniformly distributed over $\left[0, \bar{\theta}\right]$. $B's$ capacity valuation varies according to how compatible he is with the supplier. For a given compatibility, $c =$ "high" (H) or $c =$ "low" (L), the buyer's valuation, θ_B^t, is uniformly distributed over $\left[0, \bar{\theta}_c\right]$ where $0 \leq \bar{\theta}_L < \bar{\theta}_H \leq \bar{\theta}$. The higher the compatibility the higher is B's valuation on average. The supply chain lasts for T periods and the common discount factor is $\delta = 1$.*

The efficient supply channel agreement is implemented by the sequential auction characterized in Proposition 2, where members $i = S, B$ bid $b_i^t\left(\theta_i^t\right)$, receive payment $m_i^t\left(b_i^t, b_{-i}^t\right)$, are allocated capacity $x_i^*\left(\theta^t\right)$ and control rights r_i^{*t} in each period.

Table 2.1: SSS^H *Agreement:* $\theta_S^t \in [0, 100], \theta_B^t \in [0, 100], T = 5,\ Ex_B^{*t} = Ex_S^{*t} = .5$

	1	2	3	4	5
$U^{H,t}$	333.33	266.67	200.00	133.33	66.67
$\tilde{w}_B^{H,t} = \tilde{w}_S^{H,t}$	166.67	133.33	100	66.67	33.33
$r_B^{*t} = r_S^{*t}$	.5	.5	.5	.5	.5

Table 2.2: SSS^L *Agreement:* $\theta_S^t \in [0, 100]$, $\theta_B^t \in [0, 50]$, $T = 5, Ex_B^{*\tau} = .25, Ex_S^{*\tau} = .75$

	1	2	3	4	5
$U^{L,t}$	270.83	216.67	162.50	108.33	54.16
$\tilde{w}_B^t$	72.91	58.335	43.75	63.61	43.58
$\tilde{w}_S^t$	197.92	158.33	118.75	44.72	10.58
r_B^{*t}	1	1	1	.58	.33
r_S^{*t}	0	0	0	.42	.66

Tables 1 and 2 summarize the implementation of the shared supply surplus agreements, denoted by SSS^H and SSS^L respectively. Focusing first on SSS^H in Table

1, both B and S have valuations drawn from the same interval $[0, 100]$. Since they are symmetric they receive the same expected allocation, $Ex^{*t} = .5$ in each period. Row 1 indicates the expected surplus $U^{H,t}$ that B and S each receive in each period. Row 2 indicates the identical fees B and S pay to be members of the channel in period t. Each party pays half of the transfer fees because they are symmetric and have equal bargaining power. At the beginning of the agreement in period 1, each member expects to contribute 166.66 to support the agreement which promises to provide each member with expected total surplus equal to 333.33

Row 3 shows the control rights supporting the agreement are $r_B^{*t} = r_S^{*t} = .5$, reflecting the identical outside options of the parties. This specification of rights is necessary to sustain the agreement. If the supplier were given exclusive rights, $r_S^{*t} = 1$, for instance, it's easy to show that the SSS^H agreement could not be sustained, since the participation fees that B and S would pay would be insufficient.[21]

The SSS^L agreement is portrayed in Table 2. In this case $B's$ valuations vary uniformly in the interval $[0, 50]$ reflecting her decreased valuation due to the fact that B and S are relatively incompatible. As a result the expected *per period* exchange surplus is only 54.16 as reported in row 1 for period 5. The expected allocations are now $Ex_B^{*t} = .25$ and $Ex_S^{*t} = .75$. The fees that B and S pay and their control rights allocations appear in row $2 - 5$. At the beginning in period 1, B has complete control with $r_B^{H,1} = 1$. $S's$ outside option value is much greater than $B's$. The buyer is awarded absolute control to prevent S from defaulting on the agreement.

[21] To illustrate, suppose the channel agreement last for a single period, and suppose all control rights reside with the seller so that $r_S^t = 1$ and $r_B^t = 0$. Then the seller and buyer's respective willingness to pay to participate are equal to

$$\begin{aligned} E_{\theta_B^t} V\left(\theta_{Sw}^t, \theta_B^t\right) - V_S\left(\theta_{Sw}^t, 1\right) &= 6.25 \\ E_{\theta_S^t} V\left(\theta_{Bw}^t, \theta_S^t\right) &= 50.0 \end{aligned}$$

The sum of these fees, 56.25 is less than 66.66, the total transfers between the parties, so that the *SSS* can not be sustained. We reach the same conclusion when the agreement last for more than one period, as well.

$S's$ bargaining position is reduced and he is assessed larger participation fees than B.[22] As time progresses S and $B's$ outside options converge and their control rights become more balanced. By the end of the agreement, S has a larger control right and B bares a larger share of the transfers. The variation in the assessment of fees reflects the changing bargaining positions of the parties.

2.5.3 Inducing Efficient Investment

We have shown how efficient exchange of supply capacity can be implemented with standard contracts. However, it is generally thought that standard contracts can not implement both efficient ex ante investment and efficient ex post exchange. Williamson (1975) for instance, observes contracts that are renegotiated to ensure efficient exchange usually generate inefficient ex ante investment. The provisions ensuring efficient exchange are not adequate to induce the parties to invest efficiently. This disconnect between up-front investment and ex post exchange is a necessary consequence of incomplete contracting. But suppose contracting is complete, can there be both efficient ex ante investment and ex post exchange?

To answer this we extend our model to include ex ante investment by the members prior to the exchange of supply capacity. We envision that initial demands are determined by private investment the retailers undertake to increases the market value of their goods. The supplier invests initially to increase the value of inputs that he employs for his own use. The initial valuations are drawn from distributions, $F_i^1(\theta_i^t|I)$ that are differentiable functions of $I = (I_1, .., I_N)$, the vector of members' investment expenditures. Higher investments shift the distribution towards higher values so that $I' \geq I'' \Rightarrow F_i^1(z|I') \leq F_i^1(z|I'')$[23]. Two aspects of our setup are

[22] As we show below, B must compensate S for the fee that he bears, with an initial payment prior to the beginning of the agreement.

[23] We adopt the convention that

$$I' \geq I'' \Rightarrow I_i' \geq I_i''$$

noteworthy. One is the investment affects demands over the life time of the supply chain. A higher initial valuation implies future valuations are higher. A second one is that investment externalities exist in that one member's valuation may be increased by another member's investment. For instance, one member may adopt a product design that another member can also use to increase demand for its product. However, although there are investment spillovers we continue to assume members' demands are drawn independently, conditional on the initial investment, I.

The efficient investment vector, I^*, is assumed to be unique and interior with $I_i^* \in (0, \infty)$ for all i. These assumptions are convenient for ensuring the optimal investment can be implemented.[24] Given our maintained assumptions, the first order condition determining I^* is,

$$\frac{d}{dI_i}[EV^1(\theta^1|I_i, I_{-i}^*) - I_i] = 0.$$

Whether the members select the efficient investment depends on the outcome of the following investment game induced by the SSS agreement, $M\langle x^*, \tau, r^*|I^*\rangle$. The mechanism provides for the same allocations of supply capacity, as before, under the assumption members initially invest I^*. At the onset of the supply channel agreement, each i selects I_i to maximize his expected continuation profit, $W_i(I_i, I_{-i})$, which is

$$\begin{aligned} W_i(I_i, I_{-i}) &= \max_{I_i}\{EV^1(\theta^1|I_i, I_{-i}) - I_i - \Sigma_{j\neq i} I_j^*\} \\ &\quad - \frac{\tilde{w}_i^1(r^{*1}(I^*))(N-1) + \sum_{j\neq i}\tilde{w}_j^1(r^{*1}(I^*))}{N} \end{aligned} \tag{2.14}$$

The first term in (2.14) is i's expected profit net of investment costs, including the

for all i, with strict inequality for at least one i.

[24] Under weaker assumptions, there may be other investment vectors, I' satisfying first order condition that are local optima, but not global optima. In this case it is possible to implement the efficient investment as a Nash equilibrium, but other inefficient investments may also be Nash equilibria. The members must coordinate their investments to select the efficient one in this case.

projected efficient investments of members $j \neq i$ and the second term is $i's$ share of the participation fees, calculated on the assumption the members invest efficiently.

Proposition 9. *I^* is the unique Nash equilibrium to the ex ante investment game where each firm invests to maximize his expected continuation profit.*

Proposition 9 illustrates an important advantage of our SSS agreement.[25] Other *AGV-type* mechanisms are unable to implement efficient investment when there are externalities.[26] Our mechanism induces efficient investment and thereby solves the holdup problem (e.g. Rogerson (1992) and Edlin and Reichelstein (1996)) with externalities because each member is a residual claimant of the entire surplus. Each member is incented to invest efficiently even when his investment increases the demands from the other members.

2.5.4 Illustration of Supply Chain Investment

We conclude our illustration of the SSS agreement with an example demonstrating how efficient investment may be implemented in practice.

Example 10. *The supply channel compatibility, c,is determined by the private investments, (I_B, I_S). With probability $\lambda(I_B, I_S)$ the supply channel is compatible, H, and with probability $1 - \lambda(I_B, I_S)$ it is of low compatibility, L. There is a unique pair of investments, (I_B^*, I_S^*), that maximize the expected channel surplus,*

$$U(I_B^*, I_S^*) \equiv \lambda(I_B^*, I_S^*) U^{H,1} + (1 - \lambda(I_B^*, I_S^*)) U^{L,1} - I_B^* - I_S^*$$

where $U^{H,1}$ and $U^{L,1}$ is the expected surplus of a T-period SSS^c agreement for a H and L supply chain, respectively.

Under the investment agreement, each member iinitially posts a deposit of $\tilde{w}_i^o = \frac{U(I_B^*, I_S^*)}{2}$ and makes a private investment, I_i to increase the channel compatibility.

[25] Krahmer and Strausz (2006) demonstrate a similar result in the context of *AGV* mechanisms.

[26] See Bergemann and Valimaki (2002) and Arozamena and Cantillon (2004), for instance.

Once the investments are completed, B observes the outcome, determines his compatibility with the supplier, H or L, and then selects a SSS agreement from the menu below:

	SSS^H	SSS^L
S	$\tau_S^H = \tilde{w}_S^{H,1},\ M_S^H$	$\tau_S^L = \tilde{w}_S^{L,T},\ M_S^L$
B	$\tau_B^H = \tilde{w}_B^{H,T},\ M_B^H$	$\tau_B^L = \tilde{w}_B^{L,T},\ M_B^L$

The example illustrates how the efficient ex ante investment is implemented by the investment agreement. Once B observes the compatibility of the channel he selects from the menu. Each selection $c = H, L$ from the menu consists of a transfer payment–mechanism pair for S and B, denoted by $\{(\tau_S^c,\ M_S^c), (\tau_B^c,\ M_B^c)\}$. These selections provide B with the maximum expected surplus, $U^{c,1}$ corresponding to the true supply channel compatibility, c. For instance if $c = H$ and B selects SSS^H, then he receives initial payment $\tau_S^H = \tilde{w}_B^{H,1}$ and he expects to earn total surplus equal to $U^{H,1} - \tilde{w}_B^{H,1}$ under the M_S^H mechanism. Consequently the net surplus he earns from this selection is

$$\tilde{w}_B^{H,1} + \left(U^{H,T} - \tilde{w}_B^{H,1}\right) = U^{H,1}$$

This makes B the residual claimant to all the surplus generated by his choice of agreement SSS^H. B can do no better than to select the SSS^H agreement that is designed for high compatible channels. By a similar argument, S also becomes the residual claimant to all of the supply chain surplus when B selects SSS^H, since his pair $\left(\tau_S^H,\ M_S^H\right)$ generates net surplus equal to ,

$$\tilde{w}_S^H + \left(U^{H,1} - \tilde{w}_S^{H,1}\right) = U^{H,1}$$

S can therefore depend on B to select the agreement appropriate for the channel compatibility.

At the time of investment, B and S anticipate they will be the residual claimants to all of the surplus their investments create. Hence each member i will select an investment to

$$\max_{I_i} \lambda\left(I_i, I_j\right) U^{H,1} + \left(1 - \lambda\left(I_i, I_j\right)\right) U^{L,1} - I_i^* - I_j^* - \tilde{\omega}_i^0$$

The efficient investment pair, $\left(I_B^*, I_S^*\right)$, is the unique Nash equilibrium to this investment game. In equilibrium, each member i receives

$$\begin{aligned}
U_i^0 &= \lambda\left(I_B^*, I_S^*\right) U^{H,1} + \left(1 - \lambda\left(I_B^*, I_S^*\right)\right) U^{L,1} - I_B^* - I_S^* - \tilde{\omega}_i^0 \\
&= U\left(I_B^*, I_S^*\right) - \tilde{\omega}_i^0 \\
&= \frac{U\left(I_B^*, I_S^*\right)}{2}
\end{aligned}$$

The members split the maximum ex ante surplus generated by the supply channel.

3

Mechanism for Sustainable Quality Supply

3.1 Introduction

Sustained quality of input materials is vital for all supply chains. The problem is, for many supply chains serving the developed markets, the key material inputs are sourced from underdeveloped economies and traveled through many different value-adding parties across different countries and continents. With information asymmetry and unevenly distributed bargaining powers along the global supply chain, the suppliers at the source are often unfairly compensated, leaving them difficult to make ends meet, not to mention having the resources and abilities to invest in the production processes to maintain or improve the quality of the key material. As a result, long-term quality material supply is at risk, which may eventually terminate the business of the supply chain. Think of the coffee beans grown in Arabic farms for Starbucks, the world's largest specialty coffee retailer; water used in hundreds of plants all over the world for the products of Coca-Cola Co. – the world's largest beverage company; toys made in Southeast Asian factories for Mattel, the world's largest toy manufacturer; and i-pods made by contract manufacturers in Asia for

Apple Computers. Realizing this issue, many well known companies in the developed economies have initiated sustainability and corporate responsibility programs in the hope to develop long term reliable partnership with their suppliers to ensure the supply of premium quality and environmental friendly raw materials. One example of the C.A.F.E. (Coffee and Farmer Equity) Practices developed by Starbucks; see Duba et. al. (2007). According to these authors, in early 1990's, a worldwide oversupply of low-quality coffee beans depressed the market prices, leaving the coffee growers unable to cover their production costs, which consequently threatened the future supply of high-quality specialty coffee that Starbucks needed to support its growth. To resolve the issue, over the last decade, Starbucks developed the C.A.F.E. Practices to help the growers to improve their livelihood and production conditions and reward the growers for high-quality sustainably grown coffee. The C.A.F.E. practices is to achieve economic accountability, social responsibility, and environmental leadership.

However, not surprisingly, the implementation of C.A.F.E. faces many challenges, such as information asymmetry, misaligned incentives, and dynamically changing environment. In fact, these issues and challenges are typical for any other supply chain sustainability initiatives as well . See the Global Water Project at Coca-Cola Co. (Renge 2008) for another example. This is because that this kind of programs is truly collaborative in nature, requesting different supply chain members contributing complementary resources, expertise, and technology. Yet, different members belong to different organizations and may locate in a different countries or regions. The question is: what should be the key components and conditions of such programs – with the goal of achieving sustainable quality supply – that can meet these challenges? We explore the answers in this paper. In particular, we develop a multiperiod supply chain model with a risk neutral supplier and a risk neutral retailer. The supply chain is formed to provide the retailer access to a long term reliable supply source of materials that it transforms into final products for sale to its customers. The construction of the chain requires the supplier to make an initial investment in capacity and for the retailer to make an initial investment in product development. The investments are relationship specific and cannot be publicly observed. After investment, the supply of materials to the retailer begins. The costs of supply as well as the demand for the materials vary randomly from one period to the next due to unforeseen changes in market conditions. Prior to each period, the supplier privately learns his cost of production and the retailer privately observes his demand. The privately informed parties exchange information to determine the desired supply of materials for that period and bargain with each other to determine the terms of exchange.

We analyze the tradeoffs of quality investment and supply for each supply chain partner and construct a dynamic mechanism to incentivize all partners to collaborate to achieve sustainable quality supply. We call this mechanism *Sustainable Quality Supply Agreement* or *SQSA*. The Participation in the supply agreement is voluntary, at any time the supplier or retailer may request the agreement be dissolved and that the decision rights of the supply quality for the remaining periods are allocated between the parties as specified in the supply agreement. We show that a *SQSA* achieves the system-wide optimal supply quality level in each period. Moreover, the agreement is incentive compatible and individually rational. It is also ex post budget balancing so it is self-enforceable and does not need to involve any third party, and it can be implemented through an all-pay auction. Finally, the mechanism induces optimal ex ante supply chain investment.

There are two key elements of our mechanism design that permit such a contract with these strong properties. The first is the shared surplus type of mechanism that permits the supplier and retailer to be a residual claimant to all the additional supply chain profit it creates. As suggested in the motivating examples, the sustainability endeavor requires the collaboration of all parties. Every member in the supply chain contributes different skills and resources, demonstrating strong complementarity and dependence. Therefore, to ensure full collaboration to achieve supply chain optimality, each party should be fairly compensated according to its contribution. The first element accomplishes this goal precisely; it aligns the partners' incentives to truthfully share information and to produce and invest efficiently.

The second element is the dynamic allocation of the decision rights on supply quality. To reflect the above mentioned strong complementarity and dependence of the supply chain members, instead of a given principal-agent relationship typically assumed in the supply chain literature, we treat two players in the supply chain as "parallel" partners in the sense that the decision rights in each period are allocated between the partners, according to the current state of the system. In other words, there is no "fixed" owner of the supply chain, and the formation of the supply chain is truly a partnership. At the beginning of each period, after observing his current state of the environment, a player can choose to quit from the agreement, but the supply-demand relationship of the two players continues to the end of the horizon, following the pre-agreed responsibilities of the supplier and retailer in future periods. In particular, the payment terms are fixed. If one is awarded the decision rights for quality, this member will specify a quality level for the remaining periods, and the supply chain will continue the supply-demand relationship at this quality level and the prespecified payment terms.

An important feature in our model that contributes to the value of this second element is that we explicitly model a stochastically changing environment for both players. For the supplier, this environment may mean production (such as weather, land and labor) conditions. For the retailer, this may mean consumers' valuation of quality or general economic conditions. When these conditions change, the economics of exchanges change, which, in turn, affect the desired quality level as well as the incentives in investment and trade in the partnership. Recognizing these uncertainties, our mechanism is designed in a way so that at each stage, the retailer and supplier renegotiate the decision rights allocation in order to sustain each other's participation in the supply agreement. The principle for allocating the decision rights is to minimize the value of the supply chain's outside option or to increase the switching costs of quitting the supply agreement.

We illustrate our mechanism $SQSA$ in the context of a simplified version of the Starbucks' coffee supply chain. We show that many features of this mechanism resemble those in the C.A.F.E. Practices. For example, its reward scheme coincides with the dynamically changing price of the C.A.F.E. certified coffee beans. We also show that changing pattern of the control right allocation in different environment can provide valuable guidance to practice.

The paper is organized as follows. In Section 2, we relate our work to the existing literature. In Section 3, we describe the model setting and the mechanism structure and desired properties. In section 4, we construct the specifics of the mechanism SQSA and show that it indeed leads to the desired equilibrium properties. We also demonstrate that the mechanism can be implemented through an all-pay auction. In section 5, we show that the optimal ex ante supply chain investment is also achievable under SQSA. In section 6, we discuss the role of control rights through a simplified version Starbucks's supply chain and discuss its relationship with the C.A.F.E. Practices. Finally, in Section 7, we conclude the paper with a few remarks.

3.2 Literature Review

Sustainability is an increasingly popular topic in many fields(for specific definitions on sustainability see van Marrewijk and Werre 2003 and Goodland 1995) especially in operations related fields. The core concept that helps to operationalize sustainability is the triple bottom line approach: to achieve a minimum performance in economical, environmental and social aspects (Seuring and Müller 2008). Specifically, in supply chain management, the definition of sustainability is given as '*the management of material, information and capital flows as well as cooperation among companies along the supply chain while taking goals from all three dimensions of sustainable development, i.e., economic, environmental and social, into account which are derived from customer and stakeholder requirements*' (see Seuring and Müller 2008) (see De Burgos and Lorente 2001, Baumann et al 2002 for more extensive reviews). Although wide research have been done in green/environmental issues, research in social sustainability and the integration of the three dimensions are still rare (see Seuring and Müller 2008). Our paper provide a theoretical framework to analyze supply chain sustainability focusing on the integration of social (supplier development issues) and economical (profitability) dimensions. Most empirical studies find that there is a positive link between firms have incentives to collaborate to achieve supply chain sustainability with their economical benefits (Bowen et. al. (2001), Vachon and Klassen (2006), Rao and Holt (2005). Our paper supports the empirical findings and further show the existence of a mechanism that can algin the firms incentives to implement supply chain sustainability with their benefits.

Another stream of related literature in supply chain management economics is about how supply chain partners can share information to achieve efficiency; see, for example, Cachon and Lariviere (2001), Ha (2001) and Tomlin (2003). Within this literature, the most related works to ours are Plambeck and Taylor (2005, 2006, 2007) and Taylor and Plambeck (2007). Plambeck and Taylor (2005) analyze the performance of OEM and CM when the OEM's innovation is non-contractible. Plambeck and Taylor (2006) consider a joint production problem with two sided moral hazard and incomplete information. The repeated interaction between firms introduces dynamics and the actions have future influence on the process. They are able to construct the optimal rational contract and show that it is state dependent. Plambeck and Taylor (2007) study investment efficiency under two kinds of breach remedy:

specific performance and expectation damage. They show that there is no single breach remedy that can achieve efficient investment under all possible settings. Taylor and Plambeck (2007) consider relational contracts under two sided asymmetric information. They are able to characterize the optimal relational contract and show that the contract is static. In sum, these papers analyze how firms negotiate among themselves and with their upstream stages for capacity, when the cost of supply may not be publicly known and when these firms demand is possibly private information. Our setting has two sided information asymmetry and double moral hazard which generalizes the information structure in Plambeck and Taylor (2005, 2006, 2007) and Taylor and Plambeck (2007). Moreover, without restricting to certain contract forms, we consider complete contracts that accommodate the dynamically changing demand and supply conditions.

Our methodology for constructing the dynamic mechanism is similar to those in Kuribko and Lewis (2010) and Lewis, Liu and Song (2011). Instead of task assignment and resource allocation considered in these two papers respectively, our focus is on the supply quality decision jointly made by the supplier and the retailer. In addition, different from these two papers, our model has a non-symmetric supplier-retailer feature in which the supplier incurs cost while the retailer generates revenue. This feature allows us to obtain several new insights.

Our paper is also related to the literature on supply chain quality. In most of this literature, quality does not affect the market (see Wang and Gerchack 1996, Nahmias and Moinzadeh 1997, Bollapragada and Morton 1999, Grosfeld-Nir et al. 2000 and Rajaram and Karmarkar 2002 for some recent work). These papers consider how imperfections and deteriorations of the products affect the supply system. In our paper, the quality of the product has a positive effect on the supply chain surplus through the seller's profit return from a quality sensitive market. Some other works focus on inspection issues. For example, Starbird (1997) studies a buyer's inspection policy and finds conditions under which there are zero defects, and Reyniers and Tapiero (1995 a,b) study the buyer's inspection policy under both non-coorperative game and cooperative game. In our paper, the collaboration among firms is self-enforceable. Instead of inspection, we identify conditions under which the firms are self disciplined to follow the quality investment theme given in the contract. Some authors focus on how quality issues affect supply chain efficiency. For instance, Baiman et al. (2000) analyze the effect of contract terms such as quality enhancements and quality appraisal. Lim (2001) studies how a producer with incomplete supply quality information design a contract to maximize his profit and obtain the true information about quality. In our paper, the collaboration guarantees ex post supply-chain optimal quality investment under which the information on quality investment may or may not be fully extracted. Iyer et al. (2005) and Zhu et al. (2007) study the effect of a buyer investing in its supplier's quality improvement. In our paper, the retailer's investment in market sensitivity about quality indirectly affects the quality investment selection of the supplier. We analyze the influence of such investments on quality as well as on other sustainability issues. Finally, Kaya and Ozer (2009) identify two factors contributing quality risks in an outsourcing relationship – non-contactable quality and private supplier cost of quality – and study how to use a pricing strategy to mitigate the risks. Our model assumes the second risk but not the first. This is consistent with what we observe in the C.A.F.E. practices. There, there are third-party agencies to verify and assess the supply quality.

3.3 Model and Mechanism Structure

3.3.1 Model

Consider a two-party supply chain with a risk neutral supplier $\mathcal{S}$ and a risk neutral retailer $\mathcal{R}$, in which S provides material inputs (e.g., coffee beans) that R employs to make a product (e.g., coffee) for its retail franchise. The two parties form a partnership to work together to develop sustainable quality supplying over a period of time, $t = 1, , , T$. The T-period relationship represents a long-term commitment which is an important feature in all sustainability programs.

We measure the quality of the supply by a scalar $x \in [0, 1]$; a higher value of x means a higher quality level. For example, x can be the yield of each period's production (assuming production quantity is fixed for all periods), i.e., $1 - x$ is the defect rate. This model is also consistent with the grading and scoring methods used in practice. For instance, in the C.A.F.E. Practices initiated by Starbucks, a scoring system with detailed guidelines and weighing factors is developed. A supplier can earn up to 100 percentage points based on a set of environmental and social criteria graded by a third-party certification company. Suppliers with high scores are awarded preferential status and price premiums. Other companies such as Coca-Cola and Wal-Mart also adopt a single measurement for evaluating water quality or organic-produce quality, etc.

Supplier's Cost

In each period $t = 1, 2, , , T$, the supplier $\mathcal{S}$ can provide material inputs of quality $x^t \in [0, 1]$ at a cost of $C(x^t, \theta_s^t)$, with $C(0, \theta_s^t) = 0$. Here, $\theta_s^t \in [0, 1]$ is a parameter reflecting current production conditions, such as whether, land, and labor market conditions. We assume C is increasing and convex in x^t but increasing and concave in θ_s^t. Also, the marginal cost of quality improvement is increasing in θ_s^t, i.e., $\partial^2 C(x^t, \theta_s^t) / (\partial x^t \partial \theta_s^t) > 0$. One example is

$$C\left(x^t, \theta_s^t\right) = \theta_s^t c\left(x^t\right), \tag{3.1}$$

where $c(x)$ is the effort for providing quality x, which is strictly increasing and convex in x for $x > 0$, with $c(0) = 0$, $c(1) = 1$, $c'(0) = 0$, $c'(1) = \infty$.

Retailer's Revenue

With the material input of quality x^t, the retailer $\mathcal{R}$ generates revenue $R\left(x^t, \theta_r^t\right)$, with $R\left(0, \theta_r^t\right) = 0$. Here, the parameter $\theta_r^t \in [0,1]$ represents the demand condition such as consumers' valuation of quality and awareness of environmental issues, or the general economic conditions. A higher value θ_r implies a more favorable demand condition. We assume R is strictly increasing and concave in x^t but strictly increasing and convex in θ_s^t. Also, the marginal revenue of quality improvement is increasing in θ_s^t, i.e., $\partial^2 R\left(x^t, \theta_s^t\right) / (\partial x^t \partial \theta_s^t) > 0$. An example is

$$R(x^t, \theta_r^t) \quad = \quad \theta_r^t x^t \tag{3.2}$$

where θ_r^t is the market size in period t assuming the product's unit selling price is fixed at 1 in every period, and x^t is the proportion of customers who would purchase the product with quality level x^t, assuming the customers' valuation of quality given the price is 1 is uniformly distributed in the interval [0,1].

The parameters (θ_r^t, θ_r^t) are drawn independently from a *CDF* $F_i^t\left(\theta_i^t \mid \theta_i^{t-1}\right)$ with a bounded, strictly positive and continuous density $f_i^t\left(\theta_i^t \mid \theta_i^{t-1}\right)$, $i = r, s$. In addition, we assume the following:

Assumption 11. *θ_i^t is linear increasing in θ_i^{t-1} in the form $\theta_i^t = \nu_i^t \theta_i^{t-1} + \epsilon_i^t$, where ϵ_i^t is an independent random variable with mean 0, $i = r, s$.*

Here, ν_i^t can be understood as the exogenous impact on the two firms, such as the global economic conditions, exchange rates, etc. With these assumption, for each i, the random variable θ_i^t is governed by a first order stochastic process, with positive serial correlation. The distribution of the first period parameter, $F_i^1\left(\theta_i^1\right)$ is assumed to depend on known initial conditions that are publicly known. See Section 6 for some examples.

3.3.2 Coordinated Supply Chain

At the beginning of each period t, $\mathcal{S}$ and $\mathcal{R}$ privately observe information about costs and demand respectively. If the supply chain members coordinate and share their information, they can select the quality that maximizes the supply chain profit of the current period given the current market and production conditions. Define $\theta^t = (\theta_r^t, \theta_s^t)$. Then

$$x^{t*}(\theta^t) = x^*(\theta^t) = \arg\max_x \{R\left(x, \theta_r^t\right) - C\left(x, \theta_s^t\right)\} \tag{3.3}$$

is the supply-chain optimal quality in period t given θ^t, and the corresponding supply-chain profit of the period is

$$\Pi^t\left(\theta^t\right) = R\left(x^*\left(\theta^t\right), \theta_r^t\right) - C\left(x^*\left(\theta^t\right), \theta_s^t\right)$$

Let $\delta \in (0,1)$ be the common discount factor for $\mathcal{S}$ and $\mathcal{R}$. Because $x^*(\theta^k)$ is reachable for all k, provided θ^k is known, the expected optimal supply-chain profit, beginning at time t with market conditions θ^t, is given by

$$V^{t*}(\theta^t) = \Pi^t\left(\theta^t\right) + \sum_{k=t+1}^{T} \delta^{k-t} E[\Pi^k(\theta^k)|\theta^t].$$

And the myopic quality levels x^{t*} in (3.3), $t = 1, ..., T$, are the supply-chain optimal quality levels that maximizes the T-horizon expected discounted supply-chain profit.

For any two environmental vectors θ^t and ϕ^t, we say θ^t is "better" than ϕ^t, denoted by $\theta^t \succ \phi^t$, if $\theta_s^t < \phi_s^t$ and $\theta_r^t > \phi_r^t$. In other words, θ^t has more favorable market and production conditions than ϕ^t. We have

Proposition 12. *For any given t and θ^t:*

1) $x^{t*}(\theta^t)$ is increasing in θ_r^t but decreasing in θ_s^t.

2) $\Pi^t(\theta^t)$ is increasing and convex in θ_r^t but decreasing and convex in θ_s^t. Moreover,

$\Pi^t(\theta^t)$ *is jointly convex in* θ^t *and hence* $E[\Pi^t(\theta^t)|\theta^{t-1}]$ *is jointly convex in* θ^{t-1}.

3) If $\theta^t \succ \phi^t$, *then* $x^{t*}(\theta^t) > x^{t*}(\phi^t)$, *and the expected quality in period* $t+1$ *satisfies* $E[x^{(t+1)*}(\theta^{t+1})|x^{t*}(\theta^t)] \geq E[x^{(t+1)*}(\phi^{t+1})|x^{t*}(\phi^t)]$.

