

Invariance Theorems for Supersymmetric Yang-Mills Theories

Savdeep Sethi^{*1} and Mark Stern^{†2}

** School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA*

† Department of Mathematics, Duke University, Durham, NC 27706, USA

We consider quantum mechanical Yang-Mills theories with eight supercharges and a $Spin(5) \times SU(2)_R$ flavor symmetry. We show that all normalizable ground states in these gauge theories are invariant under this flavor symmetry. This includes, as a special case, all bound states of D0-branes and D4-branes. As a consequence, all bound states of D0-branes are invariant under the $Spin(9)$ flavor symmetry. When combined with index results, this implies that the bound state of two D0-branes is unique.

¹ sethi@sns.ias.edu

² stern@math.duke.edu

1. Introduction

The existence of normalizable vacua in supersymmetric Yang-Mills theories is a question that arises in many different contexts in string theory and field theory. Index arguments can be used to determine whether any vacua exist, but not exactly how many vacua. An index only counts the difference between the number of bosonic and fermionic vacua. To count the actual number of vacua, we need more information such as how the vacua transform under the global symmetries of the theory.

In this paper, we consider quantum mechanical Yang-Mills theories with eight supercharges and an $Spin(5) \times SU(2)_R$ symmetry. We take our theories to be dimensional reductions of $d = 6$ N=1 Yang-Mills theories coupled to matter. The question of normalizable ground states in these models arises in the study of bound states of D0-branes and D4-branes [1,2]; for example, a single D0-brane and a single D4-brane can be shown to bind using L^2 index arguments [3] generalized to theories without a gap. Other examples from string theory involve D0-branes moving on orbifolds [4], and the question of counting H-monopoles in the heterotic string [5,3].

In the following section, we describe the field content and symmetries of these gauge theories. We then show that all normalizable ground states in these theories must be invariant under the $SU(2)_R$ symmetry. The argument we give is suggested by recent work on the L^2 -cohomology of hyperKähler spaces by Hitchin [6]. Our result should have implications for defining and computing the L^2 -cohomology of instanton moduli spaces. Certain instanton moduli spaces appear as Higgs branches in gauge theories of the kind under consideration. For example, the moduli space of $U(N)$ instantons in \mathbb{R}^4 appears as the Higgs branch of the quantum mechanics describing D0-D4 systems. Although these spaces can be singular, their embedding into quantum mechanical gauge theory provides a natural regularization of the singularities. Heuristically, the wavefunction for a state corresponding to a form on the Higgs branch is smoothed out by leaking onto the Coulomb branch. It would be interesting to explore this connection further.

There is a second R -symmetry in these theories which comes from the dimensional reduction of the Lorentz group. For reductions of $d = 6$ N=1 Yang-Mills theories, this is a $Spin(5)$ symmetry. Using basically the same argument as in the case of the $SU(2)_R$ symmetry, we show that all normalizable ground states in these theories are invariant under this $Spin(5)$ symmetry. For reductions of $d = 10$ N=1 Yang-Mills theories [7], the R -symmetry group is $Spin(9)$. It is quite straightforward to argue that as a consequence of

the $SU(2)_R \times Spin(5)$ invariance theorem, all ground states in these theories with sixteen supercharges must be invariant under the $Spin(9)$ symmetry.

We can couple these invariance theorems with results from L^2 index theory [8,9]. The L^2 index for the non-Fredholm theory¹ of two D0-branes is proven to be one [8]. We also know that the L^2 index for the theory of a single D0-brane and a single D4-brane is one [3]. Our invariance results imply that all bound states in these theories are bosonic, and therefore unique. These results can also be combined with other interesting but heuristic attempts to study the L^2 index by either deforming the Yang-Mills theory [10,11], or by using insights from string theory [12] to compute the bulk and defect terms. The bulk terms for various Yang-Mills theories have been directly computed in [13,14,15]. There have also been a number of comments on the implications of invariance for the asymptotic form of particular bound state wavefunctions [16,17].

