



# How Efficient are the Brazilian Electricity Distribution Companies?

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**Abstract** During the last years, the electricity sector has experienced great changes, especially within the economic regulation. After receiving several criticisms, the rate of return regulation has been replaced by incentive regulation. The main objective of this regulation is to stimulate business efficiency. This paper proposes an alternative application of data envelopment analysis to the Brazilian case, characterized by a large territory: the use of Unit Networks in the distribution segment to regionalize the concession area and then to analyse the efficiencies separately. Many regulators use the entire distribution company as a decision-making unit for price regulation when benchmarking is applied. However, in Brazil, quality performance is measured in detail using sets of consuming units, i.e. quality is measured using small parts of the company. Given that efficiency cannot be assessed without considering various aspects of quality performance and characteristics of the underlying environment in the utility's concession area, this paper tries to find the trade-off between management, quality, environment and costs. Therefore, the main contribution of this paper is twofold: the solution for Brazilian distribution companies' heterogeneity and the choice of variables that are better measures for an effi-

ciency analysis. Some examples with Brazilian utilities are provided to show the advantages of the proposed approach.

**Keywords** Electricity power distribution · Incentive regulation · Data envelopment analysis

## Abbreviations

DEA	Data envelopment analysis
UN	Unit Network
DMU	Decision-making unit
RPI	Retail Price Index
FRM	Firm reference model
COLS	Corrected ordinary least square
SAIDI	System Average Interruption Duration Index
SAIFI	System Average Interruption Frequency Index
CRS	Constant Return to Scale
VRS	Variable Return to Scale
TINT	Total time lost due to interruptions
GIP	Gross internal product
ANOVA	Analysis of variance

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## List of symbols

$U_i$	Annual outage time (h)
$N_i$	Number of customers at load point $i$ (person)
$N$	Number of companies (unit)
$\theta$	Efficiency score (0–1)
$\lambda$	Vector of weights
$E$	Observed inputs
$M$	Observed outputs
$X$	Input matrix

$Y$	Output matrix
$x_i$	Input column vector for the $i$ th company
$y_i$	Output column vector for the $i$ th company
$z_i$	Vector of environmental variables
$\theta_i^*$	Latent variable related with the calculated efficiency score
$\beta$	Vector of parameters that represent the impact of environment

## 1 Introduction

Various reforms have been proposed for the electricity sector around the world to make utilities more efficient through competition, privatization and price mechanisms. In general, during the restructuring process, the industry is divided into four distinguished activities: generation, transmission, distribution and retailing. In generation and retailing, competition has become possible through the development of new generating technologies and by increasing the number of agents, while transmission and distribution remain regulated because of their natural monopoly characteristics. This paper focuses only on the economic regulation of the distribution companies.

One of the major problems of rate of return regulation is that companies are induced to over-capitalize to obtain higher remuneration of capital. Consequently, the tariffs paid by customers increase. The incentive regulation tries to force the companies to be more efficient (Ergas and Small 2001) and try to avoid the Averch–Johnson effect (Averch and Johnson 1962). However, quality of supply can be compromised because utilities can reduce costs indiscriminately to pursue this efficiency.

The incentive regulation uses benchmarking techniques to define the efficient companies. In general terms, this technique can be characterized as a method that compares a group of companies as they were subjected to a competitive environment (Lowry and Getachew 2009).

Results from a survey conducted among energy regulatory agencies in 40 countries in 2008 showed that there is a clear trend in the electricity industry towards the use of Data Envelopment Analysis (DEA) in both transmission and distribution (Haney and Pollitt 2009).

It is noteworthy that despite the popularity of the DEA methodology, its application is restricted mainly to European countries characterized by small territorial distances and homogeneous environmental conditions. In Brazil, conditions are different:

“There is a large variation in sizes, scopes and environmental characteristics of the Brazilian distribution companies. It seems obvious that the diversity is higher in Brazil than in most other countries where

benchmarking-based regulation has been traditionally used” (Bogetoft 2014).

During the Public Hearing that proposed the DEA methodology for the Third Price Control Review (2011–2014), the Brazilian regulatory agency received criticisms from distribution companies such as: (1) the existence of very different environments throughout the country and (2) not including the quality of supply.

Cook et al. (2013) emphasize that DEA is a methodology for evaluating the relative efficiency of a set of homogeneous decision-making units (DMUs), i.e. the companies under evaluation is comparable. In some situations, such as companies that have a wide concession area with different social, economic or environmental characteristics, the assumption of homogeneity does not apply. The absence of homogeneity may lead to an unfair comparison.

This paper proposes a new approach to solve the heterogeneity constraint and to allow the inclusion of quality and environmental aspects; the approach combines the DEA methodology with the Unit Networks (UN) concept. The UN is used for splitting a distribution company concession area into more homogeneous subgroups that are further considered as DMUs.

This paper is composed of six sections. After the introductory section, an overview of the regulation of Brazilian distribution companies is given in Sect. 2. Section 3 presents the proposed methodology. Section 4 describes the data and models used in the study. In Sect. 5, simulation results are presented. Finally, Sect. 6 concludes the paper.

## 2 Distribution Regulation

Before 1995, the Brazilian electricity sector was totally in the hands of federal and state companies (ANEEL). Reform began in that year with the sale of the distribution companies to the private sector. Meanwhile, the main guidelines of regulation were proposed, and the principal laws were enacted from 1997 to 1999. The Brazilian regulatory agency, ANEEL, was created at that time; in addition to other duties, it assumed the responsibility for pricing the transmission and distribution services along with the definition of their quality performance.

### 2.1 Price Regulation

Since 2003, the distribution companies have been regulated using a price cap model based on RPI<sup>1</sup>-X formula that is reset every 4 years. Price cap model typically specifies an average rate at which the prices that regulated companies charge for

<sup>1</sup> Retail Price Index

its services must decline, after adjusting for inflation. This rate is called the X-Factor.

The distribution segment completed two price revision periods (2003–2006 and 2007–2010) and at this writing is undergoing a third (2011–2014).

The Firm Reference Model (FRM) (Sanhueza et al. 2004) was used for accessing the efficient operational costs during the first and second price revisions. This model tries to mimic the operation of an optimal company with the same characteristics as the real company. All processes and activities are represented and priced according to the realities of the concession area.

However, during the third price revision, ANEEL changed from the bottom-up approach of FRM to top-down methods such as DEA and Corrected Ordinary Least Square (COLS). Instead of analysing each activity, the efficiency is measured comparing outputs and inputs among distribution companies.

