

# Essays in International Finance

by

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Philipp Sadowski

Dissertation submitted in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy in the Department of Economics  
in the Graduate School of Duke University  
2015

ABSTRACT

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# Abstract

This dissertation addresses three key issues in international finance and economics: the uncovered interest rate parity puzzle in exchange rates, the home bias puzzle in portfolio allocations, and the surprising lack of correlation between terms of trade shocks and output in small open economies.

The first chapter shows that the much-studied Uncovered Interest Rate Parity (UIP) puzzle, the observation that exchange rates do not adjust sufficiently to offset interest rate differentials, is more complicated than commonly understood. I show that the puzzle changes nature with the horizon. I confirm existing short-run evidence that high interest rate currencies depreciate less than predicted by the interest rate differential. But, building on Engel (2012), at longer horizons (4 to 7 years) I find a reverse puzzle: high interest rate currencies depreciate too much. Interestingly, the long-horizon excess depreciation leads exchange rates to converge to the UIP benchmark over the long-run. To address the changing nature of the puzzle, I propose a novel model, based on the mechanism of bond convenience yields, that can explain both the short and the long horizon UIP violations. I also provide direct empirical evidence that supports the mechanism.

In chapter 2, I address the puzzling observation that portfolios are concentrated in asset classes which comove strongly with the non-financial income of investors. As an explanation, I propose a framework of endogenously generated information asymmetry, where rational agents optimally choose to focus their limited attention

on risk factors that drive both their non-financial income and some of the risky asset payoffs. In turn, the agents concentrate their portfolios in assets driven by those endogenously familiar factors. I explore an uncertainty structure that implies decreasing returns to information, whereas the previous literature has focused on a setup with increasing returns. I show that the two frameworks have differing implications, which I test in the data and find support for decreasing returns to information.

In chapter 3, I address the puzzling lack of correlation between Terms of Trade (ToT) and the Small Open Economy (SOE) GDP. A SOE model typically relies on three sources of exogenous disturbances: world real interest rate, Terms of Trade (ToT) and technology. However, the empirical literature has failed to reach a consensus on the relative importance of the terms of trade as a driver of business cycles, with some papers claiming they are hugely important while others find no evidence of a relationship at all. Kehoe and Ruhl (2008) have recently shown that the weak empirical link between ToT and the GDP might be due to measurement limitations with the output series in an open economy framework. This paper merges data on national accounts with data on global trade flows for a panel of 31 countries and finds that Terms of Trade have a negligible effect on GDP but a strong effect on aggregate consumption. The evidence supports the hypothesis that ToT are important drivers of business cycles, but measurement issues with GDP obscure their relationship with real output. This further suggests that researchers should be careful when equating model output with measured GDP in an open economy setup.

To my parents and Lily.

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# Exchange Rates and UIP Violations at Short and Long Horizons

## 1.1 Introduction

The Uncovered Interest Rate Parity (UIP) condition, which equates the expected returns on domestic and foreign bonds, is central to exchange rate determination in standard open economy models. It is directly implied by risk-neutrality, and can also be derived by log-linearizing the standard no-arbitrage pricing conditions that obtain in a large class of equilibrium models. The excess return of foreign bonds over home bonds consists of two components: the interest rate difference between the two countries and the exchange rate change over the investment period. Thus, equating expected returns across countries requires that, on average, the exchange rates of high interest rate countries depreciate and offset potential gains that arise from interest rate differentials.

Despite the fundamental role this condition plays in theoretical models, its failure in the data is well established. Numerous papers have documented that currencies fail to depreciate sufficiently to offset interest rate differentials, which opens up profit

opportunities (e.g. the carry trade) and violates the UIP condition (see the surveys by Hodrick (1987); Lewis (1995); Engel (1996, 2013)).<sup>1</sup> In fact, most estimates cannot reject the hypothesis that exchange rates follow a random walk, implying that, on average, they do not offset any of the interest rate differential.<sup>2</sup> Overall, at the shorter horizons emphasized by the literature, the empirical evidence has consistently characterized the UIP puzzle as insufficient currency depreciation in response to high domestic interest rates.<sup>3</sup>

In this paper, inspired by the essential empirical insight in Engel (2012), I show that the puzzle is more complicated than commonly understood; I find that UIP violations change nature and direction with the horizon. I confirm the standard result that currencies fail to depreciate sufficiently in response to high interest rate differentials at short horizons (up to 3 years). However, at the longer horizons of 4 to 7 years, I find the reverse puzzle: high-interest rate currencies depreciate *too much*. To address the puzzle in its full complexity, I propose a novel model in which bonds act as convenience assets that help facilitate transactions (similar to money), and I show that the model can closely match both the short and the long horizon UIP violations. In addition, I provide direct empirical evidence in support of the mechanism, by verifying two of its key implications in the data.

In the first part of the paper, I test the UIP condition at different horizons by exploiting the implication that excess returns on one-month foreign bonds, over

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<sup>1</sup> See also Bilson (1981); Fama (1984); Froot and Thaler (1990), Canova and Ito (1991), Canova and Marrinan (1993), Bekaert and Hodrick (1993), Backus et al. (1993), Hai et al. (1997), Burnside (2013) and others.

<sup>2</sup> On the one hand, most UIP regressions, cannot reject the hypothesis that exchange rate changes are unpredictable. On the other, a large related literature, following Meese and Rogoff (1983), finds that it is exceedingly difficult to beat the random walk forecast out of sample (see the survey by Rossi (2013))

<sup>3</sup> The UIP puzzle, also known as the Forward Premium puzzle, is separate from the equity risk premium puzzle. The puzzle with currencies is about the time variation in excess returns and in their relationship with interest rate differentials. A constant risk premium, for example, would not resolve the puzzle.

one-month home bonds, must be zero in expectation and, hence, unforecastable at all horizons. I examine this with a series of direct predictive regressions, where I regress the one-month excess return  $k$  periods into the future on the current interest rate differential, using a panel of 18 OECD currencies. I find that the regression coefficients are significantly different from zero, and hence UIP is violated, at horizons of up to 7 years. I confirm the usual finding that the coefficients are negative, which implies that high interest rate currencies do not depreciate sufficiently, at horizons of up to 3 years. However, I find that the coefficients become positive at horizons of 4 to 7 years, implying that high interest rate currencies eventually start depreciating too much, relative to UIP. This is also a violation of UIP but constitutes the reverse puzzle.

Moreover, I find that the sum of positive UIP violations is roughly equal to the sum of negative violations. This leads to the interesting result that the excess depreciation at longer horizons offsets the initial insufficient depreciation, and thus, exchange rates converge to the UIP benchmark in the long run. This result is qualitatively consistent with and helps provide a new interpretation to previous studies that have looked at the excess returns on long-term bonds (5+ years) and have found some support for UIP in the long run.<sup>4</sup> My findings show that the UIP puzzle is not a short-run phenomenon that averages out over the long-run, but rather it changes its nature. Exchange rate movements violate the UIP condition at both short and long horizons, but the direction of the violations changes, and the effects roughly cancel out in the long-run. This has interesting, non-linear implications about the underlying exchange rate dynamics, and leads me to reject the random walk hypothesis as well.<sup>5</sup>

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<sup>4</sup> See Flood and Taylor (1996); Chinn and Meredith (2005); Chinn (2006), Chinn and Quayyum (2012). There is an analytical link between the two sets of results under risk-neutrality, but not necessarily otherwise.

<sup>5</sup> The results in this paper refer to *in-sample* analysis, and no claims are made about out-of-

The second part of the paper develops a novel two-country model of time-varying bond convenience yields and shows that it can closely match both the short-horizon and long-horizon UIP deviations. The key feature of the model is that government bonds offer both financial returns and convenience, or liquidity, benefits. Following the literature on bond convenience yields (e.g. Krishnamurthy and Vissing-Jorgensen (2012)), I refer to the convenience of a bond as the non-pecuniary benefit arising from the fact that safe and liquid government bonds can act as a substitute for money and help facilitate transactions. The convenience yield is the amount of interest investors are willing to forego in exchange for the convenience benefit.

In the model, excess currency returns arise as a compensation for differences in convenience yields across countries, and UIP deviations are driven by endogenous time variation in the convenience yield differential. A decrease in the convenience value of home bonds, relative to foreign bonds, leads to a corresponding, compensating increase in the excess financial return on home bonds, over foreign bonds. This is achieved through excess return on the home currency, which constitutes a violation of the standard UIP condition. Moreover, when the home convenience yield falls, the home interest rate tends to rise because investors require a higher financial compensation to hold the supply of home government debt, as its convenience benefit has decreased. This generates the classic UIP puzzle relationship of high-interest rate currencies earning excess returns.

The switch in the sign of UIP violations at longer horizons is driven by the interaction between monetary and fiscal policy. The dynamics of the convenience yield differential are closely linked to the relative supply of government debt, and hence the dynamics of UIP violations are determined by the joint dynamics of interest

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sample predictability. Incorporating the results and insights in an out-of-sample analysis is a future research project.

rates and debt.<sup>6</sup> In particular, I am able to show that when monetary policy is independent of fiscal considerations and tax policy is persistent, debt has cyclical, complex root dynamics that are also imparted on the convenience yield differential. This generates a cyclical profile of UIP deviations that are negative at short horizons, but turn positive at longer horizons. If tax policy reacts quickly to debt levels, or if the central bank helps through inflating debt away, the UIP violations, especially at longer horizons, decrease, and we could have a situation in which UIP violations are negative at all horizons.

As a first step, I analyze the mechanism in a stylized model that allows me to analytically characterize the main results and helps illustrate the key intuition. Next, to study the mechanism's quantitative implications, I incorporate it in a benchmark, two-country open economy model. I calibrate the parameters with standard values from the literature, independently of the implied UIP violations, and show that the model closely matches the estimated UIP deviations at both short and long horizons. Lastly, I provide direct empirical support for the mechanism by verifying two of its key implications in the data. First, I show that excess currency returns are indeed closely related to and forecastable by the level of government debt. Second, I document that currencies with a lower degree of capital controls, a common proxy for central bank independence, exhibit stronger UIP violations, especially at longer horizons, as predicted by the monetary-fiscal interplay in the model.<sup>7</sup>

My empirical analysis is inspired by and related to recent work on the relationship between the level of the exchange rate and the real interest rate by Engel (2012). One

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<sup>6</sup> The relationship between the supply of debt and the convenience yield is also emphasized in the previous literature on bond convenience yields.

<sup>7</sup> I use capital controls as a proxy for central bank independence due to the lack of direct and comparable measurements that span the time period and the countries in my data set. On the other hand, a number of papers find that countries with (i) greater central bank independence and/or (ii) lesser dependence on seignorage and inflation tax revenues are associated with lower degrees of capital controls (Alesina and Tabellini (1989), Drazen (1989), Alesina and Grilli (1994), Grilli and Milesi-Ferretti (1995), Leblang (1997), Quinn and Inclan (1997), Bai and Wei (2000)).

of Engel's findings is that when a country's real interest rate is above its mean, its currency tends to earn positive excess returns in the short-run, and negative excess returns in the longer-run. My empirical results, which are based on predicting returns using nominal interest rates and also exploit the cross-sectional variation in the data, echo his.<sup>8</sup> Additionally, Engel (2012) estimates that the expected cumulative excess returns of high-interest rate currencies, at very long horizons, are negative, rather than positive, as the classic UIP puzzle relation would dictate. I find a different result in my panel analysis using the nominal interest differential as the predictor: cumulative returns at the longest horizons are roughly zero and hence appear to be consistent with UIP.

The theoretical mechanism is novel to the literature on the UIP puzzle, which largely turns to one of two explanations: time-varying risk (Bekaert (1996), Farhi and Gabaix (2008), Alvarez et al. (2009), Verdelhan (2010); Bansal and Shaliastovich (2012); Colacito and Croce (2013) among others), and deviations from rational expectations (e.g. Gourinchas and Tornell (2004), Bacchetta and Van Wincoop (2010), Burnside et al. (2011), Ilut (2012)).<sup>9</sup> In addition, I analyze the changing nature of UIP violations at short, medium and long-horizons, whereas the literature has focused on explaining the short-run negative UIP violations. The new UIP mechanism is also complementary to existing ones and incorporating it in a model together with

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<sup>8</sup> In a new version of his paper, Engel (2012) also considers results based on nominal data. An advantage of working with nominal interest rates is that no model (such as a VAR) is needed in order to estimate expected inflation and construct ex-ante real interest rates. This allows me to utilize the method of local projections (Jorda (2005)), which is well-suited for examining long-horizon dynamics.

<sup>9</sup> Gabaix and Maggiori (2014) develop a model where the availability of liquid assets affects the risk-bearing capacity of financial intermediaries, and thus the risk-premia in exchange rate markets.

some of the previous approaches is a promising direction for future work.<sup>10,11,12</sup>

A number of papers have quantified the convenience yield in the data and documented its important role in the determination of equilibrium bond prices (for example Fontaine and Garcia (2012), Krishnamurthy and Vissing-Jorgensen (2012), Smith (2012), Greenwood and Vayanos (2014)). A related theoretical literature has explored bond convenience yields as a possible explanation for asset pricing puzzles such as the equity risk-premium, the low risk-free rate and the term premium (e.g. Bansal and Coleman II (1996); Bansal et al. (2011); Lagos (2010, 2011)). I extend the theoretical analysis of convenience yields by introducing it to an open economy setting, applying it to the UIP puzzle, and by studying the implications of time-varying convenience yield differentials.

Section 2 presents the main empirical results, Section 3 briefly introduces convenience yields and then Section 4 lays out and analyzes the analytical model, while Section 5 presents the quantitative model. Section 6 provides direct empirical evidence in support of the mechanism and Section 7 concludes.

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<sup>10</sup> For example, Lustig et al. (2013) find that transitory risk accounts for the great majority of short-horizon carry trade profits. Thus, building a model that incorporates both time varying risk and convenience yields could potentially deliver even stronger negative UIP coefficients at short horizons while preserving the positive UIP violations at longer horizons.

<sup>11</sup> A number of recent papers find that traditional risk factors cannot explain currency returns (Burnside (2007), Burnside et al. (2010), Burnside (2010, 2011), Menkhoff et al. (2012b)). My model suggests that time-varying convenience yields could have acted as omitted variables in such analyses and incorporating them in future studies could help properly quantify the effects of both risk and convenience factors.

<sup>12</sup> Hassan and Mano (2013) find that a significant portion of carry trade profits is not due to capturing time-variation in excess returns, but rather due to persistent differences in excess returns across currencies. This can also be rationalized by the model, given steady state differences in convenience yields.

## 1.2 Uncovered Interest Parity

### 1.2.1 The UIP Condition in Economic Models

This section gives a brief overview of the Uncovered Interest Parity condition and highlights the important role it plays in the determination of equilibrium exchange rates in standard open economy models.

To fix ideas, I define  $S_t$  to be the exchange rate, in terms of home currency per one unit of foreign currency (e.g. 1.25 USD per EUR), and  $i_t$  and  $i_t^*$  as the nominal interest rates on default-free bonds at home and abroad. For ease of exposition, I will refer to the US dollar as the “home” currency and the Euro as the “foreign” currency. A \$1 investment in US bonds at time  $t$  offers a return of  $1 + i_t$  dollars next period. The same \$1 invested in Euro denominated bonds would earn  $\frac{S_{t+1}}{S_t}(1 + i_t^*)$  dollars next period. That is, we first need to exchange this one dollar for Euros and obtain  $\frac{1}{S_t}$  EUR in return. Investing this amount of Euros earns a gross interest rate of  $1 + i_t^*$  that next period can be exchanged back into dollars at the rate  $S_{t+1}$ , for a total return of  $\frac{S_{t+1}}{S_t}(1 + i_t^*)$  dollars.

Assuming that the law of one price holds the fundamental theorem of asset pricing tells us that there exists a stochastic process  $M_{t+1}$ , usually referred to as the *stochastic discount factor*, such that<sup>13</sup>

$$E_t(M_{t+1}(1 + i_t)) = 1 \tag{1.1}$$

$$E_t(M_{t+1}\frac{S_{t+1}}{S_t}(1 + i_t^*)) = 1. \tag{1.2}$$

For example, in standard representative agent models with separatively additive utility over consumption the stochastic discount factor is equal to the marginal rate of

---

<sup>13</sup> The law of one price states that the price of a linear combination of assets must be equal to that same linear combination of the underlying assets' prices. In other words, any unique payoff, whether it is the payoff of a particular asset or formed by constructing a portfolio of distinct assets, must have a unique price.



substitution (adjusted for inflation because the payoffs considered here are nominal):

$$M_{t+1} = \frac{U'(C_{t+1})}{U'(C_t)} \frac{1}{\Pi_{t+1}}.$$

To obtain the Uncovered Interest Parity condition log-linearize the two equations, subtract them from one another and re-arrange to arrive at

$$E_t(s_{t+1} - s_t + i_t^* - i_t) = 0 \tag{1.3}$$

where lower case letters represent variables in logs and I have used the approximation  $i_t \approx \ln(1 + i_t)$ .<sup>14</sup>

The condition equates, up to a first order log-approximation, the expected return on foreign bonds,  $E_t(s_{t+1} - s_t + i_t^*)$ , to the expected return on the home bond,  $i_t$ . This restricts the joint dynamics of exchange rates and interest rates, and delivers strong implications for exchange rate behavior. In particular, the UIP condition states that exchange rates are expected to adjust in response to non-zero interest rate differentials and offset them. Thus, if for example the US interest rate is 1% higher than the Euro interest rate, i.e.  $i_t - i_t^* = 0.01$ , the UIP condition would imply that the Euro is expected to appreciate against the dollar by 1% as well, so that  $E_t(s_{t+1} - s_t) = 0.01$ . Hence, in expectation, there are no gains to be made by borrowing in one currency and investing in the other.

The limited set of assumptions underlying the UIP condition ensures that it obtains, at least up to a first order approximation, in a large class of models. It is a fundamental part of the overwhelming majority of open economy models and it puts important restrictions on the model-implied joint dynamics of exchange rates and interest rates. Given its theoretical importance, it is not surprising that there has been great interest in examining the condition's validity in the data, and I turn to empirical tests of the UIP condition next.

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<sup>14</sup> The log-linearization is typically done around the symmetric steady state where  $S_{t+1} = S_t = 1$  and  $i_t = i_t^*$ , because this allows us to express the condition in terms of the log-variables themselves. But the log-linearized condition holds for any arbitrary point of approximation.

### 1.2.2 The UIP Puzzle

The failure of the UIP condition in the data is one of the longest standing and best documented puzzles in international finance. This section briefly reviews the results in the existing literature and confirms that the condition is similarly violated in the data set used by this paper. The literature documenting the empirical failure of UIP is vast and is still an active area of research, with numerous studies expanding on the seminal contributions by Bilson (1981) and Fama (1984). For excellent surveys, please see Hodrick (1987); Froot and Thaler (1990); Engel (1996, 2013).<sup>15,16</sup>

Examining the UIP condition in the data is typically done by testing whether any variable in the time  $t$  information set can help forecast the return on foreign bonds relative to home bonds. As is standard in the literature I will equivalently refer to the relative return on foreign to home bonds as “excess return on foreign bonds” and also as “excess currency return”. I denote the one period excess return from time  $t$  to  $t + 1$  as  $\lambda_{t+1}$ :

$$\lambda_{t+1} \equiv s_{t+1} - s_t + i_t^* - i_t. \quad (1.4)$$

The UIP condition requires  $E_t(\lambda_{t+1}) = 0$  and hence  $\text{Cov}(\lambda_{t+1}, X_t) = 0$  for any variable  $X_t$  in the time  $t$  information set. In other words, there should be no variables known this period that can forecast the excess return that will realize next period. The bulk of the empirical work documenting the failure of the UIP condition comes down to showing that the current interest rate differential,  $i_t - i_t^*$ , can indeed forecast future excess returns.

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<sup>15</sup> See also Canova (1991); Canova and Ito (1991); Bekaert and Hodrick (1992); Backus et al. (1993); Bekaert and Hodrick (1993); Canova and Marrinan (1993); Cheng (1993); Hai et al. (1997); Bekaert (1995); Burnside (2013); Burnside et al. (2006)

<sup>16</sup> A related finding is the high profitability of the carry trade, an investment strategy that goes long high-interest rate currencies and short low-interest rate currencies, and that should yield zero average return under UIP. Some papers that document profitable currency trading strategies are Lustig and Verdelhan (2007); Villanueva (2007); Burnside et al. (2008); Brunnermeier et al. (2008); Burnside et al. (2010); Lustig et al. (2011); Menkhoff et al. (2012a)

The vast majority of the literature focuses on some version of the original regression specification estimated by Fama (1984):

$$\lambda_{t+1} = \alpha_0 + \beta_1(i_t - i_t^*) + \varepsilon_{t+1}, \quad (1.5)$$

where typically the base or “home” currency is the USD and  $i_t$  is the US interest rate. Under the null hypothesis that the UIP condition holds we should obtain  $\alpha_0 = \beta_1 = 0$  so that the average excess return is zero and not forecastable by current interest rates. Contrary to this, numerous papers have found that  $\beta_1 < 0$  which implies that currencies which are experiencing high interest rates today are also expected to earn positive excess returns in the future.<sup>17</sup> These findings form the basis for the celebrated “UIP Puzzle”.<sup>18</sup>

Next, I turn my attention to the UIP tests considered in this paper. For my empirical analysis I construct a data set of 18 major currencies, all advanced OECD countries, with the US dollar as the base currency and the data spans the period 01/01/1976 - 28/06/2013 at the daily frequency.<sup>19,20</sup> The data comes from Datastream, and forms an unbalanced panel because the Euro-legacy currencies cease to exist in 1999. As is standard in the literature, I do not enter the Euro as a separate currency but rather attach it at the end of the Deutsche Mark series. The time series for the other Euro-legacy currencies stops on January 1st, 1999, the time of Euro adoption for the currencies in my data set. Per the established convention, the interest rate differentials are computed from forward rates using the Covered Interest

<sup>17</sup> The typical frequency of the regressions is monthly or quarterly.

<sup>18</sup> The UIP Puzzle is traditionally centered on the  $\beta_1$  coefficient not being zero, and not  $\alpha_0$  because most studies find that  $\alpha_0$  are indeed insignificantly different from zero.

<sup>19</sup> The currencies are for Australia, Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

<sup>20</sup> Results at the monthly frequency, which have been more often used in the previous literature, are available in the Appendix and show that the estimates are extremely similar to the daily data ones.

Rate Parity (CIP).<sup>21</sup>

The interest rates I use are for one month (30 days), hence with daily observations on the exchange rate the standard UIP regression can be expressed as

$$s_{j,t+30} - s_{j,t} + i_{j,t}^* - i_t = \alpha_{j,0} + \beta_{j,1}(i_t - i_{j,t}^*) + \varepsilon_{j,t+1},$$

and I estimate this equation currency-by-currency, with  $j$  indexing the different currencies. The time period is one day, and  $i_t$  is the 30 day interest rate on USD and  $i_{j,t}^*$  the 30 day interest rate on currency  $j$ , hence the left-hand side variable is a 30 day excess return on the foreign currency. I use Newey-West standard errors to correct for the serial correlation induced by the overlapping periods in the dependent variable and report the results in Table 1.1. My estimates reaffirm the well established UIP Puzzle - I find that all  $\beta_1$  point estimates are negative and almost all are statistically significant at conventional levels (15 out of 18).<sup>22</sup>

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<sup>21</sup> This is the standard procedure in the literature because data on forward rates is available for more periods and more countries than reliable data on constant maturity, default-free bonds and the CIP has been found to hold very well in the data. Nevertheless, the Appendix shows that the results remain unchanged when considering interest rates on physical assets, although this cuts down on the sample size.

<sup>22</sup> I also re-affirm the standard finding that the constants in the UIP regression are generally found to be insignificant. I find that only 3 out of the 18 estimates are significant at standard levels, the pooled panel estimate is small and not significant, and I cannot reject the hypothesis that the mean of the fixed effects is different from zero in the fixed effects panel regression.

Table 1.1: UIP Regression Currency by Currency

Country	Currency	$\alpha_0$	(s.e.)	$\beta_1$	(s.e.)	$\chi^2$ ( $\alpha_0 = \beta_1 = 0$ )	$R^2$
Australia	AUD	-0.001	(0.002)	-1.63***	(0.48)	16.3***	0.014
Austria	ATS	0.002	(0.002)	-1.75***	(0.58)	9.5***	0.023
Belgium	BEF	-0.0002	(0.002)	-1.58***	(0.39)	17.5***	0.025
Canada	CAD	-0.003	(0.001)	-1.43***	(0.38)	19.1***	0.013
Denmark	DKK	-0.001	(0.001)	-1.51***	(0.32)	25.4***	0.025
France	FRF	-0.001	(0.002)	-0.84	(0.63)	1.9	0.007
Germany	DEM	0.002	(0.001)	-1.58***	(0.57)	7.9**	0.015
Ireland	IEP	-0.002	(0.002)	-1.32***	(0.38)	12.3***	0.020
Italy	ITL	-0.002	(0.002)	-0.79**	(0.33)	7.0**	0.013
Japan	JPY	0.006***	(0.002)	-2.76***	(0.51)	28.9***	0.038
Netherlands	NLG	0.003	(0.002)	-2.34***	(0.59)	16.0***	0.041
Norway	NOK	-0.0003	(0.001)	-1.15***	(0.39)	10.4***	0.013
New Zealand	NZD	-0.001	(0.002)	-1.74***	(0.39)	28.3***	0.038
Portugal	PTE	-0.002	(0.002)	-0.45**	(0.20)	5.9*	0.019
Spain	ESP	0.002	(0.003)	-0.19	(0.46)	2.8	0.001
Sweden	SEK	0.0001	(0.001)	-0.42	(0.50)	0.9	0.002
Switzerland	CHF	0.005***	(0.002)	-2.06***	(0.55)	13.9***	0.026
UK	GBP	-0.003**	(0.001)	-2.24***	(0.60)	14.2***	0.028
Panel, pooled		0.0002	(0.001)	-0.79***	(0.15)	22.3***	
Panel, fixed eff.				-1.01***	(0.21)	19.1***	

*Notes:* This table presents estimates of  $\alpha_0$  and  $\beta_1$  from the regression  $s_{j,t+1} - s_{j,t} + i_{j,t}^* - i_{j,t} = \alpha_{j,0} + \beta_{j,1}(i_{j,t} - i_{j,t}^*) + \varepsilon_{j,t+1}$ . The standard errors in single currency regressions are Newey-West errors robust to serial correlation. The standard errors for the panel estimations are computed according to the Driscoll and Kraay (1998) method that is robust to heteroskedasticity, serial correlation and contemporaneous correlation across equations. The base currency is the USD.

The evidence of negative and significant  $\beta_1$  is pervasive throughout all 18 currencies. Moreover, the actual magnitudes of the estimated coefficients are quite similar to each other which leads me to consider panel regressions as a good way to summarize the results, that also increases efficiency by leveraging the cross-sectional variation of the data.<sup>23</sup> I re-estimate the UIP regression as a panel in two ways, first by pooling all the data together and imposing  $\alpha_{j,0} = \alpha_0$  and  $\beta_{j,1} = \beta_1$  and second by allowing the constants to differ among currencies (fixed effects) but still restricting the slope to be the same. The resulting coefficient estimates and the corresponding

<sup>23</sup> The observation that UIP coefficients tend to be rather similar across countries (at least among developed economies) is a pervasive feature of the data and has been noted as early as Fama (1984)

Driscoll and Kraay (1998) errors that correct for both cross-sectional and time series dependence in the error term are presented in the last two lines of Table 1.1.<sup>24</sup> The estimated  $\beta_1$  coefficients are negative and highly significant and the constants are not statistically different from zero.<sup>25</sup>

Lastly, notice that the panel estimates imply that  $\beta_1$  is roughly  $-1$ , which implies that the exchange rates changes are unpredictable on the basis of the interest rate differential. This is also a common finding in the literature and is consistent with the hypothesis that exchange rates are a random walk. There is also a large, related literature, following the seminal contribution of Meese and Rogoff (1983), which has shown that beating the random walk forecast out of sample is exceedingly difficult. Such findings have lead to the general view that the random walk is a reasonable description of exchange rate behavior.

In summary, all results point to the fact that  $\beta_1$  is negative and hence periods of high domestic interest rates tend to forecast positive excess returns on the home bonds versus foreign bonds. This is the crux of the famous “UIP Puzzle” and has given rise to the common saying in the literature that “high-interest rate currencies earn high returns”, a fact that is further supported by the evidence on the profitability of the carry trade.<sup>26</sup>

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<sup>24</sup> Applying the Vogelsang (2012) asymptotic adjustment does not make any perceptible difference to the inference.

<sup>25</sup> In the case of the fixed effects estimation, the overwhelming majority of fixed effect estimates are found to be insignificant. I also cannot reject the hypothesis that the mean of the fixed effects is zero.

<sup>26</sup> Recent work by Hassan and Mano (2013) has shown that only a part of the carry profits can be explained by time-variation in the interest rate differential and that a significant portion is actually related to the persistent differences in interest rates across countries. Nevertheless, Burnside (2013) provides additional evidence that a large part of the carry profits is still traceable to time variation in interest rates.

### 1.2.3 The UIP at Different Horizons

In this section I test the UIP hypothesis at different horizons and find that while the UIP condition is violated at horizons of up to 7 years, the nature of the violations changes with the horizon. Similarly to the one-step ahead predictive regressions considered in the previous section, I find that high domestic interest rates today tend to forecast positive domestic currency excess return at horizons of up to 3 years. However, the forecastability pattern flips at longer horizons and it turns out that high interest rates today tend to forecast *negative* excess returns at horizons of 4 to 7 years. This finding is also a violation of the UIP condition, but constitutes the *opposite* puzzle, as compared to the standard formulation of the “UIP puzzle”. Thus, this section documents that the puzzle is more complex than commonly understood, as it reverses course at longer horizons.<sup>27</sup>

The methodology I follow here is based on forecasting 1-month excess returns  $k$  periods into the future. UIP requires that the conditional expectation of the one step-ahead excess return is zero, i.e.  $E_t(\lambda_{t+1}) = 0$ , and this must be true for all time periods  $t$ . Hence, for any arbitrary  $k > 0$  we have  $E_{t+k}(\lambda_{t+k+1}) = 0$  and by the law of iterated expectations,

$$E_t(\lambda_{t+k+1}) = 0, \tag{1.6}$$

for all  $k > 0$ . In other words, the UIP hypothesis requires that any future, one-period excess return must be unforecastable with time  $t$  information. This observation provides us with a series of conditions that we can test at any horizon  $k$ .

To test them, I implement a straightforward extension of the standard 1-step

<sup>27</sup> My findings do not contradict earlier work by Flood and Taylor (1996); Chinn and Meredith (2005); Chinn (2006) and Chinn and Quayyum (2012) that has found that UIP appears to hold better at horizons of 5+ years. The differences between my results and the previous studies is in the methodology used to test UIP at longer horizons, and in fact my results could help provide an interpretation to the earlier findings. As Appendix A.3 discusses in more details, one can view the methodology of the previous papers as approximately averaging over the violations I document, which change sign with the horizon and tend to cancel out over the long run. The relationship is exact under risk-neutrality, but not necessarily otherwise.

ahead UIP regression and estimate the equation

$$\lambda_{j,t+k} = \alpha_{j,k} + \beta_k(i_t - i_{j,t}^*) + \varepsilon_{j,t+k} \quad (1.7)$$

as a panel regression with fixed effects, where  $j$  indexes the currencies and  $k = 1, 2, \dots, 180$  indexes the horizon in months.<sup>28</sup> Since the data is at the daily frequency, the left-hand side variable for  $k = 1$  is the 30-day return that ends 30 days into the future, for  $k = 2$  this is the 30-day return that ends 60 days into the future (i.e. the return between 30 and 60 days into the future) and so on and so forth. Most importantly, the left-hand side variable in the regressions is always precisely a 1-month excess return of foreign bonds over home bonds and the horizon, indexed by  $k$ , changes by 1-month increments as well. Thus, each regression tests if a particular 1-month excess return is forecastable by the current 1-month interest rate differential, but the horizon at which the returns are being forecasted changes.

Figure 1.1 plots the estimated coefficients,  $\hat{\beta}_k$ , from the above set of regressions on the Y-axis with the horizon  $k$ , in months, on the X-axis. The solid blue line plots the point estimates and the shaded region represents the 95% confidence intervals around each estimate, computed with Driscoll and Kraay (1998) standard errors that correct for heteroskedasticity, serial correlation and spatial correlation. The red dot on the plot is the point estimate of the standard UIP regression that looks one month into the future. This dot is also the estimate reported in Table 1.1, and the focus of many previous papers on the UIP puzzle.

The plot shows three important results. First, the coefficients are negative and statistically significant at horizons of up to 36 months, a finding that corresponds to the typical statement of the UIP puzzle. Second, the graph shows that the coefficients, however, are *positive* and statistically significant at horizons between 48 and 84 months. This is a novel result which indicates that high interest rates today

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<sup>28</sup> Similarly to the results in the previous section, running separate regressions for each currency, rather than considering a single panel regression, produces similar results across all currencies.



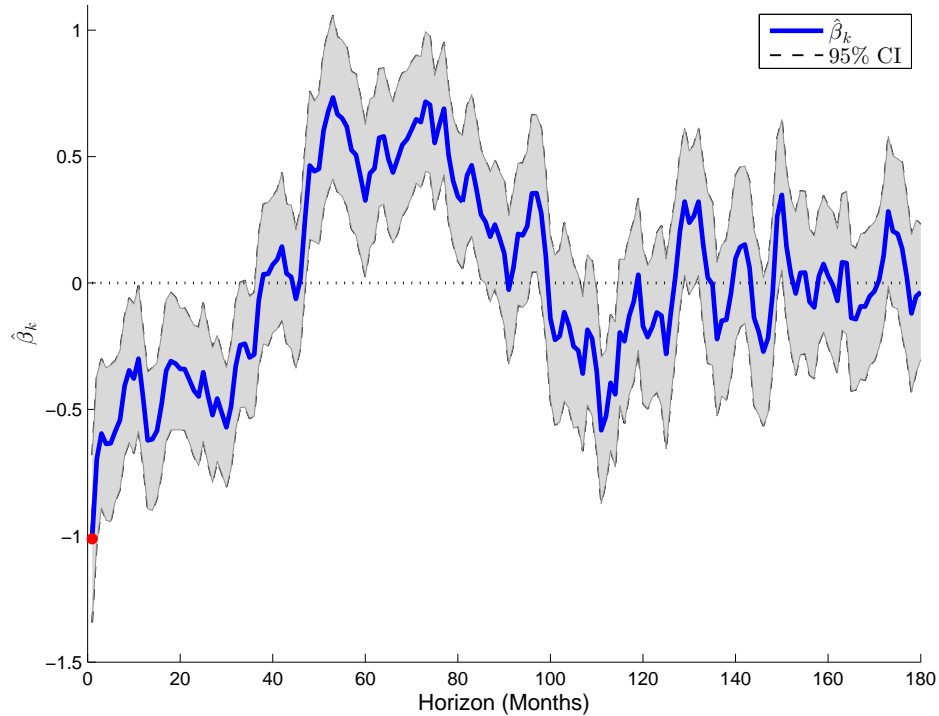


FIGURE 1.1: UIP Regression at horizons from 1 to 180 months

forecast *negative* excess returns 4 to 7 years into the future, contrary to the standard short-horizon result that high-interest rate currencies tend to earn positive excess returns. And third, after a brief spell at horizons of 100 to 120 months where the point estimates turn negative again, but only a handful are significant, the coefficients appear to converge to zero in the long-run. Overall, the UIP violations follow a clear, cyclical pattern, where they are negative at short horizons, positive at longer horizons, and gradually disappear in the long-run.

The main takeaway from the estimates is that the nature of UIP violations changes with the horizon - the short-horizon violations are characterized by negative coefficients and the longer horizon violations by positive ones. The difference is not so much in the magnitude of violations, which is roughly the same at short and long horizons, but in their fundamental nature. At short horizons we have the finding that exchange rates fail to depreciate sufficiently to fully offset the interest

rate differential, while at longer horizons we have the opposite puzzling behavior, as exchange rates in fact depreciate *too much*.<sup>29</sup> The findings show that UIP is violated at both short and long horizons, and suggest that the UIP puzzle is not driven by short-run frictions that disappear over the long-run but rather that the fundamental nature of the violations changes with the horizon. A comprehensive model of the UIP puzzle should be consistent with the full complexity of the UIP violations at all horizons.

My empirical analysis is inspired by the essential empirical insights of Engel (2012) regarding excess currency returns and real interest rates. He studies the relationship between currency returns, real exchange rate levels and real interest rate differentials, using a VAR on observed exchange rates, price levels and nominal interest rates, for each G7 country (paired with the US). The VAR is used to construct real interest differentials and model-implied ex-ante expected currency returns. Using results from the VAR, Engel (2012) shows that when a country's real interest rate is high, these expected returns are initially positive, and then, at longer horizons, negative. My results, and those in Engel (2014), show that these qualitative findings extend to the case where nominal interest rate differentials are used to predict realized returns.

There are some distinctions between the two sets of empirical results. For example, Engel (2012) finds that the forecastability pattern in excess returns changes from positive to negative at horizons of about 10 to 12 months, while I estimate that this occurs after approximately 40 months. Additionally, Engel (2012)'s results suggest that UIP is violated at very long horizons in the sense that long-horizon cumulative returns are predicted to be statistically significantly negative. I do not find an equivalent result when the nominal interest rate differential is the predictor. These different quantitative findings could be due to the specific restrictions implied

<sup>29</sup> The next section provides a detailed discussion on the implications for exchange rate behavior.

by Engel's VAR system.<sup>30</sup> Alternatively, they could be due to interesting and important differences in how nominal and real interest rates forecast excess returns at horizons longer than one year or due to the fact that my analysis also leverages the cross-sectional variation of the data. I leave disentangling those effects to future work.

#### 1.2.4 Implications for Exchange Rate Behavior

In this section I examine the implications of the changing nature of UIP violations for exchange rate behavior. I show that, following an increase in the interest rate differential, the short-horizon negative violations lead to a short-run exchange rate appreciation (rather than depreciation as UIP implies), but that the long-horizon positive violations lead to a subsequent strong depreciation. The long-horizon excess depreciation offsets the short-horizon appreciation, and in the long-run the exchange rate depreciates, and converges to the path implied by UIP. This behavior generates significant, predictable swings in the exchange rate and also lead me to reject the Random Walk hypothesis.

To begin, re-arrange the equation  $\lambda_{t+1} = s_{t+1} - s_t + i_t^* - i_t$  to express the cumulative exchange rate change from now up to  $k$  periods in the future as:

$$s_{t+k} - s_t = \underbrace{\sum_{h=1}^k (i_{t+h-1} - i_{t+h-1}^*)}_{\text{Sum of Future Int Diffs}} + \underbrace{\sum_{h=1}^k \lambda_{t+h}}_{\text{Sum of Future Excess Returns}} . \quad (1.8)$$

Thus, the cumulative exchange rate change, over any horizon, can be represented as the sum of future interest rate differentials and future excess returns. This is a standard decomposition that can be obtained for any asset price - prices move in

<sup>30</sup> The standard OLS procedure ensures that one step ahead forecast errors are uncorrelated with any of the variables included in the VAR. However, forecast errors at longer horizons are not necessarily unforecastable by the variables included in the VAR. This issue is likely to get larger as the horizon increases, since VARs are known to provide a good approximation of short-run dynamics, but less so for long-run dynamics.

response to fundamentals (interest rate differentials in this case) and excess returns.

Consider estimating the impulse response function of the cumulative exchange rate change to an innovation in the interest rate differential by using the Jorda (2005) method of local projections. This amounts to separately projecting each  $k$ -periods cumulative exchange rate change on the current interest rate differential so as to obtain

$$\text{Proj}(s_{t+1} - s_t | i_t - i_t^*) = \gamma_k (i_t - i_t^*). \quad (1.9)$$

The sequence  $\{\gamma_k\}$  forms an estimate of the impulse response function (in percentage terms) of the cumulative exchange rate change to a 1% increase in the interest rate differential  $i_t - i_t^*$ . The method of local projections is especially well suited for estimating long-run responses because of its flexible nature – there are no restrictions on the dynamics from period to period, as each response horizon is estimated via a separate projection.

Let  $\rho_k$  be the  $k$ -th autocorrelation of the interest rate differential  $i_t - i_t^*$  and use the definition of the projection coefficient  $\gamma_k = \frac{\text{Cov}(s_{t+k} - s_t, i_t - i_t^*)}{\text{Var}(i_t - i_t^*)}$  to express  $\gamma_k$  as:

$$\gamma_k = \underbrace{\sum_{h=1}^k \beta_h}_{\text{Sum of UIP Violations}} + \underbrace{\sum_{h=0}^{k-1} \rho_h}_{\text{Sum of Expected Int Diffs}}. \quad (1.10)$$

Thus the expected path of the cumulative exchange rate change is equal to the sum of UIP violations plus the sum of expected, future interest rate differentials. This showcases the important role that UIP, and deviations from UIP, play in exchange rate determination.

If UIP held, then we would have  $\beta_h = 0$  for all  $h$ , and the predicted path for the exchange rate change will be entirely determined by expectations about future interest rate differentials. In this case, exchange rate behavior will be closely linked to the underlying fundamentals, the interest rate differentials. On the other hand,

UIP violations disconnect the exchange rate from its fundamentals and can cause it to behave very differently from the UIP benchmark.

For example, the alternative hypothesis of a random walk exchange rate obtains when the UIP violations component of exchange rate dynamics exactly offsets the interest rate differentials component. Since in the data the interest rate differentials are strongly positively autocorrelated,  $\rho_h \geq 0$ , this would require that  $\beta_h$  are negative and similar in magnitude. As I show below, the short-horizon negative UIP violations indeed contribute to the fact that exchange rates tend to look like a random walk in the short-run. However, my results on the changing nature of UIP violations, i.e.  $\hat{\beta}_h > 0$  at longer horizons, suggest that at long horizons the two components may no longer offset and may even reinforce each other.

At this point, it is clear that cumulative UIP violations and cumulative expected interest rate differentials both play an important role in exchange rate behavior. To quantify their effects, I start by estimating  $\rho_k$  using similar panel regressions

$$i_{t+k} - i_{j,t+k}^* = \alpha_{j,k} + \rho_k(i_t - i_{j,t}^*) + \varepsilon_{j,t+k} \quad (1.11)$$

and then plot the partial sums of UIP violations and interest rate differential autocorrelations in Figure 1.2, for horizons ranging from 1 to 180 months.

The dashed green line plots the partial sums of  $\hat{\beta}_k$  (the estimated UIP violations), with the shaded region representing the corresponding 95% confidence error bands, and the dash-dot red line plots the partial sums of  $\hat{\rho}_k$  (I do not include the corresponding error bands to reduce clutter). The figure shows that the two components of exchange rate dynamics go in opposite directions, and thus work against each other, at horizons of up to 36 months. However, at longer horizons the positive UIP violations begin to weigh in, and the sum of  $\hat{\beta}_k$  start trending upwards. In fact, it turns out that the cumulative total of positive UIP violations is roughly equal to the cumulative total of negative UIP violations. This is manifested in the findings that

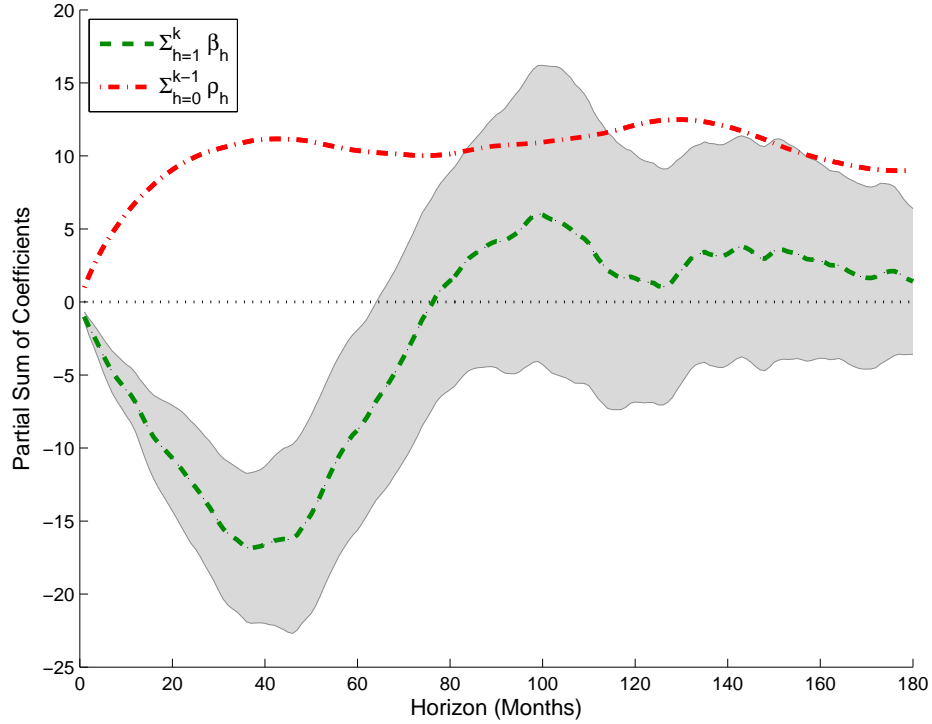


FIGURE 1.2: Cumulative Sums of the UIP Coefficients  $\beta_k$

in the long run (10+ years) the sum of UIP violations is roughly zero and is not statistically different from zero at horizons longer than six years.

The fact that the sum of UIP violations converges to zero has the interesting implication that the exchange rate path itself converges to the UIP benchmark. Thus, long-run exchange rate dynamics may appear consistent with UIP, even though UIP is in fact violated. My results suggest that the UIP puzzle is not a short-run phenomenon that disappears over the long-run; it is violated at both short and long horizons, but the nature of violations change in a way that makes long-run exchange rate behavior appear consistent with UIP.

To illustrate how these different effects impact exchange rate behavior at different horizons, in Figure 1.3 I add the estimated response to an increase in the interest rate differential,  $\gamma_k$ , with a solid blue line and its corresponding 95% error bands as the shaded region surrounding it.

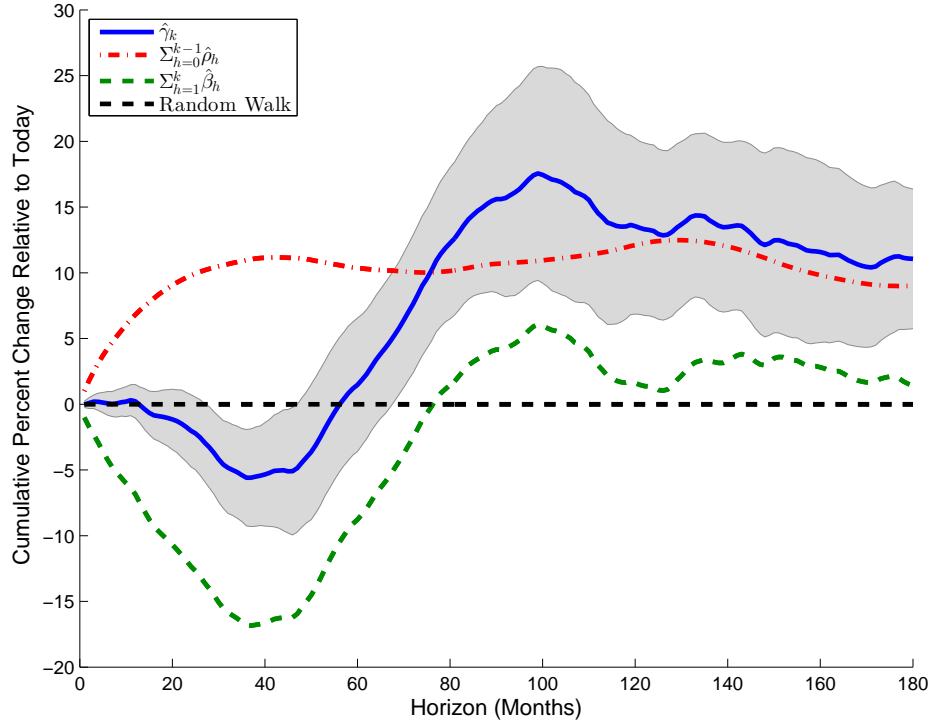


FIGURE 1.3:  $\text{Proj}(s_{t+1} - s_t | i_t - i_t^*)$

Recall that the red dash-dot line represents the UIP benchmark – the expected path of the exchange rate in case all  $\beta_h = 0$ . The disparate movements of the blue and the red line at short horizons is a manifestation of the standard formulation of the UIP Puzzle. Instead of steadily depreciating and offsetting the increase in interest rates, as predicted by UIP, the exchange rate fails to depreciate and even tends to appreciate slightly at horizons of up to 36 months. This is due to the accumulation of the short-horizon negative UIP violations, as illustrated by the strong downward trend in the green line over those horizons.

Just as the negative UIP violations do not persist at long horizons, however, the exchange rate appreciation lasts for only three to four years. At that point, the exchange rate starts responding to the accumulation of positive UIP violations and experiences a sharp depreciation at horizons of four to seven years. The depreciation is strong enough to fully offset the initial appreciation and to catch up the exchange

rate with the UIP-implied path.<sup>31</sup> This is an illustration of the fact that the cumulative sum of UIP violations is roughly zero, and hence long-run exchange rate behavior is roughly consistent with the UIP hypothesis.

But the path the exchange rate takes to get back to the UIP benchmark in the long-run, is very much in violation of UIP at every step of the way. At first, exchange rates tend to appreciate when we actually expect them to depreciate and later they reverse course and start depreciating too sharply. These cyclical movements are driven by the documented UIP violations that change sign with the horizon. Moreover, this cyclical behavior imparts interesting predictable movements in the exchange rate, especially in the long-run. Notice that the black dashed line at zero, in Figure 1.3, represents the expected path under the random walk hypothesis. The cyclical movements of the exchange rate move it away from that path, and thus the changing nature of the UIP violations also leads me to reject the random walk hypothesis.

To further illustrate the significance of the changing sign of UIP violations, Figure 1.4 plots the estimated path of the exchange rate ( $\gamma_k$ ) against a counter-factual path obtained by setting  $\beta_k = 0$  for all  $k \geq 36$ . This counterfactual exercise emulates exchange behavior in a world where UIP violations are persistently negative at short horizons (as also estimated in the data), but converge to zero monotonically, without turning positive. The main difference between the two is the long-run behavior of the exchange rate. Ignoring the positive UIP violations leads to the counterfactual implication that, following an increase in interest rates at home, the exchange rate appreciates persistently and stays high.<sup>32</sup> On the contrary, the estimated ex-

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<sup>31</sup> The finding of appreciation in the short-run and depreciation in the long-run is in line with the “delayed overshooting” result of Eichenbaum and Evans (1995), which they obtain through a VAR analysis of the response of exchange rates to identified monetary shocks.

<sup>32</sup> Note that all statements about long-run exchange rate behavior are made *relative* to today’s value. I am estimating the path of the cumulative change in the exchange rate conditional on today’s value,  $s_{t+k} - s_t$ , and I am not making a statement about the level of the exchange rate itself, i.e.  $s_t$ . I proceed in this way because the exchange rate  $s_t$  is clearly non-stationary.



change rate response features a strong depreciation at longer horizons which leaves the exchange rate depreciated, rather than appreciated in the long-run. Thus, the positive UIP violations have important implications about the long-run exchange rate behavior.

