Supply Chain Management for Digital Content Platforms

by

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Business Administration
Duke University

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Business Administration
in the Graduate School of Duke University
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Abstract

Information goods —such as books, music, and video —have long been sold via a traditional retail model involving physical media (e.g., physical books, CDs, and DVDs) sold "a la carte" via third party retailers. In recent years, however, innovations in digital distribution technology have enabled new ways of selling, distributing, and consuming information goods. For example, books can now be sold as e-books, and may be consumed via traditional a la carte methods (e.g., when a consumer buys a permanent license to an e-book) or via subscription services (e.g., when a consumer pays a monthly fee for unlimited access to a library of content without ever owning a permanent copy of any books in that library). The rapid growth and change of digital distribution technology has, however, introduced a number of challenges to the management of supply chains for information goods. We examine three problems in this area. First, we analyze a supply chain where a manufacturer produces both physical and digital goods and has a choice between selling through a single retailer who sells both formats or two format-specific retailers. We find that when the manufacturer sells through the single retailer, the supply chain achieves the centralized system outcome by selling zero physical goods and the centralized optimal quantity of the digital good. However, despite this, a manufacturer would prefer to sell through two format-specific retailers rather than through a single retailer, with a strictly positive quantity of the operationally inferior physical good, to the detriment of total supply chain efficiency. Additionally we find that consumers and society are both better off

when the manufacturer sells through independent retailers. Next, we analyze selling decisions for supply chain of a digital content platform and the two creators (low and high quality) who sell their content through the platform. The creators have a choice of selling their content a la carte and/or via subscription and being paid via revenue sharing contracts. We find that the platform cannot induce only the high quality consumer to sell via subscription which means that the subscription offerings are always weakly lower in quality than a la carte offerings. We also find that in many cases, to maximize profit, the platform should either induce only the low quality creator to sell via subscription, or it should shut down a la carte sales altogether; inducing high quality on the subscription service is excessively costly. This effect can be mitigatedand inducing high quality on the subscription service can be optimal for the platformin the presence of a large subscription only consumer segment. In the third paper, we explore the optimal design of revenue splitting rules that feasible (i.e. induces all creators to join the subscription service), fair (i.e. allocates revenue only based on amount of consumption generated by each creator) and optimal (i.e. both fair and feasible at the lowest cost possible to the platform) and allows a platform to maximize its own revenue while inducing a wide variety of high quality content on the service. We show that a splitting rule that is quadratic in the consumption of each creator's product is optimal while a linear rule is not and that a linear rule can actually perform arbitrarily bad under some circumstances.

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1
Introduction

Traditionally, information goods such as books, music, and video, have been primarily distributed via an "a la carte" sales model in which consumers purchase a permanent physical copy of each item they desire. Enabled by increasing consumer access to high speed internet connections, as well as technological advancements such as e-book readers,gaming consoles, televisions, and digital media players with internet connection capabilities, content creators and media publishers have rapidly expanded their digital distribution capabilities and made significant efforts to increase the volume of their digital channels. These efforts have been remarkably successful in all categories of media. The rise of digital formats and distribution technologies has enabled a wider variety of distribution methods for information goods, including, digital a la carte sales and subscription services. The rapid growth of digitally distributed information goods has introduced a number of challenges to the management of supply chains distributing these products.

In the second chapter, we analyze a model of supply chain design for perfectly substitutable digital and physical information goods. The physical good is characterized by positive marginal cost and positive production leadtime, while the digital good has zero marginal cost and zero leadtime. In addition, the physical good is sold to retailers via a wholesale price contract, while the digital good is sold via an agency model, i.e., a revenue sharing contract in which the manufacturer retains the decision rights for the digital good. We find the following key results. First, a centralized manufacturer (i.e., one that controls both the physical and digital distribution channels) would prefer to only sell the digital good, given its operational advantages. Second, when a manufacturer sells both products through a single retailer, the supply chain achieves the centralized system outcome by selling zero physical goods and the centralized optimal quantity of the digital good. Third, despite this, a manufacturer would prefer to sell through two format-specific retailers rather than through a single retailer, with a strictly positive quantity of the operationally inferior physical good, to the detriment of total supply chain efficiency. However, consumers and society are both better off when the manufacturer sells through independent retailers. In an extension, we verify that many of our insights continue to hold when digital and physical goods are merely partial (rather than perfect) substitutes.

In the third chapter, we analyze distribution decisions by content creators for digital information goods like books and music. There are two creators (low and high quality) that sell content through the same platform, and may choose whether to distribute their content a la carte, via a subscription service, or both. Each creator may set the price of her a la carte offerings, while the platform keeps a percentage of revenue from each sale. The platform determines the subscription service fee and what fraction of revenue to distribute to content creators, who are paid proportional to the revenue share chosen by the platform and the percentage of total "use" that they generate on the subscription service (e.g., their fraction of total pages read in a month). We show the platform cannot induce only the high quality creator to list on the subscription service; thus, subscription offerings are always weakly lower in quality than a la carte offerings. Moreover, we find that in many cases, to maximize profit, the platform should either induce only the low quality creator to sell via subscription, or it should shut down a la carte sales altogether; inducing high quality on the subscription service is excessively costly, a result which may help to explain the relative low quality of product offerings on these services in practice. We also show that this effect can be mitigated—and inducing high quality on the subscription service can be optimal for the platform—in the presence of a large "subscription only" consumer segment.

In the fourth chapter, we explore the mechanisms to allocate revenue amongst creators on subscription based digital content platforms. These platforms (ex. Kindle Unlimited and Apple Music) allocate revenue amongst content creators (i.e., authors or musicians) according to a pre-specified rule. One popular rule is a linear proportional split: subscription revenues are divided amongst content creators according to the fraction of total consumption that they account for on the platform (i.e., the fraction of total pages read or music downloads). However, although they are superficially "fair," linear rules do not necessarily allocate revenue in an optimal way, i.e., in a way that allows the platform to achieve its objective at the lowest possible cost. We show that, in general, linear revenue splitting rules allocate too much revenue to low quality creators and too little revenue to high quality creators. As a result, this makes it difficult for the platform to induce high quality creators to join a subscription service, and it must compensate by a sharing a high overall fraction of its revenue. We then determine optimal allocation rules that apportion revenue according to the amount of consumer utility generated by each creator while still being "fair" in the sense that they are a function solely of the consumption quantity of each creator's product. When consumer valuations are uniform, we show that such an optimal rule is quadratic in the consumption of each creator's product. We further quantify the performance gap between a linear rule and the optimal rule and show that, when the platform wishes to induce high quality content creators to list on its service, a linear revenue split can result in significantly lower profits for the platform than the optimal revenue split.

In total, the chapters of this dissertation help to better understand how to manage supply chains for digital content distribution. We consider both substitutable physical and digital goods, sold via a la carte and subscription models, and find that as digital content consumption becomes increasingly ubiquitous for all types of information goods, businesses need to understand not only how these digital content supply chains differ from traditional, physical supply chains but also how to manage them.

2

Supply Chain Design for Substitutable Digital and Physical Information Goods

2.1 Introduction

In the past decade, information goods such as books, music, movies, and software—traditionally distributed only via physical formats—have seen a significant rise in digital distribution. Enabled by increasing consumer access to high speed internet connections, as well as technological advancements such as e-book readers, gaming consoles, televisions, and digital media players with internet connection capabilities, content creators and media publishers have rapidly expanded their digital distribution capabilities and made significant efforts to increase the volume of their digital channels. These efforts have been remarkably successful in all categories of media. In the US music industry, sales from digital downloads surpassed CD sales for the first time in 2014 and have grown steadily since [Vincent, 2015]; in the US film and television industry, a similar transition was anticipated to occur in 2016 [Cunningham, 2016]. Other categories of information goods have experienced similar growth over the preceding decade, and digital revenues are now comparable to, or

Table 2.1: 2015 Worldwide Digital Market Shares for Different Types of Media. Source: [Statista, 2017a].

Media Type	2015 Global Digital Market Share
Movies	21%
Music	45%
Video Games	60%
Books	20%

exceed, physical revenues in many product categories worldwide; Table 2.1 provides a summary of global digital format market share for four media types in 2015.

The rapid growth of digitally distributed information goods has introduced a number of challenges to the management supply chains distributing these products. In this paper, we consider two issues in particular. The first is that digital and physical products have very different operational characteristics. Physical products have positive marginal production costs and production leadtimes—for example, for a 500 page book, one commercial printer charges about $16 per book and demands a 6-7 week leadtime.[1] Digital products, on the other hand, can be distributed instantly and at zero marginal cost. These distinct operational features imply, for instance, that physical products will potentially suffer from supply-demand mismatch (i.e., because the quantity proceeds is greater or less than the actual demand), while digital products will not. Content creators and media publishers—which we collectively refer to as "manufacturers" in the paper—thus must consider the operational advantages and disadvantages of each format when considering how to allocate sales and design their supply chains to distribute these operationally distinct goods.

The second issue we consider is that digital and physical products are sold using contrasting retail models. Physical products are frequently sold via independent retailers using either simple wholesale price contracts or variations such as buyback or returns contracts. For these products, the retailer is typically responsible for

[1] Source: http://gorhamprinting.com/, accessed March 2017.

setting the price of the physical good and managing all inventory decisions, while the manufacturer is usually responsible for setting the parameters of the contract, e.g., the wholesale price at which the good is sold to retailers. Digital goods, on the other hand, are frequently sold via the "agency model," in which a digital retailer provides the platform for distribution of the product and determines a fraction of the sales revenue it will keep; the manufacturer typically sets the retail price and keeps the remaining fraction of sales revenue. These quite dissimilar retail models imply that manufacturers may have different preferences and strategies for the sales of digital and physical products.

In this paper, we consider how these two issues jointly influence the decisions of a manufacturer selling fully substitutable digital and physical information goods. We analyze three distinct supply chain structures that a manufacturer could, potentially, use to distribute its products: a centralized system (in which the manufacturer controls both physical and digital retail channels), a single retailer system (in which the manufacturer sells both digital and physical goods through a single retailer, e.g., Amazon.com), or a dual retailer system (in which the manufacturer sells digital and physical goods through independent, format-specific retailers, such as Apple for digital goods and Target for physical goods). In this setting, we consider the following main research questions. First, given their distinct operational features, how many units of the physical and digital goods should the manufacturer sell in each supply chain structure? Second, if it is necessary to distribute through a retailer, does the manufacturer prefer to use a single retailer or dual retailers? And third, which supply chain structure is best for the retailers, the total supply chain, consumers, and society as a whole?

We find that in a centralized system, the manufacturer only sells digital good and sells no physical goods. This happens because the digital good dominates the physical good operationally: it is both less expensive and does not require a production leadtime. In a single retailer system, the same outcome is replicated, and in equilibrium no physical goods are sold; however, the manufacturer's profit is, of course, lower than in the centralized system due to the payment of some digital revenues to the retailer. In the dual retailer system, by contrast, provided the digital retailer is not "too weak," the manufacturer always sells a positive quantity of physical products through the physical retailer, and moreover, the manufacturer prefers the dual retailer system to the single retailer system, as the former allows the manufacturer to shift some sales from a powerful digital retailer (that commands a high revenue share) to a more profitable channel.

Because the dual retailer system results in positive production of the operationally dominated good, total supply chain profit is lower than in the single retailer or centralized systems; despite this, in addition to the manufacturer's preference for dual retailers, both consumers and society as a whole are better off in the dual retailer system. These results help to illustrate that the distinct operational characteristics of digital and physical goods, as well as their different retail models, can have a significant influence on the supply chain design and distribution decisions of content creators, shifting sales to a dominated version of the product (the physical good) while benefitting the manufacturer and consumers and hurting retailers.

The remainder of this paper is organized as follows. In §2.2 we review the literature on both supply chain design and digital goods. In §2.3 we introduce the components of our model. In §2.4, we analyze the equilibrium quantities, prices, and profits in the centralized, single retailer, and dual retailer systems. §2.5 discusses the implications of these equilibrium values, and determines how manufacturer, retailer, and supply chain profits compare in each, as well as consumer and overall social welfare. In §2.6 (and Appendix A), we analyze a variation of our model in which the physical and digital products are only partial (rather than perfect) substitutes, and show that while some results may be modified in this setting, a number of key insights continue to hold, such as the manufacturer's preference for a dual retailer system despite the supply chain's preference for a single retailer system. §2.7 concludes the paper with a discussion of key managerial insights and potential future research directions.

2.2 Literature Review

This paper lies at the intersection of two streams of literature: the channel structure and coordination (through contracts) literature and the digital goods literature. Both the marketing and operations management disciplines study these topics. We discuss each in more detail below.

Channel/Supply Chain Coordination Literature. The supply chain coordination literature is largely comprised of three classes of supply chain structures: bilateral monopolies (selling one product through a single retailer), one manufacturer selling to multiple retailers, or multiple manufacturers selling through one common retailer. In the marketing literature, most works do not consider the choice between structures, but rather focus on double marginalization, coordination, and the decision to vertical integrate or decentralize within a single structure (e.g., [Spengler, 1950], [Jeuland & Shugan, 1983]). [McGuire & Staelin, 1983] and [McGuire & Staelin, 1986] discuss how the inefficiencies of choosing a decentralized channel structure compared to a centralized one can be desirable in the case of strong competition. [Moorthy, 1988] looks at a similar problem with two competing manufacturers where each sells through their own retailer can benefit from decentralization to buffer price competition buffering and realize a higher profit. There is also a large body of work on contracts when multiple manufacturers sell through a common retailer through a wholesale contract, e.g. [Choi, 1991], [Trivedi, 1998], and [Lee & Staelin, 1997].

In the operations and supply chain literature, [Tsay *et al.* , 1999] and

[Cachon, 2003a] provide detailed surveys of incentive issues and coordination mechanisms under common supply chain contracts. [Lariviere & Porteus, 2001] analyze the optimal wholesale price contract when selling to a newsvendor retailer. [Pasternack, 1985] shows that supply chain coordination in a newsvendor context with a buyback or returns contract. [Cachon & Lariviere, 2005a] show that there exists an equivalent revenue sharing contract for any buy back contracts for a fixed price newsvendor retailer and for a price discount contract for a price-setting newsvendor, and that revenue sharing contracts coordinate the supply chain. [Cachon & Kök, 2010] examines contracts in a setting of multiple manufacturers selling through one retailer.

Our paper differs from the existing operations and marketing literatures on channel coordination in several key ways. First, we study the optimal supply chain structure (number of retailers through which to sell) of a manufacturer that produces two substitutable formats—that can be sold jointly or separately by retailers—with distinct operational characteristics, which none of the preceding papers do. Second, unlike the papers in this literature, we are not focused on determining a class of coordinating contracts to improve supply chain efficiency; rather, we instead adopt the conventional contracts used in the industry (wholesale price for physical goods and agency contracts for digital goods) and determine the manufacturer's preference for supply chain structure *given* the commonly observed contract types and allocation of decision rights within the supply chain.

Our work is also related to the literature on multi-channel selling. [Balasubramanian, 1998], [Chiang *et al.* , 2003], [Kumar & Ruan, 2006], and [Cattani *et al.* , 2006] look at such "mixed" channel structures, i.e., two distinct channels selling the same product, such as online/mail and physical stores, but most works in this literature focus on physical retailers facing competition from the addition of a manufacturer owned online or direct retailer. Our model considers the manufacturer choice of channel structure from "scratch," where both a physical and digital retailer exist, but one does not precede the other. Additionally, the manufacturer does not (necessarily) own the digital retailer in our model (although this is one possible interpretation of the an extreme case, i.e., the one in which the digital retail retains zero revenue from the sale of digital goods, and the manufacturer enjoys 100% of the digital revenues).

Digital Goods Literature. The literature on digital goods is more limited but has grown rapidly in recent years. [Gilbert, 2015] gives an overview of e-books and the underlying conflict between publishers and retailers over pricing, with a focus on the practices of dominant e-book retailer Amazon. Most of the existing theoretical literature on digital goods, such as [Lahiri & Dey, 2013], [August & Tunca, 2006], [Chellappa & Shivendu, 2005], and [Wu & Chen, 2008], focus on how piracy affects firms and what firms can do to react to piracy, typically using price and quality as levers. There is little work on our topic of management of digital and physical good supply chains. [Jiang & Katsamakas, 2010] look at the entry decision faced by an e-book retailer when there are existing physical book retailers. [Tan *et al.* , 2015] examines the agency model for digital goods and show that it can mitigate the double marginalization effect and coordinate the digital supply chain with multiple competing retailers. [Tan & Carrillo, 2016] uses a model of vertically differentiated goods to compare the agency model and wholesale contracts. [Johnson, 2013] and [Johnson, 2014] compare the agency model for digital goods and wholesale contracts and shows consumers being better off under the agency model. In contrast to these papers, in our model, we have perfectly substitutable goods (not vertically differentiated ones), we make contract parameters endogenous decisions of the firms in the model, and we consider manufacturer's choice of supply chain structure and whether to sell to specialty retailers or a single "dual format" retailer.

To summarize, our paper is the first to our knowledge to consider a manufacturer's decision of how structure and manage supply chains for substitutable digital and physical goods.

2.3 Model

A manufacturer (e.g., a content creator or publisher) sells two products over a single selling season: a digital product (denoted with a subscript D) and a physical product (denoted with a subscript P). These two products represent different formats of a single information good or "title," such as a book or a film. The digital good has zero marginal production cost. The physical good has a non-zero marginal cost, $c_P > 0$. The quantity of digital good sold is denoted $Q_D \geq 0$, while the quantity of the physical good sold is denoted $Q_P \geq 0$. The price of both products is assumed to be a linear function of the quantity sold, and moreover we assume that the formats are perfect substitutes, i.e., demand (and hence price) is not dependent on the format but rather on the title itself.[2] Consequently, the price of both products given their respective quantities equals

$$p(Q_D, Q_P) = A - Q_P - Q_D.$$

Throughout, we assume that $c_P < A/2$, i.e., the production cost is not excessively high; this helps to ensure non-degenerate solutions in much of our analysis.

As discussed in the introduction, we analyze three supply chain structures. In the first, the manufacturer sells both products directly to consumers. This "centralized" supply chain will serve as our baseline for comparison throughout the remainder of the paper. The only decisions in this system are the quantities of each product, Q_P and Q_D, that the manufacturer sells to the market, and the manufacturer retains all revenues and incurs all costs. In the second and third scenarios, we examine decentralized systems in which the manufacturer sells through one or two retailers.

[2] A relaxation of this assumption is discussed §2.6.

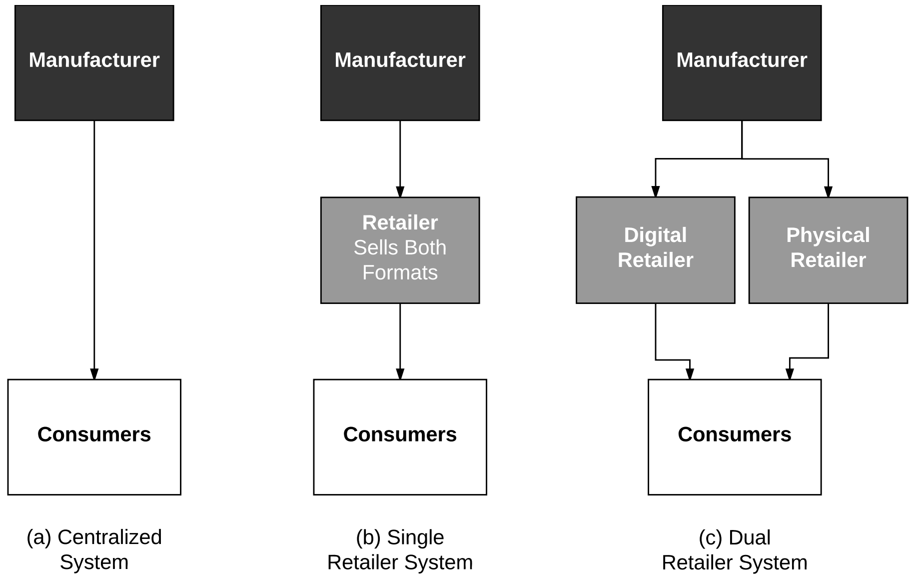


FIGURE 2.1: The three supply chain structures.

In the second scenario, the manufacturer sells both products through a single retailer (e.g., Amazon, which sells digital and physical versions of the same product); we refer to this case as the "single retailer" system. In the third scenario, the manufacturer sells through two independent retailers: one for the physical good (e.g., a brick and mortar retailer like Target) and one for the digital good (e.g., the Apple iTunes Store or the Google Play Store). We refer to this case as the "dual retailer" system. We use the subscript M to denote the manufacturer, the subscript R to denote the single retailer in the single retailer system, and the subscripts P and D to denote the physical retailer and digital retailer, respectively, in the dual retailer system. The three systems are graphically summarized in Figure 2.1. In both decentralized supply chains, consistent with examples from industry, we assume that the type of contract used for the digital product is an "agency" contract, i.e., a revenue sharing contract. In this case, the retailer retains a fraction of sales revenue $\alpha \leq \bar{\alpha}$ from sales of the digital good, while the manufacturer earns a complementary fraction of sales revenue. The parameter α is frequently called the "royalty fee" in the industry, and $\bar{\alpha}$ represents the maximum possible royalty fee that the retailer may retain; this is reflective of the relative power of the retailer, e.g., a more powerful retailer may retain a larger fraction of revenue, and hence may enjoy a larger $\bar{\alpha}$. As is typically the case in agency contracts, we assume that the manufacturer—not the retailer—is responsible for setting the price (i.e., the quantity in our model) of the digital good. Hence, in a decentralized system (with either one or two retailers) the manufacturer determines $Q_D \geq 0$ while the retailer determines $\alpha \leq \bar{\alpha}$.

For the physical good, we assume the manufacturer offers a take-it-or-leave-it wholesale price contract with parameter $w \geq c_P$ to the retailer. The retailer, after agreeing to this price, then chooses the quantity of the physical good $Q_P \geq 0$ to order from the manufacturer, and then the retailer sells its entire stock of the physical good to the market. Consistent with the operational realities of physical and digital production (in which physical goods have long production leadtimes but digital goods have zero leadtime), we assume that the physical good quantity decision must be made first, while the digital good quantity decision is made second. In other words, the retailer is a Stackelberg leader, credibly establishing the physical good quantity via its order with the manufacturer, and the manufacturer is a follower, setting the digital good quantity after the retailer has chosen the physical quantity.

Both contract decisions are made before quantities are determined, and we also assume that the digital royalty fee α is chosen first, before the wholesale price w. This is due to the fact that the royalty fee is typically a platform-wide decision made by the retailer (e.g., Amazon or Apple pick a single royalty fee for all digital products sold on their platform), and as a result is a longer-term decision, while the wholesale price is typically title-specific and is made by the manufacturer on a case-by-case basis. Consequently, the retailer acts a leader in the contract game (by setting α before the manufacturer sets w) due to the nature of the contracting process, and the retailer also acts as a leader in the quantity game (by setting Q_P before the manufacturer sets Q_D) due to the operational nature of the products. The decision

Table 2.2: Decision Rights and Sequence in the Three Supply Chain Structures

Subgame	Step	Decision	Centralized	Single Retailer	Dual Retailer
Contract	1	α	n/a	Retailer	Digital Retailer
	2	w	n/a	Manufacturer	Manufacturer
Quantity	3	Q_P	Manufacturer	Retailer	Physical Retailer
	4	Q_D	Manufacturer	Manufacturer	Manufacturer

rights and sequence of decisions are summarized in Table 2.2.[3] Observe that in the single retailer system, the same retailer makes the decisions in steps 1 and 3; in the dual retailer system, the digital retailer makes the decision in step 1, while the physical retailer makes the decision in step 3. We refer to the collection of steps 3 and 4 as the *quantity subgame*, and the collection of steps 1 and 2 as the *contract game*.

2.4 Analysis

In this section, we analyze the optimal contract and quantity decisions of each firm in the supply chain. To serve as a baseline for analysis, we first consider the centralized system, in which the manufacturer sells both the physical and digital products directly to consumers. In this case, manufacturer profit is

$$\Pi_M(Q_D, Q_P) = p(Q_D, Q_P)Q_D + (p(Q_D, Q_P) - c_P)Q_P.$$

It is easy to see from this expression that, given the fully substitutable nature of the two formats and the fact that the physical good has a strictly positive marginal cost,

[3] Note that we assume the both the physical and digital goods are sold in a "quantity setting" manner, i.e., the seller of each determines a quantity to sell, and in turn this determines a market clearing price. While a reasonable assumption for physical goods given their long production leadtimes, one might imagine that the digital good is more likely to be sold by a seller that determines a price, rather than a quantity, with which the market reacts with some quantity of demand. The two possibilities (a "quantity setting" digital seller and a "price setting" digital seller) are equivalent when both products are sold by a centralized firm; however, in a decentralized setting (e.g., a manufacturer and either one or two retailers), a quantity setting digital seller results in Cournot competition at the retail level, while a price setting digital retailer results in a mixed Cournot-Bertrand duopoly at the retail level (see [Singh & Vives, 1984]). In §2.6, we analyze this alternative model in which the digital good is sold by a price setting seller.

the digital good dominates the physical good, and as a result the optimal decisions for a centralized manufacturer are to sell no physical goods and only digital goods. What follows is a standard quantity optimization problem with a linear demand curve, yielding optimal values summarized in the following proposition:

Proposition 2.1. *In a centralized system, the optimal physical quantity is* $Q_P^* = 0$*, the optimal digital quantity is* $Q_D^* = \frac{A}{2}$*, the optimal price is* $p(Q_D^*, Q_P^*) = \frac{A}{2}$*, and the optimal manufacturer profit is* $\Pi_M^* = \frac{A^2}{4}$*.*

Proof. Omitted. □

In the preceding proposition, we have used the superscript $*$ to denote optimal values in the benchmark centralized supply chain case. In what follows, we use superscripts s and d to denote equilibrium values in the single retailer and dual retailer systems, respectively.

2.4.1 Single Retailer System

We begin with the single retailer system, in which the manufacturer sells both the digital and physical products through a single intermediary. We solve for the equilibrium in the single retailer system in reverse, beginning with step 4: the manufacturer's optimal digital quantity decision. In step 4, the manufacturer's profit is:

$$\Pi_M(Q_D; Q_P, \alpha, w) = (1-\alpha)(A - Q_D - Q_P)Q_D + (w - c_P)Q_P. \qquad (2.1)$$

Observe that a fraction α of the manufacturer's digital revenue is subtracted from the profit function (to be given to the retailer), while the manufacturer only enjoys a margin $w - c_P$ on the physical good. The manufacturer optimizes this expression over Q_D, yielding optimal quantity $Q_D^s(Q_P, \alpha, w)$, i.e., the optimal digital quantity as a function of the physical quantity (chosen in step 3), the wholesale price (chosen in step 2) and the royalty fee (chosen in step 1). Given this best response function, in step 3 the retailer optimizes its profit over Q_P, where retailer profit is given by:

$$\Pi_R(Q_P;\alpha,w) = \alpha(A - Q_D^s(Q_P,\alpha,w) - Q_P)Q_D^s(Q_P,\alpha,w) \qquad (2.2)$$
$$+ (A - Q_P - Q_D^s(Q_P,\alpha,w) - w)\, Q_P.$$

The following proposition provides the optimal quantities resulting from these two sequential optimization problems:

Proposition 2.2. *In the single retailer system, the equilibrium to the quantity subgame is, if* $A\left(\frac{1}{2}-\alpha\right) > w$,

$$Q_P^s(\alpha,w) = \frac{1}{1-\alpha}\left(A\left(\frac{1}{2}-\alpha\right) - w\right) \text{ and } Q_D^s(\alpha,w) = \frac{1}{1-\alpha}\left(\frac{A}{4}+\frac{w}{2}\right).$$

Otherwise, the equilibrium quantities are

$$Q_P^s(\alpha,w) = 0 \text{ and } Q_D^s(\alpha,w) = \frac{A}{2}.$$

Proof. Optimizing (2.1) over Q_D yields $Q_D^s(Q_P,\alpha,w) = \frac{A-Q_P}{2}$ if $Q_P \le A/2$, and zero otherwise. Next, in step 3, the retailer's profit is, if $Q_P \le A/2$,

$$\Pi_R(Q_P;\alpha,w) = \alpha\left(\frac{A-Q_P}{2}\right)^2 + \left(A - Q_P - \frac{A-Q_P}{2} - w\right)Q_P, \qquad (2.3)$$

otherwise, retailer profit is $\Pi_R(Q_P;\alpha,w) = (A - Q_P - w)\,Q_P$. Differentiating (2.3) with respect to Q_P,

$$\frac{d\Pi_R(Q_P;\alpha,w)}{dQ_P} = -2\alpha\left(\frac{A-Q_P}{2}\right) + \left(\frac{A-2Q_P}{2} - w\right)$$
$$\frac{d^2\Pi_R(Q_P;\alpha,w)}{dQ_P^2} = \alpha - 1 < 0$$

Hence, the problem is concave, and the optimal Q_P is given by $Q_P^* = \frac{A\left(\frac{1}{2}-\alpha\right)-w}{1-\alpha}$ if $A\left(\frac{1}{2}-\alpha\right) > w$, and $Q_P^* = 0$ otherwise. Observe that $\frac{A\left(\frac{1}{2}-\alpha\right)-w}{1-\alpha} < A/2$ for all $w \ge 0$, hence the maximizer of (2.3) always lies within the feasible range $Q_P \leq A/2$, i.e., the retailer never finds it optimal to force the manufacturer into zero sales of the digital good (although zero sales of the physical good may be optimal for the retailer). The equilibrium quantities in the proposition follow. □

From the proposition, it is clear that the digital good will never have zero quantity in equilibrium, although the physical good might if the wholesale price is sufficiently high. Using these equilibrium quantities, we may now proceed to analyze the contract game. In step 2, the manufacturer's profit is

$$\Pi_M(w;\alpha) = (1-\alpha)(A - Q_D^s(\alpha,w) - Q_P^s(\alpha,w))Q_D^s(\alpha,w) + (w - c_P)Q_P^s(\alpha,w), \quad (2.4)$$

where $Q_D^s(\alpha, w)$ and $Q_P^s(\alpha, w)$ are the anticipated equilibrium quantities resulting from the quantity subgame in Proposition 2.2. The manufacturer optimizes this expression over $w \geq c_P$, resulting in a best response function $w^s(\alpha)$. In step 1, the retailer anticipates the manufacturer's optimal wholesale price response, as well as the equilibrium to the quantity subgame, and optimizes its own profit over the royalty fee $\alpha \leq \bar{\alpha}$:

$$\begin{aligned} \Pi_R(\alpha) &= \alpha(A - Q_P^s(\alpha, w(\alpha)) - Q_D^s(\alpha, w(\alpha)))Q_D^s(\alpha, w(\alpha)) \\ &\quad + (A - Q_P^s(\alpha, w(\alpha)) - Q_D^s(\alpha, w(\alpha)) - w(\alpha))\, Q_P^s(\alpha, w(\alpha)). \end{aligned} \quad (2.5)$$

The resulting equilibrium is described in the following proposition:

Proposition 2.3. *In the single retailer system, the unique equilibrium is as follows:*
(i) Quantities: $Q_D^s = \frac{A}{2}$ *and* $Q_P^s = 0$.
(ii) Contract parameters: $w^s = A\left(\frac{1}{2} - \bar{\alpha}\right)$ *and* $\alpha^s = \bar{\alpha}$.
(iii) Price: $p^s = A/2$.
(iv) Manufacturer profit: $\Pi_M^s = (1-\bar{\alpha})\frac{A^2}{4}$.
(v) Retailer profit: $\Pi_R^s = \bar{\alpha}\frac{A^2}{4}$.

