Endogenous Growth, Trade, and the Environment

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
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Abstract

This dissertation presents two essays on endogenous growth and renewable resources.

The first essay explores the role of renewable resources in a tractable model of endogenous growth driven by horizontal and vertical innovation in the closed economy. The model is tractable in that it yields a complete, analytical characterization of the path of utility and the associated welfare level. This property is exploited to compare two cases of renewable resource management: open access and full property rights. The first case involves a common property problem in which agents ignore the long-term resource viability; the second fully internalizes the dynamics of the resource stock. Analysis shows that if the natural regeneration rate of the renewable resource is too low, the tragedy of the commons occurs. If, instead, the natural regeneration rate is sufficiently high, the steady-state growth rate of the economy is identical across the two management regimes. The reason is because there is no scale effect; that is, the steady-state growth rate of the economy does not depend on the labor or the resource endowment. However, the development path on which the economy transits from the developing stage (no R&D activity) to the developed stage (positive R&D activity) depends on the resource management regime. In particular, a developing economy under full property rights will cross its development threshold prior to one under open access. This threshold depends on the size of the manufacturing firms. When it becomes sufficiently large as a result of the decline in the number of firms over time, there will be an incentive for the remaining firms

to conduct R&D. Given the same number of manufacturing firms, the firm size is larger under full property rights than under open access due to higher nominal expenditure per capita. Therefore, the development threshold will be reached sooner under full property rights. In other words, the economy will start engaging in R&D activities sooner and more quickly accumulate knowledge, which is the source of long-run growth. Moreover, switching from full property rights to open access is welfare reducing due to two effects. The first is through the price of the harvest good. Although the economy initially enjoys a lower price of harvest good, the price gradually increases as the resource becomes scarcer. Secondly, the competitive household instantaneously loses the resource income and thus spends less on manufacturing goods. This decreases the incentive for manufacturing firms to conduct R&D and results in a temporary deceleration of the growth rate of TFP relative to the baseline case of full property rights. The economy therefore experiences a cumulative loss of TFP relative to the baseline, which is the novel feature of our model of endogenous innovation. This mechanism has interesting and wide-ranging implications for the role of resources in development and growth

The second essay extends the model of endogenous growth and renewable resources into the open economy framework. The paper examines the effect of trade liberalization on resource-rich countries, based on a two-country model in which the difference in endowment of a renewable resource leads to asymmetric trade. In this model, the resource-rich economy trades its harvest good and final good for the final good from the resource-poor economy. Furthermore, the renewable resource is considered to be under open access, where there is no clear ownership over the resource, leading to overexploitation. Long-term productivity, in this case, stems from endogenously-determined knowledge accumulation. Under these circumstances, analysis shows that the resource-rich country will lose from trade due to two effects. The first effect is the instantaneous loss of income. Higher demand for the harvest good, from the combined domestic and international demand, diverts labor away from the production of technological goods to the harvest sector, where rent is zero. The second effect is a scarcity effect, which becomes more severe when trade results in a greater demand for the harvest good. Overexploitation of the renewable resource today leads to falling resource stock in the future, which is then reflected in the higher price of harvest good, other things being constant. Since the harvest good is an essential input to produce the final good, given the same amount of the other inputs, the amount of final good produced will also fall in the long run.

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1

Introduction

Renewable resources are important especially for the resource-based economy. Hence, how the resource is managed will have major impact on development, growth and welfare of the economy. In the model of endogenous growth, where knowledge accumulation is important for the long-run growth to persist, the role of resources does not receive much attention. What distinguishes renewable resources from other traditional macroeconomics inputs is that the more it is extracted today, the less the stock of renewable resources will be left in the future. As a result, this dissertation will incorporate the dynamic of natural resources into the endogenous growth framework

The following two chapters are two essays related to endogenous growth model with renewable resources. The first focuses on the closed economy and compare the effect of two different resource management regimes on economic growth, welfare and development path. The other analyzes the effect of trade liberalization on growth and welfare in the resource-rich country, in which the resources are managed without property rights.

The first essay is “Endogenous Growth and Renewable Resources Under Different

Management Regimes: Impact on Development and Welfare", which address 2 questions; how different resource management regimes affect welfare and the development path. In particular how different resource management regimes contributes to the time that the economy reaches the development threshold which transit the economy to the developed state. Im specifically looking at two different resource management regimes. The first is the one with no property rights, or open access. Under this regime, the agent does not incorporate the dynamic of resource stock or more policy related, government cannot enforce the property rights toward those resources. The second is Full Property Rights where agent fully take into the account of the dynamic of the resource stock. The model assumed a closed-economy framework.

The second essay considers the open economy framework in the simplified model of endogenous growth and focuses on an open-access renewable resources. The model assumes that Home country is endowed with renewable resources and use it to produce harvest goods. Harvest goods can be used for producing domestic final goods or as an trading commodity for imported final good. The analysis compares growth and welfare level of the resource-rich country before and after trade.

2

Endogenous Growth and Renewable Resources Under Different Management Regimes: Impact on Development and Welfare

2.1 Introduction

Renewable resources play essential roles in developing countries as both productive inputs and trading commodities. Therefore, the mismanagement of these resources would inevitably impact development, growth and welfare in detrimental ways. Hardin (1968) underscores the importance of resource management through the concept of the tragedy of the commons. In particular, limited resources that are shared and do not have clear ownership are subject to overexploitation, which eventually leads to extinction.

> "What shall we do [to prevent the tragedy of the commons]? We have several options. We might sell them off as private property. We might keep them as public property, but allocate the right to enter them. The allocation might be on the basis of wealth... on the basis of merit... by lottery... [or] on a first-come, first-served basis. These, I think, are

> all the reasonable possibilities. They are all objectionable. But we must choose—or acquiesce in the destruction of the commons. . . ” Hardin (1968)

This paper aims to provide a theoretical model that internalizes the role of renewable resources within the endogenous growth framework and to emphasize the importance of how resource management regimes contribute to the development, growth and welfare of a resource-based economy.

Renewable resources are incorporated in an endogenous growth framework that takes into account the technological dynamics of manufacturing firms in terms of both vertical and horizontal innovation Peretto and Connolly (2007) . Vertical innovation occurs when existing manufacturing firms undertake R&D and learn how to increase efficiency in order to reduce the marginal cost of production. Horizontal innovation occurs when an entrepreneur designs a new product and enters the market to add variety to the manufacturing sector. The source of long-run growth is driven by vertical innovation only. In this case, however, the introduction of both vertical and horizontal innovation is necessary to explain the dynamics of the development path. In particular, an economy that engages in both horizontal and vertical innovation is considered to be developed while one that engages only in horizontal innovation is considered to still be developing. Transition along the development path depends on the firm size. When the firm size becomes sufficiently large, manufacturing firms will begin conducting R&D and start to accumulate knowledge, which is the source of long-run growth.

In the model for this paper, a renewable resource yields a harvest good that in turn serves as an essential input to produce a final good. The way that the resource is harvested depends on the resource management regime. In the literature, management policies are explored extensively and can be categorized into two extreme cases. The first case is an open access regime, in which resource ownership is ambiguous. The representative agent ignores the future availability of the resource stock and labor is allocated based solely on the current productivity. As a result, economic rent in the harvest sector dissipates to zero since the price of the harvest good does not reflect the full cost of producing it. Examples of renewable resources that tend to be managed according to this regime include ocean fisheries, wildlife populations, forests and public grazing lands. The economic theory of open access was first developed by Gordon (1991) for application to the fishery industry.

The other extreme case of resource management is the regime of full property rights. In this case, each agent has private ownership over the resource. As a result, the agents will internalize the projected future resource stock into the objective function. Alternatively, Copeland and Taylor (2009) suggest that another way to think about the resource management regime is to consider the capability of the government or regulator to monitor and enforce access to the renewable resource. If the government can effectively enforce the rights to access the resource, the regime will be equivalent to full property rights. Otherwise, it will be equivalent to open access. The paper is also in line with this concept.

The difference in resource management regime or in government enforcement of resource access leads to a difference in nominal expenditure per capita. Specifically, the economy will have a lower level of nominal expenditure per capita under open access than under full property rights. The reason is because renewable resources are overexploited under open access and rent dissipates to zero. On the contrary, if the economy has adopted full property rights, each agent internalizes the full cost of harvesting resources so positive rent is being generated in this case. The difference in nominal expenditure per capita is the core mechanism that drives the dynamics of manufacturing firms and the dynamics for the growth rate of the economy.

Although the main focus of this paper is on renewable resources, non-renewable resources are nested in the model as a special case. To be precise, non-renewable resources can be considered as natural resources with a zero natural regeneration rate. The natural regeneration rate is an important factor of the model. If it is sufficiently high, it guarantees that the growth rate of the open-access economy can be sustainable in the long run. However, if the natural regeneration rate is too low, as in the case of non-renewable resources, failure to assign full property rights on those resources would prevent the economy from sustaining its growth rate in the long run. This prediction puts pressure on policy-makers to protect natural resources.

The main contributions of this paper are in regard to three aspects: growth, development and welfare. In terms of the steady state growth of the economy, analysis shows that if the natural regeneration rate of the renewable resource is too low, as mentioned earlier, the tragedy of the commons occurs. Specifically, the resource stock converges to zero and the economy collapses. This result is applicable to oil, minerals and fossil fuels. If, instead, the natural regeneration rate is sufficiently high, the steady-state growth rate of the economy is identical across the two management regimes. The reason is because there is no scale effect; that is, the steady-state growth rate of the economy does not depend on the labor or the resource endowment.

However, resource management regimes have significant impacts on both the development path and the welfare of the economy. In particular, a developing economy under full property rights will cross its development threshold prior to one under open access. This threshold from the developing stage (no R&D activity) to the developed stage (positive R&D activity) depends on the size of the manufacturing firms. When it becomes sufficiently large as a result of the decline in the number of manufacturing firms over time, there will be an incentive for the remaining firms to conduct R&D. Given the same number of manufacturing firms, the firm size is larger under full property rights than under open access due to higher nominal expenditure per capita. Therefore, the development threshold will be reached sooner under full property rights. In other words, the economy will start engaging in R&D activities sooner and more quickly accumulate knowledge, which is the source of long-run growth. The result of the model has important implications for assigning full property rights on natural resources. Failure to assign full property rights on natural resources has an adverse effect on the development of the manufacturing sector, which in turn hinders the development path of the economy.

Moreover, this paper analyzes the entire path of the consumption level and the associated welfare level when switching from full property rights to open access. The advantage of this comparative study is that it can be derived under the closed-form solution. Switching from full property rights to open access is shown to be welfare reducing due to two effects. The first is through the price of the harvest good. Though the economy initially enjoys a lower price of harvest good, the price gradually increases as the resource becomes scarcer. In addition, the competitive household instantaneously loses the resource income and thus spends less on manufacturing goods. This decreases the incentive for manufacturing firms to conduct R&D and results in a temporary deceleration of the growth rate of TFP relative to the baseline case of full property rights. The economy therefore experiences a cumulative loss of TFP relative to the baseline, which is the novel feature of our model of endogenous innovation. This mechanism has interesting and wide-ranging implications for the role of resources in development and growth.

There is a growing number of research papers linking the role of renewable resources and endogenous growth. Aghion and Howitt (1988) were among the first to introduce renewable resources into an endogenous growth framework. They concluded that long-run growth generated from knowledge accumulation in the intermediate goods is possible. The paper formally derived conditions for such a result. Renewable resources in their model can be interpreted as environmental quality, fisheries and forests. In addition, Tahvonen and Kuuluvainen (1991) constructed a theoretical model of renewable resources and endogenous growth. Their conclusion was that long run growth is possible, although the paper did not consider the cost of extracting resources.

On the other hand, as can be seen in “Are There Limits to Growth?” , Stokey (1998) employed a simple AK model with pollution having a similar law of motion to renewable resources. In this instance, the result of the paper was that a positive growth rate of the economy cannot be sustained. The underlying intuition is that as the capital stock grows, tighter emission regulation will be imposed, which will reduce the rate of return on capital. When the rate of return becomes sufficiently low, there will be no incentive for capital accumulation and economic growth can no longer persist.

McAusland (2005) approached a similar conclusion under a closed economy model. In his model, the failure to generate long-run growth came from the fact that the government failed to adequately assign property rights. In other words, the renewable resource was assumed to be under an open access management regime. The model employed the renewable resource as a final good essential to consumers and was based on the rationale that labor is a competing input for harvesting renewable resources and creating human capital. Under open access, labor would shift too greatly toward the harvest sector, leading to resource overexploitation and a zero-growth steady state in the most optimistic scenario. If the resource stock is exploited to the point of collapse, long-run growth undoubtedly becomes impossible. Sustainable growth cannot be created in this closed system. Nevertheless, opposite results were reached when the country is assumed to open up opportunities for trade. There is a vast strand of literature that looks at the combination of endogenous growth, renewable resources and trade. However, trade is not the main purpose of this paper, so it is not factored herein.

The next part of this paper will outline the basic framework of the model. It will describe the production of final good, manufacturing goods and harvest good. Subsequently, the paper will examine the role of renewable resources under different management regimes. It will evaluate the general equilibrium and steady state growth rate of the economy in the cases of both open access and full property rights. In addition, there will be a brief discussion about the steady state variables under the two management regimes. The paper will then compare the development path and welfare level between these regimes. For the welfare comparison, the paper supposes that the economy begins at the steady state level under full property rights and then switches to open access. Finally, the paper reaches conclusion

2.2 Model

This model is set under a closed economy. Suppose an economy begins with full property rights and later changes to open access [1]. On the production side, the economy has three productive sectors: the final good sector, manufacturing sector and harvest sector. The market structure for the final good is perfectly competitive. It employs labor, harvest good and differentiated manufacturing goods. For the manufacturing sector, the model is developed from Peretto (2010) in which an incumbent firm hires labor to produce the good and conducts in-house R&D activity. This R&D generates positive externality in terms of knowledge spillover to the whole manufacturing industry and it is, in fact, an engine for long-run economic growth. The market structure in this industry is monopolistically competitive such that an incentive for firms to undertake R&D is present. Although there is an opportunity for a new entrepreneur to enter the market, there is also a constant fixed entry cost that creates a disadvantage if the newcomer produces an existing product. As a result, the newcomer will not produce the same product as an incumbent manufacturing firm. For the harvest sector, the renewable resource follows a logistic growth function and harvest technology is a well-known Schaefer production function, in which

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[1] The regime switch is chosen in this order because the transitional dynamics for open access is a lot simpler than that for full property rights.

harvest amount depends on labor, TFP of harvest production, and existing stock of the resource Schaefer (1957)[2]. In addition, a competitive household supplies labor and renewable resource in a competitive setting. It also borrows or lends in a perfectly competitive financial asset market. Moreover, it chooses the consumption path and the level of harvest production in order to maximize the present value of utility. However, the level of harvest activity will vary based on the type of management regime. The model roadmap is outlined in Figure 2.1.

