Force, Cause, and Explanation: Euler and the Metaphysics of Science

by

Michael Veldman

Department of Philosophy
Duke University

Defense Date: July 5, 2024

Approved:

Katherine Brading, Advisor

Andrew Janiak, Co-Chair

Jennifer Jhun

Jeffrey K. McDonough

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Philosophy in the Graduate School of
Duke University.
2024

ABSTRACT

Force, Cause, and Explanation: Euler and the Metaphysics of Science

by

Michael Veldman

Department of Philosophy
Duke University

Defense Date: July 5, 2024

Approved:

Katherine Brading, Advisor

Andrew Janiak, Co-Chair

Jennifer Jhun

Jeffrey K. McDonough

An abstract of a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Philosophy in the Graduate School of Duke University.
2024

Copyright by
Michael Veldman
2024

Abstract

Euler is a centrally important figure in the history of modern philosophy, having indelibly shaped the metaphysics and epistemology of Enlightenment science. This claim, however, will come across as news to almost any philosopher or historian of philosophy today. For Euler, best known for his many fundamental contributions to mathematics and physics, is little discussed in the philosophical scholarship.

This dissertation demonstrates Euler's importance to philosophy. I reveal and assess Euler's contributions to major debates, long of interest to philosophers, on causation and force, body and substance, the structure and foundations of physical theory, and teleology. I argue that Euler's philosophical interventions represent a radical rethinking of the relation between metaphysics and physics, with repercussions for all three major traditions in natural philosophy of his day – Cartesian, Leibnizian, and Newtonian.

In Chapter 1, I introduce Euler and open the question as to why he deserves the attention of philosophers, before reviewing relevant literature in philosophy and history of science and describing my own philosophical and historiographical approach. Throughout this dissertation, I employ contextual philosophical analysis of the works of Euler and other figures connected to him, aiming to move beyond traditional historiographical categories that tend to slot figures into pre-determined molds, like "Cartesian" or "Newtonian." Here, I also provide a brief historical survey of the early 18th century problem context surrounding force, causation, and explanation in physical theory, discussing the views of the three main figures mentioned above – Descartes, Leibniz, and Newton – and of key contemporaries of Euler's. The remaining three chapters then delve into specific philosophical themes.

Chapter 2 treats Euler's intervention into the complex, 18th century debates on force and causation in physics. I argue that Euler seeks to accomplish two things by conceptually engineering the physical notion of 'force': one, to resolve intra-theoretical confusion deriving from ambiguities persisting in physical theory following Newton and Leibniz, and two, to end a pernicious theoretical habit of deploying 'force' concepts to give the appearance of having genuine, *a priori* causal knowledge. By "screening" metaphysics from mechanical theory, he dealt a sharp blow to the 'Leibnizian-Wolffian' philosophers, and to an extent the 'Newtonians,' too, who all used 'force' concepts in service of vastly disparate explanatory ambitions. Philosophers and scientists through the course of the 18th century ceased talking about 'force of inertia', 'active force', or 'vis viva' as a measure of the 'force of bodies'. Euler is to be credited with supplanting them all with the notion of *impressed force*, which he did in the course of offering a radical re-conception of the separate tasks of science and metaphysics.

Chapter 3 offers a new exposition and analysis of Euler's foundations for physical theory. Euler intended to give a fundamental ontology of material body sufficient to ground mechanics, ostensibly *a priori*, since it was based on conceptual analysis, first principles, and definitions. Through close reconstruction, I ultimately find that Euler's fundamental ontology did not succeed in providing an *a priori* basis of mechanics. However, I argue that his attempt succeeded to a much greater extent than critics have given him credit for, and provided a major clarification of the concepts and logical structure of classical mechanics.

In Chapter 4, I turn to final causation, or teleology, in relation to the principle of least action (PLA). Euler and Maupertuis based the PLA on ideas of the "economy" or "simplicity" of Nature, and offered it as a foundation for physical theory alternative to the "mechanical" laws of motion descended from Newton. Some scholars have taken the PLA to evidence teleology's survival into Enlightenment physics. Others claim that PLA-based teleology was refuted by philosophical attacks on the very concept of final causation. I argue that no narratives currently on offer fully capture the philosophical interest of the demise of PLA-based teleology. Through attention to the work of Euler and Maupertuis, as well as contemporary critics like d'Alembert, d'Arcy, and König, I show how that teleological metaphysics came to be refuted because it could not be coherently modeled in the mathematics. Hence, the metaphysics built on the back of the PLA was refuted directly through mathematization, and not by philosophical argument or empirical test. This radical reshuffling of epistemic authority regarding questions of basic ontology appears to be a historical novelty.

Chapter 5 concludes and discusses directions for future work.

Table of Contents

Chapter 1. Introduction

1. Euler the polymath

Leonhard Euler is a figure who needs no introduction. His name is ubiquitous in mathematics, and attached to techniques and concepts that feature prominently in physics, economics, computer science, and engineering. Everyone will have heard of him.

That, at any rate, was what I thought for most of the period in which I was writing this dissertation. Alas, not everyone has heard of him. So it may be worth saying a little bit about the man who figures so prominently in this dissertation.[1]

Born in 1707 in Basel, Switzerland, his gifts were recognized while young and he was tutored by Johann Bernoulli of the famous family of Swiss mathematicians, with whom he remained close throughout his life. Prussian and Russian heads of state competed for his services, and as a consequence his academic career saw him moving back and forth between the scientific academies in Berlin and in St. Petersburg. He remained active until his death in St. Petersburg, in 1783, at the age of 76.

On lists of the greatest mathematicians of all time, mathematicians regularly place Euler at or near the top, alongside names like Archimedes, Pythagoras, Newton, and Gauss. He also has a claim to being the most productive mathematician or scientist of all time. The catalogue of his works, compiled by Gustav Eneström, tallies 866 texts containing countless fundamental contributions. Many of them are book length treatises on subjects including mechanics, physics, algebra, analysis, calculus of variations, optics, astronomy, and music

[1] For more, see the excellent recent biography by the intellectual historian R.S. Calinger, *Leonhard Euler: Mathematical Genius in the Enlightenment* (2016).

theory. Among his many famous results is what many regard as the most beautiful equation in mathematics, an identity which links together 5 fundamental mathematical constants[2]:

$$e^{i\pi} + 1 = 0$$

Among his many contributions to various fields of mathematics, several of which he founded: the Euler characteristic (topology), the Euler-Lagrange equations (calculus of variations), Eulerian paths (graph theory), the Euler formula (complex analysis), Euler's constant (denoted γ), Euler's theorem (number theory), Euler approximations (numerical methods), the Euler-Bernoulli beam equation (engineering), Euler's number (the base of the exponential function, denoted *e*), and Euler diagrams (logic).

It is an old joke among mathematics students that "everything is named after Euler." Even so, his name might have been attached to many further innovations of his. Today, they are so fundamental that they go unnoticed – the concept of a function, the symbols for the imaginary number *i*, and all the symbols Δ, $\sum$, π , cos, sin in their modern meanings (finite difference, summation, the ratio of a circle's circumference to its diameter, and the trigonometric functions). He did not only prove theorems; he invented a good part of the very language which modern mathematicians speak.

By all accounts, Euler was good-natured and generous of spirit. He allowed colleagues and relations to publish articles developing ideas he himself had come up with. (It seems his mind was so fecund that he simply did not have time to write up all the ideas as papers by himself.) He was raised Pietist, and his religious beliefs were personally very important to him. They do not seem to have pervasively affected the character of his thought, though they may be part of the reason he reacted strongly against the perceived

---

[2] Feynman called it "the most remarkable formula in mathematics." *Lectures on Physics*.

threat of atheism in certain philosophers like Spinoza or Wolff, and maintained that some parts of human understanding could not be touched by reason, but only by faith. In his thirties, he lost most of his sight in one eye and, in his fifties, became totally blind. Amazingly, both of these events were followed by periods of enormous scientific and mathematical productivity.

Swiss-born, his native language was German. As this was not a major academic language, he wrote in Latin, as well as in French (the official language at the Berlin Academy where he spent much of his career), and Russian (having done two stints at the St. Petersburg Academy). By proportion, the greatest part of his works are in Latin, including almost all of his work in pure mathematics, followed by a good number in French, and then Russian and German. Among his works, the *Letters to a German Princess* deserves special mention for its cultural significance. Across these three-volumes coming out in the 1760s, Euler laid out a compendious, non-technical picture of his entire thought. It became a best-seller across Europe, going into some 40 editions in multiple languages.

Last, and most importantly for this dissertation, Euler was not just a machine for churning out fundamental mathematical results. His influence was also felt deeply in *philosophy*, although this influence has since largely been forgotten or suppressed within our discipline.

2. Euler the philosopher

2.1 Did Euler do philosophy?

Few will have been prepared before reading this to acknowledge Euler's status as a philosopher. I wish to assert, loud and clear, right off the bat, that Euler must be counted as doing philosophy, provided we accept the following as issues of philosophy:

- The epistemological basis for our knowledge of nature
- The proper methods for scientific inquiry
- The relationship between metaphysics and physics as disciplines of inquiry
- The basic structure and composition of the world
- The basic laws governing natural phenomena – what they are and what kinds of forms they can take
- The logical and inferential structure of physical theory – as a deductive system, its scope and universality (unified field theories)

Of course, historians of philosophy *are* accustomed to studying what the likes of Descartes, Leibniz, Spinoza, Malebranche, or Wolff said about these questions. About, for example, the basic structure and composition of the world. Whether Descartes' pluralism of God, mind, and body/extension, Leibniz's monads, Spinoza's monistic substance, Malebranche's causally inert matter, or Wolff's simple beings, we recognize that there was a rich, early modern conversation about ontology, about what the basic constituents of the world are and what their properties and powers might be. These are classic topics of substance and causation, of dualism and the mind-body relation.

We even recognize a place for "meta-ontology," where we ask questions about the study of ontology itself (McDonough 2021). What are the right methods to use for ontology? What do historical debates show us about the possibility of ontology, of progress in that field, or of certifying it as knowledge? Works by the likes of Euler that seem to bear directly on such questions are not usually taken as data for scholars interested in these topics. To the contrary, such studies usually respect disciplinary boundaries as we have inherited them. And we have not traditionally provided much place *within* meta-ontology for the comparative study of approaches to ontology that cross perceived boundaries between philosophy and non-philosophy. Figures like d'Alembert, Du Chatelet, Euler, Maupertuis, and Newton have historically been perceived as not- or not-quite- philosophers – figures who were perhaps a little too on the science side to justify inclusion in the history of philosophy curriculum.[3] Of course, much of interest *can* be gleaned by looking at threads that wind their way through Descartes, Malebranche, Spinoza, Leibniz, and Wolff, and ignore these other figures. But this scholarly perspective is clearly partial.

A new wave of scholars have lately been seriously reckoning with the facts that inconvenience this stance. Newton, for example, intended his *Principia* to be a *replacement* for Descartes' *Principia*, an intention that was clear to his contemporaries. There is thus a clear case to be made for philosophy scholarship to read these texts in dialogue with each other, which would entail reading Newton's *Principia* as a contribution to the history of philosophy. It is easy for those of us, today, with strong sympathies for empirical science to think that Newton simply *supplanted* discursive speculation with experimental physics and mathematized theory.[4] In that case, there is no philosophical *conversation*. There was merely a monologue which was summarily interrupted by another one, which proved to be more compelling than the first. If this is how we view things, it will of course seem that there is little need for scholarly analysis of their relations.

The difficulty is that two things are true. Newton and Descartes "represent two fundamentally distinct traditions" (ibid.).[5] And yet, Newton and Descartes both "belong to

---

[3] The (absence of a) *will* to inclusion may not even be the main obstacle. For, it must also be said, the scholarly "infrastructure" that would support such inclusion is missing, so that even scholars who would like to do so face significant barriers. Canonical figures of modern philosophy have ready-to-hand literatures, scholarly editions, and interpretive traditions that make their thought hyper-accessible. By contrast, there is no ready "on-ramp" for scholars to get into Euler et al. (a situation that is rapidly changing for Du Chatelet and Newton).

[4] For a helpful overview of the complicatedness of the matter of Newton's relation to metaphysics and to traditional philosophical questions, see Chapter 1 of (Janiak 2008).

[5] The main claim I disagree with in this exposition is that the audiences for these works differed in some significant want. The implication is that the methods used in these two works by themselves sharply bifurcated natural philosophy into two intellectual communities. This picture does not seem to me to be justified. It is true that Newton's mathematical details were accessible in all their technical glory only to a select few. Yet that did not stop many from attempting to read the *Principia* for what they could understand in it, or from reading any of the countless popularizations of and polemics about the "Newtonian" philosophy. On the other side,

the seventeenth-century canon of natural philosophy" (Janiak 2008, 4). The same could be said of Euler and Wolff, or of Euler and Leibniz, or of Du Chatelet and Descartes. All of the figures named so far were deeply concerned with the nature of matter and body, how bodies act in the world, the existence of forces and other causal powers, and the grounding of natural philosophy in epistemically secure methods.

These are just the classic topics of philosophy – of substance and causation, say – under other names. Increasingly, the scholarship recognizes that these figures were all part of one conversation, even if that conversation was in the midst of a process of bifurcation just as it was happening, a process that, ultimately, is not isolable from the broader set of intellectual and cultural transformations we study today as the Enlightenment. Euler was the key contributor to the new physical understanding of the world re-shaping the culture in the period known as the Enlightenment. It is in this scholarly context that Euler emerges as a subject for philosophical study.

2.2 Airing and allaying doubts about Euler-as-philosopher

All the same, scholars faced with approaching Euler as a philosopher are likely to have their doubts and questions. This is probably true even of historians of philosophy who are already familiar with the category of 'natural philosophy' as a historically important framework for philosophical inquiry into nature.

To illustrate, and hopefully allay some of these doubts, consider the topic of chapter 3 of this dissertation. What I offer in chapter 3 is a study of what we would now regard as Euler's physics. The obvious and overarching question about that is: why is Euler's physics

---

Descartes continued to be part of the general training of educated Europeans well into the 18th century, which is why even technically proficient figures like the Bernoullis, Du Chatelet, and Euler continued to be conversant in – and converse about! – his natural philosophy.

worthy of philosophical attention today? Among the possible motivations for doubts about the merits of this endeavor, two strike me as particularly salient.

The first can be expressed thus: "In regard to physics, Euler was a mechanical philosopher. Mechanical philosophy is a movement rightly of great interest to historians of philosophy and of science, but is best appreciated through its canonical, mostly seventeenth-century defenders. By the time *Euler* was working, the impetus driving mechanical philosophy had been spent, making Euler an outlier. This makes his physics a historical curiosity, sure, but not an important, novel contribution to this way of thinking during its period of eminence."

The second set of doubts is that Euler's physics *is impossible*. His physics *failed*.[6] And not, say, in the way Newtonian gravity did, by succumbing to nuanced empirical disconfirmation after centuries of success in generating new knowledge. But immediately and logically. It never got off the ground.

These doubts have, indeed, prevailed among scholars who have turned their attention to aspects of Euler's physics. Euler's natural philosophy is a recapitulation of Descartes' basically mechanical views which was designed to find a way to "fold in" the new resources of Newtonian mechanics (Gaukroger 1982). Or else, it is thought that Euler joined in the gut-level repugnance of action-at-a-distance which is characteristic of the 'mechanical' world-picture, and stubbornly stayed on that train long after the rest of the scientific community had gotten off (Wilson 1992). Indeed, existing analyses have judged Euler harshly. Gaukroger, though (as we will see below) correct in condemning to failure Euler's attempt to give an *a priori* proof of the law of inertia, goes further by dismissing the very

[6] This has, of course, not hindered a rich scholarship on Descartes's physics, especially as begun by Garber (1992).

motive for Euler's project as mere Cartesian foundationalism. In Wilson's equally harsh judgment, alluded to above, Euler's metaphysics of nature "is impossible," its platforms raised on flawed logical pillars (1992, 416).

Let's consider these doubts, in turn. The basic idea behind the first doubt is that the philosophical motives of Euler's physics can be assimilated to the "mechanical philosophy," and that there is nothing new or of interest for philosophers in his version of it. How smoothly, however, does this assimilation go through?

Not very. (Or so I claim.) As a purported member of the mechanical tradition, Euler badly sticks out. This is reflected in the lowest as well as the highest levels of his physics. Both in (1) the motives that drive him to his basic commitments and (2) in how he builds the many particular explanations and theories of his physics out of those basic commitments, Euler differs from canonical representatives of mechanical thinking. Briefly, on (1), Euler's fundamental physical views are not evidentially grounded on an instinctive, automatic revulsion of active forces, occult qualities, and quasi-mental powers in brute matter. They are not based in a reaction against Scholastic modes of explanation. Rather, they are based on his view of the basic, conceptual needs of the science of mechanics. And it is through his orientation towards specific problems he sees in others' physical theories that he generates and grounds his own views. Needless to say, Euler was, in his day, likely the best positioned figure in Europe to make such an assessment in light of the needs of mechanics.

Moving to (2), Euler's differences from this mechanical tradition are further pointed up by their approaches to hypothesis and explanation of phenomena. Compared to the ways that, say, Descartes and his followers construct mechanically-kosher hypotheses, Euler's methods differ in a number of ways: Euler's hypotheses tend to be mathematical in construction (the model elements), tend to make deductive and mathematical linkages between features of the elements and the phenomena they generate, tend to recover important equations (e.g. equations of motion or fundamental laws), and are ultimately compared with quantitative empirical evidence. By contrast, natural philosophers working in a Cartesian vein characterize the elements of their hypotheses qualitatively, link those features to the phenomena they are supposed to generate via qualitative, ad hoc, or analogical stories, recover only qualitative behavior, and make contact with only the most generic, non-quantitative empirical evidence. Euler was, therefore, clearly doing something different from the mechanical philosophers. And yet, he was doing this *while accepting* a set of assumptions (e.g., about the nature of brute matter) that are not unfairly characterized as "mechanical." Rather than giving us license to gloss over Euler as "just another mechanical philosopher," however, I think this combination of facts makes Euler's case doubly interesting and worthy of study.

Now consider the question of the *failure* of Euler's account. First of all, as a general matter, failure, by itself, does not take away from the philosophical interest of the attempt. Indeed, Brading and Stan regard Euler's fundamental physics, together with Du Châtelet's in her *Institutions de physique* of 1740/2, as the two most promising eighteenth-century answers to the call for a "philosophical mechanics" (2024, Chapter 5). The failures of those attempts can thus speak to the philosophical meaning and promise of that conception of the project of natural philosophy. Second, I think scholars have underestimated the degree to which Euler's account succeeds, if "success" is to be measured by such criteria as logical coherence, insight into the structure of basic theory, basis in perceptive critique of prevailing assumptions, and capacity to ground or motivate fruitful research. I think that it is more internally consistent, provides more clarification of mechanical theory, constitutes a more perceptive theoretical critique, and grounded (constitutively or motivationally) more fruitful research than this traditional, pessimistic view has assumed.

Indeed, Wilson himself admits that, because of his fundamental physical views, only Euler could have made his great leaps in fluid mechanics.[7] Wilson and others have proposed a connection between Euler's fundamental (meta)physical views and his research output. Each of the proposals deserves further exploration and substantiation, and make for a strong case that inattention to, or dismissal of, Euler's metaphysics is unwarranted.

Scholars have justified the scant attention to Euler's critical project by assimilating his critiques to repugnance of action-at-a-distance and other prejudices which, today, we view as generic, and even as philosophically idle. They have not attempted to understand how Euler crafted his physical views so as to avoid the problems he identified in contemporary rival theories, whether 'mechanical' or not. As I hope I have shown in Chapter 2, undertaking this new understanding shows the seriousness of the threat that Euler's critiques posed to those theories. In Chapter 3, I provide an account of how those critiques affected the development of Euler's own theory. In particular, I argue that Euler's account is more internally consistent than the likes of Gaukroger or Wilson have allowed, in particular in section 5.3 (though in a sense exhibiting this consistency is the work of all of sections 4 and 5).[8]

---

[7] Wilson writes: "Nevertheless, I wish to argue that we owe a debt of gratitude to Euler's mistaken metaphysics… If it is asked whether, during the eighteenth century, anyone else but Euler might have originated the fundamental equations of rigid-body and fluid mechanics, the answer must surely be 'No.'" (2002, 416)

[8] There are good reasons Euler has proven such a vexing case for scholars of philosophy and of science to handle. On the one hand, he surely spent more of his time in mathematics and the sciences, and relatively speaking, his engagement in philosophical venues with traditional philosophical questions is rarer and briefer than with these other subjects. So, it is natural to assume he did not write philosophical texts of sufficient philosophical sophistication or interest. In consequence, he has been left out of the history of philosophy. On the other hand, it is precisely his *philosophical* commitments – above all, his commitment to something like 'mechanical philosophy' – that has resulted in such an impressive underappreciation of Euler's significance for *physics*. This reinforces, and is reinforced by, the view that, when it comes to the philosophical underpinnings of physics, the 18th century is Newton's. Euler is regarded as having furnished a lot of fancy mathematics and a body of technique to an already complete physical understanding which he himself accepted whole-cloth.

3. A (brief and incomplete) literature review

Euler currently lacks a dedicated and cohesive scholarship in the philosophical literature. Yet, Euler's name has never been completely suppressed within philosophy.[9] Attempts to engage with his natural philosophical thought are scattered throughout the subdisciplines, mainly history of philosophy and the (history and) philosophy of science. When gathered together, they demonstrate substantial and perennial philosophical interest in Euler's thought, though we are still missing the intellectual "infrastructure" which might organize this interest and bring it to self-awareness in philosophy.

For book-length discussions of Euler's "natural philosophy," broadly speaking, there have been a handful of treatments, so far all by Continental scholars: Speiser (1934) (in relation to German philosophy), Pulte (1989) (the principle of least action), Hakfoort (1995) (in relation to optics), and Suisky (2009) (in relation to fundamental physics). The *Leonhardi Euleri Opera Omnia* also contains many useful introductory essays.

Euler's connections to miscellaneous topics in physics, including his foundations for physics, his revision of Newton's laws, his handling of *vis viva*, attraction, and the principle of least action, have also generated a rich discussion. On these topics, see Gaukroger (1982), Hepburn (2010), Sklar (2012, Ch. 11), Stan (2017), McDonough (2021), Lyssy (2022), Brading and Stan (2024, Ch. 5), and Lin (2022).

This view has had many distinguished adherents and can be reasonably regarded as the traditional view. Cassirer, for example, implies that Euler thought that it was impossible to give up Newton's fundamental *concepts* without sacrificing his *results* (1943, 390). And Kuhn regards Euler's work, along with the other mathematical physicists of the 18th century, as solving "problems of application" of Newton's paradigm, or as "clarification by reformulation" – going so far as to call all of their work "reformulations of the *Principia*" (1962, 32-3). Recent scholarship is making the inaccuracy of these assumptions increasingly clear. See e.g. Stan (2017), Hepburn (2010), Hepburn and Biener (2022). This chapter aims to contribute to the more nuanced understanding of the 18th century that is developing from this more recent work.

[9] Fellmann nicely describes how opinion over Euler's philosophical chops and the importance of his philosophical contributions was deeply polarized from the time of his very first sallies into metaphysics (Fellmann 2007, 74-5).

Speiser (1934), somewhat famously, began a literature on the question of Euler's relation to Kant. This "Euler-Kant question" has been perhaps the principal, perennial question about Euler's significance for philosophy. This question proposes we understand it as deriving from *Kant's* significance for (theoretical) philosophy.[10] For discussions of the Euler-Kant question, see Elkana (1974), Harman (1983), Okruhlik (1983), and DiSalle (2013). Other work reads Euler's importance as instead related to the importance of Newton and Euler's revising, completing, or replacing (it depends on who you ask) of Newtonian physics (see note 6 above).

Indeed, these motives are, more often than not, joined to generate a broader one: the placing of Euler in a line "from Newton to Kant", especially in light of the question of Kant's philosophy of science and his attempted grounding of "Newtonian science." See, for example, Watkins (1997), DiSalle (2013), Stan (2017), Massimi (2017), and Hepburn and Biener (2022).

On this subject, the spectrum of opinion is wide. The debate opened with Speiser arguing that Kant *followed* Euler. Euler is alleged to be the first to declare that metaphysics is possible only as a study of the presuppositions and prerequisite concepts necessary for natural science – especially (Newtonian) mechanics. More recently, Massimi (2017) and DiSalle (2013) both find that arguments in Euler – say, arguments in favor of the validity of the concept of absolute space – that appear to prefigure Kant's later, transcendental philosophy were not genuinely "transcendental." On this reading, Kant ultimately *rejected* and had to correct some of Euler's most interesting philosophical arguments. Harman (1983)

[10] As one of the early editors of the *Opera omnia*, Speiser began to inquire into the importance of Euler to the philosophy of the later Kant. The crux idea here is that Euler inspired Kant's eventual conviction that a main project of metaphysics is to take account of, and seek to ground, the basic premises of natural science. This turns around the pre-Kantian view of metaphysics as prior to and independent of natural science, or even as prescriptive and limitative for natural science.

examines the *convergence* of Euler and Kant on their rejection of Newton's concept of '*vis inertiae*.' Yet other scholars find that Kant declined to take up key Eulerian lessons at his own peril. Stan (2013) calls attention to the noteworthy absence of the concept of *impressed force* from Kant's mechanics and from his foundations of natural science, which Stan alleges is based on Leibniz's conception of the laws of mechanics and its attendant concepts of 'force.' In my Chapter 2, I argue that one of Euler's major philosophical projects was to argue for privileging *impressed force* among all the many concepts of 'force' then in play. If this is accurate, it suggests that Kant's physics was marked by a *failure to understand* the importance of Euler's philosophical intervention. On that note, Cassirer (1943) and Calinger (1967) contain some additional commentary on the subject of Euler acting as advocate for Newtonianism in the Newton-Leibniz struggle for the soul of European science.

4. The historiographical and philosophical approach of this chapter

Some of the takes in the literature are rather dismissive of claims to Euler's importance to philosophy, if not condescending in regard to his philosophical chops.[11] Others find in him telling anticipations of nothing less than transcendental idealism. This polarized situation might be characterized as a battle between defenders of two images of Euler: Conservative Euler and Revolutionary Euler. Conservative Euler sought to preserve something like Cartesian science. Whereas Revolutionary Euler anticipated Kant's "Copernican revolution."

Notice how, in spite of their differences, champions of both Conservative Euler and Revolutionary Euler read his significance in relation to – indeed, as a kind of handmaiden to

[11] Part of the reason is doubtless that Euler does not write in a mode that philosophers are trained to read, or read as philosophy. Euler does not have the punchy, shocking theses and clever argumentation of a Hume, or a Leibniz, ready to grab polemical hold of or probe with thought experiments and intuition. Nor does he have, ironically enough, the 'geometrical' style of a Spinoza. Euler is doing philosophy, but in a different genre than what we are used to reading.

– the defining project of *another* philosopher. This suggests that we might take either of two approaches to studying Euler as a philosopher.

On the one hand, we could choose to employ the canonical modern philosophers as touchstones, to slot the different aspects of Euler's thought into familiar categories: "Cartesian", "Newtonian", "Leibnizian", etc. Using this method, Gaukroger, for example, finds nothing novel in Euler (Gaukroger 1983). He judges it to be the same "Cartesian foundationalism" that had already demonstrated its failure to produce a viable natural philosophy. Wilson (1992) concurs on the question of Euler's adherence to "mechanical" ideas about action-at-a-distance and contact forces.

In fact, this method will not turn up anything new in Euler's philosophical thought because it is designed to discover only the old. Moreover, it makes it inevitable that Euler's thought will look like a kind of hodgepodge, lacking internal unity. Here, he believes in essences and a plenum; there, he develops powerful technological advances in the Newtonian analytical mechanics to make strides in gravitational physics and astronomy; and over there, he expounds his vortex theory of attraction and leverages the principle of sufficient reason. Euler comes out looking, not merely syncretic, but confused.

On the other hand, we can follow the likes of Pulte (1989) or, for a more recent example, Brading and Stan (2024). We can start by considering Euler's thought in its own terms, work out its internal logic, and then let this be a guide to its historical-philosophical significance. Rather than simply relate Euler's views to the standard touchstones of early modern philosophy – the doctrines and theses of Descartes, Leibniz, Newton, Locke, et al. – Pulte considers the internal requirements of Euler's philosophical outlook, showing how demands in one locus exert pressures that are relieved by modifications elsewhere, until a consistent realization of Euler's project is attained.[12]

Brading and Stan (2024) explore Euler's project in a way that exemplifies a new philosophical trend: reading eighteenth-century philosophy and physics in tandem and in context. Their book reads the 18th century as engaged in the project of constructing a *philosophical mechanics* – a (metaphysical) theory of the nature of body and of force that will underwrite a union between a *causal* account of collisions and a *quantitative* theory of mechanics, and in particular the rules for collisions. Their fifth chapter considers Euler's work in light of this project. Brading and Stan ask: how does Euler seek to fulfill the demands of a philosophical mechanics of collisions? What parts of that project does he take on, and which does he reject? Why must his account ultimately fail? And what does that failure tell us about the prospects of ever finding a satisfying philosophical mechanics? They assert quite explicitly that 'philosophical mechanics' is a historiographical category. Although it is not Euler's self-assigned project, the investigation is premised on the claim that exploring the 18th century through the lens of philosophical mechanics yields fresh understanding of the period's intellectual tectonics. What are bodies? What are their powers? What is the detailed causal story of why bodies with those powers interact (through collisions) in ways that obey the established rules for collisions? The failure (of Euler and

[12] For example, in his middle period Euler thinks that the faculty of corporeal *impenetrability* is responsible for producing forces through impact and pressure, and also espouses a principle of least action. But the problem arises how impenetrability – which is a qualitative property, not a quantitative one – determines just *this* quantity of impressed force in a given collision. What principle could be extracted from the mere concept of impenetrability to determine this? Pulte thinks that Euler instead drew on the resources of the principle of least action, transfiguring it from a teleological principle to a principle of determination of repellent forces, in the form of this rule: the impenetrability of bodies always produces the *least* force that is required to prevent interpenetration.

others) to answer these questions fully is seen as driving the wedge that had lodged itself between philosophy, on the one side, and physics and mechanics, on the other.

As if it were not yet clear, the second approach is the one I will strive to take in this paper. All the same, it should be clear that a thinker like Euler raises real methodological questions about which well-meaning philosophers can disagree.

Slotting Euler's views into the logical space of possible positions on the canonical issues of early modern philosophy (realism about causation, foundationalism about epistemology, innate ideas versus *tabula rasa*, dualism and mind-body interaction, the possibility of thinking matter, and so on) may be useful as well as informative about the man Euler. But first, carrying out this exercise is effectively *bound not* to turn up new philosophical insights. Scholars will also have to ask themselves which differences matter for a given designation and which do not, and this will depend on the purposes of the scholarship and the aims and interests of the scholar. Euler clearly took himself to be *rejecting* Descartes in assigning to body not only extension, but also mobility, impenetrability, and inertia, the last two of which vied, in his thought, for pride of place in being responsible for the existence of impressed forces. Is this a non- or anti- Cartesian view? Or is it simply an amendment to a theory which is still *basically* Cartesian? There is no objective answer to these questions. To say it is merely an amendment to Descartes is to express one's disappointment that Euler did not come up with some more radical view, some more radical – and modern – *departure* from Cartesian thought. One may well nurse this type of disappointment if one's project is to uncover in history the philosophical authority for movements in the history of ideas that led to 'modernity'. In the other direction, we are not bound to take a figure's self-appraisals and declared allegiances at face value. For example, for all his rejection of characteristic Cartesian claims, Euler's matter theory surely does still bear the mark of its mechanistic predecessors.

Yet, add to this a further fact. The fact that, once one delves into the literature on Euler, one finds that he has been called, at least, a Newtonian, a Leibnizian, *and* a Cartesian.[13] On its face, this is an absurd situation. Naturally enough, these are not logical contradictories, and this could surely be explained with enough space and subtlety, Yet, given the obvious clash between these philosophical outlooks, this situation threatens to obscure more than it illuminates. To study Euler productively, I conclude, we have to be prepared to put down, for a moment, these ways of formulating and answering scholarly questions. In their place, the questions I inquire into here are of the following sort: what was the *actual* conceptual space mapped by the term 'force'? Entering the space starting in the 1730s, Euler evidently had an enormous impact on its character. How and in what ways did Euler *reshape* that conceptual space? How did this *affect the relation* between mechanics and metaphysics, between science and philosophy?

Thus, partly as against previous scholarship, though also, in part, to continue building on it, I propose that understanding Euler's fundamental physical views requires that we re-assess those views in the context of a deeper and more detailed picture of his overall scientific thought. At least as a starting point, this requires that we take seriously his stated philosophical concerns.

---

[13] In some sense, the following chapters constitute an argument that Euler *was not* a Cartesian foundationalist. This label is not apt, both for reasons immanent to his philosophy (it simply does not accurately express his philosophical temperament or commitments) and for reasons about prevailing philosophical attitudes at the time. In a word, Euler was not the only one still seeking apodictic foundations for mechanics. And we would not call (to name two very different, prominent examples) d'Alembert or Wolff "Cartesians" on that basis. There is, besides, an important philosophical reason that Euler *could not have been* a foundationalist, not in the way Descartes was. Namely, he was simply not impressed by the philosophical urgency of answering the skeptic. Like Newton, Euler "takes the possibility of our knowledge of nature for granted" (Janiak 2008, 8). This may seem like a detail, but it alters the whole thrust and purpose of his thought. At the least, it must certainly give us pause before we issue judgments that Euler was a Cartesian foundationalist. Whatever "foundations" Euler sought to give to mechanics or to physics could not be foundations in the sense of Descartes.

Euler's ideas and concerns are idiosyncratic, while at the same time deeply interwoven with the philosophical tradition. This makes him an interesting subject for the history of philosophy proper. And, to my knowledge, many of his concerns – like his worry about the compatibility of inertia with sundry 'active' principles and forces – have not been studied.[14] Taking these concerns seriously will result in a new and richer picture of the guiding issues of eighteenth-century philosophy, and so they will be the launching-off point of the analysis in this chapter.

Thus, my approach differs from the one taken by Brading and Stan (2024), though it is compatible with it. If their approach adopts a 'global' lens to scrutinize particular figures, events, breakthroughs, and so on, another possibility is to begin with a 'local' lens. That is, we might instead take Euler's self-understanding of his project as our 'North Star,' and begin by characterizing a project he would recognize as his own. From there, we can ask: What criteria of success ought *he* to accept? What further constraints was Euler trying to satisfy simultaneously in pursuing this project? How successful was he at doing so? Since, as I've mentioned, I will argue that his account of body and force failed, what got in the way of success? What goals or constraints would he have to give up to be successful? How did failure at this level affect, or else leave in place, his overall project in natural philosophy? From there, we might proceed to make connections to and draw lessons about broader movements. What was the influence of Euler's project on his contemporaries and successors? What parts were perceived as a failure or as a dead-end?

This project has importance beyond the goal of having a more accurate understanding of Euler. For, when we proceed with this analytical lens, the results put

[14] That is, not until very recently. Brading and Stan consider Euler's attempts to avoid these issues by grounding the physics of collisions on impenetrability in their (2024, §5.5).

pressure on prevalent narratives in the history and philosophy of science. Arguably, the received view of the eighteenth century is that science as a whole had to give up on the project of *explaining* properties like gravitation or magnetism, or, in the Enlightenment vernacular, of making them "intelligible." Mechanical explanations of how they operated the way they did routinely failed, and it eventually became clear that better accounts were not forthcoming. Scientists had to revert, in effect, to a scholastic metaphysics of occult qualities.[15] A common story is that doing so enabled scientists to get along with doing useful physics (and chemistry?). The idea is that they could just take attractions for granted and get along with gravity research (for example). From this perspective, metaphysical backpedaling was requisite to start physics on a forward path.

What will one inevitably make of Euler if one accepts this narrative? Of his place in the latter half of the 18th century: that he was a fossil. Of the man himself: that he should have just given up his mechanical hangups, stopped his search for a mechanical theory of gravity and other manifest powers of bodies, and gotten on with physics research. One is all but driven to the view that his work on the mechanical foundations of physics only got in the way of doing serious work.

This answer, however, is seriously puzzling. To offer it is as though to forget that Euler was the virtual engine of such research for almost half of the eighteenth century.

[15] This is Kuhn's example in Chapter 9 of *Structure*: "Gravity, interpreted as an innate attraction between every pair of particles of matter, was an occult quality in the same sense as the scholastics' 'tendency to fall' had been" (Kuhn 2012, 105) Roughly speaking, the claim he means to defend there is that paradigms-as-theories (bodies of first-order scientific claims) rise or fall hand-in-hand with the very canons of explanation (second-order epistemological standards) according to which they do or do not count as legitimately explanatory. "And as the problems change, so, often, does the standard that distinguishes a real scientific solution from a mere metaphysical speculation, word game, or mathematical play" (Kuhn 2012, 103). The case of Euler, however, appears to illustrate how the pursuit of problem solutions in the "Newtonian paradigm" – at least to the extent of accepting the law of universal gravity – could go hand in hand with a continuing rejection of the canons of explanation that it was thought to imply.

Perhaps the implication of this answer is that Euler's achievements might have been still more breathtaking than they actually were. This thesis, to my mind, lacks much prima facie interest. It is also dubious. Wilson (1992) argued, if somewhat tentatively, that Euler's fundamental physical views drove him on to obtaining many of his seminal results in rigid-body and fluid mechanics, going so far as to assert that no one but Euler could have achieved them in the 18th century.

A study of Euler thus throws up challenges to this historical narrative. First, it appears to show quite clearly that this research could get along just fine without taking attractions for granted, as 'occult qualities' or otherwise. It also directly challenges philosophical claims that have been virtually read straight off of this and related narratives. For instance, a characteristic claim of Kuhn's picture of science is that "Effective research scarcely begins before a scientific community thinks it has acquired firm answers to questions like the following: What are the fundamental entities of which the universe is composed? How do these interact with each other and with the senses?" (Kuhn 2012, 5) The research of the 18^{th} century can hardly be called ineffective. And yet the nature of the basic entity – material body – and the ways in which bodies can and do interact with each other were evidently not questions with firm answers.

That specific case from the history and philosophy of science will hopefully serve as initial justification for this study. My motive in making a study of this general kind can be described thus. Philosophy works by thinking carefully about examples. Examples standardly serve as the data or the evidence for a philosophical theory which is supposed to account for a whole slate of them. Sometimes, examples become paradigms for a sub-discipline. These perennial objects of philosophical interpretation, commentary, theorizing, reinterpretation, and so on have a place of special significance, since virtually every theory in the sub-discipline must address them if it expects even an initial hearing. Further, in connection to broader philosophical conclusions, a lot, if not everything, depends on how the example is described and interpreted. So much so that philosophical conclusions are often all but implied by the way a case is interpreted, on what is accented, left unaccented, or even omitted. Given their importance, these examples deserve to be returned to again and again, and looked at with new eyes, as long as they enjoy that status. Classical physics, and in particular the alleged transition through the 17^{th} and 18^{th} centuries to a 'Newtonian paradigm' for physics and astronomy, is, without question, such an example for the philosophy of science.

To appreciate Euler's interventions in Enlightenment philosophy, as well as how having Euler in view might upset prevalent narratives of the period, we now need to go over some history of the period. Crucially, my goal will be to contextualize early modern debates over force and body, especially as they stood immediately in the wake of Newton and Leibniz.

5. Descartes, Newton, Leibniz, and the complex variety of "force" concepts prior to Euler

With some oversimplification, natural philosophy in early modern Europe can be characterized by two key transitions. The academic world had long been dominated by "the Schoolmen," based on Christianized interpretation of Aristotelian philosophy. The first transition was from the doctrines of the Scholastics to the philosophy of the "moderns," spearheaded by Galileo and Descartes and then joined and revised by the likes of Thomas Hobbes and Robert Boyle. This latter was marked by discovery through (at least a limited class of) experiments and observations, and a *mechanistic* world-picture. According to this picture, the physical world was thought to be composed entirely of moving bits of matter, governed by laws of motion which determined the course of all events.[16]

By the end of the century, Cartesianism faced two new challengers, hitting off the second transition. The new philosophical pictures that had ascended to prominence were the Newtonian and the Leibnizian. Depending on where one wishes to place emphasis, this transition could be described in either of two ways. If one wishes to emphasize that the brief struggle between Newton and Leibniz quickly gave way to Newton's victory, then this might be viewed as the transition to the European scientific community accepting the methods and basic world-picture of Newton. This reading has a venerable history[17] and has some claim to being the "received" view. More neutrally, however, it could be viewed as a transition (back) to concepts of *force*. It was a move to (re-)introduce and re-litigate robust concepts of force, power, and active principle.

In many respects, both of these turns in the history of (natural) philosophy can be viewed as turns in the discipline's relation to ideas of *forces*[18] – indeed, this is an organizing idea for this dissertation. As a result, it will be narratively useful for me to tell the history of early modern natural philosophy with this lens. Such a history might go something like this.

In the early 17th century, Galileo was repelled by Scholastic philosophy because, in order to 'explain' natural phenomena, Scholastics cooked up suites of so-called 'forces', agencies endowed with just-so powers that conveniently let them produce the very effect

[16] Galileo's axiom was "ignorance of motion is ignorance of nature" (Swerdlow 2013). Descartes famously purported to "reduce" all phenomena to matter moving according to his laws. Certainly, there are important differences between all of these figures, but here is not the place for a detailed and nuanced account of mechanical philosophy. For a helpful description of different senses of "mechanical philosophy", see Janiak (2008, 119) and also the discussions on pp. 51-2, 87ff.

[17] Cassirer (1943) is a classic, 20th century statement, yet trumpeting the victory of Newton over Descartes and Leibniz is a tradition going back at least to Voltaire.

[18] See McDonough (2021) for a story that follows the decline, fall, and re-ascent of forces or causal powers by tracing a particular thread, one running through Descartes, Malebranche, and Leibniz.

they were invented to explain. Reacting harshly against this profligacy, Galileo decided that a proper science of nature must be formulated, to use modern terms, purely *kinematically*.[19] Once one endeavors to probe the underlying causes of these motions, he thought, one is adrift in a sea of speculation about forces, active principles, sympathies, and antipathies.[20] Better to sweep all that away. There is a more profitable suite of concepts for analyzing the motions of bodies: lines and angles, planes and spheres, speeds and accelerations. This is the stuff of *geometry* – the true language of the book of nature. In Swerdlow's words:

> "Galileo's first interest was mechanics, the application of mathematics to equilibrium states, statics, and to motion, kinematics and dynamics, the object of which is to reduce these subjects to geometry, without hidden forces or qualities." (Swerdlow 2013)

Descartes wished to carry largely this idea forward in his own project, including a complete kinematics. He famously thought that body was equivalent to 'extension'. That is to say, all there was to a body, for Descartes, was a fragment of extended space having a certain size and shape. In the *Principia philosophia*, the laws he wrote down which determined the motions of bodies made no reference to forces. The subsequent motions of a system of bodies were to be determined solely from the size, shape, and prior state of motion of those and perhaps other bodies. A philosophical movement was built on the idea that what the

---

[19] For discussion of why Galileo was "vehemently opposed" to the Renaissance natural philosophical tradition, and the role of his kinematics in paving an alternative path, see Henry (2011). In particular, what Galileo rejected is the notion of force as an agent, a cause of motion, or an existing entity. The causal action of *bodies* might still be expressed in phrases such as "the force of impact" (q.v. pp. 14-15).

[20] Henry (2011) refers to this constellation of ideas, in discussing Galileo's reaction to it, as "occult notions of force" (q.v. pp. 16-17). Jammer, in his well-known history of the force concept, writes of this web of associated notions thus: "We have also seen, in the chapters preceding Chapter 5, how the concept of force during its prescientific and semiscientific stages became loaded with a multitude of metaphysical, spiritual, and other extrascientific connotations. These connotations and associations formed an impressive psychological background, and the concept of force, when viewed against this background, seemed to be a suitable logical instrument to satisfy the human desire for causal explanation." (*The Concept of Force,* 241) Certainly, he is not attempting to write a neutral history, with his notions of pre-, semi-, and extra- scientific. It is worth noting that, outside of mechanics proper, notions of forces, virtues, and principles continued to be used throughout the 18th century in the context of theorizing about heat and fire, gravity ('attraction'), chemical reactions ('fermentations'), and living things.

Scholastics were constantly and remorselessly failing to specify was the *mechanism* of action behind the phenomena they studied. The touchstone of scientific knowledge is the mechanism. And the mechanism can be characterized entirely in terms of geometry and motion. Galileo and Descartes differed in their position with respect to a "kinematical ideal." Importantly, Descartes continued to seek causal explanations and not merely mathematically correct and complete descriptions (Schuster 2013). Nonetheless, for both, the ideal was a physics free of "forces" as causal agencies. Two observations about the pursuit of this ideal are important.

One, in spite of their efforts, their physics arguably fell short of this ideal. Force-like notions found their way back into their theoretical edifices through the cracks. At various stages, Galileo theorized notions of impressed and other forces, or 'virtues' (*virtus*), which were responsible for the occurrence of motions other than "natural motions" (Swerdlow 2013). Descartes' third law of nature in Book II of the *Principia* refers to the 'strength' or 'force' of bodies, a notion that, in context, clearly goes beyond matter understood as mere 'extension.' In many of his works, his physical accounts are couched in terms of "tendencies to motion," "force of motion," and "pressure." (Schuster 2013). All of these concepts attempt to represent these forces as direct consequences of motions themselves – as though "tendencies to motion" did not involve any facts which are, as it were, "over and above" the facts about bodies in motion. Yet, the tension inherent in staking such a position became increasingly obvious.

Historically, one response to this tension was a more thoroughgoing insistence on the passivity of matter, to be purchased by means of a new theological interpretation. Thus, Malebranche completely deprived created beings like bodies of all causal powers, and vested all of them in God. In one sense, this was a realization of the ideal that Descartes' thought imperfectly aimed it. Yet, at the same time, it failed to recover the physics – rules for motion and collision – that it was meant to (Brading and Stan 2024).

So, not surprisingly, and second, the ideal was soon abandoned by the most influential successors to Descartes. A few decades later, Newton and Leibniz each developed a new theory of mechanics (in Leibniz's case, never completed), incorporating new kinds of 'forces' – but now placed, undisguised, in the light of day, and in defiance of the austerity of their predecessors.[21] The status and fate of their various concepts of force have since been studied by scholars in tremendous depth. The literatures just on Newton's *vis impressa* or just on Leibniz's *vis viva* would each fill several volumes.[22]

Leibniz's complex taxonomy of forces distinguished not only living forces from dead ones, but also active forces from passive ones, and primitive forces from derivative ones. His system was complicated further, still, by the fact that he accepted two levels of description—physical and metaphysical. At the "bottom level of reality", monads possessed or were constituted by (or simply *were* [Jorati 2021]) active and passive primitive forces, which underlay derivative forces. But this "level" also supported a physical level populated with its own suite of similarly taxonomized forces, including a new set of distinctions between active and passive, primitive and derivative, and now also living and dead forces. Perhaps needless

---

[21] Whether in all respects Leibniz and Newton were fair to Descartes in criticizing him is a question I cannot take up here. It is worth noting that Leibniz at least sometimes qualified his criticism of Descartes on the question of forces. For instance, in a manuscript dated 1702 (G IV 393-400): "Furthermore, in conclusion, I am pleased to add that even if many Cartesians rashly reject forms and forces in bodies, Descartes, however, spoke with greater moderation, and only claimed that he found no reason for using them." (Leibniz 1989, 256).
[22] Two classic treatments are R.S. Westfall's *Force in Newton's Physics* (1971) and De Gandt's *Force and Geometry in Newton's* Principia (1995). As for Leibniz, his position is marked by complexity and subtlety, if not simply difficulty. On the one hand, he railed against Newton for threatening to bring back the "occult qualities" of the Scholastics with his unintelligible concept of at-a-distance attractions. However, he also signaled an at least partial return to Scholastic ontology via his "reintroduction of substantial forms and their concomitant qualities and powers" (McDonough 2021, 154). The issues of what causal powers it is permissible to attribute to bodies or other created beings, which ones are "occult" and impermissible, and how these questions relate to his fundamental ontology of forces and monads are complex questions. An influential recent treatment of Leibniz's metaphysics is Garber (2009).

to say, the relations between these entities purportedly living at different "levels" was unclear, a circumstance which, as I will argue in Chapter 2, Euler very perceptively latched onto in making his own anti-monadological arguments.

Newton, for his part, used the word 'force', Latin *vis*, to name a several related, but distinct entities and distinct quantities related to his conception of matter, and not just what we now know as 'impressed force'. In the context of his gravity research, there were absolute, motive, and accelerative quantities of a centripetal force (*Principia*, Definitions VI-VIII). These concepts might be understood as derivative of impressed force, yet at least two other loci of confusion remained. One is, famously, his concept of gravity as a universal force of mutual attraction. Second, in his theory of matter, one encounters not only *vis impressa*, but also notions that certainly cannot be so reduced, like *vis inertiae* and *vis insita*, the so-called "innate force of matter" (*Principia*, Definition III).

For Newton, the issues get even more complex when we consider his thinking about force and body not only as it appears in the polished, tonally reserved *Principia*, but also as it is revealed in the Queries to the *Opticks* and in unpublished manuscripts. In these texts we see highly involuted speculations marked by considerable internal tension, especially between the mechanical idea of matter as essentially *passive* and the requirements of mechanics that it have certain powers, forces, or active principles. The passivity of matter, its inertia or *Trägheit,* is a power, a faculty by which it does something: resist changes in its state of motion, an exercise of power that results in its *acting* on other bodies. Passivity is, paradoxically, transformed into activity.

In De Gandt's pithily apt appraisal of 'force' as used in 17th century natural philosophy: "The ambiguity of this force notion is very great" (De Gandt 1995, 59).[23] The notion of 'force' had not still not shed the profound ambiguity that had clung to it since the previous century. To the contrary, it became perhaps the greatest, gnarled node in the tangled web that bound metaphysics and mechanics together. Beyond their scientific or philosophical proposals and their merits, the circumstance important for my purposes is how by this stage physics, metaphysics, and mechanics had become deeply intertwined.

This is aptly illustrated by tensions thematized by McMullin (1978). McMullin notes the analogies – alongside important differences, of course – between Newton's and Leibniz's ways of reconciling the passivity and activity of matter (McMullin 1978, 32). Where, in this space, we ought to locate the concept of *impressed force*, and whether it is possible to *retain* the conceptual apparatus of Newton or Leibniz while distilling from it a purely mechanical notion of impressed force is not a trivial question. Newton's and Leibniz's conceptions of *force* were ambiguous and difficult, if not confused. So, many of their followers were left with the task of litigating their meanings, and deciding what to keep, what to alter, and what to omit in their own theories.

The stakes were a viable, rigorous natural philosophy. On the one side, theory needed to be rich enough to produce accurate, powerful, and informative accounts of natural phenomena. On the other side, many feared the risk of throwing natural science back into mystery and obscurity, the realm of Scholastic philosophy.[24] Some commentators, like Kuhn, have simply concluded that that was exactly what happened (Kuhn 2012, 105). The

[23] Chapter II of that work is an eye-opening illustration of the ambiguity of force concept.

[24] As noted above, for Leibniz, a partial return to Scholastic modes of explanation – especially substantial forms and their qualities – was an express part of his scientific thought.

eighteenth century saw a reversion to acceptance of Scholastic canons of explanation. This is allegedly evidenced by the acceptance of universal attractive forces as an "occult quality" without further explanation. Gravity and attractive forces became, of course, the most prominent controversy. But it is only the most famous of *many, many* disputes over the propriety of different modes of explanation and the legitimacy of the 'force' concepts that entered into them. "Force" remained one of the most embattled and ambiguous concepts in natural science and philosophy throughout the 18th century. In this wider context, it is less clear that Kuhn was right to conclude there occurred an, as it were, blanket, or across-the-board, reversion to previous norms of explanation.

As a final preliminary, then, I would like to canvas a small part of the continuing discussion and debate about force concepts and to exhibit further ways they appeared in philosophical-*cum*-physical theory. Admittedly, the account provided in the remainder is highly sketchy, and much more could be said. For now, I focus on a few touchstones of the 1720s to mid-century, particularly Wolff, with some brief treatment of discussions in France around the same.

Wolff, among all the followers of either Leibniz or Newton, likely became the most significant as a philosopher in his own right. Writing his major statements of ontology in the 1720s and 1730s, he came to dominate German academic philosophy in the middle of the century. Although Wolff modeled his theory of monads on Leibniz's, his realist commitments to causation and interaction, and his desire to more explicitly link together his metaphysics and physics, led him to depart from his mentor in various important ways. (On this, see also [Stan 2018].)

Like Leibniz's, his system is characterized by a multiplicity of concepts of "force," and is infamous for its difficulty, not to mention its ambition. Of particular relevance to us now, it contains protracted attempts to philosophically explicate the basic properties of bodies. Through works like the *German Metaphysics* (so-called because it was published in the vernacular rather than Latin) of 1720, the *Ontologia* of 1730, and the *Cosmologia* of 1731, Wolff attempted to give a comprehensive theory of bodies and their mechanical properties, based on a fundamental theory of simple beings. These simple beings, also called "atomic elements" – and later called "monads" by his followers, in imitation of Leibniz – were extensionless loci of force, constantly changing their states and the states of other monads. Whereas Leibniz accepted two levels of description, in Wolff's system the monads form the "bottom level" of a now *three*-tiered picture of reality. At the lowest, monad- or atom- level:

> "Since atomic elements [i.e., simple beings or monads] lack extension, the nature of atomic interaction is not spatial. It is not the case, for example, that Wolff's simple substances influence one another by physical contact and repulsion. Instead, atomic elements as unextended points of force affect, and are responsive to, degrees of change by communicating with each other in time. *The series of changes internal to a given atomic element are the result of its own power (or motor force) as well as the motor forces of other elements to which it is connected*." (Hettche and Dyck 2019, my emphasis)

So the simple beings are unextended loci of force who change their own states *and* the states of other simple beings through genuine, non-spatial causation communication. As for the second level, roughly, Wolff thought aggregates of monads formed physical atoms. These *physical* atoms are composites but are indivisible by *natural* causes.

Finally, these physical atoms in turn aggregate to form the physical bodies we experience, with the properties we observe them to have. Though Wolff somewhat disdained the "narrow" intellectual scope of mechanics, especially the rational mechanics of Newton, he was savvy enough to understand the fundamental status of the inertial property of matter. As a metaphysician, however, he wished it to be grounded ontologically in his monadology, and provided a derivation of the 'force of inertia' relying on the principle of sufficient reason. For Wolff, inertia arose first at the level of physical atoms, and also characterized, of course, physical bodies. His demonstrations of the necessity of inertia very likely inspired Euler's as well as Du Chatelet's own attempts to do the same.[25] In addition to the *force of inertia*, Wolff thought that physical atoms and physical bodies also possessed *vis motrix*, or motive force.

Besides involving various 'forces', the theories of Wolff and his associates have an important defining feature: they were meant to be all-encompassing, total accounts of being, including material body. Indeed, this was what distinguished true philosophy from other disciplines, on the Wolffian picture (Calinger 1967). They were meant to answer questions like:

- "In virtue of what does a body maintain its identity through time?"
- "Why and how do bodies undergo changes?"
- "What is the ultimate source of the changes that bodies undergo?"
- "What is the agent behind the changes produced in a collision?"
- "Are bodies themselves agents, or are they wholly passive?"
- "If bodies are agents, wherein lies their power to act on other bodies?"
- "If not, do what look like corporeal interactions simply provide occasions for God to act?"
- "Are bodies and their powers metaphysically basic, or is it instead the laws of motion which govern them and constitute their mechanical properties?"