Proof. In step 2, the manufacturer's profit is independent of w if $A\left(\frac{1}{2}-\alpha\right) < w$. Otherwise, manufacturer profit is

$$\Pi_M(w;\alpha) = \frac{1}{1-\alpha}\left[\left(\frac{A}{4}+\frac{w}{2}\right)^2 + (w-c_P)\left(A\left(\frac{1}{2}-\alpha\right)-w\right)\right]. \tag{2.6}$$

Differentiating (2.6) with respect to w,

$$\begin{aligned}\frac{d\Pi_M(w;\alpha)}{dw} &= \frac{1}{1-\alpha}\left[A\left(\frac{3}{4}-\alpha\right)-\frac{3}{2}w+c_P\right]\\ \frac{d^2\Pi_M(w;\alpha)}{dw^2} &= \frac{1}{1-\alpha}\left[-\frac{3}{2}\right] < 0\end{aligned}$$

Hence, profit is concave in w, and the optimal w is given by $w^*(\alpha) = \frac{A(3-4\alpha)+4c_P}{6}$. The condition $A\left(\frac{1}{2}-\alpha\right) \geq w$ is satisfied if $\alpha \leq -\frac{2c_P}{A}$, which can never occur. This means that the maximizer of (2.6) lies outside the feasible region, hence the manufacturer always sets w sufficiently high that the retailer orders zero quantity. The remainder of the results in the proposition follow. □

Observe that, in equilibrium, the manufacturer always sets a sufficiently high wholesale price such that the retailer orders zero physical quantity. Increasing the wholesale price has the following effects on the manufacturer: it raises his revenue on each unit of the physical good sold to the retailer, but also decreases the retailer's order quantity (and thus increases the market price of both products and raises digital goods revenue). In a single retailer system, the latter effect—in particular the increase in retail price resulting from a decrease in the physical goods quantity—always outweighs the former, and the manufacturer finds it optimal to induce zero physical good inventory.

The result occurs because in the single retailer system, the retailer's physical quantity is highly sensitive to the wholesale price. Note in particular that, from Proposition 2.3, when the physical quantity is not zero, $\frac{dQ_P^s(\alpha,w)}{dw} = -\frac{1}{1-\alpha} < -1$. As a result, the retail price is also highly sensitive to the wholesale price: $\frac{dp^s(\alpha,w)}{dw} = \frac{1}{2(1-\alpha)} > \frac{1}{2}$. This sensitivity is due to the fact that the retailer also enjoys some revenue from the digital good, and thus if the manufacturer raises the physical good wholesale price, the retailer readily reduces its physical quantity to transfer revenue to the (fully substitutable) digital product. Thus, while it may seem reasonable, *a priori*, that the manufacturer may wish to induce some positive sales of physical goods to avoid having to pay the royalty fee, this strong sensitivity to w in the quantity subgame means that this is never profitable for the manufacturer. Put another way, the wholesale price necessary to induce the retailer to order a positive quantity of the physical good is too low for the manufacturer to bear, and as a result the manufacturer prices the retailer out of the physical goods market. Consequently, the manufacturer only sells the digital good and never sells any physical goods, and because of this, the retailer will simply set $\alpha = \bar{\alpha}$ and become a *de facto* digital only retailer.

2.4.2 Dual Retailer System

We next move to the dual retailer system, in which the manufacturer sells the products through independent retailers that each specialize in one format. As in the single retailer system, we begin analyzing the equilibrium with step 4, in which the manufacturer determines the quantity of the digital good to sell (and hence the retail price). Manufacturer profit in step 4 is

$$\Pi_M(Q_D; Q_P, \alpha, w) = (1-\alpha)(A - Q_D - Q_P)Q_D + (w - c_P)Q_P. \tag{2.7}$$

The manufacturer optimizes this expression over Q_D, leading to a best response function $Q_D^d(Q_P, \alpha, w)$. The physical retailer anticipates this best response function and, in step 3, chooses a physical quantity to maximize its own profit,

$$\Pi_P(Q_P; \alpha, w) = \left(A - Q_D^d(Q_P, \alpha, w) - Q_P - w\right) Q_P. \tag{2.8}$$

The following result describes the equilibrium to this sequential quantity subgame between the manufacturer and the physical retailer:

Proposition 2.4. *In the dual retailer system, the equilibrium to the quantity subgame is, if $A/2 > w$,*

$$Q_P^d(\alpha, w) = \frac{A}{2} - w \text{ and } Q_D^d(\alpha, w) = \frac{A}{4} + \frac{w}{2}.$$

Otherwise, $Q_P^d(\alpha, w) = 0$ *and* $Q_D^d(\alpha, w) = \frac{A}{2}$.

Proof. Optimizing (2.7) over Q_D yields $Q_D^*(Q_P, \alpha, w) = \frac{A-Q_P}{2}$ if $Q_P < A$, and zero otherwise. Next, in step 3, the physical retailer's profit is

$$\Pi_P(Q_P; \alpha, w) = \left(A - \frac{A - Q_P}{2} - Q_P - w\right) Q_P$$

if $Q_P < A$. Note that any if $Q_P > A$ would necessarily result in negative profit and hence is never optimal. Optimizing over Q_P yields $Q_P^d(\alpha, w) = \frac{A}{2} - w$ if $w < A/2$, and zero otherwise. The subsequent equilibrium quantities are as given in the proposition. □

Given the equilibrium to the quantity subgame, in step 2, the manufacturer's profit is

$$\Pi_M(w; \alpha) = (1 - \alpha)(A - Q_D^d(\alpha, w) - Q_P^d(\alpha, w))Q_D^d(\alpha, w) + (w - c_P)Q_P^d(\alpha, w), \quad (2.9)$$

where Q_P^d and Q_D^d are as given in Proposition 2.4. The manufacturer optimizes this expression over $w \geq c_P$, leading to a best response function $w(\alpha)$ for the wholesale price. In turn, the digital retailer optimizes its own profit,

$$\Pi_D(\alpha) = \alpha(A - Q_D^d(\alpha, w) - Q_P^d(\alpha, w))Q_D^d(\alpha, w) \quad (2.10)$$

over $\alpha \leq \bar{\alpha}$. Note the digital retailer must anticipate the manufacturer's optimal wholesale price and digital quantity, as well as the physical retailer's optimal physical quantity, in response to its choice of the royalty fee α. Sequentially optimizing these profit functions leads to the following equilibrium outcome:

Proposition 2.5. *In the dual retailer system, if $\bar{\alpha} \le \frac{2c_P}{A}$, the single retailer equilibrium from Proposition 2.3 is replicated. If $\bar{\alpha} > \frac{2c_P}{A}$, the unique equilibrium is as follows:*

(i) Quantities: $Q_D^d = \frac{1}{2}\left(\frac{3A+2c_P}{3+\bar{\alpha}}\right)$ *and* $Q_P^d = \frac{1}{2}\left(\frac{2A\bar{\alpha}-4c_P}{3+\bar{\alpha}}\right)$.

(ii) Contract parameters: $w^d = \frac{A(3-\bar{\alpha})+4c_P}{6+2\bar{\alpha}}$ *and* $\alpha^d = \bar{\alpha}$.

(iii) Price: $p^d = \frac{A}{2} - \frac{1}{2}\left(\frac{A\bar{\alpha}-2c_P}{3+\bar{\alpha}}\right)$.

(iv) Manufacturer profit: $\Pi_M^d = \frac{(3+\bar{\alpha})(3-2\bar{\alpha})A^2-4(3-\bar{\alpha})A\bar{\alpha}c_P+12(1-\bar{\alpha})c_P^2}{4(3+\bar{\alpha})^2}$.

(v) Physical retailer profit: $\Pi_P^d = \frac{1}{2}\left(\frac{A\bar{\alpha}-2c_P}{3+\bar{\alpha}}\right)^2$.

(vi) Digital retailer profit: $\Pi_D^d = \frac{\bar{\alpha}}{4}\left(\frac{3A+2c_P}{3+\bar{\alpha}}\right)^2$.

Proof. Note that manufacturer profit is independent of w if $w > A/2$ (because zero physical quantity is ordered by the physical retailer). If $w < A/2$, using the quantity subgame equilibrium from Proposition 2.4, manufacturer profit is

$$\Pi_M(w;\alpha) = (1-\alpha)\left(\frac{A}{4}+\frac{w}{2}\right)^2 + (w-c_P)(\frac{A}{2}-w).$$

Differentiating this expression with respect to w,

$$\frac{d\Pi_M(w;\alpha)}{dw} = (1-\alpha)\left(\frac{A}{4}+\frac{w}{2}\right) + \frac{A}{2} - 2w + c_P,$$

$$\frac{d^2\Pi_M(w;\alpha)}{dw^2} = (1-\alpha)\frac{1}{2} - 2 < 0.$$

Profit is thus concave in the wholesale price, and the optimal wholesale price (provided this solution is interior to the constraint $w < A/2$) is thus $w^d(\alpha) = \frac{A(3-\alpha)+4c_P}{6+2\alpha}$. Observe that this is decreasing in α, and if $\alpha = 0$, $w^d(0) = \frac{3A+4c_P}{6}$, while if $\alpha = 1$, $w^d(1) = \frac{A+2c_P}{4} < \frac{A}{2}$ given the assumption that $c_P < A/2$. Thus, there exists a unique

$\hat{\alpha}$ such that, for all $\alpha > \hat{\alpha}$, the optimal wholesale price is interior, while for all $\alpha < \hat{\alpha}$, the optimal wholesale price is $A/2$ which induces zero physical quantity. This $\hat{\alpha}$ is determined by $\frac{A(3-\hat{\alpha})+4c_P}{6+2\hat{\alpha}} = \frac{A}{2}$, which yields $\hat{\alpha} = \frac{2c_P}{A}$. Because of this, if $\hat{\alpha} > \bar{\alpha}$, the digital retailer can only set a royalty fee that will induce zero physical quantity, and the equilibrium will proceed as in the single retailer case. If $\hat{\alpha} < \bar{\alpha}$, the digital retailer has three broad strategies: (1) Set $\alpha = \hat{\alpha}$, the maximum royalty fee that will induce zero physical quantity; (2) Set $\hat{\alpha} < \alpha < \bar{\alpha}$, which will induce positive quantities of both the digital and physical good; (3) Set $\alpha = \bar{\alpha}$, the maximum royalty fee that the manufacturer will accept.

Under the first strategy, the digital retailer's profit is $\Pi_D(\hat{\alpha}) = c_P\frac{A}{2}$. Under the second strategy, the digital retailer's profit is $\Pi_D(\alpha) = \frac{\alpha}{4}\left(\frac{3A+2c_P}{3+\alpha}\right)^2$. Differentiating with respect to α,

$$\frac{d\Pi_D(\alpha)}{d\alpha} = \frac{1}{4}\left(\frac{3A+2c_P}{3+\alpha}\right)^2 - \frac{1}{2}\alpha\frac{(3A+2c_P)^2}{(3+\alpha)^3},$$

$$\frac{d^2\Pi_D(\alpha)}{d\alpha^2} = \frac{(3A+2c_P)^2}{2(3+\alpha)^4}(-6+\alpha) < 0.$$

The optimal revenue share is thus the solution to the first order condition, $\alpha^d = 3$. However, the digital retailer cannot set an α greater than $\bar{\alpha} < 1$, hence the optimal revenue kept by the digital retailer is $\alpha^d = \bar{\alpha}$. Thus, strategy 2 reduces to strategy 3, and this clearly dominates strategy 1. The remainder of the proposition follows using $\alpha^d = \bar{\alpha}$ and $w^d(\alpha^d) = \frac{A(3-\bar{\alpha})+4c_P}{6+2\bar{\alpha}}$. □

As the proposition shows, if the digital retailer has low bargaining power (indicated by $\bar{\alpha} \leq \frac{2c_P}{A}$), the dual retailer system is identical to the single retailer system. In particular: the physical good quantity is zero, the digital good quantity, price, and profits are the same as in Proposition 2.3, and the digital retailer becomes, by default, the "single" retailer that sells a non-zero quantity. However, if the digital retailer has stronger bargaining power (implying $\bar{\alpha} > \frac{2c_P}{A}$), a different picture emerges. In this case, the retailer extracts significant revenue from the manufacturer's sales of the digital good, and in response, the manufacturer sets a lower wholesale price and sells a positive quantity of the physical good to the physical retailer.

This was never optimal in a single retailer system, because in that case the wholesale price required to induce the retailer to order a positive quantity of the physical good was too low for the manufacturer to be willing to bear; this happened because the retailer was willing to forgo physical goods revenues for digital goods revenues. In the dual retailer system, however, the physical retailer competes with the digital retailer and enjoys no digital revenues of its own; consequently, the physical retailer is willing to order positive inventory at a higher wholesale price, and overall its order quantity is less sensitive to the wholesale price ($\frac{dQ_P^d(\alpha,w)}{dw} = -1$), meaning inducing positive physical goods sales becomes profitable for the manufacturer. Put another way, the manufacturer exploits the fact that the physical retailer has no other sources from revenue from the title by charging a higher wholesale price for the physical product; in essence, the manufacturer exploits the competition between the digital and physical retailers, despite the fact that in both cases the manufacturer is the one setting Q_D and enjoying a significant fraction of the revenue from digital sales.

Interestingly, the digital retailer continues to set the highest possible royalty fee, just as in the single retailer system. One might imagine that the digital retailer could set a lower fee and effectively force the physical retailer out of the market (if $\alpha \leq \frac{2c_P}{A}$). This turns out to never be profitable for the digital retailer, because it necessitates giving up too much revenue from digital sales. In particular, the digital sales quantity is significantly higher than the physical sales quantity: since $3A + 6c_P > 2A\bar{\alpha}$ for all $\bar{\alpha} < 1$, at least 1.5 digital copies are sold for every physical copy. Consequently, the digital retailer's profit is at least 4.5 times the profit of the physical retailer. Thus, acting to aggressively push the physical goods retailer out of the market with a reduced royalty fee is never profitable for the digital retailer, even though it enjoys a leadership position in setting the royalty fee α.

Lastly, note that manufacturer profit is decreasing in $\bar{\alpha}$, but both the digital *and physical* retailer profits are increasing in $\bar{\alpha}$. While the former is to be expected, the latter is somewhat surprising; the reason for this is the manufacturer's increasing reliance on the physical retailer as the digital retailer becomes more powerful. In other words, the manufacturer responds to higher $\bar{\alpha}$ with a lower wholesale price, which thus improves the physical retailer's profit. Hence, while the manufacturer is harmed by a more powerful digital retailer, both types of retailer benefit when one of them becomes more powerful.

2.5 Implications

We now discuss several important implications of the equilibria derived in the previous section. In §2.5.1, we consider how the manufacturer is affected by the single and dual retailer systems. In §2.5.2, we discuss how the different systems impact the retailer, supply chain efficiency, consumer welfare, and social welfare.

2.5.1 The Manufacturer's Optimal Supply Chain Structure

We begin by summarizing the results from the preceding analysis in Table 2.3. Our first result in this section considers the manufacturer's preference between the two possible supply chain structures when both are characterized by the same level of digital retailer "power," i.e., by the same maximum royalty fee $\bar{\alpha}$. The manufacturer's preferences are summarized in the following proposition:

Proposition 2.6. *Holding $\bar{\alpha}$ constant in the single and dual retailer systems:*

(i) If $\bar{\alpha} \leq \frac{2c_P}{A}$, the manufacturer is indifferent between the single and dual retailer

Table 2.3: Equilibrium Values in the Three Supply Chain Structures

Parameter	**Centralized**	**Single Retailer**	**Dual Retailer** ($\bar{\alpha} > \frac{2c_P}{A}$)
α	n/a	$\bar{\alpha}$	$\bar{\alpha}$
w	n/a	$A\left(\frac{1}{2} - \bar{\alpha}\right)$	$\frac{A(3-\bar{\alpha})+4c_P}{6+2\bar{\alpha}}$
Q_P	0	0	$\frac{1}{2}\left(\frac{2A\bar{\alpha}-4c_P}{3+\bar{\alpha}}\right)$
Q_D	$\frac{A}{2}$	$\frac{A}{2}$	$\frac{1}{2}\left(\frac{3A+2c_P}{3+\bar{\alpha}}\right)$
$p(Q_D, Q_P)$	$\frac{A}{2}$	$\frac{A}{2}$	$\frac{A}{2} - \frac{1}{2}\left(\frac{A\bar{\alpha}-2c_P}{3+\bar{\alpha}}\right)$
Π_M	$\frac{A^2}{4}$	$(1-\bar{\alpha})\frac{A^2}{4}$	$\frac{(3+\bar{\alpha})(3-2\bar{\alpha})A^2-4(3-\bar{\alpha})A\bar{\alpha}c_P+12(1-\bar{\alpha})c_P^2}{4(3+\bar{\alpha})^2}$

systems.

(ii) If $\bar{\alpha} > \frac{2c_P}{A}$, the manufacturer strictly prefers the dual retailer system.

Proof. Part (i) follows immediately from Propositions 2.3 and 2.5. To see part (ii), note that profit in the dual retailer system is greater if

$$\frac{(3+\bar{\alpha})(3-2\bar{\alpha})A^2 - 4(3-\bar{\alpha})A\bar{\alpha}c_P + 12(1-\bar{\alpha})c_P^2}{4(3+\bar{\alpha})^2} > (1-\bar{\alpha})\frac{A^2}{4}.$$

Simplifying this inequality, the dual retailer system is preferred if $(\bar{\alpha}A - 2c_P)(\bar{\alpha}(3+\bar{\alpha})A - 6(1-\bar{\alpha})c_P) > 0$. The first term is always positive in case (ii), i.e., when $\bar{\alpha} > \frac{2c_P}{A}$. The second term is positive, since

$$\begin{aligned}\bar{\alpha}(3+\bar{\alpha})A - 6(1-\bar{\alpha})c_P &> \frac{2c_P}{A}(3+\bar{\alpha})A - 6(1-\bar{\alpha})c_P \\ &= 6c_P + 2\bar{\alpha}c_P - 6c_P + 6\bar{\alpha}c_P \\ &= 8\bar{\alpha}c_P > 0\end{aligned}$$

This proves the result. □

Because the single retailer equilibrium is replicated in the dual retailer system when the digital retailer is weak ($\bar{\alpha} \leq \frac{2c_P}{A}$), the manufacturer is indifferent between the systems in this case. However, when the digital retailer is strong ($\bar{\alpha} > \frac{2c_P}{A}$), the manufacturer has a *strict* preference for the dual retailer system. Figure 2.2 graphically depicts the results of Proposition 2.6; in the figure, the two systems

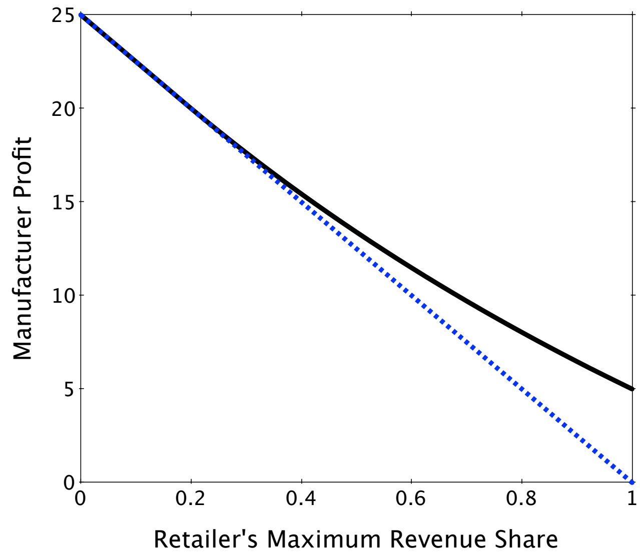


FIGURE 2.2: Manufacturer profit in the single and dual retailer systems. In the figure, $A = 10$ and $c_P = 1$, and the solid line represents profit in the dual retailer system, while the dotted line represents profit in the single retailer system.

have identical profit when the retailer's maximum revenue share $\bar{\alpha} \leq 0.2$, and above this threshold, the dual retailer system is strictly preferred by the manufacturer, increasingly so as $\bar{\alpha}$ increases. Note that the manufacturer's preference for the dual retailer system occurs despite the fact that, in equilibrium, the manufacturer will produce a positive quantity of the operationally inefficient physical good in the dual retailer system (something that is never optimal in a centralized supply chain; see Proposition 2.1). The manufacturer benefits from the presence of a separate physical retailer in this case, because he is able to offload some revenue to the retailer at a high wholesale price, avoiding the payment of a high royalty fee to the digital retailer on at least part of the total sales. This illustrates that for manufacturers or content creators, even when digital and physical products are perfect substitutes (which gives the strongest incentive to only sell digital goods) and the physical product is operationally inferior to the digital product, it is always better to sell via dual retailers than via a single retailer in both formats.

Another implication of this result is that, in addition to consumer-related factors (e.g., a simple consumer preference for physical goods), a key factor slowing the adoption of digital formats (which are operationally and environmentally more efficient) could be the relative power of digital retailers. Strong digital retailers who command high royalty fees cause manufacturers to shift some production to physical products, moving the system away from the centralized optimal allocation of production. Note that this effect arises because of the unique combination of operational distinctions between digital and physical goods (i.e., that physical goods have non-zero costs and production leadtimes), as well as the sequence of decisions that results from common industry practices (i.e., the royalty fee being a platform wide decision but the wholesale price being a title-by-title decision).

A key assumption in Proposition 2.6 is that the single and dual retailer systems have digital retailers with precisely the same level of bargaining power, which manifests as the same upper bound on the royalty fee, $\bar{\alpha}$. In reality, retailers may have different levels of power, and that power may depend on whether they sell digital goods, physical goods, or both. The following proposition examines the manufacturer's preference between supply chain structures when this is the case. In particular, we consider the possibility that the single retailer system has a more powerful retailer, which is capable of charging a higher fee than a digital only retailer; this frequently the case in practice (e.g., Amazon charges higher royalty fees than Apple on digital goods, and often places more onerous restrictions on content creators; [Giammatteo, 2015]). In this scenario, we have the following result:

Proposition 2.7. *Suppose the single and dual retailer systems have maximum royalty fees equal to $\bar{\alpha}_s$ and $\bar{\alpha}_d$, respectively. If $\bar{\alpha}_d < \bar{\alpha}_s$, the manufacturer always strictly prefers the dual retailer system.*

Proof. Suppose $\bar{\alpha}_d < \bar{\alpha}_s$. Then, the dual retailer system is preferred if

$$\frac{(3+\bar{\alpha}_d)(3-2\bar{\alpha}_d)A^2 - 4(3-\bar{\alpha}_d)A\bar{\alpha}c_P + 12(1-\bar{\alpha}_d)c_P^2}{4(3+\bar{\alpha}_d)^2} > (1-\bar{\alpha}_s)\frac{A^2}{4}.$$

Because $\bar{\alpha}_d < \bar{\alpha}_s$, it follows that $(1-\bar{\alpha}_d)\frac{A^2}{4} > (1-\bar{\alpha}_s)\frac{A^2}{4}$. Hence, the dual retailer system is always preferred if $\frac{2c_P}{A} < \bar{\alpha}_d < \bar{\alpha}_s$, per Proposition 2.6. If $\bar{\alpha}_d < \frac{2c_P}{A} < \bar{\alpha}_s$ or $\bar{\alpha}_d < \bar{\alpha}_s < \frac{2c_P}{A}$, the dual retailer system reduces to a single retailer system with a lower royalty fee; as a result, the manufacturer clearly prefers the system with the lower fee, i.e., the dual retailer system. Hence, the manufacturer always strictly prefers the dual retailer system. □

In other words, if the retailer is more powerful by virtue of its ability to sell both formats (and can command a higher royalty fee), the manufacturer's preference for the dual retailer system is only further strengthened, and in the regions of indifference from Proposition 2.6, the manufacturer now has a strict preference for the dual retailer system.

Naturally, in the opposite scenario, i.e. if $\bar{\alpha}_d > \bar{\alpha}_s$, it is possible for the digital retailer to command such a high royalty fee that the dual retailer system is no longer preferred to the single retailer system. In other words, there exists some threshold royalty fee in the single retailer system, below which the manufacturer strictly prefers the single retailer. This observation is summarized in the following proposition:

Proposition 2.8. *Suppose the single and dual retailer systems have maximum royalty fees equal to $\bar{\alpha}_s$ and $\bar{\alpha}_d$, respectively. If $\bar{\alpha}_s < \bar{\alpha}_d$, the manufacturer strictly prefers the single retailer system if and only if*

$$\bar{\alpha}_s < \frac{3\bar{\alpha}_d(3+\bar{\alpha}_d)A^2 + 4(3-\bar{\alpha}_d)\bar{\alpha}_d A c_P - 12(1-\bar{\alpha}_d)c_P^2}{A^2(3+\bar{\alpha}_d)^2}.$$

Table 2.4: The Manufacturer's Supply Chain Preference with Heterogeneous $\bar{\alpha}$

	Single Retailer, Low $\bar{\alpha}_s$	**Single Retailer, High $\bar{\alpha}_s$**
Dual Retailer, Low $\bar{\alpha}_d$	Dual Retailer	Dual Retailer
Dual Retailer, High $\bar{\alpha}_d$	Single Retailer	Dual Retailer

Proof. The manufacturer prefers the single retailer system if

$$\frac{(3+\bar{\alpha}_d)(3-2\bar{\alpha}_d)A^2 - 4(3-\bar{\alpha}_d)A\bar{\alpha}c_P + 12(1-\bar{\alpha}_d)c_P^2}{4(3+\bar{\alpha}_d)^2} < (1-\bar{\alpha}_s)\frac{A^2}{4}.$$

Rearranging this expression, the single retailer system is optimal if

$$\bar{\alpha}_s < \frac{3\bar{\alpha}_d(3+\bar{\alpha}_d)A^2 + 4(3-\bar{\alpha}_d)\bar{\alpha}_d A c_P - 12(1-\bar{\alpha}_d)c_P^2}{A^2(3+\bar{\alpha}_d)^2},$$

which proves the result. □

Taken together, these propositions show that, roughly speaking, the manufacturer's preferences for the various supply chain structures align with the two-by-two matrix in Table 2.4. If the single retailer is powerful, the manufacturer always prefers the dual retailer system. In addition, if the digital retailer in the dual system is weak, the manufacturer also prefers the dual retailer system. The single retailer system is only preferred if the digital only retailer is powerful while the single retailer is weak.

2.5.2 Retailer Profit, Supply Chain Profit, Consumer Welfare, and Social Welfare

Next, we consider the impact of the possible supply chain structures on several other key agents in this system, notably the retailers, the entire supply chain, consumers, and society as a whole.