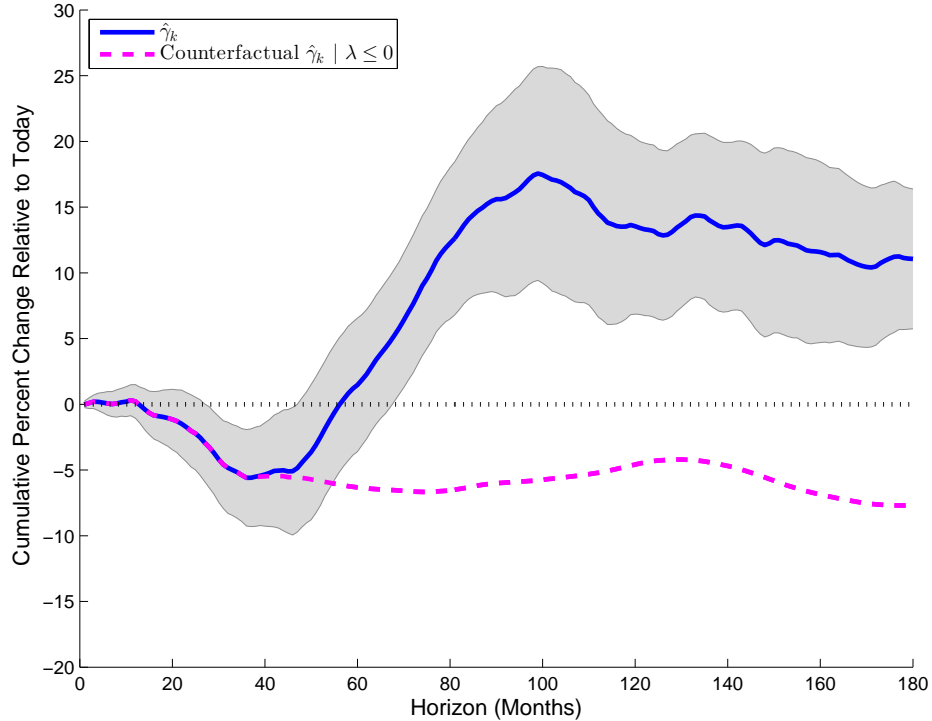


FIGURE 1.4:  $\text{Proj}(s_{t+1} - s_t | i_t - i_t^*)$  and Counterfactual

A model that fully captures the empirical regularities must account for both the negative and the positive UIP violations, and feature cyclical dynamics in the exchange rate. Given the novel nature of my empirical results, it is perhaps not surprising that existing models have largely focused on explaining the short-horizon UIP violations and exchange rate behavior, and tend to generate UIP deviations that converge monotonically to zero and cannot deliver the changing nature of UIP violations. To address this, the rest of the paper develops and analyzes a novel model, relying on the mechanism of bond convenience yields, and shows that it can closely match the full complexity of the UIP violations and exchange rate behavior

at both short and long horizons.

### 1.3 Time-Varying Convenience Yields and Excess Currency Returns

The UIP condition is derived under two important implicit assumptions. First, it abstracts from risk considerations (due to the log-linearization) and second it assumes that financial returns are the only benefit to holding bonds. Risk-premia have been extensively analyzed as a potential resolution to the UIP puzzle in the previous literature and in this paper instead I focus on relaxing the second assumption. I explore the implications of introducing a non-pecuniary (convenience or liquidity needs motivated) benefit to holding bonds and show that this can help generate UIP violations that closely match the empirical evidence.

A growing empirical literature documents a significant, time-varying “convenience yield” component to government bond yields and show that it is related to the outstanding amount of government debt (see Reinhart et al. (2000), Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2012), Greenwood and Vayanos (2014)). The convenience yield is defined as the market price of the non-pecuniary benefit of owning government debt (e.g. safety, liquidity) and is the amount of interest investors are willing to forego in exchange for the non-pecuniary benefits. The literature motivates and relates the convenience yield to the fact that government debt is extremely (nominally) safe and liquid, and could serve as a substitute to money, a special convenience asset that investors are willing to hold at a zero equilibrium rate of return. The empirical literature has documented that yields on government bonds, and US Treasuries in particular, indeed vary with the degree to which the bonds could provide some of the special services of money (e.g., facilitate transactions, store value, etc.).

There is also a sizable theoretical literature that has incorporated convenience yield mechanisms in equilibrium models and has shown that they can help account

for a diverse set of asset pricing puzzles, such as the low equilibrium risk-free rate, the equity risk premium, and the term premium (Bansal and Coleman II (1996), Bansal et al. (2011), Lagos (2010, 2011), Acharya and Viswanathan (2011)). In the following two sections I build on this literature by introducing the convenience yield mechanism into an international framework, and show that it can account for both the short and the long horizon UIP deviations I documented previously. In a later section, I also show that two of the key implications of the model are well supported by the data, by providing novel results that document the significant relationship between excess currency returns and the outstanding amount of government debt, and the relationship between UIP violations and capital controls, which serve as a proxy for monetary policy independence.

I proceed in two steps. First, I develop an intentionally stylized model that allows me to analytically characterize the main results and showcase the key intuition. Next, I relax the simplifying assumptions of the analytical model and set the mechanism in a benchmark, two-country open economy model in the modern tradition of Obstfeld and Rogoff (1995); Chari et al. (2002); Clarida et al. (2002) and the subsequent literature. I calibrate the model using standard parameters from the existing literature and show that it can closely match the documented UIP violations and exchange rate behavior at all horizons.<sup>33</sup>

## 1.4 Analytical Model

In this section, I setup and analyze an intentionally stylized model which allows me to derive several analytical results that are useful in understanding and illustrating

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<sup>33</sup> The model abstracts from trade in forward contracts, however, Appendix A.2 shows that introducing forward markets does not change the results. The intuition is that buying foreign currency forward is long foreign currency and short home currency, hence it creates a synthetic, zero-cost position in the underlying home and foreign bonds, which earns the convenience yield differential and violates UIP. The UIP violations show up in forwards data just as they do in the relationship between exchange rates and interest rates.

the main mechanism behind the UIP violations. There are two countries, a large home country and a small (measure zero) foreign country that is negligible in world equilibrium. The assumption of a small foreign country greatly simplifies the model, as the equilibrium is determined entirely by the actions of the home agents (the foreign agents are measure zero).<sup>34</sup>

There is a representative home household that consumes a single consumption good, in fixed supply, and has access to money, and home and foreign nominal bonds. In terms of the preference for liquidity, I keep things simple and transparent by following the standard approach of money-in-the-utility, assuming that real money balances provide utility to the household. Additionally, bond holdings (both home and foreign) also provide convenience (i.e. liquidity) benefits and for simplicity those are also modeled directly through the utility function. The idea is that government bonds, through their safety and liquidity, are a close substitute for money and can provide some of the same transaction benefits. For example, Treasuries are the main type of collateral accepted in repo markets, a key source of short-term financing for financial intermediaries and other investors, and are often used to mitigate counterparty risk, which helps facilitate complex financial transactions.<sup>35</sup>

The goal of this paper is to study the implications of the existence of convenience yields on equilibrium exchange rates, and not the structural microfoundations that generate the convenience yield in the first place. To this end, I follow much of the literature on bond convenience yields and assume a flexible, reduced form specification for the convenience benefits; for a structural approach to money demand see for

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<sup>34</sup> The quantitative model relaxes this assumption and studies a fully specified two country economy.

<sup>35</sup> Modeling liquidity demand through preferences allows for cleaner analytical expressions, as compared to modeling it through real transaction costs that affect the agent's budget constraint, but imply virtually the same optimality and equilibrium conditions. The analytical tractability is useful for the purposes of this section, but the quantitative model uses instead a transaction cost specification, as is standard in the existing literature (e.g. Bansal and Coleman II (1996); Bansal et al. (2011)).

example Lagos (2010, 2011). Endogenizing the convenience yield is a future research project.

Finally, there is a government that sets the growth rate of money, and finances a fixed level of expenditures by levying lump-sum taxes and issuing government bonds. The government is the sole supplier of the bonds that the households trade, and hence I will use the terms “government debt”, “government bonds” and “bonds” interchangeably.<sup>36</sup>

#### 1.4.1 *The Household*

The household is infinitely lived and has additively separable preferences over consumption, real money balances and real holdings of home and foreign bonds, with the instantaneous utility function<sup>37,38</sup>

$$u(c_t, m_t, b_{ht}, b_{ft}) = \ln(c_t) + \alpha_m \ln(m_t) + \alpha_b \ln(b_{ht} + \delta b_{ft}) \quad (1.12)$$

where  $c_t$  is consumption,  $m_t$  are the real money balances of the household, and  $b_{ht}$  and  $b_{ft}$  are its holdings of home and foreign bonds respectively, and  $\alpha_m > 0$ ,  $\alpha_b > 0$  and  $\delta \in [0, 1)$  are constants. The underlying assumption is that through their safety and liquidity, government bonds are a close substitute for money and provide some of the same liquidity benefits. Moreover,  $\delta \in [0, 1)$  models the idea that nominal government bonds denominated in foreign currency are not a perfect substitute for home government bonds, and are in general an inferior substitute for domestic money. I will use “liquidity benefits” and “convenience benefits” interchangeably throughout

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<sup>36</sup> The government is kept simple for analytical tractability, but is generalized in the quantitative model.

<sup>37</sup> Separability of preferences is not important for the results - the quantitative model uses generalized, non-separable preference for liquidity. Separability, however, helps provide tractability and this is the reason it is used in this section.

<sup>38</sup> The log-utility specification of preferences is not important, and is chosen simply for ease of exposition. The Appendix shows the results hold for any concave utility that is separable in the effects of consumption, money and bonds (and not necessarily separable in the two types of bonds).

the discussion, as is common in the literature.<sup>39</sup>

The household faces the following budget constraint at date  $t$

$$c_t + m_t + b_{ht} + b_{ft} = y - \tau_t + \frac{m_{t-1}}{\Pi_t} + b_{h,t-1} \frac{(1 + i_{t-1})}{\Pi_t} + b_{f,t-1} \frac{(1 + i_{t-1}^*)}{\Pi_t} \frac{S_t}{S_{t-1}} \quad (1.13)$$

where  $y$  is the endowment of the consumption good, which is constant over time,  $\tau_t$  are the real lump-sum taxes,  $\Pi_t$  is the gross inflation rate,  $i_t$  and  $i_t^*$  are the domestic and foreign nominal interest rates, and  $S_t$  is the exchange rate, expressed in terms of home currency units per unit of foreign currency (i.e. the domestic currency cost of foreign currency). This yields the following three Euler equations:

$$1 = \underbrace{\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{1}{\Pi_{t+1}} \right]}_{\text{Discounted Financial Return on Money}} + \underbrace{\alpha_m \frac{c_t}{m_t}}_{\text{Convenience Benefit of Money}} \quad (1.14)$$

$$1 = \underbrace{\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{1 + i_t}{\Pi_{t+1}} \right]}_{\text{Discounted Financial Return on Home Bonds}} + \underbrace{\alpha_b \frac{c_t}{b_{ht} + \delta b_{ft}}}_{\text{Convenience Benefit of Home Bonds}} \quad (1.15)$$

$$1 = \underbrace{\beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-1} \frac{S_{t+1}}{S_t} \frac{1 + i_t^*}{\Pi_{t+1}} \right]}_{\text{Discounted Financial Return on Foreign Bonds}} + \underbrace{\delta \alpha_b \frac{c_t}{b_{ht} + \delta b_{ft}}}_{\text{Convenience Benefit of Foreign Bonds}} \quad (1.16)$$

Each of the Euler conditions equates the real cost of one unit of investment (in money or bonds) to the discounted, expected benefit. The cost is simply the one unit of foregone consumption today and the payoffs are composed of both financial returns and convenience benefits (i.e. direct utility effect). Take the first Euler equation for example. An additional unit of money offers the financial return  $\frac{1}{\Pi_{t+1}}$ , which is simply the devaluation due to inflation, and also provides the direct utility benefit (expressed in consumption good terms)  $\alpha_m \frac{c_t}{m_t}$ . The real payoff to an extra

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<sup>39</sup> Again, the focus of the paper is to examine the overall implications of convenience yields for UIP violations, and I leave the study of the particular mechanisms generating the convenience yields to future work.

unit of investment is the sum of the two - it includes both financial and non-pecuniary (convenience) benefits.

The other two Euler equations follow a similar structure: each asset offers both financial and direct utility returns. For future reference, I define the following notation for the convenience returns on money, home and foreign bonds:

$$\Psi_{m,t} = \alpha_m \frac{c_t}{m_t}$$

$$\Psi_{b_h,t} = \alpha_b \frac{c_t}{b_{ht} + \delta b_{ft}}$$

$$\Psi_{b_f,t} = \delta \alpha_b \frac{c_t}{b_{ht} + \delta b_{ft}}$$

#### 1.4.2 The Government

The government controls money supply and taxes. It implements monetary policy by setting the growth rate of the nominal money supply

$$\ln(M_t) - \ln(M_{t-1}) = v_t$$

where  $v_t$  is an exogenous, white noise monetary shock. On the fiscal side, it faces a constant level of real expenditures  $g$  and the budget constraint

$$b_t^G + \tau_t + m_t^s - \frac{m_{t-1}^s}{\Pi_t} = \frac{(1 + i_{t-1})}{\Pi_t} b_{t-1}^G + g$$

where  $b_t^G$  is real government debt and  $m_t^s$  is real money supply. I follow the literature on the interaction of monetary and fiscal policy and assume that the lump-sum taxes are set according to the simple linear rule,<sup>40</sup>

$$\tau_t = \rho_\tau \tau_{t-1} + (1 - \rho_\tau) \kappa_b b_{t-1}^G,$$

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<sup>40</sup> See for example Leeper (1991); Chung et al. (2007); Davig and Leeper (2007); Bianchi and Ilut (2013).

where  $\rho_\tau \geq 0$  is the tax smoothing parameter and  $\kappa_b \geq 0$  controls how strongly taxes respond to debt levels.<sup>41</sup> This rule is a simple and tractable way of modeling the general idea that the government adjusts tax revenues to stay solvent, but that tax policy is (possibly) smoothed over time. Neither the monetary policy rule, nor the fiscal policy rule, are meant to capture optimal policy, but rather model government behavior in a tractable and reasonable way.

### 1.4.3 *Equilibrium Relations*

The foreign country is negligibly small and does not affect world markets, hence equilibrium in the goods market implies that the home household's consumption is constant over time:

$$c_t = c + g = y.$$

Constant consumption and the separable utility specification imply that the marginal rate of substitution between consumption today and consumption tomorrow is constant, and hence the equilibrium risk-free rate is also constant. This is useful to remember in interpreting results on the Euler equations that are to follow.

The small size of the foreign country also implies that foreign bonds are in zero net supply,  $b_{ft} = 0$ , and that home agents must hold the whole supply of home bonds:

$$b_{ht} = b_t^G.$$

Lastly, the money market is in equilibrium when money demand equals money supply so that  $m_t = m_t^s$ .

I solve the model by log-linearizing around the zero inflation steady state. Start by log-linearizing the monetary policy rule to obtain

$$\hat{\pi}_t = v_t - \Delta \hat{m}_t,$$

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<sup>41</sup> Bohn (1998) documents that US primary surpluses indeed respond to the level of government debt.



where hatted variables represent log-deviations from steady state. Next, log-linearize the Euler equation for money holdings and use the fact that consumption is constant to obtain

$$-\Psi_m \hat{m}_t - \beta E_t(\hat{\pi}_{t+1}) = 0.$$

where  $\Psi_m$  is the steady state marginal convenience value of money. The left hand side is the real return on money, in log-deviations from steady state, which is the sum of its convenience value and the devaluation due to inflation. Again, this is the sum of money's non-pecuniary and financial returns, respectively. The right-hand side (RHS) is the log-linearized real risk-free rate, which is constant, hence the RHS is zero.

To solve for equilibrium real money holdings, substitute out expected inflation using the inflation equation and solve forward to arrive at:

$$\hat{\pi}_t = v_t$$

$$\hat{m}_t = 0$$

In this simple economy, inflation is white noise and adjusts one-for-one with movements in the money supply, which leaves real money holdings constant over time.

Next, log-linearize the Euler equation for home bonds to obtain the following expression for the equilibrium interest rate:

$$\underbrace{\hat{i}_t - E_t(\hat{\pi}_{t+1})}_{\text{Real Interest Rate}} + \underbrace{\frac{\Psi_{b_h}}{\beta(1+i)} \hat{\Psi}_{b_h,t}}_{\text{Convenience Yield}} = 0$$

where  $\hat{\Psi}_{b_h,t}$  is the log-linearized marginal convenience value of home bonds. The left hand side of the equation is the log-linearized, real return on home government debt: the real interest rate plus the convenience yield. The convenience yield is the amount of interest agents are willing to forego in exchange for the non-pecuniary benefits of

the bonds. Naturally, there is a clear negative relationship between the convenience yield and the equilibrium interest rate - the higher the convenience yield, the lower the equilibrium interest rate the household requires in order to hold the outstanding supply of government debt.

Log-linearizing the expression for the convenience benefit of the home bond, using the equilibrium condition  $b_{ft} = 0$ , yields

$$\hat{\Psi}_{b_h,t} = -\hat{b}_{ht}$$

which shows two important things. First, the marginal convenience value of bonds is decreasing in the total amount of bonds owned by the household.<sup>42</sup> Second, the convenience yield is entirely determined by the supply of government bonds (recall that in equilibrium  $b_{ht} = b_{ht}^G$ ). This is due to the simplifying assumptions of constant consumption and additive separability in the utility function.<sup>43</sup> While this is an extreme case, it serves to highlight one of the key ingredients of the mechanism: movements in government debt. This is a feature the model shares with the previous literature, which has also emphasized the relationship between the supply of debt and the equilibrium convenience yield.

Furthermore, combining this with the previous result of white noise inflation,  $E_t(\hat{\pi}_{t+1}) = 0$ , we obtain

$$\hat{i}_t = -\frac{\Psi_{b_h}}{\beta(1+i)}\hat{\Psi}_{b_h,t} = \frac{\Psi_{b_h}}{\beta(1+i)}\hat{b}_{ht}.$$

This equation shows two important and intuitive results. First, we see that the interest rate is negatively related to the convenience yield. The higher the convenience benefits provided by the home bond, the lower is the equilibrium interest rate that investors require to hold the outstanding supply of debt. This negative relationship

<sup>42</sup> The liquidity preferences exhibit diminishing marginal utility of convenience assets.

<sup>43</sup> Both of which will be relaxed in the quantitative model.

between the convenience yield and the interest rate differential is a fundamental property of the mechanism and will be important later. Second, the equation also defines a downward sloping demand for government bonds – the higher is the outstanding supply of debt, the higher the equilibrium interest rate.

#### 1.4.4 Currency Returns and UIP Violations

Log-linearize the Euler condition for foreign bonds around the symmetric, zero inflation steady state where  $S = 1$ , and combine it with the equation for the home interest rate to obtain a modified version of the UIP condition:

$$E_t \underbrace{(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t)}_{=\lambda_{t+1}} = \underbrace{\frac{\Psi_{b_h}}{\beta(1+i)} \hat{\Psi}_{b_h,t} - \frac{\Psi_{b_f}}{\beta(1+i^*)} \hat{\Psi}_{b_f,t}}_{\text{Conv Yield Differential}}. \quad (1.17)$$

Uncovered Interest Rate Parity does not hold in this model and there are predictable, excess returns on cross-country investment. The predictable returns arise as a compensation for differences in the convenience yields on home and foreign bonds. The left hand side of the equation above gives the expected excess financial return of foreign bonds relative to home bonds. In equilibrium, the expected excess returns are equal to the difference (in log-deviations from steady state) in the convenience values of home and foreign bonds. Thus, when the home bond's convenience value increases, the foreign bond is compensated with higher expected financial returns and vice versa. Furthermore, if we remove the convenience yield mechanism there will be no forecastable currency returns and no UIP violations.

Moreover, since foreign bond holdings are constant, we have that

$$\hat{\Psi}_{b_f,t} = -\hat{b}_{ht}$$

and defining  $\eta_b \equiv \frac{\Psi_{b_h}}{\beta(1+i)} - \frac{\Psi_{b_f}}{\beta(1+i^*)}$  we have

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = -\eta_b \hat{b}_{ht}.$$

Thus, in this stylized model the convenience yield differential and the predictable excess returns are driven solely by movements in home government debt. Moreover, since  $\delta \in [0, 1)$  (i.e. foreign debt is an imperfect substitute for home debt) we can show that  $\eta_b > 0$ . Thus, the convenience yield differential and the expected excess returns on the foreign bond are decreasing in government debt. This is due to the diminishing marginal convenience value of debt and the fact that the home convenience yield is more responsive to movements in home debt, than the foreign convenience yield. When government debt increases, the convenience value of home bonds drops faster than the convenience value of foreign bonds and this creates a convenience yield differential across countries.

To fully characterize exchange rate dynamics and the implied UIP regression coefficients, we also need to model the foreign interest rate. For simplicity, I assume that the foreign central bank follows a symmetric monetary policy, resulting in an iid inflation process  $\hat{\pi}_t^*$ . Furthermore, assume that Purchasing Power Parity holds, so that the consumption good prices at home and abroad are equal when expressed in the same currency, i.e.  $P_t = S_t P_t^*$ , and thus

$$\hat{\pi}_{t+1} = \hat{s}_{t+1} - \hat{s}_t + \hat{\pi}_{t+1}^*.$$

Taking the conditional expectations on both sides yields  $E_t(\Delta \hat{s}_{t+1}) = 0$  and hence the exchange rate follows a random walk.<sup>44</sup> Using this result it follows immediately that the expected excess currency returns are equal to the negative of the interest rate differential:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = -(\hat{i}_t - \hat{i}_t^*).$$

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<sup>44</sup> The random walk exchange rate is an artifact of the simple monetary policy, which is assumed here for tractability. The analytical model is meant to illustrate the dynamics of the excess currency returns and UIP deviations, and not necessarily capture the behavior of the exchange rate itself. The quantitative model, on the other hand, relies on Taylor-rule monetary policy and this allows it to closely match both the UIP violations and the cyclical behavior of exchange rates, which appreciate over the short-run after an increase in the interest rate, but then depreciate strongly at longer horizons.

This implies that any forecastability in the excess return necessarily comes from the interest rate dynamics, and not the exchange rate. This is an implication of the intentionally stylized nature of the analytical model and is relaxed in the full blown model. The analytical model is meant to capture the key intuition that excess returns are driven by the convenience yield differential and the important role played by government debt dynamics in the convenience yields themselves. Generalizing the framework so that exchange rate adjustments, and not the interest rate differentials, are the main source of forecastability is straightforward, and is accomplished in the quantitative model by introducing Taylor rule monetary policy, which generates interest rate dynamics that match the data. The important takeaway here is the equilibrium relation that equates forecastable excess returns to the convenience yield differential, which is a fundamental part of the mechanism and is exactly the same in the quantitative model.

Combining the results on the equilibrium excess return and interest rate differential, leads us to the observation that they are both proportional to the supply of home bonds:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = -\eta_b \hat{b}_{ht}^G$$

$$(\hat{i}_t - \hat{i}_t^*) = \eta_b \hat{b}_{ht}^G$$

But while the interest rate differential increases with the supply of government debt, the excess currency return falls. This induces a negative relationship between excess returns and the interest rate differential (at least at short horizons), and it is immediate that estimating the now familiar set of UIP regressions,

$$\hat{s}_{t+k} - \hat{s}_{t+k-1} + \hat{i}_{t+k-1}^* - \hat{i}_{t+k-1} = \alpha_0 + \beta_k (\hat{i}_t - \hat{i}_t^*) + \varepsilon_{t+1},$$

on data generated by this model, would yield  $\beta_1 = -1$  regardless of parametrization. Ostensibly, the convenience yield differential acts as an omitted variable that is

perfectly negatively correlated with the regressor (at the one period horizon), the interest rate differential, hence high interest rates at home are associated one-for-one with positive excess returns on the home bond. Thus, even though interest rates themselves are not an equilibrium determinant of excess returns, the regression yields a significant negative coefficient.<sup>45</sup>

The UIP deviations are a direct consequence of the convenience yield mechanism – without it the model would yield  $\beta_k = 0$  for all  $k \geq 0$ . The UIP violations and the corresponding regression coefficients are driven by the time-variation in the convenience yield differential across countries, and the tractable nature of the model allows us to characterize the coefficients at any horizon  $k$ :

$$\beta_k = -\frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} = -\frac{\text{Cov}(\hat{b}_{h,t+k-1}, \hat{b}_{h,t})}{\text{Var}(\hat{b}_{h,t})}$$

However, to solve for the full set of regression coefficients  $\beta_k$  we need to fully characterize the equilibrium dynamics of the model, which is done in the next section.

#### 1.4.5 Model Solution

Let  $\gamma_b \equiv \frac{\Psi_{b_h}}{\beta(1+i)} > 0$ , enforce the equilibrium conditions  $b_{ht}^G = b_{ht}$  and  $m_t^s = m_t$ , and substitute the inflation, money holdings and interest rate equations into the log-linearized government budget constraint to obtain:

$$\hat{b}_{ht} + \frac{\tau}{b} \hat{\tau}_t = (1+i)(1+\gamma_b) \hat{b}_{h,t-1} - (1+i + \frac{m}{b}) v_t$$

Combine this equation with the tax-policy rule to arrive at the following system of two linear difference equations:

<sup>45</sup> For ease of exposition, the benchmark model abstracts from trade in forward exchange rate contracts, however, Appendix A.2 shows that the mechanism generates equivalent UIP violations in the forwards data. The intuition is that buying foreign currency forward creates a synthetic position that is long in foreign bonds and short home bonds, and hence earns the convenience yield differential. The mechanism operates in much the same way in forwards data.

$$\begin{bmatrix} b_{ht} \\ \tau_t \end{bmatrix} = \begin{bmatrix} (1+i)(1+\gamma_b) - (1-\rho_\tau)\kappa_b & -\rho_\tau \frac{\tau}{b} \\ (1-\rho_\tau)\kappa_b \frac{b}{\tau} & \rho_\tau \end{bmatrix} \begin{bmatrix} b_{h,t-1} \\ \tau_{t-1} \end{bmatrix} + \begin{bmatrix} -(1+i+\frac{m}{b}) \\ 0 \end{bmatrix} v_t. \quad (1.18)$$

For simplicity of notation, re-label the system as

$$x_t = Ax_{t-1} + Bv_t.$$

A stationary solution for the two endogenous variables  $\{\hat{b}_{ht}, \hat{\tau}_t\}$  exists if and only if the eigenvalues of the autoregressive matrix  $A$  are both smaller than 1 in magnitude, which is true when the parameters of the tax rule,  $\kappa_b$  and  $\rho_\tau$ , obey the restrictions derived in Lemma 1 below. When the solution exists it is also unique.

**Lemma 1 (Existence and Uniqueness).** *A stationary solution to the system of difference equations (1.18) exists if and only if the following two conditions are satisfied*

$$(i) \quad \kappa_b \in (K-1, \frac{1+\rho_\tau}{1-\rho_\tau}(K+1))$$

$$(ii) \quad \rho_\tau \in [0, \frac{1}{K})$$

where  $K = (1+i)(1+\gamma_b)$ . When the solution exists, it is unique.

Moreover, the stronger restriction  $\kappa_b \in (K-1, \frac{K}{1-\rho_\tau})$  ensures  $\hat{b}_t$  and  $\hat{\tau}_t$  are positively autocorrelated.

*Proof.* See Appendix A.1.1. □

The conditions impose restrictions on the coefficients of the tax policy rule that have clear and intuitive economic meaning. First, the conditions specify a lower and an upper bound for  $\kappa_b$ , the coefficient that controls how strongly tax policy responds to the outstanding stock of government debt. The lower bound assures that the tax response is sufficiently strong to keep debt levels stationary and the government

solvent. This is particularly straightforward to see in the case of  $\rho_\tau = 0$  when the system of two equations reduces to

$$\hat{b}_{ht} = (K - \kappa_b)b_{h,t-1} - (1 + i + \frac{m}{b})v_t.$$

In order for debt to be stationary, we need  $\kappa_b > K - 1 > 0$ .<sup>46</sup> The constant  $K = (1 + i)(1 + \gamma_b)$  characterizes the cost of servicing outstanding government debt, taking into account the fact that an increase in government debt decreases the convenience yield and hence pushes the equilibrium interest rate up (this works through the term  $(1 + \gamma_b)$ ). An extra dollar of debt issued today would require  $K > 1$  dollars to repay tomorrow and thus, to stay solvent, the government must increase taxes accordingly.

The upper bound on  $\kappa_b$  ensures that the government does not respond too strongly, and introduce a negative root that is larger than one in absolute value. However, negative roots are not relevant empirically, as the data displays strong positive autocorrelation, and for the rest of the analysis I will restrict attention to  $\kappa_b < \frac{K}{1 - \rho_\tau}$ , which ensures that the endogenous variables are positively autocorrelated.<sup>47</sup>

In addition to the restrictions on  $\kappa_b$ , Lemma 1 also imposes a restriction on the persistence of the tax rule,  $\rho_\tau$ . This is needed because a policy of smoothing taxes over time,  $\rho_\tau > 0$ , makes taxes an infinite moving average of past debt levels and this introduces feedback effects into the system. A sufficiently high smoothing coefficient would in fact lead to complex roots and oscillating (i.e. cyclical) dynamics which become unbounded when  $\rho_\tau$  is too high. The distinction between real and complex roots has important implications not only about the dynamics of the system, but

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<sup>46</sup> The last inequality follows because  $i > 0$  and  $\gamma_b > 0$ , hence  $K > 1$ .

<sup>47</sup> Intuitively, the stricter upper bound ensures that the immediate tax response is always smaller than the change in debt, so that an increase in debt does not lead to an overly large increase in taxes that actually completely reverses the original increase in debt.



also for the nature of the UIP deviations.

To formalize this, I turn attention to Impulse Response Functions (IRF) to a monetary shock, the only exogenous variable. Using the Wold decomposition of  $x_t$ ,

$$x_t = Bv_t + ABv_{t-1} + A^2Bv_{t-2} + A^3Bv_{t-3} + \dots,$$

and the particular structure of the matrices  $A$  and  $B$ , I obtain

$$\begin{aligned}\hat{b}_{ht} &= -(1 + i + \frac{m}{b})(v_t + a_{11}^{(1)}v_{t-1} + a_{11}^{(2)}v_{t-2} + a_{11}^{(3)}v_{t-3} + \dots) \\ \hat{\tau}_t &= -(1 + i + \frac{m}{b})(a_{21}^{(1)}v_{t-1} + a_{21}^{(2)}v_{t-2} + a_{21}^{(3)}v_{t-3} + \dots)\end{aligned}$$

where  $a_{j1}^{(k)}$  is the  $(j,1)$  element of the matrix  $A^k$ . Define  $a_{11}^{(0)} = 1$ ,  $a_{21}^{(0)} = 0$  and let  $\tilde{a}_{j1}^{(k)} = -(1 + i + \frac{m}{bG})a_{j1}^{(k)}$ , then the sequence  $\{\tilde{a}_{11}^k\}_{k=0}^{\infty}$  defines the IRF of government debt and the sequence  $\{\tilde{a}_{21}^k\}_{k=0}^{\infty}$  is the IRF of taxes. Lemma 2 derives the conditions under which the roots of the system are real, as opposed to complex, and characterizes the differing behavior of the IRFs.

**Lemma 2 (Impulse Response Functions).** *Let  $\kappa_b \in (K - 1, \frac{K}{1-\rho_\tau})$  and define*

$$\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b+1-K)+K-2\sqrt{K\kappa_b(\kappa_b+1-K)}}{(1+\kappa_b)^2} > 0. \text{ Then,}$$

- (i) *If  $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$  the autoregressive matrix  $A$  has two real, non-negative eigenvalues and the Impulse Response Functions never cross the steady state, i.e.*

$$\tilde{a}_{j1}^{(k)} \leq 0 \text{ for } j \in \{1, 2\}, \text{ and } k = 1, 2, 3, \dots$$

- (ii) *If  $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K})$  the autoregressive matrix  $A$  has a pair of complex conjugate eigenvalues which can be written as  $\lambda_k = a \pm bi$  for  $k \in \{1, 2\}$ , and the corresponding conjugate eigenvectors are of the form  $\vec{v}_k = [x \pm yi, 1]'$ , where*

$a, b, x, y$  are real numbers and  $i$  is the imaginary unit. Furthermore, the Impulse Response Functions follow increasingly dampened cosine waves:

$$\tilde{a}_{11}^{(k)} = -(1 + i + \frac{m}{b})|\lambda|^k \sqrt{1 + (\frac{x}{y})^2} \cos(k\phi + \psi - \frac{\pi}{2}), \text{ for } k = 1, 2, 3, \dots$$

$$\tilde{a}_{21}^{(k)} = -(1 + i + \frac{m}{b})|\lambda|^k \frac{x^2 + y^2}{y} \cos(k\phi - \frac{\pi}{2}), \text{ for } k = 1, 2, 3, \dots$$

$$\tilde{a}_{12}^{(k)} = (1 + i + \frac{m}{b})|\lambda|^k \frac{1}{y} \cos(k\phi - \frac{\pi}{2}), \text{ for } k = 1, 2, 3, \dots$$

where  $\phi = \arctan(\frac{b}{a}) = \arctan(\frac{\sqrt{4K\rho_\tau - (K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2}}{K - (1 - \rho_\tau)\kappa_b + \rho_\tau}) \in (0, \frac{\pi}{2})$  and  $\psi = \arctan(\frac{y}{x})$ . Moreover,  $\tilde{a}_{j1}^{(1)} \leq 0$  for  $j \in \{1, 2\}$ .

*Proof.* See Appendix A.1.2. □

Lemma 2 tells us several important things. First, the dynamics of the system are governed by real roots as long as taxes are not too persistent, and by complex roots otherwise. Second, the dynamics under real roots are characterized by Impulse Response Functions that converge back to steady state gradually, without overshooting it, while the IRFs under complex roots are cyclical cosine functions that overshoot the steady state before converging. Third, in both cases a positive (expansionary) monetary shock pushes both taxes and debt below steady state for at least two periods.

Consider the dynamics under real roots first. A positive monetary shock does not affect taxes on impact, because they only respond to time  $t - 1$  variables, but debt falls because of two main forces. First, the increase in money supply raises inflation, which decreases the real-value of outstanding government debt and also lowers debt servicing costs by reducing the real interest rate. And second, the increase in money supply raises seignorage revenues. The two effects combine to reduce government

outlays and increase government revenues which improves the budget and leads to a decrease in debt. In the following periods taxes decrease as well, because the government has a lower stock of outstanding debt to service. When the system is characterized by real roots both variables converge back to steady state without overshooting (i.e. going above steady state levels).

In the case of complex roots, the behavior on impact is similar but the transition dynamics back to steady state are different. The dynamics of both variables are characterized by cosine curves with a frequency of oscillation controlled by  $\phi = \arctan(\frac{b}{a}) \in (0, \frac{\pi}{2})$ , where  $a$  and  $b$  are the real and the imaginary parts of the complex roots. The larger  $\phi$  is, the higher the frequency of oscillations, but its upper bound of  $\frac{\pi}{2}$  ensures that each cycle takes at least 4 periods to complete and that the IRFs remain negative for at least one period after the shock. The cyclical nature of the cosine dynamics, however, guarantees that the endogenous variables will turn positive at some point in the future, with the number of periods needed for this first crossing to occur depending on the magnitude of  $\phi$ .

This cyclical behavior arises when tax policy is smoothed over time, which makes taxes an infinite moving average of past debt levels, with higher levels of  $\rho_\tau$  putting more weight on debt levels further in the past. When taxes are sufficiently smooth, as defined by the condition  $\rho_\tau > \underline{\rho}(\kappa_b)$ , they remain low even as debt approaches steady state because taxes responds to past, lower debt levels. The persistently low taxes keep pushing debt higher and it overshoots steady state, giving rise to the cyclical dynamics formalized by the cosine curves specified in Lemma 2.

With these results in hand, we can now characterize the UIP regression coefficients implied by the model. Given the cyclical nature of the IRFs, it is perhaps not surprising that the regression coefficients  $\beta_k$  are negative at all horizons when the roots of the system are real, but have a cyclical profile and are negative at short horizons, and positive at longer horizons when the roots are complex. Theorem 1

formalizes this result.

**Theorem 1 (UIP Violations).** *Let  $\kappa_b \in (K - 1, \frac{K}{1 - \rho_\tau})$ . The UIP regression coefficients  $\beta_k$  are equal to*

$$\beta_k = \frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} = -(a_{11}^{(k-1)} + \delta a_{12}^{(k-1)})$$

where  $a_{jl}^{(k)}$  is the  $(j,l)$  element of the matrix  $A^k$  for  $k = 1, 2, 3, \dots$ ,  $a_{11}^{(0)} = 1$  and  $a_{12}^{(0)} = 0$ , and  $\delta = (1 - \rho_\tau)\kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1+\rho_\tau) - \kappa_\tau)}{1 + \rho_\tau(1 + 2\kappa_\tau - K(1 + \rho_\tau))}$ . Furthermore,

(i) If  $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ , the roots of the system are real and

$$\beta_k < 0, \text{ for all } k = 1, 2, 3, \dots$$

(ii) If  $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K})$ , then the eigenvalues of the autoregressive matrix  $A$  are complex and of the form  $\lambda_k = a \pm bi$ ,  $k \in \{1, 2\}$  with corresponding eigenvectors of the form  $\vec{v}_i = [x \pm yi, 1]'$ , and

$$\beta_k = \begin{cases} -1 & , k = 1 \\ -|\lambda|^{k-1} \sqrt{1 + \chi^2} \cos((k-1)\phi + \psi - \frac{\pi}{2}) & , k = 2, 3, \dots \end{cases}$$

where  $\chi = \frac{x - \delta(x^2 + y^2)}{y}$ ,  $\phi = \arctan(\frac{b}{a})$ ,  $\psi = \arctan(\frac{1}{\chi}) + \pi \mathbb{I}(\chi < 0)$ .

*Proof.* See Appendix A.1.3. □

Theorem 1 derives the main results of the analytical model. First, it shows that the model always implies  $\beta_k < 0$  at short horizons and is thus qualitatively consistent with the standard formulation of the UIP Puzzle under all parametrization (as long as a solution exists).<sup>48</sup> Second, it shows that when the persistence of the tax rule is small

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<sup>48</sup> Theorem 1 can also be extended to the case where the solutions are not necessarily positively autocorrelated and we would still have  $\beta_1 = -1$ .

and the dynamics are governed by real roots, the model cannot generate positive  $\beta_k$  at any horizon but rather implies  $\beta_k < 0$  for all  $k$ . Under those conditions, the model is similar to existing models of the UIP puzzle and implies that the UIP coefficients are negative at all horizons. This pattern is illustrated by the green dashed line in Figure 1.5.

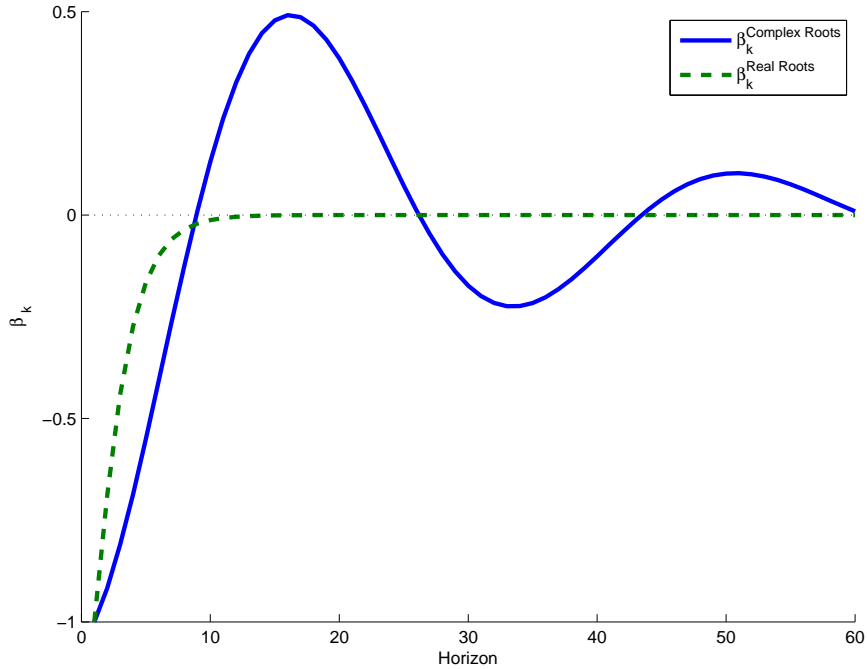


FIGURE 1.5: Model Implied UIP Regression Coefficients

Third, Theorem 1 shows that when the tax rule is sufficiently persistence, i.e.  $\rho_\tau \geq \underline{\rho}_\tau(\kappa_b)$ , the model implies a pattern of UIP regression coefficients that starts out negative and then follows a cyclical profile along an increasingly dampened cosine curve. In particular, this mean that the model generates  $\beta_k < 0$  at short horizons and  $\beta_k > 0$  at longer horizons, with the coefficients converging to zero on a vanishing cyclical path. This alternative pattern of UIP coefficients is illustrated by the solid line in Figure 1.5. The curve starts out negative, becomes positive at longer horizons and briefly dips back into negative territory before largely dieing out. The pattern is

strongly reminiscent of the empirical estimates and the next section shows that the quantitative model closely matches the data.

In this stylized model, the UIP deviations are entirely determined by the dynamics of government debt  $\hat{b}_t$ . Recall that today's interest rate is negatively related to  $\hat{b}_t$  while the expected excess foreign bond return is positively related to  $\hat{b}_t$ . Under both real and complex roots, a contractionary (negative) shock to money growth leads to an increase in government debt. The increased supply of home government debt reduces its marginal convenience value (each bond is now less special) and thus increases the home interest rate, due to the downward sloping demand for debt. Moreover, the decrease in the home convenience yield leads to compensating excess financial returns on the home currency (recall equation (1.17)).<sup>49</sup> This highlights the fact that the negative UIP coefficients are a fundamental part of the mechanism, as movements in the convenience yield push interest rates and excess currency returns in opposite directions. Lastly, the effects are persistent, under both real and complex roots, and we have  $\beta_k < 0$  for several periods after the initial shock.

In the case of real roots, debt levels, and hence the convenience yield, converge monotonically back to steady state and  $\beta_k < 0$  for all  $k$ . On the other hand, under complex roots the debt dynamics are cyclical. Similarly to the case of real roots, the initial shock increases debt levels on impact and taxes follow suit the next period. However, with  $\rho_\tau \geq \underline{\rho}(\kappa_b)$  taxes sluggish and slow to adjust, hence it takes a longer time for debt to be brought back down. Moreover, taxes can be represented as an infinite moving average of past debt levels and with a higher  $\rho_\tau$  debt levels further into the past play a larger role. Thus, even when debt approaches its steady state level taxes remain high, as they are responding to the high debt levels further in the past. This leads debt to overshoot its steady state level, go below it, and converge

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<sup>49</sup> There is a corresponding move in the foreign convenience yield as well, because home and foreign debt are substitutes, but  $\delta < 1$  ensures that the convenience yield differential moves proportionally to the home convenience yield.

to steady state on a cyclical path. The convenience yield follows a similar trajectory, and eventually goes above steady state (when debt is below steady state), which pushes excess home currency returns below steady state (while they were above steady state at short horizons) and generates positive UIP regression coefficients,  $\beta_k > 0$ , at longer horizons. The oscillations eventually die out and hence  $\lim_{k \rightarrow \infty} \beta_k = 0$ .

The UIP violations in the model occur because monetary and fiscal policy do not coordinate to keep the supply of government debt constant.<sup>50</sup> In particular, when fiscal policy does not accommodate monetary policy, monetary shocks lead to movements in the real supply of government debt which affects the supply of convenience assets to the economy. This causes movements in the convenience yield differential across countries, which in turn lead to predictable, compensatory excess currency returns and thus UIP violations.<sup>51</sup>

Monetary policy shocks are not unique in generating cyclical UIP violations – a large variety of policy shocks (expenditure, tax, etc.) have similar implications. For example, a positive government expenditures shock increases government debt, and unless the central bank coordinates with the fiscal authority, and loosens monetary policy to inflate the new debt away and offset the fiscal shock, this generates UIP violations. Moreover, the violations behave in exactly the same manner, with  $\beta_k < 0$  at all horizons when the roots are real and  $\beta_k < 0$  at short horizons and  $\beta_k > 0$  at longer horizons when the roots are complex. For the sake of brevity, this section analyzed only monetary shocks but different types of policy shocks can generate the results, as long as monetary and fiscal policy do not coordinate to keep debt constant. In this sense, the model is robust to the source of exogenous shocks.

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<sup>50</sup> Note that studying optimal policy is beyond the scope of this paper and is left to future research. This discussion only intends to highlight how monetary and fiscal policy interaction (or lack of) can affect exchange rate movements and deviations from the UIP.

<sup>51</sup> The key is actually movements in the *relative* supply of convenience assets across countries, but here the foreign country is negligible.

## 1.5 Quantitative Model

To examine the quantitative performance of the mechanism I start with a benchmark nominal, two country model (Obstfeld and Rogoff (1995); Chari et al. (2002); Clarida et al. (2002)) and add convenience benefits to holdings bonds and fiscal policy. I calibrate the model using standard parameters from the literature and then show that it can closely match the empirical evidence on UIP deviations at both short and long horizons. I start by giving a brief overview of the model.

There are two symmetric countries, home (H) and foreign (F). Households have access to a complete set of Arrow-Debreu securities and consume both a domestically produced final good and a foreign final good. Each country produces both a final good and a continuum of intermediate goods. The final good is produced by a representative firm that aggregates the intermediate goods produced domestically. There is also a mass of intermediate goods firms equal to the total population in each country, that produce differentiated goods, compete monopolistically and face Calvo (1983) frictions in setting nominal prices. Lastly, there is also a government that implements monetary policy by setting the interest rate and finances spending via lump-sum taxation and issuing government bonds.

Next I present the decision problems of the different agents in the home country, with the stipulation that the agents in the foreign country are symmetric.

### 1.5.1 *Households*

I use the same household preferences as in Clarida et al. (2002) and Gali and Monacelli (2005). The representative household maximizes the expected infinite stream of utility:<sup>52</sup>

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<sup>52</sup> Complete asset markets allows me to appeal to a representative household structure



$$E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{C_{t+j}^{1-\sigma}}{1-\sigma} - \frac{N_{it}^{1+\phi}}{1+\phi} \right)$$

Consumption is a CES aggregate of home (H) and foreign (F) final goods,

$$C_t = \left( a_h^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

where  $\eta$  is the elasticity of substitution between the two goods and the weights  $a_h$  and  $a_f$ , normalized to sum to 1, determine the degree of home bias in consumption.  $C_{Ht}$  and  $C_{Ft}$  are the amount of the home final good and the foreign final good that the household purchases. It follows that the corresponding consumption price index is

$$P_t = \left( a_H P_{Ht}^{1-\eta} + a_F P_{Ft}^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

where  $P_{Ht}$  and  $P_{Ft}$  are the prices of the home and foreign good in terms of the home currency.

To motivate the demand for liquidity, I assume that the household incurs transaction costs,  $\Psi_t$ , in purchasing consumption, a standard approach in both the monetary literature (e.g. Feenstra (1986)) and the literature on bond convenience yields (Bansal and Coleman II (1996), Bansal et al. (2011)). I model the transaction costs with a flexible CES function that includes both real money balances and real bond holdings as convenience assets:

$$\Psi(C_t, m_t, b_{ht}, b_{ft}) = \bar{\psi} c_t^{\alpha_1} h(m_t, b_{ht}, b_{ft})^{1-\alpha_1}$$

The transaction cost function has two components, the level of transactions  $C_t$  and a bundle of transaction services  $h(m_t, b_{ht}, b_{ft})$ , which is generated by the three

convenience assets: real money balances  $m_t$  and the real holdings of home and foreign nominal bonds  $b_{ht}$  and  $b_{ft}$ . The elasticity parameter  $\alpha_1 > 1$ , hence transaction costs are increasing in the level of purchases ( $C_t$ ) and decreasing in the level of transaction services. The transaction services  $h(\cdot)$  are a CES aggregator of real money balances and a bundle of transaction services generated by bonds:

$$h(m_t, b_{ht}, b_{ft}) = (m_t^{\frac{\eta_m - 1}{\eta_m}} + \tilde{h}(b_{ht}, b_{ft})^{\frac{\eta_m - 1}{\eta_m}})^{\frac{\eta_m}{\eta_m - 1}}$$

where

$$\tilde{h}(b_{ht}, b_{ft}) = \kappa_b (a_b b_{ht}^{\frac{\eta_b - 1}{\eta_b}} + (1 - a_b) b_{ft}^{\frac{\eta_b - 1}{\eta_b}})^{\frac{\eta_b}{\eta_b - 1}}$$

The dual structure of the transaction services function aims to capture the idea that money and bonds are two separate classes of convenience assets and allows for different elasticity of substitution between money and the bundle of bonds, and between home and foreign bonds.<sup>53</sup> The elasticity of substitution between money balances and the bundle of bonds is given by  $\eta_m$ , while  $\eta_b$  is the elasticity of substitution within bonds. The parameter  $\kappa_b$  shifts the overall level of transaction services provided by bonds and allows me to control the relative importance of bonds as convenience assets. Lastly, the parameter  $a_b$  controls the relative importance of home to foreign bonds.

The budget constraint of the household is

$$C_t + \int \Omega_{H,t}(z_{t+1}) x_t(z_{t+1}) dz_{t+1} + \Psi(c_t, m_t, b_{ht}, b_{ft}) + m_t + b_{ht} + b_{ft} =$$

$$w_t N_{it} + \frac{x_{t-1}(z_t)}{\Pi_t} - \tau_t + d_t + \frac{m_{t-1}}{\Pi_t} + b_{h,t-1} \frac{(1 + i_{t-1})}{\Pi_t} + b_{f,t-1} \frac{(1 + i_{t-1}^*)}{\Pi_t} \frac{S_t}{S_{t-1}}$$

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<sup>53</sup> The structure is flexible enough to also allow me to treat all convenience assets as equally substitutable. None of the results I will report later depend on the flexible structure of transaction costs and can also be delivered by a simple Cobb-Douglas function of the type  $\Psi_t(C_t, m_t, b_{ht}, b_{ft}) = \bar{\psi} c_t^{\alpha_1} m_t^{\alpha_2} b_{ht}^{\alpha_3} b_{ft}^{\alpha_4}$ . However, a Cobb-Douglas formulation has counter-factual implication about certain features of the implied money demand and I consider it only as a robustness check.

where  $\Omega_{H,t}(z_{t+1})$  is the price, in terms of home currency, of the Arrow-Debreu security that pays off in the state of nature  $z_{t+1}$  tomorrow and  $x_t(z_{t+1})$  is the amount of this security that the home household has purchased. The household spends money on consumption, Arrow-Debreu securities, transaction costs, money holdings, and home and foreign nominal bonds. It funds its purchases with the money balances it carries over from the previous period, the real wages  $w_t$  it receives for its labor, the profits of the intermediate good firms  $d_t$ , and payoffs from its holdings of contingent claims, and home and foreign bonds. Lastly, it also pays lump-sum taxes  $\tau_t$  to the domestic government.

This first-order necessary conditions for home and foreign nominal bond holdings are:

$$1 = \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + \Psi_c(c_t, m_t, b_{ht}, b_{ft})}{1 + \Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1}, b_{f,t+1})} \frac{1}{\Pi_{t+1}} \frac{1 + i_t}{1 + \Psi_{b_h}(c_t, m_t, b_{ht}, b_{ft})} \quad (1.19)$$

$$1 = \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + \Psi_c(c_t, m_t, b_{ht}, b_{ft})}{1 + \Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1}, b_{f,t+1})} \frac{S_{t+1}}{\Pi_{t+1} S_t} \frac{1 + i_t^*}{1 + \Psi_{b_f}(c_t, m_t, b_{ht}, b_{ft})} \quad (1.20)$$

where the term  $\Psi_x = \frac{\partial \Psi}{\partial x}$  is the derivative of the transaction costs in respect to the variable  $x$ . The terms  $\Psi_{b_h}$  and  $\Psi_{b_f}$  are the marginal transaction benefit of holding an extra unit of home and foreign bonds respectively. Similarly to the analytical model, these marginal benefits determine the convenience yields and will generate UIP deviations.

