Price Customization and Content Provision in Media Markets

by

Jinzhao Du

Business Administration
Duke University

Date: ____________________

Approved:

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Wilfred Amaldoss, Supervisor

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Devavrat Purohit

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Woochoel Shin

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Business Administration
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Abstract

Multi-sided media market is a rapidly evolving phenomenon where practice is ahead of research. In this market, a media platform usually manages the interaction between multiple groups of participants, such as consumers, advertisers, and content suppliers. Because of strong cross-side externalities, when making important decisions such as pricing and content provision, it is crucial for a media platform to deeply understand the likely strategic responses of all other sides. In this dissertation, I explicitly model these strategic interactions and examine a media platform's price customization strategy (Essay 1) and content provision strategy (Essay 2).

Essay 1 studies whether a competing platform should customize its consumer price by offering an option of paying for not seeing advertisements. I further examine how the possibility of agent multihoming would affect a platform's pricing strategy and profits. This analysis helps one to better appreciate the pricing strategy of media platforms such as Pandora and Spotify, and extends the literature on price discrimination in one-sided markets to two-sided markets. Essay 2 explores media platforms' content provision strategy and source of profits. I propose a model where a media platform interacts with three sides: content suppliers, consumers, and advertisers, and examine how a platform allocates its limited space between content and advertising. This analysis offers insights on how a platform's choice of content provision strategy and source of profits vary with the the structure of the content market as well as the platform market. Collectively, these two essays address man-

agerial questions on media platforms' pricing strategy and content provision strategy, and will help better understand the strategic interactions in media markets and serve as a useful theoretical framework for empirical research on media firms.

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List of Abbreviations and Symbols

Symbols for Chapter 2

λ	Consumer's type, where $\lambda \in \{H, L\}$
θ_I^λ	The location of type λ consumer indifferent between the two platforms
θ_{13I}^λ	The location of type λ consumer indifferent between joining Platform 1 and joining both
θ_{23I}^λ	The location of type λ consumer indifferent between joining Platform 2 and joining both
θ_J	The location of the advertiser indifferent between the two platforms
θ_{13J}	The location of the advertiser indifferent between joining Platform 1 and joining both
θ_{23J}	The location of the advertiser indifferent between joining Platform 2 and joining both
θ_I	'Average' marginal consumer
v_I	Consumers' base valuation for his ideal platform
v_J	Advertisers' base valuation for his ideal platform
t_I	Consumers' sensitivity to platform characteristics
t_J	Advertisers' sensitivity to platform characteristics
γ_I^λ	Type λ consumer's sensitivity to advertisements
γ_I	The average sensitivity to advertisements of consumers
γ_J	Advertiser's desire for consumers

p_{kI}	Price for consumers if Platform k chooses the uniform pricing strategy
p_{kI}^{λ}	Prices for type λ consumer if Platform k chooses the customized pricing strategy
p_{kJ}	Price for advertisers of Platform k
α_I	Market size of consumers
α_J	Market size of advertisers
α_I^{λ}	Market size of type λ consumers
δ_I	The incremental value of a consumer's multi-homing
δ_J	The incremental value of an advertiser's multi-homing
κ	Indicator of platform's pricing strategy.
Π	Platform's profit

Symbols for Chapter 3

v	Consumers' desire for content
γ_A	Advertisers' desire for consumers
t	Consumers' transportation cost
α	The proportion of a platform's bandwidth allocated for content
α_f^M	The proportion of a monopoly platform's bandwidth allocated for content under a free-content strategy
α_f^D	The proportion of a duopoly platform's bandwidth allocated for content under a free-content strategy
α_{pa}^M	The proportion of a monopoly platform's bandwidth allocated for content under a paid-content-with-ads strategy
α_{pa}^D	The proportion of a duopoly platform's bandwidth allocated for content under a paid-content-with-ads strategy
n_i	The number of consumers that join platform i
p_S	The content supplier's price for content

τ_M	The threshold of content price at which a monopoly platform transitions from adopting a paid-content-with-ads strategy to using a free-content strategy
τ_D	The threshold of content price at which a duopoly platform transitions from adopting a paid-content-with-ads strategy to using a free-content strategy
p_{fS}^M	The (interior) equilibrium content price if the monopoly platform uses a free-content strategy
p_{fS}^D	The (interior) equilibrium content price if a duopoly platform uses a free-content strategy
p_{paS}^M	The (interior) equilibrium content price of if the monopoly platform offers paid-content-with-ads
p_{paS}^D	The (interior) equilibrium content price if a duopoly platform offers paid-content-with-ads
$\overline{p_S}$	The "average" price of content in a duopoly model of content suppliers
p_C	Consumer price for joining the platform and accessing content
p_A	Advertising price
Π_P^M	Profits of the monopoly platform
Π_{iP}^D	Profits of the duopoly platform i
Π_S	Profits of a monopoly content supplier
Π_{iS}	Profits of duopoly content supplier i
κ_i	In the presence of duopoly content suppliers, the effort that supplier i exerts to make consumers prefer its content
h	In the presence of duopoly content suppliers, the ideal proportion of content that consumers prefer from supplier 1
h_i	In the presence of duopoly content suppliers, the fraction of content the platform i from supplier 1
γ_C	Consumers' dislike for advertisements

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1

Introduction

Multi-sided media markets, and more generally multi-sided markets, are growing in commercial significance. Media platforms such as streaming music platforms, newspapers, mobile apps, bring consumers and advertisers together and generate revenues from one side or both sides of the market. The rise of Internet and consumers' migration to digital platform, create firms unprecedented opportunities to advertise through media outlet. However, our intuitions honed on one-sided markets need not necessarily extend to these emerging markets because of cross-side externalities (Rochet and Tirole (2003, 2006)). In this dissertation, I closely scrutinize the interaction among the multiple strategic players in media markets: content suppliers, advertisers, media platforms, and consumers, and study two important managerial questions confronting a media platform: First, when is it optimal for a media platform to adopt price customization? Second, how should a media platform allocate its limited bandwidth or space for content and advertising, and relatedly what should be a platform's source of profits? By theoretically examining the two questions, this dissertation clarifies why one's intuition on one-sided market may not apply to mutli-sided media market, and addresses important managerial questions on media platform's pricing strategy and content provision strategy.

Essay 1 (Chapter 2): Price Customization in Two-sided Media Markets

Media platforms generate revenue by bringing consumers and advertisers together. Sometimes consumers and advertisers join a single media platform (single-home) but other times they join multiple platforms (multi-home). In this essay, I examine media platforms' equilibrium pricing strategy in the presence of cross-side externalities between consumers and advertisers. Even when consumers are heterogeneous in their sensitivity to advertising, I find that competing media platforms do not simultaneously adopt a customized pricing strategy for consumers and at least one platform pursues a uniform pricing strategy if agents single-home on both sides of the market. However, multi-homing agents induce the platforms to adopt a symmetric customized pricing strategy when consumers are quite heterogeneous. Finally, when only advertisers multi-home, I observe a symmetric customized pricing strategy (unlike in a single-homing model), asymmetric pricing strategies (unlike in a multi-homing model) or a symmetric uniform pricing strategy depending on the relative size of the cross-side network effects.

Essay 2 (Chapter 3): Media Platforms' Content Provision Strategy and Source of Profits

We see media platforms earning their profits from both consumers and advertisers (e.g., the New York Times), advertisers only (e.g., the Huffington Post), or consumers only (e.g., Tidal). This essay theoretically investigates two important strategic issues confronting a media platform: what proportion of its limited bandwidth or space should a platform allocate for content (instead of advertising)? and what should be the source of a platform's profits? To facilitate this analysis, I propose a model where a media platform interacts with three sides: content suppliers, consumers, and advertisers. In a perfectly competitive content market, my analysis shows that competing platforms will adopt a free-content strategy even in circumstances where a monopoly platform adopts a paid-content-with-ads strategy. However, the result can get reversed if the content supplier is a monopoly. Counter to conventional wisdom, inter-platform competition helps a platform to earn more profits when they adopt a free-content strategy. Next, despite paying a lower price to content suppliers, a media platform may still get hurt. Furthermore, though advertisers' higher valuation for consumers benefits a media platform, it can hurt a content supplier's profits when a monopoly supplier sells content to a platform using paid-content-with-ads strategy or when duopoly suppliers can shape consumers' preference at a low marginal cost and sell to a platform using free-content strategy. Finally, if advertising is quite annoying, even while a monopoly platform shuns the advertising market, duopoly platforms may cater to advertisers.

2

Price Customization in Two-sided Media Markets

2.1 Introduction

Media platforms bring consumers and advertisers together and often generate revenues from both sides of the market. Examples of media markets include streaming music platforms (e.g., Spotify, Pandora), newspapers (e.g., Wall Street Journal, New York Times), streaming videos (e.g., Hulu, Youtube), and mobile apps (e.g., Flappy Bird, Jump). While advertisers strive to reach more consumers, consumers dislike the advertisements on these platforms. Recognizing the heterogeneity in consumers' dislike for advertisements, some media platforms offer customized prices for consumers. Kindle users, for example, pay \$119 for a base e-reader with advertisements and \$139 for an ad-free e-reader. Spotify's customers pay nothing to listen to music streamed along with advertising, or pay \$9.99 per month to enjoy music devoid of any advertisement. Likewise, one can watch streaming videos on Hulu without the nuisance of advertisements by paying \$11.99, or with advertisements by paying \$7.99 per month. However, not all media platforms offer customized prices. Until recently, even Hulu charged a uniform price of \$7.99 for all its viewers, and it did not give them the option of avoiding advertisements. The New York Times does not provide its subscribers with an ad-free version of the newspaper. In practice, a media platform needs to carefully evaluate an important trade-off when deciding whether to use a customized pricing strategy or a uniform pricing strategy: even though customized prices help a platform to earn more profits on the consumer side of the market, the profits from the advertiser side of the market could decrease because advertisers might want to pay less for reaching fewer consumers. This trade-off raises the question: when should a media platform adopt a customized pricing strategy, and when should it pursue a uniform pricing strategy?

Another feature of a two-sided media market is the nature of homing. A consumer (or an advertiser) is said to single-home if she joins only one platform but multi-home if she joins multiple platforms. In the case of streaming music, consumers typically join either Pandora or Spotify but not both. On the other hand, many consumers subscribe to multiple newspapers, such as the Wall Street Journal and New York Times. Similarly, advertisers can also choose to place advertisements in one platform or multiple platforms. This feature of two-sided markets poses another question: how does the scope for multi-homing on one or both sides of the market alter a platform's pricing decisions?

In this paper, we seek to theoretically examine how the characteristics of the two sides of a media market affect a platform's pricing strategy. In particular, we investigate how the pricing equilibrium might be affected by the relative strength of cross-side externalities and the scope for multi-homing. For this purpose, we consider a model of competing platforms that is faithful to the two-sided structure of media markets. Our model captures the two types of cross-side externalities observed in media markets: consumers' dislike for advertisements and advertisers' desire for consumers.

We begin the analysis by considering a media market where consumers single-home on one side and advertisers single-home on the other side of the market. The consumers are of two types; H-type consumers have a high dislike for advertising and L-type consumers have a low dislike for advertising. The conventional wisdom is that if consumers are heterogeneous, a firm could offer customized prices and earn more profits. Yet, even when the two types of consumers are quite heterogeneous in their sensitivity to advertising, we do not observe both platforms simultaneously adopting a customized pricing strategy in equilibrium. On the contrary, platforms adopt either asymmetric pricing strategies (i.e., one platform using a uniform price and the other offering customized prices) or a symmetric uniform pricing strategy (i.e., both platforms setting a uniform price for both types of consumers). To understand why, first note that in the asymmetric pricing equilibrium, the platform offering customized prices protects H-type consumers from advertisements and thus attracts more H-type consumers while the platform using a uniform price serves more advertisers since it provides them with access to H-type consumers. This permits each platform to focus on one side of the market and differentiate itself from the competitor. When advertisers' desire for consumers is low and H-type consumers' dislike for advertisements is high, the benefit of differentiation is high enough that we observe an asymmetric pricing equilibrium. Otherwise, both platforms adopt a uniform pricing strategy in equilibrium. Note that a symmetric customized pricing cannot be observed in equilibrium for the following reasons. On the one hand, if consumers' dislike for advertisement is low, it is not feasible for both platforms to use customized prices because H-type consumers will not choose the price intended for them. On the other hand, when consumers' dislike for advertising is high, even if a symmetric customized pricing is feasible, a symmetric uniform pricing strategy yields more profits than a symmetric customized pricing strategy. This is because the dislike for advertising makes it less attractive to acquire an additional advertiser and thus softens the competition on the advertiser side of the market.

Next, when consumers and advertisers can multi-home, we observe a very different pattern of results. Although both platforms do not adopt a symmetric customized pricing strategy when agents single-home, they use such a strategy when agents multi-home. Moreover, we observe asymmetric pricing strategies when agents single-home, but not when agents multi-home. This is because, when agents multi-home, it eliminates the competition between the platforms; specifically, the two platforms do not compete for the same marginal agent on either side of the market. With the elimination of competition, the (indirect) strategic effect of consumers' dislike for advertising and advertisers' desire for consumers is wiped out, and the choice of pricing strategy is determined by the direct effect of these cross-side externalities. Moreover, the strategy that is best for one platform is also the best for the other platform when they are not competing for the same marginal consumer.

At times, multi-homing might be confined to one side of the market. For example, advertisers might multi-home while consumers single-home. On examining such a situation, we find that the equilibrium outcome crucially hinges on the relative size of the two externalities. Now, when advertisers' desire for consumers is quite strong compared to consumers' sensitivity to advertising, it intensifies the inter-platform competition for consumers. Consequently, the competition moderating role of cross-side externalities gains significance, and in equilibrium advertisers' desire for consumers hurts a platform's profits while consumers' dislike for advertising improves profits (as is also the case where consumers and advertisers single-home). We obtain the reverse results when advertisers' desire for consumers is quite low compared to consumer sensitivity to advertising, because now the competition on the consumer side of the market is weaker. Thus, the strategic effect of cross-side externalities is weaker, and the direct effect dominates the equilibrium outcome. Hence, a platform's profits increase with advertisers' desire for advertising but decrease with consumers' dislike for advertising (as is also the case where consumers and advertisers multi-home). Yet, when the two cross-side externalities are of a comparable size, both of them have a positive effect on platform's equilibrium profits. Recall that when agents single-home, platforms do not adopt a symmetric customized pricing strategy, whereas when they are allowed to multi-home, platforms do not use asymmetric pricing strategies. However, when advertisers multi-home and consumers single-home, we observe all of the three possible configurations of pricing strategies in equilibrium. The asymmetric pricing equilibrium arises because when consumers single-home, they induce competition between the platforms, motivating the platforms to adopt differentiated pricing strategies to soften the inter-platform competition. The symmetric customized pricing equilibrium is observed because multi-homing by advertisers eliminates the competition on the advertiser side of the market, encouraging both platforms to simultaneously adopt a customized pricing strategy (instead of asymmetric pricing strategies) when H-type consumers' sensitivity to advertising is high (and thus the IC constraints can be satisfied).

This paper examines the impact of cross-side externalities on the equilibrium pricing strategies of competing platforms under different types of homing, and makes useful contributions to the literature. First, it extends the research on price discrimination to two-sided markets by studying customized pricing in a media market where consumers are heterogeneous in their sensitivity to advertising. In particular, we identify the conditions under which competing platforms may customize prices for different segments of consumers and when they might choose not to customize prices. Second, the paper shows how a platform's motivation to adopt customized pricing strategy systematically changes across different types of homing: single-homing, multi-homing, and asymmetric homing on both sides of the media market.

The rest of the paper is organized as follows. In the next section, we relate our work to prior literature. Section 2.3 lays out the structure of a two-sided media market. Section 2.4 analyzes a duopoly platform where both consumers and advertisers single-home. Section 2.5 examines the shift in pricing strategy when both consumers and advertisers multi-home. Section 6 studies a model of multi-homing advertisers and single-homing consumers. Finally, Section 2.7 concludes the paper. The proofs for all the claims made in the paper can be seen in the appendices.

2.2 Related Literature

Our work builds on prior literature on two-sided markets. The seminal work of Caillaud and Jullien (2003) and Rochet and Tirole (2006) provides a theoretical foundation for investigating two-sided markets. Caillaud and Jullien (2003) consider a platform as a matchmaker that facilitates transaction between buyers and sellers, and show that the side which single-homes will be treated favorably compared to the side that multi-homes. Specifically, the side that multi-homes will have all its surplus extracted. Rochet and Tirole (2003) examine how price is allocated between buyers and sellers in a two-sided market. They also define a two-sided market as one where the total transaction volume depends not only on the total price borne by the two sides of the market but also on how much each side of the market pays (see Rochet and Tirole (2006)). Armstrong (2006) considers a market where one side single-homes while the other side multi-homes. He finds that in equilibrium the price for the single-homing side is low compared to the price for the multi-homing side. Weyl (2010) shows that a platform can reach a unique equilibrium in a two-sided market by choosing the participation rates and then translating them to prices for the two sides of the market. Most of the above-mentioned literature considers a uniform price for each side of the market. In contrast, we consider the possibility a platform could charge customized prices for different segments on the same side of the market in the presence of cross-side externalities. Thus, our research extends the literature on price discrimination in one-sided markets to two-sided markets (see Armstrong (2006) for a review on one-sided market analyses). More recently, Liu and Serfes (2013) examine first-degree price discrimination in a buyer-seller market, which however cannot be implemented in practice. We, on the other hand, investigate customized pricing which media platforms can implement.

In our formulation, the agents within a segment are homogeneous in their sensitivity to externality but heterogeneous across segments. Moreover, differentiated platforms compete against each other. Ambrus and Argenziano (2009) investigate a model where agents are heterogeneous in their valuation of the externality. They find that a monopolist platform can increase its profits by establishing two asymmetric platforms with each platform primarily catering to one side of the market. Similarly in a duopoly, each platform can focus on one side of the market and improve its profits in equilibrium. Our goal is different from Ambrus and Argenziano (2009). We analyze single-homing, multi-homing and asymmetric-homing on the two sides of the market, and identify the conditions where symmetric uniform pricing, symmetric customized pricing, and asymmetric pricing will be observed in equilibrium.

Our research is related to the literature on media markets. Gal-Or and Dukes (2003) investigate the competition between two broadcasters using a two-sided model. Contrary to conventional wisdom, they show that competing broadcasters may offer minimally differentiated programs. This is because when programs are minimally differentiated, advertisers choose lower levels of advertising. The lower levels of advertising soften the competition in the product market, help advertisers earn higher profits, and in turn enable broadcasters to earn higher payments for advertising space. Dukes and Gal-Or (2003) show that broadcasters can benefit from offering exclusive advertising contracts. Note that exclusive advertising reduces the levels of advertising. This leaves consumers less informed about competing products and helps advertisers earn higher product margins on their products; recognizing this, broadcasters charge a higher price for advertising. Anderson and Coate (2005) show that the advertising level in a two-sided media market can be lower than the socially optimal level. This is because each platform does not fully internalize the nuisance costs of advertising and sets a high price for advertisers due to its local monopoly power. In addition, as in Gal-Or and Dukes (2003) and Dukes and Gal-Or (2003), each platform may strategically hold down the advertising level in their competition for consumers (see also Peitz and Valletti (2008)). One might assume that platforms would set a low price for content in an attempt to draw more consumers, and leverage the larger consumer base to charge a higher price for advertisers. However, Godes et al. (2009) show that in a duopoly, because of increased competition for advertisers and the resulting lower price per impression, competing platforms charge a higher price for content (compared to a monopolist). In contrast to prior papers, Ambrus et al. (2016) allow for the possibility that consumers could multi-home. They find that in the presence of multi-homing consumers, platforms have an incentive to strategically increase the level of advertising. Note that multi-homing consumers are exposed to advertisements on both platforms, and advertisers find them less valuable compared to single-homing consumers. Thus a platform does not generate as high an advertising revenue from the multi-homing consumers as it does from the exclusive consumers. Given this reality, by increasing the level of advertising, both platforms can reduce the body of multi-homing consumers and earn higher profits. As discussed so far, prior literature primarily focuses on the advertising level. We, however, examine the equilibrium pricing strategies and how they are affected by single-homing, multi-homing, and asymmetric homing on the two sides of a media market.

Our work also builds on the empirical literature on two-sided media markets. Kaiser and Wright (2006) examine German magazine data in light of Armstrong (2006). In keeping with Armstrong (2006), readers are subsidized. And unlike Armstrong (2006), but consistent with our framework, advertisers view the magazine as differentiated. Wilbur (2008) estimates demand on both the advertiser and the consumer sides of television industry. He finds that a 10% decrease in advertising level increases audience size by 25%. Our formulation reflects this distaste for advertising and explores its implications for the pricing strategy and profits of a platform.

2.3 Model

In this section, we present a duopoly model of a two-sided media market. The two competing platforms offer consumers media content, and host the promotional material of advertisers. Below we describe consumers, advertisers, and platforms in order. We use the subscript I to denote the consumer side of the market and the subscript J to indicate the advertiser side of the market.

2.3.1 Consumers

Let v_I denote the base value that consumers derive from the contents of a media platform. Consumers are heterogeneous in their preference for the two platforms, and we let consumers be uniformly distributed on a Hotelling line of unit length with Platform 1 located at the left end of the line and Platform 2 at the other end of the line. The intrinsic utility that a consumer located at θ on the Hotelling line derives from joining Platform 1 is $v_I - t_I\theta$, where t_I is the consumer's sensitivity to platform characteristics. However, if the consumer were to join Platform 2, the corresponding intrinsic utility will be $v_I - t_I(1-\theta)$. Because each platform offers a single product for consumers, the differentiation captured in the model reflects the differentiation in content and the way in which the content is delivered to consumers.

Consumers are also heterogeneous in their dislike for advertising. We consider two types of consumers: H-type consumers have a high sensitivity to advertising (denoted by γ_I^H), whereas L-type consumers have a low sensitivity to advertising (denoted by γ_I^L with $\gamma_I^H > \gamma_I^L$). For expositional reasons, we assume that the mass of each type of consumers is $\frac{1}{2}$. Our analysis holds if we allow β fraction of consumers to be of L-type and the remaining $(1-\beta)$ fraction of consumers to be of H-type with $0 < \beta < 1$. Furthermore, without loss of generality, we normalize $\gamma_I^L = 0$, implying that L-type consumers do not mind receiving advertisements, and that the two-sided market thus will not degenerate to a one-sided market. The indirect utilities that a consumer of type $\lambda \in \{H, L\}$, who is located at θ on the Hotelling line, derives from joining Platform 1 and Platform 2 are given respectively by:

$$U_{1I}^{\lambda}(\theta) = v_I - t_I\theta - \kappa\gamma_I^{\lambda}\alpha_{1J}^{e} - p_{1I}^{\lambda} \tag{2.1}$$

$$U_{2I}^{\lambda}(\theta) = v_I - t_I(1-\theta) - \kappa\gamma_I^{\lambda}\alpha_{2J}^{e} - p_{2I}^{\lambda}, \tag{2.2}$$

where α_{kJ}^{e} is the expected number of advertisers in Platform $k \in \{1, 2\}$, p_{kI}^{λ} is the price set by Platform k for a consumer of type λ, and

$$\kappa = \begin{cases} 1 & \text{if the consumer is exposed to advertisements} \\ 0 & \text{otherwise.} \end{cases} \tag{2.3}$$

One can also view α_{kJ}^{e} as the expected intensity of advertising in Platform k emanating from the unit mass of advertisers by virtue of their decision to join the platform. As we will discuss later, the platform may allow consumers to avoid advertisements by offering a menu of prices.

2.3.2 Advertisers

Advertisers join a platform to advertise their products and services. We assume that advertisers are heterogeneous in their preference for the platforms, and let advertisers be uniformly distributed on a Hotelling line of unit length with mass one. Consider the advertiser located at θ. If this advertiser expects to reach α_{kI}^{e} consumers through Platform k, then the value of reaching those consumers is $\gamma_J\alpha_{kI}^{e}$ where $\gamma_J(> 0)$ represents the advertiser's desire for consumers. The disutility the advertiser experiences because of lack of fit with Platform 1 is $t_J\theta$, and that with Platform 2 is $t_J(1-\theta)$, where t_J is a measure of the advertiser's sensitivity to platform characteristics. Moreover, the advertiser derives some base value, v_J, if it joins a platform and if the expected number of consumers in the platform is positive. Thus, the (indirect) utilities that the advertiser at θ derives on joining Platform 1 and Platform 2 are as follows:

$$U_{1J}(\theta) = v_J - t_J\theta + \gamma_J \alpha_{1I}^e - p_{1J} \tag{2.4}$$

$$U_{2J}(\theta) = v_J - t_J(1-\theta) + \gamma_J \alpha_{2I}^e - p_{2J}, \tag{2.5}$$

where p_{kJ} is the price Platform k sets for advertisers.

2.3.3 Platforms

Each platform earns profits from consumers and advertisers. In choosing its pricing strategy for the consumer side of the market, a platform has two options: it can either charge a uniform price for all consumers or offer a customized price for each segment of consumers. When a platform offers customized prices, consumers can choose to pay the high price and avoid advertisements or pay the low price and be exposed to advertisements. Thus, consumers self-select the price they want to pay. On the advertiser side of the market, each platform always sets a uniform price for all advertisers. We normalize each platform's marginal cost to zero so that a platform's profit maximization problem reduces to one of maximizing its revenue. Then, the profits of Platform k are given by,

$$\Pi_k = \begin{cases} \alpha_{kI} \cdot p_{kI} + \alpha_{kJ} \cdot p_{kJ}, & \text{under the uniform pricing strategy} \\ \alpha_{kI}^L \cdot p_{kI}^L + \alpha_{kI}^H \cdot p_{kI}^H + \alpha_{kJ} \cdot p_{kJ}, & \text{under the customized pricing strategy} \end{cases} \tag{2.6}$$

where α_{kI} and α_{kJ} represent the realized demand from each side of the market.

2.3.4 Decision sequence

The game unfolds in three stages. In the first stage, both platforms simultaneously decide whether to adopt a uniform pricing strategy or a customized pricing strategy for the two segments of consumers. In the second stage, each platform sets the prices for consumers (p_{kI} or $\{p_{kI}^H, p_{kI}^L\}$) and advertisers (p_{kJ}) depending on the pricing strategy chosen in the first stage of the game. This reflects the reality that media firms can often change their prices without changing the broad pricing strategy. Then in the third stage, consumers and advertisers simultaneously decide whether to join both platforms or join one platform (and if so, which platform to join). Consumers make this decision based on the observed prices and the expected level of advertisements in each platform, whereas advertisers choose the platform to join depending on the observed prices and the expected number of consumers joining each platform. We assume that consumers and advertisers form rational expectations, implying that the expectations are fulfilled in equilibrium. Furthermore, the two sides of the market are fully covered in that consumers and advertisers join at least one platform. To study the two platforms' pricing decisions, we examine the subgame perfect equilibrium. When there are multiple equilibria in the platforms' pricing strategy setting game, we focus on the equilibrium yielding the highest profits for both platforms.

In the next section, we start our analysis by examining the simplest case where both consumers and advertisers join only one platform (i.e., single-home). In subsequent sections, we explore the implications of agents multi-homing on one or both sides of the platform.

2.4 Single-Homing Analysis

Suppose both consumers and advertisers join only one platform. To fully analyze this case, we examine all three possible configurations of pricing strategies for the two platforms in the following three subgames: Subgame 1 where both platforms use a uniform pricing strategy, Subgame 2 where both platforms follow a customized pricing strategy, and Subgame 3 where one platform adopts a uniform pricing strategy and the other pursues a customized pricing strategy. Based on the equilibrium outcomes of the three subgames, we examine the platforms' decisions on the pricing strategies. We use the single-homing analysis provided in this section as a benchmark to better understand other homing conditions analyzed in later sections.

2.4.1 Subgame 1: Symmetric Uniform Pricing Strategy

Recall that $\lambda \in \{L, H\}$ denotes the type of consumers. If both platforms set a uniform price for consumers, the location of the λ-type consumer who is indifferent between joining Platform 1 and Platform 2 is given by:

$$\theta_I^\lambda = \tfrac{1}{2t_I}(p_{2I} - p_{1I} + \gamma_I^\lambda(\alpha_{2J}^e - \alpha_{1J}^e)) + \tfrac{1}{2}. \tag{2.7}$$

Since H-type consumers and L-type consumers pay the same price under a uniform pricing strategy, we can view the two segments as a composite market with the "average" marginal consumer being

$$\begin{aligned}\theta_I &= \tfrac{\theta_I^L + \theta_I^H}{2} \\ &= \tfrac{1}{2t_I}(p_{2I} - p_{1I} + \gamma_I(\alpha_{2J}^e - \alpha_{1J}^e)) + \tfrac{1}{2},\end{aligned} \tag{2.8}$$

where γ_I is the weighted average of γ_I^L and γ_I^H, implying $\gamma_I \equiv \frac{\gamma_I^H}{2}$ (since $\gamma_I^L = 0$). Furthermore, we can treat γ_I as the "average" consumer's dislike for advertisements.

The advertiser who is indifferent between joining Platform 1 and Platform 2 is given by

$$\theta_J = \tfrac{1}{2t_J}(p_{2J} - p_{1J} - \gamma_J(\alpha_{2I}^e - \alpha_{1I}^e)) + \tfrac{1}{2} \tag{2.9}$$

In equilibrium, we have $\alpha_{1I}^e = \theta_I$, $\alpha_{1J}^e = \theta_J$, $\alpha_{2I}^e = 1 - \theta_I$ and $\alpha_{2J}^e = 1 - \theta_J$. Let $\mathbf{p}$ denote the price vector $(p_{1I}, p_{1J}, p_{2I}, p_{2J})$. Then, each platform's profits can be written as:

$$\Pi_1^{ss1} = \alpha_{1I}(\mathbf{p}) \cdot p_{1I} + \alpha_{1J}(\mathbf{p}) \cdot p_{1J} \tag{2.10}$$

$$\Pi_2^{ss1} = \alpha_{2I}(\mathbf{p}) \cdot p_{2I} + \alpha_{2J}(\mathbf{p}) \cdot p_{2J}, \tag{2.11}$$

where the superscript "ss" denotes single-homing consumers and single-homing advertisers, the superscript "1" indicates Subgame 1 in which both platforms offer a uniform price to consumers, and the subscript $k \in \{1, 2\}$ indicates the identity of the platform.

On solving for the Nash equilibrium prices, we obtain:

$$p^*_{1I} = p^*_{2I} = t_I - \gamma_J \tag{2.12}$$

$$p^*_{1J} = p^*_{2J} = t_J + \gamma_I \tag{2.13}$$

It follows that $\alpha_{I1} = \alpha_{I2} = \alpha_{J1} = \alpha_{J2} = \frac{1}{2}$. Then each platform's profits are given by:

$$\Pi_1^{ss1*} = \Pi_2^{ss1*} = \tfrac{1}{2}(t_I + t_J) + \tfrac{1}{2}(\gamma_I - \gamma_J) \tag{2.14}$$

Detailed derivation of the equilibrium solution and the proofs for the claims made in the paper are presented in the appendix.

2.4.2 Subgame 2: Symmetric Customized Pricing Strategy

Here we consider the case where competing platforms customize the price for each segment of consumers. Recall that consumers can either pay a higher price and avoid advertisements, or pay a lower price and tolerate advertisements. Therefore, when both platforms adopt a customized pricing strategy, advertisers reach only consumers who pay the low price. The platforms charge a uniform price to advertisers.

In our analysis, we first solve for the equilibrium solution assuming that H-type consumers self-select to pay the high price whereas L-type consumers self-select to pay the low price in both platforms. Later, we derive the conditions under which the incentive-compatibility (IC) constraints for both types of consumers are satisfied. Due to consumers' ability to self-select, the original two-sided market is divided into a one-sided market comprised of only H-type consumers, and a two-sided market involving L-type consumers and advertisers. Therefore, the equilibrium solution for the one-sided market consisting of only H-type consumers is derived as in the standard Hotelling model, whereas the equilibrium solution for the two-sided market involving L-type consumers and advertisers is similarly derived as in the previous section. Below we present the equilibrium prices for consumers and advertisers, relegating details of the derivation to the appendix:

$$p_{1I}^{H*} = p_{2I}^{H*} = t_I \tag{2.15}$$

$$p_{1I}^{L*} = p_{2I}^{L*} = t_I - \gamma_J \tag{2.16}$$

$$p_{1J}^{*} = p_{2J}^{*} = t_J \tag{2.17}$$

where p_{kI}^{H*} is the price for H-type consumers, p_{kI}^{L*} is the price for L-type consumers, and p_{kJ}^{*} is the price for advertisers ($k = 1, 2$). In equilibrium, each platform's share of H-type consumers, L-type consumers and advertisers are as follows: $\alpha_{1I}^{H*} = \alpha_{2I}^{H*} = \alpha_{1I}^{L*} = \alpha_{2I}^{L*} = \frac{1}{4}$ and $\alpha_{1J}^{*} = \alpha_{2J}^{*} = \frac{1}{2}$.

Given the equilibrium solution, we assess whether the IC constraints are satisfied. It is sufficient to check the IC constraints for consumers joining Platform 1 because the same conditions apply for consumers joining the other platform as well.[1] Since L-type consumers joining Platform 1 would prefer to pay p_{1I}^{L*} and tolerate advertisements instead of paying p_{1I}^{H*} for an advertisement-free content, the IC constraint is given by $p_{1I}^{L*} + \gamma_I^L \alpha_{1J}^{*} < p_{1I}^{H*}$, or equivalently, $p_{1I}^{L*} < p_{1I}^{H*}$ (since $\gamma_I^L = 0$). On the other hand, H-type consumers would prefer to access the platform without the nuisance of advertisements, suggesting $p_{1I}^{H*} < p_{1I}^{L*} + \gamma_I^H \alpha_{1J}^{*}$. Together, we have:

$$p_{1I}^{L*} < p_{1I}^{H*} < p_{1I}^{L*} + \gamma_I^H \alpha_{1J}^{*} \tag{2.18}$$

On substituting (2.15) and (2.16) into (A.22), the IC constraints can be simplified to:

$$0 < \gamma_J < \frac{\gamma_I^H}{2}. \tag{2.19}$$

[1] More precisely, IC constraints include conditions where neither H-type consumers pay the low price nor L-type consumers pay the high price of the other platform. Given symmetric prices, these conditions are automatically satisfied as long as (A.22) holds because moving to the other platform decreases the utility by an amount equal to the transportation cost.

Note that the left-hand side inequality always holds. Thus, L-type consumers always choose the low price and tolerate the advertisements. The right-hand side inequality requires that γ_I^H be sufficiently large so that H-type consumers prefer to pay the high price and avoid the advertisements. If the right-hand side inequality fails, both L-type and H-type consumers would pay the low price and tolerate advertisements, implying that the equilibrium in which both platforms use customized pricing will not exist.

Finally, each platform's equilibrium profits are given by:

$$\Pi_1^{ss2*} = \Pi_2^{ss2*} = \frac{1}{2}(t_I + t_J) - \frac{1}{4}\gamma_J \tag{2.20}$$

where "2" in the superscript indicates Subgame 2 in which both platforms set customized prices for the two segments of consumers. It is useful to note that γ_I^H does not affect a platform's equilibrium profits. This is because when H-type consumers are insulated from advertisers, the price advertisers pay is not affected by γ_I^H.

2.4.3 Subgame 3: Asymmetric Pricing Strategies

Now we turn to the case where one platform implements a uniform pricing strategy while the other platform uses a customized pricing strategy for consumers. Without loss of generality, assume that Platform 1 uses a uniform pricing strategy whereas Platform 2 adopts a customized pricing strategy.[2] Both platforms charge a uniform price for advertisers.

Because Platform 2 uses customized prices, H-type consumers can insulate themselves from advertising by choosing Platform 2. The resulting smaller customer base makes the platform less attractive to advertisers. We find that the marginal consumer

[2] Given that both platforms are symmetric, the equilibrium results will remain the same if the strategy choices of the two platforms are reversed.

of type $\lambda \in \{H, L\}$ who is indifferent between the two platforms is as follows:

$$\theta_I^L = \frac{1}{2t_I}(p_{2I}^L - p_{1I}) + \frac{1}{2} \tag{2.21}$$

$$\theta_I^H = \frac{1}{2t_I}(p_{2I}^H - p_{1I} - \gamma_I^H \alpha_{1J}^e) + \frac{1}{2}, \tag{2.22}$$

and the marginal advertiser who is indifferent between the two platforms is given by:

$$\theta_J = \frac{1}{2t_J}(p_{2J} - p_{1J} - \gamma_J(\alpha_{2I}^e - \alpha_{1I}^e)) + \frac{1}{2}. \tag{2.23}$$

Furthermore, because the expectations are rational we have $\alpha_{1I}^e = \frac{\theta_I^L + \theta_I^H}{2}$, $\alpha_{1J}^e = \theta_J$, $\alpha_{2I}^e = \frac{1-\theta_I^L}{2}$ and $\alpha_{2J}^e = 1 - \theta_J$. Then each platform's profits are given by:

$$\Pi_1^{ss3} = \alpha_{1I}(\mathbf{p}) \cdot p_{1I} + \alpha_{1J}(\mathbf{p}) \cdot p_{1J} \tag{2.24}$$

$$\Pi_2^{ss3} = \alpha_{2I}^L(\mathbf{p}) \cdot p_{2I}^L + \alpha_{2I}^H(\mathbf{p}) \cdot p_{2I}^H + \alpha_{2J}(\mathbf{p}) \cdot p_{2J}, \tag{2.25}$$

where $\mathbf{p} = (p_{1I}, p_{2I}^L, p_{2I}^H, p_{1J}, p_{2J})$, $\alpha_{2I}^L = \frac{1-\theta_I^L}{2}$ and $\alpha_{2I}^H = \frac{1-\theta_I^H}{2}$. In the above profit expression, the superscript "ss3" indicates single-homing consumers, single-homing advertisers, and Subgame 3. Using these profits and assuming that IC constraints are satisfied, we derive the Nash equilibrium. Details of the derivation, equilibrium prices, and profits can be seen in the appendix.

Given the equilibrium solution, the IC constraint that permits Platform 2 to implement two-tier pricing is similar to that in (A.22): $p_{2I}^{L*} < p_{2I}^{H*} < p_{2I}^{L*} + \gamma_I^H \alpha_{2J}^*$, which can be rewritten as follows:

$$-9\gamma_J^3 - 6(\gamma_I^H + t_I)\gamma_J^2 + (\gamma_I^{H^2} + 6t_I\gamma_I^H + 72t_It_J)\gamma_J + 72t_It_J\gamma_I^H > 0 \tag{2.26}$$

$$9\gamma_J^3 - 6(2\gamma_I^H - t_I)\gamma_J^2 + (8\gamma_I^{H^2} - 18t_I\gamma_I^H - 72t_It_J)\gamma_J - 7\gamma_I^{H^3} + 72t_It_J\gamma_I^H > 0 \tag{2.27}$$

Note that if any of these IC conditions is not satisfied, then we will not observe an equilibrium in this subgame.

2.4.4 Platform Pricing Strategy

Based on the equilibrium profits of the preceding three subgames, we investigate the pricing strategy that competing platforms might adopt in a two-sided market. To facilitate this analysis, we focus on the region where both platforms cater to both sides of the two-sided market (see Lemma A.4 in the appendix for specific conditions). Below we discuss the existence of equilibrium in the first-stage supergame and then examine how consumers' and advertisers' sensitivities to each other affect a platform's pricing strategy. When two segments of consumers are heterogeneous in their sensitivity to advertising, one might expect platforms to offer customized prices. Yet we have the following result on customized pricing strategy when agents single-home.

Proposition 2.1. *In equilibrium, both platforms will not simultaneously adopt a customized pricing strategy.*

The rationale for this finding is two fold. First, when it could be profitable for both platforms to adopt a customized pricing strategy, the IC constraint is violated and thus a symmetric customized pricing cannot be implemented. Second, when customized pricing could be implemented by platforms, it is Pareto dominated.

To appreciate the first rationale, note that given competing platform's decision to adopt a customized pricing strategy, a platform will prefer to offer customized prices (instead of a uniform price) if γ_I^H is small but γ_J is large. To see this, consider the case where one platform chooses a uniform pricing strategy while the other adopts a customized pricing strategy. In this case, many H-type consumers will join the platform offering customized prices and insulate themselves from advertisements, but many advertisers will be motivated to join a platform that charges consumers a uniform price and provides them with access to H-type consumers. Then, by focusing on a different side of the market, the two platforms can avoid intense competition and earn more profits. However, the gains from this differentiation are reduced when γ_I^H is small and γ_J is large. This is because a small γ_I^H weakens H-type consumers' preference for the platform offering customized prices, thus making both platforms focus on the L-type consumers and advertisers. In addition, a large γ_J strengthens advertisers' preferences for the platform with more consumers, thus intensifying competition for L-type consumers. Therefore, the differentiation benefit is lost when γ_I^H is small and γ_J is large. In such a case, the platform is better off adopting a customized pricing strategy when the competing platform employs a customized pricing strategy. However, this is not feasible because when γ_I^H is small and γ_J is large, the IC constraint for the H-type consumers will not be satisfied. Specifically, when γ_J is large, $p_{1I}^L(= t_I - \gamma_J)$ is very small compared to p_{1H}^H. Moreover, because γ_I^H is small, H-type consumers are less sensitive to advertisements and are willing to pay the low price and be exposed to advertisements rather than pay the high price and avoid advertisements: $p_{1I}^L + \gamma_I^H \alpha_{1J} < p_{1H}^H$. Therefore, it is not feasible for both platforms to adopt a customized pricing strategy.

To see why customized pricing strategy is Pareto dominated, focus on the situation where platforms cannot pursue asymmetric pricing strategies because the IC constraint for an asymmetric strategy is violated. In this case, both platforms can use either a symmetric uniform pricing strategy or a symmetric customized pricing strategy. However, a symmetric uniform pricing strategy yields higher profits than a symmetric customized pricing strategy, whenever platforms can customize prices for the two types of consumers (that is, $\frac{\gamma_I^H}{2} > \gamma_J$ as given in equation A.23). To understand this, note that under symmetric uniform pricing strategy, consumers' dislike for advertisements reduces a platform's incentive to compete for an extra advertiser. This is because when more advertisers join a platform, it decreases the utility consumers derive from the platform and thus fewer consumers join the platform. As given in equation (2.13), the price for advertisers is $t + \gamma_I$, which rises with consumers' dislike for advertisements. Thus, the profits of both platforms increase with γ_I^H. On the other hand, under symmetric customized pricing strategy, since

H-type consumers are insulated from advertisers, their dislike for advertisement is irrelevant to the equilibrium profits of the platforms. Therefore, when γ_I^H is large enough, by implementing a uniform pricing strategy, platforms can provide advertisers with access to H-type consumers and thereby dramatically soften the competition on the advertiser side of the market. Consequently, both platforms will not choose a customized pricing strategy even when it is feasible.

Given the above impossibility result, we identify in the next proposition what pricing strategies the competing platforms can adopt in equilibrium.

Proposition 2.2. *Suppose $t_J \geqslant \frac{8}{7}t_I$. Then there exist thresholds γ_I^{H0} and γ_J^0 such that (1) when $\gamma_I^H > \gamma_I^{H0}$ and $\gamma_J < \gamma_J^0$, competing platforms adopt asymmetric pricing strategies with one platform using a uniform price and the other offering customized prices; (2) otherwise, both platforms pursue a symmetric uniform pricing strategy.*

Recall from Proposition 1 that both platforms will not offer customized prices in equilibrium. Now Proposition 2 implies that when $t_J \geqslant \frac{8}{7}t_I$, it is possible for one platform to offer customized prices if the other platform sets a uniform price for consumers. To follow the intuition for this finding, first note that when $t_J \geqslant \frac{8}{7}t_I$, advertisers are less price sensitive than consumers (see also Iyer and Soberman (2000)). At a given price, when advertisers become less sensitive to price, their demand is likely to grow, exerting a greater negative externality on H-type consumers. Then H-type consumers are likely to pay the high price and avoid advertisements rather than pay the low price and tolerate the advertisements. This implies that H-type consumers' IC constraint is more likely to be satisfied in such a situation, permitting one firm to offer customized prices and earn more profits from consumers while the other firm offers a uniform price to both types of consumers and earns more profits from advertisers.[3] Therefore, when $t_J \geqslant \frac{8}{7}t_I$, the two platforms can

[3] Observe that the left-hand side of (A.49) increases with t_J when $\gamma_J < \gamma_I^H$, which always holds

implement different pricing strategies in equilibrium.