Proposition 2.9. *Holding $\bar{\alpha}$ constant in the single and dual retailer systems:*
(i) If $\bar{\alpha} \leq \frac{2c_P}{A}$, both the single and dual retailer systems achieve the centralized system profit, and retailer profit, consumer welfare, and social welfare are identical in the

single retailer and dual retailer systems.

(ii) If $\bar{\alpha} > \frac{2c_P}{A}$, the single retailer system remains identical to the centralized system, but the dual retailer system achieves strictly lower supply chain profit than either. In addition, total retailer profit is lower, and consumer and social welfare are both higher, in the dual retailer system than in the single retailer system.

Proof. (i) Follows immediately from Propositions 2.1, 2.3, and 2.5. (ii) When $\bar{\alpha} > \frac{2c_P}{A}$, it is clear that the dual retailer system must result in lower profit than the centralized system. In any system, total consumer welfare is given by the following expression: $CW = \left(\frac{A-p}{2}\right)(Q_P + Q_D)$. Hence, in the centralized/single retailer systems, $CW^* = \left(\frac{A-\frac{A}{2}}{2}\right)\left(0 + \frac{A}{2}\right) = \frac{A^2}{8}$, while in the dual retailer system, $CW^d = \left(A + \left(\frac{A\bar{\alpha}-2c_P}{3+\bar{\alpha}}\right)\right)^2/8$. Observe this is greater than CW^* since $\bar{\alpha} > \frac{2c_P}{A}$. Lastly, social welfare is defined to be the sum of consumer welfare and total supply chain profit. In the centralized and single retailer systems, this is equal to $SW^* = \frac{A^2}{8} + \frac{A^2}{4} = \frac{3A^2}{8}$. In the dual retailer system, social welfare is

$$SW^d = \frac{\left(A + \left(\frac{A\bar{\alpha}-2c_P}{3+\bar{\alpha}}\right)\right)^2}{8} + \frac{1}{2}\left(\frac{A\bar{\alpha}-2c_P}{3+\bar{\alpha}}\right)^2 + \frac{\bar{\alpha}}{4}\left(\frac{3A+2c_P}{3+\bar{\alpha}}\right)^2$$
$$+ \frac{(3+\bar{\alpha})(3-2\bar{\alpha})A^2 - 4(3-\bar{\alpha})A\bar{\alpha}c_P + 12(1-\bar{\alpha})c_P^2}{4(3+\bar{\alpha})^2}.$$

This reduces to

$$SW^d = \frac{(3+2\bar{\alpha})(9+2\bar{\alpha})A^2 - 4(3+2(3-\bar{\alpha})\bar{\alpha})Ac_P + 4(11-4\bar{\alpha})c_P^2}{8(3+\bar{\alpha})^2}$$

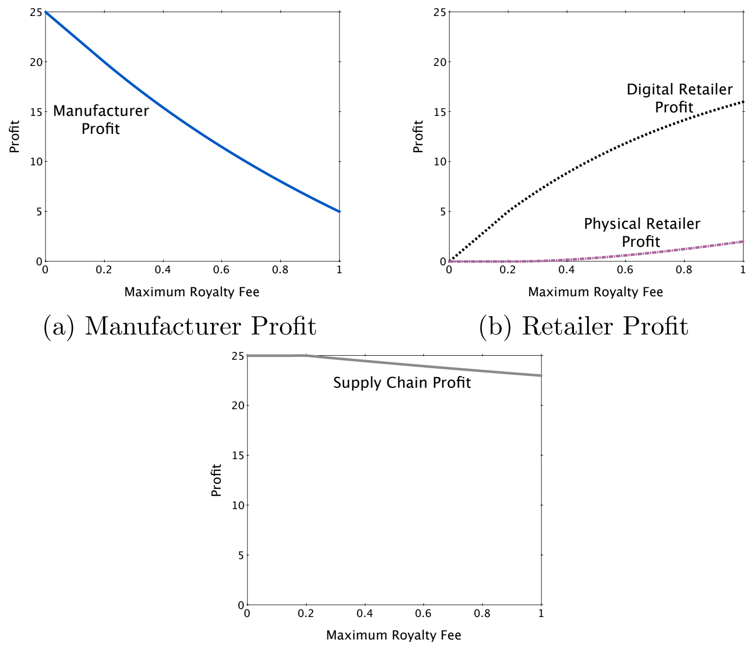
Hence,

$$SW^d - SW^* = \frac{(\bar{\alpha}A - 2c_P)((6+\bar{\alpha})A - 22c_P + 8\bar{\alpha}c_P)}{8(3+\bar{\alpha})^2}$$

This is positive if and only if $\frac{A}{2c_P} > \frac{11-4\bar{\alpha}}{6+\bar{\alpha}}$. Observe that the right hand side is decreasing in $\bar{\alpha}$, and $\bar{\alpha}$ can be any value between $\frac{2c_P}{A}$ and 1. Hence, the maximum value of the right hand side is $\frac{11A-8c_P}{6A+2c_P}$, and the minimum value is 1. The condition holds under the maximum value of the right hand side if $\frac{A}{2c_P} > \frac{11A-8c_P}{6A+2c_P}$, which reduces to $(3A-4c_P)(A-2c_P) > 0$. Recall our condition that $A > 2c_P$. This implies that the $(3A-4c_P)(A-2c_P) > 0$ holds, and hence, social welfare is always higher under the dual retailer system. □

When retailer bargaining power is low (case (i) of the proposition), there is no difference between the systems for any stakeholder in the system—this follows since the dual retailer equilibrium replicates the single retailer equilibrium in this case. When retailers have stronger bargaining power, the supply chain and retailers (in the sense of total retailer profit) are strictly worse off in the dual retailer system. This is precisely the scenario in which the manufacturer prefers the dual retailer system. Because the manufacturer typically would retain the decision rights for the supply chain structure itself, i.e. by determining the channels through which it would sell its products, this suggests that the manufacturer would voluntarily choose to establish a supply chain structure that is suboptimal for the entire supply chain. This preference conflict between the manufacturer and its retailers cannot be alleviated unless the retailers agree to a smaller royalty fee, i.e., a smaller $\bar{\alpha}$.

These results are graphically illustrated in Figure 2.3. In the figure, the single retailer and centralized systems achieve a supply chain profit of 25, and the manufacturer prefers the dual retailer system for all $\bar{\alpha} > 0.2$. As the figure shows, this is precisely when the supply chain profit is less than the centralized optimal of 25. Interestingly, the proposition also shows that consumer and social welfare are both greater in the dual retailer system than in the single retailer system in case (ii) of the proposition. Consumers benefit from lower prices, and the gain for consumers from lower prices outweighs the loss for the supply chain from producing an operationally inefficient good. Thus, despite the fact that the manufacturer chooses a



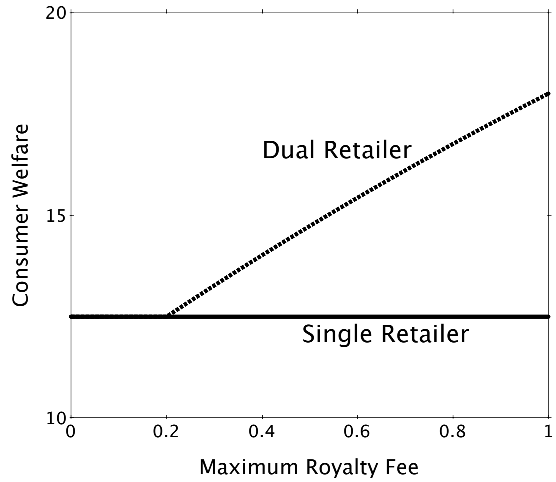
(a) Manufacturer Profit (b) Retailer Profit

(c) Supply Chain Profit

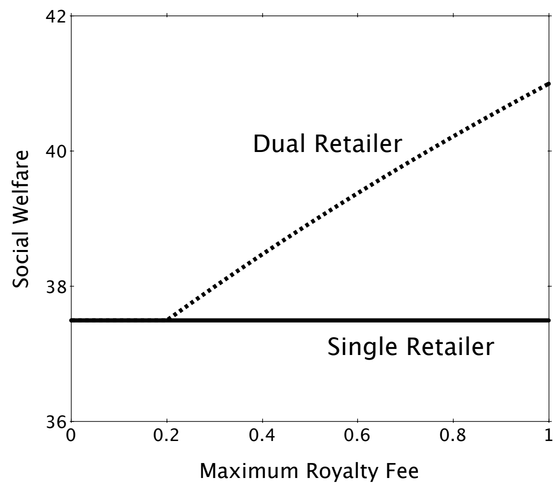
FIGURE 2.3: Supply Chain Efficiency in the Dual Retailer System. In the figure, $A = 10$ and $c_P = 1$.

supply chain structure that is inefficient in terms the total retailer and supply chain profit, it chooses one that is best for itself, consumers, and society as a whole.

The welfare results are graphically illustrated in Figure 2.4. The figure uses the same parameter values as Figure 2.3, and hence manufacturer prefers the dual retailer system for all $\bar{\alpha} > 0.2$. As the figure shows, this is precisely when the consumers and society benefit from the dual retailer system. Thus, provided the manufacturer has the option of determining the supply chain structure (i.e., there is not a single retailer monopolist that distributes both formats), consumers and society are better off when digital retailers are more powerful and can demand a higher royalty fee; this, in turn, causes the manufacturer to divert more production to the physical good, which raises consumer and social welfare.



(a) Consumer Welfare



(b) Social Welfare

FIGURE 2.4: Welfare in the Single and Dual Retailer Systems. In the figure, $A = 10$ and $c_P = 1$.

2.6 Extension: Partial Substitution and Mixed Cournot-Bertrand Duopoly

In our base model, we have modeled the decisions of the manufacturer and retailer as quantity decisions, resulting in a Stackelberg game between the retailer (the first mover in the quantity subgame) and the manufacturer (the second mover). In reality, however, digital goods are not sold based on a predetermined quantity released to the market, but rather based on a price which, in the agency model, is set by the manufacturer. This suggests a mixed Cournot-Bertrand type model in which the first mover (the retailer in our case) chooses a quantity of the physical good, and the second mover (the manufacturer in our case) chooses a price for the digital good, in the spirit of [Singh & Vives, 1984]. In this section, we analyze a variation of our model that incorporates this difference, and discuss how it modifies the key insights of our base model.. Here, we summarize the key insights from this extension; a full analysis is contained in Appendix A. For tractability, throughout this extension, we assume that the production cost of the physical good is zero, i.e., $c_P = 0$.

To begin, we note that a differentiated duopoly model essentially requires that the digital and physical products be partial substitutes, rather than perfect substitutes; the reason for this is that perfect substitution with a price-setting follower (the manufacturer) will result in intense Bertrand-like competition and zero marginal cost pricing, in equilibrium, leading to pathological outcomes. Hence, we also incorporate partial substitution in this extension. Specifically, the degree of substitution between formats is denoted by $\gamma \in (0, 1)$, and the prices of the two products given their respective quantities are

$$P_P(Q_D, Q_P) = (A - Q_P - \gamma Q_D)^+$$
$$P_D(Q_D, Q_P) = (A - \gamma Q_P - Q_D)^+.$$

The decision variables are P_D and Q_P; a chosen P_D will lead to an induced Q_D which, in turn, determines the market price of the physical good, P_P. We now summarize how the this variation affects the main insights of our base model.

First, a key insight gleaned from Proposition 2.1 was that a centralized system strictly prefers the operationally efficient (digital) product to the physical product, and hence at optimality will produce zero physical good. When digital and physical products are no longer perfect substitutes, this is clearly no longer the case, and as shown in Proposition A1 in the appendix, in general the centralized system will sell a positive quantity of both the physical and digital goods. Moreover, for tractability, we have assumed that $c_P = 0$, meaning that the physical good is no longer operationally inferior to the digital good; as a result, the optimal quantities for a centralized firm are equal, i.e., the centralized firm sells an identical quantity of physical and digital goods. If we restored positive physical good production costs, then the firm will lower the physical quantity and raise the digital quantity, and it could be true that zero physical quantity and positive digital quantity are optimal, as in our base model; nevertheless, partial substitution means the centralized firm always has an incentive to produce at least some amount of the physical good (in the absence of significant production cost differences). The stronger the substitution effect, the less incentive the firm will have to produce the physical product, and in the extreme (when $\gamma \to 1$) the firm will sell only the (cheaper) digital product.

Second, in Proposition 2.3, we showed that when selling both digital and physical goods via a single retailer, the decentralized system achieves the centralized optimal allocation of sales between the digital and physical versions. This is no longer true with partial substitution and a price setting, rather than quantity setting, digital good seller, as shown in Proposition A3. This is not surprising, as neither the centralized system nor the single retailer system result in the "boundary case" equilibrium with zero physical product sales, due to the fact that imperfect substitution necessitates some sales of both products in equilibrium. In general, the sales quantities of the digital and physical good are both distorted from the centralized system optimal values in the single retailer system (as are the sales quantities in the dual retailer system, as shown in Proposition A5).

Finally, in Propositions 2.6 and 2.9, we showed that the manufacturer always weakly prefers the dual retailer system, but the retail profit and supply chain profit are always weakly greater in the single retailer system. Propositions A6 and A7 show that this continues to be true in most cases when products are imperfect substitutes and the digital seller is a price setter rather than a quantity setter. In other words, it remains true that the manufacturer prefers selling through dual retailers as it intensifies retail competition and increases the manufacturer's profits, but this happens at the expense of retail and overall supply chain profit.

To summarize, imperfect substitution and price setting, rather than quantity setting, behavior by the seller of the digital good does indeed affect our key results. Most notably, in the absence of perfect substitution, it may be profitable for the manufacturer, the retailers, and the supply chain to sell some quantity of the "inefficient" good, i.e., the physical product; in turn, this impacts the degree to which the single retailer system can replicate the centralized optimal division of sales between the physical and digital goods. Nevertheless, it also remains true in most instances that the single retailer system is closer to the centralized optimal profit and hence would be preferred by the retailers and the overall supply chain, but still the manufacturer prefers the dual retailer system, highlighting an inefficiency that can arise when firms sell substitutable digital and physical information goods through different supply chain structures.

2.7 Conclusion

The technological changes which have made digital information goods available and effectively costless to produce on the margin are changing inventory and supply chain decisions. In this paper, we explored some consequences of this evolution of the distribution model for information goods, and have examined whether a manufacturer who is capable of producing both physical and digital goods should choose to sell those goods through a single retailer or two specialized retailers. From this exploration, we have identified three key insights.

First, when the digital and physical goods are perfect substitutes, a centralized manufacturer would sell only the operationally efficient digital good. Second, a single retailer system replicates the centralized optimal allocation of sales between the physical and digital goods, and hence results in the highest overall supply chain profit. Third, if the retailers are sufficiently strong (i.e., if they are capable of demanding a high royalty fee, $\bar{\alpha}$), the manufacturer typically prefers to sell through two format-specific retailers, rather than through a single retailer that sells both formats. While this reduces total supply chain profits, it is best for the manufacturer, consumers, and society as a whole—in other words, all stakeholders benefit, except for the retailer in a single retailer system. This effect is exacerbated when a single retailer is more powerful than a digital retailer in the dual system. For instance, Amazon charges a higher royalty fee than Apple on most media. This means the advantages of selling via a digital only retailer are even more pronounced than when the royalty fees are equal. Conversely, if the digital retailer in the dual system is more powerful than the multi-format single retailer, the manufacturer's decision is more nuanced, and may involve a preference for the single retailer system in some cases. We have also shown that some (but not all) of these insights extend to the case when digital and physical goods are imperfect substitutes and competition between products resembles a mixed Cournot-Bertrand type model in which the seller of the digital good competes via price while the seller of the physical good competes via quantity.

There are a number of interesting possibilities for extending this work, including the following potential avenues. First, we could incorporate other sources of value for both physical and digital goods in an effort to endogenize partial substitutability of the formats and more precisely model their consequences for consumers. For instance, digital goods may possess some value from their ability to be updated in the future (e.g., books can have content added, and video games can have bugs fixed), updates that may be limited or non-existent for physical formats. On the other hand, as is especially common with software, after a period of time a digital good may no longer be supported because it is considered too old or the firm has gone out of business; hence, its value may decrease relative to a physical good due to uncertainty in the durability or future use of the product. In a different vein, physical goods may offer some intangible value to consumers that digital goods cannot, such as the social capital associated with a physical product, e.g., from lending titles to friends and acquaintances, or potential value from a secondary market for used goods at some point in the future. Future work could incorporate these and other sources of value for each good, to gain a more nuanced view of the manufacturer's choice of supply chain structure.

Second, we may also consider a model of a physical resale (used goods) market model in which consumers strategically choose whether to buy goods on the new market or what for a used good on the resale market. In addition, we could incorporate multiple periods of pricing for the digital or physical goods, and allow consumers to choose their purchase periods; it is possible that the digital and physical products have different consequences for forward-looking consumer purchasing incentives (given their different marginal costs and the rigidity of their inventory levels, or lack thereof) and these differences add a new dimension to the manufacturer's decision of how to distribute these two different formats.

In sum, our analysis illustrates some of the challenges of simultaneously distributing substitutable digital and physical products to consumers. These products typically differ from one another in an operational sense (production leadtimes and marginal costs) and in a retail sense (distribution via simple quantity or wholesale price contracts versus distribution via the agency model). These differences result in complicated tensions between partners in the information good supply chain, and, our results suggest, can lead to inefficiency in the distribution of the product. As digital information goods continue to grow in importance and market share, a careful understanding of these tensions will become increasingly critical for manufacturers and retailers alike to successful manage supply chains for substitutable digital and physical information goods.

3

Revenue Sharing and Subscription Platforms for Digital Content Distribution

3.1 Introduction

With the rise of digital content distribution in the last two decades, sellers and creators of information goods such as books and music have access to a wider range of selling strategies than ever before. Books, for example, have long been sold as a la carte physical goods, with readers typically purchasing each book that they wish to read, a single unit at a time. Recent advances in e-book technology, most notably Amazon's release of the Kindle e-reader device in 2007, have enabled a new selling strategy: authors or publishers—whom we jointly refer to as content creators—may still sell their content as a la carte digital goods, but they may also include their books in a subscription service, such as Amazon's Kindle Unlimited or Rakuten's Kobo. In such a service, subscribers pay a fixed monthly fee in exchange for unlimited reading access to a library of content from multiple content creators who have all chosen to participate by listing their products on the subscription service. This allows subscribers to consume as much content (e.g., read as many books) from

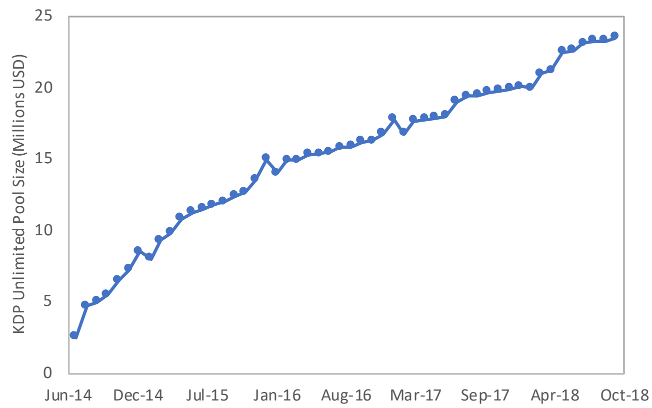


FIGURE 3.1: Growth of the monthly Kindle Direct Publishing (KDP) Unlimited Funding Pool from July 2014 to October 2018. Source: [The Digital Reader, 2018].

the library as they want each month for a flat fee. In addition, by eliminating the per-unit cost of consumption, a subscription model also allows consumers to "try out" content of uncertain or variable quality that they might not have consumed had they been required to pay a la carte for each product, particularly works by lesser known or newer content creators. Aside from books, a similar model can be found in music (e.g., Apple and Amazon both offer music subscription services) and video (e.g., Netflix and Hulu). As shown in Figure 3.1, since launching in July 2014, the Kindle Unlimited funding pool (the amount of money dispersed to authors) for creators publishing directly via Amazon (i.e., not including authors working with a publisher) has steadily increased and now exceeds $20 million per month. With subscription services for digital content, platforms typically collect monthly fees from all active subscribers and keep a fixed percentage of these revenues. The remaining amount, also known as a "funding pool," is distributed amongst content creators who have listed on the service based on a relative metric of how much their product was consumed by users, e.g., the proportion of total page reads that their book accounted for in a month, or proportion of total downloads their song accounted for in a month. This is attractive for content creators because it allows them to reach more consumers, including some who would not have otherwise been revenue generating for these content creators. For some readers, the a la carte price of a book may deter them from purchasing a book by a new, unknown author who they may or may not like: once an a la carte purchase has been made, the consumer may discover that they do not like the book. However, with a subscription service, the marginal cost of trying a new book is zero. Lesser known authors are thus more likely to be read, and if their books turn out to be compatible with the consumer, then the consumer will read a large portion of the book or to completion. As a result, the author with the most enjoyable content—content that generates the most volume on the subscription platform—will receive more revenue. As summarized in recent article from *The Atlantic* [Semuels, 2018], "readers who might not be willing to pay outright for books by unknown writers will read those books on Kindle Unlimited, where they feel free." The platform is interested in this kind of service because it may attract a larger population of customers and authors by promising a high variety of content and a substantial collective funding pool for royalties, respectively. This creates a virtuous cycle: more content creators (and consequentially more books) on the platform makes the flat fee appear more attractive to consumers, and more consumers means a larger funding pool which then further attracts content creators. Over the long term, the platform maximizes its profit by increasing the number of subscribers, since it takes a percentage cut of total subscription fees.

Despite their seeming advantages, subscription services for digital goods also lead to some challenges for platforms and content creators. In many traditional business models that involve bundling or subscription (for example, bundle of different software licenses or a set of cleaning products), the seller chooses both the contents and the price of the bundle. These choices are frequently made because of complementary consumption behaviors or to "even out" unknown individual consumer valuations for each item in the bundle. Subscription platforms for digital goods that we consider in this paper differ in that the platform sets the subscription price and sells directly to consumers, but does not (directly) choose what is in the subscription bundle. Instead, the platform can only offer the option of listing in the subscription service to content creators, and can attempt to influence precisely how content creators choose to distribute their products by setting the revenue sharing parameters for both the a la carte and subscription channels.

Creators, on the other hand, must individually choose how best to sell their own products: on the subscription platform, a la carte, or both. Because some consumers may utilize both the subscription and a la carte channels, for a content creator, while listing on a subscription service can be an additional source of demand, it can also result in cannibalization of a la carte sales. In particular, listing a product on the subscription service may result in a consumer who would have otherwise paid a full a la carte price consuming it via subscription instead. Because of this, many content creators choose not to list on the subscription service; on Amazon's Kindle Unlimited, for instance, 1.4 million books are included in the subscription service, but 3.7 million books are sold a la carte in the Kindle Store [Kowalczyk, 2017]. In addition, on the subscription service, creators must share revenue not just with the platform but also with one another, and the sharing fraction *between* creators is directly proportional to their relative levels of consumption. As a result, the introduction of this new distribution strategy raises several interesting challenges for the creators and platforms involved in the digital content supply chain.

In this paper, we explore precisely these issues as we consider how content creators should sell their work, how platforms should set revenue sharing parameters for a la carte and subscription sales, and what these choices mean for the overall digital content supply chain. We study the following main research questions. First, for content creators, given a set of revenue shares for the a la carte and subscription channels: should a product be sold via a traditional a la carte strategy, a subscription strategy, or a combination of both? Second, for platforms, how should revenue shares of the subscription and a la carte options be set? Does the answer to this question change depending on whether the platform is myopically maximizing immediate expected profits, versus if it seeks to promote the subscription service by inducing content creators to list on the service? And third, for platforms and creators, how should contracts be structured to maximize the efficiency of the digital content supply chain?

To answer these questions, we develop a stylized model of content creators who choose how to sell their products, consumers that choose how to buy them, and a platform that chooses how to price its services and share its revenue with creators. Specifically, we model two heterogeneous content creators selling via a single platform that controls both the a la carte and subscription channels. One content creator is "high quality" and sells a product that yields high consumer value, while the other is "low quality" and sells a product with lower consumer value. The platform in our model first chooses the share of revenue from a la carte and subscription sales to give to the creators. After these revenue shares have been set, content creators choose whether to sell via one channel, both channels, or neither. After distribution channels have been chosen, content creators set the price of any goods they are selling a la carte, while the platform sets the price of the subscription service. Finally, consumers arrive and choose whether to subscribe to the service and whether to consume either of the creators' products. Consumers in our model are one of two types: *a la carte only* consumers and *hybrid* consumers. A la carte only consumers do not consider a subscription and will only purchase content individually. These consumers may represent those with lower consumption quantities or infrequent consumption rates who would not benefit from a subscription model; alternatively, these could be consumers who experience some intrinsic value from "owning" a copy of the product rather than "renting" via a subscription service. Hybrid consumers, on the other hand, are willing to pay for a subscription, but will also purchase products they desire a la carte if they are not available on the subscription service. However, if a product is available on both the subscription and a la carte channels, hybrid consumer will only consume from the subscription channel.

Our main findings are as follows. First, the platform cannot induce only the high type to list on subscription. This means that the average quality of the subscription library cannot exceed the average quality of all content creators who sell through this platform. Second, the profit maximizing choice for the platform is to always induce only the low type to list on the subscription service, or to effectively "shut down" the a la carte channel entirely and induce both creators to sell via subscription. Third, in order to induce the high type to list on subscription while also selling a la carte, the platform has to offer a subscription profit share rate that is substantially higher than the a la carte rate. This rate increases as consumer tastes become more dispersed and rapidly increases for small and moderate differences between the high and low type's quality. Hence, the cost of inducing high quality creators to join the subscription service may be significant, perhaps explaining why many subscription services tend to exclude the most famous and popular content creators.

The remainder of this paper is organized as follows. In §3.2 we review the related literature on bundling and subscription goods, digital distribution, and supply chain coordination through contracts. In §3.3 we introduce the components of our model. In §3.4, we analyze the equilibrium selling strategies for content creators. §3.5 discusses platform decisions given the content creators' selling strategies. §3.6 extends the model to include a "subscription only" consumer segment and analyzes the creators' optimal selling strategies and platform decisions and contrasts it with results in the main model. §3.7 concludes the paper with a discussion of key managerial insights.

3.2 Literature Review

This paper lies at the intersection of three streams of literature: bundling and subscriptions, digital content distribution, and supply chain coordination via contracts.

Bundling and subscriptions. The literature on bundled and subscription (which may thought of as inter-temporally bundled) goods literature discusses many benefits of bundling, including cost savings during production and transaction, complementary value of bundled elements, and improving pricing power by sorting consumers based on their valuations. The last of these is most closely related to our work. [Stigler, 1963] first discusses how bundling two movie rights could increase sellers' profits when consumer valuations for the two goods are negatively correlated. [Adams & Yellen, 1976] show with two goods that commodity bundling under monopoly allows for greater extraction of consumer surplus through comparing unbundled sales to sales under pure bundling and mixed bundling. [Schmalensee, 1984] models reservation prices as a bivariate Gaussian distribution and shows that pure bundling reduces the heterogeneity of the consumer population, allowing sellers to extract more consumer surplus. [Hanson & Martin, 1990] consider how a monopolist facing segmented consumer demand should determine product line breadth (similar to bundling) and pricing using a mixed integer linear program.
[Bakos & Brynjolfsson, 1999] expand this to large bundles of goods and offer a framework for modeling value of bundling. [Bakos & Brynjolfsson, 2000] extend upon that work by including various forms of competition, including competition between two bundlers and competition between a bundler and a single-good seller. [Bakos & Brynjolfsson, 2001] generalize the framework from
[Bakos & Brynjolfsson, 1999] by adding in the cost of digital distribution over a network to compare pricing strategies under aggregation and disaggregation.
[Hitt & Chen, 2005] look at pricing strategies when the bundle is a consumers se-

lected subset of a larger pool of available goods. Our work differs from these in several ways. Unlike these papers, our seller (the platform) does not decide what *content* is in the bundle (the subscription service), because this is typically the decision of the content creator. Moreover, these content creators, who effectively decide the content of the bundle, are in direct competition with one another for the limited pool of subscription fees that they share based on their relative popularity. In addition, in our model, since the platform only offers one subscription plan, there is also no consumer-customized bundling; instead, all items listed on the subscription service are bundled together. Lastly, the platform in our model does not set all of the prices for the products it sells; reflective of actual practice (e.g., book sales on Amazon), the content creator sets the price on the unbundled (a la carte) good and the platform sets the bundled (subscription) price.

Digital distribution. Most of the existing digital distribution (and specifically e-book) literature focuses on a la carte sale of e-books. [Gilbert, 2015] gives an overview of e-books and the underlying conflicts in pricing between publishers and retailers with an in-depth examination of Amazon, but focuses on a la carte pricing. More broadly, much of the existing work on digital goods, such as [Lahiri & Dey, 2013], [Chellappa & Shivendu, 2005], and [Wu & Chen, 2008], focus on how piracy affects firms and how firms can react to piracy. We do not consider piracy in our work, and instead focus on selling digital content through subscription and/or a la carte channels, where both vertical and horizontal differentiation in quality determines pricing decisions. There are a few works related to digital retailing, specifically a common contract called the "agency model" where publishers set prices for e-books and then the electronic retailer takes a fixed percent of sales in exchange for listing the e-book. [Tan *et al.* , 2016] examines the agency model for digital goods and shows how it can mitigate the double marginalization effect and coordinate the digital supply chain with multiple competing retailers. [Tan & Carrillo, 2017], [Johnson, 2013], and [Johnson, 2017] compare the agency model and wholesale contracts for digital goods. Our paper differs in that we are not comparing contracts, but taking as given that agency contracts are used (i.e., the content creator sets the a la carte price of the e-book while the platform takes a fraction of the revenue) and the listing platform sets subscription prices for the library of opt-in e-books. Lastly, [Lei & Swinney, 2018b] analyze supply chain design and coordination for substitutable digital (zero marginal cost) and physical (positive marginal cost) information goods, i.e., e-books and paper books; by contrast, we do not consider physical information goods in the present analysis.