2.2.1 Final Output Market

Assuming wage is normalized to 1, a competitive final-output firm employs labor from the household, harvest good and manufacturing goods in order to produce final good (Y), with the production technology

$$Y = AL_Y^{\beta} H^{\alpha} \int_0^N X_i^{1-\alpha-\beta} \tag{2.1}$$

The technology is Cobb-Douglas, which exhibits a decreasing return to scale for each individual input but a constant return to scale for all factors of production. $A > 0$ represents a total factor of production (TFP) for the production of final output, whileβ, α and $1 - \alpha - \beta$ are the factor income shares related to labor, harvest and manufacturing goods, respectively. In the competitive setting, harvest good and labor are paid up to their marginal products. A representative final-output firm maximizes its profit by equating net marginal products to their factor prices. The demand for each factor of production is as follows.

$$L_Y^D = \beta P_Y Y \tag{2.2}$$

$$H^D = \frac{\alpha P_Y Y}{P_H} \tag{2.3}$$

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[2] The main results do not change when applying other functional forms. They only generate more mathematical complication.

$$X_i^D = \left[\frac{(1-\alpha-\beta) A P_Y L_Y^{\beta} H}{P_{Xi}} \right]^{\frac{1}{\alpha+\beta}} \tag{2.4}$$

2.2.2 Intermediate Manufacturing Market

The model for manufacturing goods is developed from Peretto (2008), which allows for both horizontal and vertical innovation. Horizontal innovation includes an expansion of products or number of firms while the latter is generated by an increase in quality of products within the existing firms. This model was chosen because it is tractable yet rich enough to capture the reality of the economy in the manufacturing sector. Manufacturing goods are produced under a monopolistic competition, where an existing firm can enjoy the profit margin which in turn creates an incentive for the firm to conduct R&D. When the firm conducts R&D, common knowledge will be created for the society that reduces the cost of future R&D. An entrepreneur who wishes to enter the market can do so by comparing the net present value of expected future profit with the entry cost. Given the nature of the market (Bertrand price competition and positive entry cost), the new entrant would experience losses if it decides to produce an existing product. As a result, each manufacturing firm will produce only one type of product. This section will analyze the optimal price strategy for an existing firm and then characterize an entry.

Incumbents

At a particular point in time, R&D technology exists for the economy to produce N differentiated manufacturing goods (X_i). The production function for each firm is given by

$$X_i = Z_i^{\theta}(L_{X_i} + \phi) \tag{2.5}$$

where Z_i represents firm-specific knowledge, θ is TFP of knowledge, L_{X_i} is labor employed to produce manufacturing good i, and ϕ is the fixed labor cost. The production function exhibits an increasing return to scale. Moreover, a firm accumulates knowledge according to R&D technology.

$$\dot{Z}_i(t) = \kappa K L_{Zi} \tag{2.6}$$

$$K = \frac{1}{N}\int_0^N Z_i \tag{2.7}$$

(2.6) illustrates the dynamics of firm-specific knowledge that depends on the amount of labor hired in the R&D sector (L_{Z_i}) and its marginal productivity (κK), which is the product of an exogenous parameter (κ) and the common knowledge (K). When allowing for firm symmetry, the common knowledge is approximated as the average knowledge of the manufacturing industry.

An existing manufacturing firm chooses the price of manufacturing good (P_{X_i}) and labor allocated to the R&D sector (L_{Z_i}) in order to maximize the present value of the returns from improving the quality of the product ($V_i(t)$) given the demand for good X_i (2.4) from the final output producers and research technology (2.6).

$$V_i(t) = \int_t^\infty e^{-\int_t^s (r(v)+\delta)dv}\Pi_{X_i}(s)\,ds \tag{2.8}$$

where $e^{-\delta t}$ is an instantaneous probability of death. This parameter is introduced in order to allow firms to exit the market during the transitional dynamics.[3]

$$\Pi_{X_i} = (P_{X_i} - Z_i^\theta)X_i^D - \phi - L_{Z_i} \tag{2.9}$$

Solving Current Value Hamiltonian (CVH) and imposing firm symmetry (see Appendix for *Proof*)

$$P_{X_i} = \frac{1}{(1-\alpha-\beta)Z_i^\theta} \tag{2.10}$$

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[3] It is analogous to an investment depreciation rate.

$$r_{Z_i} = \kappa \left[\frac{(1-\alpha-\beta)^2 \theta P_Y Y}{N} - \frac{L_Z}{N} \right] - \delta \tag{2.11}$$

Entrants

This paper assumes that there is free entry into the manufacturing industry. However, before entering the sector, an entrepreneur has to pay a sunk entry cost ($\psi P_{X_i} X_i$), which is proportional to the expected revenue. As a result, an entrepreneur will only enter the sector if the present value of net cash flow outweighs the entry cost. To be specific, if $V_i(t) > \psi P_{X_i} X_i$, an infinite amount of resources would be dedicated to the expansion of product variety; hence, this condition cannot hold in the equilibrium. On the other hand, if $V_i(t) < \psi P_{X_{i,}} X_i$, no resources would be devoted to the diversification of manufacturing products, so the number of firms will be constant over time. Thus, the free-entry condition suggests that the present value of net cash flow has to equal the entry cost.

$$V_i(t) = \psi P_{X_i} X_i \tag{2.12}$$

In equilibrium, $r_{N_i} = \frac{\Pi_X}{\mathrm{V}_i} + \frac{\dot{V}_i}{V_i}$, the rate of return on a new product is (see Appendix for *Proof*)

$$r_{N_i} = \frac{1}{\psi} \left[\alpha + \beta - \frac{N}{(1-\alpha-\beta) P_Y Y} \left(\phi + \frac{L_Z}{N} \right) \right] + \frac{\dot{\overline{P_Y Y}}}{P_Y Y} - \frac{\dot{N}}{N} - \delta \tag{2.13}$$

After imposing firm symmetry, (2.12) becomes

$$V(t) = \psi P_X X = \gamma \frac{P_Y Y}{N} \tag{2.14}$$

where $\gamma \equiv \psi(1-\alpha-\beta)$.

2.2.3 Renewable Resources

Regardless of management regime, a competitive household harvests renewable resource from an existing stock, denoted $S(t)$. For simplicity, the natural growth of the renewable resource, denoted $G(S(t))$, is assumed to follow a logistic growth function. This function is widely adopted in the theoretical literature and extensively applied to fisheries.

$$G(S(t)) = \eta S(t) \left(1 - \frac{S(t)}{\bar{S}}\right) \tag{2.15}$$

where η represents the intrinsic growth rate, and $\bar{S}$ is the carrying capacity, the maximum amount of resource stock a country can possess when there is no harvest activity.

$$\dot{S}(t) = G(S(t)) - H(t) \tag{2.16}$$

$$H(t) = BL_H(t)S(t) \tag{2.17}$$

The dynamics of the stock depend on the function $G(S(t))$ and the amount of harvest good being produced $(H(t))$. Harvest production follows a Schaefer production function (Schaefer 1957).[4] The special feature of this production function is that harvest production does not only depend on labor$(L_H(t))$ and TFP of harvest production (B), but also relies on the existing stock of renewable resource at a given period $(S(t))$. The production function exhibits an increasing return to scale for all factors of production. B is constant and can be viewed as theTFP of harvest production or referred to as a "catchability coefficient".

When $G(S(t)) > H(t)$, the natural growth of the resource is greater than the harvest amount. As a result, the resource stock will increase. The result is opposite when the harvest amount exceeds the natural growth of the resource. Finally, when

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[4] The model is sometimes referred to as the "Gordon Schaefer" model.

$G(S(t)) = H(t)$, the resource stock remains constant. In other words, the natural growth of the resource allows a sustainable yield that can be harvested while the economy maintains the same level of resource stock S.

It is obvious that the standard model of renewable resources involves a lot of simplifications. However, the benefit from simplification is that the overall model becomes more tractable such that the main economic implications are easier to understand.

2.2.4 Household and Resource Managers

A competitive household is endowed with L units of labor, and has to allocate its labor into five different sectors: final good, manufacturing goods, harvest good, R&D innovation and start-up activities. Without loss of generosity, the model assumes no population growth and no preference for leisure. The utility function for the household is given as

$$U(t) = \int_0^{\infty} e^{-\rho t} \log C(t)\, dt \tag{2.18}$$

where $\rho > 0$ is the rate of time preference and $C(t)$ is the aggregate real consumption at time t

Households earn income from assets, labor and revenue from the harvest sector. Assets are in the form of ownership claims on capital such as stocks issued by manufacturing firms. Total income from financial assets is therefore the product of the rate of return per unit of asset ($r(t)$) and the amount of assets ($a(t)$) that the households possess. Households also earn wage income from labor. Since the labor market is competitive, they receive the same wage rates across sectors. In addition to labor and asset incomes, households receive revenue from selling harvest goods. Total income is spent on consumption goods ($P_C(t)C(t)$) or used to accumulate more assets. As a result, the aggregate wealth constraint for each household is

$$\dot{a}(t) = r(t)a(t) + L - L_H + P_H(t)H(t) - P_C(t)C(t) \tag{2.19}$$

A competitive household chooses the consumption path and harvest amount that maximizes its lifetime utility (2.18) subject to a flow budget constraint (2.19) and the dynamics of the resource stock (2.16) [5].

Solving Current Value Hamiltonian for the household, the optimality conditions for a competitive household can be written as:

$$\frac{1}{P_C C} = \lambda_a \tag{2.20}$$

$$r\lambda_a = \rho\lambda_a - \dot{\lambda}_a \tag{2.21}$$

$$\lambda_a \left[P_H - \frac{1}{BS} \right] \lesseqgtr \lambda_S \tag{2.22}$$

$$\lambda_a \frac{H}{BS^2} + \lambda_S \eta \left(1 - \frac{2S}{\bar{S}} \right) = \rho\lambda_S - \dot{\lambda}_S \tag{2.23}$$

where λ_a and λ_S represent shadow values of financial assets and resource stock, respectively. (2.20) explains that a household equates marginal utility of consumption to the shadow value of assets. (2.22) represents a tradeoff between resource stock and financial assets. To be specific, the shadow value of assets has to be equal to the shadow value of resource stock adjusted for profit per unit of producing harvest good. After some manipulation from (2.20) and (2.21), we get a familiar Euler Equation

$$r_a = \rho + \frac{\dot{C}}{C} + \frac{\dot{P}_C}{P_C} \tag{2.24}$$

The equation suggests that households choose consumption so as to equate the rate of return on saving, r_a, to the rate of return on consumption.

So far, the layout of the problem is the same regardless of resource manage-

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[5] Note that $\Pi_H = P_H H - W_H L_H$

ment regime. However, the next section will explain the differences between the two extreme cases.

2.3 Resource Management Regimes

2.3.1 Open Access Regime

An agent in an open access economy does not consider the dynamics of the resource stock. Agents will harvest resource until profit from the harvest sector is equal to zero. Rent dissipates to zero accordingly, which implies that the shadow value of resource stock is equal to zero. Letting"*oa*" denote open access and setting $\lambda_S = 0$, (2.23) is eliminated and in equilibrium, (2.22) becomes

$$P_{H,oa}(t) = \frac{1}{BS_{oa}(t)} \qquad \forall t \tag{2.25}$$

Note that if the price of harvest good is higher than its unit cost, there will be an infinite amount of labor in the harvest sector which would halt the economy. On the other hand, if the price is lower than the unit cost, nobody will harvest the resource. But since harvest good is an essential input for the economy, it is rational to only consider the interior solution for the harvest production function. Therefore, in equilibrium, the price of harvest good must equal the unit cost of harvesting at every point in time. Given the demand for harvest good from the final good producers (2.3), a temporary equilibrium for harvest good is

$$H_{oa}(t) = \alpha BS_{oa}(t)P_{Y,oa}(t)Y_{oa}(t) \qquad \forall t \tag{2.26}$$

General Equilibrium for Open Access Regime

The general equilibrium conditions are as follows:

Final Good Market Clearing Condition

$$Y_{oa} = C_{oa} \tag{2.27}$$

Due to the setting of the economy, final output is only used for consumption. In equilibrium, the amount of final good produced in the economy must equal the amount of consumption.

Labor Market Clearing Condition

$$L_{oa} = L_{Y,oa} + L_{H,oa} + L_{X,oa} + L_{Z,oa} + L_{N,oa} \tag{2.28}$$

Given a perfectly inelastic labor supply (no preference in leisure), (2.28) states that in equilibrium, full employment must be fulfilled. In particular, given the endowment of labor, a household must allocate its labor among the various sectors in the economy: final-output production, manufactured-goods production, harvest-good production, knowledge production and entrepreneurship (see Appendix for *Proof).*

Asset Market Clearing Condition

$$r_{oa} = r_{a,oa} = r_{Z,oa} = r_{N,oa} \tag{2.29}$$

(2.29) states that the rate of return from equity issued by manufacturing firms must equal the rate of return from financial assets that consumers hold. It should also equal the rate of return from knowledge and the rate of return on developing a new product. If this condition does not hold, there will be some arbitrage opportunity and market mechanisms will adjust these rates until they are equalized.

Another way to represent an equilibrium in the asset market is to have the value of the assets held by households be equivalent to the value of the stocks issued by firms, such that

$$a_{oa} = N_{oa} V_{oa} = \gamma P_{Y,oa} Y_{oa} \tag{2.30}$$

After imposing firm symmetry, the benefit of using this model is that one dynamic equation can be eliminated so the problem can now be solved analytically. To be specific, substituting (2.24) and (2.30) into (2.19)[6], a constant rate of saving can be expressed as

$$\frac{P_{Y,oa}Y_{oa} - L}{P_{Y,oa}Y_{oa}} = \gamma\rho \quad \forall t \tag{2.31}$$

Defining $y_{oa} \equiv \frac{P_{Y,oa}Y_{oa}}{L}$ as per capita expenditure, (2.31) can be written as

$$y_{oa} = \frac{1}{1 - \gamma\rho} \tag{2.32}$$

(2.32) states that nominal expenditure per capita for an open access economy is constant at every point in time. This result appears from the Cobb-Douglas production function. It is very useful in analyzing the steady state and transitional dynamics of variables of interest. In particular, when y_{oa} is constant, it implies that $r = \rho$.

Proposition 1: If the number of existing entrepreneurs is sufficiently large, the growth rate of knowledge in the manufacturing sector will be zero

Proof : Using (2.11) and imposing firm symmetry, the return on knowledge is

$$\hat{Z}_{oa} = \kappa\frac{L_{Z,oa}}{N_{oa}} = \begin{cases} \kappa(1-\alpha-\beta)^2\theta\frac{y_{oa}}{n_{oa}} - (\rho+\delta) & if\ n_{oa} < \bar{n}_{oa} \\ 0 & if\ n_{oa} \geqslant \bar{n}_{oa} \end{cases} \tag{2.33}$$

where $n_{oa} \equiv \frac{N_{oa}}{L_{oa}}$ and $\bar{n}_{oa} = \kappa\theta(1-\alpha-\beta)^2\frac{y_{oa}}{(\rho+\delta)}$, which is the threshold for the economy to start engaging in R&D activities. (2.33) demonstrates that the growth rate of knowledge will be positive if and only if the firm size or market share for each manufacturing firm (y/n) is relatively large. In contrast, if there are a lot of manufacturing firms per capita in the economy such that the firm size or market share is relatively small, the profit margin for each firm will be smaller. As a result, there will be no incentive for those firms to conduct R&D and the growth rate of knowledge will be zero. Substituting (2.24), (2.27) and (2.33) into (2.13), and making it in terms

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[6] In equilibrium, profit from the harvest sector becomes zero.

of per capita, the expansion of product variety or number of manufacturing firms can be solved as

$$\hat{n}_{oa} = \begin{cases} \frac{1}{\psi}\left[\alpha + \beta - \theta(1-\alpha-\beta) - \frac{1}{(1-\alpha-\beta)}\left(\phi - \frac{(\rho+\delta)}{\kappa}\right)\frac{n_{oa}}{y_{oa}}\right] - (\rho+\delta) & if\ n_{oa} < \bar{n}_{oa} \\ \frac{1}{\psi}\left[\alpha + \beta - \frac{\phi}{(1-\alpha-\beta)}\frac{n_{oa}}{y_{oa}}\right] - (\rho+\delta) & if\ n_{oa} \geqslant \bar{n}_{oa} \end{cases} \tag{2.34}$$

Note that the growth rate of product variety is negatively related to the entry cost and fixed labor cost of producing good X.