The two-stage DEA model was used to take the environmental aspects of the distribution service into account. The model outputs were network length, energy delivered and number of customers. The inputs were operational costs. As environmental variables, it considered the local wage level, the precipitation rates, the customer density and a complexity index. The wage level measures the differences in labour costs at the utilities determined by the local markets. The complexity index measures the difficulty faced by each utility in reducing non-technical losses.

From this comparison with actual data from the utilities, the regulator sets different X-Factors for passing operational costs to customers through tariffs according to the average efficiency of the sector. The X-factor is applied on the value of the Parcel B<sup>2</sup> of distribution companies. Thus, for more efficient companies, it is possible to have earnings above actual costs, while for less efficient ones there are deficits not allowed to pass through to consumers (ANEEL 2006; Matos et al. 2012).

### 2.2 Quality of Supply Regulation

In Brazil, the quality performance analysis is carried out based on divisions of the concession area called sets of consuming units. Thousands of sets are created; performance comparisons, formerly done company by company, changed to set by set (Tanure et al. 2006).

One set of consuming units is composed of the units fed by the same distribution substations. The central idea is that the sets are more comparable than the distribution companies as a whole because the concession areas in Brazil usually cover a wide range of social, economic and environmental characteristics.

<sup>2</sup> Controllable costs composed by operational costs, capital remuneration and depreciation.

After defining the sets, a clustering process is carried out based on the characteristics of the sets. This is necessary because there are approximately 6000 sets to analyse and for which to establish quality performance targets.

Quality of supply is assessed for each cluster using the collective indicators System Average Interruption Duration Index (SAIDI) and System Average Interruption Frequency Index (SAIFI) (Billinton and Allan 1984). The first index measures the mean time during the observation period for which there was discontinuity in the electricity supply, as in Eq. (1).

$$SAIDI = \frac{\sum U_i * N_i}{\sum N_i} \tag{1}$$

where:

- $U_i$ : Annual outage time;
- $N_i$ : Number of customers at load point  $i$ .

This indicator is used in this paper as a quality measure, after multiplication by the number of customers at the load point  $i$ .

### 2.3 Combined Price and Quality Regulation

Regarding price regulation, the Brazilian regulator bases its analysis on the company as a whole, i.e. the DMUs are the distribution companies. However, for quality regulation, the regulator bases its analysis on the set of consuming units, which are divisions of the concession area. These perspectives are depicted in Fig. 1.

Given that price regulation cannot be disconnected from the quality of the service, the company approach and the set of consuming units approach must converge to the same base.

Consider the case of the Brazilian company CEMIG. Its distribution network is over 460,000 km in length (CEMIG). The company operates in the Minas Gerais state that has an area of approximately 586,528 km<sup>2</sup>, larger than countries

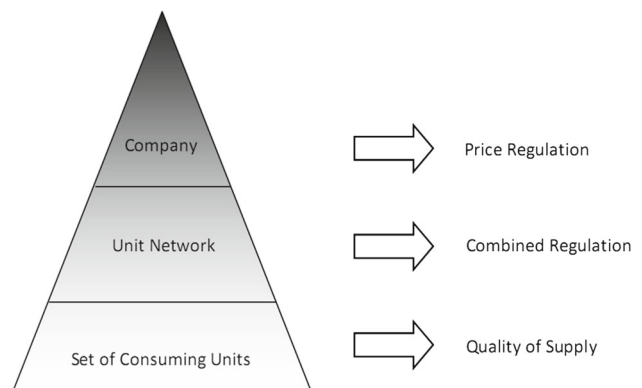


Fig. 1 Regulatory perspective

such as France, Spain and the UK (IBGE). For example, the average lightning rate, which may affect the continuity of supply, varies from 0.085 to 5.971 per km<sup>2</sup> per year within the concession area. All of these peculiarities shape the characteristics of CEMIG’s distribution network, which requires different treatment for each region.

The use of sets of consuming units as DMUs considerably increases the number of DMUs. Moreover, the DMUs should represent organizational units, whereas the sets of consuming units represent portions of the electrical distribution network.

The UN concept introduced in this paper tries to minimize the distance between the price regulation and the quality of supply regulation. The boundaries of UNs have strong connections to the regional organizations that are usually present at the distribution companies. Therefore, the regulator can consider the same unit of analysis both for the quality of supply and for price regulation. Additionally, the regulator may determine whether the cost reduction is being done to the detriment of the quality of supply.

### 3 Methodology

#### 3.1 Data Envelopment Analysis

DEA is a nonparametric methodology that uses real data to measure the relative efficiency of a DMU. It was proposed by Charnes et al. (1978) to address the efficiencies of companies operating in constant returns to scale (CRS) and further extended by Banker et al. (1984) to variable returns to scale (VRS).

This efficiency analysis can be focused on input reduction or output expansion. The result from an input-oriented model is the maximum reduction possible in the inputs level for a given level of output. With an output-oriented focus, the model seeks the maximum output quantities that can be generated by the actual level of inputs used by the company. The efficiency scores can vary from 0 to 1, where 1 denotes the efficient company.

The majority of the DEA models consider either constant (Charnes et al. 1978) or variable returns to scale (Banker et al. 1984). For constant returns to scale (CRS), outputs and inputs increase (or decrease) by the same proportion along the frontier. Where the technology exhibits increasing, constant or decreasing returns to scale along different segments of the frontier, the variable returns to scale (VRS) model is indicated (Subhash and Chen 2010).

The CRS model assesses the overall technical and scale efficiency, while a VRS model measures only the technical efficiency.

The efficiency score of the *i*th company of *N* companies in CRS models takes the form specified in Eq. (2) where  $\theta$  is a scalar (equal to the efficiency score) and  $\lambda$  is

**Table 1** Equation parameters

Sample	Unit Networks	Distribution companies
Number of DMU ( <i>N</i> )	70	10
Observed inputs ( <i>E</i> )	4	4
Observed outputs ( <i>M</i> )	2	2
Input matrix ( <i>X</i> )	4 × 70	4 × 10
Output matrix ( <i>Y</i> )	2 × 70	2 × 10

a  $N \times 1$  vector that represents the weight of each decision-making unit in the construction of the reference company (Giannakis et al. 2005). Assuming that the companies use *E* inputs and *M* outputs, *X* and *Y* represent  $E \times N$  input and  $M \times N$  output matrices, respectively. The input and output column vectors for the *i*th company are represented by  $x_i$  and  $y_i$ , respectively. In Eq. (2), company *i* is compared to a linear combination of sample companies which produce at least as much of each output with the minimum possible amount of inputs. The Eq. 2 is solved once for each company.