The first two parts of the proposition indicate that both the supply-chain optimal quality and profit are increasing when either the current market or production conditions improve. In addition, the optimal profit increases at an increasing rate as these conditions improve. The third part shows that a current higher optimal quality level can lead to a higher expected optimal quality level in the next period, because a better current environment leads to a stochastically better future environment (Assumption 1). Thus, even though the optimal quality levels are selected myopically, they can be positively correlated between periods through the changes in the underlying environment. This is consistent with the observation in Starbucks' practice in which the quality is selected based on the grower's cost and technology, but a grower who can provide high quality coffee beans this period is likely to provide high quality coffee beans in the next period because her production technology has been improved, i.e. the level of the quality is consistent over periods.

3.3.3 Mechanism Design for Sustainability

We now design a dynamic mechanism to facilitate the long-term commitment of $\mathcal{S}$ and $\mathcal{R}$ to achieve supply-chain optimal quality. Because the environment changes dynamically and information is asymmetric, the cost and benefit of a particular quality level in each period has different implications to different players, the mechanism is an agreement between the partners that specifies the quality of the of materials to be supplied in each period and each member's compensation. These specifications depend on the information that $\mathcal{S}$ and $\mathcal{R}$ report about their private market conditions. In addition, to ensure voluntary participation in the partnership and at the same time discourage the myopic behavior that a member with a favorable current status may wish to quit before seeing the true benefit of the long-term commitment, the mechanism specifies how to allocate the decision rights on supply quality if any member chooses to quit in the interim. The allocation of the decision rights too depends on the information that $\mathcal{S}$ and $\mathcal{R}$ report about their private market conditions. The basic principle of the decision-rights allocation is to limit the value of outside options or to increase the switching costs when quitting from the partnership.

When the supply agreement is formed, the initial distribution $F_i^1(\theta_i^t)$ for $i = s, r$ is common knowledge. Thereafter, at the beginning of each period t, $\mathcal{S}$ and $\mathcal{R}$ privately observe θ_s^t and θ_r^t respectively. Since θ^t are determined by a Markov process, member $i's$ private history of conditions to time t is completely summarized by $h_i^t = \theta_i^{t-1}$. By the revelation principle we are, without loss of generality, able to restrict attention to direct mechanisms that ask each member i to report its condition $\tilde{\theta}_i^t \in [0,1]$.[1] The reports may or may not be truthful, that is, $\tilde{\theta}_i^t$ need not equal θ_i^t. Let $\tilde{h}^0 = (F_s^1(\cdot),\ F_r^1(\cdot))$ be the initial distribution of $\mathcal{S}$ and $\mathcal{R}$. The subsequent history of reports can be recursively defined as $\tilde{h}^t = (\tilde{h}^{t-1}, \tilde{\theta}^t\ )$. Denote $\tilde{H}^t$ as the set of possible period t reported histories.

The multi-period direct mechanism, denoted by

$$M\left\langle x^t, \tau^t, \rho^{t+1}\right\rangle_{t=1}^{t=T} \tag{3.4}$$

is a sequence of quality levels, x^t, transfer payments, τ^t, and decision rights, ρ^{t+1}, with the following properties. First, the mapping

$$x^t : \tilde{H}^t \rightarrow [0,1] \tag{3.5}$$

specifies the quality of materials supplied at time t. Second, after the materials are

[1] The revelation principle is due to Myerson(1986). It states that any allocation that can be implemented by some process that depends on the parties types, as described by their private information, can also be implemented by a direct mechanism in which the parties are induced to truthfully report their types.

supplied, each member i, makes (or receives) a payment τ_i^t, according to the mapping

$$\tau^t : \tilde{H}^t \rightarrow \mathbb{R}^2 \tag{3.6}$$

Third, following the completion of period t's material supply and transfer of payments, the mapping

$$\rho^{t+1} : \tilde{H}^t \rightarrow [0,1]^2 \ \ s.t. \ \rho_r^{t+1} + \rho_s^{t+1} = 1. \tag{3.7}$$

assigns decision rights when the contract ends in the next period. It means that the right to decide the quality is awarded to member i with probability ρ_i^{t+1} if the contract is terminated in period $t+1$.

The decision rights ensure each member's participation in the supply agreement is voluntary. Any member can dissolve the supply agreement at the beginning of a period and request the assets of the supply agreement be distributed according to the decision rights allocation, ρ^{t+1}.[2] The allocation requires *all* of the decision be distributed only to the members. This provision prevents the diversion of authority to a third party who could act as a budget breaker to prevent the supply agreement from dissolving. The provision, of course, makes it more difficult to sustain the agreement by reducing a member's private cost of terminating the agreement.

Note that when the contract ends in period t, no more market information exchange will take place from t on, so the chain will become an uncoordinated one. In this situation, the most viable contract form is to specify a constant quality level x that $\mathcal{S}$ needs to supply and a constant transfer payment τ from $\mathcal{R}$ to $\mathcal{S}$ across all periods from t through T. We assume that the contract specifies $\tau = \tau_m$ to be the market medium price (as of the time when the contract is signed), but the value of

[2] It is reasonable to assume that the rights are allocated independent of who initiates the breakup of the supply chain agreement. Practically, courts are only able to observe the dissolution of an agreement, but not the member(s) who precipitated the breakup. This is similar to a "contracting at will" standard where courts only enforce agreements that are contingent on the level of trade, but not on who precipitates the trade disruption.

x is subject to the decision by the member who is awarded the decision right. Under this scheme, $\mathcal{S}$'s and $\mathcal{R}$'s profit in period k, $k = t, .., T$, for a given market condition θ^k are respectively

$$\begin{aligned} \Pi_s^k \left(x, \theta^k\right) &= \tau_m - C\left(x, \theta_s^k\right) \\ \Pi_r^k \left(x, \theta^k\right) &= R\left(x, \theta_r^k\right) - \tau_m. \end{aligned}$$

Therefore, the expected profit of $\mathcal{S}$ and $\mathcal{R}$ from period t on, assuming period t market conditions θ^t and a given quality level x from t through T, is

$$\begin{aligned} V_s^t(x, \theta^t) &= \Pi_s^t \left(x, \theta^t\right) + \sum_{k=t+1}^{T} \delta^{k-t} E[\Pi_s^k \left(x, \theta^k\right) |\theta^t] \\ V_r^t(x, \theta^t) &= \Pi_r^t \left(x, \theta^t\right) + \sum_{k=t+1}^{T} \delta^{k-t} E[\Pi_r^k \left(x, \theta^k\right) |\theta^t] \end{aligned}$$

For convenience, we normalize $\tau_m = 0$. Let x_i^o be the quality level member i selects if he/she is awarded the decision rights when the agreement terminates. Note that $C(x, \theta)$ is decreasing in x, so in the event that $\mathcal{S}$ is awarded the right, she will select to supply quality $x_s = 0$ and therefore receive its maximum profit of $V_s^t\left(0, \theta_s^t\right) = 0$. If, on the other hand, $\mathcal{R}$ is awarded the decision right, because $R(x, \theta)$ is increasing in x, he will select its preferred quality $x_r = 1$ and will receive profit equal to $V_r^t\left(1, \theta^t\right)$. Thus, in the rest of the paper, when we refer to x^o, we mean

$$x_s^o = 0 \quad \text{and} \quad x_r^o = 1. \tag{3.8}$$

Also, from the above discussion, under this mechanism, the expected value of the outside option for member i if the contract ends in period t with market condition θ^t is

$$O_i^t(\theta_i^t) \equiv \rho_i^t V_i^t\left(x_i^o, \theta_i^t\right) + \rho_{-i}^t V_i^t\left(x_{-i}^o, \theta_i^t\right). \tag{3.9}$$

Throughout the paper, when $i = r$, $-i = s$, and vise versa.

One way to interpret $O_i^t(\theta_i^t)$ is through buyout options. When the partnership dissolves, $\mathcal{S}$ and $\mathcal{R}$ can buyout their rights and let the government serve the agreement after the government pays them their respective expected profit under the current decision right allocation.

The dynamic mechanism, $M \left\langle x^t, \tau^t, \rho^{t+1} \right\rangle_{t=1}^{t=T}$, is implemented as a two stage process in each period:

Decision to Participate Stage: *Each $i = s, r$ privately observes θ_i^t and decides to remain in or quit the supply agreement. If a member(s) quit, the agreement is dissolved and i receives the exclusive rights to decide supply quality with probability ρ_i^t. Otherwise the agreement moves to the next stage.*

Reporting Stage*: Each i reports $\tilde{\theta}_i^t \in [0, 1]$ and the quality–payments–rights provisions*
$\left\langle x\left(\tilde{h}^t\right), \tau_i^t\left(\tilde{h}^t\right), \rho_i^{t+1}\left(\tilde{h}^t\right)\right\rangle$ *are implemented. If $t = T$, the supply agreement ends. Otherwise, if $t < T$ the agreement moves on to period $t + 1$.*

3.3.4 Properties of the Equilibrium: Meeting All the Challenges

The mechanism M induces a sequential simultaneous move game, wherein the members choose a reporting strategy in each period to that maximizes their expected profit.

Assuming the supply chain is governed by the mechanism for all $t = 1, 2, , T$, the expected payoff for member $i = s, r$ from reporting a sequence of conditions

$\tilde{\theta}_i = \left\{\tilde{\theta}_i^t\right\}$, given member $-i$ reports $\tilde{\theta}_{-i} = \left\{\tilde{\theta}_{-i}^t\right\}$, is

$$E\sum_{t=1}^{T}\left[\delta^{t-1}\left(\tau_s^t\left(\tilde{h}^{t-1},\tilde{\theta}_s^t,\tilde{\theta}_r^t\right) - C\left(x^t\left(\tilde{h}^{t-1},\tilde{\theta}_s^t,\tilde{\theta}_r^t\right),\theta_s^t\right)\right)\right] \tag{3.10}$$

$$E\sum_{t=1}^{T}\left[\delta^{t-1}\left(\tau_r^t\left(\tilde{h}^{t-1},\tilde{\theta}_r^t,\tilde{\theta}_s^t\right) + R\left(x^t\left(\tilde{h}^{t-1},\tilde{\theta}_r^t,\tilde{\theta}_s^t\right),\theta_r^t\right)\right)\right] \tag{3.11}$$

for $\mathcal{S}$ and $\mathcal{R}$ respectively. Member i selects a reporting strategy,

$$\tilde{\theta}_i^t : \left[\tilde{h}^{t-1},\theta_i^t\right] \to [0,1]$$

which is a mapping from the history of reported conditions and $i's$ current condition into a current condition report, to solve the following sequential maximization problem. Let $W_i^t\left[\tilde{h}^{t-1},\theta_i^t\right]$ be $i's$ maximized profit starting in period t given the history of reported conditions and $i's$ period t condition. Then $W_i^t\left[\tilde{h}^{t-1},\theta_i^t\right]$ is determined as the solution to the following dynamic program for $i = s, r$,

$$\begin{aligned} W_s^t\left[\tilde{h}^{t-1},\theta_s^t\right] &= \max_{\tilde{\theta}_s^t} E_{\tilde{\theta}_r^t}\left\{\tau_s^t\left(\tilde{h}^{t-1},\tilde{\theta}_s^t,\tilde{\theta}_r^t\right) - C\left(x^t\left(\tilde{h}^{t-1},\tilde{\theta}_s^t,\tilde{\theta}_r^t\right),\theta_s^t\right)\right. \\ &\quad \left. + \delta E W_s^{t+1}\left[\tilde{h}^t,\theta_s^{t+1} \mid \theta_s^t\right]\right\} \end{aligned}$$

$$\begin{aligned} W_r^t\left[\tilde{h}^{t-1},\theta_r^t\right] &= \max_{\tilde{\theta}_r^t} E_{\tilde{\theta}_r^t}\left\{\tau_r^t\left(\tilde{h}^{t-1},\tilde{\theta}_r^t,\tilde{\theta}_s^t\right) + R\left(x^t\left(\tilde{h}^{t-1},\tilde{\theta}_r^t,\tilde{\theta}_s^t\right),\theta_r^t\right)\right. \\ &\quad \left. + \delta E W_r^{t+1}\left[\tilde{h}^t,\theta_r^{t+1} \mid \theta_r^t\right]\right\} \end{aligned}$$

where $EW_i^{t+1}\left[\tilde{h}^t,\theta_i^{t+1} \mid \theta_i^t\right]$ is $i's$ expected continuation surplus in period $t+1$ given θ_i^t.

Of course our goal is to find the right specifications of M – the terms x^t, τ^t, and ρ^{t+1}, for all t – so that they induce an equilibrium of this dynamic game that achieves sustainable quality supply. As such, these specifications should be designed to meet the main challenges in carrying out sustainability initiatives. These chanllenges are: a) difficult to ensure information sharing among privately informed members, b) difficult to ensure voluntary participation because of the dynamically changing environment, and c) difficult to build self-enforceable system that operates without financial support from a third party. In other words, we would like the equilibrium to achieve the following characteristics: i) supply chain transparency, ii) voluntary participation, iii) self-enforceability, and iv) sustainable quality supply. The following provide more details of these characteristics.

Supply Chain Transparency

Supply chain transparency requires that each member truthfully reports his/her private information. Thus, we would like the reporting strategies of members $i = s, r$ comprise a *Bayesian incentive compatible* equilibrium, (BIC), provided that truth telling is a mutual best response for all i. By the one stage deviation principle it is sufficient to show that provided the other member $-i$ is following a truth telling strategy, then the solution to the following dynamic program for member $i = s, r$,

$$\begin{aligned} W_s^t\left[\tilde{h}^{t-1}, \theta_s^t\right] = \quad & \max_{\tilde{\theta}_s^t} E_{\tilde{\theta}_r^t}\left\{\tau_s^t\left(\tilde{h}^{t-1}, \tilde{\theta}_s^t, \tilde{\theta}_r^t\right) - C\left(x^t\left(\tilde{h}^{t-1}, \tilde{\theta}_s^t, \tilde{\theta}_r^t\right), \theta_s^t\right)\right. \\ & \left. + \delta E W_s^{t+1}\left[\tilde{h}^t, \theta_s^{t+1} \mid \theta_s^t\right]\right\} \end{aligned}$$

$$\begin{aligned} W_r^t\left[\tilde{h}^{t-1}, \theta_r^t\right] = \quad & \max_{\tilde{\theta}_r^t} E_{\tilde{\theta}_r^t}\left\{\tau_r^t\left(\tilde{h}^{t-1}, \tilde{\theta}_r^t, \tilde{\theta}_s^t\right) + R\left(x^t\left(\tilde{h}^{t-1}, \tilde{\theta}_r^t, \tilde{\theta}_s^t\right), \theta_r^t\right)\right. \\ & \left. + \delta E W_r^{t+1}\left[\tilde{h}^t, \theta_r^{t+1} \mid \theta_r^t\right]\right\} \qquad \text{(BIC)} \end{aligned}$$

is $\tilde{\theta}_i^t = \theta_i^t$ for all i.[3]

[3] According to the single stage deviation principle, it is necessary and sufficient that member i does not wish to deviate once from truth telling, in order for truth telling to be a best reply to truthful

Supply Chain Optimal Quality

The supply chain archives sustainable quality supply when the optimal quality is selected in every period based on the realization of the supply chain environment θ^t. That is, given M, the equilibrium is *ex post efficient* (EE):

$$x^t\left(\theta^t\right) = x^{t*}\left(\theta^t\right) = x^*\left(\theta^t\right). \tag{EE}$$

Voluntary Participation

The supply agreement is voluntary to participate if each member decides to participate even if they are given the right to quit. That is, given M, the equilibrium is *interim individual rational* (IIR) implies: for all i and θ_i^t,

$$W_i^t\left[\tilde{h}^{t-1}, \theta_i^t\right] \geq O_i^t(\theta_i^t). \tag{IIR}$$

The inequality means that each $i's$ expected payoff $W_i^t[\cdot]$ exceeds its outside option payoff, $O_i^t(\theta_i^t)$ from quitting. This is a minimal requirement of any long term arrangement that is entered into voluntarily.

Self-Enforceability

The supply agreement is self-enforceable if it is financed entirely by the member without assistance of a third party, i.e., it does not require a budget breaker to subsidize or divert resources from the members. That is, the transfer payments in the equilibrium are *ex post budget balance* (BB): for all t and $\tilde{h}^t$

$$\tau_r^t\left(\tilde{h}^t\right) + \tau_s^t\left(\tilde{h}^t\right) = 0. \tag{BB}$$

This is a reasonable expectation of any private contract arrangement.

reporting by other agents. The single stage deviation principle applies in all settings, including our model, where the member's types follow a first order Markov process.

3.4 Sustainable Quality Supply Agreement (SQSA)

In this section, we show that we can indeed identify elements of M that will lead to an equilibrium with all the desired properties described in Subsection 3.3.4. We term the resulting mechanism *Sustainable Quality Supply Agreement* (SQSA). We further show that SQSA can be carried out through an all-pay auction, which offers great potential for implementations in practise.

3.4.1 Constructing the Mechanism

To induce supply chain transparency, i.e., to induce each member to truthfully report its current market condition, we restrict the contract form to an AVG-Arrow mechanism (see Lewis et al. (2011) for a more detailed discussion of this type of mechanism). This form endows each member as a residual claimant. That is, it allows each member to receive the expected supply chain profit which is dependent on the current market condition, upon paying a participation fee that is independent the current market condition. In particular, member i's expected profit in period t, $i = s, r$, given θ_i^t is of the form

$$W_i^t\left[\theta^{t-1}, \theta_i^t\right] = E_{\theta_{-i}^t} V^{t*}\left(\theta^t\right) - K_i\left(\theta^{t-1}\right). \tag{3.12}$$

where $K_i\left(\theta^{t-1}\right)$ is a constant depending only on the previous period conditions, θ^{t-1}.[4]

To ensure voluntary participation, i.e., (IIR), we need to have

$$\begin{aligned}
& W_i^t\left[\theta^{t-1}, \theta_i^t\right] \\
& = E_{\theta_{-i}^t} V^{t*}\left(\theta^t\right) - K_i\left(\theta^{t-1}\right) - O_i^t(\theta_i^t) \\
& \geq 0.
\end{aligned}$$

[4] Note the history of reported demands is $\tilde{h}^{t-1} = h^{t-1}$ on the equilibrium path since all members truthfully report their costs each period. h^{t-1} is completely summarized by θ^{t-1} since demand conditions are first order Markov. Consequently we can represent $i's$ period t expected profit as, $W_i^t\left[\tilde{h}^{t-1}, \theta_i^t\right] = W_i^t\left[\theta^{t-1}, \theta_i^t\right]$, $x^t(\tilde{h}^t)$ as $x^t(\tilde{\theta}^t)$ and $\rho^t(\tilde{h}^t)$ as $\rho^t(\tilde{\theta}^t)$.

Define

$$\hat{w}_i^t(\rho^t, \theta_i^t) = E_{\theta_{-i}^t} V^{t*}\left(\theta^t\right) - O_i^t(\theta_i^t).$$

Then $\hat{w}_i^t(\rho^t, \theta_i^t)$ is a measure of member i's willingness to pay to participate in the contract. Define

$$\theta_{iw}^t(\rho^t) = arg \min_{\theta_i^t} \left\{ \hat{w}_i^t(\rho^t, \theta_i^t) \right\}.$$

Then

$$w_i^t\left(\rho^t\right) = \hat{w}_i^t(\rho^t, \theta_{iw}^t(\rho^t))$$

is member i's willingness to pay under the "worst off" condition. In other words, under this worst off condition, member i has the least willingness to participate, or has the biggest incentive to quit. This may happen, for instance, when the market condition is unfavorable to $\mathcal{R}$, so $\mathcal{R}$ has incentive to terminate the long term commitment. Similarly, when the production condition turns out to be very bad, $\mathcal{S}$ may consider it is too difficult to provide specified quality so she may consider to quit. To ensure (IIR), we then have:

Lemma 13. *(Voluntary Participation) M is IIR provided $K_i\left(\theta^{t-1}\right) \leq w_i^t\left(\rho^t\right)$*

Note that $w_i^t\left(\rho^t\right)$ can also be interpreted as member i's switching cost (to its outside options) in period t. To sustain the long term commitment and to avoid the myopic behavior of some member may want to quit in the interim, it is intuitive that we should choose ρ^t to maximize the switch costs of the members (i.e. to maximize the willingness to participate). This ρ^t is then a device that the designers of the sustainability programs can employ to lock in strategic and high-potential suppliers. We can indeed show the following:

Lemma 14. *(Long-Term Commitment) The decision rights, ρ^{t*},that maximize members' total participation fee, $w_s^t\left(\rho^t\right) + w_r^t\left(\rho^t\right)$, minimize the members' total outside option value $\sum_i \rho_i^t \left[V_i^t\left(x_i^o, \theta_{iw}^t\left(\rho^t\right)\right) + V_{-i}^t\left(x_i^o, \theta_{-i,w}^t\left(\rho^t\right)\right)\right]$.*

Note that $w_i^t(\rho^t)$ depends on θ^{t-1}, therefore, the above constructed ρ^{t*} depends on θ^{t-1}. We next show ρ^{t*} also ensue that the agreement is ex ante self-enforceable, i.e., (BB).

Lemma 15. *(Ex ante Budget Balancing)*

$$B^t\left(\rho^{t*}\right) \equiv w_r^t\left(\rho^{t*}(\theta^{t-1})\right) + w_s^t\left(\rho^{t*}(\theta^{t-1})\right) - E\left[V^{t*}\left(\theta^t\right) \mid \theta^{t-1}\right] > 0 \tag{3.13}$$

Because of the above properties, we will use ρ^{t*} to construct the remaining part of the mechanism, noting that there are other decision right allocations that would ensure mechanisms under the same equilibrium as well.

Recall that we would like the supply-chain optimal quality to be achieved, so we need to restrict $x^t(\theta^t) = x^*(\theta^t)$. Then what is left is to see how to construct the transfer payment terms τ^t, based on the properties of ρ^{t*} and $x^*(\theta^t)$.

Take any period t. Given $\tilde{h}^t$, let $\tilde{p}_i\left(\tilde{h}^t\right)$ denote the net payment to i in period t, defined by

$$\tilde{p}_i^t\left(\tilde{h}^t\right) = \tilde{\tau}_i^t\left(\tilde{h}^t\right) - \tilde{w}_i^t\left(\rho^{t*}\left(\tilde{\theta}^{t-1}\right)\right) + \delta E_{\tilde{\theta}_i^t}\tilde{w}_i^{t+1}\left(\rho^{(t+1)*}\left(\tilde{\theta}^t\right)\right) \tag{3.14}$$

where

$$\tilde{\tau}_s^t\left(\tilde{h}^t\right) = \underbrace{E_{\tilde{\theta}_r^t} R\left(x^*\left(\tilde{\theta}^t\right), \tilde{\theta}_r^t\right)}_{\mathcal{S}\text{'s AGV transfer}}, \quad \tilde{\tau}_r^t\left(\tilde{h}^t\right) = -\underbrace{E_{\tilde{\theta}_s^t} C\left(x^*\left(\tilde{\theta}^t\right), \tilde{\theta}_s^t\right)}_{\mathcal{R}'s\text{ AGV transfer}}, \tag{3.15}$$

$$\tilde{w}_i^t\left(\rho^{t*}\left(\tilde{\theta}^{t-1}\right)\right) = \underbrace{\left(w_i^t\left(\rho^{t*}\left(\tilde{\theta}^{t-1}\right)\right) - \frac{B^t\left(\rho^{t*}\left(\tilde{\theta}^{t-1}\right)\right)}{2}\right)}_{\text{i's expected participation payment for } t} \tag{3.16}$$

$$\delta E_{\tilde{\theta}_{-i}^t}\tilde{w}_i^{t+1}\left(\rho^{(t+1)*}\left(\tilde{\theta}^t\right)\right) = \delta E_{\tilde{\theta}_{-i}^t}\underbrace{\left(w_i^{t+1}\left(\rho^{(t+1)*}\left(\tilde{\theta}^t\right)\right) - \frac{B^{t+1}\left(\rho^{(t+1)*}\left(\tilde{\theta}^t\right)\right)}{2}\right)}_{\text{i's expected participation payment for } t+1} \tag{3.17}$$

These transfer payments have the following implications:

a) *Sustainable Quality Supply* The first transfer, $\tilde{\tau}_i^t\left(\tilde{h}^t\right)$, is the expected profit created by the other member. By transferring $\tilde{\tau}_i^t\left(\tilde{h}^t\right)$ to i, he obtains the expected supply chain profit in period t. This transfer ensures that i has the incentive to select the supply chain optimal quality.

b)*Self-Enforceability* The second transfer, $\tilde{w}_i^t\left(\rho^{t*}\left(\tilde{\theta}^{t-1}\right)\right)$, is the participation fee that member i must pay to the system in order to balance the budget in period t. This transfer ensures self-enforceability.

c) *Supply Chain Transparency* The third transfer, $\delta E\tilde{w}_i^{t+1}\left(\rho^{(t+1)*}\left(\tilde{\theta}^t\right)\right)$, is a payment from the system that equals the expected participation fee that member i will be asked to pay in period $t+1$. This transfer ensures supply chain transparency, i.e. member i does not benefit from misreporting his current period costs, in order to reduce his future participation fee.

Note that $E\left[\tilde{p}_s^t\left(\tilde{h}^t\right)+\tilde{p}_r^t\left(\tilde{h}^t\right)\right]=0$, implying that the *expected* transfer payments are balanced. The following adjusted net transfers for member i ensures ex post budget balancing:

$$\tau_i^{t*}\left(\tilde{h}^t\right)=\tilde{p}_i^t\left(\tilde{h}^t\right)-\tilde{p}_{-i}^t\left(\tilde{h}^t\right)+E_\theta\tilde{p}_{-i}^t\left(\tilde{h}^t\right). \tag{3.18}$$

Because $\tilde{w}_i^t\left(\rho^{t*}\left(\tilde{\theta}^{t-1}\right)\right)$ can be calculated through

$$\begin{aligned}
&\tilde{w}_i^t\left(\rho^{t*}\left(\tilde{\theta}^{t-1}\right)\right)\\
=\;&\frac{1}{2}\left(\min_{\theta_i^t}\{E_{\theta_{-i}^t}V^{t*}(\theta^t)-O_i^t(\theta_i^t)\}-\min_{\theta_{-i}^t}\{E_{\theta_i^t}V^{t*}(\theta^t)-O_{-i}^t(\theta_{-i}^t)\}+E[V^{t*}(\theta^t)]\right),
\end{aligned}$$

where all the terms on the right-hand side can be calculated directly from definition.

To summarize, we have

Theorem 16. *The dynamic mechanism SQSA as specified in* $M\left\langle x^{*}\left(\tilde{\theta}^{t}\right), \tau^{t*}\left(\tilde{h}^{t}\right), \rho^{(t+1)*}\left(\tilde{\theta}^{t}\right)\right\rangle_{t=1}^{T}$ *leads to an equilibrium that achieves the supply-chain optimal quality level* x^{*}*, ensures supply chain transparency, voluntary participation and self-enforceability.*

Theorem 16 shows that sustainable quality supply can be achieved under two sided asymmetric information and information updating. The payment τ_i^{t*} satisfies:

$$\tau_i^{t*} = \underbrace{\tilde{\tau}_i^t - \tilde{\tau}_{-i}^t}_{\text{i's first transfer}} + \underbrace{\tilde{w}_i^t - \tilde{w}_{-i}^t + E_\theta \tilde{p}_{-i}^t\left(\tilde{h}^t\right)}_{\text{i's second transfer}} + \underbrace{\delta E\tilde{w}_i^{t+1} - \delta E\tilde{w}_{-i}^{t+1}}_{\text{i's third transfer}}. \tag{3.19}$$

The second transfer is a constant that does not vary with the outcome of the environment. The first and third transfers depend on the realization of the market condition θ_i^t and are hence related to the agreed optimal quality. In particular, the first transfer reflects the value of the current period collaboration while the third transfer reflects the value of the future collaboration. This payment structure is consistent with two-part nonlinear tariffs commonly seen in practice. For instance, in the C.A.F.E. Practices, according to Duba et. al. (2007), the grower receives a bottom line price fixed at the beginning of every crop cycle, as well as a real-time payment based on the quality of the crop, such as a bonus payment per pound if the coffee is scored above 90. This real-time payment is a combination of the reward to the high quality in the current period and the signal for being a potential preferred grower in the future. The real-time payment ensures the fairness of the transaction while the bottom line price ensures the participation of the growers. This payment structure (bottom line plus bonus payment) can also be observed in the practices of Wal-Mart and Coca-Cola Co.

Theorem 16 implies that the sustainability programs with long-term commitment should dynamically allocate the decision rights over time instead of keeping it fixed. This can be implemented by modifying the buyout options both parties need to pay to quit the partnership. When the partnership dissolves, $\mathcal{S}$ and $\mathcal{R}$ can buyout their rights from the government to serve the agreement after the government pays them their expected profit under the current decision right allocation. These buyout options updates dynamically according to the cost and market condition. By having dynamic buyout options in the system, the mechanism can help implement programs involving long-term goals.

The following Corollary emphasis on the properties of the transfer payments that reflects the practice stated above.

Corollary 17. *The transfer payment $\tilde{\tau}_s^t(\tilde{h}^t)$ is decreasing in θ_s^t and $\tilde{\tau}_r^t(\tilde{h}^t)$ is increasing in θ_r^t. Both $\tau_r^{t*}(\tilde{h}^t)$ is decreasing in $x^{t*}(\theta^t)$ and $\tau_s^{t*}(\tilde{h}^t)$ is increasing in the optimal quality $x^{t*}(\theta^t)$.*

Next we develop an algorithm to calculate the contract terms recursively from the last period.

Initial Settings

$$V^{T+1}(\theta^{T+1}) = 0, \quad \tilde{w}_s^{T+1}(\theta_s^{T+1}) = 0, \quad \tilde{w}_r^{T+1}(\theta_r^{T+1}) = 0$$

Period t

Step 1: Calculate $x^{t*}(\theta^t) = \arg\max_x\{R(x, \theta_r^t) - C(x, \theta_s^t)\}$, $E_{\theta_i^t}[V^{t*}(\theta^t)]$, $E[V^{t*}(\theta^t)]$ and $O_i^t(\theta_i^t)$.

Step 2: Calculate $w_i^t(\rho^t) = \min_{\theta_i^t}\{E_{\theta_{-i}^t}[V^{t*}(\theta^t) - \rho_r^t O_r^t(\theta_r^t) - \rho_s^t O_s^t(\theta_s^t)]\}$ and

$\theta_{iw}^t(\rho^t) = \arg\min_{\theta_i^t}\{E_{\theta_{-i}^t}[V^{t*}(\theta^t) - \rho_r^t O_r^t(\theta_r^t) - \rho_s^t O_s^t(\theta_s^t)]\}$ as a function of ρ^t.

Step 3: Calculate $\rho^{t*} = \arg\max_{\rho_s^t + \rho_r^t = 1}\{w_s^t(\rho^t) + w_r^t(\rho^t)\}$

Step 4: Calculate $B^t = w_s^t(\rho^t) + w_r^t(\rho^t) - E[V^{t*}(\theta^t)]$.