2.3.2 Full Property Rights Economy

In a full property rights regime, households take into account the dynamics of the renewable resource stock. As a result, the shadow value of resource has to be positive, and there is a tradeoff between holding assets and conserving resources. Letting "*pr*" denote property rights, a household's Euler equation, optimality conditions and budget constraint (2.19) become

$$r_{pr} = \rho + \frac{\dot{C}_{pr}}{C_{pr}} + \frac{\dot{P}_{C,pr}}{P_{C,pr}} \tag{2.35}$$

$$\frac{1}{P_{C,pr}C_{pr}} = \lambda_{a,pr} \tag{2.36}$$

$$\frac{1}{P_{Y,pr}Y_{pr}}\left(P_{H,pr} - \frac{1}{BS_{pr}}\right) = \lambda_{S,pr} \tag{2.37}$$

$$\frac{1}{\left(P_{H,pr} - \frac{1}{BS_{pr}}\right)}\left(\frac{H}{BS_{pr}^2}\right) + \eta\left(1 - \frac{2S_{pr}}{\bar{S}}\right) = \rho - \frac{\dot{\lambda}_{S,pr}}{\lambda_{S,pr}} \tag{2.38}$$

$$\dot{a}_{pr} = r_{a,pr}a_{pr} - L - \left(\frac{H_{pr}}{BS_{pr}}\right) + P_{H.pr}H_{pr} - P_{C,pr}C_{pr} \tag{2.39}$$

Unlike in an open access economy, (2.37) implies that the profit from the harvest sector is now positive due to positive values of $\lambda_{S,pr}$ and $P_{Y,pr}Y_{pr}$

General Equilibrium for Full Property Rights Regime

With a normalized wage, the general equilibrium conditions are the same as those in an open access regime. To gain further insight about how a full property rights economy behaves, the analysis will begin similarly to that for open access. It is easy to see that the value of assets from the household side must again equal the total value of stocks issued by the manufacturing firms.

$$a_{pr} = N_{pr}V_{pr} = \gamma P_{Y,pr}Y_{pr} \tag{2.40}$$

Taking log and differentiating (2.40) with respect to time, and substituting into (2.39),

$$y_{pr}(t) = \left(\frac{1}{1-\alpha-\gamma\rho}\right)\left(1-\frac{H_{pr}(t)}{LBS_{pr}(t)}\right) = \left(\frac{1}{1-\alpha-\gamma\rho}\right)\left(1-\frac{L_{H,pr}(t)}{L}\right) \tag{2.41}$$

Note that the parameter restriction $LBS_{pr}(t) > H_{pr}(t)$ has to hold for all points in time in order to ensure a positive value of nominal expenditure per capita in a full property rights regime.

(2.41) indicates that nominal expenditure per capita in this case consists of two components: the first is income gain from economic activities and the second is income forgone at present from not overexploiting the resource stock. This can be viewed as another type of savings in terms of conserving renewable resources for the future rather than holding financial assets. Unlike in an open access economy, the nominal expenditure per capita in this economy is no longer constant at every point in time. It depends on the labor employed in the harvest sector, which is determined by the ratio of harvest good produced to existing stock of resource in the economy $\left(\frac{H_{pr}}{S_{pr}}\right)$.

2.4 Steady State Analysis

2.4.1 Open Access Economy

Let "*" denote a variable of interest evaluated at the steady state. At the steady state, the growth rate of both the renewable resource and the number of manufacturing firms is equal to zero.

Proposition 2: Under an open access regime

i) The steady state stock of renewable resource is positive if and only if $\eta(1-\gamma\rho) > \alpha BL$*. If not, then the tragedy of the commons occurs* ($S = H = 0$).

ii) Given a positive value of nominal expenditure per capita, and the parameter restriction in i) holds, the steady stock of renewable resource is stable and unique.

Proof : Using (2.15) and (2.26), and setting (2.16) to zero, the steady state stock of renewable resource in an open access economy becomes

$$S^*_{oa} = \bar{S}\left(1 - \frac{\alpha BLy^*_{oa}}{\eta}\right) > 0 \quad iff \tag{2.42}$$

Substituting y^*_{oa}[7], the parameter restriction that generates a positive steady state value of resource stock becomes

$$\eta > \frac{\alpha BL}{1-\gamma\rho} \tag{2.43}$$

The steady state level of harvest good produced is obtained by replacing (2.42) in (2.26)

$$H^*_{oa} = \alpha BLy^*_{oa}\bar{S}\left(1 - \frac{\alpha BLy^*_{oa}}{\eta}\right) \tag{2.44}$$

If (2.43) is not satisfied, the renewable resource will become extinct and the unique steady state will be at $S = H = 0$. This result would halt the economy because the harvest good is an essential input for producing the final good. Therefore,

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[7] Since y_{oa} is a constant, $y_{oa} = y^*_{oa}$

this condition is a crucial assumption for the economy to generate long-run growth. In particular, the intrinsic growth rate of renewable resource should be sufficiently high relative to labor endowment, nominal expenditure per capita, income share of harvest production and capital used in the harvest sector.

For the manufacturing sector, setting (2.34) to zero, the steady state level of number of manufacturing firms in the economy is

$$n_{oa}^{*} = \begin{cases} \frac{\kappa(1-\alpha-\beta)(\alpha+\beta-\theta(1-\alpha-\beta)-(\rho+\delta)\psi)}{\phi\kappa-(\rho+\delta)} y_{oa}^{*} & if \ \frac{\alpha+\beta-\theta(1-\alpha-\beta)-(\rho+\delta)\psi}{\phi\kappa-(\rho+\delta)} < \frac{\theta(1-\alpha-\beta)}{(\rho+\delta)} \\ \frac{(1-\alpha-\beta)(\alpha+\beta-(\rho+\delta)\psi)}{\phi} y_{oa}^{*} & if \ \frac{\alpha+\beta-\theta(1-\alpha-\beta)-\rho\psi}{\phi\kappa-(\rho+\delta)} \geqslant \frac{\theta(1-\alpha-\beta)}{(\rho+\delta)} \end{cases} \tag{2.45}$$

$$\frac{y_{oa}^{*}}{n_{oa}^{*}} = \frac{(\phi - \frac{(\rho+\delta)}{\kappa})}{(1-\alpha-\beta)(\alpha+\beta-\theta(1-\alpha-\beta)-(\rho+\delta)\psi)} \tag{2.46}$$

Considering positive horizontal and vertical innovation, the steady state growth of knowledge is

$$\hat{Z}_{oa}^{*} = \frac{\kappa\theta(1-\alpha-\beta)(\phi - \frac{(\rho+\delta)}{\kappa})}{(\alpha+\beta-\theta(1-\alpha-\beta)-(\rho+\delta)\psi)} - (\rho+\delta) \tag{2.47}$$

Consequently, the steady state growth rate of the economy under an open access management regime becomes

$$g_{oa}^{*} = (1-\alpha-\beta)\theta\hat{z}_{oa}^{*} \tag{2.48}$$

$$g_{oa}^{*} = (1-\alpha-\beta)\theta\left(\kappa\theta(1-\alpha-\beta)^{2}\frac{y_{oa}^{*}}{n_{oa}^{*}} - (\rho+\delta)\right) \tag{2.49}$$

Note that the steady state growth rate of the economy does not depend either on labor endowment (L) or renewable resource endowment (S). However, both endowments play an important role in economic growth along the transition path.

2.4.2 Full Property Rights Economy

At the steady state, (2.41) becomes

$$y^*_{pr} = \varphi\left(1 - \frac{H^*_{pr}}{BLS^*_{pr}}\right) \tag{2.50}$$

where $\varphi = 1/(1 - \alpha - \gamma\rho)$.

Using (2.3) for $P_{H,pr}$ and substituting y^*_{pr} into (2.37), the function is quadratic in H but only one value of H is valid (see Appendix for *Proof*),

$$H^*_{pr} = \frac{1}{2}\left[\frac{(1-\gamma\rho)}{\lambda_{S,pr}} + BLS_{pr} - \sqrt{\left(\frac{(1-\gamma\rho)}{\lambda_{S,pr}} + BLS_{pr}\right)^2 - \frac{4\alpha BLS_{pr}}{\lambda_{S,pr}}}\,\right] \tag{2.51}$$

Setting $\dot{\lambda}_S = \dot{\lambda}_a = 0$, (2.16) and (2.38) can be written as[8]

$$H^*_{pr} = \eta S^*_{pr}\left(1 - \frac{S^*_{pr}}{\bar{S}}\right) \tag{2.52}$$

$$\frac{H^{*2}_{pr}}{\alpha\varphi BLS^{*2}_{pr} - (\alpha\varphi + 1)H^*_{pr}S^*_{pr}} - \eta\left(1 - \frac{2S^*_{pr}}{\bar{S}}\right) = \rho \tag{2.53}$$

Substituting (2.52) in (2.53), the function is quadratic in S and again, the positive value of S^*_{pr} is chosen.

When S^*_{pr} is constant, (2.50) and (2.52) infer that harvest good produced and nominal expenditure per capita are also constant. As a result, growth analysis in the steady state will be carried out like in the open access case.

Given that nominal expenditure per capita is constant ($r = \rho$), (2.41) becomes

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[8] Note that $\lambda_S = 0$ can be a steady state level in this full property rights regime. However, focus will be on a positive value of λ_S; otherwise, the steady state value will be the same as in the open access regime.

$$y^*_{pr} = \left(\frac{1}{1-\alpha-\gamma\rho}\right)\left(1-\frac{L^*_{H,pr}}{L}\right) \tag{2.54}$$

Again imposing firm symmetry, the return on knowledge now depends on the ratio of nominal expenditure to number of manufacturing firms per capita

$$\hat{Z}_{pr} = \kappa\frac{L_{Z,pr}}{N_{pr}} = \begin{cases} \kappa(1-\alpha-\beta)^2\theta\frac{y^*_{pr}}{n^*_{pr}} - (\rho+\delta) & if\ n_{pr} < \bar{n}_{pr} \\ 0 & if\ n_{pr} \geqslant \bar{n}_{pr} \end{cases} \tag{2.55}$$

where $\bar{n}_{pr} = \kappa\theta(1-\alpha-\beta)^2\frac{y^*_{pr}}{(\rho+\delta)}$. Moreover, the growth of the number of manufacturing firms becomes

$$\hat{n}_{pr} = \begin{cases} \frac{1}{\psi}\left[\alpha+\beta-\theta(1-\alpha-\beta)-\frac{1}{(1-\alpha-\beta)}\left(\phi-\frac{(\rho+\delta)}{\kappa}\right)\frac{n^*_{pr}}{y^*_{pr}}\right]-(\rho+\delta) & if\ n_{pr} < \bar{n}_{pr} \\ \frac{1}{\psi}\left[\alpha+\beta-\left(\frac{\phi}{(1-\alpha-\beta)}\right)\frac{n^*_{pr}}{y^*_{pr}}\right]-(\rho+\delta) & if\ n_{pr} \geqslant \bar{n}_{pr} \end{cases} \tag{2.56}$$

Setting $\hat{n}_{pr} = 0$, it can be shown that

$$\frac{y^*_{pr}}{n^*_{pr}} = \frac{\phi\kappa-(\rho+\delta)}{\kappa(1-\alpha-\beta)(\alpha+\beta-\theta(1-\alpha-\beta)-(\rho+\delta)\psi)} \tag{2.57}$$

$$\hat{Z}^*_{pr} = \kappa\theta(1-\alpha-\beta)^2\frac{y^*_{pr}}{n^*_{pr}} - (\rho+\delta) \tag{2.58}$$

$$g^*_{pr} = (1-\alpha-\beta)\theta\hat{Z}^*_{pr} = (1-\alpha-\beta)\theta\left(\kappa\theta(1-\alpha-\beta)^2\frac{y^*_{pr}}{n^*_{pr}} - (\rho+\delta)\right) \tag{2.59}$$

At the steady state, the growth rate of the economy is the same under the two regimes of renewable resource management.

2.5 Steady State Comparison

Proposition 3: Considering the steady state variables between the two regimes,

i) $S^*_{oa} < S^*_{pr}$

The steady state level of resource stock in the open access economy is lower than in the full property rights one. The outcome is very intuitive because under full property rights, agents harvest the renewable resource by taking into account the long-run availability of the resource stock. On the other hand, open-access leads to uncontrolled exploitation.

ii) $H^*_{oa} > H^*_{pr}$

The steady state harvest level in the open access economy is higher because the rent associated with resource stock dissipates to zero, and the price of harvest good equates to the unit cost. As a result, the steady state price of harvest good in the open access regime is lower than in the full property rights regime. Consequently, the level of harvest activity in the open access regime is higher.

Proof: Assuming that *i)* is true, substitute (2.51) into (2.50) and impose a non-negative value of nominal expenditure

iii) $y^*_{oa} < y^*_{pr}$

Intuitively, the nominal expenditure per capita in the open access economy is lower because profit from the harvest sector is equal to zero (rent dissipates to zero instantaneously at every point in time). In the full property rights economy, conserving resource stock is considered to be one kind of savings for the future, and thus results in higher nominal expenditure per capita at the steady state.

Proof: Assuming that *i)* and *ii)* are true, *iii)* can be proven by contradiction

iv) $n^*_{oa} < n^*_{pr}$

The number of manufacturing firms is positively related to the level of nominal expenditure per capita. Since the steady state nominal expenditure per capita is lower in the open access economy, the number of manufacturing firms per capita in that regime is correspondingly smaller.

Proof: Derive from *iii)*

v) $\frac{y^*_{oa}}{n^*_{oa}} = \frac{y^*_{pr}}{n^*_{pr}}$

At the steady state, the ratio of nominal expenditure to number of manufacturing firms per capita is equal between the two regimes.

Proof: Set (2.34) and (2.56) to zero

vi) $\hat{Z}^*_{oa} = \hat{Z}^*_{pr}$

Recall that the steady state growth rate of knowledge only depends on the ratio of nominal expenditure to number of manufacturing firms or products per capita $\left(\frac{y^*}{n^*}\right)$*. Since the ratio is equivalent in both regimes, the steady state growth rate of knowledge is also the same.*

vii) $g^*_{oa} = g^*_{pr}$

*Since the growth rate of the resource stock and the growth rate of the number of manufacturing firms are both zero at the steady state, the growth rate of the economy is then proportional to the growth rate of knowledge, and is the same across the two regime*s.

2.6 Development Path

Recall that nominal expenditure per capita in the open access regime is fixed at every point in time. Now assume that the ratio of harvest good produced to existing stock of renewable resource (H/S) in the full property rights regime is constant such that nominal expenditure per capita in this regime is also fixed. From a developing stage (no R&D activity), the time it takes for the economy to transition to the developed stage (positive R&D activity) is dependent on the resource management regime and the initial number of manufacturing firms per capita. The development threshold is reached when the firm size becomes sufficiently large as a result of the decline in the number of firms over time. As a result, there will be an incentive for the remaining firms to conduct R&D. To be specific,

$$\hat{Z}_J = \kappa \frac{L_{Z,J}}{N_J} = \begin{cases} \kappa(1-\alpha-\beta)^2\theta\frac{y_J}{n_J} - (\rho+\delta) & if\ n_J < \bar{n}_J \\ 0 & if\ n_J \geqslant \bar{n}_J \end{cases} \tag{2.60}$$

$$\hat{n}_J = \begin{cases} \frac{1}{\psi}\left[\alpha+\beta-\theta(1-\alpha-\beta)-\frac{1}{(1-\alpha-\beta)}\left(\phi-\frac{(\rho+\delta)}{\kappa}\right)\frac{n_J}{y_J}\right]-(\rho+\delta) & if\ n_J < \bar{n}_J \\ \frac{1}{\psi}\left[\alpha+\beta-\left(\frac{\phi}{(1-\alpha-\beta)}\right)\frac{n_J}{y_J}\right]-(\rho+\delta) & if\ n_J \geqslant \bar{n}_J \end{cases} \tag{2.61}$$

$$\bar{n}_J = \kappa\theta(1-\alpha-\beta)^2\frac{y_J}{(\rho+\delta)} \tag{2.62}$$

Figure 2.2 demonstrates the differential equation for the number of manufacturing firms. When the number of firms is above the threshold (at Point A), there will be no growth of knowledge $(\hat{Z} = 0)$ and the rate of decline in the number of firms will correspond to the second line of $\hat{n}_J$. In contrast, when the number of firms is below the threshold (at Point B), there will be a positive growth of knowledge while the rate of change in the number of firms will correspond to the first line of $\hat{n}_J$.