Now to understand the conditions when an asymmetric pricing strategy and a uniform pricing strategy respectively emerge in equilibrium, suppose Platform 1 uses a uniform price while Platform 2 adopts customized prices. Notice that Platform 2 appeals to H-type consumers because it can shield them from advertisements. This gives Platform 2 an opportunity to charge a higher price for H-type consumers than what it would charge under a uniform pricing strategy. Thus, Platform 2 earns higher profits from H-type consumers by offering customized prices rather than a uniform price for the two segments of consumers. On the other hand, advertisers will find Platform 2 less attractive because it provides them with limited access to consumers. Recognizing this, Platform 2 charges advertisers a lower price and earns lower profits on the advertiser side of the market (compared to the case where Platform 2 charges a uniform price to consumers). From Platform 2's standpoint, therefore, the relative profitability of adopting a customized pricing strategy (instead of a uniform pricing strategy) depends on these two countervailing forces. When γ_I^H grows larger, Platform 2's profits from the consumer side of the market increase because the utility that H-type consumers derive from Platform 1 is lower. In this case, the gain on the consumer side of the market outweighs the loss on the advertiser side. On the other hand, when γ_J becomes larger, the loss on the advertiser side of the market increases because the competing platform (i.e., Platform 1) can offer advertisers a higher utility. Moreover, the loss on the advertiser side of the market may outweigh the gain on the consumer side of the market. Therefore, when Platform 1 adopts a uniform pricing strategy, it is more profitable for Platform 2 to choose a customized pricing strategy if γ_I^H is large but γ_J is small. Otherwise, it is optimal for both firms to follow a uniform pricing strategy.

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when asymmetric strategies constitute an equilibrium (i.e., when $\gamma_I^H > \gamma_I^{H0}$ and $\gamma_J < \gamma_J^0$). Furthermore, L-type consumers' IC constraint is always satisfied as long as the equilibrium demand

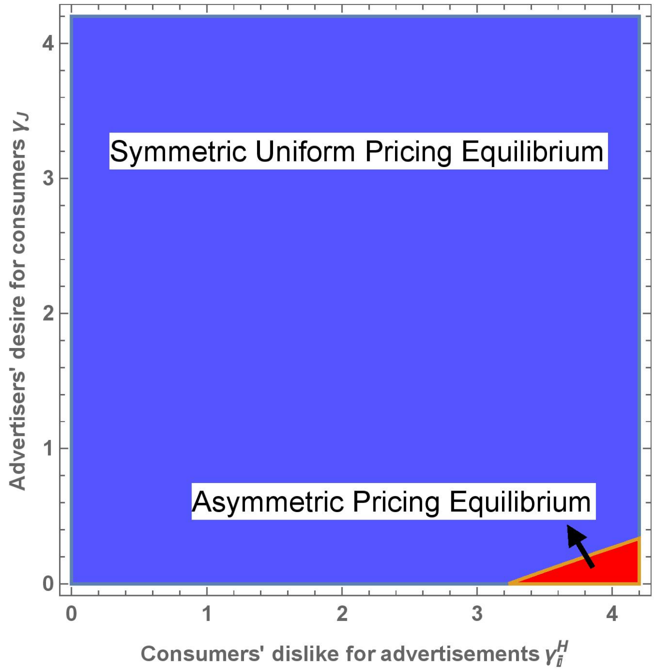

FIGURE 2.1: Pricing Strategy when Consumers and Advertisers are Single-homing ($t_I = 1$, $t_J = 5$).

Figure 2.1 illustrates how a platform's choice of pricing strategy varies with γ_I^H and γ_J. In the blue-colored region we observe the symmetric equilibrium where both platforms adopt a uniform pricing strategy, whereas in the red-colored region the asymmetric equilibrium prevails. Having considered single-homing agents thus far, we advance to investigate the equilibrium outcome when agents can multi-home on one or both sides on the market.

2.5 Multi-Homing Analysis

We see consumers reading multiple newspapers, such as the Wall Street Journal and New York Times, and firms releasing advertisements in both newspapers. This section considers a model where both advertisers and consumers have the option of joining either one platform (single-homing) or two platforms (multi-homing). Us-

for each platform is nonnegative.

ing the model, we explore how the prospect of multi-homing influences equilibrium pricing strategies.

Suppose both consumers and advertisers could join both platforms. When a consumer single-homes, her utility from joining each platform is as given in equations (B.37) and (B.38), whereas the corresponding utility for a single-homing advertiser is given by equations (2.4) and (2.5). If a consumer chooses to join both platforms, she obtains not only the intrinsic utility of joining one platform (v_I) but also the additional benefit of joining the second platform (δ_I). To appreciate the incremental utility captured in δ_I, consider a consumer already subscribing to the Wall Street Journal. By subscribing to the New York Times as well, the consumer could benefit from the additional news and analysis covered in the New York Times that do not appear in the Wall Street Journal. A downside to a consumer of joining both platforms is that she experiences disutility from the total number of advertisements in both platforms. Thus, the indirect utility derived from joining both platforms is given by:

$$U_{3I}^{\lambda}(\theta) = v_I + \delta_I - t_I - \gamma_I^{\lambda}(\kappa_1 \alpha_{1J}^e + \kappa_2 \alpha_{2J}^e) - p_{1I} - p_{2I} \tag{2.28}$$

where $\lambda \in \{L, H\}$ is the type of consumer, $\kappa_k \in \{0, 1\}$ indicates whether or not consumers are exposed to advertisements in Platform k ($k = 1, 2$), and α_{1J}^e and α_{2J}^e are the expected amount of advertisements in each platform. Moreover, $\alpha_{1J}^e + \alpha_{2J}^e \geqslant 1$ since advertisers are allowed to multi-home. In addition, p_{1I} and p_{2I} are the prices set by the platforms. Recall that when a platform sets two prices, the price that a consumer pays is denoted by p_{kI}^{λ} where $k \in \{1, 2\}$ and $\lambda \in \{H, L\}$. Furthermore, the utility derived by a multi-homing consumer does not depend on her location (θ). This is because, regardless of her location, the consumer should travel to both ends of the Hotelling line to reach both platforms: $\theta + (1 - \theta) = 1$.

Similarly, the utility of advertising in both platforms can be derived as:

$$U_{3J}(\theta) = v_J + \delta_J - t_J + \gamma_J(\alpha_{1I}^e + \alpha_{2I}^e) - p_{1J} - p_{2J}, \tag{2.29}$$

where δ_J is the incremental intrinsic value from the second platform, and α_{1I}^e and α_{2I}^e are the number of consumers that the advertiser expects to reach through each platform. As noted earlier, $\alpha_{1I}^e + \alpha_{2I}^e \geqslant 1$ since consumers are allowed to multi-home.

As in the previous section, we first discuss the three subgames: Subgame 1 where both platforms use a uniform pricing strategy, Subgame 2 where both platforms adopt a customized pricing strategy, and Subgame 3 where one platform uses a uniform pricing strategy while the other uses a customized pricing strategy. Then, we examine the equilibrium pricing strategies of both platforms.

2.5.1 Subgame 1: Symmetric Uniform Pricing Strategy

Suppose both platforms adopt a uniform pricing strategy for consumers. Since consumers are allowed to either multi-home or single-home, we have two marginal consumers: one indifferent between joining Platform 1 only and joining both platforms (θ_{13I}^λ), and the other indifferent between joining Platform 2 and multi-homing (θ_{23I}^λ). Denote the utility that a consumer located at θ derives from joining Platform 1, Platform 2 and both platforms by $U_{1I}^\lambda(\theta)$, $U_{2I}^\lambda(\theta)$, and $U_{3I}^\lambda(\theta)$ respectively. Then by solving $U_{1I}^\lambda(\theta) = U_{3I}^\lambda(\theta)$ and $U_{2I}^\lambda(\theta) = U_{3I}^\lambda(\theta)$, we obtain θ_{13I}^λ and θ_{23I}^λ as follows:

$$\theta_{13I}^\lambda = 1 - \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I^\lambda \cdot \alpha_{2J}^e - p_{2I}) \tag{2.30}$$

$$\theta_{23I}^\lambda = \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I^\lambda \cdot \alpha_{1J}^e - p_{1I}). \tag{2.31}$$

Now to ensure that at least one consumer multi-homes, we should have $\theta_{13I}^\lambda \leqslant \theta_{23I}^\lambda$, $\lambda \in \{H, L\}$. Hence we focus attention on the parameter space where this condition holds. Given this, consumers in the interval $[0, \theta_{13I}^\lambda)$ join only Platform 1, consumers in the interval $[\theta_{13I}^\lambda, \theta_{23I}^\lambda]$ join both platforms, and consumers in the interval $(\theta_{23I}^\lambda, 1]$ join only Platform 2 where $\lambda \in \{H, L\}$. Then, as in the previous section, we define the "average" marginal consumers $\theta_{13I} \equiv \frac{\theta_{13I}^H + \theta_{13I}^L}{2}$ and $\theta_{23I} \equiv \frac{\theta_{23I}^H + \theta_{23I}^L}{2}$ as follows:

$$\theta_{13I} = 1 - \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I \cdot \alpha_{2J} - p_{2I}) \tag{2.32}$$

$$\theta_{23I} = \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I \cdot \alpha_{1J} - p_{1I}) \tag{2.33}$$

where $\gamma_I = \frac{\gamma_I^H}{2}$.

Similarly in the advertiser market, we consider two marginal advertisers:

$$\theta_{13J} = 1 - \tfrac{1}{t_J} \cdot (\delta_J + \gamma_J \cdot \alpha_{2I}^e - p_{2J}), \tag{2.34}$$

$$\theta_{23J} = \tfrac{1}{t_J} \cdot (\delta_J + \gamma_J \cdot \alpha_{1I}^e - p_{1J}). \tag{2.35}$$

Note that marginal advertiser θ_{13J} is indifferent between joining Platform 1 and multi-homing while θ_{23J} is indifferent between joining Platform 2 and multi-homing. Thus, advertisers in the interval $[0, \theta_{13J})$ join only Platform 1, advertisers in the interval $[\theta_{13J}, \theta_{23J}]$ join both platforms, and advertisers in the interval $(\theta_{13J}, 1]$ join only Platform 2. Also note that to ensure the existence of multi-homing advertisers, we consider the parameter space satisfying $\theta_{13J} \leqslant \theta_{23J}$.

Then we obtain the demand functions by solving $\theta_{23I} = \alpha_{1I}^e$, $\theta_{13I} = 1 - \alpha_{2I}^e$, $\theta_{23J} = \alpha_{1J}^e$, and $\theta_{13J} = 1 - \alpha_{2J}^e$. The profits of the two platforms are as given in equations (2.10) and (2.11). The resulting equilibrium prices and profits are as follows:

$$p_{1I}^* = p_{2I}^* = \frac{2t_I t_J \delta_I - (\gamma_I + \gamma_J) t_I \delta_J + (\gamma_I - \gamma_J)\gamma_J \delta_I}{4t_I t_J - (\gamma_I - \gamma_J)^2} \tag{2.36}$$

$$p_{1J}^* = p_{2J}^* = \frac{2t_J t_I \delta_J + (\gamma_J + \gamma_I) t_J \delta_I + (\gamma_J - \gamma_I)\gamma_I \delta_J}{4t_I t_J - (\gamma_I - \gamma_J)^2} \tag{2.37}$$

$$\Pi_1^{mm1*} = \Pi_2^{mm1*} = \frac{t_J \delta_I^2 + t_I \delta_J^2 - (\gamma_I - \gamma_J)\delta_I \delta_J}{4t_I t_J - (\gamma_I - \gamma_J)^2}, \tag{2.38}$$

where the superscript $mm1$ denotes Subgame 1 in which both consumers and advertisers multi-home.

2.5.2 Subgame 2: Symmetric Customized Pricing Strategy

Now suppose both platforms set customized prices for consumers. As in the single-homing case, the two-sided market is divided into a one-sided market comprised of only H-type consumers and a two-sided market with L-type consumers on one side and advertisers on the other side.

Assuming that H-type consumers self-select to pay the high price and avoid advertisements while L-type consumers choose to pay the low price and tolerate advertisements, we derive the equilibrium prices of this game as follows:

$$p_{1I}^{H*} = p_{2I}^{H*} = \frac{\delta_I}{2} \tag{2.39}$$

$$p_{1I}^{L*} = p_{2I}^{L*} = \frac{\delta_I(4t_It_J - \gamma_J^2) - 2t_I\gamma_J\delta_J}{8t_It_J - \gamma_J^2} \tag{2.40}$$

$$p_{1J}^{*} = p_{2J}^{*} = \frac{t_J(\gamma_J\delta_J + 4t_I\delta_J)}{8t_It_J - \gamma_J^2} \tag{2.41}$$

where p_{1I}^{H*} and p_{2I}^{H*} are the prices for H-type consumers, p_{1I}^{L*} and p_{2I}^{L*} are the prices for L-type consumers, and p_{1J}^{*} and p_{2J}^{*} are the prices for advertisers as set by Platforms 1 and Platform 2, respectively. Given these equilibrium prices, L-type consumers always choose to pay p_{1I}^{L*} and tolerate the advertisements. On the other hand, H-type consumers choose to avoid advertisements and pay p_{1I}^{H*} if and only if $\gamma_J < 2\gamma_I^H$. It follows that the equilibrium demand of each platform on each side of the market is given by: $\alpha_{1I}^{H*} = \alpha_{2I}^{H*} = \frac{\delta_I}{4t_I}$, $\alpha_{1I}^{L*} = \alpha_{2I}^{L*} = \frac{2t_J\delta_I + \gamma_J\delta_J}{8t_It_J - \gamma_J^2}$, and $\alpha_{1J}^{*} = \alpha_{2J}^{*} = \frac{\gamma_J\delta_I + 4t_I\gamma_J}{8t_It_J - \gamma_J^2}$. By definition, we should have $\alpha_{kI}^{\lambda} \leqslant \frac{1}{2}$ and $\alpha_{kJ} \leqslant 1$ for $k \in \{1, 2\}$ and $\lambda \in \{H, L\}$ in equilibrium. Furthermore, to ensure at least one multi-homing consumer and advertiser, we have $\alpha_{kI}^{\lambda*} \geqslant \frac{1}{4}$ and $\alpha_{kJ}^{*} \geqslant \frac{1}{2}$. When these conditions are satisfied, the equilibrium profits are given by,

$$\Pi_1^{mm2*} = \Pi_2^{mm2*} = \frac{16t_I(t_J\delta_I^2 + t_I\delta_J^2) + \gamma_J\delta_I(8t_I\delta_J - \gamma_J\delta_I)}{8t_I(8t_It_J - \gamma_J^2)}, \tag{2.42}$$

where the superscript $mm2$ denotes Subgame 2 in which both consumers and advertisers multi-home.

2.5.3 Subgame 3: Asymmetric Pricing Strategies

Next we turn attention to the subgame where one platform (Platform 1) implements a uniform pricing strategy while the other platform (Platform 2) adopts a customized pricing strategy for consumers. Given that Platform 2 is offering customized prices, as in the single-homing case, H-type consumers can choose Platform 2 and insulate themselves from advertisements or choose Platform 1 and tolerate advertisements. Irrespective of the chosen platform, however, L-type consumers will be exposed to advertisements. In this setting, advertisers have access to both types of consumers if they choose Platform 1 but only to L-type consumers if they choose Platform 2. It is useful to note that because agents can multi-home, H-type consumers can also join both platforms, in which case they will be exposed to advertisements only in Platform 1. L-type consumers and advertisers can also join both platforms.

As in the previous section, we obtain the equilibrium prices of this game as follows:

$$p_{1I}^{*} = \frac{2(\gamma_I^H - 2\gamma_J)\gamma_J\delta_I + t_I\{8t_J\delta_I - 2(\gamma_I^H + 2\gamma_J)\delta_J\}}{16t_It_J - (\gamma_I^H - 2\gamma_J)^2} \tag{2.43}$$

$$p_{2I}^{H*} = \frac{\delta_I}{2} \tag{2.44}$$

$$p_{2I}^{L*} = \frac{\delta_I(4t_It_J - \gamma_J^2) - 2t_I\gamma_J\delta_J}{8t_It_J - \gamma_J^2} \tag{2.45}$$

$$p_{1J}^{*} = \frac{(2\gamma_J - \gamma_I^H)\gamma_I^H\delta_J + 2t_J\{4t_I\delta_J - (\gamma_I^H + 2\gamma_J)\delta_I\}}{16t_It_J - (\gamma_I^H - 2\gamma_J)^2} \tag{2.46}$$

$$p_{2J}^{*} = \frac{t_J(\gamma_J\delta_I + 4t_I\delta_J)}{8t_It_J - \gamma_J^2} \tag{2.47}$$

Observe that the prices that Platform 1 charges remain identical to those in Subgame 1 where both platforms adopt a uniform pricing strategy (after applying $\gamma_I = \frac{\gamma_I^H}{2}$), while the prices that Platform 2 sets are exactly the same as those of Subgame 2 where both platforms adopt a customized pricing strategy. This observation implies that the IC constraints of this game are identical to those in Subgame 2, and thus equivalent to $\gamma_J < 2\gamma_I^H$. It also implies that the equilibrium demand of Platform 1 in each market is identical to that in Subgame 1 while that of Platform 2 is identical to that in Subgame 2. Finally, the equilibrium profits are given as follows:

$$\Pi_1^{mm3*} = \frac{4\delta_I(t_J\delta_I+\gamma_J\delta_J)+2\delta_J(2t_I\delta_J-\gamma_I^H\delta_I)}{16t_It_J-(\gamma_I^H-2\gamma_J)^2)} \tag{2.48}$$

$$\Pi_2^{mm3*} = \frac{16t_I(t_J\delta_I^2+t_I\delta_J^2)+\gamma_J\delta_I(8t_I\delta_J-\gamma_J\delta_I)}{8t_I(8t_It_J-\gamma_J^2)}, \tag{2.49}$$

where the superscript $mm3$ denotes the third subgame when both consumers and advertisers multi-home. Again, note that $\Pi_1^{mm3*} = \Pi_1^{mm1*}$ and that $\Pi_2^{mm3*} = \Pi_2^{mm2*}$.

2.5.4 Platform Pricing Strategy

Now using the equilibrium profits of the three subgames, we examine the supergame where platforms decide on the pricing strategy for consumers. In particular, we explore whether a platform should adopt a uniform pricing strategy or pursue a customized pricing strategy. On studying the supergame, we have the following results on the existence and the condition of the pricing equilibrium.

Proposition 2.3. *When both consumers and advertisers multi-home, the two competing platforms will not use asymmetric pricing strategies in equilibrium. However, they can simultaneously adopt a customized pricing strategy.*

It is important to note that this finding is different in significant ways from the single-homing results. First, when agents single-home, platforms can use asymmetric pricing strategies (see Proposition 2.2); but when agents multi-home, we do not observe asymmetric pricing strategies in equilibrium. Second, both platforms cannot simultaneously adopt a customized pricing strategy when agents single-home (see Proposition 2.1), but they can if agents multi-home.

To understand the proposition, note that multi-homing by consumers and advertisers fully softens the competition between the two platforms. To further appreciate this, observe that a platform's demand on both sides of the market is a function of only its own prices, not the other platform's prices: for example, in the case of symmetric uniform pricing by both platforms, we have:

$$\alpha_{1I} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_I - p_{1I})t_J - (\delta_J - p_{1J})\gamma_I] \tag{2.50}$$

$$\alpha_{1J} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_J - p_{1J})t_I - (\delta_I - p_{1I})\gamma_J] \tag{2.51}$$

$$\alpha_{2I} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_I - p_{2I})t_J - (\delta_J - p_{2J})\gamma_I] \tag{2.52}$$

$$\alpha_{2J} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_J - p_{2J})t_I - (\delta_I - p_{2I})\gamma_J] \tag{2.53}$$

The reason why a platform's demand is not affected by its competitor's price is because each platform's demand is determined by the marginal agents (marginal consumer as well as marginal advertiser) who are indifferent between joining only the competing platform and joining both platforms. For example, the total consumer demand for Platform 1 is determined by the marginal consumer θ_{23J} who is indifferent between joining only Platform 2 and joining both Platform 1 and Platform 2. These marginal agents decide whether or not to join the focal platform while always joining the competing platform. Thus, their decision is driven by the price of the focal platform, not the price of the competing platform, implying that each platform's demand on each side of the market is determined only by its own price. Therefore, when consumers and advertisers multi-home, they effectively turn the duopoly market into two local monopolies. Consequently, the profits of a platform are not affected by the competing platform's pricing strategy. Thus, if a platform finds a pricing strategy (either uniform or customized) more profitable, the other platform also finds the same strategy more profitable. In equilibrium, therefore, both platforms always choose identical pricing strategies. On further examining when we will observe symmetric uniform pricing strategy and symmetric customized pricing strategy, we have the following result:

Proposition 2.4. *When both consumers and advertisers multi-home, there exists*

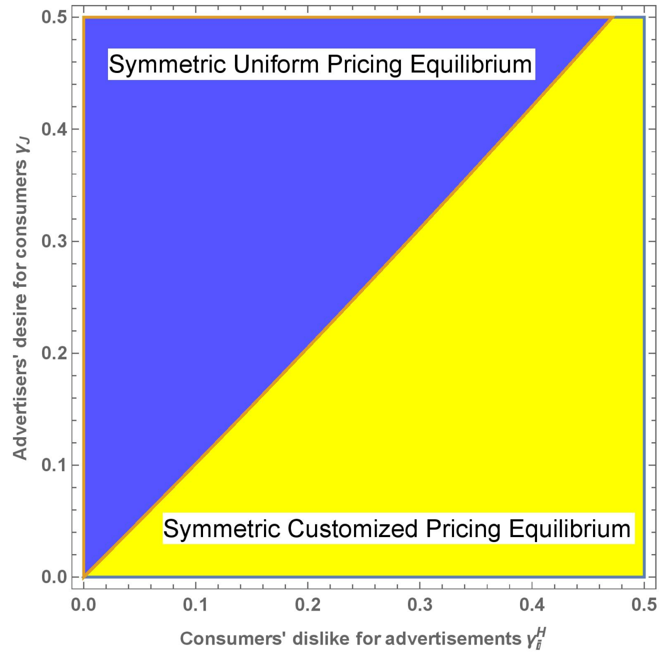

FIGURE 2.2: Pricing Strategy when Consumers and Advertisers are Multi-homing ($t_I = 1$, $t_J = 5$, $\delta_I = 1.5$, $\delta_J = 7.5$).

a threshold γ_I^{H00} such that if $\gamma_I^H > \gamma_I^{H00}$ both platforms adopt a customized pricing strategy; otherwise both platforms implement a uniform pricing strategy.

This proposition clarifies that both platforms will adopt a customized pricing strategy when H-type consumers' dislike for advertisements is above a threshold, but adopt a uniform pricing strategy otherwise. The rationale for this finding is as follows. When γ_I^H is large, H-type consumers dislike advertisements so much that they will not join a platform implementing a uniform pricing strategy because it will expose them to advertisements. Then, a platform can draw more consumers by offering customized prices and insulating H-type consumers from advertisements. However, advertisers will find this platform less attractive because it does not provide access to H-type consumers. Nevertheless, when $\gamma_I^H > \gamma_I^{H00}$, the gain from the consumer side is large enough to compensate for the loss on the advertiser side of the market. Therefore, it is more profitable for a platform to adopt a customized pricing strategy.

Because a platform's profits do not depend on the decision of the other platform, both platforms adopt a customized pricing strategy when $\gamma_I^H > \gamma_I^{H00}$. In contrast to the single-homing case analyzed in the previous section, now the IC constraints do not contradict the condition for platforms to implement a customized pricing strategy. Under the IC constraints, H-type consumers would choose to pay the high price because they strongly dislike advertisements. In such a case, shielding H-type consumers from advertisements by offering customized prices is more profitable for the platform.

Next, if γ_I^H is not large enough, insulating H-type consumers from advertisers would just reduce the demand on the advertiser side of the market without a large increase in the demand on the consumer side of the market. Consequently, both platforms find it more profitable to pursue a uniform pricing strategy. To help appreciate these results, Figure 2.2 illustrates how a platform's choice of pricing strategy varies with γ_I^H and γ_J. In the blue-colored region both platforms adopt a uniform pricing strategy, while in the yellow-colored region both platforms use a customized pricing strategy.

2.6 Analysis of One-Sided Multi-Homing

Section 4 examined the pricing strategy of duopoly platforms when both consumers and advertisers single-home, whereas Section 5 investigated the pricing strategy when agents on both sides of the market multi-home. In practice, we also observe cases where only one side of the market multi-homes. For example, in the streaming music market, consumers are most likely to join either Spotify or Pandora but not both, whereas advertisers could promote their products in one or both platforms. In this section we consider a two-sided media market where advertisers multi-home but consumers single-home.[4] This analysis helps us to understand how the strategic implications of one-sided multi-homing are different from those of two-sided multi-homing (section 2.5) and two-sided single-homing (Section 2.4).

Consider the case where advertisers multi-home but consumers do not. The utility that consumers derive on joining a platform is the same as that given in Section 2.4 (see equations (B.37) and (B.38)). Likewise, the utility that advertisers derive from joining a single platform is as given in equations (2.4) and (2.5). However, when an advertiser joins both platforms, its utility is given by equation (A.162). As discussed earlier, the total consumer demand for the two platforms adds up to 1 (i.e., $\alpha^e_{1I} + \alpha^e_{2I} = 1$) because consumers single-home, but the total advertiser demand could exceed 1 (i.e., $\alpha^e_{1J} + \alpha^e_{2J} \geqslant 1$) because advertisers may multi-home.

As in the previous two sections, we start our analysis by deriving the equilibrium solution for each of the three subgames. Because the analysis is very similar to those reported in the previous two sections, we do not repeat it here but present it in the appendix. Furthermore, we focus attention on the key results that are different from the single-homing analysis reported in Section 4 and the multi-homing analysis reported in Section 5.

In this section, we first examine how the profits of the platforms are affected by H-type consumers' sensitivity to advertising and advertisers' desire for consumers. For clarity, we focus on the subgame where both platforms use a uniform pricing but qualitative results also hold for the other subgames. To begin, we note that when both consumers and advertisers single-home, each platform's profits decrease in γ_I but increase in γ_J. This is because a greater γ_I softens competition for an additional advertiser while a greater γ_J intensifies competition for an additional consumer (see also Armstrong 2006). When both consumers and advertisers multi-home, on the

[4] It is less likely that when consumers can multi-home, advertisers only single-home. However, the analysis of such a market yields qualitatively similar results as those reported in this section, and it can be availed upon request.

other hand, each platform's profits increase in γ_I but decrease in γ_J. This is because multi-homing agents effectively turns a duopoly into two local monopolies and thus, the competition-moderating effect of cross-side network externalities has no ground.[5] We now examine the platforms' profits when consumers single-home but advertisers multi-home.

Proposition 2.5. *When only advertisers multi-home and both platforms set a uniform price for consumers, a platform's profits (a) increase in γ_I but decrease in γ_J when $\gamma_I < \frac{1}{3}\gamma_J$, (b) decrease in γ_I but increase in γ_J when $\gamma_I > 3\gamma_J$, (c) increase in both γ_I and γ_J when γ_I is between $\frac{1}{3}\gamma_J$ and $3\gamma_J$.*

Notice that Part (a) of the above result is akin to what we observe in the single-homing model, whereas Part (b) is similar to what we see in the multi-homing model. This is because when one side of the market single-homes while the other side multi-homes, the equilibrium result depends on the balance of two forces: the intensity of competition on the single-homing side of the market (i.e., the consumer side), and the market expansion on the multi-homing side of the market (i.e., the advertiser side). The size and direction of these forces depend on the relative size of the two externality parameters.

When $\gamma_I < \frac{1}{3}\gamma_J$, H-type consumers are less sensitive to advertising and advertisers have a higher desire for consumers. In this case, each platform can build a larger base of advertisers. Moreover, because now an additional consumer could increase the willingness to pay of its entire base of advertisers, the competition on the consumer side of the market is in general, very intense. In this context, an increase in γ_J expands the advertiser side of the market for each platform, thereby intensifying the competition for consumers. This hurts each platform's profits. On the other hand, an increase in γ_I^H (and thus in γ_I) makes H-type consumers more sensitive

[5] See Claims A.1 and A.2 in the Appendix for the proof of the single-homing and the multi-homing cases.

to advertisements and thus renders expanding the advertiser side of the market less attractive for both platforms. This attenuates the competition for an additional consumer, and thus increases both platforms' profits.

We observe the opposite result when $\gamma_I > 3\gamma_J$. To appreciate this finding, note that when H-type consumers are more sensitive to advertising and advertisers care less for consumers, it is harder for each platform to build a large base of advertisers, in which case the value of attracting an additional customer is low. Hence the two platforms do not fight intensely for an additional customer on the consumer side of the market. In this situation, an increase in γ_J increases advertisers' utility and thus expands the advertiser market for both platforms, but this market expansion minimally affects the competition for consumers, since the advertiser base is relatively small. Hence we see an increase in each platform's profits. On the other hand, an increase in γ_I makes consumers more sensitive to the number of advertisements, shrinks the advertiser side of the market, and decreases each platform's profits.

For intermediate values of advertiser's sensitivity to advertising ($\frac{1}{3}\gamma_J \leqslant \gamma_I \leqslant 3\gamma_J$), we find that an increase in γ_I softens competition for consumers without substantially shrinking the advertiser market. Moreover, an increase in γ_J expands the market for advertisers without intensifying competition on the consumer side of the market. Consequently, an increase in either γ_I or γ_J improves a platform's profits.

Shifting attention to the equilibrium pricing strategy, first note that three configurations of pricing strategies are possible: both platforms using a uniform pricing strategy, both platforms using a customized pricing strategy, and platforms using different pricing strategies. Recall that when consumers and advertisers single-home, both platforms cannot simultaneously adopt customized pricing in equilibrium (See Proposition 2.1). Also recall that when consumers and advertisers multi-home, they cannot use asymmetric pricing strategies (See Proposition 2.3). When only advertisers multi-home, we have the following result.

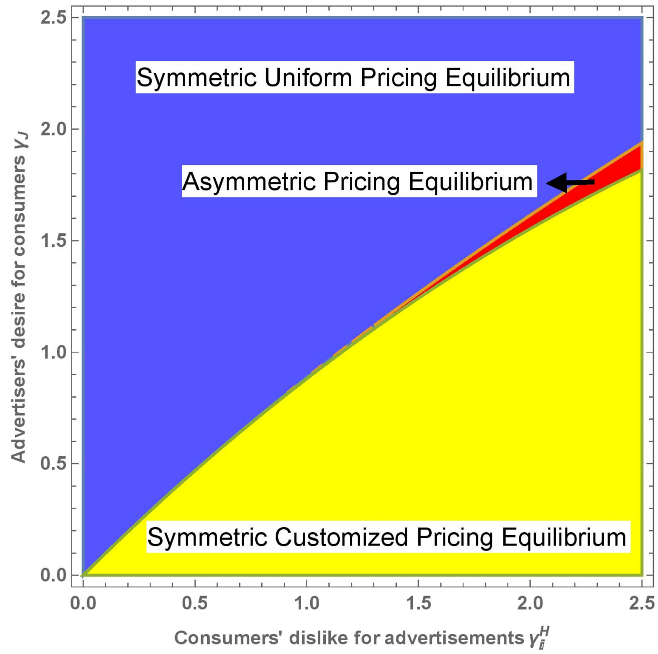

FIGURE 2.3: Pricing Strategy when only Advertisers are Multi-homing ($t_I = 1$, $t_J = 5$, $\delta_J = 7.5$).

Proposition 2.6. *When advertisers multi-home and consumers single-home, both platforms can adopt a customized pricing strategy and asymmetric pricing strategies in equilibrium.*

To appreciate the intuition, let us examine how changes in γ_I^H affect equilibrium behavior (while keeping γ_J fixed). Define $\gamma_I^{H(1)}$ and $\gamma_I^{H(2)}$ as thresholds in γ_I^H at which one platform changes the pricing strategy. First, when $\gamma_I^H < \gamma_I^{H(1)}$, a uniform pricing strategy is more attractive to both platforms. This is because if platforms were to adopt customized prices, they will protect H-type consumers from advertisers, and shrink the advertiser market without substantially increasing the revenue from H-type consumers (compared to the case of adopting a uniform pricing strategy). Next, when $\gamma \in [\gamma_I^{H(1)}, \gamma_I^{H(2)}]$, adopting a customized pricing strategy becomes more attractive. Now, one of the two platforms is willing to switch to a customized pricing while the other stays with a uniform pricing strategy. This is because differentiated pricing strategies help the two platforms avoid intense price competition on the consumer side of the market. Recall that when both consumers and advertisers multi-home, we do not observe such an asymmetric pricing equilibrium because the two platforms are not competing for the same marginal consumer. Finally, when $\gamma_I^H > \gamma_I^{H(2)}$, the benefit of adopting differentiated pricing strategies is outweighed by the relative attractiveness of using a symmetric customized pricing strategy. Therefore, the second platform also switches to a customized pricing strategy. Recall that in the single-homing analysis, we do not obtain such a symmetric customized pricing equilibrium, because the incentive compatibility constraints are not consistent with the condition for the equilibrium. Now the IC constraints hold whenever both platforms choose a customized pricing strategy. Figure 2.3 illustrates how the equilibrium pricing strategy changes with γ_I^H and γ_J. We observe a symmetric uniform pricing equilibrium in the blue-colored region, an asymmetric pricing equilibrium in the red-colored region, and a symmetric customized pricing equilibrium in the yellow-colored region. Thus, when only advertisers multi-home, we observe in equilibrium all the three possible configurations of pricing strategies.

2.7 Conclusion

The goal of the paper is to investigate the issue of price customization in a two-sided media market. Some of our intuition might suggest that a platform should offer customized prices if two segments of consumers are quite heterogeneous in their sensitivity to advertisements. Yet, when two platforms compete for single-homing consumers on one side and single-homing advertisers on the other side of the market, it is not possible for both platforms to adopt a symmetric customized pricing strategy. In fact, often both platforms will adopt a symmetric uniform pricing strategy. This is because when both platforms adopt a uniform pricing strategy they can leverage H-type consumers' dislike for advertising to soften the competition on the advertiser side of the market. Even in instances where advertisers' desire for consumers is low and H-type consumers' dislike for advertising is high, only one platform will offer customized prices. This is because by adopting asymmetric pricing strategies, the platform with uniform prices can focus on profiting from the advertiser side of the market, whereas the platform with customized prices can earn more profits from the consumer side of the market, and this differentiated strategy helps to soften inter-platform competition.

When both consumers and advertisers multi-home, however, competing platforms do not adopt asymmetric pricing strategies. Moreover, both platforms might adopt a symmetric customized pricing strategy. To understand why, recall that when consumers multi-home, they eliminate the competition between the two platforms. Then, cross-side externalities no longer play a role in moderating competition and there is also no need to soften competition by adopting differentiated pricing strategies. Hence, both platforms behave as though they are local monopolists. Therefore, the direct effects of the cross-side externalities determine the pricing decisions of both platforms: when advertisers' desire for consumers is higher, both platforms use a symmetric uniform pricing strategy; whereas when H-type consumers' dislike for advertising is higher, both platforms pursue a symmetric customized pricing strategy.

Sometimes, multi-homing is observed on only one side of the market. In a market where consumers single-home and advertisers multi-home, we obtain a very different pattern of results. First, in contrast to the multi-homing model, both platforms can pursue asymmetric pricing strategies. This is because when consumers single-home, they make the platforms compete head to head for the marginal consumer, and thus platforms strive to soften the competition by employing differentiated pricing strategies. In this case, one platform adopts a customized pricing strategy while the other platform uses a uniform pricing strategy. Second, in contrast to the single-homing model, both platforms may adopt a symmetric customized pricing strategy. We obtain this result because when advertisers multi-home, platforms no longer compete for the same marginal advertiser and hence differentiated pricing strategies become less attractive to both platforms. Thus, platforms use a symmetric customized pricing strategy when H-type consumers' sensitivity to advertising is high. Moreover, the IC constraints are consistent with the condition for the symmetric customized pricing equilibrium.

Our results have important implications for the pricing strategies of media platforms. For example, when both consumers and advertisers can multi-home, as in the case of newspapers such as the New York Times and Wall Street Journal, competing platforms might optimally choose the same pricing strategy given any pair of externalities. Thus, in the absence of the information on cross-side externalities, it is optimal to follow the pricing strategy of the competing platform. However, when consumers rarely multi-home, as in the case of music streaming services such as Spotify and Pandora, it is possible for competing platforms to adopt asymmetric pricing strategies. Finally, the changes in the magnitude of the externalities may motivate firms to alter their pricing strategies. Perhaps, Hulu's recent change from a uniform to a customized pricing strategy could be ascribed to an increase in consumers' sensitivity to advertising. A decrease in the advertisers' desire for consumers could have also motivated Hulu to adopt a uniform pricing strategy. Thus, it is important for a media platform to carefully examine the characteristics of both sides of the market and its competitor's action when formulating a pricing strategy.

Two-sided media markets constitute a thriving business. In this paper, we have theoretically analyzed some relevant issues and many others await further research. For example, future work could consider extending the model to include more than two sides. While our model allowed for two consumers segments, there was no strategic interaction between them. Further research could explore the implications of such a strategic interaction. There is an opportunity to empirically validate our model predictions using field data (e.g., Tucker and Zhang (2010), Sen et al. (2012) as well as lab experiment (e.g., Lim and Ho (2007) , Amaldoss and Shin (2011)).

3

Media Platforms' Content Provision Strategy and Source of Profits

3.1 Introduction

A media platform, such as Spotify, the Wall Street Journal or Hulu, leverages the content produced by artists, columnists, or studios to attract consumers. Advertisers are keen to reach these consumers to promote their products and services. This gives the media platform an opportunity to earn profits from the two sides of its market: consumers and advertisers. We see platforms, such as the paid newspapers and paid TV channels, generating revenue from both sides of the market. Some other platforms, such as the Huffington Post, offer free content to consumers and earn all their profits from advertisers. In contrast, ad-free platforms, such as the streaming music platform Tidal, earns its entire profits from consumer subscription. Tidal explicitly announces to the public that it "does not generate revenue through advertising placement in its service." [1] Thus media platforms employ a variety of strategies, and this divergence in practice raises an interesting question. When should a media platform adopt each of these strategies: paid content with advertising, paid

[1] http://tidal.com/soc/

content with no advertising, and free content with advertising? Underlying this issue is an inherent tradeoff that media platforms face. On the one hand, media platforms need to allocate space or bandwidth for content so that more consumers are drawn to the platform. On the other hand, the space or bandwidth could be used to generate advertising revenue, but too much advertising can deter consumers from visiting the platform.

Prior research on two-sided markets has not focused attention on how a platform allocates its limited space (or bandwidth) for content and advertising. This allocation decision is crucially related to a platform's source of profits. The literature simply assumes the way a platform earns profits, without making the source of profits an endogenous outcome of the model. For example, in Rochet and Tirole (2006) it is assumed that the platform charges both consumers and advertisers, whereas in Anderson and Coate (2005) the platform is assumed to operate on both sides of the market but earns all its profits from advertisers. The literature ignores the fact that some platforms, such as Netflix, strategically choose to forgo advertising revenue.

In this paper, we take a step toward theoretically examining a media platform's strategic allocation of bandwidth for content and advertising, and its source of profits. Consider a media market where consumers are on one side and advertisers are on the other side. To cater to its consumers, a media platform procures content from independent content supplier(s). Upon analyzing this three-sided market, we find that as long as advertising does not significantly annoy consumers, a platform earns profits from advertisers by allocating a proportion of its space for advertisements. Whether or not a platform earns profits from consumers profits, however, depends on the price of content. If content price is low, a platform purchases a large amount of content and charges consumers for it. But if the content is expensive, a platform purchases the bare minimum content, provides it for free and generates all its profits from advertisers. On studying how a platform's source of profits is moderated by the structure of both the content market and the platform market, we observe interesting results.

First, one may expect inter-platform competition to force a platform to allocate a higher proportion of space for content, and earn less profits from advertisers. In contrast to this intuition, our analysis of a perfectly competitive content market shows that inter-platform competition motivates a platform to rely more on advertising profits by switching from paid-content-paid-content strategy to free-content strategy. By strategically reducing the proportion of content, a duopoly platform reduces the competition for consumers. Yet, the opposite can happen in monopoly content market: inter-platform competition may motivate a platform to switch from a free-content strategy to a paid-content-with-ads strategy. To understand the rationale for this reversal in result, note that the price of content is its marginal cost in a perfectly competitive content supplier market. A monopoly content supplier, however, sets different prices for content depending on whether it is facing a monopoly platform or a duopoly platform. The monopoly content supplier recognizes that a duopoly platform is more sensitive to the content price than a monopoly platform because hosting less content reduces a duopoly platform's competition for consumers. Additionally, if consumers' desire for content is low, a duopoly platform is even more price sensitive than a monopoly platform. As a result, if consumers' desire for content is sufficiently low, the monopoly supplier charges a low content price for a duopoly platform and it motivates the platform to use a paid-content-with-ads strategy, whereas the monopoly content charges a higher content price for a monopoly platform and it induces the platform to adopt a free-content strategy.

Second, one may expect a platform to earn higher profits when paying a lower price for the content it procures from the supplier. Counter to view, we find that a lower content price can hurt a platform. This happens when platforms compete. To appreciate the result, notice that though a lower content price helps a competing platform save on the cost of content, it motivates them to compete for consumers by allocating more bandwidth for content, thereby reducing advertising revenue. Now if the original content price is high, then the cost saving may not compensate for the loss in advertising revenue, thus hurting the profits of competing platforms.

Third, the conventional view is that a monopoly platform can earn more profits than a duopoly platform. Yet our analysis shows that a monopoly platform can earn less profits. In particular, a monopoly content supplier extracts all the profits of a monopoly platform but not those of a duopoly platform when platforms adopt a free content strategy. To follow the intuition, note that when a monopoly platform pursues a free-content strategy, it strives to attract consumers so that it can profit from the advertisers (who are interested in reaching those consumers). In this case, the platform provides the bare minimum content to sustain its customer base. Interestingly, this minimum level of content is inelastic to content price, helping the content supplier to extract all the surplus from the media platform. The result, however, is very different when media platforms compete. Now each platform carefully balances the revenue from allocating more bandwidth for content against the foregone advertising revenue as well as its cost (i.e., the content price). Hence, each competing platform's demand for content is sensitive to content price, and this helps the platform to earn positive profits.

Fourth, when advertisers' valuation for consumers increases, one may think that a platform will host more advertising. This intuition, however, is incorrect when a platform adopts a free-content strategy. To see why, notice that if the platform is a monopolist, it provides consumers the minimal amount of content and thus cannot further reduce the content to accommodate more advertising. If the platform is a duopolist, it hosts even less advertising because each consumer can bring more advertising revenue as advertisers' valuation of consumers increases. Then to compete for these valuable consumers, a duopoly platform ends up offering a higher proportion of content, thus reducing the advertising in its platform. Consistent with our intuition, when a platform adopts a paid-content-with-ads strategy, its hosts more advertising. This is because when consumers become more valuable, now a platform can attract them consumers with a lower price. This flexibility enables a platform to host more advertising and take advantage of advertisers' higher willingness to pay.

Finally, as advertisers' higher valuation for consumers benefits a media platform, one may believe that may also benefit a content supplier. But we identify circumstances when a content supplier is hurt by an increase in advertisers' valuation of consumers. First, when a monopoly supplier sells content to a platform using a paid-content-with-ads strategy, then for a given content price the platform purchases less content if advertisers valuation of consumers increases . This resulting inward shift of the content demand curve hurts the monopoly content supplier's profits. Next, when a duopoly content supplier sells to a platform using a free-content strategy, the duopoly supplier earns more revenue from selling content because it can charge a higher content price (if it faces a monopoly platform), or because the content demand curve shifts outward (if it faces duopoly platforms). This increase in revenue, however, motivates each supplier to exert more effort toward shaping consumers' preference for its content. Now if the marginal cost of shaping consumers' preference is small, each supplier spends more on shaping consumes' preference, and the resulting the increase in content revenue does not make up the incremental expenditure. Consequently, each supplier is worse off.

Related Literature. The seminal work on two-sided platforms highlights the cross-side externalities in buyer-seller markets and provides a theoretical framework for analyzing these markets (e,g., Caillaud and Jullien (2003), Rochet and Tirole (2003), Rochet and Tirole (2006), Armstrong (2006) , Armstrong and Wright (2007)). Researchers have examined various issues such as platform competition (Rochet and Tirole (2006), Armstrong (2006)), pricing (Weyl (2010) , Chao and Derdenger (2013), and network asymmetry (Ambrus and Argenziano (2009)) in these two-sided markets.

Our research is directly related to the literature on two-sided media markets. Empirical evidence suggests that consumers dislike advertisements (Wilbur (2008)), whereas advertisers like to reach consumers (Argentesi and Filistrucchi (2007)). Dukes and Gal-Or (2003) show that broadcasters can benefit from offering exclusive advertising contracts. This is because exclusive advertising contracts soften the competition in the product market and help advertisers earn more profits, which permits the broadcasters to charge higher prices for advertisers. Anderson and Coate (2005) show that depending on the extent to which consumers regard advertising as a nuisance, competing platforms may over-provide or under-provide advertisements (compared with the socially optimal level of advertising). Godes et al. (2009) find that competition for advertisers makes a competing platform less willing to undercut the price for consumers. Amaldoss et al. (2016) identify the conditions under which competing platforms may offer consumers the option of paying a high price in order to avoid advertisements and when they would not offer consumers such an option. Research shows that consumers who switch between channels are less valuable than exclusive consumers (Ambrus et al. (2016) and Athey et al. (2013)). As the proportion of such multi-homing consumers increases, each platform increases the advertising level and earns less profits. This body of literature focuses on the consumer-side and the advertiser-side of the media market but abstracts away from the content-side of the market. The platform in our model connects three sides of the media market: consumers, advertisers, and content suppliers. Moreover, unlike most existing studies that only capture consumer's dislike for advertisements and advertiser's desire for consumers, our framework captures consumers' desire for content. Furthermore, we examine how a platform should allocate its limited bandwidth for content and advertising.