Indeed, this use of the word "besides" is inapt. Some idea of 'force' was a key ingredient in virtually all of these questions – not only in Wolff's answers to them, but also as versions

[25] See Stan (2018) for an argument for the Wolffian origins and character of Du Chatelet's physics and metaphysics. For a view of some ways Du Chatelet importantly breaks with Wolff, especially on the principle of sufficient reason, see Wells (2023).

of them were tackled by other natural philosophers. The second important observation about these questions is how clearly they illustrate the close proximity, or even the non-distinction, between metaphysics and mechanics, or physics. In questions like these, a modern reader is likely to still hear a distinctly metaphysical tone.[26]

My intention here is not to catalogue all possible such questions, or even to canvas Euler's, or anyone else's, answers to them. It is, rather, for one thing, to exhibit the diversity of questions that could be asked about material bodies. This helps to bring home to a modern audience how the subject matters of metaphysics, physics, and mechanics once overlapped. In this zone of overlap serious problems could arise about what so much as counts as a genuine answer to one of these questions. These problems were the stuff of major debates about the *scope* of philosophy as a discipline – which, as ever, was itself a *philosophical* question, a question *for* philosophy. Euler, I will argue in Chapter 2, staked one position, among many possibilities, that involved distinguishing between metaphysical and mechanical causes.

For another thing, I wish to emphasize that these kinds of questions drove the dialectic in interesting directions insofar as they served as inspiration even in the heart of "mathematical physics." Indeed, though there is some truth to the claim that Wolffian philosophy was unique in the universal scope of its ambition, these kinds of questions were actively discussed in philosophical circles throughout Europe. Especially vexing were precisely the concepts of *force* and *inertia*. Reflecting on them consistently pushed natural

[26] Many of these would have counted as questions of Physics, in the sense of many prominent figures of the time, like Musschenbroek. Yet, conceptions of physics included questions that *we* might readily regard as metaphysical.

philosophy and its sub-disciplines (as they were then conceived) like physics and mechanics in new directions.

One of these major new directions in mechanics was opened by Jean le Rond d'Alembert, with his *Traité de dynamique* of 1743. As Schmit argues, d'Alembert rejected the "obscure" concept of force in favor of a mechanics based only on the motion of bodies, which reflects the influence on his thinking of Malebranchian arguments (Schmit 2017). Specifically, occasionalists following Malebranche argued that we can form no intelligible idea of how there could be real, particular causal communication between bodies. The only real cause is God. Debate raged in France well into the 18th century, producing no clear answers to basic questions about causation in the world – a situation that has been called "the crisis of causality" (Le Ru 2003). D'Alembert appears to have been among those for whom the notion of "force" as an idea of a real cause of corporeal motion should simply not have a place in the science of mechanics.

Though not in France himself, Euler was closely connected to the Paris Academy of Sciences, entering its essay competitions and keeping up fruitful intellectual relationships with French academicians like d'Alembert, Alexis Clairaut, and Pierre-Louis Moreau de Maupertuis. As such, he was working adjacent to this discursive context. We have already seen some of what Wolff was up to in Prussia. So, it is worth getting a bit more of the story of the French scene on questions of body, motion, and causation up to the 1720s.

French occasionalists had begun in the prior century by criticizing Descartes for his notion of the "force of rest" of bodies. Properly, they thought, this is only a manner of speaking, and bodies, as purely passive, lack dynamical activity. Jean-Pierre Crousaz was a prominent defender of this idea, winning the 1720 essay competition with a discourse on the subject. In his *Essay on movement* of 1726, an elaboration of his prizewinning essay, Crousaz took as fundamental to his analysis of corporeal motion the idea that "Body of its nature is a Uniquely passive being [*etre Uniquement passif*], which does not become active except by the motion that arises in it, which it does not give itself, and which it receives from elsewhere" (1726, 149). Crousaz pushed this line of critique against Descartes (and siding with Malebranche), who he thought had been insufficiently thorough in his rejection of dynamism, insofar as he had ascribed a certain "force" or dynamical efficacy to the state of rest[27]:

> "But for some time the taste for absurdity has seized some scientists of the first rank, and from there has spread like a kind of fashion. They were tired of clarity, and attributed to Rest a certain Force, of which we have no idea, and which, moreover, is incompatible with the notion of rest and all that we know of its nature."[28] (Crousaz 1726, 168)

But Crousaz's critique does not have Descartes as his only target. The passage continues with an explicit attack on the *Newtonian* notion of 'force of inertia':[29]

> "…and to make this hypothesis seem more wonderful, they have indulged themselves in calling this imaginary force by the paradoxical name of VIS INERTIAE. This paradox had enough charm to make one not notice the contradiction." (Crousaz 1726, 168)

From Newton, then, we seem to be getting an idea of the passivity of matter that is paradoxically combined with an idea of activity – indeed, of passivity-as-activity. Though described in detail by McMullin (1978), it was evidently already something that bothered his contemporaries. These conceptual headaches, and the critiques they triggered, set the stage for Euler. Euler's own arguments about "force of inertia" will echo with Crousaz's key terms

[27] For more details, see Schmit (2009, §1), to whom I am indebted for the account given here.
[28] In the original French, the full paragraph reads: "Mais depuis quelque temps le goût de l'absurdité s'est emparé de quelques sçavans du premier ordre, et dès là s'est répandu comme une espece de mode ; on étoit las de la clarté, on à attribuè au Repos une certaine Force, dont on n'a aucune idée, & qui de plus est incompatible avec la notion de repos, & tout ce qu'on connoit de sa nature et pour donner plus de Merveilleux a cette hypothese, on s'est fait plaisir à appeller cette force imaginée du nom paradoxe de VIS INERTIAE. Ce paràdoxe a eu assez de charmes pour faire négliger la contradiction." (Crousaz 1726, 168)
[29] For more details, see Schmit (2009, §2). Recall that, at around the same time in 1720s Germany, Wolff was also incorporating *force of inertia* into his own physics.

of criticism – contradiction, incompatibility, and paradox. However, Crousaz conceptualizes it as a force of *rest*,[30] whereas Euler conceives of it as a force of *conservation of state* and will criticize it on that basis.

[30] Kant will later also criticize *vis inertiae* so understood, attacking Wolff's conception (Stan 2013).

Chapter 2. Force and Explanation: Euler's Demarcation of Metaphysics and Mechanics

1. Introduction

Newton and Leibniz are often credited with reintroducing *forces* into the natural sciences, evidently with great success. But their doing so left the wider field of natural philosophy in a state of profound confusion and disarray. What are forces? What kinds of explanatory work do they do? Who gets to decide what kinds of forces exist, and what their nature and powers are? All of these questions and more were suddenly thrown into sharp relief. Since neither Newton nor Leibniz provided clear answers to them, this resulted in a general condition of pervasive confusion over the notion of 'force' in philosophy and the natural sciences.

In this paper, I argue that Leonhard Euler resolved this confusion. More modestly, I claim he offered a *way* of resolving it, that his attempt was insightful, and that it was philosophically useful – arguably persisting as such ever since. Our modern understanding of the 'force' concept in physics is, largely, *Euler's* understanding.[31] This is the legacy of his resolution.

How did he do it? First, Euler brought clarity to *mechanics* by redefining its core concepts, most pertinently *impressed force* and *inertia*. This intervention is deceptively simple and is liable to appear straightforward to us in retrospect. Yet, crucially, it is arguably the first time any figure clearly defined 'force' not only to *include* everything that we think of today as

[31] More specifically, at least, the concept of force in *classical* physics. In the quantum regime, the 'impressed force' concept and its aptness for physical description may become hazy, for example because the picture of an "object" subject to an impressed force which causes the object to "accelerate" itself becomes hazy.

As a usage note, I will use double-quotes to represent speech, as when I am attributing the quoted words to a particular figure, or else, on occasion, in the sense of scare-quotes as in the previous sentence. Inverted commas are used to talk about concepts, terms, or words. For example, this whole paper is about 'force' concepts, and Euler complained often about the ambiguity in the concept of 'force of inertia'. Occasionally, italics may also be used to mention concepts, as in: "*impressed force* is a clear concept, and *force of inertia* is unclear." Failures of consistency are my own errors.

forces, but also to *exclude* all those posits that were once regarded as forces and which we do not count as forces today. Although that would already be a major achievement, these moves of Euler's had implications beyond mechanics alone.

Second, namely, they show Euler seeking a new *demarcation* between mechanics and metaphysics. If this is correct, then, in contrast to existing interpretations of his philosophical thought, Euler should be seen as staking out an insightful, principled position on a crucial philosophical issue. Additionally, this demarcation is itself of philosophical importance, as it resolved a confused, conceptually fraught situation in ways that have persisted as useful ever since his intervention.

As my sources of evidence, I focus on several sites of conceptual intervention: Newton's 'force of inertia', Leibniz's 'vis viva' paired with Cartesian 'quantity of motion' as a measure of the 'force of bodies,' and Wolff's 'active force'. I understand these interventions as acts of conceptual engineering. Whereas conceptual analysis seeks to define and understand the concepts we have, taken as given, a philosophical project of conceptual engineering seeks to formulate concepts that we *ought* to have, or which best fulfill a theoretical goal. There is thus a normative component to Euler's intervention.

It is significant that Euler carries out this intervention across a variety of types of texts. Sometimes they are easily recognizable to us as works of philosophy or philosophical polemics. Others are quite different: we would normally regard them, today, as part of the history of *physics* or *mechanical theory*. Yet, in his *mechanical* discussions of force and inertia, ostensibly metaphysical considerations about causation and explanation are never far away. This is both a reflection of the period as well as indicative of Euler's project. The period's disciplinary borders were not placed where they are today and did not operate the same way. This frequently put metaphysicians and mathematicians in proximity to each other on questions of natural philosophy. As for Euler, his aim was to leverage a conceptual re-engineering of 'force' to generate a new demarcation between explanatory projects in metaphysics and mechanics. This required him to frequent these borderlands. So, while we are unaccustomed to it today, it is often from texts in "mechanics" that I will pull evidence of Euler's broader philosophical moves for analysis.

1.1 Outline of the chapter

(Section 2) I begin by looking at Newton and Leibniz to motivate the idea that what you find in their concepts of force are persistent ambiguities. My primary goal is to argue that this set the stage for persistent conceptual confusion in the first decades of the 18th century, both within mechanics as well as over what the explanatory projects of their theories were supposed to be.

(Section 3) I think Euler has a way of resolving these confusions and ambiguities. To show this, I propose an interpretation of Euler whereby he conceives of inquiry into the natural world as having three levels or stages, the mechanical, the physical, and the metaphysical. The middle level, that of "physics" or "natural philosophy," has the least sharp boundaries and the greatest tendency to bleed into the other two. We can sidestep this complication, because the most important distinction for my purposes is the one between the mechanical and the metaphysical, or the distinction between what I call mechanical causes and metaphysical causes. Mechanical causes are changes in the states of motion of bodies, and metaphysical causes are primarily the sources of or reasons for the fundamental properties of bodies—for example, inertia, an important property for Euler's argument. The former class coincides with *impressed forces*, which Euler additionally declares to be the only things we should regard as true forces. Why does this matter? Many inquirers declare certain *forces* to be metaphysical causes or else label such causes as 'forces' of various kinds. Euler responds by saying that these uses of 'force' to do metaphysical work are illegitimate, a judgment he enforces by declaring that they do not actually refer to forces. The search for metaphysical causes must proceed by other means.

The remainder of this section is dedicated to clarifying my proposal about what Euler's move is. It may look like a strange move. A theorist, according to a natural thought, is free to posit whatever concepts she likes, including concepts of 'force,' as long as she defines them clearly. By what right could Euler tell Newton that his force of inertia was not really a force, or tell Leibniz that his living force was not really a force? – He might as well be rebuking someone flaunting their new *crepe de chine* dress, by telling them they are not really wearing a crepe! – I argue that the best way to understand this move is that Euler is offering a way to resolve the conceptual morass described in section 2. This conceptual morass, I remind you, will turn out to have two aspects: it manifests as confusion about the status of the 'force' concept *within* mechanical theory, and it also manifests as a mixing together of, or interference between, the explanatory projects of mechanics and of metaphysics. The fulcrum of Euler's resolution of both of these aspects is precisely this regimentation of the notion of force so that it does just mean *impressed force*. So, while Euler's accusation may *seem* question-begging, we should understand it as a gesture of conceptual engineering that offers a significant philosophical payoff.

(Section 4) As a preliminary to the argument to follow, I canvass a wide range of works to show that Euler is consistently sensitive to a demarcation between metaphysics and mechanics. Characteristically, while in the middle of a mechanical-physical discussion, he will bring up a question and make a remark which has the effect of observing that his discussion is drifting into "metaphysical" territory. Evidently, he perceives some kind of a borderline between these subjects. And we will begin to see Euler demonstrating his sense that metaphysicians *encroach* on the territory of mechanics. The main result of this section, however, is that it is important to Euler's understanding that metaphysics is about the underlying *causes* or *sources* of the mechanical properties of bodies.

(Section 5) I next look at a set of particular cases of 'force' concepts where Euler polices usage of 'force' concepts. He declares certain uses illegitimate on grounds that they are incompatible with the conception of 'force' as *impressed force* – which he declares to be the only "true" forces (*Lettres*, Vol.1, Letter 60). In some cases, this is because they simply fail to satisfy the criteria in the definition of impressed forces: they fail to be causes of changes in the states of motion of bodies. In other cases, namely of monadological active forces, it is because they are incompatible with the law of inertia; since this is a definitional correlate of the concept of impressed force, this makes them likewise incompatible with the "true" forces.

(Section 6) My goal, in running through this evidence that Euler consistently *policed* usage of 'force' concepts, is to infer a distinctive motive: that enforcing this definition will bring clarity by restoring the metaphysics-mechanics demarcation that these usages tend to cross or to blur. Put otherwise, Euler was reacting to a sense of *interference* between metaphysics and mechanics, especially in that metaphysics would often *encroach* on mechanics. And his main intervention was to regiment usage of the notion of 'force.'

To establish this, I analyze the cases discussed in section 5. I observe that these cases share three features. First, Euler sensed that these uses of 'force' concepts reflected an interference between metaphysics and mechanics. Second, Euler intervened by policing usage of these 'force' concepts. And third, that important to these interventions was clarifying or dissolving some ambiguity or confusion about the mechanical causes in play in a relevant physical scenario. This ambiguity or confusion was due to the presence of metaphysical explanatory aims which it would be better to purge the notion of 'force' of.

(Excursus) Monadology is the case where it is most clear that what Euler is concerned about is not merely interference between metaphysics and mechanics, but the *encroachment* of metaphysics on the territory belonging rightly to mechanics. Because the case of the monadology dispute is so rich and complex, it calls for deeper analysis. I place this analysis in this excursus, to make it clear that the argument in the rest of the paper should be able to be followed without taking this detour. The first task is to provide the background for the controversy between Euler and the monadologists. This context makes it clear why Euler would perceive them to be encroaching on mechanics.

I then give a novel analysis of Euler's philosophical attack on monadology. I read Euler as noticing that the monadologist, especially Wolff, purports to have explanatory authority over the same domain as mechanics, namely bodies and their motions, and us using this to press onto him a *dilemma*. On the one hand, the monadologist will face an *incompatibility problem*, whereby his ontology is incompatible with bodies having properties that mechanics ascribes to them, principally inertia. The monadologist can avoid the incompatibility problem, primarily by severing or obscuring the connection between the "bottom-floor" ontology of simple beings, with their active forces, and the phenomenal level of material bodies, whose motions are governed by inertia and impressed forces. In that case, however, they face the *explanatory exclusion* problem: for once this connection is severed, it becomes unclear how the doctrine of simple beings and their active forces is doing any explanatory work in relation to bodies and their motions – phenomena which it is, explicitly, the goal of monadology to contribute to explaining. This argument is both interesting and forceful, particularly because it abstracts from the details of any particular version of the monadology and so does not depend on a minutely “correct” reading of any particular monadologist’s theory. If correct, my reading shows that Euler’s philosophical interventions are significantly more sophisticated than he has been given credit for.

(Section 7) Above, I said that Euler insists that the search for metaphysical causes must proceed by *other* means than by the “discovery” of so many *forces*. Once we stop allowing (early modern) metaphysicians to use notions of ‘force’ in their explanations, they are thrown back on themselves. What resources are they allowed to use? Euler is, indeed, not simply against metaphysics, and takes there to be a real scientific task for them, as became clear in section 4. But the argument would be seriously incomplete if Euler did not at least propose some example of how metaphysical causes might be pursued. What are those other means?

This is a large question, and its full answer requires nothing short of a treatment of his overall epistemology. Yet we can infer from his texts that an important component will be to show how the laws of mechanics follow from first principles of knowledge. Euler’s own “deduction” of the law of inertia from the principle of sufficient reason is a case in point. Although his deduction may not ultimately be valid (this is considered in detail in Chapter 3), it illustrates a method by which Euler thinks metaphysics may proceed, in the absence of ‘force’ talk.

(Section 8: Conclusion) At this point I have already written more than enough words, so I briefly conclude.

2. ‘Force’ ambiguity inherited from Newton and Leibniz

First, we need to set the stage. Today, in late modernity, the ‘force’ concept in science is widely taken as fundamental. But it was not always so. In early modernity, medieval thought became notorious for its purportedly dubious explanations of physical phenomena, and especially for its use of numerous ideas of 'forces' and adjacent notions of powers, active principles, virtues, impetus, sympathies, and antipathies. By the early- to mid-17th century, Galileo and Descartes had ridiculed these concepts of "force" and related notions. They called for banishing them from science and philosophy on grounds of being mysterious and 'metaphysical', the occult implements of Scholastic obscurantists.[32]

According to an optimistic reading of this history, a few decades later, Newton and Leibniz reintroduced forces into physics with great éclat. Each of them initiated their own, productive lineage of research in mathematical physics. Much was left to be litigated about the exact nature of these 'forces'. Their inconvenient nomenclature together with headline controversies (like the '*vis viva* dispute') created snags, for example. But, over the ensuing decades, the dust gradually settled. The shroud of mystery and obscurity which had once hung over the idea of force dissipated, and the indispensability of the concept of 'force' to practicing physicists could no longer be contested.

This rather tidy story, useful for some purposes, is nonetheless oversimplified and therefore potentially misleading. One reason is that Newton and Leibniz promulgated their doctrines of forces in a context in which the relations between mechanics, physics, and metaphysics had once again become problematic. Indeed, their doctrines belonged squarely within and partly created this context. The first 'force' concept in Newton's *Principia*, for example, occurs in Definition III:

> "The *vis insita*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line. This force is always proportional to the body

---

[32] See Chapter 1. On Galileo and "forces," see Henry (2011) and Swerdlow (2013). On Descartes, see Schuster (2013).

> whose force it is and differs nothing from the inactivity of matter, but in our manner of conceiving it […].”

He immediately follows that with Definition IV:

> “An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line. This force consists in the action only, and remains no longer in the body when the action is over.”

Already, we have been told that one kind of force – *vis insita* or *vis inertiae* – literally *is* inactivity, and immediately after, we have been told another kind of force – impressed force – is *an action*. Finally, as Definition IV continues:

> “For a body maintains every new state it acquires by its inertia only. But impressed forces are of different origins, as from percussion, from pressure, from centripetal force.”

There follows the definition of centripetal force, as well as its absolute, motive, and accelerative quantities. In this last passage, what is most significant for my purposes is that Newton gives these three options as different “origins” of force. Immediately, it would at least not be obvious to a reader how ‘centripetal force’ could be understood as a physical *source* of force the way percussion or pressure are easily thought of.

As for Leibniz, the story most conveniently begins with the *Brief Demonstration*[33], where the idea is that *force* is estimated from the effect that a body can produce. That is, forces are forces *of bodies*, particularly bodies in motion. Generally speaking, Leibniz regarded *all* forces as “inherent powers of bodies tied inextricably to their ability to move and be moved” (McDonough 2021). The quantity or measure of the force of a body in (actual) motion is the size of the “effect” it can produce. Examples of such effects might include leaving an impression on an impacted surface, transferring motion to another body, or raising a weight against gravity. Leibniz’s primary aims in the *Brief Demonstration* are to show

[33] Full title: “A Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law.”

that "force" in this sense is not equivalent to quantity of motion, in the Cartesian sense of bulk times speed, but that *vis viva* is an adequate measure of force.

Later, Leibniz developed and complicated this picture in "A Specimen of Dynamics,"[34] part I of which was published in 1695 in *Acta Eruditorum*. Here, *vis viva* is reaffirmed as the "measure of a body's ability to bring about effects in virtue of its motion," and is once again defined as mv^2 (McDonough 2021, §3.2). Here, though, *vis viva* now has a contrast class, *vis mortua* or 'dead force.' This is the force of a (resting) body's heaviness, or the force of a rubber band when held in a stretched position. There is no motion – *yet* – but there is a 'solicitation' or 'conatus' to motion. Both living and dead force represent, in different ways, capacities which bodies have to "produce effects," in some sense. Hence, both are classed as "active forces."

If there are active forces, then by rights there ought to be passive ones, as well. And indeed there are. In "On Nature Itself"[35], published in *Acta Eruditorum* in 1698, Leibniz explains that bodies have also 'passive force,' which encompasses a suite of (passive) powers, or faculties of resisting certain kinds of action (Leibniz, 161). The two most basic passive forces are "natural inertia" and "solidity" (McDonough 2021, §2.3). Leibnizian natural inertia is resistance to *motion*, rather than changes in motion; it is because bodies have "inertia" that active force is required to set them in motion. Solidity, or sometimes impenetrability, is a body's resistance to being collocated with other bodies.

As if the multiplicity just in Leibniz and in Newton were not enough – and as if the apparent contradiction in terms represented both by Newton's "force of inertia" and

---

[34] Sub-title: "Toward Uncovering and Reducing to Their Causes Astonishing Laws of Nature Concerning the Forces of Bodies and Their Actions on One Another." In Leibniz (1989), pp. 117-38.
[35] Subtitle: "Or, on the Inherent Force and Actions of Created Things, Toward Confirming and Illustrating Their Dynamics." In Leibniz (1989), pp. 155-67.

Leibniz's "passive force" were not enough – these same features showed up in the major philosophers who took cues from them. This includes the likes of Christian Wolff and his followers. And it includes philosophers in the Cartesian tradition, who, in spite of their forefather, nonetheless had to reckon with the appearance of the new force concepts. As I explain in Chapter 1, the number and variety of notionally overlapping and yet distinct concepts of 'force' in the era of Newton, Leibniz, and Wolff was rather impressive – possibly even comparable to that which we find in the period of the Scholastics.

The menagerie of 'force'-like concepts, whether deployed nominally in metaphysics or in physics, pressed as many difficult philosophical questions as they ever had, ranging from the abstract and general, to the concrete and mundane. What kinds of forces exist? Where do they come from? Wherein do they reside? Do they all derive from mechanical, contact action? Since bodies act on each other, do all bodies then have an inherent force? If bodies are 'inert' and passive, by means of what faculty does one body *act* on another? Must force, as the power to act, come ultimately from God? Does he keep the total amount of force constant, and does he have to renew it? What is the relation between force and quantity of motion? Why do bodies retain their shape under pressure?

So far, this is largely a short trip over ground covered in Chapter 1. And, *prima facie*, these questions all look like just that: new *questions*. New research questions or problems for mechanics, physics, or metaphysics.

At this stage, though, Leonhard Euler started to do something that, to his contemporaries, could have seemed rather outrageous. Newton's "force of inertia"? – *Not a force!*, says Euler. – What about living forces? – *Those aren't forces, either!* – Wolff's active force? *Nope, sorry, not a force!* – One by one, Euler turned to almost every concept of a "force" that philosophers were positing in their theories – all but *one,* that is -- and declared that they were not forces at all.[36]

At first glance, this looks to be a truly bizarre pattern of behavior. Assuming we are reading in context, "forces" could be whatever philosophers theorized them to be. Surely a philosopher has a right to posit whatever entities she likes in order to construct her account of the phenomenon she is interested in? Force of inertia, living force, impressed force, active force, passive force, centrifugal force, and so on. As long as I define what I mean, I have every right to posit these concepts and regard them as species of force. In this light, the move that Euler then started to make was strange and philosophically remarkable.

What Euler is doing is accusing others of making a conceptual mistake on the basis that his own account is the correct one. it is a very broad-brush criticism he's putting forth

Of course, one feeling one might have is this one: yes, Euler is of course correct, but he's stating the obvious. Inertia and the like are obviously not impressed forces. But this appearance of obviousness would be based on an understanding of "force" that presupposes the result of his very intervention. It is *because* Euler's intervention was successful that we now take it to be obvious that there are only impressed forces.

Was this just an act of conceptual aggression, so to speak? If so, it must be acknowledged that Euler won that fight. Today, we side with Euler. From the perspective of contemporary physics, "forces" are just impressed forces, or one of the fundamental sources

[36] What about fundamental attractive forces? Did he declare that these, too, were not "genuine" forces? Here, it is more accurate to say that Euler simply denied that such forces could exist, rather than that the phenomena of attraction was not a matter of forces. What appeared as attractions had to be real forces insofar as they produced changes in motion. The issue was to find the true "physical seat" of the force.

of (particular) impressed forces. None of these other concepts are regarded as species of force any longer.[37]

Evidently, then, Euler's intervention had reverberating effects. What, though, was the ground of this intervention? Euler started to *declare* that various philosophers' posits were not forces, and even that philosophers were *mis-using* the term 'force' in so calling them. From what perspective is he saying this, and how does it entitle him to issue these edicts? Certainly not by availing himself of our modern perspective. We have remarked that it was just this perspective that his work enabled. Then on the basis of what background of general agreement about the meaning of the word 'force' could he make such claims? Was it from any such background at all, or was it a bare declaration? If not a bare declaration, then by what right did he make it?

The short answer is that Euler's entitlement to these judgments is grounded in his offer to resolve conceptual confusion. And indeed, I have claimed that the situation resulting from Newton's and Leibniz's reintroduction of forces was one of conceptual confusion. *Prima facie*, though, all we have seen so far are new questions. The presence of questions does not necessarily spell confusion. So, wherefore *confusion* about forces?

To demonstrate this directly would require a great deal of space. It would involve such tasks as canvassing a wide, representative range of theorists and writings from the first decades of the 18th century, cataloguing the diversity of uses of the concept of "force,"

---

37 Even *gravity*, once one of the "fundamental forces," is no longer regarded as a force, strictly speaking. It is a member of the more general category of "fundamental interactions." – "Well, that's just luck," you might say, "he surely did not anticipate the geometrization of space-time with general relativity." – The question must be handled delicately. It would certainly be too far to say Euler anticipated general relativity. However, Euler's criticism of fundamental attraction was not based merely on revulsion of distance actions. It was based on his view that a force of mutual attraction inherent in matter itself was incompatible with the inertia of matter. The possibility that Euler latched onto a conceptual issue that was not resolved until inertial and gravitational mass were clearly conceptually separated, and Einstein produced his theory of general relativity, deserves to be taken seriously.

analyzing them for their incompatibilities and ambiguities, showing how ambiguities led theorists into diverse kinds of error, and pulling out discussions by contemporary figures reporting their own senses of the confused state of the field. I have done a little bit of this in Chapter 1, but I admit that adducing a truly thorough and direct evidence base would require much more space. Instead of this approach, I will rely on three pieces of evidence that are, perhaps, more "indirect."

First, more recent philosophical commentators have issued just this kind of judgment. McMullin, for example, analyzes at length the confusion latent in Newton's 'force' and 'matter' concepts (McMullin 1978). Core Newtonian concepts like 'force of inertia' embodied, in his judgment, a kind of atavistic call-back to bygone physical notions: "it [the *vis insita*] has overtones of the *impetus* of earlier physics: its measure is similar, and so is its function" (McMullin 1978, 36). Further, McMullin explains this confusion and ambiguity in part by drawing Newton's concepts of force into comparison with *Leibniz's* concepts, implicating the latter philosopher's theories, as well. "From the beginning, his [Newton's] notion of force exhibited the same troublesome ambivalence about passivity and activity as that of Leibniz" (McMullin 1978, 32).

Second, I rely on Euler's statements as testimony in support of the claim. This, of course, assumes that Euler is not a philosophical muckraker, but a competent and reliable witness, as well as a good faith investigator motivated by his interest in theoretical clarity. I propose that we provisionally accept this. In that case, his repeated, explicit statements – which I quote and discuss in detail below – that the theory of forces and matter is "confused" must be taken as strong evidence that this was in fact the case.

Third, analysis of the sampling of concepts I have placed on the table, their logic and their explanatory roles, already shows the beginnings of a pattern of conceptual confusion.

At the time, it was unanimously agreed that the concept of a force was a concept of a kind of *cause*, hence supposed to contribute to *explanations*. Under scrutiny, though, it emerges that it was no longer clear what *kind* of an explanation was being offered, in theoretically positing the presence of a 'force'. Even a brief comparison of some of the concepts whose definitions were provided above – say, the "force of inertia", "living force," and "impressed force" – shows that these concepts are all meant to explain phenomena or specify causes in very different senses and all belong to different ontological categories. That much was already clear from the brief analysis given. The case is bolstered by two further substantive claims that I read Euler as making, whose substantiation I leave for the sections to come. In many cases it could be doubted whether a given theorist's 'force' posit was explaining *anything at all*. And further, because 'forces' were invoked in explanations of different kinds, it appeared that 'force' explanations could clash with each other regarding the *same* explanandum. These are not desirable features of a concept playing an explanatory role.

As a bonus, this conceptual confusion about forces, causes, and explanation can also be appreciated from a disciplinary angle: *who* was to provide the answers to the above questions about forces? Most importantly, who was to determine their nature and powers, and to determine where and whether they are present? The answer was not clear. It was not settled who would provide the answer to each of these questions, or who had authority to do so.

Although mechanics and metaphysics were already distinct disciplines, practiced largely by different people, there was considerable overlap in the subject matter of their inquiries. In particular, inquiry into bodies, their motions, and their causal intercourse was claimed by diverse parties. As a result, questions involving forces did not sort themselves neatly into different categories of inquiry, with mathematicians (*qua* mathematicians) having exclusive authority on some, physicists over others, and metaphysicians over the remainder.

This was a real, and not merely a nominal, problem. For the issue of *who* should address these questions is intimately related to the issues of how to address them, what counts as a solution, and what conceptual tools are available to answer them. All of these issues were in dispute.[38] This "disciplinary" angle on the situation brings the state of disarray into still sharper relief.

3. Euler and the structure of inquiry into nature

We have been looking at the theory of forces and bodies in the wake of Newton and Leibniz in the first decades of the 18th century. At this stage in the argument, I take myself to have provided evidence that that theory was plagued by conceptual confusion. This provided the context for Euler's *prima facie* remarkable and even bizarre philosophical moves of repeatedly declaring various concepts of force then in play to be misconceptions, to be not concepts of force at all.

I now turn to describing the proposal Euler offered for a new, restricted conception of 'force.' But first, the motives for this proposal. Obviously, "resolving confusion" is as good a motive as any. But, more specifically, Euler was picking up on key differences in the kinds of explanations offered by, and causes referred to by, different participants in this inquiry. Thus, an important upshot of this paper is that Euler perceived metaphysicians to be stepping on his turf: mechanics. Throughout his life, Euler was prepared to acknowledge a place for metaphysics, and admitted to making tentative forays into metaphysics himself.

[38] This is also an upshot of recent work by Brading and Stan (2024). The domains of and relations between metaphysics, physics, and mechanics in the time of Newton and Leibniz were not as we think of them today, but were in flux and up for grabs.

The issue here is instead the more delicate one of carefully demarcating the line between the questions it is the job of mechanics to answer from those it is the job of metaphysics to answer, and of ensuring that the metaphysician's box of conceptual tools is not mixed up together with that of the theorist of mechanics.

In line with this, I propose that Euler was working with a multi-level picture of inquiry into nature. When it comes to the study of corporeal nature, Euler appears to think we must distinguish at least these three levels.[39]

The first level is the changes in motions of bodies. These are explained in terms of impressed forces, and providing those explanations is the task of mechanics. The type of account given is a mathematical description of the motions of one or more bodies and of the impressed forces involved in producing them. How can we tell that Euler sees this as an isolated level unto itself? It is demonstrated by his commitment to a restricted definition of 'force' as a cause of change in the state of motion of a body – and this how I will understand the term *mechanical cause* in this paper. It is also clear from the fact that he provides many exemplars of such accounts in his countless works of mechanics. Significantly, among those accounts are, for example, treatments of celestial mechanics. When discussing gravity, Euler consistently and explicitly brackets the question of the causes of the universal attractive force while accepting the law of universal gravity as a mathematical description of the *impressed forces at work*.

[39] I take this conception of the structure of natural philosophy to be implicit in Euler's presentations of the subject matter, and makes the most sense of how he organizes his expositions of mechanics and physics. Although, as presented, it is distinct from what Brading and Stan (2024) term a 'philosophical mechanics,' I take it to be compatible with the claim that Euler was pursuing exactly that. In other words, Euler's project is *one* way of pursuing a philosophical mechanics, although not every project that attempts to provide a philosophical mechanics will look like Euler's. In Chapter 3, I examine Euler's approach to the explanatory tasks at the second and third levels in more detail.

At the second level, we inquire into the explanans at the first level: the existence, origin, and nature of impressed force. Euler's explanation is couched in terms of the basic, mechanical properties of bodies. Where – in whose wheelhouse – does this inquiry fall? It may overlap with mechanics, but it is more broadly an inquiry for what we might consider physics or natural philosophy. One important exemplar is the "Recherches sur l'Origine des Forces," where Euler carries out a kind of inquiry that is distinctive from the kinds of first-order mechanical accounts alluded to in the last paragraph. In that work, on the assumption that bodies have the properties of extension, mobility, impenetrability, and inertia, Euler undertakes to deduce that impressed forces must arise in a certain, very general physical setting – that of contact between two bodies, whether in collisions or through pressure. This demonstrates the origin of forces, says Euler.

The distinction between the first and second levels is admittedly somewhat artificial, since Euler sees the tasks involved these two levels, as I have described them, as linked. Jointly they are the task of "natural science," as he thematizes that project in the first chapter of the *Anleitung zur Naturlehre*. There, the task is to determine the general and the particular properties of material body and to deploy them in explaining why bodies undergo the changes they undergo (§8). Picking up the thread in Chapter 6, he writes:

> "The whole of natural science [*Naturlehre*] therefore consists in showing, for every one of the changes that occur, in what state the bodies had first been and that, because of the bodies' impenetrability, just that change that actually occurred had to occur. Whoever is in a position to explain, in this fashion, the changes that take place in nature has completely satisfied [the aim of] natural science, by deducing the true causes from their basic and irrefutable grounds [*ersten und unumstösslichen Gründen*]." (*Anleitung*, §50)[40]

[40] The existing English translation by E. Hirsch renders *Gründen* as "principles," which I believe to be misleading.

Thus, the task of constructing the mathematical account of the forces and motions and the task of showing how they arise from the properties of body, especially from impenetrability, are continuous with each other and conceived of as two components of 'natural science'. Nevertheless, this apparent complication for my schema is immaterial to my argument. I think the distinction is useful and has application, because Euler himself sometimes compartmentalizes tasks from each level. For example, Euler was prepared to carry out mechanical analyses of the motions of the planets (mechanical level), while leaving aside treatment of the question how the attractive forces themselves arise from impenetrability (physical level). That second, physical project might be addressed in other contexts, or simply bracketed. Whether it applies across the board is of secondary importance. For the important separation is between the first two levels and the third.

The third level repeats the pattern, for it involves explaining the explanantia used in the second level. Why do bodies have the basic, mechanical properties they do? Since inertia can be regarded as a property or as a law, this can also be understood as a request for the explanation of the laws. [41] It is here, at this third level, that a role for metaphysics emerges,

---

[41] Curiously, passages in Leibniz suggest a picture very mjch like this one. In a manuscript of 1702, he wrote:

> "However, I agree with Democritus and Descartes, against the multitude of Scholastics, that the exercise of motive power [*potential motricis*] and the phenomena of bodies can always be explained mechanically, except for the very causes of the laws of motion which derive from a higher principle, namely, from entelechy, and cannot be derived from passive mass [*massa*] and its modifications alone." (Leibniz 1989, 250).

Mechanical principles explain the phenomena of bodies – presumably, explains their motions in terms of the laws of motion – but the *causes* of the laws of motion are left for a higher principle, which seems to come from metaphysics. However, the context of the passage points up the difference between Leibniz and Euler. Immediately preceding the above-quoted passage is the following:

> "Furthermore, with Plato and Aristotle, and against Democritus and Descartes, I acknowledge a certain active force or entelechy in body. […] I believe that every body always has motive force, indeed, actual intrinsic motion, innate from the very beginning of things" (Leibniz 1989, 250).

From an Eulerian point of view, it has become once again unclear a) how this amounts to a clean ascent (or descent, as the case may be) to another level of explanation, insofar as we are still talking about forces of motion, which seems eminently mechanical (equally confusingly, the motive forces themselves are just "actual intrinsic motions") and b) how this alleged explanation *is explanatory* of the laws of motion. A defender of Leibniz may be able to reply to these objections, but I cannot go into this further here. My purpose is restricted according to Euler, and I will call what answers these questions *metaphysical causes*. Importantly, as Euler makes clear from his definition of 'force', forces do not number among the tools that may be used at this level of explanation: forces are not metaphysical causes.

I ought to say a bit more about these terms 'mechanical cause (or explanation)' and 'metaphysical cause (or explanation)', since they form an important part of my interpretation of what Euler is up to. Mechanical causes answer questions as to why (say) a particular body moves a particular way in particular circumstances. This type of question is *causal,* insofar as it asks for the cause of a particular *motion* of a particular body, or with a more general purview, of some manifest phenomenon or change. For example, why does some projectile on earth move along a parabola with a flattened front, rather than a symmetric (i.e. a genuine) parabola? Answer: because collision with and drag from air particles impress forces that impede its forward motion, which is slower on the descent than on the ascent. Or more simply, why has this ball flown up into the air? Because my hand impressed a vertically-directed force on it. These are now classic topics for the science of mechanics. And this is just how Euler wanted us to think of that science.

Another class of questions is causal in the sense that they ask about the (metaphysical) sources of the basic properties of bodies, and of the laws governing them. For example, what is the ground or metaphysical source of a body's identity through time? [42] What is the ground of its inertial resistance to having its state changed, or equivalently, what

---

to explaining what Euler's position *was*, and to providing *motivation* for it. It is beyond the scope of this chapter to answer all possible objections and to conclusively demonstrate that Euler hit his target squarely.

[42] Or, for another example, what is the ground of the unity of an object which is a composite of smaller entities, or perhaps even simple entities? A typical Wolffian answer can be found in §532 of the *Ontologia*, where Wolff gives the following criterion of unity: "The parts of which a composite being is composed constitute a composite through the link which makes the many parts taken together a unit of a definite kind." (Quoted in Hettche and Dyck 2019).

is the ground of the law of inertia? In virtue of what in the body does it continue in its motion? It may seem that the questions in this last class also, by rights, belong to mechanics. After all, mechanics is about bodies, and these questions are about the properties of bodies. But it is precisely when these sorts of questions are broached that Euler so often explicitly flags that the discussion has entered "metaphysical" territory. I argue for this directly in section 4.

From this, I infer that Euler perceives a difference between two kinds of causal or explanatory questions. I will call the first type questions about *mechanical causes*, and the second type questions about *metaphysical causes*.[43] The long and the short of the problem Euler thought was plaguing natural philosophy was this. Philosophers and physicists in the early 18th century would use the word 'force' and cognates for causes falling in *both* categories. This includes even figures who, like Newton, were taken as standard-bearers for empiricism.

It is one thing to say that the variegated uses of the 'force' concept by metaphysicians, physicists, and mathematicians belied general conceptual confusion. Yet, insofar as the distinction has purchase on philosophical discourse as it existed in Euler's time, it allows us to see something further. For, if Euler's claim is true, then it is plausible that such patterns of usage might obscure the fact that an inquiry was moving between

---

[43] The reader might wonder why "physical cause" does not make for a third type. For example, what is the physical source of the force in my hand which presses on the ball? Arguably, something could be said for this having a distinctive place in Euler's thought. However, the distinction or separation of interest in this paper is metaphysics as opposed to mechanics. Although he does make a contrast between Mechanics and Physics, it is not one that he often bothers to flag, suggesting its lower importance. Hence, when, in the course of presenting his mechanics, the discussion drifts away from the mathematical, he flags it as the intrusion of Metaphysics, suggesting that this is a crucial distinction for him. See the *Mechanica*, *Anleitung*, and *Methodus inveniendi*, and quotations below. In this connection, it is perhaps also relevant that Euler discusses even Attraction as a Metaphysical question in the *Letters*. This contrasts with others who ask whether Attraction is a genuine Physical cause of changes in motion. (But even this is somewhat complicated; in the *Mechanica* gravity is treated as a mechanical force.)

different levels or kinds of questioning, from mechanical to metaphysical and back again. By implication, the claim Euler's work was driving at, and which we are mooting here, is that 'force' became the nexus of widespread disagreement and confusion about the purview of metaphysics as such – understood as a distinctive form of (perhaps *a priori*) cognition of nature.

Let's bracket that claim for a moment (we will see some evidence for it below) in order to get Euler's positive proposal on the table. Which are the mechanical causes? They will be nothing other than *impressed forces*. Conversely, the only 'forces' with any place in natural philosophical inquiry will be mechanical causes or, extensionally-speaking, the impressed forces. Euler's proposal, in other words, is to reserve all concepts of 'force' for mechanics, in which domain the concept would be understood only to mean *impressed force*, which was thereby privileged over competing notions of force in physics and mechanics.[44]

This gesture, as I've said, might seem obvious in retrospect. It might seem a truism, without content and so without risk. Yet, in making it, Euler was sticking his neck out philosophically. How so? First, Euler has thereby declared diverse concepts that philosophers deployed in their *(meta)physics* – like 'force of inertia' and 'living force' – to be misconceptions.

Second, the move, in turn, implies and attempts to enforce a new demarcation between mechanics and metaphysics as domains of inquiry, and has the effect of depriving metaphysics of its notions of 'force.' They key point is about metaphysics as an *activity*, and not simply "metaphysicians" as a certain set of thinkers. In particular, it places serious

[44] Here, I speak of concepts of force as used in natural philosophy or theories of nature, inclusive in particular of metaphysics, mechanics, and physics. This is as opposed to, say, metaphorical uses, uses in jurisprudence or the law, etc. Nothing more is intended by it.

restrictions on then-prevalent theories of forces *both* in the broadly 'Newtonian' tradition, widely taken to be the precursor to mainstream classical mechanics and natural science, *as well as* in the broadly 'Leibnizian' and 'Wolffian' traditions, which are often regarded as exemplars of classical metaphysics and speculative philosophy. Why? In spite of the divergent careers of these traditions, all attempted to leverage 'forces' for explanatory projects that Euler locates within metaphysics (like *explaining the nature of body*), in addition to tasks that Euler locates within mechanics (like constructing *analyses of the behavior and motions of bodies*). They all, thus, run afoul of Euler's new line of disciplinary separation.

Specifically, and summarizing, I argue for the following. Euler proposed a resolution of the conceptual morass created by the manifold notions of "force" in play in the early 18th century, a resolution that:

(1) staked out a distinctive position on the domains of and relationship between metaphysics and mechanics, and one that

(2) requires us to respect a distinction between mechanical causes and metaphysical causes that many figures fail to observe.

My interpretation involves seeing his motives as simultaneously first-order theoretical and also, in part, meta-philosophical. I think this makes the most sense of his interventions. Otherwise, we would have to accept them as what I have said, at first glance, they look to be: simply bizarre.

In the remainder of the chapter, I will pull on a variety of Euler's texts where he, on the one hand, confronts metaphysics and its status and, on the other hand, engages with several cases of 'force' concepts and intervenes in the ways alluded to previously. My treatment of this evidence aims to establish the following.

(a) These passages will make it clear that Euler has *picked up on a dividing line* where, in his view, inquiry into bodies and their motions shades from the mechanical into the metaphysical, in accordance with (1) and (2) above.

(b) These passages will repeatedly exhibit Euler using his definition of force (as impressed force) to *police usage of 'force' concepts* in philosophy and physics.

(c) These acts of conceptual 'policing' as in (b) are best made sense of in light of the demarcation line as in (a).[45]

4. Metaphysics versus mechanics

It may surprise a philosophical audience to hear that Euler referenced metaphysics regularly across a wide range of works. This of course includes the handful of his works directly engaging with "philosophical" questions, as well as the magisterial summary of his overall thought, the *Letters to a German Princess*. But it also encompasses a significant number of works in mechanics and physical theory. Nor are these appearances of metaphysics easily explained away as uses of the word "metaphysics" in the dismissive, pejorative sense.

Though not straightforwardly *pejorative*, Euler's uses of the word "metaphysics" nonetheless have a *critical* function. Let's begin with Newton's concept of 'force of inertia.'

[45] Euler's proposal raises an important question: if metaphysics cannot avail itself of "forces" as conceptual tools, what, if any, tools are left to metaphysics in pursuit of its projects? I consider this question in section 7. A second important residual question that may occur to the reader is: how conceptually "just" was Euler across his acts of policing – in particular, with respect to gravity and universal attractive forces, which he famously rejected? The question arises because, insofar as his criteria for "genuine" forces led him to *reject* attractive forces, it might seem that those criteria led him to make at least one wrong call. Is that true, and if so, does that undermine the claim to philosophical insight of his overall intervention? I do not think so. Attractions raise a number of interesting questions because a) Euler did not think that fundamental attraction was explanatory of anything, b) he did not think fundamental attractions could be genuine impressed forces, because c) he took such forces to contradict the law of inertia. However, these issues take us well beyond the scope of this already lengthy chapter, and I cannot address them here. In Appendix B, I canvass Euler's criticisms of fundamental attractions as a preliminary to future work on this question.

Euler believed that this concept led natural philosophers to make mistakes when theorizing about the forces belonging to or associated with bodies. In his early work in analytic mechanics, the *Mechanica* (1736), he wrote:

> "Indeed, it is less than suitable to attribute the cause of this conservation to something named a 'force', since it is not of the same kind as other forces properly so called, such as the force of gravity, nor can it be compared with them. Many often fall into this error, especially in Metaphysics, having been deceived by the ambiguity of the word." (*Mechanica*, Preface; authors' translation)

Metaphysicians, then, are particularly apt to make these kinds of conceptual mistakes, Euler thought. Why would that be? Assuming it is true, then I think that, most plausibly, it would have to do with the nature of the explanatory projects that metaphysicians pursue. Note that the explanatory gesture Euler is highlighting here: *causal attribution* of the inertial property of matter to something called a 'force.' This will be important to my argument later on. For now, note simply that Euler has apparently associated Metaphysics with the task of determining the underlying causes or sources of a certain mechanical property of material body.

Next consider a mathematical text, the *Methodus inveniendi* of 1744. This is the treatise in which Euler laid out the basics of the branch of mathematics now known as variational calculus. It may be surprising that the work contains further illustrations of the demarcation between Metaphysics and Mechanics that Euler was working with. In an important Appendix, Euler considers applications to mechanics, particularly in the form of what is now known as the principle of least action (sometimes: principle of stationary action). He began by observing that Nature always obeys "some law of maximum or minimum." On that basis, he carried out a mechanical analysis of various physical systems using one such law: the principle of least action. The question, however, arises: *why* does this principle hold? What it is about bodies or about Nature in virtue of which the motions of bodies are characterized by this principle? That is a question for Metaphysics:

> "The force of this reasoning, though not yet sufficiently understood, nonetheless can, I do not doubt, since it agrees with the truth, be brought to greater evidence with the aid of sounder metaphysical principles; I leave this business to others who profess Metaphysics."

Here, Euler notes the borderline between Mechanics and Metaphysics in a gesture of deference. He has carried out his analysis assuming the truth of the principle, according to his duties as a mathematician, but leaves it to Metaphysicians to explain why the principle is true. This sentiment is echoed later in a 1750 article, "Recherches sur les plus grands et plus petits qui se trouvent dans les actions des forces" (Research on the largest and smallest [quantities] that occur in the actions of forces). On the same question as previously, he writes:

> "This is an investigation that belongs not so much to Mathematics as to Metaphysics, since it is about the end which nature proposes to itself in its operations. And it would be to carry that science to its highest degree of perfection if we were in a position to assign, for each effect that nature produces, that quantity of action which is the smallest, and were able to deduce it from the first principles of our knowledge. But I believe we are still quite far from this degree of perfection, and also that it will be almost impossible to arrive at it unless we discover, for a great number of different cases, the formulas that become a maximum or minimum. […] In this way, we will know *a posteriori* the formulas that express the quantity of action, and so it will no longer be as difficult to demonstrate their truth by the principles known in Metaphysics." (Euler 1750, §4).

Note how Euler here sketches a kind of *collaborative* picture of metaphysics in relation to mechanics. Importantly, though, mechanics comes first, at least methodologically, a point he strengthens in a paper on space and time. Before turning to that, I want to shift attention to a rather different context where Euler marks the crossing of the discussion from mechanics into metaphysics, while still showing considerable deference to metaphysics: the debate over fundamental attractive forces. In the *Lettres*, he writes of universal attraction:

> "The happy explanation of a large portion of the phenomena of nature sufficiently proves that this supposition is very solidly grounded, such that we can regard it as the best-confirmed fact that all bodies actually attract each other. It now remains to go deeper into the sources of these attractive forces, which belongs rather to Metaphysics than to Mathematics; and [so] I could not flatter myself to succeed there as happily." (*Lettres*, Volume 1, Letter LXVIII)

In the remainder of the letter, Euler summarizes the terms of the familiar debate, and in the following letter notes that this "Metaphysical dispute" about attractions cannot be furthered without a deeper discussion of the nature of body in general. Euler acknowledged, in the just-quoted letter, that his expertise was as a mathematician, first and foremost. Yet his modesty should not mislead: he had deeply held convictions that he was prepared to defend against the arguments of metaphysicians.

This view of metaphysics – cognition seeking rational explanations of the fundamental nature of body – was long Euler's understanding of that discipline, or at least of one of its key tasks. For instance, at the beginning of his "Reflexions sur l'espace et le tems", he opens the question of the principles of mechanical theory and the prospects of determining their metaphysical foundation. "Reflexions" is, as the title indicates, principally about space and time rather than bodies and forces, but constitutes a significant sally in his broader dispute, *qua* practitioner of mechanics, with metaphysicians. As I discuss in more detail below, Euler took metaphysicians to "reproach" mechanics for relying on concepts of absolute space and time that metaphysics had shown to be "fictions." In response, Euler notes that the principles of mechanics are secure and well-grounded *whether or not* philosophers were in a position to deduce them from general metaphysical ones. Nonetheless, he does admit:

> "it is absolutely necessary that [the principles of mechanics] are grounded [*fondées*] in the nature of bodies; and as it is Metaphysics that occupies itself with researching the nature and properties of bodies, the knowledge of these truths [of Mechanics] will be

> able to serve as a guide in these thorny researches [i.e., in Metaphysics]." ("Reflexions," §2)

That is, the ontological ground of the laws of mechanics is the nature of body, and Metaphysics studies the basic nature and properties of body.

Yet, while he and mainstream metaphysicians agree on this proposition, they evidently draw two very different lessons from it. Metaphysicians assume that they can achieve the knowledge of the nature of body independently, and then use this knowledge to inform or constrain Mechanics and its first principles. By contrast, Euler thinks that the independent security of Mechanics has provided Metaphysics with a great boon, since it means the laws of mechanics stand as a result which the Metaphysics should *recover*. The "first principles of Metaphysics must arrive at these conclusions [of mechanics]," which may be a useful constraint for helping to determine the correct first principles or "first ideas" of metaphysics ("Reflexions," §2). This position frames the extended engagement with metaphysical arguments about the nature of space and time in the remainder of this very rich paper.

It is worth emphasizing the epistemological and methodological point Euler is pressing here. The reasonings of metaphysicians have an effervescent quality in that, however apodictic they may seem at one moment, it is often the case that their power to convince – or even be adequately understood – vanishes in another moment. The propositions of mechanics, by contrast, are both clear and evidentially stable. The consequences are striking: it is really metaphysics that ought to "regulate" its own reasoning by means of the propositions of mechanics:

> "The principles of mechanics have already been established on such a sound basis that one would greatly err if he wished to encourage any doubt as to their validity. […] it will be by these conclusions, that the principal ideas of metaphysics will be necessarily regulated and determined." ("Reflexions", §§I-II)

In effect, Euler proposed to reverse the order of priority that metaphysics had arrogated to itself at least since Descartes. Metaphysics is not entitled to constrain or limit what assumptions or assertions mechanics may make about the natural world. Instead, metaphysics should moderate its own reasonings by the fundamental sciences – at the very least, by ensuring that its conclusions never run afoul of the basic propositions of a well-grounded science like mechanics.

The last four passages, from the "Recherches sur les plus grands et plus petits," the *Lettres*, and the "Reflexions," all point up this drastic reversal with particular clarity. They all suggest a "mechanics-first" approach to natural scientific inquiry, where metaphysics proceeds by taking the result of mechanics as *givens*. The laws of mechanics and the mechanical properties of bodies are "established on a sound basis," are among "the best-confirmed facts," and are "known *a posteriori*." It is then left to metaphysics to *assume* their truth, and then seek to uncover their *sources*. Plausibly, these will reside in the first principles of our knowledge, whatever those turn out to be (and *not* – and this is a main point of the chapter, argued for below – in 'forces').

The details of Euler's view, the strength of his argument for it, its implications, and its relation to later philosophy all merit further consideration. However, I am getting a bit ahead of myself. I leave discussion of these passages, having used them for the narrow purpose of showing Euler's interest in a demarcation between metaphysics and mechanics and indicating, in preliminary fashion, the character of the demarcation he seeks.

Summarizing, at this point, I take myself to have shown four things: i) Euler acknowledges metaphysics and recognizes metaphysical inquiry has a role in natural philosophy; ii) Euler sees, and seeks, a demarcation or border separating mechanical from metaphysical inquiry; iii) *prima facie*, an important part of this demarcation is that metaphysics is distinctively concerned with the *sources* of the principles, laws, and properties used in mechanics, and plausibly also iv) these mechanical properties and laws are importantly to be understood as "givens" for metaphysical inquiry, insofar as it is the job of metaphysics to recover them from first principles of knowledge, rather than to challenge them through *a priori* argument.[46]

5. Police 'force'

5.1 Linking 'force' to metaphysics and explanatory ambiguity

That was a lot of talk about metaphysics generally, but to return the focus to our primary subject, how do *forces* enter the story? Euler was a major contributor to classical mechanical theory, so he of course talked a lot about forces. But, while it would be one thing if Euler talked about metaphysics (as I showed above) *in addition* to talking about forces, it is quite another thing to show that these concerns of his are *linked*, and in a philosophically important way. That is what I aim to show here.

An implication of section 4 is that metaphysicians are to take the mechanical properties of bodies and the laws of nature that mechanics discovers as *givens*, and then

[46] An important scholarly task is to provide an account of Euler's overall epistemology, and interesting puzzles about it arise almost immediately. For example, according to Euler, our knowledge comes from three sources – experience or sense, reason or intellect, and testimony (including historical reports) – and he appears to hold the position that each item of a given subject's knowledge has precisely one source of warrant (Lettres, Vol. 2, No. 119). These sources of knowledge are "essentially distinct"; each item of knowledge – again, for a given knower – can fall in at most one class. Yet he also says that the law of inertia, for example, should be taken as an item of knowledge *both* because it is confirmed by experience *and* because it is provable through a priori argument. This would seem to place it simultaneously in two classes.

Although I do not purport to give an account of Euler's overall epistemology in this chapter, or even in this dissertation, this analysis helps us to begin answering such puzzles. Specifically, the analysis I give here shows one way of solving this puzzle. The true source of warrant for the law of inertia is experience. Given that we know it through experience, we are able to uncover its sources in first principles. But this does not give us an independent means of coming to know it, because it is only after taking it as known through experience that the search for a deduction from first principles comes into question. In other words, if the law of inertia were not confirmed through experience, we would not have grounds for regarding it as known, a priori arguments notwithstanding.

inquire into their sources in first principles. Additionally, as I show in this section, Euler thinks they should not use notions of 'force', which is a concept importantly reserved for mechanical explanations. The reason is that the resulting metaphysical explanations generally fail to be genuinely explanatory and have the further undesirable result that the conceptual structure of mechanics is thrown into confusion.