Step 5: Calculate $\tilde{w}_i^t(\rho^{t*}) = w_i^t(\rho^{t*}) + B^t/2$, $\tilde{\tau}_r^t = -E_{\theta_s^t} C(x^{t*}(\theta^t), \theta_s^t)$ and $\tilde{\tau}_s^t = E_{\theta_r^t} R(x^{t*}(\theta^t), \theta_r^t)$.

Step 6: Calculate $\tau_i^t = \tilde{\tau}_i^t - \tilde{w}_i^t + E[\tilde{w}_i^{t+1}] - (\tilde{\tau}_{-i}^t - \tilde{w}_{-i}^t + \delta E[\tilde{w}_{-i}^{t+1}]) + E_{\theta^t}(\tilde{\tau}_{-i}^t - \tilde{w}_{-i}^t + \delta E[\tilde{w}_{-i}^{t+1}])$

Step 7: Calculate $W_s^t = \tau_s^t - C(x^{t*}(\theta^t), \theta_s^t) + \delta E[W_s^{t+1}]$ and $W_r^t = \tau_r^t + R(x^{t*}(\theta^t), \theta_r^t) + \delta E[W_r^{t+1}]$.

Step 8: Calculate $E_{\theta^{t-1}}[\tilde{w}_i^t]$.

Step 9: If $t > 1$, move to period $t-1$, else end.

3.4.2 Initial Decision Rights

Before the supply agreement is formed, the initial decision right may be exogenously determined by the size of the members, market power and so on. The following proposition characterizes the property of the domain of the initial decision rights that can induce SQSA.

Proposition 18. *There exists $0 \leq \underline{\rho}_r^1 < \bar{\rho}_r^1 \leq 1$, such that when $\rho_r^1 \in [\underline{\rho}_r^1, \bar{\rho}_r^1]$ mechanism M exists.*

When $\rho_r^1 < \underline{\rho}_r^1$ or $\rho_r^1 > \bar{\rho}_r^1$, M is no longer feasible and the only way to achieve sustainable quality supply is by centralizing the supply chain, i.e. one member sells his ownership to the other member.

Proposition 18 illustrates the applicability of the mechanism when ρ_r^1 is exogenously given. The feasible domain for the decision rights is an interval. The results predict that the Equity Agreement is implementable only if the power of $\mathcal{S}$ and $\mathcal{R}$ is balanced. In practice, a retailer with a large market share, such as Starbucks, has big power and is willing to sustain the high quality supply, and therefore strategically selects suppliers who have potential to produce high quality coffee beans. The suppliers who have the low production cost or potential for lower production cost for high quality coffee beans possess greater power over those who do not, and hence are more likely to become partners of Starbucks. Similar difference in supply chain ownership structure due to different supply chain power allocation are also observed in Wal-Mart's supply chain. Wal-Mart's Direct Farm Program practise is implemented differently between Europe and Asia. The European farmers have greater power than the Asia farmers. Consequently, Wal-Mart collaborate with European farmers based on supply contracts without taking control over the local land and production process; while in Asia, Wal-Mart purchase farms, design production process and hire farmers for organic produce supply.

3.4.3 Implementing SQSA as a Sequential Auction

Even though we find a mechanism that ensures sustainable quality supply, it is difficult to implement such mechanism due to two reasons: first, the private information can be complicated and hard to measure. Second, it is unrealistic to have a third party financially involving in implementing transfer payments. In practise, instead of revealing market condition and production cost rate, $\mathcal{S}$ and $\mathcal{R}$ announce and negotiate on price. Then based on the agreed price, $\mathcal{S}$ and $\mathcal{R}$ trade directly with each other. In this section, we show that the Agreement is equivalent to a sequential auction and hence can be implemented by conventional transactions.

Consider the auction where in each period member $i = s, r$ submits a sealed bid based on its privately known cost or market condition, $b_i^t(\theta_i^t)$. We conjecture that there is a bidding equilibrium to this game. At the beginning of period t, before θ_i^t is observed, each member pays an entry fee,

$$\mathcal{C}_i^t(\rho^{t*}) = \tilde{w}_i^t(r^{t*}),$$

which is member i's participation fee in period t. After observing θ_i^t, member i decides to quit the agreement and receive $O_i^t(\theta_i^t)$ or stay in the partnership and submits a bid that is strictly increasing in θ_i^t. In equilibrium, all members stay in the partnership. Given this the bid functions can be inverted to determine the party's type. The resource allocation that is assigned for period t is therefore determined by $x^*\left(\theta_i^t\left(b_i^t\right)\right)$ where $\theta_i^t\left(b_i^t\right)$ is the inverse of the bid function, b_i^t. Member i receives a total payment,

$$m_i^t\left(b_i^t, b_{-i}^t\right) = b_i^t - b_{-i}^t + E_{\theta^t} b_{-i}^t \tag{3.20}$$

equal to the weighted sum of bids minus $i's$ share of switching costs. This is an all-pay sequential auction where all bidders receive some payment in each round.

Proposition 19. *A sequential all-pay auction with payments,*

$$m_i^t\left(b_i^t, b_{-i}^t\right) = b_i^t - b_{-i}^t + E_{\theta^t} b_{-i}^t + \delta E \tilde{w}_i^{t+1}\left(\rho^{(t+1)*}\left(\theta^t\right)\right)$$

implements the efficient supply chain agreement. In equilibrium each member bids

$$b_r^t\left(\theta_r^t\right) = E_{\theta_s^t}\left(-C\left(\theta_s^t, x^*\left(\theta^t\right)\right)\right) \tag{3.21}$$

$$b_s^t\left(\theta_s^t\right) = E_{\theta_r^t}\left(R\left(\theta_r^t, x^*\left(\theta^t\right)\right)\right) \tag{3.22}$$

The derivation of the sequential auction allows us to clarify how the membership functions to efficiently allocate profit. In equilibrium members signal their relative efficiency by the amount they bid. The signal may be used (a) to allocate decision rights and participation fee for the following period and (b) to select quality to maximize supply chain profit. Members may bid strategically to signal their participation fee in the future. To prevent this type of signaling each member's expected profit from continuing in the supply agreement does not depend on that member's current period bid.[5] On the other hand, members do bid to signal their value on quality. The higher value the member has (large θ_r or small θ_s) will result in higher quality.

[5] The expected profit of each member after the period t auction is $EV^{(t+1)*}\left(\theta^{t+1}\right)$ which does not depend on the period t bid(s) of any of the member(s).

In practise, the bidding scheme can be applied to reduce the complexity of the information system. Instead of reporting every cost measures, $\mathcal{R}$ can report a single price (the bid) to indicate his cost condition. The payments then can be calculated through a pre-made chart which corresponds the bids to the payments. This reduces the information redundancy and makes the construction of the information system to become possible.

3.5 Supplier Development: Optimal Ex-Ante Investment

The agreement we constructed ensures ex post optimal quality supply. However, is it possible to have ex ante investment and ex post exchange both be optimal? This section explores this possibility.

In practise, Starbucks invests not only to develop its end customer market but also to help the growers develop their farming technology and improve their living conditions. However, the investments are relationship specific and cannot be verified. The question is, with all the hidden information, is it possible to have $\mathcal{S}$ and $\mathcal{R}$ invest optimally under SQSA? This section explores this possibility.

We extend our model to allow investments priori to the construction of the supply agreement and focus on the possibility to achieve system optimality. To explore this possibility, we modify our model by assuming that before the supply chain is formed, $\mathcal{S}$ invests $I_s \geq 0$ in supply capacity and $\mathcal{R}$ invests $I_r \geq 0$ in initial market development. For example, in the coffee supply chain, I_s can be the grower's investments for improving land condition, using organic fertilizers and hire qualified workers. I_r can be Starbuck's investments on advertising for premium quality coffee beans. The members' actual investments, $I \equiv (I_r, I_s)$, affect each member's initial state, θ_i^1, which is drown independently from distributions $F_i(\theta_i^1|I)$ in period 1, with future period condition drawn from distributions $F_i(\theta_i^t|\ \theta_i^{t-1})$, as before. Notice investment externalities may arise since the distribution of first period state, θ_i^1, depends on the entire investment profile I, not just on separate member's investment I_i. However, we retain the private values setting, since θs are stochastically independent, given I.

Before period 1, members privately select investment anticipating the supply agreement will be governed by the efficient mechanism, $M\langle x^{t*},\ \tau^{t*}, \rho^{(t+1)*}|\ I^*\rangle_{t=1}^T$, which presumes members invest efficiently such that $I_i = I_i^*$. The efficient investment, $I^* \equiv (I_r^*, I_s^*)$, assumed to be unique with $I_i^* \in [0, \infty)$, is determined by the condition,

$$I_i^* = \arg\max_{I_i}[EV^{1*}(\theta^1|I_i, I_{-i}^*) - I_i]$$

Whether the members select the efficient investment depends on the outcome of the following *investment game* that is induced by the mechanism, $M\langle x^{t*},\ \tau^{t*}, \rho^{(t+1)*}|\ I^*\rangle_{t=1}^T$. Member i selects I_i to maximize his/her expected continuation profit, $W_i(I_i, I_{-i})$, which is

$$W_i(I_i, I_{-i}) = \max\{EV^{1*}(\theta^1|I_i, I_{-i}) - I_i\} - \frac{\sum_j \tilde{w}_j(\rho^{1*}(I^*))}{2},$$

where the first term is member i's expected profit and the second term is his share of the switching costs, calculated on the assumption the members invest efficiently.

Proposition 20. *I^* is the unique Nash equilibrium to the ex ante investment game where each member invests to maximize its expected continuation profit.*

Proposition 20 shows that the supply chain's optimal investments are achieved independent of whether one member's investment is complements or substitutes of the other member. This is different from the conclusion by Iyer et. al. (2005) in their setting. This is because our mechanism design endows each partner to be a residual claimant of the entire supply chain profit. Since each partner is the beneficiary of all the cost saving he creates, he is incented to act efficiently even in extreme cases where his investment only reduces the costs of other member. This is indeed interesting.

Remark 21. ***Extension: Periodic Investment*** *Consider $\mathcal{S}$ and $\mathcal{R}$ are allowed to make investments $I^t = (I^t_s,\ I^t_r)$ at the beginning of every period. I^t affects the distribution of θ^t along with θ^{t-1}. Specifically, θ^t_i is drown from distribution $F^t_i(\theta^t_i|\theta^{t-1}_i,\ I^t)$. When $I^t = (0,0)$, $F^t_i(\theta^t_i|\theta^{t-1}_i,\ I^t)$ degenerates to $F^t_i(\theta^t_i|\theta^{t-1}_i)$. Here we assume the investments only directly affect the current period market condition, however, they also indirectly affects the future market conditions through the underlying connections of the market conditions. Similar to the initial investments, $I^t_i,\ i = s, r$ are relationship specific and cannot be verified. At the beginning of period t, the efficient mechanism continuing from period t is $M^t\langle x^{t_0*}, \tau^{t_0*}, I^{t_0*}, \rho^{(t_0+1)*}|\ I^{t*}\rangle^T_{t_0=t}$, where $I^{t*} = (I^{t*}_s, I^{t*}_r)$ is the optimal investment that maximizes the expected profit in period t. We assume I^{t*} is unique with $I^{t*}_i \in [0,\infty)$, is determined by the condition,*

$$I^{t*}_i = \arg\max_{I^t_i}[EV^{t*}(\theta^t|I^t_i, I^{t*}_{-i}, \theta^{t-1}) - I^t_i] = 0.$$

Each member anticipating to follow M^t after they invest. The mechanism M^t induces an investment game. Member i selects investment I^t_i to maximize his expected profit, $W^t_i(I^t_i, I^t_{-i})$, in period t.

$$W^t_i(I^t_i, I^t_{-i}) = \max\{EV^{t*}(\theta^t|I^t_i, I^t_{-i}) - I^t_i, \theta^{t-1}\} - \frac{\sum_j \tilde{w}^t_j(\rho^{t*}(I^{t*}))}{2},$$

Parallel to Proposition 20, the optimal investments I^{t*} is the unique Nash equilibrium to the period t ex ante investment game.

The following example shows the properties of the investments for supplier development under long term commitment.

Example 22. *Consider a well developed retailer $\mathcal{R}$, i.e. Starbucks who has a fixed market condition $\theta^t_r = 1$ for all $t = 1, ..., T$, decides to invest in its under developed supplier $\mathcal{S}$, i.e. the grower. The grower's production cost is uncertain due* *to uncertainty in technology and innovation. In each period $\mathcal{R}$ can invest a positive amount $I^t \in [0,1]$ to reduce the grower's cost, the reduction is linear satisfying $\theta_s^t = \nu_s^t \theta_s^{t-1} - I^t + \epsilon$. According to Proposition 20, $\mathcal{R}$ selects $I^t = I^{t*}$ which maximizes the expected supply chain profit.*

The following proposition shows the investment trend of $\mathcal{R}$ under mechanism M^t.

Proposition 23. *Assume $V^{T+1}(\theta^{T+1}) = 0$.*

1) $I^{t*}(\theta_s^{t-1})$ is achieved at the boundary, i.e. either invest the full amount $I^{t*} = 1$ or not invest $I^{t*} = 0$.

2) $I^{t*}(\theta_s^{t-1})$ is decreasing in θ_s^t. In each period there exists a θ_{s0}^t such that $I^{t*}(\theta_s^{t-1}) = 1$ if $\theta_s^t < \theta_{s0}^t$ and $I^{t*}(\theta_s^{t-1}) = 0$ if $\theta_s^t \geq \theta_{s0}^t$.

3) $I^{t*}(\theta_s^{t-1})$ is decreasing in t, and therefore θ_{s0}^t is increasing in t.

Proposition 23 shows that the investment is increasing as the number of period increases. This implies that the longer the commitment is, the more $\mathcal{R}$ will invest in $\mathcal{S}$. This matches the observation under most supplier development contexts. However, since $I^{t*}(\theta_s^{t-1})$ is decreasing in θ_s^t, when the production cost is above a certain threshold θ_{s0}^t, $\mathcal{R}$ will not invest in $\mathcal{S}$. As the number of periods increases, θ_{s0}^t increases implying that $\mathcal{R}$ would invest on a larger possible set of suppliers.

3.6 Illustration and Connection to the C.A.F.E. Practices

In this section, we illustrate our dynamic mechanism through a simplified version of Starbucks's supply chain. The examples will show how the supply-chain optimal quality level and transfer payments, as well as the allocation of decision rights are dynamically changing according to the changing environment. We also emphasize the role of the decision rights in the long-term commitment to develop sustainable quality supply, and discuss its implications to the implementation the C.A.F.E. Practices.

Suppose Starbucks' supply chain consists only one grower, $\mathcal{S}$, and one retailer $\mathcal{R}$

– Starbucks. The two parties make a long term commitment to develop and supply premium quality coffee beans to the market place over a period of T years. In each period $t = 1, 2, , , T$, the grower can provide coffee beans of quality $x^t \in [0, 1]$ at a cost $C\left(x, \theta_s^t\right) = \theta_s^t c\left(x\right)$, where $\theta_s^t \in [0, 1]$ is a parameter reflecting current growing conditions and the effort cost $c(\cdot)$ satisfies the conditions specified in Section 3.1.

The assumed cost conditions ensure that it is always efficient for the grower to provide beans of intermediate quality provided $\theta_s^t > 0$. With regards to demand, Starbucks generates revenues from the sale of its coffee, $R\left(x^t, \theta_r^t\right) = x^t\theta_r^t$, where $\theta_r^t \in [0, 1]$ represents the demand conditions.

3.6.1 Shared Decision Rights

Suppose the coffee supply chain is operating in a stationary environment, so that the demand conditions $\{\theta_r^t\}$ and the supply conditions $\{\theta_s^t\}$ are i.i.d. sequences. This is appropriate when the sustainable quality concept is well known to the consumers and the technology of growing premium coffee beans has been efficient and stabilized. In this case, according to the previous analysis, SQSA exists. It can be shown:

Proposition 24. *If the distributions of θ_r^t and θ_s^t are identical. The optimal decision rights are given by:*

$$\rho_r^{t*}(\theta_{rw}^t;\ \theta^{t-1}) = \frac{1-(\delta\lambda)^{T-t}}{1-\delta\lambda} E[x^*(\theta_{rw}^t, \theta_s^t)]$$

and they satisfy these properties:

a) Shared decision rights: $\rho_r^{t*} \in (0, 1)$ for all t.

b) Equal supply chain outside options at the worst off condition: $V_s^t(1, \theta_w^t) + V_r^t(1, \theta_w^t) = V_s^t(0, \theta_w^t) + V_r^t(0, \theta_w^t)$.

c) ρ_r^{t*} is increasing in θ_{rw}^t.

d) The decision-rights allocations are sequentially independent.

Proposition 24 indicates that under stationary environment with identically distributed supply and demand conditions, when the partnership is formed under SQSA, the quality decision rights are always shared in each period the decision rights allocation to a member increases as the member contributes more to the supply chain profit under the worst off condition, i.e. the retailer with a larger θ^t_{rw} or the supplier with a smaller θ^t_{sw}. However, the member who has a larger decision right in this period may not be rewarded with a larger decision right in the future. The purpose of independently distributing the decision rights over time is to minimize the supply chain's aggregate outside option to prevent the members from abandoning the partnership.

The condition that θ^t_r and θ^t_s are identically distributed implies that $\mathcal{R}$ and $\mathcal{S}$ are almost identical in terms of size and power. In the real Starbucks' supply chain, however, typically, $\mathcal{R}$ has dominant power over $\mathcal{S}$.

3.6.2 Extreme Decision Rights

Let us now consider the situation of non i.i.d. market conditions. Suppose the expected demand condition for premium coffee beans demonstrates a jump in an interim period t_0 and stays at that level in the remaining periods. We can interpret the time before and at t_0 as the market development stage, and the time afterward the developed stage. Assume the expected market condition follows constant step functions as listed below:

$$E[\theta^t_r] = \begin{cases} \mu_r & t \leq t_0 \\ \mu_r + \varepsilon & t > t_0 \end{cases} \qquad E[\theta^t_s] = \begin{cases} \mu_s & t \leq t_0 \\ \mu_s - \varepsilon & t > t_0 \end{cases}$$

Note that from Lemma 14, the difference of the outside options is the key to indicate the value of the decision rights. We analyze the difference below to see when the decision rights take extreme values.

Define $N_l(t)$ be the number of years that the system is in the developing stage from period t on. Then $N_l(t) = [t_0 - t]^+$. Define $N_h(t)$ be the number of years that the system is in the developing stage from period t on, then $N_h(t) = [T - t_0 - [t_0 - t]^-]^+$. By definition, $N_l(t) + N_h(t) = [T - t]^+$.

In year t, given the supplier's cost rate and the retailer's market condition, their expected outside options are

$$\begin{aligned} V_r^t(1, \theta_r^t) &= \theta_r^t + (N_l(t) + N_h(t))\mu_r + N_h(t)\varepsilon \\ V_s^t(1, \theta_s^t) &= -\theta_s^t - (N_l(t) + N_h(t))\mu_s + N_h(t)\varepsilon \\ V_r^t(0, \theta_r^t) &= 0 \\ V_s^t(0, \theta_s^t) &= 0 \end{aligned}$$

and the supply chain outside profits are:

$$\begin{aligned} V^t(1, \theta^t) \equiv V_r^t(1, \theta_r^t) + V_s^t(1, \theta_s^t) &= \theta_r^t - \theta_s^t + (N_l(t) + N_h(t))(\mu_r - \mu_s) + 2N_h(t)\varepsilon \\ V^t(0, \theta^t) \equiv V_r^t(0, \theta_r^t) + V_s^t(0, \theta_s^t) &= 0, \end{aligned}$$

Then the normalized difference is

$$\frac{V^t(1, \theta^t) - V^t(0, \theta^t)}{\mu_r - \mu_s} = \frac{\theta_r^t - \theta_s^t}{\mu_r - \mu_s} + \Delta_{r-s}^t,$$

where

$$\Delta_{r-s}^t = (N_l(t) + N_h(t)) + 2N_h(t)\frac{\varepsilon}{(\mu_r - \mu_s)}$$

is the normalized difference of the number of years in the developing and developed stages.

Note that when $\varepsilon = \mu_s - \mu_r$, Δ_{r-s}^t degenerates to $N_l(t) - N_h(t)$. In this case, Δ_{r-s}^t is equivalent to the difference of the number of years in the developing stage to the developed stage. In the developing years, the cost of $\mathcal{S}$ is high if $\mathcal{R}$ selects the quality, while $\mathcal{R}$'s revenue is constant zero independent of the quality. Therefore, $\mathcal{S}$ has a higher incentive to control quality. Alternatively, in the developed stage, the revenue for $\mathcal{R}$ is low if $\mathcal{S}$ selects the quality, and hence $\mathcal{R}$ has a higher incentive to control quality. Therefore, $N_l(t) - N_h(t)$ characterizes the relative incentive to control quality between $\mathcal{S}$ and $\mathcal{R}$. When Δ is large, $\mathcal{S}$ has more incentive and when Δ is small, $\mathcal{R}$ has more incentive.

Example 25. *Suppose Starbucks and the grower agree to jointly develop C.A.F.E. practices. The parameters are taken according to the table below. Assume θ_r^t and θ_s^t are distributed according to a continuous, strictly increasing c.d.f. F_r^t and F_s^t respectively and are i.i.d. in the first 7 years and i.i.d. in the last 3 years. For this example* [6]

$$\rho_r^{t*} \left\{ \begin{array}{ll} = 1 & if\ \Delta_{r-s}^t > 1.25 \\ = \tilde{\rho}^t \in [0,1] & if\ \Delta_{r-s}^t \in (-1.25, 1.25) \\ = 0 & if\ \Delta_{r-s}^t < -1.25 \end{array} \right\} ; \rho_s^{t*} = 1 - \rho_r^{t*}$$

The normalized difference of the number of years in the developing and developed stages and the corresponding optimal quality rights for the retailer in periods $t = 1, ..., 10$ are reported in the Table below.

Table 3.1: $T = 10$, $t_0 = 7$, $u_r = 0$, $\mu_s = 0.8$, $\varepsilon = 0.8$

Year	1	2	3	4	5	6	7	8	9	10
Δ_{r-s}^t	3	2	1	0	-1	-2	-3	-2	-1	0
ρ_r^{t*}	1	1	$\tilde{\rho}$	$\tilde{\rho}$	$\tilde{\rho}$	0	0	0	$\tilde{\rho}$	$\tilde{\rho}$
ρ_s^{t*}	0	0	$1-\tilde{\rho}$	$1-\tilde{\rho}$	$1-\tilde{\rho}$	1	1	1	$1-\tilde{\rho}$	$1-\tilde{\rho}$

According to Table 1, as in years 1 and 2, when the supplier has an advantage exceeding 1.25, if he decides to quit, he is awarded no decision rights. The rationale

[6] For the case where $\Delta_{r-s}^t \in (-1.25, 1.25)$ the exact calculation of $\tilde{\rho}$ requires that we specify specific distributions for F_r^t and F_s^t, and it depends on t. Without this specification we only know that $\tilde{\rho} \in [0,1]$.

for this allocation of decision rights is to prevent the supplier who has a greater incentive than the retailer to decide the supply quality from dissolving the partnership. However, as the supplier's cost falls in the following year 2, the supplier begins accruing larger decision rights that eventually become absolute once the retailer's revenue gets sufficiently large as it does in year 6. Again the rationale for this allocation of decision rights is to distribute the rights to the supplier who now has the smaller incentive to decide the quality. The variation in decision right allocations across different stages of the project is therefore an attempt to prevent one member from dissolving the partnership to pursue his outside option. [7]

In the following, we illustrate that the allocation of B^t is flexible depending on the bargaining power of each party.

Example 26. *Continuing Example 1, suppose the market condition* θ_r *follows distribution* $Pr(\theta_r = 0) = 1$ *from year 1 to year 7 and* $Pr(\theta_r = 0) = 0.2$ *and* $Pr(\theta_r = 1) = 0.8$ *from year 8 to year 10. The grower's cost rate* θ_s *follows distribution* $Pr(\theta_s = 0) = 0.2$ *and* $Pr(\theta_s = 1) = 0.8$ *from year 1 to year 7 and* $Pr(\theta_s = 0) = 1$ *from year 8 to year 10. Then, the decision rights as well as the other contract terms can be specified in the table below:*

In Table 2, the decision rights are determined given the distributions of the market conditions. However, it is possible to have multiple decision right allocations that maximizes the aggregated participation fee. For example, in year 4, the optimal allocation can be any division between $\mathcal{R}$ *and* $\mathcal{S}$*. This is because* $\mathcal{R}$ *has as much incentive as* $\mathcal{S}$ *to decide quality and the allocation of decision rights does not effect the supply chain's total outside option value. However, this does not imply that the members are indifferent with the allocation. Indeed, the decision rights do affect the*

[7] Our findings are consistent with the legal analysis of Gilson et al (09) who similarly argue that the ongoing adjustment in control and property rights in development contracts that they examine is explained as a response of collaboration parties to continually changing conditions that require adjustment to maintain a stable partnership.

Table 3.2: $\theta_r^t = 0$ for $t \leq 7$ and $Pr(\theta_r^t = 0) = 0.2$ and $Pr(\theta_r^t = 1) = 0.8$ for $t > 7$.

$Pr(\theta_s^t = 0) = 0.2$ and $Pr(\theta_s^t = 1) = 0.8$ for $t \leq 7$ and $\theta_s^t = 0$ for $t > 7$.

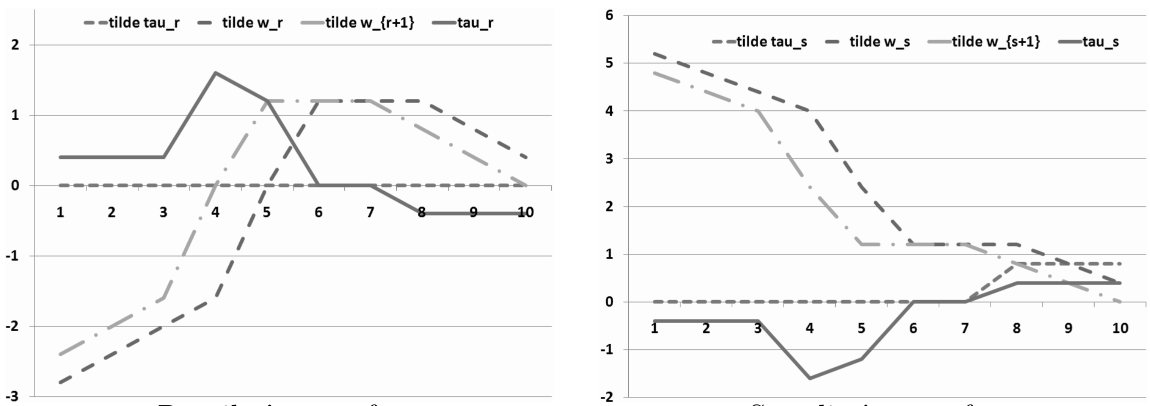
Year	1	2	3	4	5	6	7	8	9	10
x^{t*}	0	0	0	0	0	0	0	1	1	1
ρ_r^{t+1*}	1	1	1	any	0	0	0	0	0	any
ρ_s^{t+1*}	0	0	0	any	1	1	1	1	1	any
τ_r^{t*}	0.4	0.4	$2.8 - 2.4\rho_r^4$	$2.4\rho_r^4$	0	0	-0.4	-0.4	-0.4	0
τ_s^{t*}	-0.4	-0.4	$-2.8 + 2.4\rho_r^4$	$-2.4\rho_r^4$	0	0	0.4	0.4	-0.4	0
W_r^t	4.8	4.4	4	$1.2 + 2.4\rho_r^4$	1.2	1.2	1.2	1.6	1.2	0.8
W_s^t	-2.4	-2	-1.6	$1.2 - 2.4\rho_r^4$	1.2	1.2	1.2	0.8	0.4	0
$B^t(\rho^{*t})$	5.6	4.8	4	3.2	2.4	2.4	2.4	2.4	1.6	0.8
b_r^t	0	0	0	0	0	0	0	0	0	0
b_s^t	0	0	0	0	0	0	0	0.8	0.8	0.8

expected profit of each member and hence the allocation of the decision rights need to be determined through negotiation.

Member i's bid is the expected surplus of the other party given member i's private information. The bid signals member i's private information to the other party. For example, on the one hand, from year 1 to year 7, $\mathcal{S}$ bids zero because the expected surplus of $\mathcal{R}$ is zero. From year 8 to year 10, since $\mathcal{S}$'s production cost lowered to zero, therefore, $x^* = 1$ and the expected surplus of $\mathcal{R}$ increased to 0.8. Consequently, $\mathcal{S}$ announces a larger bid of 0.8 signaling his production cost has reduced. On the other hand, $\mathcal{R}$'s bids stay constant at zero because the expected surplus of $\mathcal{S}$, i.e. the production cost, stays at constant zero through the entire contract.

Assume that B^t is distributed evenly between $\mathcal{R}$ and $\mathcal{S}$, implying an equal bargaining power of the partners. The following graphs shows τ^t and its first, second and third transfers.

However, the bargaining power may not be equally distributed due to various reasons, such as different outside options. When the grower has outside option o, he will obtain bargaining power that allows him to receive at least his outside option. One common assumption is that $o = 0$. In particular, if $\rho_t^4 = 0$, from year 1 to year



Retailer's transfer Supplier's transfer

FIGURE 3.1: Retailer's and Supplier's transfer payments

3, B^t needs to be redistributed so that the grower is willing to participate. Assume B^t is allocated so that the grower's expected profit is equal to his outside profit, the table below lists the corresponding contract terms from year 1 to year 4 after the adjustment:

Year	1	2	3
ρ_r^{t*}	1	1	1
ρ_s^{t*}	0	0	0
τ_r^{t*}	0	0	1.2+$[\min\{-o, 3.2\}]^+$
τ_s^{t*}	0	0	-1.2-$[\min\{-o, 3.2\}]^+$
W_r^t	$2.4 + [\min\{-o, 3.2\}]^+$	$2.4 + [\min\{-o, 3.2\}]^+$	$2.4 + [\min\{-o, 3.2\}]^+$
W_s^t	$-[\min\{-o, 3.2\}]^+$	$-[\min\{-o, 3.2\}]^+$	$-[\min\{-o, 3.2\}]^+$
$B^t(\rho^{*t})$	4.8	4	3.2

There are two factors that affect the profit allocation: the grower's outside profit o and the negotiable profit B^t. o determines the magnitude of the profit the grower can receive while B^t puts an upper bound on the grower's profit. In particular, the upper bound binds in year 3 at $B^3 = 3.2$. If $o \leq -3.2$, the outside profit for the grower is low. From the collaboration with R, the grower receives a profit that is greater than his outside profit by threatening to dissolve the partnership. When $-3.2 < o \leq 0$, the grower is indifferent in joining the partnership and receive his outside profit. when $o > 0$, the grower will choose not to join the partnership.