In a related paper, Prasad et al. (2003) consider a two-sided model where consumers are heterogeneous in their valuation for the platform's program, and advertisers are heterogeneous in their profits margin. They show that a monopoly platform is better off offering consumers two options, namely "higher-price-fewer-ads" and "lower-price-more-ads. We consider a three-sided market involving consumers, advertisers and content suppliers. Moreover, we allow inter-platform competition as well as inter-supplier competition. In contrast to Prasad et al. (2003), we show when a platform will adopt a free-content strategy, a no-ad strategy, and a paid-content-with ads strategy, and how a platforms choice of strategy varies with the structure of the content market as well as the platform market.

Our work also adds to the literature on content provision by media firms (see Peitz and Reisinger (2015)). Gal-Or and Dukes (2003) show that competing platforms might offer minimally differentiated content. This reduces the intensity of advertising and thereby the competition among advertisers in the product market, which helps the platforms to charge more for advertising. Upon empirically examining the content quality of 3000 bloggers in China, Sun and Zhu (2013) report that bloggers, who partake in the advertising revenue of the platform, write about popular topics such as the stock market and celebrity news. Yao et al. (2017) find that within-show TV commercials creates a better viewing experience for highly-rated content but not for poorly-rated content. In contrast to this body of work where a platform is assumed to produce its own content, our work separates the content supplier and the media platform. In doing so, our work offers insights on how a supplier prices its content, and how a supplier's ability of capturing a platform's profits varies with the platform's content provision strategy. Furthermore, the separation of the two parties permits us to understand how the market environment, such as advertisers' desire for consumers, affects a media platform and a content supplier differently.

The rest of the paper is organized as follows. Section 3.2 lays out the model structure. Section 3.3 analyzes a perfectly competitive content market. Section 3.4 examines a monopoly content market. We extend the model in section 3.5, and analyze a moderately competitive content market where suppliers need to put effort to shape consumers' preference. We also explore the possibility of no-ad strategy. Finally, Section 3.6 concludes the paper. The proofs for all the claims made in the paper can be seen in the appendix.

3.2 Model

Consider a media market with three sides, namely consumers, advertisers, and content suppliers. The platforms procure content from suppliers, allocate a proportion of bandwidth for content to attract consumers, and host the promotional messages of advertisers. Below we describe the three sides of the market and the platform.

3.2.1 Consumers

Consumers join a platform to enjoy its contents. The utility that a consumer derives from the contents of platform i is given as:

$$v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2) \tag{3.1}$$

where v represents the strength of consumers' desire for content and α_i is the proportion of platform i's bandwidth or space allocated for content ($\alpha_i \in [0, 1]$). This formulation captures in a parsimonious way the reality that as the proportion of content increases, the utility of the incremental content is likely to be lower because consumers do not value all content equally, and because it takes more effort to process the incremental content. As α_i is the proportion of the platform's space allocated for content, the remaining $(1-\alpha_i)$ proportion is allocated for advertising. An important implication of this formulation is that when deciding on the proportion of space for content, a platform needs to recognize that the marginal utility of content declines, and carefully balance the potential revenue from consumers and that from advertising. Furthermore, consumers are heterogeneous in their preference for a platform (for reasons such as interface and service), and we capture the heterogeneity by assuming that consumers are uniformly distributed on a Hotelling line of unit length. Consider a consumer located at distance x from platform i. The disutility this consumer experiences because of lack of fit with the platform is given by tx, where t denotes consumers' sensitivity to platform characteristics. Thus, the consumer derives the following (indirect) utility on joining platform i:

$$U_{iC}(x) = \underbrace{v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2)}_{\text{utility from content}} - \underbrace{t \cdot x}_{\text{disutility from lack of fit}} - \underbrace{p_{iC}}_{\text{price}} \tag{3.2}$$

where p_{iC} is the price paid by the consumer for joining the platform.

3.2.2 Advertisers

Advertisers join a platform to promote their products and services to the consumers. Let γ_A denote advertisers' valuation of a consumer. In other words, γ_A is the value advertisers place on a consumer's eyeball. When an advertiser can reach n_{iC} consumers through platform i, the utility the advertiser derives from joining platform i is given by $\gamma_A \cdot n_{iC}$. Given that advertising space is scarce, we assume that a platform can capture the entire surplus from advertisers. Consistent with our assumption the advertising price is often as high as advertisers' full valuation in competitive advertising markets, such as advertising auctions (e.g. Shin (2015)). Moreover, the assumption allows us to focus on the issue of content provision while abstracting away from the dynamics of the advertising market. Thus a platform sets the advertising price for one unit of space as:

$$p_{iA} = \gamma_A \cdot n_{iC}. \tag{3.3}$$

Then, platform i's advertising revenue from its $(1 - \alpha_i)$ unit of advertising space is given by $\gamma_A \cdot n_{iC} \cdot (1 - \alpha_i)$.

3.2.3 Content Suppliers

Independent content suppliers produce the content. The platform buys the content from the suppliers and makes them available to consumers, and thus serves as a channel for suppliers to reach consumers. We use c to denote the marginal cost of producing content. Since platform i allocates α_i proportion of its bandwidth for content, the content supplier earns the following profits from the platform(s):

$$\Pi_S = p_S \cdot \Sigma_i \alpha_i - c \cdot \Sigma_i \alpha_i \quad (3.4)$$

where p_S is the price the supplier charges for a unit of content.

3.2.4 Media platforms

While media platforms pay the supplier for the content, they can potentially earn profits from both consumers and advertisers. Platform i's profits are given by:

$$\Pi_{iP} = n_{iC} \cdot p_{iC} + (1 - \alpha_i) \cdot p_{iA} - \alpha_i \cdot p_S. \quad (3.5)$$

In this setting, a platform faces two key decisions: 1) what proportion of its limited bandwidth should it allocate for content? and 2) what prices should it charge consumers? In its attempt to maximize profits, a platform could pursue one of the following strategies:

- *"Free-content" strategy*, where the platform allocates a fraction of its bandwidth for content and offers the content for free to consumers, implying $0 < \alpha_i < 1$ and $p_{iC} = 0$. In this strategy, the platform chooses to earn all its profits from advertisers.

- *"No-ad" strategy*, where the platform allocates all its bandwidth for content and charges consumers a price for the content, implying $\alpha_i = 1$ and $p_{iC} > 0$. Now the platform earns its entire profits from consumers and eschews the advertisers. More precisely, the platform is adopting a "paid-content-with-no-ad" strategy. However, for the sake of brevity, we refer to it as a "no-ad" strategy.

- *"Paid-content-with-ads" strategy*, where the platform allocates a fraction of its bandwidth to content, and charges a positive price for consuming the content, suggesting $0 < \alpha_i < 1$ and $p_{iC} > 0$. In this case, the platform earns profits from both sides of the market.

3.2.5 Decision Sequence

The decisions are taken in three stages. In the first stage, the content supplier sets a price for content (p_S). In the second stage, after observing the content supplier's price, each media platform decides on the proportion of content (α_i) and the price for consumers (p_{iC}). Finally, in the third stage, after observing the price for consumers and the proportion of content, consumers decide which platform to join.

To understand the strategic behavior of market participants, we examine the subgame perfect equilibrium of this game. Furthermore, to help us better appreciate the effect of market structure on equilibrium outcome, we focus on the parameter space where platform(s) choose to cover the entire consumer market. The precise conditions for the market to be fully covered are specified in the appendix (See Lemmas A.2, A.6, and A.9)

3.3 Analysis of a Perfectly Competitive Content Market

We begin our analysis by considering a content market where an infinite number of homogeneous content suppliers engage in Bertrand competition. Because of intense competition, the equilibrium content supllier price is driven down to the marginal cost of producing content:

$$p_S = c \tag{3.6}$$

Recognizing that content is sold by suppliers at marginal cost, media platform i sets the consumer price p_{iC} and chooses the proportion of content α_i. Next we examine the strategic interaction between the content suppliers and a monopoly platform as well as duopoly platforms.

3.3.1 Monopoly Platform

Assume that the monopoly platform, say Platform 1, is located at zero on the Hotelling line. In this case, the consumer located at distance x from the platform will join the platform only if $U_{1C}(x) \geqslant 0$, where

$$U_{1C}(x) = v \cdot (\alpha_1 - \frac{1}{2}\alpha_1^2) - t \cdot x - p_{1C} \tag{3.7}$$

Let the marginal consumer who is indifferent between joining and not joining the platform be x_0. Because our analysis focuses on the parameter space where the market is fully covered, the number of consumers in the platform is $n_{1C} = x_0 = 1$. Therefore, the price the monopoly platform would charge consumers is given by:

$$\begin{aligned} p_{1C} &= v \cdot (\alpha_1 - \frac{\alpha_1^2}{2}) - t \cdot x_0 \\ &= v \cdot (\alpha_1 - \frac{\alpha_1^2}{2}) - t, \end{aligned} \tag{3.8}$$

whereas the price for advertisers is:

$$\begin{aligned} p_{1A} &= \gamma_A \cdot n_{1C} \\ &= \gamma_A \end{aligned} \tag{3.9}$$

Upon substituting p_S, p_{1C} and p_{1A} in (3.5), we obtain the monopoly platform's profits:

$$\begin{aligned} \Pi_P^M &= p_{1C} \cdot n_{1C} + p_{1A} \cdot (1 - \alpha_1) - p_S \cdot \alpha_1 \\ &= -t - c \cdot \alpha_1 + (1 - \alpha_1) \cdot \gamma_A + (\alpha_1 - \frac{1}{2}\alpha_1^2) \cdot v, \end{aligned} \tag{3.10}$$

where the superscript "M" indicates that the platform is a monopolist. The platform chooses α_1 to maximize Π_p^M subject to $0 \leqslant \alpha_1 \leqslant 1$ and $p_{1C} = v \cdot (\alpha_1 - \frac{\alpha_1^2}{2}) \geqslant 0$.

Upon solving this optimization problem, we obtain the monopoly platform's optimal proportion of content:

$$\alpha_1 = \begin{cases} \alpha_f^M = \frac{v-\sqrt{v\cdot(v-2t)}}{v}, & \text{if } c \geqslant \tau_M \\ \alpha_{pa}^M = \frac{-c-\gamma_A+v}{v}, & \text{if } c < \tau_M \end{cases} \tag{3.11}$$

where $\tau_M = -\gamma_A + \sqrt{v\cdot(v-2t)}$. Note that α_f^M and α_{pa}^M give the demand for content from the monopoly platform when it adopts a free-content strategy and a paid-content-with-ads strategy, respectively[2].

3.3.2 Duopoly Platforms

Now consider a media market with two platforms. Assume that Platform 1 is located at $x = 0$ while platform 2 is located at $x = 1$ on the Hotelling line. Given that the content supplier's price $p_S = c$, each platform chooses the consumer price p_{iC} and the proportion of content α_i. Consider a consumer located at distance x from Platform 1. The indirect utility that the consumer derives on joining Platform 1 is given in equation (3.7), and the corresponding utility the consumer derives from joining platform 2 is as follows:

$$U_{2C}(x) = v\cdot(\alpha_2 - \frac{1}{2}\alpha_2^2) - t\cdot(1-x) - p_{2C} \tag{3.12}$$

The marginal consumer who is indifferent between joining the two platforms is given by:

$$x_0 = \frac{1}{2} + \frac{V(\alpha_1)-V(\alpha_2)}{2t} - \frac{p_{1C}-p_{2C}}{2t} \tag{3.13}$$

where $V(\alpha_i) = v\cdot(\alpha_i - \frac{1}{2}\alpha_i^2)$ is the gross utility a consumer derives from joining a platform which has α_i fraction of content and $(1-\alpha_i)$ fraction of advertisements. We assume that consumers single-home. Then the consumer base of platform 1 is given

[2] Note that τ_M is not necessarily positive. If $\tau_M \leqslant 0$, then free-content strategy is the equilibrium strategy because $c \geqslant \tau_M$ always hold.

by $n_{1C} = x_0$ and corresponding base of platform 2 is $n_{2C} = 1 - x_0$. Thus, depending on its customer base, each platform charges advertisers:

$$p_{iA} = \gamma_A \cdot n_{iC} \tag{3.14}$$

Substituting p_S, n_{iC}, p_{iA} ($i = \{1, 2\}$) in(3.5), we can obtain platform i's profits Π_{iP}^D as a function of its two decision variables $\{\alpha_i, p_{iC}\}$. Both platforms simultaneously choose α_i and p_{iC} to maximize their own profits Π_{iP}^D subject to $0 \leqslant \alpha_i \leqslant 1$ and $p_{iC} \geqslant 0$. Upon solving the equilibrium, we find that each platform's proportion of content is as follows:

$$\alpha_i = \begin{cases} \alpha_f^D = 1 - \frac{t(2c+\gamma_A)}{\sqrt{t \cdot v \gamma_A (2c+\gamma_A)}}, & \text{if } c \geqslant \tau_D \\ \alpha_{pa}^D = \frac{-2c-\gamma_A+v}{v}, & \text{if } c < \tau_D \end{cases} \tag{3.15}$$

where and $\tau_D = \frac{1}{2} \cdot (-\gamma_A + \frac{t \cdot v}{\gamma_A})$. Moreover, α_f^D and α_{pa}^D are the demand for content from a duopoly platform when it pursues a free-content strategy and a paid-content-with-ads strategy, respectively.[3]

3.3.3 Platform's Strategy

Recall that a platform could adopt one of the three different strategies: a free-content strategy, a paid-content-with-ads strategy, or a no-ad strategy. However, from (3.11) and (B.40), we find that neither a monopoly platform nor a duopoly platform adopts no-ad strategy. To understand why, note that advertising reduces the space available for content and thus reduces the utility consumers derive from joining the platform. In the current formulation, advertising does not annoy consumers and directly induce a disutility. As a result, a platform can always allocate a proportion of its space for advertising and earn from profits from advertisers. One could argue, however, that consumers dislike advertising because it is a nuisance, and we explore this possibility in Section 6.

[3] Similarly, τ_D is not necessarily positive. If $\tau_D \leqslant 0$, then free-content strategy is the equilibrium strategy because $c \geqslant \tau_D$ always hold.

Given that a platform is not motivated to adopt a no-ad strategy, it could potentially adopt a free-content strategy or a paid-content-with-ad strategy. From (3.11) and (B.40), we know that both a monopoly platform and a duopoly platform use a free-content strategy if the marginal cost of producing content is large, otherwise pursue a paid-content-with-ads strategy. To appreciate this result, recall that content price is c as suppliers face an intensely competitive market. As content price c increases, it becomes costly for a platform to attract consumers by offering them more content. Hence, a platform allocates a lower proportion of its space for content, and comes to rely more on profits from advertisers. When the content price is sufficiently high, a platform allocates too little space for content and generate all its profits from advertising. Thus after a content price threshold a platform transitions from pursuing paid-content-with-ads strategy to adopting a free-content strategy.

So far, we have examined some similarities in the strategy of a monopoly platform and a duopoly platform. However, their strategies are not identical. The following result summarizes how inter-platform competition influences a platform's strategy and the source of its profits:

Proposition 3.1. *The presence of inter-platform competition can motivate a platform to change its strategy from offering paid-content-with-ads to providing free-content, but not the other way round.*

To appreciate the intuition for this finding, first consider a situation where content price is low enough that both a monopoly platform and a duopoly platform uses a paid-content-with-ads strategy. As the marginal cost of content increases, the platform responds to the increase in content price by procuring less content and allocating a larger proportion of its space to advertising. Furthermore, when content is expensive, a duopoly platform finds it less appealing to compete for consumers by offering more content and this softens the price competition for consumers. Note that for a monopoly platform $\frac{\partial \alpha_{pa}^{M}}{\partial c} = -\frac{1}{v}$ whereas for a duopoly platform $\frac{\partial \alpha_{pa}^{D}}{\partial c} = -\frac{2}{v}$, implying that a duopoly platform turns more rapidly toward the advertising market as the marginal cost of producing content price. Consequently, even when a monopoly platform pursues paid-content-with-ads strategy, a duopoly platform may adopt a free-content strategy.

3.3.4 Platform's Profits:

In this section, we examine how the marginal cost of producing content affects a platform's profits. One might expect a platform's profits to increase as content price decreases. Yet we have the following result.

Proposition 3.2. *A reduction in the marginal cost of producing content can hurt a competing platform's profits when the cost is above a threshold.*

When the marginal cost of producing content is lower, platforms pay less for the content and this cost saving is a direct positive effect. However, as content becomes cheaper, each platform competes more aggressively for consumers by purchasing more content, and this is an indirect negative effect. The overall effect of a lower marginal cost of content depends on the relative size of these two effects.

When the marginal cost of content is sufficiently high, a platform allocates only a small proportion of space for content. In this context, a reduction in content cost results in a small cost saving, but this direct effect is dominated by the negative effect of intensified competition for consumers. This is why a decrease in content cost can come to hurt a platform's profits when the cos=t is sufficiently high.

3.4 Analysis of a Monopoly Content Market

The last section examines a perfectly competitive content market where content suppliers have no market power and hence set the price equal to marginal production cost. In this section, we examine a monopoly content market where the monopoly content supplier chooses the content price p_S to maximize its own profits. As a monopolist, the content supplier does not have to set the price at the marginal cost c. For expositional reasons, we normalize the cost to zero, that is $c = 0$. We next examine the strategic interaction between the monopoly content supplier and a monopoly platform as well as duopoly platforms.

3.4.1 Second stage of the game

In contrast to the perfectly competitive content market described in the previous section, the content price is p_S (instead of c). On solving for the equilibrium content offered by a monopoly and duopoly platform, we obtain the following.

- Monopoly Platform:

$$\alpha_1(p_S) = \begin{cases} \alpha_f^M = \frac{v-\sqrt{v\cdot(v-2t)}}{v}, & \text{if } p_S \geqslant \tau_M \\ \alpha_{pa}^M = \frac{-p_S-\gamma_A+v}{v}, & \text{if } p_S < \tau_M \end{cases} \tag{3.16}$$

  where $\tau_M = -\gamma_A + \sqrt{v \cdot (v-2t)}$. Note that α_f^M and α_{pa}^M are the demand for content from a monopoly platform when it adopts a free-content strategy and a paid-content-with-ads strategy, respectively.

- Duopoly Platform:

$$\alpha_i(p_S) = \begin{cases} \alpha_f^D = 1 - \frac{t(2p_S+\gamma_A)}{\sqrt{t\cdot v\gamma_A(2p_S+\gamma_A)}}, & \text{if } p_S \geqslant \tau_D \\ \alpha_{pa}^D = \frac{-2p_S-\gamma_A+v}{v}, & \text{if } p_S < \tau_D \end{cases} \tag{3.17}$$

  where and $\tau_D = \frac{1}{2} \cdot (-\gamma_A + \frac{t\cdot v}{\gamma_A})$. Moreover, α_f^D and α_{pa}^D are the demand for content from a duopoly platform when it pursues a free-content strategy and a paid-content-with-ads strategy, respectively.

3.4.2 First stage of the game

The monopoly content supplier foresees the demand for its content and accordingly chooses the optimal content price to maximize its own profits. Specifically, in the presence of a monopoly platform, the content supplier's profit maximization problem is as follows:

$$\max_{p_S} \Pi_S^M = p_S \cdot \alpha_1(p_S) \tag{3.18}$$

While in the presence of duopoly platforms, the content supplier's profit maximization problem is given by:

$$\max_{p_S} \Pi_S^D = p_S \cdot \sum_{i=1}^{2} \alpha_i(p_S) \tag{3.19}$$

3.4.3 Platform's Strategy

Broadly, both a monopoly platform and a duopoly platform use a free-content strategy if advertisers' desire for consumers is high; otherwise, they pursue a paid-content-with-ads strategy. To further appreciate a platform's choice of strategy note that when advertiser's desire for consumer is weak, the platform cannot earn much revenue from advertisers by allocating a higher proportion of space for advertising. Hence, a platform allocates more space for content pursues a paid-content-with-ads strategy in such a situation. On the other hand, when advertisers' desire for consumers is strong, the platform sees an opportunity to earn more profits from advertisers. Hence, the platform offers less content and earns all its profits from advertising by pursuing a free-content strategy.

According to Proposition 1, if the content market is perfectly competitive, inter-platform motivates a platform to rely more on advertising profits. However, when the content supplier is a monopolist who endogenously chooses the optimal content price, we observe a different result.

Proposition 3.3. *In a monopoly content market, the presence of inter-platform competition can motivate a platform to switch from a free-content strategy to a paid-content-with-ads strategy when v is small.*

To follow the intuition for this finding, first recall that in a perfectly competitive content market, content suppliers offer the content at marginal cost to the platforms. The content price is the same for both a duopoly platform and a monopoly platform. In this context, a duopoly platform is more motivated to increase the proportion space for advertising as it not only increases advertising revenue but also softens the competition for consumers. In contrast, in a monopoly content market the supplier sells content at different prices to a monopoly platform and a duopoly platform. In particular, when v is small, the monopoly content supplier set a high content price $p_S \geqslant \tau_M$ for a monopoly platform. This is because when consumers' desire for content is low, the platform chooses to earn all its profits from the advertising and offer consumers minimal content. Recognizing this, the monopoly content supplier chooses to charge a high price for content. On the other hand, the monopoly content supplier charges a low content price $p_S < \tau_D$ for a duopoly platforms. We observe this because when consumers' desire for content is low, duopoly platforms find the consumer market for content less attractive and moreover their demand for content is quite sensitive to price. Specifically, $\frac{\partial \alpha_f^D}{\partial p_S} = -\sqrt{\frac{t}{v\gamma_A(2p_S+\gamma_A)}}$ and $\frac{\partial \alpha_{pa}^D}{\partial p_S} = -\frac{2}{v}$. Therefore, the monopoly content supplier offers content at lower price to duopoly platforms and induces to adopt a paid-content-with-ads strategy (even though it charge a monopoly platform a high content price and induces it to pursue a free-content strategy).

3.4.4 Platform's Profits

It is natural to think that a monopoly platform would earn more profits than a duopoly platform. However, as we hinted in explaining the intuition for the previous proposition, this belief may not be always valid. Specifically, we have the following finding.

Proposition 3.4. *A monopoly platform may lose all its profits to the content supplier even when a duopoly platform earns positive profits.*

To follow the intuition for this finding, focus attention on a free-content strategy. Further recall a monopoly platform's demand for content under a free-content strategy is:

$$\alpha_f^M = \frac{v - \sqrt{v \cdot (v - 2t)}}{v} \tag{3.20}$$

Thus, under a free-content strategy, a monopoly platform's demand for content is inelastic to content supplier's price. This is because when the platform offers free content, it provides content solely to build a consumer base that can be leveraged to attract advertisers. Hence, the platform offers the minimal amount of content to minimize its cost. Recognizing that the platform needs to offer this minimal amount regardless of the content supplier's price, the content supplier raises its price until it extracts all the profits of the monopoly platform.

According to the second part of the proposition, however, a duopoly platform can earn positive profits when a monopoly platform cannot do so. Intuitively, platforms compete for consumers by offering consumers a higher proportion of content. In doing so, they carefully balance the benefit of allocating additional bandwidth for content against its cost, which includes not only the foregone advertising revenue but also the payment to the content supplier. Under a free-content strategy, both platforms offer more than the minimal amount of content, and moreover a platform's demand for content is sensitive to the content supplier's price:

$$\alpha_f^D = 1 - \frac{t(2p_S + \gamma_A)}{\sqrt{t \cdot v\gamma_A(2p_S + \gamma_A)}}. \tag{3.21}$$

Hence, if the content supplier's price increases, a platform may compete less for consumers and allocate more bandwidth for advertising. Thus the elastic demand for content prevents a competing platform from losing all profits to the content supplier under a free-content strategy.

The above proposition only pertains to a free-content strategy. This may raise the question whether the content supplier can extract all the profits of a monopoly or a duopoly platform under a paid-content-with-ads strategy. To understand this issue, note that a monopoly platform can earn positive profits when it adopts a paid-content-with-ads strategy. In this case, the amount of content provided by the platform is more than the minimum level, α_f^M. In particular, the platform's demand for content under a paid-content-with-ads strategy is given by,

$$\alpha_{pa}^M(p_S) = \frac{-p_S - \gamma_A + v}{v}. \tag{3.22}$$

Observe that the platform's demand for content is now sensitive to the content supplier's price. This implies that if the content supplier raises its price, the platform can reduce the proportion of content and allocate more bandwidth for advertising. Thus, the elastic demand for content protects a platform from losing all its profits to the content supplier. Second, even when platforms compete, a platform's demand for content remains sensitive to content supplier price:

$$\alpha_i(p_S) = \frac{-2p_S - \gamma_A + v}{v}. \tag{3.23}$$

Thus both a monopoly and duopoly platform earn positive profits under a paid-content-with-ads strategy.

3.5 Model Extension

In the previous two sections, we considered two polar content markets where consumers' preference for content is exogenous to the model. Consumers view the contents from all suppliers as perfectly substitutable in the perfectly competitive content market, whereas consumers view the content as non substitutable in the monopoly content market. In Section 5.1, we consider a duopoly content supplier market where consumers' preference for content is endogenous to the model. Using this model, we explore content suppliers' effort to shape consumer preference and, in turn, platforms' content provision strategy and profits. Next one could argue that consumers regard advertising as a nuisance (e.g., Wilbur 2008), implying that advertising directly decreases the utility consumers derive from joining a platform. In Section 5.2, we explore the strategic implications of such a possibility.

3.5.1 Moderately Competitive Content Market

Consider a content market with two suppliers, namely 1 and 2. Consumers' preference for each supplier's content depends on the supplier's effort toward shaping consumer preference. Let supplier j exert effort κ_j at a cost $\phi\kappa_j^2$ to make consumers prefer its content. Given $\{\kappa_1, \kappa_2\}$, the ideal proportion of content that consumers prefer from supplier 1 is given by:

$$h(\kappa_1, \kappa_2) = \frac{1}{1 + \exp(\kappa_2 - \kappa_1)} \tag{3.24}$$

It follows that the ideal proportion of content that consumers prefer to consume from supplier 2 is $1 - h(\kappa_1, \kappa_2)$. In the first stage of the game, each content supplier chooses the effort level κ_j and the content price p_{jS}.

In the second stage of the game, each platform decides on the proportion of space to allocate for content α_i and the consumer price for content p_{iC}. In addition, each platform decides on the fraction of content to procure from each of the content suppliers. Let h_i denote the fraction of content that platform i purchases from supplier 1. In the third stage of the game, each consumer decides on which platform to join. Consumers are distributed on a Hotelling line of unit length. The utility that the consumer located at x derives on joining platform 1 is given by:

$$U_{1C}(x) = v \cdot (1 - |h - h_1|) \cdot (\alpha_1 - \frac{1}{2}\alpha_1^2) - t \cdot x - p_{1C} \tag{3.25}$$

Note that when the fraction of content from content supplier 1 is misaligned with the ideal proportion of content from the supplier, consumers experience a disutility. We let the strength of consumers' desire for content v shrink by the factor $(1 - |h - h_i|)$. But for this shrinkage in consumers' desire for content, the utility formulation is identical to the one in our original model (See Section 2.1). The corresponding utility that the consumer derives from joining platform 2 is as follows:

$$U_{2C}(x) = v \cdot (1 - |h - h_2|) \cdot (\alpha_2 - \frac{1}{2}\alpha_2^2) - t \cdot (1 - x) - p_{2C} \tag{3.26}$$

Next we examine the conduct of a monopoly platform and duopoly platforms in this market. In the presence of a monopoly platform, a consumer located at x join platform 1 if $U_{1C}(x)$ is positive. However, in the presence of duopoly platforms, the consumer compares $U_{1C}(x)$ and $U_{2C}(x)$ and joins the platform that yields a higher utility. Let n_{iC} denote the number of consumers that join platform i. Then platform i's profits are given by:

$$\Pi_{iP} = \underbrace{n_{iC} \cdot p_{iC}}_{\text{revenue from consumers}} + \underbrace{(1 - \alpha_i) \cdot p_{iA}}_{\text{revenue from advertisers}} - \underbrace{\alpha_i \cdot h_i \cdot p_{1S}}_{\text{cost of content from supplier 1}} - \underbrace{\alpha_i \cdot (1 - h_i) \cdot p_{2S}}_{\text{cost of content from supplier 2}} \tag{3.27}$$

Based on each content supplier's price, the cost of effort toward shaping consumer preference, and the demand for the supplier's content from the platforms, we have the following profits:

$$\Pi_{1S} = p_{1S} \cdot \Sigma_i \Big(\alpha_i \cdot h_i\Big) - \phi \cdot \kappa_1^2 \tag{3.28}$$

$$\Pi_{2S} = p_{2S} \cdot \Sigma_i \Big(\alpha_i \cdot (1 - h_i)\Big) - \phi \cdot \kappa_2^2 \tag{3.29}$$

We assume that the cost parameter ϕ is above a certain threshold so that each supplier's profit function is concave in κ_i and well behaved (see Appendix for the details).

Recall that in a monopoly content market, when a platform adopts a free-content strategy, the monopoly content supplier's profits increase with advertisers' valuation of consumers. This result holds for a a monopoly platform as well as duopoly platforms.[4] To understand why we obtain this result in the case of a monopoly platform, note that when advertisers' valuation of consumers increases, the monopoly platform can earn more advertising revenue. Then, the monopoly content supplier extracts the increased advertising revenue by raising the price of content. The rationale for the result in the case of a duopoly platform is slightly different. When advertisers' valuation of consumers increases, a duopoly platform wants to compete more aggressively for consumers so that it could earn more advertising revenue by giving advertisers access to the valuable consumers. Given that platforms offer consumers the content for free, they cannot lower the price any further, and hence they compete for consumers by offering a higher proportion of content. Thus the monopoly content supplier earns more profits as γ_A increases even when platforms compete. This result, however, does not hold in a duopoly content supplier market, and we have the following proposition:

Proposition 3.5. *Under free-content strategy, in contrast to a monopoly content supplier, a duopoly content supplier earns lower profits as γ_A increases when ϕ is below a threshold.*

In a duopoly content market, each supplier maximizes its profits by setting the price for its content (p_{jS}) and by shaping consumer preference for its content (k_j). We find that as γ_A increases, platforms are motivated to offer consumers more content, and this increases the revenue of the duopoly content suppliers. The higher revenue, in turn, encourages each content supplier to exert more effort to increase consumers's preference for its content. The resulting increase in cost has a negative

[4] See Corollary 1 in the Appendix for the proof.

effect on profits. We find that when the cost of shaping consumer preference, ϕ, is sufficiently low, the negative effect of an increase in γ_A may dominate its positive effect, and competing content supplier's profits may reduce. This qualitative result holds irrespective of whether the content suppliers are providing content to a monopoly platform or a duopoly platforms.

To clearly see the positive and negative effects of an increase in γ_A, consider the case where the duopoly content suppliers provide content to a monopoly platform. Now as γ_A increases, each supplier's revenue from selling content increases at the rate of $\frac{\partial \text{Revenue}}{\partial \gamma_A} = \frac{\sqrt{v(v-2t)}}{2v}$, whereas its cost of shaping consumers' preference increases at the rate of $\frac{\partial \text{Cost}}{\partial \gamma_A} = \frac{(v-2t)\gamma_A}{32v \cdot \phi}$. Clearly, the negative effect of an increase in γ_A outweighs its positive effect on the content supplier's profits when $\phi < \frac{\sqrt{v(v-2t)}\gamma_A}{16v}$. In the Appendix, we present the corresponding threshold ϕ below which the content suppliers' profits will hurt when they provide content to duopoly platforms.

The above proposition may lead one to wonder whether a similar result will be observed under a paid-content-with ad strategy. Recall that under a paid-content-with-ads strategy, a monopoly content supplier's profits decrease with γ_A irrespective of whether it supplies content to a monopoly platform or duopoly platforms.[5]. This is because when a platform charges consumers a positive price, it can compete for consumers by setting a lower price or offering more content. This flexibility permits the platform to attract consumers with a lower price but host a higher proportion of advertising and take advantage of advertisers' higher valuation of consumers. Consequently, for any given content price, a platform's demand for content decreases as γ_A increases. The resulting inward shift in the demand curve hurts the content supplier's profits. However, in a duopoly content supplier market, we obtain a different result.

[5] See Corollary 1 in the Appendix for the proof.

Proposition 3.6. *Under paid-content-with-ads strategy, in contrast to a monopoly content supplier, a duopoly content supplier earns higher profits as γ_A increases when ϕ is below a threshold.*

In a duopoly content market, on the one hand, the content suppliers' revenues decrease as γ_A increases because platforms buy less content (but host more advertising and charge consumers a lower price). On the other hand, content suppliers reduce their expenditures on shaping consumer preference as γ_A increases. The resulting cost savings may dominate the loss in revenue when ϕ is below a threshold. We obtain this result irrespective of whether the content suppliers are providing content to a monopoly platform or duopoly platforms. To take a closer look at the net positive effect, consider the case where duopoly content suppliers cater to a monopoly platform. Now if γ_A increases, each content supplier's revenue decreases at the rate of $\left|\frac{\partial \text{Revenue}}{\partial \gamma_A}\right| = \frac{2(v-\gamma_A)}{9v}$, while the cost of shaping consumers' preference decreases at the rate of $\left|\frac{\partial \text{Cost}}{\partial \gamma_A}\right| = \frac{(v-\gamma_A)^3}{324v^2 \cdot \phi}$. Given that $\gamma_A < v$ is a necessary condition for paid-content-with-ads strategy (as noted in the Appendix), the cost savings dominate the loss in revenue when $\phi < \frac{(v-\gamma_A)^2}{72v}$. This qualitatively result also holds when duopoly content suppliers face duopoly platforms, and we prove this claim in the Appendix. It is useful to clarify that though a content supplier's profits may increase or decrease with γ_A depending on ϕ and the platform's strategy, a platform's profits(weakly) increases with γ_A.[6]

3.5.2 No-ad Strategy

In developing our model, we assumed that consumers are not annoyed by advertising. Consequently, media platforms never shun the profits from the advertisers. Now we entertain the possibility that advertising could be viewed as nuisance by consumers

[6] When a platform is a monopoly and adopts free-content strategy, it earns zero profits; but in other situations, a platform strictly benefits from a larger γ_A.

and explore its implications for platform's choice of strategy. To facilitate exposition, we focus on the perfectly competitive content supplier market where the marginal cost of producing content is c.[7]

Recall that platform i allocates α_i proportion of its space for content, implying that advertising accounts for the remaining $1-\alpha_i$ proportion of space. Let γ_C denote consumer's consumers' dislike for one unit space of advertisements. Then a consumer is located at distance x from platform i will derive the following utility on joining the platform:

$$U_{iC}(x) = \underbrace{v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2)}_{\text{utility from content}} - \underbrace{\gamma_C \cdot (1 - \alpha_i)}_{\text{distuility from ads}} - \underbrace{t \cdot x}_{\text{disutility from lack of fit}} - \underbrace{p_{iC}}_{\text{price}} \quad (3.30)$$

As a duopoly platform needs to compete for its customer base, one might think that a duopoly platform is likely to allocate a lower proportion of space for content compared to a monopoly platform. This reasoning may lead one to believe that duopoly platform is more likely to adopt a no-ads strategy. On closing scrutinizing this issue, we obtain the following result.

Proposition 3.7. *Even when a monopoly platform adopts a no-ad strategy, a duopoly platfiorm may not find it a profitable strategy.*

Consistent with one's intuition, a platform adopts no-ad strategy when consumer's dislike for advertising (γ_C) is large. Interestingly, when γ_C is moderately large $\gamma_C \in [\gamma_A + c, \gamma_A + 2c)$, a monopoly platform adopts a no-ad strategy whereas a duopoly platform shifts to catering both sides of the market. To appreciate this finding note that a duopoly platform earns more advertising revenue by allocating a higher proportion of space for advertising and in addition it can also soften the inter-platform competition for consumers (and the resulting desire to offer more content).

[7] Note that similar qualitative results prevail in a monopoly content market as well as a moderately competitive content market.

Therefore, when consumer's dislike for advertising is moderately large, even when a monopoly adopts a no-ad strategy, a duopoly platform turns to the advertising market because of the added benefit.

3.6 Conclusion

The purpose of the paper is to theoretically examine a media platform's content provision strategy, and its implications for a platform's profits and a content supplier's profits. Toward this goal, we build a model where a media platform interacts with three sides of the market — content suppliers, consumers, and advertisers. On examining a platform's content provision strategy, we find that unless consumers are sufficiently annoyed with advertising, a platform never shuns the advertising market; thus pursues either a free-content strategy or a paid-content-with-ads strategy. Broadly, if the price of content is above a certain threshold, a platform purchases the least amount of content, offers it to consumers for free, and earns all its profits from advertisers. But if the price of content content price is below the threshold, a platform purchases more content, charges consumers for the content, and generates additional revenue from advertisers by providing them access to its consumer base. Upon further analyzing how a platform's strategy changes with the structure of the platform market and the structure of the content market, we offers useful insights on a few questions of managerial significance.

- *How does inter-platform competition affect a platform's content provision strategy?*

  One may expect platforms to compete for consumers by offering them more content, and thus host less advertising and earn a lower advertising revenue. In a perfectly competitive content market, however, we obtain the opposite result. Specifically, inter-platform competition motivates a platform to offer minimal content and pursue a free-content strategy (instead of adopting a paid-content-with-ads strategy). We observe this because if duopoly platforms allocate a larger proportion of space for advertising, they can afford to compete less aggressively for consumers. Thus, when content becomes expensive, we see a duopoly platform adopting a free-content strategy even in a situation where a a monopoly platform adopts a paid-content-with-ads strategy.

The results are reversed, however, if the content supplier is a monopolist. Now inter-platform competition can motivate a platform to switch from a free-content strategy to a paid-content-with-ads strategy. To appreciate why, note that a supplier in a perfectly competitive content market sells the content at marginal cost to both a monopoly platform and a duopoly platform, whereas a monopoly content supplier sells the content at different prices to a monopoly platform and a duopoly platform. In particular, the monopoly content supplier carefully considers the price sensitivity of a monopoly platform and that of the duopoly platforms, and accordingly sets the prices. Given a duopoly platform's desire to soften the competition for consumers by allocating a higher proportion of space for advertising, a duopoly platform is more price sensitive than a monopoly platform. On top of this, if consumers' desire for content is small, a duopoly platform becomes even more sensitive to price than a monopoly platform. Recognizing this, the monopoly content supplier sells content to a duopoly platform at a lower price (compared to its price for a monopoly platform). Because of its lower content price, a duopoly platform adopt a paid-content-with-ads strategy (whereas a monopoly platform adopts a free-content strategy because of its higher content price).

- *Can a media platform earn more profits when it pays a lower price to the content supplier?*

Interestingly, we find that in the presence of inter-platform competition, a lower content price can hurt a platform's profits. To appreciate this result, first note that a lower content price has two opposing effects. On the one hand, it reduces a platform's payment to the content supplier (holding the amount of content fixed). But, on the other hand, it motivates a platform to purchase more content to compete for consumers. The increased proportion of content reduces the space for advertising and, in turn, the advertising revenue. In this context, if the price of content is sufficiently high and each platform purchases a small amount of content, then the cost saving due to a lower content price is not enough to make up for the loss in advertising revenue. Consequently, a duopoly platform can earn lower profits despite paying a lower price to the content supplier.

- *Can a media platform leverage its market power to earn more profits?*

  One naturally expects a media platform to benefit from its market power. Surprisingly, we find that a monopoly platform can lose all its profits to the content supplier, even in a situation where a duopoly platform earns positive profits. To see this, note that when pursuing a free-content strategy, the monopoly platform allocates minimal space for content regardless of the price of content, implying the demand for content is inelastic to price. The inelastic demand permits the content supplier to charge a high price and extract the entire surplus of the monopoly platform. However, a duopoly platform's demand for content is elastic to price because a duopoly platform uses the proportion of content as a strategic lever to attract consumers. It is the elastic demand of a duopoly platform that makes it difficult for the content supplier to extract all the surplus from the platform.

- *When advertisers valuation for consumers increases, should a platform allocate* *a higher proportion of its space for advertising?*

One may be inclined to answer yes to this question. The answer, however, is no when platforms adopt a free content strategy. Recall that when a monopoly platform is not charging consumers for joining the platform, it offers them the minimal amount of content. Thus, under the free-content strategy, there is no opportunity for the monopoly platform to further reduce the proportion of content and host more advertising. Even in the case of a duopoly platform, the answer to the question remains no. In contrast to a monopoly platform, a duopoly platform provides more than the minimal content in its pursuit to compete for consumers. Now as advertisers' valuation of consumers increases, each consumer brings in more advertising revenue. To attract these more valuable consumers, a duopoly platform is motivated to offer a higher proportion of content.

Consistent with our intuition, under a paid-content-with-ads strategy, a platform allocates a higher proportion of space for advertising as advertisers' valuation of consumers increases. The key reason why we observe this result in the case of paid content-with-ads strategy is that now a platform can make it attractive for consumers to join its platform either by lowering the price of content or increasing the proportion of content. Given this flexibility, a platform takes advantage of an increase in advertisers' valuation of consumers by allocating higher proportion of space for advertising but charging consumers a lower price.

- *When a media platform earns more profits from the advertising market, can the content supplier also earn more profits?*

Counter to some of our intuition, a content supplier may be hurt by an increase in advertisers valuation of consumers in two specific instances. First, consider the case of a monopoly content supplier providing content to a platform adopting a paid-content-with-ads strategy. In this case, an increase in advertisers' valuation of consumers, encourages a platform to allocate a higher proportion of space for advertising to increase advertising revenue but attract consumers by offering a lower price. The resulting lower demand for content hurts the monopoly content supplier's profits. Second, consider the case a duopoly content supplier providing content to a platform pursuing a free-content strategy. On the one hand, the competing content suppliers can earn more revenue because of the increased demand for its content or the ability to charge a higher price, but on the other hand the content suppliers also compete to influence consumer preference for their content. If the marginal cost of shaping consumer preference is small, then the incremental effort on shaping consumers' preference can be so high that a content supplier's expenditure on it can exceed the increase in its revenue. It is useful to clarify that consistent with our intuition, a content supplier can benefit from an increase in advertiser's valuation of consumers in two situations: when a monopoly content content supplier caters to a platform adopting free-content strategy, and when a duopoly content supplier caters to a platform pursuing paid-content-with-ads strategy. Both these instances the content supplier can extract some of the increased revenue of the platforms.

In this paper, we investigate a content supplier's pricing decision and its implications for a platform's content provision strategy and the source of its profits. In practice, the content supplier also makes important decisions on the quality of content as well as the distribution of content (either through a traditional media platform or a new aggregator). Future research can explore these strategic decisions. More broadly, multi-sided media market is a rapidly evolving phenomenon where practice is ahead of research, and it presents several interesting opportunities for both theoretical and empirical research.

4

Conclusion

Multi-sided media market is a rapidly evolving phenomenon. Research on media markets is burgeoning, but still falls behind the industry development. The complexity comes from the market's multi-sidedness feature, and the strategic interaction between multiple players, including content suppliers, advertisers, media platforms, and consumers. This dissertation explicitly models these strategic interactions, and answers important managerial questions on a media platform's pricing strategy and content provision strategy.

- Essay 1 examines when it is optimal for a media platform to adopt price customization. It also examines how the nature of consumer/advertiser homing changes a platform's incentive of offering customized prices. If agents single-home on both sides of the market, even when consumers are quite heterogeneous in their sensitivity to advertising, I find that least one platform pursues a uniform pricing strategy, as it can leverage consumer's dislike for advertising to soften platforms' competition for consumers. However, when both sides multi-home, the competing platforms adopt a symmetric customized pricing strategy when consumers are quite heterogeneous, but never adopt asymmetric pricing strategies. Finally, when only advertisers multi-home, depending on the relative size of the cross-side network effects, competing platforms can adopt a symmetric customized pricing strategies (unlike in a single-homing model), asymmetric pricing strategies (unlike in a multi-homing model), or a symmetric uniform pricing strategy.

- Essay 2 proposes a novel model where a media platform interacts with three sides: content suppliers, consumers, and advertisers, and examines a platform's content provision strategy and source of profits. First, I find that the structure of content market and platform market critically influence a platform's content provision strategy and its choice of source of profits: In a perfectly competitive content market, inter-platform competition motivates a platform to switch from paid-content-with-ads to free-content strategy. However, if the content supplier is a monopoly, this result can get reversed. Second, I show how inter-platform can protect a platform from losing all profits to the content supplier. Next, I explore how market environment affects a content supplier's profits and a media platform's profits differently. Specifically, though advertisers' higher valuation for consumers benefits a media platform, it can hurt a content supplier's profits when a monopoly supplier sells content to a platform using paid-content-with-ads strategy or when duopoly suppliers can shape consumers' preference at a low marginal cost and sell to a platform using free-content strategy. Finally, on examining the use of no-ad strategy, I find that compared to duopoly platform, a monopoly platform is more likely to shun the advertising market.

Appendix A

Proof for Chapter 2

In this appendix, we provide the proofs for propositions in chapter 2. We first state the lemmas that describe the equilibrium outcome of each subgame and define the parameter space. We then prove propositions and claims based on these lemmas.

A.1 Single-Homing Analysis

Lemma A.1. *If both platforms set a uniform price for consumers, the equilibrium profits of each platform are* $\Pi_1^{ss1*} = \Pi_2^{ss1*} = \frac{1}{2}(t_I - \gamma_J + t_J + \gamma_I)$*, where* $\gamma_I \equiv \frac{\gamma_I^H}{2}$.