For VRS models, a convexity constraint  $\sum \lambda = 1$  is added that ensures that the company is compared against other companies of a similar size.

$$\begin{aligned}
 &\min_{\theta, \lambda} \theta \\
 &\text{s.t.} \\
 &y_i \leq Y\lambda \\
 &\theta x_i \geq X\lambda \\
 &\lambda \geq 0
 \end{aligned} \tag{2}$$

In the context of this paper, we have two different samples: (1) Unit Networks sample and (2) distribution companies sample. Table 1 presents the parameters for each sample in Eq. 2 and  $\theta, \lambda$  they are the parameters to be calculated by linear program.

We have four inputs and two outputs variables: network length ( $x_1$ ), transformer capacity ( $x_2$ ), number of employees ( $x_3$ ), quality measure ( $x_4$ ), energy delivered ( $y_1$ ) and number of customers ( $y_2$ ).

If company *i* has the  $\theta$  value equal to 1 means that the company uses the minimum values for inputs, and it is considered efficient. Otherwise, if the value of  $\theta$  is less than 1 means that the company is using more inputs resources than the necessary, and it is considered inefficient.

Banker et al. (1984) state that one of the most important advantages of this methodology is that the efficiency score is obtained directly, without the need to specify the production function in advance. The methodology deals directly with multiple outputs and inputs, and the linear programming model facilitates the implementation and the convergence process to solve the problem.

The traditional DEA models consider that inputs can be reduced and outputs can be increased by DMU in a short time. However, there are variables that are beyond DMU control, which are known as environmental variables. There are many ways to include these variables using the DEA methodology (Subhash 1988; Simar and Wilson 2007), such as the two-stage model used in this paper.

*A Two-Stage DEA model* Two-stage analysis is one of the most popular techniques in the literature to take environmental variables into account.

We employed this technique as follows: in the first stage, we determined the technical efficiency performances of the unit networks (UNs) or distribution companies using DEA. In the second stage, treating these calculated efficiency scores as dependent variables, we used a regression technique to determine the environmental variables that may explain the efficiency scores. This approach is advocated by Chilingerian and Sherman (2004), Subhash (2004) and Ruggiero (2004).

Efficiency scores calculated from DEA take values between 0 and 1, making the dependent variable in the second stage limited. The Tobit model (Tobin 1958) is frequently used to address such a limited dependent variable and is followed in this study.

The calculated efficiency score in the first stage ( $\theta_i$ ) will be corrected by environmental variables ( $z_i$ ) in this second stage. Therefore, a latent (unobserved) variable ( $\theta_i^*$ ) is calculated as in Eq. 3:

$$\theta_i = \begin{cases} \theta_i^*; & 0 \leq \theta_i^* \leq 1 \\ 0; & \theta_i^* < 0 \\ 1; & \theta_i^* > 1 \end{cases}$$

$$\theta_i^* = z_i \beta + \varepsilon_i \tag{3}$$

Here,  $z_i$  is an  $(r \times 1)$  vector of environmental variables and  $\beta$  is an  $(r \times 1)$  vector of parameters to be estimated.

In the context of this paper, we have three environmental variables: number of lightning ( $z_1$ ), customer density ( $z_2$ ) and ownership ( $z_3$ ).

### 3.2 Unit Network

Traditional efficiency analyses usually consider DMUs to be the distribution companies. Because some distribution companies have large concession areas with different characteristics and different quality indices, this paper suggests the use of Unit Networks as decision-making units. The UNs aggregate the sets of consuming units forming regions within the distribution concession area (Lima et al. 2011).

The definition of a UN is a twofold process. The first step is to define the domain areas of each connection point between

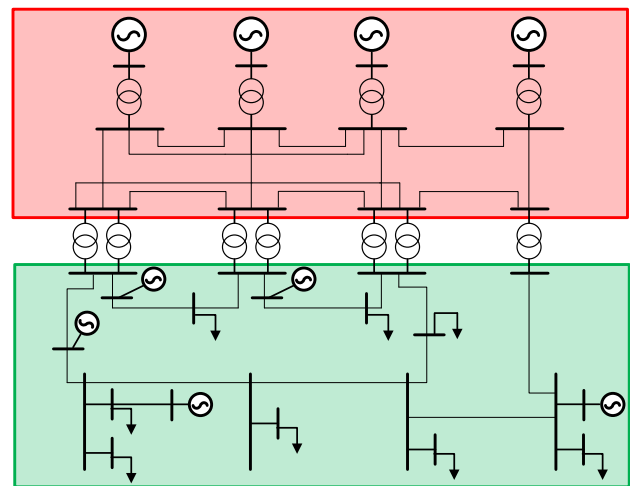


Fig. 2 Transmission and distribution grids connection

the transmission and distribution networks. The domain area of a connection point is defined as the set of buses that are reached by the power flow that cross the border transformer. The second step couples domain areas based on strong and weak links through network equivalents. In the presence of strong links, two or more UNs can be grouped to form a larger UN. Connections are strong if they have a low equivalent impedance value and are weak if the impedance is high.

### 3.3 Example of Unit Network Definition

Consider the system as depicted in Fig. 2. The red box represents the transmission grid and green box represents the distribution grid. Usually, the flow direction in the border transformers, which connects the grids, is from transmission to distribution. If a virtual generator is considered at the primary bind of the border transformer, it is possible to determine the domain of this connection point using the concept of a generator’s domain introduced by Kirschen et al. (1997).

The domain area of the connection point is the set of buses that are reached by the power flow that crosses the border transformer. The power flow reaches a specific bus if it is possible to find a path on the network going from the connection point to the bus where the flow direction remains unchanged. An example of the domain area for four connection points is depicted in Fig. 3.

Some medium-voltage distribution networks have a mesh topology, so it is possible to have overlap between domain areas where the connection points to transmission grid are close, as seen in Fig. 3. When this is the case, the second step determines whether these two or more domain areas should be coupled, using the concept of Thevenin equivalent impedance. As represented in Fig. 4, the equivalent impedance between the secondary bind of the border transformers is computed on a two-by-two basis.