3.7 Concluding Remarks

We have developed a two-party supply chain model and designed a dynamic mechanism to facilitate the long-term commitment of supply-chain partners to collaborate to achieve sustainable quality supply. The mechanism specifies transfer payments and the allocation of decision rights dynamically according to the current supply and demand conditions. This mechanism allows voluntary participation and leads to supply-chain optimal quality. It is also self-enforceable. In addition, it induces optimal ex ante investment which leads to efficient supplier development. There are two key elements that enable such strong properties. The first is to to make each partner the residual claimant of the supply chain surplus. The second is the dynamic allocation of the decision rights, which in effect minimizes the supply chain's outside options so to facilitate the long-term commitment.

On the one hand, our mechanism agrees with many elements of the C.A.F.E. Practices. For example, the pattern of the dynamic changing transfer payments matches that of the dynamically changing price for C.A.F.E. certified coffee beans between each crop cycle. The higher the quality selected by the mechanism, the more the growers are rewarded, which in practice, indicating a fair trade environment.

Our mechanism also agrees with the supplier development practice before C.A.F.E. is implemented. We model supplier development as early investments. The investments are made optimally even when they are relationship specific and allow externalities. This result coincides with the supplier development project Starbucks is implementing to help the growers, both financially and technologically, to build a better living and working environment.

On the other hand, our study also indicates some potential risks in the implementation of the C.A.F.E. Practices: In this program, the growers have the full possession of decision rights on the supply quality. Without a mechanism that dynamically allocates the decision rights may lead to inappropriate switching costs which may cause the termination of the partnership in the interim. Our analysis can provide a tool to guide companies, such as Starbucks, to foresee reasonable allocation of decision rights along the development course of a sustainability initiative like the C.A.F.E. Practices.

Our effort in this paper is only a first, though important, step towards a bigger goal of developing tools and theory to guide long-term partnerships to achieve supply chain sustainabilty. Several extensions are worth pursue, such as more complex supply network structures involving multiple and competing suppliers or retailers.

4

Dealing with Local Autonomy in Global Production Networks

4.1 Introduction

The case

A Fortune 500 US company with its headquarters located in the center of the U.S. operates a domestic home plant ($\mathcal{H}$) and a foreign branch ($\mathcal{B}$) in Asia. $\mathcal{B}$ is partly owned by the company and is responsible for the final product assembly and the emerging market in Asia. In order to be adaptive to the fasting changing market conditions, the headquarters have given $\mathcal{B}$ the autonomy to make independent decisions on local production and logistics, including inventory, capacity and assembly decisions. $\mathcal{H}$ is fully owned by the company and is responsible for the production of parts and components and the related scheduling and expediting issues. The headquarters also delegate authority to let $\mathcal{H}$ manage its production and capacity. However, due to strict requirements in technology and production, as well as proprietary concerns, a key and complex component must be produced at $\mathcal{H}$ and shipped to $\mathcal{B}$.

These settings have three implications:

- First, $\mathcal{B}$ incurs a high setup cost k associated with the overseas ordering and shipping of the key component, including specialized human resources and international telecommunication services, such as custom clearance, special shipment packaging and insurance, phone conferences, and faxes. For this reason, $\mathcal{B}$ uses an (s, t) replenishment policy for the key component, where t is the order interval and s the order-up-to level. The fixed replenishment interval helps the branch to schedule the complex international ordering process. To spread out the large k, it is desirable for $\mathcal{B}$ to choose a long replenishment interval t (see §3), which may be as long as half a year.

- Second, because $\mathcal{H}$ is the exclusive supplier of a key component, the headquarters require $\mathcal{H}$ to fulfill the branch's orders with 100% service level. When an order exceeds the inventory level at $\mathcal{H}$, $\mathcal{H}$ expedites the shortfall through overtime production or from another $\mathcal{H}$ and uses air shipping. This practice implies a convex production cost of t at $\mathcal{H}$ (see §3 for more detail). The large t imposed by $\mathcal{B}$, therefore, is costly for $\mathcal{H}$. For instance, the air transportation cost alone may amount hundreds and thousands of dollars per order cycle.

- Third, the headquarters still has the veto power on possible external changes between facilities, such as initiating cost sharing contracts, modifying the amount of transfer payments, transferring ownership of the components/parts etc, even though each facility has full authority on their internal decisions. The role of the headquarters, on the one hand, is to ensure the collaboration does not lead to significant supply chain efficiency losses; while on the other hand, provides enough flexibility for $\mathcal{B}$ and $\mathcal{H}$ to be adaptive to the environment. (see Arnold (1999))

This combination of decentralized and centralized features made this system clearly suboptimal. The system would be more effective if a smaller t is chosen. Recognizing that it is the different responsibilities of costs that induce different preferences in choosing t, a cost-sharing scheme may prove to be effective.

Semi-Centralized System

A semi-centralized system is a production supply network that has both centralized and decentralized features. On the one hand, each facility has its own autonomy to make internal decisions, due to needs for product localization, local knowledge and expertise, as well as incentives to grow. On the other hand, a headquarters control the collaboration between facilities and has the veto power on cross-facility decisions. Moreover, because all facilities belong to the same parent company, they share certain information vertically with the headquarters and obtain exclusive support, such as key components supply.

In this paper we study the collaboration scheme under the specific semi-centralized structure given in the case. By investigating several ways to coordinate the system, we find features that can be extended to a more general setting

One possibility for coordinating the supply chain is for the headquarters to act as a central planner to reallocate k between the two parties. In §4, we assume that the headquarters designs the cost sharing contract. We show that this can indeed coordinate the supply chain despite the fix cost is public information or private information. However, in either case, the contract is difficult to implement. When the fix cost is public information, while the optimal allocation of the fix cost always benefits $\mathcal{B}$, it is not so for $\mathcal{H}$. Therefore, the solution is not always implementable. When the fix cost is private information, the contract let $\mathcal{B}$ pay the full production cost at $\mathcal{H}$ allowing $\mathcal{B}$ to obtain full power over $\mathcal{H}$. This is not implementable.

An alternative is to delegate the coordination effort to $\mathcal{H}$, i.e., to have $\mathcal{H}$ initiate a cost-sharing contract with $\mathcal{B}$, resolving the above difficulties. In §5 and 6 we investigate the effectiveness of such an delegation. §5 assumes that $\mathcal{H}$ has full knowledge of the fixed order cost k and determines the cost sharing percentage. We show that the optimal contact exists. This contract benefits both $\mathcal{B}$ and $\mathcal{H}$ as well as the entire supply chain (denoted by $\mathcal{C}$) and it reduces the replenishment interval significantly and hence the role of the headquarters diminishes completely. Specifically, the payment from $\mathcal{H}$ to $\mathcal{B}$ consists of two parts: the operational compensation and the informational rent. However, this contract cannot lead to full supply chain coordination. On the other hand, numerical examples indicate that in some situations the contract can lead to near optimal performance. Also, the split of the benefit between the two parties depends highly on the cost structure of $\mathcal{H}$. When demand is deterministic, we obtain explicit expressions for the optimal sharing percentage as well as percentage gains of all parties ($\mathcal{B}$, $\mathcal{H}$, and $\mathcal{C}$).

§6 assumes that $\mathcal{H}$ has only a distribution knowledge about k and offers a menu of replenishment interval and the corresponding compensation to the branch. We show that the optimal contract exits and the compensation consists of two parts: the operational compensation and the informational rent. Moreover, under certain conditions, the role of the headquarters diminishes completely. Under deterministic demand, we obtain explicit expressions for the contract terms, which allow us to gain deeper insights. Numerical results indicate that the optimal contract does not necessarily benefit $\mathcal{H}$ on every realization of k, even though the expected cost of $\mathcal{H}$ is reduced.

§7 compares the performance of the contracts under asymmetric and full information. Interestingly, the optimal contract under asymmetric information can sometimes lead to significantly better supply chain performance than that under symmetric information.

We provide concluding remarks and discuss future research directions in §8. All proofs are in the Appendix.

4.2 Literature Review

Supply chain contract research has been very active in the last couple of decades. While most of the work in this area focuses on coordination of decentralized systems (see Cachon 2003, Chen 2003), we aim to coordinate a semi-centralized system. In addition, our paper appears to be among the very few that analyze supply chain contracts in the presence of fixed order costs.

One way of coordinating a decentralized system is through a central planner, as in Chen (1999), Porteus (2000) and Shang et al. (2009), among others. This approach assumes a single decision maker with full information and control of the entire entire supply chain. In our benchmark analysis, we employ a fixed-cost allocation scheme in a similar spirit to Shang et al. (2009).

In the absence of a central planner, the literature on decentralized supply chains study various contracts between independent parties. Two lines of research in this literature are most relevant to our work. One line is on cost sharing contracts, such as in Cachon and Zipkin (1999), who analyze backorder-cost sharing contracts. Our fixed-cost sharing contract under full information is inspired by their work. The other line is on asymmetric cost information, of which Corbett (2001) is the most related. While Corbett also considers the coordination of a two-party supply chain with fixed order cost and uncertain demand, the details are very different from ours. First, he assumes there is only one inventory location, at the downstream location in the supply chain, whereas we allow inventory at both locations. Second, he assumes that the supplier incurs the holding cost while the retailer incurs the backorder cost, whereas we assume that each location is responsible for its own inventory and backorder costs. These differences imply different incentives, which in turn require different mechanisms for coordination. In particular, in Corbett's setting, the the supplier prefers large but infrequent orders. He suggests the supplier design a contract menu consisting of order batch sizes and the corresponding compensations, so that the retailer would choose a larger order size for a higher compensation. In our case, the supplier incurs production and expediting costs and therefore prefers a smaller replenishment interval. We suggest the supplier offer a contract menu with replenishment intervals and corresponding compensations, in which a smaller reorder interval is associated with a higher compensation. In addition, in our model, we need to impose constraints to reflect the semi-centralized feature. To gain insights from this new type of optimization problem, we employ a methodology developed in Blackorby and Szalay (2007). In our numerical studies, we use measures similar to those used in Chao et al. (2009) to quantify the value of information and the value of a menu of contracts.

We have not seen any work in the supply chain literature that is specifically on semi-centralized systems. The closest concept is perhaps that of the degree of centralization in the industrial organization literature. Even there, the concept is not formally given. For example, in Palokangas (1991) the degree of centralization stands for the number of industries in the multi-regional environment, but in Kolmar (2001), the notion is characterized as the cost of exclusion of the regional government (decentralization) and taxation of the central government (centralization). Based on these definitions, some works in this literature study the optimal degree of centralization; see, e.g., Bucovetsky(2005), Janeba and Wilson (2005). Although most of the work on this topic is related to political science, using empirical and case studies, Arnold (1999) uses the notion of degree of centralization to analyze the structural aspect of global purchasing. This is different from our focus on operational aspects of a global production network.

Our work is also related to the literature on internal coordination. Kouvelis and Lariviere (2000) focuses on coordination through transfer pricing, assuming there are internal and external markets. Zimmerman (1979) is one of the first papers that studies cost allocations between facilities. The paper pointed out that the facility managers have incentives to maximize the facility's profit due to responsible accounting (the performance of the manager is evaluated based on the profit of the facility he/she is responsible for). Nonetheless the cost allocation can reduce inefficiency of the firm especially when the facilities have hard to measure costs. Some of the internal coordination papers focus on fixed cost allocations. Wei (2004) and Toktay and Wei (2009) study the fixed cost allocation under symmetric information and show that it can lead to supply chain efficiency while cost allocations that depends on quantity will result in double marginalization. Edlin and Reichelstein (1995) study fixed cost allocation under asymmetric information with negotiable transfer pricing. They are able to construct a mechanism that leads to supply chain efficiency. However, all of the papers ignored the initial state of the system. This restricts the feasibility of the contract because managers would deny the contract if it hurts the facility's performance compared to the initial state. In our paper, we focus on the coordination that facilities have authority to stay at the initial state if the contract will not benefit them.

Inventory models with (s, t) policies have been discussed in many papers. For example, Graves (1996) provides an approach to evaluate the cost for distribution systems under a virtual allocation rule. Shang and Zhou (2010) discuss echelon (r, nQ, T) policies with fixed costs in a centralized serial system. They find near-optimal heuristics by generating upper and lower bounds and provide properties of the optimal policy. Our two-stage system is different from these papers, especially in that the upstream system is required to provide 100% service to the downstream stage. Also, our focus is different. Rao (2003)and Liu and Song (2010a) study the properties of the (s, t) policy in a single location. Our analysis of the foreign branch problem is built on these properties.

Huggins and Olsen (2003) discuss a two-node supply chain with guaranteed delivery. They assume no fixed cost for the downstream retailer but a positive fixed cost at the upstream stage if expediting occurs. They find the optimal policy is structured as follows: the upstream stage follows a base stock policy, and the downstream stage follows an (s, S) policy. In our setting, the downstream incurs a fixed order cost and follows an (s, t) policy, while the upstream needs to guarantee a 100% service level with a finite production capacity and a finite shelf space. Liu and Song (2010b) characterize the optimal inventory and expediting policy for the home plant, and Liu and Song (2010c) shows that under the optimal policy, the home plant's cost is increasing and convex in the review cycle. We employ these results in §3.

4.3 Semi-Centralized System

We consider a global production network of a company comprising a home plant ($\mathcal{H}$), a foreign branch ($\mathcal{B}$) and a headquarters. We focus on the semi-centralized features observed from the inventory replenishment process of a key component that is supplied by $\mathcal{H}$ to $\mathcal{B}$. In this section, we describe the detailed operational characteristics of each facility, reflecting the requirements from the company's headquarters, and derive their respective long-run average costs.

4.3.1 Foreign Branch

Let $D(\tau)$ be the cumulative demand of the key component at $\mathcal{B}$ within time interval $(0, \tau]$. We assume the demand process $\{D(\tau), \tau \geq 0\}$ is nonnegative, stationary, and has independent increments. The stationary independent increment property implies that $\{D(\tau),\ \tau \geq 0\}$ is a semi-group, i.e. the demand can be decomposed into the sum of two independent parts: $D(\tau_1 + \tau_2) =_{st} D(\tau_1) + D(\tau_2)$ for all $\tau_1,\ \tau_2 \geq 0$. Furthermore, $D(\tau)$ is *stochastically increasing and linear by sample path* (SIL(sp)) in τ. That is, for any $\tau_1 < \tau_2 < \tau_3 < \tau_4 \in [0,\ \infty)$ with $\tau_1 + \tau_4 = \tau_2 + \tau_3$, we can construct four random variables D_i, $D_i =_{st} D(\tau_i),\ i = 1, 2, 3, 4$ to satisfy 1) $D_1 + D_4 = D_2 + D_3$

a.s. and 2) $\max[D_2 + D_3] \leq D_4$ a.s. (see Shaked and Shanthikumar 1994). Examples include stationary Poisson and compound Poisson processes.

Assume that the delivery leadtime from $\mathcal{H}$ to $\mathcal{B}$ is a fixed constant L and that unsatisfied demand at $\mathcal{B}$ is backlogged. For every replenishment order, there is a fixed order cost k. There is also a linear inventory holding cost at a per-unit rate h and a linear backorder cost at a per-unit rate b. $\mathcal{B}$ follows an (s, t) policy to replenish the inventory of the key component, where t is the reorder interval and s is the base stock level. That is, for every time interval t, $\mathcal{B}$ places an order from $\mathcal{H}$ to bring the inventory position back to s. This type of policy is very common in practice, and is particularly favored when there is a need to coordinate the replenishment processes of many products (see Rao 2003) and thus suitable in a global setting. In addition, when demand follows a Poisson process, Rao (2003) demonstrates that the (s, t) policy compares well with the continuous-review (r, q) policy, which is known to be optimal among all policies under this demand process and cost structure.

Given the demand and cost parameters, $\mathcal{B}$ chooses s and t to minimize its long-run average cost:

$$\min_{s,t \geq 0} B(s, t|k) = k\theta(t) + hE[s - D(L + U(0, t))]^+ + bE[s - D(L + U(0, t))]^-, \quad (4.1)$$

where $U(0, t)$ is a random variable with the uniform distribution on $[0, t]$, and

$$\theta(t) = Pr(D(t) > 0)/t \quad (4.2)$$

is the order frequency under any given reorder interval t. Here, for convenience of the later analysis, we use k to index the cost function to highlight its dependance on the fixed cost.

Fix any k, let $s(t|k) = \arg\min_s B(s, t|k)$ be the optimal base stock level for any given order interval t. Similarly, $t(s|k) = \arg\min_t B(s, t|k)$ is the optimal reorder interval for any given base stock level s. To proceed, we need the following result:

Lemma 27. *(Liu and Song 2010a) For any given* k*,* $B(s,t|k)$ *is coordinatewise convex and submodular in* (s,t)*. Also,* $s(t|k)$ *increases in* t *and* $t(s|k)$ *increases in* s*.*

In addition, we have

Lemma 28. *For any given* k*,* $\lim_{t\to\infty} B(s(t|k),t|k) = \infty$*.*

Hence, for any given k, there exists a least minimizer of $B(s(t|k),t\ |k)$, denoted by $(s(k),t(k))$, which is also the global minimizer of $B(s,t|k)$. Consequently, $B(s(k),t(k)|k)$ is the minimum long-run average cost of $\mathcal{B}$. Define

$$B^u(k) = B(s(k),t(k)|k)$$

as the optimal cost for $\mathcal{B}$ in the uncoordinated system.

Note that $\theta(t)$ is decreasing and convex in t and is in the form of $\theta(t) = (1-e^{-\rho t})/t$, where ρ is a positive constant depending on distribution of $D(t)$ (see Liu and Song 2010c). Also, if demand is continuous, $Pr(D(t) > 0) = 1$ for all $t > 0$, then $\theta(t) = 1/t$. We can show:

Lemma 29. *a) The optimal replenishment interval* $t(k)$ *increases in* k*. b)* $B^u(k)$ *is increasing and concave in* k*. c) As* $k \to 0$*,* $t(k) \to 0$*, so the system reduces to a continuous-review system with base stock policy. d) For continuous demand,* $k > 0$ *implies* $t(k) > 0$*.*

Deterministic Demand

According to Rao (2003), the optimal reorder interval $t(k)$ is relatively insensitive to demand variability. In particular, using the optimal reorder interval based on deterministic demand, which has a closed-form solution, and optimizing over s, the resulting long-run average branch cost is within 6.125% of optimal. For this reason, it is interesting and insightful to examine the behavior of the system, especially that of the reorder interval, under deterministic demand. We shall discuss this special case throughout the paper. For simplicity, whenever we consider this special case, including all the numerical studies later, we assume $L = 0$. This assumption merely changes a constant term in both the average cost and the base-stock level.

Suppose the demand is deterministic with rate λ, i.e., $D(\tau) = \lambda\tau$, $\tau \geq 0$. Then the branch's problem reduces to an EOQ problem with planned backorders (see Zipkin 2000, Chapter 3).

Denote

$$\gamma = \lambda hb/[2(h+b)]. \tag{4.3}$$

The optimal policy and cost are

$$t(k) = \sqrt{k/\gamma}, \qquad s(k) = 2\gamma t(k)/h, \qquad B^u(k) = 2\sqrt{\gamma k} = 2\gamma t(k). \tag{4.4}$$

All these quantities are increasing in k and proportional to $\sqrt{k}$.

4.3.2 Home Plant

For any given replenishment interval t, $\mathcal{H}$ faces a discrete-time i.i.d. demand stream $\{X_n = D((n-1)t, nt), n \geq 1\}$, where the X_n have the same distribution as $D(t)$.

As mentioned in the introduction, under the semi-centralized regime, $\mathcal{H}$ has the following supply restrictions: First, the headquarters requires $\mathcal{H}$ to provide 100% fulfillment to $\mathcal{B}$. Second, $\mathcal{H}$ has finite capacity and has many other products to produce. Hence $\mathcal{H}$ can only make to stock (instead of make to order) in anticipation of the orders from $\mathcal{B}$. When the orders from $\mathcal{B}$ exceeds its inventory on hand, to meet the 100% service level, $\mathcal{H}$ needs to expedite production either within its own production facility or from other home plants within the company. These features are also considered in Huggins and Olsen (2003, 2009). Assume the unit regular production cost is c and the unit expediting cost is c_e, where $c_e > c$. Let h_p be the unit finished-goods inventory holding cost rate.

There are two kinds of capacities at $\mathcal{H}$. The first is the *production capacity* μ, which is the maximum production rate of this component at $\mathcal{H}$. The second is the *shelf space capacity* ν, which is the maximum finished-goods inventory of this component that can be held at $\mathcal{H}$. The shelf space capacity is particularly relevant when the component is bulky and when $\mathcal{H}$ produces many different products. This parameter is independent of time and the production cycle.

Under this environment, Liu and Song (2010b) show that the optimal control policy for $\mathcal{H}$ is to produce up to a fixed base-stock level in every production cycle (of t periods) and expedite to clear the shortfall at the end of production cycle. Let $H(t)$ be the optimal long-run average cost at $\mathcal{H}$. Then, according to Liu and Song (2010c), we have

Lemma 30. *$H(t)$ is increasing and convex in t.*

Deterministic Demand

When demand is deterministic at rate λ, so $D(t) = \lambda t$, the home plant cost $H(t)$ is piecewise linear (see Liu and Song 2010c):

$$H(t) = \big(c_e(\lambda - \mu)^+ + c(\lambda \wedge \mu)\big)\, t 1_{\{t \leq \nu/(\lambda\wedge\mu)\}} + (c_e\lambda t - (c_e - c)\nu)\, 1_{\{t>\nu/(\lambda\wedge\mu)\}}. \quad (4.5)$$

Here, $x \wedge y = \min\{x, y\}$, and 1_U is the indicator function of event U. Clearly, $H(t)$ is increasing and convex in t. Moreover, the coefficient of t is increasing in c_e. Figure 1 shows $H(t)$ under several different values of λ.

In some ranges of the parameters, we obtain a linear cost function $H(t) = at$, where a is a positive constant. For example, from equation (4.5), $H(t) = at$ with $a = c_e\lambda$ when the shelf capacity constraint is binding ($\mu = \infty$) and $a = c_e(\lambda-\mu)+c\mu$ when the production capacity is binding ($\mu < \lambda$ and $\nu = \infty$). In other parameter ranges, we may approximate $H(t)$ well by a quadratic function. This fact will be used in the numerical studies in the later sections.

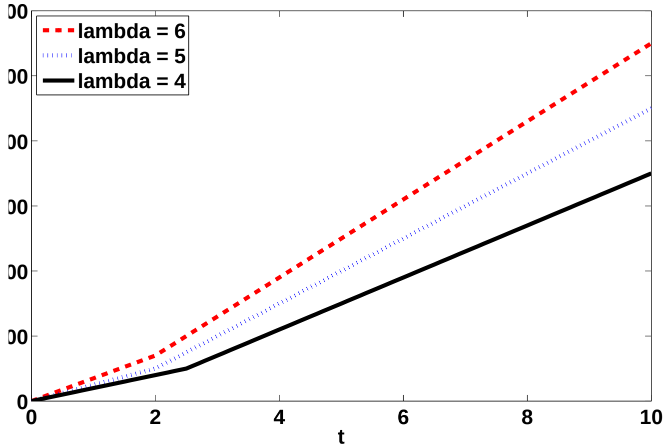


FIGURE 4.1: Deterministic $H(t)$: $c_e = 10,\ c = 5,\ \mu = 5,\ \nu = 10,\ \lambda = 4,\ 5,\ 6$

4.4 Headquarters Initiated Cost Sharing

In this section, we assume the headquarters takes full authority on initiating the contract and acquires full information on both $\mathcal{H}$ and $\mathcal{B}$. Note that even though the headquarters has the fully authority on the contract, it cannot interfere with the internal decisions each facility makes. Moreover, it cannot force each facility to participate in the contract. We first provide a bench mark that characterizes the best supply chain performance for the headquarters. Then we analyze two cost sharing contracts when k is either observable or unobservable to the Headquarters.

4.4.1 A Benchmark: The Best Performance of the Supply Chain

According to Huggins and Olsen (2003), the optimal policy for a two echelon system with guaranteed delivery is a (s, S) policy for the down stream and base stock policy for the upstream. The policy degenerates to multi-echelon base stock policies when the expediting fixed cost is zero. Therefore, the set of policy selected in our setting can optimize the supply chain performance and hence the best supply chain performance for the headquarters occurs when (s, t) are selected to minimize the total supply chain cost

$$\widehat{C}(s,t|k) = B(s,t|k) + H(t).$$

In this case we have:

Proposition 31. *For any given k, the supply chain cost $\widehat{C}(s,t|k)$ is coordinatewise convex and submodular in (s,t). The optimal replenishment interval $t^*(k)$ and base-stock level $s^*(k)$ satisfy the first order condition. Moreover, $t(k) \geq t^*(k)$ and $s(k) \geq s^*(k)$.*

Thus, in this system, $\mathcal{B}$ replenishes more frequently and uses a lower base-stock level. This, in turn, leads to smoother demand at $\mathcal{H}$, making its operations more efficient.

4.4.2 Headquarters Initiated Cost Sharing: Full Information

One possible incentive mechanism to reduce the reorder interval is to require $\mathcal{H}$ to share part of $\mathcal{B}$'s fixed order cost k. That is, for each replenishment order from $\mathcal{B}$, let $\mathcal{H}$ pay a portion of $1-\alpha$ of the fixed order cost to $\mathcal{B}$, $0 \leq \alpha \leq 1$. As mentioned in §1, this is equivalent to asking $\mathcal{H}$ to be responsible for some parts of the reorder process. Under this arrangement, the cost function for $\mathcal{B}$ becomes $B(s,t|\alpha k)$ and that for $\mathcal{H}$ becomes $\widetilde{H}(k,\alpha,t) = H(t) + (1-\alpha)k\theta(t)$.

The headquarters wants to selects an α such that both facilities would agree to choose the optimal replenishment interval.

$$\begin{aligned} \min_{\alpha\in[0,1]} \quad & B(s^*,t^*|\alpha k)+\widetilde{H}(k,\alpha,t^*) \\ s.t. \quad & (IC_B) \qquad (s^*,t^*)\in \arg\min_{0<s,t} B(s,t|\alpha k) \\ & (IC_H) \qquad t^*\in \arg\min_{0<s,t} \widetilde{H}(k,\alpha,t) \\ & (IR_H) \qquad \widetilde{H}(k,\alpha,t^*)\le H(t) \\ & (IR_B) \qquad B(s^*,t^*|\alpha k)\le B(s(k),t(k)|k) \end{aligned}$$

The first two constraints guarantees that the two facilities agree on the selection of the replenishment interval, the last two constraints characterizes the individual rationality of the two facilities.

Without the last two constraints, this problem degenerates to a topical fixed cost allocation problem in Accounting literatures. Wei (2004) show that the fixed cost allocation can lead to supply chain efficiency when each facility selects independent investments. In our setting, both facilities decide on one parameter: the replenishment interval. We show that, dropping the individual rationality constraints, i.e. assuming the headquarters has the power to force each facility to take the contract, the cost sharing contract can lead to a consensus on the replenishment interval that achieves supply chain efficiency. The following proposition shows the possible selection of α.

Proposition 32. *For any k, there exists an $\alpha^*=\alpha^*(k)$, such that the system is coordinated. That is, $t(\alpha^* k)=t^h(k,\alpha^*)=t^*(k)$ and $s(\alpha^* k)=s^*(k)$. Moreover, if $Pr(D(t)>0)=1$ for all $t>0$, then $\alpha^*<1$*

For any given k and reorder interval $\widetilde{t}(k)$, define

$$C(\widetilde{t}(k))=B(s(\widetilde{t}(k)|k),\widetilde{t}(k)|k)+H(\widetilde{t}(k)). \tag{4.6}$$

Then $C(\widetilde{t}(k))$ is the supply chain cost as a function of $\widetilde{t}(k)$, provided that $\mathcal{B}$ uses the optimal order-up-to level for $\widetilde{t}(k)$.

The following result is immediate from Lemma 29 and Lemma 30:

Corollary 33. *For any given k, $s(\alpha k)$, $t(\alpha k)$, $B^u(\alpha k)$ and $C(t(\alpha k))$ are increasing in α.*

According to Corollary 33, both the costs of $\mathcal{B}$ and $\mathcal{C}$ are lower under the cost sharing scheme. However, this may not necessarily be true for $\mathcal{H}$. The cost incurred by sharing k with $\mathcal{B}$ may outweigh the gain from a smoother demand, leaving $\mathcal{H}$ reluctant to follow the headquarters' plan. The next proposition identifies sufficient conditions for $\mathcal{H}$ to benefit from the coordination.

Proposition 34. *$\mathcal{H}$ has a positive gain in the coordinated system, i.e., $\widetilde{H}(k, \alpha^*, t^*(k)) \leq H(t(k))$ if*

$$-\theta'(t^*(k))(t(k) - t^*(k)) \geq \theta(t^*(k)). \tag{4.7}$$

When $Pr(D(t^*(k)) > 0) = 1$, (A.28) reduces to $t^*(k) \leq t(k)/2$.

Thus, if the bench mark solution does not reduce the original reorder interval by more than a half, the solution may not exist and the supply chain will remain in its original form. Moreover, the headquarters needs to know the demand distribution as well as all cost parameters at both $\mathcal{H}$ and $\mathcal{B}$. Collecting this information can be hard or impossible, especially in a global environment in which the two facilities are operating in different countries with different languages and cultures. In addition, getting access to these data may be challenging or unwise, because for various reasons the company may be better off by giving each facility enough autonomy and the freedom to keep certain information private. Taking all these factors into account, it is worth investigating whether the headquarters should delegate the coordination effort to $\mathcal{H}$.

4.4.3 Headquarters Initiated Cost Sharing: Asymmetric Information

Assume the fix cost k is a random variable privately observed by $\mathcal{B}$, it has a continuous strictly increasing distribution $F(\cdot|\ t_0)$ on a finite support $[\underline{k}, \bar{k}]$, where t_0 is the initial replenishment interval and k is stochastically increasing in t_0. The distribution function and the initial replenishment interval t_0 are publicly known to the headquarters and $\mathcal{H}$.

The sequence of events is as follows: The headquarters designs a contract based on $F(\cdot|\ t_0)$. After $\mathcal{B}$ observes k, both $\mathcal{B}$ and $\mathcal{H}$ decide to participate in the contract or quit the contract. If both decide to participate, $\mathcal{B}$ reports k. The replenishment interval is selected and transfer payments are received based on the reported k. If either of them decides to quit, then the replenishment interval is selected based on the break-up rule.

The headquarters objective is to design a contract to minimize the expected supply chain cost. Consider a contract $\mathcal{M} = \langle T^{HA}(\cdot), \tau(\cdot), r(\cdot)\rangle$.

The first element $T^{HA}(\cdot)$ is the replenishment interval. It is a mapping from all possible reports to R^+:

$$T^{HA} : [\underline{k},\ \bar{k}] \rightarrow R^+.$$

The second element $\tau(\cdot) = (\tau_B(\cdot), \tau_H(\cdot))$ is the transfer payments each subordinate receives. It is a mapping from all possible reports to R^2:

$$\tau(\cdot) : [\underline{k},\ \bar{k}] \rightarrow R^2.$$

Note that the transfer payments can be either positive or negative. Positive transfer payments imply that subordinate is receiving a transfer, while negative transfer payments imply that subordinate is making a transfer.