Proof. Let p_{1I} and p_{2I} denote the prices set by Platform 1 and Platform 2, respectively, for consumers. Further let p_{1J} and p_{2J} denote the corresponding prices for advertisers. For a λ-type consumer who is located at θ, the utility of joining Platform 1 is

$$U_{1I}^{\lambda}(\theta) = v_I - t_I\theta - \gamma_I^{\lambda}\alpha_{1J}^{e} - p_{1I} \tag{A.1}$$

where $\lambda \in \{L, H\}$. The utility derived on joining Platform 2 is

$$U_{2I}^{\lambda}(\theta) = v_I - t_I(1-\theta) - \gamma_I^{\lambda}\alpha_{2J}^{e} - p_{2I} \tag{A.2}$$

The λ -type consumer who is indifferent between the two platforms is given by

$$\theta_I^{\lambda} = \tfrac{1}{2t_I}(p_{2I} - p_{1I} + \gamma_I^{\lambda}(\alpha_{2J}^e - \alpha_{1J}^e)) + \tfrac{1}{2} \tag{A.3}$$

By defining $\gamma_I \equiv \frac{\gamma_I^H}{2}$, we obtain 'average' marginal consumer $\theta_I \equiv \frac{\theta_I^L+\theta_I^H}{2}$. Furthermore,

$$\theta_I = \tfrac{1}{2t_I}(p_{2I} - p_{1I} + \gamma_I(\alpha_{2J}^e - \alpha_{1J}^e)) + \tfrac{1}{2}. \tag{A.4}$$

The marginal advertiser who is indifferent between the two platforms is similarly given by

$$\theta_J = \tfrac{1}{2t_J}(p_{2J} - p_{1J} - \gamma_J(\alpha_{2I}^e - \alpha_{1I}^e)) + \tfrac{1}{2}. \tag{A.5}$$

In equilibrium, $\alpha_{1I}^e = \theta_I$, $\alpha_{1J}^e = \theta_J$, $\alpha_{2I}^e = 1 - \theta_I$ and $\alpha_{2J}^e = 1 - \theta_J$. On solving the system of equations, we have the following demand functions pertaining to each platform for each side of the market:

$$\alpha_{1I} = \tfrac{1}{2(t_I t_J + \gamma_I \gamma_J)}[-t_J(p_{1I} - p_{2I}) + \gamma_I(p_{1J} - p_{2J})] + \tfrac{1}{2} \tag{A.6}$$

$$\alpha_{1J} = \tfrac{1}{2(t_I t_J + \gamma_I \gamma_J)}[-t_I(p_{1J} - p_{2J}) - \gamma_J(p_{1I} - p_{2I})] + \tfrac{1}{2} \tag{A.7}$$

$$\alpha_{2I} = \tfrac{1}{2(t_I t_J + \gamma_I \gamma_J)}[-t_J(p_{2I} - p_{1I}) + \gamma_I(p_{2J} - p_{1J})] + \tfrac{1}{2} \tag{A.8}$$

$$\alpha_{2J} = \tfrac{1}{2(t_I t_J + \gamma_I \gamma_J)}[-t_I(p_{2J} - p_{1J}) - \gamma_J(p_{2I} - p_{1I})] + \tfrac{1}{2} \tag{A.9}$$

Profit function for each platform is

$$\Pi_1^{ss1} = \alpha_{1I} \cdot p_{1I} + \alpha_{1J} \cdot p_{1J} \tag{A.10}$$

$$\Pi_2^{ss1} = \alpha_{2I} \cdot p_{2I} + \alpha_{2J} \cdot p_{2J} \tag{A.11}$$

Then the hessian of Platform 1's profit function is given by:

$$H(\Pi_1^{ss1}) = \begin{bmatrix} \frac{\partial^2 \Pi_1^{ss1}}{\partial {p_{1I}}^2} & \frac{\partial^2 \Pi_1^{ss1}}{\partial p_{1I} \partial p_{1J}} \\ \frac{\partial^2 \Pi_1^{ss1}}{\partial p_{1J} \partial p_{1I}} & \frac{\partial^2 \Pi_1^{ss1}}{\partial {p_{1J}}^2} \end{bmatrix} = \begin{bmatrix} -t_J & \frac{\gamma_I - \gamma_J}{2} \\ \frac{\gamma_I - \gamma_J}{2} & -t_I \end{bmatrix} \cdot \frac{1}{t_I t_J + \gamma_I \gamma_J}. \tag{A.12}$$

The hessian of Platform 2's profit function is the same.

To ensure that both profit functions are concave, the hessian needs to be negative definite. The sufficient and necessary condition is given by

$$4t_I t_J - (\gamma_I - \gamma_J)^2 > 0 \tag{A.13}$$

By simultaneously solving $\frac{\partial \Pi_1^{ss1}}{\partial p_{1I}} = 0$, $\frac{\partial \Pi_1^{ss1}}{\partial p_{1J}} = 0$, $\frac{\partial \Pi_2^{ss1}}{\partial p_{2I}} = 0$, and $\frac{\partial \Pi_2^{ss1}}{\partial p_{2J}} = 0$, we obtain the following unique equilibrium prices:

$$p_{1I}^* = p_{2I}^* = t_I - \gamma_J \tag{A.14}$$

$$p_{1J}^* = p_{2J}^* = t_J + \gamma_I. \tag{A.15}$$

At the equilibrium prices, platforms evenly split both the consumer market and the advertiser market: $\alpha^{1I} = \alpha^{2I} = \alpha^{1J} = \alpha^{2J} = \frac{1}{2}$. The resulting profits of each platform are given by:

$$\Pi_1^{ss1*} = \Pi_2^{ss1*} = \frac{1}{2}(t_I - \gamma_J + t_J + \gamma_I) \tag{A.16}$$

□

Lemma A.2. *If both platforms set customized prices for the two segments of consumers, the equilibrium profits each of platform are* $\Pi_1^{ss2*} = \Pi_2^{ss2*} = \frac{1}{2}(t_I + t_J) - \frac{1}{4}\gamma_J$.

Proof. First note that if both types of consumers choose either the high price or the low price, then it is equivalent to both platforms adopting a uniform pricing strategy. Thus, we solve for the equilibrium solution assuming that H-type consumers choose the high price while L-type consumers choose the low price in both platforms. Later we derive the conditions under which the corresponding IC conditions hold in equilibrium.

When both platforms set customized prices for the two segments of consumers, the original two-sided market is divided into a one-sided market consisting of H-type consumers, and a two-sided market consisting of L-type consumers and advertisers. Therefore, we can separately solve the equilibrium of the one-sided market and that of the two-sided market.

The market consisting of H type consumers is a stylized Hotelling model, and the equilibrium price is consumer's transportation cost. Thus

$$p_{1I}^{H*} = p_{2I}^{H*} = t_I. \tag{A.17}$$

The equilibrium solution for the two-sided market consisting of L-type consumers and advertisers is obtained in the same way as in Lemma A.1. In this two-sided market, the sufficient and necessary condition for each platform's profits function to be concave is given by

$$4t_It_J - \tfrac{1}{2}(\gamma_I^L - \gamma_J)^2 = 4t_It_J - \tfrac{1}{2}\gamma_J^2 > 0 \tag{A.18}$$

The corresponding equilibrium prices are given by:

$$p_{1I}^{L*} = p_{2I}^{L*} = t_I - \gamma_J \tag{A.19}$$

$$p_{1J}^{*} = p_{2J}^{*} = t_J. \tag{A.20}$$

Then in equilibrium, platforms evenly split the H-type consumer market, L-type consumer market, and the advertiser market. Then, each platform's profits are given by:

$$\Pi_1^{ss2*} = \Pi_2^{ss2*} = \tfrac{1}{2}(t_I + t_J) - \tfrac{1}{4}\gamma_J \tag{A.21}$$

Finally, given the equilibrium solution, we derive conditions where the IC constraints are satisfied. Note that it is sufficient to establish the IC constraints for the consumers joining Platform 1 because the same conditions apply for the other platform. Since L-type consumers joining Platform 1 would prefer to pay p_{1I}^L and tolerate advertisements instead of paying p_{1I}^H for an advertisement-free platform, the IC constraint is given by $p_{1I}^L + \gamma_I^L \alpha_{1J} < p_{1I}^H$. On the other hand, H-type consumers would prefer to access Platform 1 without the nuisance of advertisements, suggesting $p_{1I}^H < p_{1I}^L + \gamma_I^H \alpha_{1J}$. Together, since $\gamma_I^L = 0$, we have:

$$p_{1I}^L < p_{1I}^H < p_{1I}^L + \gamma_I^H \alpha_{1J} \tag{A.22}$$

On substituting the equilibrium solutions into (A.22), the IC constraints can be simplified to

$$0 < \gamma_J < \frac{\gamma_I^H}{2}. \tag{A.23}$$

□

Lemma A.3. *If Platform 1 sets a uniform price while Platform 2 sets customized prices for consumers, the equilibrium profits of each platform are*

$$\begin{aligned}\Pi_1^{ss3*} = & \frac{1}{4t_I(288t_It_J-7{\gamma_I^H}^2+18\gamma_I^H\gamma_J-39\gamma_J^2)^2} \cdot [165888t_I^4t_J^2 + 1152t_I^3t_J(144t_J^2 - 7{\gamma_I^H}^2 + 12t_J(\gamma_I^H - 7\gamma_J) + 19\gamma_I^H\gamma_J - 35\gamma_J^2) \\ & + \gamma_J^2(26{\gamma_I^H}^4 - 77{\gamma_I^H}^3\gamma_J + 146{\gamma_I^H}^2\gamma_J^2 + 51\gamma_I^H\gamma_J^3 - 90\gamma_J^4) - 2t_I^2(-49{\gamma_I^H}^4 + 266{\gamma_I^H}^3\gamma_J - 856{\gamma_I^H}^2\gamma_J^2 + 1350\gamma_I^H\gamma_J^3 - 1215\gamma_J^4 \\ & + 1152t_J^2({\gamma_I^H}^2 - 9\gamma_I^H\gamma_J + 21\gamma_J^2) + 24t_J(7{\gamma_I^H}^3 - 175{\gamma_I^H}^2\gamma_J + 357\gamma_I^H\gamma_J^2 - 501\gamma_J^3)) - t_I\gamma_J(119{\gamma_I^H}^4 - 534{\gamma_I^H}^3\gamma_J \\ & + 1540{\gamma_I^H}^2\gamma_J^2 - 1938\gamma_I^H\gamma_J^3 + 1485\gamma_J^4 + 24\gamma_J(5{\gamma_I^H}^3 + 8{\gamma_I^H}^2\gamma_J + 129\gamma_I^H\gamma_J^2 - 174\gamma_J^3))]\end{aligned} \tag{A.24}$$

$$\begin{aligned}\Pi_2^{ss3*} = & \frac{1}{4t_I(288t_It_J-7{\gamma_I^H}^2+18\gamma_I^H\gamma_J-39\gamma_J^2)^2} \cdot [165888t_I^4t_J^2 + 1152t_I^3t_J(144t_J^2 + 60t_J\gamma_I^H - 7{\gamma_I^H}^2 - 132t_J\gamma_I + 17\gamma_I^H\gamma_J - 41\gamma_J^2) \\ & + \gamma_J(-14{\gamma_I^H}^5 + 65{\gamma_I^H}^4\gamma_J - 200{\gamma_I^H}^3\gamma_J^2 + 296{\gamma_I^H}^2\gamma_J^3 - 318\gamma_I^H\gamma_J^4 - +45\gamma_J^5) - 2t_I^2(-49{\gamma_I^H}^4 + 238{\gamma_I^H}^3\gamma_J - 859{\gamma_I^H}^2\gamma_J^2 \\ & + 1392\gamma_I^H\gamma_J^3 - 1674\gamma_J^4 + 144t_J^2(13{\gamma_I^H}^2 - 18\gamma_I^H\gamma_J + 153\gamma_J^2) + 24t_J(77{\gamma_I^H}^3 - 326{\gamma_I^H}^2\gamma_J + 789\gamma_I^H\gamma_J - 900\gamma_J^3)) + t_I(49{\gamma_I^H}^5 \\ & - 315{\gamma_I^H}^4\gamma_J + 1219{\gamma_I^H}^3\gamma_J^2 - 2773{\gamma_I^H}^2\gamma_J^3 + 3984\gamma_I^H\gamma_J^4 - 3060\gamma_J^5 - 4t_J(7{\gamma_I^H}^4 - 177{\gamma_I^H}^3\gamma_J + 237{\gamma_I^H}^2\gamma_J^2 - 387\gamma_I^H\gamma_J^3 - 648\gamma_J^4))]\end{aligned} \tag{A.25}$$

Proof. Let p_{1I} denote the price that Platform 1 charges its consumers, and let p_{2I}^H and p_{2I}^L denote the prices that Platform 2 charges its H-type and L-type consumers, respectively. Similarly, let p_{1J} and p_{2J} denote the prices for advertisers charged by Platform 1 and Platform 2, respectively. Then the utility that a H-type consumer at θ derives from joining Platform 1 and Platform 2 are as follows:

$$U_{1I}^H(\theta) = v_I - t_I\theta - \gamma_I^H\alpha_{1J}^e - p_{1I}^H \tag{A.26}$$

$$U_{2I}^H(\theta) = v_I - t_I(1-\theta) - p_{2I}^H. \tag{A.27}$$

The utilities that a low-type consumer at θ derives from joining each platform are given as in (A.1) and (A.2). Then the λ-type consumer indifferent between the two platforms is given by:

$$\theta_I^H = \frac{1}{2t_I}(p_{2I}^H - p_{1I} - \gamma_I^H\alpha_{1J}^e) + \frac{1}{2} \tag{A.28}$$

$$\theta_I^L = \frac{1}{2t_I}(p_{2I}^L - p_{1I}) + \frac{1}{2}. \tag{A.29}$$

Likewise, the advertiser indifferent between the two platforms (see Lemma A.1) is given by:

$$\theta_J = \frac{1}{2t_J}(p_{2J} - p_{1J} + \gamma_J(\alpha^e_{2I} - \alpha^e_{1I})) + \frac{1}{2}. \tag{A.30}$$

In equilibrium, we have $\alpha^e_{1I} = \frac{\theta^L_I + \theta^H_I}{2}$, $\alpha^e_{1J} = \theta_J$, $\alpha^{Le}_{2I} = \frac{1-\theta^L_I}{2}$, $\alpha^{He}_{2I} = \frac{1-\theta^H_I}{2}$ and $\alpha^e_{2J} = 1 - \theta_J$. Upon solving this system of equations, we can rewrite the demand functions α_{1I} , α_{1J}, α^L_{2I}, α^H_{2I} and α_{2J} as functions of the prices (p_{1I}, p^L_{2I}, p^H_{2I}, p_{1J}, and p_{2J}) and the parameters (t_I, t_J, γ^H_I, and γ_J). Given these demand functions, the profits of each platform are given by:

$$\Pi^{ss3}_1 = \alpha_{1I}(\mathbf{p}) \cdot p_{1I} + \alpha_{1J}(\mathbf{p}) \cdot p_{1J} \tag{A.31}$$

$$\Pi^{ss3}_2 = \alpha^L_{2I}(\mathbf{p}) \cdot p^L_{2I} + \alpha^H_{2I}(\mathbf{p}) \cdot p^H_{2I} + \alpha_{2J}(\mathbf{p}) \cdot p_{2J}, \tag{A.32}$$

where $\mathbf{p} \equiv (p_{1I}, p^L_{2I}, p^H_{2I}, p_{1J}, p_{2J})$.

Then the hessian of Platform 1's profit function is given by

$$H(\Pi^{ss3}_1) = \begin{bmatrix} \frac{\partial^2 \Pi^{ss3}_1}{\partial {p_{1I}}^2} & \frac{\partial^2 \Pi^{ss3}_1}{\partial p_{1I} \partial p_{1J}} \\ \frac{\partial^2 \Pi^{ss3}_1}{\partial p_{1J} \partial p_{1I}} & \frac{\partial^2 \Pi^{ss3}_1}{\partial {p_{1J}}^2} \end{bmatrix} = \begin{bmatrix} \frac{-32t_I t_J + 2\gamma^H_I \gamma_J}{4t_I} & -8t_I \\ -8t_I & \gamma^H_I - 3\gamma_J \end{bmatrix} \cdot \frac{1}{8t_I t_J + \gamma^H_I \gamma_J} \tag{A.33}$$

The hessian of Platform 2's profit function is given by

$$H(\Pi^{ss3}_2) = \begin{bmatrix} \frac{\partial^2 \Pi^{ss3}_2}{\partial {p_{2I}^L}^2} & \frac{\partial^2 \Pi^{ss3}_2}{\partial p^H_{2I} \partial p^L_{2I}} & \frac{\partial^2 \Pi^{ss3}_2}{\partial p_{2J} \partial p^L_{2I}} \\ \frac{\partial^2 \Pi^{ss3}_2}{\partial p^L_{2I} \partial p^H_{2I}} & \frac{\partial^2 \Pi^{ss3}_2}{\partial {p_{2I}^H}^2} & \frac{\partial^2 \Pi^{ss3}_2}{\partial p_{2J} \partial p^H_{2I}} \\ \frac{\partial^2 \Pi^{ss3}_2}{\partial p^L_{2I} \partial p_{2J}} & \frac{\partial^2 \Pi^{ss3}_2}{\partial p^H_{2I} \partial p_{2J}} & \frac{\partial^2 \Pi^{ss3}_2}{\partial {p_{2J}}^2} \end{bmatrix} = \begin{bmatrix} -\frac{8t_I t_J + \gamma^H_I \gamma_J}{2t_I} & \frac{\gamma^H_I \gamma_J}{2t_I} & -2\gamma_J \\ \frac{\gamma^H_I \gamma_J}{2t_I} & -4t_J & \gamma^H_I - \gamma_J \\ -2\gamma_J & \gamma^H_I - \gamma_J & -8t_I \end{bmatrix} \cdot \frac{1}{8t_I t_J + \gamma^H_I \gamma_J} \tag{A.34}$$

To ensure that both profit functions are concave, the hessian of each profit function needs to be negative definite. The corresponding sufficient and necessary conditions are:

$$64t_I t_J - {\gamma^H_I}^2 + 2\gamma^H_I \gamma_J - 9\gamma^2_J > 0 \tag{A.35}$$

$$64t^2_I t^2_J + 8t_I t_J \gamma^H_I \gamma_J - {\gamma^H_I}^2 \gamma^2_J > 0 \tag{A.36}$$

$$32t_I t_J - {\gamma^H_I}^2 + 2\gamma^H_I \gamma_J - 5\gamma^2_J > 0. \tag{A.37}$$

Assuming these conditions are satisfied, we derive the equilibrium prices by simultaneously solving $\frac{\partial \Pi_1^{ss3}}{\partial p_{1I}} = 0$, $\frac{\partial \Pi_1^{ss3}}{\partial p_{1J}} = 0$, $\frac{\partial \Pi_2^{ss3}}{\partial p_{2I}^L} = 0$, $\frac{\partial \Pi_2^{ss3}}{\partial p_{2I}^H} = 0$ and $\frac{\partial \Pi_2^{ss3}}{\partial p_{2J}} = 0$. The unique equilibrium prices are then given by:

$$p_{1I}^* = \frac{288t_I^2 t_J + 2\gamma_J({\gamma_I^H}^2 - 5\gamma_I^H \gamma_J + 15\gamma_J^2) - t_I(7{\gamma_I^H}^2 - 16\gamma_I^H \gamma_J + 45\gamma_J^2 + 24t_J(\gamma_I^H + 9\gamma_J))}{288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2} \tag{A.38}$$

$$p_{2I}^* = \frac{96t_I^2 t_J(12t_J + 3\gamma_I^H + \gamma_J) + \gamma_I^H \gamma_J(2{\gamma_I^H}^2 - \gamma_I^H \gamma_J + 9\gamma_J^2) - t_I(24t_J({\gamma_I^H}^2 + 7\gamma_J^2) + \gamma_I^H(7{\gamma_I^H}^2 - 16\gamma_I^H \gamma_J + 33\gamma_J^2))}{4t_I(288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2)} \tag{A.39}$$

$${p_{2I}^L}^* = \frac{288t_I^2 t_J + \gamma_J(8{\gamma_I^H}^2 - 14\gamma_I^H \gamma_J + 33\gamma_J^2) - t_I(7{\gamma_I^H}^2 - 17\gamma_I^H \gamma_J + 30\gamma_J^2 + 12t_J(\gamma_I^H + 21\gamma_J))}{288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2} \tag{A.40}$$

$${p_{2I}^H}^* = \frac{288t_I^2 t_J + t_I(-7{\gamma_I^H}^2 + 60t_J(\gamma_I^H - 3\gamma_J) + 23\gamma_I^H \gamma_J - 36\gamma_J^2) + \gamma_J(9{\gamma_I^H}^2 - 20\gamma_I^H \gamma_J + 24\gamma_J^2)}{288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2} \tag{A.41}$$

$$p_{2J}^* = \frac{96t_I^2 t_J(12t_J + 3\gamma_I^H - \gamma_J) + \gamma_I^H \gamma_J(2{\gamma_I^H}^2 - 11\gamma_I^H \gamma_J + 6\gamma_J^2) + t_I(\gamma_I^H(-7{\gamma_I^H}^2 + 23\gamma_I^H \gamma_J - 48\gamma_J^2) + 4t_J({\gamma_I^H}^2 + 9\gamma_I^H \gamma_J - 36\gamma_J^2))}{4t_I(288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2)} \tag{A.42}$$

At the equilibrium prices, the demand for each platform in the two markets are given by:

$$\alpha_{1I}^* = \frac{576t_I^2 t_J + \gamma_J(4{\gamma_I^H}^2 + 7\gamma_I^H \gamma_J - 3\gamma_J^2) - 2t_I(24t_J \gamma_I^H + 7{\gamma_I^H}^2 - 16\gamma_I^H \gamma_J + 27\gamma_J^2)}{4t_I(288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2)} \tag{A.43}$$

$$\alpha_{2I}^* = \frac{3((3\gamma_I^H - 7\gamma_J)\gamma_J + 4t_I(12t_J + \gamma_J))}{288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2} \tag{A.44}$$

$${\alpha_{2I}^L}^* = \frac{288t_I^2 t_J + t_I(-7{\gamma_I^H}^2 - 12t_J(\gamma_I^H - 3\gamma_J) + 17\gamma_I^H \gamma_J - 54\gamma_J^2) + \gamma_J(-6{\gamma_I^H}^2 + 4\gamma_I^H \gamma_J - 3\gamma_J^2)}{4t_I(288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2)} \tag{A.45}$$

$${\alpha_{2I}^H}^* = \frac{288t_I^2 t_J + t_I(60t_J \gamma_I^H - 7{\gamma_I^H}^2 - 36t_J \gamma_J + 23\gamma_I^H \gamma_J - 48\gamma_J^2) + \gamma_J(2{\gamma_I^H}^2 - 11\gamma_I^H \gamma_J + 6\gamma_J^2)}{4t_I(288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2)} \tag{A.46}$$

$$\alpha_{2J}^* = \frac{144t_I t_J - 7{\gamma_I^H}^2 - 12t_I \gamma_J + 9\gamma_I^H \gamma_J - 18\gamma_J^2}{288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H \gamma_J - 39\gamma_J^2} \tag{A.47}$$

Note that the demand has to be properly bounded in equilibrium: $0 \leqslant {\alpha_{2I}^L}^* \leqslant \frac{1}{2}$, $0 \leqslant {\alpha_{2I}^H}^* \leqslant \frac{1}{2}$, and $0 \leqslant \alpha_{2J}^* \leqslant 1$ for Platform 2. Because the market is fully covered, these conditions also ensure that the demand for Platform 1 is bounded as well: $0 \leqslant \alpha_{1I}^* \leqslant 1$, and $0 \leqslant \alpha_{1J}^* \leqslant 1$.

In equilibrium, the IC constraints that need to be satisfied are: $p_{1I}^L < p_{1I}^H < p_{1I}^L + \gamma_I^H \alpha_{1J}$. On substituting equilibrium solutions into these inequalities, the IC constraints can be simplified to

$$-9\gamma_J^3 - 6(\gamma_I^H + t_I)\gamma_J^2 + ({\gamma_I^H}^2 + 6t_I\gamma_I^H + 72t_It_J)\gamma_J + 72t_It_J\gamma_I^H > 0 \quad \text{(A.48)}$$

$$9\gamma_J^3 - 6(2\gamma_I^H - t_I)\gamma_J^2 + (8{\gamma_I^H}^2 - 18t_I\gamma_I^H - 72t_It_J)\gamma_J - 7{\gamma_I^H}^3 + 72t_It_J\gamma_I^H > 0 \quad \text{(A.49)}$$

Finally, given the equilibrium demand and prices, each platform's equilibrium profits can be calculated as in (A.24) and (A.25) □

Lemma A.4. *Define the parameter space Θ^{ss} as the pair (γ_I^H, γ_J) satisfying second-order conditions and demand conditions of the three subgames as well as $\gamma_J \leqslant \min\left\{\frac{\gamma_I^H + 3\{\sqrt{t_I(t_I+4t_J)} - t_I\}}{2}, 3\{\sqrt{t_I(t_I+4t_J)} - t_I\}\right\}$. Then in Θ^{ss}, neither platform completely abandons advertisers and serves only consumers.*

Proof. We establish this claim in four parts. In part (a), we first specify the parameter space Θ^{ss}. Then in Part (b) and Part (c), we solve for the equilibrium of two potential subgames where one platform abandons the advertiser side of the market. Finally, in Part (d), we show that neither platform has an incentive to abandon advertisers by ruling out these two subgames.

Part (a). Based Lemmas A.1-A.3, we define the parameter space Θ^{ss} as the pair (γ_I^H, γ_J) satisfying the following three sets of conditions:

1. In this parameter space, both platforms' profits are concave with respect to their own prices across all three subgames. Hence, the parameter space satisfies the second-order conditions given in (A.13), (A.18), (A.35), (A.36), and (A.37).

2. Recall that the equilibrium demand of each platform when both platforms use a symmetric uniform pricing strategy or a symmetric customized pricing strategy is $\frac{1}{2}$ on each side of the market (See Lemma A.1 and Lemma A.2). However, when competing platforms adopt asymmetric pricing strategies, the equilibrium demand of each platform varies with model parameters (See Lemma

A.3). Furthermore, the equilibrium demand should be within proper bounds: $0 \leqslant {\alpha_{2I}^{L}}^{*} \leqslant \frac{1}{2}$, $0 \leqslant {\alpha_{2I}^{H}}^{*} \leqslant \frac{1}{2}$, and $0 \leqslant \alpha_{2J}^{*} \leqslant 1$ for Platform 2. Because the market is fully covered, these conditions also ensure that the demand for Platform 1 lies with proper bounds: $0 \leqslant \alpha_{1I}^{*} \leqslant 1$, and $0 \leqslant \alpha_{1J}^{*} \leqslant 1$.

3. Finally, in the parameter space, the following inequality holds:

$$\gamma_J \leqslant \min\left\{\frac{\gamma_I^H + 3\{\sqrt{t_I(t_I + 4t_J)} - t_I\}}{2}, 3\{\sqrt{t_I(t_I + 4t_J)} - t_I\}\right\}. \tag{A.50}$$

Part (b). Here we derive the equilibrium profits in the subgame where Platform 1 abandons advertisers to serve only consumers while Platform 2 chooses to serve both sides of the market and uses a uniform pricing strategy for consumers. First note that there is competition only on the consumer side of the market. Given the absence of competition from Platform 1 on the advertiser side of the market, Platform 2 takes the entire advertiser market, implying $\theta_J = 0$, and charges the price $p_{2J} = v_J - t_J + \alpha_{2I} \cdot \gamma_J$. Given the competition on the consumer side of the market, the marginal consumer θ_I is given as in (A.4) with $\alpha_{1J}^{e} = \theta_J = 0$ and $\alpha_{2J}^{e} = 1 - \theta_J = 1$. Also, note that $\alpha_{1I} = \theta_I$ and $\alpha_{2I} = 1 - \theta_I$. Then the profit functions of the two platforms are as follows:

$$\Pi_1^{sd1} = \theta_I \cdot p_{1I} \tag{A.51}$$

$$\Pi_2^{sd1} = (1 - \theta_I) \cdot p_{2I} + (1 - \theta_J) \cdot p_{2J}, \tag{A.52}$$

where the superscript $sd1$ denotes the first subgame pertaining to a one-sided market strategy in the single-homing model. Both profit functions are concave with respect to their own price for consumers, since the second-order conditions are $\frac{\partial^2 \Pi_1^{sd1}}{\partial p_{1I}^2} = \frac{\partial^2 \Pi_2^{sd1}}{\partial p_{2I}^2} = -\frac{1}{t_I} < 0$. On solving this game, we obtain the following equilibrium prices:

$$p_{1I}^{*} = t_I + \frac{\gamma_I^H - 2\gamma_J}{6} \tag{A.53}$$

$$p_{2I}^{*} = t_I - \frac{\gamma_I^H + 4\gamma_J}{6} \tag{A.54}$$

and the following equilibrium profits:

$$\Pi_1^{sd1*} = \frac{(6t_I+\gamma_I^H-2\gamma_J)^2}{72t_I} \tag{A.55}$$

$$\Pi_2^{sd1*} = \frac{(6t_I+\gamma_I^H-2\gamma_J)^2}{72t_I} + \frac{3v_J-3t_J-\gamma_I^H+2\gamma_J}{3} \tag{A.56}$$

Note that for the equilibrium demand to be within proper bounds (i.e., $0 \leqslant \theta_I^{H*} \leqslant 1$ and $0 \leqslant \theta_I^{L*} \leqslant 1$), we should have $\gamma_J \leqslant 2\gamma_I^H - 3t_I$ and $\gamma_J \leqslant 3t_I - \gamma_I^H$.

Part (c). In this part, we derive the equilibrium profits pertaining to the second subgame where Platform 1 abandons advertisers while Platform 2 caters to both sides of the market and uses a customized pricing strategy for consumers. On the advertiser of the market, as in Part (a), we have $\theta_J = 0$ and $p_{2J} = v_J - t_J + \alpha_{2I}^L \cdot \gamma_J$. On the consumer side of the market, the marginal consumer θ_I^λ is given as in (A.30) with $\alpha_{1J}^e = \theta_J = 0$. Furthermore, we have $\alpha_{2J}^e = 1$, $\alpha_{1I} = \theta_I$, $\alpha_{2I}^L = \frac{1}{2}(1-\theta_I^L)$ and $\alpha_{2I}^H = \frac{1}{2}(1-\theta_I^H)$. Then the profit functions of the two platforms are given by:

$$\Pi_1^{sd2} = \theta_I \cdot p_{1I} \tag{A.57}$$

$$\Pi_2^{sd2} = \frac{1}{2}(1-\theta_I^L) \cdot p_{2I}^L + \frac{1}{2}(1-\theta_I^H) \cdot p_{2I}^H + (1-\theta_J) \cdot p_{2J}, \tag{A.58}$$

where the superscript $sd2$ denotes the second subgame pertaining to a one-sided market strategy in the single-homing model. Both profit functions are concave with respect to their own price for consumer because $\frac{\partial^2 \Pi_1^{sd1}}{\partial p_{1I}^2} = -\frac{1}{t_I} < 0$ and the hessian of Π_2^{sd2} is

$$H(\Pi_2^{sd2}) = \begin{bmatrix} \frac{\partial^2 \Pi_2^{s2}}{\partial {p_{2I}^L}^2} & \frac{\partial^2 \Pi_2^{s2}}{\partial p_{2I}^L \partial p_{2I}^H} \\ \frac{\partial^2 \Pi_2^{s2}}{\partial p_{2I}^H \partial p_{2I}^L} & \frac{\partial^2 \Pi_2^{s2}}{\partial {p_{2I}^H}^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2t_I} & 0 \\ 0 & -\frac{1}{2t_I} \end{bmatrix} \tag{A.59}$$

which is negative definite. On solving this game, we obtain the following equilibrium prices:

$$p_{1I}^{*} = t_I - \tfrac{\gamma_J}{6} \tag{A.60}$$

$$p_{2I}^{L*} = t_I - \tfrac{7\gamma_J}{12} \tag{A.61}$$

$$p_{2I}^{H*} = t_I - \tfrac{\gamma_J}{12}, \tag{A.62}$$

and the following equilibrium profits:

$$\Pi_1^{sd2*} = \tfrac{(6t_I-2\gamma_J)^2}{72t_I} \tag{A.63}$$

$$\Pi_2^{sd2*} = v_J + \tfrac{t_I}{2} - t_J + \tfrac{13\gamma_J^2}{288t_I} + \tfrac{\gamma_J}{6} \tag{A.64}$$

Note that for the equilibrium demand to be within proper bounds (i.e., $0 \leqslant \theta_I^{L*} \leqslant 1$ and $0 \leqslant \theta_I^{H*} \leqslant 1$) and for the IC conditions to be satisfied (i.e., $p_{2I}^{L*} \leqslant p_{2I}^{H*} \leqslant p_{2I}^{L*} + \gamma_I^H \alpha_{2J}^e$), we should have $\gamma_J \leqslant 2\gamma_I^H$ and $\gamma_J \leqslant \frac{12}{5}t_I$.

Part (d). Based on the equilibrium profits derived in Parts (b) and (c) above, we rule out the possibility that a platform will adopt a one-sided market strategy where it abandons the advertiser side of the market. First, when the competing platform offers a uniform price, neither platform has an incentive to adopt the one-sided market strategy if $\Pi_1^{ss1*} \geqslant \Pi_1^{sd1*}$, where Π_1^{ss1*} is as given in (A.16) and Π_1^{s1*} is as given in (A.55), which is equivalent to

$$\gamma_J \leqslant \frac{\gamma_I^H + 3\{\sqrt{t_I(t_I+4t_J)} - t_I\}}{2}. \tag{A.65}$$

Similarly, when the competing platform offers customized prices, neither platform has an incentive to adopt the one-sided market strategy if $\Pi_1^{ss2*} \geqslant \Pi_1^{sd2*}$, where Π_1^{ss2*} is given in (A.21) and Π_1^{s2*} is as in (A.63), which is equivalent to

$$\gamma_J \leqslant 3\{\sqrt{t_I(t_I + 4t_J)} - t_I\}. \tag{A.66}$$

Since in Θ^{ss}, we have $\gamma_J \leqslant \min\left\{\frac{\gamma_I^H + 3\{\sqrt{t_I(t_I+4t_J)} - t_I\}}{2}, 3\{\sqrt{t_I(t_I + 4t_J)} - t_I\}\right\}$, neither platform has any incentive to adopt a one-sided market strategy where it abandons the advertiser side and serves only the consumer side of the market.[1]

□

A.1.1 Proof of Proposition 2.1

Proof. We prove the proposition in two parts. In Part (a), we prove that such an equilibrium does not exist when the asymmetric pricing equilibrium is feasible, by showing a profitable unilateral deviation to the uniform pricing strategy. In Part (b), we show that even when the asymmetric pricing equilibrium is not feasible, both platforms' using the customized pricing strategy will not be the Pareto dominant equilibrium.

Part (a) In this part, we consider the case where the asymmetric pricing equilibrium is feasible (i.e., the IC constraints of the asymmetric pricing strategy are satisfied). We prove that there does not exist an equilibrium where both platforms adopt a customized pricing strategy by showing $\Pi_1^{ss2*} - \Pi_1^{ss3*} < 0$ whenever the IC constraint of the customized pricing strategy ($\gamma_I^H > 2\gamma_J$) holds.

First, by Lemma A.4, note that the parameter space Θ^{ss} satisfies the following two conditions:

1. One of the second-order conditions of the asymmetric pricing equilibrium given in (A.37) : $32t_I t_J - {\gamma_I^H}^2 + 2\gamma_I^H - 5\gamma_J^2 > 0$ (Call it C1)

2. One of the demand conditions of the asymmetric pricing equilibrium given in (A.47): $\alpha_{2J}^* = \frac{144t_I t_J - 7{\gamma_I^H}^2 - 12t_I\gamma_J + 9\gamma_I^H\gamma_J - 18\gamma_J^2}{288t_I t_J - 7{\gamma_I^H}^2 + 18\gamma_I^H\gamma_J - 39\gamma_J^2} > 0$ (Call it C2).

It is easy to see that the denominator of α_{2J}^* is positive whenever both C1 and $\gamma_I^H > 2\gamma_J$ hold. Thus, C2 becomes equivalent to the numerator being positive:

[1] In fact, when $t_J < 2t_I$, the condition in (A.65) is automatically satisfied in the space where the demand conditions are satisfied. In addition when $t_J < \frac{14}{25}t_I$, the condition in (A.66) is satisfied in the space where the demand conditions are satisfied. Thus, the condition specified in the lemma is sufficient but not necessary.

$144t_It_J - 7\gamma_I^{H^2} - 12t_I\gamma_J + 9\gamma_I^H\gamma_J - 18\gamma_J^2 > 0$, which in turn, is equivalent to

$$t_J > \frac{7\gamma_I^{H^2}+12t_I\gamma_J-9\gamma_I^H\gamma_J+18\gamma_J^2}{144t_I} (\equiv t_{J0}) \tag{A.67}$$

Thus, from this point on, we confine our attention to the case where (A.67) holds, that is, $t_J > t_{J0}$.

Now, based on profits derived in Lemma A.2 and Lemma A.3, we have

$$\Pi_1^{ss2*} - \Pi_1^{ss3*} = \frac{At_J^2+Bt_J+C}{4t_I(288t_It_J-7\gamma_I^{H^2}+18\gamma_I^H\gamma_J-39\gamma_J^2)^2}, \tag{A.68}$$

where

$$\begin{aligned}
A =& 2\{-13824t_I^3(\gamma_I^H - \gamma_J) - 1152t_I^2(5\gamma_I^{H^2} - 3\gamma_J^2)\} \\
B =& 2t_I\{49\gamma_I^{H^4} - 192\gamma_I^H 3\gamma_J + 966\gamma_I^{H^2}\gamma_J^2 + 144\gamma_I^H\gamma_J^3 - 567\gamma_J^4 - 576t_I^2\gamma_J(\gamma_I^H + 4\gamma_J) \\
& + 24t_I(7\gamma_I^{H^3} - 91\gamma_I^{H^2}\gamma_J + 141\gamma_I^H\gamma_J^2 - 33\gamma - J^3)\} \\
C =& \gamma_J\{\gamma_J(-26\gamma_I^{H^4} + 77\gamma_I^{H^3}\gamma_J - 146\gamma_I^{H^2}\gamma_J{}^2 - 51\gamma_I^H\gamma_J^3 + 90\gamma_J^4) + 4t_I^2(7\gamma_I^{H^3} + 7\gamma_I^{H^2}\gamma_J \\
& - 27\gamma_I^H\gamma_J^2 + 153\gamma_J^3) + t_I(70\gamma_I^{H^4} - 282\gamma_I^{H^3}\gamma_J + 670\gamma_I^{H^2}\gamma_J^2 - 534\gamma_I^H\gamma_J^3 - 36\gamma_J^4)\}.
\end{aligned}$$

First observe that the denominator of (A.68) is positive as shown above. Then the sign of (A.68) is determined by the sign of the numerator of (A.68). Now, define the numerator of (A.68) as a function of t_J: $N(t_J) \equiv At_J^2 + Bt_J + C$ for $t_J > t_{J0}$. Since $A < 0$ holds when $\gamma_I^H > 2\gamma_J > 0$, we have $N''(t_J) = A < 0$ and thus, $N(t_J)$ is a concave function of t_J.

We now evaluate $N(t_J)$ and $N'(t_J)$ at $t_J = t_{J0}$. In this calculation, since $\gamma_I^H > 2\gamma_J$, we let $\gamma_I^H = k\gamma_J$ with $k > 2$. First, notice that

$$N(t_J = t_{J0}) = -\frac{1}{72}\{7\gamma_I^{H^2} + 3(8t_I - \gamma_J)\gamma_J\}^2\{(36+24k)t_I\gamma_J + (-18-15k+13k^2)\gamma_J^2\} < 0, \tag{A.69}$$

since $-18 - 15k + 13k^2 > 0$ when $k > 2$.

In addition, we have

$$\begin{aligned}
N'(t_J = t_{J0}) =& -6(384+576k)t_I^3\gamma_J^2 - 6(-408-264k+376k^2+168k^3)t_I^2\gamma_J^3 \\
& - 6(45+24k-138k^2-56k^3+77k^4)t_I\gamma_J^4 < 0,
\end{aligned} \tag{A.70}$$

since $k > 2$ implies that $384 + 576k > 0$, $-408 - 264k + 376k^2 + 168k^3 > 0$ and $45 + 24k - 138k^2 - 56k^3 + 77k^4 > 0$.

By the above calculations, we have $N(t_J = t_{J0}) < 0$ and $N'(t_J = t_{J0}) < 0$. Since $N(t_J)$ is a concave function of t_J, we have $N(t_J) < 0, \forall t_J > t_{J0}$, which implies that $\Pi_1^{ss2*} - \Pi_1^{ss3*} < 0$ holds in Θ^{ss}.

Part (b) In this part, we consider the case where the asymmetric pricing equilibrium is not feasible (i.e., the IC constraints of the asymmetric pricing strategy are not satisfied). In this case, both platforms' adopting the customized pricing strategy is an equilibrium, since there is no potential for unilateral deviation for any platform. At the same time, however, for the same reason, both platforms' using the uniform pricing strategy can be an equilibrium. Given the multiplicity of equilibria, the platforms will choose the equilibrium that generates higher profits for both platforms (that is, Pareto dominant equilibrium). From the profits derived in Lemma A.1 and Lemma A.2, we have

$$\Pi_i^{ss1*} - \Pi_i^{ss2*} = \frac{1}{4}(\gamma_I^H - \gamma_J) > 0, \forall i = 1, 2, \tag{A.71}$$

since $\gamma_I^H > 2\gamma_J > 0$. Therefore, the two platforms will choose the equilibrium where both platforms use the uniform pricing strategy. This completes the proof. □

A.1.2 Proof of Proposition 2.2

Proof. We prove the proposition in two parts. In Part (a), we prove the existence of the asymmetric pricing equilibrium and the symmetric uniform pricing equilibrium. Then in Part (b), we prove the existence of the thresholds γ_I^{H0} and γ_J^0.

Part (a) First, the symmetric uniform pricing equilibrium always exists in the parameter space Θ^{ss}, since IC condition is not required in this type of equilibrium. We now prove the existence of the asymmetric pricing equilibrium by showing the existence of a pair of (γ_I^H, γ_J) satisfying $\Pi_2^{ss1*} \leqslant \Pi_2^{ss3*}$ in the intersection of Θ^{ss} and the region where the IC constraints hold. Call this intersection Φ.

Suppose γ_J is an infinitesimally small positive number, i.e., ϵ. As ϵ approaches to zero, of all conditions in Φ, the IC condition for H type (i.e., $\gamma_I^H < 6\sqrt{\frac{2t_It_J}{7}}$) is binding if $t_J < \frac{63}{50}t_I$. Otherwise, the demand condition $\alpha_{2I}^H \leqslant \frac{1}{2}$ ($\Leftrightarrow \gamma_I^H \leqslant \frac{6\{\sqrt{t_J(56t_I+25t_J)}-5t_J\}}{7}$) is binding. Below we consider these two cases.

<u>Case 1.</u> When $t_J < \frac{63}{50}t_I$, at ($\gamma_I^H = 6\sqrt{\frac{2t_It_J}{7}} - \epsilon, \gamma_J = \epsilon$), we have

$$\lim_{\epsilon\to 0} \Pi_2^{ss1*} - \Pi_2^{ss3*} = \frac{2\sqrt{14t_It_J}-7t_J}{42} \leqslant 0, \tag{A.72}$$

if and only if $t_J \geqslant \frac{8t_I}{7}$. Since $\frac{8t_I}{7} < \frac{63t_I}{50}$, we have $\Pi_2^{ss1*} \leqslant \Pi_2^{ss3*}$ at ($\gamma_I^H = 6\sqrt{\frac{2t_It_J}{7}} - \epsilon, \gamma_J = \epsilon$) for a sufficiently small ϵ, when $\frac{8}{7}t_I \leqslant t_J < \frac{63}{50}t_I$.

<u>Case 2.</u> When $t_J \geqslant \frac{63}{50}t_I$, at ($\gamma_I^H = \frac{6\{\sqrt{t_J(56t_I+25t_J)}-5t_J\}}{7}, \gamma_J = \epsilon$), we have

$$\lim_{\epsilon\to 0} \Pi_2^{ss1*} - \Pi_2^{ss3*} = \frac{t_J\{2548t_I^2-1125t_J(\sqrt{t_J(56t_I+25t_J)}-5t_J)-35t_I(\sqrt{31t_J(56t_I+25t_J)}-335t_J)\}}{175\{\sqrt{t_J(56t_I+25t_J)}-5t_J\}} \leqslant 0$$

if and only if $t_J \geqslant \frac{1183t_I}{1125}$. Since $\frac{1183t_I}{1125} < \frac{63t_I}{50}$, $\Pi_2^{ss1*} - \Pi_2^{ss3*} \leqslant 0$ always holds at ($\gamma_I^H = \frac{6\{\sqrt{t_J(56t_I+25t_J)}-5t_J\}}{7}, \gamma_J = \epsilon$) when $t_J \geqslant \frac{63}{50}t_I$.

