Going beyond Axisymmetry: 2.5D Vector Electromagnetics

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Introduction: Linear wave propagation through inhomogeneous structures of size $R \gg \lambda$ (Fig.1) is a computationally challenging problem, in particular when using finite element methods, due to the steep increase of the number of degrees of freedom as a function of R/λ. Fortunately, when the geometry of the problem possesses symmetries, one may choose an appropriate basis in which the stiffness matrix of the discretized problem is block-diagonal. A particular scenario is the case of a cylindrically-symmetric geometry, where an appropriate basis is the set of cylindrical waves with all possible azimuthal numbers (*m*). Each of the excited cylindrical harmonics propagate through the structure independently of all other harmonics, and therefore the fields associated with that harmonic can be found by solving an essentially two-dimensional PDE problem in the ρ-z (half)-plane. The cylindrical waves have a prescribed dependence on the azimuthal angle variable (φ), hence the name – 2.5D electromagnetics. This novel approach is applied to the problem of cloaking and wave scattering off a spherical nanoparticle on metallic and/or dielectric substrates.

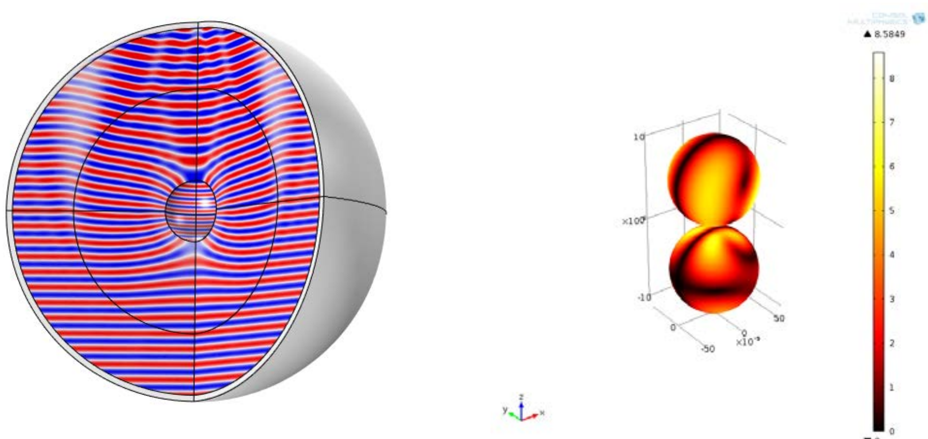

Figure 1. Illustration of the 2.5D concept: wave scattering off a cylindrically-symmetric object. Left: Wave propagation through a large-volume directional cloak of invisibility [1-3]. Right: Wave scattering off a plasmonic nanoparticle dimer. Note that the fields are not axially symmetric here, both due to the polarization and the incidence angle with respect to the axis of revolution.

Use of COMSOL Multiphysics: In this work, we implement the modified Helmholtz equation for the three-component vector field formulation for frequency-domain electromagnetics in COMSOL Multiphysics, which enables us to solve the wave propagation problem with non-zero azimuthal numbers. Even in the simplest-case scenario, for a linearly polarized wave incident along the axis of revolution of the system, the fields are not azimuthally symmetric, but rather dipolar, due to their transverse polarization. Many problems, in particular the scattering problem of electromagnetics and optics, benefit from the *Scattered Field* formulation, a unique feature of COMSOL's RF Module. To make use of this powerful approach, we obtain analytic formulas for the expansion of a polarized (transverse) electromagnetic plane wave in terms of cylindrical harmonics. Our implementation builds on the standard 2D-axisymmetry in the RF Module and adds the necessary azimuthal-number dependent terms to the weak-form volumetric equations as well as the weak-form contribution of the *Scattering* boundary condition.

Results: We have applied our 2.5D EM formulation to two problems with very different spatial scales and materials, which possess only one common feature: the contribution of non-axially symmetric fields is critical to the proper function of the otherwise cylindrically-symmetric system.

Problem A is the problem of wave propagation through inhomogeneous (graded) dielectric medium that spans an optically large ($R \gg \lambda$) volume (Fig.2). As shown in Ref.[1], the index distribution of this 3D structure with cylindrical symmetry can be chosen such as to enable optical invisibility [2] for a particular view angle. To quantify the performance of this cloaking system, we introduce a figure of merit based on a plane-averaged norm of the scattered field past the cloak. It is shown that, with respect to this measure of visibility, the cloak allows angular divergence of the incident beam up to approximately 3 degrees.