The third element $r(\cdot) = (r_B(\cdot), r_H(\cdot))$ is the probabilistic decision right of the replenishment interval decision if either subordinate decides to quit,

$$r(\cdot):[\underline{k},\ \bar{k}] \to [0,1]^2, r_B + r_H = 1.$$

Control of the replenishment interval is allocated to $\mathcal{B}$ with probability r_B and $\mathcal{H}$ with probability r_H. The decision rights are allocated exclusively to the two subordinates.

Since the headquarters has limited power over its subordinates, the mechanism needs to be design in a way to meet all the requirements from $\mathcal{B}$ and $\mathcal{H}$. We characterize the requirements from $\mathcal{B}$ and $\mathcal{H}$ as follows:

The Break-up Rule

When either party decides to terminate the contract, the decision to select the replenishment interval is allocated through the decision rights. If $\mathcal{B}$ receives the power to select the replenishment interval, he will select $t(k)$ to minimize his cost. If $\mathcal{H}$ selects the replenishment interval, he will choose the smallest replenishment interval possible, and hence causing the supply chain to incur a significant cost at $\mathcal{B}$. In order to prevent this unstable event, the headquarters regulate the selection of replenishment interval to be above a minimum level t_{min}.

Incentive Compatibility

The mechanism should ensure that $\mathcal{B}$'s best response is to report truthfully. That is

$$B(s(T^{HA}(k)), T^{HA}(k), k) - \tau_B(k) \le B(s(T^{HA}(\tilde{k})), T^{HA}(\tilde{k}), k) - \tau_B(\tilde{k})$$
$$\forall \tilde{k} \neq k, \qquad \tilde{k},\ k \in [\underline{k},\ \bar{k}]$$

Voluntary Participation

The mechanism should ensure that both $\mathcal{B}$ and $\mathcal{H}$ prefer participate to quit. That is

$$B(s(T^{HA}(k)), T^{HA}(k), k) - \tau_B(k) \le r_B B(s(k), t(k), k) + r_H B(s(t_{min}), t_{min}, k)$$
$$E[H(T^{HA}(k)) - \tau_H(k)] \le r_B E[H(t(k))] + r_H H(t_{min}).$$

Budget Balance

The mechanism should ensure that the system is self-supporting. Both $\mathcal{B}$ and $\mathcal{H}$ can coordinate without the headquarters lending or receiving payments from either of them. That is

$$\tau_B(k) + \tau_H(k) = 0.$$

The headquarters problem becomes

$$\begin{aligned}
\min_{T^{HA}(),\tau(),t_{min}} \quad & E[B(s(T^{HA}(k)), T^{HA}(k), k) + H(T^{HA}(k))] \\
s.t. \quad & (IC_B) \quad B(s(T^{HA}(k)), T^{HA}(k), k) - \tau_B(k) \\
& \quad \leq B(s(T^{HA}(\tilde{k})), T^{HA}(\tilde{k}), k) - \tau_B(\tilde{k}) \quad \forall \tilde{k} \neq k, \quad \tilde{k},\ k \in [\underline{k},\ \bar{k}] \\
& (IR_B) \quad B(s(T^{HA}(k)), T^{HA}(k), k) - \tau_B(k) \\
& \quad \leq r_B B(s(k), t(k), k) + r_H B(s(t_{min}), t_{min}, k) \\
& (IR_H) \quad E[H(T^{HA}(k)) - \tau_H(k)] \leq r_B E[H(t(k))] + r_H H(t_{min}) \\
& (BB) \quad \tau_B(k) + \tau_H(k) = 0.
\end{aligned}$$

Proposition 35. *Let $\bar{t}$ be the replenishment interval that satisfies: $E[H(t(k))] = H(\bar{t})$. We have the following results:*

1) There exists a feasible decision right allocation that induces an efficient contract $T^{HA}(k) = t^*(k)$ for all k, i.e. the supply chain is coordinated. The contract makes each subordinate the residual claimant of the supply chain.

2) For any feasible decision rights allocation r_B and r_H that induce supply chain coordination, the transfer payments are:

$$\tau_B(k) = -H(t^*(k)) + E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))]/2 - (O_B - O_H)/2$$
$$\tau_H(k) = H(t^*(k)) - E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))]/2 + (O_B - O_H)/2$$

where

$$O_B = \max_k \{B(s(t^*(k)), t^*(k), k) + H(t^*(k)) - r_B B(s(k), t(k), k) - r_H B(s(t_{min}), t_{min}, k)\}$$

and $O_H = E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))] - r_B E[H(t(k))] - r_H H(t_{min})$.

3) Specifically, when $t_{min} \geq \bar{t}$*, there exists* $r_H^0 \geq 0$ *such that the feasible decision rights satisfies* $r_H \geq r_H^0$.

Proposition 35 implies that supply chain can be coordinated by efficiently allocating decision rights. The transfer payments allow $\mathcal{B}$ to bare all the inventory-production cost and $\mathcal{H}$ a fixed payment. This payment structure implies that $\mathcal{B}$ takes full responsibility of the inventory-production cost while receiving a fixed compensation from $\mathcal{H}$. Theoretically the mechanism can be implemented similar to Corbett (2001), that $\mathcal{B}$ controls the production planning at $\mathcal{H}$ and $\mathcal{H}$ holds consignment stock for $\mathcal{B}$. However, even though the consignment stock policy can coordinate the semi-centralized supply chain, in practise, $\mathcal{H}$ will not its production planning power.

In the next two sections, we analyze the effectiveness of such delegation. In both sections, we assume $\mathcal{H}$ knows $\mathcal{B}$'s holding cost rate h, backorder cost rate b, and the distribution of the demand process $D(\tau)$. This is plausible, because $\mathcal{H}$ knows the value of the key component, and it is reasonable to have the headquarters to require $\mathcal{B}$ to share demand forecasts with $\mathcal{H}$. The two sections differ by whether $\mathcal{H}$ has full information about the fixed cost k.

4.5 Dedicate the Contract Design to $\mathcal{H}$

4.5.1 Home-Plant Initiated Cost Sharing: Full Information

In this section, we assume $\mathcal{H}$ has full knowledge of the fixed order cost k of $\mathcal{B}$ (which may be provided by the headquarters) and designs a fixed cost sharing contract with $\mathcal{B}$ to reduce its inefficiency. However, in order to make the contract implementable, both facilities need to agree on the payments. Moreover, the contract needs to receive approval from the headquarters.

The sequence of events of the Stackelberg game in the semi-centralized system is as follows:

1) $\mathcal{H}$ offers a sharing proportion $1-\alpha$, $\alpha \in [0,1]$ of the fixed order cost. The system moves to stage 2.

2) $\mathcal{B}$ decides whether or not to participate. If $\mathcal{B}$ decides to not participate, then the game ends and the system will operate under the original setting. If $\mathcal{B}$ decides to participate, then the system moves to stage 3.

3) $\mathcal{B}$ decides on its base stock level and the order interval based on the announced α.

4) The headquarters decide to accept or deny the contract. If the headquarters deny the contract, the system will operate under the original setting. If the headquarters accept the contract, the system will operate under the contract.

For any given $k \in (0,\infty)$ and $\alpha \in [0,1]$, $\mathcal{B}$'s set of strategies is: $\mathcal{S}_{\mathcal{B}} = \{(s,t)|\ s : (0,\infty) \times [0,1] \rightarrow R^{+}, t : (0,\infty) \times [0,1] \rightarrow R^{+}\}$. $\mathcal{B}$ selects its best response by minimizing its cost:

$$(s(\alpha k), t(\alpha k)) \quad = \quad \arg\min_{0<s,\ t} B(s,t|\alpha k)$$

Note that $\mathcal{B}$ participates only if $B^u(\alpha k) \leq B^u(k)$.

Denote the corresponding optimal base stock level and reorder interval for $\mathcal{B}$ are $(s(\alpha k), t(\alpha k))$. Then $\mathcal{B}$'s best response is $(s(\alpha k), t(\alpha k))$.

$\mathcal{H}$'s set of strategies is: $\mathcal{S}_{\mathcal{H}} = \{\bar{\alpha}|\ \bar{\alpha} : \mathcal{S}_{\mathcal{B}} \rightarrow [0,1]\}$

Knowing $\mathcal{B}$'s best response, $\mathcal{H}$'s problem is:

$$\begin{array}{lll} \min\limits_{0\leq\alpha\leq 1} & & \widetilde{H}(k,\alpha,t(\alpha k)) \\ s.t. & (IR*) & B^u(\alpha k) \leq B^u(k) \\ & (HA) & t(\alpha k) \leq t(k). \end{array}$$

The two constraints reflect the semi-centralized features: The individual rationality constraint (IR^*) ensures the contract not hurt $\mathcal{B}$. The headquarters agreement constraint (HA) ensures the contract not hurt $\mathcal{C}$. This is because for any $t > t(k)$, $B(s,t) \geq B^u(k)$ and $H(t) \geq H(t(k))$, and therefore the total supply chain cost $\widehat{C}(s,t|k) \geq \widehat{C}(s,t(k)|k)$. However, from Corollary 33, both of these constraints are automatically satisfied.

Because $\widetilde{H}(k,\alpha,t(\alpha k))$ is continuous in α and the optimization is over a closed and bounded region $[0,1]$, by the Weierstrass Theorem, there exists an optimal solution $\alpha^h(k)$. We have:

Proposition 36. *For any fixed k, the optimal fixed cost sharing percentage $\alpha^h = \alpha^h(k)$ satisfies $\alpha^h(k) \geq \alpha^*(k)$. Hence, $t^*(k) \leq t(\alpha^h k) \leq t(k)$, and $\widehat{C}(s(\alpha^h k), t(\alpha^h k)|k) \leq \widehat{C}(s(k), t(k)|k)$. If the division cost functions are strictly convex, then all the above inequalities are strict. This implies the role of the headquarters diminishes under full information.*

Thus, under the cost sharing contract led by $\mathcal{H}$, the supply-chain cost will be reduced. However, in general, the contract cannot lead to a fully coordinated supply chain.

Deterministic Demand

To gain more insights into the contract and its impact, in this subsection we assume the demand is deterministic, i.e. $D(\tau) = \lambda\tau$ for all $\tau \geq 0$. We also assume a linear home plant cost, $H(t) = at$, for some positive constant a. Correspondingly, we have $H(t(k)) = at(k)$ and the supply-chain cost is $\widehat{C}(s(k), t(k)|k) = (a + 2\gamma)t(k)$, where $t(k)$ is given in (4.4). Thus, in this case, the costs of all parties are linear in the reorder interval.

Denote the cost ratio

$$\eta \;=\; \frac{H(t(k))}{B^u(k)} = \frac{dH(t(k))/dk}{d[B^u(k)]/dk} = \frac{a}{2\gamma}. \tag{4.8}$$

In particular, when $a = c_e\lambda$, $\eta = c_e(1/h + 1/b)$. When $a = c_e(\lambda - \mu) + c\mu$, $\eta = (c_e - \mu/\lambda(c_e - c))(1/h + 1/b)$. We have

Proposition 37. *Suppose $D(t) = \lambda t$, $H(t) = at$, and $\mathcal{H}$ offers a contract to pay a proportion of $1 - \alpha$ of the fixed cost to the foreign branch. Then:*

a) For any given α, the long-run average cost of $\mathcal{H}$ is

$$\widetilde{H}(k, \alpha, t(\alpha k)) = \left(a\sqrt{k/\gamma} - \sqrt{\gamma k}\right)\sqrt{\alpha} + \frac{1}{\sqrt{\alpha}}\sqrt{\gamma k}.$$

This is convex in $\sqrt{\alpha}$, thus there exists an unique α^h that minimizes $\mathcal{H}$'s cost.

b) α^h is independent of k. In particular, if $\eta \leq 1$, then $\alpha^h = 1$, i.e., it is optimal for $\mathcal{H}$ not *to share the fixed order cost with $\mathcal{B}$. Otherwise, if $\eta > 1$, then $\alpha^h = \gamma/(a - \gamma)$. Thus, the optimal percentage $(1 - \alpha^h)$ is increasing in a, decreasing in λ, b and h, and independent of k.*

c) When $\eta > 1$, all parties ($\mathcal{H}$, $\mathcal{B}$ and $\mathcal{C}$) benefit from the cost sharing contract. Under the optimal sharing percentage α^h, the percentage reduction in the reorder interval (and hence in all parties' costs) is $1 - \sqrt{\alpha^h} = 1 - \sqrt{\gamma/(a - \gamma)}$, which is increasing in a but decreasing in λ, b and h.

d) The optimal centralized cost sharing percentage is $\alpha^* = \gamma/(a+\gamma)$, which is also independent of k.

e) When $a = c_e\lambda$ (i.e., the shelf space capacity is binding), $\eta > 1$ is equivalent to $c_e > hb/(h+b)$. Moreover, $\alpha^h = [2c_e(1/b + 1/h) - 1]^{-1}$ and $\alpha^* = [2c_e(1/b + 1/h) + 1]^{-1}$. Both α^h and α^* are independent of λ.

f) When $a = c_e(\lambda - \mu) + c\mu$ (i.e., the production capacity is binding), $\eta > 1$ is equivalent to $c_e - \mu/\lambda(c_e - c) > hb/(h+b)$. Moreover, $\alpha^h = [2(c_e - \mu/\lambda(c_e - c))(1/b + 1/h) - 1]^{-1}$ and $\alpha^* = [2(c_e - \mu/\lambda(c_e - c))(1/b + 1/h) + 1]^{-1}$. Both α^h and α^* depend on λ only through μ/λ, the percentage of demand covered by the production capacity.

Property 37 (b) indicates that whether or not $\mathcal{H}$ benefits from the contract only depends on the relative magnitudes of the cost rate a at $\mathcal{H}$ and the parameter ratio γ at $\mathcal{B}$, not on k. Because $\alpha^h > \alpha^*$, under the cost-sharing contract, $\mathcal{H}$ shares a smaller portion of the fixed cost than in the centralized system. Consequently, the reductions in both the reorder interval and supply-chain cost are less than those under the centralized coordination. However, the difference between the two coordination schemes decreases as a increases (e.g., it is more expensive to expedite at $\mathcal{H}$), because $\alpha^h/\alpha^* = (a+\gamma)/(a-\gamma)$ is increasing in a.

Note that the cost rate ratio η measures the relative cost changes of $\mathcal{H}$ and $\mathcal{B}$ as k changes. It is increasing in a and decreasing in b, h, λ, but is independent of k. When $\eta > 1$, $\mathcal{H}$ has a greater increment in unit cost than $\mathcal{B}$ as k increases. Thus, $\mathcal{H}$ has an incentive to share some of k to yield a reduced reorder interval. The contract is a form of compensation for the branch for its loss due to such a reduction. When $\eta < 1$, the decrement of the reorder interval brings greater savings to $\mathcal{B}$ than $\mathcal{H}$, the cost of sharing the fixed cost will surpass the benefit of a smaller reorder interval, so $\mathcal{H}$ does not offer the contract.

We now quantify the split of the contract benefits between the two divisions. Let $\Delta H(k)$, $\Delta B(k)$ and $\Delta C(k)$ be the cost savings of $\mathcal{H}$, $\mathcal{B}$, and $\mathcal{C}$ under the optimal cost sharing contract, respectively. That is, $\Delta H(k) = H(t(k)) - \widetilde{H}(k, \alpha^h, t(\alpha^h k))$, and the others are defined similarly. Define $\xi_H = \Delta H/\Delta C$ and $\xi_B = \Delta B/\Delta C = 1 - \xi_H$ to be the savings ratio for $\mathcal{H}$ and $\mathcal{B}$, respectively. We have

Proposition 38. *Suppose $\eta > 1$. Then*

$$\xi_H = \frac{\eta - \sqrt{2\eta - 1}}{1 + \eta - \sqrt{2\eta - 1} - \frac{1}{\sqrt{2\eta - 1}}}.$$

Both ξ_H and ξ_B are independent of k and depend on the other parameters only through the ratio η. In particular, ξ_H is increasing and concave in η. When $\eta = 2.5$, we have $\xi_H = \xi_B$, i.e., $\mathcal{H}$ and $\mathcal{B}$ evenly split the benefit. And $\xi_H > \xi_B$ if and only if $\eta > 2.5$, i.e., $\mathcal{H}$ reaps more benefit only when the expediting cost is very high.

Finally, we take a closer look of the meaning of $\eta > 1$ to the supply chain with a specific example. Consider the case $a = c_e \lambda$, where $\eta > 1$ is equivalent to $c_e > hb/(h + b)$. This is because c_e is the unit saving of expediting one unit less at $\mathcal{H}$, and $bh/(b + h)$ is the expected loss of having one unit less at $\mathcal{B}$. Together $c_e > hb/(h + b)$ implies that it is beneficial for the supply chain that the two parties coordinate. This further implies that the cost sharing contract at $\mathcal{H}$ is justified only if it benefits the supply chain as well.

Numerical Study

When demand is stochastic, we can no longer obtain explicit formulas for the contract terms and cost functions. To gain insights into the effect of demand variability on the contract effectiveness, we now present a numerical study for the stochastic demand case and compare it with the findings in the deterministic demand case analyzed in the previous subsection. We assume $D(\tau)$ is a random variable with the normal distribution with mean $\lambda\tau$ and variance $\lambda\tau$. This is a good approximation of the cumulative demand when customer demand arises according to a Poisson process with rate λ. Because we consider large reorder interval cases, $Pr(D(t) > 0)$ is close to 1, so we assume $Pr(D(t) > 0) = 1$. We set $h = 1$, $b = 8$, $\lambda = 1000$. Assume the cost function of $\mathcal{H}$ is $H(t) = 10t$. We vary the fixed order cost k by taking 100 sample points in $[10,000, 20,000]$ with increment of 100.

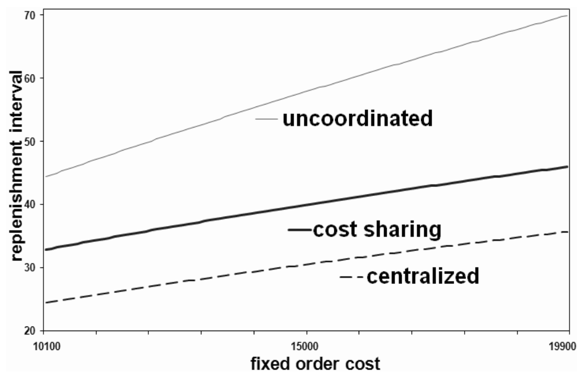


FIGURE 4.2: Replenishment Intervals

Effect on Replenishment Interval Figure 4.2 compares the optimal reorder intervals under three settings: the original uncoordinated system ($t(k)$), the centralized system ($t^*(k)$), and under the cost sharing contract ($t(\alpha^h k)$). Not surprisingly, as k increases, the reorder intervals of all settings increase. Also, the cost sharing contract significantly reduces the reorder interval, although not to the level of the centralized system. Unlike in the deterministic demand case, however, the reduction of $t(k) - t(\alpha^h k)$ is not independent of k. It increases in k. Thus, with demand variability, the impact of the cost sharing contract is more significant for large k. This is reasonable, because demand variability amplifies upward along the supply chain (the so-called bullwhip effect) and batching is known to be a key factor of this effect (Lee et al. 1997). For a bigger k, the need for batching is higher.

Effect on Costs In the first part of Figure 4.3, we compare the long-run average costs of $\mathcal{B}$ 1) in the original uncoordinated system, $B^u(k)$, 2) in the centralized system, $B(s^*(k), t^*(k), |k)$, and 3) under the cost sharing contract, $B^u(\alpha^h k)$. In the second part of Figure 4.3, we plot the counterparts for $\mathcal{H}$, namely, $H(t(k))$, $H(t^*(k))$,

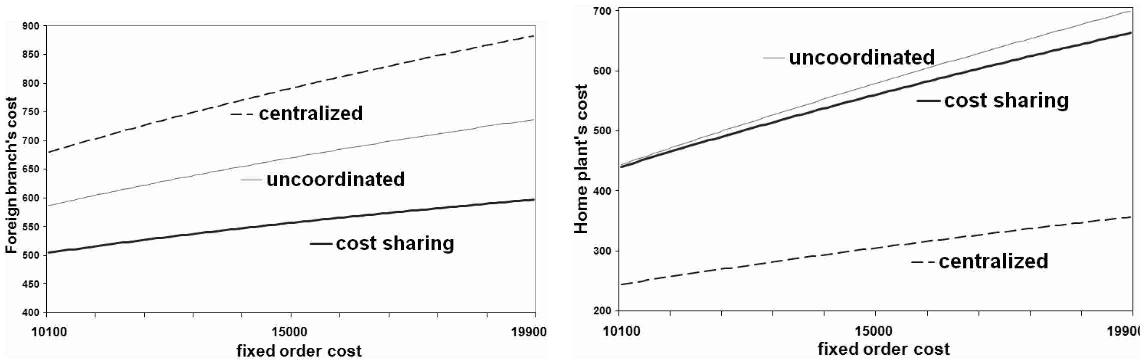


FIGURE 4.3: Costs of $\mathcal{H}$ and $\mathcal{B}$

and $\widetilde{H}(k, \alpha^h, t(\alpha^h k))$. As in the deterministic demand case, under the cost sharing contract α^h, both the $\mathcal{H}$ and $\mathcal{B}$ are better off. Unlike in the deterministic demand case, in which all parties achieve the same percentage savings, here, $\mathcal{B}$'s cost improves more than that of $\mathcal{H}$. The reason that $\mathcal{H}$ does not benefit much from the contract in this example is mainly due to the linear form of the cost function of $\mathcal{H}$. With this cost structure, the cost improvement at $\mathcal{H}$ is linear in that of the reorder interval. If this function were strictly convex, especially when the first order derivative of $\mathcal{H}$'s cost function sharply increases, the reduction of the reorder interval would lead to more cost savings. (See more discussion at the end of this subsection.)

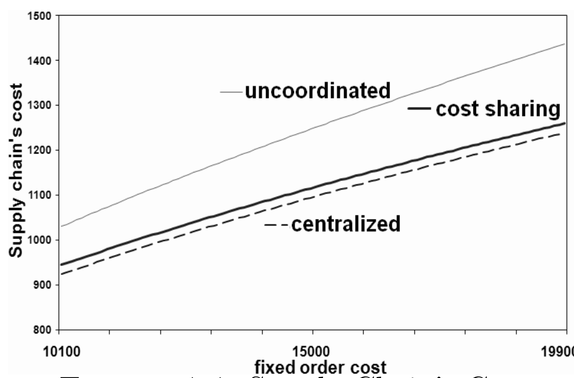
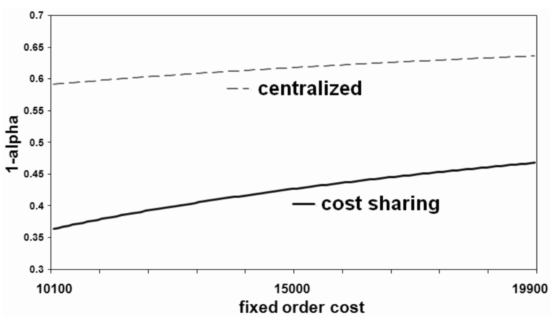


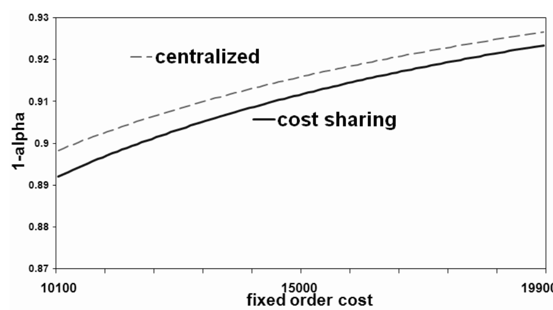
FIGURE 4.4: Supply Chain's Cost

Figure 4.4 compares the supply chain cost under the three settings examined above. It is interesting to observe that the performance of $\mathcal{C}$ improves significantly under the cost sharing contract, and the cost improvement is increasing in k. This latter observation is again different from the deterministic demand case, reflecting the bigger impact of the contract with demand variability. Moreover, the supply chain cost is reduced almost to that of the centralized system. This, along with Figure 4.3, implies that the cost sharing contract (α^h) can be effective, especially when demand is stochastic. Thus, delegating the coordination authority to $\mathcal{H}$ can be worthwhile when demand is random and the fixed cost is large.

Effect of Home-Plant Cost Structure So far, we have assumed linear $\mathcal{H}$ cost. Next, we examine the effect of the cost structure of $\mathcal{H}$. We compare the contract terms and effectiveness under two different production cost functions of $\mathcal{H}$: $H(t) = 10t$ (linear) and $H(t) = 5t^2$ (quadratic).



Linear Production Cost



Quadratic Production Cost

FIGURE 4.5: $\mathcal{H}$'s Cost Sharing Percentage $(1 - \alpha^h)$

Figure 4.5 compares the value of $\mathcal{H}$'s fixed-order-cost sharing percentage under the contracts designed by a headquarters, $1-\alpha^*$, and by $\mathcal{H}$, $1-\alpha^h$. Observe that as k increases, these sharing percentages increase, but the gap between them narrows . So, with demand variability, $\mathcal{H}$ incurs a larger share of the fixed cost to reduce the reorder interval when that cost is bigger. In addition, under the quadratic cost structure, the gap of the sharing percentages is much smaller. In this case, the cost sharing contract results in near optimal system performance.

Figure 4.6 illustrates $\mathcal{H}$'s cost savings over the total supply chain cost savings (ξ_H) under both cost structures. We can see that the $\mathcal{B}$ receives most of the benefit

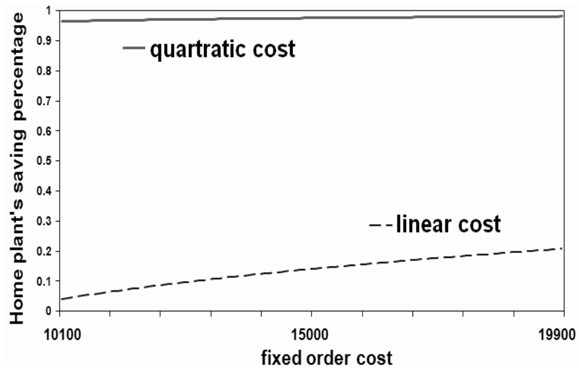


FIGURE 4.6: $\mathcal{H}$'s Saving Percentage

when $\mathcal{H}$ has a linear cost structure, but $\mathcal{H}$ receives most of the benefit when it has a quadratic cost structure.

4.5.2 Home-Plant Initiated Cost Sharing: Asymmetric Information

In this section, we assume k is a random variable privately observed by $\mathcal{B}$ before the initiation of this contract. k is drawn from a distribution $F(\cdot|\, t_0)$ with density $f(\cdot|\, t_0)$ and finite support $[\underline{k}, \bar{k}]$, where t_0 is the replenishment interval. We assume $F(\cdot|\, t_0)$ and t_0 are common knowledge and k is stochastically increasing in t_0. This informational asymmetry with regard to k mostly stems from frictions of the information flow between $\mathcal{H}$ and $\mathcal{B}$, due to lack of streamlined information system, inaccurate information on customs and transportation fees (especially the local transportation cost in the foreign country), as well as local personnel costs. Cultural differences and privacy of $\mathcal{B}$ are also contributors. The distributional knowledge of k may be gained from the headquarters through its experience with other home plant and foreign branch subsystems. The branches in these subsystems are usually located in countries different from the current one of interest. These countries may be developed countries or emerging markets. The knowledge of the fixed order costs in these subsystems can be used to infer the distribution of k in the current system.

Given the distributional knowledge of the branch's type, $\mathcal{H}$ would like to design a contract menu to the foreign branch to induce the right reorder interval, to increase efficiency. This section is organized as follows. §6.1 formulates the contract design problem. §6.2 shows the existence of an optimal contract menu and discusses its composition. §6.3 assumes deterministic demand and obtains closed-form expressions for the contract terms to enhance understanding.

We assume that the cost functions of both $\mathcal{H}$ and $\mathcal{B}$ are strictly convex. For any function $g(x, y)$, let g_x, g_{xx} and g_{xy} denote $\partial g/\partial x$, $\partial^2 g/\partial x^2$ and $\partial^2 g/\partial x \partial y$, respectively.

Contract-Design Formulation

The sequence of events is as follows:

1. At the beginning of the period, a mechanism $\mathcal{M} =< t_0|\ T(x), A(x) >$ is announced by $\mathcal{H}$. Then the system moves to the next phase.

2. $\mathcal{B}$ observes its k and decide to quit or not. If $\mathcal{B}$ quits, then $\mathcal{B}$ gets to update its replenishment interval according to the observed k and get no support from $\mathcal{H}$. If $\mathcal{B}$ decides to stay in the contract, the system moves to the next phase.

3. $\mathcal{B}$ announces k and the corresponding replenishment interval $T(k)$ and the compensation $A(k)$ are calculated.

4. The headquarters decides to vote to/against the contract. If the headquarters vote against the contract, $\mathcal{B}$ gets to update its replenishment interval according to the observed k. If the headquarters vote for the contract, the payments are transferred.

Stage 2) and 4) show the characteristics of a semi-centralized system. 2) allows $\mathcal{B}$ to quit after observing its state k. Moreover, the performance of both parties if the contract dissolves is closely tight up together. Stage 4) allows the headquarter to have the veto power on the contract this makes the coordination more difficult.

The contract requires, first the reporting strategy of $\mathcal{B}$ is incentive compatible,

$$(IC) \qquad \widehat{B}(k, T(k)) - A(k) \leq \widehat{B}(k, T(x)) - A(x) \quad \text{for any } x \in [\underline{k}, \bar{k}].$$

Second, $\mathcal{B}$ is individually rational,

$$(IR*) \qquad \widehat{B}(k, T(k)) - A(k) \leq B^u(k)$$

Third, the headquarter approves the contract,

$$(HA) \qquad T(k) \leq t(k).$$

According to the revelation principle (see Fudenberg and Tirole 1991), in the search for the optimal contract, $\mathcal{H}$ can restrict the contract menu to items that provides the branch the incentive to truthfully reveal its fixed order cost. Its objective is to find the contract menu that minimizes its own cost. Thus, $\mathcal{H}$'s problem is:

$$(\mathcal{P}_{\mathcal{H}}) \qquad V = \min_{(T(\cdot), A(\cdot))} \int_{\underline{k}}^{\bar{k}} [H(T(x)) + A(x)] f(x \mid t_0) dx$$

$$\begin{aligned} s.t. \quad (IC) \quad & \widehat{B}(k, T(k)) - A(k) \leq \widehat{B}(k, T(x)) - A(x) \text{ for all } x \in [\underline{k}, \bar{k}] \\ (IR*) \quad & \widehat{B}(k, T(k)) - A(k) \leq B^u(k) \\ (HA) \quad & T(k) \leq t(k). \end{aligned}$$

As in the standard principal-agent problems, the incentive compatibility constraint (IC) ensures that the type-k $\mathcal{B}$ will not have incentive to announce a different fixed order cost x. The individual rationality constraint (IR*) guarantees that $\mathcal{B}$ participating in the cost sharing contract will do at least as well as in the case without the contract. Note that (IR*) is a revised version of the standard individual rationality constraints. In the standard models, the right-hand-side of the constraint is a constant across all types. Here, it depends on the branch type, reflecting the fact that $\mathcal{H}$ is the only supplier available for $\mathcal{B}$, so the best possible cost without the contract for the type-k $\mathcal{B}$ is $B^u(k)$.