The forces that Euler talks about, as cases that provide warrant for these claims, include force of inertia, living force (à la Leibniz, especially), and active force (à la Wolff, especially).[47] My aim is to understand how Euler received and reacted to these ideas as part of a pattern of reaction against perceived interference between metaphysics and mechanics. Viewed from a complementary angle, this reaction was part of an effort to purify mechanical inquiry of causal questions extraneous to its pursuit of accounts of the causes of motion.

At the outset, I would like to head off a deflationary reading of Euler's intervention. Insofar as Euler is read to declare merely that these concepts *are not really forces*, it may seem that Euler is saying that philosophers are free to pursue explanations of these kinds, just not under the mantle of mechanics – and therefore, preferably, using different terminology for their concepts. Euler's point, according to this interpretation, is merely terminological. This interpretation is supported by a superficial reading of various passages in Euler, where he phrases his point as an observation about our "designations" or "nomenclature" (from Euler's German, *Benennung*, and French, *dénomination*).

[47] Readers of Leibniz will know that *vis viva* is a species of active force. Why do I not treat them at the same time? The reason is that Euler's response to 'active force,' as such, is different from his treatment of *vis viva*. In his view, the former reflects a metaphysical conceptualization of the faculties of bodies that conflicts with inertia, and so should be rejected. The latter concept can be taken to have a place as a *physical* concept, but, as he sees it, the error is in taking it to have a kind of fundamentality that it does not actually have, as his account of (true, i.e. impressed) forces shows. See sections 5.3 and 5.4 below for details.

In response, there are three things to say. First, a general point. Interventions that have to do with language or usage are not, by that fact, not philosophical. A correction to or observation about language intended to dissolve a philosophical dispute or confusion is, historically, a perennial and widely appreciated philosophical gesture, which philosophers from Leibniz to Hume to Russell to Wittgenstein have been engaged in.

Second, given his definitions, the word 'force' now has a specific content, and so in declaring the "nomenclature" of force to be inappropriate with respect to these other concepts, Euler is necessarily also saying that that content does not apply to them. That is, in declaring them not to be forces, he *must* also be declaring them not to be causes of a certain kind. This is certainly not a merely semantic point. This observation is particularly important, because Euler himself does not generally make it clear.

Third, all of these concepts *constitutively* are meant to bridge gaps between mechanics, physics, and metaphysics, so Euler's intervention, in dismantling these attempts, at the very least creates trouble for their purport at each of these levels. On the strongest reading, Euler is really saying not merely that they are not explanations of the mechanical kind, but that they are not successfully explaining anything at all. For example, *vis viva* is both a core component of Leibniz's metaphysics and physics[48], as well as intended to serve as a law of mechanics which enables the solution of mechanical problems. In the context of saying that *vis viva* – and, for that matter, *quantity of motion* – should not be regarded as forces[49], he re-analyzes these concepts *in terms of* impressed force in a way that vitiates their claim to

---

[48] How *exactly* the metaphysical is related to the physical in Leibniz is, of course, one of the most difficult and important questions in the Leibniz scholarship, but I take it to be clear that *force* spans these levels, and there is at least supposed to be a story in Leibniz about how to coordinate concepts of force operating at the metaphysical, physical, and mechanical levels. I say more on this below – though there, admittedly, more in relation to Leibniz's disciple Wolff.

[49] The context is the debate between Cartesians and Leibnizians over the correct measure of the 'force of bodies' that lasted the better part of a century, starting with Leibniz's *Brevis demonstratio* in 1686.

fundamental explanatory importance in physics. Since our understanding of and belief in their metaphysical role is in large part based on their playing a certain physical role, this also vitiates their metaphysical importance.

In this section, as evidence I consider passages where Euler is *policing* usage involving notion of 'force.' We have already seen a first intimation of this, in the passage from Euler's early text, the *Mechanica*, quoted above. In the remainder of section 5, my goal is to detail several cases of Euler's intervention into 'force' usage. The first, adumbrated above, is a major element of the 'Newtonian' conception of forces, the conception of 'force of inertia.' Following that, I consider two other, highly influential appearances of force concepts, the treatment of active force in monadology (especially in Wolff's version), and remarks on *vis viva* in the context of the controversy over the measure of the 'force of bodies.'

In each case, Euler declares these uses of the word 'force' and its correlates to be somehow flawed: not "suitable," not "proper." Often, he explicitly notes that they cause confusion or are ambiguous. The upshot of all of this for my argument can be put in the following way. There are two ways of making sense of what Euler means when he talks about "confusion" and "ambiguity" in talk about forces:

(1) On the one hand, these may be ambiguities which lead to errors or confusion in mechanics or in the understanding and application of mechanical theory.

(2) On the other hand, these may be ambiguities in causal concepts or an ambiguous adoption of distinct explanatory projects under the heading of one course of inquiry.

Calling out each of these tendencies has a somewhat different valence. Sense (1) belongs to the first goal I described Euler as having – conceptual clarification within mechanics. If all goes well, it will be more or less immediately in evidence in the passages I will adduce, given the examples of errors that Euler presents.

The second of these, sense (2), is made available by my interpretation and is meta-philosophical. This is the sense that, according to my reading, drives Euler to seek a new demarcation between metaphysics and mechanics. My claim is that *both* represent things that Euler is up to. It is the second one that really requires explanation and further substantiation. I turn to this in section 6. The eventual upshot is that, surprisingly, Euler's desire to partition mechanics and metaphysics is part of the ground of his own attack not merely on Leibniz or Wolff, but even on elements of the Newtonian conception of forces.

Finally, if Euler, as I read him, is correct that the problem described in (2) does have application to any of these cases, it would be natural to have the following sort of worry. Insofar as we are unclear about the actual nature of our explanatory project, could it be that the explanations we end up producing fail to be genuinely explanatory? I think one natural implication of Euler's intervention is precisely this finding. Even bracketing their objective accuracy, concepts like 'active force', and perhaps also 'force of inertia,' simply *fail* to be explanatory of important targets of theirs – corporeal motion, in the one case, and conservation of state, in the other. Elaborating here on this ambitious claim would involve a considerable detour. I revisit this question in section 6.3 in connection to 'active force.'

5.2 'Force of inertia'

In Newton's writings, the property of inertia is conceived of as a 'force', the *vis inertiae*. In the Definitions for all three editions of his *Principia*, Newton calls inertia the "inherent force of matter," the *vis insita*. He adds that it "can be called by *the very significant name* of force of inertia" (my emphasis). Natural philosophers who took up the Newtonian approach also generally adopted his concepts and language, including the concept of 'force of inertia.' Indeed, the term 'force of inertia' was widely adopted even outside of the network of self-described Newtonians – for instance, it was used by Wolff, whom I will discuss more below.

As we have seen, from very early in his career, Euler reacted against the 'force of inertia' concept.[50] In the *Mechanica,* which came out a decade after the third edition of the *Principia*, he wrote

> "Indeed, it is less than suitable to attribute *the cause of this conservation* to something named a 'force', since it is not of the same kind as other forces properly so called, such as the force of gravity, nor can it be compared with them. Many often fall into this error, especially in Metaphysics, having been deceived by the ambiguity of the word." (*Mechanica*, Preface; authors' translation and italics)

Note that, here, Euler singles out *Metaphysics* as the place where the idea of a 'force' is characteristically invoked to explain, by specifying the *cause* of, the property bodies possess of "conserving" their states of motion.[51] Of course, seemingly all Euler is claiming, here, is that it is unsuitable to adopt a certain *manner of speaking*. For, the term is misleading, to wit: 'force of inertia' may mislead one into thinking that inertia is a quantity that can be compared with forces like gravity or mechanical pushes and pulls. (An idea he will revisit, as we will see.)

Significantly, though, he is also already proposing that the class of forces "properly so called" are the *impressed forces*, namely, things that cause changes in states of motion. The passage of time should not mislead us into thinking this would have been accepted as a truism when he wrote it. It was a *prima facie* audacious move to declare that force of inertia, or vis viva, or the active force described above, are *not forces* properly so-called.[52] In his

---

[50] Scholars have previously pointed out comments by Euler regarding 'force of inertia' in isolation. For example, Caparrini and Fraser (2017, 360) write: "in the 'Investigations on the origin of forces', for example, [Euler] criticized the locution 'vis inertiae' as denoting something which is not a force." However, here as elsewhere, given their purposes the authors do not consider the full range of texts in which Euler engages on this and related issues at the intersection with metaphysics. This is what I do here, which puts me in a position to interpret Euler's interventions in light of a broader philosophical project.

[51] On this, McMullin is in agreement: "the term 'innate force', on its face, appears to denote some sort of causal agency" (1978, 38).

[52] It must be admitted that, chronologically speaking, a puzzle soon emerges. As readers of Euler may have noticed, he appeared to reverse his position, only to apparently reverse it again still later. In papers of the 1740s, it looks, to all appearances, as though Euler is happily invoking 'force of inertia'. Still later, in the 1750s and 60s, Euler once again criticizes use of the notion 'force of inertia'. This appears to make Euler out to hold

intellectual and discursive context, these were all perfectly acceptable concepts to theorize under the heading of 'force.'

In a paper titled "Recherches sur l'Origine des Forces" (1752b, read in 1750), Euler engaged at length with issues at the intersection of mechanics and metaphysics. On the subject of 'force of inertia', he wrote:

> "At the occasion of this definition of the term 'force', I remark that it is very improper that some call inertia the *force of inertia*. Because the effect of inertia consists in the conservation of the same state, and the effect of forces is to tend to change that state, it is clear that these two effects are directly opposed to each other, and that inertia denotes, instead, something completely opposed to the idea of forces. This remark seems all the more necessary, given that this improper denomination has contributed not a little to muddling the theory of the first principles of bodies and of motion. For in the majority of Books that treat this matter, one finds so many obscurities and contradictions, that one is obliged to renounce them altogether, when one wants to apply oneself with success to the study of Mechanics." (§9)

Not only is the 'force of inertia' incommensurable with forces properly so called, but they have opposite effects. Evidently, the combination of ideas represented by 'force of inertia' is unclear. Yet mechanical theory demands "clear concepts" – a point which Euler makes repeatedly.[53] By implication, Euler must be taking the concept of *impressed force*, which he

---

inconsistent opinions, or to be strangely shifty in his considered philosophical views. Why this middle period, in which he accepted 'force of inertia' as a legitimate theoretical notion? I think his position is consistent, but so as not to lose the thread, I will relegate my explanation to Appendix A. In short, I claim that we can make the most sense of what Euler is up to by viewing his uses of "force of inertia" in the mid-1740s as deference to prevailing usage for the sake of argument, rather than as an endorsement of the conceptualization of inertia as a force that '*vis inertiae*' represents. Importantly, he consistently eschews "force of inertia" in all of his texts devoted to mechanics, generally using the phrase only in polemical philosophical works. Assuming this interpretation is correct, a highly consistent position comes into view.

[53] Aside from the passage just quoted, see, for example, the *Gedancken von den Elementen der Cörper* (§10): "However in Science it is dangerous to draw conclusions from such phenomena that are merely perceived, and it is absolutely necessary that in advance one formulates clear concepts regarding these phenomena, and examines carefully how and under what conditions these phenomena occur." The reference to "clear concepts" is likely to call Descartes to mind. I do not think that we should understand Euler here as referring to or relying on the criterion of clear and distinct ideas. This question, however, belongs to the project of understanding Euler's overall epistemology, which I cannot go into here. I discuss this a little more in Chapter 5, and look forward to exploring this in future work.

defines as the cause of a change in the state of motion of a body, to be clear – at least, the clearest among the 'force' concepts then in use.

This is made yet more apparent in the manuscript *Anleitung zur Naturlehre* (Introduction to Natural Science), usually dated to the mid-1750s[54]. In the work, Euler attempts to build up mechanical theory on the basis of a fundamental inquiry into the nature and properties of material body. The context of the passage I will shortly quote (and which is discussed at greater length in Chapter 3) is an elaboration of the mathematical definition of impressed forces by means of their effects on the speed of bodies. Euler deduces[55] generic methods for measuring forces, the second law of mechanics, and a pair of equations relating impressed forces to changes in Cartesian 'quantity of motion' and Leibnizian 'vis viva'. It is in the context of this swirl of overlapping and miscellaneous concepts that he remarks:

> "Others estimate the mass of a body from the so-called force of inertia [*Kraft der Trägheit*], which is in complete agreement with the present concept [persistence], where we replace this uncomfortable/inconvenient nomenclature by the term *persistence* [*Standhaftigkeit*]. Here we find at once an incorrect consequence of this nomenclature, since some maintain that no force is able to put a body in motion unless it is greater than [that body's] force of inertia. But apart from the fact that it has been shown that this property cannot in any way be regarded as a force, and thus cannot be drawn into comparison with other forces, we recognize from the preceding argument that even the smallest force is able to put the largest body into motion or otherwise to change its state." (*Anleitung zur Naturlehre*, §55)

At first glance, Euler may not seem to be adding much to my argument in this passage. Is it not just an "inconvenient nomenclature", after all? Yet, he clearly does not regard it as a

[54] In dating this manuscript, two important reference points are (a) the property Euler identifies as the "essence" of matter and (b) Euler's development of the tools of fluid and solid body mechanics. In texts of the 1740s, the essence of matter and the source of force is *inertia*. By the "Recherches" (first read in 1750) and continuing in the *Lettres*, it is *impenetrability*. Chapters 20 and 21 also begin to develop the laws of fluid motion that Euler only published elsewhere in the 1750s, themselves relying on the generalization of "Newtons' second law" that he published in 1752 in "Découverte d'un nouveau principe de mécanique" (Euler 1752a). See also the discussion in the Foreword to *Leonhardi Euleri Opera Omnia*, Series III Band I, pages VIII-IX, for affirmation that it was not composed prior to 1745.

[55] That is, in the non-factive sense of 'deduce'; he takes himself to be deducing these results, but it is a question whether the argument is deductively valid. Chapter 3 is concerned with the question of the extent to which Euler's theory really is deductively valid.

mere inconvenience to assign inertia the *name* of 'force'. As he says, "this property *cannot in any way be regarded as* a force". In other words, it is not just that it ought not to be *named* 'force', but that it can only through error be *conceptualized as a force*.

Why are natural philosophers tempted by – or more neutrally put, interested in – conceptions of inertia as a force in the first place? It is hard to say precisely; different figures will have different motivations, and these are not usually given in the form of an argument. The intuitive idea appears to be something like this. First, bodies in motion left unperturbed will persist in the states of motion they are already in. This is a capacity of bodies which seems to call for explanation. Further, we know that bodies are acted on by external causes that seek to change their motions all the time. Experience makes it abundantly clear, however, that they always *resist* attempts to have their motions altered. It is only "with effort" that their motions are changed. When pushed on, they "push back." These, too, appear as capacities or powers of bodies to *do something*, hence as 'forces' that they possess. It is in virtue of possessing an inherent force of inertia that bodies persist in their motion and mount resistance to attempts to change them.

Whatever the force of this reasoning, the implication is that we have at least two kinds of things categorized as 'forces'. Since both are also quantities, this suggests that it should be possible to compare them. What happens when we "draw into comparison" the force of inertia with other forces? We get a picture whereby the force of inertia enters into the balance of forces on a body. Though these quantities are dimensionally different, this seems not to present too great a conceptual obstacle. After all, the force of inertia is a power by which a body resists changes to its state. If I press upon a body, I exert a force on it, and that force I exert is resisted by the force of inertia of that body. If these two 'forces' can resist each other, surely they ought to be mathematically comparable. Yet, if this is how we conceive of the basic physics of the situation, then, per the laws of mechanics, it stands to reason that the subsequent motions should depend on the *resultant* of those two forces. Hence the conclusion that, if the force of inertia is 'greater' than the force I exert, the body will not move.[56]

As Euler points out, this way of conceptualizing the physics of the situation is hopelessly confused. If we draw a standard force diagram for some given body, representing the forces acting on the body with arrows, the 'force of inertia' is not among them. On the misconception Euler described, it is. In fact, this is only one of many possibilities for confusion or ambiguity. For another thing, (impressed) forces are *actions*, and only exist when they are acting. By contrast, the force of inertia is intended as a permanent, inherent property of a body. One thing that seems to be going on with the misconception is that force of inertia is viewed as an inherent faculty which can *generate* forces of resistance, of fixed magnitude. However, even those forces of resistance are (mis)conceived as forces which act by mitigating or vitiating the power of the (impressed) force acting on the body. Yet, from the point of view of Euler's (and classical) mechanics, insofar as a body subject to an impressed force "reacts" with forces of its own, those forces are impressed on the agent acting on it and, in accord with the third law, these forces are always equal in magnitude.

It is important to be careful in specifying the strength of the criticism. Euler does not think that it is already a misconception simply to think that the body, because of its inertia,

[56] Euler does not cite a specific text where he judged that the author was making this error, and I have not found a text where this conclusion is specifically stated. It is not unreasonable to take Euler at his word, since it is independently plausible that some philosophers drew such conclusions. After all, the conclusion sounds very similar to some of Descartes' rules of collision, if couched in different terms. See the discussion of Descartes' laws and rules in (Slowik 2023, §4). The author notes, for example, that "Astonishingly, Descartes claims that a smaller body, regardless of its speed, can never move a larger stationary body" – due to its greater force of resistance. Cartesians remained a prominent presence in natural philosophy will into the 18th century, and certainly were so in Euler's formative years.

opposes the action of impressed forces. Euler consistently affirmed, as he put it in the *Lettres*, that "when an external force is changing the state of some body, the *inertia*…opposes itself to the action of the force." (*Lettres*, Vol. 1, No. 74).

But the 'force of inertia' concept makes it natural to take this too far. Inertia "opposes" force just in the sense that, for a force of a given magnitude, the effect which that force has on a body (i.e., the acceleration it produces) *scales inversely* with the quantity of that body's inertia. As Euler continues in the same letter:

> "bodies are endowed with inertia insofar as they contain matter. It is, indeed, by the *inertia*, or the resistance that they oppose to every change of state, that we judge the quantity of matter of a body… We also know that *it requires more force* to change the state of a larger body than of a smaller one; and *it is from this that we judge the larger body to have more matter* than the smaller one" (*Lettres*, Vol. 1, No. 74; my italics).

Hence, it is from the circumstance that more force is needed to move a body that we judge the larger body to have more inertia or resistance. The fact of the "resistance" that is at issue here is nothing more than this inverse scaling. That is: yes, bodies "oppose" and "resist" the action of forces, but not, as it were, by a head-to-head struggle between commensurate "forces." The sense of opposition or resistance that is in play is *entirely* specified by the proportionality between impressed force and inertia, itself equivalent to 'quantity of matter', that is recorded in the second law of mechanics. Euler then reiterates his by-now familiar message in letter 76, insisting again that the only "true forces" are impressed forces.

Second, the misconception Euler described in the passage from the *Anleitung* quoted above is clearly not an *inevitable* consequence of the bare use of the *term* 'force of inertia.' Newton himself may be a case in point. The progenitor of the concept was never misled into drawing such evidently false conclusions. This may make it seem as though the confusion Euler resolved was not serious.

By way of reply, I do not think it can reasonably be denied that this conception nonetheless strongly and erroneously suggests that inertial and impressed forces are commensurable, as Euler feared. The implied critique of Newton is not that he necessarily fell into such errors himself, but that his way of conceptualizing the physics made such errors natural to make, if only for those less careful than himself (which, for that matter, is a large class of people). All the same, while Newton may not have been led to erroneous *results*, this does not mean that his concepts of *vis inertiae* never guided him to reason in suspect ways. McMullin, for example, finds that it led him to make certain mistakes which puzzled even Newton himself. Namely, in the proof of Book 1, Prop. 1, Corollary 1, Newton erroneously compound centripetal forces impressed on a body with that body's own 'force of inertia' as though they were comparable quantities (McMullin 1978, 34-5).

As Euler understood its motivation, the 'force of inertia' concept was intended to name the *cause* of the inertial behavior of matter, the fact that bodies conserve their states. This is plausible, and Euler is not alone: "the term 'innate force', on its face, appears to denote some sort of causal agency" (McMullin 1978, 38). Yet, the cause of inertia is not like a cause of a (change in) motion. So, Euler re-categorized '(force of) inertia' so as not to fall in the category of 'forces'. In so doing, Euler was conceptually tidying up *mechanics* in such a way that errors like those canvassed above could be avoided. But, I argue, he was doing more than just that. Namely, he cottoned onto the circumstance that ambiguous uses of the notion of 'force' arose, here, from the temptation to mix together different kinds of explanatory projects. However, I leave fuller defense of this second interpretive claim to section 6.

5.3 '*Vis viva*' and 'quantity of motion'

A key moral of these passages is that the impressed force concept is the privileged notion of 'force', so all quantities not commensurable with it must go by a different name. As it happens, this standard is also brought to bear on Leibniz's concept of *vis viva*. For that matter, Cartesian 'quantity of motion' is also swept in for a general commentary on the two players in the so-called '*vis viva* controversy'. The reader will recall that this dispute over the proper measure of the 'force of bodies' raged between Leibnizians and Cartesians from the publication of Leibniz's *Brief Demonstration* in 1686 and continued until the middle of the next century.

The 'force of bodies' was conceived as a body's capacity to "produce effects," in some sense – that sense itself being folded into the disputed subject matter. In broadest terms, these effects were couched in terms of the motions of bodies, originally changes in the speed a body. Leibniz also famously brought into play the matter of their vertical positions in the earth's gravitational field, and the ability to compress a given spring to a given length. Roughly, the disputants argued over which of two quantities measured the capacity of bodies to act: the product of bulk and speed, or the product of bulk (or mass) and the square of speed. In other words, the question was again about *causes*: the right measure of the "force of bodies" would accurately specify a body's capacity to bring about effects.

In Euler's analysis, the only kinds of things that can be mechanical causes are impressed forces. The basic effect of a force is a change in state of motion, and so it is these which measure the magnitude of a force. The effect of a force can also be measured indirectly with quantity of motion and living force, but these represent two ways of measuring *accumulations* of those effects on a body assumed to be initially at rest. Namely, quantity of motion represents the cumulative effect of a force through a time, and living force represents the cumulative effect through a distance. In §60 of the *Anleitung*, Euler treats this whole suite of concepts with marvelous clarity:

> "The first [*Mv*] is called the *quantity of motion* and the other [*Mvv*] the *living force*. Although these designations are arbitrary, the latter does nonetheless not justifiably [*füglich*] have place here once we have fixed a definite concept for the word 'force.' For, first, the product *Mvv* can in and of itself be regarded as a force as little as the other, *Mv*, can; and insofar as this very concept [*Mvv*] is equal to the product *2nps*, where *p* refers to a true [i.e., impressed] force, so also can it not simply be drawn into comparison with a force, but must rather be compared with the product of a force with a path, that is, through a line, in just the same way that the quantity of motion, *Mv*, stands in comparison with the product of a force through a time. Thus, if one chose a proper name for such a product, so could it also be settled upon for the product *Mvv*. However, one must take care that this can be done really only insofar as one imagines that the product *Mvv* was brought about in a body at rest through a force. In this case one therefore sees that the force *p* multiplied with the time *t* denotes the quantity of motion generated, whereas the force *p* multiplied by the path denotes the living force. Apart from that, if one holds to the determinate concepts given here, all the difficulties which emerge in the controversy over living forces fall away of themselves, and the two formulas just found must in all cases point to the truth." (*Anleitung zur Naturlehre*, §60)

Euler echoes language he uses elsewhere in stating that the name is "arbitrary." Yet, this should not be mistaken for flippancy. Emphatically, it does not mean that either the dispute or his intervention into it is "merely semantic" and therefore lacking in content. The analysis given in the text, and whose results are described in this passage, amounts to rendering the two concepts of the "force of bodies" in terms of the notion of impressed force which Euler privileges.

The purport of the analysis is to reduce both *vis viva* and quantity of motion – understood as *causal* concepts – to the notion of force. This does not mean these other quantities are deprived of physical importance, *tout court*. Significantly, they can still be the subject of conservation laws, as Leibnizians and Cartesians wanted them to be, so they retain physical *significance*. What they lose is their status as the fundamental *causal* concepts of mechanics. They are no longer to be understood as fundamental measures of causal agency, in the form of measures of a body's inner capacity to act. They are at most quantities that can be associated *to a body* only when it is understood as accelerated from rest by an impressed force.

This is an important lesson. If *vis viva*, for instance, had turned out to be the basic causal agency in collisions, it should be the case that knowing the *vires vivas* of the bodies in a collision would allow one to determine the outcome of the collision. The same would go for quantity of motion. Yet, the history of mechanics had shown that neither concept by itself was sufficient to determine the outcomes of collisions.

As a corollary to their shared loss of causal significance, the dispute over their *relative* significance is also dissolved, as Euler remarks ("the difficulties which emerge in the controversy over living forces fall away of themselves"). For, from the point of view of Euler's mechanics, there is no important, physically fundamental causal notion of the "force of bodies" that *either* of them could be said to capture.

5.4 Wolffian monadology and 'active force'

Background to Euler's dispute with monadology

The monadology is most famously the cornerstone metaphysical doctrine of Leibniz. In the hands of his disciple Wolff, by the 1740s it had become one of the most influential philosophical doctrines on the Continent. And concepts of 'force' were the cornerstone of the monadology.

This made it ripe for targeting by Euler, as we will soon see. Indeed, the monadology and its use of 'force' concepts is particularly illustrative among the cases I examine. In this case, above all others, it is clear that the growing institutional power and doctrinal renown of monadology was leading to an increasingly felt encroachment on the intellectual territory that – from Euler's perspective – belonged to mechanics.

This case is somewhat different from the preceding, because the concept of active force *as such* was not itself a concept used in mechanics, as *vis viva* and (force of) inertia were. At least as Euler understood it, it was only used as an ingredient in *metaphysical* accounts of bodies and their motions. So, Euler's intervention here does not carry the valence of bringing a mechanical concept to theoretical clarity (in section 5.1, discussed as sense (1)). However, although this aspect is not present, by compensation, the sense of resistance to the encroachment of metaphysics into mechanics is particularly evident.

As it happens, Euler resisted the monadologists on several fronts and for several reasons. One of these was the atheistic tendencies associated with Wolffianism. Another – as illustrated briefly above – was the doctrine of the ideality of space (and time) that standardly came along with the monads as a package deal. Here, in line with this section's goal of examining Euler's conceptual "policing," I look at his response to the use of concepts of 'active force' in monadology.

Before I do so, what about *passive force*? Indeed, it is curious that Euler does not attack this companion concept to active force. Passive force, as we saw in section 2, was supposed to be the ground of a body's *natural inertia* and *impenetrability*. At least in regard to natural inertia, 'passive force' thus plays a similar role to 'force of inertia' – an inner faculty that explains or causes inertial behavior. One noteworthy difference is that natural inertia is conceived of as resistance to *motion*, and not just to changes in motion. In "On Nature Itself" (published 1698 in *Acta Eruditorum*) for instance, Leibniz writes:

> "we must admit, rather, that matter resists being moved through a certain *natural inertia* it has (as Kepler nicely named it), so that it is not indifferent to motion and

> rest…it is in this very passive force of resisting…that I locate the notion of primary matter" (Leibniz 1991, 161).

In any case, it seems that Euler should decry this concept with just as much vigor, and for much the same reasons, as he did 'force of inertia.' As far as I have seen, it is always active force that Euler is vexed by. I admit I am not sure why, so I leave this as an observation for further consideration.

A final preliminary: I will focus mainly on Euler's engagement with *Wolff's* version of the monadology, as opposed to Leibniz's. This was the most influential offshoot of the original, and theoretically most developed version. Moreover, Wolffianism was at the height of its influence in Germany.[57] It was the most celebrated philosophical doctrine in Euler's milieu at the Berlin Academy in the 1740s, when he began to engage with "metaphysical" topics. It was in this context that the Berlin Academy, in a gesture of respect for the older and decorated Wolff, had opened its essay competition for 1747 with the topic of monads. Opened in 1745, the competition asked entrants to derive the existence of simple beings, or monads, and to explain all natural phenomena in terms of them, or else refute the monadology. All of this makes Euler's dispute with Wolff a particularly live question.

Euler, himself at the Berlin Academy at the time, had watched Wolff's coterie gain in influence and stature in the Academy. He resisted this development from early on. It seems that, at the beginning, his resistance was based largely on the above-mentioned "atheistic"

---

[57] Wolff had ascended to pre-eminence in German-language philosophy soon after Leibniz's death, and a powerful academic coterie formed around the Wolffian philosophy. Wolff had, in effect, consolidated the different strains of German philosophy into an all-encompassing system (Calinger 1969). But Leibnizian themes – monadology, the principles of sufficient reason and of contradiction as the bases of knowledge – were especially pronounced.
Wolff's star had initially fallen, when he was ejected from his university post and exiled on pain of hanging in 1723, due to fearmongering about his atheistic tendencies. But the brouhaha that brought his philosophy only more attention and renown. Later, his star rose again when Frederick the Great later offered him his old post in Halle, and he returned there triumphant, in 1740. In the 1740s, the Wolffian (or 'Leibnizian-Wolffian') wing of the German academy was at the height of its influence.

tendencies in Wolff's thought, as Euler was a devout Pietist himself. But the monadology affair triggered a new, more forceful reaction. Euler penned an extended philosophical polemic attacking monadology from the ground up.

Indeed, it is evident that Euler took monadology to constitute a serious affront to mechanics, if for no other reason than that he continued to cast aspersions on the monadology's ideas of force in his scientific writings, both technical and popular, for years. The episode apparently stuck with him for decades. The most important document in this episode is the *Gedancken von den Elementen der Cörper* (Thoughts on the elements of bodies; hereafter *Gedancken*), an anti-monadological pamphlet which Euler published in 1746 to influence the Berlin Academy essay competition for the following year. Yet remarks on monads can be found in many other works, including the "Reflexions" (§19) and the *Anleitung* (§49). Letter 76 of the *Lettres une princesse d'Allemagne* (Vol. 1) is devoted specifically to the Wolffian system of monadology. And the monadology is taken back up in Vol. 2, letters 125-132, where Euler focuses on the question of the divisibility of matter, which I do not take up here, rather than on motion and force, which is my focus here. In these texts, Euler recapitulates versions of arguments given in the *Gedancken*.[58]

Euler's attack on Wolff's active forces

Given my theme, the aspect of the conflict between Euler and the monadologists I will focus on is the monadologists' handling of the concept of *force*. (The other main aspect was the question of the divisibility of body, which has an only ancillary role in my analysis.) 'Force' was the key explanatory concept for not only mechanics, but also Wolffian ontology.

[58] Still other texts speak to questions that are related to the debate here. The "Enodatio quaestionis," published in a collection of miscellaneous works called the *Opuscula varii argumentii* in 1746, speaks about the (in)compatibility of inertia with other candidate properties of material body. That same collection also contained the "Recherches physiques sur la nature des moindres parties de la matière."

In this section, I restrict myself to a brief exposition of Euler's basic complaint about the monadologists use of 'force' in the form of *active* or *motive* forces, as ascribed to monads and to bodies. Euler's full argument, as I interpret him, is actually quite sophisticated and so takes significant space to explain. So as not to lose the thread, I leave this fuller analysis to an excursus in section 6.3.

According to Wolff, the principle of sufficient reason demands that each monad or simple being be, or be endowed with, an *active* or *motive force.*[59] As Euler understood it, the argument for the forces of monads began with the observation that bodies are subject to constant changes in their motions. By the principle of sufficient reason, there must be a reason for those changes, and this reason is located in the bodies themselves. Since everyone agrees that a cause capable of producing a change is, by definition, a *force,* these internal causes of change are dubbed the body's 'active *force*.' Separately, an argument from repeated division compels us to concede that bodies must be 'composed', in some suitable sense, of indivisible, hence simple, things. Finally, if bodies have active forces, then, it is alleged, these simple components must be endowed with forces of the same kind.[60] In this fact lies the explanation of the phenomena of the world.

The crux of Euler's complaint is that the monadologist attempts to explain motions in bodies by attributing to them internal forces of changing their own states. But bodies have

---

[59] Recall that, for Wolff, the simple substances are unextended loci of force. Motive forces occur at the third level, that of empirically accessible material bodies. It is somewhat complicated to distinguish how we know each kind of force exists and what their inferential relations are to each other. Here, my aim is to gloss Euler's understanding, and it is certainly possible that Wolff would find inaccuracies. Since I think that the gloss is accurate enough to make for a compelling argument, I have chosen not to go into the weeds on Wolff interpretation here.

[60] Note that Euler, in his summary of the monadology in his main polemic, mostly glosses over the 'mind-like' nature of monads (see e.g. *Gedancken*, §10). He first brings up the mind-like nature of monads in §43, after the main arguments are completed. Since the force of bodies as a faculty of representation is a feature primarily of Leibniz's version of the theory, not Wolff's, it, once again, makes sense to read Euler as responding principally to Wolff.

inertia, which means that they do not change their states of themselves, but only when suffering an external, impressed force. Ergo, there is an apparent contradiction between the analyses of mechanics and the conclusions of the monadologists. As he puts the argument succinctly in the *Anleitung* (§49):

> "We now also see that these changes are a necessary consequence of the impenetrability and persistence [i.e. inertia] of the bodies, a view distinct from that of some scholars [i.e., the monadologists], who concluded from the incessant changes in the world that bodies must be equipped with forces enabling them to change their state, notwithstanding the fact that persistence is in clear contradiction to such forces."

So, active forces are incompatible with the law of inertia. Since inertia is a definitional correlate of the concept of impressed force, this makes them likewise definitionally incompatible with the "true" forces.

6. Euler was responding to perceived interference between metaphysics and mechanics

6.1 Some criteria for establishing this

In the previous section, I take myself to have exhibited Euler's wide-ranging project of policing philosophical usage of the concept of 'force.' I also take it to be established that this intervention into philosophical usage at least plausibly had salutary effects on understanding of the basic concepts of mechanics. For example, we saw how regimenting the definition of force as Euler proposed would clearly mark errors in which impressed forces are *added to* 'forces of inertia' as such. We also saw that it led to a plausible case for dissolving the controversy between Leibnizians and Cartesians over the measure of the "force of bodies."

It was the second component of my interpretation that requires further substantiation: In regimenting usage of the notion of 'force,' Euler was in part motivated to maintain a demarcation between metaphysics and mechanics that existing usages tended to cross or to blur. That is, Euler was reacting to a sense of *interference* between metaphysics and mechanics, especially in that metaphysics would often *encroach* on mechanics. This interpretation relies on the truth of the following claims:

1) Euler sensed that certain uses of 'force' concepts reflected an encroachment of metaphysics into mechanics, or an interference between metaphysics and mechanics
2) Euler intervened by policing usage of 'force' concepts
3) Important to these interventions was to disentangle ambiguity or confusion in senses of cause (mechanical and metaphysical) that was repeatedly embodied in those uses of 'force' concepts

A clarification regarding 1: I have used the word 'encroachment' because this does seem to accurately capture Euler's sense of the monadologists, in particular, over-stepping their proper bounds and claiming the authority to pronounce on mechanical topics. Admittedly, though, 'encroachment' is perhaps too strong a word for the case of 'force of inertia,' since this is not a case where institutional Metaphysicians were responsible for stepping into the territory of mechanics. After all, the concept is due to Newton. Yet, all the same, this is a case of interference between, or intermixing of, metaphysical and mechanical projects. It is this broader sense of problematic crossover that I am interested in here.

As for 2, I take it that this has largely been established in section 5. The present section is dedicated mostly to showing that we should understand those interventions in light of a motivation to disentangle ambiguity or confusion in the senses of cause in play in the theories that employed these concepts, that is, criterion 3. I now turn to completing the justification of the above claims in relation to each of the cases at hand: 'force of inertia', 'vis viva', and 'active force.'

6.2 Confirming that the criteria are satisfied

'Force of inertia'

According to an influential interpretive tradition, Newton not only established a new paradigm for mechanics but set physics on an empiricist path of disengagement from philosophy and metaphysical speculation.[61] As for Euler, he has been described as an embattled defender of Newtonianism in the philosophically hostile environment of the Berlin Academy (Calinger 1969, 321ff.). In this light, it becomes particularly interesting that Euler's desire to partition mechanics and metaphysis is part of the ground of his own attack on elements of the 'Newtonian' conception of forces.

So, to start with the case probably least likely to support my thesis, it does seem to me accurate to say that the theoretical motivation even underlying the Newtonian concept of 'force of inertia' was to specify the *cause* of the inertial behavior of bodies. It may be doubted: was this Newton's *actual* motivation?

In response, I note first that this is actually not strictly necessary to my interpretation of Euler. My point about Euler depends only this being *Euler* read the situation. And that is unambiguous, as the passages already in the early *Mechanica* show (section 5.2). Of course, it helps that it very plausibly *was* Newton's motivation. Newton was admittedly a bit cagey about such things. But this reading makes sense given what else he says about the concept, that it is in virtue of the force of inertia that bodies resist changes and persist in their motions, and plausibly is part of what he meant when he said, of '*vis inertiae*,' that it was a "very significant name." Further, this is how Newton interpreters, like McMullin, have read it (quoted in sections 2 and 5.2).

[61] This is the first of three interpretive approaches described by Janiak in his (2008), and prominently associated with the work of I.A. Cohen and Howard Stein. Cf. note 16.

That Euler took this to be a question about metaphysical causes can be inferred from the way he attempted, himself, to give an alternative explanation of the inertial behavior of bodies by deducing it from the principle of sufficient reason, and which makes no invocation of inner 'forces' of bodies. I provide more details about this deduction in section 7, below, and in Chapter 3. Suffice it to say here that the conceptual tools used in the new explanation are now quite separate from the canons of mechanical explanation. This very deduction, I offer, clinches the case that taking away the 'force' label from 'force of inertia' is meant to make way for its proper metaphysical explanation.

To add to the case that this is a problematic ambiguity, it may be helpful to refer to the scheme, from the section 3, of 'levels' of explanation in mechanical theory. Suppose, as Euler claims, that the *error* is to think that these two concepts of 'force' as of the same kind and as commensurable. It thus makes sense that this error would follow from a methodological failure to respect a division between domains of inquiry – as I have described it, a division between mathematical mechanics and metaphysics. For impressed forces arise as explanations or causes of changes in the motions of bodies. The force of inertia arises as an attempt to explain the mechanical properties or behavior of bodies. The implication is that the way of conceptualizing inertia embodied by the term 'force of inertia' mixes together levels 1 and 2. The deception that results from this double-use of force is that metaphysicians may think that (impressed) forces are causes in the same sense as an inner power, *vis*, or 'force' is a cause of the "conservation of state" that bodies manifest. Euler himself drives this point home in alluding to the boundary line separating himself and his project in the *Mechanica* from those "in Metaphysics."

It may turn out that inertia is one of the necessary faculties or powers by which bodies generate impressed forces and act on other bodies as in collisions. Inertia would then be part of the explanation of the origin or nature of these impressed forces. Yet, this would make inertia a character in an explanation at the second level of inquiry (as described in section 3), and it would be at best circular to then declare that this is because inertia itself *is* a 'force.' In resisting this move, and advocating for a separation of metaphysical from mechanical questions – and for a separation of their conceptual tools and principles – Euler "modernized" the conception of inertia.

'Vis viva' (and quantity of motion)

As a measure of the so-called 'force of bodies,' *vis viva* was supposed to explain a body's capacity to produce effects. This plausibly makes *vis viva* – and quantity of motion, for that matter – a concept of a metaphysical cause, according to Euler's scheme. Of course, it is also true that, in the systems of Leibniz and Descartes, these concepts explicitly had roles in fundamental ontology.

The consequence of Euler's intervention here was quite basic. His redefinition of *vis viva* (and quantity of motion) in terms of integrals of impressed force deprived these concepts, *qua* measures of the 'force of bodies,' of fundamental causal significance *tout court*. The upshot (at least purportedly) is to simply *dissolve* the dispute over physical-*cum*-metaphysical causes represented by the controversy over the measure of the force of bodies.

Wolffian 'active force'

Euler certainly felt that Wolffian monadology was encroaching on the territory of mechanics, as it was offered, in significant part, as a theory of material bodies and their motions. Indeed, I take Euler to hold that active forces as Wolff deploys them are not explanatory of bodies and their motions, certainly not in the way they are intended to be. They do not successfully point to the causes of changes in bodies. And part of the reason is that they collapse the kinds of cause, or levels of inquiry, as I described them in section 3.

The full argument for this will take more space to spell out, for I think that Euler's argument is actually more sophisticated than might be expected of him. So, I leave this task for the excursus on the monadology dispute in the next subsection, 6.3. To preserve momentum, I will provide motivation for the idea here; the details can be sought in that excursus. In effect, the monadologist and Euler agree on three key claims:

(1) Two bodies which collide change their states of motion.

(2) Changes in the states of motions of bodies must have a cause.

(3) The causes of changes in material bodies must ultimately reside within bodies.

At this point, the monadologist will say:

> (Monadologist) "Good, so I have all I need, and we should be in *complete* agreement! When A collides with another body, (3) tells us that the cause of the change in body A resides *in body A*. Causes of such changes are forces. So, body A has a force that produces the changes in itself. That's just what I am calling body A's 'active force.' What could possibly your problem with this?"

In brief, the way I read Euler's position, he would respond in something like the following way:

> (Eulerian) "Your understanding of (3) is exactly the issue. The immediate causes of changes in bodies' states of motion are *impressed forces*. I agree that the causes of the changes in material bodies ultimately reside within bodies. But when I say that, I mean that the origin and operation *of impressed forces* must, *in turn*, be explained in terms of the basic properties of bodies. The impressed force which causes the change in A does not have to reside within A, and quite generally, it will not."

In effect, I read Euler as maintaining that the monadologist is collapsing together different stages or levels of the total explanation of change, by moving from the phenomenon of change (in body A) *directly* to active force (in body A). Indeed, it is evident that he wishes to separate these (at least) two levels. This fact *implies* that, from his point of view, the monadologist's direct appeal to 'active force' to explain corporeal change collapses these levels.

At this stage, many questions are likely to arise about the details of this reply. I provide this brief description of it here, to try to maintain balance and continuity with the rest of the section and paper, and leave the full analysis, which is considerably more detailed, to the excursus in section 6.3.

Consolidating

If the preceding analysis is correct, Euler's regimentation of the concept of 'force' in natural philosophy has had lasting effects on the conceptual structure of classical mechanics. But not only that, it has also had reverberating effects on philosophy, both by landing a heavy blow to the viability of major doctrines of traditional, early modern philosophy and also by constraining the tools and ambit of the discipline of metaphysics.

Specifically, according to the analysis of Euler's position that I have given so far, a normative implication follows concerning the domains and methods appropriate to metaphysics and mechanics, respectively. The implication is that metaphysicians are to take the mechanical properties of bodies and the laws of nature that mechanics discovers as *givens*, and then inquire into their sources in first principles (cf. the conclusion to section 4). Additionally, they should not use notions of 'force.' Positing concepts of 'force' to serve in metaphysical explanations tends to undermine the neat division of labor that Euler proposes (section 3). These uses are in tension or contradiction with the mechanical notion of 'force', and it is for mechanical explanations that ‘force’ is importantly reserved (section 5). The justification for the normative force of this division, and the specific norm about use of ‘force’ concepts, is that the resulting metaphysical explanations throw the conceptual structure of mechanics into confusion, and generally fail to be genuinely explanatory (cf. the statement in section 5.1). The preceding (section 6.2) was intended to provide evidence that this regimentation of usage of ‘force’ has in fact served to dissolve confusions regarding concepts both as they are properly used in mechanics and also as they are deployed in metaphysical projects nearby to mechanical theory.

The matter of Euler’s engagement with Wolffian monadology and with ‘active force’, however, is particularly rich. The controversy itself is quite large. Euler’s argument here is also, I claim, more sophisticated than he has typically been given credit for making. The dialectic also promises to illustrate a bolder claim (mentioned first at the end of 5.1), that a metaphysical ‘force’ posit may, in effect, mislead one into taking oneself to have a genuine explanation of a target where none has been given. I turn to a closer analysis of the monadology dispute in section 6.3, before considering, in section 7, the question as to how one ought to pursue metaphysical explanations à la Euler, without invoking ‘forces’ as metaphysical causes.

6.3 *Excursus*: The Berlin Academy dispute and Euler’s dilemma for the monadologist

The Wolffian metaphysics of body, space, and motion

Metaphysicians have long concerned themselves with asking and answering what I have called questions about metaphysical causes. (No surprises there.) However, metaphysicians in the community around Wolff, in answering their questions, purported to identify which concepts of force, space and time, and body were rationally legitimate in themselves (as opposed to "imagined" or "fictional"), and therefore permissible for use in other forms of inquiry. This view of the metaphysics-physics relation, which sees metaphysics as prior to or more general than physical theory – and hence as *constraining* it – is traditionally associated with Descartes. Yet it also characterized the philosophers linked to monadology and to Wolff, including Wolff himself, who were then the most influential group of metaphysicians in the German-speaking lands.

Their metaphysical accounts of space, time, motion, and body in general did not jibe with mechanics as practiced by the likes of Euler. Where was the conflict? For one thing, besides claiming to have established the ultimate nature of all that exists in the world—as well as the ultimate causes of all corporeal phenomena – Wolff and his followers likewise took themselves to have settled the nature of space and time. Leibniz had famously argued that space and time are nothing more than ways of denominating relations between existents and are not themselves existents. Space and time taken as entities in their own right are at most "ideal beings"; they lack the real and absolute existence that Newton and his followers attributed to them.

Wolff, who corresponded heavily with Leibniz and succeeded him as doyen of German philosophy, developed a view similar to Leibniz's.[62] Although it did not simply recapitulate that view – in particular, his view of time was somewhat more 'realist' – his account of space was likewise broadly idealist. Specifically, Wolff defined a concept of "general space" much as Leibniz had defined space itself, as the order of coexistence of bodies. Wolff then sought to explain how we come to acquire the idea of this space. According to his story, the thinking subject comes to perceive this general space as a

[62] For the account of Wolff's views canvassed in this paragraph, I am indebted to Hettsche and Dyck (2019, §5.2).

consequence of being conscious of pluralities of things outside of itself, all of which exist at the same time. In the *German Metaphysics*, he writes:

> "In that there are many things now which exist at the same time and which are presented apart (and yet at the same time different) from each other, such things come into being under a certain order. And as soon as we perceive this order we perceive space." (*German Metaphysics*, §46)

Wolffian space is thus an abstraction from our perception of many things existing outside of us all at once. As such, it is merely a *mental* representation and lacks "objective reality." Wolff's views of space, time, and body, and his version of the monadology, gained him many allies in the academic world, like Formey, Thümmig, Baumbarten, and König, forming a circle of Wolffians that helped his ideas spread in Germany and beyond.

For present purposes, the two most important features are, first, that Euler took the Wolffians to have levied a *reproach* against practitioners of mechanics. Euler took the monadologists to have challenged the predominant way of treating space in mechanics by offering an account of space that treated it as an abstraction or ideal being. Specifically, they implied that key concepts of the science of mechanics as practiced were non-referring, illusory, or otherwise erroneous. Euler took this challenge seriously enough to pen an extended reply, in the form of his "Reflexions sur l'espace et le tems," published in the *Mémoires* of the Berlin Academy in 1750 (discussed briefly in section 4 above). Though Euler does not talk about monadology by name here, the identity of the "Metaphysicians" he addresses, and his attitude towards them, is clear from the first paragraphs. He writes:

> "And the Metaphysicians…reproach Mathematicians for improperly attaching these principles [of Mechanics] to ideas of space and of time which are only imaginary and destitute of all reality." (§3)

Euler suggests, against the "*Metaphysiciens*", that theoretical concepts like space and time cannot refer to things existing only in mathematicians' imaginations since, according to well-founded mechanical theory, space and time play indispensable roles in regulating the behavior of bodies according to mechanical laws:

> "For there is no doubt that bodies, in regulating themselves in accordance with these principles [of Mechanics] are not regulating themselves in accordance with things that only exist in our imagination; rather, it is certain that these are very real things which are referred to by the laws and which bodies follow in the conservation of their state." (§4)

The second important feature of this episode is that the Wolffians had demonstrated a high degree of theoretical ambition. They put themselves in the position of offering the ultimate explanations of the existence and nature of material bodies, of their properties, and of the causes of their motions and changes. As explained in (Hettsche and Dyck 2019):

> "Wolff's analysis of physical bodies is given from two different perspectives. First is the 'bottom-up' metaphysical account of bodies, where bodies are defined as aggregates of simple substances, and second is the 'top-down' mechanistic description, where the reality of bodies, given by the testimony of the senses, is explained in terms of interacting primitive *corpuscula* (or corpuscles)."

Epistemically, the monadologist *relies evidentially* on the empirical facts of bodies in motion and corporeal change as evidence for the necessity of active or motive forces in monads. Metaphysically (and perhaps also epistemically), the monadologist purports to provide the ultimate *explanation* for the changes that occur in material bodies.

For Wolff, then, the theory of simple substances was very expressly meant ultimately to be an account of body. This makes his case somewhat different than Leibniz, for whom the connections between the metaphysical and the physical – the monads and the world of bodies – is difficult enough to specify to remain a subject of intense scholarly disagreement even today. Since it will be discussed throughout, it is worth briefly outlining Wolff's version of "monadology."[63]

---

[63] For the summary here, I am once again indebted to Hettsche and Dyck (2019). The primary Wolffian texts are the *German Metaphysics* appearing in its first edition in 1720, the *Ontologia* of 1730, and the *Cosmologia* of 1731.

In Wolff's picture, are three levels to reality, or perhaps levels of description: 1) the level of unextended simple substances, or atomic elements, 2) the level of physical atoms, and 3) the level of phenomena or appearances. The simple substances, or genuine atoms – which Wolff's followers would eventually start to call monads – are "unextended points of force" undergoing constant change. Though resembling the original, Leibnizian monads, there are two important respects in which Wolff's differ. The force with which Wolffian monads are endowed is *not* to be conceived of as a mindlike faculty of representation, or *vis representativa*, and they not "windowless", but in causal communication with each other."[64] At the second level come corpuscles, or material atoms. Strictly speaking, these corpuscles are composite – "an aggregate of atomic elements" (Hettsche and Dyck 2019) – but earn the title of 'material atom' because natural causes cannot divide them. Wolff thinks that material properties like extension, filling of space, and possessing the "force of inertia" arise at this second level in virtue of the manner in which the simple atomic substances are arranged together. However, we may not have empirical access to their measures. The third level is the level of empirically accessible phenomena. The entities are arrangements of groups of physical atoms, making them "composites of composites" (Hettsche and Dyck 2019). At this level, properties of bodies like extension and 'force of inertia' become measurable empirically.

Tensions mount

A new instigating factor to tension between Euler and the Wolffian faction emerged in the 1740s.[65] This was the 1747 prize competition, mentioned above, proposed by the

[64] For more on these differences between Leibniz and Wolff – and, with Wolff, Du Chatelet – see Stan (2018).
[65] Others who offered versions of a monadological theory of active and passive forces, such as Emilie Du Châtelet (in her *Institutions de physique*), seem not to have been targeted by Euler or embroiled in the controversy, in spite of the fact that they were quite aware of each other's existence. The reasons for this, and the relation between Du Châtelet and Euler, call for further investigation. It may be noted that bare geographical distance

Metaphysics section of the Berlin Academy, and which involved all four of its sections in the judgment. As quoted in (Broman 2012),[66] the relevant part of the question reads:

> "after having proved [the existence of] Monads, to deduce from them an intelligible explanation of the principal phenomena of the Universe, and in particular of the origin and movement of bodies."

The question was a very public affirmation of the monadology's theoretical ambition and provoked a strong reaction in all sections of the Academy – not least in Euler's. On its face, the question presupposes that *monadology* is a candidate for an ultimate theory of bodies and their motions. By implication, the discipline to which we should turn for such an account is *metaphysics*. Yet, the mathematical discipline concerned with studying bodies and their motions was *mechanics*.

Conflict erupts

Prima facie, this question seems to invite metaphysicians to contribute to causal accounts of the motions of bodies. Understood in this way, we can see how the Prize Question by the Metaphysics class could have struck Euler as an audacious attempt to speak with authority on the subject matter reserved for mechanics. In the remainder, I will frequently drop the "as it struck Euler" qualifier, and speak directly of "encroachment," with the understanding

---

seems not to account for it. French academics like Formey were present in Berlin and participated in the dispute, and Wolffians like Samuel König had traveled in France, and even tutored Du Châtelet early in her philosophical career.

[66] Originally: "après avoir prouvé les Monades, d'en déduire une explication intelligible des principaux phénomènes de l'Univers, et en particulier de l'origine et du mouvement des corps." Broman comments: "Almost as soon as the question was published, a pamphlet war erupted in which disputants argued over the relevance of monads for natural philosophy Significantly, two of the major protagonists in the exchange were members of the Académie itself, with the mathematician Leonhard Euler attacking monads as a physical impossibility and the Académie's secretary, Samuel Formey, countering Euler's pamphlet with one of his own defending the metaphysics of Christian Wolff, whose philosophy was closely allied with Leibniz's" (Broman, 2012; p. 3). Prize questions were issued and judged by one of the four Academic 'classes', and normally went out two years before deadline for submission. The monadology question for 1747 was issued by the philosophy class, but it attracted so much controversy that all four classes ended up judging the competition.

that my aim of making sense of Euler's moves requires seeing through Euler's perspective – although that perspective is also highly plausible.

Thus, we have seen the encroachment of metaphysics onto mechanics take two forms. The first is the accusation and "reproach" that certain key concepts of mechanics are fictional, and that mathematicians are in error in taking themselves to be theorizing about realities. This is demonstrated by metaphysical argument from first principles that shows, in particular, that the concepts of space and time that mechanics uses are fictions. The second is that monadology is put forward as a genuine candidate for providing the explanation of bodies and their motions. Indeed, the claim is stronger: that it provides the only intelligible explanation.

Euler's response, too, had two parts. The first part of his response (cf. section 4) was a counterchallenge. Any metaphysician who casts doubt on concepts used in mechanics would have to explain why their arguments are more secure than the most secure propositions we know, namely the laws of mechanics themselves. Insofar as those explanations are not forthcoming, we should defer to the requirements of mechanics in assessing which concepts are legitimate or refer to "real" things. The second part of Euler's response was directed specifically to monadologists.

In effect, Euler presented the monadologist with a dilemma. It homed in on the relation which their theory posits between monads and material bodies, the sense in which monads "compose" bodies. As we will see, how this relation is spelled out is the nexus of the issue of whether monadology conflicts with the laws of mechanics. As a preliminary, recall that 'active force' (or 'motive force', as it was sometimes called at the phenomenal level) was meant to be the faculty inhering in bodies which produces the changes in motion that they undergo, and hence explains those motions.

This provides *prima facie* evidence for the case that monadology interferes with the project of mechanics. After all, the active forces are certainly distinct from impressed forces. It may turn out that there is not "room" for both of them to explain changes in motion – at least, not without problematic causal overdetermination. Call this the 'explanatory exclusion problem.' Second, it may be that the monadological forces are not compatible with what is known about the mechanical properties of bodies. Call this the 'incompatibility problem.' Does the evidence that these problems are genuine, i.e. go beyond the surface? At this stage, the dialectic seems as though it could pull in either of two directions.

In one direction, it might be replied that the conflict is at most semantic, resulting from an unfortunate use of the same word, 'force,' for different theoretical purposes. Monadology seeks to use its concepts of force to explain different things than mechanics seeks to explain with its concept of impressed force. So there is no threat of explanatory exclusion. Moreover, the monadologist's forces simply occupy a different level of analysis than the one of concern in mechanics. One is concerned with the fundamental level – or rather, with all three levels, but it is the fundamental one where the offending 'force' concept arises – whereas the other is only concerned with the phenomenal level. As such, there is no possibility of real incompatibility. What's more, the monadologist will graciously acknowledge a role for mechanics in explaining – in *some* sense of that word – worldly happenings. After all, no monadologist would claim to have "explained" the motions of bodies in a way that *leaves no room* for further accounts of the kind given in, say, celestial mechanics. She could accept that it is up to mechanics, not metaphysics, to calculate the motion of particular bodies like the planet Saturn, for example.