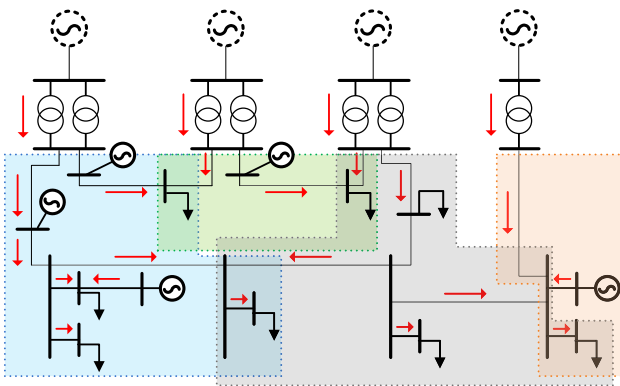


Fig. 3 Connection point domain area

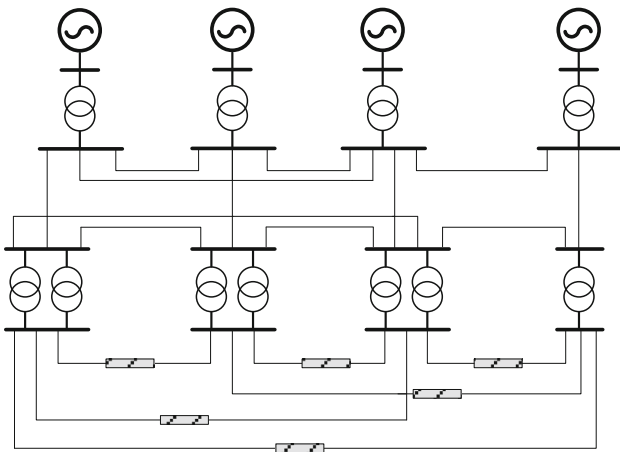


Fig. 4 Equivalent impedance between two connection points

The equivalent impedance represents the electrical proximity of the two buses. If the equivalent impedance is small, there is a strong link between the two connection points. Therefore, they should be coupled to form a unique UN. Otherwise, if the equivalent impedance is large, they should remain separate. The concept of small or large impedance depends on the system characteristics (Lima et al. 2011).

## 4 Data and Models Specification

### 4.1 Choice of Variables

Choosing the input–output variables is an important step in DEA methodology. In the DEA context, problems related to discrimination between efficient and inefficient DMUs often arise, particularly if there are a large number of variables (Dyson et al. 2001). Therefore, the researcher need to be parsimonious in the number of variables and choose those that best describe the scenario evaluated.

There is no firm consensus on which variables best describe the operation of distribution companies

(Giannakis et al. 2005). Jamasb and Pollitt (2001) outline the most widely used variables in 20 benchmarking studies of electricity distribution companies. Number of employees, transformer capacity and network length are among the most commonly used inputs in the models. The most widely used outputs include energy delivered and number of customers.

The distribution company requires labour and capital inputs. The labour input was considered via number of employees (proxy). Capital input was taken into account by other two variables: network length and transformer capacity. Regarding to the outputs, we considered number of customers and energy delivered. We use physical measures of these inputs and outputs applied in benchmarking studies (Jamasb and Pollitt 2001; Estache et al. 2004; Pombo and Taborada 2006; Çelen 2013) together with quality of supply and environmental variables.

Many authors (Giannakis et al. 2005; Yu et al. 2009; Cambini et al. 2012; Growitsch et al. 2009; Jamasb et al. 2012) have incorporated quality performance in the DEA analysis using the Total Time Lost Due To Interruptions (TINT) indicator as input instead of SAIDI directly. The TINT is calculated by multiplying SAIDI values (Eq. 1) by the number of customers.

The most relevant environmental variables for efficiency analysis are customer density (to identify rural and urban areas), frequency of lightning (to identify climate influence) and ownership (represented by a binary variable that is zero for state-owned company and 1 for a private company).

### 4.2 Brazilian Example

This paper compares the performance of 10 distribution utilities in the Brazil in the period from 2006 to 2007. The data can be found on the ANEEL website<sup>3</sup>, where it was considered the latest consistent sample available for this period.

This sample comprises the states of São Paulo, Rio de Janeiro, Minas Gerais and Rio Grande do Sul. These four states are responsible for 61 % of the Brazilian Gross Internal Product (GIP) (IBGE). The ten companies that operate in these four states supplied approximately 56 % of the total load of Brazil (ANEEL).

These distribution companies have 712 sets of consuming units. They were grouped into 70 UNs using the method of Sect. 3.2.

Each set of consuming units has the following attributes: network length ( $x_1$ ), transformer capacity ( $x_2$ ), number of employees ( $x_3$ ), TINT ( $x_4$ ), energy delivered ( $y_1$ ), number of customers ( $y_2$ ), number of lightning, ( $z_1$ ), customer density ( $z_2$ ) and ownership ( $z_3$ ). The attributes  $x_E$  (for  $E = 1, 2, 3, 4$ ) are inputs, the  $y_M$  (for  $M = 1, 2$ ) are outputs and the  $z_r$  (for  $r = 1, 2, 3$ ) are environmental variables.

<sup>3</sup> Available at: [www.aneel.gov.br](http://www.aneel.gov.br).

**Table 2** Brazilian Unit Networks (2006/2007)—statistic summary

Descriptive statistics					
Description	Unit	Minimum	Maximum	Mean	SD
Network length ( $x_1$ )	km	284	53,456	9576	13,740
Transformer capacity ( $x_2$ )	kVA	14,866	12,577,411	1,160,368	2,057,648
Number of employees ( $x_3$ )	Person	9	9131	867	1545
TINT ( $x_4$ )	Hours	171,980	40,862,936	4,356,342	5,808,652
Energy delivered ( $y_1$ )	MWh	26,191	24,763,333	1,839,310	3,592,334
Number of customers ( $y_2$ )	Person	4988	4,850,254	391,979	706,657
Lightning ( $z_1$ )	Lightning/year	561	169,954	38,696	42,433
Customer density ( $z_2$ )	Person/km <sup>2</sup>	2	1631	147	313

**Table 3** Correlation coefficient among inputs and outputs

Variables	x1	x2	x3	x4	y1	y2
x1	1					
x2	0.49	1				
x3	0.44	0.88	1			
x4	0.54	0.90	0.88	1		
y1	0.35	0.98	0.85	0.89	1	
y2	0.44	0.98	0.90	0.94	0.99	1

With respect to the numbers of employees, the UNs' geographical limits are closely similar to the areas of activity of each utility's regional management offices. Therefore, it was not difficult to allocate the number of employees to each UN.