The foreign household faces a symmetric set of first order conditions, and since the Arrow-Debreu securities are traded internationally we can combine the optimality conditions of the two of agents to arrive at the following risk-sharing condition:

$$\begin{aligned} & \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + \Psi_c(c_t, m_t, b_{ht}, b_{ft})}{1 + \Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1}, b_{f,t+1})} \\ &= \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1 + \Psi_c(c_t^*, m_t^*, b_{ft}^*, b_{ht}^*)}{1 + \Psi_c(c_{t+1}^*, m_{t+1}^*, b_{f,t+1}^*, b_{h,t+1}^*)} \frac{Q_t}{Q_{t+1}} \end{aligned}$$

where  $Q_t = \frac{S_t P_t^*}{P_t}$  is the real exchange rate. Complete financial markets ensure that the marginal rates of substitution (MRS) are equalized across countries, however in this model the MRS is also adjusted for the liquidity needs of the households. This adjustment

acts as a wedge between the consumption growth differential and the real exchange rate depreciation, and breaks the typical result that those two variables are perfectly correlated. In particular, a log-linear approximation of this equation around the steady state yields

$$\sigma(\Delta c_t - \Delta c_t^*) = \Delta q_t + \frac{\Psi_c}{1 + \Psi_c}(\Delta \hat{\Psi}_{c,t}^* - \Delta \hat{\Psi}_{c,t})$$

which clearly shows that the consumption growth differential is a function of both real exchange rate changes and the differential in marginal transaction costs. This suggests that the model, which is built with an eye towards explaining the UIP Puzzle, a phenomenon in nominal exchange rates, can also shed some light on the imperfect correlation between consumption growth differentials and changes in the real exchange rate, a major puzzle in the dynamics of the real exchange rates.

### 1.5.2 Final Goods Firms

There is a home representative final goods firm which uses the domestic continuum of intermediate goods and the following CES technology to produce total output  $Y_{H,t}$ :

$$Y_{H,t} = \left( \int_0^1 Y_{it}^{\frac{\lambda_H-1}{\lambda_H}} di \right)^{\frac{\lambda_H}{\lambda_H-1}}.$$

Profit maximization yields the standard type of demand for intermediate goods

$$Y_{it} = \left( \frac{P_{it}}{P_{Ht}} \right)^{-\lambda_H} Y_{Ht},$$

and the price index (GDP Deflator)

$$P_{H,t} = \left( \int_0^1 P_{it}^{1-\lambda_H} di \right)^{\frac{1}{1-\lambda_H}}.$$

### 1.5.3 Intermediate Goods Firms

Intermediate goods firms use a production technology linear in labor input  $N_{it}^D$ ,

$$Y_{it} = A_t N_{it}^D,$$

where  $A_t$  is an exogenous productivity process. The firms face a Calvo (1983) friction in setting prices and have a probability  $1 - \theta$  of being able to adjust prices in any given period. Firms that can adjust choose their optimal price  $\bar{P}_{it}$  so as to satisfy the first-order condition

$$\sum_{j=0}^{\infty} E_t \Omega_{Ht}(z_{t+j}) Y_{i,t+j} \theta^j \left( \bar{P}_{it} - \frac{\lambda_H}{\lambda_H - 1} (1 - \tau) \frac{w_{t+j} P_t}{A_{t+j}} \right) = 0. \quad (1.21)$$

Firms that do not get to re-optimize keep their prices constant and adjust their labor input to satisfy demand. The law of large numbers then implies that the price of the home final good evolves according to

$$P_{Ht} = (\theta P_{H,t-1}^{1-\lambda_H} + (1 - \theta) \bar{P}_{it}^{1-\lambda_H})^{\frac{1}{1-\lambda_H}} \quad (1.22)$$

#### 1.5.4 Government

The government consists of a Monetary Authority (MA), which I will also refer to interchangeably as the Central Bank (CB), and a Fiscal Authority (FA).<sup>54</sup>

The Monetary Authority sets monetary policy according to a standard Taylor rule (in log approximation to steady state):

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1 - \rho_i) \phi_\pi \hat{\pi}_t + v_t$$

where  $\pi_t$  is CPI inflation and  $v_t$  is an iid monetary shock.

The MA issues and underwrites the supply of domestic currency,  $M_t^s$ , and backs a fraction  $\frac{1}{\mu}$  of it with holdings of domestic government bonds,

$$M_t^s \frac{1}{\mu} = B_{ht}^M,$$

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<sup>54</sup> Because the mechanism of the model is tightly linked to the net amount of bonds available to the private agents, I model the two authorities as separate in order to account for the Central Bank's balance sheet and any effects coming from open market operations. The distinction between Monetary and Fiscal Authorities turns out to not be quantitatively important, however, and the Appendix shows that the main results remain largely unchanged if we move to a single, consolidated government.

where  $B_{ht}^M$  is the nominal amount of domestic government bonds held by the CB.<sup>55</sup> As is true in reality, the Monetary Authority transfers all seignorage revenues to the fiscal Authority and faces the following budget constraint

$$T_t^M = M_t^s - M_{t-1}^s + B_{h,t-1}^M(1 + i_{t-1}) - B_{ht}^M,$$

where  $T_t^M$  is the money transferred to the Fiscal Authority.

The Fiscal Authority collects taxes, the transfer of seignorage from the MA and issues government bonds to fund government expenditures and has the the budget constraint

$$B_{ht}^G + T_t + T_t^M = B_{h,t-1}^G(1 + i_{t-1}) + G_t$$

where  $B_{ht}^G$  is the nominal amount of government bonds issued and  $T_t$  are the nominal, lump-sum taxes collected. For brevity and simplicity, in much of the analysis I will abstract from movements in government spending and assume it is constant,  $G_t = G$ .<sup>56</sup>

Lastly, I follow the literature on the interaction between monetary and fiscal policy (e.g. Leeper (1991), Davig and Leeper (2007), Bianchi and Ilut (2013)) and assume that the fiscal authority follows a simple taxation rule which adjusts lump-sum taxes (as a percentage of GDP) in response to the government's debt to GDP ratio<sup>57</sup>

$$\frac{P_t \tau_t}{P_{H,t} Y_{H,t}} = \rho_\tau \frac{P_{t-1} \tau_{t-1}}{P_{H,t-1} Y_{H,t-1}} + (1 - \rho_\tau) \kappa_b \frac{P_{t-1} b_{h,t-1}^G}{P_{H,t-1} Y_{H,t-1}}$$

### 1.5.5 Excess Bond Returns and and UIP violations

Log-linearize equations (1.19) and (1.20) around the symmetric, zero-inflation steady state and subtract them from one another to obtain

<sup>55</sup> I allow for  $\mu > 1$  to be able to match the fact that the bond holdings of the Central Bank are in fact less than the total amount of money available in the economy at any given point in time (primarily because of the money multiplier).

<sup>56</sup> The Appendix discusses an extension with both fiscal and tax shocks and shows that these type of shocks also generate empirically relevant UIP deviations. Since results are so similar, in the interest of conciseness I relegate this discussion to the Appendix.

<sup>57</sup> The results do not change materially whether I express the tax rule in terms of Tax-to-GDP or simply in terms of real taxes  $\tau_t$ .

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \left| \frac{\Psi_{b_h}}{1 + \Psi_{b_h}} \right| \hat{\Psi}_{b_h,t} - \left| \frac{\Psi_{b_f}}{1 + \Psi_{b_f}} \right| \hat{\Psi}_{b_f,t} \quad (1.23)$$

where hatted variables represent log-deviations from steady state. The terms  $\hat{\Psi}_{b_h,t}$  and  $\hat{\Psi}_{b_f,t}$  are the log-deviations from steady state of the marginal transaction benefits of home and foreign bond holdings, and  $\Psi_{b_h}$  and  $\Psi_{b_f}$  are the corresponding steady state values.

Note that at the symmetric steady state  $\Psi_{b_h} = \Psi_{b_f}$  and define  $\gamma_b \equiv \left| \frac{\Psi_{b_h}}{1 + \Psi_{b_h}} \right| = \left| \frac{\Psi_{b_f}}{1 + \Psi_{b_f}} \right|$ .

If there was another home currency denominated nominal bond that did not provide convenience benefits, its interest rate  $\tilde{i}_t$  would be

$$\tilde{i}_t = \hat{i}_t + \gamma_b \hat{\Psi}_{b_h,t}$$

Hence  $\gamma_b \hat{\Psi}_{b_h,t}$  is the equilibrium convenience yield (in log-deviations from steady state) - the amount of interest the agents are willing to forego because home government bonds also offer convenience benefits. A similar relationship can also be derived for the foreign country and hence  $\gamma_b \hat{\Psi}_{b_f,t}$  is the convenience yield on the foreign bond.

Thus, equation (1.23) shows us that the expected excess currency return is equal to the convenience yield differential; an increase in the home convenience yield would result in a compensating, expected increase in the excess financial return on the foreign bond. The excess return is a function of time  $t$  variables (the convenience yields), and is thus forecastable which constitutes a violation of the UIP condition. The violations at different horizons are driven by the dynamics of the convenience yield differential, in a similar manner to the analytical results derived earlier. Moreover, we can further simplify equation (1.23) to show that

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \frac{\gamma_b}{\eta_b} (\hat{b}_{ft} - \hat{b}_{ht}),$$

which showcases the close connection between the relative holdings of government bonds and the convenience yield differential. The more abundant are home bonds, relative to foreign bonds, the lower is the marginal convenience value of holding an extra unit of the home bonds, relative to holding an extra unit of foreign bonds, and thus the lower is the

convenience yield differential. This follows from the fact that the equilibrium convenience yield is equal to the marginal convenience value of each bond, and the marginal convenience value itself is decreasing in the holdings of the bond.

The more home bonds, relative to foreign bonds, an investor is holding, the lower is the convenience yield of the home bond relative to the foreign bond. This follows from the fact that the marginal benefit of each bond is decreasing in the holdings of that bond.

We can also rewrite equation (1.23) as

$$E_t(\hat{s}_{t+1} - \hat{s}_t) = \hat{i}_t - \hat{i}_t^* + \gamma_b(\hat{\Psi}_{b_h,t} - \hat{\Psi}_{b_f,t})$$

to highlight how exchange rate dynamics are affected. Unlike in a model where the standard UIP condition holds, in this model exchange rates need to offset not just the interest rate differential across countries but the combined effect of the differentials in interest rates and convenience yields. A positive interest rate differential would not necessarily imply a depreciating interest rate, because we would need to factor in the current value of the convenience yield differential as well. Clearly, the convenience yield differential acts as an omitted variable in the standard UIP regression. In the next section, I calibrate the model, quantify the introduced bias, and show that the model is able to closely match the full complexity of UIP violations at both short and long horizons.

### 1.5.6 Main Results

I log-linearize the model's equations around the symmetric, zero-inflation steady state and solve the resulting system of difference equations using standard techniques. Then I calibrate the model's parameters, using standard values from the literature whenever possible and matching relevant relevant moments of the data otherwise (but always moments independent of the UIP regressions), and compute the UIP regression coefficients implied by the model.

#### *Calibration*

The benchmark calibration is summarized in Table 1.2 and below I detail the calibration process. In general, the results are very robust to the great majority of coefficients.

As is standard in the literature, one period in the model represents one quarter. In terms of consumer preferences, I set the coefficient of risk aversion  $\sigma$  equal to 2 and the



inverse Frisch elasticity of labor supply  $\phi = 1.5$ , both of which are standard values in the RBC literature. Estimates of the elasticity of substitution between home and foreign goods vary a lot, but most estimates fall in the range from 1 to 2 and I follow Chari et al. (2002) and set  $\eta = 1.5$ . I choose the degree of home bias  $a_h = 0.8$ , a common value in the literature that is roughly in the middle of the range of values for the G7 countries.

To calibrate the transaction cost function I proceed as follows. I calibrate  $\alpha_1, \eta_m, \kappa_b, \bar{\psi}$  to match the interest rate semi-elasticity of money demand, the income elasticity of money demand, money velocity and the average convenience yield. I target an interest rate semi-elasticity of money demand of 7, which is in the middle of most estimates which range from 3 to 11 (see discussion and references in Burnside et al. (2011)). I set the income elasticity of money demand to 1, a widely accepted value, and then set the money velocity equal to 7.7, which is the average value for the *M1* money aggregate in the US for the time period 1976 – 2013.<sup>58</sup> Next, I target a steady state annualized convenience yield of 1%, which is in the middle of the range of estimates in the literature.<sup>59</sup> Finally, I choose  $a_b$  so that foreign bonds constitute 10% of the total bond portfolios of the agents, implying a strong home bias in accordance with the data.<sup>60</sup>

Lastly, there is little prior literature guidance in choosing  $\eta_b$ , the elasticity of substitution between home and foreign bonds in the transaction cost function. I set it equal to 0.25 in order to match the volatility of foreign bond holdings to GDP. In the model, increasing  $\eta_b$  makes the home and foreign bonds better substitutes and increases the overall volatility of foreign bond holdings, as agents are more likely to substitute into foreign holdings following shocks. The value of 0.25 is also about 2.5 times higher than the elasticity of substitution between money and bonds as a whole, indicating that bonds are better substitutes for each other, than they are for cash.

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<sup>58</sup> It makes little difference if I calibrate the model instead to money velocity as calculated from the *M2* or *MZM* money aggregates.

<sup>59</sup> The estimates of Krishnamurty and Vissing-Jorgensen (2012) imply that over 1969-2008 the average convenience yield difference between Treasuries and AAA and BAA corporate bonds has been 85 and 166 basis points respectively. Using a different methodology, Krishnamurty (2002) estimates an average Treasury convenience yield of 144 basis points.

<sup>60</sup> On the home bias in international bond portfolios see Warnock and Burger (2003), Fidora et al. (2007), Kyrychenko and Shum (2009) and Coeurdacier and Rey (2013a)

Table 1.2: Calibration

Param	Description	Value	Param	Description	Value
$\sigma$	Risk Aversion	2	$\frac{G}{Y}$	Gov Exp to GDP	0.22
$\phi$	Inv Labor Elast	1.5	$\frac{b^D}{Y}$	Gov Debt to GDP	0.5
$\eta$	Elast Subst Cons	1.5			
$a_h$	Home Bias in C	0.8	$\phi_\pi$	TR Infl Resp	1.5
$\beta$	Time Discount	0.9901	$\rho_i$	TR Smoothing	0.9
			$\sigma_v$	StdDev Mon Shock	0.0033
$\alpha_1$		19			
$\eta_m$		0.091	$\rho_\tau$	Tax Smoothing	0.92
$\kappa_b$		0.35	$\kappa_b$	Tax Resp to Debt	0.48
$\bar{\psi}$		6.75E-19			
$a_b$		0.9998	$\rho_a$	Autocorr TFP	0.97
$\eta_b$		0.25	$\sigma_a$	StdDev TFP	0.0078
$\mu$	Money Multiplier	2.17			

Next, I set the money multiplier  $\mu = 2.17$ , the average ratio of the money base to the M1 money aggregate in the US for the 1976-2013 time period. This parametrization also has the independent implication that the Central Bank holds 15% of government bonds in steady-state, which is in line with the US data. This gives further support for the overall parametrization of the liquidity preferences of the households. Lastly, I choose  $\beta = 0.9901$  which implies a steady state real interest rate of 3% per year.

I calibrate the steady state ratio of government spending to GDP to 22% and the ratio of government debt to GDP to 50%, the average values of total federal spending to GDP and total federal debt to GDP, respectively, in the US data. For the Taylor rule response to inflation I pick  $\phi_\pi = 1.5$ , the original value proposed by Taylor which has become a common choice in the literature. I then set  $\rho_i = 0.9$  to match the high persistence of interest rate differentials and the high relative volatility of exchange rate changes to interest rate differentials.<sup>61</sup> I estimate the coefficients on the taxation rule,  $\rho_\tau$  and  $\kappa_b$ , using US data on federal taxes to GDP and the federal debt to GDP, and obtain  $\rho_\tau = 0.92$  and  $\kappa_b = 0.48$ . Following Corsetti et al. (2010) the Calvo parameter is set to  $\theta = 0.75$ , a standard value in the literature.

In order to parametrize the technology process I used John Fernald's TFP data and

<sup>61</sup> Bianchi and Ilut (2013) also estimate a value of 0.9 for the Taylor Rule smoothing parameter.

estimated an AR(1) process in logs which yielded  $\rho_a = 0.97$  and  $\sigma_a = 0.0078$ . I back out the standard deviation of the Taylor rule shock from the US data as well, using data on the federal funds rate, CPI inflation and the calibrated parameters of the Taylor rule to construct a series of residuals. The standard deviation of the residuals gives me  $\sigma_v = 0.0033$ . This value also implies that a one standard deviation monetary shock results in a 76 basis points (in annualized terms) response on impact by the nominal interest rate, which matches the empirical estimate in Eichenbaum and Evans (1995).<sup>62</sup>

### *Quantitative Results*

In this section I examine the model's quantitative ability to match the UIP deviations in the data. I do this by computing the implied regression coefficient of the series of UIP regressions:

$$\hat{\lambda}_{t+k} = \alpha_k + \beta_k(\hat{i}_t - \hat{i}_t^*) + \varepsilon_{t+k}$$

where as always  $\hat{\lambda}_{t+k} = \hat{s}_{t+k} - \hat{s}_{t+k-1} + \hat{i}_{t+k-1}^* - \hat{i}_{t+k-1}$  is the one period excess foreign bond return. These are the same regressions that were also considered in the empirical section and I will compare the model implied sequence of coefficients  $\beta_k$  with the empirical estimates.

I start by considering the implied regression coefficients conditional on the model being driven by only one type of shock at a time. I will show that both monetary and technology shocks are able to generate empirically relevant UIP deviations at both short and long horizons. Both types of shocks are able to match the cyclical profile of UIP deviations and the timing at which the UIP deviations switch signs. However, monetary shocks are in general able to generate stronger UIP violations at all horizons, as compared to technology shocks. After establishing these results, I will show that a model driven by both shocks is able to closely match the magnitude of the empirical estimates of UIP deviations at both short and long horizons.

The solid line in Figure 1.6 plots the  $\beta_k$  coefficients as implied by the model when only monetary shocks are active, and the dashed line plots the empirical estimates.<sup>63</sup> Figure

<sup>62</sup>  $\sigma_v = 0.0033$  is also among the range of common estimates obtained by the prior literature, e.g. Davig and Leeper (2007) estimate 0.0036 and Galí and Rabanal (2005) estimate 0.003

<sup>63</sup> The empirical estimates plotted here use 3-month interest rates as the predictive variable, rather

1.7 shows the same plot, but for the case when the model is only driven by technology shocks. The main message of the two figures is that the cyclical profile of UIP deviations are fundamental feature of the model's mechanism and are robust to the source of exogenous variation. Both types of shocks are able to generate negative UIP coefficients at short horizons and positive coefficients at longer horizons, with the switch in signs occurring around 3 years (12 quarters) in the future, just as in the data. Monetary shocks are relatively more successful at matching the exact amplitude in the UIP deviations, implying stronger violations at both short and long horizons that align closely with the exact empirical estimates.

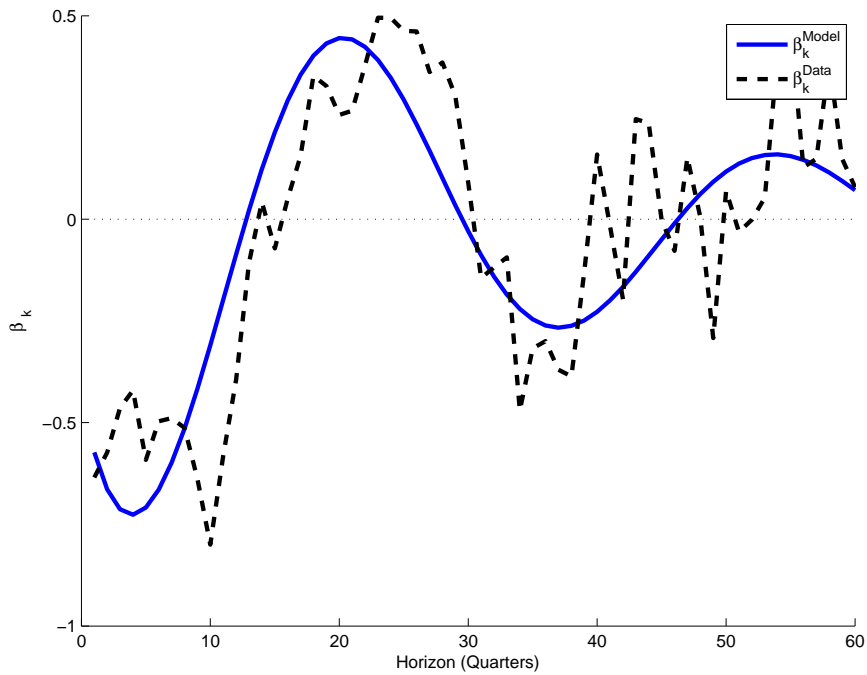


FIGURE 1.6: UIP Regression Coefficients, Monetary Shock

As will be explained in greater detail in the next section, the cyclical profile of UIP deviations is a fundamental feature of the model that is closely linked to the cyclical dynamics of government debt, which is in turn driven by the smoothing in tax policy. The cyclicity of UIP deviations is not unique to just technology and monetary shocks, but can be generated by a number of other shocks, such as fiscal, tax, and liquidity shocks. In

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than 1-month interest rates as in the empirical section, in order to be directly comparable with the model-implied regressions. The model is calibrated to a quarterly frequency and hence the one-period interest rate in the model itself is a 3-month interest rate.

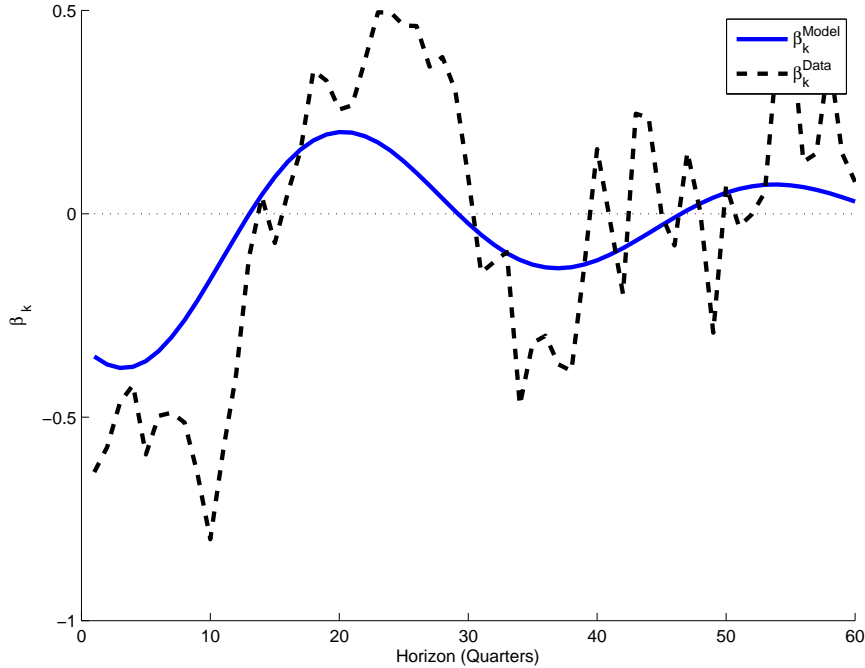


FIGURE 1.7: UIP Regression Coefficients, Technology Shock

the interest of conciseness, here I have only detailed the effects of monetary and technology shocks and have relegated analyzing other sources of exogenous variation to the Appendix.

Lastly, Figure 1.8 plots the resulting UIP coefficients when both shocks are active, and shows that in its full complexity, the model is able to closely match the empirical estimates at all horizons.

I conclude the section by examining the model’s implications about exchange rate behavior. In particular, I estimate direct projections of cumulative exchange rate changes on the interest rate differential as was done in the empirical section:

$$\hat{s}_{t+k} - \hat{s}_t = \alpha_k + \gamma_k(\hat{i}_t - \hat{i}_t^*) + \varepsilon_{t+k}$$

Recall that the sequence  $\{\gamma_k\}$  provides an estimate of the Impulse Response Function of the cumulative exchange rate change, relative to today, to an innovation in the interest rate differential, i.e.  $\text{Proj}(s_{t+k} - s_t | i_t - i_t^*) = \gamma_k(i_t - i_t^*)$ . The model-implied coefficients  $\gamma_k$  are plotted against their empirical counterparts in Figure 1.9. They show that the model does an excellent job of matching the initial appreciation of the exchange rate, the turning point at which it starts depreciating and also the level of the long-run depreciation.<sup>64</sup>

<sup>64</sup> Given that the model matches the UIP deviations well, this graph tells us that it also matches

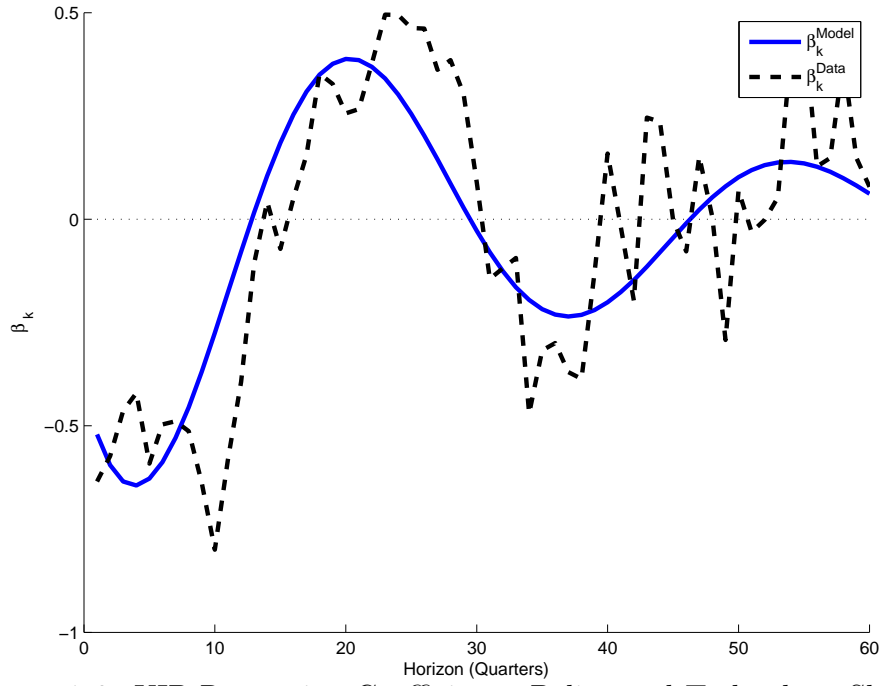


FIGURE 1.8: UIP Regression Coefficients, Policy and Technology Shocks

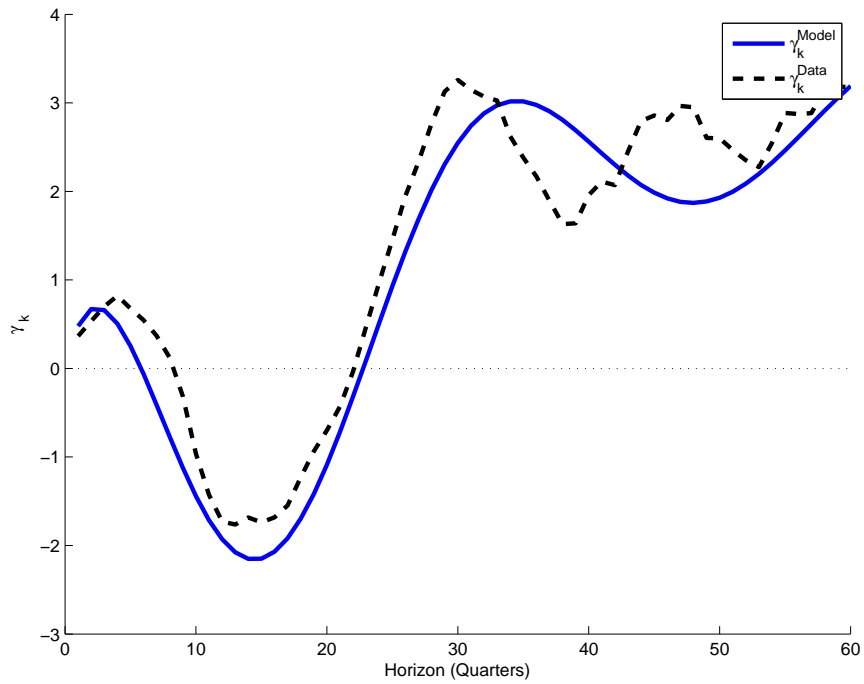


FIGURE 1.9: Direct Projection:  $\gamma_k = \frac{\partial E_t(\hat{s}_{t+k} - \hat{s}_t)}{\partial(\hat{i}_t - \hat{i}_t^*)}$

the dynamics of the underlying interest rate differentials. The forecastability pattern of the excess

### *The Mechanism Explained*

The mechanism behind the cyclical UIP violations is very similar to the one in the analytical model considered earlier, with the difference that the quantitative model features two extra channels in addition to cyclical government debt dynamics. Namely, the Open Market Operations of the central bank and the international spillover of shocks also contribute to the UIP deviations. This section will describe all three channels at work and how they propagate monetary and technology shocks. To help frame this discussion, recall that the equilibrium expected excess returns on foreign bonds is driven by changes in the households' portfolio holdings of home and foreign bonds:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \frac{\gamma^b}{\eta^b} (\hat{b}_{f,t} - \hat{b}_{h,t})$$

I start by discussing the effects of monetary shocks. A contractionary home country monetary shock raises interest rates and lowers inflation and GDP. The drop in inflation works the same way as before: it re-values existing real debt upwards, lowers seignorage revenues and increases the real cost of servicing the existing debt. On the other hand, the drop in GDP lowers tax revenues. All of these forces tighten the government's budget constraint and contribute to an increase in debt. The rise in the supply of home debt leads to a proportionate increase in the private agents' holdings of home government bonds, which lowers the home convenience yield relative to the foreign one. This leads to compensating, financial returns on the home bonds, which are achieved by an exchange rate appreciation at short horizons and this generates negative UIP coefficients.

On the other hand, taxes are quite sluggish ( $\rho_\tau = 0.92$ ) which introduces a complex root into the system and generates cyclical government debt dynamics. The reasons for the oscillations is the same as in the analytical model - persistent taxes can be expressed as a moving average of past debt levels, hence they are relatively slow to adjust to current government debt conditions. The cyclical dynamics of government debt also transfer to the private holdings of government bonds, home bond holdings eventually fall below steady state and generate a positive convenience yield differential. This gives rise to a corresponding positive excess return on the foreign bond and thus positive UIP violations at longer returns arises from the cyclical movements in the exchange rate and not from the interest rate differential. The Appendix provides further details on this point.

horizons.

The monetary shock also creates UIP deviations through Open Market Operations. A contractionary monetary shock that increases the interest rate today is accompanied by Open Market Operations in which the central bank reduces money supply by selling some of its holdings of government bonds. This, increases the net supply of home government bonds and through similar reasoning as above, leads to a negative convenience yield differential and negative UIP violations at short horizons. Equilibrium money holdings, however, do not display strong cyclicity and this channel does not contribute much to the positive violations at long horizons.

Lastly, the effects of the monetary shock also spill over to the foreign country. A contractionary home monetary shock leads to a persistent exchange rate appreciation, which increases inflation and output abroad and leads to an increase in the foreign interest rate. The rise in inflation and output loosen the foreign government's budget constraint and leads to a decrease in foreign debt. The decrease in net supply of foreign debt carries over to the private agents' portfolios, and the holdings of foreign bonds decrease as the holdings of home bonds are increasing due to the effects operating through the other two channels. Thus, the spillovers reinforce the domestic channels and make the convenience yield differential even more negative. The persistence in the foreign taxation policy also leads to similar reinforcing effects to the positive UIP deviations at longer horizons.

Now consider a negative technology shock. It decreases output and thus taxes, and increases inflation and the interest rate today. The tax and inflation effects on the government budget constraint roughly cancel out and lead to a negligible change in government debt on impact. However, the technology shock is persistent and leads to a prolonged drop in output, and hence taxes, and thus an eventual increase in government debt. Similarly to before, the increase in home government debt leads to a negative convenience yield differential and generates negative UIP coefficients. The systematic component of tax policy again reacts only sluggishly to the increase in debt and leads to the type of cyclicity in government debt that leads to positive UIP deviations at longer horizons.

The spillover of the shock operates in a similar manner and again reinforces the UIP deviations at both short and long horizons. The Open Market Operations channel, however, only generates negative UIP deviations in the case of technology shocks. The negative TFP shock generates a persistent drop in money supply, but does not impart any cyclicity on



the money process. The sum total of all three effects lead to UIP deviations that are similarly cyclical, but dampened in magnitude, as compared to the monetary shock.

In this generalized framework it is also true that the UIP deviations can be traced back to the fact that monetary policy does not accommodate fiscal policy (and vice versa), and this leads to cyclical fluctuations in the relative supply of liquid assets to the economy. If the central bank was committed to stabilizing the net supply of domestic government debt this would eliminate the two domestic channels that generate UIP deviations. The international spillover channel will still be operative, however, and some UIP deviations would be present unless there was cooperation between policy institutions internationally as well.

## 1.6 Testing the Model's Implications

### 1.6.1 Government Debt and UIP Violations in the Data

Government debt dynamics are central to the main mechanism of the model and in this section I provide direct empirical evidence that government debt affects excess currency returns in the way predicted by the model.

Recall that the excess return of foreign over home bonds in the model are equal to the convenience yield differential, which is in turn a function of the relative holdings of home and foreign bonds:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \left| \frac{\Psi_{b_h}}{1 + \Psi_{b_h}} \right| \frac{1}{\eta_b} (\hat{b}_{f,t} - \hat{b}_{h,t}).$$

In the model, forecastable excess returns and hence UIP deviations stem from movements in the relative portfolio composition of the private agents. A direct way to test this implication would be to re-estimate the standard UIP regression and control for the portfolio composition of investors. We would expect to find a significant negative coefficient on home bond holdings and a significant positive coefficient on foreign bond holdings, and an insignificant coefficient on the interest rate differential.

Data on private portfolio compositions is not readily available, but in the model household bond holdings are highly correlated with the supply of government bonds. Moreover, the supply of government debt is a common proxy for the convenience yield in the empirical literature (e.g. Krishnamurthy and Vissing-Jorgensen (2012)). This considerations lead me

to estimate an augmented version of the UIP regression by adding the US Debt-to-GDP ratio to the right hand side.<sup>65</sup>

I also consider two additional control variables: the outstanding amount of USD denominated Commercial Paper (standardized by US GDP) and the VIX index of implied volatility. I include the stock of dollar denominated Commercial Paper to control for possible substitution effects between public and high quality private debt.<sup>66</sup> A number of papers argue, both theoretically and empirically, that private debt could act as a substitute for government debt in the demand for safe and liquid assets (e.g. Bansal et al. (2011), Krishnamurthy and Vissing-Jorgensen (2012)). In fact, even a cursory look at the data is quite suggestive of this substitution effect. See for example Figure A.5 in the Appendix, which plots the evolution of both US government debt-to-GDP and commercial paper-to-GDP and shows a clear negative relationship, with a correlation coefficient of  $-0.87$ .

On the other hand, the VIX index is commonly seen as a proxy for the “global risk appetite”, with higher values indicating the market is less willing to accept risk, and I include it to proxy for any possible risk-premia that could be generating UIP deviations. A potential solution to the standard UIP puzzle at short horizons is that high interest rate currencies are riskier than lower interest rate currencies, and hence earn higher excess returns. To control for this effect, I sign the VIX index with the sign of the interest-rate differential, i.e.  $\text{sign}(i_t^* - i_t)VIX_t$ , so that the risk-factor is always properly signed and goes in the direction of the higher interest rate currency.

The general version of the updated UIP regression I consider is

$$\lambda_{j,t+1} = \alpha_j + \beta(i_{j,t} - i_{j,t}^*) + \gamma \ln \left( \frac{DEBT_t^{US}}{GDP_t^{US}} \right) + \delta_1 \ln \left( \frac{CP_t^{US}}{GDP_t^{US}} \right) + \delta_2 \text{sign}(i_t^* - i_t)VIX_t + \varepsilon_{j,t+1}.$$

Due to the availability of data on the stock of Commercial Paper the time period under consideration is shortened to January 1993 to July 2013. Moreover, I drop all

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<sup>65</sup> In the benchmark estimation I only include US Government debt because of data availability issues for higher frequency (i.e. quarterly) series on foreign government debt. Adding foreign debt (at the annual frequency) does not affect results significantly.

<sup>66</sup> Commercial Paper is very short short-term (less than 1 year in maturity) unsecured debt of large banks and corporations with excellent credit ratings. It is also a very safe investment, with default rates that are basically zero.

Euro legacy currencies (e.g. French Franc, Italian lira etc.) except the Deutsch Mark because now there is only at most 6 years of data for each. I keep the Deutsch Mark because the Euro is appended to it and hence there is a long time series in this case.<sup>67</sup>

Table 1.3: Modified UIP Regression

Variable	(1)	(2)	(3)	(4)	(5)	(6)
$i_t - i_t^*$	-2.18*** (0.76)	-2.29*** (0.76)	-2.26*** (0.83)	-1.24 (0.81)	-1.54*** (0.74)	-0.82 (0.82)
$\ln \frac{DEBT_t}{GDP_t}$		-0.53 (0.62)		-3.54** (1.60)		-3.36** (1.62)
$\ln \frac{CP_t}{GDP_t}$			0.13 (0.54)	-2.93** (1.31)		-2.75** (1.32)
$sign(i_t - i_t^*)VIX_t$					0.009** (0.0045)	0.007 (0.0045)
T	245	245	245	245	245	245
N	10	10	10	10	10	10

*Notes:* This table presents estimates of versions of the modified UIP regression  $\lambda_{j,t+1} = \alpha_j + \beta(i_{j,t} - i_{j,t}^*) + \gamma \ln\left(\frac{DEBT_t^{US}}{GDP_t^{US}}\right) + \delta_1 \ln\left(\frac{CP_t^{US}}{GDP_t^{US}}\right) + \delta_2 \text{sign}(i_t^* - i_t)VIX_t + \varepsilon_{j,t+1}$ . The regression is estimated as a panel with fixed effects and the reported standard errors are robust to both contemporaneous correlation (within time period and across currencies) and serial correlation.

Table 1.3 presents the estimation results, with each column representing a different specification. The first column estimates a regression where the interest rate differential is the only regressor, the second column adds  $\ln\left(\frac{DEBT_t^{US}}{GDP_t^{US}}\right)$ , and so on. Column (1) shows us that the UIP puzzle is very much true in this subsample as the standard UIP coefficient is  $-2.18$  and highly significant. Look next in column (4), which shows that augmenting the regression with the two measures of public and private debt changes the results. The two debt variables have significant negative coefficients, as predicted by the theory, while the coefficient on the interest rate differential is cut roughly in half and becomes insignificant. This suggests that the interest rate differential itself is not intricately linked to the UIP deviations, but rather shows up significant in standard UIP regressions because it is correlated with debt.

The table also shows that controlling for the private supply of debt is indeed important.

<sup>67</sup> Including the short-sample currencies makes no significant difference, just decreases the efficiency of the estimates. More details are available in the Appendix.

Neither government debt nor Commercial Paper are significant regressors when included by themselves, but are both significant and of the right sign when included together. This suggests that the substitution effect might indeed be playing a significant role.

Lastly, the addition of the signed VIX index has no major effect on the estimated coefficients of the debt variables, which remain negative and significant. The coefficient of the VIX itself is positive, which is the expected sign, but is not significant. Including it, however, does help to reduce the coefficient on the interest rate differential even more, which is suggestive evidence that there may be a small risk-premium effect as well. But the forecastability in the excess returns appear to be mainly driven by the debt variables.<sup>68</sup>

### *1.6.2 Monetary Policy Independence and UIP Violations in the Data*

Another strong prediction of the model is that central bank independence is positively associated with UIP deviations. In this section, I will show that this is also true in the data, by using the degree of capital controls as a proxy for monetary policy independence. I use this proxy for a lack of high quality, objective measurements of monetary policy independence that are comparable across currencies and span a significant portion of my data set (both in the cross-section and in the time dimension). On the other hand, a number of papers have shown that the degree of capital controls is a good proxy for monetary policy independence, see for example Alesina and Tabellini (1989), Drazen (1989), Alesina and Grilli (1994), Grilli and Milesi-Ferretti (1995), Leblang (1997), Quinn and Inclan (1997), and Bai and Wei (2000). Moreover, data on capital controls is readily available, has a long time series and is directly comparable across all currencies. I document that UIP violations are strongly negatively associated with the degree of capital controls, i.e. the countries that are most open financial exhibit the largest UIP violations, especially in regards to the positive violations at long horizons. Note that, if anything, a priori we might have expected UIP violations to be positively associated with capital controls because one of the implicit assumptions of the UIP hypothesis is free movement of capital across countries.<sup>69</sup>

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<sup>68</sup> The Appendix also presents results where I estimate the regression on a currency-by-currency basis. The resulting coefficients tell very much the same story as the panel estimates presented above, but as could be expected the estimates are noisier and less significant.

<sup>69</sup> Flood and Rose (1996), one of very few existing papers that have looked at the relationship between UIP violations and capital controls, also finds evidence that countries with fixed exchange rate regimes tend to exhibit smaller UIP violations at horizons of one and three months.

In both the analytical and the quantitative models, the dynamics of government debt are a key channel through which UIP violations are generated, especially the positive violations at longer horizons. In the analytical model, in particular, government debt dynamics are the only such channel and the central bank could fully eliminate UIP violations by committing to a monetary policy that stabilizes government debt. In the big model there are two additional channels, but monetary policy could still greatly weaken UIP violations by stabilizing government debt.<sup>70</sup> In Appendix A.4 I discuss two different experiments in which the foreign central bank is committed to helping the foreign fiscal authority manage its debt, and does so through lowering interest rates in response to high debt, which lowers the debt servicing costs and also helps inflate debt away. In both experiments the home central bank remains independent of the home fiscal authority, and the results show that it is enough to just change the behavior of the foreign central bank to greatly reduce UIP violations. Please see Appendix A.4 for more details on how the model works; for the rest of this section I will focus on the empirical results.

As discussed earlier, I will use the degree of capital controls as an empirical proxy for monetary policy independence and will measure them with the Chinn and Ito (2006) index of de jure capital controls, a standard procedure in the literature. The index is constructed from the binary dummy variables that characterize a country's restriction on capital flows as reported by the IMF's "Annual Report on Exchange Arrangements and Exchange Restrictions" and higher values mean less capital controls. It is a comprehensive measure, which accounts for both impediments to the free float of exchange rates and varying frictions in actual movements of financial capital in and out of the particular country.

Even though my sample is composed mainly of advanced economies there is a significant amount of cross-sectional variation in the degree of capital controls between countries, especially in the early part of the sample. Moreover, since the index is annual and not at the daily frequency as the exchange rates data, I focus on exploiting the cross-sectional variation in the degree of capital controls. In particular, I take the time-average of the Chinn-Ito index for each country and differentiate between currencies in their average level

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<sup>70</sup> By doing so, the central bank will in fact be reducing two channels: government debt dynamics themselves and the channel operating through the international spillover of shocks. This is because the spillovers mainly serve as an amplification of the effects coming from cyclical debt dynamics.

of capital controls.<sup>71</sup> To start, in Figure 1.6.2 I plot the average Chinn-Ito index of each currency against its estimated 1-month horizon UIP regression coefficient  $\beta_1$ .

The figure displays a strong negative relationship with a correlation coefficient of  $-0.72$  that is significant at the 1% level. It shows that countries with higher degrees of capital controls exhibit smaller UIP violations than countries with open capital markets. *Prima facie*, this result is counter-intuitive as one of the implicit assumptions underlying the derivation of the UIP condition is that capital is free to chase the highest available return.<sup>72</sup> However, this negative correlation is fully consistent with the prediction of the convenience yield model, when capital controls are viewed as a proxy for monetary policy independence. In the convenience yield model considered earlier, UIP violations are a direct result of the interaction between independent monetary and fiscal policies, and are greatly diminished in situations where the monetary policy is subordinate to the goals of managing the government's budget.

Next, I examine how the degree of capital controls is related to UIP deviations at different horizons. To do this, I split the sample of currencies into two subgroups based on their average Chinn-Ito index. I split the sample around around a Chinn-Ito value of 1, because there is a large gap between the two currencies to each side of 1 which makes for a natural break point.<sup>73</sup> As can be seen from Figure 1.6.2 this leaves me with a subsample of five currencies with “more capital controls” and another subsample of 13 currencies with “less capital controls”.<sup>74</sup>

I re-estimate the whole set of UIP regressions (1.7) on the two different subsamples of currencies. The estimated coefficients are plotted in the two panels of Figure 1.11,

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<sup>71</sup> I also consider a couple of different ways to control for capital controls dynamically despite the differences in frequencies of the observations. The results and the overall conclusion are very similar and are relegated to the Appendix for the sake of brevity.

<sup>72</sup> The results are also counter-intuitive from a risk-premium perspective, considering that historically many of the largest losses to the carry trade strategy stem from the re-valuation of fixed exchange rates.

<sup>73</sup> This is illustrated in Figure 1.6.2 where the average Chinn-Ito index is on the X-axis and the red dashed line marks the index value of 1.

<sup>74</sup> I have considered a number of robustness checks, including ways of controlling for capital controls dynamically by including them directly in the regression specification, and the results do not change. Splitting the sample is most straightforward and I present it as the benchmark here, and relegate the alternative specifications to the Appendix.

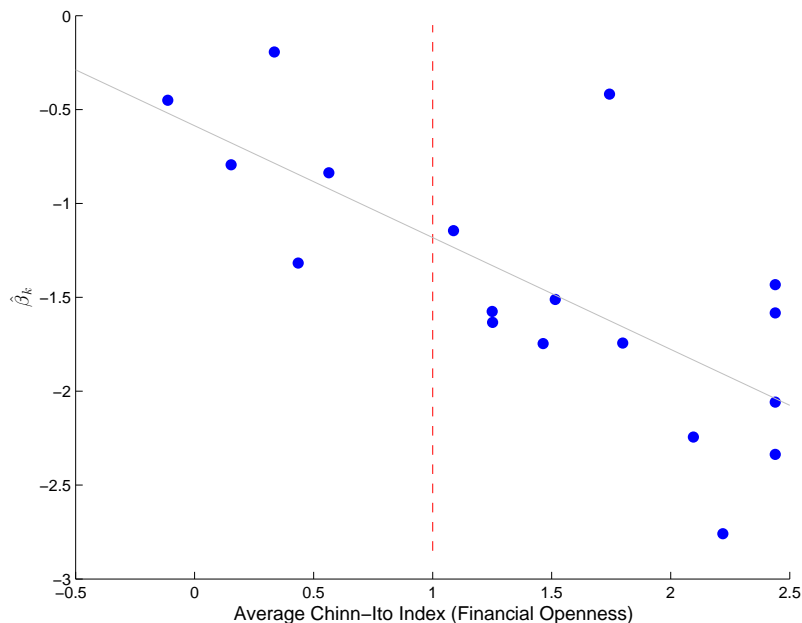


FIGURE 1.10: UIP Coefficients and Financial Openness

with the top panel displaying estimates for the subsample of the currencies with fewer capital controls. The currencies with freer capital accounts clearly display stronger UIP violations at all horizons. In fact, the group of currencies with “more capital controls” exhibit statistically significant UIP violations only at very short-horizons and are most consistent with the standard formulation of the UIP Puzzle, which emphasizes the negative short-horizon violations. Even at the short horizons, the magnitudes of the deviations are almost three times as small as the short-horizon deviations for the subset of currencies with open capital markets. The difference is even starker at longer horizons, where the currencies with stronger capital controls display no statistically significant UIP violations.

Overall, the results point to a strong, negative relationship between capital controls and UIP violations. The higher the degree of capital controls, the lower are the estimated UIP violations, especially at longer horizons. If we interpret the capital controls as a proxy for monetary policy independence, the findings support the implications of the model that UIP violations are increasing in the independence of monetary policy. Moreover, a different interpretation of the results is not straightforward, as ex-ante we would expect capital controls to actually be positively, and not negatively related to UIP violations.

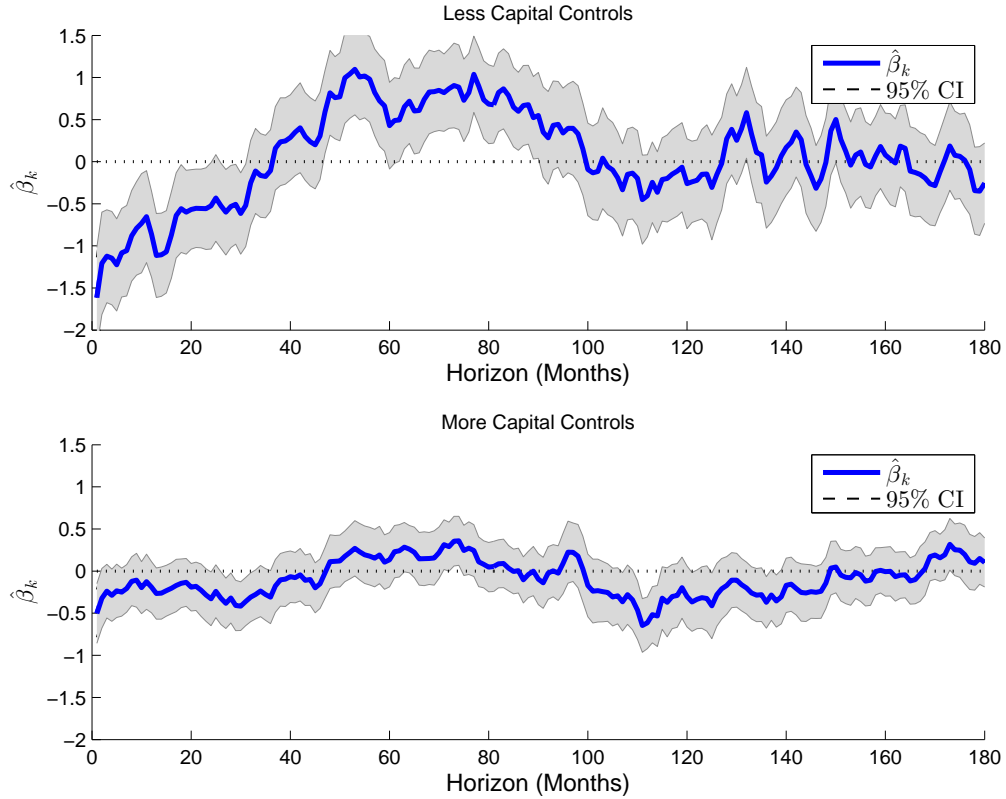


FIGURE 1.11: UIP Regressions, 1 to 180 months

## 1.7 Conclusion

This paper makes two main contributions. First, it shows that the famous Uncovered Interest Parity puzzle is more complex than typically understood. The main findings confirm previous results at short horizons which show that exchange rates fail to depreciate sufficiently to offset interest rate differentials. However, I document new evidence that at the longer horizons of 3 to 7 years exchange rates exhibit the opposite puzzling behavior - they depreciate too much. The changing nature of UIP deviations have important implications about exchange rate behavior, as they introduce cyclical dynamics and also lead to exchange rates converging to the UIP benchmark over the long-run.

The second contribution of the paper is to develop a novel model that can capture the full complexity of the UIP deviations at both short and long horizons. Unlike previous models which have primarily focused on explaining the short horizon negative UIP violations, I explore a novel mechanism that can also deliver the changing nature of UIP deviations at longer horizons. The model relies on the mechanism of the bond convenience



yield and the interaction of monetary and fiscal policies.

The negative short-horizon UIP violations are a fundamental feature of the convenience yield mechanism, which implies a negative, contemporaneous relationship between interest rates and convenience yields. The long-horizon UIP violations are in turn tied to the interaction between monetary and fiscal policy. In the model, the central bank is independent of the fiscal authority and raises interest rates to fight inflation without regard to the upward pressure this puts on government debt. On the other hand, the fiscal authority smooths taxes over time, which introduces cyclical dynamics in government debt that play a key role in the positive long-horizon violations. Lastly, I also show direct empirical evidence that supports the model's mechanism.