In the other direction, it might be agreed that, while the active forces of simple beings are assuredly meant to be different kinds of entities than impressed forces, this does not mean that the dispute here was no more than semantic. The active forces are meant to furnish the ultimate explanation of all the changes that occur in the world. For Euler, the proper explanation of the changes in states of motion of bodies resides in *impressed forces*, and it is mechanics which deals in impressed forces. If we agree with this, then there is, in fact, *no room leftover* for further accounts of change in terms of the internal, active or motive forces of either bodies or monads. Further, while the monadologist and the mechanician are focused on different levels of analysis, everyone agrees that the levels must connect *some*how. If that is the case, then there will be an avenue for generating real conflict. The explanatory exclusion and incompatibility worries are genuine.

These two worries can help us see that Euler's philosophical criticism has two inter-locking parts. By establishing the *incompatibility problem*, his general epistemic principle allows him to reject the *conclusion* of the monadologist – the doctrine of unextended simple beings endowed with active forces. But he additionally rejects the *inference* to that conclusion, as well, on grounds of *explanatory exclusion*.[67] The parts interlock in the form of a dilemma with which Euler confronts the monadologist, forcing her to face one problem or the other.

> *Euler's dilemma*: The monadologist can grant that the level of simple beings is connected with the level of empirically accessible bodies in a straightforward way, by aggregation. In that case, she runs into the incompatibility problem: there is a conflict between active forces and inertia, because inertia is an aggregative property. Alternatively, she can maintain that the level of simple beings is connected to the level of empirically accessible bodies in a more complicated way. But then, in making the connection more abstruse or obscure, she cannot claim to provide *explanations* for

[67] He also offers an argument based on the divisibility of bodies and the difficulties in conceiving how monads "compose" bodies, especially if monads are conceived of as point-sized.

the phenomena she seeks to explain – the changes in motions of bodies. Monads and their active forces are excluded from explanations by well-founded alternatives.

How successful is Euler in pressing this dilemma onto the monadologists? To assess this, let's first acknowledge that the monadologists put in play several distinctions among kinds of force – living and dead, primitive and derivative, active and passive – and these vary from figure to figure. This makes monadological forces something of a tricky target. I have pointed out, in particular, important differences between the Leibnizian and the Wolffian versions of monadology. Leibniz's is arguably both subtler and more abstruse, and as I have indicated I will mainly be talking about Wolff's, before indicating later how Leibniz's complications are folded into the argument. Iin any case, though, all of these notions belong to a conception whereby a monad (and hence, potentially, also a body) is endowed with a faculty for producing changes in its own state (*Gedancken*, §10).

I read Euler as pressing the incompatibility problem in the following way. Core 'forces' ascribed to monads amount to a faculty for producing changes in that entity's state. But bodies, however the details are worked out, are in *some* sense composed of monads. And so, at least *prima facie*, it seems that the properties which monads have, other than their simplicity itself, would also be properties of the bodies. That means bodies have a faculty for producing changes in their own states. The law of inertia, however, asserts exactly that bodies, left to their own devices, will not change their state. The active or motive forces of monadology appear to contradict the law of inertia.

The crux is the connection between the law of inertia, a law governing material *bodies*, to the forces with which *monads* are supposed to be endowed. The monadologist is talking about *monads*. And Euler is complaining that their properties are not compatible with the properties of *bodies*. Indeed, the monadologist does not stipulate that monads are endowed with inertia (and not in the sense Euler understands 'inertia,' see 5.4) – which would rule out their being endowed with forces to change their own states.[68] It might be said, against Euler, that monads are simply not required to bear the properties of *bodies* any more so than angels or minds are. The natural follow-up, to someone familiar with monadology, will be that Euler has assumed that monads are themselves something like physical atoms. But they are assuredly *not* meant to be physical atoms; to the contrary, they are meant to occupy something like a "lower metaphysical level" than the physical. Given that, Euler's complaint is levied against a strawman.

This produces something of an interpretive puzzle: given the obviousness of the distinction here, how could Euler have taken himself to have just given a knock-down argument against monadology? Perhaps Euler just did not appreciate the project of monadology and what metaphysicians are up to in constructing it. But this is too quick. As we have seen, in Wolffian monadology, monads really *are* thought to compose, first, material atoms, and next phenomenal bodies, by something like aggregation. Indeed, this makes sense given the (Wolffian) monadologist's other goals.

> Epistemically, the monadologist *relies evidentially* on the empirical facts of bodies in motion and corporeal change as evidence for the necessity of active or motive forces in monads.

---

[68] If he argued in that way, he would plausibly be begging the question against the monadologist, who believes he has identified the fundamental entities whose properties ground and explain the (higher-level) properties of material bodies. The properties at the more fundamental level might well be different. And perhaps they should be, since one hasn't purchased much by way of ground or explanation for a property by repairing to that very same property.

It has to be admitted that his language is quite loose. In some passages he appears sensitive to the need to bridge a connection between the levels of monads and bodies (*Gedancken*, §41). But, in another, he will appear to gloss this over entirely – by saying, for instance, that monadologists "ascribe to *bodies* a force to change their state perpetually" (*Gedancken*, §32). My reading of Euler as implicitly pressing a dilemma on the monadologist is meant in part to reconcile these apparent differences. For it forces the monadologist to either make a relatively direct connection between the properties and forces of monads and those of bodies, or else face explanatory inertness in relation to the phenomena of motion.

> Metaphysically (and perhaps also epistemically), the monadologist purports to provide the ultimate *explanation* for the changes that occur in bodies.

Whether or not it is granted that Euler has given a completely accurate gloss of the argument for monadology, it cannot be denied that the monadologist begins evidentially with the phenomenal facts of change in bodies. Second, the monadologist purports to give the explanation of those phenomena at the lowest "metaphysical level." As Euler put it, "a theory is specifically asked for, that could explain the cause of all the happenings [*Begebenheiten*] in the world" (§40). As such, it is clear that the monadologist wishes to see her account as having relevance for the knowledge of material bodies and their motions. It is both *epistemically grounded in* our knowledge of bodies, and meant to *explain* the motions they are subject to.

In particular, it is an aim of Wolff's theory to recover the mechanical properties of bodies at the highest, phenomenal level, and these resources provide a route to the goal of explaining the motions of bodies. If monads are the substrate for bodies in a way that allows the monadology to recover the mechanical properties of bodies, and mechanics tells a story about how those properties explain motions, then monadology has contributed (to) an explanation of motions. What this means is that the explanatory goal is an internal pressure pushing the monadologist to an "aggregative" conception of the monad-body relation. If that is the case, she runs into a problem:

> "36. Since instead of being endowed with a force perpetually to change their state, bodies are endowed with a quite oppositely directed force, namely to remain with their state unchanged, we hope nobody will accept the following conclusion: *All bodies are endowed with a force to remain with their state unchanged. Consequently the simple things, from which bodies are composed, must be endowed with a force to change their state all the time.*"

As Euler adds, with almost affected simplicity:

"37. Instead the following conclusion must be correct: *Since all bodies are endowed with a force to remain with their state unchanged, but since the bodies, being composite things, cannot have this force unless there is a place for a similar force in the simple things, these simple things, of which the bodies consist [aus welchen die Körper bestehen], must also be endowed with a force to remain in their state, or to maintain themselves therein.*"

The present point is that, on the "aggregative" conception of the monad-body relation, the conditional claim is clearly true.[69] Any version of the monadology which takes the relation of monads to bodies to be straightforward composition, or aggregation, will run into one horn of Euler's dilemma: contradiction with the law of inertia. This is the incompatibility problem.

But what if the monad-body relation is not one of simple aggregation? The monad-body composition relation can be theorized in exceedingly subtle ways, ways which may avoid the force of Euler's argument so far. On some models of the composition relation, for example, the monads are infinitesimally small, and each constitutes only an infinitesimal part of the body. In that case, it may be reasonable to conclude that they have no finite, positive quantity of inertia, and so there may be no conflict with their possessing internal forces of self-change. Another possibility might be to insist that, while the monads compose bodies by something like aggregation, the *manner* in which they are put together makes a difference to the nature and properties of the higher-level aggregate – both the mid-level material atoms and also the top-level phenomenal bodies. Or, if not the manner, then the *link*: "The parts of which a composite being is composed constitute a composite through the link which makes the many parts taken together a unit of a definite kind." (Quoted in Hettche and Dyck 2019). Admittedly, it is notoriously difficult to work out in detail how one of these alternatives can

[69] Clearly true, that is, but *not* trivially so: it is correct specifically because inertia is an aggregative property of matter. The inertia of a body was standardly taken to be simply proportional to the 'quantity of matter'. By putting together two bodies each with one unit of inertia, one gets a body with two units of inertia. Vice versa, divide a body with two units of inertia into two equal pieces, and you have two bodies with one unit each. So, the thought goes, by passing to the simple beings, and supposing one gets bodies as such by "putting together" some monads, then it stands to reason that those simple constituents must also have inertia, in proportion to their contribution to the total.

be constructed so as to be internally consistent and consistent with what else a given philosopher – whether Leibniz, Wolff, or one of their followers – has said. Still more difficult is showing how it can explain all it is supposed to explain. But I do not want to get bogged down here, because my aim is not to present airtight statements of Wolff's (or Leibniz's) views as such. [70] My aim is to bring out the interest of Euler's offensive.

In light of that, the thing of note about proposals of this sort is just that, however we work out the details, they all turn away from a picture of composition by straightforward aggregation. For Wolff, it is "composition-plus" – composition *plus* a "link" of some sort. However, if that is the case, it is hard to see how this link, whatever it is, could be of a kind to obviate the requirement that the simple constituents of bodies must also have inertia, given that the composites do.

If we wish to get still more subtle or obscure, this might be considered roughly as moving further in the direction of Leibniz. Wolff expressly wished for his theory to connect to the realm of physical bodies – indeed, in many ways it *was* a theory of physical body – and his monads were not infinitesimal. Though it is hard to say exactly what Leibniz's view was (and it is doubtful he had just one view throughout his career), it is reasonable to say that the monad-body connection was more subtle and dubitable than in Wolff.[71] There certainly was supposed to be a connection between the metaphysical and the physical in Leibniz. The nature of that connection remains the subject of scholarly debate.

---

[70] Du Chatelet's version is interesting because it [attempts to preserve for the physicist the right to posit physical atoms, if these prove to be useful in her theories and predictions, while at the 'bottom' level, what exists will still be extensionless, infinitely-varied simple beings. It is interesting that one of the main morals of Du Chatelet's version of the theory—that what we need to find is a way to preserve the non-interference between mechanics and metaphysics—resonates strongly with what I have been describing in this chapter as Euler's project.

[71] This is the subject of continuing debate in Leibniz scholarship. For an influential treatment, see Garber (2009). For a recent challenge to this account, see Jorati (2019).

The brilliance of Euler's argument is that his dilemma sweeps both the Wolffian and Leibnizian versions into a single dialectical net. The basic question is: how *does* the monadology help explain the motions of bodies without *preempting* the causal accounts of mechanics? If monads straightforwardly compose bodies, then they run afoul of the law of inertia, hence preempt mechanics (incompatibility problem). And if the monad-body relation is nuanced? Euler himself surely understood that auxiliary metaphysical hypotheses can always be introduced, even if he did not say so in these terms. If the monadologist wishes to avoid the conflict with mechanics, she will have to come up with some alternative, and it is here that Euler is waiting with the second horn of the dilemma.

The second horn of the dilemma rears its head when the need to avoid contradiction with mechanics forces the monadologist to a revised picture of the monad-body relation – one that makes it impossible for monadology to perform that explanatory task it has set itself (explanatory exclusion). In effect, the accusation is this. To avoid the incompatibility problem, the monadologist must obscure the relation between the plane of monads and the phenomenal world of material bodies. But that very act undermines the monadologist's ability to contribute to explanations of the target phenomena: bodies and their motions.

Why is that? The readily available and successful explanations of bodies and their motions with proven power are those found in mechanics, based on the mechanical properties of bodies and impressed forces.[72] According to this class of explanations, the immediate causes of the phenomena – changes in motion – are found in bodies adjacent to the one given that impress forces on the given body. (It might be objected that they could also be found in fundamental attractive forces, but this carries no water for the

[72] I recognize that this is itself another claim. For justification of it, see Appendix C.

monadologist, who rejects fundamental attractions.) The success of these explanations means that the best chance monadology has to contribute explanations of the target phenomena is to somehow *join* the explanations offered in mechanics. So as not to bury the lead: this might be, as Euler proposes, by providing deeper (second-order) explanations of the conceptual resources that mechanics uses in constructing its (first-order) explanations. Yet, by obscuring the connection between monads and bodies, or between the metaphysical and the physical, the monadologist forestalls herself from doing just that, short-circuiting her own attempt to participate in the project of giving "an intelligible explanation of the principal phenomena of the Universe, and in particular of the origin and movement of bodies" (Broman 2012). This is the *explanatory exclusion problem*.

It thus becomes unclear what the monadology is explaining about the motions of bodies, or what further content they are adding to the overall explanation of those motions. As a result, the explanations of mechanics preclude, and hence exclude, those of the monadologist.

The monadology staked a major claim to provide a privileged account of bodies and their motions. It was both epistemically grounded in the phenomena of corporeal motion and intended to provide the metaphysical explanation of those motions. Euler pressed a dilemma on the monadologist, arguing that she either faces a problem of incompatibility with the science of mechanics, or else a problem of explanatory exclusion, where the monadologist's purported explanations of motions are pre-empted by those of mechanics. This reasoning is more sophisticated that philosophers have traditionally been prepared to give Euler credit for.

The normative implication is that mechanics has circumscribed a domain of inquiry for itself and created certain tools to pursue this inquiry in generating its explanations, such that metaphysics lacks warrant for offering its own, competing explanations. The domain is material bodies and their motions, and the tools include the notions of mass and inertia, acceleration and state of motion, and force. The Prize Question assigned by the philosophy class, and the leading theorists of Monads, appeared, to Euler, as an audacious attempt to cross that line of demarcation, and the notion of 'active force', an attempt to arrogate the tools of mechanics. Metaphysics, according to Euler, is not in the business of giving first-order explanations of the motions of bodies.

Against this background, Euler's attempt to legislate that 'force' should mean only *impressed force* has significance beyond "cleaning up" the basic conceptual framework of classical physics. It is his attempt to influence an inter-disciplinary struggle for authority. Specifically, it encapsulates his effort to disentangle metaphysics from mechanics by rejecting uses of 'force' concepts that attempt to metaphysically explain mechanical phenomena in ways that interfere with the explanations of those phenomena in mechanics itself.[73]

7. Euler's demarcation and the place of metaphysics in it

Euler rejects the monadologist's approach to metaphysics. However, he does not reject *metaphysics*. While it is true that his references to the tasks of metaphysics are, on occasion, touched by irony, it would be too quick to conclude that he does not think there is a task there.[74] Many projects will be left to metaphysics.

Euler's position is distinct from two familiar patterns often read from the histories of philosophy and science. One of these familiar positions is the broadly "empiricist" one – the

[73] This doctrine, which had been at the center of physics discourse on the Continent for decades, appears to have soon after fallen out of relevance to the metaphysics of science. It was at best nominally relevant, in the form of Kant's "physical monads," which, despite sharing a name with the holders of 'active force', were sources of impressed force.

[74] If anything, they read as comments on the abilities of his metaphysician contemporaries. He once wrote of "the partisans of monads, helmed by the great and famous M. de Wolf, who claimed to be no less infallible in his decisions than the Pope." (*Lettres*, Vol. 2, No. 125).

opinion that science must recognize the abstruse questions of metaphysics to be idle or meaningless, and ignore them to get on with the useful work of empirical science.[75] A second, familiar view is that science must recognize certain puzzles of metaphysics to be irresolvable, to point to unsolvable mysteries, and that scientists, in order to move forward, must agree to cease to be bothered by the lack of an answer to them.[76]

Neither of these is Euler's view. While Euler worries about metaphysics *interfering* with the useful work of empirical science, he does not think that there are no meaningful questions of metaphysics, or that science ought to ignore metaphysics. As we saw in section 4, one of these projects might be to intelligibly exhibit the metaphysical ground of the mechanical properties of bodies. Another might be to explain why there always appears a "law of maximum or minimum" in Nature. The crucial thing is that to frame metaphysical projects in these ways serves to *acknowledge* the sovereignty of mechanics over its basic explanatory project, rather than to compete or interfere with it. But how is the metaphysician's task to be legitimately executed? First, let's use Wolff to illustrate how it may *not* be done.

7.1 How not to do metaphysics

As Euler interprets Wolffian ontology, an important goal is to theorize about the world as it actually is. Even more specifically, the point is to account for changes occurring in material

---

[75] This attitude may have reached its apotheosis in logical positivism, with its total repudiation of metaphysics as a pseudo-inquiry in propositions which are strictly empty of cognitive content. Yet it has also had currency in philosophical interpretations of the 18th century. For example, Janiak glosses a perennially influential interpretation of Newton thus: "Newton fundamentally eschewed the very metaphysical issues that animated Descartes's work, focusing instead on empirical and mathematical topics that could be solved using recognizably modern scientific techniques" (Janiak 2008, 12).

[76] An example of this kind of view in relation to the 18th century is arguably found in Kuhn, according to whom scientists had to stop fretting about action at a distance and finding an intelligible account of attraction in order to get on with useful research in gravitational astronomy. Kuhn appears to hold the view that these philosophical worries were not so much recognized as misguided as they were identified as unanswerable. In effect, scientists threw up their hands and agreed to treat gravitational attraction as an "occult quality." (Kuhn 1962, 105).

bodies (e.g. *Gedancken*, §§I.3, II.4). Recall that the prize question about monadology literally asks for theories of the "principal phenomena of the universe," and the "origin and the movement of bodies." The monadology was thus supposed to have empirical content, connecting up with and bearing on the subject matter of mechanics. If it is to have such a role, then, given the existence of mechanics, this posit must either feature in a "direct" explanation of bodily motions that *competes* with mechanics, or it must feature in an explanation of the basic concepts of mechanics itself.

Since there is no threat of real competition on the first point, this leaves one role for the monadology: explaining the properties of body that feature in the explanations of mechanics. As we have seen (section 4), this is indeed a task that Euler regards as belonging to metaphysics. The claim, then, would have to be that monads and their active forces – and possibly also passive forces – *are* the intelligible metaphysical ground of the mechanical properties of bodies, which Euler thinks include extension, mobility, inertia, and with them the faculty of exerting impressed forces on other bodies. This would be to make them out to be the *metaphysical causes* (in the sense explicated earlier) of those properties.

It is an upshot of the previous section (6.3) that this appears to be a strained reading of the metaphysics on offer. Why is that? The active forces of monads are meant to be internal faculties, and their ability to produce the changes occurring in monads and ultimately the changes we observe in bodies is not mediated by the mechanical properties which generate external, impressed forces. Indeed, there are obstructions to this in principle. (I analyze them in terms of a dilemma between an *incompatibility problem* and an *explanatory exclusion problem* in 6.3.) At least as Euler (plausibly) reads the monadologists, active forces are derived using the principle of sufficient reason in a way that expressly skirts the explanatory path marked out by externally impressed forces. They directly explain motions and changes in motion; this makes them explanatory *alternatives* to impressed forces. But the basic mechanical properties of bodies are, according to Euler, meant to lie one step back on the same explanatory path as impressed force (cf. the schema in section 3). So, in avoiding impressed forces, the monadologist cannot make contact with the (true) mechanical properties of bodies. Additionally, again according to Euler, 'active forces' are incompatible with the inertia of bodies (section 5.4). In this way, it is not just incompatible with the mechanical concept of 'force.'

The question Euler urges against the monadology in many contexts is what is the *point* of positing active force and its ilk? What does it get us? Euler is, unfortunately, not as explicit about active force as he is about other questions. For example, the monadologist's proposal that monads, as indivisible, simple beings, are the sufficient reason for, and explain the possibility of, extended body – for, if bodies were infinitely divisible, their existence would lack a sufficient reason.

> "It would be well to hope that such a thin [*leger*] argument was capable of illuminating such an important question; for me, I must avow that I understand nothing at all of this pretty argument….Yet, it is necessary, without a doubt, always to examine and acquaint ourselves with a question before responding to it. Here, they give the response before having so much as formed the question" (*Lettres*, Vol. 2, No 128).

The monadologist asks a question – or at least, purports to ask a (real) question – hence points to a place where an explanation must go. Indeed, Euler, in these passages, complains that the principle of sufficient reason only affirms *that* a reason is called for, and by itself cannot identify *what* the reason for anything is. The response, therefore, which names some new theoretical entity ('monad' or 'active force'), ends up really pointing, at best, to a mere placeholder for an explanation, rather than an explanation with genuine substance.

Thus, one of his main concerns is that while metaphysics asks questions that deserve answers, in practice the replies of (these) metaphysicians characteristically provide only the illusion of answering the question. What is more, they give the illusion of having provided the answer *a priori*, of having procured an item of *a priori* knowledge of nature.

I have not encountered Euler making parallel claims expressly about 'active force.' Yet, I take Euler's view to naturally carry the implication that 'active force' is also a plausible candidate for this charge. Strictly speaking, though, this is my interpretation of Euler's position.

Namely, the concept of 'active force' helps to give the appearance of having discovered an item of *a priori* knowledge of nature: that active or motive forces number among the fundamental entities in nature, and are drivers of observed changes. However, the concept of force as it is used by monadologists is, in effect, only a placeholder. It stands for that cause, whatever it may be, which answers the demand of the principle of sufficient reason, as applied to changes in some body. This makes it explanatorily inert or vacuous – a "faculty, I know not what," responsible for the change. And then, next, by a move which from Euler's perspective is certainly invalid, it is assumed that this faculty which immediately causes the change must reside in the body itself.

Why, then, are forces as used in mechanics not prey to this same charge of vacuity? That is because they are both identified and *quantified* by their observable effects, and also because they are only the first stage of a multi-stage inquiry into the cause of motions. They are not a "faculty, I know not what," because their identity conditions are really 'quantification conditions', and are provided by the observed change they account for. And this theoretical scheme provides, second, for further questions about the physical sources of those forces. At a still later stage, it allows for the metaphysical explanation of the properties and laws involved in the description of the physical sources. 'Active force' purports to explain it all in one go, but fails to be genuinely explanatory. 'Impressed force' provides only the first layer of a total explanation of the phenomena of bodies and their motions, but just this restricted scope allows it to make a genuine contribution to that explanation. This is what distinguishes "force" as a genuine explanatory concept in mechanics, as opposed to a non-explanatory concept metaphysics.

7.2 Illustration of finding metaphysical causes without inventing 'forces'

Euler's acceptance of a place for "metaphysics" is clear in his works that provide foundations for mechanics. The context of the monadology dispute helps to clarify the procedures of the *Anleitung*. The monadologist attempted to derive the properties fundamentally explanatory of motion and change from motion and change themselves. This is almost exactly Euler's project in the *Anleitung* and elsewhere, a project I describe and assess in detail in Chapter 3. In effect, he is trying to show the monadologist how this is done properly.

As it happens, we can exhibit Euler pursuing an alternative approach, one that he takes to provide more suitable or fruitful or informative answers to the metaphysical question: why do bodies move inertially? His choice of means, it must be said, are not original with him, and their sources do raise additional questions about Euler's philosophical outlook. For they are in the vein of broadly Leibnizian or Wolffian rationalizations of the law of inertia that were current in his period. Importantly, though, they do not terminate in an idea of inertia as a 'force of inertia' or as residing in 'passive force.'

Namely, in the *Anleitung*, Euler accounts for the inertia (or 'persistence') of bodies by means of the principle of sufficient reason. I discuss this in detail in Chapter 3, but the general idea is this. A sufficient reason is required for a body to change its state of motion.

For example, suppose a body in motion changes direction. We can ask why it changed in *this* direction rather than any other. If we suppose that impressed forces are absent, Euler argues, then there would be no cause or reason for the body to change to that direction – assuming that spatial directions in themselves are all alike. Anachronistically, we could say that the isotropy of space means that space cannot "provide a sufficient reason" for one direction over another. Similar remarks apply to changes in speed. In consequence, in the absence of impressed forces, the body will maintain its state of motion. So it turns out that the law of inertia is the expression of a fundamental, causal principle of metaphysics: the principle of sufficient reason. With this explanation of the mechanical property of inertia, mechanics and metaphysics are each allowed their own tools, their own concepts and principles, for pursuing their questions about bodies. This is not to say that the answers that will be found, including Euler's answers, will necessarily satisfy or be correct. But even if false, they will at least no longer be methodologically confused, muddled, and contradictory.

Importantly, however, this contains a crucial lesson for metaphysics about the application of the PSR. Recall that the monadologist also made use of the PSR to derive active force, a move which Euler rejected. Apparently, then, Euler thinks that some applications of the PSR are legitimate and some are illegitimate. Wherein lies the difference? Euler's remarks on the PSR suggest that he thinks of it this way. The PSR is incapable of generating positive knowledge of the cause or "reason" behind any given phenomenon. In No. 128 of the *Lettres*, Vol. 2, for example, Euler describes the PSR as a kind of vacuous truism. The Wolffians, in particular, use it to assert their claims to a kind of empty knowledge about matters regarding which "Philosophers frankly really ought to recognize their ignorance." It is too weak to generate genuine knowledge of causes, since it states only that such a cause or reason must exist. It is, as Euler wrote, a way "for them to respond to a certain '*why?'* without explaining [why]."

Thus, to use the PSR to derive the existence of "forces" is merely to pretend to prove the existence of a particular class of entities as real causes. Hence, it is to generate *a priori* a necessarily *illusory* item of positive causal knowledge. It allows them to "respond" to the why-question without "explaining" the 'why' of it. To use the PSR to derive a "force" of inertia would be to do the same. This contrasts with the use of the PSR to derive the claim that bodies must behave inertially, i.e., to derive the law of inertia.

My claim is emphatically *not* that the derivation of the law of inertia from the PSR is correct. But, significantly, this use of the PSR to specify the metaphysical cause or explanation of the property of inertia does not terminate in a 'force.' This makes it an importantly different way of using the PSR than the use that was made of it to derive active force. According to Euler, all forays into metaphysics which seek to explain laws of nature and properties of body by deducing them from first principles must respect this norm, on pain of interfering with mechanical theory and, ultimately, simply failing to genuinely explain.

8. Conclusion

Euler's engagement with his predecessors' conceptions of force was framed by a consistent anxiety about metaphysics. Though it is not in itself new for a 'scientist' to have metaphysical views, or on the other hand, to view metaphysics with suspicion, Euler's approach to the relation between physics and metaphysics is distinctive. His concern in some of his most influential interventions, or projects in 'conceptual engineering,' was the task of demarcating metaphysics from mechanics, and of making sure that these projects can be conducted without mutual interference. I have argued that underlying this stance is a view about the nature of the explanatory projects of metaphysics and mechanics.

Euler's simple, oft-repeated idea to the effect that the important notion of 'force' is *impressed force* encapsulates a basic distinction about the kinds of causal questions that metaphysics and mechanics answer. The kinds of causes mechanics deals in are impressed forces, and the effects are changes in the states of motion of bodies. Metaphysics is free to pursue other questions – like "what is the cause of the persistence of bodies in their states?" But these are questions of a different order. If metaphysicians (or scientists in a metaphysical mood) deploy the notion of force to answer them, they proceed at their own peril, and run the risk of being found in contradiction, confusion, or unprovable conjecture.

However, where this leaves Euler is in the position of facing the reasonable demand to provide a metaphysics of nature, or metaphysics of science, of his own. We saw him begin to undertake a part of this project in section 7.2. What is his overall metaphysical project, and how far does he take it? To what extent does he succeed, given the limitations on the appropriate means of metaphysical explanation which were established in this chapter? This is what I propose to examine in Chapter 3.

Chapter 3. Euler's construction of bodies and force

1. Introduction

At some point early in the 18th century, physics, and in particular mechanics, had come to rely on a suite of concepts of material body, mass, and force. As I described in Chapter 2, Euler thought that the best available understandings of these concepts, offered by the likes of Leibniz, Newton, Wolff, and their followers, were plagued with difficulties and conceptual confusions. At their core, Euler's critiques of Leibniz, Newton, and Wolff are all based on claims to the effect that *their understandings of forces are actually in contradiction with the essence of body*. Whether his critique was, in the final analysis, logically airtight, remains open to debate. Regardless, I argued in that chapter that Euler made a powerful and philosophically interesting case for these claims, and in so doing provided important conceptual clarification to foundational issues in the physics of the time.

All the same, that is to say that we have been concerned so far with the *negative* part of Euler's interventions into foundational and conceptual issues in physics. But Euler was not just a critic. His theories of a diverse range of physical phenomena and mechanical systems advanced our knowledge of the natural world as much as or more than any other figure of the 18th century. Indeed, as Radelet-de-Grave and Speiser (2004) note, although remembered as a mathematician, Euler's writings on physics and mechanics make up more than half the volumes of the *Opera omnia* cataloguing his complete works. Accordingly, I wish to bracket the success of his critical project for the time being, in order to consider another one made urgent by those very criticisms:

What would a good, or correct, or adequate, understanding of forces and bodies look like?

In this chapter, I turn to consider Euler's account of bodies, their motions, and the forces responsible for changing them. This is his *Naturlehre*. Or, roughly speaking, his natural philosophy or his "physics." His theory is of interest for a number of reasons. Most immediately, here: because it was intended to address the deep conceptual problems, discussed in Chapter 2, which Euler identified in the theories of the likes of Newton and Leibniz.

Accordingly, two goals of this chapter are, first, to lay out Euler's account of bodies and forces; second, to examine how successful this account is at navigating the issues implied by his own criteria of theoretical success. The third and final goal will be to take a step back, and to draw lessons about Euler's unique methodological approach to "foundational" physical questions.

Euler evidently subjects his account of matter, body, motion, and force to a rigorously austere standard flowing from his conception of the property of inertia. This leads him to reject a range of properties and powers that were often attributed to body. His aim is a conception of body that incorporates the insights of predecessors in its inventory of corporeal properties, but that weaves its way around the pitfalls of self-contradiction (à la 'force of inertia') and explanatory impotence (à la Cartesian matter). I will find that Euler's account has many advantages, and that it even looks as though Euler's account succeeded in resolving or sidestepping key problems he identified in others.

First, the advantages. Euler's account represents a conceptual clarification over the program of mechanical theory offered by Newton in the *Principia*. It does not contain the conceptually troublesome ideas of 'force of inertia' or 'action-at-a-distance.' Further, Euler sketches a plausible account of how a wide range of changes in bodies could be explained solely in terms of externally impressed forces. This also eliminates the need to posit the internal, active forces of self-change favored by the likes of Leibniz and Christian Wolff – a road that arguably leads to insisting that mind-like properties persist all the way down our ontology.

Nonetheless, further scrutiny reveals that the account does not succeed in providing the *a priori* derivation of the laws of mechanics that it promised to be. Section 4 is expository rather than critical. Yet, the exposition will bring us to certain gaps in Euler's argument which will lead us to conclude that his project is less reductive than promised. It will require more *a posteriori* content to patch up those gaps – especially one about the 2nd law of mechanics – than he was prepared to acknowledge, which this tells us something about the structure of mechanical theory. Nonetheless, Euler can still claim a significant achievement. For he succeeds in showing how a large part of mechanical theory can be made to rest on a combination of definitions and accepted first principles. Empirical content is needed eventually, but this is later than might have been expected.

However, the account does face difficulties. In section 5, I present and consider a slate of deeper problems with the basic conceptual architecture. I offer replies on Euler's behalf for many of them, but argue that Euler does not have promising resources for reply to all of them. First is a deeper problem about the alleged *a priori* derivation of inertia, which appears to fail, as others have observed. I suggest a plausible line of reply on Euler's behalf, but conclude that it can be regarded as successful only assuming the truth of Euler's reductive explanation of the motions of bodies in terms of inertia and collision, or impact. Second, however, comes a suite of objections to the success of that very reduction, and the force of these objections, I argue, is unavoidable. The most damning piece here is that the account does not satisfy Euler's own methodological constraints on acceptable physical theories. Third, I consider some miscellaneous objections from the secondary literature – for these, I will suggest Euler does have some plausible lines of response.

Nevertheless, although Euler does not achieve everything he set out to do, the legacy of his attempt is not mere failure. One reason is that it represents a considerable conceptual clarification of mechanical theory. Another reason traces to the methodological *approach* Euler takes, irrespective of his results. The first of these is discussed elsewhere (Chapter 2).

As for the second matter, method, I argue that the strategy Euler pursues in his theory of bodies and forces is to shift attention away from ontology and towards epistemology. Though I must leave a fuller analysis for future work, it is worth saying a bit more here.

First, this is not an unfamiliar move for early moderns. Descartes, for example, is reputed to have initiated an 'epistemic turn' by basing his metaphysics on the criterion of clear and distinction perception, or intuition, a faculty by which a human consciousness enjoys direct contact with fundamental truths. For Leibniz, what contingent facts did or could obtain was constrained by a principle of knowledge, viz. the principle of sufficient reason, which can be regarded as an insistence that the world be strictly "rational" or "intelligible," in some sense.[77] And for Locke, to name a third figure in this class, the claim on which his metaphysics was based (or by which metaphysics was to be constrained) was that all that we can know about reality derives from our perception of simple ideas and the mental operations we perform on them.

[77] In Leibniz's own words, the principle of sufficient reason "must be considered one of the greatest and most fruitful of all human knowledge, for upon it is built a great part of metaphysics, physics, and moral science" (G VII 301/L 227).

My claim is that Euler's approach to the foundations of mechanics is an unacknowledged further step in the 'epistemic turn' that is widely thought to constitute the broad sweep of early modern philosophy, from Descartes through Locke, Leibniz, Hume, and Du Chatelet and onto Kant. This makes not only Euler himself, but theorizing by scientists on the foundations of mechanics, important to the history of philosophy.

Although Euler takes on, to varying degrees, the just-mentioned notions (clear perception, principles of knowledge like the PSR, limitations of human understanding from its reliance on experience), they are transformed and further differentiated as Euler continues to thematize our epistemic capacities – in particular, our conceptual capacities. In comparison, Cartesian intuition takes on the aspect of unmediated metaphysical speculation, or reasoning directly about the nature of body and force. Regardless of his success or failure in resolving the stated problems in the conception of body and force, then, I claim that Euler's approach constitutes a distinct moment in the epistemic turn of modern philosophy.[78] However, I must defer further exploration for future work.

---

[78] Granting that a gradual epistemic turn is characteristic of the whole early modern period – roughly spanning Descartes to Kant – the claim is that Euler's thought represents a distinct and important moment within that larger turn. The three themes just mentioned -- our conceptual capacities, limitations of or constraints on human understanding, and principles of knowledge – all have philosophical precedents. Descartes, Leibniz, and Locke doubtless loom the largest among the previous generations. Hume and Du Châtelet are significant representatives among Euler's contemporaries. I cannot carry out an exhaustive comparison which would distinguish Euler from all of these figures. In any case, Descartes and Leibniz figure largest in Euler's own intellectual formation. About Euler in relation to these two, what I would like to claim is the following:

Euler's approach goes further along the direction begun by Descartes in ways incompatible with Cartesianism. Euler's method of conceptual analysis, especially of the concept of body, bears a loose resemblance to Cartesian 'intuition' but Euler cannot be regarded simply as invoking clear and distinct perception.

Similarly, his use of "Leibnizian" principles like the PSR by degrees gives them a (de-ontologized) significance that Leibniz would be quite unhappy with. Euler's relationship to Leibnizian metaphysical principles – add to the PSR the principle of the identity of indiscernibles, the law of continuity, and principles of optimality – is complex and problematic.

Though acknowledging there is a risk of reading history teleologically, it could be said that Euler's moves foreshadowed, in important ways, the characteristic concerns and moves of the later Kant.

Summarizing: Goal number one, setting out Euler's theory, will proceed in three steps across sections 2, 3, and 4. Since Euler's philosophical thought is so little known, section 3 prepares for delving into Euler specifically; there, I recount the context and constraints inherited from the debate studied in greater depth in Chapter 2, and describe the relevant texts. In section 3, I present an outline which exhibits the multi-tiered structure of Euler's overall project for natural philosophy. Section 4, following these preliminaries, provides the promised exposition of Euler's account of the "general concept of material body" and its relation to forces – the first tier of his grand project for natural philosophy. Goal number two is assessing the success or failure of Euler's account in resolving the problems implied by his own constraints, which I tackle is section 5.

2. Euler on the concepts of matter needed for physics

2.1 Context: questions and constraints

What is material body, and what are its properties, powers, actions, and effects? These questions lay at the heart of both the physics and philosophy of the early 18th century.[79] Euler began to wrestle with them from his earliest contributions to physics, and continuously revisited them, through to his most mature philosophical reflections, decades later.

At the time Euler was forming his fundamental physical views, the continuously raging debate about the nature of material body as a subject matter for inquiry reflected an embattled and fluctuating disciplinary division. Radically different approaches to answering this question were possible. *What* kind of inquiry is this? What does success look like? And by *whom* is the inquiry rightly conducted? This was up for grabs among several candidate

[79] For a useful summary, see Brading and Stan (2024, Chapter 1).

disciplines, with the likes of “natural philosophy,” “physics,” “metaphysics,” and “mechanics” being the most prominent. The boundaries of the academic disciplines were not settled then the way they more or less are today. The differences between them, or whether, say, physics and natural philosophy even *were* distinct inquiries, as opposed to different names for identical or overlapping inquiries, did not have consensus answers. Lines were in the process of solidifying as state institutions took shape to support the sciences. As I argued in Chapter 2, Euler’s own natural philosophy was guided by the aim of respecting and reinforcing a division of inquiry which he attempted to create between metaphysics on the one hand, and the sciences of mechanics and physics on the other. And his view of bodies and forces, as well as what and how we can know about them, reflects this division.

Euler’s reflections on the nature of body were motivated by a set of auxiliary questions. It is useful to understand why these questions are in the background, and how they motivated Euler’s views. Three of the most important are: (1) How austere should our general concept of material body be? (2) How does material body relate to or interact with force? (3) By what means do we properly come to know the essence or properties of material body? In Appendix A, I provide some background regarding the sources and significance of these questions, and canvas some of the prevailing responses to them (which did not always mean accepting their terms). For present purposes, though, the most pressing questions and constraints are the ones coming out of the story presented in my Chapter 2.

At a minimum, our account of force must be *consistent* with the essence of body. We can see Euler explicitly stating this constraint early on – for example, in his *Enodatio quaestionis*.[80] Call this the Weak General Constraint (WGC) on our account of forces.

[80] Here he argues that the faculty of thought cannot be attributed to body, because it is incompatible with the known properties of bodies. The principle is here stated:

Evidently, Euler took the WGC to be uncontroversial. It is the primary methodological-*cum*-metaphysical principle that he wielded against the Newtonian and the monadological accounts of forces.

When Euler got around to attempting his own account of forces, what he seems to have demanded of himself is actually a bit stronger. As I put it in Chapter 2, he wanted "an understanding of forces and hence of attraction that is based exclusively on the essence of body." Call this the Strong General Constraint (SGC) on our account of forces. Euler's commitment to the SGC is on display, for example, in the *Anleitung zur Naturlehre*.[81] The SGC is surely also more controversial than the WGC (one reason why using the WGC is dialectically more effective for attacking other theories). Famously, in the Queries to the *Opticks,* Newton vehemently denied that he viewed attraction as "innate, inherent, or essential" to body. On one way of interpreting Newton's words, this amounts to saying that attraction *could not* be based on the essence of body. Others who claimed the broadly "Newtonian" mantle, like Locke, did not see a problem with supposing that God had endowed bodies with powers of thought or of mutual attraction, powers which were not essential to them but which God had simply – i.e., by his mere Will – super-added to them. Thus, attractive forces were placed in bodies by the *arbitrary* decree or will of God.

> "Si enim ista facultas [i.e., of thought] ita deprehendatur comparata, ut uni alterive proprietati corporum cognitae e diametro adversetur neque sine contradiction cum iis consistere possit, tum utique agnoscendum erit en a Deo quidem materiae facultatem cogitandi tribui et infundi posse." (quoted from *Opera Omnia*, Series 3, Vol. 2, 367-372; p. 368).

In the translation of J. McAlhany: "For if upon examination this faculty is found to be diametrically opposed to one or more of the known properties of bodies, so that it cannot exist together with them without contradiction, then we will have to admit that the faculty of thought cannot be attributed or imparted to matter, even by God."

[81] As in, for example, §2 of Chapter 1, "On Natural Science in General": "All changes involving material bodies must arise from the essence and from the properties of the bodies themselves."

The required sense of "based on" in the SGC is ambiguous, and in particular it varies depending on how much content is to be packed into the sought "understanding" of forces. One thing you might want to understand is how forces could arise at all. For this, Euler seeks an account that is based on the essence of body in the sense of being *logically deducible* from that essence alone. A further thing you might want to understand is how observed forces of a specific character – above all, forces that are apparently attractive – do or could arise. This latter type of understanding, by contrast, may require additional data or assumptions, or else may be inferable in some weaker sense than strict logical deducibility. It might be, for instance, inferable as a probable hypothesis, or else as the result of an abductive inference. Each of these senses of "understanding of forces" will become important in turn. The basic contrast to be emphasized now is the one between the SGC, however strongly it is understood, and views according to which specific forces either need have no logical connection to the essence of bodies, or else could be in logical tension with that essence. That is, views that forces could either be *arbitrary* in relation to the essence of body (hence not understandable by human cognizers at all), or worse, even in ostensible (to human cognizers) *contradiction* to the essence of body.[82]

So, to satisfy his own implied criteria, Euler needs to provide an understanding of how forces, including (apparent) attractions, could be based on the essence of body. The question of the "essence of body" is itself not one with a consensus answer that Euler can just take off the shelf, as it were. To provide the needed understanding of forces, he obviously also has to provide an account of the essence of body. This task will engage Euler in additional philosophically interesting moves which illustrate his approach via epistemology

[82] The second type of view, of course, also violates the WGC.

(see section 6). Laying out his account, and how that leads to his account of forces, will be the work of section 4. In the remainder of the current section, I detail the texts from which I glean this account, and then set it in the context of his larger vision of a research program for natural philosophy.

2.2 Texts

The most express and comprehensive statement of Euler's conception of material body is likely his unpublished manuscript *Anleitung zur Naturlehre*, usually translated as 'Introduction to Natural Science.' An "Anleitung" (a word whose use has fallen off dramatically since the 18th century) is something like an introductory guide or tutorial, or else a manual or set of instructions, suggesting an emphasis on introducing a student into a practice, rather than on exposition of known experimental and theoretical results or on providing physical explanations of particular phenomena.[83] "Naturlehre" can be rendered as "natural science."[84] The work is valuable because it fleshes out the philosophical underpinning of Euler's picture of body and force more than his published articles do. The core elements of the picture are adumbrated and occasionally contained in texts that preceded it: *Mechanica (1736)*, *Enodatio*

[83] Euler is also clearly responding to and correcting Descartes, suggesting he was preparing to offer it to readers at least in part with the wish to displace texts like the *Principia* and subsequent Cartesian textbooks. Like the *Principia*, which sought to replace traditional Scholastic texts, Euler was dismissive of the classic, unresolvable questions subject to "disputation amongst teachers." For example, as in §18, the question whether movement should be categorized as a 'property' or as an 'accident'.

[84] It may be remembered that the usual encompassing term in German is 'Naturwissenschaft.' 'Naturwissenschaft' and 'Naturphilosophie' were also available terms, but it is difficult to judge exactly what Euler intended to convey by opting for 'Naturlehre.' In this connection, it is noteworthy that the German translation of Du Châtelet's *Institutions de phsyqieu* was *Der Frau Marquisinn von Chastellet Naturlehre an ihren Sohn* (1743).

The word 'Lehre' can mean *theory* or *doctrine*, but also comes from the verb form 'lehren', *to instruct*, and can thus also mean *teaching*. One possibility for a more concrete interpretation is indicated by Baumgartner (1842). According to the classification in that work, *Naturwissenschaft* is subdivided into *Naturlehre* and *Naturgeschichte* (natural history) (1842, Introduction, p. 4). This suggests that the 'Lehre' is more theoretical and predictive – and relatedly, for that matter, *mathematical*. He then further divides *Naturlehre* into "reine" (pure) and "Erfahrungs-" (experimental), according as they are or are not independent of experience.
This understanding of the specific meaning of 'Naturlehre', if accurate, should serve to further inform our sense of what Euler is up to in this and similar texts.

(1746a), *Gedancken (1746c)*, *Recherches (1752b)*. The *Letters to a German Princess* (written 1759-61, publication completed in 1768) was likely written only a few years after the *Anleitung* was composed, and provides yet more perspective on his philosophical stance, setting his fundamental physics in the context of a compendious overview of his thought. The basic physical picture is presented once again in depth, in the *Theoria motus corporum solidorum* (1765a), which is significant because, after giving his fundamental *(meta)physical* account of material body, Euler presents and synthesizes his latest, seminal contributions to the *mechanical* theory of solid bodies and of rigid-body motion.

Some evolution (less charitably: increasingly conspicuous ambivalence) in his attitudes about particular issues is perceivable, regarding, for example, the principle of sufficient reason, and the relative status of impenetrability and force of inertia as the 'essence' of body. A survey of these texts confirms that the views expressed in the *Anleitung* about matter, force, universal attraction, and monads largely reflect Euler's mature outlook, implying the stability of the main aspects of that outlook over roughly a quarter century. Besides that, the *Anleitung* offers one additional benefit that other texts do not seem to offer, deriving from the fact that it is a pedagogical text. It is not a popular word, like the *Letters*, intended for wide consumption, but serves as a kind of textbook introduction to the study of natural science. It thus provides useful perspective on his goals for mechanical and physical theory, and the principles and procedures one ought to use to carry out research.

3. The project: inferring Euler's blueprint for natural philosophy

Little has been written about Euler's philosophy, and as far as I know, no one has attempted to characterize his overall project for natural philosophy.[85] Accordingly, it should be not only

[85] This is not to say there has not been commentary or assessment. Judgments of his natural philosophy run the gamut from 'failed attempt to provide a Cartesian ground for Newtonian physics' (Gaukroger 1982) to

prudent but, hopefully, also of more general utility to start by characterizing Euler's theoretical aims in prosecuting natural philosophy. One reason is to mitigate the obstacle to clear understanding posed by the fact that 18th century opinion about the definition, scope, and aims of the different disciplines was widely divided. Understanding the specific view on that issue which Euler stakes out is needed to ensure that the questions or disagreements that arise are pitched at the right level. Another important reason is that Euler's "fundamental" physical views about bodies and forces is only a part of an overall research program for natural philosophy, one which cannot be adequately appreciated without a sense of the whole project, which I will now try to provide.[86]

Euler wanted a concept of body sufficient for a mathematical theory of motion and equilibrium which was powerful enough to accurately treat the actual, observed motions of bodies (celestial mechanics, acoustics, hydrodynamics, etc.). In consequence, he needed to spell out its nature and causal powers in a way that would underwrite such a concept. Whereas "most scholars" think the essence of body is "unfathomable" – an allusion to Lockean doctrine – determining their essence and properties should nonetheless be the "first and foremost effort of natural science" (*Anleitung*, §3).

This entails, first, determining the *general concept of body*, the properties of matter in general. These include any property which is either necessary or sufficient for something to count as matter (*Anleitung*, Ch. 1). The general concept should also make it clear how matter as such can be subject to the laws of motion as they are framed in that work, and should be

'visionary first attempt to construct a unified field theory for physics' (Radelet-de-Grave and Speiser 2004). However, it seems to me that the assessments that have been offered are based on partial perspectives of Euler's overall (natural) philosophical project.

[86] One reason that existing scholarship on Euler's natural philosophy has tended to judge him harshly is, I think, due to its tendency to focus on his lowest-level physical views in (logical, if not psychological) isolation from the broader project.

amenable to representation in the mathematics of analysis that Euler was then advancing by leaps and bounds. This influences the nature of the properties Euler will find to define bodies, or at least the formulation of those properties. Some properties, like impenetrability, appear to be absolute and non-quantifiable. So Euler will have to be careful to conceptualize it in such a way that it can be related to quantities. Ultimately, indeed, Euler is more concerned that our discursive understanding of matter should respect its canonical mathematical representation (in geometry and analysis) than he is with questions like the 'unity and identity of bodies through time,' questions which exercised philosophers like Leibniz (*Anleitung*, Ch. 2). Finally, it is characteristic of this stage, as it is undertaken in most of his works, that it proceeds *a priori*.[87] These general properties will make it possible to understand many kinds of actual physical phenomena by themselves. But not all such phenomena (§50). And this reflects one place of 'empiricism' in Euler's scientific epistemology – some of the properties of bodies necessary to account for empirical phenomena cannot be known *a priori*.

So, second, this general concept must be supplemented with analyses of *special* classes of matter (e.g. 'subtle matter') that identify their distinctive, 'special' properties and causal powers (Chapters 12, 13, and 14). The move from the 'general' to the 'special' is advertised as simply that, a move from the genus to the species. Indeed, Euler advertises the special properties as the result of adding "certain stipulations and constraints" to the essence of body (§7). However, and although it is not thematized as such, this step also involves a transition in method, from the *a priori* to the *a posteriori*. Whereas the general concept was arrived at through an unspooling of concepts we already possess, generating the 'special'

[87] It is thus perhaps an example of what Baumgartner categorized as '*reine Naturlehre*' as opposed to '*Erfahrungsnaturlehre*'. See previous note.

concepts appears to require more obviously empirical data. These empirical facts can be quite familiar and obvious: the fact that bodies have pores, that the matter in the pores at one moment can be replaced with different matter, or that bodies can only be divided by the application of more or less force. Conclusions about the 'subtle matter' and its properties are arrived at through rational analysis of these empirical facts, supplemented with probable hypotheses. Although the general concept of matter was supposed to be known with certainty, Euler self-consciously offered subtle matter as a *hypothesis*, not as a necessary or *a priori* truth. Indeed, in an early letter to Du Chatelet, he praised her well-known analysis of the utility of hypotheses in her *Institutions de physique,* and explicitly related his thinking on this very question to her view of hypotheses.[88]

What about the third and final stage? One might expect the final stage to be analysis of particular systems and phenomena, like the solar system, the sun-moon-earth system, the phenomena of the tides, the heating of the earth by the sun, the flow of liquid through a tube, or the free-fall of a body under air resistance. As I read Euler, this is basically accurate. However, before we get there, Euler seems to proceed first through an intermediate stage. I will label it "*stage two-B*" or "*stage two-and-a-half*." in order to emphasize its intermediate character, since it partly shares in the aim and method of the prior and also of the subsequent stages. In brief, stage two-and-a-half is about the constitution of matter. But the

---

[88] Euler first wrote Du Chatelet in February of 1740, thanking her for the copy of her *Institutions de physique* that she had sent with Maupertuis. In a later letter of May 1742, he wrote again, at this point having read the work. He may have had the chance to read it in its second edition, which had just come out. Euler praised the work highly:

> "Mais surtout le Chapitre sur les hypothèses m'a fait le plus grand plaisir, voyant, que Vous combattez, Madame, si fortement et si solidement quelques Philosophes Anglais, qui ont voulu bannir tout à fait les hypothèses de la Physique qui sont pourtant à mon avis le seul moyen de parvenir à une connaissance certaine des causes physiques."

We only have an unfinished version of this letter. However, a finished version apparently did reach her, because she replied to its contents in a letter of 1744. Their correspondence is catalogued and summarized in *LEOO* Series IVA, Vol. 1, items 383-385. The text of Euler's letters can be found in the 1963 volume *Леонард Ейлер. Письма к ученым*, edited by T.N. Klado et al., pp. 277-281.

'constitution' in question is not fundamental, as is the distinction between coarse and subtle matter. Instead, the aim is to use the resources of the fundamental characterization of types of matter – coarse versus subtle – in order to construct explanations of the characteristic properties of various classes of matter. In several cases, they are about what we now call different *states* of matter – as in 'solid' versus 'liquid'. In the *Anleitung*, Euler addresses, in particular, the liquid state of matter (Ch. 15), the hardness and softness of bodies (Ch. 16), and the strength of matter in its solid state (Ch 17). But he also endeavors to explain important (though, for Euler, not *fundamental*) properties of matter, like compressibility and elasticity (Ch. 18). Because we are talking about the constitution of matter, rather than its complex behavior in time, this stage shares something with stage 2. But it is not about the fundamental or elementary categories of matter. Rather, the properties in question are derived from those fundamental categories, together with additional information obtained *a posteriori*, and begins to require mathematical analysis, making this stage akin to stage 3.

After this intermediate, we arrive at the final stage, stage 3, where these conceptual and mathematical tools are to be applied to particular systems and phenomena which we encounter in the world. This application gives us an explanation (*Erklärung*) of those phenomena, i.e. an understanding of the causes of particular events, or else the causal structures of regular phenomena, or an insight into the unobserved or unobservable behaviors of complex systems. It is characteristic of this stage that here emerge *equations of motion* (or of equilibrium). But this stage amounts to an ongoing research program: no single work even approaches a complete realization of this aim. In the *Anleitung,* Euler leverages the analysis of elasticity to derive equations for the force of gravity, understood as a consequence of an elastic aether (Ch. 19), then analyzes equilibrium in fluids (Ch. 20) as well as fluid flow (Ch. 21).[89] It is left to other works to begin tackling such diverse phenomena as light, heat, sound, climate, lunar theory, and so on.[90]

The goals for this paper entail a focus on Euler's first stage, with some attention to the second. Not very much will be said about stages two-and-a-half or three. [91]

4. The general concept of material body

As we have said, Euler thinks that natural science ought to begin with a study of the essence and properties of bodies. For Euler, this is within the grasp of human cognition, *pace* philosophers who take this to be forever obscure to us. However, if it is knowable to us, it remains to be seen *how* we come to acquire knowledge of it. As it happens, Euler is prepared to recognize two "routes to the recognition of the general properties of bodies" (at least in the *Anleitung*, §4). The first route is, in effect, by determining what all bodies have in common through observation and induction. (The classic reference point is Newton's Rules

[89] I do not presume to have given an adequate characterization of the oftentimes complicated, iterative process of bringing physical theory to bear on empirical reality via mathematical analysis. History shows that the process is not linear. The third stage, besides not being as simple as this passage suggests, may well demand that scientists go back to "prior" stages and revise their conclusions there. My claim is only that these three stages (together with the intermediate half-stage between the second and third) characterize *Euler's* picture of how natural science proceeds. But even he was prepared to acknowledge the hypothetical and hence revisable character of the earlier stages (even if he did not in fact, during his career, see fit to revise them).

[90] These stages are categories for historical analysis and not Euler's own labels. However, I take them to be justified. Importantly, they are marked by Euler's use of genuinely different methods with different epistemic bases. They are distinguishable even if in application his use of one method can bleed into his use of another. Charity should lead us to assume that they are adopted with intention, chosen because of the epistemic task they are meant to tackle. Ultimately, though, these categories are justified by what they help us to perceive in Euler's project, and I make no claim for their importance beyond this.

[91] English-language scholarship on Euler' physics has typically stopped at what I call the second stage, or even at the first stage. For example, Gaukroger (1982) and Suisky (2009) largely restrict themselves to the four general properties of body, as in the first 5 chapters of the *Anleitung* or introduction to the *Theoria motus corporum solidorum*. Breidert (2007) gives a more expansive overview, but the only aspects treated in any detail are again these questions about the essence of material body. Wilson (2002) considers Euler's views of material body, though this time in the context of his subtle matter explanation of gravity and his fluid mechanics.