An overview of a summary of key statistics of the data for the 70 UNs is presented in Table 2 in the form of minimum, maximum, mean and standard deviation values.

To validate DEA model, Table 3 was constructed from the correlation coefficients between the inputs and outputs. Its goal is to verify whether an increase in some input does not result in a reduction in some output (assumptions of monotonicity).

Although there is a high correlation between energy delivered and number of costumers, both variables are kept in the analysis. It is possible for two UNs to deliver same amount of energy to distinctly different numbers of consumers (Neu-berg 1977).

To support the choice of variables, a statistical analysis was carried out. Four distinct linear regressions were per-

formed, one for each dependent variable (network length, transformer capacity, number of employees and TINT). The independent variables were energy delivered and number of customers. It is important to emphasize that network length and transformer capacity are proxies for capital inputs, and the number of employees is a proxy for labour inputs.

Table 4 presents the statistical parameters evaluated to ascertain the relevance of the choice of variables for accessing the performance of UN.

$R^2$  values in Table 4 indicate that 41 % of the variation in network length, 97 % of the variation in transformer capacity, 86 % of the variation in number of employees and 11 % of the variation in TINT were subjected to the two independent variables: energy delivered and number of customers.

The ANOVA (Fisher 1918) results are also shown in Table 4 with independent variables that indicate F ratios of 47.95, 2147.68, 412.82 and 8.50 for the dependent variables network length, transformer capacity, number of employees and TINT, respectively. In the proposed model, the variables network length, transformer capacity and number of employees are well explained by the independent variables chosen ( $p < 0.05$ ).

### 4.3 Model Specifications

There are three different models as shown in Table 5 that are all based on DEA considering input orientation and variable returns to scale (VRS).

In Model 1, three inputs and two outputs were considered: network length, transformer capacity and number of employees were treated as inputs and energy delivered and number

**Table 4**  $R^2$  and Anova results

Aspect	Dependent variable	$R^2$	Adjusted $R^2$	F value	Significance
Capital input	Network length	0.41	0.40	47.95	1.63338E-16
Capital input	Transformer capacity	0.97	0.97	2147.68	3.7225E-104
Labour input	Number of employees	0.86	0.86	412.82	9.95182E-59
Quality of supply	TINT	0.11	0.10	8.50	0.000331945

**Table 5** Summary of evaluated models

Variables	Models		
	Model 1	Model 2	Model 3
Network length	I	I	I
Transformers capacity	I	I	I
Number of employees	I	I	I
TINT		I	I
Energy delivered	O	O	O
Number of customer	O	O	O
Lightning			EV
Customer density			EV
Ownership			EV

*I* input, *O* output, *EV* environmental variable

of customers as outputs. It is noteworthy that in this model, quality of supply can be compromised because utilities can reduce labour and capital inputs indiscriminately to pursue this efficiency.

In Model 2, the TINT indicator was added as input based on the notion that DMUs should minimize the duration of interruptions (undesirable output).

Model 3 used the same input and output variables as Model 2, but the environmental variables were included. This model tries to capture the extent to which the results are influenced by environmental variables.

## 5 Practical Results

The proposed methodology was applied to the three models defined in Sect. 4.3 using data provided by ten Brazilian distribution companies (Aes Sul, Bandeirante, CEEE, CEMIG, Elektro, Eletropaulo, Light, Paulista, Piratininga and RGE).

Two analyses were made: one treated the Unit Networks as DMUs and the other treated the companies as DMUs.

### 5.1 Unit Network-Oriented Analysis

The technical efficiency scores were calculated for the 70 UNs over the period 2006 to 2007. Models 1 and 2 were carried out based on a one-stage DEA, whereas Model 3 was based on a two-stage DEA.

For the last Model, in which environmental variables are included, the Tobit analysis described in Sect. 3.1 was applied; Table 6 presents the estimation results.

The lightning rate was statistically significant and produced a negative coefficient in the model. A one-unit increase in lightning leads to 0.04 decrease in the efficiency score. The effect of lightning on efficiency of distribution companies was also confirmed by [Jamasp et al. \(2012\)](#).

Customer density is statistically significant also and produces a positive coefficient. A one-unit increase in customer density leads to 0.07 increase in the efficiency score. A positive effect of customer density on the efficiency of distribution companies was also confirmed by [Çelen \(2013\)](#). The ownership variable was statistically insignificant for this example, and it was not considered.

Table 7 presents the variable returns to scale efficiency scores (VRS), SAIDI index and environmental characteristics.

By evaluating the environmental variables of Table 7, two types of heterogeneity can be identified:

- External heterogeneity is related to the different characteristics of distribution companies. For example, Light is predominantly urban with a high customer density, and CEMIG is predominantly rural with a low customer density;
- Internal heterogeneity is related to the different characteristics within a single distribution company. For example, Aes Sul has high, medium and low customer densities and various levels of lightning incidence.

The results indicate that the UNs are, on average, technically efficient by approximately 0.75 under Model 1, 0.79 under Model 2 and 0.79 under Model 3; these numbers reflect that there is room for improvement.

**Table 6** Tobit analysis results—Unit Network

Variable	Parameter	Coefficient	<i>t</i> ratio	<i>p</i> value
Constant	$\beta_0$	0.80	48.34	<0.00001***
Lightning	$\beta_1$	−0.04	−2.84	0.00455***
Customer density	$\beta_2$	0.07	3.19	0.00141***
Dummy for ownership	$\beta_3$	−0.03	−0.84	0.39885
Number of observations		140		
Censored observations		0		
Log-likelihood		61.33		