# Endogenous Information Asymmetry and Portfolio Bias

## 2.1 Introduction

Empirical evidence indicates that investors fail to hold diversified portfolios and appear to leave themselves exposed to substantial amounts of, otherwise avoidable, idiosyncratic risk. Furthermore, portfolios are concentrated in assets that are strongly *positively* correlated with investors' non-financial income, and thus appear to provide little hedging benefits. In contrast to this, traditional economic theory suggests that rational agents will hold a large variety of assets exposed to different sources of risk, and in particular favor assets which are not positively correlated with their non-financial income. These systematic differences between observed and model-predicted portfolios is typically referred to as "portfolio bias", and is a long-standing puzzle in both financial and international economics.

The empirical literature has documented a number of portfolio biases, with the "equity home bias" being perhaps the best known example. The term refers to the observation that aggregate national portfolios are strongly biased towards holding domestic equity assets and generally exhibit a low propensity for investing in foreign equity assets (see among others French and Poterba (1991), Tesar and Werner (1998), and Ahearne et al. (2004)).

A number of other portfolio biases have also been identified at the individual investor level. Huberman and Sengmueller (2004), Poterba (2003), Benartzi (2001), find that investors overinvest in the stock of their own employer or in the stock of other companies in the same industry. Ivković and Weisbenner (2005) and Huberman (2001) find that investors exhibit a “local bias” – the tendency to overinvest in assets that are geographically close to the investor’s place of residence. Massa and Simonov (2006) show that investors tilt their portfolios away from the market portfolio, and towards assets with a significantly higher correlation with their non-financial income. Goetzmann and Kumar (2008) show that the stocks individual investors hold are highly correlated with each other, which suggests that they are driven by the same risk factors.

This paper proposes a framework of endogenously generated information asymmetry that can help resolve the puzzle. The information asymmetry arises because the agents choose to be differentially informed about the existing risk factors, where they choose to pay more attention to the factors that end up being the most important drivers of their consumption. Which are those important factors is not exogenously given, but is determined through the endogenous choices of the agents and the resulting home bias in portfolios is a major reason why “home” factors are the most important for the home agents and vice versa. In some sense there is a vicious cycle where agents want to apportion their limited attention to the risk factors that are most closely related to their consumption, but in shifting their attention to such factors they also tilt their portfolio decisions and end up exposing their consumption even more heavily to these same risk factors. This prompts another round of attention shifts, portfolio shifts and etc.<sup>1</sup>

The key ingredients to the model are non-diversifiable non-financial income (incomplete-insurance) and information processing constraints.<sup>2</sup> Households can observe signals that

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<sup>1</sup> This amplification loop was first described by Van Nieuwerburgh and Veldkamp (2009) and is a crucial component of any model of endogenous information acquisition and portfolio choice

<sup>2</sup> Non-diversifiable labor income plays a fundamental role in Nieuwerburgh and Veldkamp (2006) in much the same way it does here. A key difference is that their framework exhibits increasing returns to information acquisition, while I introduce unlearnable uncertainty which brings about decreasing returns to information acquisition. Analyzing the interaction between non-diversifiable

provide noisy information about the future states of the world but unlike the traditional signal extraction literature the structure of the signals is not imposed exogenously but is determined endogenously. The households choose the precision of their signals, subject to the constraint that the signals can only contain a finite amount of information. The information acquisition framework is a version of the Rational Inattention framework developed by Sims (2003). The information constraint compels agents to focus their attention on information that is highly pertinent to their particular situation. It turns out that the agents are most sensitive to factors that affect both their non-financial and their financial incomes and hence focus most of their attention on them. In turn, this information acquisition behavior generates information asymmetry where agents end up possessing information of different quality about the different factors that can affect asset payoffs. As a consequence, the optimal portfolio choice of the agents leads them to concentrate their holdings in assets closely related with their non-financial income. It is important to note that the portfolios are the result of optimal behavior, even though they may look inefficient when viewed through the lenses of a standard, symmetric information model.

The idea that information asymmetry can generate concentrated portfolios is quite intuitive, and goes back to Merton (1987) who imposed the information structure exogenously. Recent work by Van Nieuwerburgh and Veldkamp (2009, 2010) has revitalized this literature by developing a model where the agents make optimal information acquisition choices and the information structure arises endogenously. That framework relies on increasing returns to information acquisition which leads to the strong, and somewhat restrictive, implication that the information choices of agents always serve to amplify any pre-existing information differences. The endogenous information choice acts as a powerful amplification mechanism but the direction of the resulting information asymmetry is determined entirely by the exogenous a-priori information differences that are assumed by the particular model. Moreover, the resulting information asymmetry is not robust to certain model perturbations (as first pointed out by Van Nieuwerburgh and Veldkamp

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labor income and unlearnable uncertainty are at the core of the current paper.

(2008)) and has counter-factual implications, as will be shown in this paper. I extend the literature by developing a model in which endogenous information choice can generate information asymmetry without having to rely on increasing returns to information and on prior information differences. In this setting, the endogenous information choice acts as both an amplifier and the original source of information asymmetry. I find that this new type of framework provides results that are more robust to model perturbations and avoid the counter-factual implications of previous setups.

An important implication of the model is that portfolio concentration has a non-monotonic relationship with an agent's capacity to process information. Agents with a low capacity find it optimal to specialize in acquiring information about only one asset and thus allocate any extra information capacity to the same asset, while agents with a high capacity choose to spread their efforts across a variety of different assets. Thus, the resulting information asymmetry and hence the degree of portfolio concentration is increasing in the capacity for information acquisition when the capacity is low, and decreasing when it is high. On the other hand, existing models that allow for optimal information acquisition but are based on exogenous prior information differences imply that the agents always choose to specialize their information acquisition in a single asset. In such models, the relationship between portfolio concentration and the capacity for acquiring information is unambiguously positive. This allows me to empirically differentiate the frameworks and I show that the empirical evidence rejects the implications of models based on exogenous prior differences but is consistent with the model of this paper. I conclude that a model of endogenous information asymmetry is key for an information based explanation of portfolio concentration, as models based on exogenous information differences lead to counterfactual implications.

The traditional home bias literature has focused on the determination of static (steady state) aggregate portfolios. However, more recent work has emphasized the related questions of the evolution of home bias (Coeurdacier and Rey (2013b), Hau and Rey (2008a), Hau and Rey (2008b)) and the dynamic behavior of international capital flows (Maggiori (2013a),

Maggiore (2013b)). And while this paper is targeted at the static dimension of the home bias puzzle, the heterogeneous information setup being studied here could have interesting implications about the dynamic behavior of portfolios and capital flows. Brennan and Cao (1997), Albuquerque et al. (2007) and Albuquerque et al. (2009) show that frameworks of exogenously specified heterogeneous information structures can explain a number of puzzling features of the dynamic behavior of international capital flows. Endogenizing the information structure of such frameworks along the lines of the model presented here is a fruitful avenue of future research and could provide a unified model of the joint determination of steady state portfolios and capital flow dynamics.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the details of the model and Section 4 presents the solutions and derives the main analytical results of the paper. Lastly, Section 5 presents the empirical evidence and Section 6 concludes.

## 2.2 Information Frictions, Nontradeable Labor Income and Portfolio Choice

This paper is related to three different strands of literature: the literature on portfolio choice under exogenous information asymmetry, the literature on portfolio choice and endogenous information acquisition, and the open economy macroeconomics literature relating home bias and labor income.

Assuming an extreme form of information asymmetry exogenously, Merton (1987) shows that when investors have heterogeneous information sets individual portfolios can differ significantly from the benchmark representative agent portfolio. A number of subsequent papers (Gehrig (1993), Brennan and Cao (1997); Brennan et al. (2005), Hatchondo (2008), and others) expand on this insight and apply it specifically to the home bias equity puzzle by developing two-country models, where home and foreign agents receive signals about the future payoffs of assets, but home agents are assumed to receive more precise sig-

nals about the home asset. The major drawback of this approach is summarized by Pástor (2000), who shows that for sufficient home bias to exist, the home agents must possess very strong prior information advantages. It is not clear why agents who start with a large information advantage over domestic asset would not spend resources to inform themselves about the highly uncertain (in relative terms) foreign asset. This paper provides an answer to this criticism by developing a framework where information asymmetry is generated by the endogenous learning choice of investors.

Van Nieuwerburgh and Veldkamp (2009, 2010) provide a key step forward by developing a model where the information structure arises endogenously, through the optimal information acquisition choices of the agents, rather than imposing it exogenously.<sup>3</sup> Their framework exhibits increasing returns to information acquisition and thus the optimal attention allocation strategy is the corner solution where agents only learn about one of the assets. This allows them to derive the important result that endogenous information acquisition serves as a powerful amplification mechanism, a seminal result of the literature which also provides the foundation of a number of subsequent papers (see for example Mondria (2010), Mondria and Wu (2011)). Generating portfolio bias inside this framework depends on both the increasing returns to information acquisition and the assumption of exogenous prior information asymmetry, unfortunately as I show in this paper, the increasing returns to information acquisition lead to counter-factual implications. In this work, I extend the literature by studying a mechanism that does not rely on increasing returns to information or exogenous prior differences, and is thus able to endogenously generate information asymmetry in a way that agrees with the data.

Mondria and Wu (2011) extend Mondria (2010) by considering imperfectly integrated international financial markets. In their framework, information asymmetry can also arise endogenously, and is driven by the fact that agents face transactions costs only when they trade foreign assets, but not when they trade home assets. The results depend on the

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<sup>3</sup> Their framework has also been used as a stepping stone by Mondria (2010) and Mondria and Wu (2011) and a number of other recent papers

asymmetric transaction costs associated with home and foreign assets, which could be justified in an international setting but are harder to motivate in a closed economy setting. Furthermore, Mondria and Wu (2011) use the previous literature’s information acquisition framework, which always exhibits increasing returns to information acquisition, and thus has counter-factual implications.

Lastly, the paper is also related to the open macroeconomics literature on the home bias, and specifically the strand of literature that considers the importance of labor income in the determination of international portfolios. Coeurdacier and Gourinchas (2011) and Heathcote and Perri (2007) develop two distinct frameworks where the joint determination of the equilibrium real exchange rate, labor income and asset returns generates a positive labor income-hedging demand for the home equity asset. This paper shares the literature’s key insight that labor income plays an important role in the formation of home biased portfolios, but the mechanisms are fundamentally different. In this work, labor income does not provide a positive hedging demand, but rather is the reason that the agents decide to bias their information acquisition strategy towards the home asset. The two frameworks are complimentary and would amplify each other’s results if put together. An important difference, however, is that the current model can simultaneously explain the portfolio concentration in both international and domestic portfolios, while the hedging motive identified by the macro literature only operates in an international environment with volatile exchange rates.

## 2.3 Model Framework

This paper models the information acquisition decision and asset demands of rational agents who face information processing constraints and take prices as given. For ease of exposition, I will present the model in an international setting, assuming that agents choose between “home” and “foreign” assets. This allows for the unambiguous terminology “home” and “foreign” but all results carry over to a closed, multi-sector (region) economy



where an agent has the option of investing in different sectors (regions) of the economy.

There is a continuum of agents of mass 1 that live in the home country, earn their non-financial income there,<sup>4</sup> and can trade a riskless bond and “home” and “foreign” risky securities. The agents live for 3 periods – they make information acquisition choices in period 0, observe informative but noisy signals and then choose their portfolio allocations in period 1 and in period 2 shocks are realized and the agents consume all of their wealth.<sup>5</sup> Agents maximize utility over period 2 consumption, and do not value leisure. Therefore agents supply their whole labor endowment, normalized to 1, at the wage rate they face. Thus each agent faces the following period 2 budget constraint,

$$c_2 = \delta w + x_h y_h + x_f y_f + bR, \quad (2.1)$$

where  $w$  is the wage rate,  $y_h$  is the payoff of the home asset,  $y_f$  is the payoff of the foreign asset,  $R$  is the gross return on the riskless bond, and  $x_h$  and  $x_f$  are the quantities of the home and foreign asset the agent chose in period 1. The first term in the above equation,  $\delta w$ , is the non-financial (wage) income of the individual, and the rest is his financial based income. The parameter  $\delta$  controls the relative weight of labor income in the agent’s total income. When  $\delta = 0$  the agent has no labor income and period 2 consumption relies entirely on financial income, and the importance of labor income increases as  $\delta$  grows. The existence of labor income is central to the mechanism of the model, and I will inspect how the relative size of non-financial to financial income affects the results by varying  $\delta$ .

The focus of the paper is on the information acquisition behavior of the households (investors) and is silent on the equilibrium determination of wage and rent rates, hence I model the wage and the payoffs to the risky assets as exogenous stochastic processes. I

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<sup>4</sup> In this paper I model the non-financial income as labor income, but in actuality this is the income stream coming from all other sources, but the publicly traded financial investments of an agent. Hence, this also includes the profits/dividends received from any privately-owned companies, private investments and etc.

<sup>5</sup> Given the static nature of the model, the agents do not make a decision on changing the country (industry) they work in (for) or the amount of time spent on working. An infinite horizon extension of the model which allows for these considerations is left to future work.

assume that the 3 payoff processes are given by:

$$\begin{aligned}
 w &= z_h + \varepsilon_w \\
 y_h &= z_h + \varepsilon_h \\
 y_f &= z_f + \varepsilon_f
 \end{aligned}
 \tag{2.2}$$

where all variables are normally distributed with  $[z_h, z_f]' \sim N(\mu, \Sigma)$ ,  $\mu$  is a column vector with  $\mu_1 = \mu_2$ , and all epsilons i.i.d.  $N(0, \sigma_{\varepsilon_j}^2)$ ,  $j \in \{w, h, f\}$ . Each payoff is driven by two stochastic factors, and the factor  $z_h$  drives both labor income,  $w$ , and the payoff to home equity  $y_h$ . This captures the idea that labor income and home equity are both affected by some of the same forces.<sup>6</sup> However, the two are not perfectly correlated as the factors  $\varepsilon_w$  and  $\varepsilon_h$  are independent from each other (this ensures labor income is not traded and hence is not perfectly diversifiable). For simplicity, the framework also assumes  $\Sigma$  is diagonal and hence there is no relationship between home and foreign payoffs. It is straightforward to introduce a “global” factor which would drive all three processes and thus introduce correlation across countries (or sectors), however I found this to have no qualitative effect on the results. For ease of exposition, therefore, I present the model without such a “global” factor.

In period 1, an agent chooses her portfolio allocations subject to her initial budget constraint,

$$A = p_h x_h + p_f x_f + b, \tag{2.3}$$

where  $A$  is the initial wealth of agent  $i$ ,  $p_h$  is the price of the home asset and  $p_f$  is the price of the foreign asset. The paper analyzes the agents’ optimal information and portfolio choices in a partial equilibrium setup and keeps the prices fixed. It is straightforward to obtain the equilibrium prices following the results in Admati (1985), however I found that

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<sup>6</sup> One can think of  $z_h$  as home country/sector TFP and of  $z_f$  as the foreign TFP.

this has no effect on the optimal behavior of the agents, which is the main focus of the paper. Hence, for ease of exposition, I abstract from the equilibrium determination of the asset prices.<sup>7</sup>

Prior to making the portfolio allocation choice the agents receive informative, but noisy, signals about  $\{z_h, z_f\}$  but not about any of the  $\varepsilon_j$ . This captures the idea that payoffs are subject to two types of uncertainty – uncertainty that an agent can reduce by acquiring and processing information available today (newspapers, Federal Reserve announcements, Analyst reports, etc.) and uncertainty that the agents cannot reduce with any of today’s information. I will continuously refer to the first type as “learnable” uncertainty and to the other one as “unlearnable”. Without such a dual structure of uncertainty the model would imply that an agent with sufficiently large information processing capacity could forecast the future arbitrarily well. Given how hard it is to forecast macroeconomic and, especially, financial series, this seems unrealistic. Later sections will study how the existence of unlearnable uncertainty affects the results in full detail.

In particular, the agents receive unbiased signals of the form:

$$\eta_h = z_h + u_h \tag{2.4}$$

$$\eta_f = z_f + u_f \tag{2.5}$$

where  $u_h$  and  $u_f$  are independent of each other and  $u_h \sim N(0, \sigma_{u_h}^2), u_f \sim N(0, \sigma_{u_f}^2)$ .

Moreover, the agents are allowed to choose  $\sigma_{u_h}^2$  and  $\sigma_{u_f}^2$ , which control the amount of noise in the signals  $\{\eta_h, \eta_f\}$ . This choice is made subject to the constraint that the total amount of information carried in the chosen signals is limited by the following entropy

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<sup>7</sup> In a dynamic setting, however, the equilibrium determination of the price will play a key role. In that setting the agents would want to forecast future prices, which are determined simultaneously with the information choices of the market participants, and thus there are interesting feedback effects between agents’ decisions and the market price. The dynamic case is thus different, but is outside of the scope of this paper and left to future work.

reduction constraint:

$$H(z_h, z_f) - H(z_h, z_f | \eta_h, \eta_f) \leq \kappa, \quad (2.6)$$

where  $H(X)$  is the entropy of random variable  $X$  and  $H(X|Y)$  is the entropy of  $X$  conditional on knowing  $Y$ .<sup>8</sup> Entropy is the standard measure of uncertainty in information theory, and consequently reductions in entropy measure information flow. The expression  $H(z_h, z_f) - H(z_h, z_f | \eta_h, \eta_f)$ , measures the information about  $\{z_h, z_f\}$  carried by  $\{\eta_h, \eta_f\}$  and the above constraint states that agents can only choose signals that carry no more than a total of  $\kappa$  bits of information.

The independence of  $u_h$  and  $u_f$  implies that agents can only reduce the posterior variance of  $z_h$  and  $z_f$  but cannot choose a correlation structure for their posterior beliefs – the correlation structure is taken as given from their priors. Intuitively, this amounts to assuming that learning about  $z_h$  and  $z_f$  are independent and unrelated activities that the agents carry out separately from one another, which appears to be a natural assumption given that the factors  $z_h$  and  $z_f$  themselves are assumed to be independent.<sup>9</sup> After observing the described signals, the agents use Bayesian updating with the correct prior to form their posterior beliefs. As opposed to other papers in the literature, I assume the agents have identical priors over both the home and foreign factors. In this sense, there is no exogenously imposed information asymmetry and the rest of the paper focuses on showing that even without prior exogenous information advantages, the home agents' utility maximizing decisions lead to ex-post information asymmetry.

Lastly, I assume that the agent's preferences over period 2 consumption are summarized

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<sup>8</sup> Entropy is defined as  $H(X) = -E(\ln(f(x)))$ , where  $f(x)$  is the probability density function of  $X$ .

<sup>9</sup> I could relax the assumption that signals are independent by allowing the agents to observe a linear combination of the factors  $z_h$  and  $z_f$  as in Mondria (2010), but I would lose the analytical tractability of the model. On the other hand, numerical solutions show that the main conclusions of the model and the nature of the mechanism examined remain unaffected. Thus, I maintain the assumption of independent signals.

by the following utility function:

$$E_0(-\ln(-E_1(-\exp(-\gamma c_2)))) \tag{2.7}$$

This is the “mean-variance” utility function used by the previous literature on information acquisition and portfolio choice (e.g. Van Nieuwerburgh and Veldkamp (2009, 2010)). It is a CARA utility function with an added desire for early resolution of uncertainty. This follows from results in Kreps and Porteus (1978) who show that if  $u(c)$  is a standard Von Neumann Morgenstern utility function and  $T(X)$  is a convex operator, then  $U = E_1(T(E_2(u(c))))$  is a version of the original utility function which exhibits a desire for early resolution of uncertainty, while preserving the original utility function’s risk aversion characteristics. The natural log is the typical choice for the convex operator  $T(X)$  in the literature as it allows for tractable analytical solutions.

The desire for early resolution of uncertainty is key in generating endogenous information asymmetry in this model but is not necessary in general. What is needed is for the agent to value information about his future non-financial income and the desire for early resolution of uncertainty is a tractable way to achieve this. This tractable setup was specifically chosen because it allows us to really put the model’s mechanism under the microscope and to derive a number of analytical results. Moreover, I have computed numerical solutions to several richer setups that do not rely on the desire for early resolution of uncertainty (such as CRRA utility or a dynamic model with a savings decision) and found that all of the results and insights of this paper remain true in these other setups as well.<sup>10</sup>

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<sup>10</sup> I find the richer frameworks interesting in their own right and I am studying them in greater detail in on-going work. However, they do not lend themselves to the clean analytical analysis that I am able to carry out in this paper and one must rely on numerical solutions. Interestingly, the numerical solutions overwhelmingly confirm the analytical results of this model which has convinced me that the tractable mechanism highlighted in this paper successfully captures the most important features of the interaction between learning and portfolio choice.

## 2.4 Model Solution

### 2.4.1 Period 1: Portfolio Allocation

It is convenient to define the following notation for the posterior and prior variances of  $z_h$  and  $z_f$ :  $\text{Var}(z_h|\eta_h) = \sigma_h^2$ ,  $\text{Var}(z_f|\eta_f) = \sigma_f^2$ , and  $\text{Var}(z_h) = \text{Var}(z_f) = \sigma_z^2$ . Having observed signals  $\{\eta_h, \eta_f\}$ , the agents use Bayesian updating and their prior beliefs to obtain the

posterior distribution  $\vec{z}|\vec{\eta} \sim N(\hat{\vec{\mu}}, \hat{\Sigma})$ , where  $\hat{\vec{\mu}} = E(\vec{z}|\vec{\eta})$  and  $\hat{\Sigma} = \begin{bmatrix} \sigma_h^2 & 0 \\ 0 & \sigma_f^2 \end{bmatrix}$ . Taking the

conditional distribution as given, the solution to an agent's period 1 problem yields the standard CARA utility portfolio choice rules:

$$x_h^* = \frac{\hat{\mu}_h - p_h R}{\gamma(\sigma_h^2 + \sigma_{\varepsilon_h}^2)} - \delta \frac{\sigma_h^2}{\sigma_h^2 + \sigma_{\varepsilon_h}^2} \quad (2.8)$$

$$x_f^* = \frac{\hat{\mu}_f - p_f R}{\gamma(\sigma_f^2 + \sigma_{\varepsilon_f}^2)} \quad (2.9)$$

The expressions show two things. First, an agent has an additional hedging motive to trade the home asset (the second term in the expression for  $x_h$ ) that does not factor into the demand for the foreign asset. Secondly, one can already see how portfolio concentration can be obtained if  $\sigma_h^2 \neq \sigma_f^2$ . The optimal choice of  $\sigma_h^2$  and  $\sigma_f^2$  is the focus of the next section.

### 2.4.2 Period 0: Information Acquisition Choice

In period 0 the agent takes into account the form of his optimal portfolio choice allocations in period 1 and chooses the utility maximizing posterior variances for the factors  $z_h$  and  $z_f$ , such that the signals he receives carry no more than  $\kappa$  bits of information.<sup>11</sup> The main

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<sup>11</sup> Technically, the agent chooses the variances of the error terms in his signals,  $\sigma_{u_h}^2$  and  $\sigma_{u_f}^2$ . However, there is a one-to-one relationship between the posterior variances of the  $z$  factors and the variance of the error terms,  $\sigma_h^2 = (\frac{1}{\sigma_z^2} + \frac{1}{\sigma_{u_h}^2})^{-1}$  and  $\sigma_f^2 = (\frac{1}{\sigma_z^2} + \frac{1}{\sigma_{u_f}^2})^{-1}$ , and it is more convenient to parameterize the problem in terms of  $\sigma_h^2$  and  $\sigma_f^2$ .

result of the paper is that the optimal information acquisition choice is always such that the agent acquires more information about  $z_h$  than about  $z_f$ . This is formalized in Proposition 1 below.

**Proposition 1.** *Let i)  $\delta > 0$  and ii)  $\sigma_{\varepsilon_h}^2 > 0$ . Then the unique solution to the information acquisition problem,  $\{\sigma_h^{*2}, \sigma_f^{*2}\}$ , is such that  $\sigma_h^{*2} < \sigma_f^{*2}$ , i.e. a home agent always pays more attention to the home forecastable factor than to the foreign forecastable factor.*

*Proof.* The proof is in the Appendix. □

Proposition 1 summarizes the main result of the paper. In the presence of non-diversifiable labor income, the optimal signal about the home factor has higher precision than the optimal signal about the foreign factor. The model endogenously generates the information asymmetry which is typically introduced exogenously by the previous literature.

The key for the result is the existence of a non-diversifiable, forecastable component of non-financial income. Condition i) states that the agent has any non-financial income at all ( $\delta > 0$ ), and condition ii) ensures that the forecastable component is non-diversifiable given the available financial instruments. To see the second point, imagine that ii) did not hold. Then the agent will be able to trade the risk factor  $z_h$  and since non-traded income is linear in  $z_h$  he will be able to perfectly hedge all forecastable risk in it. The only remaining uncertainty in the non-financial income will then be unrelated to the forecastable factors  $z_h$  and  $z_f$  and will thus have no effect on the information acquisition decision of the agent.

It is important to highlight that while crucial, non-financial income does not do all the heavy lifting by itself. An amplification mechanism similar to the one first identified by Van Nieuwerburgh and Veldkamp (2010) is at work in the following way. The forecastable component of non-financial income induces the agent to shift some of his attention toward the home factor, which in turn translates to lower posterior variance of the home risky asset return as compared to the foreign risky asset. This compels the agent to rebalance his portfolio towards the home asset but this increases his dependence on the home factor

even more. This provides him with an even greater incentive to shift attention toward the home factor, which leads to further portfolio rebalancing and so forth.

A direct implication of Proposition 1 is that portfolio allocations will diverge from what traditional theory would consider as a “well diversified” portfolio. Formally we have the following Corollary:

**Corollary 1.** *Let  $\{x_h^B, x_f^B\}$  be the mean portfolio allocations of an economy of Bayesian agents who receive equally precise signals about  $\{z_h, z_f\}$  in period 1. Let  $\{x_h^O, x_f^O\}$  be the allocations under optimal attention allocation, where the signals are chosen so that they carry the same amount of total information as in the previously defined economy. Then,  $x_h^O > x_h^B$  and  $x_f^O < x_f^B$  – optimal information acquisition biases portfolios towards the home asset.*

*Proof.* Follows directly from Proposition 1 and the portfolio choice expressions. □

This Corollary is not surprising since Proposition 1 tells us the model generates information asymmetry and the previous literature has already formalized the relationship between information asymmetry and portfolio concentration. This paper introduces a mechanism that generates the information asymmetry endogenously, but does not affect the portfolio choice in any other way.

### 2.4.3 *The Size of The Endogenous Information Asymmetry*

In this section I characterize the size and behavior of the generated information asymmetry over the whole parameter space and find conditions under which it achieves its maximum and minimum values.

A natural measure for information asymmetry is the difference in the amount of information learned about the home forecastable shock,  $z_h$ , and the amount of information learned about the foreign forecastable shock,  $z_f$ .<sup>12</sup> Call this difference in information flows

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<sup>12</sup> The unit of measure for information is bits, as is standard in information theory.



$\Lambda$ :

$$\begin{aligned}\Lambda &= (H(z_h) - H(z_h|\eta_h)) - (H(z_f) - H(z_f|\eta_f)) \\ &= \frac{1}{2}(\ln(\sigma_f^2) - \ln(\sigma_h^2))\end{aligned}\tag{2.10}$$

By Proposition 1,  $\Lambda > 0$ , or in other words, the agents always process more information about home shocks than about foreign shocks. Further,

$$\frac{\sigma_f}{\sigma_h} = \exp(\Lambda)\tag{2.11}$$

thus there is a one-to-one positive relationship between the variable  $\Lambda$  and the ratio of posterior standard deviations for the two forecastable shocks. Thus, increasing  $\Lambda$ , the measure of information asymmetry, decreases  $\sigma_h$  relative to  $\sigma_f$ .

Before we study the behavior of  $\Lambda$  itself, I derive the following Proposition.

**Proposition 2.** *The objective function  $U$  is convex in home information when  $\sigma_h^2 \geq \sigma_e^2$  and concave otherwise. Similarly it is convex in foreign information when  $\sigma_f^2 \geq \sigma_e^2$  and concave otherwise.*

*Proof.* The proof is in the Appendix. □

This result tells us that an agent enjoys increasing returns to information acquisition whenever the size of the remaining learnable uncertainty (either  $\sigma_h^2$  or  $\sigma_f^2$ ) is greater than the size of unlearnable uncertainty  $\sigma_e^2$ . This suggests that there are regions of the parameter space where the model has corner solutions and the optimal strategy of the agents is to allocate all available attention to only one of the factors – this is the relevant case for models in the spirit of Van Nieuwerburgh and Veldkamp (2010). However, there will also be regions of the parameter space where the problem is concave and the agent will acquire positive amounts of information about both factors. In particular, specialization pays off whenever information acquisition can reduce the majority of the uncertainty about an asset. In cases

where information acquisition can only reduce a minor part of the present uncertainty, the optimal strategy is to learn some about both factors. The model's implications are rich in the sense that different conditions (e.g. amounts of uncertainty being faced, size of the information capacity etc.) would imply different optimal information acquisition strategies. The next few propositions characterize how the optimal information acquisition strategy changes with the salient parameters of the model.

The following proposition characterizes how information asymmetry changes with  $\kappa$ .

**Proposition 3.** *Let  $\delta$ ,  $\gamma$  and  $\sigma_{\varepsilon_h}^2 = \sigma_{\varepsilon_f}^2 = \sigma_e^2$  be given (symmetric assets), and satisfy the conditions of Proposition 1. Then,*

- (i) *There exists a value  $\bar{\kappa}(\gamma, \delta, \sigma_e^2)$  such that  $\Lambda$  is an increasing function of information processing capacity,  $\kappa$ , whenever  $\kappa < \bar{\kappa}(\gamma, \delta, \sigma_e^2)$  and a decreasing function of  $\kappa$  for  $\kappa \geq \bar{\kappa}(\gamma, \delta, \sigma_e^2)$ .*
- (ii) *When  $\kappa \leq \bar{\kappa}(\gamma, \delta, \sigma_e^2)$  agents choose to devote their whole attention to the home forecastable factor, hence  $\Lambda = \kappa(\gamma, \delta, \sigma_e^2)$ ,  $\sigma_h^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$  and  $\sigma_f^2 = \sigma_z^2$ . The agent starts to allocate some of his attention capacity to the foreign factor once  $\kappa > \bar{\kappa}(\gamma, \delta, \sigma_e^2)$ .*
- (iii) *As  $\kappa \rightarrow \infty$ ,  $\Lambda \rightarrow \frac{1}{2} \ln(\frac{A}{B}) > 0$ , where  $A$  and  $B$  are constants defined in the appendix.*

*Proof.* The proof is in the Appendix. □

Proposition 3 develops three results that are useful in determining the size of the generated information asymmetry. First, it tells us that information asymmetry is a non-monotonic function of the information processing ability of an agent, where it is increasing for low values of information capacity and decreasing for large ones. Second, it informs us that some agents (ones for which  $\kappa \leq \bar{\kappa}(\gamma, \delta, \sigma_e^2)$ ) feel so constrained in terms of information capacity, that they allocate all of it to the home forecastable factor and ignore information about the foreign factor. Third, information asymmetry converges to a positive limit as information capacity becomes infinite – thus information asymmetry exists even in the limit of unbounded information processing ability.

Figure 2.4.3 plots  $\Lambda$  and the ratio  $\frac{\sigma_f}{\sigma_h}$  on  $\kappa$ . One can clearly see the two regions defined by Proposition 3. At first, the agent devotes all attention to the home asset and  $\Lambda$  is growing linearly with  $\kappa$ . After a certain point he starts paying attention to the foreign asset as well, information asymmetry unravels and eventually converges to a positive number. The second plot shows the ratio of the posterior standard deviations of  $z_f$  and  $z_h$ . This ratio starts at 1 as  $\kappa$  is 0 and at first rises, then falls, and eventually converges to a number strictly greater than 1. Thus, the value of  $\kappa$  is key to determining the size of the generated information asymmetry. At the two extremes of scarce and abundant information capacity information asymmetry is smallest, and is highest for moderate values of  $\kappa$ .

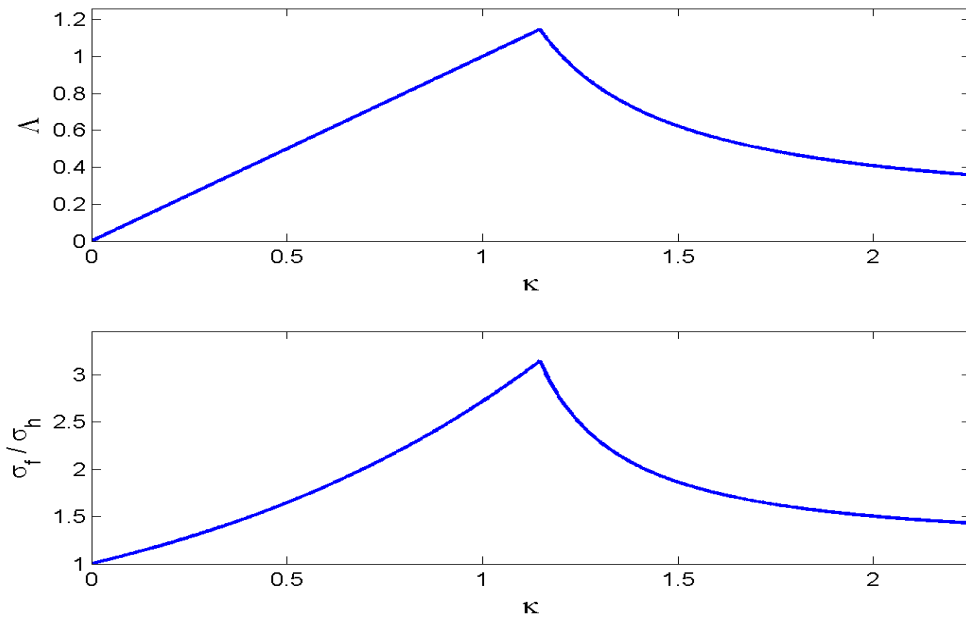


FIGURE 2.1: Information Asymmetry and Information Capacity

The result that  $\Lambda$  is first increasing and then decreasing with information capacity is intuitive. It is easy to imagine that people with little time and/or information resources would focus their attention on issues important to their particular situation, and ignore information about other things. For example, imagine that you had only five minutes to check the web page of your favorite news provider. Odds are you will go to the page for domestic/local news and simply scan the headlines for the major news, as this will give

you the best overall picture of the things that matter most to you that can be obtained in five minutes. Now imagine that you had twenty minutes to read the news. You will still go to the domestic/local news page first and scan the headlines, but then you will also most likely read through an entire article or two. This is an example of the increasing returns to information acquisition when information processing abilities are scarce. Reading just the title might give you an interesting piece of information like “The Economy is Close to a Recession”, but the body of the article would be even more useful (increasing returns). Finally, imagine that you have several hours to spend reading the news. Most likely, in such a large amount of time you will drive down the marginal benefit of domestic/local information to the point where you move on to the news page of a country that is of particular interest to you, for example possibly one in which you have invested some money. This is an example of the decreasing returns to information acquisition which occur when  $\kappa$  is sufficiently large.

The next Proposition derives results about how information asymmetry and the shape of the graph in Figure 2.4.3 change with risk aversion ( $\gamma$ ), the size of labor income ( $\delta$ ) and the total amount of unlearnable uncertainty ( $\sigma_\varepsilon^2$ ).

**Proposition 4.** *Let the conditions of Proposition 1 be satisfied. Then,  $\bar{\kappa}(\gamma, \delta, \sigma_\varepsilon^2)$  is an increasing function of  $\gamma$ ,  $\delta$  and  $\alpha$ , where  $\sigma_z^2 = \alpha\sigma_y^2$ . Moreover,  $\Lambda$  - the overall amount of information asymmetry - is an increasing function of  $\gamma$  and  $\delta$ .*

*Proof.* The proof is in the Appendix. □

More risk averse agents and agents for whom labor income is a more important source of funds have stronger incentives to skew their attention towards the home factor. Moreover, in a world where most of the uncertainty agents face cannot be reduced through acquiring the information available today (i.e. low  $\alpha$ ), the payoffs to information specialization are low.<sup>13</sup> Hence, the attention tipping point  $\bar{\kappa}$  occurs at a smaller value of  $\kappa$ , as compared to

<sup>13</sup> The parameter  $\alpha$  allows me to study what happens when we shift some uncertainty from the forecastable factors  $z$  to the unforecastable  $\varepsilon$ , while keeping the total amount of uncertainty  $\sigma_y^2 = \sigma_z^2 + \sigma_\varepsilon^2$  fixed.

environments where the agents can use the information they acquire to reduce most of the uncertainty they face.

Figure 2.4.3 plots  $\Lambda$  and  $\frac{\sigma_f}{\sigma_h}$  on  $\kappa$  for three different values of the risk aversion coefficient  $\gamma$ . The graph illustrates the result of Proposition 4 that information asymmetry increases with risk aversion, and in particular, one can see that increasing risk aversion has three different effects. First, it increases the value of  $\kappa$  at which the tipping point occurs. Second, it increases the information asymmetry at any given  $\kappa$ . And third, it increases the value to which information asymmetry converges as capacity is made limitless. The last result is not mentioned explicitly in Proposition 4, but it can easily be verified from the proof of Proposition 3.

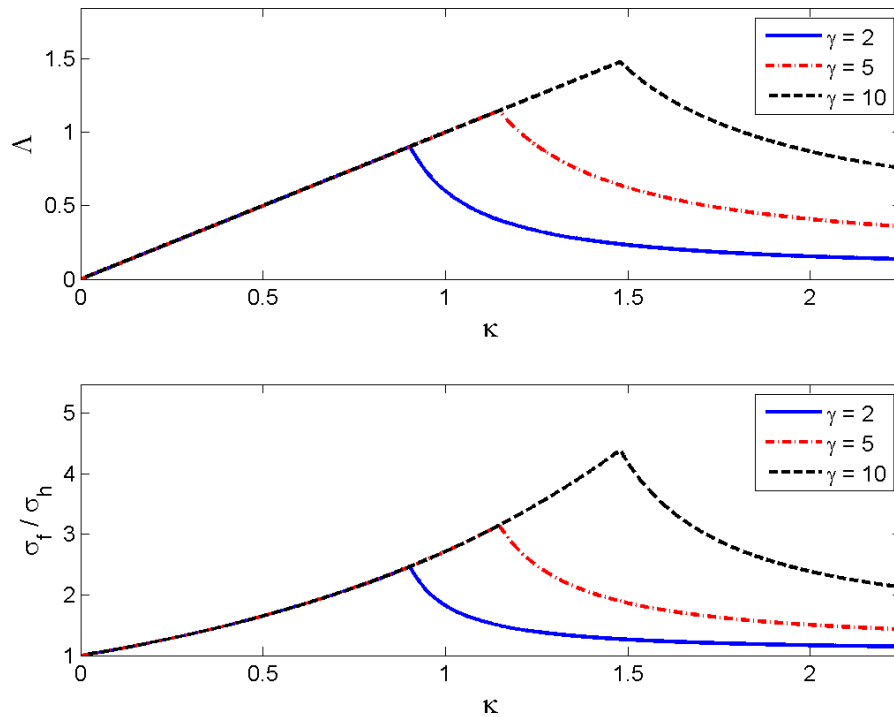


FIGURE 2.2: Information Asymmetry as a Function of Risk Aversion

Figure 2.4.3 plots the same quantities, but considers changes in  $\delta$  instead of  $\gamma$ . The resulting picture is very similar -  $\delta$  varies both the tipping point and the amount of information asymmetry for any given  $\kappa$ . Increasing  $\delta$  increases the share of labor income in an

agent's total income, making labor income risk a larger concern. Since non-diversifiable labor risk is what drives the information asymmetry, it is not surprising to see that the skew in attention allocation increases as  $\delta$  (and hence the labor/non-financial share of income) increases.

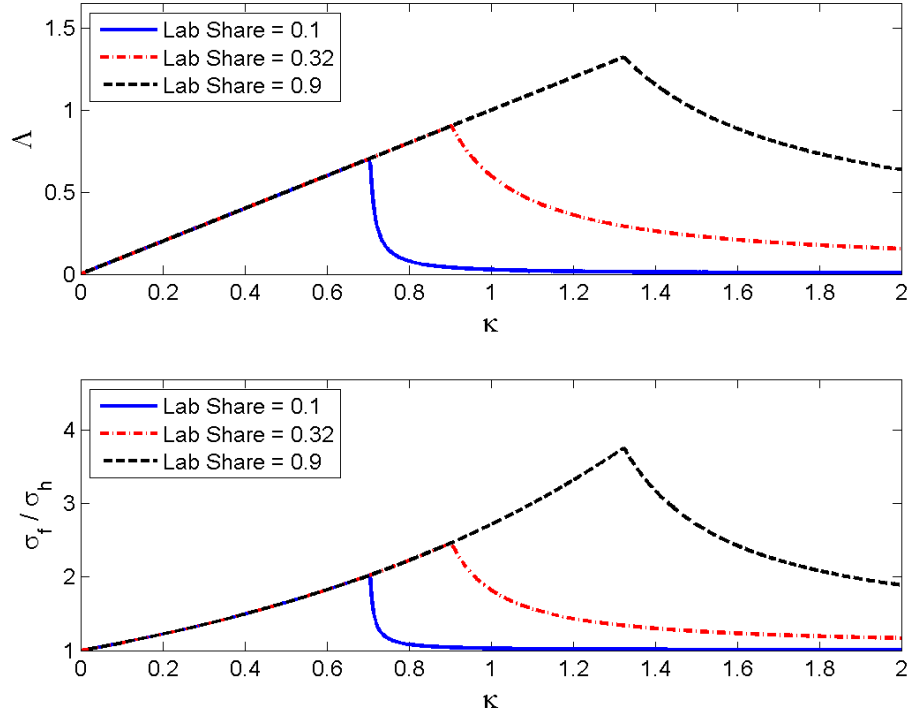


FIGURE 2.3: Information Asymmetry and Labor Income

Lastly, Figure 2.4.3 presents plots for three different values of  $\alpha$ . One can see that the attention tipping point  $\bar{\kappa}$  increases as  $\alpha$  increases - this is because the incentives for specialization increase as the learnable information can now reduce a larger fraction of the total uncertainty. However, unlike  $\gamma$  and  $\delta$ ,  $\alpha$  also has another effect. Increasing  $\alpha$  decreases the terminal value of information asymmetry because  $\alpha$  also controls the correlation between the home asset and labor income. A higher  $\alpha$  implies a higher correlation and makes the home asset a better hedge for labor income. In turn, this results in a lower exposure to non-diversifiable labor income risk, which then makes home information relatively less valuable. This second effect dominates for high values of  $\kappa$ , and hence we observe the crossing of the

lines in Figure 2.4.3. This result is also not explicitly mentioned in Proposition 4, but can be easily verified from the expressions for the constants  $A$  and  $B$  cited in Proposition 3.<sup>14</sup>

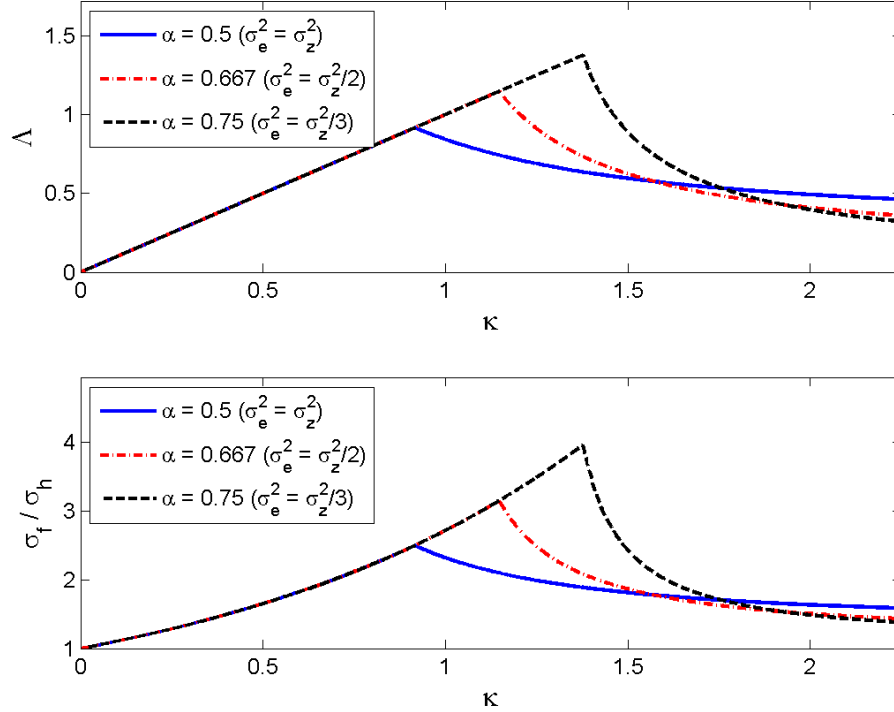


FIGURE 2.4: Information Asymmetry and Unlearnable Uncertainty

The effect of the relative size of unlearnable to learnable uncertainty on the information acquisition choice is quite intuitive in its own right. As with everything else, specialization in information acquisition is only appealing when additional information on the same topic could potentially lead to a relatively high payoff. If  $\alpha$  is close to zero, even perfect information about either  $z_h$  or  $z_f$  offers little reward. If  $\alpha$  is close to one, however, precise information about one of the forecastable factors is very valuable to the individual, as it comes very close to perfectly revealing the future.

<sup>14</sup> In any case, Proposition 1 holds, and some information asymmetry always exists.

#### 2.4.4 *Endogenous vs Exogenous Information Asymmetry*

The seminal work by Van Nieuwerburgh and Veldkamp (2009, 2010) develops framework in which endogenous information acquisition generates information asymmetry by amplifying any prior exogenous information advantages. The exogenously given prior information advantage is key, as the model does not generate any information asymmetry without it, but it can be arbitrarily small. This is a fundamental result in the literature that has served as the foundation of a number of subsequent works, but unfortunately does not hold in a setup with unlearnable uncertainty ( $\sigma_e^2 > 0$ ). The fact that unlearnable uncertainty breaks the increasing returns and hence the amplification result is first mentioned in the working paper Van Nieuwerburgh and Veldkamp (2008), but does not appear in the published version of that paper and the subsequent literature has exclusively focused on the case of increasing returns. In this section, I will compare and contrast the models with and without unlearnable uncertainty in detail and will show that a mechanism of endogenous information asymmetry appears to be necessary for a robust information-based explanation of portfolio bias.

A benchmark model in the spirit of the existing literature is given by setting  $\delta = 0$  and  $\sigma_e^2 = 0$ , and assuming that the home agents have a prior information advantage over the home risk factor (i.e. they have tighter priors over  $z_h$ ).<sup>15</sup> I will refer to this specification as the benchmark model of exogenous information differences and analyze the implications of introducing  $\sigma_e^2 > 0$  and  $\delta > 0$  which would lead us to the model of this paper.

In an environment with no labor income ( $\delta = 0$ ) and no unlearnable uncertainty ( $\sigma_e^2 = 0$ ) the model exhibits increasing returns to information acquisition over the whole parameter space. Thus, the optimal strategy of the agents is to always exhaust their whole information processing capacity on one and only one of the forecastable factors. Which of the two assets the agent will choose to specialize in depends on the differences in his prior beliefs. If one endows the agent with symmetric prior information, as this paper does,

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<sup>15</sup> For example this would be the model in Corollary 2 of Van Nieuwerburgh and Veldkamp (2010)



then the solution is indeterminate - the agent will be indifferent between focusing only on the home factor or focusing only on the foreign factor. But if we endow the agents with even just an arbitrarily small information advantage in one of the factors, then the unique optimal strategy is to pay attention only to that factor. This result can be then used to conclude that small prior information advantages are not only sustained, but amplified through endogenous information acquisition.

This result does not hold in a more general setting where  $\sigma_e^2 > 0$  as this introduces decreasing returns to information acquisition. In particular, it is the case that the objective function will become concave when  $\kappa > \bar{\kappa}$ . At that instant, the agent's optimal strategy changes so that he pays attention to both risk factors and he does so in such a way as to undo any and all prior information advantages. Given a large enough capacity, any prior information advantage will be eliminated and not amplified.<sup>16</sup> Figure 2.4.4 shows how the size of information asymmetry behaves in the benchmark exogenous information asymmetry model, in a model that simply adds unlearnable uncertainty and finally the full model of this paper.

The blue dashed line shows the benchmark exogenous differences framework, the red dash-dot line augments that model with unlearnable uncertainty only and the solid green line is the full model developed here. When the agent has no capacity for processing information ( $\kappa = 0$ )  $\Lambda$  is trivially zero in all models, and then it increases linearly with  $\kappa$ . This is due to the increasing returns to information acquisition that would exist in all models for small values of  $\kappa$ . However, if  $\sigma_e^2 > 0$ , once  $\kappa$  becomes bigger than  $\bar{\kappa}$  the optimal attention allocation strategy changes abruptly. Instead of only acquiring information about the home forecastable factor, the agents find it optimal to learn about both home and foreign factors, and in fact in the model with no labor income ( $\delta = 0$ ) agents end up focusing most of their attention on the *foreign* factor. This is because the agents face a concave problem and the optimal solution is to have equally precise posteriors about both the

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<sup>16</sup> This result also holds for models that extend the Van Nieuwerburgh and Veldkamp (2010) framework along the lines of Mondria (2010). Such models allow the agents to observe an optimal linear combination of the two factors, rather than separate signals for each.

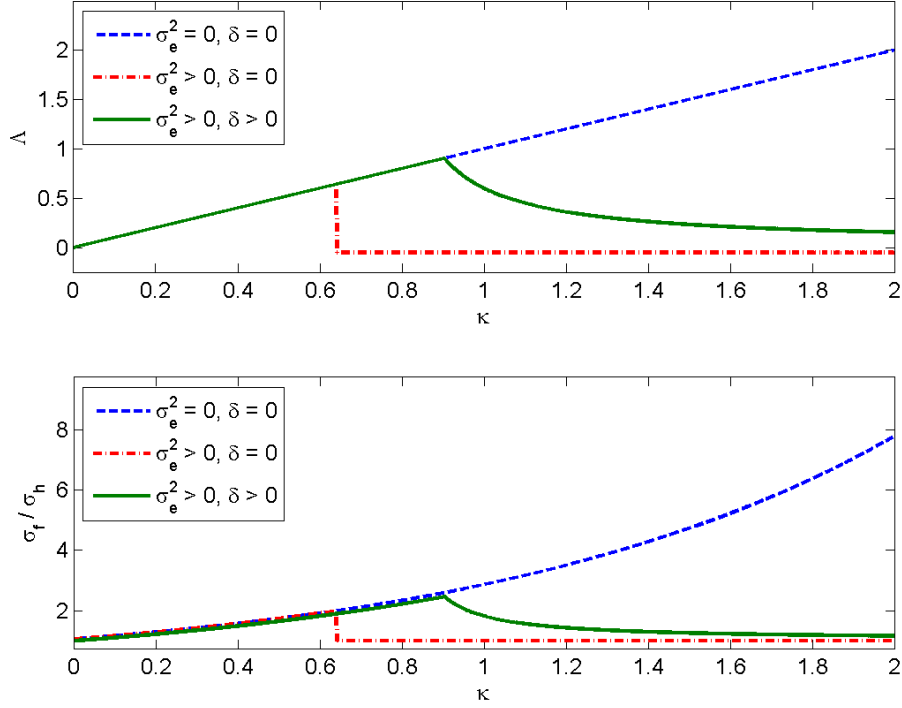


FIGURE 2.5: Models of Exogenous Information Asymmetry

home and foreign fundamentals. But to get there they first need to undo the exogenously assumed prior information asymmetry, and hence spend most of their information capacity on the foreign factor. The second graph makes this clear by plotting the ratio of posterior standard deviations  $\frac{\sigma_f}{\sigma_h}$  as a function of  $\kappa$  - notice the jump down to 1 for  $\kappa > \bar{\kappa}$ . The following proposition formalizes these results and more.