2. The Field Content and Symmetries

2.1. The vector multiplet supercharge

The argument we wish to make requires reasonably little explicit knowledge of the gauge theory. There is a $Spin(5) \times SU(2)_R$ symmetry which commutes with the Hamiltonian H . Since we are considering a gauge theory, we must have at least one vector multiplet. It contains five scalars x^μ with $\mu = 1, \dots, 5$ transforming in the $(\mathbf{5}, \mathbf{1})$ of the symmetry group. These scalars transform in the adjoint representation of the gauge group G . Let p^μ be the associated canonical momenta obeying,

$$[x_A^\mu, p_B^\nu] = i\delta^{\mu\nu}\delta_{AB}, \quad (2.1)$$

where the subscript A is a group index.

Associated to these bosons are eight real fermions λ_a where $a = 1, \dots, 8$ transforming in the $(\mathbf{4}, \mathbf{2})$ representation of the symmetry group. These fermions are also in the adjoint representation of the gauge group. The eight supercharges also transform in the $(\mathbf{4}, \mathbf{2})$ representation. These fermions obey the usual quantization relation,

$$\{\lambda_{aA}, \lambda_{bB}\} = \delta_{ab}\delta_{AB}. \quad (2.2)$$

¹ By non-Fredholm, we mean a theory without a gap.

Let γ^μ be hermitian real gamma matrices which obey,

$$\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}. \quad (2.3)$$

Appendix A includes an explicit basis for these gamma matrices along with a discussion of the symmetry group action.

The supercharge takes the form,

$$Q_a^v = (\gamma^\mu p_A^\mu \lambda_A)_a + \frac{1}{2} f_{ABC} (\gamma^{\mu\nu} \lambda_A x_B^\mu x_C^\nu)_a + D_{abA} \lambda_{bA}, \quad (2.4)$$

where f_{ABC} are the structure constants and $\gamma^{\mu\nu} = (1/2)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. The real anti-symmetric matrix D does not involve momenta. The D -term transforms in the $(\mathbf{1}, \mathbf{3})$ representation of the symmetry group, and in the adjoint representation of the gauge group. The precise form of D is not important for our argument. In general, there can be many vector multiplets. In that case, the terms in the supercharge (2.4) generalize in an obvious way.

2.2. The hypermultiplet supercharge

A hypermultiplet contains four real scalars which we can package into a quaternion q with components q^i where $i = 1, 2, 3, 4$. This field transforms as $(\mathbf{1}, \mathbf{2})$ under the symmetry group, and in some representation T of the gauge group. We again introduce canonical momenta p_i satisfying the usual commutation relations. Now $SU(2)_R \sim Sp(1)_R$ is the group of unit quaternions. We choose $SU(2)_R$ to act on a hypermultiplet q by right multiplication by a unit quaternion. The gauge symmetry commutes with the $SU(2)_R$ symmetry and acts by left multiplication on q . See Appendix A for a more detailed discussion.

The superpartner to q is a real fermion ψ_a with $a = 1, \dots, 8$ satisfying,

$$\{\psi_a^R, \psi_{bS}\} = \delta_{ab} \delta_S^R. \quad (2.5)$$

These fermions transform in the $(\mathbf{4}, \mathbf{1})$ representation, and the R, S subscripts index the T representation of G . For n hypermultiplets, the gauge group G acts via a subgroup of the $Sp(n)_L$ symmetry. In terms of the s^j operators given in Appendix A, the hypermultiplet charge takes the form

$$Q_a^h = s_{ab}^j \psi_b p_j + I_{ab} \psi_b. \quad (2.6)$$

We have lumped all the interactions into the non-derivative operator I which transforms in the $\mathbf{2}$ of $SU(2)_R$. We also need to note that I is proportional to $x^\mu \gamma^\mu$ with a proportionality constant that commutes with the $Spin(5)$ generators. We have also suppressed gauge indices. Note that since the s^j implement right multiplication by a quaternion, they commute with γ^μ . Again, there can be many hypermultiplets in different representations of the gauge group. In that case, the hypermultiplet supercharge (2.6) generalizes in a straightforward way. The full Hermitian supercharge is the sum of the vector and hypermultiplet supercharges,