Indeed, for detailed attention to Euler's whole physics, readers have long had the introductory commentaries in the *Opera omnia* as their only resource. Fortunately, some recent philosophical work has begun to delve deeper. See Stan (2017), Hepburn (2013). Nonetheless, no one has attempted to sketch the panoramic view. I believe this is necessary in order to understand Euler's natural philosophy. Unavoidably, this will involve going over some of the same ground as others have. However, the context provided by a view of Euler's overall project sheds new light on even this familiar territory.

of Reasoning, especially rule number three.) It is significant that Euler is prepared to recognize, at least in principle, an *a posteriori* path to knowledge of the fundamental properties of bodies. His reluctance to take that path derives from the great uncertainty of such an inference, given the paucity of bodies we are able to examine.

The second route to this knowledge which he canvasses here relies on our *concept* of body. "Since we presuppose a concept of bodies…" both in our everyday dealings and in mechanics, we can draw out what is contained in this concept (§4). The premise is this: "For a property which is such that when it is absent in a thing, we would not regard the thing as a body, can justly be regarded as a general property of bodies" (§4). The properties of bodies are to be revealed by testing what is, for us, criterial for body-hood. Any property that follows logically from the possession of some other general property will, in Euler's approach, itself count as a general property of bodies. That, at any rate, is the method. The goal was originally to uncover all the general properties of body. Curiously, though, Euler adds on top of this an even stronger, metaphysical posit. Namely, he posits that among the properties of body there is a unique property, called the *essence* of body, that is *sufficient* for body-hood. All general properties must therefore be "founded in" the essence, that is, follow as logical consequences of it. We will see below to what extent these conditions obtain for his analysis. At the moment, I note only that nothing in Euler's *method* of conceptual analysis requires that there be an 'essence' in this sense.[92]

[92] Why does Euler make this additional posit? It does not follow from the fact that properties may be logically related to each other that there must be one from which all others can be derived. It does not seem to be required or even motivated by his other theoretical goals – an account of the general and specific properties of body that accounts for the applicability of laws of mechanics; determining the origin of forces; explaining the invalidity of action-at-a-distance hypotheses; eliminating non-rigorous senses of 'force' from science. It also seems that dropping the assumption that there is an essence (in Euler's sense) would leave the rest of his account in place, allowing him to say everything else that he wants to say. The obvious answer is: Euler is emulating Descartes, the most visible early modern poster child for essentialism about body; Euler agrees that there is an essence, and merely disagrees about what it is.

For Euler, body as such is extended, mobile, impenetrable, and possesses inertia, or the property of persisting in its state of motion or rest absent external causes of change.[93] Among these, impenetrability is singled out as the *essence* of body, and so extendedness, mobility, and inertia must follow logically from impenetrability. These are sufficient to fully

---

Descartes' influence doubtless *does* loom here. There are important differences, in addition to the similarities, which I cannot go into in depth here. Descartes does have a concept of 'primary attribute' which constitutes the nature of a thing:

> "To constitute the nature of a thing, then, is to stand to that nature as form to substance, to be the ground of the properties that follow from having that nature. Extension by itself is not the nature of corporeal substance. That nature also includes being divisible, mobile, capable of figure. But all those properties follow, or so Descartes believes, from extension." (Des Chene 1996, 365)

The meaning of this is evidently similar to Euler's role for the 'essence'. Further:

> "…Descartes does not merely mean to claim that bodies are essentially extended, but that the entire nature of body follows from its being extended. Body has every characteristic that is implicit in the idea of an extended thing, and just as importantly, it has no other real characteristics." (Zepeda 2009, 54).

In a sense, then, Euler is following in a pretty close lineage from Descartes, who is himself following Aristotle. Specifically, they use the same metaphysical schema according to which body is characterized essentially by one property which implies all the properties contained in the nature of body.

Euler differs from Descartes on important claims. For one, Euler would not countenance a derivation of the other properties from extension alone. Euler knew that "although Descartes considered the essence of bodies to be in their mere extent, he nevertheless considered impenetrability to be connected with extent" (*Anleitung*, §35). But he faulted Descartes for thinking that the connection was strong enough that impenetrability could be derived from extension. This is Descartes' view:

> "When we conceive of a concrete extended thing, which exists merely as an extended thing, we can, according to Descartes, introspectively recognize that it will necessarily have certain properties, notably divisibility, mobility and impenetrability." (ibid., 54)

But moving from "extension" to "extended thing" involves adding the content of 'thing-ness,' or existence as a unified object, which is not contained in the concept of extension alone (a problem that bothered Leibniz). Further, even if we are prepared to acknowledge divisibility or mobility as plausible implications of 'existing-as-extended', impenetrability is not. As Euler says, we have the concepts of ghosts, of shadows, and even of empty space – things that are extended but penetrable. Moreover, Euler does not think that body has no real characteristics other than those "implicit in" the essence of body. What Euler does say is that the other characteristics of bodies arise through "constraining" the essence. Admittedly, the difference between them on this point may seem subtle. But the further determinations that arise from these "constraints" are not deducible from the essence, as they arguably are for Descartes.

The contrast between Descartes' and Euler's conclusions on extension and impenetrability point up the difference between Descartes' method of introspective intuition and Euler's method of attending to the content of our concepts. This is a topic in its own right, but I say a little more on it in section 6.

[93] Compare Newton, for whom the universal qualities of bodies are "extension, hardness, mobility, impenetrability, and inertia" (quoted in [McMullin 1978, 8]). Newton takes this to define material body. But he "never says that all other properties can be derived from them; he says that with them and a variety of forces and active principles, only some of which are as yet known, all natural phenomena can be explained." (ibid., 26).

characterize the "general concept" of body. Euler leaves it open whether there are several species within the genus 'matter,' each defined by further properties characteristic to it.[94]

As we proceed through the contours of his exposition, this striking feature of Euler's development of this account of body, namely its reliance on what looks like conceptual analysis, will come into greater relief – in addition to the places where this method is insufficient, by itself, to generate the required properties.

4.1 Extension

Euler's reliance on "our concepts" as an argumentative resource is already manifest in his analysis of extension, the property the meaning of which would seem to provide the least material for controversy (*Anleitung,* Ch. 2). The first candidate concept of body Euler entertains is the Cartesian one, according to which material body is identical with a flexible, changeable portion of extended space.[95] Indeed, it is clear that Euler, in all his natural philosophical commentary, is responding to Descartes, whose ideas were still influential on the Continent in Euler's early career.

Against this proposal, Euler observes that, though empty space may not in fact be possible, or realized anywhere in the universe, its *concept* at least is possible. Since we can conceive of space void of matter, it follows that matter is not identical with extended space. Hence, Descartes' definition of body is not adequate to our actual concept of body and of

---

[94] For example, the subtle matter or aether may be distinguished from coarse matter by having a different fundamental density (namely, a much lower one); or again, perhaps subtle matter possesses a fundamental elastic force, manifest upon compression, which coarse matter lacks. Like Newton, Euler does not think all other properties of matter can be derived directly from them. Instead of supplementing the account with "active forces," however, Euler thinks these further properties arise through "constraining" the essence of body in characteristic ways. What this "constraining" amounts to is an interesting question I cannot take up here.

[95] From Meditation II: "Let us attentively consider this, and, abstracting from all that does not belong to the wax, let us see what remains. Certainly nothing remains excepting a certain extended thing which is flexible and movable." (AT VII: 31). (Latin: "Attendamus, &, remotis iis quae ad ceram non pertinent, videamus quid supersit: nempe nihil aliud quàm extensum quid, flexibile, mutabile.")

space. Which concept, or whose concept, is this, and where did it come from? Given the early stage in the exposition at which this analysis occurs, it cannot be the concept of experts, or a technical concept. The appeal Euler seems to be making throughout is to our ordinary concepts and conceptual capacities.[96] Brading and Stan make the further claim that, for Euler, the concept of body is one "at which we arrive from experience" (2024, "Euler on the extension of bodies"). This is plausible, though I do not find that Euler has all that much to say about where this concept comes from – whether from experience or innately, or through some combination of experience and innate, intellectual capacities. Grounding the concept on experience also creates a new puzzle. Euler reasons on the basis of "our concept" of body. If that concept is based in experience, why should Euler not simply reason *directly* on the basis of what is known of bodies through sense experience and induction? The "concept of body" becomes a needless middle term. Indeed, Euler arguably *needs* the concept of body *not* to derive entirely from experience, on pain of running into the same trouble that Newton did: how could we reconcile the imperative to follow the Rules of Reasoning (including "induction") to determine the properties of all bodies with the claim that all bodies are fundamentally hard or impenetrable? These facts seem to beyond the reach of our sensory faculties, if not apparently refuted by them.

The ultimate goal is a deeper understanding of Euler's epistemology. This might allow us to see how concepts arrived at via experience (alone) could be the ground of demonstrative truths – if demonstrative they are meant to be. This work is yet to be undertaken. In the meantime, this is what Euler thinks we have: an ordinary concept of body

[96] Both Euler and Du Chatelet seek to answer epistemological and metaphysical questions by way of an analysis of our cognitive capacities and processes. But Euler, who we know to have read the *Institutions*, sounds here very much as though he rejected her theory of the way we come to acquire a representation of space. The similarities and differences between their respective turns towards human cognitive capacities is an interesting question, not least because of the question of their relation to Kant. It is one I cannot take up further here.

which is a sufficiently adequate representation of reality to support robust, if non-specific, conclusions about the nature of body and the possibility of void.

Euler will later argue, in chapter 12, that there is, as a matter fact, no vacuum in nature. All of space is filled with either coarse or subtle matter. So this makes it noteworthy that he argues here that empty space is conceptually possible. Purportedly, his arguments later in the text against void are to be empirical. But in those same passages, he also alludes to "all the arguments that are leveled against empty space" (Ch. 12, §95). It is not clear to which specific arguments he is referring, or whether he 'ought to' find those arguments convincing. Given that he is content to allude to them, he likely has in mind the most famous such arguments, many of which were aprioristic, relying on assumptions about nature's "abhorrence of a vacuum" or on some version of the principle of sufficient reason.[97] So, on the one hand, Euler thinks the "concept" of empty space is possible, whereas on the other hand, he appears ready to regard arguments for a plenum, some of which are quite *aprioristic*, as having some force. This makes discerning his actual view something of a puzzle. However, all of these constraints can be accommodated. Euler could hold that vacuum is possible in the sense of being conceivable, while being an empirical impossibility on principled grounds – for example, because it would imply that some empirical events would have to happen without a sufficient reason.

The property of extension, being commonly accepted, was not developed in his earlier articles like inertia and impenetrability were. What remained controversial about

[97] Arguments relying on the assumption of nature's "abhorrence" of a vacuum continued to feature in explanations of experimental observations. See Manzo (2003) on the debate over void in the seventeenth century.

extension were the *implications* of the property, not whether it was indeed a fundamental property of body.

In particular, natural philosophers had long fretted over the implications of cutting up bodies into bits. Some have thought the following. As we divide, or imagine dividing, bodies into smaller and smaller parts, this process of division must end in *atoms* – particles which are themselves not further divisible, yet which are extended. But, goes the reply, how could something *extended* also *fail* to be divisible? Surely even if we mortals cannot split it up, it could at least notionally be divided, or divided by God, if nothing else. But, goes a consequent worry, if that is the case, does that not mean that matter would be "divided to dust," i.e. separated into extensionless parts? And if *that* were true, we have a new problem. Matter is supposed to be composed of a "dust" of extensionless parts, but how could putting together two or more extensionless parts ever add up to space-filling *body*?[98]

Euler had thoughts about all of these questions, which he began to share in his *Gedancken*, developed in the *Reflexions*, and also discussed in Chapter 2 of the *Anleitung*. See also No. 123 of *Lettres*, Volume 2. The crux of his thinking is that the spatial extendedness of body is accurately represented as a solid (Euclidean) geometrical figure, potentially an extremely complex one, and that all facts about its divisibility follow from a rigorous application of this fact. Indeed, it is possible to read into his comments a peculiarly strong

---

[98] This was a particular concern of Leibniz, who thought that the idea of a continuum composed of points was absurd. For example, in the *New System of Nature* published in 1695, he wrote: "Now, a multitude can derive its reality only from *true unities*, which have some other origin and are considerably different from [[mathematical]] points [[which are only the extremities and modifications of extension,]] which all agree cannot make up the *continuum*." (Leibniz 1989, 139). The problem of how extended being could be composed of dimensionless points or monads is sometimes call the "composition question" or "composition problem," and became a concern of not only Euler, but also Du Chatelet, Boscovich, Baumgarten, and Kant. See also Brading and Stan (2024, §5.1)
Note that Euler himself would not avow that bodies or continua are, in the last analysis, composed of points in a sense that would allow us to speak of them as "the" (ultimate) parts of bodies. However, he does allow for a different use of 'points' – the mathematical representation of macroscopic bodies as points.

theory of mathematical representation. We represent bodies (the earth, for example) as geometrical objects (e.g., an oblate or elongate spheroid). For Euler, that means we have assimilated bodies into the *genus* of 'extended things,' characterized by the key property of spatial extension. The science of spatial extension is geometry. Anything that is learned about extension in geometry automatically applies to anything that falls under the genus 'extended thing,' simply as a matter of the "rules of logic" (§10).[99] Thus, body is infinitely divisible. After any finite division of a body – into halves, thirds, or millionths – the parts that result must have some finite size. In that case, they can be divided again. That is all that 'infinite divisibility' amounts to.

In this respect, he sides with Descartes against monadologists and atomists. Respectively, they thought that all things consisted, in the last analysis, of indivisible, small monads, or else of physical atoms or corpuscles. Much philosophical hand-wringing turned around the issue of the nature of the "last parts" or "simple parts" of bodies – what we arrive at once we can divide no further. Regarding extended atoms, Euler dismisses them as a logical contradiction: an extended thing that is not divisible. If these indivisible parts are supposed to lack extension, then "since all parts have no extent, it should be possible to bring them together in a point, which contradicts experience and logic" (§13). But Euler likewise rejects the alleged implication that the "simple constituent parts" of matter must, in that case, be *points*, or else *nonextended*.

[99] Alternatively, we could say that he does not view the relationship between body and geometrical object as one of *representation*. Bodies simply *are* members of the genus 'extended thing,' and "the properties of an extended thing quite generally must be regarded as also the property of material bodies." (§10). Euler does not appear to think that anything metaphysically important is involved in the transition from space to our idea of space, as Leibniz and Du Chatelet had.

Euler's most fundamental move is to go after what he sees as the key presupposition of the monadologists, who take themselves to be avoiding the vulgar contradiction of the atomists. This is the notion of 'last parts.' Euler's claim is that the very idea of the 'last parts' or 'simple parts' rests on a mistake. Those who reject atoms imagine that there is, all the same, a final division, which would necessarily be into infinitely many, "infinitely small" parts. But to say that a thing is infinitely divisible is precisely to *deny* that there are "last parts" to it. All that can arise from such an idea is philosophical confusion. In consequence, he emphatically rejects the proposition *"where there are composite things, there must also be simple ones*."[100]

Going further, Euler declares that the expression "the parts of a body" is *ill-defined* unless one specifies the nature and number of the divisions that were undertaken or imagined to be undertaken (§14). This point – *prima facie* a claim about the metaphysics of the part-whole relation as applied to bodies – is at the same time pragmatic and methodological. In fact, it converts the notion of 'parts' to a pragmatic rather than a real category. "The" parts of a body will be determined, to the extent they can be, by the configuration of the problem or question in which it figures when we consider it.

---

[100] In light of early analytic philosophy, this reads as a piece of philosophical "therapy" *avant la lettre*. Euler's cautions sound remarkably like Wittgenstein's late-career disavowal of his early logical atomism. That view, eventually repudiated, required that language, in the final analysis, be composed of names that referred to the (absolutely) simple constituents of reality, or in the *Tractatus*'s terms, 'objects'. In reply to this demand that there be absolute simples, and correlatively, an absolute sense of being 'composite', Wittgenstein in effect replied that the question whether something is simple or composed of parts will vary according to the context and intent behind that question. "The question 'Is what you see composite?' makes good sense if it is already established what kind of complexity – that is, which particular use of the word – is in question." (*Philosophical Investigations*, §47). The implication is analogous for the question "And what are its simple component parts?"

It should, nonetheless, be noted that Euler's focus on physical ontology is distinct from Wittgenstein's. The latter, given his immersion in logical empiricism and the 'linguistic turn', is talking about the parts of 'visual images' in a discussion mediated through the lens of language.

Summarizing: bodies are extended, from which it follows that they can be infinitely divided and that they do not consist of simple parts which are themselves partless.

4.2 Mobility and motion

How do we get these bodies moving? A Cartesian thought is that the mobility of an extended thing is knowable simply through introspection (Zepeda 2009, 54). If by this is meant some simple mental act of recognition, then although Euler's argument is also reflective or introspective, Euler's approach is distinguished in at least this respect: that there is a further rational structure to the introspection, which proceeds at each stage by attention to the contents of our concepts.

First, we have to acknowledge that space and body are distinct, as we did in the previous chapter on the extendedness of bodies. Then, we on reflection come to realize that there is nothing in our concept of a body that makes reference to a particular location in space. It is therefore conceivable that a body which occupies one location at one time, could occupy another location at another, later time.[101] To occupy different locations at successive times is precisely what we call *motion*. Of course, nothing we have identified so far determines that body should move in one way or another, or move at all. But we are licensed to conclude that body is moveable, or mobile.

So far so good, perhaps, but establishing the bare fact that bodies are mobile will not get us very far in natural philosophy. The remainder of the chapter manifests another of Euler's concerns: to show that a sufficiently powerful mathematical account of shape and motion is applicable to real, material bodies.

---

[101] Euler has not, by this point in the exposition, separately developed an account of space or time, or said anything about their structure.

The first step in showing this is to elaborate what is to count as the *complete description* of the motion of a body. Bodies have all kinds of complex motions, including rotations and commotion among their internal parts. How to describe them mathematically was not (is not) a trivial problem. Newton, for instance, was very far from having the tools for such a description, even for more or less 'ordinary' solid bodies. Euler's own *Mechanica* of 1736 began to develop these tools, but only got as far as point masses. Around the time of composing the *Anleitung*, Euler is more or less in the thick of developing the mathematics to deal with solid bodies.

Here, with more sophisticated tools not quite in hand, he makes do by restricting himself to what is needed for the "general investigation of motion": he will be concerned only with the relation of the body's outer circumference to ambient space (§17). The results of this investigation can then be applied to the body's parts, such as we take them to be in some context.[102] To do this successfully, we must first be assured of having access to the complete description of the motion of the circumference of a body. This entails finding the trajectory of each point: the line described by each point together with the time at which that point of the body traverses each point of the path. (Evidently, this involves assumptions about the continuity of the motion of bodies and its representation by a "line," some of which assumptions are unpacked below.) This threatens to be an unwieldy amount of information. So, Euler will begin, in this work, with a further simplification: bodies that are point-sized (harking back to the *Mechanica*). The first justification for this simplification is that, in certain, well-known cases, extended bodies can be treated as though point-sized.[103]

[102] This, too, was an ongoing research program, one to which Euler was in the middle of making major contributions. On his contributions to the theory of rigid body motion, see Stan (2017).
[103] It seems that Euler must be thinking of point-sized bodies as no more than an approximation, since otherwise there would be bodies without extension.

At least in this analysis here in the *Anleitung*, though, he is, to an extent, being cagey. The second justification is in effect a promissory note that this will segue into further tools for rigid-body motion –discoveries that were, at best, on the horizon. (They first appeared in several texts published in 1765, the earliest of which was composed in 1758.)

Summarizing: Euler has shown that material bodies are mobile. Modulo two assumptions – a constitutive assumption about the continuity of motion and a practical assumption (or 'promissory note') about the tractability of mathematical representations of complex motions – he has also set the stage for the application of analysis to corporeal motions.

Questions and puzzles arise next, for the view, including some traditional philosophical puzzles about motion. The argument of the text here can appear rather out of place in the context of the *Anleitung* as a whole, because it seems to depart from its overall natural-philosophical bent, relating instead to traditional metaphysics and its categories and puzzles. However, in the context in which Euler's was writing, metaphysical arguments were offered, and heard, as direct challenges to the validity of conceptions of motion deployed in natural science. Euler's purpose can be understood as two-fold: demonstrating that his account can satisfy the demands of metaphysicians on their own terms, and also that it is not subject to metaphysical flaws that render its constitutive concepts inapplicable to the real world.

First is an old dispute about the categorization of motion. We attribute motion to bodies. Does that make motion a *property* of a body, or else an *accidental* characteristic? If nothing else, the question shows how Euler was concerned to have convincing responses to questions coming from metaphysicians. This one is resolved by observing that motion and changes in motion are not features of or changes in the body itself, but concern the relation of the body, specifically the body's outer circumference, to space.[104] A body's state of motion is not a fact about that body alone; *a fortiori*, it is neither simply a property nor an accident of it. It is something like a "relational accident." Whether this will satisfy a metaphysician who wants the world to fit into the categories he has prepared for it will depend on the answers to further questions, like whether space, or a location in space, can be a relatum in a relation to a body.[105]

The second puzzle is addressed at more length. Euler raises what appears to be some version of Zeno's paradoxes. This gesture, too, might seem to be offered only for the satisfaction of the metaphysicians. He may have that audience in mind, but the analysis also serves his purposes, as described above: to ensure that the motion of bodies is fit to be treated mathematically (in this case, with the calculus).[106] Puzzles about bodies in motion, in one way of taking them, amount to metaphysical worries about the "possibility" of motion. Taken another way, they raise doubts about the ability of motion to be treated mathematically, by means of such concepts as "point," "line," "interval," "distance," "infinitely small," and so on. The doubt needs to be addressed for Euler's project to succeed, since bodies and their motions must be amenable to mathematical treatment. The account is thus not directed merely towards philosophical worries about motion, worries that hardheaded mathematicians might dismiss as idle – although resisting the incursions of metaphysics into mechanics was surely one of Euler's perennial aims.

---

[104] As noted, Euler has not yet provided an account of space or specified its properties.

[105] One important controversy this categorization does seem to bear on is the *vis viva* dispute. If motion is relational and accidental, then *vis viva* will not provide a frame-independent measure of 'the force of bodies.'

[106] As noted above in n. 26, these puzzles exercised Leibniz to the end of his life. Though I cannot pull on this thread further here, it is likely that these puzzles continued to be a subject of discussion between mathematicians and metaphysicians in the years after his death, and that Euler was exposed to these controversies early in life.

Now for the puzzle. Suppose a body moves from a point A to a point B, a finite distance away, on a path that passes through C. Then there is a moment at which the body is at C. But the body cannot *be at* the point C unless it is there for some duration, however short. Further, goes the thought, this must apply to the path between A and C, as well as between C and B, yielding further points, say D and E. What we have said about point C applies equally to D and E. We quickly find that the intermediate locations multiply without end, and since the body has to be at each of them for some duration, "it is clear that the body would never leave its original location." (§19) It will not do, so goes this argument, to say that the duration spent at each intermediate point is "infinitely short," because that is just to say it is no duration at all, amounting to a denial of the premise that the body resides for some time at the point C (as well as D, E, etc.). The conclusion is that continuous motion is impossible. In consequence, bodies would have to "leap" or "teleport", or else not move at all. The framing of the problem is in essence Zeno's Dichotomy paradox. But it is notable that the conclusion is better understood as saying that continuous motion is impossible, rather than that motion as such is impossible.

Indeed, in responding to the puzzle, Euler does not claim, or need to claim, that all motion is continuous. And in fact he does not claim this. The motion he is interested in for the purposes of mechanics is continuous, as is clear from the wording of §19, and so he wants to clear its name. But he does not assume that nature is subject to a *lex continui*, as Leibniz thought.[107] Indeed, he expressly asserts that the proverbial 'change through a leap' is

[107] This departure from Leibniz ramified elsewhere in his theory. For example, in the *Specimen Dynamicum* of 1695, Leibniz rejected absolute hardness on the grounds that, in a collision between two absolutely hard bodies, an instantaneous, finite change in velocity would occur, violating the law of continuity. Euler also wanted to reject absolute hardness. But since he did not endorse a law of continuity, the reason had to come from elsewhere. According to Euler, absolutely hard bodies cannot exist because their collisions would require a finite change in velocity take place in zero time; this would demand infinitely large forces. Note that this is now a physical impossibility, whereas the law of continuity was plausibly supposed to be a metaphysical law.

not only logically possible, but may actually occur in some real, physical motions: "It is possible to imagine such a hopping motion, interrupted by small periods of rest, and this may indeed occur in some cases" (§19). What he needs to argue is only that continuous motion is possible.

Euler's way out of the paradox is partly conceptual or discursive, and partly mathematical. All the same, it is a negative or burden-shifting argument, rather than a positive, constructive argument. (This should not be surprising, because an adequate construction of continuous motion only emerged out of 19th century analysis.) What he rejects is that continuous motion is accurately depicted by the assumptions which generated the paradox, a picture he glosses as a "succession of short residences at intermediate locations." First, Euler claims that the notion that continuously moving bodies are *at* each point for some duration is a misconception. When a body moves continuously, it does not "spend time" at the intermediate points, not even an "infinitesimally short time" (in his view,

---

The exact connections of the law of continuity to the principle of sufficient reason are somewhat complicated and vary from figure to figure. Van Strien (2014) notes that there were two rather different ways of "deriving" the law of continuity from the PSR. One argument is "global": only a world respecting the law of continuity could be the best possible, giving God a sufficient reason to select that world as actual. According to a second, "local" argument, any particular change through a leap would be a change that occurred without a sufficient reason; by the PSR, this is impossible, so all changes are gradual. According to Van Strien, and also Rutherford (1995, 29), Leibniz actually endorsed the first kind of argument. It was only others, including Johann Bernoulli and d'Alembert, who endorsed the second kind. Du Chatelet is arguably another figure who endorsed the second argument:

> "For if there was a possible state between the present state and the one immediately preceding it, nature would have left the first state without already being determined by the second state to abandon the first; there would thus be no sufficient reason why it would pass to that state rather than any other possible state" (1740, 31; my translation).

The *Encyclopédie* entry by Formey and D'Alembert in effect quotes this passage without attribution. Compare Johann Bernoulli, quoted by Van Strien (2014, §5):

> "If nature could pass from one extreme to another, for example from rest to motion, from motion to rest, or from a motion in one direction to a motion in the opposite direction, without passing through all the imperceptible motions that lead from the one to the other, it should be the case that the first state is destroyed, without nature knowing what new state it must conform itself to…"

Euler, as we have seen, did not endorse the law of continuity, and eventually came to discount the principle of sufficient reason. The connections between these positions and their relation to the rest of his thought merits further investigation, which I cannot undertake here. For now, we can observe that Euler must not have found these arguments convincing. And his reasons for rejecting these arguments are hinted at in the passages discussed in the present section.

a nonsense idea), but only "passes through" them. So he denies that bodies must spend a duration of time at any point. Second, he denies that sense can be given to the idea of "counting up" the intermediate locations, or assigning a number to them: "it would be inappropriate to attempt counting all possible intermediate locations."

The concepts of continuous motion and continuity, Euler concludes, are *sui generis*. It is a mistake to assume continuous motion reduces to, or that it must, or even can, be conceptualized as equivalent to a succession of very small, very quick hops from location to location. And it *does not need* to be so reduced in order to be a legitimate and mathematically well-defined concept. When a body moves continuously, there are always intermediate points, points which the body *passes through*, but without resting at them.

> "On this rests the concept of continuity, which applies to extent as well as to movement. It allows one to think of parts, but these are not regarded as separated from each other. In continuity everything hangs together and there is no actual subdivision that would allow parts to be counted." (§19)

This is not a trivial insight. Nonetheless, it is not a positive argument for the mathematical legitimacy of such a concept of continuous motion. That is, it is not actually enough to show *how* to make good mathematical sense of a motion where a body spends literally zero time at each point in its trajectory, and yet which as a whole takes positive (i.e., non-zero) time. From the perspective of contemporary mathematics, then, his argument is unsuccessful in that he left much still to be demonstrated. In particular, he needed a clearer understanding of the distinction between continuity and differentiability, not to mention transfinite cardinality and measure. Though Euler is reaching after the notion of a continuum, he is not there. Rather than go into the details here, I explore these issues a bit further in Appendix C, in relation to the much later analysis that was able to make all of this sensible.

All the same, Euler has pointed quite accurately at what it is that *needs* to be made sense of, in order for continuous motion to make sense ("on this rests the concept of continuity"). This is the criterion of "passing through." One way of understanding the lesson here is this. Euler came to a correct conclusion, even if his argument for it was incomplete, and even if he only did so "by chance." The conclusion is this: that there is a mathematically respectable concept of continuous motion that is genuinely distinct from discrete motion, does not require that a body spend some positive duration at any point, and hence is not prey to paradoxes of infinity that would show such motion to be impossible.

Indeed, I believe we should view the takeaway in an even more positive light. Namely, not that Euler "got lucky", but that he is illustrating a methodological approach with wider importance. If a natural philosopher or mathematician finds she would like a concept of continuous motion, she can avail herself of the idea of a "passing through" motion – a motion which passes through any given point on its trajectory, without spending positive time at any point. The imagination may balk at the task of *picturing* what it would mean to satisfy this criterion (after all, it may balk at the task of picturing a genuine point). But the concept (of continuous motion) that Euler wants is constructed by its stipulation.[108] This construction is *permissible* because the notion, however strange it may seem, is not prey to self-contradiction. And it is *advantageous* because it allows us to either a) resist metaphysical paradoxes about the impossibility of motion, or b) make use of representations of motion which are useful in the science of mechanics. It is therefore prima facie warranted. This is so

[108] The task of "rigorously" constructing this criterion of "passing through" out of concepts that are (at least supposed to be) still *more* basic required another century or so of highly non-trivial mathematical labor.

even absent a modern construction of the real line.[109] Evidently, this methodological approach is also one that cuts against the grain of traditional metaphysics.

4.3 Inertia, or persistence

Now that our bodies are mobile, the next matter to take up is their inertia [*Trägheit*] or persistence [*Standhaftigkeit*], their respective tendencies to remain in their states of motion (*Anleitung*, Ch. 4). The phenomenon of relevance here is thus changes in bodies, or the absence of such changes. The cause of or reason for these changes will be sought. It will no longer be sufficient to use a 'pure' analysis of concepts in order to show that bodies possess inertia. Specifically, the principle of sufficient reason (PSR) will be brought in to secure this conclusion.[110]

Before we can apply the PSR, we need to identify the change for which a reason is sought. The change that we will ultimately be interested in is change in state of uniform motion, i.e. acceleration. But it is worth asking whether the importance of this concept can be derived merely through the unfolding of the concept of body. It is reasonable to think that what is of general, physical importance will be changes to a body's state. But what is to count as the body's "state"? At this stage in the exposition, we know that bodies at least have figure, volume, position, and are mobile. A priori, any one or more of these should be able to contribute to the full specification of a body's "state". Figure and volume might be

[109] Indeed, "rigorous" constructions of the real line of the Dedekind-cut kind did not put to rest all philosophical objections. Holdouts remained long after the constructions were widely accepted. Euler's approach is

[110] Derivations of the law of inertia from the PSR were not uncommon in the 18th century. As Gaukroger has succinctly observed, it is easy to appreciate that inertia – understood as preservation of state of rectilinear motion – *could not* be obtained solely from our concept of body, analogously to the other properties. "We may not be able to conceive of a body being penetrable or unextended, but we can surely conceive of it not obeying Newton's law of inertia; indeed, people had been doing so for thousands of years prior to the seventeenth century" (1982, 142-3). This problem will be scrutinized in section 5.1, where I will attempt to sketch an Eulerian response.

understood in terms of the relative situation of parts of the body and may be subsumed under position. If that works out, then it would naively seem reasonable to think that the body's state is at least partly specified by its location. Hence, it would be natural to suppose that *change in location* is what calls for explanation via a sufficient reason. But this is not the type of change to which the PSR will be applied. It will be used to demand reasons for changes in velocity. This will raise a problem for the claim to derive inertia from first principle and concepts, but before describing it, let us see how Euler proceeds.

Euler first divides the law of inertia into three parts. Assuming no external cause acts on the body: (1) a body at rest will remain at rest; (2) a body in motion will maintain its direction; (3) a body in motion will maintain its speed. These are the claims to be argued for.

Start with (1). Euler notes that, for a body at rest to begin to move, it must move in some direction. However, absent an external cause, there could not be a sufficient reason to choose one direction rather than another.[111] Ergo, the body will remain at rest. [112]

---

[111] Here lies a key difference between Euler's way of thinking and monadology that presses on a division internal to the monadological camp. To see this, let us bracket the differences between the monad concept and the concept of material body in general. (Euler has an argument for this, which I discuss in Chapter 2.) And observe, for the sake of completeness, that there is an option which would block the conclusion that the entity would remain at rest, yet which does not involve sufficient reasons for motion in any direction: randomness. The monad or body could be constituted so as to initiate motions (or changes in motion) at random. Of course, this idea itself would be incompatible with then-dominant views of the world as mechanism and also of the world as created by an intelligent God. It would be widely accepted that a reason for *this* kind of change is called for. Supposing, then, that we required a reason, where could such a reason come from? What we need is some information about which direction is to be chosen. It is hard to understand how we could locate such information within the body or monad itself without thereby ascribing some kind of faculty of representation to it. Hence, if it is truly the body or monad in isolation that is set in motion, it is hard to see how to avoid viewing the reason as a *vis representativa* of some kind. This is not a problem for Leibniz. However, the Wolffian does not wish to inscribe her fundamental entities merely with forces of representation, as discussed in Stan (2018).

As such, the Wolffian does seem to face a problem in the form of a dichotomy that does not make room for non-representational active forces. The upshot of Euler's argument is that one must either adopt a Leibnizian view of the forces of monads, or else be driven to the view that monads/bodies contain no internal active forces at all. Though I do not claim that exactly this argument is made in such texts as, for instance, the *Enodatio* of 1746 (which expressly considered the question whether "mental" powers could be attributed to bodies), I think it is a natural reading of his writings on these topics taken as a whole.

[112] Sprinkled through the *Anleitung* are curious didactic remarks that are apparently meant as correctives to erroneous views that then had some currency (and which might lead a student astray). In that vein, a further curiosity of this chapter is §27, where Euler considers and rejects another proposed explanation of the

Importantly, this argument assumes that space itself cannot supply the missing sufficient reason, either. So, for example, space could not have the properties implied by the Aristotelian system of the world. In the terms of Miller (2014), Aristotle's system uses a "centered representation of space," with the preferred direction being the vertical extending from the center of the earth to its surface. As the argument of Miller's book makes clear, conceptions of "unforced" or "natural" or "the same" motion evolved in tandem with representations of the structure of space. Although he did not make this explicit, Euler must have been assuming that space itself lacks a structure for indicating 'preferred directions.'

Parts (2) and (3) of the proposition are handled analogously after some further preparatory remarks. For (2), Euler reiterates, or stipulates, that the body we are considering is assumed to be point-sized, and "we must begin our investigation with these" (§28). Now, it is not immediately clear how this assumption contributes to deriving the conclusion. Considering the context, the idea is perhaps this: if the body is larger than point-sized, it will possess internal structure, and this internal structure might provide resources for distinguishing among the otherwise-indifferent directions in space. These might provide grounds for preferring some direction, hence a reason for a change in direction. If the body is point-sized, by contrast, there is nothing *internal* to the body to provide such resources, just because the body, being point-sized, has no internal structure. Further, we are, by

---

circumstance that a body at rest will remain at rest: a body has an internal tendency to motion, but this tendency is of equal strength in all directions, hence neutralized by default. Collision is understood thus: when body B collides with body A, B neutralizes the internal tendency of A in the direction of B, allowing the internal tendency in the opposite direction to express itself.

The reason some posit these "internal tendencies" is that, without them, they do not "understand how during impact one body could act on another and set it in motion." Euler's reply is first, that if these tendencies are in their default state neutralized, this amounts to their not existing at all. Second, the explanation does not help, because now the "force required to neutralize the tendency" to motion has to be explained, "so that the explanation of movement imparted through impact would not be made any easier." It is interesting that Euler would launch a critique of this flavor here, since in other circumstances he appears to provide explanations of which much the same could be said.

hypothesis, putting aside external circumstances. Since the external and the internal exhaust our options, there can be no grounds for a change in direction.

Of course, for Euler, there are no point-sized bodies, since he views matter as a continuous, divisible plenum with no ultimate parts. Further, the text does not contain a "part two" of the argument, in which Euler relaxes the assumption that the bodies are point-sized, and thereby proves the general conclusion. It is, therefore, not immediately clear how he thinks the move to the general case is justified. One idea is that the argument in the last paragraph could be naturally extended to bodies that can be represented as a point. However, representability as a point is not a feature of a body *per se*, but only of a body in the context of a particular mechanical problem. Perhaps more convincing is this: finite-sized bodies can, in general, have variable relations among their parts. These relations among the parts may result in asymmetries in the body as a whole that provide reasons for determinate changes in direction or speed. Considering the case of points allows us to set aside these variable, internal relations. But, strictly speaking, the same argument should apply to finite-sized bodies that are homogeneous and lack variable internal structure. It is plausible to think that this is what Euler had in mind, given his distinction between 'internal state' and 'external state'. The internal state "represents the kind and composition of the parts comprising the body" whereas the external state "represents the relations of the body with space" (*Anleitung*, §30). Considering points amounts to bracketing the internal states of bodies.

This leaves us with the conclusion that a body not subject to external cause of change will not change its direction. One might think that the proof, however convincing it is, is at this point complete. Yet Euler is not satisfied to leave it there. To finish the proof, he observes:

> "The nature of the body must contain a certain instruction that determines the continuance of the movement, or it must contain a reason why the movement continues in one way rather than another." (§29)

The body needs a reason to continue the way it does. Euler is apparently not satisfied to say that the body *lacks* a reason to *change*, and hence will continue in the same direction without change. He wants a *positive* reason for what is observed. Where could this come from? A body in motion has a state of motion, comprising a speed and direction. In a manner of speaking, the body needs "information" to determine its speed and direction at any time. Lacking external determinants contributing further information about what it ought to do, the only determinant of the speed and direction in the next moment is its speed and direction now. In that case, the state of motion in the next moment will simply be what it is now. That is just to say that the body will move inertially in the sense of the law of inertia. This argument is very similar to what we have already seen. It differs mainly in attempting a positive gloss. The speed and direction of motion of a body at a time is a "reason" which, given the absence of further determinants of state, is allowed full expression in determining the state of the body in the next moment.

As a final note, it is significant that, at this stage, Euler has not introduced the concept of *(impressed) force*. He has only spoken of the causes of change in the states of bodies as just that: causes of change, and external causes, at that. We will return to this issue in section 4.5. At this point, he has mentioned 'forces' of only one kind: the active, *internal* 'forces' that the monadological tradition ascribes to simples in order to explain the changes they undergo. And he mentions them only to dismiss them, on grounds that inner faculties of self-change are incompatible with inner faculties of conservation of state.

Summarizing: Euler attempts a derivation of the property of inertia, or 'persistence', from the concept of body. Importantly, Euler's analysis is now supplemented with the principle of sufficient reason. This property – inertia or persistence – is incompatible with monadological forces, but it has not yet been brought into connection with the idea of impressed force.

4.4 Impenetrability

The fourth property of body in general is impenetrability [*Undurchdringlichkeit*]. Euler purports to derive this property once again through a 'pure' analysis of concepts – that is, without the PSR. He claims that our general concept of body includes within it the idea of impenetrably occupying space, or excluding other bodies from occupying the same space at the same time. Why? If we imagine something moving towards a body, and then proceeding to enter and pass right through it, we would not count such a thing as itself a body. We would judge that either it or the purported body must, in fact, have been an apparition, a shadow, or something else of the sort.

Euler further claims that the *only* kind of thing that occupies space in such a way that (other) material bodies cannot simultaneously occupy the same space is itself just that: a material body. Possessing the property of impenetrably occupying space is *sufficient* to be a material body. This, again, is based on the presumed fact that we would judge, based on the concept with which we all find ourselves equipped, that anything that we observe to impenetrably occupy space, such that other bodies are unable to penetrate it, is itself a material body of some sort.

Impenetrability thus captures the necessary and sufficient condition of body-hood. It is not so with the other properties. Shadows and mirror images have extension, are mobile, and even (Euler thinks) have persistence, but they are not bodies. Since impenetrability is sufficient for body-hood, this singles out impenetrability from the other properties: it is the *essence* of body.

Per Euler's account of essences, this means that impenetrability must logically imply the other properties (*Anleitung*, §38). The reasoning for this goes as follows. For something to be impenetrable, it must occupy a region of space of non-zero dimension, otherwise there is nothing for its being impenetrable to amount to. Hence it has spatial extent, as well as location in space.[113] This immediately implies the possibility of being located elsewhere in space (because, as before, nothing internal to the concept precludes this), i.e. mobility. And from mobility follows persistence, via the principle of sufficient reason, as we saw in the previous subsection. Scholars have criticized Euler's claim that these properties follow from impenetrability (Wilson 1992). I discuss such criticisms below, in section 5.

As for the history of Euler's arriving at this view, important aspects of it were already settled by the writing of his *Mechanica* (1736). But Euler seems to have arrived at his mature formulation of these ideas in his *Recherches*, read to the Berlin Academy in 1748 and published in 1750, around the time he published his first work towards laying the foundations for rigid body motion and rotation, the "Principe nouveau." He then reiterated this analysis in the *Anleitung*, in the *Lettres* (published in the late 1760s but written 1760-1), and once again in the introductory sections of his major treatise, the *Theoria motus corporum solidorum* of 1765.

[113] Perhaps curiously, Euler thinks that impenetrability only applies to objects with length, breadth, and depth, and that "one cannot say of a line or surface that it is impenetrable" (§38). Wilson seems to find this very dubious (2002). But this makes sense given an understanding of contact action that requires the surfaces of bodies to touch. In a weak sense, that contact might involve "mutual penetration," but does not violate genuine impenetrability, which prevents other bodies from reaching the interior. Do "real" surfaces and lines, like bubbles or fishing lines, constitute counterexamples to Euler's claim? Arguably not. Euler can point out that these are not geometrical lines or surfaces. In fact, no real (part of a) body can have zero dimension, as discussed in section 4.1; there are no "infinitely small" parts of bodies.

4.5 Material body in general and force

The above completes Euler's account of the general concept of *body*. But what is sought is not merely a catalogue of the properties of bodies in general. Namely, because it is only with the concept of force that the theory can become explanatory of actual motions, a key part of the epistemic project of mechanics. This provides us with a particular way of *characterizing* the motions of bodies (for example, when "natural", when "forced") and, further, with the resources for a *causal* account of the behaviors of bodies (to identify the physical sources of their motions). This is the ultimate goal for the account, but in fact, it is only one of two matters on the table. The second issue is still prior to putting the account to use in mechanical problems. In effect, the account of body is not actually finished by describing the four properties above. Why?

When Euler deduces that bodies have "inertia" or "persistence," it is not already coupled to the concept of impressed force. As I will explain further below, this means, in effect, that the concept he deduces is *not yet* the full-blooded concept of inertia as used in mechanics. Introducing forces, it turns out, is prerequisite to completing the account of the concept of body, if that is supposed to be a concept usable in mechanics. There are two tasks or puzzles Euler must address, in turn, in order to move from the account of the general concept of material body to this more fleshed out account of body as having inertial mass and being subject to impressed forces.

1. Deducing the existence of impressed forces: How do we know that impressed forces exist? In what manner do they exist? Where do they come from? How do they come into existence?

2. Showing that bodies follow the laws of mechanics: How does Euler deduce that bodies have inertia, in the full-blooded sense, and generally follow the laws of mechanics, from the *a priori* resources he has allowed himself so far?

First Task

The first task is to provide the ontological origin-story. How can we be sure that impressed forces exist? Where do these forces come from? What generates them? How do they arise or come into existence? No one doubted that "forces," in the most general, vaguest, and non-technical sense, act in the universe. Changes occur, events are caused, and so "forces" must be at work. But in the proprietary sense of *impressed force*, these are theoretical entities with distinctive properties, and one would naturally desire that the argument for their existence should be secure.

These kinds of questions emerged already in reaction to Newton, and he was generally *not* taken to have provided convincing answers. It is true that some of his later interpreters have thought that, for Newton, "force" belonged to the Aristotelian category of quantity and were not entities at all. As one commentator has put it, "it makes no sense to talk of forces as individuated entities or substances" (Smith 2008, §4). The merits of this view are worth considering, but this will have to be done elsewhere. The present point is that, in context, Newton was taken to have introduced a new class of entities. And further, many of his readers felt that he did not provide convincing answers to the above questions. Even some six decades later, at the same time as Euler was writing, d'Alembert, too, thought it important to acknowledge skepticism about the "obscurity" of the concept of force and to adopt a kind of nominalist, anti-realist posture.[114]

In Euler's context, the stakes for answering this question were high. As we have noted, forces will (obviously) be important explanatory resources for a causal account of why bodies move and change their motions in the ways they do. But they were not the only live options. Impressed forces, as a theoretical tool and as Euler theorized them, would have to compete against different resources favored by other philosophers. These other resources would include, prominently, fundamental attractions, as favored by philosophers taking themselves to follow Newton. Another influential camp were the monadologists, who proposed still other resources. They offered a variety of related schemata for the powers of change internal to bodies or else to fundamental beings, the notion of *vis viva* being only the most easily recognizable.[115] Euler's argument for impressed forces is logically related to his refusal to accept the use of these other resources.

It might seem strange to count forces of attraction as "other," since Newton both theorized impressed force and regarded attractions as sources of impressed force. However, given the way Euler theorizes impressed force, fundamental attractions represent a genuine departure from that explanatory project. Asked to explain why attractive forces arise or where they come from, the attractionist had two options: insist that attraction is essential to body or trace them to the arbitrary Will of God. For many, including Euler, the first answer was incompatible with predominant views of the nature of body, and the second amounted to denying the need to provide an explanation at all. Attractions could, therefore, only

---

114 In the *Traité de dynamique* (1743), d'Alembert writes: "since we don't wish to represent by this word [force] a supposed being that resides in the Body, we only use it as an abbreviated manner of expressing a fact." (Preface, xviii)

115 D'Alembert's attempt to sketch a nominalist position is yet another approach, but I will not discuss it further.

superficially be regarded as alternative explanatory resources. Genuine explanations require a theory of force that is sufficient for the actual motions of bodies *without* recourse to fundamental attractions.

Metaphysicians – most especially, in the German context, Leibniz, Wolff, and their followers – constituted another influential camp. They agreed with Euler that attractionists, in passing the buck onto God, failed to give an informative or intelligible story about how forces are generated. But they insisted that the answer must be sought in the 'forces' described by monadology. Many among them viewed these forces as more fundamental, hence more deeply or ultimately explanatory, than notions like 'impressed force,' which were relevant only at the phenomenal level.[116] Euler found this metaphysical theory to be objectionable on several grounds. Most damningly, monadology, including its story about the origin of forces of change, is founded on a suite of logical errors. (Or so Euler argued – as I discuss in Chapter 2.)

Thus, to avoid being driven to either camp, Euler sought an in-house explanation of how forces are generated. He wished to keep this explanation within mechanics – or perhaps, at worst, within physics[117] – and avoid appeals to metaphysics or theology, as his opponents had. This implies that all the forces we need should be grounded in the known properties of material bodies. To see how Euler accomplishes this, let's continue following the train of his exposition in the *Anleitung*. There, we find that 'force' is first mentioned in the sense of an external cause of change in §40. That section aims to show that:

[116] Just how the metaphysical level relates to the physical level for these figures is a notoriously thorny issue. For a helpful recent discussion of Leibniz's views on this topic, see McDonough (2016).

[117] Brading and Stan interpret Euler's account of the origin of forces in impenetrability as a physical account, where the goals of physics are reconceptualized so as to only need a qualitative, causal story of how collisions occur (2024, §5.5).

> "Since bodies through their essence are impenetrable, no force, however large, is able to compress two bodies such that, even in their smallest parts, a real interpenetration can occur." (*Anleitung*, §40)

At this stage, 'force' has not yet been defined, nor is it defined in the immediately following passages. So, it appears that Euler is beginning with an intuitive, ordinary notion of force – in just the sense in which an ordinary speaker might say that two bodies were "rammed together with great force." His purpose for using a non-technical notion here is likely that he takes it to be all he needs to elucidate the consequences of the property of impenetrability.

But Euler cannot rely on a non-technical sense of 'force' for long. By §49, for instance, Euler claims that all forces of change in bodies originate in the impenetrability of bodies. Although the precise definition and the consequent mathematical theory has still not been laid out, it is clear that Euler already means to be talking about *impressed force*. By what logic did we arrive there? It would be reasonable to assume that light is shed on that in the intervening passages, §§41-48. Unfortunately for us, those sections have been lost. As a consequence, we must do our best to sketch a picture with information gleaned from other passages and texts. From those other sources, we can gather that Euler brings together two lines of reasoning.

First, it is an empirical fact that bodies change their states. These changes in corporeal states must have causes, and we will call causes from this class "forces." *This is a matter of definition*. We understand by "force" the cause of a change in motion of a body. Persistence tells us that bodies will not change their states of themselves (§33). The causes of each of these changes must, therefore, be sought outside the body in which that change occurs (§34). Thus, if changes occur in any body, there must be forces coming from outside that body which change its state of motion.

Second, we examine why bodies in fact *must* change their states, which will tell us about the nature and origin of the forces involved. Bodies, being mobile, may enter onto a collision course with each other. Their inertia or persistence would have them continue uniformly, but doing so would require them to pass through each other. This violates their impenetrability. It follows that, as the bodies come into contact and risk interpenetration, they must be redirected. That is, forces must be evoked which redirect the bodies along paths that avoid interpenetration. Indeed, Euler makes the stronger claim that, but for impenetrability, bodies would never have any reason to change their states:

> "As long as there is a distance between the bodies, none of them will prevent the others from remaining in their respective states; in fact, if the bodies could freely interpenetrate each other, the state of none would be disturbed by the others." (§49)[118]

So, we know that bodies can press and push against each other in virtue of the fact that they are mutually impenetrable. These pushes cause bodies to change each other's states of motion. Since one body can cause another body to change its state of motion, by definition, it can generate impressed forces. And since these forces prevent interpenetration, they must be either distance forces or forces acting at the point of contact of the bodies' surfaces. But they are not distance forces, because bodies separated by distance are not at risk of interpenetration, and so we have no reason to think that forces *must* be evoked. From this, Euler concludes that *all* forces derive from impenetrability and are evoked only in collisions through contact, echoing a then-standard analysis. All forces are therefore repulsive forces acting upon the contact of two bodies.

The last part of this argument is hard for us to swallow today. We will put critical pressure on these further claims below, in section 5. All the same, note that there is an

---

[118] This basic story is also presented in the *Recherches*.

argument from epistemic security, here, which is not totally without merit. Today, we sometimes balk at the moderns' difficulties with representing action-at-a-distance forces. After all, we can *imagine* arbitrary, at-a-distance forces acting between or on bodies. There seems nothing particularly difficult about this; and if they are imaginable or conceivable, they are, prima facie, possible. But we can imagine a lot of things. For example, we can imagine some object in our vicinity abruptly self-annihilating and disappearing from the world without a trace. Collisions are different. For collision is the only situation we can conceive of where forces *must* arise. These forces are therefore the only ones we know *must* exist. Accordingly, goes the thought, we should, to the furthest possible extent, theorize only on the basis of forces arising in collision. Those forces are repulsive, contact forces. So, to theorize securely, we should proceed as far as we can on the basis of only these forces. Naturally, this is only an argument in favor of the attempt. If adopting this restriction seriously hobbles, say, the progress of celestial mechanics, that is a count against it.

With that said in favor of the argument, observe that the argument for the existence of impressed forces (and their status as forces of impenetrability) has now been given. Still, the earlier questions have not all been answered. One would like a little more clarity on how these forces act and in what manner they arise. For instance, in Euler's view, when it comes to impressed forces, *to exist* is *to act*, hence to produce an effect on the motions of a body.[119] According to the argument, before bodies come in contact in collision, forces are not acting. Therefore, they do not yet exist, meaning that, at some point, they must "arise" to do the

[119] It is true that notions which seem to foreshadow later concepts of "field of force" or "potential" had been in play, arguably at least since Newton's "absolute quantity" of a central force in the Definitions. For such concepts, the force (field) "exists" even when it is not acting. Euler himself contributed to the development of the potential concept. This is itself something of an interpretive puzzle, given his philosophical commitments. Collisions do not lend themselves to representation through a potential. If all impressed force derives from impenetrability, there is a gap that needs to be bridged, to arrive at something like a "field" or "potential" concept.

necessary work of staving off interpenetration. Where do they come from? What generates them, or is their source? This question was a persistent concern of Euler's, not least because of the context described above – that the 'impressed force' concept had to out-compete major alternatives in terms of its intelligibility and epistemological bona fides.

The early Euler found that *inertia* was the source of the forces evoked in collision. The position is manifest, for instance, in an early study of collisions, *De la force de percussion et sa véritable mesure*, published in the Memoirs of the Berlin Academy in 1746. There, he writes that "the inertia by which each body remains in its state cannot properly be called Force, although a genuine force can sometimes result from it" (1746d, 22). His explanation of how inertia generates forces through collision is the one we have already seen. It invokes impenetrability as a necessary *condition* for the arising of forces in collision, although not as the *source* of the forces themselves.

By about 1750, beginning with his publication *Recherches sur l'origine des forces*, Euler had decided to identify *impenetrability* as the source of forces.[120] Why the switch? After all, prima facie, it appears that both inertia and impenetrability are necessary ingredients for the story Euler repeatedly gives about how forces must arise in collisions in order to divert two bodies on a collision course. If bodies lacked inertia, could they not, as it were, spontaneously change direction and avoid collision without an impressed force causing this change? Could they not simply come to a halt (Gaukroger 1982, 148)? If they came to a stop prior to contact, then the requirement that action requires contact would imply that neither

[120] Curiously, some of Euler's language in the *Recherches* suggests that it is certain forces which are necessary for maintaining the impenetrability of bodies, or that the non-penetration is "causally downstream" of the forces. In section 19, he writes that the "impenetrability could not subsist without forces sufficient to change the state [of the other body in a collision]." And in section 20: "Insofar as the bodies are impenetrable, they are endowed with certain forces which are necessary to maintain this property" (author's translation). Nonetheless, Euler always returns to saying that the impenetrability is the "source" or "origin" of these forces.

body could exert forces on the other. More generally, for bodies to lack inertia means that there would be no need for forces to explain these changes. So, both inertia and impenetrability are necessary for forces to arise. If that is true, then it is unclear why we should single out *impenetrability* as their source, rather than inertia.

Previous scholars have noticed this. Gaukroger, for example, in scrutinizing Euler's shift from inertia to impenetrability, raises doubts that inertia can really be sidelined the way Euler seems to want (1982, 147-8). He points out that Euler is unclear whether he thinks it is impenetrability alone that is supposed to be "dynamically effective," and also doubts whether this is in fact the case – that is, whether inertia could really be discounted. There is more to say about the need to independently posit inertia, and I will do so below. For now, it is important to understand, at least, how Euler takes the logic to work.

There is both a reason why Euler *needs* impenetrability, not inertia, to be the source of forces, as well as a reason that *justifies* this selection. Euler arguably needs impenetrability to be the source of forces because otherwise the account risks being founded on a logical circle. In truth, this may be a "want" rather than a "need. The circularity that is thereby risked is not obviously vicious, but I think it does, at least, give Euler good reasons for *wanting* impenetrability, not inertia, to produce forces. Wherein does it lie? Recall that the argument that shows that forces must arise in collisions is based on "persistence" and impenetrability. As I have claimed, and will discuss shortly as the "second task" facing Euler, this notion of "persistence" is not yet *inertia*, at least as it is used in mechanics. Indeed, with forces in place, Euler only *subsequently* argues for the proportionalities relating persistence and force as quantities. It is those proportionalities that constitute persistence *as* inertia. Epistemologically, then, inertia comes *after* force. For the account epistemologically to mirror the ontology of the situation, inertia should not be the source of forces. This is, admittedly, not an absolute theoretical requirement, but provides Euler with a motive for privileging impenetrability.