**Proposition 5.** *Let  $\delta = 0$  and the agents be endowed with a free signal about the home fundamentals,  $s = z_h + u$ , where  $u \sim N(0, \sigma_u^2)$ . Then,*

- *If  $\sigma_e^2 = 0$ , agents choose to devote their whole attention to the home factor, and hence we obtain  $\Lambda = \kappa$ .*
- *If  $\sigma_e^2 > 0$ , then there exists a  $\bar{\kappa}(\sigma_e^2, \sigma_u^2)$  such that agents allocate their whole attention to the home factor for  $\kappa < \bar{\kappa}$  but focus most of their attention on the foreign factor when  $\kappa > \bar{\kappa}$ . When  $\kappa < \bar{\kappa}$ ,  $\Lambda = \kappa$  and  $\frac{\sigma_f}{\sigma_h} = \exp(2\kappa)$ , and when  $\kappa > \bar{\kappa}$ ,  $\Lambda < 0$  and*

$\frac{\sigma_f}{\sigma_h} = 1$ . Also,  $\bar{\kappa}(\sigma_e^2, \sigma_u^2)$  is a decreasing function of  $\sigma_e^2$  and an increasing function of  $\sigma_u^2$

*Proof.* The proof is in the Appendix. □

Proposition 5 formalizes the intuition displayed in Figure 2.4.4. In the presence of unlearnable uncertainty, agents exploit the benefits of increasing returns to information acquisition and only allocate attention to the home factor when the information capacity constraint is tight. However, if the agents possess sufficient information processing capacity, they exhaust the increasing returns to information and instead prefer to be equally well informed about both factors. To do so, they need to unravel the prior information advantage they have over the home factor, and thus for high values of the capacity constraint the agents always pay more attention to the foreign factor rather than the home one.<sup>17</sup>

The last point in Proposition 5 states that the tipping point  $\kappa$  is an increasing function of  $\sigma_u^2$ . The reason is that larger prior information advantage (lower  $\sigma_u^2$ ) implies that the agents need to expend less of their capacity before they reduce the posterior uncertainty  $\sigma_h^2$  to the point where they lose the increasing returns to information acquisition. In particular, this implies that if agents start with a sufficiently large prior information advantage the interval of increasing returns to information acquisition would not exist at all, and the agents will immediately undo it by paying most of their attention to the foreign fundamentals.

This result connects to one of the main criticisms of information asymmetry explanations of portfolio concentration – why agents do not learn about what they are most uncertain about. Non-diversifiable labor income can provide an answer, as it gives the agents an endogenous reason to value home information more than foreign information. The solid green line in Figure 5 represents the model with labor income. The graph shows it can generate home bias in information acquisition even for high values of  $\kappa$ , where the

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<sup>17</sup> In fact these results are even stronger if we generalize the information structure to allow the agents to observe a linear combination of the factors as in Mondria (2010). In that setting, agents acquire information more efficiently and in fact display a foreign bias in their information acquisition for all  $\kappa$ .

other model implies the opposite - a foreign bias. Most importantly, whereas the model without labor income implies that  $\sigma_h^2 = \sigma_f^2$  for high values of  $\kappa$ , the model of this paper always produces  $\sigma_h^2 < \sigma_f^2$ . This latter result is crucial to explaining portfolio concentration, because in these frameworks the agents would only bias their portfolios towards one of the assets if they perceive it to be less risky, i.e. one needs  $\sigma_h^2 < \sigma_f^2$ .

Thus a model based solely on exogenous prior information differences is not able to generate portfolio home bias when  $\sigma_e^2 > 0$  and  $\kappa > \bar{\kappa}$ . More importantly, in Section 6 I present empirical evidence that the most likely description of the real world is indeed  $\kappa > \bar{\kappa}$ . This should not be surprising, because finding otherwise would suggest that investors enjoy increasing returns to information acquisition. But imagine a straightforward extension of the model, where instead of keeping  $\kappa$  as a fixed parameter we allow agents to choose their optimal  $\kappa$  subject to a simple, linear information cost function. In that case, the agents would never choose a  $\kappa < \bar{\kappa}$  because it is optimal to exhaust all available increasing returns to information. The empirical findings in Section 6 confirm this intuition. Hence, a theory of endogenous information asymmetry is necessary for an information based explanation of portfolio concentration.

## 2.5 Empirical Evidence

Section 5.4 shows that under the parametric restriction of  $\kappa < \bar{\kappa}$ , the exogenous information advantage model delivers the same results and implications as the endogenous information asymmetry model developed in this paper. In light of this, it is fair to ask if a model of endogenous information asymmetry is empirically relevant and if such theory is needed in order to fully understand the phenomenon of portfolio concentration. Here I address this question by exploiting the fact that whenever  $\kappa \geq \bar{\kappa}$ , the two frameworks have markedly different implications for relationship between portfolio concentration, the level of information capacity, and the relative size of non-financial income.<sup>18</sup>

<sup>18</sup> Another clear difference between the models arises in an N-asset framework. In an exogenous information advantage model, the agents only acquire information about the home asset and thus

The empirical restrictions examined in this section are derived from Propositions 3, 4 and 5. Proposition 5 shows that in exogenous information advantage models, agents exhaust their entire information acquisition ability on the home asset, and hence the implied information asymmetry is increasing in the capacity constraint  $\kappa$ . This paper's framework, however, has a different prediction. According to Proposition 3, the magnitude of the information asymmetry *decreases* with  $\kappa$ , whenever  $\kappa \geq \bar{\kappa}$ . This suggests that one way to differentiate the two frameworks is to look at the empirical relationship between portfolio bias and information processing constraints – a negative relationship will be at odds with the exogenous information advantage models but consistent with the framework of this paper.

Another differentiating feature is the relationship between the relative size of non-financial income and portfolio bias. In an exogenous information advantage model the non-financial income affects portfolio choice only through the hedging component of the portfolio. Under the assumption that non-financial income and the home asset returns are positively correlated this hedging effect is negative and hence we would also expect the relative size of labor income to be negatively related with portfolio bias. In the endogenous information asymmetry model, however, non-financial income also affects the information acquisition decision of the agents. Proposition 4 shows that agents with a relatively high non-financial incomes use more of their information capacity on home fundamentals and this results in a more acute information asymmetry and hence a stronger informational motive for loading up on the home asset. Clearly, this second effect of non-financial income is positively related to home bias and thus is of the opposite sign of the original hedging effect (which is still operational in this model as well). We now have two competing effects

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have symmetric information over all foreign assets and hold all foreign assets in the same proportion. On the other hand, in an endogenous information asymmetry model, the agents stagger foreign information acquisition and thus for any  $\kappa$ , they possess information of different quality for the different foreign assets, which leaves them holding the foreign assets in different proportions. Empirical evidence shows that countries do not treat all foreign assets as equal, and the composition of the foreign equity portfolio varies greatly across countries. This empirical regularity could also be used to differentiate the two models but the formal treatment is left to future work.

and is unclear whether we should expect a positive or a negative relationship, but we can at least say that a positive relationship would be consistent with the endogenous information asymmetry model but not with the model exogenous information advantage.

To examine these relationships I have obtained aggregate data on 35 OECD countries for the time period between 2001 and 2008. I have data on portfolio positions, proxies for  $\kappa$ , labor income and financial wealth per capita, the Chinn-Ito index of financial openness and real PPP GDP per capita. The time period choice is restricted by the availability of the IMF's Coordinated Portfolio Investment Survey (CPIS), the highest quality international portfolio positions data and all monetary values are in PPP terms. The data series come from the IMF, the OECD and the World Bank and full details are given in the Appendix.

The data set is in a panel format but since the models I am trying to differentiate are both static I conduct the benchmark analysis on cross-sectional regressions of time-averaged variables. In particular, I regress the degree of home bias (EHB) on proxies for  $\kappa$ , the relative size of non-financial income, and other controls.

$$\overline{EHB}_i = const + \beta_k \kappa_i + \beta_\delta \delta_i + \beta' \overline{X}_i + \varepsilon_i \quad (2.12)$$

As a benchmark proxy for information capacity,  $\kappa$ , I use the number of Internet users per 100 people, which I obtain from the World Bank. The availability of the Internet is a natural proxy for information processing capacity for two reasons. First, Internet technology greatly reduces the time associated with collecting information, and second, an Internet connection also presupposes the availability of a computer, which further enhances a person's ability to sift through and analyze information. It is straightforward that for any fixed period of time, an Internet user can process more bits of information than a person without an Internet connection. Lastly, for the time period at hand - 2001 through 2008 - the Internet was readily available to all 35 countries in the sample, but at the same time the countries exhibit significant variation in the extent to which the Internet had penetrated

their populations.<sup>19</sup>

I base my proxy for non-financial income on total labor compensation. What matters in the model, is not the total size of labor income but rather its size relative to capital income and I consider several different proxies that try to capture this relationship. First, I use the standard calculation of labor share of aggregate income:

$$\delta_i = \frac{\text{Total Labor Compensation}_i}{\text{GDP}_i} \quad (2.13)$$

This first proxy is most relevant in a closed economy setup, where the total income of the representative agent is comprised by the GDP. In an open economy setup, however, it is more appropriate to also factor in any income earned on foreign investments. To control for this, I compute a second version of Labor Share as follows:

$$\text{Labor Share}_2 = \frac{\text{Total Labor Compensation}_i}{\text{Total Labor Compensation}_i + \text{Total Financial Assets}_i * r_{US}^{3m}} \quad (2.14)$$

Here I express the labor share as the ratio of labor income and total income, where total income also includes the capital income flow from the Financial Assets (both domestic and foreign) of the representative agent. Unfortunately, I do not have data on the total capital income but only on the portfolio position, hence I proxy for the flow capital income by multiplying the portfolio position by the yield on the 3-months US Treasury. I have analyzed a number of other possible portfolio returns instead of the 3-months US Treasury and found that the actual return used has negligible effects on the results. The big assumption here is that the relevant return does not vary too much *across* countries, as then it will be an omitted variable in my regression.

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<sup>19</sup> I am in the process of running the analysis with two alternative proxies for  $\kappa$  - cell phones per capita and computers per capita. Unfortunately, the availability of this data and its quality is worse than the data on Internet users per capita and I am still working on obtaining a dataset with good coverage. Preliminary regressions, show that the coefficient on  $\kappa$  remains negative but is less significant but this is still work in progress

Lastly, I also include Total Labor Income and Total Financial Assets as regressors and put no restrictions on their two coefficients. Since I do not have a direct observation of  $\delta$ , this third strategy aims to control for the relative size labor income without making explicit functional assumptions about the relationship between  $\delta$  and the observable income proxies. For this specification, the regression is given by:

$$\overline{EHB}_i = const + \beta_k \kappa_i + \beta_l \ln(\overline{LabIncome}_i) + \beta_f \ln(\overline{FinWealth}_i) + \beta' \overline{X}_i + \varepsilon_i \quad (2.15)$$

The vector of controls has two elements. First, I include the Chinn-Ito Index, which measures the degree of existing capital controls in a given country (Chinn and Ito (2006)), in an attempt to control for home bias which arises purely from institutional barriers to foreign investment. Second, I also include the initial period (year 2001) level of real PPP GDP per capita which controls for initial conditions, and especially ensures that the coefficient on Internet Users per 100 people does not simply pick up the fact that richer countries tend to be more diversified.<sup>20</sup>

The coefficients of primary interest are  $\beta_k$  and  $\beta_\delta$ . The first empirical restriction I test can be summarized by  $H_0 : \beta_k > 0$  vs  $H_a : \beta_k \leq 0$ , and the second is given by  $H_0 : \beta_\delta < 0$  vs  $H_a : \beta_\delta \geq 0$ . A rejection of the first null hypothesis would mean that the degree of home bias is decreasing in information processing capacity and a rejection of the second hypothesis would imply that home bias is increasing in the relative size of labor (non-financial) income. Technically both hypotheses are one-sided, however, to be conservative all results are reported at the standard 2-sided confidence levels.<sup>21</sup>

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<sup>20</sup> The Appendix includes regressions which consider additional control variables, such as non-tradable consumption (Stockman and Tesar (1995), Pesenti and Van Wincoop (2002), Lewis (1999)), but I do not include them in the main body of the paper for the sake of parsimony of the empirical model and because the results remain unchanged.

<sup>21</sup> The Appendix includes results from a regression that includes the square of  $\kappa$  to capture potential non-linearities (which is, strictly speaking, the model's prediction). This more flexible setup does find the non-monotonic relationship predicted by Prop. 3 – EHB is increasing in  $\kappa$  for countries with low  $\kappa$  and decreasing otherwise (only 5 countries are found to be in the increasing region).



Table 2.1: Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Internet Users	-0.158*** (0.039)	-0.130*** (0.039)	-0.161*** (0.037)	-0.095* (0.057)	-0.075 (0.049)	-0.090** (0.045)	-0.079* (0.045)	-0.113** (0.049)
$\delta \left( \frac{\text{Labor Income}}{\text{Total Assets}} \right)$		0.176*** (0.047)				0.120*** (0.048)		
$\delta (\text{Labor Share})$			0.016 (0.057)				0.057 (0.044)	
Log Labor Income				-0.033 (0.062)				0.259** (0.101)
Log Fin Wealth				-0.035* (0.019)				-0.019 (0.015)
Chinn-Ito Index	-0.072*** (0.024)	-0.067*** (0.019)	-0.072*** (0.023)	-0.065** (0.024)	-0.049** (0.021)	-0.055*** (0.019)	-0.049*** (0.017)	-0.060*** (0.018)
Log RGPD <sub>2001</sub> p.c.					-0.148*** (0.036)	-0.086** (0.041)	-0.162*** (0.034)	-0.327*** (0.095)
N	35	35	35	35	35	35	35	35
Rsqr	0.585	0.659	0.586	0.638	0.651	0.674	0.659	0.684

Table 2.1 presents OLS estimates with the corresponding heteroskedasticity robust standard errors in parentheses. The first column shows results when I do not include any proxy for  $\delta$  and only include the Chinn-Ito Index as an additional control. Columns (2)-(4) add alternatively the three different ways I attempt to proxy for  $\delta$ . Lastly, columns (5)-(8) repeat the regressions in (1)-(4) by adding the initial period real GDP per capita as well.

The main result is that all estimate of  $\beta_k$  are found to be negative and statistically significant at standard levels in 7 out of the 8 specifications. The only regression which shows up with an insignificant  $\beta_k$  coefficient does not include a proxy for  $\delta$  and hence is misspecified. The data is conclusive that information processing capacity is negatively related to the degree of home bias and this leads to a rejection of the first null hypothesis being tested.<sup>22</sup> Therefore, an information asymmetry model of the home bias that is based on increasing returns to learning, such as the standard exogenous information advantage model, has counter-factual implications. If increasing returns to learning generate home bias, then it must be the case that higher learning capacity leads to a higher degree of home bias and the data points to the opposite. This leads me to conclude that a theory of endogenous information asymmetry is key to an information based explanation of the home bias.

The empirical results are not quite as inconclusive in regards to the relationship between home bias and the relative size of labor income. I find a positive and significant  $\beta_\delta$  when I use the ratio of labor income to total income as a proxy for  $\delta$ , however when I use the standard labor share I find a positive but insignificant coefficient. Lastly, when I include labor income and total financial assets in the full specification of the model (column (8)) I find that the labor income coefficient is positive and significant, while the coefficient on financial wealth is negative but insignificant. Overall, the evidence against the second null

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<sup>22</sup> Mondria and Wu (2010) use a slightly different proxy for  $\kappa$ , which is a measure of IT technology per \$1000 of economic activity rather than IT technology per capita, and find the opposite result that the Home Bias is positively related with their measure of  $\kappa$ . In the Appendix I reestimate my regressions using the Mondria and Wu (2010) measure of  $\kappa$  and again find a statistically significant negative relationship. There I also discuss three possible reasons for the difference in the results: the different data sets, the extra controls I include, and differences in methodology.

hypothesis is not overwhelming but the estimates are consistently pointing towards a slight positive relationship between labor income and portfolio bias.

Interestingly, this is a phenomenon present in micro-level data as well. Using data on each individual investor's labor income, financial income and portfolio composition, Massa and Simonov (2006) find that investors actively tilt their portfolios towards assets that are positively correlated with their labor income process. They find that while the average correlation between the stock market as a whole and an individual's labor income is roughly zero, the average correlation between an individual's labor income and his portfolio proceeds is positive and statistically significant. Moreover, they also find evidence that investors act deliberately in skewing their portfolios towards assets that are positively correlated with their labor incomes. They show on average the stocks that investors buy increases the correlation of portfolio returns and non-financial income, while the stocks they sell decrease this correlation. Most importantly, they find that the size of portfolio bias is negatively related to the investor's total wealth. They find that on average, the bias amounts to 41% of the total risky asset holdings of low wealth investors and for 10% of the risky assets of wealthy investors. This suggests that on the micro level, investors that are more dependent on non-financial income have more concentrated portfolios, and this is again evidence against the second empirical restriction again.<sup>23</sup>

Thus, this appears to be a pattern found both in micro and macro data. Coeurdacier and Gourinchas (2011) develop an open economy macroeconomic model where the presence of real exchange rate uncertainty and non-tradable labor income can also deliver the result that home bias is positively related to the size of labor income. It is hard to differentiate that framework from the endogenous information asymmetry model when only using regressions on aggregate data. However, their mechanism relies on 1) real exchange rate risk, 2) the availability of a local-currency denominated bond with returns that co-move strongly with the local price level and 3) equity returns which are negatively related with

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<sup>23</sup> This is also in accordance with a large part of the empirical literature on under-diversification which documents that wealthier investors hold better diversified portfolios.

labor income, conditional on the bond returns. Coeurdacier and Gourinchas (2011) and others show that the necessary conditions are generally satisfied by the joint distribution of international price levels, bonds and equities. It is harder, however, to imagine that such locality-tied nominal bond instruments exist inside a single country while the mechanism of this paper operates both inside a single country and on the international level. The endogenous information asymmetry mechanism is thus interesting because it is not specific to the domestic or international settings and provides an explanation of the overall phenomenon of portfolio concentration. Moreover, the two mechanisms are not rival but in fact amplify each other when put together. Exploring frameworks where both mechanisms are active would be an interesting avenue for future research, as it could help explain why international portfolios appear to be a lot more concentrated than their domestic counterparts.

Lastly, note that this paper considers only technological measures of information processing capacity. While in the 21<sup>st</sup> century information technology is certainly a very important, if not the chief, determinant of information processing ability, human capital is also likely to play an important role. Examining the empirical relationship between portfolio concentration and investors' human capital is beyond the scope of this paper, but there already exists an empirical literature on this issue. The interested reader is directed to Goetzmann and Kumar (2008) who show that portfolio concentration is decreasing in investors' educational level, investing experience and sophistication, and also to, Kimball and Shumway (2010) who document that the international home bias is decreasing in a number of different measures of investor sophistication.

In conclusion, this section set out to use data to differentiate between the model derived in this paper and the exogenous information advantage model used in previous work. The empirical evidence rejects two key implications of the exogenous information asymmetry model and is consistent instead with an endogenous information asymmetry framework. This suggests that there are empirically relevant differences in the two frameworks, and hence endogenous information asymmetry is likely important for a complete understanding

of how information imperfections affect the phenomenon of portfolio concentration.

## 2.6 Conclusion

This paper addressed one of the major puzzles to financial and international economics - the systematic difference between theoretically optimal portfolios and portfolios observed in the data. It developed a framework in which information asymmetry arises endogenously and the resulting information sets are skewed towards the risk-factors driving the agents' non-financial income. This leads to portfolio holdings that look like they are "under-diversified" or "biased" from the viewpoint of traditional portfolio choice theory. In particular, the theory can account for the international finance phenomenon of the "home bias" in equities and could also explain the micro data observation that individual investor's tend to concentrate their holdings in assets that are positively related to their non-financial income.

It was shown that exogenous information advantage models have counter-factual implications which can be avoided when we move to a framework of endogenous information asymmetry. Thus, a key message of the paper is that a theory of endogenous information asymmetry appears to play an important role in information-based explanations of portfolio concentration. Here I showed that non-financial income considerations are one way to generate information asymmetry endogenously but there are surely many more. The important thing is to think about channels through which information about asset returns also has an effect on the other decisions of the agents (e.g. housing choice, labor choice, etc.), but I leave the analysis of such additional channels to future work.

Another fruitful avenue for future work is to introduce dynamics into the framework and think about the determination of international capital flows at business cycle frequencies. A number of papers (see for example Brennan and Cao (1997) and Albuquerque et al. (2007)) argue that setups of exogenously specified heterogeneous information can explain a number of puzzling features in the dynamic behavior of capital flows (both inflows and outflows). That literature, however, has only considered exogenously given information

structure. A model that implements the endogenous information structure of this paper in a dynamic framework could inform us about the joint determination of the home bias and the behavior international capital flows.

## Do World Prices Affect Small Open Economies?

### 3.1 Introduction

The Small Open Economy (SOE) literature has long used the terms of trade - the relative price of exports over imports - as an important source of output fluctuations (Mendoza (1995); Kose (2002)). The defining characteristic of the small open economy model is the assumption that the economy is a price taker on the world markets, hence international interest rates and the terms of trade are viewed as exogenous to domestic disturbances. This allows SOE Real Business Cycle models to rely on exogenous international interest rates and the country's terms of trade in addition to the typical productivity shocks as driving forces for output fluctuations. The high variability displayed by both interest rates and terms of trade in general helps the models in producing realistic fluctuations in national accounts variables such as GDP, Investment and Consumption (Mendoza (1995); Kose (2002); Correia et al. (1995)).

On an intuitive level, we can think of international trade as a linear production mechanism which takes exports as inputs and outputs imports, where the terms of trade are the “productivity” parameter of this production technology. The higher the terms of trade, the more imports the country obtains for the same amount of exports. To illustrate, consider

the following simple example. Imagine a country that produces a single good, which it both consumes and exports, and in the production of which the country uses intermediate goods imported from abroad. If the terms of trade increase a few things happen. First, the value-added index of production increases simply due to the change in relative price. Value added production is of course gross output minus the value of intermediate inputs:

$$Y_t = Q_t - \frac{m_t}{tot_t}$$

where  $Q_t$  is gross output,  $m_t$  are the imported intermediate goods and  $tot_t$  are the terms of trade. As  $tot_t$  goes up  $Y_t$  increases without the need of any changes in  $Q_t$  or  $m_t$ . In this paper, this direct effect of the terms of trade will be referred to as the first order effect.

On top of this first order effect, a change in the terms of trade will also have a second order effect due to the re-alignment in the factors of production that the change in the relative price brings. Cheaper imports will induce firms to increase their use of intermediate goods, raising the marginal product of labor and hence the market wage rate, which in turn increases labor. Thus, through increased labor effort and intermediate goods consumption, output will be further increased, and this additional movement I will call the second order effect of an increase in the terms of trade. This simple example already hints at how similar the effects of terms of trade changes are to the effects of a technology shock, and in fact it can be shown that the terms of trade have effects analogous to those of productivity shocks and are often viewed precisely as productivity shocks in the literature.<sup>1</sup>

The recent paper Kehoe and Ruhl (2008), however, challenges the notion that this relationship would hold in observed data. The authors analyze the methods national statistics agencies, such as the Bureau of Economic Analysis (BEA), use to construct measured GDP figures and conclude that such measurements will not capture variations

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<sup>1</sup> This is so common that some of the first Small Open Economy models, such as Mendoza (1991), did not model international trade separately but included the terms of trade shocks in the production function productivity shock. Also, as pointed out by Finn (2000) it is a “popular” idea to view increases in the relative prices of imported energy (e.g. oil prices) as negative technology shocks.



due to movements in the terms of trade. The reason is that while strong functional form assumptions allow us to write down the value-added production function inside a theoretical general equilibrium model, it is not clear if a closed form representation of value added even exists in practice. We do not know the functional forms of the gross output production functions of the different industries of an economy and cannot backwards engineer the aggregate real value-added production number. Instead, national statistics agencies settle for approximating the *changes* in the actual value-added output by removing the effect of changing prices from the collected information on the nominal value-added production. Unfortunately, the terms of trade are formed as the ratio of two prices - the price of exports, over the price of imports. Thus, because real GDP is constructed by removing the effect of price changes, the measured GDP numbers published by national statistics agencies will omit the first order effect of a change in the terms of trade.

Kehoe and Ruhl (2008) formalize this intuition in several different setups of increasing complexity and then ask the question if a typical SOE model has propagation mechanisms strong enough to match the observed empirical relationship between GDP and the terms of trade. They find this is not the case, and pose the question for finding the missing propagation mechanisms for future research. Unfortunately, the focus of Kehoe and Ruhl (2008) is squarely on analyzing national accounting methods and their empirical analysis of the relationship between GDP and terms of trade is somewhat lacking. They conclude that the relationship between the two variables is strong solely based on the high contemporaneous correlations between annual GDP, Total Factor Productivity (TFP) and the terms of trade of Mexico and the United States, each for the period 1970-2007.<sup>2</sup> The question whether or not a propagation mechanism is missing from the canonical SOE is an interesting one, but a more careful empirical analysis is needed before we answer it conclusively. The aim of this paper is to provide this empirical analysis.

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<sup>2</sup> In actuality only Mexico can be viewed as a small open economy, hence their conclusion is really based on data from just one country. The correlation between US GDP and terms of trade could very plausibly be due to the fact that US domestic disturbances affects the world market prices and not the other way around.

A number of preceding papers have looked at this issue, but have failed to reach a consensus with estimates of the contribution of terms of trade to output fluctuations ranging from just above zero to almost 90%. This variability is at least in part due to the fact that the majority of the existing papers only focus on data from a small number of countries and the choice of countries varies greatly across the papers. The few papers that analyze a broad panel of countries, such as Broda (2004); Edwards and Levy Yeyati (2005); Levy-Yeyati and Sturzenegger (2003), focus on studying other questions and address the effect of terms of trade on domestic fluctuations only in passing. This paper will attempt to unify the literature by considering a broad panel of countries and focusing the analysis squarely on the effect of terms of trade on the domestic economy.

The existing literature can be subdivided into three broad subcategories. One uses model-free techniques such as VAR and regression analysis, another relies on calibrated general equilibrium models and the most recent of the three utilizes structurally estimated general equilibrium models. Using a structural VAR approach with the identification assumption that the terms of trade are exogenous with respect to domestic macroeconomic shocks and a panel of 75 developing countries, Broda (2004) finds that the terms of trade account for only 9.6% of output fluctuations in countries with flexible exchange rate regimes and up to 30% in countries with fixed exchange rate regimes.<sup>3</sup> Hoffmaister et al. (1997) use data on 15 Asian and 17 Latin American countries to estimate a structural VAR with the restriction that only supply shocks, and not demand shocks, have long run effects on macro variables and find that only about 6% to 7% of the variability of output could be attributed to the terms of trade. Hoffmaister et al. (1998) use similar techniques and data from sub-Saharan Africa and find contributions of up to 15%, and Ahmed and Murthy (1994) use data on Canada and find the contribution of terms of trade to be about 3%. In another type of contribution to the model-free literature, Levy-Yeyati and Sturzenegger (2003) and Edwards and Levy Yeyati (2005) use growth regressions to find that terms of trade have a statistically significant positive relationship with both output and output

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<sup>3</sup> See also Broda (2001)

volatility.

In another strand of the literature, Mendoza (1995) uses a calibrated general equilibrium Small Open Economy model and finds that terms of trade fluctuations explain about 30% of output fluctuations for G-7 countries and up to 56% for a group of developing countries. On the other hand, Kose (2002) expands on this type of analysis by differentiating between imported capital and imported intermediate goods, calibrates his model to a group of 28 non-fuel exporting developing countries, and finds that up to 90% of output fluctuations are attributable to movements in the terms of trade. A few more recent papers also use structural general equilibrium models, but rely on Bayesian methods for estimation of parameters rather than on calibration. Lubik and Teo (2005) estimate their model with data from Australia, Canada, New Zealand, Mexico and Chile and find that typically no more than 1-2% of output fluctuations are due to the terms of trade. Da Silva (2010) uses a similar technique and data from Brazil to find that 13% of output variability could be attributed to fluctuations in the terms of trade

Thus the existing literature leaves us with no clear answer. There is evidence that the terms of trade account for virtually none of the output fluctuations of a typical small open economy, that they account for a moderate (10-15%) amount and also evidence that they are the major driver of said fluctuations (up to 90% in Kose (2002)). Fortunately, there are a few ways to potentially strengthen and unify the existing literature. The broad-based panel studies of Broda (2004) and Edwards and Levy Yeyati (2005) focus on estimating the difference in the impact of terms of trade shocks between countries with fixed and floating exchange rate regimes. Their studies do not attempt to consider what variables are mainly responsible for the output fluctuations in a small economy and are thus not directly comparable to the canonical SOE models considered by Mendoza (1995); Kose (2002); Lubik and Teo (2005), and Da Silva (2010). For example, Broda (2004) does not include any measure of the real interest rate in his VAR analysis while the world interest rate is a major driving force in the theoretical SOE models. A part of the model-free literature does consider VARs that are directly comparable with the SOE models, but

then includes data on large developing countries like Brazil and China (Hoffmaister et al. (1997)) which most likely have considerable power on world markets and are unlikely to have exogenous terms of trade. Or only considers data on a single country (Ahmed and Murthy (1994)), or analyzes countries whose economies are most likely not well described by a real business cycles general equilibrium model which was developed for a stable country like the United States (the sub-Saharan Africa sample of Hoffmaister et al. (1998)).

To alleviate such issues, the current paper will consider a structural VAR setup which is deliberately designed to be directly comparable with the canonical SOE model and uses a broad and long sample of annual observations for 31 countries for the time period 1950 to 2007. Because of this comparability with the structural models, this paper is able to judge whether the high or the low estimates arising in the structural literature are supported by the data. In turn, this will inform us whether the SOE model is indeed missing important propagation mechanisms. The exogeneity of the terms of trade are a crucial identification assumption. To ensure its validity, data on the trade statistics for the considered countries has been collected from the United Nations' Handbook of International Trade and Development Statistics for all available periods (1995-2009).

The results of this paper support the findings of the previous model free literature, finding that while the terms of trade and output are contemporaneously correlated, with a statistically significant average correlation of 0.20, their contribution to overall output fluctuations are small. When the VAR is estimated on a country-by-country basis, the average contribution of terms of trade to GDP fluctuations is about 6%. While, when the VAR is estimated on the panel of all countries, with the restriction that all coefficients are the same across countries, the contribution of terms of trade drops to just about 2%.<sup>4</sup> These results remain unchanged through an array of robustness checks.

The variance decomposition results are not due to small estimates of terms of trade variability. In fact, the estimates for the variance of the terms of trade shocks are five times bigger than the estimates for the variance of productivity disturbances. However,

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<sup>4</sup> Allowing for fixed country effects does not change the results

the dynamic response functions point to a very small response, with a 1% increase in the terms of trade resulting in just about a 0.06% increase in output in the first year, no effect in subsequent periods. These results support the estimates of Lubik and Teo (2005) who find that terms of trade have almost no role in driving business cycles, as determined by their effect on measured GDP.

However, this is not to say that terms of trade fluctuations have no effect on macroeconomic variables and the general welfare of small countries. In fact, in light of the arguments presented by Kehoe and Ruhl (2008), it is expected that terms of trade have no effect on *measured* GDP even though they affect the actual, unobserved value-added production index. In addition, welfare is related to measured GDP only as far as this construct is able to capture the actual consumption possibilities of a given country. To test whether or not terms of trade do affect macroeconomic variables other than GDP significantly, the same VAR setup was estimated with aggregate consumption replacing GDP. In this formulation, it was found that terms of trade account for up to 14% of the fluctuations in consumption, and that there are economically significant dynamic responses of consumption to terms of trade shocks. A 1% increase in the terms of trade lead to a 0.2% rise in aggregate consumption. Hence, while no significant relationship between the terms of trade and GDP is found, there is evidence that terms of trade are an important determinant of aggregate consumption. Thus, the theoretical argument of Kehoe and Ruhl (2008) is affirmed in that *measured* GDP does not capture movements in true value-added production due to terms of trade fluctuations, but these effects show up in aggregate consumption, which is strongly affected by the the unobserved true value-added production.

The rest of the paper proceeds as follows. Section 2 describes the data used and examines the exogeneity of the terms of trade, Section 3 presents the structural VAR that is at the heart of this empirical analysis, Section 4 considers a number of robustness checks, and Section 5 concludes.

## 3.2 Data and Descriptive Statistics

### *3.2.1 Terms of Trade Shocks and Aggregate Output*

Before we proceed to analyzing the assembled data and the discovered empirical relationships, it is important to briefly consider what kind of results are to be expected, as predicted by the theoretical literature. Economists examining development and economic growth questions have long been interested in the effects of terms of trade on economic performance. This interest dates back to at least Prebisch (1950) and Singer (1950), who argue that developing countries have experienced a persistent deterioration in their terms of trade, which has contributed to the slow long term economic growth of these countries. Modern Growth Economics continues to be interested in this issue. Mendoza (1997) and Van Wincoop (1992), for example, study how terms of trade shocks affect the exchange rate constraints facing developing countries, how this relates to factor movement and savings decisions, and ultimately how it influences growth. This literature finds that terms of trade changes are positively related with long-term economic growth.

In another strand of literature, Dynamic Stochastic General Equilibrium (DSGE) models are used to analyze the relationship between terms of trade and output fluctuations at the Business Cycle frequencies. Basu and McLeod (1991), for example, develop a model with imported intermediate goods that is very similar in spirit to the simple example given in the previous section. In their model, intermediate goods are complimentary to capital, hence increases in the terms of trade improve capital's productivity and lead to higher output. Moreover, Mendoza (1991, 1995); Kose (2002) all develop theoretical DSGE models that feature a strong positive relationship between terms of trade and aggregate output.

Thus, one everpresent conclusion of the theoretical treatment of the terms of trade is that they are positively related to output in both the long and the short term. Hence, if there is any relationship between measured GDP and the terms of trade, it is expected to be positive.

### *3.2.2 Data Description*

The sample consists of annual observations on national accounts variables of 31 different countries for the years 1950-2007, observations on the annualized yield of the 3-month U.S. Treasury Bond, and the U.S. Consumer Price Index (CPI). The national accounts data comes from the Penn World Table 7.0 (PWT 7.0)<sup>5,6</sup> U.S. interest rates were collected from the Federal Reserve and CPI numbers are from the Bureau of Labor Statistics (BLS).

The Penn World Table 7.0 has data on 189 countries for the time period 1950-2009. This data set assigns grades from A to D (A is the highest) on the data quality for each country and this study includes only the 31 countries that received either 'A' or 'B'. Moreover, the last two available years, 2008 and 2009, are excluded in case the global financial crisis of these years potentially obscure the true relationship between domestic output and the terms of trade.<sup>7</sup> The list of countries and the span of data availability for each can be found in Table A.1 in the Appendix. The specific national accounts variables collected for each country are real GDP per capita, real aggregate Consumption per capita, the percent of government spending in GDP, the ratio of the sum of imports and exports to GDP and deflators for exports, imports and GDP. The real national accounts variables and price indexes in PWT 7.0 are expressed in terms of international dollars, which are equivalent to the U.S. dollar in 2005. Detailed description of the creation of these variables can be found in the technical appendix to Summers and Heston (1991) and the technical appendices of subsequent versions of PWT.

The 31 countries in the sample are predominantly developed countries, with well established and stable legal systems, and in most cases, long history of free markets. It is a much different sample from the ones considered by the majority of previous model free studies,

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<sup>5</sup> Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 7.0, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, May 2011.

<sup>6</sup> An earlier version of this work is described in detail in Summers and Heston (1991)

<sup>7</sup> Nevertheless I checked if the results are affected by the inclusion of those 2 years of data and they are not

such as Broda (2004) and Hoffmaister et al. (1997), whose samples consist mostly of developing countries. However, more developed countries are better suited for modeling using Real Business Cycle models and thus it is unsurprising to see that this sample is much closer to the ones considered by DSGE studies such as Mendoza (1995) and Lubik and Teo (2005).



Table 3.1: Descriptive Statistics

Country	$\sigma_y$	$\sigma_c$	$\sigma_{tot}$	$\rho_{y,tot}$	$\rho_{c,tot}$
Argentina	2.9	4.7	14.5	0.178	0.182
Australia	1.7	1.6	4.5	0.078	0.200
Austria	1.3	1.2	1.5	0.119	0.151
Belgium	1.2	1.1	1.6	-0.030	0.279**
Canada	1.4	1.2	1.9	0.274**	0.251*
Switzerland	1.9	0.9	2.2	-0.462***	-0.104
Chile	3.5	5.0	8.2	0.380***	0.353***
Denmark	1.5	1.7	1.8	0.506***	0.578***
Spain	2.0	1.8	4.1	-0.261*	-0.185
Finland	2.2	2.2	3.5	0.429***	0.462***
France	1.0	0.9	2.3	-0.072	-0.036
UK	1.1	1.4	2.1	-0.255*	-0.013
USA	1.5	1.1	2.1	0.109	0.295**
Germany	1.1	1.0	2.2	0.073	0.424***
Greece	1.9	1.6	2.1	-0.036	0.141
Hong Kong	3.1	2.6	1.2	-0.005	0.105
Ireland	1.8	1.9	2.6	0.128	0.160
Israel	2.9	2.7	5.8	-0.055	-0.020
Italy	1.2	1.1	3.2	-0.440***	0.013
Iceland	3.3	4.3	3.1	0.504***	0.592***
Japan	1.4	1.2	4.4	-0.044	0.077
Korea	2.5	2.6	3.7	0.346***	0.051
Luxembourg	2.2	1.5	3.6	-0.067	0.117
Netherlands	1.7	1.6	0.9	0.279**	0.356***
New Zealand	2.1	2.8	6.1	0.234*	0.370***
Norway	1.0	1.5	4.3	-0.179	-0.173
Poland	3.1	3.5	2.8	0.334**	0.264
Portugal	2.5	2.4	3.6	0.349***	0.560***
Singapore	2.6	2.4	0.3	0.158	-0.003
Sweden	1.0	1.1	1.8	0.067	0.037
Uruguay	3.8	4.6	9.6	0.425***	0.557***

Note: The table lists percentage standard deviations of logged and HP(6.25) filtered series and simple correlations.

\* indicates 10% level of significance

\*\* indicates 5% level of significance

\*\*\* indicates 1% level of significance

Table 3.1 presents some descriptive statistics for the sample at hand. The series considered in the table are first logged and then HP filtered using a smoothing coefficient of 6.25, the value Ravn and Uhlig (2002) suggest to use for annual data<sup>8</sup>. Two things

<sup>8</sup> The results are qualitatively the same if instead I use a value of 100, which is the value used by

stick out in this table. First, the terms of trade are found to be about twice as volatile as output for the typical country. Second, the contemporaneous correlation between GDP and the terms of trade is generally positive, but there are five notable exceptions that exhibit a strong negative relationship. This negative correlation could be a signal that the country in question is not a price taker on the world markets. The logic is that if a country is major exporter of a commodity, like Chile and copper, then a negative shock to domestic productivity would have a significant negative impact on the worldwide supply of this commodity. The negative supply shock will increase the world price of the commodity, and *ceteris paribus*, the terms of trade of this country would rise during a time in which GDP is falling, resulting in a negative contemporaneous correlation. Interestingly, all countries with significant negative correlations exhibit correlations between terms of trade and consumption that are always more positive compared to the corresponding correlation with output, and in fact not statistically different from zero. Moreover in 25 out of the 31 countries, the correlation between the terms of trade and consumption is more positive than the corresponding correlation with output. Thus, the first pass of the data largely delivers the expected result of a positive contemporaneous association between output and the terms of trade, and moreover hints that the relationship between terms of trade and consumption is usually more positive.

The moments reported in Table 3.1 are similar in spirit to the ones presented in Mendoza (1995), but the two are not directly comparable because there the aggregate series are computed at import prices and the time period considered is vastly different. Nevertheless, it is important to point out that this sample has very different characteristics than Mendoza's. While in Mendoza (1995), output is generally more volatile than the terms of trade, in my sample the terms of trade are twice as volatile than output for the typical country. Moreover, Mendoza (1995) mostly reports correlations between output and terms of trade vastly greater in magnitude than the ones found in my sample. Broda (2004) also find that terms of trade are more volatile than output, contrary to the results in Mendoza

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most previous papers

(1995).

### *3.2.3 Exogeneity of the Terms of Trade*

As was mentioned in the introduction, the exogeneity of the terms of trade is a crucial identification assumption for the empirical analysis of this paper. Moreover, as perhaps the reader has already noticed, the 31 country sample includes a number of large countries (e.g. the U.S.) for which this assumption almost surely does not hold. In order to arrive at a subsample of countries for which this assumption would be more plausible, I have obtained trade statistics data from the United Nations' Handbook of International Trade and Development Statistics for the years 1996-2009. Unfortunately, the United Nations' trade database does not go further back in time so I will be making the assumption that countries who are found to be major players in the world markets in the period 1996-2009 have also had similar market power in the past.

The UN trade database holds annual data on the value of exports and imports for over 300 different commodities, both on a country-by-country basis and summed over all countries. Thus, for every year, there is a total value for the worldwide exports of each commodity, and also the value of the exports for each country and similar data for the imports. From this data, I am able to determine how much of the worldwide export or import market for each commodity a given country has accounted for. Such information can inform us about the market share that each country exercises in all of the considered markets.

Table 3.2 below summarizes this information. Each column lists what percentage of that country's exports or imports were made in markets where the country had more than 5%, 10% or 15% market share respectively. The figure is for the total amount of trade in the period from 1996 to 2009. The cutoff points of 5%, 10% and 15% are not completely arbitrary. Broda (2004) uses the same database (but only data for the years 1996-1997) to do a similar adjustment and considers countries with over 15% of the world market in a commodity to be significant players for which the exogeneity of the terms of trade most

likely does not hold. Since no theoretical argument supports this particular choice, however, I have decided to look at a few other cutoff points as well for the sake of robustness.

The data in Table 3.2 provides a better picture of what country can be assumed to be a price taker in world markets than a simple GDP size comparison. For example, if one would determine terms of trade exogeneity based on sheer GDP size then countries like the U.S., Germany, Japan, U.K., France and Italy would most likely be excluded from the sample and smaller (by GDP size) countries like Chile and Australia would remain in the sample. But in fact, as can be seen from the table, in some ways Chile and Australia are a lot less similar to a price taker than the U.K. or France. While the U.K. and France are big countries, they do not account for a particularly large portion of the different exports and imports markets. They appear to be big countries with well diversified exports and imports, and thus have no significant power over any single one. Chile on the other hand, supplies about 15% of the world's copper exports every year. Despite its small size, the world copper prices are very much affected by Chilean domestic disturbances.

This trade data could be used to better understand the instances of significant negative correlations between output and the terms of trade found in Table 3.1. Simple theory postulates that price taking countries should have positive correlation, and significant players in the world markets should have a negative correlation between output and their terms of trade. The four countries with significant negative correlations are Switzerland, Spain, the U.K. and Italy. With the possible exceptions of Italy and the U.K., none of them can be characterized as significant players in the world markets. Perhaps the negative correlations found in Table 3.1 stem from something else and not only market power.

To carry this preliminary investigation a little further, I regressed the cross-section of correlations between output and the terms of trade on 3 controls and a constant. The first two regressors are the average share of government expenditure in GDP and the average ratio of trade to GDP, where averages are taken for each country over the full sample

Table 3.2: Trade Statistics

Country	% of Exp. in Markets with			% of Imp. in Markets with		
	> 15% sh	> 10% sh	> 5% sh	> 15% sh	> 10% sh	> 5% sh
Argentina	2.7	9.13	17.96	0	0	0
Australia	12.86	20.09	35.38	0.018	0.01	0.067
Austria	0.23	0.95	1.77	0.18	0.28	0.48
Belgium	2.28	7.36	31.41	1.80	5.49	16.36
Canada	4.84	12.87	31.15	0	1.26	9.01
Switzerland	2.35	7.27	16.12	0.05	0.38	2.14
Chile	10.08	11.97	28.14	0	0	0.07
Denmark	0.3	1.08	4.43	0	0.01	0.09
Spain	0.27	2.64	9.15	0	0.11	3.25
Finland	0.11	3.19	8.57	0	0.03	0.31
France	0.11	3.19	8.57	0.03	0.53	48.15
UK	1.25	4.71	37.74	0.55	2.06	63.35
USA	23.95	69.89	95.82	68.27	92.73	99.49
Germany	15.76	71.84	95.45	2.53	9.90	97.35
Greece	0	0	0.64	0.01	0.24	0.81
Hong Kong	1.98	5.17	19.65	0.12	0.84	15.46
Ireland	3.98	6.67	19.4	0.12	0.84	15.46
Israel	6.36	7.87	15.74	0	2.72	7.91
Italy	3.37	8.35	48.22	0.34	1.5	20.29
Iceland	0	1.58	3.45	0	0	0
Japan	11.88	46.31	86.78	5	22.99	62.09
Korea	2.34	3.85	17.32	0.23	0.89	18.71
Luxembourg	0.08	0.08	0.08	0.16	0.16	0.69
Netherlands	2.21	4.99	26.83	0.01	0.87	8.78
New Zealand	0.92	2.55	11.31	0	0	0.01
Norway	1.79	6.18	44.55	0.4	0.67	1.30
Poland	0.11	0.24	2.48	0	0	0.21
Portugal	0.59	0.59	1.10	0.03	0.13	0.17
Singapore	1.61	6.49	21.98	0.02	0.78	12.22
Sweden	0	0.21	7.32	0	0	0.07
Uruguay	0	0	0	0	0	0

Note: The table lists the percentage of total exports and imports for which each country had more than 15,10 or 5% of the world market. The numbers are totals for the period 1996-2007.

1950-2007. The third regressor is the sum of the third and sixth columns in Table 3.2. The regression results in estimates that are insignificant, except for the coefficient on the “market power” variable, which is negative and significant on the 10% level. Running the same regression, but if instead of using the sum of the third and sixth column, I use the

sum of the second and the fourth, then all coefficients become insignificant. Thus, while it appears that there is a weak positive association between being a significant player in world markets and observing a negative correlation between output and the terms of trade, there must be more important determinants of this correlation.

As a last check on the exogeneity assumption, I have tested whether the GDP of each country in the sample Granger-causes its terms of trade. Following Broda (2004) the Granger causality tests include 4 lags. Changing the number of lags does not have an overly large effect on the results. Table 3.3 lists the countries whose test statistics have p-values of less than 10%.

Table 3.3: Granger Causality Tests

Country	p-value
Switzerland	0.016
Chile	0.000
U.K.	0.022
Iceland	0.065
Norway	0.049
Singapore	0.054

Notice that the countries singled out in Table 3.3, with the exception of Iceland, largely correspond to countries that were also found to be significant players in the world import or export markets.

### 3.3 Empirical Model and Results

#### 3.3.1 Panel VAR Results

This paper's empirical results are based on a structural VAR analysis of the relationship between the real interest rate, the terms of trade and GDP. The specific form of the VAR is as follows:

$$\Gamma_0 Y_t = C + \Gamma_1 Y_{t-1} + B_0 X_t + B_1 X_{t-1} + u_t \quad (3.1)$$

where  $Y_t = [r_t, \ln(\text{tot}_t), \ln(\text{gdp}_t)]$  is the vector of endogenous variables and  $X_t$  is a vector of exogenous controls. The variables listed in  $Y_t$  are the world real interest rate,  $r_t$ , the log of the terms of trade (Price of Exports over Price of Imports) and the log of the real Gross Domestic Product (GDP). All variables are HP filtered with a smoothing parameter of 6.25 (as per Ravn and Uhlig (2002)), and in the case of the logged variables, the log was applied before the HP filter. The exogenous variables are chosen so that this VAR is directly comparable to the canonical SOE model, which is driven by exogenous shocks to the world interest rate, the terms of trade and technology shocks.

The terms of trade and GDP data come straight from the Penn World Table 7.0. The world interest rate is defined as the annualized, secondary market yield of the 3-month U.S. Treasury bill minus the current year change in the U.S. CPI.<sup>9</sup> By all means, this is the real interest rate faced by the U.S. and is not necessarily the same for other countries because of exchange rate movements and differences in the composition of consumption baskets. Unfortunately, long series CPI data does not exist for the broad panel of countries that I am considering and prevents me from using a more country-specific measure of the real interest rate. In any case, if Purchasing Power Parity holds at least approximately on the aggregate, and the consumption baskets across countries are not vastly different, then my measure of the real interest rate should not be too much different from a preferable country-specific measure.<sup>10</sup>

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<sup>9</sup> The US Treasury rate is chosen as the world nominal interest rate because of data availability over a long time span. The LIBOR rate, which has recently been the most popular measurement of a worldwide interest rate, was only created in the mid 1980s which would cut the length of the sample in half. Nevertheless, I plan to carry out future robustness checks on a shorter sample using the LIBOR as the nominal interest rate instead.

<sup>10</sup> Section 4 describes several attempts at examining the robustness of the results to different choices of real interest rate measurements. I consider two alternative methods of deflating the yield of the US T-Bill. One is to use the international dollar inflation quoted by the World Penn Table, and the second one is to use the inflation in the price index of exports. The first alternative is

The vector of controls is defined as  $X_t = [g_t, trade_t, wbc_t]$ , where  $g_t$  is the percentage of real government consumption expenditures in real GDP and  $trade_t$  is the ratio of real trade (real exports plus real imports) to real GDP, and both variables have been HP-filtered with smoothing coefficient 6.25. These two variables are the only control variables found to be significant by Broda (2004). The variable  $wbc_t$  is meant to represent the world business cycle and is motivated by Kose et al. (2003) who find that there is a significant common world component present in the macroeconomic fluctuations of a large cross section of countries (including all 31 considered in this paper). The effect of this “world business cycle”, in the words of Kose et al. (2003), is the strongest for the more developed countries, hence in this paper,  $wbc_t$  is the average of the HP-filtered log GDP of the G-7 countries.<sup>11</sup> This world business cycle proxy is meant to control for any important developments in the world business environment over and above what is already priced in the world real interest rate. The idea is that a high real interest rate would affect a small economy differently during times of a global expansion as opposed to during a global recession.<sup>12</sup>

The number of lags of  $Y_t$  and  $X_t$  were selected based on the Bayesian Information Criterion. To achieve identification of the structural shocks  $u_t$ , I will assume that  $var(u_t) = D$  is a diagonal matrix and that  $\Gamma_0$  is lower triangular with 1’s on the main diagonal. This identification scheme requires a specific ordering of the endogenous variables, where the top variables will be unaffected contemporaneously by shocks to lower seeded variables. As mentioned previously, the assumption of the exogeneity of a small economy vis-a-vis the outside world plays a crucial role. As is common in the literature, I will assume that domestic GDP is positioned last, hence it is affected by same period disturbances to both the terms of trade and the real interest rate, but it could possibly affect them only with

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motivated by the fact that the Price Level in WPT is quoted relative to the U.S. dollar hence it gets closer to measuring domestic inflation in terms of U.S. dollars (and the nominal interest rate used is in U.S. dollars). The second alternative is suggested and used by Kose (2002).

<sup>11</sup> The G-7 countries are Canada, France, Germany, Italy, Japan, the U.K. and the U.S.

<sup>12</sup> In general I found that excluding this variable has virtually no effect on the estimation



a lag.<sup>13</sup> The relative ordering of the interest rate and the terms of trade is less clear and there is less guidance available in existing studies. I will follow the established convention and position the real interest rate at the very top in my baseline model, but as discussed in Section 4, all results remain virtually unchanged when the terms of trade are positioned on the top instead. And lastly, the estimation method used is Full-Information Instrumental Variables (FIVE), which is a generalization of the Seemingly Unrelated Regressions (SUR) method used by Broda (2004)<sup>14, 15</sup>.