$$Q_a = Q_a^v + Q_a^h.$$

2.3. The $SU(2)_R$ currents

The three generators of $SU(2)_R$ correspond to right multiplication by I, J, K and are given in terms of the gauge invariant rotation generators,

$$W_{ij} = q_i p_j - q_j p_i. \quad (2.7)$$

Again here and in the subsequent discussion, we generally suppress gauge indices. In accord with prior notation, we denote the three $SU(2)_R$ generators by \tilde{s}^i :

$$\begin{aligned} \tilde{s}^2 &= W_{12} - W_{34} + \frac{i}{2} \lambda s^2 \lambda \\ \tilde{s}^3 &= W_{13} + W_{24} + \frac{i}{2} \lambda s^3 \lambda \\ \tilde{s}^4 &= W_{14} - W_{23} + \frac{i}{2} \lambda s^4 \lambda. \end{aligned} \quad (2.8)$$

As they should, these generators act on the bosons of the hypermultiplet and the fermions of the vector multiplet. Adding either more vector multiplets or more hypermultiplets is straightforward: we simply need to sum the contributions to the three currents (2.8) from each multiplet.

2.4. The $Spin(5)$ currents

The ten generators of $Spin(5)$ act on the bosons of the vector multiplet and all fermions in the problem. The generators are given by:

$$T^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu - \frac{i}{4} \gamma_{ab}^{\mu\nu} (\lambda_a \lambda_b + \psi_a \psi_b). \quad (2.9)$$

Adding either more vector multiplets or more hypermultiplets is again straightforward.

3. An Invariance Argument for the $SU(2)_R$ Symmetry

3.1. Relating the $SU(2)_R$ currents to the supercharge

A key point in the argument is a relation between the supercharge and the $SU(2)_R$ currents. For some choice of v_a^i , we want to show that:

$$\tilde{s}^i = \sum_a \{Q_a, v_a^i\}. \quad (3.1)$$

Let us start with the vector multiplet. We take a candidate gauge singlet,

$$(v_1)_a^i = (s^i \gamma^\nu \lambda)_a x^\nu. \quad (3.2)$$

First note that this choice anti-commutes with Q^h because λ anti-commutes with ψ . It also anti-commutes with the D -term in (2.4). To see this, we compute:

$$\sum_a \{D_{ab} \lambda_b, (v_1)_a^i\} = x_A^\nu \text{tr} (s^i \gamma^\nu D_A^T), \quad (3.3)$$

However, we can immediately see that (3.3) vanishes by noting that the operator $s^i \gamma^\nu D^T$ does not contain a singlet under $Spin(5)$. The trace of the operator therefore vanishes. Our choice for v_1 anti-commutes with $\frac{1}{2} f_{ABC} (\gamma^{\mu\nu} \lambda_A x_B^\mu x_C^\nu)_a$ for the same reason: the resulting trace does not contain a singlet of $Spin(5)$.

What remains is the following anti-commutator which is not hard to compute,

$$\sum_a \{(\gamma^\mu p^\mu \lambda)_a, (v_1)_a^i\} \sim i \lambda s^i \lambda. \quad (3.4)$$

The exact proportionality constant does not matter for this argument. The important point is that we can use (3.2) to generate the terms in the $SU(2)_R$ currents which act on vector multiplets.

For the hypermultiplet, we take the following candidate gauge singlet:

$$(v_2)_a^i = (s^i s^l \psi)_a q^l. \quad (3.5)$$

Note that v_2 anti-commutes with Q^v because λ anti-commutes with ψ . It is also not too hard to argue that the anti-commutator of v_2 with the interaction term I in (2.6) must vanish. We see that,

$$\sum_a \{I_{ab} \psi_b, (v_2)_a^i\} \sim q^l \text{tr} (s^i s^l I), \quad (3.6)$$

but $s^i s^l I$ does not contain a singlet under the $Spin(5)$ action on fermions because I is proportional to γ^μ so the trace vanishes.