Across the two cases, it is possible to have $\Pi_2^{ss1*} - \Pi_2^{ss3*} \leqslant 0$ at some pairs of (γ_I^H, γ_J), if $t_J \geqslant \frac{8}{7}t_I$. Therefore, when $t_J \geqslant \frac{8}{7}t_I$, there exists an asymmetric equilibrium.

<u>**Part (b)**</u> To begin with, note that we prove the proposition when $\gamma_J > 0$, and $\gamma_I^H > 0$ holds (by definition) and in the parameter space Θ^{ss}. First, the IC constraint

of the asymmetric equilibrium is equivalent to $\gamma_J < \gamma_J^0$, where

$$\gamma_J^0 \equiv \frac{1}{18}\Bigg\{ - t_I + 8\gamma_I^H + (4(2t_I(t_I + 54t_J) + 19t_I\gamma_I^H - 4\gamma_I^{H\,2})) \cdot \Bigg[(-8t_I^3 + \frac{407}{2}\gamma_I^{H\,3} + 60t_I\gamma_I^H(-27t_J + 5\gamma_I^H) - 6t_I^2(108t_J + 19\gamma_I^H)$$

$$+ \frac{1}{2}\sqrt{-32(2t_I(t_I + 54t_J) + 19t_I\gamma_I^H - 4\gamma_I^{H\,2\,3} + (16t_I^2(t_I + 81t_J) + 12t_I(19t_I + 270t_J)\gamma_I^H - 600t_I\gamma_I^{H\,2} - 407\gamma_I^{H\,3\,2}}\Bigg]^{-\frac{1}{3}}$$

$$+ 2^{\frac{2}{3}}\Bigg[- 16t_I^3 + 407\gamma_I^{H\,3} + 120t_I\gamma_I^H(-27t_J + 5\gamma_I^H) - 12t_I^2(108t_J + 19\gamma_I^H)$$

$$+ \sqrt{-32(t_I(t_I + 54t_J) + 19t_I\gamma_I^H - 4\gamma_I^{H\,2})^3 + (16t_I^2(t_I + 81t_J) + 12t_I(19t_I + 270t_J)\gamma_I^H - 600t_I\gamma_I^{H\,2} - 407\gamma_I^{H\,3})^2}\Bigg]^{\frac{1}{3}}\Bigg\} \quad \text{(A.73)}$$

Given $\gamma_J < \gamma_J^0$, it is easy to see that $\Pi_2^{ss1*} < \Pi_2^{ss3*}$ is equivalent to $\gamma_I^H > \gamma_I^{H0}$ where γ_I^{H0} is implicitly defined by $\Pi_2^{ss1*} = \Pi_2^{ss3*}$. (To confirm the direction of the inequality, it helps to note that (1) at $(\gamma_I^H = \epsilon, \gamma_J = 0)$, $\Pi_2^{ss1*} - \Pi_2^{ss3*} = \frac{3\epsilon t_J\{2304t_I^2t_J+21\epsilon^3-8\epsilon t_I(90t_J+7\epsilon)\}}{2(288t_It_J-7\epsilon^2)^2} > 0$ and (2) at $(\gamma_I^H = 6\sqrt{\frac{2t_It_J}{7}}, \gamma_J = 0)$, $\Pi_2^{ss1*} - \Pi_2^{ss3*} = \frac{2\sqrt{14t_It_J}-7t_J}{42} < 0$ since $t_J \geqslant \frac{8}{7}t_I$ holds.) Thus, the asymmetric equilibrium is obtained when $\gamma_J < \gamma_J^0$ and $\gamma_I^H > \gamma_I^{H0}$.

On the other hand, we know from Proposition 2.1 that symmetric customized pricing strategy is not an equilibrium. Then, noting that the symmetric equilibrium with a uniform price does not require any condition other than the second-order condition (which is already satisfied in Θ^{ss}), we find that the symmetric uniform pricing strategy is an equilibrium, if and only if $\Pi_2^{ss1*} \geqslant \Pi_2^{ss3*}$ or the asymmetric equilibrium's IC condition is not satisfied. Therefore, both platforms adopt a symmetric uniform pricing strategy in equilibrium when $\gamma_J \geqslant \gamma_J^0$ or $\gamma_I^H \leqslant \gamma_I^{H0}$. □

A.2 Multi-Homing Analysis

Lemma A.5. *If both platforms set a uniform price for consumers, the equilibrium profits of each platform are* $\Pi_1^{mm1*} = \Pi_2^{mm1*} = \frac{t_J\delta_I^2+t_I\delta_J^2-(\gamma_I-\gamma_J)\delta_I\delta_J}{4t_It_J-(\gamma_I-\gamma_J)^2}$, *where* $\gamma_I \equiv \frac{\gamma_I^H}{2}$.

Proof. For a λ-type consumer who is located at θ, the utility of joining only Platform 1 is given by (A.1), and the utility of joining only Platform 2 is given by (A.2). If the consumer joins both platforms, the utility derived is as follows:

$$U_{3I}^{\lambda}(\theta) = v_I - t_I\theta - \gamma_I^{\lambda}\alpha_{1J}^{e} - p_{1I} + \delta_I - t_I(1-\theta) - \gamma_I^{\lambda}\alpha_{2J}^{e} - p_{2I} \tag{A.74}$$

$$= v_I - t_I - \gamma_I^{\lambda}(\alpha_{1J}^{e} + \alpha_{2J}^{e}) - p_{1I} - p_{2I} + \delta_I \tag{A.75}$$

where $\lambda \in \{L, H\}$.

Since consumers can join two platforms, there are two marginal consumers: θ_{13I} and θ_{23I}. The marginal consumer θ_{13I} is indifferent between joining Platform 1 alone and joining both platforms, while the marginal consumer θ_{23I} is indifferent between joining Platform 2 alone and joining both platforms. Then, θ_{13I} is given by solving $U_{1I}^{\lambda}(\theta) = U_{3I}^{\lambda}(\theta)$:

$$\theta_{13I}^{\lambda} = 1 - \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I^{\lambda} \cdot \alpha_{2J}^{e} - p_{2I}) \tag{A.76}$$

and θ_{23I} is given by solving $U_{2I}^{\lambda}(\theta) = U_{3I}^{\lambda}(\theta)$:

$$\theta_{23I}^{\lambda} = \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I^{\lambda} \cdot \alpha_{1J}^{e} - p_{1I}) \tag{A.77}$$

To ensure that at least one consumer multi-homes in each segment, we assume $\theta_{13I}^{\lambda} \leqslant \theta_{23I}^{\lambda}$, $\lambda \in \{H, L\}$, and later derive the conditions under which this assumption holds. This assumption implies that consumers in the interval $[0, \theta_{13I}^{\lambda})$ join only Platform 1, consumers in the interval $[\theta_{13I}^{\lambda}, \theta_{23I}^{\lambda}]$ join both platforms, and consumers in the interval $(\theta_{23I}^{\lambda}, 1]$ join only Platform 2.

We define the "average" marginal consumers $\theta_{13I} \equiv \frac{\theta_{13I}^{L} + \theta_{13I}^{H}}{2}$ and $\theta_{23I} \equiv \frac{\theta_{23I}^{L} + \theta_{23I}^{H}}{2}$ as follows:

$$\theta_{13I} = 1 - \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I \cdot \alpha_{2J}^{e} - p_{2I}) \tag{A.78}$$

$$\theta_{23I} = \tfrac{1}{t_I} \cdot (\delta_I - \gamma_I \cdot \alpha_{1J}^{e} - p_{1I}). \tag{A.79}$$

Similarly, the advertiser who is indifferent between joining Platform 1 alone and joining both platforms is given by

$$\theta_{13J} = 1 - \tfrac{1}{t_J} \cdot (\delta_J + \gamma_J \cdot \alpha_{2I}^{e} - p_{2J}), \tag{A.80}$$

and the advertiser who is indifferent between joining Platform 2 alone and joining both platforms is given by

$$\theta_{23J} = \tfrac{1}{t_J} \cdot (\delta_J + \gamma_J \cdot \alpha^e_{1I} - p_{1J}). \tag{A.81}$$

To ensure the existence of a multi-homing advertiser, we assume $\theta_{13J} \leqslant \theta_{23J}$ and later derive the condition under which this assumption holds. Thus, advertisers in the interval $[0, \theta_{13J})$ advertise only on Platform 1, advertisers in the interval $[\theta_{13J}, \theta_{23J}]$ advertise on both platforms, and advertisers in the interval $(\theta_{23J}, 1]$ advertise only on Platform 2.

Then, in equilibrium, $\alpha^e_{1I} = \theta_{23I}$, $\alpha^e_{1J} = \theta_{23J}$, $\alpha^e_{2I} = 1 - \theta_{13I}$ and $\alpha^e_{2J} = 1 - \theta_{23J}$. On solving this system of equations, the demand functions for each platform are given by:

$$\alpha_{1I} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_I - p_{1I})t_J - (\delta_J - p_{1J})\gamma_I] \tag{A.82}$$

$$\alpha_{1J} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_J - p_{1J})t_I - (\delta_I - p_{1I})\gamma_J] \tag{A.83}$$

$$\alpha_{2I} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_I - p_{2I})t_J - (\delta_J - p_{2J})\gamma_I] \tag{A.84}$$

$$\alpha_{2J} = \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \cdot [(\delta_J - p_{2J})t_I - (\delta_I - p_{1I})\gamma_J] \tag{A.85}$$

It can be easily seen that each platform's demand is only affected by its own prices for consumers and advertisers, but not affected by its rival's prices.

Each platform's profit function is as follows:

$$\Pi^{mm1}_1 = \alpha_{1I} \cdot p_{1I} + \alpha_{1J} \cdot p_{1J} \tag{A.86}$$

$$\Pi^{mm1}_2 = \alpha_{2I} \cdot p_{2I} + \alpha_{2J} \cdot p_{2J} \tag{A.87}$$

The hessian of Platform 1's profit function is given by:

$$H(\Pi^{mm1}_1) = \begin{bmatrix} \frac{\partial^2 \Pi^{mm1}_1}{\partial {p_{1I}}^2} & \frac{\partial^2 \Pi^{mm1}_1}{\partial p_{1I} \partial p_{1J}} \\ \frac{\partial^2 \Pi^{mm1}_1}{\partial p_{1J} \partial p_{1I}} & \frac{\partial^2 \Pi^{mm1}_1}{\partial {p_{1J}}^2} \end{bmatrix} = \begin{bmatrix} -2t_J & \gamma_I - \gamma_J \\ \gamma_I - \gamma_J & -2t_I \end{bmatrix} \cdot \tfrac{1}{t_I t_J + \gamma_I \gamma_J} \tag{A.88}$$

The hessian of Platform 2's profit function is the same.

The hessian needs to be negative definite so that both platform's profit functions are concave. The corresponding sufficient and necessary condition is given by:

$$4t_I t_J - (\gamma_I - \gamma_J)^2 > 0 \tag{A.89}$$

Then on solving $\frac{\partial \Pi_1^{mm1}}{\partial p_{1I}} = 0$, $\frac{\partial \Pi_1^{mm1}}{\partial p_{1J}} = 0$, $\frac{\partial \Pi_2^{mm1}}{\partial p_{2I}} = 0$, and $\frac{\partial \Pi_2^{mm1}}{\partial p_{2J}} = 0$, we obtain the following equilibrium prices:

$$p_{1I}^* = p_{2I}^* = \frac{2t_I t_J \delta_I - (\gamma_I + \gamma_J) t_I \delta_J + (\gamma_I - \gamma_J)\gamma_J \delta_I}{4t_I t_J - (\gamma_I - \gamma_J)^2} \tag{A.90}$$

$$p_{1J}^* = p_{2J}^* = \frac{2t_J t_I \delta_J + (\gamma_J + \gamma_I) t_J \delta_I + (\gamma_J - \gamma_I)\gamma_I \delta_J}{4t_I t_J - (\gamma_I - \gamma_J)^2}, \tag{A.91}$$

Given the above prices, the equilibrium market sizes are given by:

$$\alpha_{1I}^* = \alpha_{2I}^* = \frac{2t_J \delta_I - (\gamma_I - \gamma_J)\delta_J}{4t_I t_J - (\gamma_I - \gamma_J)^2} \tag{A.92}$$

$$\alpha_{1J}^* = \alpha_{2J}^* = \frac{2t_I \delta_J - (\gamma_I - \gamma_J)\delta_I}{4t_I t_J - (\gamma_I - \gamma_J)^2} \tag{A.93}$$

Note that by definition, $0 \leqslant \alpha_{kI}^* \leqslant 1$ and $0 \leqslant \alpha_{kJ}^* \leqslant 1$ for $k \in \{1, 2\}$. In addition, multi-homing by advertisers and consumers implies that $\alpha_{kI}^* \geqslant \frac{1}{2}$ and $\alpha_{kJ}^* \geqslant \frac{1}{2}$ for $k \in \{1, 2\}$ because $\alpha_{1I}^* = \alpha_{2I}^*$ and $\alpha_{1J}^* = \alpha_{2J}^*$. When these conditions and (A.89) hold, the equilibrium profits of both platforms are given by:

$$\Pi_1^{mm1*} = \Pi_2^{mm1*} = \frac{t_J \delta_I^2 + t_I \delta_J^2 - (\gamma_I - \gamma_J)\delta_I \delta_J}{4t_I t_J - (\gamma_I - \gamma_J)^2}. \tag{A.94}$$

Finally, note that $\theta_{13I}^\lambda \leqslant \theta_{23I}^\lambda$ ($\lambda \in \{H, L\}$) and $\theta_{13J} \leqslant \theta_{23J}$ are respectively equivalent to $\alpha_{kI}^* \geqslant \frac{1}{2}$ and $\alpha_{kJ}^* \geqslant \frac{1}{2}$ for $k \in \{1, 2\}$. □

Lemma A.6. *If both platforms set customized prices for the two segments of consumers, the equilibrium profits of each platform are* $\Pi_1^{mm2*} = \Pi_2^{mm2*} = \frac{16t_I(t_J\delta_I^2 + t_I\delta_J^2) + \gamma_J\delta_I(8t_I\delta_J - \gamma_J\delta_I)}{8t_I(8t_I t_J - \gamma_J^2)}$.

Proof. Following a similar procedure as in Lemma A.5, we derive the profits of both platforms as follows:

$$\Pi_1^{mm2} = \alpha_{1I}^H \cdot p_{1I}^H + \alpha_{1I}^L \cdot p_{1I}^L + \alpha_{1J} \cdot p_{1J} \tag{A.95}$$

$$\Pi_2^{mm2} = \alpha_{2I}^H \cdot p_{2I}^H + \alpha_{2I}^L \cdot p_{2I}^L + \alpha_{2J} \cdot p_{2J} \tag{A.96}$$

where

$$\alpha_{1I}^{L} = \frac{\delta_I - p_{1I}^{L}}{2t_I} \tag{A.97}$$

$$\alpha_{1I}^{H} = \frac{\delta_I - p_{1I}^{H}}{2t_I} \tag{A.98}$$

$$\alpha_{2I}^{L} = \frac{\delta_I - p_{2I}^{L}}{2t_I} \tag{A.99}$$

$$\alpha_{2I}^{H} = \frac{\delta_I - p_{2I}^{H}}{2t_I} \tag{A.100}$$

$$\alpha_{1J} = \frac{\gamma_J(\delta_I - p_{1I}^{L}) + 2t_I(\delta_J - p_{1J})}{2t_I t_J} \tag{A.101}$$

$$\alpha_{2J} = \frac{\gamma_J(\delta_I - p_{2I}^{L}) + 2t_I(\delta_J - p_{2J})}{2t_I t_J} \tag{A.102}$$

Then both platforms' profit functions are concave if and only if

$$8t_I t_J - \gamma_J^2 > 0 \tag{A.103}$$

On solving this game, we obtain the following equilibrium prices:

$$p_{1I}^{H*} = p_{2I}^{H*} = \frac{\delta_I}{2} \tag{A.104}$$

$$p_{1I}^{L*} = p_{2I}^{L*} = \frac{\delta_I(4t_I t_J - \gamma_J^2) - 2t_I \gamma_J \delta_J}{8t_I t_J - \gamma_J^2} \tag{A.105}$$

$$p_{1J}^{*} = p_{2J}^{*} = \frac{t_J(\gamma_J \delta_I + 4t_I \delta_J)}{8t_I t_J - \gamma_J^2}, \tag{A.106}$$

and the following equilibrium market shares:

$$\alpha_{1I}^{H*} = \alpha_{2I}^{H*} = \frac{\delta_I}{4t_I} \tag{A.107}$$

$$\alpha_{1I}^{L*} = \alpha_{2I}^{L*} = \frac{2t_J \delta_I + \gamma_J \delta_J}{8t_I t_J - \gamma_J^2} \tag{A.108}$$

$$\alpha_{1J}^{*} = \alpha_{2J}^{*} = \frac{\gamma_J \delta_I + 4t_I \gamma_J}{8t_I t_J - \gamma_J^2} \tag{A.109}$$

Note that by definition, $0 \leqslant \alpha_{kI}^{H*} \leqslant \frac{1}{2}$, $0 \leqslant \alpha_{kI}^{L*} \leqslant \frac{1}{2}$, and $0 \leqslant \alpha_{kJ}^{*} \leqslant 1$ for $k \in \{1, 2\}$. Because both consumers and advertisers can multi-home and $\alpha_{1I}^{H*} = \alpha_{2I}^{H*}$, $\alpha_{1I}^{L*} = \alpha_{2I}^{L*}$, and $\alpha_{1J}^{*} = \alpha_{2J}^{*}$, we have $\alpha_{kI}^{H*} \geqslant \frac{1}{4}$, $\alpha_{kI}^{L*} \geqslant \frac{1}{4}$, and $\alpha_{kJ}^{*} \geqslant \frac{1}{2}$ for $k \in \{1, 2\}$. Finally, according to the IC constraints, we have:

$$p_{1I}^{L*} < p_{1I}^{H*} < p_{1I}^{L*} + \gamma_I^H \alpha_{1J}^{*} \tag{A.110}$$

The left-hand side inequality is equivalent to $\frac{\gamma_J(\gamma_J\delta_I+4t_I\delta_J)}{2(8t_It_J-\gamma_J^2)} > 0$, which always holds as long as (A.103) is satisfied. The right-hand side inequality is equivalent to $2\gamma_I^H > \gamma_J$, which holds given that (A.103) is satisfied. When all of these conditions are satisfied, the equilibrium profits of each platform are given by:

$$\Pi_1^{mm2*} = \Pi_2^{mm2*} = \frac{16t_I(t_J\delta_I^2+t_I\delta_J^2)+\gamma_J\delta_I(8t_I\delta_J-\gamma_J\delta_I)}{8t_I(8t_It_J-\gamma_J^2)} \tag{A.111}$$

□

Lemma A.7. *If Platform 1 sets a uniform price while Platform 2 sets customized prices for the two segments of consumers, their equilibrium profits are* $\Pi_1^{mm3*} = \frac{4\delta_I(t_J\delta_I+\gamma_J\delta_J)+2\delta_J(2t_I\delta_J-\gamma_I^H\delta_I)}{16t_It_J-(\gamma_I^H-2\gamma_J)^2}$ *and* $\Pi_2^{mm3*} = \frac{16t_I(t_J\delta_I^2+t_I\delta_J^2)+\gamma_J\delta_I(8t_I\delta_J-\gamma_J\delta_I)}{8t_I(8t_It_J-\gamma_J^2)}$.

Proof. Following a procedure similar to that used in the previous two lemmas, we derive the profits of the two platforms as follows:

$$\Pi_1^{mm3} = \alpha_{1I}\cdot p_{1I} + \alpha_{1J}\cdot p_{1J} \tag{A.112}$$

$$\Pi_2^{mm3} = \alpha_{2I}^H\cdot p_{2I}^H + \alpha_{2I}^L\cdot p_{2I}^L + \alpha_{2J}\cdot p_{2J} \tag{A.113}$$

where

$$\alpha_{1I} = \frac{2t_J(\delta_I-p_{1I})-\gamma_I^H(\delta_J-p_{1J})}{2t_It_J+\gamma_I^H\gamma_J} \tag{A.114}$$

$$\alpha_{2I}^L = \frac{\delta_I-p_{2I}^L}{2t_I} \tag{A.115}$$

$$\alpha_{2I}^H = \frac{\delta_I-p_{2I}^H}{2t_I} \tag{A.116}$$

$$\alpha_{1J} = \frac{2\gamma_J(\delta_I-p_{1I})+2t_I(\delta_J-p_{1J})}{2t_It_J+\gamma_I^H\gamma_J} \tag{A.117}$$

$$\alpha_{2J} = \frac{\gamma_J(\delta_I-p_{2I}^L)+2t_I(\delta_J-p_{2J})}{2t_It_J} \tag{A.118}$$

Both platforms' profit functions are concave if and only if the following conditions hold:

$$16t_It_J-(\gamma_I^H-2\gamma_J)^2 > 0 \tag{A.119}$$

$$8t_It_J-\gamma_J^2 > 0 \tag{A.120}$$

On solving this game, we obtain the following equilibrium prices:

$$p_{1I}^{*} = \frac{2(\gamma_I^H - 2\gamma_J)\gamma_J\delta_I + t_I\{8t_J\delta_I - 2(\gamma_I^H + 2\gamma_J)\delta_J\}}{16t_It_J - (\gamma_I^H - 2\gamma_J)^2} \tag{A.121}$$

$$p_{2I}^{L*} = \frac{\delta_I(4t_It_J - \gamma_J^2) - 2t_I\gamma_J\delta_J}{8t_It_J - \gamma_J^2} \tag{A.122}$$

$$p_{2I}^{H*} = \frac{\delta_I}{2} \tag{A.123}$$

$$p_{1J}^{*} = \frac{(2\gamma_J - \gamma_I^H)\gamma_I^H\delta_J + 2t_J\{4t_I\delta_J - (\gamma_I^H + 2\gamma_J)\delta_I\}}{16t_It_J - (\gamma_I^H - 2\gamma_J)^2} \tag{A.124}$$

$$p_{2J}^{*} = \frac{t_J(\gamma_J\delta_I + 4t_I\delta_J)}{8t_It_J - \gamma_J^2}, \tag{A.125}$$

and the following equilibrium market shares:

$$\alpha_{1I}^{*} = \frac{8t_J\delta_I - 2(\gamma_I^H - 2\gamma_J)\delta_J}{16t_It_J - (\gamma_I^H - 2\gamma_J)^2} \tag{A.126}$$

$$\alpha_{2I}^{L*} = \frac{2t_J\delta_I + \gamma_J\delta_J}{8t_It_J - \gamma_J^2} \tag{A.127}$$

$$\alpha_{2I}^{H*} = \frac{\delta_I}{4t_I} \tag{A.128}$$

$$\alpha_{1J}^{*} = \frac{2(2\gamma_J - \gamma_I^H)\delta_I + 8t_I\delta_J}{16t_It_J - (\gamma_I^H - 2\gamma_J)^2} \tag{A.129}$$

$$\alpha_{2J}^{*} = \frac{\gamma_J\delta_I + 4t_I\delta_J}{8t_It_J - \gamma_J^2} \tag{A.130}$$

Note that by definition, $0 \leqslant \alpha_{1I}^{*} \leqslant 1$, $0 \leqslant \alpha_{2I}^{H*} \leqslant \frac{1}{2}$, $0 \leqslant \alpha_{2I}^{L*} \leqslant \frac{1}{2}$, and $0 \leqslant \alpha_{kJ}^{*} \leqslant 1$ for $k \in \{1, 2\}$. Furthermore, $\alpha_{1I}^{*} + \alpha_{2I}^{L*} + \alpha_{2I}^{H*} \geqslant 1$ and $\alpha_{1J}^{*} + \alpha_{2J}^{*} \geqslant 1$ because both consumers and advertisers can multi-home. Note that the IC constraints are given by $p_{1I}^{L*} < p_{1I}^{H*} < p_{1I}^{L*} + \gamma_I^H\alpha_{1J}$. The first inequality is satisfied when (A.120) holds and the second inequality is equivalent to $2\gamma_I^H > \gamma_J$ when (A.120) holds. Then, the equilibrium profits of the two platforms are given by

$$\Pi_1^{mm3*} = \frac{4\delta_I(t_J\delta_I + \gamma_J\delta_J) + 2\delta_J(2t_I\delta_J - \gamma_I^H\delta_I)}{16t_It_J - (\gamma_I^H - 2\gamma_J)^2)} \tag{A.131}$$

$$\Pi_2^{mm3*} = \frac{16t_I(t_J\delta_I^2 + t_I\delta_J^2) + \gamma_J\delta_I(8t_I\delta_J - \gamma_J\delta_I)}{8t_I(8t_It_J - \gamma_J^2)} \tag{A.132}$$

□

Lemma A.8. *Define the parameter space Θ^{mm} as the pair (γ_I^H, γ_J) satisfying second-order conditions and demand conditions of the three subgames. Then in Θ^{mm}, neither platform completely abandons advertisers and serves only consumers.*

Proof. We prove the lemma in four parts (see also Lemma A.4). In Part (a), we specify the parameter space Θ^{mm}. Then in Part (b) and Part (c), we derive the equilibrium profits of two subgames where one platform chooses to completely abandon advertisers. Finally, in Part (d), we prove that such a one-sided strategy is not possible in equilibrium.

Part (a). Based on Lemmas A.5-A.7, we define the parameter space Θ^{mm} as one in which the following conditions hold:

1. Both platforms' profits are concave with respect to their own prices across all three regions. Thus, (A.89), (A.103), (A.119), and (A.120) are satisfied in Θ^{mm}.

2. The demand conditions as well as the multi-homing conditions are satisfied across all three subgames.

Part (b). In this part, we derive the equilibrium profits where Platform 1 abandons advertisers while Platform 2 serves both sides and sets a uniform price for both segments of consumers. Since there is no competition on the advertiser side, Platform 2 caters to all advertisers with the marginal advertiser being located at $\theta_J = 0$ and the price for advertisers being $p_{2J} = v_J - t_J + \alpha_{2I}\gamma_J$.

Because the two platforms compete on the consumer side of the market and because consumers can multi-home, there are two marginal consumers in each segment. The first marginal consumer in each segment is indifferent between joining Platform

1 and multi-homing, and they are as follows:

$$\theta_{13I}^{L} = 1 + \frac{p_{2I} - \delta_I}{t_I} \tag{A.133}$$

$$\theta_{13I}^{H} = 1 + \frac{\gamma_I^H \alpha_{2J} + p_{2I} - \delta_I}{t_I} \tag{A.134}$$

The second marginal consumer in each segment is indifferent between joining Platform 2 and multi-homing, and are given by:

$$\theta_{23I}^{L} = \frac{\delta_I - p_{1I}}{t_I} \tag{A.135}$$

$$\theta_{23I}^{H} = \frac{\delta_I - p_{1I}}{t_I} \tag{A.136}$$

Based on this, in equilibrium, we have $\alpha_{1I} = \frac{1}{2}\theta_{23I}^L + \frac{1}{2}\theta_{23I}^H$, $\alpha_{2I} = \frac{1}{2}(1 - \theta_{13I}^L) + \frac{1}{2}(1 - \theta_{13I}^H)$, $\alpha_{2J} = 1$. Then, the profit functions of the two platforms are:

$$\Pi_1^{md1} = \alpha_{1I} \cdot p_{1I} \tag{A.137}$$

$$\Pi_2^{md1} = \alpha_{2I} \cdot p_{2I} + \alpha_{2J} \cdot p_{2J} \tag{A.138}$$

Both profit functions are concave with respect to their own price for consumers, since $\frac{\partial^2 \Pi_1^{md1}}{\partial p_{1I}^2} = \frac{\partial^2 \Pi_2^{md1}}{\partial p_{2I}^2} = -\frac{2}{t_I} < 0$. On solving the game, we find that the equilibrium prices are:

$$p_{1I}^* = \frac{\delta_I}{2} \tag{A.139}$$

$$p_{2I}^* = \frac{2\delta_I - 2\gamma_J - \gamma_I^H}{4} \tag{A.140}$$

and the equilibrium profits are,

$$\Pi_1^{md1*} = \frac{\delta_I^2}{4t_I} \tag{A.141}$$

$$\Pi_2^{md1*} = v_J - t_J + \frac{(\gamma_I^H - 2\gamma_J - 2\delta_I)^2}{16t_I}, \tag{A.142}$$

where the superscript "md1" represents the first subgame of the one-sided market strategy in the multi-homing model.

Part (c). Here we derive the equilibrium profits when Platform 1 abandons advertisers while Platform 2 serves both sides of the market and sets a customized price for each segment of consumers. As in Part (a), we have $\theta_J = 0$. Because advertisers can only reach low type consumers, the price Platform 2 charges advertisers is given by $p_{2J} = v_J - t_J + \alpha_{2I}^L \gamma_J$.

On the consumer side of the market, the marginal consumer in each segment (H and L-types), who is indifferent between joining Platform 1 and multi-homing is given by:

$$\theta_{13I}^L = 1 + \frac{p_{2I}^L - \delta_I}{t_I} \tag{A.143}$$

$$\theta_{13I}^H = 1 + \frac{p_{2I}^H - \delta_I}{t_I} \tag{A.144}$$

Likewise, the marginal consumer in each segment who is indifferent between joining Platform 2 and multi-homing is given by:

$$\theta_{23I}^L = \frac{\delta_I - p_{1I}}{t_I} \tag{A.145}$$

$$\theta_{23I}^H = \frac{\delta_I - p_{1I}}{t_I} \tag{A.146}$$

Based on this, in equilibrium, we have $\alpha_{1I} = \frac{1}{2}\theta_{23I}^L + \frac{1}{2}\theta_{23I}^H$, $\alpha_{2I}^L = \frac{1}{2}(1 - \theta_{13I}^L)$, $\alpha_{2I}^H = \frac{1}{2}(1 - \theta_{13I}^H)$ and $\alpha_{2J} = 1$. The profit function of each platform is:

$$\Pi_1^{md2} = \alpha_{1I} \cdot p_{1I} \tag{A.147}$$

$$\Pi_2^{md2} = \alpha_{2I}^L \cdot p_{2I}^L + \alpha_{2I}^H \cdot p_{2I}^H + \alpha_{2J} \cdot p_{2J} \tag{A.148}$$

Both profit functions are concave with respect to their own price(s) for consumers, since $\frac{\partial^2 \Pi_1^{md2}}{\partial p_{1I}^2} = -\frac{2}{t_I} < 0$ and the hessian of Π_2^{md2} is

$$H(\Pi_2^{md2}) = \begin{bmatrix} \frac{\partial^2 \Pi_2^{md2}}{\partial {p_{2I}^L}^2} & \frac{\partial^2 \Pi_2^{md2}}{\partial p_{2I}^L \partial p_{2I}^H} \\ \frac{\partial^2 \Pi_2^{md2}}{\partial p_{2I}^H \partial p_{2I}^L} & \frac{\partial^2 \Pi_2^{md2}}{\partial {p_{2I}^H}^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{t_I} & 0 \\ 0 & -\frac{1}{t_I} \end{bmatrix} \tag{A.149}$$

On solving the game, we obtain the following equilibrium prices:

$$p_{1I}^* = \frac{\delta_I}{2} \tag{A.150}$$

$$p_{2I}^{L*} = \frac{\delta_I - \gamma_J}{2} \tag{A.151}$$

$$p_{2I}^{H*} = \frac{\delta_I}{2} \tag{A.152}$$

Then the IC constraints, $p_{2I}^{L*} < p_{2I}^{H*} < p_{2I}^{L*} + \gamma_I^H \alpha_{2J}$, are satisfied if and only if $\gamma_J < 2\gamma_I^H$. Finally, the equilibrium profits are given by:

$$\Pi_1^{md2*} = \frac{\delta_I^2}{4t_I} \tag{A.153}$$

$$\Pi_2^{md2*} = v_J - t_J + \frac{\gamma_J^2 + 2\gamma_J\delta_I + 2\delta_I^2}{8t_I} \tag{A.154}$$

Part (d). In this part, we prove that Platform 1 never abandons the advertiser side in equilibrium. To see this, first note that from (A.94) and (A.141), we have

$$\Pi_1^{mm1*} - \Pi_1^{md1*} = \frac{(\gamma_I^H\delta_I - 2\gamma_J\delta_I - 4t_I\delta_J)^2}{4t_I(16t_It_J - (\gamma_I^H - 2\gamma_J)^2)} > 0, \tag{A.155}$$

which implies that a platform has no incentive to adopt the one-sided market strategy when its competitor operates on two sides and sets a uniform price for consumers.

Second, from (A.111) and (A.153), we have

$$\Pi_1^{mm2*} - \Pi_1^{md2*} = \frac{(\gamma_J\delta_I + 4t_I\delta_J)^2}{8t_I(8t_It_J - \gamma_J^2)} > 0, \tag{A.156}$$

which implies that a platform has no incentive to adopt the one-sided market strategy when its competitor operates in two sides and sets customized prices for the two segments of consumers.

Therefore, in Θ^{mm}, no platform abandons the advertiser side.

□

Lemma A.9. *If $t_I \leqslant \delta_I \leqslant 2t_I$ and $t_J \leqslant \delta_J \leqslant 2t_J$, then $\Theta^{mm} \neq \varnothing$.*

Proof. To prove that Θ^{mm} is a non-empty set, it suffices to show that there exists a point that satisfies all conditions pertaining to Θ^{mm}. Consider a point $(\gamma_I^H, \gamma_J) = (\epsilon, \epsilon)$, where ϵ is a very small positive number. In Subgame 1 (where both platforms use the uniform pricing strategy), at this point, as ϵ approaches to zero, we have:

1. $\frac{1}{2} \leqslant \alpha_{kI}^* \leqslant 1$ $(k = 1, 2)$ is equivalent to $t_I \leqslant \delta_I \leqslant 2t_I$.

2. $\frac{1}{2} \leqslant \alpha_{kJ}^* \leqslant 1$ $(k = 1, 2)$ is equivalent to $t_J \leqslant \delta_J \leqslant 2t_J$.

3. (A.89) is equivalent to $\frac{4}{t_I t_J} > 0$, which always holds.

In Subgame 2 (where both platforms use the customized pricing strategy), at $(\gamma_I^H, \gamma_J) = (\epsilon, \epsilon)$, as ϵ approaches to zero, we have:

1. $\frac{1}{4} \leqslant \alpha_{kI}^{H*} \leqslant \frac{1}{2}$ $(k = 1, 2)$ is equivalent to $t_I \leqslant \delta_I \leqslant 2t_I$.

2. $\frac{1}{4} \leqslant \alpha_{kI}^{L*} \leqslant \frac{1}{2}$ $(k = 1, 2)$ is equivalent to $t_I \leqslant \delta_I \leqslant 2t_I$.

3. $\frac{1}{2} \leqslant \alpha_{kJ}^{*} \leqslant 1$ $(k = 1, 2)$ is equivalent to $t_J \leqslant \delta_J \leqslant 2t_J$.

4. (A.103) is equivalent to $\frac{2}{t_I t_J} > 0$, which always holds.

In Subgame 3 (where the two platforms use distinct pricing strategies), at $(\gamma_I^H, \gamma_J) = (\epsilon, \epsilon)$, as ϵ approaches to zero, we find:

1. $0 \leqslant \alpha_{1I}^{*} \leqslant 1$ is equivalent to $\delta_I \leqslant 2t_I$.

2. Both $0 \leqslant \alpha_{2I}^{H*} \leqslant \frac{1}{2}$ and $0 \leqslant \alpha_{2I}^{L*} \leqslant \frac{1}{2}$ are equivalent to $\delta_I \leqslant 2t_I$.

3. $0 \leqslant \alpha_{kJ}^{*} \leqslant 1$ $(k = 1, 2)$ is equivalent to $\delta_J \leqslant 2t_J$.

4. $\alpha_{1I}^{*} + \alpha_{2I}^{L*} + \alpha_{2I}^{H*} \geqslant 1$ is equivalent to $\delta_I \geqslant t_I$.

5. $\alpha_{1J}^{*} + \alpha_{2J}^{*} \geqslant 1$ is equivalent to $\delta_J \geqslant t_J$.

6. (A.119), and (A.120) are respectively equivalent to $\frac{4}{t_I t_J} > 0$ and $\frac{2}{t_I^2 t_J} > 0$, both of which always hold.

Therefore, if $t_I \leqslant \delta_I \leqslant 2t_I$ and $t_J \leqslant \delta_J \leqslant 2t_J$, we have $(\gamma_I^H, \gamma_J) = (\epsilon, \epsilon) \in \Theta^{mm}$ for a very small ϵ. This implies that Θ^{mm} is not an empty set. □

A.2.1 Proof of Proposition 2.3

Proof. We prove the two claims of the proposition in the following two parts.

Part (a). First, we show that the two platforms cannot use asymmetric pricing strategies in equilibrium. Suppose there exists an equilibrium where Platform 1 uses a uniform pricing strategy while Platform 2 uses a customized pricing strategy. This implies both $\Pi_1^{mm3*} \geqslant \Pi_1^{mm2*}$ and $\Pi_2^{mm3*} \geqslant \Pi_2^{mm1*}$. However, since by Lemmas A.5-A.7, $\Pi_1^{mm3*} = \Pi_2^{mm1*}$ and $\Pi_1^{mm2*} = \Pi_2^{mm3*}$ but $\Pi_1^{mm3*} \neq \Pi_2^{mm3*}$, both inequalities cannot simultaneously hold, which is a contradiction. Therefore, there does not exist such an asymmetric equilibrium.

Part (b). The existence of the symmetric customized pricing equilibrium can be shown by an example. Suppose $t_I = 1$, $t_J = 5$, $\delta_I = 1.5$, and $\delta_J = 7.5$. Since $t_I \leqslant \delta_I \leqslant 2t_I$ and $t_J \leqslant \delta_J \leqslant 2t_J$, by Lemma A.9, $\Theta^{mm} \neq \varnothing$ (i.e., the parameter space is a non-empty set). In fact, Θ^{mm} is given as $0 < \gamma_J \leqslant 0.63941$. Further, let $(\gamma_I^H, \gamma_J) = (0.5, 0.1) \in \Theta^{mm}$. For this pair, $\Pi_1^{mm2*} - \Pi_1^{mm3*} = 0.1096 > 0$, which implies that both firms will choose the customized pricing strategy when the competitor also does so. Moreover, the IC constraint for customized pricing ($\gamma_J < 2\gamma_I^H$) holds at this pair. Therefore, there exists an equilibrium where both firms simultaneously adopt a customized pricing strategy. □

A.2.2 Proof of Proposition 2.4

Proof. First, the condition for each symmetric equilibrium is given as $\Pi_2^{mm1*} \geqslant \Pi_2^{mm3*}$ (for the symmetric uniform pricing equilibrium) and $\Pi_1^{mm2*} \geqslant \Pi_1^{mm3*}$ (for the symmetric customized pricing equilibrium).

Note that $\Pi_2^{mm1*} \geqslant \Pi_2^{mm3*}$ is equivalent to $\Pi_1^{mm3*} \geqslant \Pi_1^{mm2*}$, since $\Pi_2^{mm1*} = \Pi_1^{mm3*}$ and $\Pi_2^{mm3*} = \Pi_1^{mm2*}$. In addition, these inequalities are equivalent to $\gamma_I^H \leqslant$

γ_I^{H00}, where

$$\gamma_I^{H00} \equiv \frac{1}{\gamma_J^2\delta_I^2 - 16t_I^2\delta_J^2 - 8t_I\delta_I(2t_J\delta_I + \gamma_J\delta_J)} \cdot$$

$$2\Big\{\gamma_J^3\delta_I^2 - 16t_I^2\delta_J(2t_J\delta_I + \gamma_J\delta_J) - 4t_I\gamma_J\delta_I(4t_J\delta_I + \gamma_J\delta_J) +$$

$$2\sqrt{t_I(\gamma_J\delta_I + 4t_I\delta_J)^2(-t_J\gamma_J^2\delta_I^2 + 2t_I(8t_J^2\delta_I^2 + 4t_J\gamma_J\delta_I\delta_J + \gamma_J^2\delta_J^2))}\Big\}$$

Therefore, when $\gamma_I^H \leqslant \gamma_I^{H00}$, the symmetric uniform pricing equilibrium is obtained but otherwise, the symmetric customized pricing equilibrium is obtained.

Finally, note that $\gamma_I^{H00} \geqslant \frac{\gamma_J}{2}$ holds for any $\gamma_J \in [0, 4\sqrt{t_I t_J}]$ and that in Θ^{mm}, $\gamma_J < 4\sqrt{t_I t_J}$ always holds by (A.103). This implies that whenever the customized pricing is an equilibrium strategy for both platforms (i.e., when $\gamma_I^H \geqslant \gamma_I^{H00}$), the IC constraints for customized pricing ($\gamma_I^H > \frac{\gamma_J}{2}$) also holds. □

A.3 Analysis of Multi-Homing Advertisers

Lemma A.10. *If both platforms set a uniform price for consumers, the equilibrium profits of each platform are* $\Pi_1^{sm1*} = \Pi_2^{sm1*} = \frac{1}{2}t_I + \frac{1}{16t_J}\{4\delta_J^2 - (\gamma_I^2 + \gamma_J^2 - 6\gamma_I\gamma_J)\}$, *where* $\gamma_I \equiv \frac{\gamma_I^H}{2}$.

Proof. Because consumers single-home, they only join one platform. As in Lemma A.1, λ-type consumer who is indifferent between the two platforms is given by

$$\theta_I^\lambda = \tfrac{1}{2t_I}(p_{2I} - p_{1I} + \gamma_I^\lambda(\alpha_{2J} - \alpha_{1J})) + \tfrac{1}{2} \tag{A.157}$$

and the 'average' consumer $\theta_I \equiv \frac{\theta_I^L + \theta_I^H}{2}$ is

$$\theta_I = \tfrac{1}{2t_I}(p_{2I} - p_{1I} + \gamma_I(\alpha_{2J} - \alpha_{1J})) + \tfrac{1}{2}, \tag{A.158}$$

where $\gamma_I \equiv \frac{\gamma_I^H}{2}$.

Because advertisers can multi-home, we have two marginal advertisers who are denoted by θ_{13J} and θ_{23J}. The marginal advertiser at θ_{13J} is indifferent between advertising only on Platform 1 and advertising in both platforms, and the marginal advertiser at θ_{23J} is indifferent between advertising only on Platform 2 and advertising in both platforms.

Advertiser's utility from advertising only on Platform 1 is:

$$U_{1J}(\theta) = v_J - t_J\theta + \gamma_J \alpha_{1I}^e - p_{1J}, \tag{A.159}$$

the utility from advertising only on Platform 2 is:

$$U_{2J}(\theta) = v_J - t_J(1-\theta) + \gamma_J \alpha_{2I}^e - p_{1J}, \tag{A.160}$$

and the utility from advertising on both platforms is:

$$U_{3J}(\theta) = v_J - t_J\theta + \gamma_J \alpha_{1I}^e - p_{1J} + \delta_J - t_J(1-\theta) + \gamma_J \alpha_{2I}^e - p_{1J} \tag{A.161}$$

$$= v_J - t_J + \gamma_J(\alpha_{1I}^e + \alpha_{2I}^e) - p_{1J} - p_{2J} + \delta_J, \tag{A.162}$$

By solving $U_{1J}(\theta) = U_{3J}(\theta)$ and $U_{2J}(\theta) = U_{3J}(\theta)$, we obtain:

$$\theta_{13J} = 1 - \tfrac{1}{t_J}(\delta_J + \gamma_J \cdot \alpha_{2I}^e - p_{2J}) \tag{A.163}$$

$$\theta_{23J} = \tfrac{1}{t_J}(\delta_J + \gamma_J \cdot \alpha_{1I}^e - p_{1J}) \tag{A.164}$$

To ensure the existence of a multi-homing advertiser, we assume $\theta_{13J} \leqslant \theta_{23J}$ and later derive the condition under which this assumption holds. This assumption implies that advertisers in the interval $[0, \theta_{13J})$ advertise only on Platform 1, advertisers in the interval $[\theta_{13J}, \theta_{23J}]$ advertise on both platforms, and advertisers in the interval $(\theta_{23J}, 1]$ advertise only on Platform 2.