As was discussed at length previously, not all 31 countries satisfy the identification assumption that their terms of trade are exogenous vis-a-vis their domestic GDP. But since there is no clear answer as to which countries satisfy this assumption and which do not, I have carried out the estimation for six different subsamples. Subsample 1 includes all countries which derive less than 10 % of their exports or imports from commodities for which they account for more than 15% of the international demand or supply, respectively. Subsamples 2 and 3 are similarly defined to be only countries which derive no more than 10% of their exports or imports from commodities in which they have no more than 10% or 5% of the world market, respectively. Subsamples 4,5 and 6 are respectively Subsamples 1,2 and 3 further restricted to exclude all countries which were found to have a significant Granger Causality Test in Table 3.3. The subsamples were selected so that going from 1 to 3, I implement an increasingly stricter definition of what constitutes a price taking economy, and then 4, 5 and 6 are each even more restricted than their corresponding base

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<sup>13</sup> In fact, the SOE assumption in its strong form asserts that domestic GDP should have no effect on the world interest rate and the terms of trade at any lag. This would suggest that  $\Gamma_1$  is also lower triangular, but I will not restrict it to be so here. A planned future extension is to test whether  $\Gamma_1$  is indeed lower triangular as an additional test of the exogeneity assumption. As you will see in the results below, this is almost surely the case because the variance decompositions show that GDP innovations account for less than 1% of the total variance of the terms of trade.

<sup>14</sup> I impose the restriction that all coefficients are the same across countries, including the constants, but relaxing the assumption of equal constants has virtually no effect on the results

<sup>15</sup> The estimation technique is basically the multiple equation Generalized Method of Moments with the assumption of homoskedastic variance. Details can be found in Hayashi (2000)

subsamples.<sup>16</sup>

The main question explored in this paper is whether world prices are a significant determinant of the business cycles of small economies or not. As a first crack at it, Table 3.4 presents the estimated variance decompositions of the three endogenous variables in each of the 6 subsamples. Due to data availability, Subsamples 1, 2, 4 and 5 are estimated over the time span 1960-2007 and Subsamples 3 and 6 are estimated over the full sample 1950-2007. Re-estimating Samples 3 and 6 over 1960-2007 does not have any major effect on the results neither quantitatively, nor qualitatively.

Table 3.4: Variance Decomposition Panel VAR

Sample	$Var(r_t)$ due to:			$Var(tot_t)$ due to:			$Var(gdp_t)$ due to:		
	$u^r$	$u^{tot}$	$u^{gdp}$	$u^r$	$u^{tot}$	$u^{gdp}$	$u^r$	$u^{tot}$	$u^{gdp}$
Subsample 1	99.99	0.01	0.00	0.12	99.87	0.01	0.12	1.60	98.28
Subsample 2	99.99	0.01	0.00	0.11	99.88	0.01	0.10	1.61	98.29
Subsample 3	99.93	0.05	0.02	0.28	99.32	0.40	0.23	3.54	96.22
Subsample 4	99.99	0.01	0.00	0.11	99.88	0.01	0.10	1.61	98.29
Subsample 5	99.99	0.01	0.00	0.11	99.88	0.01	0.10	1.61	98.29
Subsample 6	99.93	0.05	0.02	0.49	99.11	0.40	0.19	3.30	96.51

The table details what percentage of the variance of the real interest rate, the terms of trade and GDP is attributable to each of the exogenous shocks listed in the second line.

All six different groupings of “price takers” tell the same story: the terms of trade do not appear to have an important effect on GDP fluctuations. The estimates of their contribution to the variance of GDP is about 2-3% in all of the subsamples, providing strong support for the theoretical claim of Kehoe and Ruhl (2008) and more. In fact, the argument in Kehoe and Ruhl (2008) suggests that terms of trade have no first order effect on GDP, but can still affect it through altering the equilibrium labor and investment decisions. The evidence in Table 3.4, however, suggests that such secondary effects are also small, thus this first crack at the data implies that terms of trade have no economically

<sup>16</sup> Poland should appear in all subsamples but data is available for it only starting at 1970, which considerably reduces the length of the balanced panel. Hence it is omitted in the sake of longer timer series. Re-running the estimation on the shorter panel including Poland does not have important effects on the results. Moreover, there could be even bigger issues with Polish data since its economy was centrally planned prior to the early 90s.

meaningful impact on *measured* GDP at the business cycle frequencies.<sup>17</sup>

The results are meaningfully different from previous model-free studies who generally find that the terms of trade account for about 7% to 15% of the variance of GDP. It is possible that this difference is arising from a sample selection issue. For the most part, previous studies have not been able to use the U.N. trade statistics database (available only since 1996) to determine which countries could plausibly be viewed as price takers in the world markets and have had to rely on simply selecting countries with relatively small GDPs. The motivation of this approach would be that a small country is also an insignificant player in the world markets. However, such an assumption is far from universally true because many small countries exercise a power disproportionate to their size over certain international markets, like for example the market for copper and Chile. It may be reasonable to expect that simply selecting countries with small GDP's would yield many nations for which the important identifying assumption of exogenous terms of trade does not hold and this would bias the overall results. As compared to previous DSGE based studies, the evidence in this paper favors the findings of the structural estimation of Lubik and Teo (2005) over the calibrated models of Mendoza (1995) and Kose (2002).

The reason terms of trade fluctuations account for a small portion of GDP fluctuations is because the estimated impulse response of GDP to a shock to the terms of trade is small, and not because of a small estimate of  $var(u_t^{tot})$  relative to  $var(u_t^{gdp})$ . In fact, across all six subsamples  $var(u_t^{tot})$  is on average 3-4 times larger than  $var(u_t^{gdp})$ . On the other hand, the dynamic response of GDP following a shock to  $u_t^{tot}$  is negligible across all samples with the possible exception of 3 and 6. Figure 3.3.1 plots the impulse responses for the six subsamples described above. Notice that all six impulse responses are fairly similar, with the strongest effect coming in the year of the impact. All responses appear to be inconsequential (as also evidenced by the variance decompositions) , however, with a 1% increase in the Terms of Trade bringing no more than a 0.09% corresponding increase in

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<sup>17</sup> Excluding  $wbc_t$  from the regression has virtually no effect on the results and for the sake of brevity a table with these results is not presented

real GDP for Subsamples 3 and 6 and just 0.04% in the other samples. One standard deviation of  $u_t^{tot}$  is equal to just 3.32%, hence a one standard deviation shock to the terms of trade would lead to about 0.3 % increase in real GDP. When comparing this with the standard deviation of the HP filtered, logged GDP across all samples, which is 2.25% one sees that shocks to the terms of trade really do not cause any serious movements in real GDP.

The evidence presented so far supports the hypothesis that the terms of trade have no significant effect on GDP fluctuations over the business cycle. But the question remains then, whether the terms of trade have no effect over the *business cycle* itself or if the lack of a relationship is mostly due to arguments in the spirit of Kehoe and Ruhl (2008). To cast some light on this question, I have considered estimating the same panel VAR relationship as above, but substituting real aggregate consumption per capita for the GDP per capita series in the  $Y_t$  vector. I find that the terms of trade contribute a meaningful amount to the fluctuations of aggregate consumption. I believe this is evidence that terms of trade are indeed important determinants of the business cycles of small countries, but due to limitations in our ability to measure true value added output, they are unrelated to the index national statistics agencies call Gross Domestic Product. Table 3.5 summarizes the variance decomposition estimates for the same six subsamples as above, where the last variable in  $Y_t$  is now the HP(6.25) filtered, logged aggregate consumption per capita.

One major difference between Table 3.5 and Table 3.4 is the second-to-last column. In all subsamples the estimate for the contribution of terms of trade to the variance of aggregate consumption is about 3 to 4 times larger than the contribution of terms of trade to the variance of GDP. In fact, in the case of Subsamples 3 and 6, which are assembled under the strictest definition of what constitutes a price taking economy, this estimate goes as high as 14%, which indicates an economically significant relationship.

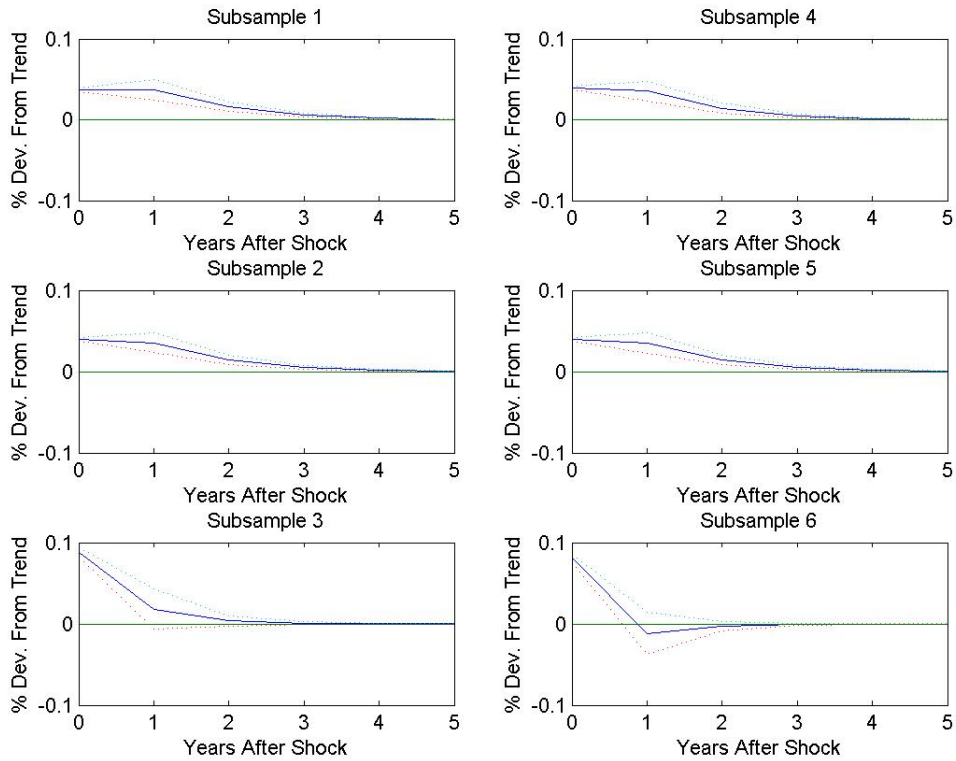


FIGURE 3.1: Real GDP response to 1% Increase to Terms of Trade

Table 3.5: Variance Decomposition Panel VAR (Consumption)

Sample	$Var(r_t)$ due to:			$Var(tot_t)$ due to:			$Var(c_t)$ due to:		
	$u^r$	$u^{tot}$	$u^c$	$u^r$	$u^{tot}$	$u^c$	$u^r$	$u^{tot}$	$u^c$
Subsample 1	100.00	0.00	0.00	0.15	99.84	0.00	0.04	5.63	94.33
Subsample 2	100.00	0.00	0.00	0.14	99.86	0.00	0.06	5.88	94.06
Subsample 3	99.93	0.07	0.00	0.28	99.72	0.00	0.15	14.12	85.73
Subsample 4	100.00	0.00	0.00	0.14	99.86	0.00	0.06	5.88	94.06
Subsample 5	100.00	0.00	0.00	0.14	99.86	0.00	0.06	5.88	94.06
Subsample 6	99.90	0.08	0.02	0.45	99.54	0.01	0.25	12.65	87.10

The table details what percentage of the variance of the real interest rate, the terms of trade and real consumption is attributable to each of the exogenous shocks listed on the second line.

Figure 3.3.1 plots the dynamic response of real aggregate consumption to a positive 1% shock to the terms of trade. The responses on impact are at least two times bigger than in Figure 3.3.1, across all Subsamples, and also appear a lot more persistent. While the impulse responses estimated from Subsamples 1, 2, 4 and 5 still do not feature strong

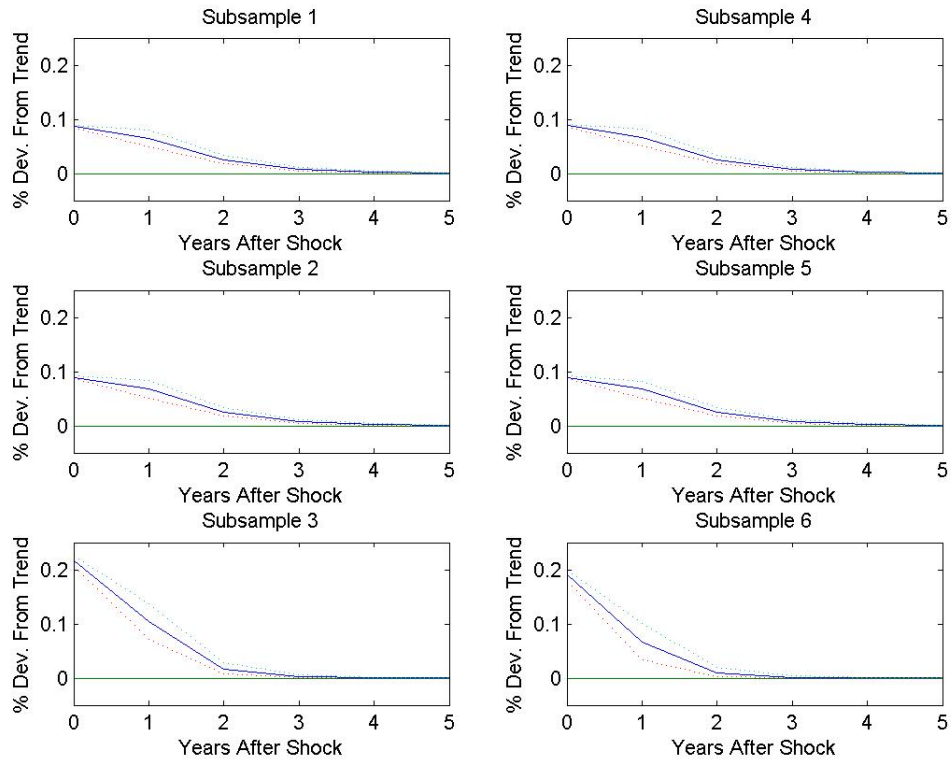


FIGURE 3.2: Real Consumption response to 1% Shock to Terms of Trade

instantaneous response, they at least do not become negligible in less than two years. On the other hand, the estimated responses from Subsamples 3 and 6 (again the ones for which the exogeneity assumption is most plausible) feature both strong instantaneous responses and longevity of the effect. In their case, a 1% increase to the terms of trade leads to 0.2% contemporaneous increase in consumption and a 0.1% increase in the year following the shock. Moreover, the standard deviation of the terms of trade is again estimated to be about 3%, hence a 1 standard deviation increase in the terms of trade leads to about one third of a standard deviation increase in aggregate consumption, a much more significant effect than in the case of GDP.

This evidence shows that the terms of trade have an important economic role in determining the fluctuations of aggregate consumption at the business cycles frequencies. Combined with the fact that they appear to have only a negligible role in explaining the

fluctuations of measured GDP, this result suggests that the theoretical arguments in Kehoe and Ruhl (2008) are borne out by the data as well. While terms of trade certainly have important effects on a small country's business cycle, as showcased by their effects on aggregate consumption, the said effects do not demonstrate themselves in the measured GDP series. The potential differences between the value-added output inside a DSGE open economy model and the GDP construct measured by national statistics agencies is not only a theoretical possibility, but is an empirical fact. Hence it will be prudent for future researchers to not directly equate inside-the-model output with GDP, but be careful about adjusting model variables to be consistent with the way GDP is measured in real life.<sup>18</sup> Another possibility would be to use series like aggregate consumption and investment instead of GDP and avoid the issue altogether.

### *3.3.2 Country By Country Results*

The previous section presented results of estimating the structural VAR by imposing the restriction that the regression coefficients are the same across all countries. This technique has the advantage of lending precision to the estimates but could potentially obscure important heterogeneity among the cross section of countries. This section attempts to shed some light on how much heterogeneity there is by estimating the VAR separately for each country with no cross-country coefficient restrictions. The results suggest that there is some difference among the countries, but the overarching conclusions of the previous section remain unchanged.

Table 3.6 presents the average and median Variance Decompositions for Subsamples 1,2,3.<sup>19</sup> For more individual country results, the reader is advised to consult with Table A-2 in the Appendix which lists the variance decompositions for all 31 countries.

The average variance decomposition numbers suggest that terms of trade explain about

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<sup>18</sup> In fact Rotemberg and Woodford (1996) provides an early, but unfortunately also lonely, example of empirical researchers being careful about adjusting their model variables to the GDP construct.

<sup>19</sup> Results for Subsamples 4,5 and 6 are omitted for the sake of brevity since they are virtually the same as those for 1, 2 and 3

Table 3.6: Variance Decomposition Summary for Country by Country VAR

Sample	$Var(r_t)$ due to:			$Var(tot_t)$ due to:			$Var(gdp_t)$ due to:		
	$u^{rt}$	$u^{tot_t}$	$u^{gdp_t}$	$u^{rt}$	$u^{tot_t}$	$u^{gdp_t}$	$u^{rt}$	$u^{tot_t}$	$u^{gdp_t}$
Mean Subs. 1	90.36	6.29	3.36	5.29	91.11	3.60	6.54	7.53	85.94
Mean Subs. 2	90.92	5.61	3.47	5.38	90.93	3.69	6.73	7.42	85.86
Mean Subs. 3	87.41	6.91	5.67	6.61	89.75	3.64	6.04	7.44	86.52
Med. Subs 1	95.01	2.40	1.49	4.17	90.85	1.94	4.58	5.84	87.22
Med. Subs 2	95.27	2.12	1.70	4.17	90.83	2.06	4.85	5.70	86.48
Med. Subs 3	93.54	1.08	3.17	5.27	89.39	2.23	1.99	5.70	89.22

The table details what percentage of the variance of the real interest rate, the terms of trade and GDP is attributable to each of the exogenous shocks listed in the second line.

7 to 8% of the total variation of GDP, which is somewhat different from the 2-3% number estimated from the panel VAR. However, the median of the variance decompositions suggests a 5 to 6% figure which is a little bit closer to the panel numbers. The average numbers are most likely a bit skewed by large outliers and a look at Table A-2 confirms this suspicion. Out of all 31 countries only 8 countries have double digit estimates for the contribution of terms of trade. Countries like Belgium, Finland and Portugal (all appearing in the 3 Subsamples of price taking countries) have estimates ranging from 16% to 24% while the other countries typically have estimates of 4-5%. If we omit these three countries from the average calculation, then the mean contribution of terms of trade drops down to 5%. Thus, while there is some heterogeneity among the countries, it appears that aside of a few outliers the typical country does not display a strong relationship between measured GDP and terms of trade fluctuations, as also concluded from the panel VAR results.

Table 3.7 presents the variance decomposition results from running the VARs with real aggregate consumption instead of real GDP. In case of Subsamples 1 and 2, the average and median estimates are two times higher than the estimates from the panel regressions and the results for Subsample 3 and virtually identical with the panel numbers. The estimated contributions of the terms of trade are also meaningfully higher than the ones presented in Table 3.6, providing further support for the hypothesis that the terms of trade fluctuations do not affect measured GDP simply because of accounting limitations, and not because they are a unimportant driver of business cycles.



Table 3.7: Variance Decomposition Summary for Country-by-Country VAR (Consumption)

Sample	$Var(r_t)$ due to:			$Var(tot_t)$ due to:			$Var(c_t)$ due to:		
	$u^{r_t}$	$u^{tot_t}$	$u^{c_t}$	$u^{r_t}$	$u^{tot_t}$	$u^{c_t}$	$u^{r_t}$	$u^{tot_t}$	$u^{c_t}$
Mean Subs. 1	90.35	6.57	3.08	4.91	91.97	3.12	6.42	11.30	82.29
Mean Subs. 2	91.05	5.84	3.10	5.01	91.94	3.05	6.45	11.62	81.93
Mean Subs. 3	89.55	6.85	3.60	6.31	91.32	2.37	6.15	13.32	80.53
Med. Subs 1	95.45	2.26	1.49	3.33	93.93	1.48	5.50	10.06	80.89
Med. Subs 2	95.92	2.10	1.13	3.43	94.58	1.44	5.36	11.15	80.70
Med. Subs 3	94.99	0.89	2.25	5.12	93.29	1.40	3.80	12.26	80.70

The table details what percentage of the variance of the real interest rate, the terms of trade and GDP is attributable to each of the exogenous shocks listed in the second line.

Next consider the impulse response function of real GDP to a 1% positive shock to the terms of trade that are generated from the country-by-country VARs. Figure 3.3.2 presents six plots. The first three take the impulse responses of each country in Subsamples 1, 2 and 3 respectively and averages them pointwise across countries. The second three do the same, but take the median instead of an average.

All three are fairly similar to the panel impulse responses in that they all show negligible responses to the terms of trade shock. On the other hand, Figure 3.3.2 plots the impulse response functions of real consumption to a 1% positive shock to the terms of trade. The responses are not hugely different, but across all six plots the general theme is that the response of real consumption to a shock to the terms of trade is somewhat stronger than the corresponding response of GDP.

Thus, the conclusion from the results of this section is that while there is some heterogeneity between countries (the origins of which could be an interesting topic of a future study), the typical country's terms of trade have a negligible effect on GDP fluctuations, but exhibit a stronger relation with real consumption.

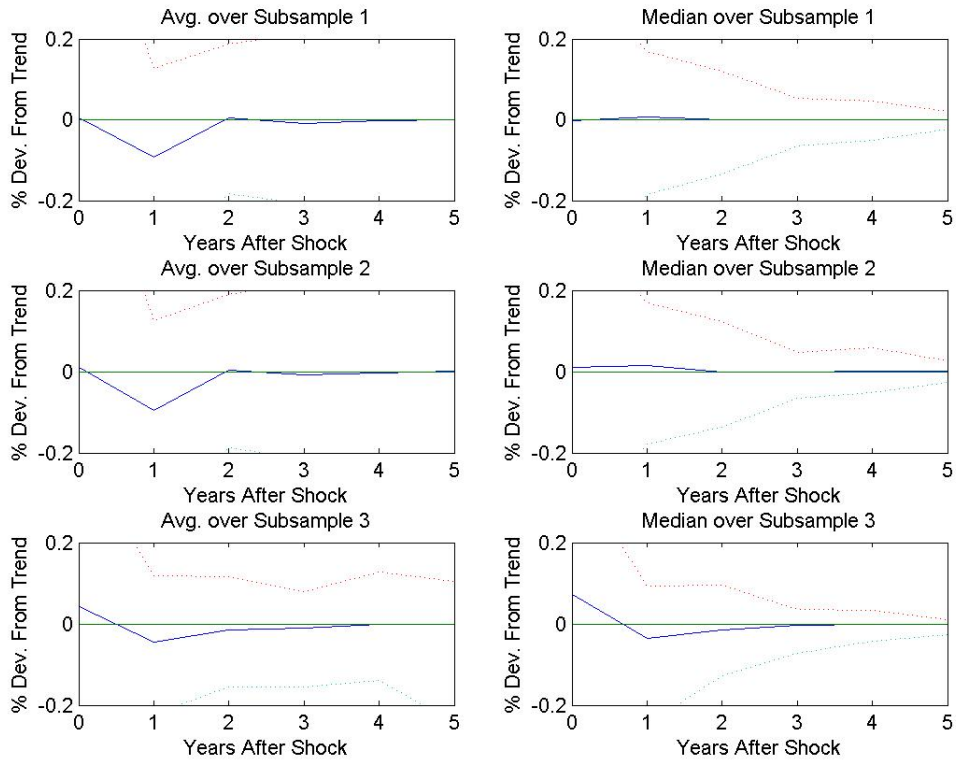


FIGURE 3.3: Real GDP response to 1% Shock to Terms of Trade (Averaged over Countries)

### 3.4 Sensitivity Analysis

This section analyzes the robustness of the results to different recursive ordering of  $r_t$  and  $tot_t$ , time periods, definitions of the world real interest rate and ways of addressing potential non-stationarity. None of the main findings of the paper are changed by using such alternative specifications.

First, I will address the choice of recursive ordering of the endogenous variables. The benchmark model specifies that the real interest rate affects the terms of trade contemporaneously but is itself not affected contemporaneously by the terms of trade. In this

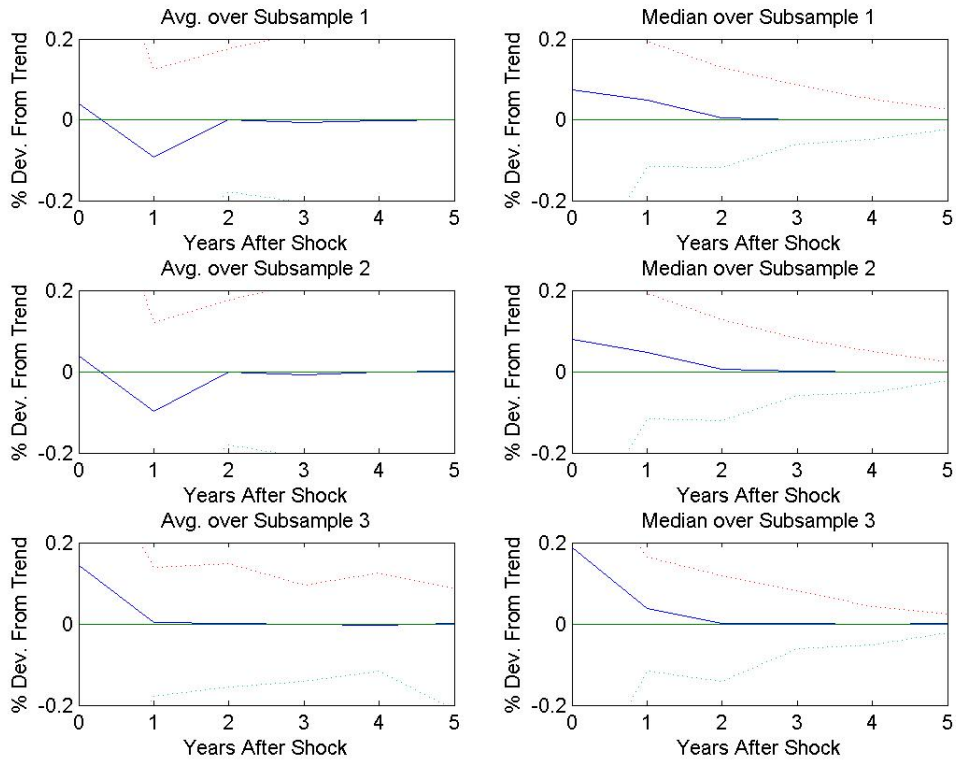


FIGURE 3.4: Real Consumption response to 1% Shock to Terms of Trade (Averaged over Countries)

section I consider the opposite ordering, where the terms of trade are positioned at the very top of  $Y_t$  and the real interest rate goes in the middle. The main results remain virtually unchanged and are listed in columns 1 and 2 of Table 3.8. Column 1 reprints the terms of trade contribution to the variance of GDP (top 6 lines) and to the variance of real aggregate consumption (bottom 6 lines) from the benchmark model examined in Section 3. Column 2 lists the corresponding estimates when the variables in  $Y_t$  are re-ordered so that the real interest rate comes in the middle and the terms of trade goes on top. The results are neither quantitatively nor qualitatively different, with the relationship between terms of trade and GDP being negligible. On the other hand, the relationship between terms of trade and aggregate consumption appears to be economically meaningful, with the terms of trade potentially explaining as much as 14% of the total variation in real

consumption. For complete disclosure, Figures 3.4 and 3.4 display the impulse response functions estimated under this alternative specification. There are no glaring differences with previous estimates in both magnitude and shape of the dynamic responses.

Table 3.8: Sensitivity Analysis: Variance Decompositions

$Var(gdp_t)$ due to $u_t^{tot}$ :								
Sample	Benchmark	ToT 1 <sup>st</sup>	HP(100)	$\Delta$	$r_t^2$	$r_t^3$	1970-	1980-
1	1.60	1.58	0.90	0.79	0.71	1.41	2.53	1.91
2	1.61	1.60	0.78	1.22	0.80	1.75	2.27	8.14
3	3.54	3.56	1.58	1.72	2.22	4.92	2.96	9.38
4	1.61	1.60	0.78	1.22	0.80	1.75	2.27	8.14
5	1.61	1.60	0.78	1.22	0.80	1.75	2.27	8.14
6	3.30	3.35	1.75	2.49	2.60	5.24	2.16	11.64
$Var(c_t)$ due to $u_t^{tot}$ :								
Sample	Benchmark	ToT 1 <sup>st</sup>	HP(100)	$\Delta$	$r_t^2$	$r_t^3$	1970-	1980-
1	5.63	5.60	9.76	3.76	3.63	7.03	10.32	20.95
2	5.88	5.84	10.05	4.83	3.95	7.42	8.25	5.54
3	14.12	14.07	14.77	10.72	11.55	19.06	14.91	21.42
4	5.88	5.84	10.05	4.83	3.95	7.42	8.25	5.54
5	5.88	5.84	10.05	4.83	3.95	7.42	8.25	5.54
6	12.65	12.60	12.86	9.65	11.50	17.80	12.36	24.32

The table details what percentage of the variance of GDP (top 6 lines) and Consumption (bottom 6 lines) is attributable to variation in the terms of trade.

Next, let's consider two different approaches of dealing with the potential non-stationarity of the macroeconomic series under study. This paper chooses to HP filter all series with a smoothing coefficient of 6.25, where the value of the coefficient was chosen as per the findings of Ravn and Uhlig (2002). Nevertheless, the previous literature has overwhelmingly opted for a smoothing coefficient of 100 and some papers, like Broda (2004), turn to first differencing instead of the HP filter altogether. Columns 3 and 4 in Table 3.8 list the estimates for the contribution of the terms of trade to the fluctuations of GDP and real consumption if all of the variables were HP(100) filtered or first differenced, respectively. There are no major differences. The relationship with GDP is again found to be weak while the relationship with consumption is about 10 times higher and of the same order of

magnitude as in the benchmark model.

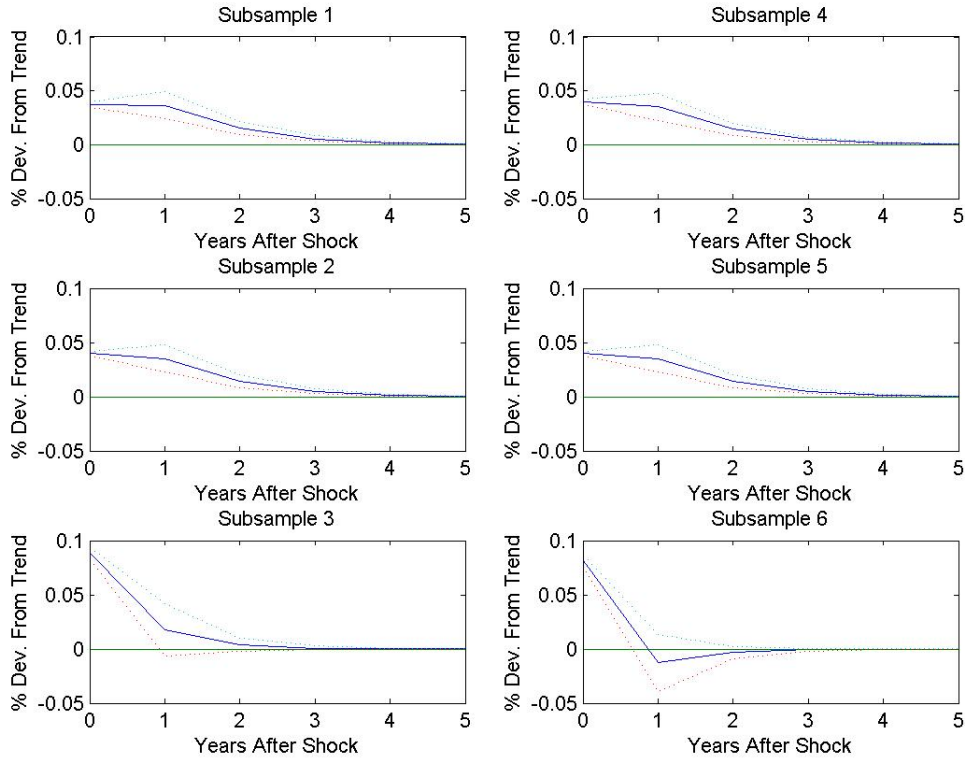


FIGURE 3.5: Real GDP response to 1% Shock to Terms of Trade (ToT ord. 1<sup>st</sup>)

Another possible modification is choosing a new definition for the real interest rate. As detailed in Section 2, the ideal measure for the interest rate will be one that makes use of local Consumer Price Index (CPI) data, which is unfortunately unavailable. The benchmark model instead equates the real world interest rate for all countries to the U.S. real interest rate, hoping that fluctuations at the business cycle frequencies in the U.S. CPI are not much different from the converted to U.S. dollars CPI of other countries.<sup>20</sup> Here I will consider two different specifications of the world real interest rate. Recall that

<sup>20</sup> As discussed previously, there is sufficient reason to believe the benchmark case holds in a world where the nominal interest rate are expressed in U.S. dollars and exchange rates are flexible. In times of high domestic inflation, relative the US inflation, the exchange rate with the dollar (local currency for 1 USD) would most likely rise and bring dollar-wise domestic inflation closer to the US figure. Hence a local-to-country real interest rate could be proxied for by the U.S. real interest rate

the Penn World Table 7.0 express the national account variables for all countries in international dollars, which by construction are chained to the U.S. dollar. One international dollar in 2005 is equivalent to one U.S. dollar in 2005 and while the relationship between the two in other years is not entirely clear and quite complicated, the Penn World Table makes a deliberate attempt to use the U.S. dollar and U.S. prices as numeraires for all of its calculations. Hence, the price indexes reported in PWT, which are over international dollars, can be used as proxies for a local CPI expressed in U.S. dollars. Thus, one alternative reasonable definition of the world real interest rate would then be to take the yield of the 3-month U.S. Treasury Bill and deflate it with the Price Indexes published in the PWT. This specification of the real world interest rate I will call  $r_t^2$ . Yet another option would be to follow Kose (2002) and deflate the US Treasury Bill by the Exports Price index of a country. This makes sense in a world where the export goods and the consumption goods are distinct (as in the model of Kose (2002)) and all international debts have to be settled with export goods. Columns 5 and 6 detail the results of using these two alternative measures of the world interest rate. No significant differences arise.

Next I analyze the robustness of my results to estimating the VAR over two alternative time periods. In the benchmark model Subsamples 1, 2, 4, and 5 are estimated from 1960 to 2007 and Subsamples 3 and 6 from 1950 to 2007. As an alternative, I re-estimate all samples first starting from 1970 and then starting from 1980. Results are given in the last two columns of Table 3.8 and again there are no significant qualitative differences from the Benchmark model.

Lastly, please note that I have also considered carrying out this analysis using quarterly data on OECD countries from OECD.Stat for the period 1995-2007. The analysis of this data is only preliminary for now as there is an issue with the availability of export and import price indices for the years before 1995, but the first look at the data set supports the findings of this paper. Currently, I am also considering using quarterly data from the IMF or the World Bank databases which have longer time series data on price indices.

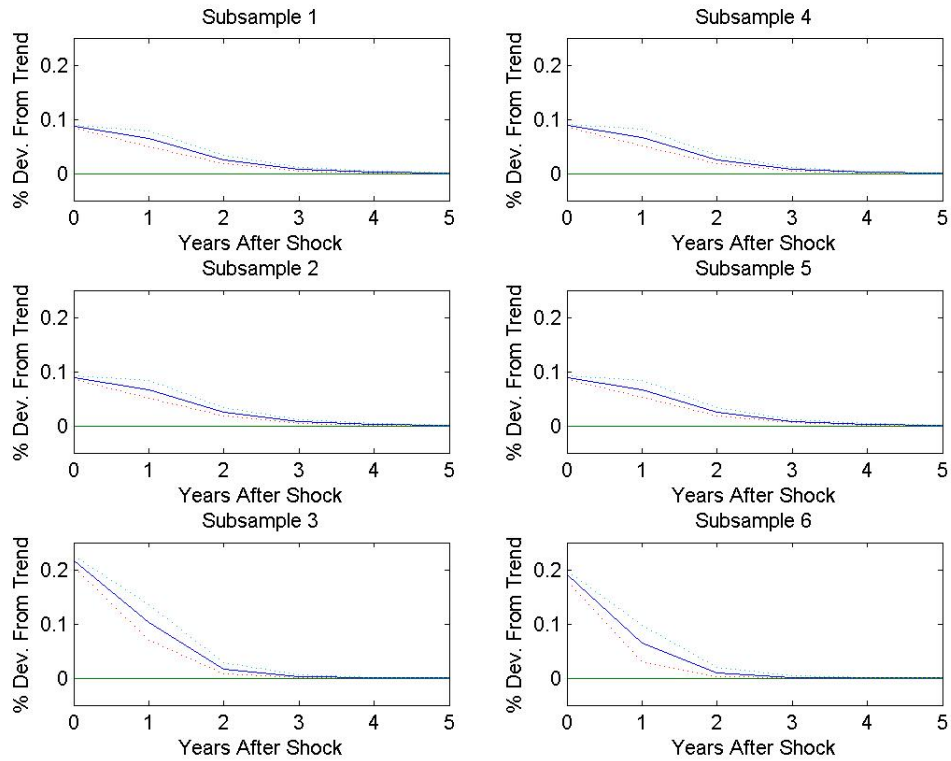


FIGURE 3.6: Real Consumption response to 1% Shock to Terms of Trade (ToT ord. 1<sup>st</sup>)

### 3.5 Conclusion

The theoretical small open economy literature has long used world prices of exports and imports as important, exogenous driving forces of the business cycles in the typical small economy. However, the empirical evidence on the relationship between terms of trade fluctuations and real GDP is inconclusive, with the estimated contribution of the terms of trade to GDP fluctuations ranging from a low of almost 0% up to 90%. Moreover, recent research by Kehoe and Ruhl (2008) complicates the question by noting that due to practical limitations in accounting for real output, the terms of trade should have no effect on *measured* GDP even when they are modeled to be important business cycle drivers. The main contribution of this paper is to provide a broad and robust analysis of the available

data, which hopefully provides a unified final answer.

The paper presents strong evidence in support of the hypothesis that the terms of trade have no important effects on real GDP at the business cycle frequencies. It uses detailed trade data not considered by previous literature to carefully select a sample of countries that satisfy the identifying assumptions, and in turn this precise selection mechanism allows it to deliver results that are robust to a number of alternatives. The main results of the paper strongly reject the conjecture of Kehoe and Ruhl (2008) that the canonical Small Open Economy model is missing important propagation mechanisms.

That is not to say, however, that the paper claims the terms of trade have no effect on business cycles. On the contrary, they are found to have a meaningful impact on real aggregate consumption and it is thus concluded that the lack of a relationship with GDP is due to the theoretical arguments of Kehoe and Ruhl (2008) and not because the terms of trade are not important in determining business cycles. Therefore, a potential pitfall for empirical researchers is identified in that while terms of trade should definitely be modeled as having an effect on the actual real value-added output, such effects would not show up in measured GDP series. This could lead to a misalignment between the empirical series researchers use as representations of actual model variables. Hence, it is prudent for researchers to adjust their in-model value-added output to be consistent with the way national statistics agencies measure GDP or avoid the issue altogether by utilizing other national accounts variables such as consumption and investment instead of GDP.



# Appendix A

## Appendix for Chapter 1

### A.1 Proofs

#### A.1.1 Proof of Lemma 1:

**Lemma 1 (Existence and Uniqueness).** *A stationary solution to the system of difference equations (1.18) exists if and only if the following two conditions are satisfied*

$$(i) \quad \kappa_b \in (K - 1, \frac{1+\rho_\tau}{1-\rho_\tau}(K + 1))$$

$$(ii) \quad \rho_\tau \in [0, \frac{1}{K})$$

where  $K = (1 + i)(1 + \gamma_b)$ . When the solution exists, it is unique.

Moreover, under the stronger restriction  $\kappa_b \in (K - 1, \frac{K}{1-\rho_\tau}]$  the solutions to  $\hat{b}_t$  and  $\hat{\tau}_t$  have non-negative autocorrelation.

*Proof.* There exists a covariance stationary solution for  $x_t$  if and only if the eigenvalues of the autoregressive matrix  $A$  are inside the unit circle, i.e. are less than 1 in magnitude. Thus, in the proof I will show that conditions (i) and (ii) are necessary and sufficient for the eigenvalues of  $A$  to be smaller than 1 in absolute value.

I will prove the if direction first. To this end, let  $\kappa_b \in (K - 1, \frac{1+\rho_\tau}{1-\rho_\tau}(K + 1))$  and  $\rho_\tau \in [0, \frac{1}{K})$  and define the useful notation  $K = (1 + i)(1 + \gamma)$ . The two eigenvalues of  $A$  are

$$\lambda_{1,2} = \frac{K - (1 - \rho_\tau)\kappa_b + \rho_\tau \pm \sqrt{(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau}}{2}.$$

The eigenvalues are complex conjugates when  $(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau < 0$ . The left-hand side of this equation defines a quadratic expression in  $\rho_\tau$  that is convex and crosses zero at the following two points

$$\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b + 1 - K) + K - 2\sqrt{K\kappa_b(\kappa_b + 1 - K)}}{(1 + \kappa_b)^2} < 1$$

$$\bar{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b + 1 - K) + K + 2\sqrt{K\kappa_b(\kappa_b + 1 - K)}}{(1 + \kappa_b)^2} > 0$$

For  $\kappa_b = K - 1$  we have  $\underline{\rho}(\kappa_b) = \bar{\rho}(\kappa_b) = \frac{1}{K}$  and for  $\kappa_b > K - 1$  we have  $\bar{\rho}(\kappa_b) > \frac{1}{K}$ . To prove the second fact we will proceed in two steps. First, we will evaluate  $\bar{\rho}(\kappa_b)$  at  $\kappa_b = (K + 1)\frac{1+\rho_\tau}{1-\rho_\tau}$  and show that it is greater than  $\frac{1}{K}$ . Next, we will solve for the first order conditions for the minimization of  $\bar{\rho}(\kappa_b)$  in respect to  $\kappa_b$  and show that the interior solution is also greater than  $\frac{1}{K}$ .

Start with evaluating  $\bar{\rho}(\kappa_b)$  at  $\kappa_b = (K + 1)\frac{1+\rho_\tau}{1-\rho_\tau}$

$$\begin{aligned} & \bar{\rho}\left((K + 1)\frac{1 + \rho_\tau}{1 - \rho_\tau}\right) - \frac{1}{K} \\ &= \frac{2(K(K - 1 + (K^2 - 1)\rho_\tau + K(1 + K)\rho_\tau^2 + (1 - \rho_\tau)\sqrt{2K(1 + K)(1 + \rho_\tau)(1 + K\rho_\tau)}) - 2)}{K(2 + K + K\rho_\tau^2)} \end{aligned}$$

Call the numerator of the above quantity  $D$  and notice that

$$\begin{aligned} \frac{\partial D}{\partial \rho_\tau} &= K(K^2 - 1) + 2K(1 + K) - \sqrt{2K(1 + K)(1 + \rho_\tau)(1 + K\rho_\tau)} \\ &\quad + (1 - \rho_\tau) \frac{2(K(1 + K)(1 + 2\rho_\tau K + K))}{\sqrt{2K(1 + K)(1 + \rho_\tau)(1 + K\rho_\tau)}} \end{aligned}$$

and since  $\sqrt{2K(1 + K)(1 + \rho_\tau)(1 + K\rho_\tau)} \leq 2\sqrt{K}(1 + K) \leq 2K(1 + K)$  it follows that  $\frac{\partial D}{\partial \rho_\tau} > 0$ . Therefore, it is enough to show that  $D > 0$  for  $\rho_\tau = 0$  to know that  $D > 0$  for all  $\rho_\tau \in (0, 1)$ . Computing the expression with  $\rho_\tau = 0$  yields:

$$D = 2(K(K - 1) + K\sqrt{2K(1 + K)} - 2) \geq 0$$

where the inequality follows from  $K\sqrt{2K(1 + K)} > 2K\sqrt{K} > 2$ , and these results imply  $\bar{\rho}((K + 1)\frac{1+\rho_\tau}{1-\rho_\tau}) > \frac{1}{K}$ .

Now evaluate the derivative  $\frac{\partial \bar{\rho}(\kappa_b)}{\partial \kappa_b}$  to find that the only solution to  $\frac{\partial \bar{\rho}(\kappa_b)}{\partial \kappa_b} = 0$  is  $\kappa_b = K$ . At the local optimum  $\bar{\rho}(K) = \frac{4K}{(1+K)^2}$  and notice that

$$\frac{4K}{(1 + K)^2} - \frac{1}{K} = \frac{(K - 1)(2K + 1)}{K(1 + K^2)} > 0$$

Thus, we have shown that the minimum of  $\bar{\rho}(\kappa_b)$  is obtained at  $\kappa_b = K - 1$  and is equal to  $\frac{1}{K}$ . Hence for  $\kappa_b \in (K - 1, \frac{1+\rho_\tau}{1-\rho_\tau}(K + 1))$  we have that  $\bar{\rho}(\kappa_b) > \frac{1}{K}$ .

Now go back to the discriminant in the expression for the eigenvalues  $(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau$ , and notice that the quadratic polynomial in  $\rho_\tau$  is convex and thus

$$(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau \geq 0, \text{ for } \rho_\tau \in [0, \underline{\rho}_\tau]$$

$$(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau \leq 0, \text{ for } \rho_\tau \in [\underline{\rho}_\tau, \bar{\rho}_\tau]$$

If  $\rho_\tau \in [\underline{\rho}_\tau, \bar{\rho}_\tau]$  the eigenvalues of  $A$  are complex and in that case their magnitude is

$$\begin{aligned} |\lambda_k| &= \frac{1}{2} \left( (K - (1 - \rho_\tau)k + \rho_\tau)^2 + [4K\rho_\tau - (K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2] \right)^{\frac{1}{2}} \\ &= \frac{1}{2} \sqrt{4K\rho_\tau} \\ &= \sqrt{K\rho_\tau} \end{aligned}$$

and hence in this case  $|\lambda_k| < 1$  if and only if  $\rho_\tau < \frac{1}{K}$ . Thus  $\rho_\tau \in [\underline{\rho}_\tau, \frac{1}{K})$  results in eigenvalues that are inside the unit circle.

Now consider the case  $\rho_\tau \in [0, \underline{\rho}_\tau]$ , which results in real eigenvalues. For  $\kappa_b = K - 1$  we have

$$\begin{aligned} \lambda_1 &= \frac{1}{2} (1 + \rho_\tau K + \sqrt{(1 + \rho_\tau K)^2 - 4K\rho_\tau}) \\ &= \frac{1}{2} (1 + \rho_\tau K + \sqrt{(1 - \rho_\tau K)^2}) \\ &= \frac{1}{2} (1 + \rho_\tau K + (1 - \rho_\tau K)) \\ &= 1 \end{aligned}$$

Next, notice that when  $\kappa_b < \frac{K + \rho_\tau}{1 - \rho_\tau}$  we have  $K - (1 - \rho_\tau)\kappa_b + \rho_\tau > 0$  and thus  $\lambda_1 > 0$ .

Moreover,

$$\begin{aligned} &\frac{\partial \lambda_1}{\partial \kappa_b} \\ &= -\frac{1 - \rho_\tau}{2\sqrt{(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau}} (\sqrt{(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau} + K - (1 - \rho_\tau)\kappa_b + \rho_\tau) \\ &< 0 \end{aligned}$$

and since at  $\kappa_b = K - 1$  we have  $\lambda_1 = 1$  it follows that for  $\kappa_b \in (K - 1, \frac{K + \rho_\tau}{1 - \rho_\tau})$  we have

$\lambda_1 \in (0, 1)$ . Meanwhile since  $(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau \geq 0$ , it follows that

$$0 < \lambda_2 < \lambda_1$$

and hence  $\lambda_2 \in (0, 1)$  as well.

If on the other hand  $\kappa_b \in \left[ \frac{K + \rho_\tau}{1 - \rho_\tau}, \frac{(K + 1)(1 + \rho_\tau)}{1 - \rho_\tau} \right]$  and

$$\begin{aligned} \lambda_1 &= \frac{K - (1 - \rho_\tau)\kappa_b + \rho_\tau + \sqrt{(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau}}{2} \\ &\leq \frac{K - (1 - \rho_\tau)\kappa_b + \rho_\tau + |K - (1 - \rho_\tau)\kappa_b + \rho_\tau|}{2} \\ &\leq 0 \end{aligned}$$

and thus  $\lambda_2 < \lambda_1 \leq 0$ . Moreover,

$$\begin{aligned} & \frac{\partial \lambda_2}{\partial \kappa_b} \\ &= -\frac{1 - \rho_\tau}{2\sqrt{(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau}} (K - (1 - \rho_\tau)\kappa_b + \rho_\tau - \sqrt{(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau}) \\ & < 0 \end{aligned}$$

and at  $\kappa_b = \frac{(K + 1)(1 + \rho_\tau)}{1 - \rho_\tau}$  we have

$$\lambda_2 = -1$$

Therefore, for  $\kappa_b \in \left( K - 1, \frac{1 + \rho_\tau}{1 - \rho_\tau}(K + 1) \right)$  and  $\rho_\tau \in [0, \underline{\rho}_\tau]$  the eigenvalues of  $A$  are real and  $|\lambda_k| < 1$ . And for  $\rho_\tau \in [\underline{\rho}_\tau, \frac{1}{K}]$  the eigenvalues are complex and again less than 1 in modulus. This completes the proof that the two conditions are sufficient for a stationary solution.

To prove they are necessary, notice that if  $\rho_\tau \in [\frac{1}{K}, \bar{\rho}_\tau]$  then the resulting complex

eigenvalue will be outside of the unit circle. On the other hand, now I will show that  $\rho_\tau \in [\bar{\rho}_\tau, 1)$  results in real eigenvalues that are bigger than 1 in absolute value.

At  $\rho_\tau = \bar{\rho}_\tau$  the bigger eigenvalue is

$$\lambda_1(\bar{\rho}_\tau) = \frac{K + \sqrt{K\kappa_b(\kappa_b + 1 - K)}}{1 + \kappa_b}$$

and I will show that  $\lambda_1(\bar{\rho}_\tau) \geq 1$ . Start with observing that since  $K \geq 1$  we have  $K\kappa_b \geq \kappa_b \geq 1 + \kappa_b - K$  and since  $1 + \kappa_b - K \geq 0$  for  $\kappa_b \geq K - 1$  we have

$$\begin{aligned} K\kappa_b &\geq 1 + \kappa_b - K \\ \Rightarrow K\kappa_b(1 + \kappa_b - K) &\geq (1 + \kappa_b - K)^2 \\ \Rightarrow \sqrt{K\kappa_b(1 + \kappa_b - K)} &\geq (1 + \kappa_b - K) \end{aligned}$$

and therefore

$$\begin{aligned} \lambda_1(\bar{\rho}_\tau) &= \frac{K + \sqrt{K\kappa_b(\kappa_b + 1 - K)}}{1 + \kappa_b} \\ &\geq \frac{1 + \kappa_b}{1 + \kappa_b} \\ &\geq 1 \end{aligned}$$

Therefore, there exist no stationary solutions for  $\rho_\tau \geq \bar{\rho}_\tau$ . This proves that  $\rho_\tau \in [0, \frac{1}{K})$  is a necessary condition.

Now consider  $\kappa_b < K - 1$ . Above we showed that  $\frac{\partial \lambda_1}{\partial \kappa_b} < 0$  and that  $\lambda_1 = 1$  when  $\kappa_b = K - 1$ , hence  $\lambda_1 \geq 1$  for any  $\kappa_b \leq K - 1$ . Similar reasoning and the facts that  $\lambda_2$  is a decreasing function of  $\kappa_b$  for  $\kappa_b > \frac{K + \rho_\tau}{1 - \rho_\tau}$  and  $\lambda_2 = -1$  for  $\kappa_b = (K + 1)(1 + \rho_\tau)1 - \rho_\tau$  lead to the conclusion that  $\kappa_b = (K + 1)(1 + \rho_\tau)1 - \rho_\tau$  implies  $\lambda_2 < -1$ . This completes the necessary direction of the proof.