Again what remains is the anti-commutator,

$$\sum_a \left\{ s_{ab}^j \psi_b p_j, (v_2)_a^i \right\}. \quad (3.7)$$

It is easy to check that the $\psi\psi$ terms in the anti-commutator vanish because,

$$\sum_k \psi \{s^k\}^T s^i s^k \psi = 0.$$

With a little additional work, we find that (3.7) gives precisely the bosonic terms in (2.8) up to an overall non-vanishing constant. We therefore conclude that for appropriately chosen constants α_1 and α_2 , the choice

$$v_a^i = \alpha_1 (v_1)_a^i + \alpha_2 (v_2)_a^i \quad (3.8)$$

satisfies (3.1).

3.2. Rotating a ground state

We assume there exists a normalizable ground state Ψ which is not a singlet under $SU(2)_R$. Under some $SU(2)_R$ rotation, we obtain another non-trivial L^2 zero-energy state. What does L^2 imply? Let us collectively denote all the bosonic coordinates x and q by y^i where $i = 1, \dots, D$. Normalizability requires that,

$$\langle \Psi, \Psi \rangle = \int d^D y \Psi^\dagger(y^i) \Psi(y^i) < \infty.$$

For some \tilde{s}^i , the state $\tilde{s}^i \Psi$ is a non-trivial ground state. It satisfies the relation,

$$Q_a (\tilde{s}^i \Psi) = Q_a \Psi = 0, \quad (3.9)$$

for each a by definition of a ground state. Using (3.1), we find that

$$\begin{aligned} \tilde{s}^i \Psi &= \sum_a \{Q_a, v_a^i\} \Psi, \\ &= \sum_a Q_a (v_a^i \Psi). \end{aligned} \quad (3.10)$$

The new ground state looks Q -trivial. To show that it really is physically trivial, we need to check that it has zero norm. Since Q is Hermitian and kills $\tilde{s}^i \Psi$, the norm of $\tilde{s}^i \Psi$ vanishes if we can integrate by parts. To integrate by parts, we argue as in Jost and Zuo [18,6]: in terms of $y = |y^i|$, we can cutoff of the integral using a smooth bump function $\rho_R(y)$ which vanishes for $y > 2R$, satisfies $|d\rho_R| < 4/R$ and is one for $y < R$,

$$\langle \tilde{s}^i \Psi, \tilde{s}^i \Psi \rangle = \lim_{R \rightarrow \infty} \langle \rho_R(y) \tilde{s}^i \Psi, \tilde{s}^i \Psi \rangle .$$

Using (3.9) and (3.10), we see that

$$\begin{aligned} \langle \tilde{s}^i \Psi, \tilde{s}^i \Psi \rangle &= \lim_{R \rightarrow \infty} \langle \rho_R(y) \tilde{s}^i \Psi, \sum_a \{Q_a, v_a^i\} \Psi \rangle, \\ &= \lim_{R \rightarrow \infty} \sum_a \langle [Q_a, \rho_R(y)] \tilde{s}^i \Psi, v_a^i \Psi \rangle . \end{aligned} \tag{3.11}$$

We see that $[Q_a, \rho_R(y)]$ is $O(1/y)$ and vanishes for $y < R$ and $y > 2R$. Since v_a^i is $O(y)$ at worst, the right hand side of (3.11) vanishes. The $SU(2)_R$ symmetry therefore acts trivially on all normalizable ground states.