Resource Management Regimes

It can be seen from (2.60), (2.61), (2.62) that $\bar{n}_{pr} > \bar{n}_{oa}$.

Proof: $\bar{n}_J = \kappa\theta(1-\alpha-\beta)^2\frac{y_j^*}{(\rho+\delta)}$ and $y_{pr}^* > y_{oa}$

The threshold for accumulating knowledge depends on the number of manufacturing firms per capita. If the number becomes sufficiently small, the firm size becomes relatively large, providing an incentive for the firms to conduct R&D and accumulate knowledge, which is the source of long-run growth. Given the same number of manufacturing firms, the firm size is larger under full property rights than under open access due to higher nominal expenditure per capita. Therefore, under full property rights, the development threshold occurs at a higher number of firms and is reached sooner on the development path.

Suppose the parameters are such that the growth rate of knowledge has a unique and positive steady state growth rate, the development paths for the two resource management regimes are depicted in Figure 2.3. In particular,

Case i : where $n(0) > \bar{n}_{pr}$

The economy starts with no R&D innovation regardless of the resource management regime. Under full property rights, the economy will begin to engage in R&D activities and enjoy positive growth of knowledge after the number of manufacturing firms falls below point A. However, under open access, the economy would begin the R&D process later, once the number of manufacturing firms falls further below point B.

Case ii: where $\overline{n}_{pr} > n(0) > \bar{n}_{oa}$

The economy under full property rights enjoys positive growth of knowledge from the start, but the economy under open access does not. In the latter case, the dynamics of the number of manufacturing firms will move along the dashed-red line until it reaches point B. Only at that time would the economy initiate R&D and enjoy positive growth of knowledge as well.

Case iii: where $n(0) < \bar{n}_{oa}$

The economy under both regimes is in the positive growth of knowledge stage.

2.7 Transitional Dynamics in an Open Access Case

2.7.1 *Dynamics for Number of Manufacturing Firms Per Capita (n)*

Assuming the interior steady state involves both vertical and horizontal innovation and positive steady state resource stock, the transitional dynamics for the number of manufacturing firms in the economy can be written as a logistic equation[9]

$$\hat{n}_{oa}(t) = \chi_0 - \chi_1 n_{oa}(t) \tag{2.63}$$

where $\chi_0 = \frac{1}{\psi}(\alpha + \beta - \theta(1 - \alpha - \beta)) - (\rho + \delta)$, $\chi_1 = \frac{\left(\phi + \frac{(\rho+\delta)}{\kappa}\right)}{\psi(1-\alpha-\beta)y^*_{oa}}$

which has the solution

$$n_{oa}(t) = \frac{n^*_{oa}}{1 + \left(\frac{n^*_{oa}}{n_{oa}(0)} - 1\right) e^{-\chi_0 t}} \tag{2.64}$$

where $n_{oa}(0)$ denotes the initial number of manufacturing firms in the open access economy.

When the number of firms is higher than the steady state level, there are too many firms in the economy. As a result, there will be a decrease in the rate of return on knowledge and the present value of manufacturing firms, so some firms will no longer make a profit and exit the market. The opposite result occurs when the number of firms is lower than the steady state level.

2.7.2 *Dynamics for Renewable Resource Stock*

Given a positive steady state level of renewable resource along with the Schaefer production function, the dynamics for the resource stock can also be displayed in the form of a logistic function.

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[9] See Banks (1994)).

$$\hat{S}_{oa}(t) = \nu - \frac{S_{oa}(t)}{\bar{S}}, \quad \nu = \eta - \frac{\alpha L}{1-\gamma\rho} \tag{2.65}$$

The solution yields

$$S_{oa}(t) = \frac{S^*_{oa}}{1 + \left(\frac{S^*_{oa}}{S_{oa}(0)} - 1\right) e^{-\nu t}} \tag{2.66}$$

where $S_{oa}(0)$ is the initial level of resource stock.

(2.66) states that if the initial stock of the renewable resource is lower than the steady state level, the price of harvest good, which is negatively related to the stock, will be high. This condition lowers the demand for harvest good, which in turn allows the resource stock to accumulate to the steady state level. If the initial stock is higher than the steady state level, the opposite occurs.

2.7.3 Dynamics for the Growth Rate of the Economy

The growth rate of the economy is equal to the growth rate of consumption, which is equivalent to the growth rate of the final output. Log differentiating (2.1), the growth rate of the economy becomes

$$\hat{g}_{oa} \equiv \hat{c}_{oa} \equiv \widehat{\left(\frac{Y_{oa}}{L}\right)} \tag{2.67}$$

$$\hat{g}_{oa} = \beta\hat{L}_{Y,oa} + \alpha\hat{S}_{oa} + (1-\beta)\hat{n}_{oa} + (1-\alpha-\beta)\theta\hat{Z}_{oa} \tag{2.68}$$

(2.68) states that the dynamics of economic growth depend on the growth rate of i) labor employed in final output (which is zero in this setting), ii) stock of renewable resource, iii) number of entrepreneurs, and iv) knowledge. At the steady state, $\hat{S} = \hat{n} = 0$. The dynamics of economic growth can behave as follows:

Case 1: If the economy starts with a low (high) level of resource stock and a low (high) level of manufacturing products, economic growth will increase (decrease) until it reaches the steady state growth rate.

(2.68) suggests that if $S(0)$ and $n(0)$ are lower (higher) than the steady state values, the growth rate of the economy will start above (below) the steady state rate. This is because when the economy has a low (high) level of resource stock, the price of harvest good will be high (low). As a result, the final good producers will demand a small (large) amount of harvest good and allow the resource to accumulate (deplete) or $\hat{S} > 0$ ($\hat{S} < 0$).

A similar principle applies to the manufacturing sector. When $n(0)$ is relatively low (high), the growth rate of knowledge is high (low), which provides an incentive (disincentive) for new entrepreneurs. Consequently, the number of manufacturing firms will grow (shrink) but at a decreasing rate until it converges to the steady state number.

Case 2: If the economy starts with a high (low) level of resource stock and a low (high) level of manufacturing products, the dynamics of economic growth are ambiguous. The dynamics will depend on the relative magnitudes of the two opposing forces, namely the growth rate of the resource stock and the growth rate of the number of manufacturing firms.

2.8 Welfare analysis

Suppose the economy starts out at the steady state level of full property rights and continues until period $t = 0$. At that time, the economy switches to an open access regime due to the failure of property rights. There are two kinds of change that follow. The first is an instantaneous change in the welfare level at the time of regime switch ($t = 0$) and the other is a transitional change for the economy to converge to a new steady state level.

2.8.1 Baseline Consumption Path

Before further analysis will take place, it is useful to characterize some important variables related to the welfare analysis. Suppose prior to the regime switch, the economy was carrying on at the steady state level of consumption under full property rights. At time $t = 0$, if there is no change in policy, the consumption per capita under full property rights can be written as:

$$c_{pr}(t=0) = \Phi y^*_{pr}(P^*_{H,pr})^{-\alpha}(n^*_{pr})^{\alpha+\beta}(Z^*{}_{pr})^{\theta(1-\alpha-\beta)} \tag{2.69}$$

where $\Phi = A\beta^\beta\alpha^\alpha(1-\alpha-\beta)^{2(1-\alpha-\beta)}L^{(\alpha+\beta)}$. The consumption path under no switch of regime is referred to as the "baseline consumption path" and it grows at the rate g^*_{pr}.

2.8.2 Instantaneous Change in Consumption

Proposition 4: Consumption per capita drops after the regime switch from full property rights to open access if and only if

$$\frac{y^*_{pr}}{P^{*\alpha}_{H,pr}} > \frac{y_{oa}}{P^{\alpha}_{H,oa}(0)} \tag{2.70}$$

Proof: At $t = 0$, the the resource management regime switches to open access and the consumption level becomes

$$c_{oa}(t=0) = \Phi y_{oa}(P_{H,oa})^{-\alpha}(n^*_{pr})^{\alpha+\beta}(Z^*_{pr})^{\theta(1-\alpha-\beta)} \tag{2.71}$$

Notice that the only differences between (2.69) and (2.71) are nominal expenditure per capita and the price of harvest good because the number of manufacturing firms and knowledge stock are state variables that can only adjust over time. For nominal expenditure per capita, it was proven in the previous section that

$y^*_{pr}(0) > y_{oa}(0)$ for $\forall t$. However, the price of harvest good deserves additional consideration. Specifically,

$$P^*_{H,pr}(0) = \lambda^*_S y^*_{pr} + \left(\frac{1}{BS^*{}_{pr}}\right) \tag{2.72}$$

When the regime switches to open access, $\lambda_S = 0$, so the price of harvest good then decreases instantaneously.

$$P^*_{H,oa}(0) = \left(\frac{1}{BS^*_{pr}}\right) \tag{2.73}$$

Switching from full property rights to open access results in a tradeoff between nominal expenditure per capita and price of harvest good. Under open access, nominal expenditure per capita is lower because the rent generated from the harvest sector now disappears. However, agents can now harvest the resource at a higher rate, which then reduces the price of harvest good. As a result, the instantaneous change in consumption after the regime switch is indeterminate. It depends on the magnitude of the fall in nominal expenditure per capita relative to the fall in price of harvest good. Figure 2.4 shows the transitional dynamics for nominal expenditure per capita and price of harvest good. It is important to note that the economy will later experience a gradual increase in the price of harvest good as the renewable resource becomes scarcer, which will be a factor in reducing the welfare of the economy in the long run.

2.8.3 Transitional Change in Consumption Path

After the initial instantaneous change in consumption, the economy will start to transition to the steady state of consumption growth under an open access regime, which has the same slope as that under full property rights. In order to evaluate the transitional path, substitute (2.73) into (2.71) and take log

$$\log c_{oa}(t) = \log \Phi + \log y_{oa} + \alpha \log B + \log D(t) \tag{2.74}$$

where $D(t) = S(\mathrm{t})^{\alpha} n(t)^{(\alpha+\beta)} Z(t)^{\theta(1-\alpha-\beta)}$. This will determine the transitional dynamics of TFP from full property rights to the steady state open access economy.

$$\hat{D} = \theta(1-\alpha-\beta)\hat{Z} + (\alpha+\beta)\hat{n} + \alpha\hat{S} \tag{2.75}$$

At the steady state,

$$\hat{D}^{*} = \theta(1-\alpha-\beta)\hat{Z}^{*}_{oa} = \theta(1-\alpha-\beta)\hat{Z}^{*}_{pr} = g^{*}_{pr} = g^{*}_{oa} \tag{2.76}$$

$$\log D(t) = (\alpha+\beta)\log n(t) + \alpha \log S(t) + \theta(1-\alpha-\beta)\log Z(t) \tag{2.77}$$

Figure 2.5 displays the transitional dynamics for the growth rate of manufacturing firms ($\hat{n}$), growth rate of resource stock $\left(\hat{S}\right)$, growth rate of knowledge ($\hat{Z}$), and transitional growth rate of consumption $\left(\hat{D}\right)$.

Notice that $\hat{Z}$ increases until it reaches the steady state level because i) $\hat{Z} = fn(\frac{y}{n})$, ii) y_{oa} is constant at every point in time, and iii) $n(t)$ decreases as time increases. $\hat{S}$ and $\hat{n}$ are negative for they have to approach the lower steady state levels. Combining all the information qualitatively, two cases could happen with differing consumption paths.

Case i): The slope of $\log D(t)$ begins with a negative value, which implies that consumption will fall at the beginning but eventually rise in order to get to the steady state level of consumption growth. In other words, the slope of the consumption path will be negative at first and then increase to the positive steady state rate. This case occurs when initial $\hat{S}$ and $\hat{n}$ are negative enough to outweigh the positive growth of Z. Intuitively, at $t \geqslant 0$, agents will harvest more than the optimal amount and cause resource stock to fall ($\hat{S} < 0$). Moreover, as nominal expenditure per capita decreases, the number of entrepreneurs starts to fall ($\hat{n} < 0$). The corresponding fall in number of products hurts consumption level but causes an increase in knowledge stock over time. As a result, in the early stages after the regime switch, the level of consumption starts to fall deeper from a downturn in both the resource stock and number of manufacturing firms. Later, however, the downturn effect becomes smaller and the positive effect of having higher knowledge starts to outweigh it. At that point, consumption starts to increase until the steady state level of consumption growth at g^* is achieved.

Case ii): The slope of $\log D(t)$ begins with a positive value but less than g^*. The consumption path will increase slowly at first and then converge to the same slope as in the baseline path. Intuitively, the negative effect from a downturn in both n and S is not strong enough to hinder the positive effect from the growth of knowledge. Consumption per capita still increases but at a lower rate. As the economy converges to a steady state, consumption grows faster until it reaches the steady state level of consumption growth.

Figure 2.6 illustrates the possible consumption paths after an initial instantaneous drop in consumption level following the change in resource management regime.

If consumption instantaneously drops after a regime switch, it can be seen that the resulting consumption path is strictly below the baseline path. Consequently, it can be implied that the welfare level is strictly lower when the economy changes regime from full property rights to open access due to two effects. The first effect is a cumulative loss of TFP (the slope of D is lower than that of the baseline) and the second is an increase in the price of harvest good in the long run.

However, when (2.70) does not hold, the economy initially enjoys a higher level of consumption because the decrease in the price of harvest good when switching from full property rights to open access is very drastic. Nevertheless, as the stock of renewable resource becomes scarcer later on, the price of harvest good increases and the consumption level eventually falls below the baseline consumption path before finally increasing to converge on the same steady state growth rate (Figure 2.7). Therefore, even with an initial rise in consumption level, it is apparent that the consumption path and welfare level is lower in the long run when switching to open access.

2.9 Conclusion

This paper's contribution is that it endogenizes the role of renewable resources, which is an important factor of production for developing countries and mostly suppressed in modern models of endogenous growth. The paper sets up two cases with identical labor and resource endowment. Labor, the harvest good and a variety of manufacturing goods are employed to produce a final good. The source of long-run growth stems from vertical innovation. The only difference between the two cases is in regard to the resource management regime. The first case is an open access regime where resource ownership is ambiguous. A negative externality arises in the sense that agents only consider the unit cost of harvesting without incorporating the cost of depleting the future resource stock or, as represented in this model, $\lambda_S = 0$. The other case is an optimal management regime of full property rights. Resource owners, households in this case, internalize the dynamics of the resource stock in order to determine the optimal level of harvest production. The model analyzed the balanced growth path of the economy, transitional dynamics and welfare comparison under the two different management regimes.

One of the main results of this paper is that, as long as the steady state resource stock is positive, the two economies enjoy the same steady state growth rate . This result derives from the fact that there are no "growth stimulating factors" such as R&D in the resource sector. The steady state growth rate of the economy depends solely on the steady state growth rate of knowledge, which is in turn dependent on the ratio of nominal expenditure to number of manufacturing firms per capita. Although nominal expenditure per capita in the steady state is higher under full property rights than under open access, so is the number of manufacturing firms per capita. As a result, this ratio is the same in the steady state regardless of management regime. It can therefore be concluded that the steady state growth rate of the economy is equal under the two different management regimes.

Nonetheless, resource management does play an important role in the transitional dynamics. In regard to the development path, a developing economy will cross the development threshold sooner under full property rights than under open access. In the context of this paper, the developing stage only involves the expansion of product variety, which does not contribute to long-run growth. On the other hand, the developed stage involves both expansion of product variety and R&D activities that lead to knowledge accumulation, which is the source of long-run growth. Manufacturing firms will conduct R&D if and only if the firm size is large enough to provide an incentive for them to do so. The specific firm size that compels firms to initiate R&D represents the development threshold. The more quickly firm size grows to reach this threshold, the sooner the economy will engage in R&D activities and accumulate knowledge. Coming from a developing stage, firm size grows as the number of manufacturing firms declines over time. However, the firm size is larger under full property rights than under open access, given the same number of firms. As the result, the economy will take off and transition to the developed stage earlier under full property rights.