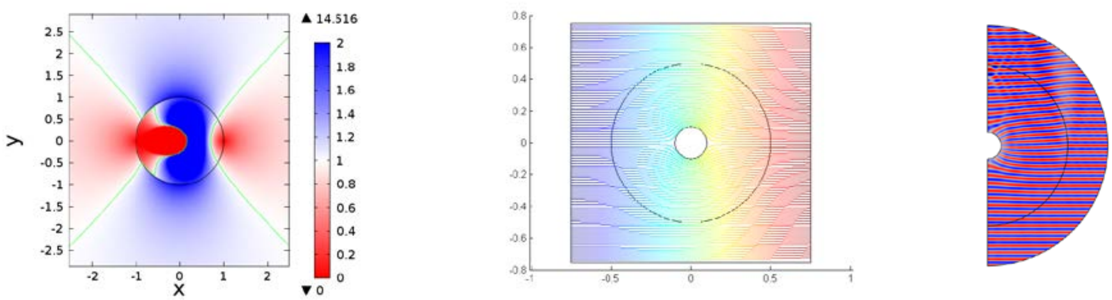

Figure 2. Directional cloak introduced in [1] based on the conformal mapping from Ref. [2]. Left to right: refractive index profile; ray tracing obtained with COMSOL Particle Tracing Module; 2.5D full-wave simulation based on the 3-component vector E-field formulation.

Problem B is a wave scattering problem with a deeply subwavelength plasmonic nanoparticle situated on a large-area (not necessarily subwavelength) substrate (Fig.3). The substrate can be another plasmonic material [4] or a dielectric. Since the 2.5D methodology reduces the problem to one-half of a 2D plane, we are able to model a gigantic (relative to λ^3) volume around the particle and still maintain sub-nanometer resolution in the vicinity of the particle.

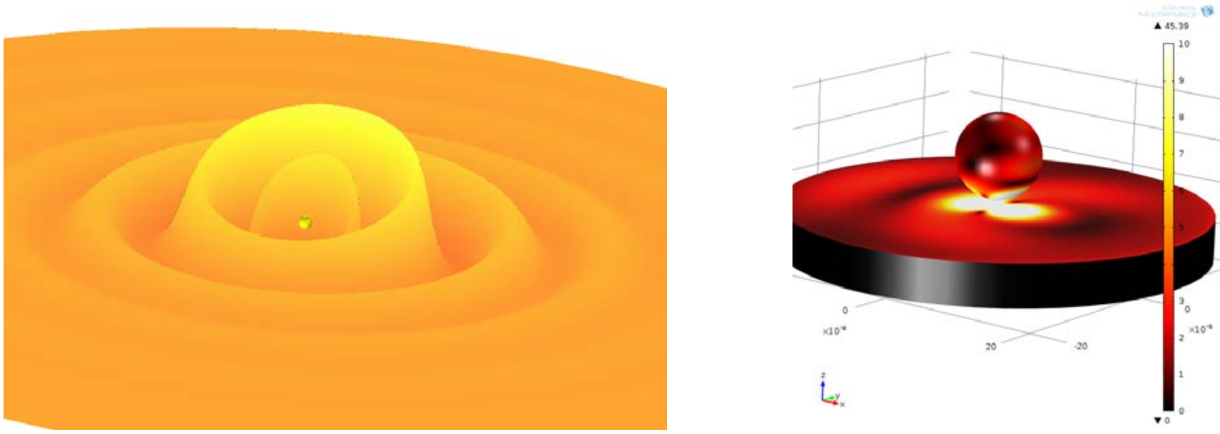

Figure 3. 2.5 D vector-field simulations of a gold nanoparticle held above a gold film substrate with a thin (1nm) self-assembled polymer layer [4]. Left: The surface charge wave associated with the surface plasmon polariton of the film is visualized using the Deformed Surface plotting tool in COMSOL. Right: a more conventional color surface plot showing the actual film and the particle.

Conclusion: While the acoustic (scalar pressure wave) implementation of 2.5D formalism [1] is already available in COMSOL Multiphysics as a part of the Acoustics module, vector electromagnetics and tensor elastodynamics implementations of this broad concept are still lacking due to their increased complexity. In this work, we show that a full three-component vector Helmholtz equation required for full-wave modeling of electromagnetic phenomena can be implemented in 2.5 dimensions using the basic platform of COMSOL Multiphysics, as a modification of the RF Module equations. The implications of this work go well beyond the two problems discussed and extend to various low-frequency magnetics [5,6], RF and microwave electromagnetics [7-8], and optics [8] problems involving cylindrically-symmetric objects and non-symmetric vector fields.

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