Then in equilibrium, we have $\alpha_{1I}^e = \theta_I$, $\alpha_{1J}^e = \theta_{23J}$, $\alpha_{2I}^e = 1 - \theta_I$, and $\alpha_{2J}^e = 1 - \theta_{13J}$. These lead to the following system of demand:

$$\alpha_{1I} = \tfrac{1}{2} + \tfrac{1}{2(t_I t_J + \gamma_I \gamma_J)} \cdot [\gamma_I(p_{1J} - p_{2J}) - t_J(p_{1I} - p_{2I})] \tag{A.165}$$

$$\alpha_{2I} = \tfrac{1}{2} + \tfrac{1}{2(t_I t_J + \gamma_I \gamma_J)} \cdot [\gamma_I(p_{2J} - p_{1J}) - t_J(p_{2I} - p_{1I})] \tag{A.166}$$

$$\alpha_{1J} = \tfrac{1}{2t_J} \cdot (2\delta_J + \gamma_J - p_{1J}) - \tfrac{1}{2t_J(t_I t_J + \gamma_I \gamma_J)}[t_I t_J p_{1J} + \gamma_I \gamma_J p_{2J} + t_J \gamma_J (p_{1I} - p_{2I})] \tag{A.167}$$

$$\alpha_{2J} = \tfrac{1}{2t_J} \cdot (2\delta_J + \gamma_J - p_{2J}) - \tfrac{1}{2t_J(t_I t_J + \gamma_I \gamma_J)}[t_I t_J p_{2J} + \gamma_I \gamma_J p_{1J} + t_J \gamma_J (p_{2I} - p_{1I})] \tag{A.168}$$

Then each platform's profit function is given by:

$$\Pi_1^{sm1} = \alpha_{1I} \cdot p_{1I} + \alpha_{1J} \cdot p_{1J} \tag{A.169}$$

$$\Pi_2^{sm1} = \alpha_{2I} \cdot p_{2I} + \alpha_{2J} \cdot p_{2J} \tag{A.170}$$

The hessian of Π_1^{sm1} is

$$H(\Pi_1^{sm1}) = \begin{bmatrix} \frac{\partial^2 \Pi_1^{sm1}}{\partial p_{1I}{}^2} & \frac{\partial^2 \Pi_1^{sm1}}{\partial p_{1I} \partial p_{1J}} \\ \frac{\partial^2 \Pi_1^{sm1}}{\partial p_{1J} \partial p_{1I}} & \frac{\partial^2 \Pi_1^{sm1}}{\partial p_{1J}{}^2} \end{bmatrix} = \begin{bmatrix} -t_J & \frac{\gamma_I - \gamma_J}{2} \\ \frac{\gamma_I - \gamma_J}{2} & -\frac{2 t_I t_J + \gamma_I \gamma_J}{t_J} \end{bmatrix} \cdot \frac{1}{t_I t_J + \gamma_I \gamma_J} \tag{A.171}$$

The hessian of Platform 2's profit function is the same. The hessian needs to be negative definite so that each platform's profit function is concave. The corresponding sufficient and necessary condition is given by:

$$8 t_I t_J - (\gamma_I^2 + \gamma_J^2 - 6 \gamma_I \gamma_J) > 0 \tag{A.172}$$

Then by solving $\frac{\partial \Pi_1^{sm1}}{\partial p_{1I}} = 0$, $\frac{\partial \Pi_1^{sm1}}{\partial p_{1J}} = 0$, $\frac{\partial \Pi_2^{sm1}}{\partial p_{2I}} = 0$, and $\frac{\partial \Pi_2^{sm1}}{\partial p_{2J}} = 0$, we have the following equilibrium prices:

$$p_{1I}^* = p_{2I}^* = t_I - \frac{\gamma_J}{4 t_J}(-3\gamma_I + \gamma_J + 2\delta_J) \tag{A.173}$$

$$p_{1J}^* = p_{2J}^* = \frac{1}{4}(\gamma_I + \gamma_J + 2\delta_J). \tag{A.174}$$

Given this, the equilibrium market size of each platform is given as,

$$\alpha_{1I}^* = \alpha_{2I}^* = \frac{1}{2} \tag{A.175}$$

$$\alpha_{1J}^* = \alpha_{2J}^* = \frac{1}{4 t_J}(-\gamma_I + \gamma_J + 2\delta_J) \tag{A.176}$$

Note that by definition, $\alpha_{kJ}^* \leqslant 1$ should hold for $k \in \{1, 2\}$. In addition, multi-homing by advertisers implies $\alpha_{kJ}^* \geqslant \frac{1}{2}, \forall k$, since $\alpha_{1J}^* = \alpha_{2J}^*$. These conditions lead to:

$$2(t_J - \delta_J) + \frac{1}{2}\gamma_I^H \leqslant \gamma_J \leqslant 2(2t_J - \delta_J) + \frac{1}{2}\gamma_I^H. \tag{A.177}$$

Note that these conditions lead to $\theta_{13J} \leqslant \theta_{23J}$, implying some advertisers indeed multi-home in equilibrium. Finally, when both (A.172) and (A.177) hold, the equilibrium profits of each platform is given by:

$$\Pi_1^{sm1*} = \Pi_2^{sm1*} = \frac{1}{2} t_I + \frac{1}{16 t_J}\{4\delta_J^2 - (\gamma_I^2 + \gamma_J^2 - 6\gamma_I \gamma_J)\} \tag{A.178}$$

□

Lemma A.11. *If both platforms set customized prices for the two segments of consumers, the equilibrium profits of each platform is:* $\Pi_1^{sm2*} = \Pi_2^{sm2*} = \frac{1}{2}t_I + \frac{1}{64t_J}(16\delta_J^2 - \gamma_J^2)$.

Proof. Using a procedure similar to that used in Lemma A.10, we derive the profits of the two platforms as follows:

$$\Pi_1^{sm2} = \alpha_{1I}^H \cdot p_{1I}^H + \alpha_{1I}^L \cdot p_{1I}^L + \alpha_{1J} \cdot p_{1J} \tag{A.179}$$

$$\Pi_2^{sm2} = \alpha_{2I}^H \cdot p_{2I}^H + \alpha_{2I}^L \cdot p_{2I}^L + \alpha_{2J} \cdot p_{2J} \tag{A.180}$$

where

$$\alpha_{1I}^L = \frac{1}{4} + \frac{p_{2I}^L - p_{1I}^L}{4t_I} \tag{A.181}$$

$$\alpha_{1I}^H = \frac{1}{4} + \frac{p_{2I}^H - p_{1I}^H}{4t_I} \tag{A.182}$$

$$\alpha_{2I}^L = \frac{1}{4} + \frac{p_{1I}^L - p_{2I}^L}{4t_I} \tag{A.183}$$

$$\alpha_{2I}^H = \frac{1}{4} + \frac{p_{1I}^H - p_{2I}^H}{4t_I} \tag{A.184}$$

$$\alpha_{1J} = \frac{4\delta_J + \gamma_J - 4p_{1J}}{4t_J} - \frac{\gamma_J(p_{2I}^L - p_{1I}^L)}{4t_It_J} \tag{A.185}$$

$$\alpha_{2J} = \frac{4\delta_J + \gamma_J - 4p_{2J}}{4t_J} - \frac{\gamma_J(p_{1I}^L - p_{2I}^L)}{4t_It_J} \tag{A.186}$$

Both platforms' profit functions are concave if and only if

$$16t_It_J - \gamma_J^2 > 0 \tag{A.187}$$

Then the equilibrium prices for L-type consumers and advertisers are given by

$$p_{1I}^{L*} = p_{2I}^{L*} = t_I - \frac{\gamma_J}{8t_J} \cdot (\gamma_J + 4\delta_J) \tag{A.188}$$

$$p_{1J}^* = p_{2J}^* = \frac{1}{8} \cdot (\gamma_J + 4\delta_J). \tag{A.189}$$

In equilibrium, the demand for each platform in each market is given by $\alpha_{1I}^{H*} = \alpha_{2I}^{H*} = \alpha_{1I}^{L*} = \alpha_{2I}^{L*} = \frac{1}{4}$, and $\alpha_{1J}^* = \alpha_{2J}^* = \frac{1}{8t_J}(\gamma_J + 4\delta_J)$. Note that since $\alpha_{1J}^* = \alpha_{2J}^*$, multi-homing by advertisers implies that $\frac{1}{2} \leqslant \alpha_{kJ}^* \leqslant 1$ for $k \in \{1, 2\}$. This leads to the following condition:

$$4(t_J - \delta_J) \leqslant \gamma_J \leqslant 4(2t_J - \delta_J). \tag{A.190}$$

Finally, IC constraints require

$$p_{1I}^{L*} < p_{1I}^{H*} < p_{1I}^{L*} + \gamma_I^H \alpha_{1J} \tag{A.191}$$

The left-hand side inequality is equivalent to $\frac{\gamma_J(\gamma_J + 4\delta_J)}{8t_J} > 0$ and it always holds. The right-hand side inequality is equivalent to $\gamma_I^H > \gamma_J$. When these conditions hold, the equilibrium profits of the two platforms are given by

$$\Pi_1^{sm2*} = \Pi_2^{sm2*} = \tfrac{1}{2}t_I + \tfrac{1}{64t_J}(16\delta_J^2 - \gamma_J^2). \tag{A.192}$$

□

Lemma A.12. *If Platform 1 sets a uniform price for consumers while Platform 2 sets customized prices for consumers, their equilibrium profits are*

$$\begin{aligned}
\Pi_1^{sm3*} = & \frac{1}{t_J A^2}\Big\{294912 t_I^5 t_J^5 - 3072 t_I^4 t_J^4 (3{\gamma_I^H}^2 - 44\gamma_I^H\gamma_J + 26\gamma_J^2 + 16\gamma_I^H\delta_J - 16\gamma_J\delta_J - 48\delta_J^2) - \\
& t_I t_J \gamma_J^2 \big(32{\gamma_I^H}^5\delta_J + 4\gamma_J^4\delta_J(7\gamma_J + 16\delta_J) + 4\gamma_I^H\gamma_J^3(14\gamma_J^2 - 40\gamma_J\delta_J - 119\delta_J^2) - \\
& 2{\gamma_I^H}^4(16\gamma_J^2 + 71\gamma_J\delta_J + 48\delta_J^2) + {\gamma_I^H}^3\gamma_J(142\gamma_J^2 + 233\gamma_J\delta_J + 112\delta_J^2) + {\gamma_I^H}^2\gamma_J^2(-216\gamma_J^2 + 19\gamma_J\delta_J + 420\delta_J^2)\big) - \\
& 32 t_I^3 t_J^3 \big(48{\gamma_I^H}^3(2\gamma_J - \delta_J) + 8\gamma_I^H\gamma_J(121\gamma_J^2 - 82\gamma_J\delta_J - 176\delta_J^2) + {\gamma_I^H}^2(-682\gamma_J^2 + 304\gamma_J\delta_J + 128\delta_J^2) + \\
& \gamma_J^2(-217\gamma_J^2 + 400\gamma_J\delta_J + 1184\delta_J^2)\big) + t_I^2 t_J^2 \gamma_J\big(-64{\gamma_I^H}^4(4\gamma_J - \delta_J) + 4\gamma_I^H\gamma_J^2(577\gamma_J^2 - 680\gamma_J\delta_J - 2144\delta_J^2) + \\
& 16{\gamma_I^H}^3(90\gamma_J^2 + 31\gamma_J\delta_J - 24\delta_J^2) + 4\gamma_J^3(-49\gamma_J^2 + 264\gamma_J\delta_J + 740\delta_J^2) + \\
& {\gamma_I^H}^2\gamma_J(-3761\gamma_J^2 + 1104\gamma_J\delta_J + 3721\delta_J^2)\big) + \gamma_I^H\gamma_J^3\big(2{\gamma_I^H}^4\delta_J(\gamma_J + \delta_J) - 4\gamma_J^4\delta_J(\gamma_J + 2\delta_J) - \\
& {\gamma_I^H}^3\gamma_J(\gamma_J^2 + 9\gamma_J\delta_J 12\delta_J^2) + {\gamma_I^H}^2\gamma_J^2(4\gamma_J^2 + 11\gamma_J\delta_J + 13\delta_J^2) + \gamma_I^H(-4\gamma_J^5 + 14\gamma_J^3\delta_J^2)\big)\Big\}
\end{aligned} \tag{A.193}$$

$$
\begin{aligned}
\Pi_2^{sm3*} =& \frac{1}{t_J A^2}\Big\{294912 t_I^5 t_J^5 - 3072 t_I^4 t_J^4 \big(8{\gamma_I^H}^2 + 41\gamma_J^2 + 16\gamma_J\delta_J - 48\delta_J^2 - 8\gamma_I^H(7\gamma_J + 2\delta_J)\big) - \\
& 32 t_I^3 t_J^3 \big(25{\gamma_I^H}^4 - 4{\gamma_I^H}^3(83\gamma_J + 34\delta_J) + 2\gamma_J^2(281\gamma_J^2 + 244\gamma_J\delta_J - 520\delta_J^2) - 8\gamma_I^H\gamma_J(229\gamma_J^2 + 166\gamma_J\delta_J - 140\delta_J^2) + \\
& 4{\gamma_I^H}^2(309\gamma_J^2 + 254\gamma_J\delta_J + 4\delta_J^2)\big) + \gamma_I^H\gamma_J^2\big({\gamma_I^H}^5(\gamma_J + \delta_J)^2 + 4\gamma_J^5(\gamma_J^2 + 3\gamma_J\delta_J + 2\delta_J^2) - \\
& 2\gamma_I^H\gamma_J^4(6\gamma_J^2 + 14\gamma_J\delta_J + 5\delta_J^2) + {\gamma_I^H}^2\gamma_J^3(5\gamma_J^2 + 9\gamma_J\delta_J + 6\delta_J^2) - {\gamma_I^H}^4\gamma_J(5\gamma_J^2 + 12\gamma_J\delta_J + 7\delta_J^2) + \\
& {\gamma_I^H}^3\gamma_J^2(6\gamma_J^2 + 17\gamma_J\delta_J + 11\delta_J^2)\big) + t_I^2 t_J^2\big(192{\gamma_I^H}^5\gamma_J + 16\gamma_I^H\gamma_J^3(411\gamma_J^2 + 509\gamma_J\delta_J - 286\delta_J^2) - \\
& 64\gamma_J^4(15\gamma_J^2 + 19\gamma_J\delta_J - 32\delta_J^2) + {\gamma_I^H}^4(-945\gamma_J^2 - 1248\gamma_J\delta_J + 64\delta_J^2) - 4{\gamma_I^H}^2\gamma_J^2(2061\gamma_J^2 + 2762\gamma_J\delta_J + 372\delta_J^2) + \\
& 4{\gamma_I^H}^3\gamma_J(939\gamma_J^2 + 1342\gamma_J\delta_J + 416\delta_J^2)\big) + t_I t_J \gamma_J\big(12{\gamma_I^H}^6\gamma_J + 4{\gamma_I^H}^5(13\gamma_J^2 + 10\gamma_J\delta_J - 4\delta_J^2) \\
& + 16\gamma_J^5(\gamma_J^2 + 2\gamma_J\delta - \delta_J^2) - 4\gamma_I^H\gamma_J^4(72\gamma_J^2 + 145\gamma_J\delta_J + 6\delta_J^2) + 4{\gamma_I^H}^2\gamma_J^3(141\gamma_J^2 + 273\gamma_J\delta_J + 109\delta_J^2) \\
& + {\gamma_I^H}^4\gamma_J(-22\gamma_J^2 + 55\gamma_J\delta_J + 128\delta_J^2) - {\gamma_I^H}^3\gamma_J^2(280\gamma_J^2 + 639\gamma_J\delta_J + 532\delta_J^2)\big)\Big\} \qquad \text{(A.194)}
\end{aligned}
$$

where $A \equiv 768 t_I^2 t_J^2 - 8 t_I t_J (2{\gamma_I^H}^2 - 16\gamma_I^H\gamma_J + 13\gamma_J^2) + \gamma_J(2{\gamma_I^H}^3 - 3{\gamma_I^H}^2\gamma_J - 7\gamma_I^H\gamma_J^2 + 2\gamma_J^3)$.

Proof. Let p_{1I}, p_{2I}^H, p_{2I}^L, p_{1J}, and p_{2J} denote the prices charged to consumers of Platform 1, H-type consumers of Platform 2, L-type consumers of Platform 2, advertisers of Platform 1, and advertisers of Platform 2, respectively. Then, using a similar procedure as in the previous two lemmas, we derive the profits of the two platforms as follows:

$$\Pi_1^{sm3} = \alpha_{1I} \cdot p_{1I} + \alpha_{1J} \cdot p_{1J} \qquad \text{(A.195)}$$

$$\Pi_2^{sm3} = \alpha_{2I}^H \cdot p_{2I}^H + \alpha_{2I}^L \cdot p_{2I}^L + \alpha_{2J} \cdot p_{2J} \qquad \text{(A.196)}$$

where

$$\alpha_{1I} = \frac{t_J(p_{2I}^H + p_{2I}^L - 2p_{1I} + 2t_I) + \gamma_I^H(p_{1J} - \delta_J)}{4t_I t_J + \gamma_I^H\gamma_J} \qquad \text{(A.197)}$$

$$\alpha_{2I}^L = \frac{1}{4} + \frac{p_{1I} - p_{2I}^L}{4t_I} \qquad \text{(A.198)}$$

$$\alpha_{2I}^H = \frac{4t_I t_J(p_{1I} - p_{2I}^H + t_I) - \gamma_I^H\{t_I(4p_{1J} - 3\gamma_J - 4\delta_J) - \gamma_J(p_{1I} - p_{2I}^L)\}}{4t_I(4t_I + \gamma_I^H + \gamma_J)} \qquad \text{(A.199)}$$

$$\alpha_{1J} = \frac{\gamma_J(p_{2I}^H + p_{2I}^L - 2p_{1I} + 2t_I) - 4t_I(p_{1J} - \delta_J)}{4t_I t_J + \gamma_I^H\gamma_J} \qquad \text{(A.200)}$$

$$\alpha_{2J} = \frac{1}{4} + \frac{4t_I(-p_{2J} + \delta_J) + \gamma_J(p_{1I} - p_{2I}^L)}{4t_I t_J} \qquad \text{(A.201)}$$

Both platforms' profit functions are concave if and only if the following conditions hold:

$$32t_It_J - (\gamma_I^H - 2\gamma_J)^2 > 0 \tag{A.202}$$

$$64t_I^2t_J^2 + \gamma_I^H\gamma_J(16t_It_J - \gamma_I^H\gamma_J) > 0 \tag{A.203}$$

$$64t_I^2t_J^2 + 4t_It_J\gamma_J(4\gamma_I^H - \gamma_J) - \gamma_I^H\gamma_J^2(\gamma_I^H + \gamma_J) > 0. \tag{A.204}$$

Then the equilibrium prices are given by

$$p_{1I}^* = \frac{1}{t_JA}\Big\{768t_I^3t_J^3 + 8t_I^2t_J^2(28\gamma_I^H\gamma_J - 31\gamma_J^2 - 8\gamma_I^H\delta_J - 40\gamma_J\delta_J) - \gamma_I^H\gamma_J^3(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + 2\delta_J)) +$$
$$t_It_J\gamma_J(8{\gamma_I^H}^2(2\gamma_J + \delta_J) + 2\gamma_J^2(7\gamma_J + 16\delta_J) - \gamma_I^H\gamma_J(47\gamma_J + 44\delta_J)\Big\} \tag{A.205}$$

$$p_{1J}^* = \frac{1}{A}\Big\{96t_I^2t_J^2(\gamma_I^H + 2\gamma_J + 4\delta_J) + \gamma_I^H\gamma_J(-\gamma_I^H\gamma_J^2 + {\gamma_I^H}^2\delta_J - \gamma_J^2(2\gamma_J + 5\delta_J)) + t_It_J(16{\gamma_I^H}^2(\gamma_J - \delta_J) -$$
$$2\gamma_J^2(7\gamma_J + 18\delta_J) + \gamma_I^H\gamma_J(25\gamma_J + 56\delta_J))\Big\} \tag{A.206}$$

$$p_{2I}^{L}{}^* = \frac{1}{t_JA}\Big\{768t_I^3t_J^3 - 8t_I^2t_J^2({\gamma_I^H}^2 - 34\gamma_I^H\gamma_J + 28\gamma_J^2 + 4\gamma_I^H\delta_J + 44\gamma_J\delta_J) - \gamma_I^H\gamma_J^3(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + \delta_J)) +$$
$$+ t_It_J\gamma_J(-6{\gamma_I^H}^3 - 18\gamma_I^H\gamma_J(3\gamma_J + 4\delta_J) + 8\gamma_J^2(2\gamma_J + 5\delta_J) + {\gamma_I^H}^2(29\gamma_J + 28\delta_J))\Big\} \tag{A.207}$$

$$p_{2I}^{H}{}^* = \frac{4t_It_J + \gamma_I^H\gamma_J}{t_JA}\Big\{192t_I^2t_J^2 - 2t_It_J(7{\gamma_I^H}^2 - 22\gamma_I^H\gamma_J + 22\gamma_J^2 - 20\gamma_I^H\delta_J + 20\gamma_J\delta_J) + (\gamma_I^H - \gamma_J)\gamma_J(\gamma_I^H(\gamma_J + \delta_J) -$$
$$2\gamma_J(\gamma_J + 2\delta_J))\Big\} \tag{A.208}$$

$$p_{2J}^* = \frac{1}{A}\Big\{96t_I^2t_J^2(\gamma_J + 4\delta_J) + {\gamma_I^H}^2\gamma_J(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + \delta_J)) - t_It_J(16\gamma_J^2(\gamma_J + 3\delta_J) - 10\gamma_I^H\gamma_J(\gamma_J + 6\delta_J)$$
$$+ {\gamma_I^H}^2(\gamma_J + 8\delta_J)\Big\} \tag{A.209}$$

where $A = 768t_I^2t_J^2 - 8t_It_J(2{\gamma_I^H}^2 - 16\gamma_I^H\gamma_J + 13\gamma_J^2) + \gamma_J(2{\gamma_I^H}^3 - 3{\gamma_I^H}^2\gamma_J - 7\gamma_I^H\gamma_J^2 + 2\gamma_J^3)$.

At the equilibrium prices, the demand for each platform in both markets are given by

$$\alpha_{1I}^* = \frac{1}{A}\Big\{384t_I^2t_J^2 + 4t_It_J(16\gamma_I^H\gamma_J - 7\gamma_J^2 - 8\gamma_I^H\delta_J + 8\gamma_J\delta_J) - \gamma_J(-2{\gamma_I^H}^2\delta_J + \gamma_J^2\delta_J + \gamma_I^H\gamma_J(2\gamma_J + \delta_J)\Big\} \tag{A.210}$$

$$\alpha_{1J}^* = \frac{1}{t_JA}\Big\{-96t_I^2t_J^2(\gamma_I^H - 2(\gamma_J + 2\delta_J)) + \gamma_I^H\gamma_J^2(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + 2\delta_J)) +$$
$$t_It_J\gamma_J(-16{\gamma_I^H}^2 - 2\gamma_J(7\gamma_J + 18\delta_J) + \gamma_I^H(39\gamma_J + 40\delta_J))\Big\} \tag{A.211}$$

$$\alpha_{2I}^{H\,*} = \frac{1}{A}\Big\{192t_I^2t_J^2 - 2t_It_J(7{\gamma_I^H}^2 - 22\gamma_I^H\gamma_J + 22\gamma_J^2 - 20\gamma_I^H\delta_J + 20\gamma_J\delta_J) + (\gamma_I^H - \gamma_J)\gamma_J(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + 2\delta_J))\Big\} \tag{A.212}$$

$$\alpha_{2I}^{L\,*} = \frac{1}{A}\Big\{192t_I^2t_J^2 - 2t_It_J({\gamma_I^H}^2 - 10\gamma_I^H\gamma_J + 16\gamma_J^2 + 4\gamma_I^H\delta_J - 4\gamma_J\delta_J) + \gamma_J(2{\gamma_I^H}^3 + 7\gamma_I^H\gamma_J\delta_J - 2\gamma_J^2\delta_J - {\gamma_I^H}^2(4\gamma_J + 5\delta_J))\Big\} \tag{A.213}$$

$$\alpha_{2J}^{*} = \frac{1}{t_JA}\Big\{96t_I^2t_J^2(\gamma_J + 4\delta_J) + {\gamma_I^H}^2\gamma_J(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + 2\delta_J)) - t_It_J(16\gamma_J^2(\gamma_J + 3\delta_J) - 10\gamma_I^H\gamma_J(\gamma_J + 6\delta_J) + {\gamma_I^H}^2(\gamma_J + 8\delta_J))\Big\} \tag{A.214}$$

Note that the demand has to be properly bounded in equilibrium: $0 \leqslant \alpha_{1I}^* \leqslant 1$, $0 \leqslant \alpha_{1J}^* \leqslant 1$, $0 \leqslant \alpha_{2I}^{L\,*} \leqslant \frac{1}{2}$, $0 \leqslant \alpha_{2I}^{H\,*} \leqslant \frac{1}{2}$, and $0 \leqslant \alpha_{2J}^* \leqslant 1$. In addition, because advertisers can multi-home, we have $\alpha_{1J}^* + \alpha_{2J}^* \geqslant 1$.

Next, given the equilibrium solution, the IC constraints also need to be satisfied: $p_{2I}^{L*} < p_{2I}^{H*} < p_{2I}^{L*} + \gamma_I^H\alpha_{2J}$. On substituting equilibrium solutions into these inequalities, we have

$$\frac{1}{t_JA}\Big\{{\gamma_I^H}^2\gamma_J^2(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + 2\delta_J)) - 48t_I^2t_J^2({\gamma_I^H}^2 - 2\gamma_I^H(\gamma_J + 2\delta_J) - \gamma_J(\gamma_J + 4\delta_J)) + t_It_J\gamma_J(-8{\gamma_I^H}^3 - 2\gamma_I^H\gamma_J(\gamma_J - 6\delta_J) - 8\gamma_J^2(\gamma_J + 3\delta_J) + {\gamma_I^H}^2(19\gamma_J + 16\delta_J))\Big\} > 0 \tag{A.215}$$

$$\frac{1}{t_JA}\Big\{48t_I^2t_J^2(\gamma_I^H - \gamma_J)(\gamma_I^H + \gamma_J + 4\delta_J) + {\gamma_I^H}^2(\gamma_I^H - \gamma_J)\gamma_J(\gamma_I^H(\gamma_J + \delta_J) - 2\gamma_J(\gamma_J + 2\delta_J)) + t_It_J({\gamma_I^H}^3(7\gamma_J - 8\delta_J) + 8\gamma_J^3(\gamma_J + 3\delta_J) - 2\gamma_I^H\gamma_J^2(7\gamma_J + 30\delta_J) + {\gamma_I^H}^2\gamma_J(-9\gamma_J + 44\delta_J))\Big\} > 0 \tag{A.216}$$

Finally, when all of these conditions are satisfied, given the equilibrium demand and prices, each platform's equilibrium profits are calculated as in (A.193) and (A.194).

□

Lemma A.13. *Define the parameter space* Θ^{sm} *as the pair* (γ_I^H, γ_J) *satisfying second-order conditions and demand conditions of the three subgames as well as* $\underline{\gamma_J} \leqslant \gamma_J \leqslant$

$\overline{\gamma_J}$ *where*

$$\underline{\gamma_J} \equiv \max\left\{0, \frac{48t_It_J+27t_I\gamma_I^H+8t_J\gamma_I^H-6\sqrt{2}\sqrt{t_I(4t_J({\gamma_I^H}^2+4{\delta_J}^2)+t_I(32t_J^2+24_J\gamma_I^H+9({\gamma_I^H}^2+2\delta_J^2)))}}{2(9t_I+8t_J)}\right\}$$

$$\overline{\gamma_J} \equiv \min\left\{\frac{48t_It_J+27t_I\gamma_I^H+8t_J\gamma_I^H+6\sqrt{2}\sqrt{t_I(4t_J({\gamma_I^H}^2+4{\delta_J}^2)+t_I(32t_J^2+24_J\gamma_I^H+9({\gamma_I^H}^2+2\delta_J^2)))}}{2(9t_I+8t_J)},\right.$$

$$\left.\frac{12\sqrt{4t_It_J+\sqrt{16t_I^2t_J^2+9t_I^2\delta_J^2+8t_It_J\delta_J^2}}}{9t_I+8t_J}\right\}.$$

Then in Θ^{sm}, neither platform completely abandons advertisers and serves only consumers.

Proof. As in Lemma A.4, we prove the lemma in four parts. We first establish the parameter space Θ^{sm}. Then we derive the equilibrium profits when one platform abandons the advertiser side of the market. Finally, we show that neither platform finds it profitable to do so.

Part (a). Based Lemmas A.10-A.12, we define the parameter space Θ^{sm} where the following conditions hold:

1. In Θ^{sm}, (A.172), (A.187), (A.202), (A.203), and (A.204) are satisfied. Thus, both platforms' profits are concave with respect to their own prices in all the three subgames.

2. The demand conditions as well as the multi-homing conditions are satisfied across all three subgames in Θ^{sm}.

3. In Θ^{sm}, $\underline{\gamma_J} \leqslant \gamma_J \leqslant \overline{\gamma_J}$ is also satisfied (with $\underline{\gamma_J}$ and $\overline{\gamma_J}$ as defined above).

Part (b). Suppose Platform 1 abandons advertisers to serve only consumers while Platform 2 serves both sides of the market and sets a uniform price for consumers. Note that because consumers single-home, this subgame is the same as the one in Part (a) of Lemma A.4. Therefore, the equilibrium profits of each platform are:

$$\Pi_1^{sd1*} = \frac{(6t_I + \gamma_I^H - 2\gamma_J)^2}{72t_I} \tag{A.217}$$

$$\Pi_2^{sd2*} = \frac{(6t_I + \gamma_I^H - 2\gamma_J)^2}{72t_I} + \frac{3v_J - 3t_J - \gamma_I^H + 2\gamma_J}{3} \tag{A.218}$$

For the equilibrium demand to lie within proper bounds, we need the following conditions: $\gamma_J \leqslant 2\gamma_I^H - 3t_I$ and $\gamma_J \leqslant 3t_I - \gamma_I^H$.

Part (c). Now suppose Platform 1 abandons advertisers to serve only consumers while Platform 2 serves both sides of the market and adopts a customized pricing strategy for consumers. Again, this subgame is the same as the one in Part (b) of Lemma A.4. Hence, the equilibrium profits of each platform are:

$$\Pi_1^{sd2*} = \frac{(6t_I - 2\gamma_J)^2}{72t_I} \tag{A.219}$$

$$\Pi_2^{sd2*} = v_J + \frac{t_I}{2} - t_J + \frac{13\gamma_J^2}{288t_I} + \frac{\gamma_J}{6} \tag{A.220}$$

To ensure that the demand is within proper bounds and IC conditions are satisfied, we need the following conditions: $\gamma_J \leqslant 2\gamma_I^H$ and $\gamma_J \leqslant \frac{12}{5}t_I$.

Part (d). In this part, we show that it is not profitable for a platform to abandon the advertiser side of the market. First, when a platform's competitor operates on two sides of the market and uses a uniform pricing strategy for consumers, the platform has no incentive to adopt the one-sided market strategy if $\Pi_1^{sm1*} \geqslant \Pi_1^{sd1*}$. This implies

$$\underline{\gamma_J} \leqslant \gamma_J \leqslant \widetilde{\gamma}_J, \tag{A.221}$$

where $\underline{\gamma_J} = \max\{0, \frac{48t_I t_J + 27t_I\gamma_I^H + 8t_J\gamma_I^H - 6\sqrt{2}\sqrt{t_I(4t_J({\gamma_I^H}^2 + 4{\delta_J}^2) + t_I(32t_J^2 + 24_J\gamma_I^H + 9({\gamma_I^H}^2 + 2\delta_J^2)))}}{2(9t_I + 8t_J)}\}$

and $\widetilde{\gamma}_J = \frac{48t_I t_J + 27t_I\gamma_I^H + 8t_J\gamma_I^H + 6\sqrt{2}\sqrt{t_I(4t_J({\gamma_I^H}^2 + 4{\delta_J}^2) + t_I(32t_J^2 + 24_J\gamma_I^H + 9({\gamma_I^H}^2 + 2\delta_J^2)))}}{2(9t_I + 8t_J)}$.

Similarly, when a platform's competitor uses a customized pricing strategy, the platform has no incentive to abandon advertisers if $\Pi_2^{sm2*} \geqslant \Pi_2^{sd2*}$. This is equivalent to:

$$\gamma_J \leqslant \widetilde{\widetilde{\gamma}}_J \tag{A.222}$$

where $\tilde{\tilde{\gamma}}_J \equiv \frac{12\sqrt{4t_It_J+\sqrt{16t_I^2t_J^2+9t_I^2\delta_J^2+8t_It_J\delta_J^2}}}{9t_I+8t_J}$. Now let $\overline{\gamma_J} \equiv \min\{\tilde{\gamma}_J, \tilde{\tilde{\gamma}}_J\}$. Since in Θ^{sm}, $\underline{\gamma_J} \leqslant \gamma_J \leqslant \overline{\gamma_J}$ holds, neither platform completely abandons the advertiser side of the market. □

Lemma A.14. *If $t_J \leqslant \delta_J \leqslant 2t_J$, then $\Theta^{sm} \neq \varnothing$.*

Proof. Suppose $t_J \leqslant \delta_J \leqslant 2t_J$, and consider a point (ϵ, ϵ) in the (γ_I^H, γ_J) space, where ϵ is a very small positive number. As ϵ approaches to zero, this point satisfies both (A.172) and (A.177), which are the second-order condition and the demand conditions (including the multi-homing conditions) of Subgame 1 where both platforms use a uniform pricing strategy. Similarly, it is easy to see that, as ϵ approaches to zero, (ϵ, ϵ) satisfies (A.187) and (A.190), which are the second-order condition and the demand conditions of Subgame 2 where both firms use a customized pricing strategy. Next, at (ϵ, ϵ) with ϵ being close to zero, we have $0 \leqslant \alpha_{1I}^* \leqslant 1$, $0 \leqslant \alpha_{1J}^* \leqslant 1$, $0 \leqslant {\alpha_{2I}^L}^* \leqslant \frac{1}{2}$, $0 \leqslant {\alpha_{2I}^H}^* \leqslant \frac{1}{2}$, $0 \leqslant \alpha_{2J}^* \leqslant 1$, and $\alpha_{1J}+\alpha_{2J} \geqslant 1$, which are the conditions pertaining to Subgame 3 (where both platforms use asymmetric pricing strategies). In addition, (A.202), (A.203) and (A.204) are satisfied at this point. Finally, note that we have $\lim_{\epsilon\to 0} \overline{\gamma_J} = \frac{6\{4t_It_J+\sqrt{t_I\{16t_It_J^2+(9t_I+8t_J)\delta_J^2\}}\}}{9t_I+8t_J} > 0$ and $\lim_{\epsilon\to 0} \underline{\gamma_J} = 0$ (where $\overline{\gamma_J}$ and $\underline{\gamma_J}$ are as defined in Lemma A.13). Then the point (ϵ, ϵ) also satisfies the condition for Lemma A.13. Therefore, if $t_J \leqslant \delta_J \leqslant 2t_J$, (ϵ, ϵ) is included in Θ^{sm} and thus, Θ^{sm} is not an empty set. □

Claim A.1. *When consumers and advertisers single-home and both platforms set a uniform price for consumers, each platform's profits decrease in γ_J but increase in γ_I.*

Proof. By Lemma A.1, each platform's profits are $\Pi_1^{ss1*} = \Pi_2^{ss1*} = \frac{1}{2}(t_I - \gamma_J + t_J + \gamma_I)$. The claim immediately follows. □

Claim A.2. *When consumers and advertisers can multi-home and both platforms set a uniform price for consumers, each platform's profits increase in γ_J but decrease in γ_I.*

Proof. By differentiating the equilibrium profits in Lemma A.5 with respect to γ_I and γ_J, we obtain:

$$\frac{\partial \Pi_1^{mm1*}}{\partial \gamma_I} = \frac{\partial \Pi_2^{mm1*}}{\partial \gamma_I} = -\frac{2t_J\delta_I-(\gamma_I-\gamma_J)\delta_J}{4t_It_J-(\gamma_I-\gamma_J)^2} \cdot \frac{2t_I\delta_J-(\gamma_I-\gamma_J)\delta_I}{4t_It_J-(\gamma_I-\gamma_J)^2}$$
$$= -\alpha_{1I}^* \cdot \alpha_{1J}^* < 0, \tag{A.223}$$

and

$$\frac{\partial \Pi_1^{mm1*}}{\partial \gamma_J} = \frac{\partial \Pi_2^{mm1*}}{\partial \gamma_J} = \frac{2t_J\delta_I-(\gamma_I-\gamma_J)\delta_J}{4t_It_J-(\gamma_I-\gamma_J)^2} \cdot \frac{2t_I\delta_J-(\gamma_I-\gamma_J)\delta_I}{4t_It_J-(\gamma_I-\gamma_J)^2}$$
$$= \alpha_{1I}^* \cdot \alpha_{1J}^* > 0, \tag{A.224}$$

because the market sizes are positive (that is, $\alpha_{1I}^* = \alpha_{2I}^* \in (\frac{1}{2}, 1)$ whereas $\alpha_{1J}^* = \alpha_{2J}^* \in (\frac{1}{2}, 1)$ by Lemma A.5). □

A.3.1 Proof of Proposition 2.5

Proof. Using the profits derived in Lemma A.10, we find that $\frac{\partial \Pi_1^{sm1*}}{\partial \gamma_I} = \frac{\partial \Pi_2^{sm1*}}{\partial \gamma_I} = \frac{1}{8t_J} \cdot (-\gamma_I + 3\gamma_J) > 0$ which is equivalent to $\gamma_I < 3\gamma_J$. Furthermore, $\frac{\partial \Pi_1^{sm1*}}{\partial \gamma_J} = \frac{\partial \Pi_2^{sm1*}}{\partial \gamma_J} = \frac{1}{8t_J} \cdot (3\gamma_I - \gamma_J) > 0$ which is equivalent to $\gamma_I > \frac{1}{3}\gamma_J$. Hence the results follow. □

A.3.2 Proof of Proposition 2.6

Proof. It suffices to show an example of both possibilities in equilibrium. Given the two platforms' profits derived in Lemmas A.10-A.12, we define $\delta_{13} \equiv \Pi_2^{sm1*} - \Pi_2^{sm3*}$ and $\delta_{23} \equiv \Pi_1^{sm2*} - \Pi_1^{sm3*}$.

Now suppose $t_I = 1$, $t_J = 5$, and $\delta_J = 7.5$. Because $t_J \leqslant \delta_J \leqslant 2t_J$, the parameter space is a non-empty set according to Lemma A.14. Further let $\gamma_J = 2$. Then the parameter space Θ^{sm} is given by $0 \leqslant \gamma_I^H \leqslant 7.3056$. Moreover, in Θ^{sm}, $\delta_{13} \geqslant 0$ is equivalent to $\gamma_I^H \leqslant 2.5994$ while $\delta_{23} \geqslant 0$ is equivalent to $\gamma_I^H \geqslant 2.9151$. Then we have the following equilibria:

1. The Subgame 1 equilibrium where both platforms use uniform pricing exists when $0 \leqslant \gamma_I^H \leqslant 2.5994$.

2. The Subgame 2 equilibrium where both platforms use customized pricing exists when $2.9151 \leqslant \gamma_I^H \leqslant 7.3056$. Note that in this interval, the IC condition: $\gamma_I^H > 2$ is satisfied.

3. The Subgame 3 equilibrium where both platforms use distinct pricing strategies exists when $2.5994 \leqslant \gamma_J \leqslant 2.9151$. Note that the IC constraints do not alter this condition. This is because the IC condition for L-type consumers is always satisfied in Θ^{sm} and that for H-type consumers is satisfied when $\gamma_I^H \geqslant 2.0158$.

Thus, when $t_J \leqslant \delta_J \leqslant 2t_J$, both platforms can adopt a customized pricing strategy and asymmetric pricing strategies in equilibrium. $\square$

Appendix B

Proof for Chapter 3

B.1 Analysis of a Perfectly Competitive Content Market

We first characterize the equilibrium solution corresponding to the monopoly and duopoly models of platforms in a series of lemmas, and then use them to prove the main propositions.

B.1.1 Monopoly Platform

Lemma B.1. *The equilibrium is characterized as follows:*

- *If $p_S \geqslant \tau_M$, the platform adopts a free-content strategy and*

$$\alpha_1^* = \alpha_f^M \equiv 1 - \frac{t(2c+\gamma_A)}{\sqrt{t \cdot v\gamma_A(2c+\gamma_A)}}, \qquad p_C^* = p_{fC}^M \equiv 0. \tag{B.1}$$

- *If $p_S < \tau_M$, the platform adopts a paid-content-with-ads strategy and*

$$\alpha_1^* = \alpha_{pa}^M \equiv \frac{-c-\gamma_A+v}{v}, \qquad p_C^* = p_{paC}^M \equiv \frac{v^2-2vt-c^2-2\gamma_A \cdot c-\gamma_A^2}{2v}. \tag{B.2}$$

where $\tau_M = -\gamma_A + \sqrt{v \cdot (v-2t)}$.

Proof. Because the content market is perfectly competitive, the content price is equal to the marginal cost of producing it

$$p_S = c \tag{B.3}$$

Given the assumption that the market is fully covered ($n_{1C} = 1$), as given in equation (3.8) in the main paper, the price charged to consumers is

$$p_{1C} = v \cdot (\alpha_1 - \frac{\alpha_1^2}{2}) - t \tag{B.4}$$

Moreover, the monopoly platform's profits, as give in equation (10) in the main paper, are

$$\Pi_P^M\big|_{n_{1C}=1} = -t - c \cdot \alpha_1 + (1 - \alpha_1) \cdot \gamma_A + (\alpha_1 - \frac{1}{2}\alpha_1^2) \cdot v \tag{B.5}$$

The condition $p_{1C} \geqslant 0$ is equivalent to

$$\alpha_1 \geqslant \alpha_f^M \equiv \frac{v - \sqrt{v \cdot (v - 2t)}}{v} \tag{B.6}$$

where α_f^M is the proportion of content the platform provides when adopting a free-content strategy ($p_{1C} = 0$). Notice that $\alpha_f^M \in (0, 1)$.

Now the platform chooses $\alpha_1 \in [\alpha_f^M, 1]$ to maximize its profits. The monopoly platform's profits $\Pi_P^M\big|_{n_{1C}=1}$ is concave in α_1. On solving $\frac{\partial \Pi_P^M}{\partial \alpha_1} = 0$, we obtain:

$$\alpha_1 = \alpha_{pa}^M \equiv \frac{-c - \gamma_A + v}{v} \tag{B.7}$$

Note that $\alpha_{pa}^M < 1$ always holds. Then depending on content supplier price ($p_S = c$), we could have one of the following two situations: (1) $\alpha_{pa}^M \leqslant \alpha_f^M$ and (2) $\alpha_f^M < \alpha_{pa}^M < 1$, which are respectively equivalent to (1) $c \geqslant \tau_M$, and (2) $c < \tau_M$, where

$$\tau_M \equiv -\gamma_A + \sqrt{v \cdot (v - 2t)}. \tag{B.8}$$

Next, we proceed to analyze the three cases mentioned above.

Case L1.1: $c \geqslant \tau_M$

Because $\alpha_{pa}^M \leqslant \alpha_f^M$ and $\Pi_P^M\big|_{n_{1C}=1}$ is a concave function of α_1, we know that for any $\alpha_1 \in [\alpha_f^M, 1]$, $\Pi_P^M\big|_{n_{1C}=1}$ decreases with α_1. Further, since the platform chooses $\alpha_1^* = \alpha_f^M$ to maximize its profits, we have $p_{1C}^* = p_{fC}^M \equiv 0$, implying the platform adopts a free-content strategy.

Case L1.2: $c < \tau_M$

Since $\alpha_f^M < \alpha_{pa}^M < 1$ and $\Pi_P^M\big|_{n_{1C}=1}$ is maximized at $\alpha_1 = \alpha_{pa}^M$, the platform chooses $\alpha_1^* = \alpha_{pa}^M$. In this case, we have from (B.4) that $p_{1C}^* = p_{paC}^M \equiv \frac{v^2-2vt-c^2-2\gamma_A\cdot c-\gamma_A^2}{2v}$. Now because $p_{1C}^* > 0$ and $\alpha_{pa}^M < 1$, the platform adopts a paid-content-with-ads strategy.

□

Lemma B.2. *The market is completely covered if the following conditions simultaneously hold:* $\gamma_A > t$ *and* $v \geqslant \max\{4t, \frac{2t\cdot\gamma_A^2}{\gamma_A^2-t^2}\}$.

Proof. If the platform sells to cover n_{1C} consumers, from equation (3.8),(3.9), and (3.10) of the main paper, we have the platform's price for consumers $p_{1C} = v \cdot (\alpha_1 - \frac{\alpha_1^2}{2}) - t \cdot n_{1C}$, the platform's price for advertisers $p_{1A} = \gamma_A \cdot n_{1C}$, and the platform's profits $\Pi_P^M = -n_{1C}^2 \cdot t - c \cdot \alpha_1 + n_{1C}\Big(\gamma_A(1-\alpha_1) - \frac{1}{2}(-2+\alpha_1)\cdot\alpha_1\cdot v\Big)$.

The market is completely covered in equilibrium if and only if the monopoly platform chooses $n_{1C} = 1$. It is optimal for the monopoly platform to do so if Π_P^M increases with n_{1C} for $n_{1C} \in [0, 1]$. Thus, we find a condition under which $\frac{\partial \Pi_P^M}{\partial n_{1C}} = -\frac{v}{2}\cdot\alpha_1^2 + (-\gamma_A + v)\cdot\alpha_1 + \gamma_A - 2t\cdot n_{1C} \geqslant 0$ holds for any $n_{1C} \in [0, 1]$ and any $\alpha_1 \in [\frac{v-\sqrt{v\cdot(v-2t\cdot n_{1C})}}{v}, 1]$ (noting that, $p_{1C} = v\cdot(\alpha_1 - \frac{\alpha_1^2}{2}) - t\cdot n_{1C} \geqslant 0$ is equivalent to $\alpha_1 \geqslant \frac{v-\sqrt{v\cdot(v-2t\cdot n_{1C})}}{v}$).