4. Invariance Under the $Spin(5)$ Symmetry

4.1. Relating the $Spin(5)$ currents to the supercharge

We want to use essentially the same argument as in the $SU(2)_R$ case. For some choice of $v_a^{\mu\nu}$, we want to show that:

$$T^{\mu\nu} = \sum_a \{Q_a, v_a^{\mu\nu}\}. \tag{4.1}$$

Let us start with the vector multiplet. We take a candidate gauge singlet,

$$(v_1)_a^{\mu\nu} = \{\gamma^\mu x^\nu - \gamma^\nu x^\mu\}_{ab} \lambda_b. \tag{4.2}$$

Again this choice anti-commutes with Q^h because λ anti-commutes with ψ . The anti-commutator with $\frac{1}{2} f_{ABC} (\gamma^{\mu\nu} \lambda_A x_B^\mu x_C^\nu)_a$ results in a trace of three gamma matrices and so vanishes. It also anti-commutes with the D -term in (2.4). To see this, we compute:

$$\sum_a \{D_{ab} \lambda_b, (v_1)_a^{\mu\nu}\} = D_{ab} \{\gamma^\mu x^\nu - \gamma^\nu x^\mu\}_{ab}. \tag{4.3}$$

However, this combination does not contain a singlet under $Spin(5)$ so (4.3) vanishes.

We are left with the following anti-commutator which we need to compute quite carefully,

$$\sum_a \{(\gamma^\mu p^\mu \lambda)_a, (v_1)_a^{\mu\nu}\} = 8(x^\nu p^\mu - x^\mu p^\nu) + 2i\lambda\gamma^{\mu\nu}\lambda. \quad (4.4)$$

This computation is sensitive to the size of the γ matrix. We obtain precisely the right ratio between the bosonic and fermion terms in (4.4) because the theory is reduced from six dimensions. We would not obtain the right ratio had we considered a theory reduced from ten dimensions with a $Spin(9)$ symmetry. Again, we can use (4.2) to generate the terms in the $Spin(5)$ currents which act on vector multiplets.

For the hypermultiplet, we take the following choice:

$$(v_2)_a^{\mu\nu} = (\gamma^{\mu\nu} s^i \psi)_a q^i. \quad (4.5)$$

Again v_2 anti-commutes with Q^ν because λ anti-commutes with ψ . In much the same way as before, we can argue that the anti-commutator of v_2 with the interaction term I in (2.6) must vanish. We see that,

$$\sum_a \{I_{ab}\psi_b, (v_2)_a^{\mu\nu}\} \sim q^i \text{tr}(I\gamma^{\mu\nu} s^i), \quad (4.6)$$

but $I\gamma^{\mu\nu} s^i$ again does not contain a singlet under $Spin(5)$ so the trace vanishes.

The remaining anti-commutator involves the kinetic term in the hypermultiplet charge,

$$\sum_a \left\{ s_{ab}^j \psi_b p_j, (v_2)_a^{\mu\nu} \right\} = -i\psi\gamma^{\mu\nu}\psi. \quad (4.7)$$

Again we conclude that for appropriately chosen constants α_1 and α_2 , the choice

$$v_a^{\mu\nu} = \alpha_1 (v_1)_a^{\mu\nu} + \alpha_2 (v_2)_a^{\mu\nu} \quad (4.8)$$

satisfies (4.1). A straightforward repeat of the argument given in section 3.2 then implies that the $Spin(5)$ symmetry acts trivially on all normalizable ground states.

4.2. Theories with sixteen supercharges

For theories obtained by reduction from ten dimensions, the previous argument does not apply directly to the $Spin(9)$ symmetry for reasons mentioned earlier. These theories contain scalars y^i where $i = 1, \dots, 9$ transforming in the adjoint representation of the gauge group. The superpartners to these scalars are real fermions η_α where $\alpha = 1, \dots, 16$ also in the adjoint representation.