Another main result of this paper is in regard to welfare comparison. Assume that the country is in the developed stage where R&D activities and expansion of product variety both take place. If full property rights collapse and the economy switches to an open access regime, the ensuing welfare path is ambiguous. In particular, however, if the fall in nominal expenditure per capita sufficiently outweighs the fall in price of harvest good, the welfare level will be significantly lower after switching to open access. After an initial drop in consumption level, the slope of the ensuing welfare path (transitional growth rate of consumption) can be either negative or slightly positive at the beginning before finally converging to the same slope as the baseline bath prior to the regime switch. Either way, the welfare level is strictly below the baseline. Thus, switching to open access is welfare reducing because of a cumulative loss of TFP. Moreover, as the resource stock is depleted, the price of harvest good will eventually rise, further reducing welfare in the long run. Even in the case where there is an initial rise in consumption level after the regime switch, the depletion of resource stock and increase in price of harvest good would eventually cause the consumption level to fall below the baseline path. Therefore, the welfare level would still be lower in the long run in this case as well.

This model serves the main objectives of the paper in the sense that it is tractable and relatively simple to follow, allowing the audience to understand how renewable resources and endogenous growth intertwine. Nevertheless, this simplicity comes with the costs of foregoing other real world considerations. For example, the model does not consider technological development in the resource sector, which obviously happens in reality. Moreover, the two resource management policies in this model are at very extreme ends of the spectrum. In the real world, policies tend to be in between these two extremes. The model also refrains from acknowledging concerns about policy implementation, such as enforcement effort involving governance and other institutions. It would be interesting if there is an extension of the model that can capture these other pieces of reality.

2.10 Appendix

2.10.1 Proof for Current Value Hamiltonian (CVH)

$$CVH = \left[P_{Xi} - WZ_i^{\theta}\right] X_i - W\phi - WL_{Zi} + z_i\kappa K_i L_{Zi} \tag{2.78}$$

where z_i is the shadow value of Z_i. Differentiate (2.78) with respect to P_{Xi}, L_{Zi} and Z_i respectively.

$$P_{Xi} = \frac{W_i}{(1-\alpha-\beta)Z_i^{\theta}} \tag{2.79}$$

$$W_{Zi} = z_i\kappa K \tag{2.80}$$

If $W_{Zi} > z_i\kappa K$, there will be no demand for labor in the R&D sector, and vice versa.

$$\frac{\dot{z}_i}{z_i} = r_{Zi} - \kappa K\theta Z_i^{-\theta-1}X_i - \delta \tag{2.81}$$

Substitute (2.4) into (2.81) to get (2.11).

2.10.2 Proof for the Rate of Return on Entrepreneurship

Impose firm symmetry, and substitute $P_X X = \frac{(1-\alpha-\beta)P_Y Y}{N}$ into (2.12).

$$V_i = V = \psi(1-\alpha-\beta)\frac{P_Y Y}{N} \tag{2.82}$$

Take log and time derivative of (2.8) and (2.82) to get

$$r_N = \frac{\Pi_X}{V} + \frac{\dot{V}}{V} \tag{2.83}$$

$$\frac{\dot{V}}{V} = \frac{\dot{W}}{W} + \frac{\dot{\overline{P_Y Y}}}{P_Y Y} - \frac{\dot{N}}{N} \tag{2.84}$$

Substitute (2.9) and (2.84) into (2.83) to get (2.13).

2.10.3 Proof for Equilibrium Amount of Harvest under Full Property Rights

For (2.40) to be positive, $BLS^*_{pr} > H^*_{pr}$. Therefore, (2.51) is the only solution because

$$H^*_{pr} = H^*_{pr} = \frac{1}{2}\left[\frac{(1-\gamma\rho)}{\lambda_{S,pr}} + BLS_{pr} + \sqrt{\left(\frac{(1-\gamma\rho)}{\lambda_{S,pr}} + BLS_{pr}\right)^2 - \frac{4\alpha BLS_{pr}}{\lambda_{S,pr}}}\right] > BLS^*_{pr} \tag{2.85}$$

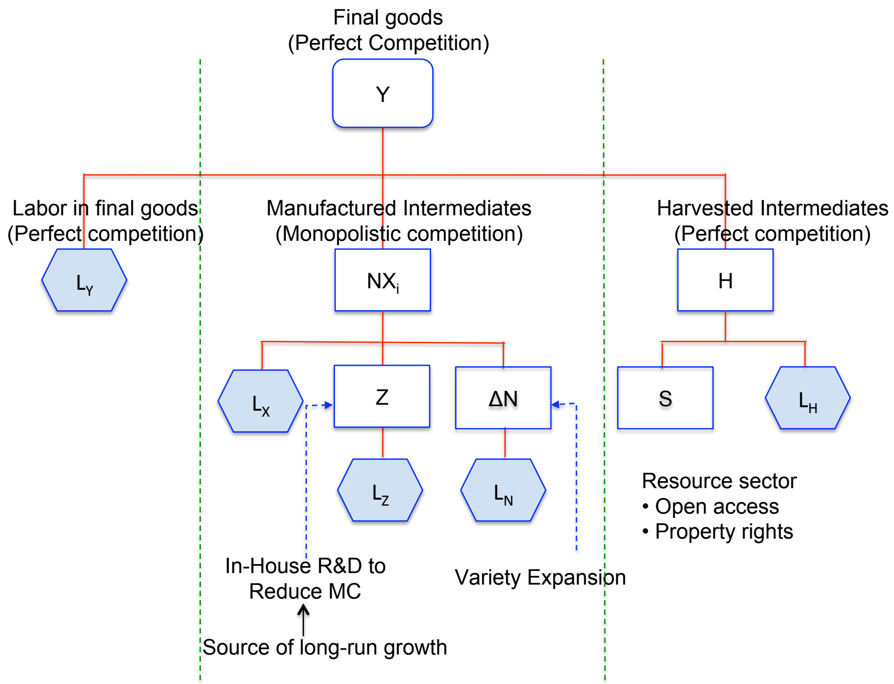

Figure 2.1: Model Roadmap

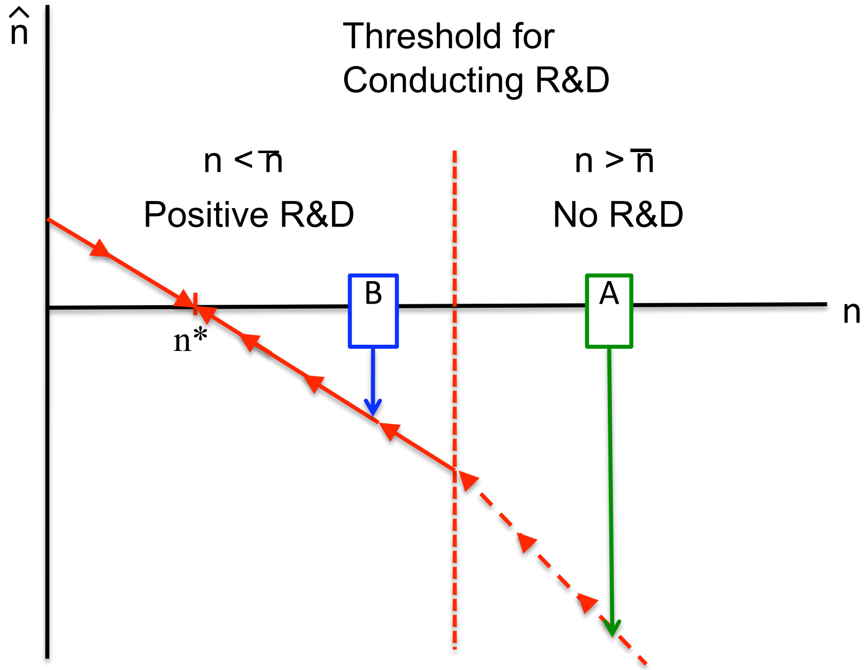

Figure 2.2: Differential Equation for Number of Manufacturing Firms

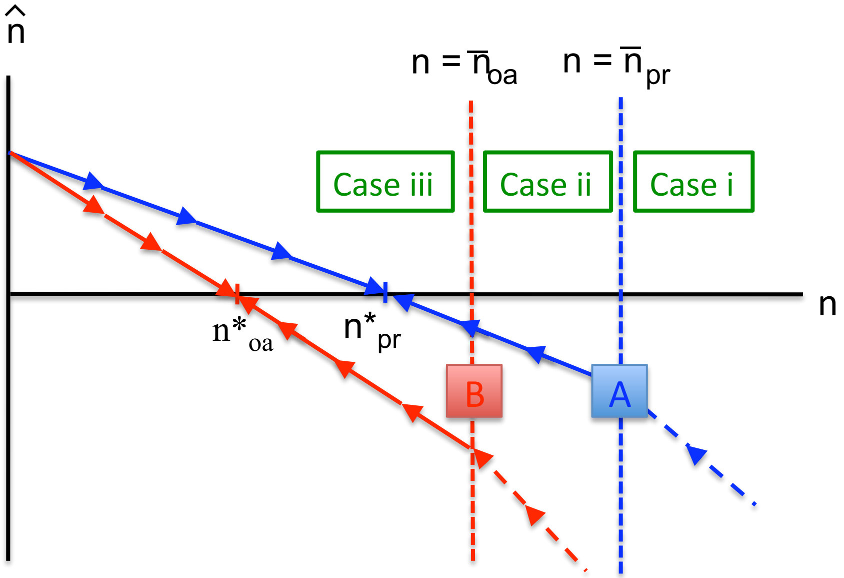

FIGURE 2.3: Development Paths under Different Management Regimes

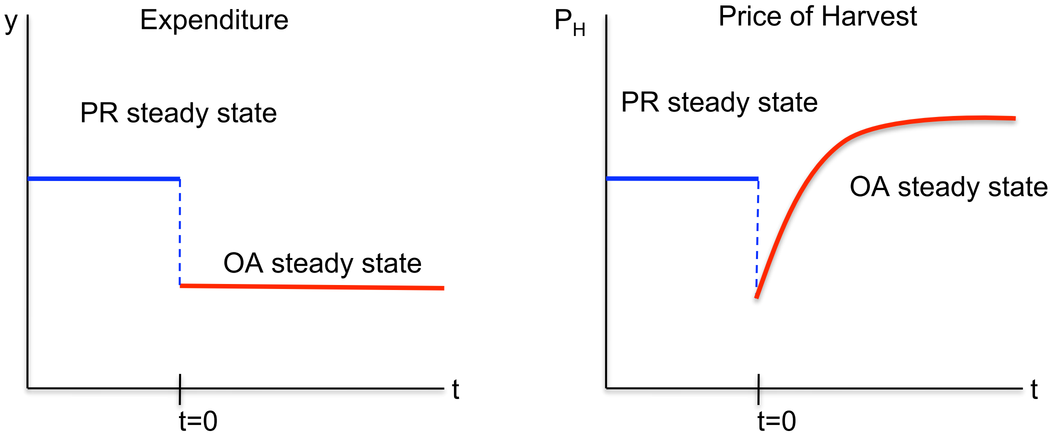

FIGURE 2.4: Transitional Dynamics for y and P_H

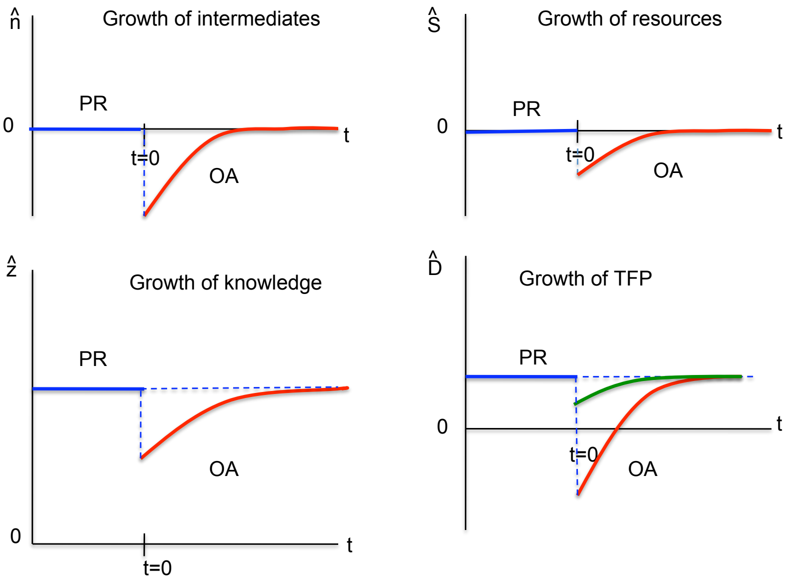

FIGURE 2.5: Transitional Dynamics for $(\hat{n})$, $\left(\hat{S}\right)$, $(\hat{Z})$ and $\left(\hat{D}\right)$

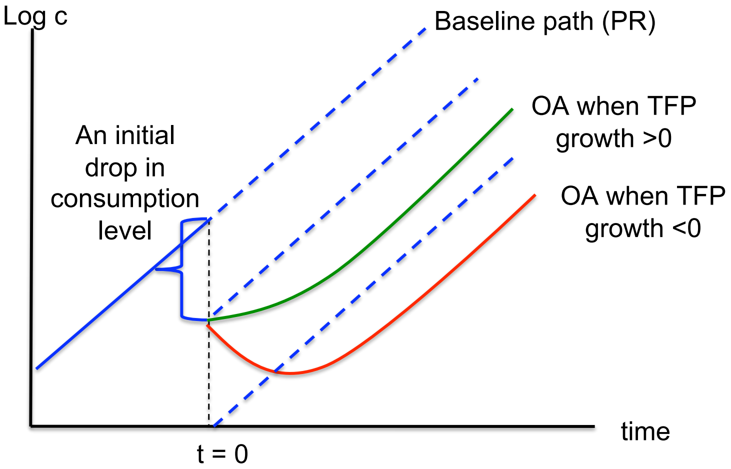

FIGURE 2.6: Consumption Paths after Initial Drop in Level of Consumption

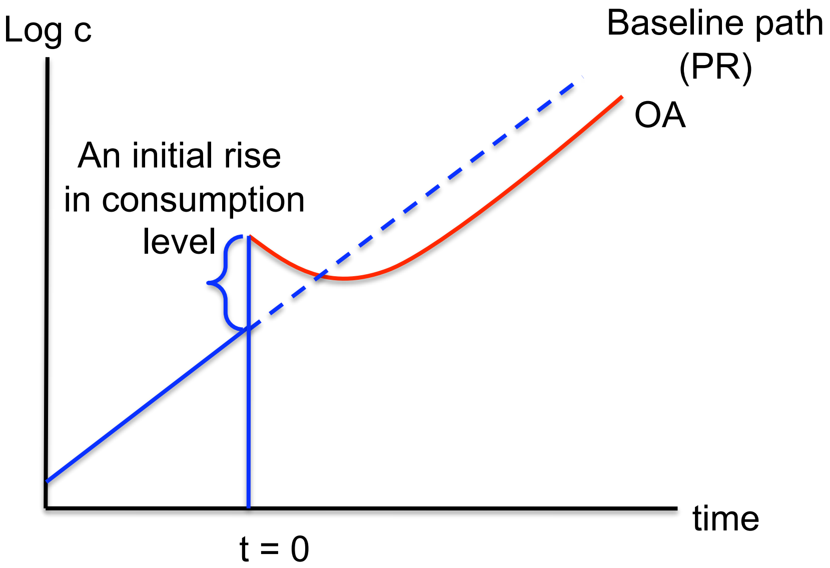

FIGURE 2.7: Consumption Path after Initial Jump in Level of Consumption

3

Asymmetric Trade, Growth and Open Access Renewable Resources

3.1 Introduction

The relationship between trade and welfare in the presence of open access renewable resources has been discussed extensively in the literature. Brown (1995) and Lopez (1997) empirically found that trade liberalization decreases welfare in the long run. Sachs and Warner (1995), Rodriguez and Sachs (1999) , and Gylfason (2001) also discovered a negative relationship between natural resource abundance and growth rate of the economy. Sachs and Warner (1995) suggest that a greater abundance of resources may cause economies to shift away from sectors that generate ongoing growth. Rodriguez and Sachs (1999), meanwhile, suggest that resource-rich countries are likely to live beyond their means during a transitional phase while their resource stocks are being depleted. Finally, Gylfason (2001) associated resource-rich countries with a more volatile exchange rate, which in turn is likely to prevent investment and growth.