To prove the second result let  $\kappa_b \in (K - 1, \frac{K}{1-\rho_\tau}]$  and start by computing the variance on both sides of the tax policy rule to obtain

$$\text{Var}(\tau_t) = \frac{b^2 k_\tau^2}{(1 + \rho)\tau^2} (1 - \rho_\tau) \text{Var}(b_t) + 2 \frac{bk\rho_\tau}{(1 + \rho_\tau)\tau} \text{Cov}(\tau_t, b_t)$$

and then combine with

$$\begin{aligned} \text{Cov}(\hat{\tau}_t, \hat{b}_t) &= \text{Cov}(\rho\tau_{t-1} + a_{21}b_{t-1}, a_{11}b_{t-1} + a_{12}\tau_{t-1} + b_{11}v_t) \\ &= -\rho_\tau^2 \frac{\tau}{b} \text{Var}(\hat{\tau}_t) + a_{11}^{(1)} (1 - \rho_\tau) \kappa_b \frac{b}{\tau} \text{Var}(\hat{b}_t) + a_{11}^{(1)} \rho_\tau - (1 - \rho_\tau) \kappa_b \rho_\tau \text{Cov}(\hat{b}_t, \hat{\tau}_t) \end{aligned}$$

to obtain

$$\text{Cov}(\hat{\tau}_t, \hat{b}_t) = \underbrace{(1 - \rho_\tau) \kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1 + \rho_\tau) - \kappa_\tau)}{1 + \rho_\tau(1 + 2\kappa_\tau - K(1 + \rho_\tau))}}_{=\delta} \text{Var}(\hat{b}_t).$$

Now work with  $\text{Cov}(\hat{\tau}_{t+1}, \hat{\tau}_t)$  to obtain

$$\begin{aligned} \text{Cov}(\hat{\tau}_{t+1}, \hat{\tau}_t) &= \text{Cov}(\rho_\tau \hat{\tau}_t + (1 - \rho_\tau) \frac{b}{\tau} \kappa_b \hat{b}_t, \hat{\tau}_t) \\ &= \rho_\tau \text{Var}(\hat{\tau}_t) + (1 - \rho_\tau) \frac{b}{\tau} \kappa_b \delta \text{Var}(\hat{b}_t) \end{aligned}$$

Combine again with the expression for  $\text{Var}(\hat{\tau}_t)$  and simplify to arrive at

$$\text{Cov}(\hat{\tau}_{t+1}, \hat{\tau}_t) = \frac{b^2 k^2}{\tau^2} \frac{(1 - \rho_\tau) \rho_\tau (1 + K - (1 - \rho_\tau)(\kappa_b - K\rho_\tau))}{1 + \rho_\tau(1 + 2\kappa_b - K(1 + \rho_\tau))} \text{Var}(\hat{b}_t)$$

Let

$$\tilde{\rho} = \frac{b^2 k^2 (1 - \rho_\tau) \rho_\tau (1 + K - (1 - \rho_\tau)(\kappa_b - K\rho_\tau))}{\tau^2 (1 + \rho_\tau(1 + 2\kappa_b - K(1 + \rho_\tau)))}$$

Taxes are non-negatively autocorrelated as long as  $\tilde{\rho} \geq 0$ . Notice that the denominator is always positive and hence this expression is positive only if the numerator is positive. The numerator itself is positive if and only if

$$\kappa_b < \frac{K+1}{1-\rho_\tau} + \rho_\tau K$$

which satisfies our initial assumption.

Now let's turn our attention to  $\text{Cov}(\hat{b}_{t+1}, \hat{b}_t)$ . It is straightforward to show that

$$\text{Cov}(\hat{b}_{t+1}, \hat{b}_t) = a_{11} \text{Var}(\hat{b}_t) + a_{12} \text{Cov}(\hat{\tau}_t, \hat{b}_t) = (a_{11} + \delta a_{12}) \text{Var}(\hat{b}_t)$$

where  $a_{kl}$  is the (k,l) element of the matrix A. The coefficient in front of  $\text{Var}(\hat{b}_t)$  can be simplified down to

$$(a_{11} + \delta a_{12}) = K - \frac{\kappa_b(1-\rho_\tau)(1+\rho_\tau + \kappa_b\rho_\tau)}{1+\rho_\tau(1+2\kappa_b - K(1-\rho_\tau))}$$

which is non-negative if and only if

$$K(1+\rho_\tau(1+2\kappa_b - K(1-\rho_\tau))) - \kappa_b(1-\rho_\tau)(1+\rho_\tau + \kappa_b\rho_\tau) \geq 0$$

the above expression is a convex quadratic polynomial in  $\kappa_b$ , which crosses zero at the points

$$\underline{\kappa} = \frac{\rho_\tau^2 + 2K\rho_\tau - 1 - \sqrt{(1-\rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1-\rho_\tau)\rho_\tau}$$

$$\bar{\kappa} = \frac{\rho_\tau^2 + 2K\rho_\tau - 1 + \sqrt{(1-\rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1-\rho_\tau)\rho_\tau}$$

First, I will show that  $\underline{\kappa} < 0$  so that under my assumed conditions  $\kappa_b \geq \underline{\kappa}$ . This is trivially true if  $\rho_\tau^2 - 2K\rho_\tau - 1 < 0$ . Assume that  $\rho_\tau$  is high enough so that is positive and



assume

$$\rho_\tau^2 + 2K\rho_\tau - 1 > \sqrt{(1 - \rho_\tau^2)^2 + 4K^2\rho_\tau^4}$$

Take squares on both sides, re-arrange and simplify to arrive at the condition

$$4K\rho_\tau(1 - \rho_\tau^2)(K\rho_\tau - 1) > 0$$

but since  $\rho_\tau < \frac{1}{K}$  we have reached a contradiction, and thus  $\underline{\kappa} < 0 < \kappa_b$ . Now, work with  $\bar{\kappa}_b$ :

$$\begin{aligned} \bar{\kappa}_b &= \frac{\rho_\tau^2 + 2K\rho_\tau - 1 + \sqrt{(1 - \rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1 - \rho_\tau)\rho_\tau} \\ &= \frac{\rho_\tau^2 - 1 + \sqrt{(1 - \rho_\tau^2)^2 + 4K^2\rho_\tau^4}}{2(1 - \rho_\tau)\rho_\tau} + \frac{K}{(1 - \rho_\tau)} \\ &\geq \frac{\rho_\tau^2 - 1 + (1 - \rho_\tau^2)}{2(1 - \rho_\tau)\rho_\tau} + \frac{K}{(1 - \rho_\tau)} \\ &\geq \frac{K}{(1 - \rho_\tau)} \end{aligned}$$

therefore under our assumptions  $\kappa_b < \bar{\kappa}_b$  and hence, rolling everything back, this implies that

$$\text{Cov}(\hat{b}_{t+1}, \hat{b}_t) \geq 0$$

and we are done. □

*A.1.2 Proof of Lemma 2:*

**Lemma 2 (Impulse Response Functions).** *Let  $\kappa_b \in (K - 1, \frac{K}{1 - \rho_\tau})$  and define  $\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b + 1 - K) + K - 2\sqrt{K\kappa_b(\kappa_b + 1 - K)}}{(1 + \kappa_b)^2} > 0$ . Then,*

(i) *If  $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$  the autoregressive matrix  $A$  has two real, non-negative eigenvalues*

and the Impulse Response Functions never crosses the steady state, i.e.

$$\tilde{a}_{k1}^{(j)} \leq 0 \text{ for } k \in \{1, 2\}, \text{ and } j = 1, 2, 3, \dots$$

(ii) If  $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K})$  the autoregressive matrix  $A$  has a pair of complex conjugate eigenvalues which can be written as  $\lambda_k = a \pm bi$  for  $k \in \{1, 2\}$ , and the corresponding conjugate eigenvectors are of the form  $\vec{v}_k = [x \pm yi, 1]'$ , where  $a, b, x, y$  are real numbers and  $i$  is the imaginary unit. Furthermore, the Impulse Response Functions follow the increasingly dampened cosine waves:

$$\tilde{a}_{11}^{(j)} = -(1 + i + \frac{m}{bG})|\lambda|^j \sqrt{1 + (\frac{x}{y})^2} \cos(j\phi + \psi - \frac{\pi}{2}), \text{ for } j = 1, 2, 3, \dots$$

$$\tilde{a}_{12}^{(j)} = (1 + i + \frac{m}{bG})|\lambda|^j \frac{1}{y} \cos(j\phi - \frac{\pi}{2}), \text{ for } j = 1, 2, 3, \dots$$

$$\tilde{a}_{21}^{(j)} = -(1 + i + \frac{m}{bG})|\lambda|^j \frac{x^2 + y^2}{y} \cos(j\phi - \frac{\pi}{2}), \text{ for } j = 1, 2, 3, \dots$$

where  $\phi = \arctan(\frac{b}{a})$  and  $\psi = \arctan(\frac{y}{x})$ . Moreover,  $\tilde{a}_{k1}^{(1)} \leq 0$  for  $k \in \{1, 2\}$ .

*Proof.* (i) The first part follows directly from the proof of Lemma 1. The eigenvalues of  $A$  are real when

$$(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2 - 4K\rho_\tau \geq 0$$

which is true for  $\rho_\tau \in [0, \underline{\rho}_\tau(\kappa_v)]$ , using the fact that we restrict attention to  $\rho_\tau \geq 0$ .

Moreover, when  $\kappa_b \leq \frac{K}{1-\rho_\tau}$  it is immediate that  $K - (1 - \rho_\tau)\kappa_b + \rho_\tau > 0$  and hence  $\lambda_1 > 0$ . Moreover,

$$\begin{aligned}\lambda_2 &\geq K - (1 - \rho_\tau)\kappa_b + \rho_\tau + \sqrt{(K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2} \\ &\geq 0\end{aligned}$$

Hence both eigenvalues are non-negative.

To characterize the Impulse Response Function note that the Wold Decomposition of  $x_t$  is

$$x_t = Bv_t + ABv_{t-1} + A^2Bv_{t-2} + \dots$$

Use the fact that

$$B = \begin{bmatrix} -(1 + i + \frac{m}{bG}) \\ 0 \end{bmatrix} v_t$$

to obtain

$$\hat{b}_t = -(1 + i + \frac{m}{bG})(v_t + a_{11}^{(1)}v_{t-1} + a_{11}^{(2)}v_{t-2} + a_{11}^{(3)}v_{t-3} + \dots)$$

$$\hat{\tau}_t = -(1 + i + \frac{m}{bG})(a_{21}^{(1)}v_{t-1} + a_{21}^{(2)}v_{t-2} + a_{21}^{(3)}v_{t-3} + \dots)$$

where  $a_{kl}^{(j)}$  is the  $(k,j)$  element of the matrix  $A^j$ . Define  $a_{11}^{(0)} = 1$  and  $a_{11}^{(0)} = 0$  and the transformation

$$\tilde{a}_{kl}^{(j)} = -(1 + i + \frac{m}{bG})a_{kl}^{(j)}.$$

The sequences  $\{\tilde{a}_{k1}\}_0^\infty$  define the Impulse Response Functions of  $\hat{b}_t$  (for  $k = 1$ ) and  $\hat{\tau}_t$  (for  $k = 2$ ).

First, I will show that  $a_{k1} \geq 0$  for all  $j = 1, 2, 3, \dots$  and  $k = 1, 2$  when the matrix  $A$  is diagonalizable, and then I will handle the case when the eigenvalue is repeated

and  $A$  is not diagonalizable (the only other case we need to worry about for a two by two matrix).

Assuming that  $A$  is diagonalizable, define

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

as a matrix with the two eigenvalues of  $A$  on its diagonal ordered like  $\lambda_1 > \lambda_2$  (remember we are handling the case of real eigenvalues right now) and  $P$  as a matrix that has the eigenvectors of  $A$  as its columns. Since we have assumed  $A$  is diagonalizable, we have  $A = P\Lambda P^{-1}$  and in particular

$$\begin{aligned} A^j &= (P\Lambda P^{-1})^j \\ &= (P\Lambda P^{-1})(P\Lambda P^{-1})(P\Lambda P^{-1})\dots \\ &= P\Lambda^j P^{-1} \end{aligned}$$

and since  $\Lambda$  is diagonal

$$\Lambda^j = \begin{bmatrix} \lambda_1^j & 0 \\ 0 & \lambda_2^j \end{bmatrix}$$

and thus if we expand the expression for  $A^j$  we obtain that

$$a_{11}^{(j)} = \frac{p_{11}p_{22}\lambda_1^j - p_{12}p_{21}\lambda_2^j}{|P|}$$

where  $|P|$  is the determinant of the matrix of eigenvectors  $P$  and  $p_{kl}$  is its  $(k, l)$ -th

element. Computing the eigenvectors of  $A$  I obtain

$$p_{1k} = \frac{\tau \lambda_k}{\sqrt{\tau^2 \lambda_k^2 + b^2(K - \lambda_k)^2}}$$

$$p_{2k} = \frac{b(K - \lambda_k)}{\sqrt{\tau^2 \lambda_k^2 + b^2(K - \lambda_k)^2}}$$

Since both of the eigenvalues are positive and are ordered so that  $\lambda_1 > \lambda_2$  it follows that  $|P| > 0$  and hence

$$\frac{p_{11}p_{22}\lambda_1^j - p_{12}p_{21}\lambda_2^j}{|P|} > 0.$$

This proves that  $a_{11}^{(j)} \geq 0$  for all  $j$  and hence  $\tilde{a}_{11}^{(j)} \leq 0$  for all  $j$ . On the other hand,

$$a_{21}^{(j)} = p_{21}p_{22}(\lambda_1^j - \lambda_2^j)$$

and since  $\lambda_k < 1 < K$  it follows that  $p_{21}p_{22} > 0$ ,  $a_{21}^{(j)} \geq 0$  and thus  $\tilde{a}_{21}^{(j)} \leq 0$ , which completes the proof for diagonalizable  $A$ .

Now assume that  $A$  is not diagonalizable.  $A$  can still be written as  $A = P\Lambda P^{-1}$  but now

$$\Lambda = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_2 \end{bmatrix}$$

and the columns of  $P$  are the generalized eigenvectors of  $A$ . In this case, there is only one linearly independent eigenvector associated with the eigenvalue of  $\lambda$ , call it  $\vec{p}$ , and thus the second generalized eigenvector, call it  $\vec{u}$ , is a 2x1 vector that solves

$$(A - \lambda I)\vec{u} = \vec{p}$$

Thus  $P = [\vec{p}, \vec{u}]$ . We know that the  $\vec{p}$  is a regular eigenvector and hence must satisfy the relationship we derived before:

$$p_1 = \frac{\tau\lambda}{(K - \lambda)b}p_2.$$

Using this fact, we can solve  $(A - \lambda I)\vec{u} = \vec{p}$  for  $\vec{u}$  and show that the generalized eigenvector is of the form  $[1, 0]$ . With these results in mind we can show that for  $j \geq 1$ :

$$\begin{aligned} a_{11}^{(j)} &= \lambda^{j-1}(\lambda - j\frac{p_2 p_1}{|P|}) \\ &= \lambda^{j-1}(\lambda + j\frac{p_1}{u_1}) > 0 \end{aligned}$$

where the second equality follows from the fact that  $|P| = -u_1 p_2$  (remember  $P = \begin{bmatrix} p_1 & u_1 \\ p_2 & u_2 \end{bmatrix}$ ) and the inequality on the third line follows from the fact that  $u_1 \geq 0$ ,

$p_1 \geq 0$ ,  $\lambda \geq 0$ . On the other hand,

$$\begin{aligned} a_{21}^{(j)} &= -j\frac{p_2^2}{|P|}\lambda^{j-1} \\ &= j\frac{p_2}{u_1}\lambda^{j-1} > 0 \end{aligned}$$

and thus we are done with the case when  $A$  is not diagonalizable. This completes the proof of part (i).

- (ii) From the proof of Lemma 1 we know that  $\rho_\tau \in (\underline{\rho}_\tau, \frac{1}{K})$  implies that the eigenvalues of  $A$  are complex. We can express them as  $\lambda_1 = a + bi$  and  $\lambda_2 = a - bi$  where  $a = \frac{1}{2}(K - (1 - \rho_\tau)\kappa_b + \rho_\tau) > 0$ ,  $b = \frac{1}{2}\sqrt{4K\rho_\tau - (K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2} \geq 0$  and  $i$  is the imaginary unit. The two conjugate eigenvectors can be written as  $\vec{p}_k = [x \pm yi, 1]'$ ,

where

$$x = \frac{(K - (1 - \rho_\tau)\kappa_b - \rho_\tau)\tau}{2b(1 - \rho_\tau)\kappa_b}$$

$$y = \frac{\tau\sqrt{4K\rho_\tau - (K - (1 - \rho_\tau)\kappa_b + \rho_\tau)^2}}{2b(1 - \rho_\tau)\kappa_b}$$

With two conjugate complex eigenvalues  $A$  is diagonalizable and can be expressed as  $A = P\Lambda P^{-1}$  where again  $P$  is a similarity matrix with the eigenvectors of  $A$  as its columns and  $\Lambda$  is a diagonal matrix with the eigenvalues on the diagonal. Next remember that Euler's formula allows us to express complex numbers in exponential form, so that we can write  $\lambda_1 = a + bi = |\lambda|e^{j\phi}$  where  $\phi = \arctan(\frac{b}{a})$ . This formulation is convenient because it is easy to take powers of the eigenvalues, e.g.

$$\lambda_1^j = |\lambda|^j e^{j\phi i}$$

and hence it is easy to compute powers of the eigenvalue matrix  $\Lambda$ . Using this, Euler's formula and the fact that  $A^j = P\Lambda^j P^{-1}$  it is straightforward to compute

$$a_{11}^{(j)} = |\lambda|^j (\cos(j\phi) + \frac{x}{y} \sin(j\phi))$$

$$a_{12}^{(j)} = -|\lambda|^j \frac{x^2 + y^2}{y} \sin(j\phi)$$

$$a_{21}^{(j)} = |\lambda|^j \frac{1}{y} \sin(j\phi)$$

where  $\phi = \arctan(\frac{b}{a})$ .

Moreover we can rewrite these expressions as

$$\begin{aligned}
a_{11}^{(j)} &= |\lambda|^j \left( \cos(j\phi) + \frac{x}{y} \sin(j\phi) \right) \\
&= |\lambda|^j \sqrt{1 + \left(\frac{x}{y}\right)^2} \sin(j\phi + \psi) \\
&= |\lambda|^j \sqrt{1 + \left(\frac{x}{y}\right)^2} \cos\left(j\phi + \psi - \frac{\pi}{2}\right)
\end{aligned}$$

where  $\psi = \arctan\left(\frac{y}{x}\right) + \pi \mathbb{I}\left(\frac{y}{x} < 0\right)$ . The second equality follows from the formula for linear combinations of trig functions, and the third is simply an application of  $\cos\left(\theta - \frac{\pi}{2}\right) = \sin(\theta)$ . And similarly

$$\begin{aligned}
a_{12}^{(j)} &= -|\lambda|^j \frac{x^2 + y^2}{y} \cos\left(j\phi - \frac{\pi}{2}\right) \\
a_{21}^{(j)} &= |\lambda|^j \frac{1}{y} \cos\left(j\phi - \frac{\pi}{2}\right)
\end{aligned}$$

By the definition of the  $\arctan(\cdot)$  function and the virtue of  $a \geq 0, b \geq 0$  it follows that  $\phi \in [0, \frac{\pi}{2})$ . Therefore,  $j\phi - \frac{\pi}{2} < \frac{\pi}{2}$  for  $j \leq 2$  and hence

$$\cos\left(j\phi - \frac{\pi}{2}\right) \geq 0, \text{ for } j \in \{1, 2\}$$

and hence  $\tilde{a}_{21}^{(j)} \leq 0$  for  $j \in \{1, 2\}$ . On the other hand, if  $x \geq 0$  then  $\psi \leq \frac{\pi}{2}$  and this case  $\cos\left(j\phi + \psi - \frac{\pi}{2}\right) \geq 0$  for at least  $j = 1$ . On the other hand  $x < 0$  implies that  $\kappa_b > \frac{K - \rho\tau}{1 - \rho\tau}$ . Then using the formula for addition of arctangent I obtain,



$$\arctan\left(\frac{b}{a}\right) + \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\frac{b}{a} + \frac{y}{x}}{1 - \frac{by}{ax}}\right).$$

where  $1 - \frac{by}{ax} > 0$ . Working with the numerator I get  $\frac{y}{x} = \frac{b}{a - \rho_\tau}$  and given  $\kappa_b \in (\frac{K - \rho_\tau}{1 - \rho_\tau}, \frac{K}{1 - \rho_\tau})$  it follows that  $\frac{\rho_\tau}{2} < a < \rho_\tau$  and thus

$$\left|\frac{y}{x}\right| > \frac{b}{a}.$$

This shows that  $\frac{b}{a} + \frac{y}{x} < 0$  and therefore  $\arctan\left(\frac{\frac{b}{a} + \frac{y}{x}}{1 - \frac{by}{ax}}\right) \in (-\frac{\pi}{2}, 0)$ . Therefore, we again reach the conclusion that  $\cos(j\phi + \psi - \frac{\pi}{2}) \geq 0$  for at least  $j = 1$ . This completes the proof of Lemma 2. □

### A.1.3 Theorem 1:

**Theorem 1 (UIP Violations).** *Let  $\kappa_b \in (K - 1, \frac{K}{1 - \rho_\tau})$ . The UIP regression coefficients  $\beta_j$  are equal to*

$$\beta_j = \frac{\text{Cov}(\hat{\lambda}_{t+1}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} = -(a_{11}^{(j)} + \delta a_{12}^{(j)})$$

where  $a_{kl}^{(j)}$  is the  $(k, j)$  element of the matrix  $A^j$  for  $j = 1, 2, 3, \dots$ ,  $a_{11}^{(0)} = 1$  and  $a_{12}^{(0)} = 0$ , and  $\delta = (1 - \rho_\tau)\kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1 + \rho_\tau) - \kappa_\tau)}{1 + \rho_\tau(1 + 2\kappa_\tau - K(1 + \rho_\tau))}$ . Furthermore,

[(i)]

1. If  $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ , the roots of the system are real and we have

$\beta_j < 0$ , for all  $j = 1, 2, 3, \dots$

2. If  $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K}]$ , then the eigenvalues of the autoregressive matrix  $A$  are complex and of the form  $\lambda_k = a + \pm bi$ ,  $k \in \{1, 2\}$  with corresponding eigenvectors of the form  $\vec{v}_i = [x \pm yi, 1]'$ , and we have

$$\beta_j = \begin{cases} -1 & , j = 1 \\ -|\lambda|^j \sqrt{1 + \chi^2} \cos(j\phi + \psi - \frac{\pi}{2}) & , j = 2, 3, \dots \end{cases}$$

where  $\chi = \frac{x - \delta(x^2 + y^2)}{y}$ ,  $\phi = \arctan(\frac{b}{a})$ ,  $\psi = \arctan(\frac{1}{\chi}) + \pi \mathbb{I}(\chi < 0)$ . Thus, while  $\beta_1 < 0$ , there exists a  $j > 0$  such that  $\beta_j > 0$ .

*Proof.* To derive the expression for the UIP regression coefficients notice that  $E_t(\hat{b}_{t+j}) = [1, 0]A^j x_t$ , hence

$$\text{Cov}(E_t(\hat{b}_{t+j}), b_t) = a_{11}^{(j)} \text{Var}(\hat{b}_t) + a_{12}^{(j)} \text{Cov}(\hat{\tau}_t, \hat{b}_t). \quad (\text{A.1})$$

Compute the variance on both sides of the tax policy rule to obtain

$$\text{Var}(\hat{\tau}_t) = \frac{b^2 k_\tau^2}{(1 + \rho_\tau)\tau^2} (1 - \rho_\tau) \text{Var}(\hat{b}_t) + 2 \frac{bk\rho_\tau}{(1 + \rho_\tau)\tau} \text{Cov}(\hat{\tau}_t, \hat{b}_t)$$

and then combine with

$$\begin{aligned} \text{Cov}(\hat{\tau}_t, \hat{b}_t) &= \text{Cov}(\rho\hat{\tau}_{t-1} + a_{21}\hat{b}_{t-1}, a_{11}\hat{b}_{t-1} + a_{12}\hat{\tau}_{t-1} + b_{11}v_t) \\ &= -\rho_\tau^2 \frac{\tau}{b} \text{Var}(\hat{\tau}_t) + a_{11}^{(1)} (1 - \rho_\tau) \kappa_b \frac{b}{\tau} \text{Var}(\hat{b}_t) + a_{11}^{(1)} \rho_\tau - (1 - \rho_\tau) \kappa_b \rho_\tau \text{Cov}(\hat{b}_t, \hat{\tau}_t) \end{aligned}$$

to obtain

$$\text{Cov}(\hat{\tau}_t, \hat{b}_t) = \underbrace{(1 - \rho_\tau) \kappa_\tau \frac{\bar{b}^G}{\bar{\tau}} \frac{(K(1 + \rho_\tau) - \kappa_\tau)}{1 + \rho_\tau(1 + 2\kappa_\tau - K(1 + \rho_\tau))}}_{=\delta} \text{Var}(\hat{b}_t).$$

Substituting this back in (A.1) yields  $\text{Cov}(E_t(\hat{b}_{t+j}), b_t) = (a_{11}^{(j)} + \delta a_{12}^{(j)}) \text{Var}(\hat{b}_t)$  and hence

$$\beta_j = -(a_{11}^{(j)} + \delta a_{12}^{(j)}).$$

This gives us the general expression for the UIP regression coefficients, next I characterize them further under real and complex roots specifically.

To prove (i) let  $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$  and remember that

$$\hat{b}_t = \tilde{a}_{11}^{(0)} v_t + \tilde{a}_{11}^{(1)} v_{t-1} + \tilde{a}_{11}^{(2)} v_{t-2} + \tilde{a}_{11}^{(3)} v_{t-3} + \dots$$

and then

$$\text{Cov}(E_t(\hat{b}_{t+j}), \hat{b}_t) = \tilde{a}_{11}^{(j)} \tilde{a}_{11}^{(0)} + \tilde{a}_{11}^{(j+1)} \tilde{a}_{11}^{(1)} + \tilde{a}_{11}^{(j+2)} \tilde{a}_{11}^{(2)} + \tilde{a}_{11}^{(j+3)} \tilde{a}_{11}^{(3)} + \dots$$

Lemma 2 gives us  $\tilde{a}_{11}^{(j)} \leq 0$  for  $j = 0, 1, 2, 3, \dots$  and thus all pairs in the above sum are non-negative, and hence  $\text{Cov}(E_t(\hat{b}_{t+j}), \hat{b}_t) \geq 0$ . This gives us that

$$\beta_j = -\frac{\text{Cov}(E_t(\hat{b}_{t+j-1}), \hat{b}_t)}{\text{Var}(\hat{b}_t)} \leq 0, \text{ for } j = 1, 2, 3, \dots$$

To prove (ii) let  $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{1}{K}]$  and notice that Lemma 2 tells us that the eigenvalues of  $A$  are of the form  $\lambda_k = a \pm bi$  for  $k \in \{1, 2\}$ , and the corresponding conjugate eigenvectors are of the form  $\vec{v}_k = [x \pm yi, 1]'$ , where  $a, b, x, y$  are real numbers and  $i$  is the imaginary unit. Using the expressions for  $a_{11}^{(j)}$  and  $a_{12}^{(j)}$  found in Lemma 2 I get that for  $j > 1$

$$\begin{aligned}
\beta_j &= -(a_{11}^{(j)} + \delta a_{12}^{(j)}) = -|\lambda|^j \left( \cos(j\phi) + \frac{x}{y} \sin(j\phi) - \delta \frac{x^2 + y^2}{y} \sin(j\phi) \right) \\
&= -|\lambda|^j \sqrt{1 + \chi^2} \cos\left(j\phi + \psi - \frac{\pi}{2}\right)
\end{aligned}$$

with  $\chi = \frac{x - \delta(x^2 + y^2)}{y}$ ,  $\phi = \arctan\left(\frac{b}{a}\right) > 0$ ,  $\psi = \arctan\left(\frac{1}{\chi}\right) + \pi \mathbb{I}(\chi < 0)$ . For  $j = 1$ , trivially  $\beta_j = \frac{\text{Cov}(\hat{b}_t, \hat{b}_t)}{\text{Var}(\hat{b}_t)} = -1$ . Since the cosine function is cyclical at some  $j > 1$  it will turn negative and hence  $\beta_j > 0$ , and this completes the derivation of the regression coefficients in the case of complex roots.  $\square$

## A.2 Forward Exchange Rate Contracts and UIP Violations

In this section, I augment the model to include trade in forward contracts on currencies, and show that trading in forward contracts creates a synthetic position long one country's bond and short the other. Hence, it does not matter whether one implements carry trades through forward contracts, or through trades in the bonds themselves, as both trading strategies earn the same convenience yield differential, which violates UIP. In other words, the convenience yield mechanism generates UIP violations that emerge both when looking at exchange rates and interest rates data only, and when only looking at forward and spot exchange rates.

The key to the result is that in a model with bond convenience yields, the Covered Interest Parity (CIP) holds if and only if a covered position in foreign currency is equivalent to a position in home bonds, both in financial terms and in convenience benefits. Why is that? Notice that a covered position in EUR risk-free bonds, where the future interest rate  $(1 + i_t^*)$  has been sold forward for dollars at the equilibrium rate  $F_t$ , generates a risk-free USD payoff, and not a risk-free EUR payoff. As such, it has a comparable convenience value to the other risk-free USD asset - US Treasuries. Being a risk-free USD asset, it carries the convenience benefits of USD risk-free assets, because it allows the investor to pledge a sure, future amount of USD, and not a sure future amount of EUR. Another way to think about it, is that a covered position in EUR bonds is in fact long USD, and not long EUR. Similarly, buying foreign currency forward is a strategy long in foreign currency and short home currency. It simultaneously increases the pledgeable amount of foreign currency proceeds and decreases the pledgeable amount of home currency, hence it creates a synthetic, zero-cost position that is long home bonds and short foreign bonds, and thus in equilibrium, on average it earns the convenience yield differential.

The CIP condition states that investing in foreign risk-free bonds and using forward contracts to eliminate the exchange rate risk must yield the same rate of return as investing in home bonds. CIP has been shown to hold in the data very well, outside of a few, short-

lived episodes during times of extraordinary financial markets turbulence (e.g. some days during the recent financial crisis). The intuition behind the condition is that the covered position in foreign bonds is a risk-free asset denominated in home currency, and not in foreign currency, hence in equilibrium it must have the same rate of return as the domestic bond (another domestic risk-free asset), or otherwise there will be an arbitrage opportunity.

To be more concrete, let  $F_t$  denote the equilibrium USD-EUR forward rate, so that today we can agree to trade 1 EUR tomorrow in exchange of  $F_t$  USD. Imagine then that an investor borrows \$1 today at the interest rate  $1 + i_t$ , changes it into  $\frac{1}{S_t}$  EURs and invests it at the interest rate  $1 + i_t^*$ , and at the same time has sold forward the proceeds at the forward rate  $F_t$ . Thus, his payoff from the covered foreign position is  $\frac{F_t}{S_t}(1 + i_t^*)$  and the cost of the 1 USD is  $1 + i_t$  and CIP states:

$$1 + i_t = \frac{F_t}{S_t}(1 + i_t^*),$$

so that a position in a US Treasury has an equivalent financial return to a covered position in EUR denominated government bonds (e.g. German Bunds). A position in US Treasuries also carries the convenience benefit  $\Psi_{b_h,t}$  and the covered position in foreign bonds is another risk-free USD asset which carries the (possibly different) convenience benefit  $\tilde{\Psi}_{b_h,t}$ . Conditional on CIP, the convenience benefits of the two positions must be the same:

$$\Psi_{b_h,t} = \tilde{\Psi}_{b_h,t}.$$

This follows from the fact that an investment in US Treasuries carries a total return of  $1 + i_t + \Psi_{b_h,t}$ , the sum of the financial return and the convenience benefit, and an investment in a covered position in EUR denominated bonds similarly carries a total return of  $\frac{F_t}{S_t}(1 + i_t^*) + \tilde{\Psi}_{b_h,t}$ . The two risk-free returns must be equal, otherwise there is an arbitrage opportunity. Given that CIP restricts the financial returns to be equal, it follows that the convenience benefits must be equal as well:  $\Psi_{b_h,t} = \tilde{\Psi}_{b_h,t}$ .

Thus, when CIP holds (as it does in the data) and bonds offer convenience benefits (as is also true in the data), in equilibrium, covered position in foreign bonds, which yield a risk-free payoff in the home currency and not a payoff in foreign currency, must offer the same convenience benefits as an equivalent position in home currency bonds.

This leads to the important result that (in log-approximation) the expected return on buying foreign currency forward (a popular way of implementing the carry trade without the need to transact in bond markets) is:

$$E_t(s_{t+1} - f_t) = E_t(\Delta s_{t+1} + i_t^* - i_t) = \hat{\Psi}_{b_h,t} - \hat{\Psi}_{b_f,t}.$$

This shows that taking positions in the forwards market is akin to creating a synthetic position that is simultaneously long foreign currency bonds and short home currency bonds. This is of course, very intuitive, as buying foreign currency forward is a contract long in the foreign currency and short in the home currency. Entering into this contract reduces the amount of future USD the investor is able to pledge today as collateral (since he has already sold this USD for EURs) and at the same time increase the pledgeable amount of EUR. At the end of the day, the strategy implemented through forwards market has equivalent financial and convenience returns to a trade in the home and foreign bonds themselves, hence the forwards data would display equivalent UIP violations and the mechanism works in the same way. Due to this equivalence and for simplicity, the benchmark model abstracts from trade in forward contracts.

### A.3 Relationship to existing Long-Run UIP Tests

Previous work by Chinn and Meredith (2005), Chinn (2006) and others has found that UIP tends to hold better over the long run, especially at horizons of five or more years. It is important to note that my results, which find that UIP is violated at horizons of up to 7 years but with a change in the direction of violations, are not contradictory but rather complementary to the previous results. The difference comes from a difference in

methodologies, and in fact my results can help interpret and provide context to the existing findings.

The previous literature on the UIP hypothesis at long horizons has focused on testing whether long-run exchange rate changes (say 5+ years) tend to offset the corresponding long-run interest rate differential. The UIP condition equates the rates of return on bonds of equivalent maturities, hence if  $i_t^{(k)}$  is the yield (per month) on a  $k$ -months to maturity US Treasury bond, and  $i_t^{(k),*}$  is the corresponding yield on a foreign  $k$ -months to maturity bond then for any  $k$  the UIP condition requires:

$$E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})) = 0$$

Under UIP we would expect the long-term exchange rate change  $s_{t+k} - s_t$  to offset the corresponding  $k$ -month interest rate differential. The existing literature on UIP in the long-run has focused on testing this condition by employing bonds with maturities of up to ten years. The common result is that the empirical evidence against the UIP condition is weaker when using long term bonds with maturities of five or more years. Thus, there is some evidence that the long-run movements in exchange rates tend to offset the corresponding long-term interest rate differential and hence UIP has appeared to hold better over the long run.

In contrast, in this paper I always use one month interest rates and forecast one month excess returns, but I vary the horizon of the forecast. Rather than asking whether the cumulative, five year change in the exchange rate offsets the five year interest rate and hence the five year excess return is zero in expectation,

$$E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})) = 0,$$

I test whether all sixty, intermediate one-month excess returns in the next five years are



individually zero in expectation

$$E_t(s_{t+h} - s_{t+h-1} + i_{t+h-1}^{(1),*} - i_{t+h-1}^{(1)}) = 0 \text{ for } h = 1, 2, \dots, k$$

The two approaches are clearly related to each other. In particular, as I will show below, if the expectations hypothesis (EH) on the yield curve held, then the two approaches would have a very clear and intuitive link, where the methodology of the previous papers would amount to averaging over the UIP violations I document. Thus, my results can help us understand why the previous long-run UIP tests have tended to find support for the UIP hypothesis. My findings suggest that this happens because looking at long-run bonds and the corresponding long-run exchange rate changes tends to average over intermediate, higher-frequency UIP violations that change sign and cancel each other out when cumulated over a sufficiently long horizon. Hence, the UIP is violated at both short and long horizons, and this does not appear to be due to a short-term phenomenon that disappears over the long run. Rather, the UIP violations themselves change nature with the horizon, and as I have shown in the main body of the text, the positive violations roughly offset the negative violations in the long-run which makes it appear as if the UIP held in the long-run when only long-run evidence is considered.

To illustrate the connection between the two empirical methodologies, consider the following predictive regression that forecasts the excess return on long term,  $k$ -month bonds held to maturity

$$\frac{s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})}{k} = \alpha_k + \delta_k(i_t^{(1)} - i_t^{(1),*}) + \varepsilon_{t+k} \quad (\text{A.2})$$

To emphasize the link with my main empirical results, I use the one month interest rate differential as the predictive variable, and for the same reason I have standardized (i.e. divided by  $k$ , the holding period) the left hand side to turn it into the *per month* excess return on the  $k$ -months to maturity bonds. It is worth emphasizing that the predictive

variable I use is not the same as in the previous literature, which has rather used the corresponding long-term interest rate differential  $i_t^{(k)} - i_t^{(k),*}$  as the regressor. Here I consider instead a regression on the short-term, one-month interest rate in order to highlight the link with the main empirical results, which always the one-month interest rate differential as the regressor.

As mentioned earlier, the link between my empirical strategy and the existing studies is particularly straightforward when the expectations hypothesis on the term structure of the interest rate differential across countries holds, at least up to a constant risk-premium, so that

$$i_t^{(k),*} - i_t^{(k)} = \frac{1}{k} \left( \sum_{h=0}^{k-1} E_t(i_{t+h}^* - i_{t+h}) + c_k \right).$$

Intuitively, this means that the yield differential on long-term  $k$ -period bonds is equal to the expected average of future 1-period short-term interest rate differentials, up to a constant risk-premium.<sup>1</sup> In this case, the long holding period excess returns are equal (up to a constant) to the expected sum of the future, one-period excess returns from now until maturity:

$$\begin{aligned} E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})) &= E_t \left( \sum_{h=0}^{k-1} (s_{t+h+1} - s_{t+h} + i_{t+h}^{(1),*} - i_{t+h}^{(1)} + c_k) \right) \\ &= E_t \left( \sum_{h=0}^{k-1} \lambda_{t+h+1} \right) + c_k. \end{aligned}$$

In other words, the  $k$  period excess return on foreign bonds relative to the home bonds is akin to cumulating the  $k$  one period, intermediate excess returns. With this result in mind, it is not surprising that we can re-express the regression coefficient  $\delta_k$  as:

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<sup>1</sup> Bekaert et al. (2007) show that this may not be a bad assumption. They show that in international data the hypothesis is rejected statistically but the actual numerical deviations are quite small and they argue that they are insignificant from an economic point of view.

$$\begin{aligned}
\delta_k &= \frac{1}{k} \frac{\text{Cov}(E_t(s_{t+k} - s_t + k(i_t^{(k),*} - i_t^{(k)})), (i_t^{(1)} - i_t^{(1),*}))}{\text{Var}(i_t^{(1)} - i_t^{(1),*})} \\
&= \frac{1}{k} \frac{\text{Cov}(E_t(\sum_{h=0}^{k-1} \lambda_{t+h+1}) + c_k, (i_t^{(1)} - i_t^{(1),*}))}{\text{Var}(i_t^{(1)} - i_t^{(1),*})} \\
&= \frac{1}{k} \sum_{h=0}^{k-1} \frac{\text{Cov}(\lambda_{t+h+1}, i_t^{(1)} - i_t^{(1),*})}{\text{Var}(i_t^{(1)} - i_t^{(1),*})} \\
&= \sum_{h=1}^k \frac{\beta_h}{k}
\end{aligned}$$

In the above expression  $\beta_h$  are the coefficients obtained in the the main set of regressions, defined in equation (1.7), where we regressed one-month excess returns at different horizons on the current one-month interest rate differential. Thus, the coefficient in a long-run UIP regression that uses long-term,  $k$ -period excess returns as the dependent variable, can be viewed as the average of the  $k$  regression coefficients from the one-period UIP regressions at the intermediate horizons from 1 to  $k$  periods in the future.

This suggests that UIP regressions using long-term bonds and returns tend to average over the high-frequency UIP violations within the full holding period of the long-term bond. Combined with my previous finding that the high-frequency violations change sign at longer horizons, this can help rationalize why previous studies have found strong evidence against the UIP in the short-run but much weaker evidence in the long-run. Moreover, the weaker evidence in long-run studies is not because the UIP condition tends to hold over horizons of five years or more – Figure 1.1 shows that in fact the UIP is violated at horizons of up to 7 years. Instead, my results suggest that long-run UIP test might find weaker evidence against the UIP because the fundamental nature of the underlying, higher-frequency UIP violations change sign at longer horizons. A regression with a sufficiently long-period cumulative return on the left hand side would average over negative and positive violations

that cancel each other out and it may look like UIP holds.

To illustrate the fact that the positive and negative violations roughly cancel out at long horizons, Figure 1.2 plots  $\tilde{\delta}_k = \sum_{h=1}^k \beta_h$ , the cumulative sum of the UIP coefficients  $\beta_k$  against the horizon in months on the X-axis. The solid blue line shows the value of the partial sum up to horizon  $k$  and the shaded area represents the 95% confidence interval. The partial sums are at first decreasing, because the short-horizon violations are all negative, but eventually start rising and at the longest horizons appear to be roughly zero. This shows that the cumulative total of the positive UIP violations is approximately equal to the cumulative total of the negative violations.

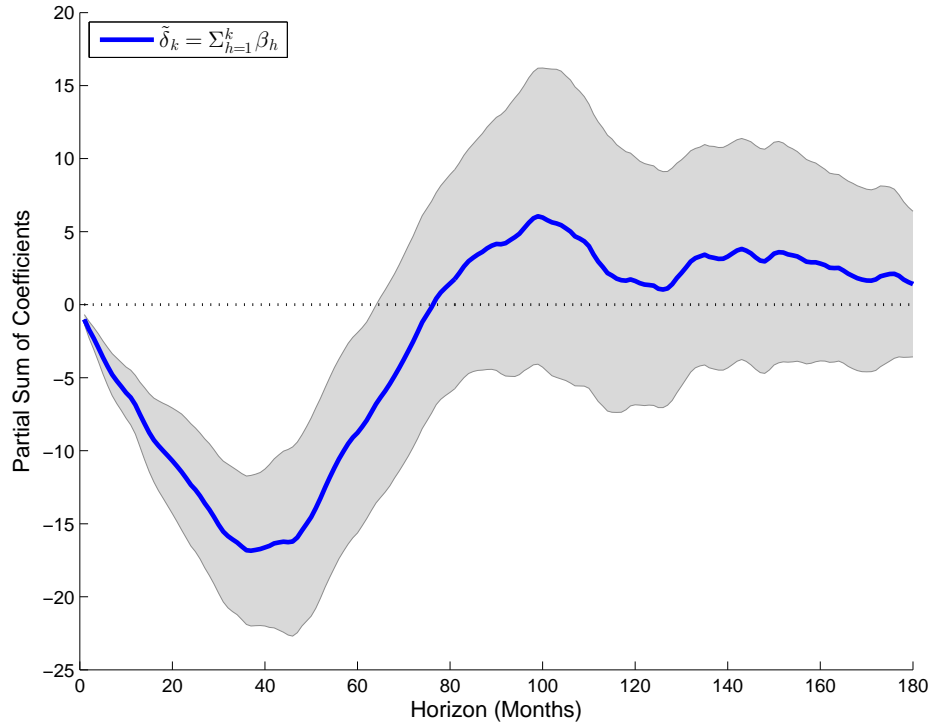


FIGURE A.1: Cumulative Sums of the UIP Coefficients  $\beta_k$

Thus, the results of this paper support the earlier findings that the exchange rate behavior in the long run, on average, is consistent with the UIP hypothesis. But this is not because UIP violations manifest themselves only in the short-run and get washed out in the longer-run. The reason is instead that the high-frequency violations change sign at different horizons, and thus tend to cancel each other over the long-run. Interestingly,

the UIP condition is still violated in both the short and the long run, but nevertheless exchange rate behavior in the long run appears to be consistent with Uncovered Interest Parity.

## A.4 Monetary Independence and UIP Violations in the Model

In this section I will illustrate how monetary independence is related to UIP violations in the (quantitative) model. The relationship is trivial in the analytical model, as keeping debt constant in that model removes any time variation in excess returns. For the quantitative model, I will consider two different experiments, and in both I will only be changing the behavior of the foreign central bank but not the home one. In the first experiment, I will augment the Taylor rule of the bank to include the outstanding stock of government debt and in the second I will consider an Active Fiscal/Passive Monetary policy mix, in which case the fiscal authority does not raise taxes to pay off debt and the central bank is left to induce inflation and keep the government solvent by inflating it away.

For the first experiment, I will keep the assumption that the home central bank follows a standard Taylor rule and is committed to fighting inflation, but will assume that part of the mandate of the foreign central bank is to keep foreign government debt constant. I only change the policy in the foreign country because in the data I take the USD as the benchmark exchange rate and the Federal Reserve's actions have been apparently independent of fiscal considerations for the time period at hand. I implement this in a straightforward way, by augmenting the foreign Taylor rule to respond to the outstanding amount of foreign government debt

$$\hat{i}_t^* = \rho_i \hat{i}_{t-1}^* + (1 - \rho_i) \phi_\pi^* \hat{\pi}_t^* - (1 - \rho_i) \phi_b^* \hat{b}_{f,t}^* + v_t^*,$$

where  $\phi_b^* > 0$ . With this parametrization, when foreign government debt rises, the foreign central bank lowers interest rates which helps reduce the cost of servicing the debt and also puts an upward pressure on inflation, and helps inflate some of the debt away.

Overall, increasing  $\phi_b^*$  decreases the volatility of government debt.

I plot the implied UIP regression coefficients from the augmented version of the model in Figure A.2 for three different values of  $\phi_b$ . As we can see, increasing  $\phi_b$  leads to lower UIP deviations at all horizons, but the effect is particularly visible for the positive long-horizon violations. With  $\phi_b = 0.25$  the longer horizon positive violations virtually disappear, while there are still short-horizon negative violations, although they are small.

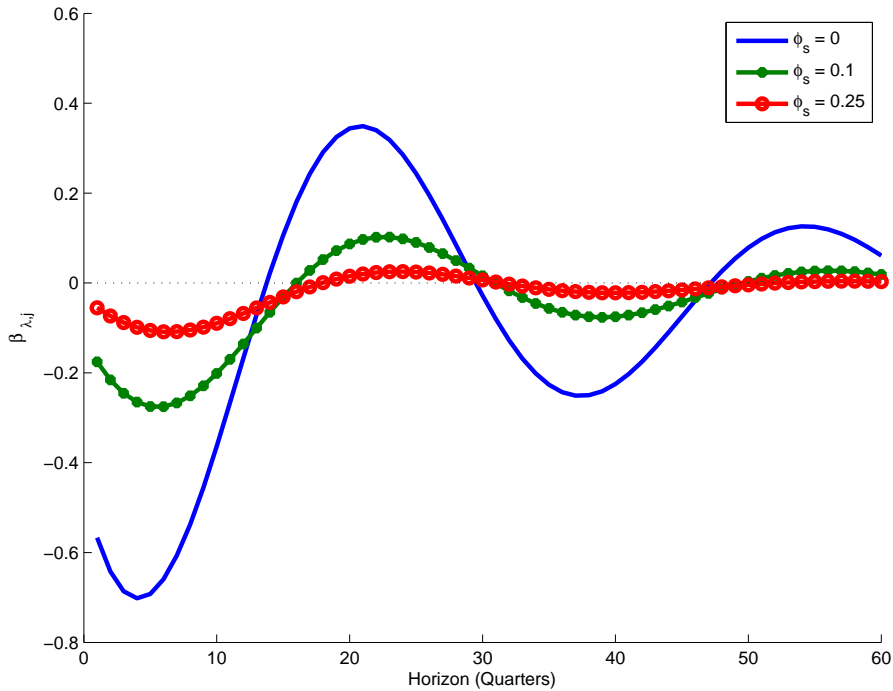


FIGURE A.2: UIP Regression Coefficients, Augmented Taylor Rule

The intuition for the results is that when the monetary authority is actively trying to stabilize debt it achieves two things. First, it makes government debt less sensitive to shocks and second its actions help any new debt to be repaid quicker, which brings debt back to steady state faster and also greatly reduces the cyclicity in its dynamics. Debt is less responsive to shocks, because concurrent with the shocks (not only monetary, but any type of shock, e.g. technology, fiscal etc.) there is a systematic, endogenous response in the monetary policy that counteracts the shock. Moreover, any new debt ends up being repaid much faster, as compared to when the fiscal authority is on its own, because the

endogenous monetary policy is persistently helping the fiscal authority pay off its debt through both reduced interest rates and increased inflation. This second effect is especially important at long horizons because the quicker repayment of debts reduces the cyclical dynamics of government debt, and this explains the complete disappearance of positive UIP violations at longer horizons. Interestingly, it is enough to have only one of the two central banks (home and foreign) targeting government debt in their policy to achieve a significant reduction in the generated UIP violations.

Another exercise I consider is moving to an Active Fiscal and Passive Monetary policy in the foreign country. I use the terms Active and Passive policy as in Leeper (1991), hence an Active Fiscal and Passive Monetary policy mix is one where the fiscal authority does not adjust taxes sufficiently to stay solvent on its own, but instead the monetary authority allows inflation to rise and inflate debt to keep the government solvent. To implement this experiment, I set  $\phi_\pi^* = 0$  and  $\kappa_b = 0$ , the common parametrization that characterizes Active Fiscal/Passive Monetary policy mix in the literature. This parametrization implies that in fact the fiscal authority does not respond to debt at all, and the monetary authority does not respond to inflation but rather adjusts interest so that inflation is allowed to rise and keep the government solvent.

The resulting implied coefficients from this experiment are plotted in A.3 and they show that UIP violations virtually disappear again. The reason is very much the same as before, the actions of the central bank are again stabilizing foreign debt supply, which makes it less responsive to shocks and also helps pay it down faster and removes the cyclical dynamics. This greatly weakens through key mechanisms through which UIP violations are generated: dynamics of government debt and spillover of shocks abroad. As a result, the implied UIP deviations become almost negligible.