However, we can always view these theories as special cases of theories with eight supercharges. We choose any 5 of the 9 scalars y^i to be the vector multiplet, and the remaining 4 scalars comprise an adjoint hypermultiplet. Of the original $Spin(9)$ symmetry, only a $Spin(5) \times SU(2)_L \times SU(2)_R$ subgroup is manifest. The scalars decompose in the following way,

$$\mathbf{9} \rightarrow (\mathbf{5}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2}). \quad (4.9)$$

The fermions decompose according to,

$$\mathbf{16} \rightarrow (\mathbf{4}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{4}, \mathbf{2}, \mathbf{1}). \quad (4.10)$$

Our invariance argument implies that all normalizable ground states are invariant under the $Spin(5) \times SU(2)_R$ symmetry. However, this is true regardless of how we embed $Spin(5) \times SU(2)_R$ into $Spin(9)$. This is only possible if the full $Spin(9)$ symmetry acts trivially on all normalizable ground states.

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Appendix A. Quaternions and Symplectic Groups

We will summarize some useful relations between quaternions and symplectic groups. Let us label a basis for our quaternions by $\{\mathbf{1}, I, J, K\}$ where,

$$I^2 = J^2 = K^2 = -\mathbf{1}, \quad IJK = -\mathbf{1}.$$

A quaternion q can then be expanded in components

$$q = q^1 + Iq^2 + Jq^3 + Kq^4.$$

The conjugate quaternion \bar{q} has an expansion

$$\bar{q} = q^1 - Iq^2 - Jq^3 - Kq^4.$$

The symmetry group $Sp(1)_R \sim SU(2)_R$ is the group of unit quaternions. Let Λ be a field transforming in the $\mathbf{2}$ of $Sp(1)_R$. If we view $Sp(1)_R$ acting on Λ as right multiplication by a unit quaternion g then,

$$\Lambda \rightarrow \Lambda g.$$

In this formalism, Λ is valued in the quaternions. Equivalently, we can expand Λ in components and express the action of g in the following way,

$$\Lambda_a \rightarrow g_{ab} \Lambda_b,$$

where g_{ab} implements right multiplication by the unit quaternion g . For example, right multiplication by I on q gives

$$\begin{aligned} q &\rightarrow qI \\ &\rightarrow q^1 I - q^2 - q^3 K + q^4 J, \end{aligned}$$

which can be realized by the matrix

$$I^R = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad (\text{A.1})$$

acting on q in the usual way $q_a \rightarrow I_{ab}^R q_b$. The matrices J^R and K^R realize right multiplication by J, K while $\mathbf{1}^R$ is the identity matrix:

$$J^R = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad K^R = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \quad (\text{A.2})$$

We define operators s^j in terms of $\{\mathbf{1}^R, I^R, J^R, K^R\}$

$$s^1 = \begin{pmatrix} \mathbf{1}^R & 0 \\ 0 & \mathbf{1}^R \end{pmatrix}, \quad s^2 = \begin{pmatrix} I^R & 0 \\ 0 & I^R \end{pmatrix}, \quad s^3 = \begin{pmatrix} J^R & 0 \\ 0 & J^R \end{pmatrix}, \quad s^4 = \begin{pmatrix} K^R & 0 \\ 0 & K^R \end{pmatrix}.$$

In a similar way, the group $Sp(2) \sim Spin(5)$ is the group of quaternion-valued 2×2 matrices with unit determinant. We will view $Sp(2)$ as acting by left multiplication on a field Ψ in the defining representation. So an element $U \in Sp(2)$ acts in the following way:

$$\Psi \rightarrow U\Psi.$$

Equivalently, in terms of components

$$\Psi_a \rightarrow U_{ab}\Psi_b.$$

Lastly, we can give an explicit form for the gamma matrices (2.3) in terms of quaternions:

$$\begin{aligned} \gamma^1 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, & \gamma^2 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & \gamma^3 &= \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \\ \gamma^4 &= \begin{pmatrix} 0 & J \\ -J & 0 \end{pmatrix}, & \gamma^5 &= \begin{pmatrix} 0 & K \\ -K & 0 \end{pmatrix}. \end{aligned}$$

In turn, $\{I, J, K\}$ can be expressed in terms of the Pauli matrices σ^i

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

as 4×4 real anti-symmetric matrices:

$$I = \begin{pmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}, \quad K = \begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{pmatrix}.$$

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