In addition to these two standard types of constraints, $\mathcal{H}$ needs the agreement from the headquarters to execute the contract, therefore it needs to ensure $\mathcal{C}$ is not worse off under all possible fixed costs. The headquarters agreement constraint (HA) serves precisely this purpose. Thus, in this formulation, both the (IR*) and (HA) constraints represent the semi-centralized feature of the system studied here. These additional requirements imply that the home plant has less power in designing the contract than a typical principal in decentralized systems.

Optimal Contract Menu

We now proceed to show the existence of an optimal contract menu. We do so by showing how to construct the optimal contract menu $(T(x), A(x))$. To that end, we first establish a relationship between $T(\cdot)$ and $A(\cdot)$ through the Lagrangian first order conditions, and therefore reduce the dimension of $\mathcal{H}$'s problem. Finally, we present conditions under which optimal contract menu can be constructed in a pointwise manner. Along this analysis, we also identify the key components of the optimal contract terms and their properties.

First, we observe that the participation constraints yield monotonicity of $T(\cdot)$. That is:

Lemma 39. *The contract reorder interval $T(x)$ is non-decreasing in x.*

The following proposition shows we can indeed solve this home-plant's optimization problem. Thus, an optimal contract menu exists for the semi-centralized system.

Moreover, we can explicitly construct this menu. Let

$$\varphi(x) = F(x)/f(x), \qquad \omega(x) = x + \varphi(x). \tag{4.9}$$

Proposition 40. *If $\omega(x)$ is nondecreasing, then there exits an optimal contract menu $\{(T(x), A(x)), x \in [\underline{k}, \bar{k}]\}$. Let*

$$\widehat{H}(k, T(k)) \;\; = \;\; H(T(k)) + A(k). \tag{4.10}$$

The optimal compensation $A(x)$ satisfies:

$$A(k) = [\widehat{B}(k, T(k)) - B^u(k)] + \int_k^{\bar{k}} [\theta(T(x)) - \theta(t(x))]dx. \tag{4.11}$$

The optimal contract reorder interval $T(x)$ satisfies:

$$T(k) = \begin{cases} \tilde{T}(k) & \text{if } H'(t(k)) + \theta'(t(k))\varphi(k) > 0 \\ t(k) & \text{if } H'(t(k)) + \theta'(t(k))\varphi(k) \le 0 \end{cases},$$

where $\tilde{T}(k) < t(k)$ and solves

$$\widehat{H}_T(x, T) \;\; = \;\; H'(T(x)) + \widehat{B}_T(x, T(x)) + \theta'(T(x))\varphi(x) = 0. \tag{4.12}$$

Note that $\mathcal{H}$ obtains the benefits from the type-k $\mathcal{B}$ by reducing the reorder interval $t(k)$ to $T(k)$ and passes down the benefit to $\mathcal{B}$ by paying a compensation $A(k)$. Expression (A.33) indicates that this compensation consists of two parts: the operational compensation $\widehat{B}(k, T(k)) - B^u(k)$ and the informational rent $r(k)$. Correspondingly, $u(k)$, the net cost for the type-k $\mathcal{B}$, constitutes of two components – the fixed operating cost $B^u(k)$, which is independent of $F(\cdot)$, and the informational reward $r(k)$. In addition, $r(k)$ and $A(k)$ are nonincreasing in k, while the net costs of the divisions $u(k)$ and $\widehat{H}(T(k))$ are nondecreasing in k.

Similarly, we can interpret the fixed cost-sharing contract in §4 in the following way: $\mathcal{H}$ pushes $\mathcal{B}$ to choose a smaller reorder interval $t(\alpha^h k)$ by paying $\mathcal{B}$ a side payment $(1-\alpha^h)k\theta(t(\alpha^h k))$ per order. The side payment consists of two parts: the operational compensation $B(s(\alpha^h k), t(\alpha^h k)|k) - B^u(k)$ and an extra compensation $B^u(k) - B^u(\alpha^h k)$.

The constraint (HA) is new to the semi-centralized system. When $T(k) = t(k)$, i.e., (HA) is binding, the contract is the least efficient. Moreover, when (HA) is binding, $\mathcal{H}$ has incentive to raise $T(k)$ above $t(k)$ to extract more benefit from the contract while pushing the supply chain into a worse condition. The headquarters hence plays a role to limit $\mathcal{H}$'s power when he has incentive conflict with the supply chain. Therefore it would be interesting to characterize when this constraint is tight at optimality and also when it is slack for *all* possible k. The condition (A.34) above already provides some characterization, but it can be strengthened under certain distributions of k. Let $\Delta(x) = H'(t(x)) + \theta'(t(x))\varphi(x)$.

Corollary 41. *Suppose $\omega(x)$ is nondecreasing.*

a) If $\Delta'(x) < 0$, then there exist k^0 such that $T(k) = t(k)$ when $k \geq k^0$ and $T(k) = \tilde{T}(k)$ when $k < k^0$.

b) If $\Delta'(x) \geq 0$, then there exist k^0 such that $T(k) = t(k)$ when $k \leq k^0$ and $T(k) = \tilde{T}(k)$ when $k > k^0$.

c) A sufficient condition for $\Delta'(x) \geq 0$ is that $\varphi(x)$ is nonincreasing.

Thus, under certain distributions of k, the binding region of (HA) can be an interval. A sufficient condition is

Condition 1. *$\omega(x)$ is nondecreasing and $\varphi(x)$ is nonincreasing.*

This holds holds when F is log-convex and $(F/f)' > -1$. Examples include the exponential distribution, the mirror image of the Pareto distribution. (For more examples and methods of constructing log-convex distributions see Aliprantis (2006))

We also have

Proposition 42. *Suppose the distribution of k satisfies Condition 1 and $\widehat{H}_T(\underline{k}, t(\underline{k})) > 0$. Then (HA) is slack for all k.*

Proposition 42 implies that when Condition 1 holds, the semi-centralized system reduces to a decentralized system. In this case, the role of the headquarters diminishes.

Deterministic Demand

In this subsection, we assume deterministic demand so to obtain closed-form expressions of the optimal contract terms. We use the same settings as in §5.2., i.e., $D(t) = \lambda t$ and $H(t) = at$, for some $\lambda > 0$ and $a > 0$, for all $t \geq 0$. Recall that $\gamma = \lambda hb/[2(h+b)]$. Let

$$\beta(k) = \sqrt{\frac{\omega(k)}{k}} = \sqrt{1 + \frac{\phi(k)}{k}}. \tag{4.13}$$

Equation (A.34) now becomes

$$a - \frac{x}{(T(x))^2} + \gamma - \frac{\varphi(x)}{(T(x))^2} = 0, \tag{4.14}$$

and the condition $\widehat{H}_T(\underline{k}, t(\underline{k})) > 0$ becomes $(\beta(\underline{k}))^2 \leq (a-\gamma)/\gamma$. (This implies $\eta > 1$.) Note that $\beta(k)$ is non-increasing in k, so $(\beta(k))^2 \leq (a-\gamma)/\gamma$ for all k.

The optimal contract menu is

$$\begin{aligned} T(k) &= \sqrt{\frac{\omega(k)}{a+\gamma}}, \\ A(k) &= k/T(k) + T(k)\gamma - 2\sqrt{k\gamma} + r(k), \quad k \in [\underline{k}, \bar{k}], \end{aligned}$$

where the information rent $r(k)$ for the type-k branch is

$$r(k) = 2\sqrt{k\gamma} - 2\sqrt{\bar{k}\gamma} + \int_k^{\bar{k}} \sqrt{\frac{a+\gamma}{\omega(x)}} dx.$$

Under the optimal contract menu, with a type-k $\mathcal{B}$, the costs of $\mathcal{B}$ and $\mathcal{H}$ are, respectively,

$$
\begin{aligned}
u(k) &= 2\sqrt{\bar{k}\gamma} - \int_k^{\bar{k}} \sqrt{\frac{a+\gamma}{\omega(x)}} dx \le B^u(k), \\
\widehat{H}(T(k)) &= \sqrt{\omega(k)}\sqrt{a+\gamma}\left(1 + \frac{k}{\omega(k)} + \frac{1}{\sqrt{\omega(k)}}\int_k^{\bar{k}} \sqrt{\frac{1}{\omega(x)}} dx\right) - 2\sqrt{\bar{k}\gamma}.
\end{aligned}
$$

Effect of Uncertainty

Suppose the fixed cost consists of two parts: $k = k_d + k_r$. The first part k_d is an increasing function of t_0, known to all parties, such as the international shipping cost. The second part k_r is independent with t_0 and is only known to $\mathcal{B}$, such as the local shipping, communication and personnel costs. We can assume $k_d = \underline{k}$, $k_r \sim F(\cdot)$ and $k_r \in [0, \Delta]$, where $\Delta = \bar{k} - \underline{k}$. The optimal contract reorder interval and informational rent for the type-k $\mathcal{B}$ are then simplified to:

$$
\begin{aligned}
T(k) &= \sqrt{\frac{k_0 + \kappa + F(\kappa)/f(\kappa)}{a+\gamma}} = \sqrt{(t^*(k_d))^2 + (T(k_r))^2}, \\
r(k) &= \sqrt{(\mathcal{B}(k_d))^2 + (\mathcal{B}(k_r))^2} - \mathcal{B}(\bar{k}) + \int_{k_d}^{\Delta} \frac{1}{\sqrt{(t^*(k_d))^2 + (T(x))^2}} dx, \quad k_d \in [0, \Delta],
\end{aligned}
$$

Thus, when $k_d >> \Delta$, e.g., when the international shipping cost has the dominate impact on the fixed cost, so $t^*(k_d) >> T(k_r)$ and $T(k) \approx t^*(k)$ for all realization of k_r hence $\mathcal{C}$ will be almost fully coordinated. If $k_d << \Delta$, e.g., when $\mathcal{B}$'s private local cost component has the dominate impact on the fixed cost, the information asymmetry causes $T(k) > t^*(k)$ in some realization of k_r, so $\mathcal{C}$ is less efficient.

Contract Effectiveness

We now use the closed-form expressions to discuss the effectiveness of the contract. Under the optimal contract menu, for any given branch type k, we have:

1) Compared with the uncoordinated system, the reorder interval is reduced by a factor of

$$T(k)/t(k) = \beta(k)\sqrt{\gamma/(a+\gamma)} \geq 1/\sqrt{3}.$$

The home-plant cost is reduced by a factor of

$$\widehat{H}(T(k))/H(t(k)) \leq \sqrt{\frac{a+\gamma}{a-\gamma}} \leq \sqrt{3},$$

and the supply chain cost is reduced by a factor of

$$\begin{aligned} C(T(k))/C(t(k)) &= \sqrt{\frac{a+\gamma}{\gamma}}\left(\frac{\beta(k)+1/\beta(k)}{2+a/\gamma}\right) \\ &\leq \sqrt{\frac{a+\gamma}{\gamma}}\left(\frac{\sqrt{(a-\gamma)/\gamma}+\sqrt{\gamma/(a-\gamma)}}{2+a/\gamma}\right). \end{aligned}$$

2) Compared with the centralized system, the reorder interval is increased by a factor of

$$t^*(k)/T(k) = 1/\beta(k) \geq \sqrt{\gamma/(a-\gamma)},$$

and supply-chain cost is increased by a factor of

$$C(T(k))/C(t^*(k)) = (\beta(k)+1/\beta(k))/2 \leq \left[\sqrt{(a-\gamma)/\gamma}+\sqrt{\gamma/(a-\gamma)}\right]/2.$$

Here, all the bounds are derived under the conditions for Proposition 42.

Set $h=1,\ b=8,\ \lambda=100,\ a=100$, and $k \sim \mathrm{U}[2000, 32000]$, Table 4.1 illustrates the contract terms and compares the reorder interval on the menu with those in the uncoordinated and centralized systems. In this example, the optimal contract menu is very effective, resulting in near optimal supply chain performance. For small k, the reduction of the reorder interval is quite significant. However, $\mathcal{H}$ may not be benefit from the contract for large k, even though its expected cost is optimized. As k increases, the information rent decreases rapidly, so does the compensation to $\mathcal{B}$. The bounds for the ratios of C are reasonably reliable, but not those for t and H.

Table 4.1: Contract Terms and Effectiveness

k	$t(k)$	$t^*(k)$	$T(k)$	$A(k)$	$r(k)$	$\frac{T(k)}{t(k)}$	$\frac{t^*(k)}{T(k)}$	$\frac{\hat{H}(T(k))}{H(t(k))}$	$\frac{C(T(k))}{C(t(k))}$	$\frac{C(T(k))}{C(t^*(k))}$
2000	6.71	3.72	3.72	772.82	666.25	0.55	1.00	1.71	0.85	1.00
4000	9.49	5.26	6.45	583.57	519.77	0.68	0.82	1.29	0.87	1.02
8000	13.42	7.44	9.84	435.55	377.97	0.73	0.76	1.06	0.88	1.04
16000	18.97	10.52	14.41	276.51	212.33	0.76	0.73	0.91	0.89	1.05
20000	21.21	11.77	16.22	218.55	150.22	0.76	0.73	0.87	0.89	1.05
24000	23.24	12.89	17.85	167.77	95.35	0.77	0.72	0.84	0.89	1.05
28000	25.10	13.92	19.34	122.08	45.69	0.77	0.72	0.82	0.89	1.05
32000	26.83	14.88	20.72	80.21	0.00	0.77	0.72	0.80	0.90	1.06
Bounds*						0.31	0.89	1.73	0.85	1.01

Bounds hold only under the condition $\hat{H}_T(\underline{k}, t(\underline{k})) > 0$

4.6 Concluding Remarks

In this paper, we discussed the concept of *a semi-centralized supply chain* and analyzed its coordination problems, motivated by our experience with a global company. The supply chain consists of a home plant and a foreign branch, both under the same parent company, but both have considerable autonomy and private information. The role of the home plant is to provide a key component to the foreign branch with guaranteed service (at 100% service level). As a result of a high fixed order cost, the branch orders the key component rather infrequently, so the home plant suffers from high inventory and expediting costs. When the headquarters take the role to initiate coordination, it is possible to have the supply chain fully coordinated. However, the home plant will passively disagree when the replenishment interval is not significantly reduced. Changing from this passive service role to a more proactive service role, the home plant considers to design a cost-sharing contract with the foreign branch so to reduce its own inefficiency. Given that the home plant is a subordinate in the company, this initiative needs approval from the headquarters. Thus, the headquarters has indirect influence on the contract. It needs to make sure that the contract will improve the entire supply chain, not only improve the efficiency of the home plant. It also needs to ensure that the home plant will not reap all the benefits of the contract by squeezing the foreign branch. These constraints are absent from previous supply chain coordination models. In this literature, either the headquarters (or a central planner) observes the problem and develops coordination mechanisms, or one party in the supply chain initiates a contract with the other party without the concern of getting a headquarters agreement.

Our study shows that it is possible for the home plant to design a cost-sharing contract that will pass the audit of the headquarters, both under full information and under asymmetric information about the fixed order cost of the foreign branch. Under symmetric information, the role of the headquarters diminishes completely and the contract form is very simple. It only specifies a cost sharing percentage, which is easy to compute. It is equivalent to agreeing to share certain steps in the replenishment order process. Also, the contract can lead to near optimal supply chain performance. Under information asymmetry, a menu of contracts can also significantly improve the supply chain performance, and sometimes lead to near optimal performance. Compared to the result under symmetric information, the supply chain performance is possible to be more efficient under asymmetric information.

Compared with a semi-centralized system, a centralized system is more efficient in cost control but has less flexibility, while a decentralized system has more flexibility but less supply chain efficiency (see Arnold 1999). However, many questions remain as to how to design a semi-centralized system: In a semi-centralized structure, what is the optimal allocation of decision power between the superior and subordinates? How to allocate coordination power to limit the internal conflict when investment in required to improve the supply chain performance? We shall leave these issues to future research. We hope our paper can help generate more interest in this general direction.

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Appendix A

Proofs of Results

A.1 Proof of Results in Chapter 2

Lemma 1 *M is* IIR provided

$$K_i\left(\theta^{t-1}\right) \leq w_i^t\left(r^t\right) \equiv \min_{\theta_i^t}\left[\bar{w}_i^t\left(r^t, \theta_i^t\right)\right]$$

PROOF: Given $K_i\left(\theta^{t-1}\right) \leq w_i^t\left(r^t\right)$, then,

$$\begin{aligned} W_i^t\left[\theta^{t-1}, \theta_i^t\right] - r_i^t V_i^t\left(\theta_i^t, 1\right) &= \bar{w}_i^t\left(r^t, \theta_i^t\right) - K_i\left(\theta^{t-1}\right) \\ &\geq \bar{w}_i^t\left(r^t, \theta_i^t\right) - w_i^t\left(r^t\right) \\ &\geq 0. \end{aligned}$$

■

Lemma 2 Assume $\theta_{iw}^t\left(r^t\right)$ is continuously differentiable. Then the control rights, r^{*t}, that maximize total willingness to pay, $\sum_i w_i^t\left(r^t\right)$, minimize the aggregate outside option value, $\sum_i r_i^t V_i^t\left(\theta_{iw}^t\left(r^t\right), 1\right)$.

PROOF: Consider the problem:

$$\max_{r^t} \Sigma_i w_i^t\left(r^t\right) \quad s.t. \begin{cases} r_i^t \geq 0 & \text{for all } i \\ \Sigma_i r_i^t = 1 & \end{cases} . \tag{A.1}$$

Given $\theta_{iw}^{t}\left(r^{t}\right)$ is differentiable, the envelop theorem holds (cf Thm 3.10.4 in Sydsaeter et al (2008)) and the Kuhn Tucker conditions for the solution to (A.1) are

$$-V_i^t\left(\theta_{iw}^t\left(r^t\right),1\right)+\mu_i^t+\rho^t=0 \tag{A.2}$$

$$\mu_i^t, r_i^t \geq 0 \,, \mu_i^t r_i^t=0,\ \sum\nolimits_i r_i^t=1,$$

where μ_i^t and ρ^t are multipliers for the constraints, $\{r_i^t \geq 0\}$ and $\{\sum_i r_i^t = 1\}$ respectively. Condition (A.2) implies:

$$V_i^t\left(\theta_{iw}^t\left(r^t\right),1\right)\begin{cases} = & \rho^t \quad \text{for } r_i^t>0 \\ \geq & \rho^t \quad \text{for } r_i^t=0 \end{cases}$$

and therefore,

$$r_i^{*t}=\begin{cases} =0 & \text{if} \quad V_i^t\left(\theta_{iw}^t\left(r^t\right),1\right) > V_j^t\left(\theta_{jw}^t\left(r^t\right),1\right) \text{ for } j\neq i \\ \in[0,1] & \text{if} \quad V_i^t\left(\theta_{iw}^t\left(r^t\right),1\right) \leq V_j^t\left(\theta_{jw}^t\left(r^t\right),1\right) \text{ for } j\neq i \end{cases}. \tag{A.3}$$

From (A.3) it follows,$r_i^{*t}>0$ implies

$$V_i^t\left(\theta_{iw}^t\left(r^{*t}\right),1\right)=\min_j V_j^t\left(\theta_{jw}^t(r^{*t}),1\right).$$

Therefore

$$\sum\nolimits_i r_i^{*t}V_i^t\left(\theta_{iw}^t\left(r^{*t}\right),1\right)=\min_i V_i^t\left(\theta_{iw}^t(r^{*t}),1\right)=\min_{r^t}\sum\nolimits_i r_i^t V_i^t\left(\theta_{iw}^t\left(r^t\right),1\right). \tag{A.4}$$

■

Lemma 3 $B^t\left(r^{*t}\right)>0$.

PROOF

Recall that condition (A.4) in the proof of Lemma 2 implies that the control rights r^{*t} are selected to minimize the aggregate outside option, such that

$$\sum\nolimits_i r_i^{*t}\left\{V_i^t\left(\theta_{iw}^t,1\right)\right\}=\min_i V_i^t\left(\theta_{iw}^t,1\right)$$

Therefore $B^t\left(r^{*t}\right)$ can be expressed as,

$$B^t\left(r^{*t}\right) = -\min_i V_i^t\left(\theta_{iw}^t, 1\right) + \sum\nolimits_i E_{\theta_{-i}^t}\left[V^t\left(\theta_{iw}^t, \theta_{-i}^t\right)\right] - (N-1)E\left[V^t\left(\theta^t\right) \mid \theta^{t-1}\right] \tag{A.5}$$

Writing out the expression for $B\left(r^{*t}\right)$ in terms of the profits functions $\pi_i\left(\theta_i^t, x_i^*\left(\theta^t\right)\right)$ we have

$$\begin{aligned} B^t\left(r^{*t}\right) = & -\min_i\left[V_i^t\left(\theta_{iw}^t, 1\right)\right] \\ & +\sum\nolimits_i E_{\theta_{-i}^t}\left[\pi_i\left(\theta_{iw}^t, x_i^*\left(\theta_{iw}^t, \theta_{-i}^t\right)\right)\right. \\ & \qquad \left.+\sum\nolimits_{k=t+1}^T \delta^{k-t} E_{\theta^k}\left[\pi_i\left(\theta_i^k, x_i^*\left(\theta^k\right)\right)\right] \mid \left(\theta_{iw}^t, \theta_{-i}^t\right)\right] \\ & +\sum\nolimits_i \sum\nolimits_{j \neq i}\left[E_{\theta_{-i}^t}\left[\pi_j\left(\theta_j^t, x_i^*\left(\theta_{iw}^t, \theta_{-i}^t\right)\right)\right.\right. \\ & \qquad \left.\left.+\sum\nolimits_{k=t+1}^T \delta^{k-t} E_{\theta^k}\left[\pi_j\left(\theta_j^k, x_i^*\left(\theta^k\right)\right) \mid \left(\theta_{iw}^t, \theta_{-i}^t\right)\right]\right]\right] \\ & -(N-1)E\left[V^t\left(\theta^t\right) \mid \theta^{t-1}\right] \end{aligned} \tag{A.6}$$

Notice the allocations $x_i^*(\theta^t)$ and $x_i^*\left(\theta^k \mid \theta^t\right)$ are inefficient relative to the allocations $x_i^*\left(\theta_{iw}^t, \theta_{-i}^t\right)$ and $\left(x_i^*\left(\theta^k\right)\right) \mid \left(\theta_{iw}^t, \theta_{-i}^t\right)$ because they are not conditioned on θ_{iw}^t. Hence substituting the allocations $x_i^*(\theta^t)$ and $x_i^*\left(\theta^k \mid \theta^t\right)$ for the efficient allocations

$x_i^*\left(\theta_{iw}^t,\theta_{-i}^t\right)$ and $\left(x_i^*\left(\theta^k\right)\right)\mid\left(\theta_{iw}^t,\theta_{-i}^t\right)$ in (A.6) we have

$$\begin{aligned}
B^t\left(r^{*t}\right) \geq & -\min_i\left[V_i^t\left(\theta_{iw}^t,1\right)\right] \\
& +\sum\nolimits_i E_{\theta_{-i}^t}\left[\pi_i\left(\theta_{iw}^t,x_i^*\left(\theta^t\right)\right)+\sum\nolimits_{k=t+1}^T \delta^{k-t}E_{\theta^k}\left[\pi_i\left(\theta_i^k,x_i^*\left(\theta^k\mid\theta^t\right)\right)\right]\right] \\
& +\sum\nolimits_i\sum\nolimits_{j\neq i}\left[E_{\theta_{-i}^t}\left[\pi_j\left(\theta_j^t,x_j^*\left(\theta^t\right)\right)\right.\right. \\
& \quad \left.\left.+\sum\nolimits_{k=t+1}^T \delta^{k-t}E_{\theta^k}\left[\pi_j\left(\theta_j^t,x_j^*\left(\theta^k\right)\right)\mid\left(\theta^t\right)\right]\right]\right] \\
& -(N-1)E\left[V^t\left(\theta^t\right)\mid\theta^{t-1}\right] \qquad \text{(A.7)}
\end{aligned}$$

$$\begin{aligned}
= & -\min_i\left[V_i^t\left(\theta_{iw}^t,1\right)\right] \\
& +\sum\nolimits_i E_{\theta_{-i}^t}\left[\pi_i\left(\theta_{iw}^t,x_i^*\left(\theta^t\right)\right)+\sum\nolimits_{k=t+1}^T \delta^{k-t}E_{\theta^k}\left[\pi_i\left(\theta_i^k,x_i^*\left(\theta^k\mid\theta^t\right)\right)\right]\right] \qquad \text{(A.8)}
\end{aligned}$$

$$\begin{aligned}
\geq & -\min_i\left[V_i^t\left(\theta_{iw}^t,1\right)\right] \\
& +\min_{(\bar{x}_1,,\bar{x}_N)}\sum\nolimits_i\left[\pi_i\left(\theta_{iw}^t,\bar{x}_i\right)+\sum\nolimits_{k=t+1}^T \delta^{k-t}E_{\theta^k}\left[\pi_i\left(\theta_i^k,\bar{x}_i\right)\right]\right] \qquad \text{(A.9)}
\end{aligned}$$

$$= \quad 0 \qquad \text{(A.10)}$$

(A.8) follows from noticing that the third and fourth expressions in (A.7) cancel out each other. (A.9) results from substituting in the constant allocations $(\bar{x}_{1,,}\bar{x}_N)$ that minimize $\sum_i \pi_i\left(\theta_{iw}^t,\bar{x}_i\right) + \sum_{k=t+1}^T \delta^{k-t}E_{\theta_i^k}\left[\pi_i\left(\theta_i^k,\bar{x}_i\right)\mid\theta_{iw}^t\right]$ into (A.8). Finally we obtain (A.10) because

$\min_i\left[V_i^t\left(\theta_{iw}^t,X\right)\right]=\sum_i \pi_i\left(\theta_{iw}^t,\bar{x}_i\right)+\sum_{k=t+1}^T \delta^{k-t}E_{\theta_i^k}\left[\pi_i\left(\theta_i^k,\bar{x}_i\right)\mid\theta_{iw}^t\right]$ since π_i is strictly concave in x_i and therefore attains a minimum at extreme values of $\bar{x}$.■

Proposition 4 The dynamic mechanism, $M\left\langle x^*\left(\tilde{\theta}^t\right),\tau^t\left(\tilde{h}^t\right),r^{*(t+1)}\left(\tilde{\theta}^t\right)\right\rangle_{t=1}^T$, implements the efficient allocation x^* as a Bayesian incentive compatible, individually rational and balanced budget equilibrium.

PROOF: The proof proceeds in three steps:

(i) M is BIC : This requires we show that truth telling is a mutual best response equilibrium for all i. Suppose members $j \neq i$ truthfully disclose. By the single stage deviation principle, it suffices to show following any history, $\tilde{h}_i^{t-1}$, $i's$ best response is to disclose truthfully in period t, if he expects to disclose truthfully in all future periods. Given M, $i's$ continuation value for the reported history and demand, $\left(\tilde{h}_i^{t-1}, h_{-i}^{t-1}, \theta_i^t\right)$, when he reports $\tilde{\theta}_i^t$ is,

$$\begin{aligned} W_i^t\left[\tilde{h}_i^{t-1}, h_{-i}^{t-1}, \theta_i^t \mid \tilde{\theta}_i^t\right] = & E_{\theta_{-i}^t}\left[\tau_i^t\left(\tilde{h}_i^{t-1}, h_r^{t-1}, \tilde{\theta}_i^t, \theta_{-i}^t\right) + \pi_i\left(\theta_i^t, x^*\left(\tilde{\theta}_i^t, \theta_{-i}^t\right)\right)\right. \\ & \left. +\delta E W_i^{t+1}\left[\tilde{h}_i^t, h_{-i}^t\right]\right]. \end{aligned} \tag{A.11}$$

Substituting for $\tau_i^t\left(\tilde{h}_i^{t-1}, h_r^{t-1}, \tilde{\theta}_i^t, \theta_{-i}^t\right)$ from (3.18) into (A.11) yields

$$W_i^t\left[\tilde{h}_i^{t-1}, h_{-i}^{t-1}, \theta_i^t \mid \tilde{\theta}_i^t\right] = E_{\theta_{-i}^t}\left[\sum_i \pi_i\left(\theta_i^t, x^*\left(\tilde{\theta}_i^t, \theta_{-i}^t\right)\right) + \delta E V^{t+1}\left(\theta^{t+1}\right)\right], \tag{A.12}$$

which is clearly maximized by truthful disclosure, $\tilde{\theta}_i^t = \theta_i^t$. Thus M is BIC.

(ii) M is BB: Note by construction the transfers $\left\{\tau_i^t\left(\tilde{h}^t\right)\right\}$ satisfy ex post budget balance, since

$$\sum_i \tau_i^t\left(\tilde{h}^t\right) = \sum_i\left[\tilde{p}_i^t\left(\tilde{h}^t\right) - \frac{\Sigma_{j\neq i}\left(\tilde{p}_j^t\left(\tilde{h}^t\right) - E_\theta \tilde{p}_j^t\left(\tilde{h}^t\right)\right)}{N-1}\right] = 0$$

(iii) M is IIR : Along the equilibrium path note,

$$\begin{aligned} & W_i^t\left[\tilde{h}_i^{t-1}, h_{-i}^{t-1}, \theta_i^t \mid \theta_i^t\right] - r_i^{*t} V_i^t\left(\theta_i^t, 1\right) \\ = \; & E_{\theta_{-i}^t} V^t\left(\theta^t\right) - r_i^{*t} V_i^t(\theta_i^t, 1) + \tfrac{B^t\left(r^{*t}\right)}{N} - w_i^t\left(r^t\right) \\ > \; & 0. \end{aligned}$$

■

Proposition 6 1) $V^t(\theta^t,\ \tilde{x}^{t-1})$ is coordinatewise linear in $\tilde{x}^{t-1}$ and hence $E_{\theta^t}[V^t(\theta^t,\ \tilde{x}^{t-1})]$ is coordinatewise linear in $\tilde{x}^{t-1}$. Thus, $\Delta_{\tilde{x}_i^t} J^t$ and $\Delta^t(\theta^t)$ are independent with $\tilde{x}^t$.

2) The optimal allocation is: if $\Delta^t(\theta^t) \geq 0$, then $\tilde{x}_j^{*t} = 1 - \sum_{i \neq j} \tilde{x}_i^t$ and $\tilde{x}_i^{*t} = \tilde{x}_i^{t-1}$ for all $i \neq j$, where $j = \arg\max_j \{\partial E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)]/\partial x_j^t\}$. If $\Delta^t(\theta^t) < 0$, then $\tilde{x}_i^{*t} = \tilde{x}_i^{t-1}$.

3) The supply chain optimal cost, $V^t(\theta^t,\ \tilde{x}^{t-1})$, given θ^t is convex in θ^t. Moreover, it is also concave in θ^t implying that $V^t(\theta^t,\ \tilde{x}^{t-1})$ is a hyperplane in θ^t.