Within the logic of his story, there is also a justification for Euler to privilege impenetrability over inertia. It is that being impenetrable is not only necessary, but also a *logically sufficient condition* for moving forces to exist. As we have seen, impenetrability is purported to imply that bodies are extended, mobile, and possess inertia (or persistence). These three properties, in turn, make possible conditions under which bodies, even though not (yet) subject to any forces, may find themselves on a collision course. Impenetrability requires that the bodies cannot stay on that inertial course. It requires that their states of motion change once they come into contact, which is just to say that it requires forces to arise. Hence impenetrability is the sufficient ground for the existence of forces of collision. The same cannot be said of inertia. Two masses which are mutually penetrable could continue on their paths by passing through each other. So while both impenetrability and inertia may be necessary for forces of collision to arise (at least in the scenario as described), only impenetrability is *sufficient*. This may provide some justification for privileging it. In any case, it appears to be why Euler himself viewed impenetrability as special.

Questions surely remain. Even as it stands, this account only shows that impenetrability is *logically* sufficient for the existence of forces. To infer that impenetrability is the *source* of these forces is a further step.[121] Still further, this story about how impenetrability conditions these forces is highly schematic and even vague. "Impenetrability" itself is an absolute property and does not admit of magnitude.[122] Yet when collisions occur, forces of

---

[121] Cf. (Wilson 1992, 416): "And, like other rationalists before him, he has confused logical implication, or something pretending to be such, with causality."

[122] "The impenetrability of bodies does not itself yield a magnitude…all [bodies] are impenetrable to the highest degree, which implies to the same degree" (§91). Indeed, the only two properties of body that admit of measure or quantification are extension and persistence.

determinate magnitudes and directions come into existence, acting on each body. How does this property which is "indeterminate in itself" (*Recherches*, §28) produce determinate quantities of force? I consider these and other questions in the next section.

Second Task

In the meantime, we have now set the stage for Euler's second task. We have the basic picture of how and why forces arise. And we know that bodies have 'persistence.' But these have not yet been constituted as quantities or put in relation via laws of mechanics. It is even legitimate to ask how *inertial mass* is derived or proved to exist from first principles. The question may seem puzzling. After all, did we not already derive 'persistence', as Euler called it, intending it to be synonymous with 'inertia'?

If the question is puzzling, it is because there is a real puzzle. We did, indeed, derive 'persistence.' Yet the property of 'persistence,' as Euler derives it, does not amount to inertial mass or the mechanical law of inertia. Persistence, in fact, is a curious property. To say that a thing has persistence means, at most, that for any change in its state of motion or rest, there must be a reason why that change happened that way rather than some other way. For someone committed to the principle of sufficient reason, it would follow that just about everything has 'persistence.' After all, for the partisans of the PSR, all changes, hence all changes in motion, must have a sufficient reason. The property of persistence appears to lack specific physical content, making it almost a logical construct. Indeed, and perhaps surprisingly, in Euler's own development of the point, persistence also applies to things in space other than bodies – like shadows and mirror images (*Anleitung*, §39). 'Persistence' is supposed to be an alternative name for inertia.[123] And above, I followed Euler in using

[123] This is not to mention another problem tied to nomenclature: the problematic term "force of inertia". I treat this in greater depth in Chapter 2. Euler acknowledges that it is the erroneous associations with the name

‘persistence’ and ‘inertia’ interchangeably. Yet just as Euler acknowledges that shadows have persistence, he would deny that they possess inertial mass or are susceptible to impressed forces.[124]

Henceforth, I will restrict use of the word ‘inertia’ to the property of material bodies specifically, whereas ‘persistence’ will refer to the more general, quasi-logical property shared with ghosts and shadows, in addition to bodies. Our puzzle is this. Evidently, mechanics will need more than mere ‘persistence’. It will need a concept that permits us to analyze the mechanical causes of the complex motions of bodies – a concept, inertia, which is thereby constitutively coupled to the concept of impressed force which identifies them as quantities which obey key proportionalities. That is the concept of *inertia*, or *inertial mass*, and there remains a logical gap between it and ‘persistence.’ Accordingly, if Euler’s *a priori* derivation is to go through, it would seem that there must be something further in the concept of body, such that persistence takes on the specific character of *inertia* in bodies, whereas it does not do so in non-corporeal entities like shadows or spirits. What could that be?

Unsurprisingly, a key idea must be that shadows, spirits, and mirror images are *not impenetrable*. Indeed, it is in light of this apparent gap in the argument that Euler’s systematic reliance on corporeal impenetrability [comes to seem most logical/makes the most sense as part of the overall logic of his theory]. Our goal will be to test whether this is enough to bridge the gap remaining in Euler’s deduction.

‘inertia’ that are the problem, and which carry the risk of misleading, rather than a deep problem with the way scientists employ the concept in mechanics (§31).

[124] Insofar as it just is an alternative name for inertia, both Gaukroger (1982) and Wilson (2002) have observed that Euler’s purported demonstration *a priori* of the law of inertia fails.

The target for that deduction is several key claims about force and inertia, or persistence. All are needed for the mechanical theory of force. Some have been treated already, but are collected here for exposition. Let us record them as the following propositions:

E1. *Impenetrability demands the existence of impressed forces; these forces are repulsive, contact forces:* "all forces that originate from impenetrability are of the nature of a pressure, through which the bodies interact at their point of contact, so each one tries to push the other away from itself." (§51)

E2. *Forces are directed*: "every force pushes with a certain effort in a certain direction" (§51)

E3. *The effect of a force is proportional to the time during which it continuously acts:* "If a moving body is pushed forwards by a force, then the increase in speed is the greater, the longer the force acts on the body" (§52)

E4. *The magnitude of a force is measured by the magnitude of its effect:* "A force twice as big must in the same time produce twice as big an effect, because just in view of that do we consider it twice as big." (§53)

E5. *The ratio of force to persistence is proportional to the effect produced (the change in motion):* "the increase in speed produced in a certain time is the larger, the smaller the persistence of the body" (§54)

E6. *Persistence is a magnitude:* "persistence falls into the category of magnitudes, and as such is accessible to measurement" (§54)

In comparison to Newton's laws, it would seem that an ingredient is missing. This is the third law, the equality of action and reaction. As it happens, this is not treated directly in the Anleitung, but it is addressed in the *Recherches*. Let us add a final proposition:

E7. *Equality of action and reaction:* "if two bodies A and B meet in such a way that they would have to interpenetrate, were they penetrable…the forces with which they each resist penetration are equal to each other and oppositely directed" (*Recherches*, §29)

Only with these claims in place can we at last redub the 'persistence' *inertia* and claim to have derived the full theory of impressed force and inertia as used in mechanics.

Recall also that Euler's project so far has been to use *a priori* resources to build his account. It is clear that Euler takes these propositions to be *a priori* truths, relying only what has already been shown perhaps in conjunction with suitable definitions. How well does Euler succeed? Can he get all of these results relying only on first principles and concepts? Or will *a posteriori* information about the nature of bodies be required? If so, at what stage in the argument does he have to – even if implicitly – make use of empirical information?[125]

Let's see how Euler demonstrates these propositions. For the equality of action and reaction, it is in §§29-31 of the *Recherches*. For the rest of them, this work is carried out in Chapters 7 and 8 in the *Anleitung*. They deal, respectively, with the effect of forces on the speed and on the direction of material bodies. I will focus on the first part. Euler's inferences are quick, and it can sometimes appear as though these propositions are simply stated, without explanation. I will aim to give flesh to the inferences where I can; in one crucial case, I find that the inference is not valid, and explain why.

The first proposition does not require much explanation; it more or less summarizes what was supposed to have been shown already. As for the second, it states that forces are

[125] Put another way: we all appreciate that at *some* point mechanics will become empirical. After all, the point is to apply it to actual bodies. But at *what* point does it become empirical? Can we derive all of the foundations a priori – the concepts of material body, inertia, and impressed force, and the basic laws – or does some part of even that require empirical input? Euler is not the only one with a stake in this kind of project. Nor is it even only someone with "Cartesian", foundationalist sympathies who would be so invested. It was widely thought that propositions that will serve as the axioms for mechanics, hence natural philosophy, ought to have a more secure epistemic basis than via empirical discovery and confirmation.

directed magnitudes. This seems to follow from the plausible idea that forces (which have been defined as causes of changes in states of motion) automatically inherit a direction from the direction of the change that they cause. If we bracket the "magnitude" part of the claim – since that is the focus of E4 – that settles E2.

What about proposition E3? Arguably, this proposition is something like a "definition in disguise." To see this, consider first that it is in the nature of forces that they must act through some duration of time, however small, to produce an effect.[126] Suppose a force acts on a body during an interval [0, 2T], and the change produced in [0, T] is different from the change produced in [T, 2T]. The situation requires us to deny at least one of the following:

a) The force on the body was constant in magnitude throughout [0, 2T].

b) The body and its properties did not change in [0, 2T]. (We would say: the mass of the body was constant through [0, 2T].)

c) The effect of a force is proportional to the time in which it acts.

To deny c) would amount to saying that there is some third factor, indexed to the time, additional to the force itself and the body acted on, which contributes to determining the effect on the body. Arguably, this amount to introducing an additional, mediating cause of these effects, besides force. But force is by definition *the* cause of changes in motion. As force was defined, any discrepancy from a constant rate of change in a body whose properties are constant automatically, by definition, counts as a change in the magnitude of the force. There is no room for additional, mediating factors, but this is not a claim with empirical content. It is how the "accounting" is defined to work.

[126] "for if a pressure is to produce an effect, it must be of some duration, however short this might be" (*Anleitung*, §52)

It *appears* that the other part of Proposition E4 should be settled very neatly, too. In fact, though, it is more complicated, and E4-E6 turn out to be interconnected. So, before moving to them, let's look at E7, which should be simpler. Curiously, though, what Euler initially has to say suggests that the equality of forces in collisions is a *consequence* of the equality of action and reaction:

> "However, since the pressure between two bodies is the same on both sides, *due to the equality between action and reaction*, it follows that the impenetrability of the one and the other of these two bodies deploys the same force, which is equal to that with which the two bodies are pressed together" (*Recherches*, §29; my emphasis)

From there, Euler draws the conclusion, quoted above, that the forces between any two bodies which are either colliding or pressing against each other are equal and oppositely directed. However, shortly thereafter, he claims the converse: that the equality of action and reaction depends on the nature of impenetrability:

> "*This equality of forces, on which depends the great principle of the equality between action and reaction, is a necessary consequence of the nature of penetration.* For, if it were possible that the body A should penetrate the body B, the body A would be penetrated by B to precisely the same extent; thus, since the danger that these bodies interpenetrate is the same on both sides, these two bodies must employ equal forces to resist penetration. […] they each deploy exactly as much force as is necessary to prevent penetration. Now, these two bodies, acting one on the other by some force, will be in the same state as they would be in if they were pressed together by that same force." (*Recherches*, §31; my emphasis)

Here, the equality of action and reaction is supposed to be deduced by a conceptual analysis of corporeal penetration. This is what must be the case if Euler is to succeed in deriving the laws of mechanics from concepts and first principles. Why did he first say that the equality of forces of impenetrability is due to equality of action and reaction? It is possible that his purpose there was still illustrative of the result to be shown, and that §31 provides the needed argument.

What is that argument? It is based on the idea that the nature of corporeal penetration is to be mutual. If one body A penetrates another body B, that means some of the matter of A is occupying the same volume of space as some matter of B. But the situation is symmetric. The apparent "active" and "passive" roles are illusory, since both of the bodies are actually penetrating each other whenever either is, and to exactly the same extent. Now, the effect of the impenetrability of A is to prevent the penetration of A, and the effect of the impenetrability of B is to prevent the penetration of B. But, as we have said, these are one and the same circumstance. Thus, the impenetrability of A and of B produce equal effects, i.e. equal forces.

Now let's return to E4, and with them E5 and E6, which deal with constituting force as a magnitude in a certain relation with the "persistence." The magnitude of forces was, of course, *defined* in terms of the magnitude of the effect they produce on a body. This makes good sense given the conceptual or ontological role constitutively assigned to forces in the theory: they are the causes of changes in bodies. There is an important caveat, however. Strictly, this definition still allows that the magnitude of a force is measured relative to a given body. That is, we can meaningfully compare the magnitudes of forces which all act on some given body. But we do not yet have a way to compare the magnitudes of forces acting on different bodies. The problem, put otherwise, is that Euler has defined forces by their *effects*, but then defined the effect as *change in motion*. Quite explicitly, Euler defines the effect of a force on a body (in a fixed duration of time) as the change it produces in its *direction* and *speed.* (§54: "In our case the effect is the increase in speed.") He did not define it as the change in what we now call 'momentum.' If he cannot somehow show this, the result will be a very different theory from the mechanics we know and love.

Euler must formulate corporeal persistence as a quantity abstracted from particular bodies. What this means is that establishing E4 is inextricably tied up with establishing E5 and E6: they all come as a package. In the *Anleitung*, the argument for this package, such as there is, is contained in a passage from §54:

> "Since it is because of its persistence that a body attempts to remain in its state, the persistence opposes all change and because of this forces are required to produce a change. The greater the persistence, the greater the force required to effect the same change in the same time, and from this follows that persistence falls into the category of magnitudes, and as such is accessible to measurement."

This argument is not convincing. To see why, let's look at another scenario. As before, let's consider a situation where E5 is placed in tension with others, so that we can try to imagine grounds for doubting it. What kinds of empirical situations would, *prima facie*, lead us to question whether this proportionality holds?

Imagine we have a source of force P1, like a mechanical push or pull – say, the rebound of a certain coiled spring. Suppose that P1 produces an effect on a body B which is twice the effect that P1 produces on body A in a fixed time. If nothing else were fishy about this, we would ordinarily conclude that B is half as massive as A. And suppose we have some other force or push P2, and that P2 produces an effect on body A which is (say) double the effect of P1 on A. The situation is summarized in this table (where 'N' represents arbitrary units of speed):

	P1	P2
Body A	N	2N
Body B	2N	??

Now, we of course would normally expect that the last experiment, testing P2 against body B, would lead us to fill the remaining cell of this table with '4N'. Imagine, though, that P2 produces an effect on body B which is *not* double, but say triple, the effect of P1 on B. We can even suppose that this was all studied under many trials. Then we have a situation as summarized in the following table of changes in states of motion, where ‘N’ refers to some units of velocity.

	P1	P2
Body A	N	2N
Body B	2N	6N

Given the truth of the other propositions, the following cannot all be true in this scenario:

i. The bodies have the same magnitude of persistence across the different trials.

ii. P1 and P2 are exerting the same magnitude of force across the different trials.

iii. The effect of a force is proportional (inversely) to the persistence.

We have to deny one of these claims, but we can ask questions analogous to those we asked earlier, about proposition E3. Which of these claims do we deny? Are there any which we can *never* deny? Evidently, we in fact never deny iii. For ease of expression, say that forces (and material bodies) are “well-behaved” if, in addition to meeting the criteria E1-E4, they also obey this proportionality and thereby satisfy E5 (and E6). Well-behaved forces and material bodies admit of formulation as abstract (real-valued) quantities. But can we, or rather can Euler, provide *a priori* argument that forces and bodies are well-behaved?

Suppose as before that, while holding onto iii., we *insist* that the bodies are not changing at all. We have thoroughly probed them, and we insist that all the evidence shows that A and B are identically the same body at the different trials. Therefore, none of their properties can have changed. Since their persistence is an innate property, it cannot have changed. And yet the empirical failure of proportionality persists. If we accepted E5 already, we would be forced to conclude that the mechanical source of one of the forces, say P2, is, for some reason, producing a *different* magnitude of force when it is applied to the different masses. But are there independent reasons that would demand that we accept E5? Are we forced to this conclusion, the way we were with the proportionality of force to time, by the meanings of the concepts as we have deduced and defined them? Or can we consistently deny iii.?

It seems that we *can* consistently deny iii. In effect, the failure of iii. would mean that a body could offer more or less "persistence" (or resistance) to different forces. It would amount to introducing a dependence of what we might call the "efficacy" of a force on the body on which it acts. *A priori*, it could turn out that forces and bodies are not well-behaved.

Could we not resist this conclusion, by arguing on grounds analogous to those we brought to bear on E3? After all, denying iii. would also introduce an additional, "mediating factor" which modulates the efficacy of a force, just like we considered above. By definition, the force is supposed to be wholly determinative of the effect (on a given body). So, there is no room for further factors. This argument is appealing, however, there is an important difference between the previous case and this one. The additional factor in the previous case is supposed to modulate the efficacy of a force acting on a body via a dependence on the *time*. This is genuinely an additional variable, and so is not permitted by the definition. However, in this current scenario, the efficacy of the force on a body would vary with *the body*. By definition, the effect of the force is *supposed* to vary with the body. After all, what we "want," namely Proposition E5, is nothing but the claim that that variation takes the form of an inverse proportion, of the effect of a force with a property of the body called 'persistence,' a relation which allows us to assign the persistence a determinate, fixed magnitude. However, we cannot *a priori* show that that variation *must* be via an inverse proportion, rather than via some other, more complicated functional relationship. In general, then, the "persistence" itself could vary with the force.

The consequence of this is that we cannot rule out *a priori* that the persistence of a body might be indexed to the agency that acts on it. How, then, do we know that forces and bodies are well-behaved in the above sense, required by mechanics? Does Euler have any other resources he might be able to use to fix the truth of E5?

Of course, Euler could appeal to *experience*. Suppose we have done all we can to uniformize the empirical conditions of the application of forces to given bodies, which we endeavor to ensure are not changing between trials. Empirically, we find that, in those conditions, the inverse proportion is obeyed. That is, we always find empirically that the "fourth cell" is to be filled in with '4N.' That does seem to settle it. Provided these proportionalities are observed, we are licensed to conclude that bodies have a fixed quantity of persistence which, as their faculty of "resistance to force," is a correlate to the defined concept of impressed force.[127] The quantity of persistence, otherwise dubbed the inertial mass, is inversely proportional to the change in velocity produced by a given force. With that, we have empirical criteria for measuring the relative inertia of different bodies, and the relative magnitudes of the force of different pushes, which is then called a measure of impressed force.[128] Importantly, though, this path uses empirical input, which changes the epistemic status of mechanics.

What about *a priori* reasons? Euler does, it seems to me, have one argument to make here. Namely, that in his ontology, both body and forces are physically homogeneous. All forces have, at bottom, the same physical nature. They are pushes or pressures deriving from

[127] I note that Euler's move from persistence in state of motion to the idea of "resisting" a force adds additional content, and it is not obvious that it is licensed. However, I will not go into that here.
[128] Note that this definition of inertial mass is different from the Definition given by Newton in the *Principia*: "Quantity of matter [or mass] is a measure of matter that arises from its density and volume jointly." Although Newton's apparently circular definition is arguably not so (insofar as density was to be measured through specific gravity), it is not the unambiguously dynamical measure of mass that Euler presents here.

the impenetrability of bodies. And Euler also thinks he can show that matter itself is of one, uniform kind. In the face of this uniformity, we would not expect for there to be variation in how different forces act on different bodies. Because, at bottom, all forces and all bodies are physically-speaking "the same." To the extent this ontology is established, it provides some metaphysical reasons for adopting the fixed inverse proportionality in E5. This route is a reasonable one to attempt, but where does it leave the epistemic status of the laws of mechanics? It will be *a priori* true to the extent that Euler's ontology is *a priori* true. As it happens, though, there are good reasons to doubt whether important features of Euler's ontology are in fact deducible from first principles. One reason is simply that the uniformity of material body, at least in its "coarse" form, is not entirely an *a priori* proposition – and Euler admits this. Another is that the subtle matter posit may spoil the argument from ontological uniformity.

4.6 Conclusions

Where does this leave us? On the one hand, Euler's deduction of the basic concepts of mechanics is not the clean, *a priori* argument he represents himself as offering. For this argument to deliver the desired physical concepts – inertial mass and impressed force – depends upon contingent empirical findings that go beyond the content of the four general properties of bodies. Those properties – extension, mobility, persistence, and impenetrability – appear to suffice only as a conceptual scaffolding for experimental facts (or "inductions" based on such) which then flesh it out, yielding the physical concepts which form the foundation of mechanics.

It is true that, alternatively, we may be able to get the result on metaphysical grounds. That is, provided we buy into Euler's ontology. However, that may be only an apparent alternative, insofar as that ontology – especially the claim about the uniformity of matter – may itself be evidentially based on empirical lines of reasoning.

Nonetheless, Euler's account has many successes. Internally, his story succeeds in elucidating some of its most surprising claims. For example, he makes it clear why entities like shadows can have 'persistence' but not inertial mass: lacking impenetrability, they are not subject to the 'pushes' that get the story off the ground. Externally, as a reconstruction and justification of mechanical theory, Euler's account helps to clarify what parts of the theory can be built up with *a priori* resources. Likewise, it helps us to pinpoint the places where empirical input is needed, and understand the nature of that input.

To appreciate this last point, recall that it is not the case that all theories or propositions requiring empirical confirmation, and all empirical input which may be adduced to confirm them, are equal, epistemically speaking. As a great deal of 20th century philosophy of science has explored, the propositions may vary across such potentially important criteria as *a priori* plausibility, simplicity, or generality. The empirical input can vary in its degree of accessibility, quantity, or generality (i.e., it is, plausibly, epistemically better when it is easy to get a lot of evidence from diverse physical contexts). And the evidential relation between them can also vary in terms of the amount of additional hypotheses required for interpreting the empirical input as evidence in favor of the proposition.[129] As these parameters vary, the epistemic status of the proposition(s) in question may also vary.

Plausibly, Euler's account here, though not *a priori*, gains strength along many of these dimensions. For instance, the inverse proportionality of E5 is plausibly simpler than

---

[129] This is intended as a relatively uncontroversial, but certainly not an exhaustive, list of some epistemic virtues of scientific theories or claims or evidence. For more on theoretical virtues in science, see Kuhn (1977), McMullin (1982), Laudan (1986), and Douglas (2014); on simplicity in particular, see Sober (2015).

alternatives which construe the efficacy of a source of force as a quantity that varies with the body. The experience of the constancy of the effects of forces appears to be relatively ready-to-hand, with forces that manifest in different ways (e.g. human pushes and pulls, springs, magnets, or simple machines). Finally, because of the way Euler developed his argument, the crucial proposition, E5, plausibly stands to relatively directly receive confirmation from such empirical input, needing relatively little in the way of further theory or auxiliary hypothesis in order for empirical input to count as confirming the inverse proportion.

Summarizing: Bracketing the question whether Euler's *a priori* derivation of the four properties of body succeeds, that derivation may not by itself secure knowledge of the existence of impressed forces as theorized in mechanics, or, correlatively, the full-blooded mechanical property of inertia (rather than merely persistence). To obtain those concepts, Euler appears to need to make indispensable use of *a posteriori* information, laid over the scaffolding provided by his earlier derivations. However, the derivation should nonetheless be judged a qualified success, as the argument secures the well-behaved-ness of forces with demands for additional, empirical information that are quite minimal. Insofar as the goal is epistemic security for mechanical theory, then, although Euler fell short of apodictic demonstration, he did succeed in establishing that theory conditional on very minimal empirical input.[130]

[130] This accords with, and provides some context for interpreting, some recent interpretations of "Newton's laws of mechanics" among philosophers of science. For example, Lange (2013) has argued that the laws of mechanics – specifically the second law – are "more necessary" than any particular law of force, either one obtaining generally in nature, like classical electromagnetism, or one describing a specific physical system, like the gravitational forces on a compound pendulum. Although these particular forces do hold with something like "physical necessity," $F=ma$ itself, as a framework for any forces at all, has even *greater* "modal strength." On this basis, Lange argued that certain propositions about *physical* systems, which can be demonstrated using only facts about forces in general, count as admitting of purely or distinctively *mathematical* explanations.

Many philosophers of science have remained dubious. Some have recoiled from the implication that we ought to regard the laws of mechanics as near to necessary or *a priori* truths. Euler's account here provides means for seeing concretely how the laws of mechanics could be epistemically more secure than particular force laws, without needing to appeal to hierarchies of metaphysical of modality and to place the laws of

5. Objections to Euler

In reconstructing Euler's theory of the nature of body and its relation to force, I have argued that it represents a qualified success. It manages to get quite far based on minimal empirical input Nonetheless, there is room for objection. Issues bearing on the fundamental physics contained in Euler's theory reside in his accounts of inertia and impenetrability. I will discuss three sets of problems.

The first set again has to do with the fact that Euler's derivation of 'persistence' does not succeed in deriving the full, mechanical law of inertia. Above, I attempted to explain how, according to Euler, we could get from 'persistence' to the full-blooded notion of 'inertia'. But there, we were already assuming that the persistence in question was persistence in *state of rectilinear motion*. Why is change of *velocity* the relevant sense of "change of state"? Why is the feature that tends to persist a body's instantaneous velocity – rather than any of the many other facts about a body that, *prima facie*, might constitute its "state," like its position? Without an in-principle explanation of why *velocity* is "the state" of a body, the argument is circular.

The second set of issues has to do with Euler's reductive account of forces in terms of the impenetrability of bodies. Above, I presented an Eulerian story that would at least make it plausible why impenetrability would be a better candidate than inertia for 'the source of forces'. But this argument was comparative. Assuming either inertia or impenetrability generates forces, it shows why impenetrability is the better candidate. That assumption was not established. Euler only established that impenetrability (and inertia) are *logical* sufficient conditions for the existence of forces. How, then, can we know that this faculty *generates*

---

mechanics beyond the reach of contingent facts and above ordinary empirical laws. For it provides a clear illustration of a possible world in which they fail to hold.

forces, and does not merely serve as a sufficient condition for their existence? Still less has Euler established that impenetrability is the source of *all* forces.

The last set of issues I will discuss encompasses miscellaneous other criticisms of Euler's account coming from the secondary literature, especially from Gaukroger (1982) and Wilson (1992).

5.1 Objection: the move from 'persistence' to 'inertia' is circular

Section 4.3 left us with a puzzle. For reasons alluded to there, the derivation Euler provided is suspect.[131] Insofar as he was successful, he derived 'persistence.' We have treated in detail what would be needed for Euler to obtain the full-blooded property of inertial mass, but there is another problem, distinct from that one. For, persistence was defined as the faculty of the preservation of a thing *in its state*. But what is "the state" of a material body? How do we know that the relevant "state" is velocity? What Euler gives us is the following assertion:

> "One says that a body remains in the same state, when it remains at rest, or when it continues its movement in the same direction at the same speed." (§30, Ch. 4).

We can all appreciate that we "want" the state to turn out to be instantaneous velocity. For, in that case, the PSR will tell us that there must be a whole class of *reasons* which explain changes in velocity. We can then call those reasons 'impressed forces.' Then we will be able to say that bodies cannot change their velocity absent an impressed force, and can follow the rest of the account presented in section 4.5. But Euler has not provided a reason from first principles why velocity should be the state of a body for purposes of applying the PSR. After all, *any* change is subject to the PSR's querying for a reason. Changes

[131] Scholars have made note of this: "mobility, even with the principle of sufficient reason thrown in as an added premiss, does not imply inertia as understood in modern physics." (Wilson 1992, 416). But this remark is unaccompanied by explanation, or any evaluation beyond the judgment that "Euler, a rationalist believing in innate ideas, has failed to examine the strict logical relations among the ideas he takes as fundamental" (ibid.).

in position, for example, or acceleration.[132] The PSR is merely a logical-epistemological tool; in itself it is applicable to any one of a large number of different conceptions of the instantaneous mechanical "state" of a body. Are non-circular arguments for singling out the velocity forthcoming? This seems doubtful.

For, note that when we apply the PSR to some given conception of state, we generate a demand for a reason or cause for changes in that state. Now suppose that we, like Euler, add the hypothesis that there are no external causes operating, and further assume there could be no internal causes of changes. Then, when we apply the PSR to any given conception of mechanical state, we generate a *preservation principle* for that state, that is, a law that says that that state will be preserved in the absence of external causes acting on the body. In this way, we might have derived a law of preservation of position, a law of preservation of velocity (i.e. the law of inertia), a law of preservation of acceleration, or even, taking a different tack, a law of preservation of circular motion.[133] (We are assuming the relevant derivatives exist, but at the time smoothness was a frequent implicit assumption.) And suppose we give the name 'force' to the category of causes which change each state, since these causes force the body out of its present state into a different one. Then the PSR

---

[132] Or even, for that matter, its instantaneous circular motion: we could take the osculating circle along a body's trajectory to define the spatial part of its instantaneous state of motion, departures from which state would require external causes.

[133] For illustration, let's follow in parallel with Euler's exposition, but supposing instead that we have taken the state of a body at a time to be its spatial position at that time. Consider a body in uniform motion, subject to no external causes. Let the body be at position x at that instant. We can ask whether it will stay in that same state (being at x) or will change position-state. If it changes, there must be a sufficient reason for this change. We could even, analogously to Euler, rule out internal grounds for change. So, there must be an external cause of the body's change in position. In the absence of such, the body ought to conserve its state – that is, to remain stationary at x. *We* know – as a matter of empirical fact, let's say – that even should a body in motion be subject to no external causes, its position-state will change, because of inertia. But, by our hypothesis, the velocity of the body, its state of motion, is not its state, hence not a property that could be maintained through a faculty of conservation-of-state which we have derived a priori. That is to say: a priori, given our choice of 'state', conservation of state ought to mean staying at x. We need an argument that this is the "wrong" choice of state. The PSR alone cannot provide one.

will generate a whole series of persistence or preservation-of-state laws, and correlative to that a whole series of concepts of 'force'. Only one of these is the mechanical law of inertia, tied to the concept of impressed force as we know it. Again, the PSR does not contain in itself the grounds to pick that one out from among the others. Hence Euler's problem.

Euler's theory of inertia will certainly not work unless it can show why the state of a body at a moment in time is the combination of its speed and direction of motion. But how can we know that this is the 'state of a body' in the relevant, physically or metaphysically important sense? This knowledge cannot come from the PSR.[134] We have not been given the *a priori* reason to think that the physically important, *general* demand for a sufficient reason will lie at the level of the second derivative (*why did it accelerate?)*, rather than the zeroth (*why is it there?*), the first (*why is it moving?*), the third (*why is it jerking around*?), or any other. No reason, that is, other than a wish for the answer to correspond to the notion of 'impressed force' that we in fact now have, rather than a panoply of such notions with no clear order of priority. The PSR, as a logical-epistemological tool, is therefore powerless to select, among possible 'mechanical states', the one important to physics – important in virtue of connecting up with the physical notion of cause as impressed force. The definition of 'the state of a body' is, rather, something additional that must be supplied in order to give the PSR the desired physical content when applied to the behavior of bodies.[135]

---

[134] It might be thought that the argument could be completed by repairing to facts about the structure of space and time – in particular, its relational structure (hence indirectly via the PSR). Roughly, the idea would be that if there are no really existing spatial locations, and space is just a system of relations, then position-states are not reasonable candidate for 'the state' of a body. But this would, at best, only partly help, since there are other candidates for 'state-hood' besides position and velocity. Further, Euler was, in principle, in favor of absolute space. So he appears to lack principled reasons to reject even position-states.

[135] For completeness, I make two observations. First, recall that Euler distinguishes between an 'internal state' and an 'external state'. Whatever its importance, this distinction does not bear on the point at issue. Second, Gaukroger levels an objection nearby to the one made here. Euler thought that it could be demonstrated *a priori* that bodies have persistence, i.e. inertia, and in this he joined others in the German-

5.1.1 How Euler might reply

Of course, Euler does arrive at "change in state of rectilinear motion" as the thing that needs a reason. As I hope I have shown, the prospects are dim for doing so without simply anticipating the laws of motion he already accepted, with a desire to arrive at the pre-conceived notion of *impressed force* as the sought class of 'reasons.' Once we have specified the velocity as the body's state, we have a plausible case for deriving the law of inertia. The problem for the account is that this was supposed to be a derivation of the law of inertia from concepts and first principles. Explaining why velocity is the body's state is part of the job.

This makes it something of a puzzle why Euler attempted to use the PSR. Indeed, it is possible that the PSR's powerlessness to obtain this result was one reason the PSR seems to have lost its sparkle for Euler later in life. Often invoked in the 1740s and 1750s, the PSR had become, by Letter No. 128 of the *Lettres*, Vol. 2, a kind of vacuous truism, and one

---

speaking tradition. (Though d'Alembert's *Traité de dynamique* (1743) makes it clear that that a mechanics with *a priori* bona-fides was not exclusively a German obsession.) As Gaukroger (1982) argued, this amounts to taking the law of inertia to be a necessary truth, but necessary truth it is not: other ways in which force relates to mass are possible. Gaukroger diagnoses the problem with his derivation thus:

> "The idea that the law of inertia could be justified in these terms was quite common in the eighteenth century, but the proposed justification was clearly question-begging. Aristotle, for example, had considered that every motion must have an external cause, so that in the absence of this cause no body will maintain its motion. This view of inertia could just as easily be based on the principle of sufficient reason, but the law of inertia that would result would clearly be different from Euler's. Everything depends on how, and under what conditions, we assign forces. Only given a particular characterization of forces does the law of inertia follow from the principle of sufficient reason." (1982, 141).

I agree with Gaukroger that everything does depend on the conditions under which we assign forces. But this is just another way of describing the case: the conditions under which we assign forces are just the conditions under which we judge that a change of state has occurred. Plausibly, the prior decision is what to count as the body's "state." On the reading of Aristotle which is offered, the "state" of any (sub-celestial) body is (absolute) rest, and any departure from this state of rest is to be explained, hence would need forces, or an Aristotelian substitute. (Alternatively, one could say that the "state" of a body is its location. Any change in its location, i.e. any motion, demands explanation.) For Euler, the "state" of a body is its speed and direction of motion. But if he has *a priori* reasons for this, he does not make them explicit.

abused, in particular, by Wolffians, who use it to assert their claims to a kind of empty knowledge about matters regarding which they really ought to confess their ignorance.[136]

That said, it is worth exploring what else Euler might have had in mind that would help his PSR-based argument go through. There is at least one plausible line of reasoning available to him which can make headway and comes via his description of the origin of forces. Here's how that argument might work:

> Bodies are in danger of interpenetrating when their motion puts them on a collision course. At the instant of collision, each body has a speed and direction of motion which, if it was maintained, would involve interpenetration. Thus, it is their speeds or directions of motion that must be changed. It is not their *positions* that must change – they could avoid interpenetration by stopping at the moment they touch. Nor does the acceleration have to change. We could imagine the velocities suffering a step-function-like change through an instantaneous impulse that suffices to prevent penetration. And the accelerations would remain the same except for this discontinuous point (the same, a fortiori, would hold of the higher derivatives). The one candidate mechanical state that definitively *must* change in a collision is the instantaneous velocity.

This gives at least some reason for a principled distinction of the velocity state of the body from the others (position, acceleration, and so on). As such, it gives some grounds for taking "the" state of motion of a body to be its velocity: the velocity is what must necessarily be altered in collision to avoid penetration.

---

[136] Euler's views on the principle of sufficient reason as a principle of knowledge evolve over time. As we have seen, he uses it as a principle in deriving his general physics of body and force. Yet, later, he accuses Wolff and other partisans of the PSR of abusing it, insofar as they use it to do work even in cases where the sufficient reason in question is not known. Whether these two positions of Euler's are compatible, and if so, how, deserves further attention, along with Euler's general epistemology.

There are two complications. It is, of course, a weakness of the argument that changes in velocity will, strictly speaking, entail changes in all derivatives of displacement, at least for smooth motions. Moreover, Euler himself did not think instantaneous changes in velocity were physically possible. However, if it can be allowed to stand as a conceptual point – and Euler was prepared to accept that discontinuous motions were at least conceivable – then it does show an asymmetry between velocity and acceleration (and indeed other derivatives of displacement). For there will be cases where a change in acceleration alone will *not* be sufficient to avoid interpenetration, whereas a change in velocity is always required. However, if we allow for discontinuities, a second potential weakness arises just on that point. For, it is at least conceivable that the bodies on a collision course discontinuously change position – just a jump to the left, say – right before they interpenetrate. This idea is even familiar to us, nowadays, at least in the quantum realm. But, at the time, the possibility of "teleporting" or "tunneling" particles was not seriously considered.[137] Indeed, the contrary was widely taken as an implicit assumption – and for some, as a quite explicit principle, in the form of the *lex continui*. Thus, the argument for focusing on changes in velocity could have had considerable force in Euler's context.

As an aside, this argument also happens to be in line with what else we have suggested about Euler's epistemological preferences. At least at the level of fundamental theory, Euler thinks we should proceed on the basis of what we can infer *must* exist or *must* occur, given what else we know, and eschew concepts or entities that are more doubtful or contingent. It is a maxim of epistemic caution. So, we ground our picture of *forces* on impenetrability and collisions, because that is at least somewhere we know forces *must* arise.

[137] It might be thought that the Lucretian *clinamen* or "swerve" is a precursor, but it seems the swerving motion was supposed to be a random, but *continuous*, deflection of the atom.

And we take *velocity* to be "the state" of a body because that is what *must* change in collisions. Because of their inevitability, these concepts are the most epistemically secure bases for fundamental theory.

Summarizing: Conditional on the success of Euler's reductive story of the origin of all forces in collisions, and the accuracy of Euler's coarse-grained causal story of how those forces arise in collisions, there are lines of reply to the challenge issued in section 5.1. In particular, it can be plausibly argued that velocity is the only derivative of displacement that *must* be changed in order to avoid interpenetration of colliding bodies, and so gives some reason for privileging velocity as the state of a body. This shows, once again, that there is more logical rigor and unity to Euler's account than he has typically been given credit for. However, it cannot be understated how appreciating this coherence requires thoroughly buying into his insistence on the basic importance of impenetrability and collisions to physics in general.

5.2 Objection: the reduction of forces to impenetrability does not succeed

The second set of issues has to do with Euler's reductive account of forces in terms of the impenetrability of bodies. Above, I presented an Eulerian story that would at least make it plausible why impenetrability would be a *better* candidate than inertia for 'the source of forces'. Of course, the reasoning we have seen so far, if it is correct, shows at most that impenetrability is *logically* sufficient for forces to be called into existence. There are at least three puzzles about Euler's development of the account of body and forces from this point onward.

One, as just noted, the basic analysis showed that inertia and impenetrability are sufficient conditions for forces to arise in collisions. But logical sufficiency is distinct from causation. How can we know that one of these faculties must *generate* forces, and not merely that they serve as logically sufficient conditions for the existence of forces? Second, Euler has not established that impenetrability is the source of *all* forces, including forces responsible for gravitation and cohesion. Yet a third problem – the most damning insofar as it reflects a contradiction internal to Euler's thinker – is that this reductive account does not appear to respect Euler's own criteria for what count as valid corporeal faculties or legitimate explanatory resources. These are the criteria I above called the Weak and the Strong General Constraints.

5.2.1 Problems with the conception of impenetrability as a 'faculty' or causal power

This first objection has already been mentioned. It is the transition from impenetrability as a logically sufficient condition for force to impenetrability as the "source" and "origin" of forces. Minimally, impenetrability could be simply regarded as a logical or logico-mathematical *constraint* on an account of material body. This would be independent of the details of the ontology. The facts about fundamental constitution, or micro-structure, or the basic building blocks if there be such, could vary. Impenetrability would only require that their constitution delivers the result that bodies cannot mutually penetrate. Euler, however, seems to go well beyond this. He speaks of the impenetrability as the *source* or *origin* of forces. Impenetrability is thus converted into a *faculty* which generates forces. It is represented not merely as a constraint, but as a kind of causal agency or power which produces the forces needed to satisfy that constraint.

There seems to be just one line of reasoning that could patch up this apparent logical gap in the argument. The story that tells us that impenetrability is sufficient to require the operation of forces purports to show that there is a constant conjunction or correlation between (a) the operation of the constraint of impenetrability and (b) the arising of forces.

Constant correlations of this kind are paradigmatically thought to require explanation in some sort of causal story. Although perhaps only named and philosophically thematized in the 20th century by Reichenbach, such a 'common cause' principle could very well implicitly be in play in Euler's reasoning and would offer an at least partial defense. It would license the following inference: either impenetrability produces forces, or forces produce impenetrability, or some third thing produces both. Additional reasons might then suggest that the first option is most likely.

What do these options mean? We roughly know what the first option entails. This is Euler's (implicit) claim that impenetrability is something like a faculty that generates forces in collision. These forces are repulsive, and their range of action is limited to contact. This is the 'impenetrability first' picture:

(IF) Impenetrability → Forces arising (conditional on collision) → Non-penetration in collision

What would it mean, conversely, to say that the forces "produce" or "cause" impenetrability? This would be like saying that bodies are only "impenetrable" because in collisions forces arise which, as a matter of fact, always prevent penetration. This would, in effect, amount to *nominalism* about impenetrability: there is no fact of impenetrability "over and above" the fact that some repelling forces always arise in circumstances of collision. Rather, 'impenetrability' is just a name which sums up that fact. If those forces, for whatever reason, failed to operate, the bodies would mutually penetrate. The 'force first' picture is:

(FF) Forces arising in collisions → Non-penetration in collision

The third option asserts that there is some third cause or faculty which produces *both* impenetrability and the repelling forces in collisions. Perhaps such a view could be maintained. But on its face it seems to be imbricated with puzzles about emergence and identity. It would be strange to maintain that this third power is the *cause* of *both* impenetrability *and* of the forces that prevent penetration. There are not *two* facts that have to be explained or caused. Surely all that its causing impenetrability amounts to is producing those forces. This third option, therefore, is really a modification of the second, by explicitly inserting a 'placeholder faculty' causally upstream of the forces. "Impenetrability" as such does not appear in the causal diagram, but leaves its mark in the nature of the force-faculty: that it is constituted so as to always produce forces sufficient to prevent penetration. This yields a 'force-faculty first' picture:

(FFF) Force-faculty → Forces arising in collisions → Non-penetration in collision

What argument could Euler have against (FF) or (FFF)? It seems his argument would have to be conceptual. The (FF) picture, as we said, is a kind of nominalism about impenetrability. If the forces which arise in collisions failed to arise, the bodies would interpenetrate. But Euler thinks that impenetrability is constitutive of the very concept of body. So to imagine this would be to imagine things to which the concept of 'body' would no longer apply. The collision is between bodies, and this injects a necessity into the scenario not accounted for in (FF). In consequence, there must be a fact "over and above" the fact that such-and-such forces happen to arise in each collision. That fact must either be corporeal impenetrability as a brute, essential property, or some other property which guarantees that interpenetration never occurs, like the force-faculty.

Either of (IF) and (FFF) could satisfy this demand, however, and there is less room for principled choice between them. Admittedly, we, with our hindsight, would certainly be more inclined to say that it is certain force-faculties (i.e., the electromagnetic force) which are the "sources" of impenetrability, such as it obtains. What could Euler's reasons be for preferring (IF), in historical context? I claim that Euler's move only makes sense in view of his reductive *project* – to explain the forces that move bodies in terms of their general basic properties.

That is, it seems his inference must, really, have been licensed by the *expectation* that forces will originate from or be grounded in the essence and general properties of bodies.[138] This is not a first-order evidential reason, but a meta-theoretical justification for choosing one path over another in the absence of decisive empirical evidence. (FFF), then, is scotched by Euler's methodological procedure. There is no question that forces must enter the picture to get a powerful, viable mechanics. And the force-faculty may in fact always produce forces sufficient to prevent interpenetration. But this fact about the force-faculty itself would need explanation, from Euler's perspective. And this explanation is only to be found in the fact that impenetrability is necessary to the *concept* of material body. Therefore, forces take on derivative significance in relation to a property which is conceptually necessary: impenetrability.

Suppose this is sufficient to bridge the apparent logical gaps so far. What other puzzles are there? One seeming puzzle, mentioned above, is that impenetrability is an absolute property and has no magnitude, yet it is responsible for producing forces of determinate magnitude. Impenetrability alone is compatible with forces of a wide range of magnitudes arising in collisions. What other factors determine the magnitude of the forces that the impenetrability will produce in order to prevent interpenetration?

Euler asserts a minimal criterion: the impenetrability will produce the *least* such force necessary to avoid interpenetration. In some texts, this economy is expressly said to be a

---

[138] That expectation, itself, relies on further assumptions. Ontologically, that 'material body' names a class of entities with a well-defined essence. Epistemologically, that we have (a priori) access to that essence. Methodologically, that it makes good sense to define the subject matter of a field of inquiry prior to and independently of the concepts, tools, and methods used to study it.

consequence of another law of nature, the principle of least action, or conversely that this economy of the use of forces is the basis for the principle of natural economy. Could this provide the explanation?

> "Ainsi dans le choc des corps leur impénétrabilité ne fournit toujours que la plus petite force, qui est capable de les garantir de la penetration et c'est sans doute sur cette circonstance, qu'est fondé ce principe si general, que tous les changemens au monde sont produits aux moindres dépens qu'il est possible, ou avec les plus petites forces, qui sont capables de cet effet." (*Recherches*, §26)
> (Thus, in the collision of bodies their impenetrability furnishes only the smallest force that is capable of safeguarding them from penetration, and it is doubtless on this circumstance that this very general principle is founded, that all changes in the world are produced at the least possible expense, or least possible force capable of producing this effect.)

Here, there is a somewhat loose invocation of the principle of least action. Later commentators have found the appropriateness of this doubtful. Much later, Jacobi, in his lectures of 1843-4 on variational mechanics, took Euler to task on this point, for equivocating on the meaning of 'least action,' since producing the least *force* in a collision does not necessarily correspond to involving the least *action* defined as the product of mass, speed, and distance.[139] Further, whether this condition is even sufficient to uniquely determine the unfolding of the forces in observed collisions is unclear, to say the least. Gaukroger, as we saw, noted that it would be economical, in *some* sense, for the forces in collisions to simply bring the bodies to a halt (1982, 148). In fact, insofar as the account of forces in collisions should also be able to handle the inelastic case, it should be possible for "prevention of interpenetration" to look like this in some cases. The account should also allow for it. So, the implied objection to Euler is that, if economy is our only aim, we should expect colliding bodies *always* to simply come to a halt.

[139] See also Pulte (1989).

This is a neat philosophical point. And it does seem to deftly rebuff the appeal to natural economy. But I think that, in fact, it does not join the problem, at least as Euler sees it. As he develops this theory in the *Recherches*, the next move is to observe that collisions reveal two kinds of bodies. Collision will, in general, involve deformation of the colliding bodies. As a consequence, bodies will come in two kinds: those that, after collision, are restored to their previous shape, and those that retain the deformation.

Though it is true that there are ways of resisting that proposal, they arguably must rely on special interpretations of 'action' and 'least action of nature' that themselves have no deeper basis than Gaukroger's intuitions about what would be "most economical." Further, while this criterion is easy enough to understand if we are thinking about hard spheres, the story will need to be more complicated if we are considering actual bodies. This is because some of the force will be directed into deforming the colliding bodies, and not simply into changing their states of motion. In general, this may involve a complicated interchange of forces with the elastic aether, which exists in the various open pores or channels as well as in the closed pockets contained within the nominal regions of the bodies. The criterion of being sufficient to avoid interpenetration seems too weak to determine the precise way that real collisions happen. So, this line of justification seems unpromising.

It may seem Euler is really in trouble, then. Euler does not actually have the resources, in the form of corporeal impenetrability, to work out all the quantitative details. This is, in effect, admitted in a recent, sympathetic look at Euler's account, one that regards it as provisionally successful. Brading and Stan claim that, by 1750, Euler came to view a "qualitative causal account of collisions—in terms of impenetrability—[as a] complete physics" of collisions (2024, §5.5). A quantitative account, or an account in terms of microlevel causes, was not necessary. On what amounted to a re-conception of the goal of a physics of collisions, Euler's account, they argue, is largely successful.

I think it is both correct and important to observe that Euler is, here as throughout his work, re-conceiving the remit, standards, and methods of physics, mechanics, and metaphysics. All the same, I think that Euler does not even have to cede that much to make his limited point here. The sentences preceding the just-quoted passaged from the *Recherches* explain the real reason for the "economy" of nature in her use of force to prevent interpenetration: once the forces have succeeded in acting so as to eliminate the risk of interpenetration, there is no longer a reason for them to continue to act.

5.2.2 Impenetrability as the origin of all manifestations of force

Euler rather audaciously concludes that impenetrability is in fact the origin of *all* forces that exist in the world: they derive from the circumstance that impenetrable bodies must shove each other out of the way when on a collision course. Impenetrability produces repulsive, externally-directed forces. But it would also have to account for manifestations of force that, prima facie, look attractive. The second way that Euler's reduction of force to impenetrability may fail is that, even if it does account for the forces generated in collisions, it may not be able to handle all manifestations of force.

Two other conceptions of 'force' which are singled out for rejection are forces that act at a distance and those which have the character of 'internal forces.' I will focus here on Euler's attempt to account for fundamental attraction. He rejected it on both of these grounds – it operates at a distance and has an 'internal' character. Impenetrability, or the "force of impenetrability," must then produce the forces that appear descriptively as attractions. That is, Euler must explain phenomenal attraction via impenetrability and without making recourse to attraction as a fundamental posit.

This means that Euler faces a standard objection to any theory that requires all apparent forces to be constituted, at the fundamental level, out of contact forces. Certain known forces prima facie have properties incompatible with forces of collision. This is especially true of the force of gravity, regarded as a universal attractive force, and magnetism, both of which show attraction, and gravity also being proportional to the masses, rather than the surface areas, of the attracting bodies. So it looks as though any theory based on the interaction of bodies at their external surfaces is doomed to fail to account for gravity. This objection is based on the premise that gravity is a genuine, universal attractive force. Euler is aware of this objection (*Anleitung*, §144). Indeed, as he himself notes, the empirical fact that bodies fall at the same rate in vacuum implies that the force of gravity experienced by a body is proportional to its mass. This scotches any attempt to account for the force of gravity as a stream of particles impacting the body, because such a force would vary with the shape and volume of the body, instead of its mass.

How else can we explain gravity, as it is observed to really act, while retaining a theory of force as communicated exclusively through local, contact action? Gravity cannot be due to a stream of particles, as we have observed. So he will opt for a 'pressure' mechanism for gravity. But note that, although Euler calls it a "pressure" (*Druck*), it cannot simply be due to a pressure gradient as ordinarily understood. Such gradients – like the ones we observe in bodies of water or in the atmosphere – themselves derive from gravity, so, on pain of circularity, gravity cannot be due to a pressure gradient of *that* kind.

Instead, the pressure is hypothesized to be due to the elastic force of the subtle matter. Euler's explanation posits that subtle matter has the distinguishing property of having an innate elastic force, and then leverages the mathematical analysis of elasticity which he gave in Chapter 18. By its effort to expand, the subtle matter presses on coarse matter. In order to derive the variation of gravity with distance, Euler finds that the presence of massive bodies (i.e. their proper part, consisting of coarse matter) must weaken the elastic force of the subtle matter as it nears them. That way, bodies near, say, the earth are pressed by the expanding aether to a greater degree at their parts farthest from the earth, producing a net acceleration towards the earth. As long as the diminution of the elasticity with distance has the right functional form, the result will be a net downward-directed force that varies as the inverse-square of distance.

This route may seem curiously circuitous. Why does the elasticity of the aether vary this way? In arriving at this new question have we really gotten any farther than we had by simply positing an inverse square mutual attraction to begin with? One may well, then, wonder why we should have taken this detour through subtle matter in the first place. Indeed, this is one of the readier objections to Euler's theory of force. Euler has to make recourse to a preferred set of faculties – 'impenetrability' and 'elasticity– where others would have used different ones, like 'active forces' or 'active principles' like attraction. The challenge is that, unless further investigations into the nature and mechanisms behind these faculties are forthcoming, or at least arguments that such investigations will be possible, the attractionist will be able to protest, with some right, that, if attractions are "occult," they are no more so than aethereal elasticity.

The distinction between Euler and the attractionist, however, is that Euler issues a promissory note for an eventual explanation in keeping with mechanical principles. The attempt can even be read as an attempt to make good on Newton's demand for an "agent," which he famously expressed in a letter to Bentley:

> "It is inconceivable that inanimate brute matter should, without the mediation of something else, which is not material, operate upon, and affect other matter without mutual contact; as it must do, if gravitation, in the sense of Epicurus, be essential and

> inherent in it. And this is the reason why I desired you would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers."

Euler's answer is that the agent of gravity is a subtle, elastic matter permeating the heavens, which can accelerate matter through contact action because there is a gradient in the degree of its elastic force. The next, important question is: how does the elastic force of the aether come to acquire this gradient?

Euler gives a 'how-possibly' explanation of the reduction in elastic force near "attracting" bodies. It is well-known that motions can weaken the elasticity of elastic materials. So, he hypothesizes that the reduction in the elasticity of the aether is due to a motion in it which is caused by proximity to large, coarse bodies. Of course, that effect itself calls for a physical explanation: why do the heavenly bodies set the aether in motion in this particular way? After the lengthy journey which brought Euler to this particular form of subtle matter hypothesis, here he starts to run out of explanations:

> "Although we have to stop here and can hardly ever hope to find the cause of the diminution of the elastic force of the aether, it is easier to resign to this [*sich damit begnügen*] than to merely maintain [*blosserdings vorgiebt*] that all bodies are by their nature endowed with a force to attract each other. For since one cannot even form an understandable concept [*einen verständlichen Begriff*] of this attraction, one can by way of contrast at least understand [*einsehen*] how it is possible that the elastic force of a liquid matter is reduced, and one also understands that this can occur in a way that is in accordance with the laws of Nature." (§146, Hirsch translation)[140]

[140] In the original German: "Denn, da man sich von diesem Anziehn nicht einmal einen verständlichen Begriff [illegible] kann, so kann man im Gegentheil zum wenigsten überhaupt einsehn, wie es möglich sei, dass die elastische Kraft einer flüssigen Materie vermindert werde, und man begreift auch, das [illegible] auf eine den Gesetzen der Natur gemässe Art geschehen könne."

Let's take stock. What has Euler achieved, and what questions are left unanswered? In one respect, it could be said, the explanation on offer does not advance us any further towards answering core questions about the nature and origins of gravitational attraction. It simply pushes the explanation back one step, since we will, of course, next want to know from where derives this other capacity of the coarse matter to exert forces on the subtle matter, which seems just as in need of explanation as the original, mutually attractive forces. And Euler is quite explicit here that the hopes for this further explanation are dim.

Does Euler nonetheless achieve something, or at least point in the direction of satisfying explanations? To answer this, it is important to acknowledge that the pessimism Euler expresses in the above passage is a real puzzle. It would be one thing if Euler admitted that he does not already have the full explanation in hand. It is curious, though, especially after coming this far, that he further insists that we "can hardly ever hope" to find it.

For, first, this explanation should have seemed eminently worth pursuing, given widespread epistemic standards to which Euler was a subscriber. Even Newton himself – at least in some passages, like the above – claimed that we should hope, ultimately, to find a mediating agent for gravitational attraction, even if he never sketched such an explanation himself. Second, nothing about the hypothesis obviously proscribes further inquiry. It seems clearly open to further empirical and theoretical investigation. If coarse matter is causing subtle matter to lose its elasticity, there must be an interaction between them. Supposing, as Euler does, that the coarse matter might do so by imbuing the subtle matter with motion, there must, in particular, be an exchange of forces. And if there is an exchange of forces, these can be quantified and theoretically investigated using the tools of mechanics that Euler went to such lengths to develop. They can then be related mathematically to the elasticity and other properties of the ether, which can be expected to have other empirical traces that can be searched for.

The puzzle, then, is to understand the reasons why Euler was so pessimistic about this inquiry. Understanding those reasons is a task I cannot undertake here. I will merely point to some possible implications of the character of those reasons for the question of what Euler's gravitational hypothesis can be said to achieve. Suppose, first, that the reasons turn out to be based on relatively contingent or pragmatic matters – like empirical access or calculational tractability. That would be all well and good. The explanation may well be on the right track, but it could turn out to be beyond the power of human faculties to push the explanation any deeper. His reasons for thinking that there are such insurmountable obstacles to human cognition could be questioned, but the philosophical questions this raises would arguably stop there.