Supply chain coordination via contracts. The current literature on supply chain coordination looks at three types of supply chain structures: bilateral monopolies, one manufacturer selling to multiple retailers, or multiple manufacturers selling through one common retailer. The last is closest to our paper. The marketing literature looks at contracts when multiple manufacturers sell through a common retailer using a wholesale price contract, e.g. [Choi, 1991], [Trivedi, 1998], and [Lee & Staelin, 1997]. However, in our model, to reflect common practice for information goods like e-books, we have a revenue sharing contract for both the subscription (bundled) and a la carte (not bundled) sales, and moreover prices are set by two different parties: the platform sets the subscription price (bundled) and the content creator sets the a la carte prices (not bundled). [Tsay *et al.* , 1999], [Cachon & Lariviere, 2005b], and [Cachon, 2003b] provide surveys of incentive issues and coordination mechanisms under common supply chain contracts. Our work is also related to models of multi-channel retail distribution, including [Balasubramanian, 1998] and [Chiang *et al.* , 2003].

To summarize, our paper is the first, to our knowledge, to consider a competitive model of content creators selling on a common platform that allows creators to choose between a la carte and subscription distribution methods, a choice which, in turn, is determined by revenue sharing contract terms established by the platform.

3.3 Model

Two content creators each have a single product to sell. The product is sold via a platform (such as Amazon's Kindle store or Apple's iTunes store) that offers both a la carte sales and a subscription service. One creator is higher quality than the other: we call the high quality creator "H" and the low quality creator "L." The type of each creator is public information, i.e., it is known to all consumers, the creators, and the platform. For instance, one creator could be an author of low-quality pulp novels, while the other is a internationally recognized bestselling author. The creators do not *directly* compete with one another (e.g., via price competition): consumers have an inherent interest in the products of both creators, and will consume those products independently if they receive non-negative utility from doing so (as described below). However, creators *indirectly* compete with one another if both sell via the subscription platform, as their share of the revenue received depends on their relative quality levels.

Consumer Population. The utility that a consumer receives from the product of a type i creator is $v_i + \epsilon_i$, where $v_H \geq v_L$ are homogeneous between consumers. The "noise term" ϵ_i is a mean zero random variable, and each consumer-creator pair realizes an iid draw from the same distribution to generate ϵ_i. Let F be the distribution function and f be the density of ϵ_i. For ease of exposition, throughout this analysis we assume that $\epsilon_i \sim U[-\ell, \ell]$. This noise term represents the personal preferences and tastes of the individual. Here, ℓ can be thought of as a measure of how "dispersed" consumer tastes are. If ℓ is small, then consumers have very homogenous tastes and they all value the L and H creators' products similarly; if ℓ is large, they have very disparate tastes, and consequently have a wide range of values for the creators' products. We thus refer to ℓ as the "dispersion" of consumer valuations throughout the paper. Note that while the H creator's product does, on average, result in higher utility for consumers than the L creator's product, it's possible for an individual consumer to prefer the L creator's product, depending on the realization of her noise term ϵ_i for each creator. This reflects the fact that, for instance, while consumers may *usually* prefer books by J. K. Rowling to those by John Grisham, it is possible for an individual consumer to prefer a *particular* Grisham novel to a particular Rowling novel.

Consumers have a homogenous outside option value of zero, and prefer to consume the product if their utility net of their consumption cost is weakly greater than zero. We assume there are two populations of consumers: "a la carte"-only consumers who only purchase goods that are sold a la carte, and "hybrid" consumers who consider both the subscription service and a la carte products and, potentially, purchase both if they receive non-negative utility from doing so. The number of a la carte only consumers is N_A, and the number of hybrid consumers is N_H. This division of the consumer population is reflective of the observation that some consumers are receptive to subscription services while some are not; for instance, in a 2015 a survey of US millennials (consumers aged 18-34), 42% purchased a la carte e-books, while only 22% used subscription services [Statista, 2015]; in 2017, 37% of all US consumers were "not at all interested" in an e-book subscription service, and only 14% were "very interested" [Statista, 2017b].

A la carte Consumption. Product i, if offered on the a la carte channel, is sold at price p_i. Consumers who purchase via the a la carte channel must buy the product before they observe their own personal ϵ_i. Because of the nature of the information goods we are considering, they may "sample" before they commit to fully consuming the good; this allows them to realize their value of ϵ_i prior to completing consumption of the product. For instance, they may read the first few chapters of a book, from this deduce their ϵ_i, and then compare their utility to their outside option (zero) and choose to either continue reading or abandon the book. We assume this initial sampling stage is instantaneous and costless to the consumer, and moreover the cost of the product (either a la carte or subscription) is sunk once this decision is made, thus they choose to fully consume the product if

$$v_i + \epsilon_i \geq 0.$$

Because of this, the fraction of users that consume the product is equal to $1-F(-v_i)$. Ex ante (before making a purchase of the a la carte book), the homogeneous population of consumers thus perceives their probability of "liking" a product (enough to fully consume it) to be $1-F(-v_i)$. The utility from consuming the product is $v_i+\epsilon_i$ if the consumer chooses to fully consume it, and zero otherwise, thus the expected utility at the time of purchase is

$$\begin{aligned} u_i &\equiv (1 - F(-v_i))E(v_i + \epsilon_i | v_i + \epsilon_i \geq 0) + F(-v_i)E(0 | v_i + \epsilon_i < 0) \\ &= (1 - F(-v_i))E(v_i + \epsilon_i | v_i + \epsilon_i \geq 0) \\ &= \int_{-v_i}^{\infty} (v_i + x) f(x) dx. \end{aligned}$$

Given the assumption of uniform noise on $[-\ell, \ell]$, $f(x) = \frac{1}{2\ell}$. Thus,

$$\begin{aligned} u_i &= \int_{-v_i}^{\ell} (v_i + x) \frac{1}{2\ell} dx \\ &= \frac{(\ell + v_i)^2}{4\ell} \end{aligned}$$

Because $v_H \geq v_L$ and the ϵ_i noise term is iid, it follows that $u_H \geq u_L$. Consumers are willing to purchase the a la carte option as long as their expected surplus is non-negative, i.e.,

$$u_i - p_i \geq 0.$$

We assume that the content creator sets the price of a la carte product (p_i), and it is clearly optimal to set $p_i = u_i$.

Subscription Consumption. Consumers who purchase via the subscription channel do so in a two step, sequential process. First, they pay the fee to subscribe. After paying the fee, they may begin to consume the products offered via subscription. Let C denote the set of creators who sell via the subscription service, i.e., $C \in \{\emptyset, L, H, L+H\}$. The price of the subscription service is p_S. Given this, expected (ex ante, at the time of signing up for the subscription service) net utility from the subscription service is

$$\sum_{i \in C} u_i - p_S.$$

If a product is sold both a la carte and via the subscription service, we assume that a hybrid consumer who has paid to use the subscription service will consume it via subscription. Consumers compare their utility from subscribing to the service versus their utility from not subscribing (which, in turn, depends on the selling strategies of the creators, e.g., whether they sell a la carte or not), and they choose to subscribe if doing so increases their utility. We assume that the platform sets the price of the subscription service (p_S) after content creators have chosen their selling strategies to maximize its own profit.

Creator Choice of Selling Strategy. The creators choose their selling strategy—i.e., which distribution channels they wish to use—simultaneously after the platform has chosen its revenue sharing parameters. Consistent with prevalent practice, we assume that there is no fixed or variable cost associated with either selling strategy (e.g., there is no per unit fee charged by the platform for each a la carte sale, and there is no "listing fee" for either distribution channel). If a creator is indifferent between using and not using a particular distribution channel, we assume the creator uses the channel in question.

Contracts and Revenue Sharing. The platform is assumed to receive a fixed fraction of the a la carte revenue, $1 - \phi_A$, leaving the content creator the remaining fraction (ϕ_A). Subscription revenue is determined as follows. First, the platform collects revenue from all consumers who pay for the service. Next, the platform keeps a fraction of this revenue, $1 - \phi_S$. Finally, the remaining fraction of revenue, ϕ_S, is divided between the content creators based on the fraction of realized "complete consumptions." For example, if the L creator realizes 5 complete reads, and the H creator realizes 10 complete reads, then the L creator receives 1/3 of the revenue while the H creator receives 2/3. Further details of the revenue split are discussed below. The platform chooses the revenue share parameters at the start of the game to maximize its own profit, possibly subject to some reservation level of revenue sharing required by the creators to participate (more on this point will be discussed in §3.5).

Sequence of Events. The sequence of events is as follows:

1. The platform determines the revenue shares, ϕ_A and ϕ_S.
2. Creators simultaneously determine whether to sell a la carte, subscription, or both.
3. Creators simultaneously determine their a la carte prices, p_i, and the platform determines its subscription price, p_S.
4. Hybrid consumers determine whether to sign up for the subscription service.
5. All consumers determine whether (and how) to consume the products of the L and H creators.

We let Π_P be the profit of the platform, and Π_i be the profit of the type i creator. We refer to step 1 as the Contracting Stage, and steps 2-5 as the Selling Stage. We focus our initial analysis on the Selling Stage before considering the Contracting Stage.

3.4 The Selling Stage

In this section, we analyze the game beginning with step 2 in the sequence of events. At this point in the game, the revenue shares have been determined, and the creators must choose how to distribute their content. Each creator has three possible selling strategies: a la carte only (AO), subscription only (SO), and both (SA). Thus, in total, between the two creators there are nine possible combinations of selling strategies that may occur. We thus analyze the game by first enumerating each of these possible outcomes and deriving creator and platform profits that result, and then determine the equilibrium choice of selling strategies by creators on the platform. We let (X, Y) refer to the outcome in which the L creator uses selling strategy X and the H creator uses selling strategy Y.

3.4.1 Possible Selling Strategies

We begin with (AO, AO). In this case, no products are sold via subscription. The optimal a la carte price is $p_i = u_i$, and both creators earn this revenue from all hybrid and a la carte only customers, yielding total profits:

$$\Pi_P = (1 - \phi_A)(N_H + N_A)(u_L + u_H)$$

$$\Pi_L = \phi_A(N_H + N_A)u_L$$

$$\Pi_H = \phi_A(N_H + N_A)u_H$$

Next, we move to (AO, SO). In this case, the L product is only sold via a la carte and the H product is only sold via subscription. The optimal price of the L product is $p_L = u_L$, and the L creator earns this revenue from all hybrid and a la carte only customers. The only subscription product is the H product, so (ex ante) net consumer utility from the subscription is $u_H - p_S$. The optimal subscription price set by the platform is thus $p_S = u_H$. Total profits are:

$$\Pi_P = (1-\phi_A)(N_H+N_A)u_L + (1-\phi_S)N_H u_H$$

$$\Pi_L = \phi_A(N_H+N_A)u_L$$

$$\Pi_H = \phi_S N_H u_H$$

Note that by selling only via the subscription platform, the H creator has lost revenue from the N_A a la carte only customers.

In (AO, SA), the L product is only sold via a la carte and the H product is sold via both channels. The optimal price of the L product is $p_L = u_L$, and the L creator earns this revenue from all hybrid and a la carte only customers. The optimal a la carte price of the H product is $p_H = u_H$, and the H creator earns this revenue from all a la carte only customers. The only subscription product is the H product, so net consumer utility from the subscription is $u_H - p_S$. Hybrid customers could forgo the subscription and instead purchase the a la carte version of the product; however, this would give them zero utility. Hence, the optimal subscription price set by the platform is $p_S = u_H$, which makes hybrid customers indifferent between consumption channels and causes them to sign up for the subscription and consume the product via that channel (per our earlier assumption). Total profits are:

$$\Pi_P = (1-\phi_A)(N_H+N_A)u_L + (1-\phi_A)N_A u_H + (1-\phi_S)N_H u_H$$

$$\Pi_L = \phi_A(N_H+N_A)u_L$$

$$\Pi_H = \phi_S N_H u_H + \phi_A N_A u_H$$

Note that for H creator, this clearly dominates (AO, SO), as it provides access to an additional pool of consumers (the N_A a la carte only consumers). Moving next to (SO, AO), this case is the opposite of (AO, SO), and so by symmetry:

$$\Pi_P = (1-\phi_A)(N_H+N_A)u_H + (1-\phi_S)N_H u_L$$

$$\Pi_L = \phi_S N_H u_L$$

$$\Pi_H = \phi_A(N_H+N_A)u_H$$

In (SO, SO), both creators sell only via the subscription service. The optimal price is thus $p_S = u_L + u_H$. After the platform takes its cut of $1 - \phi_S$, the remaining revenues for the creators are $\phi_S N_H(u_L + u_H)$. The total number of customers that read the L product is equal to $(1 - F(-v_L))N_H$ while $(1 - F(-v_H))N_H$ read the H product. Thus, the L creator is entitled to the following fraction of the shared revenue:

$$\alpha \equiv \frac{(1 - F(-v_L))}{(1 - F(-v_L)) + (1 - F(-v_H))} \tag{3.1}$$

while the H creator is entitled to a complementary fraction. Total profits are thus:

$$\Pi_P = (1 - \phi_S)N_H(u_L + u_H)$$
$$\Pi_L = \alpha\phi_S N_H(u_L + u_H)$$
$$\Pi_H = (1 - \alpha)\phi_S N_H(u_L + u_H)$$

Moving to (SO, SA), in this case both creators sell via the subscription service, but the H type sells via a la carte as well. Revenues from the subscription service follow as in the previous case, but the H-type enjoys the addition of a la carte revenues from N_A consumers. Total profits are thus:

$$\Pi_P = (1 - \phi_A)N_A u_H + (1 - \phi_S)N_H(u_L + u_H)$$
$$\Pi_L = \alpha\phi_S N_H(u_L + u_H)$$
$$\Pi_H = (1 - \alpha)\phi_S N_H(u_L + u_H) + \phi_A N_A u_H$$

Note that for H creator, this clearly dominates (SO, SO), as (once again) it provides access to the a la carte only population of customers. (SA, AO) is the opposite of (AO, SA), and so by symmetry:

$$\Pi_P = (1 - \phi_A)(N_H + N_A)u_H + (1 - \phi_A)N_A u_L + (1 - \phi_S)N_H u_L$$
$$\Pi_L = \phi_S N_H u_L + \phi_A N_A u_L$$
$$\Pi_H = \phi_A(N_H + N_A)u_H$$

Note that for L creator, this clearly dominates (SO, AO), for similar reasons as in the previously dominated cases. On the other hand, (SA, SO). is the opposite of (SO, SA), and so by symmetry:

$$\Pi_P = (1 - \phi_A)N_A u_L + (1 - \phi_S)N_H(u_L + u_H)$$
$$\Pi_L = \alpha\phi_S N_H(u_L + u_H) + \phi_A N_A u_L$$
$$\Pi_H = (1 - \alpha)\phi_S N_H(u_L + u_H)$$

For L creator, this clearly dominates (SO, SO). Finally, in (SA, SA), both creators sell via both channels. Total profits are thus:

$$\Pi_P = (1 - \phi_A)N_A(u_H + u_L) + (1 - \phi_S)N_H(u_L + u_H)$$
$$\Pi_L = \alpha\phi_S N_H(u_L + u_H) + \phi_A N_A u_L$$
$$\Pi_H = (1 - \alpha)\phi_S N_H(u_L + u_H) + \phi_A N_A u_H$$

Note that for H creator, this clearly dominates (SA, SO), while for the L creator, it dominates (SO, SA).

3.4.2 Equilibrium Analysis of the Selling Stage

Combining and summarizing the insights discussed above, we our first result:

Proposition 3.1. *For both creator types, strategy SA dominates strategy SO, regardless of the other creator's strategy.*

Proof. From the enumeration in §3.4.1, we see that for both creator types, profit is always higher under strategy SA compared to under strategy SO regardless of the other creator's strategy. □

When a content creator sells only via subscription versus selling on both subscription and a la carte, she is missing out on selling to a la carte only consumers. If she also lists on a la carte in addition to subscription and no one purchases the

Table 3.1: Payoffs for the H creator in the selling stage game.

		H Creator	
		AO	SA
L creator	AO	$\phi_A(N_H + N_A)u_H$	$\phi_S N_H u_H + \phi_A N_A u_H$
	SA	$\phi_A(N_H + N_A)u_H$	$(1-\alpha)\phi_S N_H(u_L + u_H) + \phi_A N_A u_H$

book, then she is no worse off than if she sold only on subscription. If even one a la carte only consumer purchases, then she realizes strictly greater profit; as a result, selling strategy SA (both channels) dominates SO (subscription only). However, this contrasts with a comparison between selling a la carte only versus selling both a la carte and on subscription. When the content creator sells only a la carte, she is capturing a mix of a la carte only consumers and hybrid consumers. When she adds the option of subscription, then hybrid consumers will switch to reading on subscription. Depending on the profit sharing parameter and expected profit share based on reads from subscription fees, the content creator could be better or worse off as a result. Thus, while it is always preferred for the content creator to list both on subscription and a la carte versus subscription only, the same is not always true for listing on both channels versus selling a la carte only.

Because of Proposition 3.1, we may restrict our analysis to only the AO and SA strategies, and the selling strategy choice reduces to a 2x2 normal form game in which each creator chooses either a la carte only, or subscription and a la carte. To analyze this game, we begin by considering the incentives of the H creator, whose payoffs from the selling stage game are given in Table 3.1. Conditional on the L creator playing AO, the H creator plays SA if and only if

$$\phi_S N_H u_H + \phi_A N_A u_H \geq \phi_A(N_H + N_A)u_H,$$

which reduces to $\phi_S \geq \phi_A$. Otherwise, the H creator plays AO. Conditional on the

L creator playing SA, the H creator plays SA if and only if:

$$(1-\alpha)\phi_S N_H(u_L+u_H)+\phi_A N_A u_H \geq \phi_A(N_H+N_A)u_H,$$

which reduces to

$$(1-\alpha)\left(\frac{u_L+u_H}{u_H}\right) \geq \frac{\phi_A}{\phi_S}. \tag{3.2}$$

Consider the left hand side of this inequality. An alternate way of writing this expression is:

$$(1-\alpha)\left(\frac{u_L+u_H}{u_H}\right) = \frac{\frac{(1-F(-v_H))}{(1-F(-v_H))+(1-F(-v_L))}}{\frac{u_H}{u_L+u_H}},$$

i.e., it is the ratio between the H creator's fraction of *total consumption quantity* on the subscription platform and the H creator's contribution to the fraction of *total consumer utility* from a subscription. Note that, given our distributional assumption of uniform noise,

$$\frac{(1-F(-v_H))}{(1-F(-v_H))+(1-F(-v_L))} = \frac{\ell+v_H}{\ell+v_L+\ell+v_H}$$

and

$$\frac{u_H}{u_L+u_H} = \frac{(\ell+v_H)^2}{(\ell+v_L)^2+(\ell+v_H)^2}.$$

In turn, this yields

$$(1-\alpha)\left(\frac{u_L+u_H}{u_H}\right) = \frac{(\ell+v_L)^2+(\ell+v_H)^2}{(\ell+v_H)(\ell+v_L)+(\ell+v_H)^2} \leq 1. \tag{3.3}$$

In other words, the H creator always contributes less to the *total consumption quantity* that it does to *total consumer utility* on the subscription service. Given that (3.2) must hold in order for the H creator to play SA conditional on the L creator playing SA, and the left hand side (3.2) is weakly less than 1 per (3.3), it follows

Table 3.2: Payoffs for L creator in the selling stage game.

		H Creator	
		AO	SA
L creator	AO	$\phi_A(N_H + N_A)u_L$	$\phi_A(N_H + N_A)u_L$
	SA	$\phi_S N_H u_L + \phi_A N_A u_L$	$\alpha\phi_S N_H(u_L + u_H) + \phi_A N_A u_L$

that $\phi_A < \phi_S$ is necessary (but not sufficient) for (SA, SA) to be an equilibrium. Put another way, the H creator is not compensated enough for her contribution to overall consumer utility (and hence her contribution to subscription revenue) by allocating her share of the subscription revenue based on her fraction of consumption; this, in turn, implies that in order for the H creator to be induced to sell via the subscription service, more favorable terms (in the sense of a higher overall revenue share, ϕ_S) are required.

Next, we analyze the best response of the L creator. Payoffs for the L creator in the selling stage game are given in Table 3.2. Conditional on the H creator playing AO, the L creator plays SA if and only if

$$\phi_S N_H u_L + \phi_A N_A u_L > \phi_A(N_H + N_A)u_L,$$

which reduces to $\phi_S \geq \phi_A$. Otherwise, when $\phi_S < \phi_A$, conditional on the H creator playing AO, the L creator also plays AO. Conditional on the H creator playing SA, the L creator plays SA if and only if

$$\alpha\phi_S N_H(u_L + u_H) + \phi_A N_A u_L \geq \phi_A(N_H + N_A)u_L,$$

which reduces to

$$\alpha \cdot \left(\frac{u_L + u_H}{u_L}\right) \geq \frac{\phi_A}{\phi_S}.$$

Similar to the H creator's payoffs analyzed above, the left hand side of this expression is ratio between the L creator's fraction of *total consumption quantity* on the subscription platform and the L creator's contribution to the fraction of *total consumer* *utility* from a subscription. Given our distributional assumptions,

$$\alpha \cdot \left(\frac{u_L + u_H}{u_L}\right) = \frac{(\ell + v_L)^2 + (\ell + v_H)^2}{(\ell + v_H)(\ell + v_L) + (\ell + v_L)^2} > 1,$$

i.e., the L creator always contributes *more* to consumption quantity than she does to total utility generated on the subscription service by her presence. Hence, the L creator receive a disproportionately high share of the revenue split on the subscription service, while (as discussed above) the H creator receives a disproportionately low share of the revenue split. This fact leads to the following result:

Proposition 3.2. *Define* $\beta \equiv \left((1-\alpha)\frac{u_L+u_H}{u_H}\right)^{-1} = \frac{((l+v_L)(l+v_H)+(l+v_H)^2)}{((l+v_L)^2+(l+v_H)^2)} > 1$. *Then, the equilibrium to the selling stage exists and is unique, and is characterized as follows:*

(i) If $\phi_S \geq \beta\phi_A$*, both creators sell via both distribution channels.*

(ii) If $\beta\phi_A > \phi_S \geq \phi_A$*, the low quality creator sells via both channels, while the high quality creator sells only via the a la carte channel.*

(iii) If $\phi_S < \phi_A$*, both creators sell only via the a la carte channel.*

Proof. (i) Suppose $\phi_S \geq \phi_A$ and $(1-\alpha)\frac{u_L+u_H}{u_H} \geq \frac{\phi_A}{\phi_S}$. Per the discussion in §3.4.2, the dominant action of the H creator is SA. If the H creator plays SA, the L creator responds with SA if and only if $\alpha\frac{u_L+u_H}{u_L} \geq \frac{\phi_A}{\phi_S}$. Since $\alpha\frac{u_L+u_H}{u_L} > 1 > (1-\alpha)\frac{u_L+u_H}{u_H} \geq \frac{\phi_A}{\phi_S}$, this is always true. Hence, in this case, (SA, SA) is the unique equilibrium.

(ii) Suppose $\phi_S \geq \phi_A$ and $(1-\alpha)\frac{u_L+u_H}{u_H} < \frac{\phi_A}{\phi_S}$. From §3.4.2, the L creator will play SA if the H creator plays AO. Conditional on the L creator playing SA, AO is optimal for the H creator if $(1-\alpha)\frac{u_L+u_H}{u_H} < \frac{\phi_A}{\phi_S}$; hence, (SA, AO) is an equilibrium in these conditions. The only other possible equilibrium is (AO, SA). For this to hold, it must be true that AO for the L creator is a best response to SA from the H creator. Per §3.4.2, this is true if and only if $\alpha\frac{u_L+u_H}{u_L} < \frac{\phi_A}{\phi_S}$. However, since $\alpha\frac{u_L+u_H}{u_L} > 1$ and

$\frac{\phi_A}{\phi_S} < 1$ if $\phi_S \geq \phi_A$, this cannot hold. Hence, (AO, SA) cannot be an equilibrium, and (SA, AO) is the unique equilibrium in this region.

(iii) Suppose $\phi_S < \phi_A$. Then, (AO, AO) is clearly an equilibrium, per the discussion in §3.4.2. From equation (3.2), if the L creator plays SA, the H creator will play AO. However, if the H creator plays AO and $\phi_S < \phi_A$, the L creator's best response is also AO. Hence, (AO, AO) is the unique equilibrium when $\phi_S < \phi_A$.

The remainder of the proposition follows by defining $\beta \equiv \left((1-\alpha)\frac{u_L+u_H}{u_H}\right)^{-1} = \frac{((l+v_L)(l+v_H)+(l+v_H)^2)}{((l+v_L)^2+(l+v_H)^2)} > 1$ and reframing each of the above regions. □

This proposition has several important implications. From part (iii), we see that when the subscription revenue share rate (the portion of subscription revenue given to content creators) is lower than that of a la carte sales, no content creator lists their work on subscription. The reason for this is that the entire customer population is receptive to a la carte sales; hence, creators may capture the entire market with a la carte, and if they cannot achieve a higher revenue share than a la carte sales with subscription, there is no reason for them to use the subscription service. Thus, if a content creator decides not to sell via subscription but only sell a la carte, it is not necessarily true that the other content creator chooses to list on subscription, despite the fact that she could capture the whole subscription reader base and associated profit, i.e., by yielding 100% of consumption on the subscription service. If the a la carte revenue share is higher than the subscription share, then both creators will make more per unit book consumption on a la carte than they would by capturing the entire subscriber base. Thus, by selling a la carte only and not listing on subscription, content creators can realize greater profit because they can charge a higher per unit a la carte price (indicative of their type) and/or they can retain a larger portion of a la carte sales revenue. We note that if a portion of customer demand came from "subscription only" customers, this result may no longer hold; however, our assumption of “a la carte” only customers and “hybrid” customers was meant to reflect the observation that, typically, customers that use a subscription service are also receptive to the idea of purchasing a la carte books for authors they particularly like, but the opposite is usually not true. In other words, as long as *all* consumers of e-books consider a la carte purchases and only a *subset* considers subscription, the platform cannot induce either creator to list on the subscription service if it offers a lower total revenue share than a la carte sales.

This means that to get creators to list on subscription (and, in turn, to get consumers to pay for the service), the platform must offer $\phi_S \geq \phi_A$. If the platform sets $\beta\phi_A > \phi_S \geq \phi_A$, it encourages the L creator to list on subscription; if it sets $\phi_S \geq \beta\phi_A$, both creators list on subscription. Here, $\beta > 1$ represents the reciprocal of the earlier term discussed following equation (3.2): β is the ratio between the H creator's fraction of total consumer utility and her fraction of total consumption quantity. Because she contributes more to consumer utility than she does to consumption quantity, this value is always more than one. Numerically, we observe that this value falls somewhere between 1 and slightly over 1.2. This means that if the platform wants to induce both creator types to participate in the subscription service, it will have to, depending on the relative qualities of the L and H type and the level of taste disparity among consumers, share up to 20% more revenue with creators; see Figure 3.2. In other words, while it is “easy” for the platform to induce the L creator to list on the subscription service (it need only match the revenue share of the a la carte channel, i.e., it can set $\phi_S = \phi_A$), it requires a more substantial concession to induce the H creator to sell via subscription. This additional “boost” in revenue share required to induce the high quality creator to use the subscription channel can be as much as 20%, and derives from the fact that the platform must compensate the H creator for her “unfair” share of revenue when using fraction of consumption to divide revenues between creators.