Since $\frac{\partial \Pi_P^M}{\partial n_{1C}}$ is decreasing in n_{1C} and concave in α_1, it suffices to find a condition for the following two inequalities:

$$\frac{\partial \Pi_P^M}{\partial n_{1C}}\Big|_{\{\alpha_1=\frac{v-\sqrt{v\cdot(v-2t\cdot n_{1C})}}{v}, n_{1C}=1\}} = \frac{\gamma_A \cdot v\sqrt{v(v-2t\cdot)}}{v} - t \geqslant 0$$

$$\frac{\partial \Pi_P^M}{\partial n_{1C}}\Big|_{\{\alpha_1=1, n_{1C}=1\}} = \frac{1}{2}(v-4t) \geqslant 0$$

On solving the two inequalities, we obtain:

$$v \geqslant \max\{4t, \frac{2t\cdot\gamma_A^2}{\gamma_A^2 - t^2}\} \tag{B.9}$$

$$\gamma_A > t \tag{B.10}$$

Therefore, these conditions ensure that the monopoly platform fully covers the consumer market. □

B.1.2 Duopoly Platforms

Lemma B.3. *Suppose $\gamma_A > \sqrt{(2-\sqrt{3})\cdot v\cdot t}$ holds. The equilibrium is characterized as follows:*

- *If $c \geqslant \tau_D$, the platform adopts a free-content strategy and*

$$\alpha_1^* = \alpha_2^* = \alpha_f^D \equiv 1 - \frac{t(2c+\gamma_A)}{\sqrt{t\cdot v\gamma_A(2c+\gamma_A)}}, \quad p_{1C}^* = p_{2C}^* = p_{fC}^D \equiv 0. \tag{B.11}$$

- *If $c < \tau_D$, the platform adopts a paid-content-with-ads strategy and*

$$\alpha_1^* = \alpha_2^* = \alpha_{pa}^D \equiv \frac{-2c-\gamma_A+v}{v}, \quad p_{1C}^* = p_{2C}^* = p_{paC}^D \equiv \frac{-2c\cdot\gamma_A - \gamma_A^2 + t\cdot v}{v}. \tag{B.12}$$

where $\tau_D = \frac{1}{2}\cdot(-\gamma_A + \frac{t\cdot v}{\gamma_A})$.

Proof. In this game, as described in Section 3.2 of the main paper, both platforms simultaneously choose α_i and p_{iC} to maximize their own profits Π_{iP}^D subject to $0 \leqslant$

$\alpha_i \leqslant 1$ and $p_{iC} \geqslant 0$. We first solve $\frac{\partial \Pi_{1P}^D}{\partial p_{1C}} = 0$, $\frac{\partial \Pi_{1P}^D}{\partial \alpha_1} = 0$, $\frac{\partial \Pi_{2P}^D}{\partial p_{2C}} = 0$, and $\frac{\partial \Pi_{2P}^D}{\partial \alpha_2} = 0$, and obtain the following symmetric solution:

$$\alpha_1 = \alpha_2 = \alpha_{pa}^D \equiv \frac{-2c - \gamma_A + v}{v} \tag{B.13}$$

$$p_{1C} = p_{2C} = p_{paC}^D \equiv \frac{-2c \cdot \gamma_A - \gamma_A^2 + t \cdot v}{v} \tag{B.14}$$

[1]To begin, note that $\alpha_{pa}^D \geqslant 1$ always holds, and $p_{paC}^D \leqslant 0$ is equivalent to $c \geqslant \tau_D$, where

$$\tau_D \equiv \frac{1}{2} \cdot (-\gamma_A + \frac{t \cdot v}{\gamma_A}). \tag{B.15}$$

Then, depending on the content supplier price ($p_S = c$), we could have one of the following two situations: (1) $p_S \geqslant \tau_D$, and (2) $p_S < \tau_H^D$. We analyze these two cases.

<u>Case L5.1</u>: $c \geqslant \tau_D$

In this case, since $p_{paC}^D \leqslant 0$, the constraint on the consumer price is binding. Thus, we set $p_{1C} = p_{2C} = 0$ and simultaneously solve $\frac{\partial \Pi_{1P}^D}{\partial \alpha_1} = 0$ and $\frac{\partial \Pi_{2P}^D}{\partial \alpha_2} = 0$. Then, we have:

$$\alpha_1 = \alpha_2 = \alpha_f^D \equiv 1 - \frac{t(2c + \gamma_A)}{\sqrt{t \cdot v \gamma_A (2c + \gamma_A)}} \tag{B.16}$$

The second-order condition is also satisfied as shown below. Because of symmetry, we focus attention on only one platform's second-order condition.

$$\left.\frac{\partial^2 \Pi_{1P}}{\partial {\alpha_1}^2}\right|_{(\alpha_1 = \alpha_2 = \alpha_f^D, p_{1C} = p_{2C} = 0)} = -\frac{3\sqrt{\gamma_A[4t \cdot v \cdot (2c + \gamma_A)]}}{4t} < 0. \tag{B.17}$$

[1] In this proof, we focus on the symmetric equilibrium. This is because the goal of this paper is to derive the implications of different content market structure on the platform's content provision strategy. As will be seen in the supplementary analysis (Lemma B.14), asymmetric solutions are ruled out when the content supplier is a monopoly. Therefore, we also focus on the symmetric equilibrium here so that we are making fair comparisons.

Note, however, that the above solution $\{\alpha_1 = \alpha_2 = \alpha_f^D, p_{1C} = p_{2C} = 0\}$ is a corner solution. Hence, we check whether it is possible to deviate to an interior solution. One can see that

$$\left.\frac{\partial \Pi_{1P}}{\partial p_{1C}}\right|_{(\alpha_1=\alpha_f^D, p_{1C}=0 \;\mid\; \alpha_2=\alpha_f^D, p_{2C}=0)} = \frac{2t \cdot v - \sqrt{\gamma_A[4t \cdot v \cdot (2c + \gamma_A)]}}{4 \cdot t \cdot v} \tag{B.18}$$

$$\leqslant \frac{2t \cdot v - \sqrt{\gamma_A[4t \cdot v \cdot (2\tau_D + \gamma_A)]}}{4 \cdot t \cdot v} = 0, \tag{B.19}$$

which implies that given $(\alpha_1 = \alpha_f^D,\ \alpha_2 = \alpha_f^D)$ neither platform has an incentive to unilaterally deviate from $p_{iC} = 0$. Therefore, when $p_S \geqslant \tau_D$, the equilibrium is given as $\{\alpha_1 = \alpha_2 = \alpha_f^D, p_{1C} = p_{2C} = 0\}$ and both platforms pursue a free-content strategy.

Case L5.2: $c < \tau_D$

In this case, since $p_{paC}^D > 0$ and $0 < \alpha_{pa}^D < 1$, the solution given in (B.13) and (B.14) is indeed an equilibrium solution. Note that no deviation is possible from this solution because this is an unconstrained solution. The second-order condition is satisfied as shown below:

$$\left.\frac{\partial^2 \Pi_{1P}}{\partial {\alpha_1}^2}\right|_{(\alpha_1=\alpha_2=\alpha_{pa}^D, p_{1C}=p_{2C}=p_{paC}^D)} = -\frac{4c\gamma_A + 2\gamma_A^2 + t \cdot v}{2t} < 0$$

$$\left.\frac{\partial^2 \Pi_{1P}}{\partial {p_{1C}}^2}\right|_{(\alpha_1=\alpha_2=\alpha_{pa}^D, p_{1C}=p_{2C}=p_{paC}^D)} = -\frac{1}{t} < 0$$

$$\left.\left(\frac{\partial^2 \Pi_{1P}}{\partial {\alpha_1}^2} \cdot \frac{\partial^2 \Pi_{1P}}{\partial {p_{1C}}^2} - (\frac{\partial^2 \Pi_{1P}}{\partial \alpha_1 \partial p_{1C}})^2\right)\right|_{(\alpha_1=\alpha_2=\alpha_{pa}^D, p_{1C}=p_{2C}=p_{paC}^D)} = \frac{-2c^2 + t \cdot v}{2t^2} > \frac{-2{\tau_D}^2 + t \cdot v}{2t^2} > 0$$

The last inequality holds since we impose the condition: $\gamma_A > \sqrt{(2 - \sqrt{3}) \cdot v \cdot t}$. Therefore, when $p_S < \tau_D$, the equilibrium is given as $\{\alpha_1 = \alpha_2 = \alpha_{pa}^D, p_{1C} = p_{2C} = p_{paC}^D\}$ and both platforms pursue a paid-content-with-ads strategy.

□

Lemma B.4. *The market is completely covered in equilibrium if the following conditions simultaneously hold:* $\gamma_A > t$, $v \geqslant \max\{2t, \frac{t\cdot\gamma_A^2}{\gamma_A^2 - t^2}\}$ *and* $c \leqslant \overline{\tau} \equiv \frac{(v-2t)\gamma_A}{2t}$.

Proof. In a duopoly, the market is completely covered if and only if the utility of the marginal consumer (who is indifferent between the two products) is non-negative. By the symmetry of the solution, the marginal consumer's location is given as $x = \frac{1}{2}$. Thus, we derive the condition for $U_{1C}(\frac{1}{2}) \geqslant 0$, given the equilibrium price and content demand derived in Lemma B.3 for each strategy choice.

First, under a free-content strategy, we have

$$U_{1C}(\frac{1}{2}) = -\frac{c\cdot t}{\gamma_A} + \frac{v}{2} - t \tag{B.20}$$

which is equivalent to

$$c \leqslant \overline{\tau} \equiv \frac{(v-2t)\gamma_A}{2t}. \tag{B.21}$$

This defines the upper bound of the marginal cost of producing content. Note that $\overline{\tau} \geqslant c \geqslant 0$ also implies that

$$v \geqslant 2t \tag{B.22}$$

Second, under a paid-content-with-ads strategy, we have

$$U_{1C}(\frac{1}{2}) = \frac{-c^2 + \gamma_A^2 - 3t\cdot v + v^2}{2v} \geqslant 0, \tag{B.23}$$

which is equivalent to

$$c \leqslant \frac{1}{2}\sqrt{\gamma_A^2 - 3tv + v^2} \tag{B.24}$$

Given that $c < \tau^D$ under the paid-content-with-ads strategy, the above inequality holds as long as $\tau_D \leqslant \frac{1}{2}\sqrt{\gamma_A^2 - 3tv + v^2}$, which is equivalent to

$$\gamma_A > t \quad \text{and} \quad v \geqslant \frac{t\gamma_A^2}{\gamma_A^2 - t^2} \tag{B.25}$$

Therefore, when both of (B.21), (B.22),and (B.25) simultaneously hold, the market is completely covered across the two strategies. □

B.1.3 Proof of Proposition 3.1

Proof. From Lemma B.1 and B.3, a competing platform chooses free-content strategy if and only if $c \geqslant \tau_D$, while the monopoly platform chooses paid-content-with-ads strategy f and only if $c < \tau_M$. Therefore, the presence of competition motivates a platform to change from paid-content-with-ads to free-content if and only if $c \in [\max\{\tau_D, 0\}, \tau_M)$. For this to happen, we need

$$\tau_M > \max\{\tau_D, 0\} \tag{B.26}$$

To show this is not a null set, note that $\{t = 1, v = 5, \gamma_A = 3\}$ satisfy the equality (and all the conditions in lemma B.1 - B.4). Therefore, the presence of competition can motivate a platform to change from paid-content-with-ads to free-content.

On the other hand, a competing platform chooses paid-content-with-ads strategy if and only if $c < \tau_D$, while the monopoly platform chooses free-content strategy f and only if $c \geqslant \tau_M$. Therefore, the presence of competition motivates a platform to change from free-content to paid-content-with-ads if and only if $c \in [\max\{\tau_D, 0\}, \tau_M)$. For this to happen, we need

$$\tau_D > \max\{\tau_M, 0\} \tag{B.27}$$

Next we show this inequality cannot hold. To see this, note that $\tau_D > 0$ is equivalent to $\gamma_A < \sqrt{v \cdot t}$. We also have $\frac{\partial(\tau_D - \tau_M)}{\partial \gamma_A} = \frac{\gamma_A^2 - v \cdot t}{2\gamma_A^2}$. This implies that conditional on $\tau_D > 0$, we have $\frac{\partial(\tau_D - \tau_M)}{\partial \gamma_A} < 0$. Therefore, upon solving $\tau_D > \tau_M$, we have

$$\gamma_A < \sqrt{v(-2t + v)} - \sqrt{-3tv + v^2} \tag{B.28}$$

Note from lemma B.2 that the monopoly platform's full market coverage requires $v > \frac{2t\gamma_A^2}{\gamma_A^2 - t^2}$, which is equivalent to

$$\gamma_A > t \cdot \sqrt{\frac{v}{v - 2t}} \tag{B.29}$$

Therefore, the necessary condition for (B.27) is

$$t \cdot \sqrt{\frac{v}{v-2t}} < \sqrt{v(-2t+v)} - \sqrt{-3tv+v^2} \tag{B.30}$$

However, by solving this inequality, one gets $\{v > 0, t < 0\}$ or $\{v < 0, \frac{v}{3} \leqslant t < 0\}$, neither of which is a feasible parameter set. Therefore, (B.27) does not hold, implying that the presence of competition cannot motivate a platform to change from free-content to paid-content-with-ads. □

B.1.4 Proof of Proposition B.2

Proof. When $c \geqslant \tau_D$, free-content is the equilibrium strategy. A completing platform's profits are

$$\Pi_{iP}^{D} = \frac{t \cdot (2c+\gamma_A)^2}{2\sqrt{t \cdot v\gamma_A(2c+\gamma_A)}} - c \tag{B.31}$$

Taking derivative, $\frac{\partial \Pi_{iP}^{D}}{\partial c} > 0$ if and only if $c > \frac{\gamma_A}{18} \cdot (\frac{4v}{t} - 9)$. Therefore $\frac{\partial \Pi_{iP}^{D}}{\partial c} > 0$ if $c > \max\{\tau_D, \frac{\gamma_A}{18} \cdot (\frac{4v}{t} - 9)\}$. This implies that when c is large, a decrease in c hurts a competing platform's profits. □

B.2 Analysis of a Monopoly Content Market

B.2.1 Monopoly Platform

Lemma B.5. *There exists a threshold $\widehat{\gamma_A^M}$ such that:*

- *If $\gamma_A < \widehat{\gamma_A^M}$, a paid-content-with-ads strategy is adopted;*
- *If $\gamma_A \geqslant \widehat{\gamma_A^M}$, a free-content strategy is adopted.*

Proof. The result in Lemma B.1 pertains to the second-stage equilibrium, except that the content price c is replaced by p_S. Therefore, the monopoly platform's optimal proportion of content is

$$\alpha_1(p_S) = \begin{cases} \alpha_f^M = \frac{v-\sqrt{v\cdot(v-2t)}}{v}, & \text{if } p_S \geqslant \tau_M \\ \alpha_{pa}^M = \frac{-p_S-\gamma_A+v}{v}, & \text{if } p_S < \tau_M \end{cases} \tag{B.32}$$

where $\tau_M = -\gamma_A + \sqrt{v\cdot(v-2t)}$. Note that α_f^M and α_{pa}^M give the demand for content from the monopoly platform when it adopts a free-content strategy and a paid-content-with-ads strategy respectively.

Given the second-stage equilibrium, the content supplier's profits can be rewritten as a function of p_S:

$$\Pi_S^M(p_S) = \begin{cases} \frac{p_S\ (v-\sqrt{v\cdot(v-2t)})}{v}, & \text{if } p_S \geqslant \tau_M \\ \frac{p_S(-p_S-\gamma_A+v)}{v}, & \text{if } p_S < \tau_M \end{cases} \tag{B.33}$$

First, note that, under a free-content strategy (i.e., when $p_S \geqslant \tau_M$), the content supplier can increase its price until it extracts all of the platform's profits because α_f^M is not a function of p_S. Thus, on solving $\Pi_P^M\big|_{\{n_{1C}=1,\alpha_1=\alpha_f^M\}} = 0$, we obtain

$$p_S = p_{fS}^M \equiv \frac{\gamma_A\cdot\sqrt{v\cdot(v-2t)}}{v-\sqrt{v\cdot(v-2t)}}, \tag{B.34}$$

which maximizes the content supplier's profits under the free-content strategy. Note we also need to ensure $p_{fS}^M \geqslant \tau_M$. This can be verified as $p_{fS}^M - \tau_M = \frac{v\cdot\left((\gamma_A-t)+(v-t-\sqrt{v(v-2t)})\right)}{v-\sqrt{v(v-2t)}}$ > 0. The inequality holds because $\gamma_A - t > 0$ (Lemma B.1), $v - t - \sqrt{v(v-2t)} > 0$, and $v - \sqrt{v(v-2t)} > 0$.

Second, under a paid-content-with-ads strategy (i.e., when $p_S < \tau_M$), the price p_S that maximizes the content supplier's profits is obtained upon solving $\frac{\partial}{\partial p_S}(p_S\cdot\alpha_{pa}^M) = 0$. We have:

$$p_S = p_{paS}^M \equiv \frac{1}{2}\cdot(-\gamma_A+v). \tag{B.35}$$

Note that p^M_{paS} is relevant only if $p^M_{paS} < \tau_M$, which is equivalent to

$$\gamma_A < \gamma^M_{A1} \equiv -v + 2\sqrt{v(v-2t)}. \quad \text{(B.36)}$$

We consider the following two cases depending on the region where γ_A falls: (1) $\gamma_A \geqslant \gamma^M_{A1}$, and (2) $\gamma_A < \gamma^M_{A1}$

Case L4.1: $\gamma_A \geqslant \gamma^M_{A1}$.

In this case, the profit function Π^M_S increases with p_S when $p_S \in [0, \tau_M)$, and it continues to increase with p_S when $p_S \in [\tau_M, p^M_{fS}]$. Therefore, the content supplier chooses $p^*_S = p^M_{fS}$ to maximize its profits. It is useful to note that $p^M_{fS} \equiv \frac{\gamma_A \cdot \sqrt{v \cdot (v-2t)}}{v - \sqrt{v \cdot (v-2t)}}$ is always positive. At this content supplier price, the platform pursues a free-content strategy but earns zero profits.

Case L4.2: $\gamma_A < \gamma^M_{A1}$.

In this case, since $p^M_{paS} < \tau_M$, the profit function Π^M_S is concave in p_S when $p_S < \tau^M_H$, and it increases with p_S when $p_S \in [\tau_M, p^M_{fS}]$. Moreover, when $p_S = p^M_{paS}$, it can be calculated that the monopoly platform's profits are $\Pi^M_P(p_S = p^M_{paS}) = \frac{v(v-2t)+6v(\gamma_A - t)+\gamma_A^2}{8v} > 0$, so that the platform earns positive profits. Thus, to determine the content supplier's choice of price, we compare $\Pi^M_S(p_S = p^M_{fS})$ and $\Pi^M_S(p_S = p^M_{paS})$ where

$$\Pi^M_S(p_S = p^M_{paS}) = \frac{(-\gamma_A + v)^2}{4v} \quad \text{(B.37)}$$

$$\Pi^M_S(p_S = p^M_{fS}) = \frac{\gamma_A \cdot (\sqrt{v \cdot (v-2t)}}{v}. \quad \text{(B.38)}$$

In doing so, it is useful to note that p^M_{paS} is non-negative. This is because when $\gamma_A < \gamma^M_{A1}$, we have $p^M_{paS} \equiv \frac{1}{2} \cdot (-\gamma_A + v) > \frac{1}{2} \cdot (-\gamma^M_{A1} + v) = v - \sqrt{v(v-2t)} > 0$. Given (B.37) and (B.38), when $\gamma_A < \gamma^M_{A1}$, $\Pi^M_S((p_S = p^M_{fS}) \geqslant \Pi^M_S(p_S = p^M_{paS})$ is equivalent to $\gamma_A \geqslant \gamma_A^M$, where

$$\gamma_A^M \equiv v + 2\sqrt{v(v-2t)} - 2\sqrt{v(v-2t+\sqrt{v(v-2t)})} \tag{B.39}$$

Thus, given $\gamma_A < \gamma_{A1}^M$, a free-content strategy is adopted if $\gamma_A \geqslant \gamma_A^M$, otherwise a paid-content-with-ads strategy is chosen.

Combining the two cases, in sum, a free-content strategy is adopted if and only if $\gamma_A \geqslant \gamma_A^M$. This completes the proof. □

B.2.2 Duopoly Platforms

Lemma B.6. *Then, there exists a threshold* $\widehat{\gamma_A^D}$ *such that:*

- *If* $\gamma_A < \widehat{\gamma_A^D}$, *a paid-content-with-ads strategy is adopted;*
- *If* $\gamma_A \geqslant \widehat{\gamma_A^D}$, *a free-content strategy is adopted.*

Proof. The result in Lemma B.3 pertains to the second-stage equilibrium, except that the content price c is replaced by p_S (Note that when solving $\frac{\partial \Pi_{1P}^D}{\partial p_{1C}} = 0$, $\frac{\partial \Pi_{1P}^D}{\partial \alpha_1} = 0$, $\frac{\partial \Pi_{2P}^D}{\partial p_{2C}} = 0$, and $\frac{\partial \Pi_{2P}^D}{\partial \alpha_2} = 0$, we also obtain asymmetric solutions, as we show later (in Lemma B.14). However, they generate less profits to the content supplier than the symmetric solution; hence the asymmetric solutions are ruled out in the content supplier's decision). Therefore, each duopoly platform's optimal proportion of content is

$$\alpha_i(p_S) = \begin{cases} \alpha_f^D = 1 - \frac{t(2p_S+\gamma_A)}{\sqrt{t \cdot v \gamma_A (2p_S+\gamma_A)}}, & \text{if } p_S \geqslant \tau_D \\ \alpha_{pa}^D = \frac{-2p_S-\gamma_A+v}{v}, & \text{if } p_S < \tau_D \end{cases} \tag{B.40}$$

where and $\tau_D = \frac{1}{2} \cdot (-\gamma_A + \frac{t \cdot v}{\gamma_A})$. Note that α_f^D and α_{pa}^D give the demand for content from a duopoly platform when it pursues a free-content strategy and a paid-content-with-ads strategy respectively.

Based on the second-stage equilibrium the content supplier's profits and the total channel profits can be written as a function of p_S as shown below:

$$\Pi_S^D(p_S) = \begin{cases} 2p_S(1 - \frac{t(2p_S+\gamma_A)}{\sqrt{t \cdot v\gamma_A(2c+\gamma_A)}}), & \text{if } p_S \geqslant \tau_D \\ \frac{2p_S(-2p_S-\gamma_A+\gamma_C+v)}{v}, & \text{if } p_S < \tau_D \end{cases} \tag{B.41}$$

We further define total channel profits as the sum of the profits earned by the monopoly supplier and both platforms given by $\Pi_C^D = \Pi_S^D + \Pi_{1P}^D + \Pi_{2P}^D$

$$\Pi_C^D(p_S) = \begin{cases} \frac{\sqrt{tv\gamma_A(2p_S+\gamma_A)}}{v}, & \text{if } p_S \geqslant \tau_D \\ t, & \text{if } p_S < \tau_D \end{cases} \tag{B.42}$$

As will be seen later, Π_C^D is useful in assessing each platform's IR constraint. Also note that $\Pi_C^D(p_S)$ is a continuous function of p_S, which is constant when $p_S \leqslant \tau_D$ but increases with p_S when $p_S \geqslant \tau_D$.

From the second-stage analysis, we know that no-ad strategy is not adopted. Therefore, we only consider the other two strategies. In particular, we prove the proposition by establishing the following three claims:

1. If a free-content strategy is adopted at $\gamma_A = \gamma_A^*$, then a free-content strategy is adopted at as well any γ_A great than γ_A^*.

2. There exists a γ_A at which a free-content strategy is adopted.

3. There exists a γ_A at which a paid-content-with-ads strategy is adopted.

We prove each of these claims in order.

Claim B.1. *If a free-content strategy is adopted at $\gamma_A = \gamma_A^*$, then a free-content strategy is adopted as well at any γ_A great than γ_A^*.*

Proof. Suppose the free-content strategy is adopted at $\gamma_A = \gamma_A^*$. And denote the equilibrium content price at $\gamma_A = \gamma_A^*$ by p_S^*: $p_S^* \equiv p_S(\gamma_A^*)$. Note that $p_S^* \geqslant \tau_{D*}$, where τ_{D*} is defined as τ_D evaluated at $\gamma_A = \gamma_A^*$.

When $\tau_{D*} \leqslant 0$, the claim trivially holds. This is because $\frac{\partial \tau_D(\gamma_A)}{\partial \gamma_A} = -\frac{\gamma_A^2 + tv}{2\gamma_A^2} < 0$ and thus, in this case, $\tau_D(\gamma_A) \leqslant 0, \forall \gamma_A > \gamma_A^*$, which implies that for any positive p_S, $p_S \geqslant \tau_D$ always holds. Thus, for the rest of the proof, we will focus on the case where $\tau_{D*} > 0$.

In this case, at $\gamma_A = \gamma_A^*$, the content supplier earns more profits by choosing p_S^* than any other $p_S \in (0, \tau_{D*}]$: $\Pi_S^D(p_S^*|\gamma_A^*) > \Pi_S^D(p_S|\gamma_A^*), \forall p_S \in (0, \tau_{D*}]$. First, note that, given the second-stage profits in (B.41), under a paid-content-with-ads strategy, we have

$$\frac{\partial \Pi_S^D(p_S)}{\partial \gamma_A} = -\frac{p_S}{v} < 0,\ \forall p_S < \tau_D,$$

which suggests that for any given p_S, the content supplier's profits under a paid-content-with-ads strategy decrease as γ_A increases beyond γ_A^*: $\Pi_S^D(p_S|\gamma_A^*) > \Pi_S^D(p_S|\gamma_A > \gamma_A^*), \forall p_S \leqslant \tau_D(\gamma_A)$. This implies that for a given γ_A^0 such that $\gamma_A^0 > \gamma_A^*$,

$$\Pi_S^D(p_S^*|\gamma_A^*) > \Pi_S^D(p_S|\gamma_A^0), \forall p_S \leqslant \tau_D(\gamma_A^0)). \tag{B.43}$$

Second, given the second-stage profits, under a free-content strategy, we have

$$\frac{\partial \Pi_S^D(p_S)}{\partial \gamma_A} = \frac{2p_S^2 t}{\gamma_A^{\frac{3}{2}}\sqrt{tv(2p_S + \gamma_A)}} > 0,\ \forall p_S \geqslant \tau_D,$$

which implies that if $p_S = p_S^*$, the content supplier's profits increase as γ_A increases beyond γ_A^*. Then, for any given γ_A^0 such that $\gamma_A^0 > \gamma_A^*$,

$$\Pi_S^D(p_S^*|\gamma_A^*) < \Pi_S^D(p_S^*|\gamma_A^0). \tag{B.44}$$

Combining (B.43) and (B.44), we have

$$\Pi_S^D(p_S^*|\gamma_A^0) > \Pi_S^D(p_S|\gamma_A^0),\ \forall p_S \leqslant \tau_D(\gamma_A^0)), \forall \gamma_A^0 \geqslant \gamma_A^*, \tag{B.45}$$

which implies that the content supplier would rather choose p_S^* than any $p_S < \tau_D(\gamma_A^0)$ for a given $\gamma_A^0 > \gamma_A^*$, as long as $\Pi_{iP}^D(p_S^*|\gamma_A^0) \geqslant 0$. In other words, a paid-content-with-ads strategy is ruled out if $\Pi_{iP}^D(p_S^*|\gamma_A^0) \geqslant 0$ holds.

Even if $\Pi_{iP}^D(p_S^*|\gamma_A^0) < 0$, there always exists a $p_S^{**} \in (p_S^*, \overline{\tau})$ such that

$$(1)\Pi_{iP}^D(p_S^{**}|\gamma_A^0) = 0 \tag{B.46}$$

$$(2)\Pi_S^D(p_S^{**}|\gamma_A^0) > \Pi_S^D(p_S|\gamma_A^0), \forall p_S < \tau_D(\gamma_A^0)). \tag{B.47}$$

where $\tau \equiv \frac{(v-2t)\gamma_A}{2t}$ is the highest content price the content supplier can choose (from Lemma B.3). We obtain (1) shown above because $\Pi_{iP}^D(p_S^*|\gamma_A^0) < 0$, $\Pi_{iP}^D(\overline{\tau}|\gamma_A^0) > 0, \forall \gamma_A^0$ (by Lemma B.7 given below), and $\Pi_{iP}^D(p_S|\gamma_A^0)$ is a continuous function of p_S. We obtain (2) shown above because $\Pi_C^D(p_S|\gamma_A^0)$ increases with p_S and thus, whenever (1) holds, we have $\Pi_S^D(p_S^{**}|\gamma_A^0) = \Pi_C^D(p_S^{**}|\gamma_A^0) > \Pi_C^D(\tau_D|\gamma_A^0) \geqslant \Pi_S^D(p_S|\gamma_A^0), \forall p_S < \tau_D(\gamma_A^0)$. This implies that a paid-content-with-ads strategy is still ruled out even when $\Pi_{iP}^D(p_S^*|\gamma_A^0) < 0$.

Therefore, a free-content strategy is adopted at any γ_A greater than γ_A^*. ■

Claim B.2. *There exists a γ_A at which a free-content strategy is adopted.*

Proof. First, under the free-content strategy, because $\Pi_S^D(p_S)$ is concave in $p_S \geqslant \tau_D$, the price p_S that maximizes the content supplier's profits is obtained upon solving $\frac{\partial \Pi_S^D(p_S)}{\partial p_S} = 0$ for $p_S \geqslant \tau_D$. We obtain:

$$p_S = p_{fS}^D \equiv \frac{\gamma_A\left(v - 3t + \sqrt{v(3t+v)}\right)}{9t}. \tag{B.48}$$

Note that p_{fS}^D is valid if and only if $p_{fS}^D \geqslant \tau_D$, or equivalently,

$$\gamma_A \geqslant \gamma_{A1}^D \equiv -v + \sqrt{v(v+3t)}. \tag{B.49}$$

Second, under a paid-content-with-ads strategy, since $\Pi_S^D(p_S)$ is concave in $p_S < \tau_D$, the price p_S that maximizes the content supplier's profits is obtained upon solving $\frac{\partial \Pi_S^D(p_S)}{\partial p_S} = 0$ for $< p_S < \tau_D$. We find that:

$$p_S = p_{paS}^D \equiv \frac{1}{4} \cdot (-\gamma_A + v) \tag{B.50}$$

Note that p_{paS}^D is valid if and only if $p_{paS}^D < \tau_D$, which is equivalent to:

$$\gamma_A < \gamma_{A2}^D \equiv -\frac{v}{2} + \frac{1}{2}\sqrt{v(v+8t)}. \tag{B.51}$$

On comparing γ_{A1}^D and γ_{A2}^D, we have $\gamma_{A1}^D < \gamma_{A2}^D$.

Now suppose $\gamma_A > \gamma_{A2}^D$. Then, since $p_{fS}^D > \tau_D$ and $p_{paS}^D > \tau_D$, the content supplier's profits $\Pi_S^D(p_S)$ increases in p_S when $p_S < p_{fS}^D$ and then decreases when $p_S > p_{fS}^D$. In this case, the content supplier chooses $p_S = p_{fS}^D$ as long as $\Pi_{iP}^D(p_S = p_{fS}^D) \geqslant 0$. Otherwise, it chooses $p_S = p_S^{**}(\in (p_S^*, \overline{\tau}))$ satisfying $\Pi_{iP}^D(p_S^{**}) = 0$, following the same argument in the proof of Claim 1 (see the last paragraph). Therefore, a free-content strategy is adopted when $\gamma_A > \gamma_{A2}^D$. This completes the proof. ■

Claim B.3. *There exists a γ_A at which the paid-content-with-ads strategy is adopted.*

Proof. Suppose $\gamma_A < \gamma_{A1}^D$. Then, since $p_{fS}^D < \tau_H^D$ and $p_{paS}^D < \tau_H^D$, the content supplier's profits $\Pi_S^D(p_S)$ increases in p_S when $p_S < p_{paS}^D$ and then decreases when $p_S > p_{paS}^D$. In this case, the content supplier chooses $p_S = p_{paS}^D$ as long as $\Pi_{iP}^D(p_S = p_{paS}^D) \geqslant 0$, which is equivalent to $v - 2\sqrt{tv} \leqslant \gamma_A \leqslant v + 2\sqrt{tv}$. Finally, it is easy to show that $\gamma_{A1}^D < v + \gamma_C + 2\sqrt{tv}$ always holds and that $\gamma_{A1}^D > v - 2\sqrt{tv}$ holds for some set of parameter values. Hence, when $v - 2\sqrt{tv} \leqslant \gamma_A < \gamma_{A1}^D$, the paid-content-with-ads strategy is adopted. This completes the proof. ■

Taken together, these claims complete the proof of the lemma. □

Lemma B.7. *The market is completely covered in equilibrium if the following condition hold: $\gamma_A > t$ and $v \geqslant \max\{\frac{8t}{5-\sqrt{5}}, \frac{t\cdot\gamma_A^2}{\gamma_A^2 - t^2}\}$.*

Proof. The analysis in Lemma B.4 also pertains to here, except the content price is p_S is chosen by the content supplier. Therefore, we only need to derive the condition when the equilibrium content price under the free-content strategy satisfies $p_S \leqslant \overline{\tau}$.

Note that, under a free-content strategy, the equilibrium content price is given as either p_{fS}^D (interior solution) or p_S^{**} (corner solution). First, upon solving $p_{fS}^D \leqslant \overline{\tau}$, we have

$$v \geqslant \frac{8t}{5-\sqrt{5}} \tag{B.52}$$

Next, because the platform's IR condition is binding at $p_S = p_S^{**}$ (i.e., $\Pi_{iP}^D(p_S = p_S^{**}) = 0$), note that $p_S^{**} < \overline{\tau}$ suggests $\Pi_{iP}^D(p_S = \overline{\tau}) \geqslant 0$. Note that $\Pi_{iP}^D(p_S = \overline{\tau}) = \frac{\left(2tv - t\sqrt{v(-t+v)} + v(-v+\sqrt{v(-t+v)})\right)\cdot\gamma_A}{2tv}$. Upon solving $\Pi_{iP}^D(p_S = \overline{\tau}) \geqslant 0$, we have $v > t$. This is automatically satisfied by the condition $v \geqslant 2t$.

Combining (B.52) with $\gamma_A > t$ and $v \geqslant \max\{2t, \frac{t\cdot\gamma_A^2}{\gamma_A^2 - t^2}\}$, and noting that $\frac{8t}{5-\sqrt{5}} > 2t$, we get the full market coverage condition as in the lemma.

□

B.2.3 Proof of Proposition 3.3

Proof. Define v' as the largest root of $G(v) = 0$, where

$$G(v) \equiv -17v^4 - 92t\cdot v^3 + 1242t^2\cdot v^2 + 3492t^3\cdot v - 11025t^4$$

If $v < v'$, then $\gamma_A^M < \gamma_{A1}^D$ holds because $\gamma_A^M < \gamma_{A1}^D$ is equivalent to $G(v) > 0$. Since, according to the proof of Lemma B.6 (see Claim B.3), we also have $\widehat{\gamma_A^D} \geqslant \gamma_{A1}^D$, it is easy to see that $\gamma_{A3}^M < \gamma_{A1}^D$ implies $\widehat{\gamma_A^M} < \widehat{\gamma_A^D}$. Therefore, if $v < v'$, we have $\widehat{\gamma_A^M} < \widehat{\gamma_A^D}$, which implies that in the presence of inter-platform competition, a platform switches from a free-content strategy to a paid-content-with-ads strategy but not the other way round.

□

B.2.4 Proof of Proposition 3.4

Proof. According to Lemma B.3, when $\gamma_A \geqslant \widehat{\gamma_A^M}$, a monopoly platform chooses $p_S = p_{fS}^M$ and pursues a free-content strategy. In this case, by the definition of p_{fS}^M

(see (B.34) and the paragraph above), $\Pi_P^M(p_S = p_{fS}^M) = 0$ holds. This proves the first part.

Next, according to Claim B.2 of Lemma B.6, when $\gamma_A > \gamma_{A2}^D$, a duopoly platform pursues a free-content strategy. It also chooses $p_S = p_{fS}^D$ when $\Pi_{iP}^D(p_S = p_{fS}^D) > 0$ which holds for any γ_A as long as both $v < 6.9641t$.

Therefore, when $\gamma_A > \max\{\gamma_A^M, \gamma_{A2}^D\}$ and $v < 6.9641t$, under a free-content strategy a duopoly platform earns positive profits while a monopoly platform loses all its profits to the content supplier. This completes the proof.

□

Corollary B.1. *The content supplier's profits benefit from a larger γ_A under free-content strategy, but are hurt by a larger γ_A under paid-content-with-ads strategy.*

Proof. This claim holds when the platform market is either a monopoly market or a duopoly market. We prove the result for the two markets in the following two parts:

Part 1: In the presence of a monopoly platform

Under free-content strategy, the content supplier captures all the revenue from the platform. Because the platform only earns advertising revenue, the content supplier's profits are equal to the platform's advertising revenue : $\Pi_S^{M*} = \gamma_A \cdot (1 - \alpha_f^M)$, where $\alpha_f^M = \frac{v - \sqrt{v \cdot (v-2t)}}{v}$ given in (B.32). Because α_f^M is not a function of γ_A, it immediately follows that Π_S^{M*} increases with γ_A.

Under paid-content-with-ads strategy, the demand function facing the supplier is $\alpha_{pa}^M = \frac{-p_S - \gamma_A + v}{v}$ given in (B.32). Note that

$$\frac{\partial \alpha_{pa}^M}{\partial \gamma_A} = -\frac{1}{v} < 0 \tag{B.53}$$

which implies that a increase in γ_A shifts the supplier's demand curve inwards. As a result, the supplier's equilibrium profits are hurt by a larger γ_A

Part 2: In the presence of a duopoly platforms

Under free-content strategy, the demand function facing the supplier is $2\alpha_f^D = 2\left(1 - \frac{t(2p_S+\gamma_A)}{\sqrt{t \cdot v\gamma_A(2p_S+\gamma_A)}}\right)$ given in (B.40). Note that

$$\frac{\partial 2\alpha_f^D}{\partial \gamma_A} = \frac{2p_S \cdot t}{\gamma_A\sqrt{tv\gamma_A(2p_S + \gamma_A)}} > 0 \tag{B.54}$$

which implies that a increase in γ_A shifts the supplier's demand curve outwards. As a result, the supplier's equilibrium profits benefit from a larger γ_A.

Under paid-content-with-ads strategy, the demand function facing the supplier is $2\alpha_{pa}^D = 2 \cdot \frac{-2p_S-\gamma_A+v}{v}$ given in (B.40). Note that

$$\frac{\partial 2\alpha_{pa}^D}{\partial \gamma_A} = -\frac{4}{v} > 0 \tag{B.55}$$

which implies that a increase in γ_A shifts the supplier's demand curve inwards. As a result, the supplier's equilibrium profits are hurt by a larger γ_A.

This completes the proof.

□

B.3 Model Extension

B.3.1 Analysis of a Moderately Competitive Content Market

We establish 6 lemmas to prove Proposition 3.5 and 3.6. Lemma B.8,B.9 and B.10 are the analysis in the presence of a monopoly platform, and derive each supplier's profits under different strategies. Lemma B.11, B.12 and B.13 are the analysis in the presence of duopoly platforms, and derive each supplier's profits under different strategies.

Monopoly Platform

Lemma B.8. *In the second stage, the platform's optimal choice of* $\{\alpha_1, h_1, p_{1C}\}$ *needs to satisfy the following condition*

$$h_1 = \begin{cases} 0, & \textit{if} \quad p_{1S} - p_{2S} \in (-\infty, \frac{v(2-\alpha_1)}{2}) \\ h, & \textit{if} \quad p_{1S} - p_{2S} \in [-\frac{v(2-\alpha_1)}{2}, \frac{v(2-\alpha_1)}{2}] \\ 1, & \textit{if} \quad p_{1S} - p_{2S} \in (\frac{v(2-\alpha_1)}{2}, \infty) \end{cases} \tag{B.56}$$

Proof. Given each supplier's choice of $\{\kappa_j, p_{jS}\}$, $j \in \{1, 2\}$ in the first stage, $h = \frac{1}{1+\exp(\kappa_2-\kappa_1)}$ is realized. After observing (p_{1S}, p_{2S}), the monopoly platform chooses $\{\alpha_1, h_1, p_{1C}\}$ to maximize its own profits. Given the market is fully covered, the consumer price and the platform's profits are given as

$$p_{1C} = v \cdot (1 - |h - h_1|) \cdot (\alpha_1 - \frac{\alpha_1^2}{2}) - t \tag{B.57}$$

$$\Pi_P^M = -t - \overline{p_S} \cdot \alpha_1 + (1 - \alpha_1) \cdot \gamma_A + (\alpha_1 - \frac{1}{2}\alpha_1^2) \cdot v \cdot (1 - |h - h_1|) \tag{B.58}$$

where $\overline{p_S} \equiv h_1 \cdot p_{1S} + (1 - h_1) \cdot p_{2S}$ is the "average" price of one unit of content. By calculating $\frac{\partial \Pi_P^M}{\partial h_1}$, we have

$$\frac{\partial \Pi_P^M}{\partial h_1} = \begin{cases} \frac{1}{2} \cdot \Big(-2p_{1S} + 2p_{2S} - v(-2 + \alpha_1)\Big) \cdot \alpha_1, & \text{if } h_1 < h \\ \frac{1}{2} \cdot \Big(-2p_{1S} + 2p_{2S} + v(-2 + \alpha_1)\Big) \cdot \alpha_1, & \text{if } h_1 > h \end{cases} \tag{B.59}$$

This suggests that Π_P^M is linear in h_1 when $h_1 \in [0, h)$ and when $h_1 \in (h, 1]$. Furthermore, when $p_{1S} - p_{2S} < -\frac{v(2-\alpha_1)}{2}$, we have $\frac{\partial \Pi_P^M}{\partial h_1} > 0$ for $h_1 \in [0, h)$ and $h_1 \in (h, 1]$; When $-\frac{v(2-\alpha_1)}{2} \leqslant p_{1S} - p_{2S} \leqslant \frac{v(2-\alpha_1)}{2}$, we have $\frac{\partial \Pi_P^M}{\partial h_1} \geqslant 0$ for $h_1 \in [0, h)$, while $\frac{\partial \Pi_P^M}{\partial h_1} \leqslant 0$ for $h_1 \in (h, 1]$; when $p_{1S} - p_{2S} > \frac{v(2-\alpha_1)}{2}$, we have $\frac{\partial \Pi_P^M}{\partial h_1} < 0$ for $h_1 \in [0, h)$ and $h_1 \in (h, 1]$. Also note Π_P^M is continuous in h_1. Therefore, given $\{\alpha_1, h_1, p_{1C}\}$ is the monopoly platform's optimal choice, h_1 needs satisfy the following condition:

$$h_1 = \begin{cases} 0, & \text{if} \quad p_{1S} - p_{2S} \in (-\infty, \frac{v(2-\alpha_1)}{2}) \\ h, & \text{if} \quad p_{1S} - p_{2S} \in [-\frac{v(2-\alpha_1)}{2}, \frac{v(2-\alpha_1)}{2}] \\ 1, & \text{if} \quad p_{1S} - p_{2S} \in (\frac{v(2-\alpha_1)}{2}, \infty) \end{cases} \tag{B.60}$$

This completes the proof. □

Lemma B.9. *When free-content strategy is adopted, in the symmetric equilibrium, each content supplier's profits are*

$$\Pi^*_{jS} = \frac{\sqrt{v(v-2t)} \cdot \gamma_A}{2v} - \frac{(v-2t) \cdot \gamma_A^2}{64v \cdot \phi}, \quad j \in \{1,2\} \tag{B.61}$$

Proof. In this proof, we first derive a content supplier's profits when the media platform adopts free-content strategy. Then we give a sufficient condition for free-content strategy to be adopted.

Given that we focus on the symmetric equilibrium, from Lemma B.8, we know that in equilibrium $h_1 = h$. Then the platform's profits function (B.58) becomes

$$\Pi_P^M = -t - \overline{p_S} \cdot \alpha_1 + (1-\alpha_1) \cdot \gamma_A + (\alpha_1 - \frac{1}{2}\alpha_1^2) \cdot v \tag{B.62}$$

The above profits function is identical to (B.5) if we were to replace c with $\overline{p_S}$. This implies that in the second stage, given the content suppliers' prices, the monopoly platform's demand for content is given as

$$\alpha_1(p_{1S}, p_{2S}) = \begin{cases} \alpha_f^M = \frac{v - \sqrt{v \cdot (v-2t)}}{v}, & \text{if } \overline{p_S} \geqslant \tau_M \\ \alpha_{pa}^M = \frac{-\overline{p_S} - \gamma_A + v}{v}, & \text{if } \overline{p_S} < \tau_M \end{cases} \tag{B.63}$$

where $\overline{p_S} \equiv h \cdot p_{1S} + (1-h) \cdot p_{2S}$ (because $h_1 = h$), and $\tau_M = -\gamma_A + \sqrt{v \cdot (v-2t)}$. Furthermore α_f^M and α_{pa}^M give the demand for content from the monopoly platform when it adopts a free-content strategy and a paid-content-with-ads strategy respectively.