In addition, James and Scott (1997), Brander and Scott Taylor (1997), and Brander and Scott Taylor (1998) developed a two-country model of trade with open access renewable resources, and they concluded that a resource-rich country can lose from trade in the long run due to overexploitation of the resources. On the other hand, Hannesson (2000) and Jinji (2006) extended Brander and Taylor's model and found opposite results. In particular, Hannesson (2000) added diminishing returns to the manufacturing sector and concluded that the country may gain from trade even if the renewable resource is under open access. This is because trade causes labor to divert away from the diminishing-return sector to the harvest sector, resulting in higher marginal return on labor and consequently, higher national income. In the latter case, Jinji (2006) endogenized the carrying capacity of renewable resources and suggested that free trade may increase resource stock in resource-exporting countries. Using forestry as an example, the rationale was that the higher price of forest good under free trade would increase the value of the forest stock, such that more land and labor would be committed to the forestry sector.

The papers above, however, did not incorporate an endogenous growth framework with possible welfare gain from innovation and research and development. Previously, Baldwin (1992), Grossman and Helpman (1991)G, and Taylor (1994) found that trade liberalization may enhance welfare through dynamic effects on capital accumulation or on research and development, although renewable resources were not considered in their analyses. So more recently, Peretto and Valente (2010) adopted an endogenous growth framework to explore the relationship between trade and welfare in the presence of renewable resources. Results indicated that trade has an ambiguous effect on welfare depending on the elasticity of substitution between resources and labor in the production of intermediate goods. Nonetheless, the dynamics of the resource stock and the implications of an open access resource management regime were not taken into account when evaluating the possible effects on welfare. To bridge the gap, this paper will combine elements of open access renewable resources and endogenous innovation to analyze the effect of trade liberalization on welfare.

This paper incorporates the model of James and Scott (1997) while building upon the endogenous growth framework of Peretto and Valente (2010). Only the home economy, representing the resource-rich country, is endowed with the renewable resource and produces the harvest good to be used domestically for the production of its final good and to be exported internationally. The foreign economy, representing the resource-poor country, exports its final good in exchange for the resource-rich country's final good and harvest good. The productivity of both economies stems from endogenous innovation in the manufacturing sector. This paper shows that the resource-rich country loses from trade when the rate of innovation is the same in both countries. The welfare loss from free trade is due to two effects. The first effect concerns the productive manufacturing sectors that generate ongoing growth. Specifically, higher demand for the harvest good, from the combined domestic and international demand, diverts labor away from the manufacturing sectors to the harvest sector. Since the resource is under open access, the resource-access cost is not realized and the potential profit dissipates to zero in the harvest sector. Therefore, reallocation of labor to the harvest sector lowers the income of the resource-rich country. As a result of this lower income, there is less incentive for entrepreneurs to enter the manufacturing sectors, which in turn decreases the dynamic growth rate of the economy. The second effect is consistent with James and Scott (1997). Under autarky, the resource-rich country enjoys a larger resource stock and a lower relative price of harvest good. When trade opens, harvest activity intensifies because of additional demand from the resource-poor country. This leads to a depletion of the resource stock in the resource-rich country, and as the stock falls, the price of harvest good gradually increases. Since the harvest good is an essential input to produce the final good, the amount of final good produced will also fall in the long run.

The open economy model will be explained in the next section. Subsequently, the general world equilibrium will be characterized. The paper will then analyze the welfare effect of trade liberalization when the resource-rich country is under open access.

3.2 The Open Economy Model

There are two countries in this model: home (H) and foreign (F). The home country is endowed with labor and a renewable resource, while the foreign country is only endowed with labor. Labor is homogenous and can move freely across sectors domestically but not internationally. The renewable resource is employed to produce a harvest good which, in turn, is used as one of the inputs to produce each country's final good. Each country also produces intermediate manufacturing goods that serve as the other inputs for its final good. The source of long-run growth comes from variety expansion of manufacturing products. The model is summarized in Figure 3.1

3.2.1 Household

Let $j = h, f$. A representative household of country j is endowed with L^j units of labor. Without loss of generosity, the model assumes no population growth and no preference for leisure. The lifetime utility is given by

$$U^j(0) = \int_0^\infty e^{-\rho t} \log u^j(t)\, dt \tag{3.1}$$

where $\rho > 0$ denotes the rate of time preference and $u(t)$ is the instantaneous utility function at time t, which for each country is

$$u^h = \left(C^{hh}\right)^\epsilon \left(C^{fh}\right)^{1-\epsilon} \tag{3.2}$$

$$u^f = \left(C^{ff}\right)^{\epsilon} \left(C^{hf}\right)^{1-\epsilon} \tag{3.3}$$

The household's utility for each country depends on the consumption levels of domestic and imported final goods. Let C^{jk} denote the consumption level of the final good produced by country j and consumed in country k, where $j, k = h, f$. Let ϵ denote the domestic bias, where $\epsilon \in [0,5,1)$[1]. A country gets utility from consuming imported goods, but the more the bias is, the less a representative household prefers imported goods.

Let E^j denote the aggregate expenditure for country j. As a result, the instantaneous expenditure constraint for each country at each period is

$$E^h = P_Y^h C^{hh} + P_Y^f C^{fh} \tag{3.4}$$

$$E^f = P_Y^f C^{ff} + P_Y^h C^{hf} \tag{3.5}$$

Since each country's final good is homogenous and there is no trade friction, the law of one price holds across countries. To be specific, the prices of final goods sold in the home and foreign countries are the same.

A representative household in the home country earns income from assets, labor, and profit from the harvest sector, which is absent in the foreign country. Therefore, the dynamic wealth constraints for the home and foreign countries are

$$\dot{A}^h = r^h A^h + w^h L^h + \Pi^h - E^h \tag{3.6}$$

$$\dot{A}^f = r^f A^f + w^f L^f - E^f \tag{3.7}$$

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[1] $\epsilon \neq 1$ because that would yield no trade

Overall, a representative household in the home or foreign country chooses the consumption path that maximizes the lifetime utility function (3.1), subject to (3.2)/(3.3), (3.4)/(3.5), and (3.6)/(3.7). The intratemporal allocation between domestic and imported final goods and the Euler equation are given by

$$P_Y^h C^{hh} = \epsilon E^h, \quad P_Y^f C^{fh} = (1-\epsilon) E^h \tag{3.8}$$

$$P_Y^f C^{ff} = \epsilon E^f, \quad P_Y^h C^{hf} = (1-\epsilon) E^f \tag{3.9}$$

$$\hat{E}^j = r_A^j - \rho \tag{3.10}$$

3.2.2 *Final Good Production*

A competitive final good producer in country j employs labor $\left(L_Y^j\right)$, a variety of intermediate manufacturing products $\left(X_i^j\right)$, and the harvest good $\left(H^{hj}\right)$ to maximize profit

$$\Pi_Y^j = P_Y^j Y^j - w^j L_Y^j - P_H^T H^{hj} - \int_0^{N^j} P_X^j X^j(n^j) dn \tag{3.11}$$

subject to

$$Y^j = A^j \left(L_Y^j\right)^\beta \left(H^{hj}\right)^\alpha \int_0^{N^j} \left(X^j(n^j)\right)^\gamma dn \tag{3.12}$$

where $\alpha + \beta + \gamma = 1$ and P_H^T denotes the world price of harvest good. Final good production is Cobb-Douglas, which exhibits a constant return to scale for all inputs and decreasing returns to scale for each factor of production. Letting $I^j \equiv P_Y^j Y^j$, which represents the value of final good produced, the demand schedule for each factor of production is

$$L_Y^j = \frac{\beta P_Y^j Y^j}{w^h} = \beta \frac{I^j}{w^h} \tag{3.13}$$

$$H^{hj} = \frac{\alpha P_Y^j Y^j}{P_H^T} = \alpha \frac{I^j}{P_H^T} \tag{3.14}$$

$$X_i^j(N^j) = \left[\frac{\gamma A^j P_Y^j \left(H^{hj}\right)^{\alpha} \left(L_Y^j\right)^{\beta}}{P_{Xi}^j} \right]^{\frac{1}{1-\gamma}} \tag{3.15}$$

3.2.3 Harvest Good Production and the Open Access Renewable Resource

Only the home country is endowed with a stock of renewable resource, denoted $S^h(t)$. For simplicity, natural growth of the renewable resource, denoted $G^h(S^h(t))$, is assumed to follow a logistic growth function, which is widely used in fisheries.

$$G^h(S^h(t)) = \eta S^h(t) \left(1 - \frac{S^h(t)}{\bar{S}^h}\right) \tag{3.16}$$

where η is the natural regeneration rate and $\bar{S}^h$ is the carrying capacity, the maximum stock of renewable resource a country can possess when there is no harvesting activity. For non-renewable resources, set $\eta = 0$, which is a special case of the model.

The dynamics of the resource stock depends on the function $G^h(S^h(t))$ and the amount of harvest good being produced ($H^h(t)$). Harvest production follows the Schaefer harvest production function Schaefer (1957)[2]. It depends on the existing stock of renewable resource at a given time ($S^h(t)$) as well as the labor allocated to the harvest sector ($L_H^h(t)$). The production function exhibits an increasing return

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[2] The qualitative result does not change when different harvest production functions are applied

to scale for all factors of production. B is a constant and can be viewed as the total factor of productivity in the harvest sector or referred to as a "catchability coefficient".

$$\dot{S}^h(t) = G^h(S^h(t)) - H^h(t) \tag{3.17}$$

$$H^h(t) = BL_H^h(t)S^h(t) \tag{3.18}$$

(3.17) says that if natural growth of the renewable resource is higher (lower) than the amount being harvested, the existing stock of resource is increasing (decreasing). The steady state level of resource stock occurs when $G^h(S^h(t)) = H^h(t)$.

Open Access Renewable Resource

Since the renewable resource in this model is under open access, there is no incentive for a competitive harvest firm to take the dynamics of the resource stock (3.17) into consideration. As a result, the problem of the harvest firm becomes static, such that the firm chooses labor in order to maximize its profit, subject to (3.18). This yields the price of harvest good, which is negatively related to the stock of renewable resource at a given time.

$$P_H^T(t) = \frac{1}{BS^h(t)} \qquad for \quad \forall t \tag{3.19}$$

A competitive firm takes the price of harvest good as given and produces up to the point that the market clears.

$$H^h = H^{hh} + H^{hf} \tag{3.20}$$

Substituting (3.19)into (3.14) yields the temporary equilibrium level of harvest good that the home country will produce.

$$H^* = \alpha BS \frac{\left(I^h + I^f\right)}{w^h} \qquad for \quad \forall t \tag{3.21}$$

Substituting (3.18) into (3.21), labor allocation for the harvest good becomes

$$L_H^{h*} = \alpha \frac{(I^h + I^f)}{w^h} \tag{3.22}$$

In contrast, note that if the renewable resource has well-defined property rights, the harvest firm will maximize its profit with respect to the dynamics of the resource stock (3.17).

3.2.4 Intermediate Manufacturing Sector

Incumbent Firms

Intermediate manufacturing firms in country j produce differentiated products according to technology

$$X_i^j = (Z_i^j)^\theta \left[L_{Xi}^j + \phi\right] \tag{3.23}$$

where Z_i^j is the stock of firm-specific knowledge, $\theta \in (0,1)$ is the elasticity of the firm's total factor of productivity with respect to knowledge, L_{Xi} is the labor employed in the industry, and ϕ is the fixed operating labor cost. For simplicity, assume that knowledge exogenously grows at rate z $\left(\hat{Z} \equiv \frac{\dot{Z}}{Z} = z > 0\right)$. The technology exhibits an increasing return to scale for all factors of production.

A monopolistic manufacturing firm i in country j chooses labor to maximize its lifetime profit

$$V_i^j(t) = \int_t^\infty \Pi_{X_i}^j(v) e^{-\int_t^v (r^j(s)-\delta)ds} dv \tag{3.24}$$

$$\Pi_{Xi}^{j} = \left(P_{Xi}^{j} - w^{j}\left(Z_{i}^{j}\right)^{-\theta}\right) X_{i}^{j} - w^{j}\phi \tag{3.25}$$

where Π_{Xi}^{j} is the instantaneous profit for the firm, $r^{j}(s)$ is the interest rate, and δ is the death rate of firms.

Subject to (3.15), solving Current Value Hamiltonian yields the price of each manufacturing product

$$P_{Xi}^{j} = \frac{w^{j}}{\gamma\left(Z_{i}^{j}\right)^{\theta}} \tag{3.26}$$

Entrants

To enter the manufacturing market, an entrepreneur faces an entrycost$\varphi P_{Xi}^{j} X_{i}^{j} = w^{j} L_{N}^{j}$. Once a new firm enters the market, it faces the same maximization problem as the existing manufacturing firms. Since there is an entry cost, it is not profitable for the new entrant to produce the same product as an incumbent firm. New entrants will enter the manufacturing sector to produce differentiated products until the value of the existing firms equals the entry cost.

$$V_{i}^{j} = \varphi P_{X_i}^{j} X_{i}^{j} \tag{3.27}$$

Substituting (3.26) into (3.15) and imposing firm symmetry conditions,

$$X_{i}^{j} = X^{j} = \gamma \frac{I^{j}}{P_{Xi}^{j} N^{j}} \tag{3.28}$$

Substituting (3.26) and (3.28)into (3.27), the value of the manufacturing firms is then proportional to the value of the final good, and the rate of return for an entrepreneur becomes

$$V_i^j = V^j = \varphi\gamma\frac{I^j}{N^j} \tag{3.29}$$

$$r_N^j(t) + \delta = \left[\frac{1}{\varphi}\left[1 - \gamma - \frac{\phi}{\gamma}\frac{w^j N^j}{I^j}\right]\right] + \hat{I}^j - \hat{N}^j \tag{3.30}$$

3.3 World Equilibrium

Trade Balance

Since the home country is the only one with the renewable resource, the foreign country imports both the harvest good and final good from the home country in exchange for its own final good. The trade pattern is asymmetric in that the home country has a structural deficit in final goods. The balanced trade condition becomes

$$P_H^T H^{hf} + P_Y^h C^{hf} = P_Y^f C^{fh} \tag{3.31}$$

Final Good Market Clearing

The final good produced in each country is either consumed domestically or exported internationally.

$$Y^h = C^{hh} + C^{hf} \tag{3.32}$$

$$Y^f = C^{ff} + C^{fh} \tag{3.33}$$

Income and Expenditure

Recall that E^j and I^j denote expenditure and value of final good produced (income) for country j, respectively.