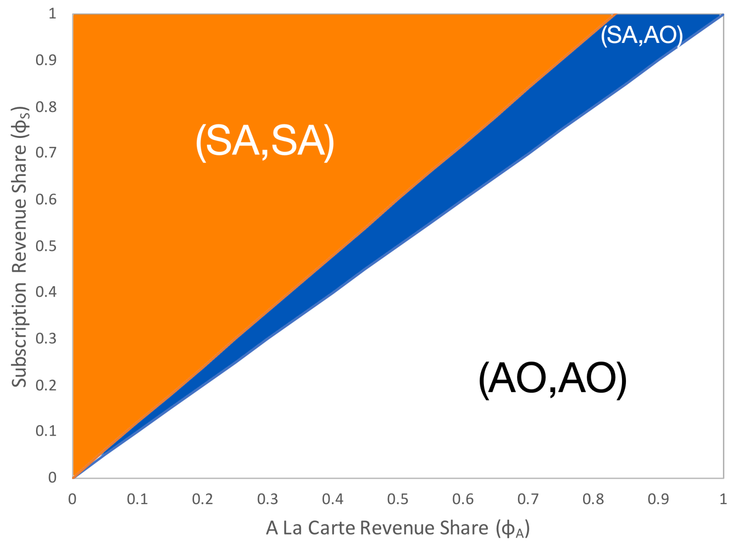


FIGURE 3.2: Selling Stage Equilibria for $l = 2, v_L = 5, v_H = 12\,(\beta = 1.2)$

This illustrates that inducing high quality content to join the subscription service can be quite costly for the platform; as a result, platforms may be unwilling to concede as much to high quality creators, and the subscription service may be filled with mostly lower quality content. The proposition further implies that, while the platform can induce (AO, AO), (SA, AO), and (SA, SA), it cannot induce (AO, SA), i.e., it cannot induce the low quality creator to sell only a la carte while the high quality creator sells via subscription. As a result, the average quality of books on the subscription service cannot exceed the average quality of all books available through any form of selling strategy on the platform, and often quality on the subscription service is lower than on the entire platform. This is consistent with the observation that bestsellers and popular authors rarely appear on the Kindle Unlimited service [Kowalczyk, 2017]; because these authors would be contributing an outsized portion of consumer utility to the service relative to the total fraction of page reads they incur, they would not be appropriately compensated, resulting in their choice to sell only a la carte e-books.

3.5 The Contracting Stage

3.5.1 The Platform's Optimal Revenue Shares

In the contracting stage, the platform seeks to maximize its profits by appropriately setting the revenue shares ϕ_S and ϕ_A for its two services. We assume that the platform must offer some minimum reservation revenue share to induce creator participation in both subscription and a la carte (conditional on the subsequent selling stage equilibrium from Proposition 3.2 resulting in at least one creator using the subscription channel), i.e. the platform must set $\phi_S > \underline{\phi_S}$ and $\phi_A \geq \underline{\phi_A}$. However, we recognize that there is more competition in the a la carte e-book business and thus the reservation revenue share for a la carte is higher than that for subscription, i.e. $\underline{\phi_A} > \underline{\phi_S}$. Amazon, for example, dominates subscription services for e-books in the US, but faces a much more competitive market for a la carte e-book sales (in which competes not only with other a la carte e-book sellers, but also sellers of physical books; if e-books become too unprofitable, then content creators will abandon e-books for physical books).

Consequently, in the contracting stage, the platform may induce an equilibrium described by Proposition 3.2 if it sets $\phi_A \geq \underline{\phi_A}$ and $\phi_S \geq \underline{\phi_S}$. If the platform sets $\phi_A < \underline{\phi_A}$, then it effectively "shuts down" its a la carte channel, as no creator will participate. However, it may continue to induce participation in the subscription service for any $\phi_S \geq \underline{\phi_S}$; clearly, in this case, the optimal revenue share given to creators will be zero, allowing the platform to extract all revenue.[1] The platform thus broadly has two options: operate only a subscription channel and extract all

[1] Note that (SO, SO) is not a viable equilibrium per Proposition 3.1 when $\phi_A = 0$. However, Proposition 3.1 assumes creator participation for any ϕ_A. If creators do not consider the a la carte channel for $\phi_A < \underline{\phi_A}$, then in this region, the only decision for creators is whether to join the subscription service, which we assume they will do for any $\phi_S \geq 0$. Consequently, by setting $\phi_S = 0$ and $\phi_A < \underline{\phi_A}$, the platform may induce both creators to list on the subscription service and given up all revenue, while inducing neither to sell a la carte. The same would be true for a minimum subscription revenue share $\underline{\phi_S} > 0$, provided $\underline{\phi_S} < \underline{\phi_A}$; for ease of exposition, we assume $\underline{\phi_S} = 0$.

revenue from content creators, or operate both channels by setting $\phi_A \geq \underline{\phi_A} > \underline{\phi_S}$ and $\phi_S \geq \underline{\phi_S}$. We assume that if the platform is indifferent between operating a channel and not operating a channel, that it chooses to operate the channel in question.

In the former case (operating only a subscription platform), platform revenue is $\Pi_P = (1 - \underline{\phi_S})N_H(u_L + u_H)$. In the latter case (inducing an equilibrium per Proposition 3.2), platform revenue is given by the expressions in §3.4.1. By comparing these expressions, we may derive the following result:

Proposition 3.3. *When the platform seeks to maximize profit,*
(i) If $\frac{N_H}{N_H+N_A} \geq \frac{\underline{\phi_A}-\underline{\phi_S}}{1-\underline{\phi_S}}$*, it sets* $\phi_A = \underline{\phi_A}$ *and* $\phi_S = \underline{\phi_A}$*, resulting in the* (SA, AO) *equilibrium.*
(ii) Otherwise, it sets $\phi_A = \phi_S = 0$ *and operates only the subscription channel.*

Proof. As noted, $\Pi_P = (1 - \underline{\phi_S})N_H(u_L + u_H)$ when inducing only subscription distribution. When inducing a la carte distribution (setting $\phi_A \geq \underline{\phi_A} > \underline{\phi_S}$), platform profit is as follows:

$$\Pi_P = \begin{cases} (1-\phi_A)(N_H+N_A)(u_L+u_H) & \phi_A > \phi_S \\ (1-\phi_A)(N_H+N_A)u_H + (1-\phi_A)N_A u_L + (1-\phi_S)N_H u_L & \beta\phi_A > \phi_S \geq \phi_A \\ (1-\phi_A)N_A(u_H+u_L) + (1-\phi_S)N_H(u_L+u_H) & \phi_S \geq \beta\phi_A \end{cases}$$

It is clear that the optimal $\phi_A = \underline{\phi_A}$, and moreover the optimal ϕ_S is any $\phi_S \in [\underline{\phi_S}, \underline{\phi_A}]$. Thus, optimal profits when inducing a la carte sales are $\Pi_P = (1-\underline{\phi_A})(N_H + N_A)(u_L + u_H)$, and this includes outcomes when only a la carte sales are induced ($\phi_S < \underline{\phi_A}$) and outcomes when (SA, AO) is induced ($\phi_S = \underline{\phi_A}$). Because we assume that the platform chooses to operate a channel when it is indifferent about doing so, this means that $\phi_S = \underline{\phi_A}$ is optimal conditional on $\phi_A \geq \underline{\phi_A}$. Comparing profits, we see that inducing (SA, AO) is preferred to inducing subscription only sales if $\frac{N_A}{N_H+N_A} \geq \frac{\underline{\phi_A}-\underline{\phi_S}}{1-\underline{\phi_S}}$. □

As the proposition shows, to maximize profit, the platform always sets the a la carte and subscription revenue shares to be equal. It either induces the low quality creator to list on the subscription service while inducing both to sell a la carte (by setting $\phi_A = \phi_S = \underline{\phi_A}$) or induces both creators to sell only via subscription while shutting down the a la carte channel ($\phi_A = \phi_S = 0$). The choice between these two strategies does not depend on consumer utilities: it is only a function of the relative sizes of the a la carte only and hybrid consumer populations N_A and N_H, and the minimum a la carte revenue share $\underline{\phi_A}$. If there are many a la carte only consumers, the platform induces (SA, AO); if not, it induces only subscription sales.[2] To give some (rough) numerical intuition for this result, recall that among US consumers, 37% are "a la carte" only purchasers, while 14% are "very interested" in subscription services [Statista, 2017b]; this suggests that a reasonable estimate of this ratio is $\frac{N_A}{N_H+N_A} \approx 72\%$. On the other hand, Amazon offers a la carte revenue shares for e-books of either 35% or 70% depending on certain criteria. Hence, it is conceivable that for e-books, case (i) of the proposition holds. The proposition also illustrates that the platform never induces both types to list on a la carte and subscription simultaneously. Doing so would require the platform to give up an even higher portion of profits than what it offers the content creators for a la carte sales ($\underline{\phi_A}$), i.e., it would have to set $\phi_S = \beta\underline{\phi_A}$ (where $\beta > 1$). This is never profitable for the platform, since the market for the H creator's content is already fully covered at a smaller revenue share via a la carte sales.

[2] Technically, in case (i) of the proposition, the platform is indifferent between operating the subscription channel and not operating it. In other words, the platform achieves the same profit if $\phi_A = \underline{\phi_A}$ and $\phi_S = 0$. However, in practice, there are many reasons why the platform may seek to operate the subscription channel in the long run that are not included in our model, including the before mentioned "virtuous cycle" from attracting more consumers and more creators to list on the service. In addition, there could be some "subscription only" customers in the population, which we have not included in our model for analytical tractability; the presence of such consumers would cause the platform to strictly prefer (SA, AO) to (AO, AO). Hence, we assume, as noted above, that if the platform is indifferent between operating a channel and not, it chooses to operate the channel; this means that (SA, AO) is the optimal equilibrium for the platform to induce.

Nevertheless, in an effort to grow their subscription service (and feed the "virtuous cycle" described earlier), it is possible that platforms may not myopically seek to maximize immediate profits, but may instead seek to induce high quality content creators to sell via subscription while also maintaining a la carte sales, i.e., induce an (SA, SA) equilibrium. Thus, it is also interesting to consider precisely how much revenue share and profit the platform must give up in order to achieve this. The following proposition analyzes this question:

Proposition 3.4. *Suppose* $\frac{N_A}{N_H+N_A} \geq \underline{\phi_A}$ *and the platform seeks to induce* (SA, SA) *to enhance the quality of the subscription offerings. Then it must offer* $\phi_A = \underline{\phi_A}$ *and* $\phi_S = \beta\underline{\phi_A}$*, yielding a profit of*

$$\Pi_P^{(SA,SA)} = \left[(1-\underline{\phi_A})N_A + (1-\beta\underline{\phi_A})N_H\right](u_L + u_H).$$

Compared to the optimal strategy in this region of inducing (SA, AO)*, this results in a profit loss of*

$$\Pi_P^{(SA,AO)} - \Pi_P^{(SA,SA)} = \frac{(l+v_L)(v_H - v_L)}{2\ell}\underline{\phi_A}N_H \geq 0.$$

Proof. The first part of the proposition follows from the proof of Proposition 3.3. Noting that

$$\Pi_P^{(SA,SA)} = \left[(1-\underline{\phi_A})N_A + (1-\beta\underline{\phi_A})N_H\right](u_L + u_H)$$

and

$$\Pi_P^{(SA,AO)} = (1-\underline{\phi_A})(N_H + N_A)(u_L + u_H),$$

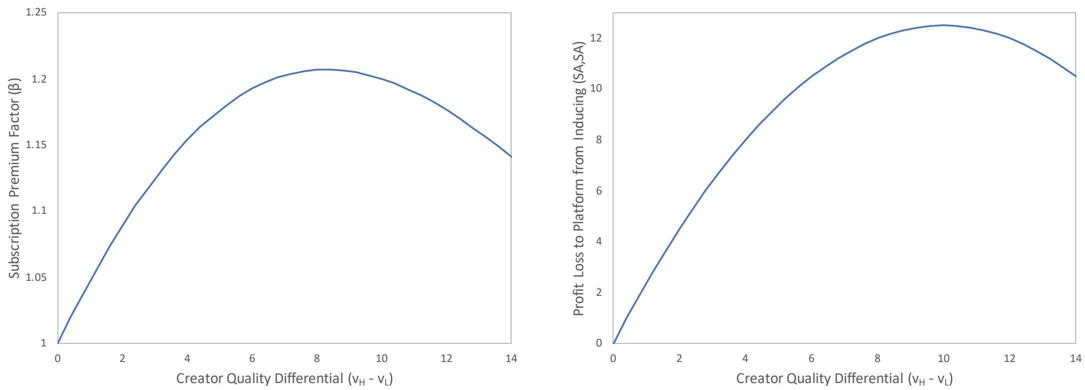
it follows that

$$\begin{aligned}\Pi_P^{(SA,AO)} - \Pi_P^{(SA,SA)} &= (1-\underline{\phi_A})(N_H + N_A)(u_L + u_H \\ &\quad - \left[(1-\underline{\phi_A})N_A + (1-\beta\underline{\phi_A})N_H\right](u_L + u_H) \\ &= (\beta - 1)\underline{\phi_A}N_H(u_L + u_H) \\ &= \frac{(l+v_L)(v_H - v_L)}{2\ell}\underline{\phi_A}N_H \geq 0.\end{aligned}$$

This proves the result. □

To improve the quality of the subscription service, the platform has to give up some immediate profit. Inducing the L type creator to list on the subscription is simple: the platform need only match the revenue share of the a la carte offering. However, at such a revenue share, the H type will still be better off selling only a la carte. In order to induce the H type to also list on subscription, the platform has to given up an even higher share of profits, i.e., it must set $\phi_S = \beta\underline{\phi_A}$, where, recall, $\beta = \frac{((l+v_L)(l+v_H)+(l+v_H)^2)}{((l+v_L)^2+(l+v_H)^2)} > 1$. This leads to several observations. Note that the platform's profit loss from inducing (SA, SA) increases in the reservation profit share $\underline{\phi_A}$ and the number of hybrid consumers, N_H. This means that in a highly competitive market for a la carte book sales, it can be very expensive to promote a subscription e-book service where both well known (H type) and new (L type) authors list instead of only new authors. It also helps explain the phenomenon of more new authors listing on subscription than seasoned ones. In addition, holding all else constant, the profit loss is increasing in v_H; consequently, very popular creators—e.g., the most popular authors or musicians—are extremely costly to lure on to the subscription service.

On the other hand, note that holding average quality $((v_H + v_L)/2)$ constant, as the disparity between the H and L type qualities $(v_H - v_L)$ grows, the platform's profit loss is non-monotonic. Considering the impact of this value on β, which we can think of as the subscription participation premium factor, illustrates the driving forces behind this observation (see Figure 3.3). The figure shows that β is concave in $v_H - v_L$, first increasing then decreasing. The intuition behind this behavior that when then the H type is slightly higher quality than the L type, the platform has to compensate her significantly to participate in the subscription platform, because, while she is higher quality than the L type, her quality is not great enough to capture a significant portion of consumption on the subscription service. However, as the H



(a) (b)

FIGURE 3.3: The impact of creator quality differential $(v_H - v_L)$ on the subscription premium factor (β) and platform cost of inducing (SA, SA). In both figures, $(v_H + v_L)/2$ is held constant and equal to 9, $\ell = 1$, $N_H = 1$, and $\underline{\phi_A} = 0.5$.

type's quality advantage over the L type continues to increase, she captures a larger and larger share of the total consumption, meaning the platform can compensate her less (in the form of a smaller β). Hence, the premium necessary to induce (SA, SA)—and hence the cost of inducing this outcome—is greatest for moderate difference in quality between the low and high quality creators. Lastly, note that the platform's profit loss is also increasing in ℓ, the dispersion of consumer tastes. If consumers are very homogeneous in tastes, the cost of inducing (SA, SA) is small, but if they have widely dispersed tastes, it is large. This further illustrates that it can be very costly to induce high quality on a subscription service if consumers have highly idiosyncratic tastes for content.

3.5.2 Supply Chain Optimal Decisions

In this section, we consider what revenue sharing parameters maximize the overall supply chain profit (defined to be the sum of platform and creator profits), and compare this to the optimal decisions of the platform derived in Proposition 3.3. The following proposition summarizes these results:

Proposition 3.5. *Supply chain profit is maximized by inducing any equilibrium that results in both creators selling a la carte (and zero, one, or both selling via subscription). Hence, the platform chooses a strictly suboptimal selling strategy for the supply chain when* $\frac{N_A}{N_H+N_A} < \underline{\phi_A}$.

Proof. Define $\Pi_{SC} \equiv \Pi_P + \Pi_L + \Pi_H$. Whenever an equilibrium results in both creators selling a la carte (i.e., in (AO, AO), (SA, AO), and (SA, SA)), $\Pi_{SC} = (N_A + N_H) \cdot (u_L + u_H)$. However, when the platform shuts down a la carte sales, $\Pi_{SC} = N_H(u_L + u_H)$, which is strictly lower than inducing a la carte sales if $N_A > 0$. □

Any outcome that results in both creators selling a la carte maximizes supply chain profits. This is because, in equilibrium, both consumer segments always consume both books, and all surplus from that consumption is extracted by optimal price. Thus, the market is fully covered in all cases, and supply chain profit is the same in all equilibria that survive, although the different equilibria result in different allocations of that profit between the platform and the creators. This implies that when the platform prefers to shut down a la carte sales and induce both creators to list on the subscription service (i.e., when $\frac{N_A}{N_H+N_A} < \underline{\phi_A}$), it necessarily results in an inefficient supply chain outcome: a la carte only consumers are no longer covered in the market, meaning total supply chain profits are reduced. In addition, when this happens, the platform extracts all surplus from creators as well as consumers; hence, the platform sacrifices supply chain efficiency in order to extract a larger share of the supply chain surplus.

We note here that one reason we see total market coverage whenever a la carte sales are induce comes from our model assumptions. Specifically, we assumed discrete types of consumers and content creators, which in turn leads to total market coverage under all of the realizable pure strategy equilibria. If consumer or creator types were continuous, it is likely that the result in Proposition 3.5 that supply chain profit is maximized for any $\frac{N_A}{N_H+N_A} \geq \underline{\phi_A}$ will no longer hold. Nevertheless, the proposition illustrates that the platform may sacrifice supply chain profit in order to maximize its own share, an observation that will likely continue to be true even under more complicated distributions of creator and consumer types.

3.6 Extension: Subscription Only Consumers

3.6.1 Possible Selling Strategies

In this section, we add in a consumer group that has only subscription of size N_S. We can think of them as using the subscription to experiment with consuming media through a digital platform or perhaps budget constrained consumers. First, let us restate all the updated profits associated with the nine selling strategies from 3.4.1.

We begin with (AO, AO). In this case, no products are sold via subscription. The optimal a la carte price is $p_i = u_i$, and both creators earn this revenue from all hybrid and a la carte only customers but no subscription only customers, yielding total profits:

$$\Pi_P = (1 - \phi_A)(N_H + N_A)(u_L + u_H)$$
$$\Pi_L = \phi_A(N_H + N_A)u_L$$
$$\Pi_H = \phi_A(N_H + N_A)u_H$$

Next, we move to (AO, SO). In this case, the L product is only sold via a la carte and the H product is only sold via subscription. The optimal price of the L product is $p_L = u_L$, and the L creator earns this revenue from all hybrid and a la carte only customers. The only subscription product is the H product which is sold to subscription only and hybric customers, so (ex ante) net consumer utility from the subscription is $u_H - p_S$. The optimal subscription price set by the platform is thus

$p_S = u_H$. Total profits are:

$$\Pi_P = (1-\phi_A)(N_H + N_A)u_L + (1-\phi_S)(N_H + N_S)u_H$$
$$\Pi_L = \phi_A(N_H + N_A)u_L$$
$$\Pi_H = \phi_S(N_H + N_S)u_H$$

Note that by selling only via the subscription platform, the H creator has lost revenue from the N_A a la carte only customers, and by selling only a la carte, the L creator has lost revenue from the N_S subscription only customers.

In (AO, SA), the L product is only sold via a la carte and the H product is sold via both channels. The optimal price of the L product is $p_L = u_L$, and the L creator earns this revenue from all hybrid and a la carte only customers. The optimal a la carte price of the H product is $p_H = u_H$, and the H creator earns this revenue from all a la carte only customers. The only subscription product is the H product, so net consumer utility from the subscription is $u_H - p_S$. Hybrid customers could forgo the subscription and instead purchase the a la carte version of the product; however, this would give them zero utility. Hence, the optimal subscription price set by the platform is $p_S = u_H$, which makes hybrid customers indifferent between consumption channels and causes them to sign up for the subscription and consume the product via that channel (per our earlier assumption). Total profits are:

$$\Pi_P = (1-\phi_A)(N_H + N_A)u_L + (1-\phi_A)N_A u_H + (1-\phi_S)(N_H + N_S)u_H$$
$$\Pi_L = \phi_A(N_H + N_A)u_L$$
$$\Pi_H = \phi_S(N_H + N_S)u_H + \phi_A N_A u_H$$

Note that for H creator, this clearly dominates (AO, SO), as it provides access to an additional pool of consumers (the N_A a la carte only consumers). Moving next to

(SO, AO), this case is the opposite of (AO, SO), and so by symmetry:

$$\Pi_P = (1-\phi_A)(N_H+N_A)u_H + (1-\phi_S)(N_H+N_S)u_L$$
$$\Pi_L = \phi_S(N_H+N_S)u_L$$
$$\Pi_H = \phi_A(N_H+N_A)u_H$$

In (SO, SO), both creators sell only via the subscription service. The optimal price is thus $p_S = u_L + u_H$. After the platform takes its cut of $1-\phi_S$, the remaining revenues for the creators are $\phi_S N_H(u_L+u_H)$. The total number of customers that read the L product is equal to $(1-F(-v_L))N_H$ while $(1-F(-v_H))N_H$ read the H product. Thus, the L creator is entitled to the following fraction of the shared revenue:

$$\alpha \equiv \frac{(1-F(-v_L))}{(1-F(-v_L))+(1-F(-v_H))} \tag{3.4}$$

while the H creator is entitled to a complementary fraction. Total profits are thus:

$$\Pi_P = (1-\phi_S)(N_H+N_S)(u_L+u_H)$$
$$\Pi_L = \alpha\phi_S(N_H+N_S)(u_L+u_H)$$
$$\Pi_H = (1-\alpha)\phi_S(N_H+N_S)(u_L+u_H)$$

Moving to (SO, SA), in this case both creators sell via the subscription service, but the H type sells via a la carte as well. Revenues from the subscription service follow as in the previous case, but the H-type enjoys the addition of a la carte revenues from N_A consumers. Total profits are thus:

$$\Pi_P = (1-\phi_A)N_A u_H + (1-\phi_S)(N_H+N_S)(u_L+u_H)$$
$$\Pi_L = \alpha\phi_S(N_H+N_S)(u_L+u_H)$$
$$\Pi_H = (1-\alpha)\phi_S(N_H+N_S)(u_L+u_H) + \phi_A N_A u_H$$

Note that for H creator, this clearly dominates (SO, SO), as (once again) it provides access to the a la carte only population of customers. (SA, AO) is the opposite of

(AO, SA), and so by symmetry:

$$\Pi_P = (1 - \phi_A)(N_H + N_A)u_H + (1 - \phi_A)N_A u_L + (1 - \phi_S)(N_H + N_S)u_L$$
$$\Pi_L = \phi_S(N_H + N_S)u_L + \phi_A N_A u_L$$
$$\Pi_H = \phi_A(N_H + N_A)u_H$$

Note that for L creator, this clearly dominates (SO, AO), for similar reasons as in the previously dominated cases. On the other hand, (SA, SO). is the opposite of (SO, SA), and so by symmetry:

$$\Pi_P = (1 - \phi_A)N_A u_L + (1 - \phi_S)(N_H + N_S)(u_L + u_H)$$
$$\Pi_L = \alpha\phi_S(N_H + N_S)(u_L + u_H) + \phi_A N_A u_L$$
$$\Pi_H = (1 - \alpha)\phi_S(N_H + N_S)(u_L + u_H)$$

For L creator, this clearly dominates (SO, SO). Finally, in (SA, SA), both creators sell via both channels. Total profits are thus:

$$\Pi_P = (1 - \phi_A)N_A(u_H + u_L) + (1 - \phi_S)(N_H + N_S)(u_L + u_H)$$
$$\Pi_L = \alpha\phi_S(N_H + N_S)(u_L + u_H) + \phi_A N_A u_L$$
$$\Pi_H = (1 - \alpha)\phi_S(N_H + N_S)(u_L + u_H) + \phi_A N_A u_H$$

Note that for H creator, this clearly dominates (SA, SO), while for the L creator, it dominates (SO, SA).

3.6.2 Equilibrium Analysis of the Selling Stage

Proposition 3.6. *For both creator types, strategy SA dominates strategy SO, regardless of the other creator's strategy.*

Proof. From the enumeration in §3.6.1, we see that for both creator types, profit is always higher under strategy SA compared to under strategy SO regardless of the other creator's strategy. □

Table 3.3: Payoffs for the H creator in the selling stage game.

		H Creator	
		AO	SA
2*L creator	AO	$\phi_A(N_H + N_A)u_H$	$\phi_S(N_H + N_S)u_H + \phi_A N_A u_H$
	SA	$\phi_A(N_H + N_A)u_H$	$(1-\alpha)\phi_S(N_H + N_S)(u_L + u_H) + \phi_A N_A u_H$

This result stands because by selling SO only, a la carte consumers are still excluded. Further adding to this is that by selling AO only now excludes all subscription only consumers, but depending on the rates for which share of profits are distributed among content creators for subscription versus a la carte sales, it may or may not be worth the tradeoff of excluding subscription only consumers in exchange for all hybrid consumers paying for content a la carte. This allows us, like in the main model, reduce our analysis to strategies AO and SA.

Let us consider the incentives for the H creator which are in Table 3.3. Conditional on the L creator playing AO, the H creator plays SA if and only if:

$$1 > \frac{N_H}{N_S + N_H} \cdot \frac{\phi_A}{\phi_S}.$$

Otherwise, the H creator plays AO. Conditional on the L creator playing SA, the H creator plays SA if and only if:

$$(1-\alpha)\frac{u_L + u_H}{u_H} > \frac{N_H}{N_S + N_H} \cdot \frac{\phi_A}{\phi_S} \tag{3.5}$$

which we can rewrite in the same fashion as in section 3.4.2 as

$$1 > \frac{(l + v_L)^2 + (l + v_H)^2}{(l + v_H)(l + v_L) + (l + v_H)^2} > \frac{N_H}{N_H + N_S} \cdot \frac{\phi_A}{\phi_S}.$$

Otherwise, the H creator plays AO. In other words, this is a similar result to our analysis that did not include a subscription only consumer population, except now

Table 3.4: Payoffs for L creator in the selling stage game.

		H Creator	
		AO	SA
2*L creator	AO	$\phi_A(N_H + N_A)u_L$	$\phi_A(N_H + N_A)u_L$
	SA	$\phi_S N_H u_L + \phi_A N_A u_L$	$\alpha\phi_S N_H(u_L + u_H) + \phi_A N_A u_L$

the relative size of the hybrid consumer base compared to the total population that could purchase via subscription (subscription only and hybrid consumers) also matters. Having a lower relative portion of hybrid consumers, or stated in other words, a greater relative portion of subscription only consumers in this population of consumers who could purchase via subscription, makes it more worthwhile to sell with strategy SA as to not exclude the subscription only consumers when selling via AO. Recall that in the main model, in order for the H creator to prefer to sell with strategy SA, it was necessary (but not sufficient) that $\phi_A < \phi_S$. With the addition of a subscription only consumer group, it is now possible for $\phi_A > \phi_S$ and to still be profitable to sell SA as long as $1 > \frac{N_H}{N_S+N_H} \cdot \frac{\phi_A}{\phi_S}$.

Next, we analyze the best response of the L creator whose pays offs are in Table 3.4. Conditional on the H creator playing AO, the L creator prefers SA if and only if:

$$1 > \frac{\phi_A}{\phi_S} \cdot \frac{N_H}{N_S + N_H}.$$

Otherwise, the L creator prefers to play AO. Conditional on the H type playing SA, the L creator prefers SA if and only if:

$$\alpha \cdot \frac{u_L + u_H}{u_L} > \frac{N_H}{N_S + N_H} \cdot \frac{\phi_A}{\phi_S}$$

which can be rewritten as

$$\frac{(l+v_L)^2+(l+v_H)^2}{(l+v_H)(l+v_L)+(l+v_L)^2} > \frac{N_H}{N_S+N_H}\cdot\frac{\phi_A}{\phi_S}$$

where

$$\frac{(l+v_L)^2+(l+v_H)^2}{(l+v_H)(l+v_L)+(l+v_L)^2} > 1.$$

Otherwise, the L creator prefers to play AO. Similar to the general model, the L creator contributes more to consumption quantity (with a weight given to the relative abundance of hybrid consumers among all consumers who can purchase via subscription) than she does to the total utility generated on the subscription service by her presence. The H creator, on the other hand, contributes more to the total utility generated on the subscription platform by her presence than the consumption quantity (with the same weight given to hybrid consumers versus all consumers who can consume via subscription) and thus receives a disproportionately low share of the revenue split. This leads to the following result:

Proposition 3.7. *Define* $\gamma \equiv (1-\alpha)\frac{u_L+u_H}{u_H} = \frac{((l+v_L)^2+(l+v_H)^2)}{((l+v_L)(l+v_H)+(l+v_H)^2)} < 1$. *Then, the equilibrium to the selling stage exists and is unique, and is characterized as follows:*
(i) If $1 > \gamma > \frac{N_H}{N_S+N_H}\cdot\frac{\phi_A}{\phi_S}$*, both creators sell via both distribution channels.*
(ii) If $1 > \frac{N_H}{N_S+N_H}\cdot\frac{\phi_A}{\phi_S} > \gamma$*, the low quality creator sells via both channels, while the high quality creator sells only via the a la carte channel.*
(iii) If $1 < \frac{N_H}{N_S+N_H}\cdot\frac{\phi_A}{\phi_S}$*, both creators sell only via the a la carte channel.*

Proof. (i) Suppose $1 > (1-\alpha)\frac{u_L+u_H}{u_H} \geq \frac{\phi_A}{\phi_S}\cdot\frac{N_H}{N_S+N_H}$. Per the discussion in §3.7, the dominant action of the H creator is SA. If the H creator plays SA, the L creator responds with SA if and only if $\alpha\frac{u_L+u_H}{u_L} \geq \frac{\phi_A}{\phi_S}\cdot\frac{N_H}{N_S+N_H}$. Since $\alpha\frac{u_L+u_H}{u_L} > 1 > (1-\alpha)\frac{u_L+u_H}{u_H} \geq \frac{\phi_A}{\phi_S}\cdot\frac{N_H}{N_S+N_H}$, this is always true. Hence, in this case, (SA, SA) is the unique equilibrium.