PROOF: 1) The statement is true at the end of period $T+1$. Assume the statement is true for $t+1$, i.e. $\partial V^{t+1}(\theta^{t+1},\ \tilde{x}^t)/\partial x_j^t$ is a constant. Then

$$\Delta_{\tilde{x}_i^t} J^t = c_1 \theta_1^t + \partial E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)]/\partial x_i^t$$

is a constant, and hence

$$\Delta^t(\theta^t) = c_1 \theta_1^t + \max_j \{\partial E[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)]/\partial x_j^t\}$$

is a constant.

The optimal policy is to allocate all the rest of the capacity to one drug dealer or not allocate any capacity to any drug dealer. If $\tilde{x}_i^t = \tilde{x}_i^{t-1}$, then

$$\partial V^t(\theta^t,\ \tilde{x}^{t-1})/\partial x_j^{t-1} = \Delta_{\tilde{x}_i^t} J^t;$$

if $\tilde{x}_i^t = 1 - \sum_{j \neq i} \tilde{x}_j^{t-1}$, then

$$\partial V^t(\theta^t,\ \tilde{x}^{t-1})/\partial x_j^{t-1} = 0.$$

This proves the statement.

2) follows directly from 1).

3) Assume $V^t(\cdot,\ \cdot)$ is convex in θ at the end of period $T+1$. Assume the statement is true for $t+1$, i.e. $V^{t+1}(\theta^{t+1},\ \tilde{x}^t)$ is convex. Since θ^{t+1} is a linear function of θ^t, $E_{\theta^{t+1}}[V^{t+1}(\theta^{t+1},\ \tilde{x}^t)]$ is convex in θ^t and $J^t(\theta^t,\ \tilde{x}^t)$ is convex in θ^t. Optimizing over $\tilde{x}^t$, we have $V^t(\theta^t,\ \tilde{x}^{t-1})$ is convex in θ^t. Following the same logic, since $V^t(\cdot,\ \cdot)$ is also concave in θ at the end of period $T+1$, thus $V^t(\theta^t,\ \tilde{x}^{t-1})$ is concave in θ^t. This proves the statement. ■

Proposition 7*A* sequential all-pay auction with entry fees and bid payments $\left\{F_i^t\left(r^{*t}\right), m_i^t\left(\tilde{b}_i^t, \tilde{b}_{-i}^t\right)\right\}_{i=1}^N$ in each period $t=1,...,T$ implements the SSS agreement. In equilibrium each member i bids,

$$b_i^t\left(\theta_i^t\right) = E_{\theta_{-i}^t}\left(\Sigma_{j\neq i}\pi_j\left(\theta_j^t, x_j^*\left(\theta^t\right)\right)\right) \tag{A.13}$$

PROOF: The proof is to show that $\{b_i^t\left(\theta_i^t\right)\}_{i=1}^N$ are strictly increasing Bayesian equilibrium bid functions for this sequential auction. Suppose all members j different from i follow the bidding strategy, $\left\{b_j^t\left(\theta_j^t\right)\right\}$. Then $i's$ best response in period t, given he intends to adopt $b_i^{t'}\left(\theta_i^{t'}\right)$ in all future periods, $t'>t$ is the solution to:

$$\begin{aligned}\max_{\tilde{b}_i^t} U_i^t\left(\theta_i^t, \tilde{b}_i^t\right) &= \max_{\tilde{b}_i^t} E_{\theta_{-i}^t}\left[\pi_i\left(\theta_i^t, x_i^*((b_i^t)^{-1}\left(\tilde{b}_i^t\right), \theta_{-i}^t\right) + m_i^t\left(\tilde{b}_i^t, b_{-i}^t\left(\theta_{-i}^t\right)\right)\right. \\ &\quad \left. + \delta E\left[U_i^{t+1}\left(\theta_i^{t+1}, b_i^{t+1}\left(\theta_i^{t+1}\right)\right) \mid \theta_i^t\right]\right] - F_i^t\left(r^{*t}\right) \qquad \text{(A.14)}\end{aligned}$$

$$\begin{aligned}&= \max_{\tilde{b}_i^t} E_{\theta_{-i}^t}\left[\sum_j \pi_j\left(\theta_j^t, x_j^*\left(\left(b_i^t\right)^{-1}\left(\tilde{b}_i^t\right), \theta_{-i}^t\right)\right) + \delta E V^{t+1}\left(\theta^{t+1}\right)\right] \\ &\quad - F_i^t\left(r^*\right) \qquad \text{(A.15)}\end{aligned}$$

where we've substituted for $m_i^t\left(\tilde{b}_i^t, b_{-i}^t\left(\theta_{-i}^t\right)\right)$ from (2.12) into (A.14) to obtain

(A.15). Differentiating $U_i^t\left(\theta_i^t, \tilde{b}_i^t\right)$ *wrt* to $\tilde{b}_i^t$ to identify the local maximum we obtain:

$$\frac{d}{d\tilde{b}_i^t} E_{\theta_{-i}^t} \sum_j \pi_j \left(\theta_j^t, x_j^*\left(\left(b_i^t\right)^{-1}\left(\tilde{b}_i^t\right), \theta_{-i}^t\right)\right) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \text{ for } \left(b_i^t\right)^{-1}\left(\tilde{b}_i^t\right) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \theta_i^t$$

Thus $i's$ best response is to bid $\tilde{b}_i^t = b_i^t\left(\theta_i^t\right)$ which is strictly increasing in θ_i^t and is therefore invertible. Hence the bid reveals the member's current period demand. Therefore each member $i's$ allocation, $x_i^*\left(\theta^t\right)$, can be determined by the members' bids which reveal their current period demand for capacity. The members with the higher bids receive the higher allocations. Moreover the payments and fees are constructed to satisfy IIR and BB. This sequential auction therefore implements the SSS agreement.■

Proposition 9 I^* is the unique Nash equilibrium to the ex ante investment game where each firm invests to maximize his expected continuation profit.

PROOF: Suppose i expects other members different from i, to invest I_{-i}^*. Then i selects I_i to

$$\max_{I_i} W_i\left(I_i, I_{-i}^*\right).$$

His best response is $I_i = I_i^*$. Since this is true for all i, I^* is a Nash equilibrium. I^* is the unique equilibrium since it is the only investment profile where each member maximizes its expected surplus, given the investments of the other partners. ■

A.2 Proof of Results in Chapter 3

Proposition 12: For any given t,

1) $x^{t*}(\theta^t)$ is increasing in θ_r^t, decreasing in θ_s^t.

2) $\Pi^t(\theta^t)$ is increasing and convex in θ_r^t, decreasing and convex in θ_s^t. Moreover, $\Pi^t(\theta^t)$ is jointly convex in θ^t and hence $E[\Pi^t(\theta^t)|\theta^{t-1}]$ is jointly convex in θ^{t-1}.

3) If the two market conditions that one is strictly better then the other, i.e. $\theta_s^{t-1} <$

ϕ_s^{t-1} and $\theta_r^{t-1} > \phi_r^{t-1}$, then $x^{(t-1)*}(\theta^{t-1}) > x^{(t-1)*}(\phi^{t-1})$, and the expected quality in period t given the realization in period $t-1$ satisfy $E[x^{t*}(\theta^t)|x^{(t-1)*}(\theta^{t-1})] \geq E[x^{t*}(\phi^t)|x^{(t-1)*}(\phi^{t-1})]$.

PROOF:

1) x^* satisfies the first order condition

$$\frac{\partial C(x^{t*}, \theta_s^t)}{\partial x} = \frac{\partial R(x^{t*}, \theta_r^t)}{\partial x}.$$

Therefore, taking derivatives on both sides, we have

$$\frac{\partial x^{t*}}{\partial \theta_r^t} = \frac{\frac{\partial^2 R(x^{t*},\theta_r^t)}{\partial x \partial \theta_r^t}}{\frac{\partial^2 C(x^{t*},\theta_s^t)}{\partial x^2} - \frac{\partial^2 R(x^{t*},\theta_r^t)}{\partial x^2}} > 0$$

and

$$\frac{\partial x^{t*}}{\partial \theta_s^t} = -\frac{\frac{\partial^2 C(x^{t*},\theta_s^t)}{\partial x \partial \theta_s^t}}{\frac{\partial^2 C(x^{t*},\theta_s^t)}{\partial x^2} - \frac{\partial^2 R(x^{t*},\theta_r^t)}{\partial x^2}} < 0.$$

2)

$$\frac{\partial \Pi^{t*}}{\partial \theta_r^t} = \frac{\partial R(x^{t*}, \theta_r^t)}{\partial \theta_r^t} > 0$$

$$\frac{\partial^2 \Pi^{t*}}{\partial \theta_r^t \partial \theta_r^t} = \frac{\partial^2 R(x^{t*}, \theta_r^t)}{\partial \theta_r^t \partial \theta_r^t} + \frac{\partial^2 R(x^{t*}, \theta_r^t)}{\partial \theta_r^t \partial x} \frac{\partial x^{t*}}{\partial \theta_r^t} > 0.$$

$$\frac{\partial \Pi^{t*}}{\partial \theta_s^t} = -\frac{\partial C(x^{t*}, \theta_s^t)}{\partial \theta_s^t} < 0$$

$$\frac{\partial^2 \Pi^{t*}}{\partial \theta_s^t \partial \theta_s^t} = -\frac{\partial^2 C(x^{t*}, \theta_s^t)}{\partial \theta_s^t \partial \theta_s^t} - \frac{\partial^2 C(x^{t*}, \theta_s^t)}{\partial \theta_s^t \partial x} \frac{\partial x^{t*}}{\partial \theta_s^t} > 0.$$

Therefore, Π^{t*} is coordinatewise convex in θ^t.

$$\frac{\partial^2\Pi^{t*}}{\partial\theta_s^t\partial\theta_s^t}+\frac{\partial^2\Pi^{t*}}{\partial\theta_r^t\partial\theta_r^t}-\left[\frac{\partial^2\Pi^{t*}}{\partial\theta_s^t\partial\theta_r^t}\right]^2=$$

$$-\frac{\partial^2 C(x^{t*},\theta_s^t)}{\partial\theta_s^t\partial\theta_s^t}\frac{\partial^2 R(x^{t*},\theta_r^t)}{\partial\theta_r^t\partial\theta_r^t}--\frac{\partial^2 C(x^{t*},\theta_s^t)}{\partial\theta_s^t\partial\theta_s^t}\frac{\partial^2 R(x^{t*},\theta_r^t)}{\partial\theta_r^t\partial x}\frac{\partial x^{t*}}{\partial\theta_r^t}$$

$$-\frac{\partial^2 R(x^{t*},\theta_r^t)}{\partial\theta_r^t\partial\theta_r^t}\frac{\partial^2 C(x^{t*},\theta_s^t)}{\partial\theta_s^t\partial x}\frac{\partial x^{t*}}{\partial\theta_s^t}>0.$$

Since the Π^{t*} is coordinatewise convex in θ^t, the above calculation shows that Π^{t*} is jointly convex in θ^t. Note that θ^t is linear in θ^{t-1}. Therefore $E[\Pi^{t*}|\theta^{t-1}]$ is jointly convex in θ^{t-1}.

Lemma 13: M is IIR provided $K_i\left(\theta^{t-1}\right)\leq w_i^t\left(\rho^t\right).$

PROOF: Given $K_i\left(\theta^{t-1}\right)\leq w_i^t\left(\rho^t\right)$, then,

$$\begin{aligned}
& W_i^t\left[\theta^{t-1},\theta_i^t\right]-O_i^t(\theta_{iw}^t(\rho^t)) \\
= \;& E_{\theta_{-i}^t}V^{t*}\left(\theta^t\right)-K_i\left(\theta^{t-1}\right)-O_i^t(\theta_{iw}^t(\rho^t)) \\
\geq \;& E_{\theta_{-i}^t}\left[V^{t*}\left(\theta^t\right)-O_i^t(\theta_{iw}^t(\rho^t))\right]-w_i^t\left(\rho^t\right) \\
\geq \;& 0
\end{aligned}$$

□

Lemma 14: The property rights, ρ^{t*},that maximize total willingness to pay to participate, $w_r\left(\rho^t\right)+w_s\left(\rho^t\right)$, minimize the members' aggregate outside option value, $\sum_i\rho_i^t\left[V_i^t\left(x_i^o,\theta_{iw}^t\left(\rho^t\right)\right)+V_{-i}^t\left(x_{-i}^o,\theta_{-iw}^t\left(\rho^t\right)\right)\right].$

PROOF Consider the problem:

$$\max_{\rho^t} \sum_i w_i^t \left(\rho^t\right) \tag{A.16}$$

$$s.t.\ \rho_i^t \geq 0,\ \sum\nolimits_i \rho_i^t = 1.$$

Given our assumption 2 on $F_i\left(\theta_i^t \mid \theta_i^{t-1}\right)$ and $E\left[\theta_i^t \mid \theta_i^{t-1}\right]$, the min in (A.16) is finite and $\theta_{iw}^t\left(\rho^t\right)$ is uniquely defined[1] by

$$\frac{d}{dz} E_{\theta_{-i}^t} V^{t*}\left(z, \theta_{-i}^t\right) - \frac{d}{dz} O_i^t(z) \left\{\begin{array}{c} \geq \\ = \\ \leq \end{array}\right\} 0 \ \Rightarrow\ \theta_{iw}^t\left(\rho^t\right) = \left\{\begin{array}{c} 0 \\ z \\ 1 \end{array}\right\}$$

By the envelop theorem the necessary Kuhn Tucker conditions for the solution to (A.16) are

$$-V_i^t\left(x_i^o, \theta_{iw}^t\left(\rho^t\right)\right) - V_{-i}^t\left(x_i^o, \theta_{-iw}^t\left(\rho^t\right)\right) + \mu_i^t + \lambda^t = 0 \tag{A.17}$$

$$\mu_i^t \rho_i^t = 0,\ \mu_i^t, \rho_i^t \geq 0, \sum\nolimits_i \rho_i^t = 1$$

where μ_i^t and λ^t are multipliers for the $\rho_i^t \geq 0$ and $\sum_i \rho_i^t = 1$ constraints respectively. Eq (A.17) implies:

$$\begin{aligned} \Sigma_j V_j^t\left(x_i^o, \theta_{jw}^t\left(\rho^t\right)\right) &= \lambda^t \text{ for } \rho_i^t > 0 \\ \Sigma_j V_j^t\left(x_i^o, \theta_{jw}^t\left(\rho^t\right)\right) &\geq \lambda^t \text{ for } \rho_i^t = 0 \end{aligned}$$

Consequently,

$$\rho_i^{t*} = \begin{cases} 0 \text{ if } \Sigma_j V_j^t\left(x_i^o, \theta_{jw}^t\left(\rho^t\right)\right) > \Sigma_i V_i^t\left(x_j, \theta_{iw}^t\left(\rho^t\right)\right) \\ \in (0,1] \ \text{ if } \Sigma_j V_j^t\left(x_i^o, \theta_{jw}^t\left(\rho^t\right)\right) = \Sigma_i V_i^t\left(x_j, \theta_{iw}^t\left(\rho^t\right)\right) \end{cases} \tag{A.18}$$

and therefore

[1] Assumption 2 part 2 the first two inequalities ensure period t's profit is strictly convex in θ_i^t; the third equality ensures the expected profit staring from period $t+1$ is strictly convex in θ_i^t.

$$\min_{\rho^t} \sum\nolimits_i \rho_i^t \left[V_i^t\left(x_i^o, \theta_{iw}^t\left(\rho^t\right)\right) + V_{-i}^t\left(x_{-i}^o, \theta_{-iw}^t\left(\rho^t\right)\right)\right]$$

$$= \sum\nolimits_i \rho_i^{t*} \left[V_i^t\left(x_i^o, \theta_{iw}^t\left(\rho^{t*}\right)\right) + V_{-i}^t\left(x_{-i}^o, \theta_{-iw}^t\left(\rho^{t*}\right)\right)\right].$$

□

Lemma 15: Under Assumption 1, $B\left(\rho^{t*}\right) > 0$

PROOF: $B^t\left(\rho^{t*}\right)$ can be written in terms of u_is:

$$\begin{aligned}
B^t\left(\rho^{t*}\right) &= \sum\nolimits_i E_{\theta_{-i}^t}\left[V^{t*}\left(\theta_{iw}^t, \theta_{-i}^t\right)\right] - E\left[V^{t*}\left(\theta^t\right) \mid \theta^{t-1}\right] \\
&\quad - \min\left\{V_r^t\left(1, \theta_{rw}^t\right) + V_s^t\left(1, \theta_{sw}^t\right), V_r^t\left(0, \theta_{rw}^t\right) + V_s^t\left(0, \theta_{sw}^t\right)\right\} \\
&= \sum\nolimits_i E_{\theta_{-i}^t} \sum_{k=t}^{T} \left(E\left[R(x^*(\theta^k), \theta_r^k) - C(x^*(\theta^k), \theta_s^k) | \theta_{iw}^t\right]\right) - E\left[V^{t*}\left(\theta^t\right) \mid \theta^{t-1}\right] \\
&\quad - \min\left\{V_r^t\left(1, \theta_{rw}^t\right) + V_s^t\left(1, \theta_{sw}^t\right), V_r^t\left(0, \theta_{rw}^t\right) + V_s^t\left(0, \theta_{sw}^t\right)\right\}
\end{aligned}$$

Note that, if we substitute $x^*(\theta^k|\theta_{iw}^t)$ by $x^*(\theta^k)$ without the condition $\theta_i^t = \theta_{iw}^t$, then we get an inequality as follows:

$$\begin{aligned}
B\left(\rho^{t*}\right) &\geq \sum_{k=t}^{T} \left(E\left[R(x^*(\theta^k), \theta_r^k) | \theta_{rw}^t\right] - E\left[C(x^*(\theta^k), \theta_s^k) | \theta_{sw}^t\right]\right) \\
&\quad - \min\left\{V_r^t\left(1, \theta_{rw}^t\right) + V_s^t\left(1, \theta_{sw}^t\right), V_r^t\left(0, \theta_{rw}^t\right) + V_s^t\left(0, \theta_{sw}^t\right)\right\}
\end{aligned}$$

Without loss of generality, assume $V_r^t\left(1, \theta_{rw}^t\right) + V_s^t\left(1, \theta_{sw}^t\right) \geq V_r^t\left(0, \theta_{rw}^t\right) + V_s^t\left(0, \theta_{sw}^t\right)$ (the other side of inequality can be proved following the same logic). From Assumption 2, $V_r^t\left(1, \theta_{rw}^t\right) + V_s^t\left(1, \theta_{sw}^t\right) \geq V_r^t\left(0, \theta_{rw}^t\right) + V_s^t\left(0, \theta_{sw}^t\right)$ if and only if $E[R(1, \theta_r^k)|\theta_{rw}^t] - E[C(1, \theta_s^k)|\theta_{sw}^t] \geq E[R(0, \theta_r^k)|\theta_{rw}^t] - E[C(0, \theta_s^k)|\theta_{sw}^t]$. Note that $E[R(x, \theta_r^k)|\theta_{rw}^t] - E[C(x, \theta_s^k)|\theta_{sw}^t]$ is concave in x, therefore, the minima is achieved at the extreme point $x = 0$. Thus,

$$\begin{aligned} & E\left[R(x^*(\theta^k), \theta_r^k)|\theta_{rw}^t\right] - E\left[C(x^*(\theta^k), \theta_s^k)|\theta_{sw}^t\right] \\ \geq \; & \min_x \left\{E\left[R(x, \theta_r^k)|\theta_{rw}^t\right] - E\left[C(x, \theta_s^k)|\theta_{sw}^t\right]\right\} \\ = \; & E[R(0, \theta_r^k)|\theta_{rw}^t] - E[C(0, \theta_s^k)|\theta_{sw}^t]. \end{aligned}$$

This proves the result. □

Theorem 16: EASQS exists. The optimal quality level x^* is Bayesian incentive compatible, individually rational and balanced budget in the dynamic mechanism $M\left\langle x^*\left(\tilde{\theta}^t\right), \tau^t\left(\tilde{h}^t\right), \rho^{(t+1)*}\left(\tilde{\theta}^t\right)\right\rangle_{t=1}^{T}$.

PROOF We show the payment constructed in part 2) satisfies (EE), (BIC), (IIR) and (BB).

(i)M is BIC : To demonstrate M is BIC it is sufficient to show that truth telling is a mutual best response equilibrium for all members i. Suppose that members $j \neq i$ always truthfully disclose. By the single deviation principle, it suffices to show that following any history, $\tilde{h}_i^{t-1}$, member $i's$ best response is to disclose truthfully in period t, if i expects to disclose truthfully in all future periods. Given M, $S's$ expected profit for the reported history and cost, $\left(\tilde{h}_i^{t-1}, h_{-i}^{t-1}, \theta_i^t\right)$, given he reports $\tilde{\theta}_i^t$ is,

$$\begin{aligned} W_s^t\left[\tilde{h}_s^{t-1}, h_r^{t-1}, \theta_i^t \mid \tilde{\theta}_i^t\right] \;=\; & E_{\theta_r^t}\left[\tau_i^t\left(\tilde{h}_s^{t-1}, h_r^{t-1}, \tilde{\theta}_s^t, \theta_r^t\right) - C\left(x^*\left(\tilde{\theta}_s^t, \theta_r^t\right)\right)\right. \\ & \left. + \delta E W_s^{t+1}\left[\tilde{h}_s^t, h_r^t, \theta_i^{t+1} \mid \tilde{\theta}_i^{t+1}\right]\right] \end{aligned} \quad \text{(A.19)}$$

For M to be BIC requires

$$W_s^t\left[\tilde{h}_s^{t-1}, h_r^{t-1}, \theta_s^t \mid \theta_s^t\right] \geq W_s^t\left[\tilde{h}_s^{t-1}, h_r^{t-1}, \theta_s^t \mid \tilde{\theta}_s^t\right] \text{ for all } \tilde{\theta}_s^t, \theta_s^t \quad \text{(A.20)}$$

Substituting for the transfer, τ_i, from (3.18) into (A.19) yields,

$$\begin{aligned} W_s^t\left[\tilde{h}_s^{t-1}, h_r^{t-1}, \theta_s^t \mid \tilde{\theta}_s^t\right] &= E_{\theta_r^t}\left[R\left(x^*\left(\tilde{\theta}_s^t, \theta_r^t\right), \theta_r^t\right) - C\left(x^*\left(\tilde{\theta}_s^t, \theta_r^t\right), \theta_s^t\right)\right. \\ &\quad \left. + \delta E W_s^{t+1}\left[\tilde{h}_s^t, h_r^t, \theta_i^{t+1} \mid \tilde{\theta}_i^{t+1}\right] + \delta E_{\theta_r^t} \tilde{w}_s\left(\rho^{(t+1)*}\left(\tilde{\theta}_s^t, c_r^t\right)\right)\right] \\ &\quad + (OT) \end{aligned} \quad (A.21)$$

where (OT) is a constant term that does not depend on $\tilde{\theta}_s^t$.

Under the maintained assumption that all members truthfully disclose for all periods after t the expected profit, $\delta E W_s^{t+1}\left[\tilde{h}_s^t, h_r^t, \theta_r^{t+1} \mid \tilde{\theta}_s^{t+1}\right]$, in the second line of (A.21) satisfies,

$$\delta E W_s^{t+1}\left[\tilde{h}_s^t, h_r^t, \theta_r^{t+1} \mid \tilde{\theta}_s^{t+1}\right]$$

$$\begin{cases} = \delta E V^{(t+1)*}\left(\theta^{t+1}\right) - \delta E_{\theta_r^t} \tilde{w}_s\left(\rho^{(t+1)*}\left(\tilde{\theta}_s^t, \theta_r^t\right)\right) & \text{if } \tilde{\theta}_s^t = \theta_s^t \\ \leq \delta E V^{(t+1)*}\left(\theta^{t+1}\right) - \delta E_{\theta_r^t} \tilde{w}_s\left(\rho^{(t+1)*}\left(\tilde{\theta}_s^t, \theta_r^t\right)\right) & \text{if } \tilde{\theta}_s^t \neq \theta_s^t \end{cases} \quad (A.22)$$

According to (A.22) the maximum expected profit is obtained provided IIR is satisfied in period $t+1$ which is guaranteed whenever agents truthfully disclose their condition in the previous period t. Otherwise when some member misreports its condition in period t, IIR may be violated which may cause the sustainable supply to dissolve in $t+1$, thus reducing the expected profit of each member. From (A.22) and (A.21) it follows that

$$\begin{aligned} & W_s^t\left[\tilde{h}_s^{t-1}, h_r^{t-1}, \theta_s^t \mid \theta_s^t\right] - W_s^t\left[\tilde{h}_s^{t-1}, h_r^{t-1}, \theta_s^t \mid \tilde{\theta}_s^t\right] \\ \geq\ & E_{\theta_r^t}\left(R\left(x^*\left(\theta_s^t, \theta_r^t\right), \theta_r^t\right) - C\left(x^*\left(\theta_s^t, \theta_r^t\right), \theta_s^t\right)\right) \\ & -E_{\theta_r^t}\left(R\left(x^*\left(\tilde{\theta}_s^t, \theta_r^t\right), \theta_r^t\right) - C\left(x^*\left(\tilde{\theta}_s^t, \theta_r^t\right), \theta_s^t\right)\right) > 0 \end{aligned}$$

thus satisfying (A.20) and proving truthful disclosure is a best response for $\mathcal{S}$ in period t.A similar argument shows that truthful disclosure is a best response for $\mathcal{R}$

in period t, provided $\mathcal{S}$ always truthfully discloses. This completes are proof of part (i).

(ii) M is BB : Note that by construction, the transfers $\left\{\tau_i^t\left(\tilde{h}^t\right)\right\}$ satisfy ex post budget balance, since

$$\tau_r^t\left(\tilde{h}^t\right)+\tau_s^t\left(\tilde{h}^t\right)=\sum_i\left[\tilde{p}_i^t\left(\tilde{h}^t\right)-\Sigma_{j\neq i}\left(\tilde{p}_j^t\left(\tilde{h}^t\right)-E_c\tilde{p}_j^t\left(\tilde{h}^t\right)\right)\right]=0$$

(iii) M is IIR : The transfers satisfy voluntary participation, along the Bayesian incentive compatible equilibrium path, since for $\mathcal{S}$

$$\begin{aligned}
&W_s^t\left[\tilde{h}_s^{t-1},h_r^{t-1},\theta_s^t\mid\theta_s^t\right]-\rho_s^{t*}V_s^t\left(0,\theta_s^t\right)\\
=\ &E_{\theta_r^t}\left(V^{t*}\left(\theta^t\right)-\rho_s^{t*}V_s^t\left(0,\theta_s^t\right)\right)-\min_{\theta_s^t}(E_{\theta_r^t}V^{t*}\left(\theta_s^t,\theta_r^t\right)-\rho_s^{t*}V_s^t\left(0,\theta_s^t\right))+\frac{B^t\left(\rho^{t*}\right)}{2}\\
>\ &0
\end{aligned}$$

A similar argument establishes $M$is IIR for $\mathcal{R}$ as well. □

Proposition 18 There exists $0\leq\underline{\rho}_r^1<\bar{\rho}_r^1\leq1$, such that when $\rho_r^1\in[\underline{\rho}_r^1,\bar{\rho}_r^1]$ mechanism M exists.

When $\rho_r^1<\underline{\rho}_r^1$ or $\rho_r^1>\bar{\rho}_r^1$, M is no longer feasible and the only way to achieve sustainable quality supply is by centralizing the supply chain, i.e. one member sales his ownership to the other member.

PROOF We show that the initial administration fee is unimodule in ρ_r^1 and it is first increasing and then decreasing.

The initial administration fee writing in terms of ρ_r^1 is as follows:

$$\begin{aligned}
w_r(\rho^1)+w_s(\rho^1)\ &=\ E_{\theta_r^1}[V^{1*}(\theta_{sw}^1,\theta_r^1)-\rho_s^1V_s^1(0,\theta_{sw}^1)-\rho_r^1V_s^1(1,\theta_{sw}^1)]\\
&\quad+E_{\theta_s^1}[V^{1*}(\theta_s^1,\theta_{rw}^1)-\rho_s^1V_r^1(0,\theta_{rw}^1)-\rho_r^1V_r^1(1,\theta_{rw}^1)]\\
&=\ E_{\theta_r^1}[V^{1*}(\theta_{sw}^1,\theta_r^1)]+E_{\theta_s^1}[V^{1*}(\theta_s^t,\theta_{rw}^1)]-V^1(0,\theta_{sw}^1,\theta_{rw}^1)\\
&\quad+\rho_r^1(V^1(0,\theta_{sw}^1,\theta_{rw}^1)-V^1(1,\theta_{sw}^1,\theta_{rw}^1))
\end{aligned}$$

$(\theta^1_{sw}, \theta^1_{rw})$ satisfies

$$\frac{d}{dy}E_{\theta^1_r}V^{1*}(y,\theta^1_r) - \rho^1_r\frac{d}{dy}V^1_s(y,1)\left\{\begin{array}{c}\geq\\=\\\leq\end{array}\right\}0 \Rightarrow \theta^1_{sw}(\rho^1) = \left\{\begin{array}{c}0\\y\\1\end{array}\right\}$$

and

$$\frac{d}{dy}E_{\theta^1_s}V^{1*}(\theta^1_s,y) - \rho^1_r\frac{d}{dy}V^1_r(y,1)\left\{\begin{array}{c}\geq\\=\\\leq\end{array}\right\}0 \Rightarrow \theta^1_{rw}(\rho^1) = \left\{\begin{array}{c}0\\y\\1\end{array}\right\},$$

implying that θ^1_{rw} is increasing in ρ^1_r and θ^1_{sw} is decreasing in ρ^1_r.

By envelope theorem,

$$\frac{\partial}{\partial\rho^1_r}(w_r(\rho^1) + w_s(\rho^1)) \quad = \quad V^1(0,\theta^1_{sw},\theta^1_{rw}) - V^1(1,\theta^1_{sw},\theta^1_{rw})$$

and $V^1(1,\theta^1_{sw},\theta^1_{rw})$ is increasing in θ^1_{rw} and decreasing in θ^1_{sw}, therefore $\frac{\partial}{\partial\rho^1_r}(w_r(\rho^1) + w_s(\rho^1))$ is decreasing in ρ^1_r.