\*\*\* Significance at the 1 % level using a two-tailed test



**Table 7** Efficiency scores for Brazilian Unit Networks—2006/2007

Utility	UN	Models			Quality	Environment		Lightning incidence	UN	Utility	Quality	Models			SAIDI	Environment		Lightning incidence
		1	2	3		SAIDI	Customer density					1	2	3		SAIDI	Customer density	
Aes Sul	1	0.39	0.41	0.42	28.6	Low	Medium	11	Cemig	13.7	Medium	0.73	0.73	0.76	13.7	Medium	Medium	
Aes Sul	2	0.39	0.41	0.49	41.6	Low	High	1	Elektro	6.8	Medium	0.45	0.84	0.86	6.8	Medium	Medium	
Aes Sul	3	0.47	0.47	0.60	32.7	Low	High	2	Elektro	11.7	High	0.59	0.64	0.63	11.7	High	Medium	
Aes Sul	4	0.62	0.62	0.75	21.9	Low	High	3	Elektro	13.3	Medium	0.52	0.59	0.58	13.3	Medium	Medium	
Aes Sul	5	0.80	0.83	0.85	19.4	Low	Medium	4	Elektro	8.7	Medium	0.46	0.70	0.74	8.7	Medium	Medium	
Aes Sul	6	0.55	0.55	0.59	24.8	Medium	Medium	5	Elektro	16.4	Medium	0.42	0.51	0.53	16.4	Medium	Medium	
Aes Sul	7	0.59	0.62	0.62	19.8	Medium	Low	6	Elektro	8.8	Low	0.45	0.80	0.82	8.8	Low	Medium	
Aes Sul	8	0.63	0.66	0.67	20.0	Medium	Medium	7	Elektro	16.2	High	0.66	0.68	0.76	16.2	Medium	High	
Aes Sul	9	1.00	1.00	1.00	16.7	Low	Low	8	Elektro	4.5	Medium	0.68	1.00	1.02	4.5	Medium	Medium	
Aes Sul	10	0.71	0.75	0.73	10.9	High	Medium	1	Eletropaulo	13.3	Low	1.00	1.00	0.83	13.3	High	Low	
Aes Sul	11	0.99	0.99	0.90	12.7	High	Low	2	Eletropaulo	8.0	Medium	1.00	1.00	0.66	8.0	High	Medium	
Aes Sul	12	1.00	1.00	1.00	41.7	Low	Low	3	Eletropaulo	7.1	Low	1.00	1.00	0.82	7.1	High	Low	
Bandeirante	1	0.72	0.76	0.72	10.8	High	Low	4	Eletropaulo	11.6	Low	1.00	1.00	0.89	11.6	High	Low	
Bandeirante	2	0.84	0.91	0.90	8.6	High	Medium	1	Light	14.5	Low	0.82	0.82	0.76	14.5	High	Low	
Bandeirante	3	1.00	1.00	0.79	10.3	High	Low	2	Light	8.6	Medium	0.81	0.89	0.90	8.6	Medium	Medium	
Bandeirante	4	0.91	0.98	0.97	7.0	Medium	Medium	3	Light	9.0	Low	1.00	1.00	0.69	9.0	High	Low	
Ceee	1	0.73	0.73	0.74	23.6	Medium	Low	4	Light	14.5	Low	1.00	1.00	0.96	14.5	High	Low	
Ceee	2	0.82	0.82	0.81	19.3	Medium	Low	5	Light	6.4	Low	0.72	1.00	0.88	6.4	High	Low	
Ceee	3	0.65	0.65	0.68	48.4	Medium	Medium	1	Paulista	6.2	High	0.90	0.90	0.95	6.2	Medium	High	
Ceee	4	0.59	0.59	0.63	28.1	Low	Medium	2	Paulista	6.5	Medium	0.85	0.91	0.92	6.5	Medium	Medium	
Ceee	5	0.69	0.69	0.77	41.5	Medium	High	3	Paulista	6.9	High	1.00	1.00	1.14	6.9	Medium	High	
Ceee	6	0.47	0.47	0.48	50.9	Low	Medium	1	Piratininga	5.0	Low	1.00	1.00	0.90	5.0	High	Low	
Ceee	7	1.00	1.00	1.00	32.6	Medium	Medium	2	Piratininga	9.6	Medium	0.82	0.82	0.82	9.6	High	Medium	
Ceee	8	0.89	0.89	0.88	35.8	Medium	Low	1	Rge	19.9	Low	0.70	0.70	0.79	19.9	Low	High	
Ceee	9	1.00	1.00	0.85	12.2	High	Low	2	Rge	20.4	High	0.55	0.56	0.61	20.4	Low	High	
Ceee	10	0.68	0.68	0.68	38.6	Medium	Low	3	Rge	8.7	Medium	0.58	0.74	0.73	8.7	Medium	Medium	
Cemig	1	0.94	0.94	0.98	15.4	Low	Medium	4	Rge	13.0	Medium	0.54	0.65	0.64	13.0	Medium	Medium	
Cemig	2	0.75	0.80	0.85	10.9	Medium	High	5	Rge	22.4	Medium	0.79	0.79	0.82	22.4	Medium	Medium	
Cemig	3	0.72	0.73	0.77	16.1	Medium	Medium	6	Rge	26.8	Medium	0.89	0.89	0.91	26.8	Medium	Medium	
Cemig	4	0.72	0.73	0.82	14.9	Medium	High	7	Rge	21.6	Medium	0.93	0.93	0.95	21.6	Medium	Medium	
Cemig	5	0.86	0.86	0.87	14.9	Medium	Medium	8	Rge	29.6	Medium	1.00	1.00	1.02	29.6	Low	Medium	
Cemig	6	0.84	0.86	0.95	9.4	Medium	High	9	Rge	26.0	High	1.00	1.00	1.06	26.0	Low	High	
Cemig	7	0.57	0.68	0.81	8.8	Low	High	10	Rge	16.9	Low	0.69	0.69	0.68	16.9	Medium	Low	
Cemig	8	0.46	0.52	0.59	13.1	Low	High	11	Rge	20.9	Medium	0.66	0.66	0.68	20.9	Medium	Medium	
Cemig	9	0.38	0.38	0.45	33.2	Low	High		Average			0.75	0.79	0.79				
Cemig	10	0.76	0.76	0.84	23.9	Low	High		Std. deviation			0.19	0.18	0.15				

The 15 UNs in Model 1 are efficient; note that nine UNs belong to an area with a high customer density. The UNs with low customer density that reached the frontier are Aes Sul (UN 9, 12) and RGE (UN 8, 9), which implies that the management is relatively good in terms of resource use.

The other UNs with low customer density had average efficiencies of 0.57. The inefficiencies of all of the low-customer-density areas may be mainly due to poor load characteristics and scattered households, which cause these areas to be expensive and challenging for a power supplier.

All of the UNs of Eletropaulo are efficient. It is noteworthy that Eletropaulo operates in an area with the highest load density in the country with low lightning incidence, in other words, a favourable area. Thus, in this model that includes no environmental variables, this distribution company appears as the most efficient.

CEMIG (UN 9) has the worst score (0.38). The UN is compared to a linear combination of Aes Sul (UN 12), Eletropaulo (UN 3) and Light (UN 4). CEMIG (UN 9) has a strong rural character, while its latter two peers have an urban characteristic. Thus, it is expected that this Unit Network will increase its efficiency in Model 3, which includes customer density. From this comparison, the model results indicate that there must be a 62 % reduction in the number of employees.