(ii) Suppose $1 > \frac{N_H}{N_S+N_H} \cdot \frac{\phi_A}{\phi_S} > \frac{((l+v_L)^2+(l+v_H)^2)}{((l+v_L)(l+v_H)+(l+v_H)^2)}$. From §3.6.2, the H creator will play AO if the L creator plays SA. Conditional on the H creator playing AO, SA is optimal for the L creator if $1 > \frac{\phi_A}{\phi_S} \cdot \frac{N_H}{N_S+N_H}$; hence, (SA, AO) is an equilibrium in these conditions. The only other possible equilibrium is (AO, SA). For this to hold, it must be true that AO for the L creator is a best response to SA from the H creator. Per §3.6.2, this is true if and only if $1 > \frac{\phi_A}{\phi_S} \cdot \frac{N_H}{N_S+N_H}$ and $\frac{(l+v_L)^2+(l+v_H)^2}{(l+v_H)(l+v_L)+(l+v_L)^2} < \frac{N_H}{N_S+N_H} \cdot \frac{\phi_A}{\phi_s}$ hold simultaneously, which is impossible because $1 < \frac{(l+v_L)^2+(l+v_H)^2}{(l+v_H)(l+v_L)+(l+v_L)^2}$. Hence, (AO, SA) cannot be an equilibrium, and (SA, AO) is the unique equilibrium in this region.

(iii) Suppose $1 < \frac{N_H}{N_S+N_H} \cdot \frac{\phi_A}{\phi_S}$. Then, (AO, AO) is clearly an equilibrium, per the discussion in §3.6.2. From equation (3.5), if the L creator plays SA, the H creator will play AO. However, if the H creator plays AO and $1 < \frac{N_H}{N_S+N_H} \cdot \frac{\phi_A}{\phi_S}$, the L creator's best response is also AO. Hence, (AO, AO) is the unique equilibrium when $1 < \frac{N_H}{N_S+N_H} \cdot \frac{\phi_A}{\phi_S}$.

The remainder of the proposition follows by defining $\gamma \equiv (1-\alpha)\frac{u_L+u_H}{u_H} = \frac{((l+v_L)^2+(l+v_H)^2)}{((l+v_L)(l+v_H)+(l+v_H)^2)} < 1$ and reframing each of the above regions. □

This proposition is very similar to Proposition 3.2 except that with the addition of a subscription only consumer base, now listing a la carte only means losing out on a segment of the consumers. Thus, it becomes important to consider what the relative size of hybrid consumers are compared to all consumers who can purchase via subscription (hybrid and subscription only). If this ratio is very high (close to 1), then it means that the decision on whether to sell a la carte only or via both subscription and a la carte rests mostly on what profit sharing rates are offered to creators by the platform. Otherwise, even if more favorable rates are offered for a la carte than subscription, if selling a la carte only meant excluding selling to a comparatively large number of consumers (because they only consume via subscription), then it may still be more profitable to sell subscription and a la carte instead of a la carte only. Part (iii) can also be written as $\phi_S < \frac{N_H}{N_H+N_S} \cdot \phi_A$ which means that even if the platform offers a lower profit share rate for subscription sales than a la carte sales, one or both creators may still elect to sell via both subscription and a la carte. But if the difference is substantially large, i.e. such that (iii) is satisfied, then it is important to note that if one content creator chooses not to sell on subscription, does not mean that the other creator chooses to list on subscription, even though she could capture the entire consumption of the subscription market. By selling a la carte only and not listing on subscription, content creators can realize greater profit because they can charge a higher per unit price, as long as subscription only consumers are not a substantial portion of the consumer base since they will be excluded from a la carte only sales.

With the presence of subscription only consumers, it is now less expensive to induce creators to list on subscription as the platform no longer needs to offer $\phi_S > \phi_A$. If the platform sets $\phi_S = \frac{N_H}{N_S+N_H}\phi_A$, it can induce the L creator to sell on subscription and a la carte. Contrast this with in the general model where we did not have a population of subscription only consumers and notice that the platform can now induce the L creator to sell on subscription at a lower profit share rate than a la carte. The higher the relative share of subscription only consumers, and consequentially the lower the relative share of hybrid consumers, the lower a rate of subscription profit shares the platform needs to offer to induce the L creator to sell on subscription because the lost sales from excluding all of the subscription only consumers increases as their relative population size increases. This means that if a sufficiently large portion of consumers are subscription only versus hybrid, then the platform can quite cheaply induce both the L and H creators to participate in the subscription service. However, like in the main model, the platform, through changing profit share rates, can induce (AO, AO), (SA, AO), and (SA, SA) but still not (AO, SA) and thus the quality of books on the subscription service still cannot exceed the average quality of all books sold on the platform.

3.6.3 The Platform's Optimal Revenue Shares

In this section, we analyze what the optimal revenue shares ϕ_A and ϕ_S the platform should set when seeking to maximize its profits. We will compare this to the optimal profit share rates in the main model and also compare to see if, like in the main model, profit maximizing choices are still suboptimal for inducing participation by creators in the subscription service. The platform can induce the selling strategy equilibria from Proposition 3.7 by choosing the following revenue sharing parameters:

(i) By setting $\phi_A = \underline{\phi_A}$ and $\phi_S = \frac{N_H}{N_S+N_H} \cdot \frac{1}{\gamma} \cdot \underline{\phi_A}$, the platform induces (SA, SA) and realizes a profit of $\Pi_P^{(i)} = (1-\underline{\phi_A})N_A(u_L+u_H)+(1-\frac{N_H}{N_S+N_H} \cdot \frac{1}{\gamma} \cdot \underline{\phi_A})(N_S+N_H)(u_L+u_H)$

(ii) By setting $\phi_A = \underline{\phi_A}$ and $\phi_S = \frac{N_H}{N_S+N_H} \cdot \underline{\phi_A}$, the platform induces (SA, AO) and realizes a profit of $\Pi_P^{(ii)} = (1-\underline{\phi_A})(N_A+N_H)u_H + (1-\underline{\phi_A})N_A u_L + (1-\frac{N_H}{N_S+N_H} \cdot \underline{\phi_A})(N_H+N_S)u_L$

(iii) By setting $\phi_A = \underline{\phi_A}$ and $\phi_S = 0$, the platform "shuts down" the subscription service and induces (AO, AO) and realizes a profit of $\Pi_P^{(iii)} = (1-\underline{\phi_A})(N_A+N_H)(u_L+u_H)$

It is important to note that while $\frac{1}{\gamma} = \frac{((l+v_L)^2+(l+v_H)^2)}{((l+v_L)(l+v_H)+(l+v_H)^2)} > 1$, in the presence of a sufficiently large subscription only consumer base (relative to the size of the hybrid consumer base), $\phi_S(SA, SA) = \frac{N_H}{N_S+N_H} \cdot \frac{1}{\gamma} \cdot \underline{\phi_A} < \underline{\phi_A}$ because $\frac{N_H}{N_S+N_H} << 1$ and thus also $\phi_S(SA, AO) = \frac{N_H}{N_S+N_H} \cdot \underline{\phi_A} < \underline{\phi_A}$. This means that the subscription rate offered to induce creator participation in the subscription service can be lower than what is offered for a la carte profit sharing. After comparing these platform profits, we arrive at the next result:

Proposition 3.8. *When optimizing for profit, the platform induces* (SA, SA) *by setting* $(\phi_A = \underline{\phi_A}, \phi_S = \frac{N_H}{N_S+N_H} \cdot \frac{1}{\gamma} \cdot \underline{\phi_A})$*, where* $\frac{1}{\gamma} = \frac{((l+v_L)^2+(l+v_H)^2)}{((l+v_L)(l+v_H)+(l+v_H)^2)} > 1$*, and realizing a profit of* $\Pi_P = (1-\underline{\phi_A})N_A(u_L+u_H)+(1-\frac{N_H}{N_S+N_H}\cdot\frac{1}{\gamma}\cdot\underline{\phi_A})(N_S+N_H)(u_L+u_H)$.

Proof. Platform profit under (ii) is less than under (i). $\Pi_P^{(ii)} - \Pi_P^{(i)} = (1-\frac{1}{\gamma})(1-\underline{\phi_A})N_H u_H + (1-\frac{1}{\gamma})u_L(N_S + (1-\underline{\phi_A})N_H) - \frac{1}{\gamma}u_H N_S < 0$. Note that each term in this expression is negative because $\frac{1}{\gamma} > 1$ and thus $1 - \frac{1}{\gamma} < 0$. Platform under (i) is greater than under (iii). $\Pi_P^{(i)} - \Pi_P^{(iii)} = \frac{1}{\gamma}N_S u_L + (\frac{1}{\gamma} - 1)(1-\underline{\phi_A})N_H(u_L + u_H) > 0$ because $1 - \underline{\phi_A} > 0$ and $\frac{1}{\gamma} - 1 > 0$. Platform profit under (ii) is greater than under (iii). $\Pi_P^{(ii)} - \Pi_P^{(iii)} = N_S u_L > 0$. Since $\Pi_P^{(i)} > \Pi_P^{(iii)}, \Pi_P^{(i)} > \Pi_P^{(ii)}$ and $\Pi_P^{(ii)} > \Pi_P^{(iii)}$ then we can conclude that $\Pi_P^{(i)} > \Pi_P^{(ii)} > \Pi_P^{(iii)}$ and thus platform profit is highest under (i) where (SA, SA) is induced. □

This result is important when we contrast it to Propositions 3.3 and 3.4 because in the presence of a subscription only consumer group, maximizing profit and growing the subscription service are no longer conflicting objectives. Previously, selling AO did not mean excluding any consumers, so since the H and L creators did not receive shares of subscription consumption commensurate with their contributions to quality, it made sense for the H type to not sell on subscription unless induced so by more favorable profit sharing terms. Here, it is costly to sell AO because it means excluding a population of subscription only consumers. As the relative size of this population grows relative to the hybrid consumer population, it becomes more and more costly to exclude them. Thus, the creators do not need a higher subscription profit share rate (compared to a la carte share rate) to sell on subscription. They will, in fact, sell SA while realizing a subscription rate that might be lower than the a la carte rate if the subscription only population is sufficiently large in comparison to the hybrid population. This means that if the platform has enough subscription only consumers compared to hybrid consumers, they can not only avoid having to "subsidize" the subscription service by paying creators higher shares of subscription profits but instead achieve profit maximizing and subscription service growing objectives at the same time.

3.7 Conclusion

In this paper, we have studied the problem of subscription and a la carte e-book selling between a platform and two competitive content creators: one high quality (e.g., an established and well-known author) and one low quality (e.g., lesser-known or new author). We showed that depending on the revenue sharing parameters set by the platform, content creators will choose different strategies. If the revenue share given to content creators for subscription reads is lower than that for a la carte sales, then no content creator will want to list on subscription, regardless of what the other content creator does. If the revenue share for subscription is weakly greater than the share for a la carte sales, then it is possible to induce the low quality creator or both creators to sell via subscription, while it is never possible to induce the high quality creator alone to do so.

These results lead to several important managerial insights. If the platform were to focus on only maximizing immediate profits, then it should never induce the high quality creator to sell via subscription; doing so is too costly and requires too much subscription revenue given to the creators, since the H creator makes an outsize contribution to consumer utility but not to the quantity of consumption. This means that the average quality of content on subscription is lower than the average quality of content creators (selling through the platform in general). The platform cannot through changing profit sharing rates on subscription or a la carte induce only the H type, and not also the L type, to list on subscription. This means that the highest quality the subscription content can be is no better than the average content quality on the general platform. This could also explain why we frequently observe that most books listed on services like Kindle Unlimited are by lesser-known or new authors and not bestselling authors.

However, given that the subscription service may have value in the future for the platform, such as access to different or new consumers, the platform may want to promote it as a service. In order to induce both types of content creators to list on subscription (in addition to a la carte), the platform will need to give up a significant portion of immediate profits in order to offer a sufficiently high subscription profit share to content creators that will induce both types to list on subscription. The amount of revenue the platform must relinquish is proportional to the a la carte revenue share, and increases with higher disparity between consumers (denoted by l in our model) and quality difference between creators ($v_H - v_L$). An important consequence of these results for platforms is that the subscription revenue share rate can be used as a lever to induce content creators into change their selling behaviors: by increasing the rate of subscription revenue sharing, platforms can induce content creators to sell via this channel and raise the overall quality of the service.

In the extension where we added a "subscription only" consumer segment, we found that while results about creator selling strategies stayed the same, the platform could achieve profit maximization by inducing both creators to sell on both subscription and a la carte. The rationale behind it is that in the presence of a "subscription only" segment, especially a comparatively large one, it was possible to induce both creators to sell on both subscription and a la carte because selling a la carte only (and thus excluding the subscription only consumers) is too costly. Thus, the creators, for a sufficiently large "subscription only" consumer segment will sell on both channels even if the subscription profit share rate was lower than that offered by the platform for a la carte sales.

To summarize, in this paper, we've shown that by choosing contract terms for a la carte and subscription sales, platforms can influence who, if anyone, participates in the subscription platform; however, this influence is not without limitations, and inducing high quality content on a subscription platform is challenging and costly, due to the way that high quality creators make an outsized contribution to consumer utility relative to the quantity of consumption. Our work thus helps to develop a more thorough understanding of the new e-commerce landscape for digital information goods, especially the growing market of subscription service for information goods. Our insights can help both platforms and the content creators understand the tradeoffs between selling via subscription and selling a la carte, and can also explain some of the most common practical observations regarding the number and quality of creators that list on these platforms.

4

Optimal Revenue Splitting Rules for Digital Content Subscription Platforms

4.1 Introduction

Traditionally, information goods such as books, music, and video, have been primarily distributed via an “a la carte” sales model in which consumers purchase a permanent copy of each item they desire. The rise of digital formats and distribution technologies, however, has enabled a wider variety of distribution methods for information goods, including, most notably, subscription services. With a subscription service, a platform, such as Amazon Kindle (for books), Spotify (for music), or Netflix (for video), allows content creators (i.e., authors or musicians) to opt in to being included as part of a library. Consumers pay a flat monthly fee for unlimited access to this library, and creators are paid based on how much their content is consumed relative to total consumption of content in the library. Subscription services ostensibly generate value to consumers by offering them a wide variety of content, while generating value to content creators by exposing them to a large audience of potential consumers. By eliminating the per-unit cost of consumption, a subscription model also allows consumers to “try out” content of uncertain or variable quality that they might not have consumed had they been required to pay a la carte for each product, particularly works by lesser known or newer content creators.

With most subscription platforms, the platform keeps a fraction of the subscription fees and then distributes the remaining portion, also known as a royalty or funding pool, among creators who are opted into the library, based on a relative measure of how much their content was consumed. In this paper, we refer to the fraction of total revenue given to creators as the *revenue share*, and we refer to the allocation of that revenue share amongst creators as the *revenue split*. Frequently, the revenue split is determined by a simple linear rule: for instance, on Amazon Kindle Unlimited, the revenue given to each creator is a constant price per page read of the creator’s content, which in turn is determined from dividing the royalty pool by the total number of pages read on the platform ([The Digital Reader, 2018]). In other words, creators are paid according to their contribution to the percentage of total content consumed and the overall amount of revenue shared with creators. An individual creator can thus earn more revenue in two ways: if they generate a larger fraction of total consumption on the platform relative to other creators, or if the platform shares a larger fraction of total revenue with all creators.

Since the platform takes a percentage of the subscription fees, it maximizes profit by attracting more consumers to subscribe to the service. Consumers, in turn, are attracted to the platform based on the number and quality of content offered in the subscription library. In the long run, the platform seeks to create a virtuous cycle where more content creators join the platform (and thus also more content is listed), which in turn makes the monthly subscription fee more attractive to consumers, which induces more consumers to join the service, increasing the size of the royalty pool and enticing more creators to join the platform, and so on. Hence, the platform’s objective is to maximize its profit by inducing a large number of high-quality content creators to list on the platform, and this may be accomplished by adjusting the revenue share and the revenue split.

In this paper, we consider this problem as we explore the optimal design of revenue splitting rules for subscription based digital content platforms. We accomplish this by analyzing a stylized model in which a platform offers a take-it-or-leave-it contract consisting of a revenue share and revenue split to a set of N creators. Creators are heterogeneous in the quality of their offerings, i.e., in the amount of consumer utility that their content generates. We begin by identifying three desirable properties of a revenue splitting rule. First, we say that a revenue split is *feasible* if it induces all content creators in the set to join the platform. Second, we say that a revenue split is *fair* if it allocates revenue based only on the amount of consumption generated by each creator. And third, we say that a revenue split is *optimal* if it is both fair and feasible at the lowest possible cost to the platform. Given this, we then study the following research questions. Does a linear revenue split—as is commonly seen in practice—satisfy the properties of feasibility, fairness, and optimality? If a linear split does not achieve these three properties, what sort of revenue split does, and how much improvement does it offer over a simple linear split? And lastly, how does the optimal revenue split change as the quality and characteristics of the creator population changes?

Our main findings are as follows. First, we show that a linear revenue splitting rule can be feasible and fair, but not also optimal. The reason for this is that high quality creators contribute more, relatively speaking, to overall consumer utility than they do consumption on the platform; as a result, to induce them to participate in the subscription service, the platform will have to offer a very high share of the overall revenue to the creator population, which violates the optimality condition by not achieving fairness and feasibility at the lowest possible cost. We then show that a quadratic revenue splitting rule—in which creators are allocated revenue based on a quadratic function of the total consumption they generate on the platform—can achieve all three properties at the same time. This means that a quadratic split can achieve the same outcome as a linear split (inducing all creators to sign up for the subscription service with a split based only on their consumption quantities) at a lower cost. We show that the quadratic rule outperforms the linear rule most when the creators' mean qualities are more heterogeneous, underscoring the importance of an appropriate revenue splitting rule when curating content from a highly diverse population of creators. Finally, under an optimal quadratic rule, we determine conditions that lead to the platform increasing or decreasing the revenue split of each individual creator, and show, for instance, that as consumer tastes become more heterogeneous, a high quality creator should be allocated a smaller split of the revenue while a lower quality creator should be allocated a greater split; this shows that heterogeneity of consumer tastes also plays a critical role in determining the optimal revenue split.

The remainder of this paper is organized as follows. §4.2 provides a brief review of the literature. §4.3 introduces our model of content creators and the subscription platform. §4.4 analyzes linear and non-linear revenue splitting rules, analyzes their behavior, and compares their relative performance. Finally, §4.5 concludes the paper.

4.2 Literature Review

There is a small but growing literature on digital content distribution and subscription services. The closest related paper in this stream is [Lei & Swinney, 2018a]; we refer readers to that paper for a discussion of references related to digital distribution and subscriptions. [Lei & Swinney, 2018a] in particular also analyze the management of a digital subscription service for information goods, with two key differences from the present paper. First, in [Lei & Swinney, 2018a], the same platform sells digital content in both subscription and a la carte formats, and induces creators to sell via one or both formats based on the terms it establishes for each. Hence, there is no a priori assumption in [Lei & Swinney, 2018a] that the platform seeks to induce all creators to list on the subscription service, as it may prefer to induce some to use a la carte and others to use subscription; this is appropriate for sellers for both a la carte and subscription content, like Apple Music or Amazon Kindle. By contrast, we assume in this paper that the platform only offers the subscription service, and hence seeks to maximize the content available on that service (e.g., as is the case with subscription-only firms like Netflix, Hulu, or Spotify). The second key difference between this paper and [Lei & Swinney, 2018a] is that, in [Lei & Swinney, 2018a], a linear revenue split is assumed throughout, due to its prevalence in practice; in this paper, we consider non-linear revenue splits as well, and show that, indeed, the linear split is not optimal, despite its widespread adoption in industry.

Beyond the digital content distribution literature, our paper lies at the intersection of several other streams of literature: nonlinear pricing, bundling and subscriptions, and revenue sharing. The literature on nonlinear pricing is focused on its use as a method of price discrimination by monopolies and oligopolies. The literature starts with [Mussa & Rosen, 1978] which analyzes a vertical differentiation model where differentiation within a monopolist's product line allows him to discriminate among buyers with various characteristics leading him to offer a broader range of products than is efficient. This is extended into a case with incomplete information by [Maskin & Riley, 1984] and into competing oligopolists with product lines by [Champsaur & Rochet, 1989]. [O'Brien & Shaffer, 1997] extends the duopoly competition by adding in a monopolistic retailer both manufacturers sell through and shows that the only market foreclosure equilibria (where the retailer sells only the goods of one manufacturer) are Pareto dominated from the perspectives of the manufacturers. [Rochet & Stole, 2002] extend this to a framework for the setting of principal-agent contracts (still similar to consumer purchasing decisions where the decisions becomes participation instead of purchasing) where the agents have non-deterministic outside options. [Fishe & McAfee, 1987] shows that quadratic contracts in auctions with one unit of good can elicit truthful valuations in situations where a winner's valuation is private during the auction but known afterwards. [Masten, 1987] looks at minimum bill contracts (a nonlinear contract, where one pays for a minimum amount even if the actual amount taken is less) and shows that they are efficient for parties when facing uncertain demand. [Spulber, 1989] evaluates nonlinear pricing in a market with free entry monopolistic competition and shows that it can result in greater variety of products.

The literature on bundled and subscription (which may thought of as intertemporally bundled) goods discusses many benefits of bundling, including cost savings during production and transaction, complementary value of bundled elements, and improving pricing power by sorting consumers based on their valuations. The last of these is most closely related to our work. [Stigler, 1963] first discusses how bundling two movie rights could increase sellers' profits when consumer valuations for the two goods are negatively correlated. [Adams & Yellen, 1976] show with two goods that commodity bundling under monopoly allows for greater extraction of consumer surplus through comparing unbundled sales to sales under pure bundling and mixed bundling. [Schmalensee, 1984] models reservation prices as a bivariate Gaussian distribution and shows that pure bundling reduces the heterogeneity of the consumer population, allowing sellers to extract more consumer surplus. [Hanson & Martin, 1990] consider how a monopolist facing segmented consumer demand should determine product line breadth (similar to bundling) and pricing using a mixed integer linear program. [Bakos & Brynjolfsson, 1999] expand this to large bundles of goods and offer a framework for modeling value of bundling. [Bakos & Brynjolfsson, 2000] extend upon that work by including various forms of competition, including competition between two bundlers and competition between a bundler and a single-good seller. [Bakos & Brynjolfsson, 2001] generalize the framework from [Bakos & Brynjolfsson, 1999] by adding in the cost of digital distribution over a network to compare pricing strategies under aggregation and disaggregation. [Hitt & Chen, 2005] look at pricing strategies when the bundle is a consumers selected subset of a larger pool of available goods. Our work differs from these in several ways. Unlike these papers, our seller (the platform) does not decide what *content* is in the bundle (the subscription service), because this is typically the decision of the content creator. Moreover, these content creators, who effectively decide the content of the bundle, are in direct competition with one another for the limited pool of subscription fees that they share based on their relative popularity. In addition, in our model, since the platform only offers one subscription plan, there is also no consumer-customized bundling; instead, all items listed on the subscription service are bundled together. Lastly, the platform in our model does not set all of the prices for the products it sells; reflective of actual practice (e.g., book sales on Amazon), the content creator sets the price on the unbundled (a la carte) good and the platform sets the bundled (subscription) price.

The literature on revenue sharing in supply chain is mostly focused on coordinating supply chains through using linear revenue sharing contracts. In addition to the works cited above, which fall under both revenue sharing contracts and nonlinear pricing, [Tsay *et al.* , 1999], [Cachon & Lariviere, 2005b], and [Cachon, 2003b] provide surveys of incentive issues and coordination mechanisms under common supply chain contracts. Our work is also related to models of multi-channel retail distribution, including [Balasubramanian, 1998] and [Chiang *et al.* , 2003]. [Salanié, 2003] provides a survey of empirical studies in contract theory that test analytical theories around various contracts and their optimality and performance in the presence of uncertainties such as asymmetric information. Specifically relating to information goods and revenue sharing for bundled music, [Shiller & Waldfogel, 2013] evaluate multiple revenue sharing schemes using econometric methods and conclude that the Shapley value is incentive compatible (better off in bundle than a la carte) under all scenarios available with their dataset. [Gong *et al.* , 2018] compares revenue and profit sharing contracts in internet platform businesses selling physical goods and what participants (platforms versus sellers/creators) prefer.

In summary, to our knowledge, this is the first analytical paper to explore why and how nonlinear revenue sharing contracts outperform linear revenue sharing contracts in digital subscription platforms.

4.3 Model

We consider a population of N heterogeneous content creators who independently choose whether to sell via a single subscription platform. The creators differ in the utility that their content generates for consumers. Let $v_i + \epsilon_i$ be the utility generated by creator i, where v_i is a constant and ϵ_i is a random "noise" term that has mean zero and is iid between individual consumers. In other words, v_i is the mean utility generated by creator i's content, while ϵ_i reflects the idiosyncratic tastes of individual consumers for that content. Without loss of generality, let the creators be numbered such that $v_1 < v_2 < ... < v_N$. For ease of analysis, we assume that the noise term is uniform and its distribution is identical across creators, i.e., $\epsilon_i \sim U[-\ell, \ell]$. The parameter ℓ thus serves to represent the dispersion of consumer tastes: higher ℓ means consumers have more heterogeneous, dispersed tastes for the content of each creator, while smaller ℓ means consumers have more homogenous tastes.

The mean utility v_i for each item is known to all content creators, the platform, and consumers; this reflects public knowledge of differences in average quality between different creators, e.g., one author may be more popular than another author, and hence have a higher v_i. Before subscribing to the service, consumers cannot observe their individual value of ϵ_i for each piece of content on the platform. However, due to the nature of subscription services (in particular the fact that consumption incurs zero marginal cost), after subscribing to the service, consumers can sample each piece of content instantaneously and costlessly (e.g., read the first chapter of a book, or listen to the first 30 seconds of a song), and observe their ϵ_i for that item. We assume that after this sampling phase, consumers complete consumption only if their total realized utility is non-negative. Consumers are infinitesimal and the number of consumers is normalized to one. Thus, the probability that an individual consumer completes consumption (and hence the total consumption quantity across all consumers) is

$$Q_i \equiv \Pr\left(v_i + \epsilon \geq 0\right).$$

We also define

$$u_i \equiv \Pr\left(v_i + \epsilon_i \geq 0\right) E\left[v_i + \epsilon_i \middle| v_i + \epsilon_i \geq 0\right]$$

to be the expected utility for product i before a consumer samples the product, i.e., before she observes her idiosyncratic noise term ϵ_i for that product.

Creators join the subscription platform if their revenue from doing so is at least as high as they revenue from selling a la carte via a competing platform, which shares a fraction ϕ_o of its revenue with the creators. Because sales on the outside option platform are "a la carte," they are not split amongst creators, i.e., each creator sells her own content directly to consumers, and moreover consumers choose to purchase before they can sample the product and learn their precise utilities. Thus, the outside option platform can thus charge, at most, u_i for creator i's product. We also assume the outside option has a market size equal to one (although this is easily relaxed), and hence creator revenue from the outside option platform is $\phi_o u_i$.

The platform subscription fee is p_S for each customer. The platform collects subscription fees from all customers and retains a fraction $1 - \phi$ of that revenue. The remainder, a fraction ϕ, is the *revenue share* given to the creators. The revenue share is split amongst the creators according to a predetermined revenue splitting rule, $S_i(Q_i, Q_{-i})$, which gives a fraction of total shared revenue to allocate to creator i. Assuming all consumers sign up for the service, creator i's resulting revenue is

$$\pi_i \equiv S_i(Q_i, Q_{-i})\phi p_S.$$

As noted in the introduction, we define the following desirable properties of a revenue splitting rule:

1. **Fairness:** a splitting rule is *fair* if it satisfies the following condition: it compensates only based on the amount of consumption of each creator, Q_i. In other words, a fair splitting rule does not allocate to the creators based on *who* they are, but rather based on *how much consumption* they generate on the platform.

2. **Feasibility:** the platform has an interest in encouraging as many creators as possible to list on its service, as this will stimulate consumer demand and increase market share. Thus, we assume that a *feasible* revenue split induces all creators to join the service; see [Lei & Swinney, 2018a] for a model in which the platform may find it optimal to induce a subset of the creators to join the service, e.g., if it also sells content a la carte from creators who elect not sell via subscription.

3. **Optimality:** lastly, we say that a revenue split is *optimal* if it is both *fair* and *feasible*, while making the participation constraint $\pi_i \geq \phi_o u_i$ binding for each creator. This ensures that the platform achieves fairness and feasibility at the minimum possible cost and hence maximizes its own profit.