The mechanism exists only if the administration fee is greater than one time of the supply chain profit and we know that the mechanism exists when ρ^1_r is chosen to maximize the participation fee. Therefore, $[\underline{\rho}^1_r, \bar{\rho}^1_r]$ is a non-empty set and the mechanism exists when the initial $\rho^1_r \in [\underline{\rho}^1_r, \bar{\rho}^1_r]$. □

Proposition 19: *A* sequential all-pay auction with payments,

$$m^t_i\left(b^t_i, b^t_{-i}\right) = b^t_i - b^t_{-i} + E_{\theta^t}b^t_{-i} + \delta E\tilde{w}^{t+1}_i\left(\rho^{(t+1)*}\left(\theta^t\right)\right)$$

implements the efficient supply chain agreement. In equilibrium each member bids

$$b^t_r\left(\theta^t_r\right) \quad = \quad E_{\theta^t_s}\left(-C\left(\theta^t_s, x^*\left(\theta^t\right)\right)\right) \qquad \text{(A.23)}$$

$$b^t_s\left(\theta^t_s\right) \quad = \quad E_{\theta^t_r}\left(R\left(\theta^t_r, x^*\left(\theta^t\right)\right)\right) \qquad \text{(A.24)}$$

PROOF: The proof is to show that the $\{b_r^t(\theta_r^t), b_s^t(\theta_s^t)\}$ are strictly increasing Bayesian equilibrium bid functions for this sequential auction. Suppose $\mathcal{S}$ follows the bidding strategy, $\{b_s^t(\theta_s^t)\}$. Then $\mathcal{R}$'s best response in period t, given he intends to follow $b_r^{t'}\left(\theta_r^{t'}\right)$ in all future periods, $t' > t$ is the solution to:

$$\begin{aligned} U_r\left(\theta_r^t\right) &= \max_b E_{\theta_s^t}\left[R\left(\theta_r^t, x^*\left(\theta_s^t, \left(b_r^t\right)^{-1}(b)\right)\right)\right. \\ &\quad \left.+m_r^t\left(b, b_s^t\left(\theta_s^t\right)\right) + \delta EW_r^t\left(b, b_s^t\right)\right] \end{aligned} \tag{A.25}$$

$$\begin{aligned} &= \max_b \; E_{\theta_s^t}\left[R\left(\theta_r^t, x^*\left(\theta_s^t, \left(b_r^t\right)^{-1}(b)\right)\right)\right. \\ &\quad \left.-C\left(\theta_s^t, x^*\left(\theta_r^t, \left(b_s^t\right)^{-1}(b)\right)\right) + \delta EV^{t*}\left(\theta^{t+1}\right)\right] + OT \end{aligned} \tag{A.26}$$

where we've substituted for $m_r^t(b, b_s^t(\theta_s^t))$ from (A.24) into (A.25) to obtain (A.26) and where OT is a constant that is independent of b. Differentiating (A.26) *wrt* to b to identify the local maximum we obtain:

$$\frac{d}{db}U_r\left(\theta_r^t\right) = \frac{d}{db}E_{\theta_s^t}R\left(\theta_r^t, x^*\left(\theta_s^t, \left(b_r^t\right)^{-1}(b)\right)\right) - C\left(\theta_s^t, x^*\left(\theta_r^t, \left(b_s^t\right)^{-1}(b)\right)\right)\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\}0$$

$$\text{for } \left(b_r^t\right)^{-1}(b)\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}\theta_r^t$$

Thus $\mathcal{R}$'s best response is to bid $b_r^t(\theta_r^t)$ which is strictly increasing in θ_r^t. The same reasoning follows for $\mathcal{S}$ as well. Hence we have shown these bid functions are mutual best responses for the two parties and therefore constitute a Bayesian equilibrium to the bidding game. Moreover the equilibrium bid function implement the efficient supply quality in each period and the payments and fees that are constructed from this equilibrium are satisfy *IIR* and *BB*. This sequential auction therefore implements the efficient supply chain arrangement. □

Proposition 20: I^* is the unique Nash equilibrium to the ex ante investment game where each member invests to maximize his expected continuation profit.

PROOF: Suppose i expects other members, $-i$, to invest I^*_{-i}. Then i's best response is to

$$\max_{I_i} W_i(I_i, I^*_{-i})$$

Member i's best response is $I_i = I_i^*$ and therefore I^* is a Nash equilibrium. I^* is the unique equilibrium since it is the only investment profile where each member maximizes its expected surplus, given the investments of the other partners. □

Proposition 23: Assume $V^{T+1}(\theta^{T+1}) = 0$.

1) $I^{t*}(\theta_s^{t-1})$ is achieved at the boundary, i.e. either invest the full amount $I^{t*} = 1$ or not invest $I^{t*} = 0$.

2) $I^{t*}(\theta_s^{t-1})$ is decreasing in θ_s^t. In each period there exists a θ_{s0}^t such that $I^{t*}(\theta_s^{t-1}) = 1$ if $\theta_s^t < \theta_{s0}^t$ and $I^{t*}(\theta_s^{t-1}) = 0$ if $\theta_s^t \geq \theta_{s0}^t$.

3) $I^{t*}(\theta_s^{t-1})$ is decreasing in t, and therefore θ_{s0}^t is increasing in t.

PROOF: The solution of the optimal investment can be obtained by solving a dynamic programming. Let $U^t(\theta_s^{t-1})$ be the expected supply chain profit at the beginning of period t. Then $U^t(\theta_s^{t-1})$ satisfies:

$$\begin{aligned} U^t(\theta_s^{t-1}) &= \max_{I^t} E_{\theta_s^t}\{\Pi(1, \theta_s^t) - I^t + \delta U^{t+1}(\theta_s^t) | \theta_s^{t-1}, I^t\} \\ &= \max_{I^t} E_\epsilon\{\Pi(1, \nu_s^t \theta_s^{t-1} - I^t + \epsilon) - I^t + \delta U^{t+1}(\nu_s^t \theta_s^{t-1} - I^t + \epsilon)\} \\ U^{T+1}(\theta_s^T) &= 0. \end{aligned}$$

Next we show that $U^t(\theta_s^{t-1})$ is decreasing and convex in θ_s^{t-1} by induction. The statement is true when $t = T + 1$, assume it is true for all periods after t. Then at the beginning of period t, we have:

$$U^t(\theta_s^{t-1}) = \max_{I^t} E_\epsilon\{\Pi(1, \nu_s^t \theta_s^{t-1} - I^t + \epsilon) - I^t + \delta U^{t+1}(\nu_s^t \theta_s^{t-1} - I^t + \epsilon)\}$$

Note that $\Pi(1, \nu_s^t\theta_s^{t-1} - I^t + \epsilon)$ and $U^{t+1}(\nu_s^t\theta_s^{t-1} - I^t + \epsilon)$ are decreasing and convex in $\nu_s^t\theta_s^{t-1} - I^t + \epsilon$. Therefore, for any given θ_s^{t-1}, I^{t*} is achieved at the boundary. Therefore, the dynamic programming can be simplified to

$$\begin{aligned} U^t(\theta_s^{t-1}) &= \max\{E_\epsilon\{\Pi(1, \nu_s^t\theta_s^{t-1} + \epsilon) + \delta U^{t+1}(\nu_s^t\theta_s^{t-1} + \epsilon)\}, \\ &\quad E_\epsilon\{\Pi(1, \nu_s^t\theta_s^{t-1} - 1 + \epsilon) - 1 + \delta U^{t+1}(\nu_s^t\theta_s^{t-1} - 1 + \epsilon)\}\}. \end{aligned}$$

Note that both terms before maximizing are all decreasing and convex in θ_s^{t-1}, $U^t(\theta_s^{t-1})$ is decreasing and convex in θ_s^{t-1}. □

A.3 Proof of Results in Chapter 4

Lemma 28: For any given k, $\lim_{t\to\infty} B(s(t|k), t|k) = \infty$.

PROOF: Note that $D[L + U(0, t)]$ is SIL(sp) in t and it has mean $(L + t/2)\lambda$. Moreover, $D[L + U(0, t)] \leq_{cx} D[L + t/2] \leq_{cx} \lambda L + D[t/2]$.

Therefore, for any s, we have $hE[s - D(L + U(0, t))]^+ + bE[s - D(L + U(0, t))]^- \geq hE[s - D(L + t/2)]^+ + bE[s - D(L + t/2)]^- \geq hE[s - D(t/2) - \lambda L]^+ + bE[s - D(t/2) - \lambda L]^-$. Substitute this inequality into $B(s, t|k)$:

$$B(k, s, t) \geq hE[s - D(t/2) - \lambda L]^+ + bE[s - D(t/2) - \lambda L]^-.$$

Let $s^0(t)$ be the minimizer of $hE[s - D(t/2) - \lambda L]^+ + bE[s - D(t/2) - \lambda L]^-$ for any given t. From Liu and Song (2010b), we have $hE[s^0(t) - D(t/2) - \lambda L]^+ + bE[s^0(t) - D(t/2) - \lambda L]^-$ is increasing in t with a rate $\sqrt{t}$. Therefore, $\lim_{t\to\infty} B(s(k, t), t|k) = \infty$. □

Lemma 29: a) The optimal replenishment interval $t(k)$ increases in k. b) $B^u(k)$ is increasing and concave in k. c) As $k \to 0$, $t(k) \to 0$, so the system reduces to a continuous-review system with base stock policy. d) For continuous demand, $k > 0$ implies $t(k) > 0$.

PROOF: a) $\partial^2 B(s,t|k)/\partial k \partial t = \theta'(t) \leq 0$, $B(s,t|k)$ is submodular in (k,t), therefore $t(k)$ in increasing in k.

b) $\partial B(s,t|k)/\partial k = \theta(t) > 0$ and from part a) $\partial^2 B(s,t|k)/\partial k^2 = \theta'(t)t'(k) \leq 0$, therefore $B^u(k)$ is increasing and concave in k.

c) $B(s,t|k) - k\theta(t)$ is increasing in t and $\theta(t)$ is decreasing in t, therefore, when $k \to 0$, from the first order condition we know that $t \to 0$.

d) When $k > 0$, we know that $\lim_{t\to 0} k\theta(t) = \infty$, therefore $t(k) > 0$. □

Proposition 31: For any given k, the supply chain cost $\widehat{C}(s,t|k)$ is coordinatewise convex and submodular in (s,t). The optimal replenishment interval $t^*(k)$ and base-stock level $s^*(k)$ satisfy the first order condition. Moreover, $t(k) \geq t^*(k)$ and $s(k) \geq s^*(k)$.

PROOF: The fact that $\widehat{C}(s,t|k)$ is coordinatewise convex and submodular is due to Lemma 27 and Lemma 2. The optimal $(s^*(k), t^*(k))$ satisfies the first order condition. Fix $s = s^*(k)$, $t^*(k)$ satisfies the first order condition of $\widehat{C}$ with respect to t:

$$\frac{\partial}{\partial t}\widehat{C}((s^*(k), t^*(k)|k) = \frac{\partial}{\partial t}B((s^*(k), t^*(k)|k) + H'(t^*(k)) = 0.$$

Because for a fixed t the optimal s for $B(s,t \mid k)$ and $C(s,t \mid k)$ is identical. Therefore, $B(s,t \mid k)$ and $C(s,t \mid k)$ has a difference of $H(t)$ after optimizing over s. Since $H(t)$ is increasing and convex, we obtain $t(k) \geq t^*(k)$. By Lemma27, $s(k) = s(t(k)|k) \geq s(t^*(k)|k) = s^*(k)$. The last equality follows because both $s(t^*(k)|k)$ and $s^*(k)$ satisfy the first order condition of $\widehat{C}$ with respect to s, i.e., $\partial\widehat{C}((s, t^*(k)|k)/\partial s = 0$.

Because $H(t)$ is increasing and convex in t, $H'(t^*(k)) \geq 0$, and hence we must have

$$\frac{\partial}{\partial t}B((s^*(k), t^*(k)|k) \leq 0.$$

From the convexity of B with respect to t, we obtain $t(k) \geq t^*(k)$. By Lemma27, $s(k) = s(t(k)|k) \geq s(t^*(k)|k) = s^*(k)$. The last equality follows because both $s(t^*(k)|k)$ and $s^*(k)$ satisfy the first order condition of $\widehat{C}$ with respect to s, i.e., $\partial\widehat{C}((s, t^*(k)|k)/\partial s = 0$. □

Proposition 32: For any k, there exists an $\alpha^* = \alpha^*(k)$, such that the system is coordinated. That is, $t(\alpha^* k) = t^h(k, \alpha^*) = t^*(k)$ and $s(\alpha^* k) = s^*(k)$. Moreover, if $Pr(D(t) > 0) = 1$ for all $t > 0$, then $\alpha^* < 1$.

PROOF: Note that $t(k) \geq t^*(k)$ and $t(\alpha k) \to 0$ when $\alpha \to 0$. $t(\alpha k)$ is continuous in α, thus there exist an α^* such that $t(\alpha^* k) = t^*(k)$.

Because the cost function of $\mathcal{H}$ under α^* is $H(k, \alpha^*, t)$, the optimal replenishment interval for the home plant must satisfy the first order condition with $\alpha = \alpha^*$. Substitute $t^*(k)$ into the equation, we have:

$$\begin{aligned} \left.\frac{\partial H(k, \alpha^*, t)}{\partial t}\right|_{t^*(k)} &= \left.\frac{\partial H}{\partial t}\right|_{t^*(k)} + (1-\alpha^*)k\left.\frac{\partial \theta(t)}{\partial t}\right|_{t^*(k)} \\ &= -\left(\left.\frac{\partial B}{\partial t}\right|_{(s^*(k), t^*(k)|k)} - (1-\alpha^*)k\left.\frac{\partial \theta(t)}{\partial t}\right|_{t^*(k)}\right) \\ &= -\left.\frac{\partial B}{\partial t}\right|_{(s(\alpha^* k), t(\alpha^* k)|\alpha^* k)} = 0. \end{aligned} \tag{A.27}$$

Thus $t^*(k) = t^h(k, \alpha^*)$ is an optimal solution for the home plant and the system is coordinated.

Note that α^* and $t^*(k)$ satisfy:

$$\frac{\partial \widehat{C}(s, t|k)}{\partial t} = 0 \qquad \frac{\partial B(s, t|\alpha^* k)}{\partial t} = 0.$$

Solving for α^*, we have:

$$\alpha^* = \left.\frac{\partial B(s,t|k)/\partial t - k\theta'(t)}{\partial \widehat{C}(s,t|k)/\partial t - k\theta'(t)}\right|_{(s^*(k),t^*(k))},$$

which is a function of $t^*(k)$ only.

Moreover, $t^h(k,1) = 0$ and since $Pr(D(t) > 0) = 1$ for all $t > 0$, we have $t(k) > 0$. Therefore, when $\alpha = 1$, $t^h(k,1) < t(k)$ and hence $\alpha < 1$. □

Proof of Corollary 33: Follows from Lemma 29. □

Proposition 34: $\mathcal{H}$ has a positive gain in the coordinated system, i.e., $\widetilde{H}(k,\alpha^*,t^*(k)) \leq H(t(k))$ if

$$-\theta'(t^*(k))(t(k) - t^*(k)) \geq \theta(t^*(k)). \tag{A.28}$$

When $Pr(D(t^*(k)) > 0) = 1$, (A.28) reduces to $t^*(k) \leq t(k)/2$.

PROOF: We only need to show $H(t(k)) \geq H(k,\alpha^*,t^*(k))$. Note that $t(k) \geq t^*(k)$. From the convexity of $H(t)$, we have

$$H(t(k)) \geq H'(t^*(k))(t(k) - t^*(k)) + H(t^*(k)). \tag{A.29}$$

Because $t^*(k)$ is the optimal replenishment interval for the home plant, it satisfies: $H'(t^*(k)) = -(1-\alpha^*)k\theta'(t^*(k))$. Thus,

$$\begin{aligned} H(t(k)) &\geq -(1-\alpha^*)k\theta'(t^*(k))(t(k) - t^*(k)) + H(t^*(k)) \\ &\geq (1-\alpha^*)k\theta(t^*(k)) + H(t^*(k)) = H(k,\alpha^*,t^*(k)). \end{aligned}$$

If $Pr(D(t^*(k)) > 0) = 1$, then $\theta(t) = 1/t$, so equation (A.28) is simplified to $t(k) \geq 2t^*(k)$. □

Proposition 36: For any fixed k, the optimal fixed cost sharing percentage $\alpha^h = \alpha^h(k)$ satisfies $\alpha^h(k) \geq \alpha^*(k)$. Hence, $t^*(k) \leq t(\alpha^h k) \leq t(k)$, and $\widehat{C}(s(\alpha^h k), t(\alpha^h k)|k) \leq \widehat{C}(s(k), t(k)|k)$. If the division cost functions are strictly convex, then all the above inequalities are strict. This implies the role of the headquarters diminishes under full information.

PROOF: For any $0 \leq \alpha \leq \alpha^*$, we have

$$B(\alpha^* k, s^*(k), t^*(k)) \geq B(\alpha k, s(\alpha^* k), t(\alpha^* k)) \geq B^u(\alpha k).$$

Because α^* coordinates $\mathcal{C}$, we have:

$$\widehat{C}(k, s(\alpha k), t(\alpha k)) \geq \widehat{C}(k, s(\alpha^* k), t(\alpha^* k)).$$

Combine the above two inequalities:

$$H(t(\alpha^* k) + (1 - \alpha^*)k\theta(t^*(k)) \leq H(t(\alpha k)) + (1 - \alpha)k\theta(t(\alpha k)),$$

which proves the statement. □

Lemma 39: The contract reorder interval $T(x)$ is non-decreasing in x.

Proof: From the participation constraint (4.9), we know that for any $x' > x$:

$$\begin{aligned} \widehat{B}(x, T(x)) - A(x) &\leq \widehat{B}(x, T(x')) - A(x'), \\ \widehat{B}(x', T(x')) - A(x') &\leq \widehat{B}(x', T(x)) - A(x). \end{aligned} \tag{A.30}$$

Adding the two inequalities we obtain:

$$\widehat{B}(x, T(x)) + \widehat{B}(x', T(x')) \leq \widehat{B}(x, T(x')) + \widehat{B}(x', T(x)) \tag{A.31}$$

Note that $\widehat{B}_{kT}(x, T) = \partial\theta(T)/\partial T \leq 0$ which implies $\widehat{B}(x, T)$ is submodular in (x, T). Thus, from (A.31), we know that $T(x') \geq T(x)$. □

Proposition 35 Let $\bar{t}$ be the replenishment interval that satisfies: $E[H(t(k))] = H(\bar{t})$. We have the following results:

1) There exists a feasible decision right allocation that induces an efficient contract $T^{HA}(k) = t^*(k)$ for all k, i.e. the supply chain is coordinated. The contract makes each subordinate the residual claimant of the supply chain.

2) For any feasible decision rights allocation r_B and r_H that induce supply chain coordination, the transfer payments are:

$$\tau_B(k) = -H(t^*(k)) + E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))]/2 - (O_B - O_H)/2$$
$$\tau_H(k) = H(t^*(k)) - E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))]/2 + (O_B - O_H)/2$$

where

$$O_B = \max_k \{B(s(t^*(k)), t^*(k), k) + H(t^*(k)) - r_B B(s(k), t(k), k) - r_H B(s(t_{min}), t_{min}, k)\}$$

and $O_H = E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))] - r_B E[H(t(k))] - r_H H(t_{min})$.

3) Specifically, when $t_{min} \geq \bar{t}$, there exists $r_H^0 \geq 0$ such that the feasible decision rights satisfies $r_H \geq r_H^0$.

PROOF:

Part 1) and 2):

Similar to Kuribko and Lewis (2010) and Lewis et al (2011), we conjecture that the contract has the structure that allows ắnd $\mathcal{B}$ to be the residual claimant of the supply chain. Let $W_B(k)$ and W_H to be the long run average cost under the contract. The conjecture implies

$$W_B(k) = B(s(t^*(k)), t^*(k), k) + H(t^*(k)) - U_B$$
$$W_H = E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))] - U_H,$$

where U_B and U_H are constants independent with the branch's type.

Under the conjecture, let $O_B(k)$ be the least transfer that need to reward to type k branch to satisfy individual rationality. Then

$$O_B(k) = B(s(t^*(k)), t^*(k), k) + H(t^*(k)) - r_B B(s(k), t(k), k) - r_H B(s(t_{min}), t_{min}, k),$$

similarly define

$$O_H = E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))] - r_B E[H(t(k))] - r_H H(t_{min}).$$

Take the maximum of $O_B(k)$, we obtain the least transfer that is needed to reward type k branch:

$$O_B = \min_k \{O_B(k)\}.$$

One necessary condition for the transfers to balance is that the least amount of cost distributed should be more than the cost generated by the contract.

$$\begin{aligned}
&E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))] - O_H \\
&+E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))] - O_B \\
&\geq E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))]
\end{aligned}$$

which is equivalent to

$$-r_B E[H(t(k))] - r_H H(t_{min}) + O_B \geq 0.$$

Note that there always exists decision rights that satisfies the above inequality. To proof this, take $r_H = 1$ and $r_B = 0$. Then $O_B = \max_k\{B(s(t^*(k)), t^*(k), k) + H(t^*(k)) - B(s(t_{min}), t_{min}, k)\}$ and the left hand side of the inequality becomes

$$-H(t_{min}) + \max_k\{B(s(t^*(k)), t^*(k), k) + H(t^*(k)) - B(s(t_{min}), t_{min}, k)\}.$$

For any $k \in [\underline{k}, \bar{k}]$, $B(s(t_{min}), t_{min}, k) + H(t_{min})$ is the system cost under replenishment interval t_{min}, which is more than the cost under the supply chain optimal replenishment interval $t^*(k)$. Therefore, $B(s(t^*(k)), t^*(k), k) + H(t^*(k)) \leq B(s(t_{min}), t_{min}, k) + H(t_{min})$ and hence the inequality holds when $r_H = 1$ and $r_B = 0$.

Select any r_B and r_H satisfies the inequality, following Kuribko and Lewis (2010) and Lewis et al (2011),

$$\tau_B(k) = -H(t^*(k)) + E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))]/2 - (O_B - O_H)/2$$
$$\tau_H(k) = H(t^*(k)) - E[B(s(t^*(k)), t^*(k), k) + H(t^*(k))]/2 + (O_B - O_H)/2$$

are the transfer payments that solves the Headquarters problem.

Part 3)

Substitute $r_B = 1 - r_H$ in $\Delta(r_B, r_H) = -r_B E[H(t(k))] - r_H H(t_{min}) + O_B$ and take derivative respect to r_H we have:

$$\frac{\partial \Delta(1 - r_H, r_H)}{\partial r_H} = E[H(t(k))] - H(t_{min})$$
$$+B(s(t(k_w)), t(k_w), k_w) - B(s(t_{min}), t_{min}, k_w),$$

where k_w is the maximizer of $O_B(k)$. Note that when $t_{min} \geq \bar{t}$, we have:

$$\begin{aligned}\frac{\partial \Delta(1 - r_H, r_H)}{\partial r_H} &= H(\bar{t}) - H(t_{min}) + B(s(t(k_w)), t(k_w), k_w) - B(s(t_{min}), t_{min}, k_w)\\ &\leq B(s(t(k_w)), t(k_w), k_w) - B(s(t_{min}), t_{min}, k_w) \leq 0,\end{aligned}$$

$\Delta(1 - r_H, r_H)$ is decreasing in r_H, thus, there exists r_H^0 such that for all $r_H > r_H^0$, $\Delta(1 - r_H, r_H) < 0$ and $r_H < r_H^0$ $\Delta(1 - r_H, r_H) \geq 0$. This proofs the statement. □

Proposition 40: If $\omega(x)$ is nondecreasing, then there exits an optimal contract menu $\{(T(x), A(x)), x \in [\underline{k}, \bar{k}]\}$. Let

$$\widehat{H}(k, T(k)) \quad = \quad H(T(k)) + A(k). \tag{A.32}$$

The optimal compensation $A(x)$ satisfies:

$$A(k) = [\widehat{B}(k, T(k)) - B^u(k)] + \int_k^{\bar{k}} [\theta(T(x)) - \theta(t(x))] dx. \tag{A.33}$$

The optimal contract reorder interval $T(x)$ satisfies:

$$T(k) = \begin{cases} \tilde{T}(k) & \text{if } H'(t(k)) + \theta'(t(k))\varphi(k) > 0 \\ t(k) & \text{if } H'(t(k)) + \theta'(t(k))\varphi(k) \leq 0 \end{cases},$$

where $\tilde{T}(k) < t(k)$ and solves

$$\widehat{H}_T(x,T) = H'(T(x)) + \widehat{B}_T(x,T(x)) + \theta'(T(x))\varphi(x) = 0. \tag{A.34}$$

PROOF :

Under the contract menu, the type-k branch's minimum net cost is

$$u(k) = \min_{x\in[\underline{k},\bar{k}]} \{\widehat{B}(k,T(x)) - A(x)\} = \widehat{B}(k,T(k)) - A(k), \tag{A.35}$$

where the second equality follows from the constraint (IC). From constraint (IR*), the informational rent to the type-k $\mathcal{B}$ (i.e. the reward for having the private information) is

$$r(k) = B^u(k) - u(k) \geq 0. \tag{A.36}$$

By the Envelope Theorem (see Fudenberg and Tirole 1991), $u'(k) = \widehat{B}_k(k,T(k)) \geq 0$. Along with constraint (HA), we have:

$$r'(k) = B^u_k(k) - \widehat{B}_k(k,T(k)) = \theta(t(k)) - \theta(T(k)) \leq 0,$$

where θ is given by equation (4.2). Hence, the informational rent is decreasing in k. Using Lagrangian multipliers, we can show that in order to minimize cost, the home plant will pay no informational rent to the "least efficient" branch – the branch with highest k. This means that under the optimal contract $r(\bar{k}) = 0$. Hence, $u(\bar{k}) = B(s(\bar{k}), t(\bar{k})|\bar{k})$, i.e., under the optimal contract, the type-$\bar{k}$ branch incurs a cost that is identical to that under the uncoordinated system. This leads to

$$r(k) = -\int_k^{\bar{k}} r'(x)dx = \int_k^{\bar{k}} [\theta(T(x)) - \theta(t(x))]dx, \quad k \in [\underline{k},\bar{k}]. \tag{A.37}$$

Using (A.35)-(A.37), we obtain

$$A(k) = [\widehat{B}(k, T(k)) - B^u(k)] + \int_k^{\bar{k}} [\theta(T(x)) - \theta(t(x))]dx.$$

With this relationship, $\mathcal{H}$'s cost with a type-k branch becomes a function of $T(k)$:

$$\widehat{H}(k, T(k)) \quad = \quad H(T(k)) + A(k).$$

Solving $T(k)$ from $\widehat{H}(k, T(k))$ we have the results.

□

Proposition 42 Suppose the distribution of k satisfies Condition 1 and $\widehat{H}_T(\underline{k}, t(\underline{k})) > 0$. Then (HA) is slack for all k.

PROOF: Substituting this expression for $A(k)$ into ($\mathcal{H}$), we obtain the following version of $\mathcal{H}$'s problem with reduced dimension:

$$(\mathcal{P}_{\mathcal{H}}{}') \qquad V = \min_{T(\cdot)} \int_{\underline{k}}^{\bar{k}} \widehat{H}(x, T(x)) f(x \mid t_0) dx \tag{A.38}$$

$$s.t. \qquad T(\cdot) \text{ satisfies participation constraints (IC),(IR*) and (HA),}$$

where

$$\widehat{H}(x, T(x)) \quad = \quad H(T(x)) + A(x).$$

In the following proposition, we provide conditions under which ($\mathcal{P}_{\mathcal{H}}{}'$) can be solved through pointwise optimization in T. Hence solving ($\mathcal{P}_{\mathcal{H}}{}'$) is equivalent to solving the unconstrained problem

$$(\mathcal{P}_{\mathcal{H}}{}'') \qquad V = \min_{T(\cdot)} \int_{\underline{k}}^{\bar{k}} \widehat{H}(x, T(x)) f(x \mid t_0) dx. \tag{A.39}$$

Using integration by parts, we have

$$\int_{\underline{k}}^{\bar{k}}\int_{k}^{\bar{k}} \widehat{B}_k(x,T(x))f(y)dxdy = \int_{\underline{k}}^{\bar{k}} \widehat{B}_k(x,T(x))\varphi(x)f(x \mid t_0)dx$$

$$= \int_{\underline{k}}^{\bar{k}} \theta(T(x))\varphi(x)f(x \mid t_0)dx,$$

$$\int_{\underline{k}}^{\bar{k}}\int_{k}^{\bar{k}} B_k(x,t(x),s(x))f(y \mid t_0)dxdy = \int_{\underline{k}}^{\bar{k}} B_k(x,t(x),s(x))\varphi(x)f(x)dx$$

$$= \int_{\underline{k}}^{\bar{k}} \theta(t(x))\varphi(x)f(x \mid t_0)dx.$$

Taking the derivative with respect to T for each fixed x in (A.39) yields

$$\begin{aligned}\widehat{H}_T(x,T(x)) &= H'(T(x)) + \widehat{B}_T(x,T(x)) + \widehat{B}_{kT}(x,T(x))\varphi(x)\\ &= H'(T(x)) + \widehat{B}_T(x,T(x)) + \theta'(T(x))\varphi(x).\end{aligned}$$

Suppose $T(x)$ satisfies $\widehat{H}_T(x,T(x)) = 0$. To establish the proposition, it is sufficient to show that $T(x)$ is increasing in x and $T(x) \leq t(x)$, and $T(x)$ and the corresponding A defined in (A.33) satisfy (IC) and (IR*).

We first verify monotonicity. Taking the derivative over x in equation (A.34)

$$T'(x) = -\frac{\widehat{B}_{xT}(x,T(x)) + \theta'(T(x))\varphi'(x)}{H''(T(x)) + \widehat{B}_{TT}(x,T(x))}.$$

The denominator is non-negative because $H(t)$ is convex in t and $\widehat{B}(x,T)$ is coordinatewise convex in T. The numerator is non-positive because $\widehat{B}_{xT}(x,T(x)) \leq 0$ and $\varphi'(x) \leq 0$. Thus, $T'(x) \geq 0$ and hence $T(x)$ is non-decreasing in x.

Next, we prove that $T(x) < t(x)$ for any x. We know that $t(x)$ satisfies $\widehat{B}_t(x, t(x)) = 0$; $T(x)$ satisfies $\widehat{H}_T(x, T(x)) = 0$, that is, $H'(T(x)) + \varphi(x)\theta'(T(x)) + \widehat{B}_T(x, T(x)) = 0$. From Assumption 1, we have $H'(t(\underline{k})) + \varphi(\underline{k})\theta'(t(\underline{k})) > 0$. Because $H(t) + \varphi(\underline{k})\theta(t)$ is convex in t, we have for any $t > t(\underline{k})$, $H(t) + \varphi(\underline{k})\theta(t) > H(t(\underline{k})) + \varphi(\underline{k})\theta(t(\underline{k}))$. Moreover, because $t(\underline{k})$ is the minimizer of $\widehat{B}(\underline{k}, t)$, we have $\widehat{B}(\underline{k}, t) \geq \widehat{B}(\underline{k}, t(\underline{k}))$. Adding the two inequalities, we have $\widehat{H}(\underline{k}, t) > \widehat{H}(\underline{k}, t(\underline{k}))$. Thus, $T(\underline{k}) < t(\underline{k})$.

For any $k > \underline{k}$, because $\varphi(k)$ is non-increasing in k, we have $0 < \varphi(k) < \varphi(\underline{k})$. Moreover, because $t(k)$ is increasing in k, we have $H'(t(k)) \geq H'(t(\underline{k})) \geq 0$ and $0 \geq \theta'(t(k)) \geq \theta(t(\underline{k}))$. Therefore, $H'(t(k)) + \varphi(k)\theta'(t(k)) > 0$. The rest of the proof follows exactly as above.

Finally, (IC) and (IR*) hold because of the way we construct $A(x)$. $\square$

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