Next we move to the first stage. Foreseeing the monopoly's demand function, each supplier's profits are

$$\Pi_{1S} = p_{1S} \cdot \alpha_1 \cdot h - \phi \cdot \kappa_1^2 \tag{B.64}$$

$$\Pi_{2S} = p_{2S} \cdot \alpha_1 \cdot (1-h) - \phi \cdot \kappa_2^2 \tag{B.65}$$

where $h = \frac{1}{1+\exp(\kappa_2 - \kappa_1)}$. Each supplier chooses its content price p_{jS} and effort κ_j to maximize its own profits.

The adoption of free-content strategy implies $\alpha_1 = \alpha_f^M$. Substituting $\alpha_1(p_{1S}, p_{2S}) = \alpha_f^M$ in (B.64) and (B.65), we have the two content supplier's profits functions as

$$\Pi_{1S}(\alpha_1 = \alpha_f^M) = \frac{p_{1S} \cdot (v - \sqrt{v \cdot (v - 2t)})}{v} \cdot h - \phi\kappa_1^2 \tag{B.66}$$

$$\Pi_{2S}(\alpha_1 = \alpha_f^M) = \frac{p_{2S} \cdot (v - \sqrt{v \cdot (v - 2t)})}{v} \cdot (1 - h) - \phi\kappa_2^2 \tag{B.67}$$

Note that each supplier's profits is a linear function of its own price. This implies that the monopoly platform must earn zero profits in equilibrium. The reason is that, if not, each supplier can continue to increase price to improve profits. Consequently, from (B.34) in Lemma B.5, we know that the equilibrium content prices pair $\{p_{1S}, p_{2S}\}$ must satisfy

$$h \cdot p_{1S} + (1 - h) \cdot p_{2S} = \overline{p_S} = p_{fS}^M \equiv \frac{\gamma_A \cdot \sqrt{v \cdot (v - 2t)}}{v - \sqrt{v \cdot (v - 2t)}} \tag{B.68}$$

On the other hand, for any given h, the content prices pair $\{p_{1S}, p_{2S}\}$ that satisfies (B.68) constitutes equilibrium, because no content supplier can further increase its own price to improve profits. Therefore, there is a continuum of equilibrium. We focus our attention on the symmetric equilibrium where $p_{1S}^* = p_{2S}^* = p_{fS}^M$.

We then solve the equilibrium κ_j. Given $h = \frac{1}{1+\exp(\kappa_2 - \kappa_1)}$, by solving $\frac{\partial \Pi_{1S}}{\partial \kappa_1} = 0$ and $\frac{\partial \Pi_{2S}}{\partial \kappa_2} = 0$ simultaneously, we obtain

$$\kappa_j^* = \frac{\sqrt{v(v - 2t)}\gamma_A}{8v\phi}, \quad j \in \{1, 2\} \tag{B.69}$$

Next we check the second order condition:

$$\left.\frac{\partial^2 \Pi_{1S}}{\partial \kappa_j^2}\right|_{\kappa_1 = \kappa_2 = \frac{\sqrt{v(v-2t)}\gamma_A}{8v\phi}} = -2\phi, \quad j \in \{1, 2\} \tag{B.70}$$

Therefore the SOC is satisfied as long as $\phi > 0$.Given $p_{1S} = p_{2S} = p_{fS}^M$ and $\kappa_1 =$

$\kappa_2 = \frac{\sqrt{v(v-2t)}\gamma_A}{8v\phi}$, each supplier's equilibrium profits under free-content strategy are

$$\Pi^*_{jS} = \frac{\sqrt{v(v-2t)} \cdot \gamma_A}{2v} - \frac{(v-2t) \cdot \gamma_A^2}{64v \cdot \phi}, \quad j \in \{1, 2\} \tag{B.71}$$

Note the first term $\frac{\sqrt{v(v-2t)} \cdot \gamma_A}{2v}$ is the revenue from selling content. The second term $\frac{(v-2t) \cdot \gamma_A^2}{64v \cdot \phi}$ is the cost of κ_j.

Lastly, we give a sufficient condition for free-content to be adopted. According to (B.63), free-content strategy is adopted if $\overline{p_S} \geqslant \tau_M$. Note that if $\tau_M \leqslant 0$, then $\overline{p_S} \geqslant \tau_M$ trivially holds, suggesting that free-content strategy is the equilibrium. Upon solving $\tau_M \leqslant 0$, we get $\gamma_A \geqslant \sqrt{v \cdot (v-2t)}$. Therefore, a sufficient condition for free-content strategy is $\gamma_A \geqslant \sqrt{v \cdot (v-2t)}$.

This completes the proof. □

Lemma B.10. *When paid-content-with-ads strategy is adopted, in the symmetric equilibrium, each content supplier's profits are*

$$\Pi^*_{jS} = \frac{(v-\gamma_A)^2}{9v} - \frac{(v-\gamma_A)^4}{1296v^2 \cdot \phi} \quad j \in \{1, 2\} \tag{B.72}$$

Proof. In this proof, we first derive a content supplier's profits when the media platform adopts paid-content-with-ads. Then we give a sufficient condition for paid-content-with-ads to be adopted.

From (B.63), when paid-content-with-ads strategy is adopted, we have $\alpha_1 = \alpha^M_{pa} = \frac{-\overline{p_S}-\gamma_A+v}{v}$. Upon simultaneously solving $\frac{\partial \Pi_{1S}}{\partial p_{1S}} = 0$, $\frac{\partial \Pi_{1S}}{\partial \kappa_1} = 0$, $\frac{\partial \Pi_{2S}}{\partial p_{2S}} = 0$,and $\frac{\partial \Pi_{2S}}{\partial \kappa_2} = 0$, we obtain

$$p_{jS} = \frac{2(v-\gamma_A)}{3}, \quad \kappa_j = \frac{(v-\gamma_A)^2}{36v\phi}, \quad j \in \{1, 2\} \tag{B.73}$$

Next we check the second condition:

$$\left.\frac{\partial^2 \Pi_{jS}}{\partial p_{jS}^2}\right|_{(p_{1S}=p_{2S}=\frac{2(v-\gamma_A)}{3},\kappa_1=\kappa_2=\frac{(v-\gamma_A)^2}{36v\phi})} = -\frac{1}{2v} < 0 \tag{B.74}$$

$$\left.\frac{\partial^2 \Pi_{jS}}{\partial \kappa_{jS}^2}\right|_{(p_{1S}=p_{2S}=\frac{2(v-\gamma_A)}{3},\kappa_1=\kappa_2=\frac{(v-\gamma_A)^2}{36v\phi})} = -2\phi < 0 \tag{B.75}$$

$$\left.\frac{\partial^2 \Pi_{jS}}{\partial p_{jS}^2}\cdot\frac{\partial^2 \Pi_{jS}}{\partial \kappa_j^2} - (\frac{\partial^2 \Pi_{jS}}{\partial pjS\partial \kappa_j})^2\right|_{(p_{1S}=p_{2S}=\frac{2(v-\gamma_A)}{3},\kappa_1=\kappa_2=\frac{(v-\gamma_A)^2}{36v\phi})} = \frac{\phi}{v} - \frac{(v-\gamma_A)^2}{144v^2} \tag{B.76}$$

Therefore, the SOC condition is satisfied if and only if $\phi > \frac{(v-\gamma_A)^2}{144v}$. Given that SOC is satisfied, each supplier's profits under paid-content-with-ads strategy is

$$\Pi_{jS}^* = \frac{(v-\gamma_A)^2}{9v} - \frac{(v-\gamma_A)^4}{1296v^2\cdot\phi} \quad j \in \{1,2\} \tag{B.77}$$

Note the first term $\frac{(v-\gamma_A)^2}{9v}$ is the revenue from selling content. The second term $\frac{(v-\gamma_A)^4}{1296v^2\cdot\phi}$ is the cost of κ_j.

Lastly, we derive a sufficient condition for paid-content-with-ads to be adopted. All we need to do is to ensure that no supplier wants to unilaterally significantly increase its price to induce a platform to switch to free-content strategy. From Lemma B.9, we know that under free-content strategy, the two content suppliers fully extract a monopoly platform's revenue. Furthermore, depending on how the two suppliers split the platform's revenue, there is a continuum of equilibrium. We assume that the two suppliers choose the equilibrium where they equally split the revenue from the platform. Note under free-content, the total revenue a monopoly platform earns is $\gamma_A\cdot(1-\alpha_f^M) = \frac{\sqrt{v(v-2t)}\cdot\gamma_A}{v}$, where α_f^M is given in (B.63). This means each supplier gets half of it: $\frac{\sqrt{v(v-2t)}\cdot\gamma_A}{2v}$. Therefore, incorporating the cost on κ_j, the profits of deviating to free-content strategy is

$$\Pi_{jS}^{Deviation} = \frac{\sqrt{v(v-2t)}\cdot\gamma_A}{2v} - \frac{(v-\gamma_A)^4}{1296v^2\cdot\phi} \tag{B.78}$$

If each supplier has no incentive to deviate, we must have $\Pi^*_{jS} \geqslant \Pi^{Deviation}_{jS}$. Upon solving this, we have

$$\gamma_A \leqslant v + \frac{9}{4}\sqrt{v(v-2t)} - \frac{3}{4}v\sqrt{\frac{9(v-2t)+8\sqrt{v(v-2t)}}{v}} \tag{B.79}$$

Therefore, a sufficient condition for paid-content-with-ads strategy is $\gamma_A \leqslant v + \frac{9}{4}\sqrt{v(v-2t)} - \frac{3}{4}v\sqrt{\frac{9(v-2t)+8\sqrt{v(v-2t)}}{v}}$. This completes the proof. □

Duopoly Platforms

Lemma B.11. *In the second stage, each platform's optimal choice of* $\{\alpha_i, h_i, p_{iC}\}, i \in \{1,2\}$ *needs to satisfy the following condition*

$$h_i = \begin{cases} 0, & \textit{if} \quad p_{1S} - p_{2S} \in (-\infty, \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}) \\ h, & \textit{if} \quad p_{1S} - p_{2S} \in [-\frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}, \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}] \\ 1, & \textit{if} \quad p_{1S} - p_{2S} \in (\frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}, \infty) \end{cases} \tag{B.80}$$

Proof. Given each supplier's choice of $\{\kappa_j, p_{jS}\}$, $j \in \{1,2\}$ in the first stage, $h = \frac{1}{1+\exp(\kappa_2-\kappa_1)}$ is realized. After observing (p_{1S}, p_{2S}), each platform chooses $\{\alpha_i, h_i, p_{iC}\}$ to maximize its own profits. The utility that the consumer located at x derives on joining platform 1 and 2 are given as:

$$U_{1C}(x) = v \cdot (1 - |h - h_1|) \cdot (\alpha_1 - \frac{1}{2}\alpha_1^2) - t \cdot x - p_{1C} \tag{B.81}$$

$$U_{2C}(x) = v \cdot (1 - |h - h_2|) \cdot (\alpha_2 - \frac{1}{2}\alpha_2^2) - t \cdot (1-x) - p_{2C} \tag{B.82}$$

By solving $U_{1C}(x) = U_{2C}(x)$, we have the marginal consumer who is indifferent between joining the two platforms:

$$x_0 = \frac{1}{2} + \frac{V(\alpha_1, h_1) - V(\alpha_2, h_2)}{2t} - \frac{p_{1C} - p_{2C}}{2t} \tag{B.83}$$

where $V(\alpha_i, h_i) = v \cdot (1 - |h-h_i|) \cdot (\alpha_i - \frac{1}{2}\alpha_i^2)$ is the gross utility a consumer derives from joining platform i. Therefore, each platform's number of consumers are $n_{1C} = x_0$

and $n_{2C} = 1 - x_0$. Consequently, platform i's profits are

$$\Pi_{iP}^D = n_{iC} \cdot p_{iC} + (1 - \alpha_i) \cdot p_{iA} - \alpha_i \cdot \overline{p_{iS}} \tag{B.84}$$

where $p_{iA} = \gamma_A \cdot n_{iC}$, and $\overline{p_{iS}} \equiv h_i \cdot p_{1S} + (1 - h_i) \cdot p_{2S}$ is the "average" content price to platform i. By calculating $\frac{\partial \Pi_{iP}^D}{\partial h_i}$, we have

$$\frac{\partial \Pi_{iP}^D}{\partial h_i} = \begin{cases} \alpha_i \cdot \Big(- (p_{1S} - p_{2S}) + \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t} \Big), & \text{if } h_i < h \\ \alpha_i \cdot \Big(- (p_{1S} - p_{2S}) - \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t} \Big), & \text{if } h_i > h \end{cases} \tag{B.85}$$

This suggests that Π_{iP}^D is linear in h_i when $h_i \in [0, h)$ and when $h_i \in (h, 1]$. Furthermore, when $p_{1S} - p_{2S} < -\frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}$, we have $\frac{\partial \Pi_{iP}^D}{\partial h_i} > 0$ for $h_i \in [0, h)$ and $h_i \in (h, 1]$; When $-\frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t} \leqslant p_{1S} - p_{2S} \leqslant \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}$, we have $\frac{\partial \Pi_{iP}^D}{\partial h_i} \geqslant 0$ for $h_i \in [0, h)$, while $\frac{\partial \Pi_{iP}^D}{\partial h_i} \leqslant 0$ for $h_i \in (h, 1]$; When $p_{1S} - p_{2S} > \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}$, we have $\frac{\partial \Pi_{iP}^D}{\partial h_i} < 0$ for $h_i \in [0, h)$ and $h_i \in (h, 1]$. Also note Π_{iP}^D is continuous in h_i. Therefore, if $\{\alpha_i, h_i, p_{iC}\}, i \in \{1, 2\}$ constitutes the second stage equilibrium, h_i needs satisfy the following condition:

$$h_i = \begin{cases} 0, & \text{if} \quad p_{1S} - p_{2S} \in (-\infty, \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}) \\ h, & \text{if} \quad p_{1S} - p_{2S} \in [-\frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}, \frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}] \\ 1, & \text{if} \quad p_{1S} - p_{2S} \in (\frac{v(2-\alpha_i)(p_{iC}+\gamma_A(1-\alpha_i))}{4t}, \infty) \end{cases} \tag{B.86}$$

This completes the proof.

□

Lemma B.12. *When Free-content strategy is adopted, in the symmetric equilibrium, each content supplier's profits are*

$$\begin{aligned} \Pi_{jS}^* =& \frac{(2v - 5t + \sqrt{v(5t+4v)})\Big(5v - \sqrt{v(5t + 8v + 4\sqrt{v(5t+4v)})}\Big) \cdot \gamma_A}{125vt} \\ &- \frac{\Big(5t(-4v + \sqrt{v(5t+4v)}) + v(2v + \sqrt{v(5t+4v)})\Big)^2 \cdot \gamma_A^2}{250000t^2v^2 \cdot \phi} \end{aligned} \tag{B.87}$$

Proof. In this proof, we first derive a content supplier's profits when media platforms adopt free-content strategy. Then we give a sufficient condition for free-content strategy to be adopted.

Given that we focus on the symmetric equilibrium, from Lemma B.11, we know that in equilibrium $h_1 = h_2 = h$. The consumer utility derived from joining two platforms, (B.81) and (B.82), then become

$$U_{1C}(x) = v \cdot (\alpha_1 - \frac{1}{2}\alpha_1^2) - t \cdot x - p_{1C} \tag{B.88}$$

$$U_{2C}(x) = v \cdot (\alpha_2 - \frac{1}{2}\alpha_2^2) - t \cdot (1-x) - p_{2C} \tag{B.89}$$

which are the same as those in a monopoly content market. Furthermore, $h_1 = h_2 = h$ implies that each content supplier faces the same "average" content price: $\overline{p_{1S}} = \overline{p_{2S}} = \overline{p_S} \equiv h \cdot p_{1S} + (1-h_1) \cdot p_{2S}$. This implies that each platform's profits function (B.84) is identical to that in a monopoly content market, if we were to replace c with $\overline{p_S}$. As a result, given the content suppliers' prices, each platform's demand for content is given as

$$\alpha_i(p_{1S}, p_{2S}) = \begin{cases} \alpha_f^D = 1 - \frac{t(2\overline{p_S}+\gamma_A)}{\sqrt{t \cdot v \gamma_A (2\overline{p_S}+\gamma_A)}}, & \text{if } \overline{p_S} \geqslant \tau_D \\ \alpha_{pa}^D = \frac{-2\overline{p_S}-\gamma_A+v}{v}, & \text{if } \overline{p_S} < \tau_D \end{cases} \tag{B.90}$$

where $\overline{p_S} \equiv h \cdot p_{1S} + (1-h) \cdot p_{2S}$ and $\tau_D = \frac{1}{2} \cdot (-\gamma_A + \frac{t \cdot v}{\gamma_A})$. Furthermore, α_f^D and α_{pa}^D give the demand for content from a duopoly platform when it pursues a free-content strategy and a paid-content-with-ads strategy respectively.

Next we move to the first stage. Foreseeing each platform's proportion of content, each supplier's profits are

$$\Pi_{1S} = p_{1S} \cdot \Sigma_i \Big(\alpha_i \cdot h_i\Big) - \phi \cdot \kappa_1^2 \tag{B.91}$$

$$\Pi_{2S} = p_{2S} \cdot \Sigma_i \Big(\alpha_i \cdot (1-h_i)\Big) - \phi \cdot \kappa_2^2 \tag{B.92}$$

where $h = \frac{1}{1+\exp(\kappa_2 - \kappa_1)}$. Each supplier chooses its content price p_{jS} and effort κ_j to maximize its own profits.

The adoption of free-content strategy implies $\alpha_i = \alpha_f^D$. Substituting $\alpha_1 = \alpha_2 = \alpha_f^D$ in (B.91) and (B.92), and simultaneously solving $\frac{\partial \Pi_{1S}}{\partial p_{1S}} = 0$, $\frac{\partial \Pi_{1S}}{\partial \kappa_1} = 0$,$\frac{\partial \Pi_{2S}}{\partial p_{2S}} = 0$,and $\frac{\partial \Pi_{2S}}{\partial \kappa_2} = 0$, we obtain

$$p_{1S} = p_{2S} = p_{jS}^* \equiv \frac{2\Big(2v - 5t + \sqrt{v(5t+4v)}\Big)\gamma_A}{25t} \tag{B.93}$$

$$\kappa_1 = \kappa_2 = \kappa_j^* \equiv \frac{\Big(5t(-4v + \sqrt{v(5t+4v)}) + v(2v + \sqrt{v(5v+4t)}\Big)\gamma_A}{500tv\phi}, \quad j \in \{1,2\} \tag{B.94}$$

Next we check the second condition:

$$\frac{\partial^2 \Pi_{jS}}{\partial p_{jS}^2}\bigg|_{(p_{1S}=p_{2S}=p_{jS}^*, \kappa_1=\kappa_2=\kappa_j^*} = -\frac{5vt\Big(15t + 7(2v + \sqrt{v(5t+4v)}\Big)}{4\Big(v(5t+8v+4\sqrt{v(5t+4v)}\Big)^{\frac{3}{2}} \cdot \gamma_A} < 0 \tag{B.95}$$

$$\frac{\partial^2 \Pi_{jS}}{\partial \kappa_{jS}^2}\bigg|_{(p_{1S}=p_{2S}=p_{jS}^*, \kappa_1=\kappa_2=\kappa_j^*} = -2\phi < 0 \tag{B.96}$$

$$\frac{\partial^2 \Pi_{jS}}{\partial p_{jS}^2} \cdot \frac{\partial^2 \Pi_{jS}}{\partial \kappa_j^2} - (\frac{\partial^2 \Pi_{jS}}{\partial pjS \partial \kappa_j})^2\bigg|_{(p_{1S}=p_{2S}=p_{jS}^*, \kappa_1=\kappa_2=\kappa_j^*)} = \frac{5vt\Big(15t + 7(2v + \sqrt{v(5t+4v)}\Big)}{2\Big(v(5t+8v+4\sqrt{v(5t+4v)}\Big)^{\frac{3}{2}} \cdot \gamma_A} \cdot \phi$$

$$- \frac{\Big(5t - 12v - 6\sqrt{v(5t+4v)} + 5\sqrt{v(5t+8v+4\sqrt{v(5t+4v)}}\Big)^2}{400v(5t+8v+4\sqrt{v(5t+4v)})} \tag{B.97}$$

Therefore, the SOC condition is satisfied if and only if

$$\phi > \frac{\sqrt{v(5t+8v+4\sqrt{v(5t+4v)}} \cdot \Big(5t - 12v - 6\sqrt{v(5t+4v)} + 5\sqrt{v(5t+8v+4\sqrt{v(5t+4v)}}\Big)^2 \cdot \gamma_A}{1000tv(15t + 7(2v + \sqrt{v(5t+4v)})} \tag{B.98}$$

Given the SOC is satisfies, each supplier's profits under free-content strategy is

$$\Pi^*_{jS} = \frac{(2v - 5t + \sqrt{v(5t+4v)})\left(5v - \sqrt{v(5t+8v+4\sqrt{v(5t+4v)}}\right) \cdot \gamma_A}{125vt} - \frac{\left(5t(-4v + \sqrt{v(5t+4v)}) + v(2v + \sqrt{v(5t+4v)})\right)^2 \cdot \gamma_A^2}{250000t^2v^2 \cdot \phi} \tag{B.99}$$

Note the first term $\frac{(2v-5t+\sqrt{v(5t+4v)})\left(5v-\sqrt{v(5t+8v+4\sqrt{v(5t+4v)}}\right)\cdot\gamma_A}{125vt}$ is the revenue from selling content. The second term $\frac{\left(5t(-4v+\sqrt{v(5t+4v)})+v(2v+\sqrt{v(5t+4v)})\right)^2\cdot\gamma_A^2}{250000t^2v^2\cdot\phi}$ is the cost of κ_j.

Lastly, we give a sufficient condition for free-content to be adopted.According to (B.90), free-content strategy is adopted if $\overline{p_S} \geqslant \tau_D$. Note that if $\tau_D \leqslant 0$, then $\overline{p_S} \geqslant \tau_D$ trivially holds, suggesting that free-content strategy is the equilibrium. Upon solving $\tau_D \leqslant 0$, we get $\gamma_A \geqslant \sqrt{vt}$. Therefore, a sufficient condition for free-content strategy is $\gamma_A \geqslant \sqrt{vt}$.

This completes the proof.

□

Lemma B.13. *When paid-content-with-ads strategy is adopted, in the symmetric equilibrium, each content supplier's profits are*

$$\Pi^*_{jS} = \frac{(v-\gamma_A)^2}{18v} - \frac{(v-\gamma_A)^2}{5184v^2 \cdot \phi} \tag{B.100}$$

Proof. In this proof, we first derive a content supplier's profits when media platforms adopt paid-content-with-ads strategy. Then we give a sufficient condition for paid-content-with-ads to be adopted.

From (B.90), when paid-content-with-ads is adopted, we have $\alpha_i = \alpha^D_{pa} = \frac{-2\overline{p_S}-\gamma_A+v}{v}$.

Upon simultaneously solving $\frac{\partial \Pi_{1S}}{\partial p_{1S}} = 0$, $\frac{\partial \Pi_{1S}}{\partial \kappa_1} = 0$, $\frac{\partial \Pi_{2S}}{\partial p_{2S}} = 0$, and $\frac{\partial \Pi_{2S}}{\partial \kappa_2} = 0$, we obtain

$$p_{jS} = \frac{v-\gamma_A}{3}, \quad \kappa_j = \frac{(v-\gamma_A)^2}{72v\phi}, \quad j \in \{1,2\} \tag{B.101}$$

Next we check the second condition:

$$\left.\frac{\partial^2 \Pi_{jS}}{\partial p_{jS}^2}\right|_{(p_{1S}=p_{2S}=\frac{v-\gamma_A}{3},\kappa_1=\kappa_2=\frac{(v-\gamma_A)^2}{72v\phi})} = -\frac{1}{v} < 0 \tag{B.102}$$

$$\left.\frac{\partial^2 \Pi_{jS}}{\partial \kappa_{jS}^2}\right|_{(p_{1S}=p_{2S}=\frac{v-\gamma_A}{3},\kappa_1=\kappa_2=\frac{(v-\gamma_A)^2}{72v\phi})} = -2\phi < 0 \tag{B.103}$$

$$\left.\frac{\partial^2 \Pi_{jS}}{\partial p_{jS}^2} \cdot \frac{\partial^2 \Pi_{jS}}{\partial \kappa_j^2} - (\frac{\partial^2 \Pi_{jS}}{\partial p_{jS}\partial \kappa_j})^2\right|_{(p_{1S}=p_{2S}=\frac{v-\gamma_A}{3},\kappa_1=\kappa_2=\frac{(v-\gamma_A)^2}{72v\phi})} = \frac{2\phi}{v} - \frac{(v-\gamma_A)^2}{144v^2} \tag{B.104}$$

Therefore, the SOC condition is satisfied if and only if $\phi > \frac{(v-\gamma_A)^2}{288v}$. Given that SOC is satisfied, each supplier's profits under paid-content-with-ads strategy is

$$\Pi_{jS}^* = \frac{(v-\gamma_A)^2}{18v} - \frac{(v-\gamma_A)^4}{5184v^2 \cdot \phi} \quad j \in \{1,2\} \tag{B.105}$$

Note the first term $\frac{(v-\gamma_A)^2}{18v}$ is the revenue from selling content. The second term $\frac{(v-\gamma_A)^4}{5184v^2 \cdot \phi}$ is the cost of κ_j.

Lastly, we derive a sufficient condition for paid-content-with-ads to be adopted. Again we only need to ensure that no supplier wants to unilaterally significantly increase its price to induce a platform to switch to free-content strategy. Because of symmetry, we only need to find the condition that supplier 1 does not want to deviate. Note from (B.90), the platforms switch to free-content if and only if $\overline{p_S} \geqslant \tau_D$. Given $p_{2S} = \frac{v-\gamma_A}{3}$, upon solving $\overline{p_S} \geqslant \tau_D$, we obtain $p_{1S} \geqslant v \cdot (\frac{t}{\gamma_A} - \frac{1}{3}) - \frac{2\gamma_A}{3}$. Therefore, supplier 1 needs to deviate to $p_{1S} \geqslant v \cdot (\frac{t}{\gamma_A} - \frac{1}{3}) - \frac{2\gamma_A}{3}$ so that platforms will switch to free-content strategy. Platform 1's profits from deviating to $p_{1S} \geqslant v \cdot (\frac{t}{\gamma_A} - \frac{1}{3}) - \frac{2\gamma_A}{3}$ is

$$\Pi_{1S}^{Deviation} = p_{1S} \cdot \alpha_f^D \cdot h$$

where $h = \frac{1}{2}$. Note $\frac{\partial^2 \Pi_{1S}^{Deviation}}{\partial p_{1S}^2} = -\frac{\sqrt{3}t^2 v\gamma_A(9p_{1S}+4v+8\gamma_A)}{4(tv\gamma_A(3p_{1S}+v+2\gamma_A))^{\frac{3}{2}}} < 0$, implying that $\Pi_{1S}^{Deviation}$ is concave in p_{1S}. Furthermore, by continuity, we know that $\Pi_{1S}^{*} \geqslant \Pi_{1S}^{Deviation}\Big(p_{1S} = v\cdot(\frac{t}{\gamma_A} - \frac{1}{3}) - \frac{2\gamma_A}{3}\Big)$. Therefore, one sufficient condition for $\Pi_{1S}^{*} \geqslant \Pi_{1S}^{Deviation}\Big(\forall p_{1S} \geqslant v\cdot(\frac{t}{\gamma_A} - \frac{1}{3}) - \frac{2\gamma_A}{3}\Big)$ is that

$$\frac{\partial \Pi_{1S}^{Deviation}}{\partial p_{1S}}\Bigg|_{p_{1S}=v\cdot(\frac{t}{\gamma_A}-\frac{1}{3})-\frac{2\gamma_A}{3}} \leqslant 0$$

Upon solving this inequality, we get

$$\gamma_A \leqslant -\frac{7}{4}v + \frac{1}{4}\sqrt{72tv + 49v^2} \tag{B.106}$$

Therefore, a sufficient condition for paid-content-with-ads strategy is $\gamma_A \leqslant -\frac{7}{4}v + \frac{1}{4}\sqrt{72tv + 49v^2}$

This completes the proof.

□

B.3.2 Proof of Proposition 3.5

Proof. Note that the statement holds when the platform market is either a monopoly market or a duopoly market. We prove the result for the two markets in the following two parts.

Part 1: In the presence of a monopoly platform

According to Lemma B.9, under free-content strategy,each supplier's equilibrium profits in the presence of a monopoly platform Π_{jS}^{*} are given in (B.61). Upon solving $\frac{\partial \Pi_{jS}^{*}}{\partial \gamma_A} < 0$ we obtain

$$\phi < \frac{\sqrt{v(v-2t)}\gamma_A}{16v} \tag{B.107}$$

Part 2: In the presence of duopoly platforms

According to Lemma B.12, under free-content strategy,each supplier's equilibrium profits in the presence of a monopoly platform Π_{jS}^* are given in (B.87). Upon solving $\frac{\partial \Pi_{jS}^*}{\partial \gamma_A} < 0$ we obtain

$$\phi < \frac{\left(125t^3 + 4v^2(2v + \sqrt{v(5t+4v)}) + 50t^2(11v - 4\sqrt{v(5t+4v)}) - 5tv(7v + 4\sqrt{v(5v+4t)})\right) \cdot \gamma_A}{1000t(2v - 5t + \sqrt{v(5t+4v)})\left(5v - \sqrt{v(5t+8v+4\sqrt{v(5t+4v)}}\right)} \tag{B.108}$$

Note from (B.98) in Lemma B.12, the SOC condition requires $\phi > \frac{\sqrt{v(5t+8v+4\sqrt{v(5t+4v)}}\cdot\left(5t-12v-6\sqrt{v(5t+4v)}+5\sqrt{v(5t+8v+4\sqrt{v(5t+4v)}}\right)^2\cdot\gamma_A}{1000tv(15t+7(2v+\sqrt{v(5t+4v)})}$. To ensure that we are not discussing a null set, we can set $\{t = 1, v = 5\}$, then (B.108) becomes $\phi < 0.0123607\gamma_A$, while the SOC (B.98) becomes $\phi > 0.00679285\gamma_A$. Therefore, (B.108) does not contradict the SOC.

This completes the proof. □

B.3.3 Proof of Proposition 3.6

Proof. Note that the statement holds when the platform market is either a monopoly market or a duopoly market. We prove the result for the two markets in the following two parts.

Part 1: In the presence of a monopoly platform

According to Lemma B.10, under paid-content-with-ads strategy,each supplier's equilibrium profits in the presence of a monopoly platform Π_{jS}^* are given in (B.72). Upon solving $\frac{\partial \Pi_{jS}^*}{\partial \gamma_A} > 0$ we obtain

$$\phi < \frac{(v-\gamma_A)^2}{72v} \tag{B.109}$$

Note that this condition does not contradict the SOC $\phi > \frac{(v-\gamma_A)^2}{144v}$ derived in Lemma B.10.

Part 2: In the presence of a duopoly platforms

According to Lemma B.13, under paid-content-with-ads strategy,each supplier's equilibrium profits in the presence of duopoly platforms Π^*_{jS} are given in (B.100). Upon solving $\frac{\partial \Pi^*_{jS}}{\partial \gamma_A} > 0$ we obtain

$$\phi < \frac{(v-\gamma_A)^2}{144v} \tag{B.110}$$

Note that this condition does not contradict the SOC $\phi > \frac{(v-\gamma_A)^2}{288v}$ derived in Lemma B.13.

This completes the proof.

□

B.3.4 No-ad Strategy

We analyze the perfectly competitive content supplier market where the marginal cost of producing content is c. In Lemma B.12 and B.13, we derive the conditions for a no-ad strategy to be adopted, in the presence of a monopoly platform or duopoly platforms respectively. Using the two lemmas we prove proposition 3.7.

Monopoly Platform

Lemma B.14. *When no-ad strategy is feasible, a monopoly platform adopts no-ad if and only if* $\gamma_C \geqslant \gamma_A + c$.

Proof. Given the content market is perfectly competitive, the content price is equal to the marginal cost of producing it: $p_S = c$. Because the consumer market is fully covered ($n_{1C} = 1$), the price the monopoly platform would charge consumers is

$$p_{1C} = v \cdot (\alpha_1 - \frac{\alpha_1^2}{2}) - \gamma_C \cdot (1-\alpha_1) - t \tag{B.111}$$

whereas the advertising price is

$$p_{1A} = \gamma_A \tag{B.112}$$

As a result, the monopoly platform's profits are

$$\begin{aligned}\Pi_P^M &= p_{1C} \cdot n_{1C} + p_{1A} \cdot (1-\alpha_1) - p_S \cdot \alpha_1 \\ &= -t - c \cdot \alpha_1 + (\gamma_A - \gamma_C)(1-\alpha_1) + \frac{1}{2}(2-\alpha_1) \cdot \alpha_1 \cdot v, \end{aligned} \tag{B.113}$$

No-ad strategy means the platform chooses the corner solution $\alpha_1 = 1$ and earns profits $\Pi_P^M(\alpha_1 = 1) = \frac{v}{2} - t - c$. Therefore, no-ad strategy is feasible only when $c \leqslant \frac{v}{2} - t$.

Given that no-ad strategy is feasible, we characterize the condition for it to be adopted. Note that Π_P^M is concave in α_1 because $\frac{\partial^2 \Pi_P^M}{\partial \alpha_1^2} = -v < 0$. Upon solving $\frac{\partial \Pi_P^M}{\partial \alpha_1} = 0$, we obtain $\alpha_1 = \frac{-c-\gamma_A+\gamma_C+v}{v}$. Therefore, no-ad is adopted if and only if $\frac{-c-\gamma_A+\gamma_C+v}{v} \geqslant 1$, which is equivalent to $\gamma_C \geqslant \gamma_A + c$. This completes the proof.

□

Duopoly Platforms

Lemma B.15. *A duopoly platform adopts no-ad strategy if and only if* $\gamma_C \geqslant \gamma_A + 2c$.

Proof. Given the content market is perfectly competitive, the content price is equal to the marginal cost of producing it: $p_S = c$. If a consumer is located at x, her utility derived from joining the two platforms are

$$U_{1C}(x) = v \cdot (\alpha_1 - \frac{1}{2}\alpha_1^2) - \gamma_C \cdot (1-\alpha_1) - t \cdot x - p_{1C} \tag{B.114}$$

$$U_{2C}(x) = v \cdot (\alpha_2 - \frac{1}{2}\alpha_2^2) - \gamma_C \cdot (1-\alpha_2) - t \cdot (1-x) - p_{2C} \tag{B.115}$$

By solving $U_{1C}(x) = U_{2C}(x)$, we obtain the marginal consumer

$$x_0 = \frac{1}{2} + \frac{V(\alpha_1) - V(\alpha_2)}{2t} - \frac{p_{1C} - p_{2C}}{2t} \tag{B.116}$$

where $V(\alpha_i) = v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2) - \gamma_C \cdot (1-\alpha_i)$. It follows that the consumer bases of the two platforms are given by $n_{1C} = x_0$ and $n_{2C} = 1 - x_0$. Thus, depending on its customer base, each platform charges advertisers:

$$p_{iA} = \gamma_A \cdot n_{iC} \tag{B.117}$$

Hence, platform i's profits are given by:

$$\Pi_{iP}^D = p_{iC} \cdot n_{iC} + p_{iA} \cdot (1-\alpha_i) - c \cdot \alpha_i \tag{B.118}$$

Note that no-ad strategy means $\alpha_1 = \alpha_2 = 1$. We first solve the consumer price under no-ad strategy, then derive the condition for both platforms to adopt no-ad strategy. Given $\alpha_1 = \alpha_2 = 0$, we solve $\frac{\partial \Pi_{1P}^D}{\partial p_{1C}} = 0$ and $\frac{\partial \Pi_{2P}^D}{\partial p_{2C}} = 0$, and obtain

$$p_{1C} = p_{2C} = t \tag{B.119}$$

Each platform earns profits $\Pi_{1P} = \Pi_{2P} = \frac{t}{2} - c$. Therefore, no-ad strategy is feasible only when $c \leqslant \frac{t}{2}$.

Given that no-ad is feasible, next we derive the condition such that $\{\alpha_1 = \alpha_2 = 1, p_{1C} = p_{2C} = t\}$ is the equilibrium. That is, no platform wants to unilaterally change its proportion of content and consumer price. Because of symmetry, we analyze platform 1's incentive of deviation.

First note that $\frac{\partial^2 \Pi_{1P}^D}{\partial \alpha_1^2} = \frac{-2\gamma_A\gamma_C + p_{1C}v + 3\gamma_A(1-\alpha_1)v}{t} < 0$. If $\{\alpha_1 = \alpha_2 = 1, p_{1C} = p_{2C} = t\}$ is the equilibrium, we must have

$$\left.\frac{\partial \Pi_{1P}^D}{\partial \alpha_1}\right|_{\{\alpha_1=\alpha_2=1, p_{1C}=p_{2C}=t\}} = \frac{1}{2}(-2c - \gamma_A + \gamma_C) \geqslant 0. \tag{B.120}$$

so that the platform has no incentive to decrease its α_1. This is equivalent to $\gamma_C \geqslant 2c + \gamma_A$. Next note that $\frac{\partial^2 \Pi_{1P}^D}{\partial p_{1C}^2} = -\frac{1}{t}$. This means the platform has no incentive to deviate its price from $p_{1C} = t$, because $p_{1C} = t$ satisfies the FOC.

Lastly, because $\left.\frac{\partial \Pi_{1P}^D}{\partial \alpha_1}\right|_{\{\alpha_1=\alpha_2=1, p_{1C}=p_{2C}=t\}} = 0$ when $\gamma_C = 2c + \gamma_A$, we also need to verify the hessian condition for this special case. This can be see as when $\gamma_C = 2c + \gamma_A$

$$\left.\Big(\frac{\partial^2 \Pi_{1P}}{\partial {\alpha_1}^2} \cdot \frac{\partial^2 \Pi_{1P}}{\partial {p_{1C}}^2} - (\frac{\partial^2 \Pi_{1P}}{\partial \alpha_1 \partial p_{1C}})^2\Big)\right|_{\{\alpha_1=\alpha_2=\alpha_{pa}^D, p_{1C}=p_{2C}=p_{paC}^D\}} = \frac{v}{2t} - \frac{c^2}{t^2} > 0 \tag{B.121}$$

To see why the inequality holds, first note that the IR condition of consumer located at $x = \frac{1}{2}$ requires $v \cdot (1 - \frac{1}{2}) - \frac{t}{2} - p_{1C} = \frac{v-3t}{2} > 0$, equivalent to $v > 3t$. Together with the condition for no-ad to be feasible ($c \leqslant \frac{t}{2}$), it follows that $\frac{v}{2t} - \frac{c^2}{t^2} \geqslant \frac{3}{2} - \frac{1}{4} = \frac{5}{4} > 0$.

This completes the proof.

□

B.3.5 Proof of Proposition 3.7

Proof. From Lemma B.12, a monopoly platform adopts no-ad strategy if and only if $\gamma_C \geqslant \gamma_A + c$. From Lemma B.13, a duopoly platform adopts no-ad strategy if and only if $\gamma_C \geqslant \gamma_A + 2c$. As a result, when $\gamma_C \in [\gamma_A + c, \gamma_A + 2c)$, a monopoly platform adopts no-ad strategy, while a duopoly platform starts to hosts advertising.

This completes the proof. □

B.4 Supplementary Analysis

In Lemma B.6, we focus on the symmetric equilibrium. However, there also exist two other asymmetric equilibria in the second stage of the game. In the next lemma, we show that these two asymmetric subgame equilibria are dominated by the symmetric subgame equilibrium, in the sense that the content supplier earns more profits under the symmetric equilibrium than under the asymmetric equilibria in the supergame.

Lemma B.16. *The content supplier earns more profits under the symmetric equilibrium than under the asymmetric equilibria.*

Proof. The two asymmetric solutions to $\frac{\partial \Pi_{1P}^D}{\partial p_{1C}} = 0$, $\frac{\partial \Pi_{1P}^D}{\partial \alpha_1} = 0$, $\frac{\partial \Pi_{2P}^D}{\partial p_{2C}} = 0$, and $\frac{\partial \Pi_{2P}^D}{\partial \alpha_2} = 0$

, that can satisfy the constraints $p_{iC} \geqslant 0$ and $0 \leqslant \alpha_i \leqslant 1$ are [2]

$$p_{1C} = \frac{-6\gamma_A^2 + 6tv + 2\sqrt{3}t\sqrt{-\frac{2\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{3t}{v}} \cdot v^{\frac{3}{2}} + \gamma_A\Big(-3\sqrt{6tv} + 3\sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v\Big)}{6v} \tag{B.122}$$

$$p_{2C} = \frac{-6\gamma_A^2 + 6tv - 2\sqrt{3}t\sqrt{-\frac{2\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{3t}{v}} \cdot v^{\frac{3}{2}} + \gamma_A\Big(-3\sqrt{6tv} - 3\sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v\Big)}{6v} \tag{B.123}$$

$$\alpha_1 = \frac{-2\gamma_A - \sqrt{6tv} + 2v + \sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v}{2v} \tag{B.124}$$

$$\alpha_2 = \frac{-2\gamma_A - \sqrt{6tv} + 2v - \sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v}{2v} \tag{B.125}$$

and

$$p_{1C} = \frac{-6\gamma_A^2 + 6tv - 2\sqrt{3}t\sqrt{-\frac{2\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{3t}{v}} \cdot v^{\frac{3}{2}} + \gamma_A\Big(-3\sqrt{6tv} - 3\sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v\Big)}{6v} \tag{B.126}$$

$$p_{2C} = \frac{-6\gamma_A^2 + 6tv + 2\sqrt{3}t\sqrt{-\frac{2\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{3t}{v}} \cdot v^{\frac{3}{2}} + \gamma_A\Big(-3\sqrt{6tv} + 3\sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v\Big)}{6v} \tag{B.127}$$

$$\alpha_1 = \frac{-2\gamma_A - \sqrt{6tv} + 2v - \sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v}{2v} \tag{B.128}$$

$$\alpha_2 = \frac{-2\gamma_A - \sqrt{6tv} + 2v + \sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}} + \frac{6t}{v}} \cdot v}{2v} \tag{B.129}$$

As these two asymmetric solutions are essentially the same, we only analyze the first solution.

To begin with, note that both the asymmetric equilibria solutions are real only

[2] There are two other asymmetric solutions. However, the solutions violate the demand constraints that $p_{iC} \geqslant 0$ and $0 \leqslant \alpha_i \leqslant 1$, so that they cannot be the candidate for second stage equilibrium

when $-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}}+\frac{6t}{v}\geqslant 0$. This implies

$$p_S \leqslant \widehat{p_S} \equiv \frac{\sqrt{6tv}}{4} \tag{B.130}$$

In other words, when $p_S > \widehat{p_S}$, the two asymmetric solutions do not exist.

Next, we prove that even when the asymmetric solutions can be the second stage equilibrium, the content supplier earns less profits under the asymmetric equilibria compared to those under the symmetric equilibrium presented analyzed in the main paper.

If the asymmetric equilibrium is played in the second stage, then the content supplier's profits are

$$\begin{aligned}\Pi_{asymmetric\_S} &= p_S \cdot (\alpha_1+\alpha_2)|_{asymmetric_equilbirium} \\ &= p_S \cdot \Big(\frac{-2\gamma_A+2\gamma_C-\sqrt{6tv}+2v-\sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}}+\frac{6t}{v}}\cdot v}{2v} \\ &+ \frac{-2\gamma_A+2\gamma_C-\sqrt{6tv}+2v+\sqrt{-\frac{4\sqrt{6}tp_S}{v^{\frac{3}{2}}}+\frac{6t}{v}}\cdot v}{2v}\Big) \\ &= p_S \cdot \frac{-2\gamma_A+2\gamma_C-\sqrt{6tv}+2v}{v}\end{aligned} \tag{B.131}$$

If the symmetric equilibrium is played in the second stage, by lemma B.6, the content supplier's profits are

$$\begin{aligned}\Pi_{symmetric\_S} &= p_S \cdot (\alpha_1+\alpha_2)|_{symmetric_equilibrium} \\ &= p_S \cdot 2\alpha_{pa}^D \\ &= p_S \cdot \frac{-4p_S-2\gamma_A+2\gamma_C+2v}{v}\end{aligned} \tag{B.132}$$

Therefore,

$$\Pi_{symmetric\_S} - \Pi_{asymmetric\_S} = p_S \cdot \frac{-4p_S+\sqrt{6tv}}{v} \tag{B.133}$$

This implies $p_S \leqslant \widehat{p_S} = \frac{\sqrt{6tv}}{4} \iff \Pi_{symmetric\_S} - \Pi_{asymmetric\_S} \geqslant 0$. The inequality strictly holds when $p_S < \widehat{p_S}$

□

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Biography

Jinzhao Du was born on November 18, 1989 in Qiqihar, China. He graduated from Tsinghua University in 2012 with a bechlor's degree in Economics. He will receive his Ph.D. in Business Administration (Marketing) from Duke University in 2018. He is a recipient of the 2017 AMA-Sheth Foundation Doctoral Consortium Fellow. He will join the Faculty of Business and Economics at the University of Hong Kong in August, 2018.