The sequence of events is as follows. First, the platform determines the subscription price, p_S, the revenue share ϕ, and the splitting rule $S_i(Q_i, Q_{-i})$. Second, creators simultaneously decide whether to join the service by comparing their profit to their outside option, and join if $\pi_i \geq \phi_o u_i$. Third, consumers decide whether to sign up for the service and subsequently consume any content that gives them positive utility. Consumers are assumed to sign up for the service if their net utility from doing so is non-negative, i.e., if $p_S \geq \sum_{j \in S} u_j$, where S is the set of creators that list on the subscription service.

4.4 Analysis

4.4.1 Linear and Optimal Revenue Splits

In this section we will derive a revenue splitting rule that satisfies all three conditions listed in the previous section. We begin our analysis by determining whether the linear revenue split, commonly seen in practice, accomplishes this. First, note that since $\epsilon_i \sim U[-\ell, \ell]$, the ex ante expected utility from creator i's content is

$$\begin{aligned} u_i &= \Pr\left(v_i + \epsilon_i \geq 0\right) E\left[v_i + \epsilon_i \middle| v_i + \epsilon_i \geq 0\right] \\ &= \frac{(v_i + \ell)^2}{4\ell}. \end{aligned}$$

Similarly, the consumption quantity of creator i's content is

$$Q_i = \Pr\left(v_i + \epsilon \geq 0\right) = \frac{v_i + \ell}{2\ell}.$$

With a linear revenue splitting rule, $S_i(Q_i, Q_{-i})$ simply allocates revenue based on the fraction of total consumption that creator i generates, i.e.,

$$S_i(Q_i, Q_{-i}) = \frac{Q_i}{\sum_j Q_j}.$$

Hence, creator i's revenue is

$$\pi_i = \frac{Q_i}{\sum_j Q_j} \phi p_S.$$

Note that this rule is fair as per the definition above, i.e., it depends only on quantity consumers for each creator. The rule is further feasible and optimal if $\pi_i = \phi_o u_i$ for all i. This leads to our first result:

Proposition 4.1. *No linear revenue split exists that is both feasible and optimal. Moreover, under any linear revenue split that is optimal (but not necessarily feasible), only the lowest quality creator can be induced to join the subscription service.*

Proof. First, note that a rule that is feasible induces all creators to join the service. Thus, total consumer utility on the service is $\sum_j u_j$, and the platform will set the optimal price to extract all consumer surplus, $p_S = \sum_j u_j$. Thus, $\pi_i = \frac{Q_i}{\sum_j Q_j}\phi \sum_j u_j$. This implies $\frac{Q_i}{\sum_j Q_j}\phi \geq \frac{u_i}{\sum_j u_j}\phi_o$ for all i. Substituting expressions for Q_i and u_i, this is the same as

$$\phi \sum_j (v_j + \ell)^2 \geq \phi_o (v_i + \ell) \sum_j (v_j + \ell) \, .$$

Now suppose that the rule is optimal for the lowest quality creator, i.e., the participation constraint binds. This implies

$$\phi \sum_j (v_j + \ell)^2 = \phi_o \sum_j (v_1 + \ell)(v_j + \ell) < \phi_o \sum_j (v_j + \ell)^2 \, ,$$

which in turn implies $\phi < \phi_o$. Next, suppose the rule is optimal for the highest quality creator. Then,

$$\phi \sum_j (v_j + \ell)^2 = \phi_o \sum_j (v_N + \ell)(v_j + \ell) > \phi_o \sum_j (v_j + \ell)^2 \, ,$$

which in turn implies $\phi > \phi_o$. This is a contradiction, hence a rule cannot be feasible and also optimal, i.e., it cannot achieve a binding participation constraint for all creators. By iterating this argument from creator N as the highest-value participating creator down to creator 2, it can be seen that the only optimal linear revenue split that can be achieved is one in which only the lowest quality creator joins the platform. □

The proposition shows that no linear revenue split can simultaneously achieve all three desirable criteria: fairness (depends only on the consumption quantity of each product), feasibility (induces all creators to join the service), and optimality (shares a total fraction of revenue that is equal to the outside option revenue sharing rate for each creator). Examination of the proof of the proposition shows that the problem arises with the highest quality participating creator: creator N will not join the subscription service if the revenue splitting rule is linear, fair, and optimal, as her contribution to consumer utility ($\frac{u_N}{\sum_j u_j}$) is greater than her share of the total consumption ($\frac{Q_N}{\sum_j Q_j}$). In fact, this is true for the highest utility creator in any participating set of creators; iterating on this argument, it follows that only the only "optimal" linear revenue split that can be achieved (in the sense of making the participation constraint binding for all creators who elect to use the platform) is the one in which only the lowest quality creator joins the subscription service.

Despite this, it is possible to design a linear rule that is fair and feasible, but not optimal. Such a rule is described in the following proposition:

Proposition 4.2. *The highest platform revenue with a fair and feasible linear revenue split is achieved when* $S_i(Q_i, Q_{-i}) = \frac{Q_i}{\sum_j Q_j}$ *and*

$$\phi = \phi_o \frac{\sum_j (v_N + \ell)\,(v_j + \ell)}{\sum_j (v_j + \ell)^2} > \phi_o.$$

Proof. A linear revenue split is fair and feasible if all creators participate, i.e., if $\pi_i \geq \phi_o u_i$ for all i. This implies

$$\frac{v_i + \ell}{\sum_j (v_j + \ell)} \phi \sum_j \frac{(v_j + \ell)^2}{4\ell} \geq \phi_o \frac{(v_i + \ell)^2}{4\ell}$$

or equivalently $\phi \sum_j (v_j+\ell)^2 \geq \phi_o (v_i+\ell) \sum_j (v_j+\ell)$. It is easy to see that if this inequality binds for creator i, it holds for all creators $1, 2, ..., i$, and it is violated for all creators $i+1, ..., N$. Hence, if a particular revenue split induces the participation of creator N, it induces participation of all creators. Suppose this is the case; then, platform revenue is $\pi_P = (1-\phi)\sum_j u_j$. Revenue is maximized at the smallest ϕ which achieves this outcome, which occurs under the condition given in the proposition. □

This seems to suggest that the platform's problem is a challenging one: with a simple linear splitting rule, it can only induce all creators to join the platform if it (potentially significantly) overshares revenue with creators. The extent to which it must overshare revenue depends on the quality of the highest quality creator: if v_N is large, then the necessary revenue share to induce creator N to participate under a linear rule may be significantly higher than the reservation revenue share rate, ϕ_o.

It is possible that the platform could achieve greater profit by breaking the feasibility condition, i.e., but not inducing all creators to participate in the platform. However, this will reduce the number of creators on the platform, lowering the willingness to pay for the service by consumers and sacrificing the growth of the subscription service. On the other hand, the proof of Proposition 4.1 suggests an alternate route: the platform could use a different revenue splitting rule that simultaneously achieves fairness, feasibility, and optimality. Indeed, such a rule exists, and it splits revenue amongst creators according a quadratic function of the consumption quantity of their content, as shown in the following proposition:

Proposition 4.3. *A revenue splitting rule in which* $S_i(Q_i, Q_{-i}) = \frac{Q_i^2}{\sum_j Q_j^2}$ *and* $\phi = \phi_o$ *is fair, feasible, and optimal.*

Proof. First, note that a rule that is feasible induces all creators to join the service. Thus, total consumer utility on the service is $\sum_j u_j$, and the optimal price is $p_S =$

$\sum_j u_j$. Thus, $\pi_i = \frac{Q_i^2}{\sum_j Q_j^2}\phi_o \sum_j u_j$. The rule is feasible if $\frac{Q_i^2}{\sum_j Q_j^2}\phi_o \sum_j u_j \geq \phi_o u_i$ for all i. Equivalently, $\frac{Q_i^2}{\sum_j Q_j^2} \geq \frac{u_i}{\sum_j u_j}$. For uniform noise, substituting expressions for Q_i and u_i,

$$\frac{(v_i + \ell)^2}{\sum_j (v_j + \ell)^2} \geq \frac{(v_i + \ell)^2}{\sum_j (v_j + \ell)^2}$$

which holds with equality for all i. □

This revenue split achieves all three desirable properties simultaneously because it compensates creators exactly for the proportion of utility they contribute to the service. This is due to the fact that, under uniform valuation noise, utility is equal to a quadratic function of consumption. In turn, a quadratic rule allows the platform to extract all surplus from *all* creators, i.e., to make their participation constraints binding. A linear rule, by contrast, can only be structured to extract all surplus from *one* creator, and all creators with lower quality than this creator have positive surplus while all creators with higher quality choose not to participate in the subscription service.

4.4.2 Behavior of the Optimal Revenue Split

Next, we analyze the characteristics of the optimal quadratic revenue split derived in Proposition 4.3. The following corollary summarizes the behavior of the optimal revenue allocation and revenue split for each creator as a function of the problem parameters:

Corollary 1. *The fair, feasible, and optimal revenue splitting rule in which* $S_i(Q_i, Q_{-i}) = \frac{Q_i^2}{\sum_j Q_j^2}$ *and* $\phi = \phi_o$ *has the following properties:*

(i) The total revenue of creator i (π_i) *is increasing creator i's mean utility* (v_i)*, is independent of the mean utility of all other creators* ($v_{j\neq i}$)*, and is increasing in the dispersion of consumer tastes* (ℓ) *if and only if* $\ell > v_i$.

(ii) The revenue split for creator i ($S_i(Q_i, Q_{-i})$) is increasing creator i's mean utility (v_i), decreasing in the mean utility of all other creators ($v_{j \neq i}$), and is increasing in the dispersion of consumer tastes (ℓ) if and only if $\sum_j (v_j - v_i)(v_j + \ell) > 0$.

Proof. Part (i) follows since $\pi_i = \frac{Q_i^2}{\sum_j Q_j^2} \phi_o \sum_j u_j = \frac{(v_i+\ell)^2}{\sum_j (v_j+\ell)^2} \phi_o \sum_j \frac{(v_j+\ell)^2}{4\ell} = \phi_o \frac{(v_i+\ell)^2}{2\ell}$ and hence $\frac{d\pi_i}{dv_{j\neq i}} = 0$, $\frac{d\pi_i}{dv_i} = \phi_o \frac{v_i+\ell}{\ell} > 0$, and $\frac{d\pi_i}{d\ell} = \phi_o \frac{\left(\ell^2 - v_i^2\right)}{2\ell^2}$. Part (ii) follows since $S_i(Q_i, Q_{-i}) = \frac{(v_i+\ell)^2}{\sum_j (v_j+\ell)^2}$ and hence

$$\frac{dS_i(Q_i, Q_{-i})}{dv_i} = \frac{2(v_i + \ell) \sum_{j \neq i} (v_j + \ell)^2}{\left(\sum_j (v_j + \ell)^2\right)^2} > 0,$$

$$\frac{dS_i(Q_i, Q_{-i})}{dv_{j \neq i}} = -\frac{2(v_i + \ell)^2 (v_j + \ell)}{\left(\sum_j (v_j + \ell)^2\right)^2} < 0,$$

$$\frac{dS_i(Q_i, Q_{-i})}{d\ell} = \frac{2(v_i + \ell)\left(\sum_j (v_j - v_i)(v_j + \ell)\right)}{\left(\sum_j (v_j + \ell)^2\right)^2}.$$

□

Part (i) of the corollary shows that each creator earns more money as her own mean utility (and hence contribution to the overall platform) increases. In addition, under the optimal revenue split, each creator's revenue is independent of the mean utilities of all other creators. The latter property, in particular, is not true under a linear split, and under the linear revenue split described in Proposition 4.2, the revenue of of creators 1 to $N - 1$ all depend on the mean utility of creator N. Hence, the optimal revenue split decouples the revenues of the creators from the mean utilities generated by all other creators. Moreover, the revenue of each creator is increasing in the dispersion of consumer tastes (ℓ) only if her mean utility is "low," i.e., less than ℓ. Generally speaking, an increase in ℓ both helps and hurts each

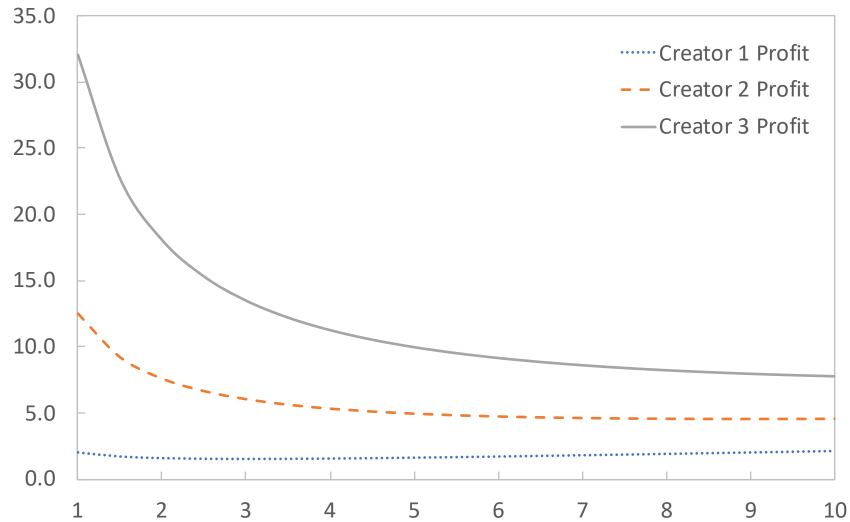


FIGURE 4.1: Creator profit (π_i) as a function of the dispersion of consumer tastes (ℓ). In this example, $\phi_o = 0.5$, $v_1 = 3$, $v_2 = 9$, and $v_3 = 15$.

individual creator: it helps by increasing the upper end of the consumer valuation distribution, but hurts by decreasing the lower end of the distribution. If v_i is small relative to ℓ, then the former dominates, and an increase in ℓ has a net positive impact on profit; if v_i is large relative to ℓ, the opposite is true, and increasing ℓ has a net negative impact on profit due to the downward distortion in utilities from consumers at the lower end of the distribution. As a result, creator profit can be non-monotonic in ℓ, as shown in Figure 4.1. In the figure, creator 1 has low mean utility ($v_1 = 3$), and her profit is first decreasing then increasing in ℓ over the plotted range. By contrast, creator 3 has high mean utility ($v_3 = 15$) and her profit is decreasing in ℓ over the entire range. Part (ii) of the corollary shows how each creator's fraction of the overall revenue—the equilibrium revenue split itself—depends on the problem parameters. Intuitively, each creator's share is increasing in her own mean utility (v_i) and decreasing in the mean utilities of all other creators ($v_{j \neq i}$). In other words, a creator earns a greater fraction of overall revenue as her quality increases, or as the quality of any other creator decreases. Note that although the creator's revenue

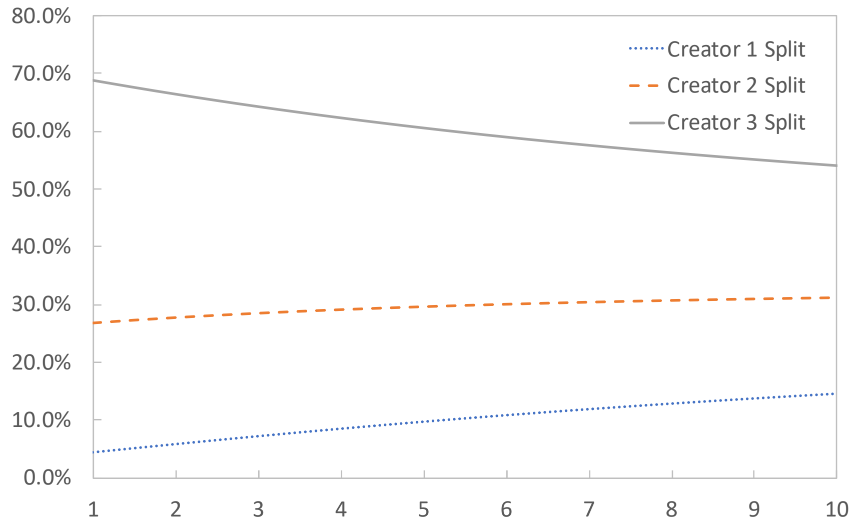


FIGURE 4.2: Optimal split for each creator ($S_i(Q_i, Q_{-i})$) as a function of the dispersion of consumer tastes (ℓ). In this example, $\phi_o = 0.5$, $v_1 = 3$, $v_2 = 9$, and $v_3 = 15$.

split is decrease in the mean utility of all other creators, part (i) showed that her overall revenue is independent of this value; thus, even though creators receive a smaller relative fraction of total revenue as other creators become higher quality, they do not receive less absolute revenue. Lastly, part (ii) also shows that the revenue split of creator i is increasing in the dispersion of consumers tastes ℓ if and only if $\sum_j (v_j - v_i)(v_j + \ell) > 0$. Note that this quantity is definitely negative for creator N, since $v_j \leq v_N$ for all j. Hence, the highest quality creator's split of the revenue is decreasing in ℓ. On the other hand, creator 1's profit is definitely increasing in ℓ, since $v_j \geq v_1$ for all j. This is because of the aforementioned opposite forces of ℓ on the creators depending on the relationship between v_i and ℓ: an increase in ℓ tends to help low quality creators and hurt high quality creators. For creators between 1 and N, profit may be increasing or decreasing in ℓ, depending on the precise value of v_i. This is depicted graphically in Figure 4.2. Taken together, the results in Corollary 1 illustrate how the platform should structure its splitting rule to apportion revenue amongst creators. Higher quality creators should be awarded with a greater relative fraction and absolute amount of revenue. Under an optimal rule, absolute revenue for a creator should not depend on the quality of other creators, although the relative fraction of revenue will depend on all creator qualities. And, as shown in Figures 4.1 and 4.2, as consumer tastes become more dispersed, low quality creators should be rewarded with a higher absolute and relative amount of revenue, while high quality creators should receive a smaller absolute and relative amount of revenue.

4.4.3 Performance of the Best Feasible Linear Split

While a quadratic splitting rule does simultaneously achieve the three desirable properties of feasibility, fairness, and optimality, it also is complex (in the sense that it is non-linear) and may be perceived by creators as favoring high-volume, popular content over more niche content. Because of this, it's possible that platforms may favor the simple linear rule. When this is the case, it is important to understand how much profit is given up by the platform when adopting the simpler linear split. Hence, in this section, we analyze the performance of the best feasible linear splitting rule determined in Proposition 4.2 relative to the optimal quadratic rule derived in Proposition 4.3.

Proposition 4.4. *Let π_P^{linear} be the revenue of the platform under the best fair and feasible linear split described in Proposition 4.2, and let π_P^{quad} be the revenue of the platform under the optimal quadratic rule from Proposition 4.3. Then, the performance ratio of the best fair and feasible linear rule is*

$$\Delta \equiv \frac{\pi_P^{linear}}{\pi_P^{quad}} = \frac{1}{1-\phi_o}\left(1-\phi_o\frac{(v_N+\ell)\sum_j(v_j+\ell)}{\sum_j(v_j+\ell)^2}\right) \leq 1.$$

Proof. From Proposition 4.2, the best fair and feasible linear rule has

$\phi = \phi_o \frac{\sum_j (v_N+\ell)(v_j+\ell)}{\sum_j (v_j+\ell)^2}$ and $S_i(Q_i, Q_{-i}) = \frac{v_i+\ell}{\sum_j (v_j+\ell)}$. Hence, revenue for creator i is

$$\begin{aligned}\pi_i &= \frac{v_i+\ell}{\sum_j (v_j+\ell)} \phi_o \frac{\sum_j (v_N+\ell)(v_j+\ell)}{\sum_j (v_j+\ell)^2} \frac{\sum_j (v_j+\ell)^2}{2\ell} \\ &= \frac{(v_i+\ell)(v_N+\ell)}{2\ell}\phi_o.\end{aligned}$$

The platform's revenue is thus

$$\pi_P^{linear} = p_S(1-\phi) = \frac{\sum_j (v_j+\ell)^2}{2\ell}\left(1 - \phi_o \frac{\sum_j (v_N+\ell)(v_j+\ell)}{\sum_j (v_j+\ell)^2}\right).$$

Under the optimal quadratic rule,

$$\pi_P^{quad} = p_S(1-\phi_o) = \frac{\sum_j (v_j+\ell)^2}{2\ell}(1-\phi_o).$$

This leads to the result. □

When the performance ratio Δ is equal to 1, it means that the best linear rule yields as much platform profit as the optimal quadratic rule. This is only achievable if all creators have identical mean utility and thus every creator is paid equal to her outside option. If even one creator has a higher or lower mean utility than the others, then the linear rule will perform worse than the optimal quadratic rule. Indeed, it can even be the case that the best fair and feasible linear results in negative platform profit, i.e., $\Delta < 0$; the same is never true for the quadratic rule, which always results in non-negative revenue for the platform.

Proposition 4.4 leads to the following corollary:

Corollary 2. *The performance ratio of the best fair and feasible linear rule behaves as follows:*

(i) $\frac{d\Delta}{d\ell} > 0$ *if and only if* $\left(\sum_j (v_j+\ell)\right)^2 > \left(\frac{N}{2} + \frac{\sum_j (v_j+\ell)}{2(v_N+\ell)}\right)\sum_j (v_j+\ell)^2$.

(ii) $\frac{d\Delta}{dv_N} > 0$ *if and only if*

$2(v_N+\ell)\left(\sum_j(v_N+\ell)(v_j+\ell)-\sum_j(v_j+\ell)^2\right) > \sum_{j\neq N}(v_j+\ell)\sum_j(v_j+\ell)^2.$

(iii) $\frac{d\Delta}{d\phi_o} < 0.$

Proof. (i) Differentiating with respect to ℓ,

$$\frac{d\Delta}{d\ell} = \frac{\phi_o}{1-\phi_o}\left(\frac{2(v_N+\ell)\left(\sum_j(v_j+\ell)\right)^2-(v_N+\ell)N\sum_j(v_j+\ell)^2}{\left(\sum_j(v_j+\ell)^2\right)^2}\right)$$

$$-\frac{\phi_o}{1-\phi_o}\left(\frac{\sum_j(v_j+\ell)\sum_j(v_j+\ell)^2}{\left(\sum_j(v_j+\ell)^2\right)^2}\right).$$

This is positive if and only if $\left(\sum_j(v_j+\ell)\right)^2 > \left(\frac{N}{2}+\frac{\sum_j(v_j+\ell)}{2(v_N+\ell)}\right)\sum_j(v_j+\ell)^2.$

(ii) Differentiating with respect to v_N,

$$\frac{d\Delta}{dv_N} = \frac{\phi_o}{1-\phi_o}\left(\frac{2(v_N+\ell)^2\sum_j(v_j+\ell)-2(v_N+\ell)\sum_j(v_j+\ell)^2}{\left(\sum_j(v_j+\ell)^2\right)^2}\right)$$

$$-\frac{\phi_o}{1-\phi_o}\left(\frac{\sum_{j\neq N}(v_j+\ell)\sum_j(v_j+\ell)^2}{\left(\sum_j(v_j+\ell)^2\right)^2}\right).$$

This is positive if and only if

$$2(v_N+\ell)\left(\sum_j(v_N+\ell)(v_j+\ell)-\sum_j(v_j+\ell)^2\right) > \sum_{j\neq N}(v_j+\ell)\sum_j(v_j+\ell)^2.$$

(iii) Differentiating with respect to ϕ_o,

$$\frac{d\Delta}{d\phi_o} = \frac{1}{(1-\phi_o)^2}\left(1-\frac{(v_N+\ell)\sum_j(v_j+\ell)}{\sum_j(v_j+\ell)^2}\right) < 0.$$

□

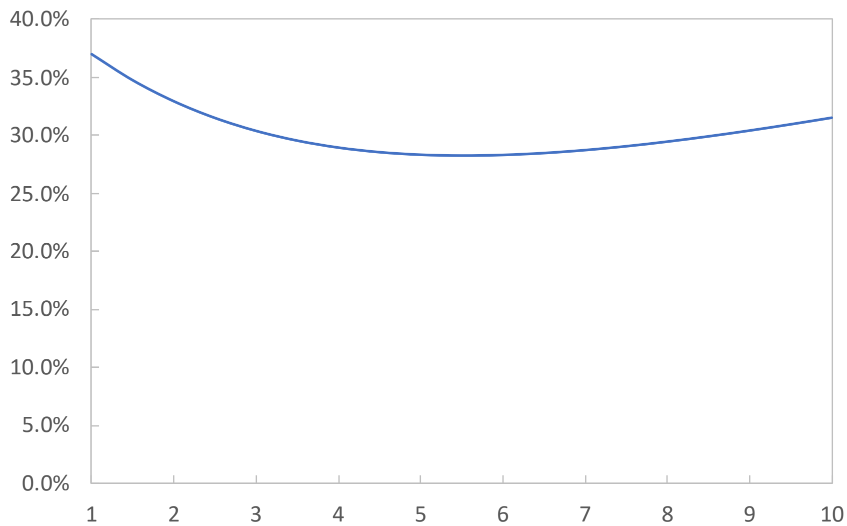


FIGURE 4.3: Performance ratio of the best linear rule (Δ) as a function of consumer taste dispersion (ℓ). In this example, $\phi_o = 0.7$, $v_1 = 1$, $v_2 = 9$, and $v_3 = 19$.

Part (i) of the corollary shows that the performance of the best linear rule may be increasing or decreasing in ℓ, the dispersion of consumer tastes. Generally speaking, the condition in Corollary 2 holds at large ℓ and may be violated for small ℓ. Hence, performance of the best linear rule tends to be minimized at intermediate ℓ, i.e., the linear rule performs worst when consumers have moderately dispersed tastes. As ℓ increases beyond this point, the best linear rule performs closer to the optimal quadratic rule. This behavior can be seen in Figure 4.3. This happens because, as consumer taste dispersion increases, the mean utility of each creator becomes less critical in determining which the consumption, revenue split, and absolute revenue of each creator, since ℓ starts to dominate each $(v_i + \ell)$ term. Put another way, as taste dispersion increases, creators become more equal contributors, and thus the linear rule is less inefficient; recall that, as discussed earlier, the linear rule performs identically to the quadratic rule when all creators have identical mean utilities. Note that the magnitude of Δ, and hence the performance of a linear rule, can be arbitrarily low (as noted above, Δ can even be negative); in the figure, the minimum performance ratio is less than 30%, meaning the platform could more than triple revenues by adopting a quadratic revenue split.

Part (ii) shows similar non-monotonic behavior of the performance ratio as a function of v_N, the mean utility of the highest value creator. There are two counteracting forces at work that generate this non-monotonicity. First, higher v_N means that under a linear rule the platform overpays all other creators by a larger amount, since the equilibrium revenue share ϕ is an increasing function of v_N. At the same time, higher v_N also means that creator N is more dominant and, hence, all other creators are less important to the platform in the sense that they account for a smaller fraction of consumption and overall consumer utility. Hence, initially, Δ decreases in v_N (since the former effect dominates), but as v_N grows very large, the latter effect dominates and Δ increases in v_N. Observe that, in the limit as $v_N \to \infty$, $\Delta \to 1$, i.e., the linear rule converges to the quadratic rule in performance; this is because, in this extreme scenario, the only creator that matters is the highest value creator, and the platform makes binding the participation constraint of this creator using either the linear or quadratic rules.

Finally, part (iii) shows that performance of the best linear split is decreasing in the revenue sharing rate of the outside option. Put another way, the optimal quadratic split offers the most value when the outside option is particularly strong; this is intuitive, as the platform is overpaying creators 1 to $N-1$ the most when ϕ_o (and hence the best linear split revenue sharing rate ϕ given in Proposition 4.2) is large. Indeed, for sufficiently high ϕ_o, Δ can even be negative, meaning the platform loses money when attempting to implement a fair and feasible linear rule. On the other hand, if ϕ_o is small, the platform cannot be overpaying very much under a linear rule, since in any case the total fraction of revenue shared with creators will be small.

4.5 Conclusion

In this paper, we have studied the optimal design of revenue splitting rules for a subscription platform that lists content from multiple creators who are heterogeneous in the qualities of their offerings. The platform seeks to implement a revenue splitting rule that is fair (in that it divides revenue solely based on consumption quantity), feasible (in that it induces all creators in the consideration set to join the platform), and optimal (in the sense that it extracts the maximum surplus from all creators and makes their participation constraints binding). We showed that a quadratic revenue splitting rule meets all three of these criteria, but a linear rule—as is commonly seen in practice—does not. In particular, a linear split cannot achieve the optimality criteria while also simultaneously being fair and feasible, because of the way that high quality creators make an outsize contribution to consumer utility but not to consumption quantity. We then proceeded to analyze the behavior of the optimal quadratic splitting rule, and showed that in equilibrium, the revenue earned by each creator depends only her own mean utility and not the mean utility of any other creators. This is not true of the best fair and feasible linear revenue split, as the revenue of each creator depends both on her own quality and the quality of the highest quality creator. Finally, we analyzed the performance of the best fair and feasible linear revenue split relative to an optimal quadratic split, and showed that a linear rule performs worst if the dispersion of consumer tastes (ℓ) and quality of the best creator (v_N) are both moderate, and if the outside option is strong (ϕ_o is large).