Under Model 2, to which quality of supply was added to the analysis, 17 UNs are efficient, and 11 UNs are located in low lightning incidence areas. The average efficiency shows that some Unit Networks rank high in Model 2 while they rank low in Model 1.

Elektro has better results. Elektro (UN 1) has an efficiency of 0.45 in Model 1, where quality is not included. In Model 2, the same UN has an efficiency of 0.84, an increase of 0.39 in efficiency score. This indicates that the Model 1 can penalize Unit Networks that are efficient in quality of supply.

Elektro (UN 1) peers are Aes Sul (UN 9), Eletropaulo (UN 3) and Piratininga (UN 1); the latter belongs to the distribution company with the lowest SAIDI in Brazil.

Thus, Elektro (UN 1) showed an efficiency increase due to quality of supply because it has a SAIDI of 6.8 h, and its peers in Model 2 have 16.7, 7.1 and 5.0 h, respectively.

Comparing UN 1 with other UNs of Elektro, it has the second smallest SAIDI of the company, surpassed only by UN 8, which operates in the most industrialized region of the concession area.

Light (UN 5) had an efficiency of 0.72 in Model 1; in Model 2, it achieved the efficient frontier, an increase of 0.28 in efficiency score. The UN has the smallest SAIDI of the company with 6.4 h; the others have SAIDIs between 8.6 and 14.5 h.

Model 1 may distort companies' incentive. For example, in Model 1, RGE (UN 4) had an efficiency of 0.54 (which would result in a high X-factor) while its efficiency score in Model 2 is 0.65.

These findings suggest that there is trade-off between labour and capital inputs and quality of supply. Thus, models with quality are more suitable for efficiency analysis (Gianakakis et al. 2005). In this way, models like Model 1 have not captured the quality of supply aspect of distribution companies.

Under Model 3, there are only seven efficient UNs that contrast with the results of Model 2. Some Units Networks have decreased their performance because they are located in a more favourable area. Some Units Networks have increased their performance because they are located in a less favourable area. For example, all four UNs of Eletropaulo have decreased performance. This is consistent with the reality that this company is in a high-density area.

Additionally, CEMIG improves its performance, but is still far from the efficient frontier. CEMIG (Unit Network 4) has an efficiency of 0.72 in Model 1 and 0.73 in Model 2, where environment is not considered. In Model 3, the same Unit Network has an efficiency of 0.82, an increase in efficiency score of 0.10 and 0.09, respectively. This change can also be explained because of its lower-density area and the lightning incidence in some of its regions. This result indicates that the Model 1 and 2 can penalize Unit Networks that are located in an adverse area.

Another interesting result from Table 7 is the differences in performance of UNs that belong to the same company. The manager can look more carefully for the worst UN and establish an improvement plan to take the UN to a better rank. For example, Aes Sul (UN 1 and 2) had an average efficiency of 0.41 in the Model 2. Their environment can explain part of this inefficiency: UN 2 has the third highest lightning incidence in the company and a density of 3 customers per km<sup>2</sup>. These environmental characteristics are reflected in the quality of supply: Aes Sul (UN 2) customers on average suffer 42 h per year without electric power. Aes Sul (UN 1) has a less adverse environment than Aes Sul (UN 2), with lower lightning incidence and 6 customers per km<sup>2</sup>.

## 5.2 Company-Oriented Analysis

The results of the three models are compared under the two approaches: (1) UN as DMU and (2) distribution companies as DMU. For the first approach, the results of Sect. 5.1 were weighted by the number of customers of each UN that belong to one company to produce a weighted average for each company.

For Model 3, in which environmental variables are included, the Tobit analysis described in Sect. 3.1 was applied, and Table 8 presents the estimation results.

The  $p$  value is greater than 0.05, which means that the variables are not significant. This result was not observed for the Unit Network-oriented approach (see Table 6 in Sect. 5.1).

**Table 8** Tobit analysis results—utilities

Variable	Parameter	Coefficient	<i>t</i> ratio	<i>p</i> value
Constant	$\beta_0$	0.92	27.86	5.48E-15***
Lightning	$\beta_1$	4.43E-08	0.48	0.64
Customer density	$\beta_2$	7.31E-05	1.36	0.19
Dummy for ownership	$\beta_3$	-0.04	-0.70	0.50
Number of observations	20			
Censored observations	0			
Log-likelihood	25.24			

\*\*\* Significance at the 1 % level using a two-tailed test

**Table 9** Comparison of aggregate approaches

Utility	Unit Network			Utility		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Aes Sul	0.68	0.70	0.72	0.95	0.95	0.95
Bandeirante	0.91	0.95	0.85	1.00	1.00	1.00
CEEE	0.83	0.83	0.79	0.84	0.84	0.84
CEMIG	0.76	0.79	0.87	0.98	0.98	0.98
Elektro	0.60	0.79	0.81	0.66	0.78	0.78
Eletropaulo	1.00	1.00	0.69	1.00	1.00	1.00
Light	0.88	0.98	0.79	0.92	0.95	0.95
Paulista	0.97	0.97	1.09	1.00	1.00	1.00
Piratininga	0.93	0.93	0.86	1.00	1.00	1.00
RGE	0.69	0.73	0.76	0.92	0.92	0.92
Average	0.83	0.87	0.82	0.93	0.94	0.94
St. deviation	0.14	0.11	0.11	0.11	0.08	0.08

One possible reason is that the environment variables are treated as averages for the entire concession area, failing to represent the diversity among regions as observed, for example, in the CEMIG concession area.

This fact is shown in Table 9. For the utility-oriented approach, the efficient scores under Model 2 and Model 3 do not differ (columns 3 and 4 in the right table), whereas this is not true for the Unit Network-oriented approach (columns 3 and 4 in the left table).

Bogetoft (2014) states that the models that ignore important environmental variables may have biased results. If environmental factors have impact on operation, such as rain and lightning, they must be part of the efficiency analysis.

This is a very important result because many regulators, including that in Brazil, use the utility-oriented approach.