But suppose, instead, that the reasons for Euler's pessimism are theoretically motivated. Then the difficult question about what this explanation is supposed to achieve returns with a vengeance. For instance, maybe it is because the demand for mechanical explanations of all material properties and all corporeal action cannot in principle be satisfied here. In that case, we may need to appeal to a bare "faculty" of coarse matter, by which it causes certain motions in subtle matter, which motions in turn produce variations in the strength of the elastic force of the required form. Is this not then an "occult" faculty? Let's not over-state the criticism. The resulting picture has *some* advantages over fundamental attraction. It may be possible to have it turn out to involve only *local* action. We have *some* mechanical understanding of how elastic force, in general, can change through motion. And, with Euler's arguments from the property of inertia in mind, it does not run afoul of the inertia of matter. On all of these counts, the elastic aether hypothesis does *better* than fundamental attraction. However, this would leave them alike in one important respect. They would each be relying on a fundamental corporeal faculty that, in principle, cannot be explained.

The reason impenetrability was so long favored in the wider community is that collision is purported to be intelligible. In Euler's case, the additional reason was that impenetrability was supposed to be the very essence of body. Epistemologically, it was as fundamental and unquestionable as the concept of body itself. Ultimately, these difficulties pose a problem for Euler's epistemology, but I must defer discussion for future work.

5.3. Responding to objections of Gaukroger and Wilson

Two of the sustained treatments of Euler's natural philosophy are the articles by Gaukroger and Wilson. I have discussed their content above, but not covered every criticism they leveraged. For the sake of completeness, in this last subsection I note some miscellaneous criticisms they leveled against Euler. As I will explain, I think that Euler has replies available to him.

Gaukroger doubted whether impenetrability could count as an 'external force', in the required sense. Specifically, he wonders whether, in a collision of bodies A and B, the forces generated by body A's impenetrability and which are responsible for changing the body B's state, must not be in some sense *internal to A*. If so, Euler cannot avoid "internal forces." On the other hand, if they are external to A, then they are external to both A and B, and would therefore seemingly have to act at a distance (1982, 145). In the face of these difficulties, Gaukroger is led to the conclusion that the existence of repulsive forces simply will not be explained on these lines; rather, these forces simply must be *posited*, in the vein of a Kant or a Boscovich (1982, 148).

The answer, to put it quite simply, is that Euler does not reject internal forces as such, but internal forces *of self-change*. The forces said to be internal to A are, in this situation, forces which tend to change the state of body B. As such, this objection does not cut against the account. Nevertheless, this does nothing to mitigate the related objection I raise in subsection 5.2.3.

Curtis Wilson uses facts about the circumstances of the *Anleitung* to motivate skepticism even about Euler's own confidence in the account contained therein. The *Anleitung*, which has already emerged as a focal point for examinations of Euler's metaphysics of nature, was of course never published, and even its date of composition is somewhat doubtful. It is not known why it was not published. The possibilities are numerous. Perhaps he came to repudiate its philosophical core and decided against publishing it. Or perhaps he simply could not find a publisher, or got carried away with other work. He soon after perfected the mathematics of fluids and solid bodies that was in preliminary form in the *Anleitung*, so he might have preferred to move on with other projects rather than to go back and update all of those parts. Within at most a few years, he was already composing his *Letters to a German Princess*, which may have replaced the *Anleitung* as a statement of his natural philosophical views, so that he saw no need to publish it. At present, we just do not know.

Wilson, however, favors the "he thought better of it" explanation. Speculating on why this was so, Wilson writes:

> "Can Euler have had doubts? He ought to have had, of course; his metaphysics is impossible. Impenetrability does not imply extension or mobility; mobility, even with the principle of sufficient reason thrown in as an added premiss, does not imply inertia as understood in modern physics. Nor can forces be derived from impenetrability; if any derivation is possible here, it will be of impenetrability in a restricted sense from forces of a certain kind. Euler, a rationalist believing in innate ideas, has failed to examine the strict logical relations among the ideas he takes as

> fundamental. And, like other rationalists before him, he has confused logical implication, or something pretending to be such, with causality." (Wilson 1992, 416)[141]

As indicated in section 5.2.1, I basically agree with the assessment that Euler confused logical implication with causality, or at a minimum, tried to gloss over his leap from the one to the other. And I agree that the PSR together with mobility does not yield inertia in the intended sense, but at most the weaker sense of 'persistence.' But I do not agree with Wilson's accusation that Euler demonstrated a failure of basic logic, in particular in regard to the connections between extension, mobility, and impenetrability. In opposition, I would propose that we let the assumption that Euler's logic is sound guide us in understanding what he intended by such concepts as 'extension', 'mobility,' and 'impenetrability.'

The claim is: "Impenetrability implies extent and mobility, and in consequence also the persistence. Therefore, if one ascribes to bodies impenetrability, one must also ascribe to them the other properties." (§38)

It is true that it is difficult to see how mobility, say, is "contained in" the concept of impenetrability, the way 'unmarried man' is sometimes alleged to be contained in 'bachelor.' But Euler was not thinking in terms of conceptual containment. Rather, he thought in terms of what we would now think of as set containment – or as he thinks about it, relationship of species to genus. What he meant, indeed very clearly, is that the class of impenetrable things is wholly contained in the class of mobile things (as well as in the class of extended things), and that this can be appreciated from consideration of those concepts together with, perhaps, other a priori resources.

---

[141] Gaukroger also grapples with the apparently puzzling nature of these inferences, and decides that we should not regard them as "inferences" at all (1982, 141). (I note that he is examining the presentation of the theory in the *Theoria motus corporum solidorum* of 1765. But, as he writes, the theory remains quite stable from the 1730s to the 1760s.)

Anything that is impenetrable is something that has extended existence in space. Assuming a three-dimensional ambient space, there is nothing that could count as penetration of something that fills less than three dimensions, since penetration implies reaching the interior of something.[142] Conceptually, it only makes sense to speak of impenetrability when there is a volume of space that cannot be penetrated. So impenetrable things are extended things in space. But there is, further, nothing in the concept of space that provides resources to pin something to a particular location. So, anything that exists at some location in space can also exist elsewhere at some other time (note: exists *in* space, hence excepting space itself). Mobility just means possible existence elsewhere in space, so the thing is also mobile.

When it comes to 'persistence,' I have already argued that this bare, quasi-logical property does follow, but that Euler indeed cannot get to the law of inertia with a priori resources. In sum, inertia aside, all of this looks pretty kosher, as far as it goes. But to repeat an earlier point: I agree that it does not go as far as Euler thought it did. He was wrong to convert impenetrability into a faculty, and nor is this complaint unfairly anachronistic. Seeing the invalidity of this inference does not depend on a modern understanding of electromagnetism as responsible for the repelling forces that keep my body from falling through my chair.

---

[142] There is admittedly some freedom of interpretation here. One could regard a point, say, as trivially 'impenetrable', just because nothing could count as penetrating a point. But in such a case, 'impenetrability' is not a faculty that prevents something that might otherwise happen. So it seems that we must take Euler's understanding of impenetrability to require something stronger – that there must actually be something that would count as penetration, otherwise impenetrability is trivialized. For another possibility, one might think that a line or surface could be penetrated in the sense that it could be broken or punctured, so that the concept of impenetrability has application to beings that lack extent in three dimensions. But this cannot be what is meant by impenetrability. A (three dimensional) body can likewise be broken or punctured, but this is no point against its being impenetrable.

6. Conclusion

In this chapter, I have provided a sympathetic reconstruction of Euler's natural philosophy, specifically his theory of body and force.

The main upshot is that Euler shows that a physical theory of forces and bodies which includes the laws of mechanics can be grounded in an intuitive conception of body together with some minimal empirical supports – e.g. as needed to establish the proportionality in the second law. As a corollary, this shows that Euler's account is more reasonable than other scholars have given him credit for and calls for a rethinking of received designations of Euler as a rationalist or foundationalist. The chapter, in conjunction with chapter 2, points to further work that is needed to explore the question of how Euler's account, as reconstructed her, illustrates his conception of the relation between mechanics and physics, on the one hand, and metaphysics, on the other.

Chapter 4: Mathematizing Metaphysics: the Case of the Principle of Least Action[143]

Abstract

Standard narratives about physical teleology say its death was a fait accompli of the Scientific Revolution, but the principle of least action (PLA) has been taken to instantiate teleology's survival into Enlightenment physics. Other scholars claim this PLA-based teleological metaphysics fell to general philosophical attacks on final causes. None of these narratives fully captures the philosophical interest of its demise. It illustrates a metaphysics being refuted because it could not be coherently modeled in mathematics, hence directly through mathematization and not by philosophical argument or empirical test.

1. Introduction

A venerable philosophical narrative says that teleological metaphysics died out in the 17th century: first banished by Bacon, Descartes, and Spinoza, it soon fell at the hands of mechanistic physics. The principle of least action (PLA), whose progenitors claimed to derive it from a metaphysical thesis about the operation of final causes in nature, renewed these philosophical controversies in the 1740s, drawing some of the Enlightenment's brightest luminaries into one of the liveliest querelles in the history of physics.

As originally stated, the PLA asserts that all physical processes realize the end of expending the smallest possible 'action of nature,' embodying the immanent final cause of efficiency, economy, or simplicity. The principle was mathematized as the assertion that

[143] This dissertation chapter was independently published as (Veldman 2024). This is an open access article distributed under the terms of the Creative Commons CC BY license, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. The text of the chapter here is virtually unchanged with respect to the published version, with the exception of a few added footnotes.

Nature chooses particle trajectories that minimize $\int mv\, ds$, the path integral of mass times velocity times distance, over the space of possible trajectories between two given points. From this infinite haystack of possible paths, Nature plucks the action-minimizing needle. This teleological interpretation played a role in the principle's early development, a fact which has been leveraged in arguing that teleology continued to enjoy a non-trivial place in Enlightenment physics, pace traditional narratives to the contrary (McDonough 2020).

Nonetheless, this metaphysics, created by Pierre Maupertuis with support from his colleague Leonhard Euler, soon encountered vehement opposition. Eventually — indeed, rather quickly — scientists rejected it. Why? This is the question for which I seek to develop a novel answer.

The core idea of this paper is that it was the act of mathematizing the original metaphysical principle that exposed it to the most serious and direct refutation. This was not simply because mathematization exposed it to precise measurement, empirical test, or other, more familiar sources of disconfirmation. Rather, the metaphysics underwriting the PLA was rejected by scientists because of obstacles emerging immanently within the process of mathematically representing or modelling 'the action of Nature' as a universally budgeted quantity. This entailed translating the implicit and explicit components of the metaphysical story into mathematical correlates. In particular, the metaphysics underwriting the principle became a set of formal demands. Purely formal labors of Maupertuis, Euler, and critics soon revealed they were jointly inconsistent. Mathematics and metaphysics clashed and, in the ensuing fray, the metaphysics lost. The PLA, newly deflated, remained an accepted principle of mathematical physics, and was even theoretically fruitful. But scientists decisively rejected its original metaphysical underpinning.

This idea has not, to my knowledge, received philosophical attention. Existing accounts of the rejection of the PLA's teleological metaphysics credit philosophical trends and arguments against final causes, or attacks from elsewhere in intellectual culture, such as Voltaire's widely-read satires, not mathematization (Pulte 1989; Schramm 1985). If anything, scholars have emphasized the role of mathematization in artificially prolonging the life of physical teleology by giving it a patina of rigor (Stöltzner 1994, p. 36).[144] And, as mentioned, some have proposed that it shows teleology not merely on artificial life support, but playing a significant role in 18th-century science (McDonough 2020).

This is surprising, because the question of 'refutation via mathematics' goes back to the dispute's origins. In a late attempt to defend Maupertuis and the PLA, Euler observed that their opponent Samuel König made a surprising claim: that the PLA and its metaphysical underpinning were not only false, but were "thoroughly overthrown by [König's] demonstrations, not Metaphysical, but Geometrical" (Euler 1753a). Euler objected that the PLA was both mathematically and metaphysically sound. I will argue that, while the PLA's teleological metaphysics may not have been overthrown by the Geometrical demonstrations of König, it was nonetheless refuted and rejected on grounds immanent to its mathematical elaboration.

This development is interesting because it shows mathematics playing a historically new role constraining metaphysics. Philosopher-scientists had previously adduced fundamental physical principles allegedly based on mathematizing metaphysical claims. But, when rejected, the constraints came from elsewhere. Descartes, for example, in Book II of his Principia, argued on metaphysical grounds that the totality of motion is conserved, since

[144] I agree that mathematization may, at first, have had this effect; but the ultimate effect was, ironically, to deliver an even more decisive refutation.

it is caused by an immutable God. He then mathematized the thesis by defining 'quantity of motion' as bulk times speed. But this theory is not plagued by mathematical inconsistencies. Indeed, Leibniz's famous 'brief demonstration' of its falsity relied on two key constraints: the (empirical) law of Galilean free fall and the (metaphysical) principle of equality of cause and effect. Conversely, prior clashes between metaphysics and mathematics could leave metaphysics unscathed. A plausible example, described by Harman (1983, p. 235), is Kant's 1747 *vis viva* essay, where he finds Leibniz's doctrine to be 'false in mathematics' and yet true metaphysically.

The PLA episode thus appears to instantiate a historically novel way of "accord[ing] ontological force to mathematical structure" (Mahoney 1998). Accordingly, philosophers interested in mathematization in science and its history or in the interaction between science and metaphysics will, hopefully, find this analysis of interest. It is partly inspired by, and hopefully provides additional perspective for, the debate over whether there can be non-causal scientific explanations of physical phenomena in which mathematics plays an indispensable explanatory role.[145]

Section 2 gives background and characterizes my objectives; section 3 gives my analysis of the immanent mathematical failure of the PLA's teleological metaphysics; and section 4 argues this led scientists to reject it. The PLA and the philosophical issues it raises are complicated, and have supported a rich scholarship through to the present.[146] It is

[145] These are often called 'distinctively mathematical explanations', following Lange's (2013) terminology. The more recent account is his (2018). For an opposing, deflationary view, see Kuorikoski (2021).
[146] On the richness of the historical debate about the PLA and physical teleology, and its interwovenness with other philosophical and cultural questions, see (Stöltzner 1994), (Schramm 1985), (Lyssy 2022), and (McDonough 2020). On the PLA as an alternative foundation for physical theory, see (Sklar 2012, Ch. 11). On its connection to theoretical
unification of physics, see (Stöltzner 2002, 2005). In relation to the metaphysics of modality, see (Butterfield 2005; Terekhovich 2017), and in relation to the metaphysics of causality, see (Ben-Menahem 2018, Ch. 6).

impossible for me to adequately address the many existing perspectives on the abandonment of physical teleology linked to the PLA, even restricting my attention to the 18th century, but I try to address the readiest objections, especially in subsection 3.2.

2. Metaphysics and the Early PLA

2.1 The Maupertuis-Euler teleological metaphysics

Appeals to principles of nature's simplicity and efficiency date back to Aristotle or even the pre-Socratics, classically under the title *lex parsimoniae*.[147] After a long, relatively dormant period, these typically vague ideas of the simplicity of nature took on mathematical form in early modern physics. Fermat used his 'principle of least time' to derive and explain the classical laws of optics. In Maupertuis's later, more general vision, the Creator instituted a principle assigning Nature the end of carrying out its processes in the simplest, most efficient way possible (Maupertuis 1748a, p. 421). The content of this metaphysical principle is admittedly vague; I will use the label "principle of the simplicity or efficiency (or economy) of nature" (PSEN).

In Maupertuis' recounting, Fermat and Leibniz had appealed to the PSEN in particular applications, most notably optics. Maupertuis followed their lead, observing that the linear propagation and reflection of light, which exhibit how light always 'takes the simplest path,' seem to depend on the PSEN. For Maupertuis, that left the question of the refraction law. He thought his predecessors has gotten refraction wrong and that their success in mathematizing the PSEN was only partial. It was left to him to finally "reconcile

---

[147] In Aristotle, related ideas ('nature does nothing in vain') appear in the *Politics* (Sections II and VIII), as well as in *Generation of Animals* (Section III) and other biological works. Yourgrau and Mandelstam (1968) stress the importance of *De Caelo*'s move from vague ideas of 'simplicity' to that of 'minimization,' a 'proto-mathematization' brought even closer to modernity by Hero of Alexandria's first-century treatment of the classical law of reflection.

[*accorder*] the law of refraction with the metaphysical principle" (Maupertuis 1748a, p. 423). Specifically, his initial project was to reconcile teleological explanation of the optical laws with what he (erroneously) believed to be the correct account of light, the Newtonian view that light consists of particles whose velocity increases with the density of the medium (Schramm 2005). The result was the PLA, which asserts that the action, or $\int mv\,ds$, associated to the possible trajectories of a particle attains a minimum at the actual trajectory (Maupertuis 1748a, p. 423).[148] This definition of the 'action' was Leibniz's, who used it to prove a conservation principle. The thesis that it is minimized is Maupertuis'. Maupertuis claimed to derive all three optical laws from the PLA, and on that basis asserted that the quantity of action was "the true expense of Nature, and that which she economizes [*ménage*] as much as possible in the movement of light" (ibid., p. 423). By recovering the laws of optics from the PLA, he believed he had demonstrated the PSEN's deep truth, having brought the phenomena into accord with that "great principle" (ibid., p. 424).

Independently (and probably chronologically prior), Euler used an equivalent principle, writing down the action integral for a particle subject to a variety of conservative forces, and expressly asserting that it is *minimized* (1744, Addit. II, §2). Nonetheless, he always ceded priority to Maupertuis. For his part, Euler did not boast of *deriving* his physical principle from metaphysical ones. Nonetheless, he expressed hope that such a derivation would be forthcoming, the fruits of an inquiry "into the innermost laws of Nature and final causes" (ibid., Addit. II, §1). Maupertuis' paper likely appeared to him as a timely and welcome contribution (Schramm 2005).

---

148 Note that in this and Euler's initial work, which treated a single particle, the mass was ignored.

The thesis is that there is a fundamental physical quantity, the *action*, which can be regarded as "the action of Nature" in the context of the claim, stemming from the PSEN, that for all changes Nature chooses the trajectory requiring the least amount of it. The PLA was thereby adduced as the physical expression of the PSEN: it represents physically what it means for a natural process to be the simplest or most efficient, as demanded by the metaphysical principle, underwriting a picture of Nature as sublimely economical (or, if you like, 'lazy'). I will refer to this theory as the 'Maupertuis-Euler teleological metaphysics', hereafter METM.[149]

Unfortunately, it is hard to specify necessary *and sufficient* conditions for the success of the METM which do not involve God or design, and I am restricting my concern to its role in physics. But Maupertuis' comments suggest at least three necessary conditions for the PLA to express the operation of the immanent final cause of efficiency.[150] These conditions turn out to be inter-related, but are conceptually distinct:

(Budget) The action is a kind of 'resource' or 'budget'

(Minimization) The action is minimized in the processes covered by the law

(Universality) The principle covers all physical processes

2.2 Varieties of teleology in physics

Philosophers interested in the PLA and teleology have emphasized diverse senses of 'physical teleology,' which, though distinct, are complexly inter-related, making it a delicate

[149] Euler's degree of commitment to the METM is a difficult question which I cannot go into deeply here. Though Maupertuis was its strongest devotee, it is equally clear that, in the period 1744-1753, Euler, across several works, endorsed, defended, and even premised arguments on the METM. That Euler's recognition of unavoidable mathematical difficulties *eventually* led him to discard the thesis is part of my argument.

[150] A fourth condition might be added: the PLA alone *uniquely determines* the actual path. This condition has the least direct textual support in Maupertuis and Euler. Lacking space, I omit it.

matter to carve out a philosophical question about just one variety. To better distinguish the one concerning me here, I will mention other, often-discussed varieties.

Lyssy (2022) views Maupertuis and Euler as carrying on a Leibnizian heritage in the philosophy of science. Leibniz's teleological principles demanded that changes in Nature be the "most determined," i.e. they are characterized by either a minimum or maximum of some quantity. This teleology is 'formal' in that metaphysical significance attaches to the property of extremization, not the quantity extremized. It implies a directive for physics research as well as a unifying schema for physical laws.[151] A related set of ideas, deriving from William of Ockham, views simplicity or economy as (desirable) properties of our theories. The 'final cause' of simplicity characterizes the activity of theorizing, not nature.

By contrast with these 'formal' approaches to teleology, the METM was intended to express the PSEN, a metaphysical claim about Nature and not (merely) a criterion for our theories about it. Maupertuis implied that his success, where his predecessors fell short, was in reasoning to "the end of Nature" and finding "the quantity that we ought to regard as her expense [*dépens*] in the production of her effects" (Maupertuis 1748a, p. 426). What is minimized is always the action, a 'substantive,' not merely 'formal,' final cause.

Maupertuis and Euler themselves attached *multiple* kinds of teleological significance to the PLA. Three aspects of the Maupertuis-Euler picture are reflected in the descriptions of (McDonough 2020, pp. 173–4, 176). First, Maupertuis took the simplicity and unifying power of the principle as the epistemic basis of an inference to *divine teleology*. Second, the PLA suggests that "teleology must operate within the order of nature," in the guise of

[151] Advocacy for using the PLA to unify physics reemerged in the late 19th and early 20th centuries, especially via Planck, though apparently only as a *formal* unification (Stöltzner 2002). For detailed analysis of varieties of 'formal teleology', see (Stöltzner 2005).

efficiency, a kind of *immanent teleology* (ibid., p. 176). Finally, McDonough there gestures at an epistemic notion of "teleological reasoning," related to the notion of "optimality" explanations discussed in (McDonough 2022, §5.4).

Though there is much to say about each of these, I focus on the METM, a thesis about immanent physical teleology, not formal teleology, divine teleology, or teleological explanation. This picture starts with the PSEN, proceeds to formulate a quasi-physical idea of the 'action' of Nature as something it always minimizes, and then mathematizes this principle as 'the PLA': the claim that $\int mvds$ is minimized in actual natural processes, over the class of possible trajectories between fixed endpoints. The METM was arguably the most significant attempt in modern history to establish teleological metaphysics within physics.

2.3 Mathematization and the metaphysics-physics relation

The mechanism of refutation of the METM that I propose is, in Euler's phrase, by "demonstrations Geometrical, and not Metaphysical." Though its falsity is not exactly proved the way geometrical theorems are, it is refuted via formal mathematical considerations. Indeed, when we remain at the discursive level of metaphysical speculation, all three conditions from section 2.1 appear jointly possible, not susceptible to *a priori* philosophical disproof. Seemingly nothing prevents us from positing that Nature has a 'budget' of some universal 'resource' that she expends to produce her effects, and that she is always frugal in her purchases. A metaphysician could not hope to decisively *refute* it, except perhaps on the basis of a prior philosophical opposition to teleology, which may simply beg the question against the project of grounding physics in a teleological world-picture. Even scholars who credit philosophical analysis of epistemological bona fides (e.g. of the teleological 'derivation' of the PLA) acknowledge that this "may not by itself be a sufficient refutation" (Yourgrau and Mandelstam 1968, p. 174).

But everything changed when the principle was *mathematized*. Or so I argue. Once the action was specified as $\int mv\, ds$, the principle was hoisted into the mathematical framework of the calculus (of variations), and the three features Budget, Minimization, and Universality had to be 'mathematized': modeled or represented in the mathematical theory of the PLA. As I reconstruct this history, complications for and criticism of the METM came on several levels, corresponding to these features.

At the bottom-most level, doubts arose about the mathematical definition of the action quantity itself. Critics questioned whether its mathematical properties justified giving it the sense of a 'budgeted resource' (Budget), and doubted whether, as defined, it could be physically or metaphysically fundamental. The middle level concerned the assertion that this (allegedly fundamental) quantity is economized (Minimization), underwriting a view of nature as "intending to be sparing." A thicket of problems soon sprang up for this claim, too.

The only hope of rectifying these difficulties was to adjust the underlying metaphysical picture. Ultimately, however, this path was blocked at the highest level, Universality, which required the PLA to be a representational tool for solving physical problems. The PLA was meant to be a single, general principle, but, in application, Maupertuis was forced to divide it into three distinct forms which lacked a unifying metaphysical basis. Only one of them embodied, in any clear way, the original metaphysics which motivated the PLA, whereas the others appeared as arbitrary in relation to it. Euler attempted to address this difficulty mathematically, with Maupertuis' blessing, but this effort led him into self-contradiction, requiring him to both affirm and deny Minimization, or nature's "intention to be sparing." Critics came to complain that substantiating the PLA's generality required arbitrary reformulations and auxiliary assumptions that belied the absence of a truly universal metaphysical foundation. Collectively, these problems led to the METM's ruin.

The remarkable feature of this route to refutation is that it was entirely formal, based on immanent mathematical problems rather than on metaphysical or empirical argument. The tribunal of mathematics had determined that the METM's component features were not compossible, giving scientists *good reasons* to reject it. In section 4, I show how members of the scientific community highlighted these reasons, and the proponent of the METM most significant to our question, namely Euler, ceased to endorse it, after his valiant but ultimately doomed effort to save its mathematical viability.

I conclude that, around the mid-18th century, *mathematization* came to operate not only as a demand on physical theories to become more precise, predictively powerful, or capable of empirical confirmation, but also as a direct constraint on the metaphysical theses purported to ground them, arising as a consequence of the use of mathematics to represent or 'model' the metaphysics. Although commentators have occasionally remarked on broadly 'mathematical' reasons for the failure of the PLA's metaphysics, none has given the full picture of these reasons. Likely because of this, none has thematized the refutation of the METM the way I do here, or highlighted its novelty. By 'direct constraint' I just mean that complications and inconsistencies arose within the process of producing and working out the model *formally*, with no express attempt to 'compare the model with the empirical world'. Indeed, doubting the empirical accuracy of Maupertuis' or Euler's results was never in question. This episode thus appears to differ from otherwise analogous cases from the history of physics in which a metaphysical picture was mathematized, but was later rejected for more traditional reasons, such as the case of Descartes mentioned above. Implicitly, then, scientists had adopted a new constraint: a metaphysics that is mathematizable must have a consistent mathematical representation or model, and is otherwise discredited or disconfirmed. The presuppositions behind and full implications of adopting such a constraint merit further attention. Here, though, I begin by offering it as a philosophically interesting development in the relationship between metaphysics and physics, mediated by a historically new role for mathematics.

3. Mathematizing the PLA, Unraveling the METM

My analysis in this section is focused on problems emerging internally to the process of working out the PLA's theory, so my attention is largely restricted to the principal early theorists, Maupertuis and Euler. One can perceive their gradually growing awareness that mathematizing the theory of 'least action in nature' invites complications for the three conditions listed above. Mathematization puts significant pressure on each individually. Jointly, it makes them untenable. Or so, at any rate, I will now argue.

3.1 (Budget) Action as 'expense,' 'budget,' or 'resource'

In a paper read in 1744, Maupertuis announce that the action is Nature's budget: "this fund or budget [*fonds*], this quantity of action, which Nature saves up [*épargne*]," representing the amount of 'resources' available for purchasing physical effects (Maupertuis 1748a, pp. 424–5). The expression $\int mv\, ds$ should support this interpretation as measuring an "expense of Nature." Maupertuis' contemporaries, too, wanted to know why that algebraic expression represented the "effort" or "expense" of nature (see section 4).

Maupertuis' discussion of the principle of least time (PLT) shows that he was sensitive to this question from the start (ibid.). The PLT was flawed, Maupertuis wrote, because it got the law of refraction wrong, incorrectly selecting time over space to be the minimized quantity. With no reason to privilege one over the other, we should expect both to appear in nature's budget (ibid., p. 16). Maupertuis further reasons, intuitively, that nature's action should be larger exactly when it moves a *greater mass*, moves it *faster*, or moves it *farther*, an explanation developed in his *Essai de cosmologie* (1750, p. 42).[152] Evidently, Maupertuis understood he needed to justify why the algebraic form of his chosen quantity comported with its teleological interpretation.

Though his explanation has at least some intuitive appeal, an appeal to intuition is no demonstration. Further, Maupertuis' references to monetary expenditure imply that the 'action' should have further mathematical features, those of a currency. Giving content to the claim of 'economizing' need not require a natural or objective choice of unit. Yet the quantity of action must still support a meaningful zero point (compare: a difference between spending positive money and no money). Likewise, there must be a difference between an expenditure of action and its opposite (compare: paying out, versus being paid).

Mathematically, these conditions were not satisfied. Euler observed, in his (1753d), that the sign of the quantity of action (together with its scale) is insignificant, and also indicated that there may be no mathematically principled way to assign to the action a meaningful absolute level, since it contains an additive constant (ibid., §XIX). It was also evident that the action integral can be mathematically transformed at will into other quantities. Since $ds = v\, dt$, a substitution shows that $\int mv\, ds = \int mv^2 dt$, yielding a time-

[152] This conception of the action comes from Leibniz. However, in that context, Leibniz generated this conception of action in order to argue that this quantity is *conserved*.

integral of *vis viva* (§§16-20). Indeed, he had already shown this in (1744), where he was evidently pleased to have shown the compatibility of Cartesian and Leibnizian concepts. What proved embarrassing to the metaphysics is that, in Leibniz's theory, vis viva is conserved. So this algebraic transformation immediately raised two difficult questions for the METM, both later pressed by d'Arcy (1756): (1) whether either quantity can be regarded as the *true* 'action of Nature,' and (2) whether nature minimizes action, or conserves it. Last, problems that could be formulated using path integrals of the purported 'budget' of $mv\,ds$ include little more than the motion of point particles, whereas Maupertuis' ambitions were universal. As I will explain more fully in section 3.3, the quantity so defined is not mathematically, or even dimensionally, apt to characterize all mechanical effects, violating Universality. Collectively, these complications make it difficult to read the action as a quantity of some resource that Nature must economize.

3.2 (Minimization) Action as minimized in natural processes

The action quantity must be *minimized* to do justice to the 'efficiency' aspect of the metaphysical ur -principle, the PSEN, which exposes the METM to the mathematical possibility of mechanical configurations that fail to minimize the action, or else its statical analogue, which Euler called the 'effort.'

In applying the PLA, one demands stationarity of the action integral, i.e. that its derivative (or variation) vanish. It is often observed that, besides minima, maxima and saddle points are also stationary, scotching a specifically minimizing interpretation. If one has not read the historical sources, one might think, implausibly, that Maupertuis failed to understand that stationarity does not imply minimization.

Maupertuis's thinking is better appreciated by comparing his (1740) with his (1748a). The first introduces the 'law of rest' – an important precursor to the PLA on which Euler also wrote several papers (1750a,b, 1753b,d). The law of rest is deduced from the stationarity of a certain expression (in modern terms, the *potential*; in Euler's, the 'effort') which is itself derived using other mechanical principles, through an analysis rooted in ideas of Daniel Bernoulli's. Maupertuis infers that its integral will attain a minimum or maximum, making it an *extremal*, not a minimal, principle. He proposes no teleological underpinning. Euler also attempted a 'metaphysical demonstration' of the law of rest, referencing fundamental facts about forces, *not* teleology (1753b). In Maupertuis' (1748a), by contrast, teleology expressly motivated a minimum condition, and *from* minimization, he inferred stationarity. Maupertuis did not posit a stationarity principle and then mistakenly infer that the action is minimized. Rather, he posited a metaphysics of least action, and mathematized it for use in mechanics, yielding the PLA. As mentioned, Euler also called for an inquiry into the final causes behind the PLA, although he did not undertake it himself.

But does the quantity of action, defined as $\int mv\, ds$, always attain a minimum? And does this hold for other classes of problems supposed to fall under the auspices of the PLA, such as static equilibrium? Mathematical analyses of cases in which the action or effort attained a *maximum* soon posed a problem, the one usually emphasized in historical and philosophical commentary on the PLA, e.g. (Stöltzner 1994; Yourgrau and Mandelstam 1968). Maupertuis never acknowledged in print that maxima of the action are also possible. Euler, as we will see, was more equivocal.

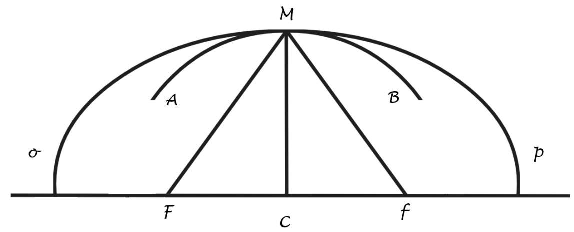
In his (1753d), Euler described the case of a rigid bar fixed at its midpoint, free to rotate, with a weight hung from one end; the opposite end is pinned frictionlessly against a wall such that the weighted end is hefted up. The configuration, in unstable equilibrium, is at a maximum of the 'effort,' an apparent counterexample obtained simply by toying around with a mathematical model.

To discount it, one might search for reasons to ignore the case as 'unphysical,' the way certain solutions to differential equations are thrown out as 'unphysical' (e.g., if they blow up in finite time.) This would preserve the consonance between physical principle and underlying metaphysics. But the case involves nothing more outlandish than 'frictionless contact,' so no such considerations became available. A different gambit for safeguarding the teleological metaphysics is to weaken the thesis slightly. The *spirit* of the METM is that Nature "has it in view to make the sum of efforts as small as possible" (ibid., §XI). Perhaps the action and effort, though not always minimized in fact, always *tend to a minimum*. This amounts to replacing Minimization with:

> (Minimization*) The action (or effort) tends to a minimum; Nature *prefers* a minimum of the action.

The implication is that cases of effort-maximization should always be *unstable*. If perturbed, Nature will not return them to that configuration, because Nature tends toward, and strives to preserve, the effort-*minimizing* equilibria. We could then reasonably conclude that Nature's goal is revealed in its *tendency* towards minima.

This gambit seems to preserve the spirit of the METM, but it also raises further questions. Does Nature have ends that it can *fail* to attain? Worse, other action-maximizing cases emerged. The most important was reflection in a concave mirror, described by d'Alembert (1752; 1754) and d'Arcy (1756); on this, see also (Schramm 2005). Suppose light travels from point *F* to point *f*, reflecting at *M* in a plane mirror. Maupertuis' analysis (1748a) showed that the principle that the path of light minimizes action implies the law of reflection. Place, instead, a concave spherical mirror *AMB* whose tangent plane at *M* coincides with the plane mirror, as in Figure 1. The light's path will not differ, but the other



parameters can be chosen so that this path locally the action. For instance, choose *F* and *f* equidistant from *M* and far enough apart that the ellipse through *M* with *F* and *f* as its foci, labeled *oMp*, passes outside the spherical mirror. By elementary properties of the ellipse, paths through points on *AMB* to the left or right of *M* have strictly lower action. (A second derivative test can be used, though the computation is somewhat laborious.) It appears neither Maupertuis nor Euler directly replied to this objection, which seems an obvious embarrassment to the view.

Figure 1: Reflection in concave mirrors can maximize action; based on (d'Arcy 1756).

Indeed, it is so obvious that it may seem to imply an objection to my account. It is trivial to see to see that the teleological reading fails, since it demands a minimum of the action. For example, Schramm (2005, p. 112) portrays the maximization of the action during reflection in a concave mirror as the single, decisive objection to the METM.[153] If this

[153] In their classic book on variational principles, Yourgrau and Mandelstam draw the same conclusion from a different problem illustrating that particles take non-action-minimizing paths, namely orbital motion under a Coulomb force (1968, p. 175). They appear especially interested in having this objection, for the METM seems to postulate a "mysterious purposive agency," and is compared with scientific bogeys which are "dangerous because of their metaphysical insinuations" (1968, p. 173). It is understandable that those who view physical teleology as a monster would want a silver bullet to kill it. My argument is merely that it was not that simple.

interpretation is correct, my account is largely redundant. Am I making much philosophical ado about nothing?

The failure of minimization with concave mirrors was, indeed, in some sense, "obvious." But this was not regarded as an automatic or direct refutation of the METM, and nor is it, logically, a sufficient refutation. One need not even retreat to weaker, Leibnizian versions of physical teleology requiring only extremization, as described, for example, in (McDonough 2022). Those wishing to retain the specific teleological metaphysics of 'efficiency' underlying the PLA could retouch the metaphysical picture without changing its spirit. As a start, this might involve adopting Minimization* to accommodate problematic cases involving unstable equilibria. If we ignore the other conditions on the METM, the concave mirror could also be accommodated. I cannot work out an auxiliary hypothesis in detail here. But here is how one possible response might go. Observe that, in the concave mirror case, no local minimum exists in the first place. Recast the PLA as the conjunction of two claims: (1)"*Whenever* the geometry of the constraints admits of a minimum of the action, then Nature will choose etc.. . . ," and (2) all geometries allow a minimum. Concave mirrors only refute the second claim, allowing defenders to hold onto the first as an expression of Nature's preference for minima. This way of avoiding the counterexample would of course lead to calls for further explanation. But the conditions required for minima had yet to be explained mathematically and so, especially in this vacuum, an enterprising metaphysician would surely be up to the task of suitably elaborating the account.

Thus, the failure of Minimization was not, by itself, a 'silver bullet.' Indeed, that view has trouble accommodating history: if correct, it would make a puzzle of the fact that scientists scrutinizing the PLA did not cite concave mirrors as an automatic and decisive refutation.[154] As exhibited in section 4, opponents nonetheless developed a rich, *formal* critique based on a plurality of mathematical considerations falling into the categories I identify and thematize.[155] All three conditions were needed to block avenues for rescuing the overall metaphysical picture: they amount to sacrificing the METM's picture of action as a universally budgeted resource. Given all three conditions, the modification sketched above is no longer available, particularly violating Universality. It was collectively that these conditions allowed mathematics to firmly grab onto the METM and hold its metaphysical feet to the fire, leading scientists of the time to conclude that the PLA expresses nothing metaphysically deep — at any rate, nothing metaphysically deeper than the accepted laws of mechanics. With that said by way of forestalling an objection, I now turn to completing the analysis.

3.3 (Universality) The PLA as a universal mechanical principle

For both Maupertuis and Euler, the importance of the PLA, and the truth of its teleological interpretation, relies on its claim to generality. Its importance "consists in its Universality" (Euler 1753e, p. 201). This is what distinguishes it from the 'least time' and 'least resistance' principles of Fermat and Leibniz. As Lyssy puts it, Fermat's optical law was allegedly "not a proper law by itself", but at best derivative of the PLA (2022, p. 134). At worst, it was only "*par un pur hasard*" that their choices of the end of Nature yielded accurate formulas. By

---

[154] Euler himself understood that the mathematics he largely invented did not fundamentally distinguish between maxima and minima (nor would Maupertuis have been ignorant of this). Yet he persisted for years supporting the METM. Scholars have noted that Euler's 'preferential treatment' towards minima is something of a puzzle (Pulte 1989, 229n). Here is not the place to attempt to resolve it definitively, but note that the 'silver bullet' view requires us to regard him as foolish or self-deceiving, whereas my account exhibits a complexity to the case that makes reasonable an evolution of his attitudes over time.

[155] Existing scholarship, though rich, generally ignores the many other mathematical considerations contained in the responses of other prominent scientists like d'Arcy and d'Alembert. The works that do discuss these figures, like Pulte (1989, pp. 225–30) or Schramm (1985, pp. 150–7), have aims that do not lead them to analyses of the kind I give here.

contrast, the PLA, in its generality, embodies "the reason for these phenomena," the universal, immanent final cause of physical events (Euler 1753e, p. 208). Universality was also why the PLA was supposed to be superior to *non*-teleological principles, like conservation of *vis viva*, which, Maupertuis observed, is "true only for certain bodies" (Maupertuis 1748b).

That the end of Nature really is to minimize action is allegedly proved by the fact that the PLA can be used to derive the laws for *all* physical processes, which Maupertuis endeavors to demonstrate in his (1748a,b), also citing (Euler 1744). [156] So, setting aside considerations of mathematical tractability, the minimization of $\int mv\, ds$ should be a criterion universally applicable to nature. By consequence, as a mathematical tool, it should be suitable for solving all mechanical problems. However, a suite of issues arose when the principle was translated into forms suitable for that task. In his (1748b), Maupertuis attempted to show that the PLA is "*si universel et si fécond*" that from it can be derived, not only the laws of optics, but also the law of the lever and the rules for elastic and inelastic collisions.

Unfortunately, the usual modeling procedure, in which one first assigns a quantity of action to each possible trajectory of the particle, is unsuited to these new problems. Take equilibrium problems. By hypothesis, the bodies are not moving. There is no nontrivial 'path', and the action is necessarily zero. There is simply no way to extract any information by imposing the PLA condition. As for collisions, the bodies are moving, and so their trajectories do possess a nonzero quantity of action. But here we are not interested in their

[156] This is their *claim*. Lagrange later objected: "But it must be admitted that these applications are too particular to serve to establish the truth of a general Principle [of mechanics]" (Lagrange 1788, p. 188).

trajectories. We seek their velocities post-collision. For that, the PLA as stated in 1744 is not a suitable representational tool.

The way Maupertuis addressed these problems looks a great deal like sleight-of-hand. He wrote two new "statements" of the principle, one each for collisions and for equilibrium: "In the Collision of Bodies, Motion is distributed in such a way that the quantity of action supposed by the change is the smallest possible. In Rest, the Bodies in equilibrium must be so situated, that if they were subject to a small Motion, the quantity of action would be least" (Maupertuis 1748b).

By what right does Maupertuis offer these as formulations *of* the PLA? It looks as though he simply asserted two *new* principles, banking on their resemblance to the original PLA to pass them off as equivalent assertions. To see this, let $\int mv\, ds$ (a functional over trajectories) be the baseline definition of 'action.' Then the version of the PLA used to analyze optical laws, PLA_O, indeed says: nature chooses the trajectory which minimizes *action*. Call the version used to analyze collisions PLA_C. The application of PLA_C in Maupertuis' text shows that it comes to this: nature chooses the final velocity that minimizes *the instantaneous rate of change of action*. Finally, the version applied to static equilibrium, PLA_E, asserts: nature chooses, as the equilibrium state, that configuration for which the *action associated to an infinitesimal virtual movement away from that configuration* is a minimum. We are no longer even talking about 'possible paths', and the principle is now, logically, a conditional: *if* the system is in equilibrium, *then* it minimizes what might be called the 'virtual infinitesimal action'. This last concept, moreover, has dubious mathematical bona fides, insofar as it is defined by integration over an infinitesimally short 'path'.

The present point is not to deny that PLA_O, PLA_C, and PLA_E are related. But they are different assertions, with different content. (Indeed, as mentioned, PLA_E, or the 'law of rest,' was not even teleological when it originally appeared in (1740).) They each assert the minimization of a different quantity relative to a different definition of the space of possibilities. And they are suitable for treating non-overlapping classes of problems. [157] Translating the metaphysical principle into a tool in the mathematical discipline of mechanics was supposed to exhibit that it is the unique teleological law governing the course of nature. But in actuality it gave rise to a problem for that metaphysics. For, we no longer have a single principle governing physical processes at all, but an ensemble of principles which at best bear a loose conceptual association to each other. The formal properties of the original PLA that were supposed to express the METM are not even shared among its new 'versions.'

As for Euler, initially, he was delighted to see the PLA find wider application. But he also noticed these difficulties. He tried to address them formally, by demonstrating that these principles are mathematically equivalent. A proof of the equivalence of the law of rest (PLA_E) and the 'proper' PLA of 1744 (PLA_O) was first sketched at the end of (1750b), then treated at greater length in (1753d). Maupertuis himself, in concluding his *Réponse* to d'Arcy's first attack, referred, one imagines with relief, to Euler's proof of the equivalence of the law of rest and the principle of least action (1754). Both of them appear to have understood that the significance of the principle and its metaphysical purport was at stake.

[157] The principle PLA_E has a further peculiarity. When it is applied to the equilibrium of the lever, for example, it gives the position x of the fulcrum that *would* leave the lever balanced. This will be the 'actual' position of the fulcrum, not by *Nature's* decree, but only should the experimenter choose to place it there.

What about this proof? The only direction treated in detail proceeds from the law of rest (PLA_E) to the 'proper' PLA (PLA_O). Euler's proof relies on a number of auxiliary assumptions, one being the principle of conservation of vis viva. Indeed, Euler already understood the need to restrict the principle to conservative systems in his (1744). Others had remarked that the need to import this independent energy constraint weakens the PLA's claim to universality. It is, indeed, difficult to see how the resources of the METM could motivate this condition. The most striking assumption in the proof, however, is a premise to the effect that action minimization *locally* implies action minimization over the whole path. As it turns out, there is no *mathematical* justification for this assumption, since it is not a mathematical theorem. Where, then, does it come from? It is a *metaphysical premise*, itself based on the METM. Euler reasoned: "For if the intention of Nature is to be as sparing as possible with the sum of efforts [in equilibrium], this [intention] must extend also to movement, provided we take the efforts, not only as they subsist in an instant, but in all the instants through which the movement lasts, taken together [i.e., the action]" (Euler 1753d, §XII).

Euler deployed a (metaphysical) premise about the intentions of Nature in an otherwise mathematical proof of the equivalence of these two principles. Even so, what he eventually proved is not that $\int mv\, ds$ is minimized, but that $Const. - \int mv\, ds$ is minimized. The constant may be determined by the particle's initial velocity, but even setting that aside, the sign is the reverse of what was intended, an awkward result. Of course, the mathematical procedure for applying the principle to find solutions to problems in mechanics is unaffected by a change of sign. Yet his excuses go further. He wrote: "But though the difference between a maximum and a minimum may seem large, *it is, however, of no consequence in Nature itself*' (Euler 1753d, §XIX, my emphasis).

This is striking. Euler had just leaned on the METM to provide a bridge from local to global minimization. This was to *justify* his equivalence proof, allowing him to rescue Universality. If, as he wrote, "the intention of Nature is to be as sparing as possible," one would naturally expect that the difference between maximizing and minimizing must be of some consequence (recall 3.1). But he was forced to deny that very claim and, in so doing, to sacrifice a core component of the teleological metaphysics. He tried to purchase Universality by sacrificing Minimization, while at the same time using 'Nature's intention to minimize' in his argument for Universality.

By now, I submit, the features of the mathematized concept of action, as prescribed by the underlying metaphysics, have given rise to severe contradictions, providing excellent reasons to reject that metaphysics. Maupertuis may have sought to give teleology new life by placing it on the solid foundation of mathematics. But in doing so, he appears, ironically, to have exposed himself to a novel and distinctively mathematical way of undermining his metaphysical project.

4. Rejection of the Teleological Metaphysics of the PLA

I conclude that, within about ten years, the advocates of the PLA faced contradictions between the metaphysical picture, motivated by the PSEN, and the mathematical resources used to model it. But did these reasons in fact operate as an effective constraint on metaphysics? I argue that they did. This is best appreciated by seeing how such considerations informed the scientific community's rejection of this metaphysics. I have already hinted at the reactions of the likes of d'Alembert and d'Arcy, and a closer look would seem to confirm that considerations of the kind I identify motivated their own critiques of the METM.

D'Alembert was no friend of teleology, and might easily have dismissed the METM for general philosophical reasons of obscurity or lack of rigor, as he did with other teleological ideas. This is not what he did. Indeed, in his article 'Final causes' (1752), d'Alembert discussed Maupertuis only to approve of his attacks on fallacious uses of final causes, staying silent on the PLA. Elsewhere, in the article 'Action,' he expressed skepticism about the algebraic from of the action quantity. The most extensive discussion of the PLA resides in the article 'Cosmologie' (1754), which contains the most measured and detailed critical analysis I have come across.[158]

D'Alembert was prepared to take Maupertuis' ideas seriously: the article praises Maupertuis for turning the vague PSEN into a precise principle of mathematical physics. Nonetheless, d'Alembert brought a sharp critical eye to the PLA and the METM. His analysis agrees with my own on many matters of detail, but more importantly, in its overall character: the METM falls victim to a suite of problems immanent to its formal elaboration, falling into the categories I identify. For reasons of space, I provide only a synopsis. Against the METM, d'Alembert cites: the non-significance and "arbitrariness" of the algebraic combination $mv\,ds$("we can make as many mathematical combinations as we like of these two things [space and time], and can call all of this *action*; but the primitive, metaphysical idea behind the word *action* will not be made any more clear" (p. 297); that certain problems Maupertuis tackles with action are more appropriately formulated via *vis viva* ("but when we substitute here the word *vis viva* for that of the *action*..." p. 296); the failure of minimization

[158] Though co-authored with Samuel Formey, d'Alembert wrote the second part, concerning the PLA.

with concave mirrors (p. 295); the need to formulate mathematically distinct versions of the principle for different problems ("the principle applies to many other cases, with some modifications that are more or less arbitrary," p. 297); and the difficulty in making sense of the action associated to changes taking place in zero time ("in the case of hard bodies, the change happening in an indivisible instant, the time is zero, and in consequence the action is null" p. 296). These considerations parallel those I raised regarding Budget, Minimization, and Universality. Evidently, he was concerned with much more than just the failure of Minimization. Though he mentions the problem of concave mirrors briefly, the majority of his analysis concerns other difficulties. By the end of the article, d'Alembert both circumscribes the domain of application of the PLA and withholds support for the METM.

D'Arcy is more polemical, but echoes many of these points (his articles and the *Encyclopédie* cite each other). Worth particular mention is his attack on the algebraic expression $mv\,ds$ as representing the 'action of Nature', the issue I called Budget. This is in part because he wishes to defend his own theory, arguing that nature's action both has a different algebraic form and that it is actually conserved. He also forcefully presses the failure of Minimization, and a potential fourth problem, also formal, which I did not discuss: that the PLA alone does not always determine a unique trajectory (see note 7).

As for Maupertuis, he did not give up on the teleological interpretation of the PLA that he invented. But the significance of his position should not be overestimated. Aside from the critics just mentioned, Samuel König dragged him into an ugly priority dispute, claiming that Maupertuis's PLA was both false and plagiarized. Voltaire penned a vicious satire of him. Clergymen attacked him for trying to supplant their preferred teleological arguments for God. Embattled, his reputation and his priority claim at risk, and his health degenerating, it was an inopportune time to publicly reconsider the significance of his discovery. Maupertuis was also most committed to the theological implications of his principle, giving him additional stake in the viability of its metaphysics. This indicates apportioning less significance to Maupertuis' attitudes on the scientific merits.

In my view, Euler's attitude shift is more significant, not only because of his preeminence in the Enlightenment, but because he was the earliest and most vocal and consistent ally of Maupertuis. Although he never, as far as I am aware, repudiated the METM in print, the evidence here is telling. As we saw, Euler had to internalize serious contradictions between the METM and the implied features of the PLA's mathematical theory. Given that, as Maupertuis' ally, it would have been awkward for him to acknowledge these tensions publicly, I regard it as indicative of Euler's attitudes that his advocacy on behalf of the PLA effectively ceases after 1753. It was around then that he made a last, valiant effort, publishing a defense of the PLA (1753e), a reply to König (1753c), and an octavo book collecting his defenses along with the original attack by König (Euler 1753a). But after this, he ceased to discuss teleology in connection to action, despite enjoying many more active research years. Nor was he without opportunities to reflect on foundational and philosophical issues in physics. Touching on the topic of action in Letter 78 (dated November 22, 1760) of the enormously successful *Lettres à une Princesse d'Allemagne* (1768), Euler remained deafeningly silent on the subject of teleology, once so intimately associated with the PLA. Happy to credit Maupertuis as the PLA's discoverer and, noting that it once elicited both extraordinary praise and extraordinary criticism, he wrote that the principle is nothing more than a consequence of the impenetrability of bodies. Somewhat slyly equivocating on the word 'action', he had more or less returned to the view pre-dating his association with Maupertuis: the PLA is merely a consequence of the properties of bodies, no longer attributed to Nature's "intention to be sparing."

Although the mathematicians who continued work on variational principles did not provide detailed excursions into metaphysical interpretation, what they do say seems to further confirm the role of mathematical considerations in warding off physical teleology. As Fraser (1983, p. 233) notes, Lagrange was comfortable speaking of 'least action' in his early correspondence with Euler, but by 1760 the phrase disappears from his official lexicon in his memoir on the delta-calculus. It reappears in some historical remarks in the *Méchanique analitique* of 1788, where Lagrange recounts the controversy over whether the action is minimized or conserved (à la d'Arcy). Dismissing the whole debate, he wrote: "as if these vague and arbitrary denominations constituted the essence of the laws of nature, and could, by some secret virtue, erect the simple results of the known laws of mechanics on top of [a foundation of] final causes" (quoted in (ibid., translation modified)).

Lagrange also complained about Maupertuis' *applications* of the PLA to diverse problems: "[T]hey have, besides, something vague and arbitrary about them, that could not but render uncertain the consequences that we might draw regarding the exactitude of this Principle" (Lagrange 1788, p. 188). The failure of Universality and what Lagrange here calls "vagueness and arbitrariness" are sides of a coin. The allegedly universal PLA had a form unsuited to static equilibrium and collisions. Maupertuis addressed this by a fudge: (vague) constructions involving 'infinitesimal virtual paths' and (arbitrary) decisions to instead minimize the *rate of change* of the action. Lagrange clarified that the true, general principle, which he himself generalized out of the work of Maupertuis and Euler, is an extremal, not a minimal, principle. He concluded: "Such is the principle I, however improperly, here give the name 'least action', and which I regard not as a metaphysical principle, but as a simple result of the laws of Mechanics."

The fourth of Jacobi's *Vorlesungen über Dynamik*, delivered in 1842-3, also invoke formal considerations for rejecting the METM. The PLA is formulated in the modern notation of variational calculus:

$$\delta \int \sqrt{2(U+h)} \sqrt{\sum_i m_i ds_i^2}$$

(where *U* is the potential and *h* an arbitrary constant). Jacobi then comments: "It is hard to find a metaphysical cause for the principle of least action when it is expressed, as it must be, in this true form" (Jacobi 1866, p. 45). The descendent variational principles of mechanics had thus quickly ceased to be interpreted teleologically by those developing the theory. And its development continued in the absence of a proof that the action is universally minimized. Indeed, Jacobi's tools (the theory of conjugate points) for determining conditions for minima likely only further undermined the proposal that minima possess fundamental (meta-)physical significance. The force of these formal reasons has been borne out by discussion of the PLA into subsequent centuries. In reference works for physicists, for example, one finds lists of considerations which, from the perspective of contemporary mathematical physics, defeat the teleological metaphysics which originally motivated the PLA (Gray 2009). Many of these notably diverse reasons rely on later theoretical developments, but they are all best understood as formal, or as immanent to the mathematics.

5. Conclusion

In the 1740s, a metaphysical thesis asserting the efficiency of Nature came to underwrite a new, general principle of physics. But, as I have argued, the process of working out these ideas soon led to contradictions between the metaphysics of 'least action in Nature' and the mathematical theory which was supposed to provide a rigorous foundation for it. As a result, this teleological metaphysics could no longer find footing in its mathematical representation. Formal, mathematical considerations — not *a priori* philosophical reasoning or empirical test — resulted in the rejection of a metaphysical thesis.

Mathematization was supposed to finally place physical teleology on a rigorous foundation. Ironically, it provided novel, scientific reasons to decisively reject it.

Chapter 5. Conclusion

The preceding chapters have focused on the implications of Euler's interventions into metaphysics. What kinds of causes are there in the world? What are bodies, and how do they act on each other? Is there room for teleology in natural science? Spelling out Euler's answers to the just-listed questions led us to observe, at various points, that his interventions in metaphysics are intimately tied up with moves in epistemology and methodology. In this chapter, I provide some perspective on the state of things in philosophical interpretation of Euler, before motivating four research questions or sets of questions that I regard as emerging from this dissertation.

The clearest work yet to be done is to provide an explicit account of Euler's epistemology and, with it, his place in the history of modern European philosophy. The analysis in the preceding chapters begins to provide resources for such an account, however, it does not attempt to tackle that job head on.