Taken in sum, these results show when it is most—and least—important for platforms to consider an optimal revenue split over a simpler linear split. Since the linear split can perform arbitrarily poorly relative to the quadratic split (it can even lead to negative profit for the platform), this is an important insight, as it can mean the difference between success and failure of the platform's subscription service. In particular, an optimal rule is important if consumer tastes and creator qualities are moderately dispersed and if creators have strong outside options; a linear rule may be sufficient if creators and consumers are identical or widely dispersed, or if outside options for creators are weak.

There are a number of ways these results may be extended in the future. First, it would be interesting to move beyond the uniform distribution of consumer tastes to a general distribution of tastes. This would likely lead to a more complicated non-linear optimal splitting rule than the quadratic rule that is optimal with uniform valuations. Understanding the behavior of the optimal mechanism under a wider range of consumer valuation distributions could yield interesting insights. Second, it would be interesting to consider how the platform can curate the consideration set of creators that used to construct the subscription service, i.e., how creators should be selected and what qualities would be preferred by the platform, given a larger population to choose from.

To summarize, our results show that a critical part of a subscription platform digital information goods is determining how revenue is split amongst the creators. Even with a uniformly distributed consumer valuations, a linear revenue split is not optimal and may perform arbitrarily poorly. This highlights that creators platforms should think carefully about revenue splitting rules when designing subscription services, and, likewise, creators should think carefully about these allocation mechanisms when deciding whether to list on subscription platforms.

5

Conclusion

The chapters of this dissertation were motivated by the supply chain challenges resulting from the rapid adoption of digital information goods by content creators, retailers, and consumers. Digital goods—which differ from physical information goods in important ways—are increasingly ubiquitous, and our focus has been on understanding how these differences impact the management of information good supply chains.

In the second chapter of the dissertation, we considered supply chain design for an information goods creator who produces both physical and digital formats of an information good. We show showed that operational differences between the two formats—specifically, the lack of marginal cost and production leadtime for digital goods—dictated what supply chain structure is preferred by each of the parties involved: the manufacturer, retailers, consumers, and the entire supply chain.

In the third chapter, we turned to a new distribution method for digital content: subscription services, wherein consumers pay a fixed fee for unlimited access to a library of content. Such services are increasingly important, both in terms of the number of creators and consumers who utilize them, and in terms of their economic impact. We studied two content creators (of high and low quality) who how to distribute their products via a single digital platform that has two channels: a la carte and subscription. We showed that the platform's choice of revenue sharing terms determines which, if any, creator will list on the subscription channel. We discussed the tradeoffs for the platform between short term profits and growing a subscription service (inducing all creators to join the service) and the tradeoffs for the creators in terms of expected revenue between choosing to sell via subscription, via a la carte, or both.

In the fourth chapter, we continued our focus on digital good subscription services, and expanded beyond simple linear revenue splits to study the optimal design of revenue splitting rules for subscription platforms listing content by multiple content creators with heterogeneous quality levels. We defined three desirable qualities of a splitting rule: fairness (divides revenue based only on consumption quantity), feasibility (induces all creators to participate), and optimality (extracts maximum surplus from all creators). We showed that even under simple uniform consumer valuations, with heterogeneous creators, a linear rule can never satisfy all three properties simultaneously and can perform arbitrarily poorly (in terms of platform revenue). We then showed that all three properties can be satisfied with a more complicated quadratic splitting rule. We discussed the behavior of the quadratic splitting rule and the relative performance of the "best" linear rule compared to the optimal quadratic rule.

In conclusion, the chapters of this dissertation answer questions that help to better understand how to manage supply chains for digital content distribution. We considered supply chains with both physical and digital information goods, and selling via a la carte and subscription channels. As consumers come to expect more information goods to be available for digital consumption and become more comfortable with digital subscription services, it becomes increasingly more important for platforms and content creators to understand both how digital content supply chains differ from traditional physical content supply chains and how to manage these new digital supply chains.

Appendices

A Appendix to Chapter 2: Partial Substitution and Mixed Cournot-Bertrand Duopoly

In contrast to our base model, suppose that the decision variables are the price of the digital good, p_D, and the quantity of the physical good, Q_P. For simplicity, assume that the physical good cost $c_P = 0$. The prices of the two products given their respective quantities are

$$P_P(Q_D, Q_P) = (A - Q_P - \gamma Q_D)^+$$
$$P_D(Q_D, Q_P) = (A - \gamma Q_P - Q_D)^+.$$

Consequently, $Q_D(P_D) = A - \gamma Q_P - P_D$. We first consider the centralized system, in which the manufacturer sells both the physical and digital products directly to consumers. Since a single firm chooses both the digital price and physical quantity, we may equivalently let the centralized manufacturer choose Q_D and Q_P via a chance of variables, and hence manufacturer profit is

$$\Pi_M(Q_P, Q_D) = (A - \gamma Q_P - Q_D)^+ Q_D + (A - Q_P - \gamma Q_D)^+ Q_P.$$

Optimizing this yields:

Proposition A1. *In a centralized system, the optimal physical quantity is* $Q_P^* = \frac{A}{2(1+\gamma)}$, *the optimal digital price is* $P_D^* = \frac{A}{2}$, *and the optimal manufacturer profit is* $\Pi_M^* = \frac{A^2}{2+2\gamma}$. *Moreover,* $Q_D^* = \frac{A}{2(1+\gamma)}$ *and* $P_P^* = \frac{A}{2}$ *in equilibrium.*

Proof. Follows from optimizing over Q_P and Q_D, then calculating the resulting optimal prices and profits. □

Moving next to the single retailer system, we solve for the equilibrium in reverse, beginning with step 4: the manufacturer's optimal digital good decision, which in this variation of the model, is the digital good price p_D. In step 4, the manufacturer's profit is:

$$\Pi_M(P_D; Q_P, \alpha, w) = (1-\alpha)(A - \gamma Q_P - P_D)P_D + wQ_P. \tag{A.1}$$

Observe that a fraction α of the manufacturer's digital revenue is subtracted from the profit function (to be given to the retailer), while the manufacturer only enjoys a margin w on the physical good. The manufacturer optimizes this expression over P_D, yielding optimal digital price $P_D^s(Q_P, \alpha, w)$, i.e., the optimal digital price as a function of the physical quantity (chosen in step 3), the wholesale price (chosen in step 2) and the royalty fee (chosen in step 1). Given this best response function, in step 3 the retailer optimizes its profit over Q_P, where retailer profit is given by:

$$\begin{aligned}\Pi_R(Q_P; \alpha, w) = {} & \alpha(A - P_D^s(Q_P, \alpha, w) - \gamma Q_P)P_D^s(Q_P, \alpha, w) \\ & + \left(A - Q_P - \gamma \cdot (A - P_D^s(Q_P, \alpha, w) - \gamma Q_P) - w\right) Q_P.\end{aligned} \tag{A.2}$$

The following proposition provides the optimal quantities resulting from these two sequential optimization problems:

Proposition A2. *In the single retailer system, the equilibrium to the subgame is, if* $\gamma < \frac{2-\frac{2w}{A}}{1-\alpha}$,

$$Q_P^s(\alpha, w) = \frac{A \cdot (1 - \frac{\gamma}{2} - \frac{\alpha\gamma}{2}) - w}{2 - \gamma^2 - \frac{\alpha\gamma^2}{2}} \text{ and } P_D^s(\alpha, w) = \frac{2A - \gamma A + \gamma w}{4 - 2\gamma^2 - \alpha\gamma^2}.$$

Otherwise, the equilibrium quantities are $Q_P^s(\alpha, w) = 0$ *and* $P_D^s(\alpha, w) = \frac{A}{2}$.

Proof. Optimizing (A.1) overP_D yields $P_D^s(Q_P, \alpha, w) = \frac{A - \gamma Q_P}{2}$. Next, in step 3, the retailer's profit is,

$$\Pi_R(Q_P; \alpha, w) = (A - \frac{\gamma A}{2} - Q_P + \frac{\gamma^2 Q_P}{2} - w)Q_P + \frac{\alpha}{4}(A - \gamma Q_P)^2, \tag{A.3}$$

Differentiating (A.3) with respect to Q_P,

$$\frac{d\Pi_R(Q_P;\alpha,w)}{dQ_P} = A(1-\frac{\gamma}{2}) - Q_P(2-\gamma^2) - w + \frac{\alpha}{2}(-\gamma A + \gamma^2 Q_P)$$

$$\frac{d^2\Pi_R(Q_P;\alpha,w)}{dQ_P^2} = -2 + \gamma^2 + \frac{\alpha\gamma^2}{2} < 0$$

Hence, the problem is concave, and the optimal Q_P and P_D are given by

$$Q_P^* = \begin{cases} \frac{A\cdot(1-\frac{\gamma}{2}-\frac{\alpha\gamma}{2})-w}{2-\gamma^2-\frac{\alpha\gamma^2}{2}} & if\ A\cdot(1-\frac{\gamma}{2}-\frac{\alpha\gamma}{2}) > w \\ 0 & otherwise \end{cases}$$

$$P_D^* = \begin{cases} \frac{A(4-2\gamma-\gamma^2)+2\gamma w}{8-4\gamma^2-2\alpha\gamma^2} & if\ A\cdot(1-\frac{\gamma}{2}-\frac{\alpha\gamma}{2}) > w \\ \frac{A}{2} & otherwise \end{cases}$$

The corresponding digital quantity and physical price are:

$$Q_D^* = \begin{cases} \frac{A(4-2\gamma-\gamma^2)+2\gamma w}{8-4\gamma^2-2\alpha\gamma^2} & if\ A\cdot(1-\frac{\gamma}{2}-\frac{\alpha\gamma}{2}) > w \\ \frac{A}{2} & otherwise \end{cases}$$

$$P_P^* = \begin{cases} \frac{A(4+2(-1+\alpha)\gamma-2(1+\alpha)\gamma^2+\gamma^3)+2(2-\gamma^2)w}{8-2(2+\alpha)\gamma^2} & if\ A\cdot(1-\frac{\gamma}{2}-\frac{\alpha\gamma}{2}) > w \\ A(1-\frac{\gamma}{2}) & otherwise \end{cases}$$

□

Using these equilibrium quantities, we may now proceed to analyze the contract game. In step 2, the manufacturer's profit is In step 2, the manufacturer's profit is

$$\Pi_M(w;\alpha) = (1-\alpha)P_D^s(\alpha,w)(A-\gamma Q_P^s(\alpha,w) - P_D^s(\alpha,w)) + w\cdot Q_P^s(\alpha,w), \quad \text{(A.4)}$$

where $P_D^s(\alpha,w)$ and $Q_P^s(\alpha,w)$ are the anticipated equilibrium quantities resulting from the quantity subgame in Proposition A2. The manufacturer optimizes this expression over w, resulting in a best response function $w^s(\alpha)$. In step 1, the retailer anticipates the manufacturer's optimal wholesale price response, as well as the equilibrium to the quantity subgame, and optimizes its own profit over the royalty fee $\alpha \leq \bar{\alpha}$:

$$\Pi_R(\alpha) = \alpha(A - \gamma Q_P^s(\alpha, w(\alpha)) - Q_D^s(\alpha, w(\alpha)))Q_D^s(\alpha, w(\alpha)) \\ + (A - Q_P^s(\alpha, w(\alpha)) - \gamma Q_D^s(\alpha, w(\alpha)) - w(\alpha))\, Q_P^s(\alpha, w(\alpha)). \tag{A.5}$$

The resulting equilibrium is described in the following proposition:

Proposition A3. *In the single retailer system, the unique equilibrium is as follows:*
(i) Quantities: $Q_D^s = \frac{A(8-2\gamma-(3+\bar{\alpha})\gamma^2)}{2(8-(5+\bar{\alpha})\gamma^2)}$ *and* $Q_P^s = \frac{2A(1-\gamma)}{8-(5+\bar{\alpha})\gamma^2}$.

(ii) Contract parameters: $w^s = \frac{A(8-8\bar{\alpha}\gamma-6\gamma^2+\gamma^3+4\bar{\alpha}\gamma^3+\bar{\alpha}^2\gamma^3)}{2(8-5\gamma^2-\bar{\alpha}\gamma^2)}$ *and* $\alpha^s = \bar{\alpha}$.

(iii) Prices: $P_P^s = \frac{A(12-4\gamma-2(4+\bar{\alpha})\gamma^2+(3+\bar{\alpha})\gamma^3)}{2(8-(5+\bar{\alpha})\gamma^2)}$ *and* $P_D^s = \frac{A(8-2\gamma-(3+\bar{\alpha})\gamma^2)}{2(8-(5+\bar{\alpha})\gamma^2)}$.

(iv) Manufacturer profit: $\Pi_M^s = \frac{A^2(12-8\gamma-\gamma^2+\bar{\alpha}^2\gamma^2+4\bar{\alpha}(-2+\gamma^2))}{4(8-(5+\bar{\alpha})\gamma^2)}$.

(v) Retailer profit: $\Pi_R^s = \frac{A^2(\bar{\alpha}^3\gamma^4-8(-1+\gamma)^2(-2+\gamma^2)+2\bar{\alpha}^2\gamma^2(-8+5\gamma^2)+\bar{\alpha}(64-84\gamma^2+8\gamma^3+21\gamma^4))}{4(-8+(5+\bar{\alpha})\gamma^2)^2}$.

Proof. In step 2, the manufacturer's profit is independent of w if $A(1 - \frac{\gamma}{2} - \frac{\alpha\gamma}{2}) < w$. Otherwise, manufacturer profit is

$$\Pi_M(w;\alpha) = w \cdot Q_P + (1-\alpha) \cdot (\frac{A - \gamma Q_P}{2})^2. \tag{A.6}$$

Differentiating (A.6) with respect to w,

$$\frac{d\Pi_M(w;\alpha)}{dw} = \frac{A(8 - 6\gamma^2 + \gamma^3 + \alpha^2\gamma^3 + 4\alpha\gamma(-2+\gamma^2)) + 2(-8+(5+\alpha)\gamma^2)w}{(-4+(2+\alpha)\gamma^2)^2}$$

$$\frac{d^2\Pi_M(w;\alpha)}{dw^2} = \frac{2(-8+(5+\alpha)\gamma^2)}{(-4+(2+\alpha)\gamma^2)^2} < 0$$

Hence, profit is concave in w, and the optimal w is given by
$w^*(\alpha) = \frac{A(8-8\alpha\gamma-6\gamma^2+\gamma^3+4\alpha\gamma^3+\alpha^2\gamma^3)}{2(8-5\gamma^2-\alpha\gamma^2)}$. The condition $A(1 - \frac{\gamma}{2} - \frac{\alpha\gamma}{2}) \geq w$ is always satisfied. This means that the maximizer of (A.6) lies inside the feasible region,

hence the manufacturer always sets w sufficiently low that the retailer always orders positive physical quantity. The remainder of the results in the proposition follow.

$$\frac{d\Pi_R}{d\alpha} = \frac{A^2(-512 + 512\gamma + 64(5+3\alpha)\gamma^2 - 4(-17+43\alpha+6\alpha^2)\gamma^4)}{4(-8+(5+\alpha)\gamma^2)^3}$$

$$+\frac{A^2(8(7+5\alpha)\gamma^5 + (-23+31\alpha+15\alpha^2+\alpha^3)\gamma^6) - 64(7+\alpha)\gamma^3}{4(-8+(5+\alpha)\gamma^2)^3} > 0$$

$$\frac{d^2\Pi_R}{d\alpha^2} = \frac{2A^2(-1+\gamma)\gamma^3(128 - 44\gamma - 4(33+4\alpha)\gamma^2}{(-8+(5+\alpha)\gamma^2)^4}$$
$$+\frac{(32+\alpha)\gamma^3 + (28+11\alpha)\gamma^4)}{(-8+(5+\alpha)\gamma^2)^4} \leq 0$$

When we set the first derivative equal to zero, we get no real roots. However, since $\frac{d\Pi_R}{d\alpha} > 0$, we can conclude that $\alpha^* = \bar{\alpha}$. □

Recall that in A2 we showed that there are two possible equilibria depending on the value of the wholesale price. However in equilibrium, the manufacturer's optimal choice of wholesale price is always sufficiently low that the retailer will want to sell positive quantities of both the physical and digital good.

We next move to the dual retailer system, in which the manufacturer sells the products through independent retailers that each specialize in one format. As in the single retailer system, we begin analyzing the equilibrium with step 4, in which the manufacturer determines the price of the digital good to sell (and hence the retail quantity). Manufacturer profit in step 4 is

$$\Pi_M(Q_D; Q_P, \alpha, w) = (1-\alpha)(A - \gamma Q_P - P_D)P_D + wQ_P. \tag{A.7}$$

The manufacturer optimizes this expression over P_D, leading to a best response function $P_D^d(Q_P, \alpha, w)$. The physical retailer anticipates this best response function and, in step 3, chooses a physical quantity to maximize its own profit,

$$\Pi_P(Q_P;\alpha,w) = \big(A-\gamma\cdot(A-\gamma Q_P - P_D^d(Q_P,\alpha,w)) - w\big)\,Q_P. \tag{A.8}$$

The following result describes the equilibrium to this sequential quantity subgame between the manufacturer and the physical retailer:

Proposition A4. *In the dual retailer system, the equilibrium to the quantity subgame is, if $A(1-\frac{\gamma}{2}) > w$,*

$$Q_P^d(\alpha,w) = \frac{A(1-\frac{\gamma}{2})-w}{2(1-\frac{\gamma^2}{2})} \text{ and } P_D^d(\alpha,w) = \frac{A(2-\gamma-\frac{\gamma^2}{2})+\gamma w}{4(1-\frac{\gamma^2}{2})}.$$

Otherwise, $Q_P^d(\alpha,w)=0$ and $P_D^d(\alpha,w)=\frac{A}{2}$.

Proof. Optimizing (A.7) over P_D yields $P_D^*(Q_P,\alpha,w) = \frac{A-\gamma Q_P}{2}$. Next, in step 3, the physical retailer's profit is

$$\Pi_P(Q_P;\alpha,w) = \left(A(1-\frac{\gamma}{2}) - Q_P(1-\frac{\gamma^2}{2}) - w\right)Q_P.$$

Optimizing over Q_P yields $Q_P^d(\alpha,w) = \frac{A(1-\frac{\gamma}{2})-w}{2-\gamma^2}$ if $w < A(1-\frac{\gamma}{2})$, and zero otherwise. The subsequent equilibrium quantities are as given in the proposition. □

Given the equilibrium to the subgame, in step 2, the manufacturer's profit is

$$\Pi_M(w;\alpha) = (1-\alpha)(A-\gamma Q_P^d(\alpha,w) - P_D^d(\alpha,w))P_D^d(\alpha,w) + w\cdot Q_P^d(\alpha,w), \tag{A.9}$$

where Q_P^d and P_D^d are as given in Proposition A4. The manufacturer optimizes this expression over w, leading to a best response function $w(\alpha)$ for the wholesale price. In turn, the digital retailer optimizes its own profit,

$$\Pi_D(\alpha) = \alpha(A-\gamma Q_P^d(\alpha,w) - P_D^d(\alpha,w))P_D^d(\alpha,w) \tag{A.10}$$

over $\alpha \le \bar{\alpha}$. Sequentially optimizing these profit functions leads to the following equilibrium outcome:

Proposition A5. *In the dual retailer system, the unique equilibrium is as follows:*

(i) Quantities:$Q_D^d = \frac{A(8-2\gamma-3\gamma^2)}{2(8-5\gamma^2+\bar{\alpha}\gamma^2)}$ *and* $Q_P^d = \frac{A(2+(-2+\bar{\alpha})\gamma)}{8-5\gamma^2+\bar{\alpha}\gamma^2}$.

(ii) Contract parameters: $w^d = \frac{8A-6\gamma^2A+\gamma^3A-4\bar{\alpha}\gamma A+2\bar{\alpha}\gamma^2A+\bar{\alpha}\gamma^3A}{2(8-5\gamma^2+\bar{\alpha}\gamma^2)}$ *and* $\alpha^d = \bar{\alpha}$.

(iii) Prices: $P_D^d = \frac{A(8-2\gamma-3\gamma^2)}{2(8-5\gamma^2+\bar{\alpha}\gamma^2)}$ *and* $P_P^d = \frac{A(12-2(2+\bar{\alpha})\gamma+2(-4+\bar{\alpha})\gamma^2+3\gamma^3)}{2(8-5\gamma^2+\bar{\alpha}\gamma^2)}$.

(iv) Manufacturer profit: $\Pi_M^d = \frac{A^2(12-8\gamma-\gamma^2+2\bar{\alpha}(-4+2\gamma+\gamma^2))}{4(8-5\gamma^2-\bar{\alpha}\gamma^2)}$.

(v) Physical retailer profit: $\Pi_P^d = \frac{A^2(2+(-2+\bar{\alpha})\gamma)^2(2-\gamma^2)}{2(8-5\gamma^2+\bar{\alpha}\gamma^2)^2}$.

(vi) Digital retailer profit: $\Pi_D^d = \frac{\bar{\alpha}A^2(-8+2\gamma+3\gamma^2)^2}{4(8-5\gamma^2+\bar{\alpha}\gamma^2)}$.

Proof. Note that manufacturer profit is independent of w if $w > A(1-\frac{\gamma}{2})$ (because zero physical quantity is ordered by the physical retailer). If $w < A(1-\frac{\gamma}{2})$, using the quantity subgame equilibrium from Proposition A4, manufacturer profit is

$$\Pi_M = (w - c_P) \cdot \frac{A(1-\frac{\gamma}{2}) - w}{2+3\gamma^2} + \frac{(1-\alpha)}{4}(A - \gamma\frac{A(1-\frac{\gamma}{2}) - w}{2(1-\frac{\gamma^2}{2})})^2.$$

Differentiating this expression with respect to w,

$$\frac{d\Pi_M(w;\alpha)}{dw} = \frac{A(1-\frac{\gamma}{2}) - 2w}{2(1-\frac{\gamma^2}{2})} + \frac{(1-\alpha)}{(4-2\gamma^2)^2} \cdot [4\gamma A - 2\gamma^2 A + \gamma^3 A + 2\gamma^2 w]$$

$$\frac{d^2\Pi_M(w;\alpha)}{dw^2} = \frac{2\cdot(1-\alpha)\gamma^2 - 16 + 8\gamma^2}{(4-2\gamma^2)^2} < 0.$$

Profit is thus concave in the wholesale price, and the optimal wholesale price (provided this solution is interior to the constraint $w < A(1-\frac{\gamma}{2})$) is thus $w^d(\alpha) = \frac{8A-6\gamma^2A+\gamma^3A-4\alpha\gamma A+2\alpha\gamma^2A+\alpha\gamma^3A}{16-10\gamma^2+2\alpha\gamma^2}$. Observe that $w^d(\alpha) = \frac{8A-6\gamma^2A+\gamma^3A-4\alpha\gamma A+2\alpha\gamma^2A+\alpha\gamma^3A}{16-10\gamma^2+2\alpha\gamma^2} < A(1-\frac{\gamma}{2})$. This means there is a unique equilibrium where both physical and digital goods will be sold. Recall that the digital retailer's profit is $\Pi_D(\alpha) = \alpha(P_D^d(\alpha, w))^2$. Differentiating with respect to α,

$$\frac{d\Pi_D}{d\alpha} = \frac{A^2(-8+2\gamma+3\gamma^2)^2(8-(5+\alpha)\gamma^2)}{4(8+(-5+\alpha)\gamma^2)^3} > 0$$

$$\frac{d^2\Pi_D}{d\alpha^2} = \frac{A^2\gamma^2(-8+2\gamma+3\gamma^2)^2(-16+(10+\alpha)\gamma^2)}{2(8+(-5+\alpha)\gamma^2)^4} \leq 0$$

Solving $\frac{d\Pi_D}{d\alpha} = 0$ for α yields $\alpha^* = \frac{8-5\gamma^2}{\gamma^2} > 1$ given our assumptions that $0 \leq \gamma \leq 1$. But since the first derivative of the digital retailer's profit with respect to α is positive, it means that profit is increasing in α and we know therefore $\alpha^* = \bar{\alpha}$. This makes sense because in the dual retailer system, the digital retailer does not realize any benefits from not claiming as high a share of the digital shares as he can. Asking for a lower share of digital profits would boost physical sales but he is in competition with the physical retailer and not only does not benefit from higher physical goods sales, but hurts in digital price from each additional unit of physical good sold. □

Comparing the two systems from the manufacturer's viewpoint, we have:

Proposition A6. *Holding $\bar{\alpha}$ constant in the single and dual retailer systems:*
(i) If $\alpha\gamma(-32+8(5-\alpha)\gamma+4(1+\alpha)\gamma^2+(-12+\alpha+\alpha^2)\gamma^3) = 0$, the manufacturer is indifferent between the single and dual retailer systems.
(ii) If $\alpha\gamma(-32+8(5-\alpha)\gamma+4(1+\alpha)\gamma^2+(-12+\alpha+\alpha^2)\gamma^3) < 0$,, the manufacturer strictly prefers the dual retailer system.

Proof. Part (i) follows immediately from Propositions A3 and A5. To see part (ii), note that profit in the dual retailer system is greater if

$$\frac{A^2(12-8\gamma-\gamma^2+2\bar{\alpha}(-4+2\gamma+\gamma^2))}{4(8-5\gamma^2+\bar{\alpha}\gamma^2)} > \frac{A^2(12-8\gamma-\gamma^2+\alpha^2\gamma^2+4\alpha(-2+\gamma^2))}{4(8-(5+\alpha)\gamma^2)}.$$

Simplifying this inequality, the dual retailer system is preferred if

$$\alpha\gamma(-32+8(5-\alpha)\gamma+4(1+\alpha)\gamma^2+(-12+\alpha+\alpha^2)\gamma^3) < 0$$

Note that given the possible domain of $\alpha, \gamma \in [0,1]$, $A \gg 0$, it is not possible for $\alpha\gamma(-32+8(5-\alpha)\gamma+4(1+\alpha)\gamma^2+(-12+\alpha+\alpha^2)\gamma^3) > 0$ to hold. The case of

$\alpha\gamma(-32+8(5-\alpha)\gamma+4(1+\alpha)\gamma^2+(-12+\alpha+\alpha^2)\gamma^3) = 0$ only happen in the degenerate cases of when goods are perfectly non-substitutable and/or when the digital retailer keeps all profit. The manufacturer is thus always better off choosing the dual retailer system. □

Finally, we can compare supply chain and retail profit in the two systems:

Proposition A7. *Holding $\bar{\alpha}$ constant in the single and dual retailer systems, retail level profit is maximized in the single retailer system. Supply chain profit is maximized in the single retailer system unless the physical and digital good are highly substitutable.*

Proof. Recall that the (combined) retailer profit under the single and dual retailers are as follows:

$$\Pi_R^s = \frac{A^2(\bar{\alpha}^3\gamma^4 - 8(-1+\gamma)^2(-2+\gamma^2) + 2\bar{\alpha}^2\gamma^2(-8+5\gamma^2)}{4(-8+(5+\bar{\alpha})\gamma^2)^2} + \frac{\bar{\alpha}(64 - 84\gamma^2 + 8\gamma^3 + 21\gamma^4))}{4(-8+(5+\bar{\alpha})\gamma^2)^2}$$

$$\Pi_P^d + \Pi_D^d = \frac{A^2(2+(-2+\bar{\alpha})\gamma)^2(2-\gamma^2)}{2(8-5\gamma^2+\bar{\alpha}\gamma^2)^2} + \frac{\bar{\alpha}A^2(-8+2\gamma+3\gamma^2)^2}{4(8-5\gamma^2+\bar{\alpha}\gamma^2)}$$

It follows that $\Pi_R^s - \Pi_P^d + \Pi_D^d > 0$ as long as $\alpha\gamma > 0$. Recall that the total supply chain profit under the single retailer and dual retailer are as follows:

$$\Pi_{SC}^s = \frac{A^2(112 - 96\gamma - 12(5+2\alpha)\gamma^2 + 8(7+2\alpha)\gamma^3 + (-3+2\alpha+\alpha^2)\gamma^4)}{4(-8+(5+\alpha)\gamma^2)^2}$$

$$\Pi_{SC}^d = \frac{A^2(112 + 16(-6+\alpha)\gamma - 4(15 - 2\alpha + \alpha^2)\gamma^2}{4(-8 + (5+\alpha)\gamma^2)^2}$$
$$+ \frac{4(14 - 6\alpha + \alpha^2)\gamma^3 + (-3 + 6\alpha)\gamma^4)}{4(-8 + (5+\alpha)\gamma^2)^2}$$

When we compare the two supply chain profits, we can see that $\Pi_{SC}^s < \Pi_{SC}^d$ when $\alpha\gamma(-1024 + 256(6+\alpha)\gamma + 768\gamma^2 - 64(29 + 8\alpha + \alpha^2)\gamma^3 + 16(7 + 2\alpha + 3\alpha^2)\gamma^4 + 4(156 + 77\alpha + 6\alpha^2 + \alpha^3)\gamma^5 - 4(30 + 5\alpha + \alpha^3)\gamma^6 + (-40 - 55\alpha - 14\alpha^2 + \alpha^3)\gamma^7) \geq 0$. (Otherwise $\Pi_{SC}^s < \Pi_{SC}^d$.) Condition 1 is met for high values of γ. Numerically, $\gamma \approx 0.9$ is the threshold above which some combinations of α, γ will result in the above condition being met. □

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