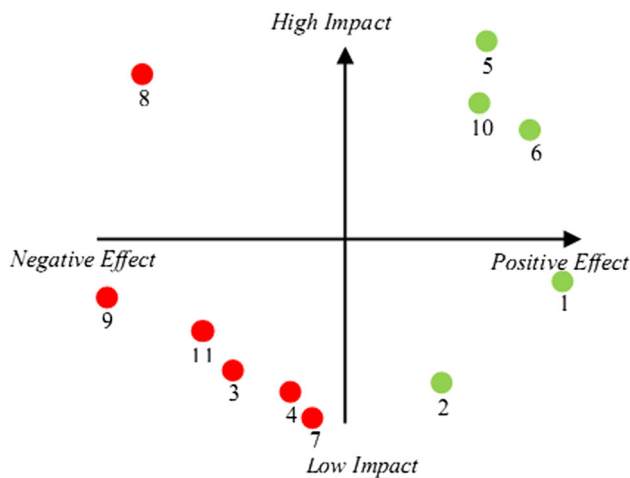
Based on the left table, CEMIG improves its position and efficiency score under Model 3. In Model 1 and 2, CEMIG occupies the seventh position, while in Model 3, the same company occupies the second position. CEMIG increased its efficiency by 0.11 compared to the Model 1 and 0.08 when compared to the Model 2. Eletropaulo leaves the efficiency frontier when compared to Models 1 and 2, with a decrease of 0.31 in its efficiency score. This is because CEMIG has a wide concession area with different characteristics, particularly the

environmental aspects. This is not observed at Eletropaulo, which has a small concession area characterized by a high-density load.

Paulista also had its efficiency increased with the addition of environmental variables. The company increased its efficiency by 0.12 compared to the Model 1 and 2. Despite an environment with medium customer density, the distribution company operates in an area with a high lightning incidence.

To evaluate the economic impact of different models presented in the left side of Table 9, a simulation was done with data from the Elektro distribution company. If we consider the Model 1, the reduction in Parcel B value is US\$ 13,257,836. When evaluating the Model 2, Elektro has to reduce US\$ 8,020,991 of the Parcel B value in the first year of the Third Price Revision, it means US \$5,236,845 less than in Model 1. Model 3 imposes a reduction of \$6,032,331 in the Parcel B value. This reduction is \$7,225,505 lower than in Model 1 and \$1,988,660 lower than in Model 2.

For a better view of the UN influence on the company performance, Fig. 5 is generated from Table 7: each UN in CEMIG is mapped according to its effect (positive or negative) and its intensity (high and low) on the efficiency score of Model 3.



**Fig. 5** Unit Networks map

For the impact intensity, the number of consumers was used as a weight to address the relative importance of one UN to the company. For the positive and negative effects, the scores were divided into quartiles; the first quartile means the best performance and the fourth the worst. In this way, the UNs in the first quadrant have high positive impact, those in the third quadrant have low negative impact, etc.

From Table 7 and Fig. 5, one can see that UNs 8 and 9 play an important role in lowering the position of CEMIG because they have an average efficiency of approximately 0.52 in Model 3.

UNs 8 and 9 are located in Southwest and Northwest of Minas Gerais state, respectively. These regions are characterized by low customer density (5 customers per km<sup>2</sup>) and high lightning incidence. This adverse environment is reflected in the quality of supply: UN 9 customers on average are without electricity 33 h per year (highest SAIDI of CEMIG).

Thus, the focus of the administration should be on UNs 8 and 9; every effort should be made to understand the problems and make the necessary adjustments to reduce the negative influence of the environment.

UNs 1 and 6 contribute positively to the company rank because they have an average efficiency of 0.96. UN 1 is located in north-eastern Minas Gerais state, which has a low customer density (6 customers per km<sup>2</sup>). UN 6 is in the central region of the state, characterized by a greater customer density than UNs 1, 8 and 9 (42 customers per km<sup>2</sup>) and high lightning incidence. It is noteworthy that UN 6 has the second best SAIDI of CEMIG.

It is important that with the UNs approach, the CEMIG administration can compare performance among their regions, extract lessons from UNs 1 and 6 and apply them UNs 8 and 9.

Some companies such as CEMIG, Elektro and Light already split the administration into regions. Each region has

its own management and the board of the company views each as independent, i.e. each can allocate resources (capital and operational costs) to accomplish the objectives of the company. Although the UN was originally formed using electrical characteristics, they try to delimit regional units by their physical aspects, which resembles the approach described in Sect. 3.2.

## 6 Conclusion

Efficiency analysis is receiving considerable attention from the regulators of the electricity power sector, more specifically in the electricity distribution segment. Because of the natural monopoly characteristics of the distribution segment, utilities are not subjected to market forces. This paper simulated a virtual competitive scenario among utilities. Data Envelopment Analysis assists in this purpose by calculating the relative efficiency of distribution companies. It constructs an efficient frontier from the input and output data of a decision-making unit. This analysis provides a framework to analyse the effect of environment on distribution performance, especially in case of countries with large territories.

The novel approach of this paper is in the use of Unit Network for split a distribution company concession area into more homogeneous subgroups that are further considered as decision-making units, being different from the traditional approach in which companies are seen as natural DMUs. Brazilian distribution companies are subject to external and internal heterogeneity due to its large concession area. This proposal solves the external and internal heterogeneity problem of Brazilian distribution companies.

Although it may seem strange to view UNs as being administratively independent, many companies with a large concession area have already created their regional units. Companies may differ in the degree of freedom of decision-making in terms given to each regional unit. This issue may also arise even for the traditional approach, because there are many distribution companies in Brazil that belong to the same holding company and would have the same guidelines in terms of administration.

Another important improvement of the proposed method is that quality and environmental characteristics can be better represented when the company is divided into UNs. We studied three different models (Models 1, 2 and 3), and two analysis were made: one treated the Unit Networks as decision-making units and the other treated the companies as decision-making units.

Considering Unit Network-oriented analysis, we found that some UN that had a poor performance in Model 1 did score high in Model 2. These findings show that it is necessary to integrate quality of supply in benchmarking models.

We find evidence of statistical significance in the relationship between environment variables and efficiency scores in Model 3. Thus, lightning and customer density in our case have an impact on the performance of UNs. The size of adjustment of efficiency scores in some UNs is remarkable.

Considering company-oriented analysis, we also found that efficiency scores are affected by the inclusion of quality. With regard to environmental variables, the effect on efficiency scores is insignificant. One possible reason is that the environment variables are treated as averages for the entire concession area, failing to represent the diversity among regions as observed.

The definition of the product “electricity” and its price cannot be disassociated from quality of supply and environment characteristics. The distribution charge must take into account location, voltage level, quality of supply and the environment. Given that DEA is used for determining the allowed revenue, the regulator cannot override these factors.

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