Proposition 1: Relative expenditure and relative income between two countries are constant at every point in time.. Likeswise, expenditure-income ratio for country j is constant at every point in time. For the home and foreign countries, (See Appendix for Proof)

$$\frac{E^h}{E^f} = \frac{1-\epsilon+\alpha\epsilon}{(1-\epsilon)(1-\alpha)}, \tag{3.34}$$

$$\frac{E^h}{I^h} = \frac{1-\epsilon+\alpha\epsilon}{1-\alpha-\epsilon+2\alpha\epsilon}, \tag{3.35}$$

$$\frac{E^f}{I^f} = 1-\alpha, \tag{3.36}$$

$$\frac{I^h}{I^f} = \frac{1-\alpha-\epsilon+2\alpha\epsilon,}{1-\epsilon} \tag{3.37}$$

The mechanisms for the results in *Proposition 1* are the Cobb-Douglas production and utility functions and rent dissipation from the open access renewable resource. The Cobb-Douglas production function suggests that the relative values of production of all factors of production, including harvest goods, are proportional to the value of final output. The Cobb-Douglas utility function implies that intratemporal allocation between domestic and imported final goods is constant, which results in constant expenditure shares. The balanced trade condition which links income and expenditure in the two countries then suggests that income and expenditure in these two countries are proportional to each other. The results in *Proposition 1* imply that the growth rates of expenditure and income in the home and foreign countries are equal to each other $\left(\hat{E^h} = \hat{E^f}\right)$. Substituting this condition into (3.10) shows that the interest rate is equal across countries, or $r^h = r^f$.

Proposition 2: Comparing income and expenditure between the home and foreign countries, (See Appendix for Proof)

i) $I^h > I^f$ *if and only if* $\epsilon > 1/2$

Intuition: If there is a weighted preference for domestic final goods, the value of final good produced in the home country is greater than in the foreign country. The underlying reason is that even though the home country has a structural deficit in final goods, strong domestic bias creates high demand for the home country's final good and leads to a higher value of final good produced or income for the country.

ii) $E^h > E^f$

Intuition: According to the balanced trade condition, the foreign country needs to import both harvest and final goods while the home country only imports the final good. Therefore, the expenditure that can be used toward consumption of final goods in the foreign country is less than in the home country.

iii) $E^h > I^h$

Intuition: Since the home country exports both harvest and final goods, a representative household can spend more than the value of final good that it produces.

iv) $E^f < I^f$

Intuition: The value of final good produced in the foreign country needs to be higher than the expenditure since some parts of its income must be used to purchase the harvest good from the home country. In particular, (3.36) *suggests that the foreign country's available expenditure for consumption is the difference between its total income and the value that is used to buy the harvest good from the home country.*

Asset Market Clearing

As indicated in the previous section, the interest rate is equal across countries due to a special feature of the model. Moreover, to prevent arbitrage opportunity within a country, the rate of return on financial assets in country j has to equal the rate of return for entering the manufacturing sector.

$$r_A^j = r_{Ni}^j \tag{3.38}$$

Substituting (3.30) in (3.10) gives

$$\hat{E}^j + \rho = \frac{1}{\varphi}\left[1 - \gamma - \frac{\phi w^j N^j}{\gamma I^j}\right] + \hat{I}^j - \hat{N}^j - \delta \tag{3.39}$$

From (3.35) and (3.36), it can be seen that $\hat{E}^j = \hat{I}^j$, so (3.39) becomes

$$\hat{N}^j = \frac{1}{\varphi}\left[(1 - \gamma - \varphi(\rho + \delta)) - \frac{\phi w^j N^j}{\gamma I^j}\right] \tag{3.40}$$

Moreover, the value of stocks issued by intermediate manufacturing firms has to equal the value of financial assets held byall the households.

$$V^h N^h = A^h \tag{3.41}$$

Proposition 3: Nominal income and expenditure in the home and foreign countries are constant at every point in time. (See Appendix for Proof)

$$I^h = L^h\left[\frac{1}{(E^h/I^h) - \rho\varphi\gamma}\right]; \tag{3.42}$$

$$E^h = \left[\frac{1 - \epsilon + \alpha\epsilon}{1 - \alpha - \epsilon + 2\alpha\epsilon}\right] I^h \tag{3.43}$$

$$I^f = \left[\frac{(1 - \epsilon)}{1 - \alpha - \epsilon + 2\alpha\epsilon}\right] I^h \equiv \mu I^h; \tag{3.44}$$

$$w^f = (1 - \alpha - \varphi\gamma\rho)\frac{I^f}{L^f}. \tag{3.45}$$

Labor Market Clearing

Labor is immobile across countries but can move freely between sectors within a country. As a result, labor in the home country is allocated among the harvest good sector, the final good sector, existing firms in the manufacturing goods sector, and new entrants to the manufacturing goods sector. On the other hand, labor in the foreign country is allocated among just the final good sector, existing firms in the manufacturing goods sector, and new entrants to the manufacturing goods sector because there is no harvest good sector.

$$L^h = L^h_H + L^h_Y + L^h_X + L^h_N \tag{3.46}$$

$$L^f = L^f_Y + L^f_X + L^f_N \tag{3.47}$$

3.4 Effects of Trade on Income, Expenditure, Growth and Welfare in the Home Country

In order to explore the question of whether trade worsens or improves growth and welfare of the economy, this section will analyze the effects of trade on macroeconomic variables, focusing on the home country.

3.4.1 The Economy with No Trade: Autarky

Household

Without trade, the domestic preference now becomes $\epsilon = 1$. A representative household in the home country maximizes the lifetime utility function

$$U^{hA}(0) = \int\limits_0^\infty e^{-\rho t} \log C^{hA}(t)\, dt \tag{3.48}$$

subject to

$$\dot{A}^{hA} = r^{hA}A^{hA} + w^{hA}L^{hA} + \Pi^{hA} - E^{hA} \tag{3.49}$$

$$E^{A} = P_Y^{A}C^{hA} \tag{3.50}$$

where $(\cdot)^{A}$ denotes autarky

Final Good and Intermediate Manufacturing Goods Production

The final good sector's demands for factors of production are proportional to the value of final good produced in autarky. Manufacturing firms still face the same optimization problem as before except that now, they take the value of final good produced in autarky instead of that with trade.

Harvest Good Production

Since there is no demand forexported harvest good, the harvest good sector will produce harvest good demanded by domestic final good producers only. The price of harvest good then becomes

$$P_H^{A} = \frac{1}{BS^{A}} \tag{3.51}$$

Proposition 4: The home country's economy in autarky is such that

i) $E^{hA} = I^{hA} = L^{h}\left[\frac{1}{1-\rho\varphi\gamma}\right]$

ii) $Y^{hA} = C^{hA}$

3.4.2 Comparison of Income and Expenditure Before and After Trade

Proposition 5: With open access renewable resources, the home country loses income and expenditure after trade. (See Appendix for Proof)

$$I^{hA} > I^{h} \tag{3.52}$$

$$I^{hA} < I^{h} + I^{f} \tag{3.53}$$

$$E^{hA} > E^{h} \tag{3.54}$$

Trade has three effects on the home country's equilibrium income and expenditure: a *specialization* effect, an *income* effect, and a *global demand* effect. The specialization effect refers to the fact that the home country is the only one with the renewable resource and hence, specializes in producing the harvest good. Due to the balanced trade condition, the home country runs a structural deficit in final goods, which implies a lower value of final good produced in the country after trade. The income effect is based on rent dissipation in the harvest sector. When the home country opens up to trade, it needs to produce more of the harvest good in order to meet both domestic and international demands. However, because the renewable resource is under open access, the price of harvest good is underestimated since a competitive harvester pays only for labor cost, but not for the cost of the resource itself. Consequently, producing more harvest good does not increase income for the representative household. In fact, it decreases the value of final good produced. In contrast to the specialization and open access effects, the global demand effect puts upward pressure on the value of final good produced because trade induces demand from abroad. So while the first two effects decrease the home country's income and expenditure after trade, the last one operates in the opposite direction. Nevertheless, the first two effects outweigh the last one because the domestic bias, or preference for domestic final goods, is relatively high. As a result, the effect from market expansion would be limited.

3.4.3 Dynamics and Steady State with Trade

Due to the special features of the model, the home economy pins down to two dynamics.

Entrepreneur in Manufacturing Sector

Dynamics of an Entrepreneur in the Manufacturing Sector

$$\hat{N^h} = \frac{1}{\varphi}\left[(1-\gamma-\varphi(\rho+\delta)) - \frac{\phi N^h(t)}{\gamma I^h}\right] \tag{3.55}$$

The dynamics of an entrepreneur in the manufacturing sector is a logistic function, which has an explicit solution

$$N^h(t) = \frac{N^{h*}}{1+\left[(\frac{N^{h*}}{N^h(0)})-1\right]e^{-\nu t}} \tag{3.56}$$

where $\nu = \frac{(1-\gamma-\varphi(\rho+\delta))}{\varphi} > 0$, $N^h(0)$ denotes the steady state number of manufacturing firms in autarky, and N^{h*} denotes the steady state number of manufacturing firms after trade.

Steady State Number of Manufacturing Firms At the steady state when $\hat{N^h} = 0$, the number of manufacturing firms becomes

$$N^{h*} = \frac{\gamma}{\varphi}\left(1-\gamma-\varphi(\rho+\delta)\right) I^h \tag{3.57}$$

The steady state number of manufacturing firms is positively related to the value of final good produced. The underlying reason stems from a simple Cobb-Douglas final good production function. The higher the value of final good produced, the more the demand for differentiated intermediate manufacturing products. This increases the incentive for new manufacturing firms to enter the market and results in a higher number of manufacturing firms.

Stock of Renewable Resource

Dynamics of Renewable Resource Substituting the global equilibrium level of harvest good (3.21) and price of harvest good (3.19) into (3.17). The dynamics of the renewable resource stock becomes

$$\hat{S^h} = \eta\left(1 - \frac{S^h(t)}{\bar{S}^h}\right) - \alpha B(I^h + I^f) \tag{3.58}$$

Again, an explicit solution can be obtained

$$S^h(t) = \frac{S^{h*}}{1 + \left[\left(\frac{S^{h*}}{S^h(0)}\right) - 1\right] e^{-\chi t}} \tag{3.59}$$

where $\chi = \eta - \alpha B(I^h + I^f) > 0$, $S^h(0)$ denotes the steady state stock of renewable resource in autarky, and S^{h*} denotes the steady state stock of renewable resource after trade.

Steady State Level of Renewable Resource Stock Equating (3.58) to zero, the steady state level of renewable resource stock becomes

$$S^{h*} = \bar{S}^h\left[1 - \frac{\alpha B}{\eta}(I^h + I^f)\right] \tag{3.60}$$

The steady state stock of renewable resource is positively related to the natural regeneration rate (η) and themaximum carrying capacity $\left(\bar{S}^h\right)$ but negatively related to the total value of the *world's* production of final goods.

Growth Rate of Real Income

Dynamics of Growth Rate of Real Income The relevant measure of real income in the economy is the utility function (3.1). Denoting g^j as the growth rate of real income in country j, the dynamics of the growth rate of real income in the home country after trade is

$$g^h(t) = \epsilon \hat{C^{hh}} + (1-\epsilon)\hat{C^{fh}} = \epsilon \hat{Y^h} + (1-\epsilon)\hat{Y^f} \tag{3.61}$$

Taking log and differentiating (3.12), the dynamics of the growth rate of real income in the home country becomes

$$g^h(t) = \alpha \hat{S^h} + (1-\gamma)\left[\epsilon \hat{N^h} + (1-\epsilon)\hat{N^f}\right] + \theta\gamma\left[\epsilon \hat{Z^h} + (1-\epsilon)\hat{Z^f}\right] \tag{3.62}$$

The dynamics of the growth rate of real income depends positively on the growth rate of renewable resource stock, growth rate of number of manufacturing firms, and exogenous growth rate of R&D activity for the incumbent manufacturing firms.

Steady State Growth Rate of Real Income Setting $\hat{S^h} = \hat{N^j} = 0$, the steady state growth rate of the home economy becomes

$$g^{h*} = \theta\gamma z \tag{3.63}$$

3.4.4 Steady State Comparison

Proposition 6: Comparing the steady state variables in the home country before and after trade, (See Appendix for Proof)

i) $S^{hA*} > S^{h*}$

Intuition: With trade, there is more demand for the harvest good stemming from the foreign country. Consequently, the steady state stock of renewable resource is *lower with trade than in autarky, which raises the price of harvest good due to greater scarcity of the renewable resource.*

ii) $H^{hA*} \gtreqless H^{h*}$

Intuition: The harvest production function is determined by the level of labor employed in the harvest sector and the existing stock of renewable resource. Hence, trade generates two opposing effects on the steady state level of harvest good. The first effect is a "demand effect" that stimulates harvest good production. Since trade induces more demand for the good, labor in the home country will reallocate itself toward the harvest sector. However, given the natural dynamics of the resource stock, the higher level of harvesting activity would lead to a decline in the existing stock of renewable resource. As a result, there is also a second effect which is a "supply effect" that constrains harvest good production. The overall result depends on the parameters underpinning these two forces. If the demand (supply) effect exceeds the supply (demand) effect, the steady state level of harvest good would rise (fall).

iii) $N^{hA*} > N^{h*}$

Intuition: As can be seen from (3.52)*, the steady state number of manufacturing firms in the home country only depends on the value of final good produced. Since the value of final good produced is lower after trade than in autarky, the steady state number of manufacturing firms is then lower after trade than in autarky as well. When the value of final good produced falls after trade, existing manufacturing firms would find it less profitable to operate in the market and therefore, some firms would exit the market until the value of the remaining firms equals the entry cost..*

iv) $g^{hA*} = g^{h*} = z^h = z^f$

Intuition: The steady state growth rate of the home economy before and after trade is the same due to the lack of scale effect and the identical growth rate of R&D innovation in the home and foreign countries. The lower value of final good producted after trade results in a lower number of manufacturing firms. As a result, the long-

run growth rate of the economy only stems from the exogenous rate of knowledge spillover from the existing manufacturing firms.[3]

3.4.5 Reallocation of Labor in Home Country

Trade liberalization results in reallocation of labor among the harvest good sector, the final good sector, existing firms in the manufacturing goods sector, and new entrants to the manufacturing goods sector. Using the labor market clearing condition (3.46), labor allocation in the home country becomes

$$L_Y^h = \beta I^h, \tag{3.64}$$

$$L_X^h = \gamma I^h + \phi N^h(t), \tag{3.65}$$

$$L_N^h = \gamma(1-\gamma-\gamma\rho)I^h - \phi N^h(t) \tag{3.66}$$

$$L_H^h = L^h - (\beta+\gamma+\gamma(1-\gamma-\gamma\rho))I^h \tag{3.67}$$

With no population growth, the amount of labor employed in the final good and harvest good sectors only depends on the total value of final good produced. As a result, the home country experiences rising employment in the harvest sector and falling employment in the final good sector after trade. Moreover, the lower value of final good produced and lower number of manufacturing firms reduce the level of employment in both the existing firms and new entrants to the manufacturing goods sector.

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[3] Note that if the growth rate of knowledge in the foreign country is higher than in the home country, the home economywould enjoy a higher steady state growth rate after trade.

3.4.6 Welfare and Growth Effect in Home Country

Consumption Path in Autarky

Suppose that prior to trade, the home economy is at the steady state level of utility. When trade occurs at time $t = 0$, the home economy's utility at time $t = 0$ can be written as

$$\log C^{hA}(t=0) = \Phi_1 + \log I^{hA} + \alpha \log S^{*hA} + (1-\gamma)\log N^{*hA} + \theta\gamma \log Z^{*hA} \quad (3.68)$$

where $\Phi_1 = \log\left(A\beta^{\beta}\alpha^{\alpha}\gamma^{\gamma}B^{\alpha}\right)$.