A more complete assessment of the meaning of Euler's philosophy for the history of philosophy might be expected to begin with a close analysis of his philosophical allegiances and influences. Such an analysis would doubtless begin, as I largely have in this dissertation, with the three largest signposts in 18th century natural philosophy. The first is Descartes and the Cartesian tradition, with further representation in the likes of Rohault or Malebranche. The second is Leibniz and the Leibnizian tradition, as it was furthered by the Bernoullis on the analytical side, and by Wolff, Formey, and König, on the philosophical side. Third is Newton and the Newtonian tradition, as represented on the Continent by Maupertuis, Clairaut, or Voltaire. Du Chatelet represents a special case for positioning herself as synthesizing scientific and philosophical thought across these traditional divisions. Of course, in this respect, she is much like Euler.

These are the reference points referred to in practically all existing commentaries on Euler's thought, but there is little agreement about where to place him. Euler has been called a Cartesian (Gaukroger 1982, Wilson 1992), an upholder of the great German Leibnizian tradition (Lyssy 2023), as well as an epigone of Newton (Radelet-de-Grave and Speiser 2004) – even one whose role was "merely" to apply and articulate the theory of the *Principia* with rare mathematical prowess (Kuhn 2012, 32-3).

Often these apparently incompatible ascriptions are made in the same work. In some cases, such dual or treble ascriptions are handled with considerable care. As advertised in the title of a recent paper, Lyssy attempts to exhibit the "Leibnizian metaphysics behind the principle of least action" which Euler had done so much to develop, while also acknowledging that Euler rejected important facets of Leibniz's metaphysics and took important cues from Newton (Lyssy 2022).

At other times, however, these dual ascriptions can have a jarring effect. For instance, in the Introduction to Series III, Volume 10 of the *Opera omnia*, the editors write that, on the one hand: "One cannot emphasize enough that Euler wants to conduct his entire investigation within the framework of Newtonian mechanics; this is cardinal to understanding his ideas" (Radelet-de-Grave and Speiser 2004, 63). On the other hand, they write that Euler was "a follower of Descartes and of his teacher Johann Bernoulli" (ibid., 112). It seems reasonable to ask: *how* is one to conduct one's entire investigation in the framework of Newtonian mechanics while remaining a follower of Descartes? At the very least, this would seem to require Euler to aggressively compartmentalize the various aspects of his most basic scientific commitments that are at odds with each other. The state of the scholarship is thus complicated by crisscrossing ascriptions of philosophical allegiance, making it hard to see where to enter this web of interpretations. If we try to sort through everything at once, it will be easy to get lost.

I. Was Euler a Cartesian?

As a practical matter, it would be ideal to start with just one figure and one question. A decent starting point would be: Was Euler a Cartesian?

One reason to start there is that scholars who have looked at Euler's philosophical disposition, as opposed to his mathematical physics, have tended to categorize him as a Cartesian more often than in another tradition. Two scholars who place particular emphasis on the Cartesian foundation of his thought are Curtis Wilson and Stephen Gaukroger. Wilson, criticizing Euler for the defects in his foundations of physics, writes that "Euler, a rationalist believing in innate ideas, has failed to examine the ideas he takes as fundamental" (Wilson 1992, 416). We could hardly ask for a more direct statement. In explicitly aligning Euler with the Cartesian doctrine of innate ideas and the Rationalist tradition that followed him, Wilson has decided that Descartes is Euler's most important philosophical inspiration. Gaukroger is in agreement. While affirming that Euler's goal is to underwrite a mechanics in a Newtonian style, Gaukroger assesses his metaphysical project thus:

> "I suggest the procedure here is Cartesian, even if the results are not. In asking why a Cartesian procedure was adopted, at a time when Cartesian mechanics had effectively met its demise, we must remember the great appeal of Cartesian method, with its procedure of building up an apodeictic mechanics on the basis of self-evident and necessary foundations.[159] […]

---

159 It is somewhat equivocal to say, in this context, that "Cartesian mechanics" is apodeictic. The rules for collisions are, of course, supposed to be apodeictic. But the physics as a whole is hypothetical.

> To simplify somewhat, for Euler as for d'Alembert it was a question of squeezing Newtonian mechanics into a Cartesian shape, a shape which, it was hoped, was to render it more certain and more fruitful. The certainty of the new mechanics was to derive from its structure, but more importantly from the nature of the foundations of that structure. *These foundations were of a rigorously Cartesian type: they could not be doubted without fear of contradiction. This, I suggest, was the essence of Euler's foundational project.* Newtonian dynamics, suitably reformulated, had to be shown not just to be true but to be necessarily true." (Gaukroger 1982, 133-4; my emphasis)

It has arguably not been emphasized enough how astonishing this conclusion would be, if it were true. Euler remained scientifically active essentially until his death in 1787. If he was a committed Cartesian, the implication is that Cartesianism had major defenders not only after Newton and Leibniz had left their mark on philosophy, but well into Kant's Critical Period (the First Critique was published in 1781)! It would be surprising, indeed, if a philosophy put forward ca. 1640 had not merely survived a century and a half, but a century and a half containing so many revolutions in scientific method.

Whatever truth may be contained in Gaukroger's explanation, it is at best incomplete. It is observed that Cartesian mechanics was obsolete while Euler continued to use the methods characteristic of Cartesian mechanics. Why? Because of "the great appeal of Cartesian method." But then, if that method still enjoyed such great appeal, why was the mechanics judged obsolete? Put another way, if both the appeal of Cartesian method and the failure of Cartesian mechanics were widely appreciated, and assuming that the consensus view is accurately reflected in the judgment that "Cartesian mechanics had effectively met its demise", why did *Euler* buck that trend? Indeed, Gaukroger doubles down in his conclusion thus:

> "The idea that in providing the foundations of a physical system we must automatically be concerned with questions of self-evidence, indeed intuitive self-evidence, and necessity, is a Cartesian myth. It cannot be denied that Euler's attempt to realize this myth is remarkable, but it is a myth nevertheless." (Gaukroger 1982, 154)

I believe my work puts serious pressure on this assessment of Euler's views, and on his classification as a Cartesian. On the one hand, if my account of Euler's view of the relation between metaphysics and science is roughly correct (as in Ch. 2, §4), then Euler clearly rejects and even reverses Descartes on the epistemic priority of *a priori* metaphysics over physics. Scholars have taken the presentation in texts like the *Anleitung* and *Theoria motum corporum solidorum* to suggest just such "a Cartesian procedure," where knowledge of physics is based on metaphysical first principles. Yet, if we look at the range of what Euler has written on the topic of metaphysics and mechanics, we can see that this order of presentation (starting with conceptual analysis and metaphysical first principles and deriving laws of mechanics) does not mirror the *epistemological* order of priority.

On the other hand, what are some key features of Euler's thought that his interpreters have read as Cartesian? Two important cases are the doctrine of innate ideas, mentioned by Wilson, and the appearance of references to "clear ideas" or "clear concepts" in Euler's work, which evoke Descartes' epistemic criterion of clear and distinct perception (discussed briefly in a note in Chapter 2, §5.2). These are taken as marks of Euler's true philosophical allegiances.

Yet, the evidence for this is thin. I think it is clear that, for Euler, the status of our concepts of, for instance, *material body* is not "revealed, innate, and apodeictic," but only "adequate for classification" (as discussed briefly in Chapter 3). He certainly does not produce his items of conceptual knowledge from the perspective of the *cogito* or ever explicitly invoke clear and distinct perception to justify his analysis of the concept of 'body'.

What's more: it makes perfect philosophical sense for him not to appeal to these canons of innate ideas or certain knowledge. Why? Descartes' epistemology is founded on skeptical doubt. And Euler is simply not motivated by responding to the skeptic the way

Descartes is. Euler appears to advocate for a kind of "naïve realism" in response to skeptical challenges in *Lettres* Vol. 2, Nos. 115-120. Insofar as he responds to skepticism, it is of the Pyrrhonist kind, not Cartesian radical doubt (Nos. 118-119). It would be curious for a philosopher who took his epistemological cues from Descartes to pass over these core ideas in his thought.

II. Euler's overall epistemology: how does he integrate 'rationalism' and 'empiricism'?

Ultimately, though, this inquiry would have to go beyond the confines of the question "Cartesian or not?" In fact, it would do so of itself. For, at least some of the reasons that Euler cannot be considered a Cartesian is because of his acceptance of key tenets of Newton's or Leibniz's thought. However, it is exactly this feature that naturally raises the next question: given the superficial appearance of eclecticism, as it were, does Euler have a theoretically unified system, and if so, what is it? Detailing Euler's epistemology of science is a large task, but it might usefully be approached by attempting to situate Euler along another major interpretive axis for modern thought: rationalism and empiricism.

In the early modern period, classical epistemology consisted of questions about the sources, scope, and certainty of human knowledge. Traditionally, the debate is understood as a battle between the Rationalists and Empiricists. The sources named by the Rationalist tend to be *a priori* and innate, where those named by the Empiricist are *a posteriori* and learned through sense experience. What were the sources of knowledge, according to Euler, and what were the kinds of certainty within human reach? How much falls within the reach of the human knowledge faculty. What falls outside it, and what options do we have to deal with what cannot be known with something like certainty?

I have suggested that, while there are Cartesian echoes in Euler's works, what he has to say explicitly about knowledge and our access to it suggests a non-Cartesian approach. But full treatment of Euler's epistemology would have to take into account the variety of his positions. For instance, his occasional use and implicit endorsement of the principle of sufficient reason, his various uses of *intelligibility* as a (negative) criterion for certain physical concepts (like action-at-a-distance), and his use of conceptual analysis. How does Euler think we acquire concepts, for instance, and what does that imply about the epistemic heft of conceptual analysis?

Consider the last issue, for example. Insofar as Euler's procedure appears to be *a priori*, and based on the contents of our concepts, it may seem to have the flavor of Cartesian intuition, as Gaukroger suggests. Or perhaps it may even seem to have the character of a classical empiricist 'inspection of our ideas.' But Euler never invokes the source of his insights like Descartes does: he makes no reference to intuition or to the divine light. Further, this "concept of body" is not a Lockean idea or Humean impression, for the reason, at least, that Euler's attention to it does not involve pushing oneself back inside one's "mind" to examine the "ideas" one perceives there.[160]

Indeed, Euler does not say much about how exactly we acquire or come to enjoy access to this concept and its features. Our conceptual capacity is taken as given, and its features are brought out by assessing our natural responses to real and imagined scenarios. That seems to mean that, for Euler, our access to *a priori* truths via our concepts does not derive from any *mental scrutiny* of the contents of our minds. It is realized or revealed in our

[160] Indeed, scrutiny of our ideas in this sense seems quite foreign to Euler's way of thinking. Relatedly, Euler has no patience for idealism (see e.g. the *Reflexions sur l'espace et le tems*), and, in spite of his willingness to entertain hypotheses, seems to countenance no separation of a layer of "appearances" from the underlying "reality." This 'flattening' tendency to his thought accords with the impression some have had that Euler moves seamlessly between metaphysics and physics – in Lyssy's estimation, a frustrating tendency (Lyssy 2023).

use of those concepts to distinguish empirically real particulars as falling into different categories corresponding to those concepts. If we imagine something moving along at uniform velocity on a collision course with a body, and then observe that it passes through the body unperturbed, we would conclude that it was not, in fact, a body.[161] (The appeal is always, grammatically speaking, to the judgment of the "we" and not, as in Descartes, to "I.") As best one can judge, this is the concept of body with which we each of us "get around." That Euler should take it to have such pervasive consequences and unimpeachable bona fides within physical theory is only the more remarkable given the reappearance of ideas of this character in the later Kant (see below).

III. Euler on scientific method and hypotheses

Related to his epistemology are Euler's views on *method* in the sciences and in philosophy. His views on hypotheses are of particular interest, especially in connection to his characteristic ways of deploying them as a practicing scientist. In the early modern period, hypotheses were a central battleground in the conflict between Cartesian, Newtonian, and Leibnizian ways of understanding the natural world. Euler, together with figures like Emilie Du Chatelet, staked out a particularly embattled position. As they saw value in each of these main traditions, they were liable to be attacked by representatives of all of them. Euler even lavished praise on her chapter 'On Hypotheses.' Further, Euler competed in, and won, the same 1738 essay competition with his own dissertation, in both of which they put forward

[161] This is not an idle metaphysical point, either, about ghosts or angels. The explanation of transparency, bodies through which light freely passes, was a point of contention on which Euler in part rested his case for the wave theory of light and against emission theories (Hakfoort 1995). If light consists of small corpuscles, then in passing through transparent bodies, they must be making their way through channels in those bodies. But light passes through from all directions equally. The body would have to be shot through with so many channels as to leave nothing left. Although competing with Euler (1752c), Du Châtelet *Dissertation on fire* (1744/1752) makes similar arguments as Euler's against a view of light as consisting of heavy, impenetrable particles in her.

their own distinct hypotheses on the nature of fire. Exploring these connections promises to add depth to our understanding of Enlightenment thought on hypothesis and methodology.

IV. Euler in connection to Du Chatelet and Kant

Euler's thought presents tantalizing connections to that of Du Châtelet and of Kant. Scholars have noted these connections but have only tentatively attended to them. These connections call for sustained investigation, for which this dissertation partly paves the way. So, as a final topic, or collection of topics, I hope to take up the question of Euler's relations to each of these figures. Insofar as the same is often said of Du Chatelet – that she prefigures Kant in various more or less significant ways – there is good cause to study all three of them in conjunction.

Euler and Du Châtelet, as mentioned, both raise the question of connection to Kant. Euler argued that core metaphysical questions (e.g. whether space is absolute or relative) should be decided by reference to the conceptual requirements of empirical physics. Du Châtelet argued that space, though in fact ideal and relational, is constructed by our minds so as to appear as a real entity with roughly the properties of Newtonian or Euclidean three-dimensional extension. These sound a lot like arguments Kant made – three or four decades later. And Kant is also known to have read them both closely. Yet, how deep his similarities go remains an open question.

Seeking an answer may even provide resources to help move long-standing interpretive debates about the meaning of Kant's philosophy. Kant's transcendental period is constantly taken up with the subject of cognition and this subject's experience. Yet, it is still a matter of disagreement among Kant scholars whether to take this subject as something like a representative of *the scientific community*, a kind of abstract and constructed subject of scientific cognition, or else as *an individual human, cognizing subject*, not at all abstract. At least in connection to the cognition of space, these two interpretations roughly correspond to an "Eulerian" gloss and a "Du Châteletian." Knowing more about the lines of influence might make one or the other of these interpretations more likely.

Next, themes from Chapter 3 suggest that closer investigation might reveal affinities between Kant and Euler that are more pervasive than their respective views on space and its ontology. For example, Euler's ca. 1750s treatise *Anleitung zur Naturlehre* shows that he recognizes a class of *a priori* insights into nature, and the essence of categories of being like 'material body'. They are based on insight into the conceptual schemes that order our experience of nature. Revealing these Kantian themes in earlier thinkers opens new vistas on Kant's later thought, which is often viewed as having Hume as its sole stimulus.

Last, but not least, there is the question of the metaphysics-science relation – but now looking forward to Kant and not merely backward to Descartes. Especially in Chapter 2, I take myself to have gone farther than the existing scholarship in laying out the nature and scope of Euler's re-conception of the relation between metaphysics and science, first broached by Speiser (1934). Although my analysis proceeded through the particular lens of the notion of 'force' and was not based on Speiser's, my conclusions broadly harmonize with his aim of connecting Euler to Kant. I do find, especially as explained in §§3-4 of Chapter 2, that Euler advocated for a reversal of the order of priority between metaphysics and mechanics that had maintained its dominance, especially on the Continent, notably through the influence of Descartes, Malebranche, Leibniz, and Wolff.

This is very redolent of Kant, who reconceived metaphysics as a source of synthetic *a priori* knowledge to be possible only insofar as it was an investigation of the conditions of possibility of cognition or experience. The *Metaphysical Foundations of Natural Science*, in particular, lays out what he takes to be those conditions for mechanics and physics. Importantly, he sought to show how to "construct" the most fundamental concepts of these sciences, and to recover the laws of mechanics (as he understood them).

Euler does not quite conceive of the task of metaphysics as a search for "conditions of possibility," a framework he never used. A (metaphysical) condition of possibility is a *sine qua non*, a (metaphysically) necessary condition. The most Euler asks of metaphysics, at least prima facie, is to deduce the laws and concepts of mechanics from its principles. This would make the metaphysical principles at most *sufficient* for the laws of mechanics. On more or less these grounds, DiSalle (2013) judged that Euler's conception was "hypothetico-deductive" rather than a true instance of Kant's "transcendental method", a method which he reads into Newton. Of course, even in the case of Kant, it is a *question*, for each condition he identified for a given form of cognition, whether it is in fact *necessary* for constituting that form of cognition or else only *sufficient* (or, for that matter, neither).[162] So it would, in any case, be open to Euler to respond to DiSalle's finding of fault: and so what?

That is, it could simply *be Euler's position* that metaphysics cannot hope to be any more than hypothetico-deductive in relation to the laws established by science, taken as the "givens," and that Kant went too far in seeking to establish that its conclusions could be

---

[162] Consider, for example, Kant's well-known argument from "incongruent counterparts," a pair of objects that are exactly alike except for the fact that one is a mirror image of the other, like a left and a right hand. The impetus for the example came from Leibniz's 3rd letter to Clarke, in which he argued that a world in which "east" turned into "west" and vice versa would be indistinguishable from the original; absolute space would treat these worlds as different, without a reason for doing so. Kant picked up on this case and thought it would be a problem for Leibniz. Specifically, he intended this to show, *pace* the relational theorist of space, that there is no relational criterion that can account for the difference between a left and a right hand. Space, therefore, cannot be explained in relational terms. Since Newtonian "absolute" space is likewise problematic, we are faced with an antinomy, says Kant.

Et voilà: transcendental idealism, our *only* way out of the antinomy. Yet, Falkenburg (2020) argues that transcendental idealism is in fact *not* necessary to escape or dissolve this antinomy, but at most sufficient. In her view, new solutions have since appeared that depend on later philosophical and scientific developments. And even aside from those developments, it is not obvious why Leibniz could not have bitten the bullet and insisted there was no difference, e.g., between our world and the world "flipped" east-west.

necessary, as well. There is, thus, not only the question of philosophical influence and transformation from Euler to Kant, but also the question of whether Euler might have reason to reject the direction taken by the later Kant.

Appendices.

Appendix A. Euler's apparent reversal on the legitimacy of the "force of inertia" concept

The works containing the apparent reversal seem to be a pair of papers published a decade after the *Mechanica*, and which represent two forays into expressly metaphysical debates. Though confusing at first sight, my solution to the puzzle will be to explain why these instances can be set aside, as not expressing his considered view. The first comes in the *Enodatio*, where Euler addresses the question whether matter could be endowed with the power of thinking, a hot topic after Locke broached the issue.[163] In that article, Euler leverages the law of inertia in service of a negative proof. He notes that all philosophers agree that bodies are endowed with a '*vis inertiae*', which serves as a force of by which bodies conserve their states. The power of thought, on the other hand, is a power an entity has to change its own states, as witness, for example, in our ever-changing thoughts and perceptions.[164] Euler reasons that if bodies, in addition to possessing a force for conserving their state, also possessed a force for altering their state, this would lead to a contradiction, since the effects of these forces are in opposition. So, since it is universally agreed that bodies have the former force, they must necessarily lack the latter. Matter cannot acquire the power of thought.

Whatever we think of this syllogism, Euler is clearly helping himself to the notion of 'force of inertia', a notion which he had just indicated should be done away with. The case is similar with another text of that year, the *Gedanken* (1746), the polemic against Leibnizian-Wolffian metaphysics that we encountered above. Recall that his strategy in his main

---

[163] In his *Essay* (IV.3.6).

[164] The resemblance to the arguments canvassed earlier against monadological 'active forces' may be noted.

argument is closely analogous to the argument of the *Enodatio*: the internal forces of monads are, analogous to the power of thought, forces of producing changes in the state of the entity. The force of inertia is a force of conservation and of resistance to changes of state. And these cannot coexist as essential properties of the same entity. Once again, to make this argument, Euler has to assume the 'force of inertia' (or *vis inertiae*):

> "16: This force of all bodies to remain in their state is called in the theory of movement the *vis inertiae*, and it is as general [*allgemeine*] a property of bodies as the extent, such that a body without this force would cease to be a body...
> 17: It is not possible to imagine this *force* that enables bodies to remain in their state, without at the same time ascribing to them a force to resist all changes. For if a body were to undergo all changes without resisting them in the slightest, one could not say that it is endowed with a force to remain in its state."

So his acceptance of the conception of body offered by *vis inertiae* certainly appears decided at this point in time. There seem to be two plausible interpretive possibilities here.

The first is to say: nonetheless, this *apparent* acceptance of 'force of inertia' is best bracketed, if we consider the argumentative purposes of these papers. First, neither is a work of mechanics or physics. Euler's target is, in the one instance, a view about the nature of minds, and of bodies in connection to their distinguishability from minds. In the other, the purpose is a refutation of monadology. And in both articles, Euler's strategy is to leverage a generally accepted proposition of mechanics, the law of inertia. So it makes good sense that he would take on board what was then the accepted and familiar expression of that proposition, rather than air his doubts about it and about the conceptual obscurities it entails. I propose that Euler should be read as accepting *vis inertiae* for the sake of argument only. He did so because his opponents in these debates were working from that assumption. Further, in the context of this debate, it did not matter whether inertia is thought of specifically as a force or else as a property in more generic terms. (If we take the reading where inertia is attributed to body only in a generic way as a property, it will still be in tension with the power of thought and with the monadological forces as Euler describes them—at least, this does not put them in any *less* tension.) Indeed, at most the rhetorical effect would suffer, insofar as the odor of contradiction is sharpened when a *force* of persistence is pitted against another *force*, one of change. So, as I indicated, I propose that we think of Euler as bracketing the controversy latent in the notion of 'force of inertia'. Neither of these argumentative contexts were appropriate for opening debate about the proper conceptualization of the first law of motion. And so we should not seek his actual theoretical commitments regarding mechanics in these texts.

The second is to accept that Euler's position on the legitimacy of the 'force of inertia' concept wavered in the 1740s. But even so, I do not think this is properly taken as evidence of lack of conviction or strong reasons. Rather, I think it was likely a reflection of the fact that he was thinking very consciously about what is expressed by calling inertia a 'force'. He was receptive to, and indeed, at one point, strongly moved by, the reasons that may well have moved Newton and others to regard inertia *as* a force—as something like a power, or a source of corporeal action. Because his eventual decision that inertia must *not* be considered a kind of force was made in full awareness of these considerations, this only gives further weight to that decision.

Appendix B. Euler's rejection of fundamental attractions

Euler rejects fundamental attractive forces as a purported explanation of gravitation, whether the fundamentality claim is understood as the claim that that such forces are essential to or inherent in bodies, or else as the claim that attraction is a 'brute fact' that can have no deeper explanation. His reasons for this rejection are adjacent to some of the complaints raised in the body of this chapter about other concepts of 'force'. In particular, he here, yet again, approaches the subject of attractive forces with caution on the basis of his sense that we are nearing a border-crossing into the territory of metaphysics:

> "It now remains, that we enquire into the cause of these attractive powers; but this research belongs rather to the province of metaphysics than of mathematics. I dare not, therefore, flatter myself with the prospect of assured success in the prosecution of it." (Letter 68)

As I have been arguing, his treatments of many topics in natural philosophy were often part of his philosophical advocacy for circumscribing metaphysics. However, these restrictions were not motivated by dogmatic empiricism so much as a desire for metaphysics to be kept in its own lane both for its sake, as well as for the sake of mechanics. And here, yet again, we see that the restrictions extended even to what could even be regarded as physical theorizing – even what we would *now* regard as physical theorizing. Indeed, it is not entirely clear to me why Euler should view inquiry into the causes of the "attractive powers" of bodies as belonging to metaphysics. Elsewhere, for example in Chapter 19 of the *Anleitung zur Naturlehre*, he attempts to offer an aethereal account of gravity

By 'universal attractive force', I will understand a force with which every material body is endowed – perhaps in virtue of being a material body or perhaps as a 'brute fact' – and by which it mutually pulls on, and is pulled on by, every other material body in the universe. Euler never doubted what we might anachronistically call the 'empirical adequacy' of Newton's Law of Universal Gravity; it will never lead us into error, to calculate forces by its means. (This is the conclusion, for instance, of Letter 54 of *Lettres*, Vol. 1, after he reports the experiment in South America that was taken to have measured the deviation of a plumb line due to the gravity of a nearby mountain.) Yet he never accepted the hypothesis that there was a genuine, primitive force of attraction being exercised between every pair of bodies in the universe – that is, at a distance and without a medium.[165] This led him to hold out for a kind of ethereal or vortex theory of gravity throughout his life, a fact which many, including confidantes of his like Bernoulli, believed was an unfortunate intellectual error unbecoming of someone of Euler's stature. How could someone so brilliant make such a grave error, if indeed an error it was? What were the bases, philosophical and scientific, of Euler's rejection of universal attractive forces?

His criticisms of universal attraction, it turns out, were many. Nonetheless, their philosophical basis was of a type with that described in the previous sections. What (celestial or terrestrial) mechanics needed was only the mathematical form of the law of universal gravity. Hypotheses about a 'real attraction' represented speculation into the causes of this behavior. The quotation which heads this section indicates that Euler counted this as a metaphysical inquiry. This would place the invocation of attractive forces to explain the phenomena of gravitation in close analogy with the invocation of 'force of inertia' to explain the persistence-behavior of bodies. However, the parallel ceases when we reach the issue of alternatives. In the case of explaining inertia, Euler believed (at least at one point) that the proprietary tools of metaphysics could do the job: the principle of sufficient reason explains inertia. When it came to the phenomenon of attraction, though, Euler actually thought that a

---

[165] Nor, for that matter, did Newton himself.

mechanical explanation would be forthcoming. In the remainder of this Appendix, I survey his criticisms of the universal-attraction hypothesis.

Many of the criticisms Euler made were already in circulation since at least the time of the Leibniz-Clarke Correspondence. Given what we have learned in the previous section about Euler's extensive attacks on one of the main edifices of Leibniz's thought, it may surprise one to hear that Euler was likely echoing many of Leibniz's criticisms in raising his own doubts about universal attraction. These doubts were aired, for example, the early *Enodatio quaestionis* of 1746, and also much later in the *Letters to a German Princess* (esp. Letter 68). In these pieces, we find a variety of complaints.

There are three complaints directed towards the interpretation whereby fundamental attractions reflect innate or essential properties of bodies. First, we find (a) very traditional complaints about 'action-at-a-distance' forces. Such forces require bodies to be able to act where they are not, which many, including Euler, took to be unintelligible. Another traditional complaint is (b) that fundamental attraction, regarded as an essential property of matter, would be an *occult quality*. The proposition is that material bodies *just do* have this power of mutual attraction. Euler replies:

> "The ancient philosophers contented themselves to explain the phenomena of ht eworld by those kinds of qualities they named occult, in saying, for example, that opium makes one sleep by an occult quality that makes it suitable for procuring sleep: this is to say nothing at all, or rather it is to wish to hide one's ignorance; we ought to regard attraction, as well, as an occult quality, insofar as we take it as an essential property of body" (Letter 68)

Next, we find (c) a kind of revulsion of the idea of attributing quasi-desiderative or quasi-intentional states to bodies, taken to be an implication of the idea that bodies tend toward other bodies with which they are not in contact.[166] This, too, seems like a traditional

[166] Incidentally, this is comparable to some of his complaints against monads which I did not discuss above.

complaint, but Euler takes it in an interesting direction. In the *Enodatio quaestionis*, these arguments are explicitly related to arguments that bodies cannot possess a faculty of thought – part of the debate over the possibility of "thinking matter" in the early 18th century. This suggests that the crux of the complaint in both cases is that these posits – a faculty of thought or of mutually attracting a distant body – run afoul of the law of inertia. Bodies possess inertia, and this is a consequence of their essence (see Chapter 3 for details of Euler's ontology of body). For Euler, this means that bodies necessarily cannot possess any faculty by which they change their own states. But, first, thought is of its nature a kind of mental self-movement. It is a faculty possessed by a thinking thing for changing its own states, and thus directly contrary to inertia. The argument is that the same is true of the idea of fundamental attraction.

If attraction belonged to the nature of body, that would be one thing, and would call for explanation if it were not to be an 'occult quality'. If it did not belong to the nature of bodies, then the attraction would be what we might today call a 'brute fact.' In the way of thinking of early modern philosophy, this means it would have to be produced through the immediate action of God. Thus, complaint (d), that universal attraction would, in that case, constitute a 'perpetual miracle.' Euler claims, very much following Leibniz, this is no different from imagining God to continually intervene in the antecedently understood physical world of bodies, which is tantamount to "*des miracles continuels*."

The passages in Letter 68 also contain (e) a thought experiment, which is perhaps only loosely worked out. Imagine a universe in which the only things that exist are two bodies, placed at some distance. Ask yourself, then: if absolutely nothing else existed, how could they interact or even 'sense' each other? As Euler observes, "*ce sont des idées qui révoltent*" (these are outrageous ideas; Letter 68). Remembering his relatively modest claims on behalf of his reasonings, the suggestion seems to be, at a minimum, that the theorist of universal attraction has an explanatory burden to address, and that she lacks obvious resources to do so. Whereas, on the other hand, an understanding of forces and hence of attraction that is based exclusively on the essence of body requires us to hypothesize or predict a kind of subtle matter or ether, since the essence of body comprehends only extension, mobility, inertia, and impenetrability. (See Chapter 3 for a treatment of Euler's ontology of body.)

This is his final argument against universal attractions: (e) a positive defense of the alternative hypothesis of subtle matter, which would undermine the need to posit fundamental attraction. Though Euler is not explicit about the ground for his views, plausibly it is supposed to be an abductive inference of the following form. First, we already have independent grounds to believe that celestial space is filled with a subtle matter. How could he assume that this is already well-established? Since Descartes, the idea of such an ether was used to explain the transmission of light and, at the time, it enjoyed a fair measure of acceptance. So, we already have a plausible candidate on the table for a mechanism for other effects, like gravity. It would then be a reasonable hypothesis that this ether, or a similar kind of subtle matter, could, in addition, be responsible for the phenomena of attraction. Since we have already confirmed an instance of this subtle matter, the thought appears to go, this hypothesis is, in particular, more near-at-hand than positing an entirely new class of entities or properties, ones which amount to 'perpetual miracles.' However, he does not attempt to work out a detailed aethereal account of gravity in the *Letters*. One fairly serious attempt is made in the manuscript *Anleitung zur Naturlehre*.[167]

[167] See Lin (2023).

All of these objections, as I have indicated, resemble, in their broad outlines, objections to universal attraction which were already in circulation. Huygens already, and infamously, maintained a vortical, 'subtle matter' theory of gravity in the face of the universal attraction posited in the *Principia*, not to mention Leibniz. Though the modern reader is still likely to be unimpressed by the argument (e), it is worth observing that there is at least somewhat more to it than a bare, blind commitment to something like the mechanical philosophy. Some – see e.g. (Stein 1989) – have expressed disappointment with the likes of Huygens and Euler for explaining gravity by such outlandish hypotheses, out of stubborn refusal to accept at-a-distance attraction. But, in fairness, the record should at least show that there was already some reason for thinking some sort of ether filled the heavens. So it was not simple pigheadedness or lack of imagination that led these figures to suppose an ether might be interacting with the planets and the bodies inhabiting them, and affecting their motions.

Appendix C. Euler's alternative to the monadology for the explanation of corporeal motion

In Chapter 2, §6.3 ('Conflict erupts'), I made the claim, on behalf of Euler, that the most successful and readily available explanations of the motions of bodies were those of mechanics, based in mechanical properties and impressed forces. I think this is eminently plausible, especially in comparison to monadology. Yet it is a claim, not a tautology. So, Euler must shoulder an explanatory burden. He must show how the resources of mechanical theory as he understands it, which do not include active forces, are both ready-to-hand and sufficient to account for the phenomena of change.

That is a tall order. It is simply not one he can fulfill in a single polemic. In effect, the project it entails amounts to an entire research program, one that Euler pursues across a number of papers, treatises, and manuscripts and, in truth, is nothing short of his entire mechanics. Not surprisingly, I cannot describe and assess that whole project here. [168]

To pin down the basic story, Euler scrutinizes a typical scene of corporeal change, the two early modern paradigms cases: collisions and pressures. When one body A collides with (or applies a pressure to) another body B, it changes B's state of motion. In Euler's analysis, we can understand the cause of that change to reside outside B, namely in A, and make good sense of this causal story by grounding it in the basic mechanical properties of body. Those properties are, besides inertia, impenetrability, extension, and mobility.

---

[168] Euler's grounding of mechanics is a version of what Brading and Stan (2024) call a "philosophical mechanics" – roughly, a science of mechanics that was not only sufficient to solve mechanical problems, but which was also grounded in a philosophical picture of the constitution of bodies that explained why its laws and principles were true and held of real bodies. In their assessment, though Euler's mechanics ultimately did not succeed as a complete philosophical mechanics, it was one of the two most promising candidates of the century. For more details, see Brading and Stan (2024, Ch. 5). It is worth also mentioning that this is only one part – if the "foundational" part – of Euler's overall mechanics.

If Euler is right, then in this paradigmatic case there is no need to call on forces *within* B in order to explain the change in B. This, at any rate, is Euler's understanding of action through contact. It is true that the monadologist may not see things that way. The claim is that this way of understanding the process provides the resources for an adequate account of the causes of changes in motion in bodies. For this idea to permit Euler to head off the inference of the monadologist, Euler has to show that this means of producing changes in bodies is sufficient to account for *all* corporeal motions.

Yet, the reader should be cautioned against thinking this makes Euler's argument flimsy. This research program was fruitful for generating new physical insight for decades, often insights that were beyond the reach of others at the time. Examples include finding the laws of rotational motion for solid bodies, and advances in the mechanics of fluid and elastic bodies. Even modern critics of Euler's philosophical views admit that those views put him in a unique position to generate crucial advances in mechanics.[169] Far from being an empty promise, the project showed considerable potential as the sought-after complete account of mechanics.

A second relevant observation is that outlining and beginning this project had a burden-shifting effect within the dialectic with the monadologists. For, if the monadologist still wished to prove the existence of active forces, she needed show that external forces of the kind Euler used would inevitably turn out to be *in*sufficient to account for all the changes that we observe to occur in bodies. Only that would justify applying the principle of sufficient reason. For that inference is based on the claim that a certain explanation (i.e., of changes in motion in bodies) is *needed* and that only a certain kind of explanatory tool is

---

[169] See Wilson (1992).

sufficient to provide it (active forces). Given the first point, that could not simply be claimed. It would require argument.

References

Primary

Crequy, Jean-Antoine de, (1752) « Explication de la nature du feu, et de sa propagation. » In *Recueils des pièces qui ont remporté le Prix de l'Académie Royale des Sciences de Paris*. Tome IV. Paris. pp. 59-82.

Crousaz, J-P de. (1726) *Essay sur le movement, ou l'on traitte de sa nature, son origine, et sa communication en général….* Groningue: Jean Coste. [Link]

D'Alembert, Jean le Rond. (1743) *Traité de Dynamique, dans lequel les Loix de l'Equilibre et du Mouvement des Corps sont réduites au plus petit nombre possible, et démontrées d'une manière nouvelle….* Paris : David.

D'Alembert, Jean le Rond (1752). Causes finales. *Encyclopédie, ou Dictionnaire raisonné des sciences, des arts, et des métiers*. Vol. 2. Paris, p. 789.

D'Alembert, Jean le Rond and Samuel Formey (1754). Cosmologie. *Encyclopédie, ou Dictionnaire raisonné des sciences, des arts, et des métiers*. Vol. 4. Paris, pp. 294–297.

D'Arcy, Patrick (1756). "Replique a un Mémoire de M. de Maupertuis". *Mémoires de l'académie des sciences de Paris*, pp. 503–519.

Descartes, René. (1996) *Meditations on First Philosophy*. Revised edition. Translated and edited by John Cottingham. United Kingdom: Cambridge University Press.

Du Fech, Louis-Antoine Lozeran. (1752) « Discours sur la propagation du feu. » In *Recueils des pièces qui ont remporté le Prix de l'Académie Royale des Sciences de Paris*. Tome IV. Paris. pp. 25-56.

Du Châtelet, Émilie. (1740) *Institutions de physiques*. Paris : Prault fils.

Du Châtelet, Émilie. (1742). *Institutions Physiques*. Amsterdam.

Du Chatelet, Emilie. (1744). *Dissertation sur la Nature et la Propagation du Feu*. Paris: Prault, Fils.

Du Chatelet, Emilie. (1752). « Dissertation sur la Nature et la Propagation du Feu. » In *Recueils des pièces qui ont remporté le Prix de l'Académie Royale des Sciences de Paris*. Tome IV. Paris. pp. 87-170.

Euler, Leonhard. (1736) *Mechanica*. 2 volumes. St. Petersburg: Imperial Academy of Sciences.

Euler, Leonhard. (1744) *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes, sive solutio problematis isoperimetrici lattissimo sensu accepti*. Lausanne & Geneva: Marcum-Michaelem Bousquet.

Euler, Leonhard (1746a). « Enodatio quaestionis utrum materiae facultas cogitandi tribui possit necne." In *Opuscula varii argumenti*, Volume 1, pp. 277-286.

Euler, Leonhard (1746b). « Recherches physiques sur la nature des moindres parties de la matière." In *Opuscula varii argumenti*, Volume 1, pp. 287-300.

Euler, Leonhard. (1746c). *Gedancken von den Elementen der Cörper*. Berlin: Haude and Spener.

Euler, Leonhard. (1746d). « De la force de percussion et sa véritable mesure. » *Mémoires de l'académie des sciences de Berlin*, Volume 1, pp. 21-53.

Euler, Leonhard. (1750a) "Recherches sur les plus grands et plus petits qui se trouvent dans les actions des forces" *Mémoires de l'académie des sciences de Berlin*, Volume 4, pp. 149-188.

Euler, Leonhard (1750b). "Réflexions sur quelques loix générales de la nature qui s'observent dans les effets des forces quelconques". *Mémoires de l'académie des sciences de Berlin* 4, pp. 189–218.

Euler, Leonhard. (1750c) "Reflexions sur l'espace et le tems." *Mémoires de l'académie des sciences de Berlin*, Volume 4, pp. 324-333.

Euler, Leonhard (1752a). « Découverte d'un nouveau principe de mécanique. » *Mémoires de l'académie des sciences de Berlin*, Volume 6, pp. 185-217.

Euler, Leonhard (1752b). « Recherches sur l'origine des forces" *Mémoires de l'académie des sciences de Berlin,* Volume 6, pp. 419-447.

Euler, Leonhard (1752c). « Dissertatio de igne in qua ejus natura et proprietates explicantur. » In *Recueils des pièces qui ont remporté le Prix de l'Académie Royale des Sciences de Paris*. Tome IV. Paris. pp. 5-21.

Euler, Leonhard (1753a). *Dissertation sur le principe de la moindre action, avec l'examen des objections de M. le Prof. Koenig faites contre ce principe*. Berlin: Michaelis.

Euler, Leonhard (1753b). "Essay d'une démonstration métaphysique du principe général de l'equilibre". *Mémoires de l'académie des sciences de Berlin* 7, pp. 246–254.

Euler, Leonhard (1753c). "Examen de la dissertation de M. le Professeur Koenig, insérée dans les actes de Leipzig, pour le mois de mars 1751". *Mémoires de l'académie des sciences de Berlin* 7, pp. 219–245.

Euler, Leonhard (1753d). "Harmonie entre les principes généraux de repos et de mouvement de M. de Maupertuis". *Mémoires de l'académie des sciences de Berlin* 7, pp. 169–198.

Euler, Leonhard (1753e). "Sur le principe de la moindre action". *Mémoires de l'académie des sciences de Berlin* 7, pp. 199–218.

Euler, Leonhard (1765a). *Theoria motus corporum solidorum seu rigidorum*. Rostock and Greifswald: A.F. Röse.

Euler, Leonhard. (1765b) « Recherches sur la connoissance mécanique des corps." *Mémoires de l'académie des sciences de Berlin*, Volume 14, pp. 131-153.

Euler, Leonhard. (1768) *Lettres a une princesse d'Allemagne sur divers sujets de physique & de philosophie*, Volume 1, pp. 1-314. St. Petersburg: Imperial Academy of Sciences.

Euler, Leonhard. (1768) *Lettres a une princesse d'Allemagne sur divers sujets de physique & de philosophie*, Volume 2, pp. 1-340. St. Petersburg: Imperial Academy of Sciences.

Euler, Leonhard. (1772) *Lettres a une princesse d'Allemagne sur divers sujets de physique & de philosophie*, Volume 3, pp. 1-404. St. Petersburg: Imperial Academy of Sciences.

Euler, Leonhard. (1862) *Anleitung zur Naturlehre*. In *Opera Postuma*, Volume 2, pp. 449-560.

Euler, Leonhard. (1963) *Леонард Ейлер. Письма к ученым [Leonard Eĭler. Pis'ma k učenym / Leonhard Euler. Letters to scholars]* ed. T. N. Klado, et al. Moskva, Leningrad.

Jacobi, C.G.J (1866). *C. G. J. Jacobi's Vorlesungen über Dynamik*, 1842-1843. G. Reimer.

Lagrange, Joseph-Louis (1788). *Méchanique analitique*. Paris: DeSaint.

Leibniz, G.W. (1989) *Philosophical Writings*. Edited by Roger Ariew and Daniel Garber. Indianapolis: Hackett Publishing.

Leibniz, G.W. and Samuel Clarke. (2002) *Correspondence*. Edited by Roger Ariew. Hackett Publishing.

Maupertuis, Pierre de (1740). "Loi du repos des corps". *Mémoires de l'académie des sciences de Paris*, pp. 170–6.

Maupertuis, Pierre de (1748a). "Accord de differentes loix de la nature qui avaient jusqu'ici paru incompatibles". *Mémoires de l'académie des sciences de Paris*, pp. 417–26.

Maupertuis, Pierre de (1748b). "Les loix du mouvement et du repos, déduites d'un principe Métaphysique". *Mémoires de l'académie des sciences de Berlin*, pp. 267–294.

Maupertuis, Pierre de (1750). *Essai de cosmologie*. Berlin. As republished in *Oeuvres* (1756) Vol. 1. Lyon: Bruyset.

Maupertuis, Pierre de (1754). "Réponse à un Mémoire de M. d'Arcy". *Mémoires de l'académie des sciences de Berlin* 8, pp. 293–298.

Newton, Isaac. (2016) *The Principia: Mathematical Principles of Natural Philosophy, the Authoritative Translation and Guide*. Translated by I. Bernard Cohen and Anne Whitman, assisted by Julia Budenz. University of California Press.

Secondary

Ben-Menahem, Yemima (2018). Causation in Science. Princeton: Princeton University Press.

Brading, Katherine and Marius Stan (2024). *Philosophical Mechanics in the Age of Reason*. Oxford: Oxford University Press.

Breidert, Wolfgang (2007). "Leonhard Euler and Philosophy." In *Leonhard Euler: Life, Work, and Legacy*, pp. 97-108. Elsevier.

Broman, Thomas (2012). "Metaphysics for an Enlightened Public." *Isis*. 103: 1-23.

Butterfield, Jeremy (2005). "Some Aspects of Modality in Analytical Mechanics". Unpublished.

Calinger, R. S. (1969). "The Newtonian-Wolffian Controversy: 1740-1759." *Journal of the History of Ideas*, 30(3), 319–330. https://doi.org/10.2307/2708560

Caparrini, Sandro and Craig Fraser (2017). "Mechanics in the Eighteenth Century." In *The Oxford Handbook of the History of Physics*. Oxford University Press.

Cassirer, Ernst (1943). "Newton and Leibniz." *Philosophical Review* 52 (4):366-391.

De Gandt, Francois (1995). *Force and Geometry in Newton's* Principia. Translated by Curtis Wilson. Princeton: Princeton University Press.

DiSalle, Robert (2013): "The transcendental method from Newton to Kant." *Studies in History and Philosophy of Science* 44: 448-456.

Douglas, Heather (2014). "The Value of Cognitive Values." *Philosophy of Science* 80.5: 796–806.

Dyck, Corey W. (2019). *Early Modern German Philosophy (1690-1750)*. Oxford: Oxford University Press.

Elkana, Y. (1974). "Scientific and Metaphysical Problems: Euler and Kant." In: Cohen, R.S., Wartofsky, M.W. (eds) *Methodological and Historical Essays in the Natural and Social Sciences*. Boston Studies in the Philosophy of Science, vol 14. Springer, Dordrecht. 277-305.

Falkenburg, Brigitte (2020). *Kant's cosmology: from the pre-critical system to the antinomy of pure reason*. Cham: Springer.

Fellmann, Emil (2007). *Leonhard Euler*. Translated by Erika Gautschi and Walter Gautschi. Basel/Boston: Birkhäuser Verlag.

Fraser, Craig (1983). "J. L. Lagrange's Early Contributions to the Principles and Methods of Mechanics". *Archive for History of Exact Sciences* 28.3, pp. 197–241.

Garber, Daniel (1992). *Descartes' Metaphysical Physics*.

Garber, Daniel (2009). *Leibniz: Body, Substance, Monad*. Oxford: Oxford University Press.

Gaukroger, Stephen (1982). "The Metaphysics of Impenetrability: Euler's Conception of force." *British Journal for the History of Science* 15 (2):132-154.

Gray, C. G. (2009). "Principle of least action". *Scholarpedia* 4.12, p. 8291. doi: 10.4249/scholarpedia.8291.

Harman, P. (1983). "Force and Inertia: Euler and Kant's *Metaphysical Foundations of Natural Science"*, in *Nature Mathematized: Historical and Philosophical Case Studies in Classical Modern Natural Philosophy*, ed. William R. Shea. pp. 229–249. Dordrecht: Reidel.

Hakfoort, C. (1995). *Optics in the age of Euler*. Cambridge: Cambridge University Press.

Henry, John (2011). "Galileo and the Scientific Revolution: The Importance of His Kinematics," *Galilaeana: Journal of Galilean Studies*, vol. 8, no. 1, pp. 3-36

Hepburn, Brian (2010). "Euler, *vis viva*, and equilibrium." *Studies in History and Philosophy of Science* 41: 120–127.

Hepburn, Brian (2017). "Euler's Galilean philosophy of science." (Unpublished presentation)

Hepburn, Brian and Zvi Biener (2022). "Mechanics in Newton's Wake." In *The Cambridge History of Philosophy of the Scientific Revolution*, eds. David Marshall Miller and Dana Jalobeanu, pp. 293 – 312. Cambridge University Press.

Hettsche, Matt and Corey Dyck (2019). "Christian Wolff", *The Stanford Encyclopedia of Philosophy* (Winter 2019 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2019/entries/wolff-christian/>.

Janiak, Andrew (ed.) (2004). *Isaac Newton: Philosophical Writings*. Cambridge, UK ;: Cambridge University Press.

Janiak, Andrew (2007). "Newton and the reality of force." *Journal of the History of Philosophy* 45 (1):127-147.

Janiak, Andrew (2008). *Newton as Philosopher*. New York: Cambridge University Press.

Jorati, Julia (2018). "Leibniz's Ontology of Force." *Oxford Studies in Early Modern Philosophy* 8:189–224.

Kuhn, Thomas (2012). *The Structure of Scientific Revolutions*. 4th Edition. Chicago and London: University of Chicago Press.

Kuhn, Thomas (1977). “Objectivity, Value Judgment, and Theory Choice.” In *The Essential Tension*. By Thomas S. Kuhn, 320–333. Chicago: University of Chicago Press.

Kuorikoski, Jaakko (2021). “There Are No Mathematical Explanations”. *Philosophy of Science* 88.2, pp. 189–212. doi: 10.1086/711479.

Lange, Marc (2013). “What Makes a Scientific Explanation Distinctively Mathematical?” *British Journal for the Philosophy of Science* 64.3, pp. 485–511. doi: 10.1093/bjps/axs012.

Lange, Marc (2018). *Because Without Cause: Scientific Explanations by Constraint.* Oxford University Press. doi: 10.1093/oso/9780198777946.003.0002.

Laudan, Larry (1986) *Science and Values: The Aims of Science and Their Role in Scientific Debate*. Berkeley: University of California Press.

Le Ru, Véronique (2003). *La Crise de la substance et de la causalité. Des petits écarts cartésiens au grand écart occasionaliste*. Paris: CNRS.

Lin, Qiu (2022). “Euler against Newtonian Gravity: “A Crude Hypothesis”? Chapter IV in *Three Essays about the Problem of Space in the Early Modern Period*. Ph.D. Dissertation. Duke University.

Lyssy, Ansgar (2022). “Maupertuis, Euler and the Leibnizian metaphysics behind the Principle of least action”. *The Berlin Academy in the reign of Frederick the Great*. Ed. by Tinca Prunea-Bretonnet and Peter R. Anstey. Oxford University Press, pp. 123–152.

Mahoney, Michael Sean (1998). “The Mathematical Realm of Nature”. *The Cambridge History of Seventeenth-Century Philosophy*. Ed. by Daniel Garber and Michael Ayers. Cambridge University Press, pp. 702–55.

Manzo, Silvia (2003). “The Arguments on Void in the Seventeenth Century: The Case of Francis Bacon.” *The British Journal for the History of Science*, Vol. 36, No. 1, pp. 43-61

Massimi, Michela (2017). “The Legacy of Newton for the Pre-Critical Kant,” in Eric Schliesser, and Chris Smeenk (eds), *The Oxford Handbook of Newton* (online edition, Oxford Academic).

McDonough, Jeffrey K. (2016). “Leibniz and the Foundations of Physics: The Later Years.” *Philosophical Review* 125 (1):1-34.

McDonough, Jeffrey K. (2020). “Not Dead Yet: Teleology and the Scientific Revolution”. *Teleology: A History*. New York: Oxford University Press, pp. 150–179.

McDonough, Jeffrey K. (2021). “Leibniz’s Philosophy of Physics,” *The Stanford Encyclopedia of Philosophy* (Fall 2021 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2021/entries/leibniz-physics/>.

McDonough, Jeffrey K. (2021). "Causal Powers and Ontology in Descartes, Malebranche, and Leibniz," in Julia Jorati (ed.), *Powers: A History*. New York: Oxford Academic.

McDonough, Jeffrey K. (2022). A Miracle Creed. Oxford University Press.

McMullin, Ernan (1978). *Newton on Matter and Activity*. University of Notre Dame Press.

McMullin, Ernan (1982). "Values in Science." *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*. Vol. 2, Symposia and Invited Papers. Edited by Peter D. Asquith and Thomas Nickles, 3–28. Chicago: University of Chicago Press.

Okruhlik, Kathleen (1983). "Kant on the Foundations of Science." In: Shea, W.R. (eds) *Nature Mathematized. The University of Western Ontario Series in Philosophy of Science*, vol 20. Springer, Dordrecht.

Pulte, Helmut (1989). *Das Prinzip der kleinsten Wirkung und die Kraftkonzeptionen der rationalen Mechanik: eine Untersuchung zur Grundlegungsproblematik bei Leonhard Euler, Pierre Louis Moreau de Maupertuis und Joseph Louis Lagrange.* Franz Steiner Verlag Wiesbaden.

Radelet-de-Grave, P. and D. Speiser (2004). "Introduction." *Leonhardi Euleri Opera Omnia*, Series III, Vol. 10: 13-120.

Rutherford, D. (1995). *Leibniz and the rational order of nature*. Cambridge: Cambridge University Press.

Schmit, Christoph (2009). "Force d'inertie et causalité." *Archives internationales d'histoire des sciences*, vol. 59 (162), pp. 97-155.

Schmit, Christoph (2017). "D'Alembert et la dynamique: Contexte et originalité." R*evue de métaphysique et de morale*, 93, 19-30.

Schramm, Matthias (1985). *Natur Ohne Sinn?: Das Ende des Teleologischen Weltbildes*. Graz: Verlag Styria.

Schramm, Matthias (2005). "The Creation of the Principle of Least Action". *Formal Teleology and Causality in Physics*. Ed. by Michael Stöltzner and Paul Weingartner. Paderborn: Mentis, pp. 99–114.

Schuster, John A. (2013). "Cartesian Physics." In *The Oxford Handbook of the History of Physics*. Edited by Jed Buchwald and Robert Fox. Oxford University Press.

Sklar, Lawrence (2012). *Philosophy and the Foundations of Dynamics*. Cambridge University Press.

Smith, George (2008). "Newton's Philosophiae Naturalis Principia Mathematica," The Stanford Encyclopedia of Philosophy (Winter 2008 Edition), Edward N. Zalta (ed.).

Sober, Elliott (2015). *Ockham's Razors*. Cambridge, UK: Cambridge University Press.

Speiser, Andreas (1934). *Leonhard Euler und die deutsche Philosophie*. Zurich.

Stan, Marius (2013). "Kant's third law of mechanics: The long shadow of Leibniz." *Studies in History and Philosophy of Science Part A* 44 (3):493-504.

Stan, Marius (2017) "Euler, Newton, and Foundations for Mechanics." In Eric Schliesser, and Chris Smeenk (eds), *The Oxford Handbook of Newton* (online edition, Oxford Academic).

Stan, Marius (2018). "Emilie du Chatelet's Metaphysics of Substance." *Journal of the History of Philosophy* 56 (3):477-496.

Stöltzner, Michael (1994). "Action Principles and Teleology". Inside Versus Outside. Ed. by Harald Atmanspacher and Gerhard J. Dalenoort. Berlin, Heidelberg: Springer, pp. 33–62.

Stöltzner, Michael (2002). "The Principle of Least Action as the Logical Empiricist's Shibboleth". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 34.2, pp. 285–318. doi: 10.1016/s1355-2198(03)00002-9.

Stöltzner, Michael (2005). "Drei Ordnungen Formaler Teleologie". *Formal Teleolgoy and Causality in Physics*. Ed. by Michael Stöltzner and Paul Weingartner. Paderborn: Mentis, pp. 199–241.

Suisky, Dieter (2009). *Euler as Physicist*. Springer Verlag.

Swerdlow, N.M. (2013). "Galileo's Mechanics of Natural Motion and Projectiles." In *The Oxford Handbook of the History of Physics*. Edited by Jed Buchwald and Robert Fox. Oxford University Press.

Terekhovich, Vladislav (2017). "Metaphysics of the Principle of Least Action". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 62, pp. 189–201. doi: 10.1016/j.shpsb.2017.09.004.

Van Strien, Marij (2014). "On the origins and foundations of Laplacian determinism. *Studies in History and Philosophy of Science*, 45(1), pp. 24–31.

Veldman, Michael (2024). "Mathematizing Metaphysics: The Case of the Principle of Least Action." *Philosophy of Science* 91 (2): 351–69.

Watkins, Eric (1997). "The Laws of Motion from Newton to Kant." *Perspectives on Science* 5 (3):311-348.

Watkins, Eric (1998). "The Argumentative Structure of Kant's Metaphysical Foundations of Natural Science." *Journal of the History of Philosophy* 36 (4):567-593.

Wells, Aaron (2023). "Science and the Principle of Sufficient Reason: Du Châtelet contra Wolff." *Hopos: The Journal of the International Society for the History of Philosophy of Science* 13 (1):24–53.

Westfall, R.S. (1971). *Force in Newton's Physics. The Science of Dynamics in the Seventeenth Century*. Macdonald, London, and Elsevier, New York.

Wilson, Curtis (1992). "Euler on action-at-a-distance and fundamental equations in continuum mechanics." In *The investigation of difficult things: Essays on Newton and the history of exact sciences in honour of D.T. Whiteside*, pages 399–420. Cambridge University Press.

Yourgrau, Wolfgang and Stanley Mandelstam (1968). *Variational Principles in Dynamics and Quantum Theory*. 3rd ed. Philadelphia: I. Pitman.

Zepeda, Joseph Raphael (2009). *Descartes and His Critics on Space and Vacuum*. PhD Dissertation. University of Notre Dame.