Consumption Path After Trade

When trade occurs, the home economy's utility not only depends on the level of final good production at home, but also relies on imported final good from the foreign country. In particular,

$$\log u^h = \epsilon \log C^{hh} + (1-\epsilon)\log C^{fh} \quad (3.69)$$

where $C^{hh} = \left(\epsilon \frac{E^h}{I^h}\right) Y^h$ and $C^{fh} = \left(1 - \epsilon \frac{E^f}{I^f}\right) Y^f$. *(See Appendix for Proof)*

After trade, the level of utility depends on the expenditure to income ratios as well as the levels of final good production in both the home and foreign countries. The path of utility after trade becomes

$$\log u^h(t=0) = \Phi_2 + \log I^h + \alpha \log S^h(t=0) + (1-\gamma)\log\left(N^h(t=0)\right)^{\epsilon}\left(N^f(t=0)\right)^{1-\epsilon} + ... \quad (3.70)$$

$$+\,\theta\gamma \log\left(Z^h(t=0)\right)^{\epsilon}\left(Z^f(t=0)\right)^{1-\epsilon} \quad (3.71)$$

where $\Phi_2 = \Phi_0 + \Phi_1 + (1-\epsilon)\log\mu$, and $\Phi_0 = \epsilon\log\left(\epsilon\frac{E^h}{I^h}\right) + (1-\epsilon)\log\left(1-\epsilon\frac{E^f}{I^f}\right)$

Instantaneous Change in Welfare after Trade The first two terms on the RHS indicate the instantaneous change in welfare after trade. If $\Phi_2 + \log I^h < \Phi_1 + \log I^{hA}$, the home economy instantaneously loses utility from opening up to trade, assuming that the autarky steady state number of manufacturing firms and level of knowledge accumulation are the same in the home and foreign countries, $N^{*hA} = N^{*fA}$ and $Z^{*hA} = Z^{*fA}$. Welfare instantaneously falls because of the income effect mentioned previously. In particular, trade liberalization exacerbates the rent dissipation problem that is a feature of open access resources. *(See Appendix for Proof)*

Dynamics of Growth Rate of Utility Path after Trade After trade, the dynamics of the growth rate of the utility path for the home economy becomes

$$g^h = \alpha \hat{S^h} + (1-\gamma)\left(\epsilon \hat{N^h} + (1-\epsilon)\hat{N^f}\right) + \theta\gamma\left(\epsilon \hat{Z^h} + (1-\epsilon)\hat{Z^f}\right) \tag{3.72}$$

Since $\hat{Z^h} = \hat{Z^f} = z$,

$$g^h = \alpha \hat{S^h} + (1-\gamma)\left(\epsilon \hat{N^h} + (1-\epsilon)\hat{N^f}\right) + \theta\gamma z \tag{3.73}$$

When the home country opens up to trade, there is more demand for the harvest good stemming from the foreign country. Given the dynamics of the renewable resource stock, the more harvest good produced today means the less resource stock in the future. As a result, the growth of renewable resource stock $\left(\hat{S^h}\right)$ decreases, leading to a higher price of harvest good. As the resource becomes scarcer and less harvest good is produced, the amount of final good that can be produced by the home country also falls. Therefore, this scarcity effect causes a decrease in welfare in the long run.

Recall from (3.40)that the dynamics of the number of manufacturing firms depends on the value of final good produced and wage.Given the same normalized wage, a fall in income or value of final good produced in the home country leads to a decrease in the growthof number of manufacturing firms $\left(\hat{N^h} < 0\right)$. On the other hand, in the foreign country, since wageis proportional to the value of final good produced, the dynamics of the number of manufacturing firms just depends on the initial number of firms prior to trade. For simplicity, assume that this initial number of firms is the same as the steady state number so that the the growth of number of manufacturing firms is zero $\left(\hat{N^f} = 0\right)$. As a result, (3.73) becomes

$$g^h = \alpha \hat{S^h} + (1 - \gamma)\,\epsilon \hat{N^h} + \theta\gamma z \tag{3.74}$$

The dynamics of growth rate of Home country can be summarized in Figure 3.2

At the steady state, $\hat{S^h} = \hat{N^h} = 0$, so the steady state growth rate of the home economy becomes

$$g^{h*} = \theta\gamma z = g^{h*A} \tag{3.75}$$

Figure 3.3 illustrates the effect of trade liberalization on the welfare of the home economy.

3.5 Conclusion

This paper presents a two-country trade model in which only one of the countries is endowed with a renewable resource in order to study the effect of trade liberalization on the welfare of the home or resource-rich economy. The paper incorporates the dynamics of the renewable resource stock into the endogenous growth framework, where long-run growth stems from R&D innovation. In the model, the renewable resource is under open access, which means that no property rights are assigned to the common pool resource. This results in overexploitation of the resource and may lead to the tragedy of the commons. The renewable resource is used to produce the harvest good, which in turn serves as an input to produce the final goods. Since only the home country is endowed with the renewable resource, it trades both its harvest good and final good for the foreign country's final good. Therefore, in order to ensure balanced trade at all times, the home country runs a structural deficit in final goods.

In this study, trade causes welfare loss in the home economy due to two effects: an income effect and a scarcity effect. The first effect is the instantaneous loss of income. Trade increases demand for the harvest good as international demand is added to domestic demand. This greater demand for the harvest good diverts labor away from the productive manufacturing sectors to the harvest sector, where rent is zero. Since the resource is under open access, the resource-access cost is not realized and the price of harvest good is underestimated. Consequently, reallocation of labor to the harvest sector decreases the value of final good produced and lowers the income of the resource-rich country. In turn, this decreases the incentive for new entrants to the manufacturing sector, which is an important sector for generating long-run growth. The second effect is the long-run decline in the amount of final good produced due to increasing scarcity of the renewable resource. Under open access, the renewable resource is overexploited and becomes scarcer as trade creates greater demand for the harvest good. Given the same amount of labor, less and less harvest good is produced, which gradually increases the price of harvest good and leads to a fall in the amount of final good produced in the long run.

3.6 Appendix

Proof of (3.30)

Take log of 3.24 and differentiate with respect to time.

$$r^h(t) - \delta = \frac{\Pi^h_{X_i}}{V^h_i} + \frac{\dot{V}^h_i}{V^h_i} \tag{3.76}$$

Substitute the profit function of manufacturing goods into the previous equation.

$$r^h(t) + \delta = \frac{\left(P^h_{Xi} - w^h \left(Z^h_i\right)^{-\theta}\right) X^h_i - w^h\phi}{V^h} + \frac{\dot{I}^h}{I^h} - \frac{\dot{N}^h}{N^h} \tag{3.77}$$

Substitute (3.28) into the previous equation to get (3.30).

Proof of Proposition 1

Substituting (3.8), (3.9) and (3.14) into the balanced trade condition, (3.34) can be derived.

For (3.35) and (3.36),multiply P^h_Y, and P^f_Y on both sides of (3.32) and (3.33), respectively.

$$P^h_Y Y^h = P^h_Y C^{hh} + P^h_Y C^{hf}, \qquad\qquad P^f_Y Y^f = P^f_Y C^{ff} + P^f_Y C^{fh} \tag{3.78}$$

Using the definition $I^j \equiv P^j_Y Y^j$, and substituting (3.8) and (3.9) into (3.78) , the equations become

$$I^h = \epsilon E^h + (1-\epsilon)E^f \tag{3.79}$$

$$I^f = \epsilon E^f + (1-\epsilon)E^h \tag{3.80}$$

Substituting (3.34) into (3.79) and (3.80), (3.35) and (3.36) can be derived.

To derive the relative value of final good produced between the home and foreign countries (3.37), substitute (3.35) and (3.36) into (3.80).

Proof of Proposition 2

For i): rearrange (3.37); $1-\epsilon \lesseqgtr 1-\alpha-\epsilon+2\alpha\epsilon$ iff $\epsilon \gtreqless 0.5$. And since by assumption $\epsilon \geqslant 0.5$, $I^h \geqslant I^f$.

ii): rearrage (3.34); the result follows because by construction $\alpha > 0$.

iii): rearrage (3.35); the result follows because by construction $\epsilon < 1$.

iv): rearrage (3.36); the result follows because by construction $\alpha < 1$.

Proof of Proposition 3

From (3.35) and (3.36), it can be shown that $\hat{E^j} = \hat{I^j}$. Substituting (3.29) into (3.41), taking log and differentiating, the equation becomes $\hat{A^h} = \hat{I^h}$. Using this term and substituting (3.10) into (3.6), (3.42) can be derived.

Substituting (3.42) into (3.35) and (3.37), (3.43) and (3.44) can be derived, respectively.

Substituting $\hat{A^f} = \hat{I^f}$ and(3.36) into the foreign country's budget constaint, (3.45) is then derived.

Proof of Proposition 4

Applying the condition $E^{hA} = I^{hA}$, and using the same method as Proof of Proposition 3, (3.4.1) is derived.

Moreover, $Y^{hA} = C^{hA}$ results from the final good market clearing condition in autarky.

Proof of Proposition 5

For home income before and after trade, compare (3.4.1) and (3.42); the only difference is the term 1 and $\frac{E^h}{I^h}$. Since $E^h/I^h > 1$, then $I^h < I^{hA}$.

For world income before after trade, rearrange (3.42), (3.44), and (3.4.1); it can be easily proved by contradiction that $I^{hA} < I^h + I^f$.

For home expenditure before and after trade, rearrange (3.4.1) and (3.43); the result follows because by construction $\epsilon < 1$.

Proof of Proposition 6

i) The only difference in the steady state level of renewable resource stock before and after trade is due to income. In particular, while the autarky level of resource stock depends on the home country's autarkic income level, the steady state level of resource stock after trade depends on the world's income. From Proposition 4, it is shown that $I^{hA} < I^h + I^f$. As a result, i) can be derived.

ii) Recall that $I^{hA} < I^h + I^f$. Expressing $H^{*A} = BL_H^{*A}S^{*A}$ in terms of autarkic income, the steady state level of harvest good before trade can be written as

$$H^{*A} = B\alpha\bar{S}I^{hA}\left[1 - \frac{\alpha B}{\eta}I^{hA}\right] \tag{3.81}$$

Differentiating the equation with respect to income, it can be seen that

$$\frac{\partial H}{\partial I}\begin{cases} > 0 & iff \quad I < \frac{\eta}{2\alpha B} \\ \leqslant 0 & \quad\quad otherwise \end{cases} \tag{3.82}$$

The steady state level of harvest good is then ambiguous, depending on the relevant parameters.

iii) Since $I^{hA} > I^h$, iii) can be derived.

iv) From (3.73), setting $\hat{S} = \hat{N} = 0$, iv) can be derived.

Proof for C^{hh} and C^{fh}

$$C^{hh} + C^{hf} = Y^{hh} + Y^{hf} \tag{3.83}$$

Using the definition of $I^j \equiv P_Y^j Y^j$, (3.35) and (3.36) become

$$\frac{E^j}{I^j} = \frac{C^{jj}}{Y^j} + (1-\epsilon)\frac{E^j}{I^j} \tag{3.84}$$

Rearrange the terms to derive C^{hh} and C^{fh}.

Proof of Instantaneous Change in Welfare after Trade

The home economy instantaneously loses from trade if and only if

$$\Phi_2 + \log I^h < \Phi_1 + \log I^{hA} \tag{3.85}$$

Since $\Phi_2 = \Phi_0 + \Phi_1 + (1-\epsilon)\log\mu$, it can be seen that the instantaneous loss occurs if and only if

$$\Phi_0 + (1-\epsilon)\log\mu + \log I^h - \log I^{hA} < 0 \tag{3.86}$$

$\Phi_0 < 0$ because by construction $0.5 < \epsilon < 1$ and $\log\mu < 0$ because by construction $\epsilon > 0.5$. The difference between the last two terms is negative since $I^{hA} > I^h$. Consequently, $\Phi_2 + \log I^h < \Phi_1 + \log I^{hA}$, which means that the economy experiences instantaneous loss of welfare after trade liberalization.

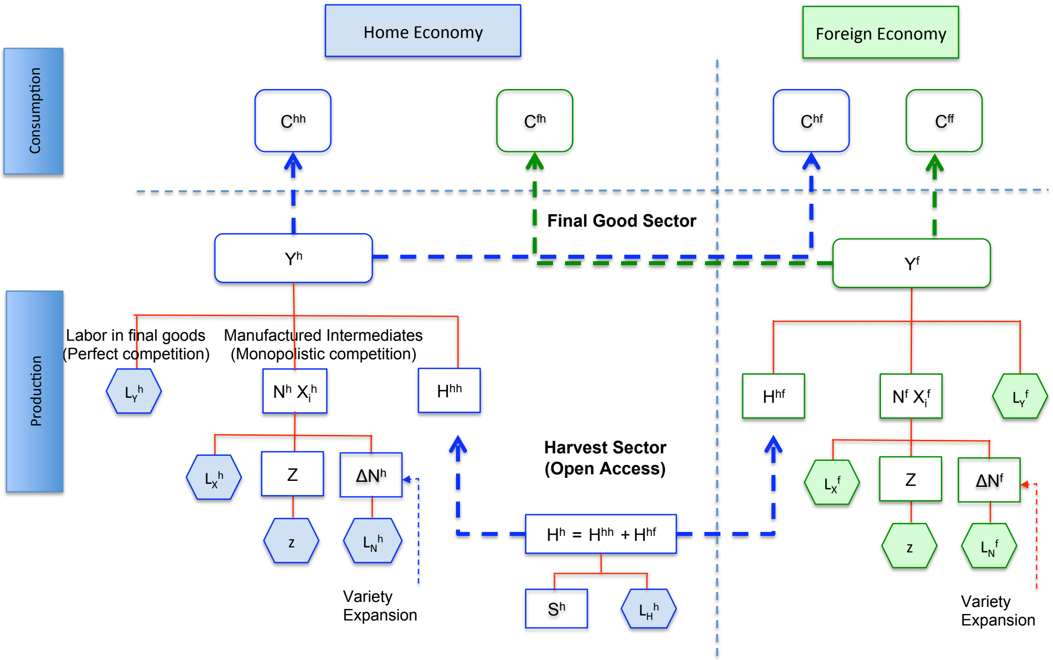

Figure 3.1: Model Roadmap

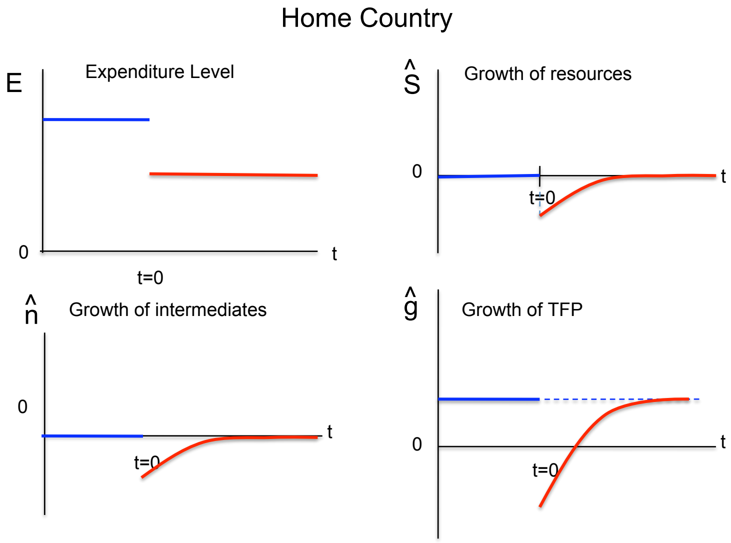

Figure 3.2: Dynamics of Growth Rate of Home Country Before and After Trade

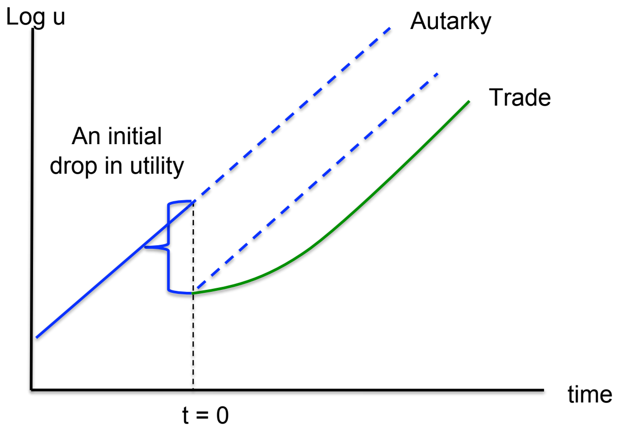

Figure 3.3: Welfare Effect After Trade Liberalization in Home Country

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Biography

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