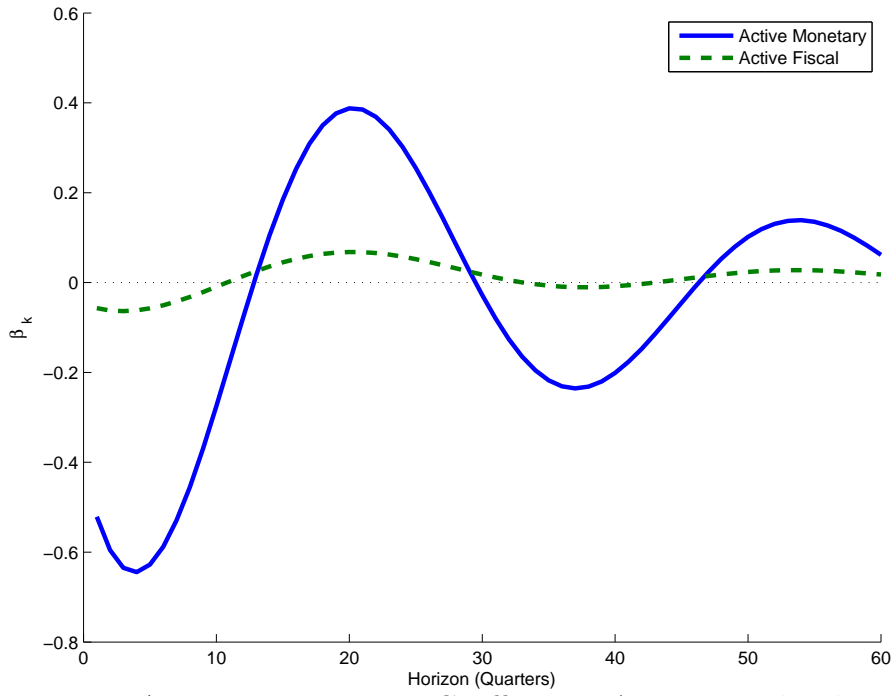


FIGURE A.3: UIP Regression Coefficients, Active Fiscal Policy

## A.5 Extra Figures



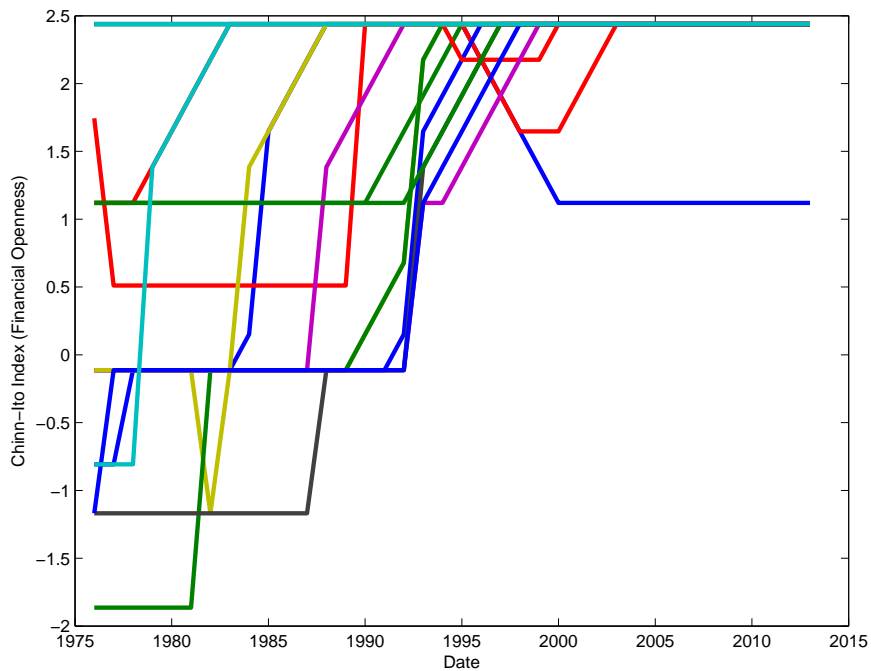


FIGURE A.4: Evolution of Capital Openness

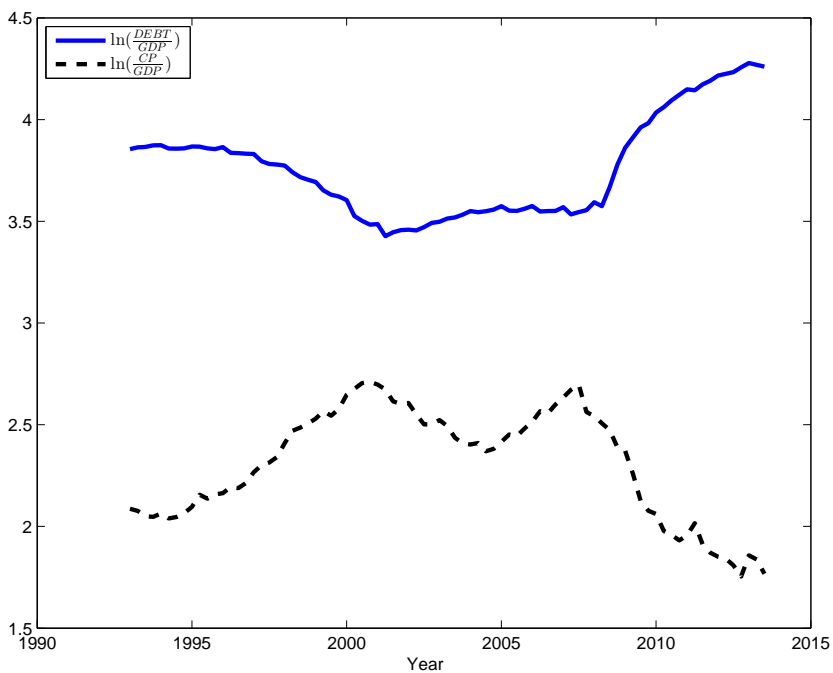


FIGURE A.5: The Evolution of Public and Private Debt

# Appendix B

## Appendix for Chapter 2

### B.1 Additional Model Details

#### *B.1.1 The Model's Key Ingredients*

Before presenting the main results, it is useful to take a closer look at the key ingredients of the model and obtain some intuition about the mechanism which generates the information asymmetry. The basic intuition is that the agent's period 2 consumption is more heavily dependent on  $z_h$  than on  $z_f$ . Thus, home information is more pertinent to the home agent than foreign information, and consequently he focuses his limited attention on it.

To see this more clearly it is useful to consider the following reparameterisation of the problem. Let  $\bar{x}_h = x_h + \delta$ , and add and subtract  $\delta y_h$  from the right hand-side of the second period budget constraint:

$$c_2 = \delta(\varepsilon_w - \varepsilon_h) + \bar{x}_h y_h + x_f y_f + bR$$

Now define  $\bar{A} = A + \delta p_h$  and rewrite the initial wealth constraint in terms of the new variable  $\bar{x}_h$ :

$$\bar{A} = p_h \bar{x}_h + p_f x_f + b$$

This re-parameterization expresses the agent's second period consumption as the sum of a modified portfolio income ( $\bar{x}_h y_h + x_f y_f + b$ ) and a modified non-financial income term ( $\delta(\varepsilon_w - \varepsilon_h)$ ). This modified non-financial income term is in fact the scaled difference between  $w$  and  $y_h$  and hence I will call it the "relative non-financial income".

From the expression above, it is clear that  $\text{Cov}(y_h, \delta(\varepsilon_w - \varepsilon_h)) = \text{Cov}(z_h + \varepsilon_h, \delta(\varepsilon_w - \varepsilon_h)) < 0$ , and hence the home asset correlates negatively with relative non-financial income. Thus, the agent has two incentives to buy the home asset: 1) it offers high expected returns (risk premium) and 2) it can help hedge relative non-financial income risk. On the other hand, the foreign asset has zero correlation with the agent's labor income, and thus the agents only buy it because of the high expected return. Because of the negative correlation between the home asset and the agent's relative non-financial income, standard portfolio choice theory tells us that  $\bar{x}_h > x_f$ . Then we can compute

$$\text{Var}(c_2) = \delta^2(\sigma_{\varepsilon_w}^2 + \sigma_{\varepsilon_h}^2) + \bar{x}_h^2 \text{Var}(y_h) + x_f^2 \text{Var}(y_f) - 2\delta\bar{x}_h\sigma_{\varepsilon_h}^2$$

and note that the volatility of  $c_2$  is affected by the variance of  $y_h$  more so than the variance of  $y_f$  - second period consumption has a higher exposure to risk related to  $y_h$  (and hence  $z_h$ ) than to risk related to  $y_f$  (and hence  $z_f$ ).

We can conclude that there are two reasons for the agent to value the information contained in the signals  $\eta_h$  and  $\eta_f$ . First, high precision signals would help the agent choose a portfolio allocation that maximizes risk-adjusted returns. Second, the signals will resolve some of the uncertainty about period 2 consumption, which is valued by the agent because the utility function exhibits a desire for early resolution of uncertainty. In regards to portfolio selection, the precision of  $\eta_h$  is just as valuable to the agent as the precision on  $\eta_f$ . The CARA investor being studied here chooses portfolio allocations that simply optimize the mean-variance properties of the available assets. The two signals are equally useful in determining the mean-variance frontier and thus have symmetric effects on his expected utility. However,  $\eta_h$  is more effective in reducing the posterior variance of  $c_2$  than

$\eta_f$ , because as we showed above  $c_2$  has a higher exposure to the factor  $z_h$  than to  $z_f$ . Thus, the optimal choice of information acquisition puts a higher precision on  $\eta_h$  than on  $\eta_f$ , because a unit of information about  $z_h$  eliminates more uncertainty about  $c_2$  than a unit of information about  $z_f$ .

Lastly, note that the desire for early resolution of uncertainty is key in generating endogenous information asymmetry in this model but is not necessary in general. All that is needed is for the agent to value information about his future non-financial income. A tractable, though a bit mechanical, way to achieve this is through the introduction of a desire for early resolution of uncertainty. Another way is to use a model where knowledge about future non-financial income informs the agent’s actions today and is thus useful in today’s optimization. For example, this happens in a dynamic model where the agent is also making a savings decision, as savings depend on the expectation of future income. It also arises under standard power utility because the agent then has decreasing absolute risk aversion, and the expectation of future non-financial income factors in the agent’s appetite for risk and thus his optimal portfolio. These setups do not yield themselves to analytical solutions, however, and I have opted for the setup of this paper precisely because it allows for the derivation of analytical results. Nevertheless, numerical solutions of the other setups yield the same qualitative results and I leave the detailed analysis of such extensions to future work.<sup>1</sup>

### *B.1.2 Quantitative Results on Portfolio Allocations*

This section analyzes the quantitative dimensions of the model and asks the question of how much portfolio concentration it can generate under a few reasonable calibrations.

I compare the model against three benchmark models of standard Bayesian agents. I refer to the first one as “Equal Information” because the agents in it receive equally

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<sup>1</sup> Regular CARA utility also yields itself to analytical solutions. In that case, I find that investors value home and foreign information equally and there is no information asymmetry. This result arises because home and foreign information are equally useful in determining the mean-variance frontier, as discussed in the previous paragraph.

informative signals with a total information content of  $\kappa$ . I also consider two models of exogenous information asymmetry. In the first model the agents are assumed to receive a home signal with 25% higher precision than their foreign signal, and in the second the home signal is twice as precise as the foreign signal.

For this numerical exercise, I follow Van Nieuwerburgh and Veldkamp (2009) and set the mean payoff of the assets to 1 and the standard deviation to 15%, and following Mondria (2010) I set the gross return of the riskless bond to 1.02 and the coefficient of absolute risk aversion  $\gamma$  to 2. I choose  $\delta$  so that on average the agents' labor share is 0.68, a standard value in the RBC literature.<sup>2</sup> Since the previous literature has not considered unlearnable uncertainty, there is little guidance for the value of  $\alpha$ . In order to better illustrate the results of the model I will consider two different values:  $\alpha = \frac{2}{3}$  and  $\alpha = \frac{3}{4}$ . I also calibrate two different values of  $p_h = p_f$ , one which implies a relatively small risk premium of 3% and another value which implies a risk premium of 6%.

Given all other parameters, I calibrate  $\kappa$  so that the model implies a set amount of attention is allocated to the foreign factor. In particular, I consider values for  $\kappa$  such that the agent allocates 5%, 15%, 25%, 35% and 45% of available capacity to the foreign factor. This range of values was chosen to cover the whole interval of likely values (0 to 50%).<sup>3,4</sup>

The results in Table B.1 indicate that the model can deliver sizable amounts of portfolio

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<sup>2</sup> The average investor is wealthier than the average person, and hence at first glance this  $\delta$  may appear poorly chosen, but this is not the case. The 2007 Survey of Consumer Finances (SCF) shows that people in the top decile for wealth get only about 50% of their income from wages, but do get about 20% from private business and self-employment, which is another source of non-financial income. Counting both wages and self-employment income as non-financial income, the SCF suggests that non-financial income accounts for about 70 – 80% of the total income for all wealth groups. Hence a non-financial income share of 0.68 is consistent with the empirical evidence.

<sup>3</sup> Weaver et al. (1984) document that major TV networks devote about 25% of their news coverage to foreign events. If that is any indication of the general populace's attention split, the most likely values for attention allocated to the foreign fundamentals is somewhere between 15% and 35%.

<sup>4</sup> Notice that the model is not compared against the exogenous information differences framework discussed in the previous section. This is because the results in the previous section showed such models cannot simultaneously deliver portfolio concentration and the implication that agents acquire some foreign information. Since I only consider  $\kappa > \bar{\kappa}$ , such frameworks would actually imply a foreign bias in information acquisition and portfolio choices further away from home bias than the considered benchmark models.

Table B.1: Holdings of the Home Asset as Percentage of the Total Portfolio Holdings

	$\alpha = \frac{3}{4}$					$\alpha = \frac{2}{3}$				
Risk Premium = 6%										
	% of Att Allocated to $z_f$ :					% of Att Allocated to $z_f$ :				
	5	15	25	35	45	5	15	25	35	45
End Info Asym	71.2	65.5	59.3	52.7	47.9	62.3	57.9	53.5	49.3	47.5
Equal Info	37.4	37.8	38.5	39.9	43.8	33.8	34.7	36.2	38.7	44.2
Ex Home Adv (25%)	41.5	41.9	42.4	43.5	46.3	38.3	39.0	40.2	42.1	46.3
Ex Home Adv (100%)	49.7	49.9	50.1	50.6	51.4	47.0	47.4	48.0	48.9	50.4
Risk Premium = 3%										
	% of Att Allocated to $z_f$ :					% of Att Allocated to $z_f$ :				
	5	15	25	35	45	5	15	25	35	45
End Info Asym	67.8	60.4	51.9	43.4	39.1	53.1	46.9	40.8	36.1	38.5
Equal Info.	18.1	19.1	20.7	23.9	33.1	3.5	6.4	10.8	18.1	33.4
Ex Home Adv (25%)	25.2	26.0	27.3	29.9	37.3	13.2	15.6	19.1	24.9	38.5
Ex Home Adv (100%)	38.5	39.0	39.8	41.3	45.5	30.4	31.8	33.8	37.3	44.5

home bias for a large number of parameters. The results are strongest when the risk premium is high (6%) and the amount of unlearnable uncertainty is low ( $\alpha = \frac{3}{4}$ ). In that case, the model delivers home bias in portfolios even when the attention of the agent is split fairly evenly among home and foreign matters, and the portfolio share of home assets rises above 60% if agents allocate 25% or less of their attention to the foreign fundamental. The results are somewhat weakened when the risk premium decreases to 3% - in this case, home bias arises if agents pay less than 25% of their attention to the foreign fundamental and home assets reach above 60% of the total portfolio once the foreign share of attention dips below 15%. On the other hand, the generated home bias is also smaller if we increase the amount of unlearnable uncertainty as this decreases the incentive for specialization in information acquisition. The model can still generate considerable amounts of home bias but in all cases the values are lower than in the case of small unlearnable uncertainty.

In particular, the model is most challenged under a parameterization where both the risk premium is low and the unlearnable uncertainty is large. In this case, one needs to calibrate  $\kappa$  so that the agent allocates less than 5% of his attention to the foreign assets in order to get a considerable amount of portfolio home bias.

The main takeaway of the table, however, is that the framework performs very well relative to the three benchmark models. It generates significantly higher amounts of home bias under most parameterizations. Perhaps even more importantly, it generates at least some home bias under most parameterizations, while the other models virtually always imply a foreign bias. The foreign bias arises because of the strong hedging incentive to sell the home asset due to its positive correlation with labor income. This paper overcomes this strong force by allowing the agents to optimally choose their information acquisition strategy. In this setting, the labor income risk does not only compel agents to short the home asset for hedging purposes, but also makes home information more valuable, and in turn generates information asymmetry which provides a positive demand for the home asset. The two forces act opposite to each other, but the table demonstrates that the information channel dominates the hedging incentive. On the other hand, the table makes clear that the models of exogenous information asymmetry are not able to overturn the hedging incentives, even though they assume significant information advantages over the home asset.

Lastly, Figure B.1.2 puts these numbers in context by comparing them to the data. It uses the values in Table B.1 to compute and graph the implied Equity Home Bias index (EHB) for the model of this paper and the strong exogenous information advantage model. The EHB index is a commonly used measure of portfolio concentration in the literature on home equity bias. It is defined as,

$$EHB = 1 - \frac{\text{Foreign Equity as a Share of the National Portfolio}}{\text{Foreign Equity as a Share of the World Market Portfolio}}$$

and measures how much a country's equity portfolio deviates from the global market port-

folio.<sup>5</sup> The index is positive when the analyzed portfolio differs from the market portfolio by holding a larger proportion of home assets (i.e. exhibits home bias), negative when it exhibits foreign bias, and is 0 when it is exactly equal to the market portfolio. In addition to the EHB index implied by the values in Table B.1, the figure also plots two horizontal lines which are the average EHB found in international data and the average EHB found in domestic data.<sup>6</sup>

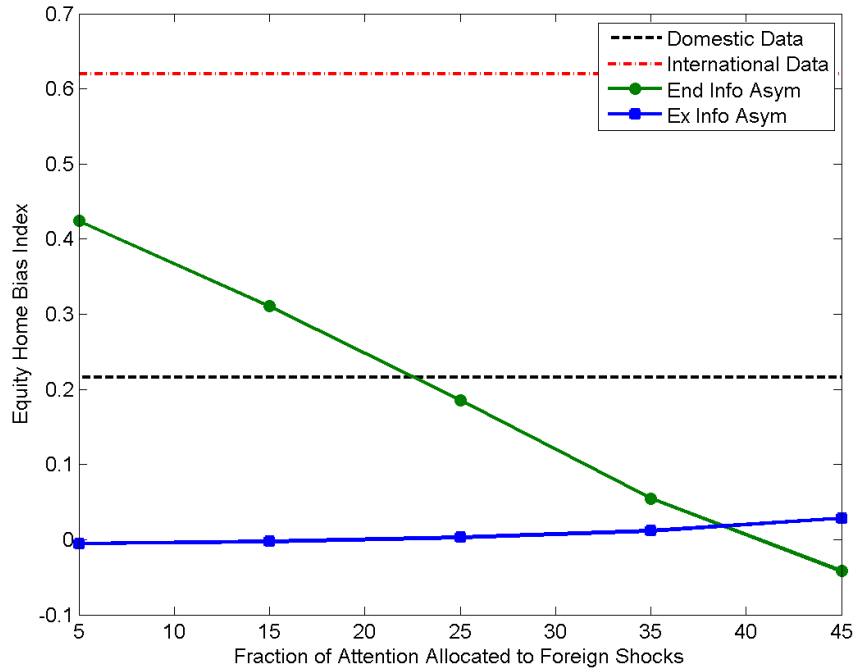


FIGURE B.1: Exogenous vs Endogenous Information Asymmetry

The graph shows that the theoretical framework in this paper can generate home bias in the neighborhood of the estimates found in domestic data, but falls short of the high values estimated from international data. This is most likely due to the fact that inter-

<sup>5</sup> The market portfolio is the equilibrium portfolio in standard CAPM theory (Sharpe (1964)) and is the most commonly used benchmark of the “fully diversified” portfolio.

<sup>6</sup> The sources of the international data are described in Section 6. The domestic EHB is calculated from data presented in Ivković and Weisbenner (2005) by classifying holdings of “local” firms as “home equity assets”, and “non-local” firms as “foreign equity assets”. In the case of domestic data, the index then measures how much is a portfolio biased towards local stocks, compared to the domestic market portfolio.



national financial markets have a lot more frictions than modeled here. The frictionless environment of the paper is likely a much better description of the domestic market and thus it is encouraging to see that the model can match the domestic data easily. In any case, the endogenous information asymmetry model represents a vast improvement over the benchmark model of exogenous information asymmetry.<sup>7</sup>

### *B.1.3 Thinking About Mutual Funds*

An information asymmetry explanation of portfolio concentration rests on the assumption that the individual investors trade only on differing private information sets. However, one could wonder why there is no mutual funds industry, presumably with a higher ability to acquire information than the individual investors, that would offer well diversified investment opportunities to the investors. After all, information asymmetry is more plausible at the household level, rather than at the fund level, because the fund managers have a lot more time and resources to spend on acquiring all relevant information. This is an important criticism on the literature explaining the home bias through information asymmetry as a whole, and in this section I outline a few different reasons for why introducing a mutual funds industry would not invalidate the results of this paper. The key intuition is that as long as the objective function or the quality of the different fund managers are not observable the investors will prefer funds that specialize and ex-ante commit to focusing either on “domestic” or “foreign” securities. Moreover, the investors will then rely on their private information sets to allocate their money between the different specialized funds, and this would again result in biased portfolios.

This result is in line with the data. Coeurdacier and Rey (2013b) show that the great

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<sup>7</sup> A criticism of the literature on information asymmetry and the home bias is that it often abstracts from the existence of investment intermediaries (e.g. mutual funds). At first glance, it seems that mutual funds could be well informed about both home and foreign events and offer the agents a well-diversified investment opportunity. However, introducing mutual funds is unlikely to change the main results of the paper because of issues such as principal-agent problems and uncertain fund quality. In Appendix A3 I discuss some reasons why an extension along these lines would not have major effects on the paper’s results, but a detailed analysis is left to future work.

majority of US Funds either have 90–100% of their assets invested domestically or 0–10%. The distribution of US funds is remarkably bi-modal, with two pronounced mass points at the two ends and almost no funds in between. This suggests that specialization is a defining feature of the US mutual funds industry and the resulting landscape basically leaves investors with a binary choice of a “home” and a “foreign” investment. Coeurdacier and Rey (2013b) also document that there is a considerable amount of heterogeneity in the fund industry across countries, but the general finding that funds tend to cluster around “domestic-only” and “foreign-only” types holds true for most countries. A careful analysis of the structure of the fund industry could perhaps help shed more light on the issue of home bias, but is beyond the scope of this Appendix. Here I will just show that specialization of funds arises naturally because the final decision of what to do with one’s money is the investor’s, and hence his private information set is never irrelevant.

An obvious issue with the introduction of mutual funds into the model is that the investors would also face a principal-agent problem. This arises because the investors cannot be sure that the objective of the fund managers coincides with their own. For example, the fund manager might just be running a Ponzi scheme and be stealing from the invested money. Or less abrasively, the fund manager might just be using a different stochastic discount factor than the investor and thus may choose an inefficient portfolio, from the view point of a single investor, despite his superior information and good intentions. In short, investing in mutual funds adds an additional layer of uncertainty from the view point of the individual investor because the fund manager’s portfolio choice is also viewed as stochastic.

In the extreme case where investors do not trust the managers to make their investment decisions at all, there will only be any demand for index funds with pre-announced portfolio allocations (e.g. a fund that mirrors the S&P 500). In this case, it will be optimal for all funds to fully specialize in “domestic only” and “foreign only”, as this will attract the most customers (I am assuming funds try to maximize the number of their customers). One way to combat this mistrust is for the mutual funds to announce particular investment mandates

in their prospectuses. For example, a fund might decide to invest “at least  $x\%$  of equity in domestic stocks”. A commitment to a particular type of investment strategy like this makes the fund manager’s portfolio choice less uncertain to the investors, and leaves a smaller scope for a principal-agent problem. And in fact mandates like this are quite common in the mutual funds industry (see Coeurdacier and Rey (2013b)) - most funds do have a clearly defined investment “type”.

However, mandates that restrict the fund managers’ portfolio choices also diminishes their ability to use their superior information to earn better returns than the individual investors. The individual investors are still not able to just turn their money over to a better informed professional, because the professional now faces extra constraints which hinder his performance. Thus, the investor’s private information set is not irrelevant. In fact, the agent will use his own beliefs to choose whether it is best to invest in funds specializing in home or foreign securities. Hence, the overall decision of whether to invest at home or abroad is still tied to the individual investor’s information set, which is biased towards home information.

Apart from the principal-agent problem, another issue arises with the introduction of mutual funds if investors are not able to observe the quality of the different managers. In a similar setting of information asymmetry, Dziuda and Mondria (2012) show that investors will prefer funds that specialize in domestic securities, because they will be better able to judge a manager’s quality. So even though the managers themselves are equally well informed about both home and foreign securities, they choose to focus on the domestic market because this attracts more customers. Hence, this is a framework in which the information asymmetry plaguing the individual investors generates home bias even in the presence of sophisticated, well informed professional fund managers.

At the end of the day, although interesting, a detailed analysis of the mutual funds industry and its implications for the home bias is beyond the scope of this paper. This sections only purports to sketch a few considerations which could leave investors reliant on their private information sets and thus holding biased portfolios, even in the presence

of well informed fund managers. And while this section only considered the issue from a theoretical point, it is important to note that in the data mutual funds do adhere to published investment mandates and tend to specialize into broadly defined "domestic" and "foreign" funds. Thus, for one reason or another, the equilibrium we observe in practice does not seem to offer an easily identifiable "optimal" fund to investors. And in such a case individual biases and information frictions are certain to play a role.

## B.2 Proofs

### B.2.1 Proposition 1:

*Proof.* By evaluating the log-normal expectation conditional on time 1 information and taking the log of the resulting expression I arrive at a non-central chi-square random variable. Then, using the formula for the expectation of chi-square variables, the agent's maximization problem becomes:

$$\begin{aligned} \max_{\sigma_h^2, \sigma_f^2} U(\sigma_h^2, \sigma_f^2) = & \gamma\delta\mu + \frac{1}{2} \left[ -2 + \frac{\sigma_z^2 + \sigma_{\varepsilon_h}^2}{\sigma_h^2 + \sigma_{\varepsilon_h}^2} \left( 1 + \frac{(\mu - p_h R)^2}{\sigma_z^2 + \sigma_{\varepsilon_h}^2} \right) + \frac{\sigma_z^2 + \sigma_{\varepsilon_f}^2}{\sigma_f^2 + \sigma_{\varepsilon_f}^2} \left( 1 + \frac{(\mu - p_f R)^2}{\sigma_z^2 + \sigma_{\varepsilon_f}^2} \right) \right] \\ & - \gamma\delta(\mu - p_h R) \frac{\sigma_h^2}{\sigma_h^2 + \sigma_{\varepsilon_h}^2} - \frac{\gamma^2 \delta^2}{2} \sigma_{\varepsilon_w}^2 - \frac{\gamma^2 \delta^2}{2} \frac{\sigma_{\varepsilon_h}^2 \sigma_{\varepsilon_w}^2}{\sigma_h^2 + \sigma_{\varepsilon_h}^2} \end{aligned}$$

s.t.

$$\frac{1}{2}(\ln(\sigma_z^4) - \ln(\sigma_h^2) - \ln(\sigma_f^2)) \leq \kappa$$

$$-\infty \leq \ln(\sigma_h^2) \leq \ln(\sigma_z^2), \quad -\infty \leq \ln(\sigma_f^2) \leq \ln(\sigma_z^2)$$

The objective function is continuous and the domain of maximization is compact hence a maximum always exists. Since all constraints are linear in the log of the posterior variances it is more convenient to find the maximum in terms of the variables  $\ln(\sigma_h^2)$  and

$\ln(\sigma_f^2)$ , rather than the posterior variances  $\sigma_h^2$  and  $\sigma_f^2$  themselves. Thus, to find the maximum reparameterize the problem in terms of  $\ln(\sigma_h^2)$  and  $\ln(\sigma_f^2)$  and setup the Lagrangian:

$$L = U(\exp(\ln(\sigma_h^2)), \exp(\ln(\sigma_f^2))) + \lambda(2\kappa - \ln(\sigma_z^4) + \ln(\sigma_h^2) + \ln(\sigma_f^2)) \\ + \psi_h(\ln(\sigma_z^2) - \ln(\sigma_h^2)) + \psi_f(\ln(\sigma_z^2) - \ln(\sigma_f^2))$$

The Karush-Kuhn-Tucker necessary optimality conditions are:

$$\frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(\ln(\sigma_h^2)), \exp(\ln(\sigma_f^2))) = -\lambda + \psi_h$$

$$\frac{\partial U}{\partial \ln(\sigma_f^2)}(\exp(\ln(\sigma_h^2)), \exp(\ln(\sigma_f^2))) = -\lambda + \psi_f$$

$$(2\kappa - \ln(\sigma_z^4) + \ln(\sigma_h^2) + \ln(\sigma_f^2))\lambda = 0$$

$$\psi_h(\ln(\sigma_z^2) - \ln(\sigma_h^2)) = 0$$

$$\psi_f(\ln(\sigma_z^2) - \ln(\sigma_f^2)) = 0$$

$$\lambda \geq 0, \psi_h \geq 0, \psi_f \geq 0$$

Before we proceed, I compute the first derivatives of the utility function for future reference:

$$\frac{\partial U}{\partial \ln(\sigma_h^2)} = A \frac{\sigma_h^2}{(\sigma_h^2 + \sigma_{\varepsilon_h}^2)^2}$$

$$\frac{\partial U}{\partial \ln(\sigma_f^2)} = B \frac{\sigma_f^2}{(\sigma_f^2 + \sigma_{\varepsilon_f}^2)^2}$$

where

$$A = -\left[\frac{1}{2}((\sigma_z^2 + \sigma_{\varepsilon_h}^2)\left(1 + \frac{(\mu - p_h R)^2}{\sigma_z^2 + \sigma_{\varepsilon_h}^2}\right)) + \gamma\delta(\mu - p_h R)\sigma_{\varepsilon_h}^2 + \frac{\gamma^2\delta^2}{2}\sigma_{\varepsilon_h}^4\right]$$

$$B = -\frac{1}{2}((\sigma_z^2 + \sigma_{\varepsilon_f}^2)\left(1 + \frac{(\mu - p_f R)^2}{\sigma_z^2 + \sigma_{\varepsilon_f}^2}\right))$$

Next, lets analyze the KKT necessary conditions. There are eight cases to consider.

Case I:

$$\lambda = \psi_h = \psi_f = 0$$

This is the case when all constraints are lax and the KKT conditions imply that at the maximum we have

$$\frac{\partial U}{\partial \ln(\sigma_h^2)} = \frac{\partial U}{\partial \ln(\sigma_f^2)} = 0$$

This is only possible if  $\sigma_h^2 = \sigma_f^2 = 0$  which will violate the entropy constraint because  $\ln(\sigma_z^4) - 2\ln(0) = \infty > 2\kappa$ . Hence, this case is infeasible.

Case II:

$$\lambda = 0, \psi_h > 0, \psi_f > 0$$

This is the case where the entropy constraint is lax, but the “no forgetting” constraints  $\ln(\sigma_h^2) \leq \ln(\sigma_z^2)$  and  $\ln(\sigma_f^2) \leq \ln(\sigma_z^2)$  are both binding. Hence, we must have  $\sigma_h^2 = \sigma_f^2 = \sigma_z^2$  and the information capacity is not used at all. However, the derivatives of the utility function  $\frac{\partial U}{\partial \ln(\sigma_h^2)}$  and  $\frac{\partial U}{\partial \ln(\sigma_f^2)}$  are both strictly negative and thus the agent can acquire strictly greater utility by setting  $\sigma_h^2 < \sigma_z^2$  or  $\sigma_f^2 < \sigma_z^2$ . Therefore, this case does not characterize the maximum of the problem either.

Case III:

$$\lambda = 0, \psi_h = 0, \psi_f > 0$$

In this case the “no forgetting” constraint on the foreign fundamental is binding, and the other two are lax. Hence  $\sigma_f^2 = \sigma_z^2$  and because the partial derivative is always strictly negative, it is optimal to exhaust the whole information constraint and set  $\sigma_h^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$ . For now note that this can indeed characterize the maximum and I will derive the specific conditions under which it does turn out to be the global optimum further below.

Case IV:

$$\lambda = 0, \psi_h > 0, \psi_f = 0$$

This is the mirror case, where the “no forgetting” constraint on the home fundamental is binding and hence  $\sigma_h^2 = \sigma_z^2$  and  $\sigma_f^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$ . This allocation always achieves utility which is strictly lower than the utility achieved in Case III, hence it cannot be optimal. The key to this result is that  $A < B$  and hence

$$\frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(\ln(\sigma^2)), x) < \frac{\partial U}{\partial \ln(\sigma_f^2)}(y, \exp(\ln(\sigma^2))), \forall x, y \quad (\text{B.1})$$

Thus, we have

$$\begin{aligned} & U\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right) - U\left(\sigma_z^2, \frac{\sigma_z^2}{\exp(2\kappa)}\right) \\ &= - \int_{\ln(\sigma_z^2) - 2\kappa}^{\ln(\sigma_z^2)} \frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(y), \sigma_z^2) dy + \int_{\ln(\sigma_z^2) - 2\kappa}^{\ln(\sigma_z^2)} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_z^2, \exp(y)) dy \\ &= \int_{\ln(\sigma_z^2) - 2\kappa}^{\ln(\sigma_z^2)} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_z^2, \exp(y)) - \frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(y), \sigma_z^2) dy \\ &> 0 \end{aligned}$$

The first equality follows from the Fundamental Theorem of Calculus and the third line inequality follows from (B.1). Thus, allocating all attention to  $z_h$  is always strictly better

than allocating all attention to  $z_f$  and hence Case III dominates Case IV.

Case V:  $\lambda > 0$ ,  $\psi_h = \psi_f = 0$

In this situation the information capacity constraint binds and the “no forgetting” constraints are lax. It follows that information acquisition decision satisfies

$$\frac{\partial U}{\partial \ln(\sigma_h^2)} = \frac{\partial U}{\partial \ln(\sigma_f^2)} = -\lambda < 0$$

$$\ln(\sigma_z^4) - \ln(\sigma_h^2) - \ln(\sigma_f^2) = \kappa$$

This can also characterize the maximum and the specific conditions will be characterized below. I will refer to this case as the “interior solution”.

Case VI:  $\lambda > 0$ ,  $\psi_h > 0$ ,  $\psi_f > 0$

This is a situation where all constraints are binding and this is impossible. Whenever  $\sigma_h^2 = \sigma_f^2 = \sigma_z^2$  it follows that the information acquisition constraint does not bind, and thus one of the complimentary slackness conditions is not satisfied. This cannot be a maximum.

Case VII:  $\lambda > 0$ ,  $\psi_h = 0$ ,  $\psi_f > 0$

This is the case where the information capacity constraint and the “no forgetting” constraint on  $z_f$  are binding and the other “no forgetting” constraint is lax. This results in  $\sigma_f^2 = \sigma_z^2$  and  $\sigma_h^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$  which are the exact same allocations as in Case III - Case III and Case VII are identical.

Case VIII:  $\lambda > 0$ ,  $\psi_h > 0$ ,  $\psi_f = 0$

This case is identical to Case IV and for the same reason, it achieves strictly lower utility than Cases III and VII, hence it cannot be optimal.

Having analyzed all possible cases, it turns out that there are only two types of information acquisition allocations which could be optimal -  $\{\sigma_h^2, \sigma_f^2\}$  satisfy either:

$$\frac{\partial U}{\partial \ln(\sigma_h^2)} = \frac{\partial U}{\partial \ln(\sigma_f^2)} = -\lambda < 0$$



$$\ln(\sigma_z^4) - \ln(\sigma_h^2) - \ln(\sigma_f^2) = 2\kappa$$

OR

$$\sigma_f^2 = \sigma_z^2, \sigma_h^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$$

The first is an interior solution where the agent acquires information about both  $z_h$  and  $z_f$  and the second is a corner solution where the agent acquires information only about  $z_h$ . Next, I analyze under what conditions we obtain one or the other.

First, I show that an allocation where  $\sigma_h^2 > \sigma_e^2$  and  $\sigma_f^2 < \sigma_z^2$  can never be optimal. In other words, unless the agent has enough capacity to drive  $\sigma_h^2$  at least as low as  $\sigma_e^2$ , the corner solution is optimal and he will never allocate any attention to the foreign fundamental  $z_f$ . The proof of this fact proceeds by contradiction and to this end let us assume that  $\sigma_h^2 > \sigma_e^2$  and  $\sigma_f^2 < \sigma_z^2$  obtains a global maximum of the utility function. This is an interior point (the corner solution where  $\sigma_h^2 = \sigma_e^2$  was already shown to never be optimal, so we only need to worry about possible interior solutions where  $\sigma_h^2 > \sigma_e^2$  and  $\sigma_f^2 < \sigma_z^2$ ), hence the two partial derivatives must be equal to each other, i.e.:

$$\frac{\partial U}{\partial \ln(\sigma_h^2)} = \frac{\partial U}{\partial \ln(\sigma_f^2)}$$

Now, if we have  $\sigma_e^2 < \sigma_h^2$  and  $\sigma_e^2 < \sigma_f^2$  it follows from Proposition 2 (proved below) that both derivatives are increasing in their respective posterior variances  $\sigma_h^2$  and  $\sigma_f^2$  and thus there are increasing returns to information acquisition. Hence if we move  $\bar{\varepsilon}$  amount of attention from  $z_h$  to  $z_f$  so that we reduce  $\sigma_h^2$  down to  $\frac{\sigma_h^2}{\exp(\bar{\varepsilon})} \geq \sigma_e^2$  and simultaneously increase  $\sigma_f^2$  to  $\sigma_f^2 \exp(\bar{\varepsilon}) \leq \sigma_z^2$  (pick  $\bar{\varepsilon}$  small enough), the derivative on the left-hand side monotonically decreases, while the one on the right increases. Thus for all  $\varepsilon \in [0, \bar{\varepsilon}]$ :

$$\frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(\ln(\sigma^2) - \varepsilon), x) < \frac{\partial U}{\partial \ln(\sigma_f^2)}(\exp(y, \ln(\sigma^2) + \varepsilon)), \forall x, y$$

Therefore,

$$\begin{aligned} U\left(\frac{\sigma_h^2}{e^{\bar{\varepsilon}}}, \sigma_f^2 e^{\bar{\varepsilon}}\right) - U(\sigma_h^2, \sigma_f^2) &= \int_{\sigma_f^2}^{\sigma_f^2 e^{\bar{\varepsilon}}} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h^2, x) d \ln(x) - \int_{\frac{\sigma_h^2}{e^{\bar{\varepsilon}}}}^{\sigma_h^2} \frac{\partial U}{\partial \ln(\sigma_h^2)}(x, \sigma_f^2 e^{\bar{\varepsilon}}) d \ln(x) \\ &= \int_{\ln(\sigma_f^2)}^{\ln(\sigma_f^2) + \bar{\varepsilon}} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h^2, \exp(y)) dy - \int_{\ln(\sigma_h^2) - \bar{\varepsilon}}^{\ln(\sigma_h^2)} \frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(y), \sigma_f^2 e^{\bar{\varepsilon}}) dy \\ &= \int_0^{\bar{\varepsilon}} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h^2, \exp(\ln(\sigma_f^2) + \varepsilon)) d\varepsilon - \int_0^{\bar{\varepsilon}} \frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(\ln(\sigma_h^2) - \varepsilon), \sigma_f^2 e^{\bar{\varepsilon}}) d\varepsilon \\ &> 0 \end{aligned}$$

In other words, transferring  $\bar{\varepsilon}$  attention from  $z_f$  to  $z_h$  achieves a higher utility than the original allocation and hence we have reached a contradiction. An allocation such that  $\sigma_e^2 < \sigma_h^2$  and  $\sigma_e^2 < \sigma_f^2 < \sigma_z^2$  cannot be the solution to the maximization problem.

On the other hand, consider the case where the agent chooses an allocation  $\sigma_f^2 \leq \sigma_e^2 < \sigma_h^2$ . I will show that the mirror information acquisition strategy, i.e.  $\{\sigma_f^2, \sigma_h^2\}$  achieves higher utility than  $\{\sigma_h^2, \sigma_f^2\}$  and thus reach another contradiction. Consider the difference in utilities from these two information acquisition choices:

$$\begin{aligned} U(\sigma_h^2, \sigma_f^2) - U(\sigma_f^2, \sigma_h^2) &= \int_{\ln(\sigma_h^2)}^{\ln(\sigma_h^2)} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_f^2, \exp(x)) d(x) - \int_{\ln(\sigma_f^2)}^{\ln(\sigma_h^2)} \frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(x), \sigma_h^2) d(x) \\ &< 0 \end{aligned}$$

The inequality follows for the same reasoning as in Case IV -  $\frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(\ln(\sigma^2)), x) < \frac{\partial U}{\partial \ln(\sigma_f^2)}(y, \exp(\ln(\sigma^2))), \forall x, y$ , and thus we have reached another contradiction. Therefore, we have proven that it is never optimal to set  $\sigma_f^2 < \sigma_z^2$  and  $\sigma_h^2 > \sigma_e^2$ . This means that an interior solution is optimal only if it sets  $\sigma_h^2 \leq \sigma_e^2$ . With this in mind, I will now show that in any interior solution it must be the case that  $\sigma_h^2 < \sigma_f^2$ . If an interior solution implies an allocation where  $\sigma_f^2 > \sigma_e^2$  it is trivial that  $\sigma_h^2 < \sigma_f^2$ , since we just proved that any interior solution must be such that  $\sigma_h^2 \leq \sigma_e^2$ . So let us consider what kind of interior solutions can obtain if  $\sigma_f^2 \leq \sigma_e^2$ . Any interior solution requires

$$A \frac{\sigma_h^2}{(\sigma_h^2 + \sigma_e^2)^2} = B \frac{\sigma_f^2}{(\sigma_f^2 + \sigma_e^2)^2}$$

Remember that  $A < B < 0$ , while the other terms are positive, hence for the equality to hold we need

$$\frac{\sigma_h^2}{(\sigma_h^2 + \sigma_e^2)^2} < \frac{\sigma_f^2}{(\sigma_f^2 + \sigma_e^2)^2}$$

However, the function  $\frac{\sigma^2}{(\sigma^2 + \sigma_e^2)^2}$  is increasing in  $\sigma^2$  for  $\sigma^2 \leq \sigma_e^2$  hence the above inequality can only hold if  $\sigma_h^2 < \sigma_f^2$ . Hence for any interior solution, it must be the case that  $\sigma_h^2 < \sigma_f^2$ . This is also trivially true for the corner solution where all attention is allocated to  $z_h$  and hence this concludes the proof that at any solution, it is always the case that  $\sigma_h^2 < \sigma_f^2$ .

Lastly, we still need to prove that the solution of the maximization problem is unique. The proof amounts to showing that for any combination of parameters there is at most 1 possible interior solution and that this interior solution achieves the exact same utility as the corner solution only in the case where they correspond to the same allocations, and are thus the same solution.

Let us start by proving there is at most only one feasible interior solution. At any interior solution, it must be the case that

$$\frac{\partial U}{\partial \ln(\sigma_h^2)} - \frac{\partial U}{\partial \ln(\sigma_f^2)} = 0$$

Using the expressions for these derivatives, it follows that

$$\begin{aligned} \frac{\partial U}{\partial \ln(\sigma_h^2)} - \frac{\partial U}{\partial \ln(\sigma_f^2)} &= A \frac{\sigma_h^2}{(\sigma_h^2 + \sigma_e^2)^2} - B \frac{\sigma_f^2}{(\sigma_f^2 + \sigma_e^2)^2} \\ &= \frac{A\sigma_h^2(\sigma_f^2 + \sigma_e^2)^2 - B\sigma_f^2(\sigma_h^2 + \sigma_e^2)^2}{(\sigma_h^2 + \sigma_e^2)^2(\sigma_f^2 + \sigma_e^2)^2} \\ &= 0 \end{aligned}$$

Which is true if and only if

$$A\sigma_h^2(\sigma_f^2 + \sigma_e^2)^2 - B\sigma_f^2(\sigma_h^2 + \sigma_e^2)^2 = 0$$

Now use the fact that the information capacity constraint is binding to get that  $\sigma_f^2 = \frac{\sigma_z^4}{\sigma_h^2 \exp(2\kappa)}$ , substitute this expression on the left hand side above, expand and combine all terms to arrive at the following:

$$(-B\sigma_z^4 + \exp(2\kappa)A\sigma_e^4)\sigma_h^4 + 2\sigma_z^2\sigma_e^2(A - B)\sigma_h^2 + \sigma_z^4(A\sigma_z^4 \exp(-2\kappa) - B\sigma_e^4) = 0$$

Notice that the left hand side is a second order polynomial in  $\sigma_h^2$ , and we are interested in its strictly positive roots. We are only looking for positive solutions because  $\sigma_h^2 = 0$  would require infinite information transfer and will violate the entropy constraint - such interior solutions are not feasible. The roots can be determined by standard root finding techniques, but a more intuitive and perhaps a bit less tedious approach is as follows. Let

$P(\sigma_h^2)$  be the quadratic polynomial of interest and notice that <sup>8</sup>

$$P(0) = \sigma_z^4(A\sigma_z^4 \exp(-2\kappa) - B\sigma_e^4) \geq 0, \text{ if } \frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} \leq \frac{B}{A}$$

$$< 0, \text{ if } \frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} > \frac{B}{A}$$

$$P'(0) = 2\sigma_z^4\sigma_e^2(A - B) < 0$$

$$P''(\sigma_h^2) = -B\sigma_z^4 + \exp(2\kappa)\sigma_e^4A \geq 0, \text{ if } \frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} \geq \frac{A}{B}$$

$$< 0, \text{ if } \frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} < \frac{A}{B}$$

First, let's handle the special case where the coefficient on  $\sigma_h^4$  is 0 and this is in fact a linear function. This happens when  $\frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} = \frac{A}{B} > \frac{B}{A}$ , hence  $P(0) < 0$ . And since the function is decreasing there is no  $\sigma_h^2 > 0$  such that  $P(\sigma_h^2) = 0$ . Second, consider  $\frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} = \frac{B}{A} < \frac{A}{B}$ . In this case,  $P(0) = 0$ , the polynomial is concave and decreasing at 0, hence there are no strictly positive solutions.

Now consider  $P''(\sigma_h^2) \neq 0$ . There are a few different cases to consider. First, let  $\frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} > \frac{A}{B} > \frac{B}{A}$ . This implies that  $P(0) < 0$  and the polynomial is convex, therefore there is exactly one  $\sigma_h^2 > 0$  such that  $P(\sigma_h^2) = 0$ . Now assume instead that  $\frac{B}{A} < \frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} < \frac{A}{B}$ . In this case,  $P(0) < 0$ , the polynomial is concave and  $P'(0) < 0$  all of which tell us that there exist no  $\sigma_h^2 > 0$  such that  $P(\sigma_h^2) = 0$ . Lastly, consider  $\frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} < \frac{B}{A}$ . In this

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<sup>8</sup> Of course while the chosen notation only makes it explicit that the polynomial is a function of  $\sigma_h^2$ , it is important to keep in mind that it depends on all other model parameters, e.g.  $\sigma_z^2, \kappa$ , etc., as well.

case  $P(0) \geq 0$  and the polynomial is concave and decreasing at 0, hence there is exactly one  $\sigma_h^2 > 0$  such that  $P(\sigma_h^2) = 0$ .

To summarize, we have the following:

$$\frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} > \frac{A}{B} \quad \Rightarrow 1 \text{ positive solution}$$

$$\frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} \in \left[ \frac{B}{A}, \frac{A}{B} \right] \quad \Rightarrow \text{No positive solutions}$$

$$\frac{\sigma_z^4}{\sigma_e^4 \exp(2\kappa)} < \frac{B}{A} \quad \Rightarrow 1 \text{ positive solution}$$

Thus, for any combination of parameters we can have at most one feasible interior solution. Next, I shall show that the global maximizer is always unique by comparing the feasible interior solution with the corner solution in the following three exhaustive cases.

Case 1: Let  $\frac{\partial U}{\partial \ln(\sigma_h^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right) = \frac{\partial U}{\partial \ln(\sigma_f^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right)$ , or in other words, this is the situation where at the corner the derivative in terms of home information is equal to the derivative in terms of foreign information. Thus, this allocation also corresponds to the only interior solution, hence the corner and the interior solutions are one and the same in this case. The unique optimal solution in this case is the corner solution of allocating all attention to the home fundamental.

Case 2: Let  $\left| \frac{\partial U}{\partial \ln(\sigma_h^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right) \right| > \left| \frac{\partial U}{\partial \ln(\sigma_f^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right) \right|$ . In this case, at the corner, the marginal benefit of an extra unit of home information is strictly higher than the marginal benefit of foreign information. Hence,

$$P\left(\frac{\sigma_z^2}{\exp(2\kappa)}\right) = \frac{\partial U}{\partial \ln(\sigma_h^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right) - \frac{\partial U}{\partial \ln(\sigma_f^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right)$$

$$< 0$$

The polynomial whose roots characterize the interior solution is negative at the corner. Now I will show that the polynomial is also negative at  $\sigma_h^2 = \sigma_e^2$  and thus because the polynomial has at most one zero for  $\sigma_h \geq 0$  there cannot be any interior solutions where  $\sigma_h^2 \leq \sigma_e^2$ . This is enough to conclude that there are no optimal interior solutions, because it was already shown that the corner solution dominates all allocations where  $\sigma_h^2 > \sigma_e^2$ .

$$\begin{aligned} P(\sigma_e^2) &= (-B\sigma_z^4 + \exp(2\kappa)A\sigma_e^4)\sigma_e^4 + 2\sigma_z^4\sigma_e^4(A - B) + \sigma_z^4\left(\frac{A\sigma_z^4}{\exp(2\kappa)} - B\sigma_e^4\right) \\ &= \sigma_e^4\sigma_z^4(-B + A\frac{\sigma_e^4 \exp(2\kappa)}{\sigma_z^4} + 2(A - B) + A\frac{\sigma_z^4}{\exp(2\kappa)\sigma_e^4} - B) \\ &= \sigma_e^4\sigma_z^4(2(A - B) + A(\frac{\sigma_e^4 \exp(2\kappa)}{\sigma_z^4} + \frac{\sigma_z^4}{\exp(2\kappa)\sigma_e^4}) - 2B) \\ &\leq \sigma_e^4\sigma_z^4(2(A - B) + 2A - 2B) \\ &= 4\sigma_e^4\sigma_z^4(A - B) \\ &< 0 \end{aligned}$$

The first inequality follows from the fact that  $y + \frac{1}{y} \geq 2$  when  $y \geq 0$  and the second inequality makes use of the fact that  $A < B < 0$ . Note that this result does not rely on any parameter restrictions - it is always the case that the derivative in respect to home information, evaluated at  $\sigma_h^2 = \sigma_e^2$ , is more negative than the derivative in respect to foreign information. Therefore, since  $P(\sigma_e^2) < 0$ ,  $P(\frac{\sigma_z^2}{\exp(2\kappa)}) < 0$  and there can be at most only

one interior solution, it follows that there is no  $\sigma^2 \in [\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_e^2]$  such that  $P(\sigma^2) = 0$  and hence there is no interior solution that could possibly be truly optimal. Thus, the corner solution is the unique maximum again.

Case 3: Lastly, consider the case when  $|\frac{\partial U}{\partial \ln(\sigma_h^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2)| < |\frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2)|$ . In this situation, at the corner solution, the marginal benefit of home information is smaller than the marginal benefit of foreign information and hence

$$\begin{aligned} P(\frac{\sigma_z^2}{\exp(2\kappa)}) &= \frac{\partial U}{\partial \ln(\sigma_h^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2) - \frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2) \\ &> 0 \end{aligned}$$

Since the derivatives are continuous functions, it follows that there exists a  $\bar{\varepsilon} > 0$  such that for all  $\varepsilon \in [0, \bar{\varepsilon}]$ :

$$\frac{\partial U}{\partial \ln(\sigma_h^2)}(\exp(\ln(\sigma_z^2) - 2\kappa + \varepsilon), x) > \frac{\partial U}{\partial \ln(\sigma_f^2)}(\exp(y, \ln(\sigma_z^2) - \varepsilon)), \forall x, y$$

Therefore it will be beneficial to shift this  $\bar{\varepsilon}$  attention from  $z_h$  to  $z_f$ . Formally, I consider going from  $\{\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\}$  to  $\{\{\frac{\sigma_z^2 \exp(\bar{\varepsilon})}{\exp(2\kappa)}, \frac{\sigma_z^2}{\exp(\bar{\varepsilon})}\}$ :

$$\begin{aligned} U(\frac{\sigma_z^2 e^{\bar{\varepsilon}}}{e^{2\kappa}}, \frac{\sigma_z^2}{e^{\bar{\varepsilon}}}) - U(\frac{\sigma_z^2}{e^{2\kappa}}, \sigma_z^2) &= \int_{\frac{\sigma_z^2}{e^{2\kappa}}}^{\frac{\sigma_z^2 e^{\bar{\varepsilon}}}{e^{2\kappa}}} \frac{\partial U}{\partial \ln(\sigma_h^2)}(x, \frac{\sigma_z^2}{e^{\bar{\varepsilon}}}) d \ln(x) - \int_{\frac{\sigma_z^2}{e^{2\kappa}}}^{\sigma_z^2} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\sigma_z^2}{e^{2\kappa}}, x) d \ln(x) \\ &= \int_0^{\bar{\varepsilon}} \frac{\partial U}{\partial \ln(\sigma_h^2)}(e^{\ln(\sigma_z^2) - 2\kappa + \varepsilon}, \frac{\sigma_z^2}{e^{\bar{\varepsilon}}}) d \varepsilon - \int_0^{\bar{\varepsilon}} \frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\sigma_z^2}{e^{2\kappa}}, e^{\ln(\sigma_z^2) - \varepsilon}) d \varepsilon \\ &> 0 \end{aligned}$$



Therefore, the corner solution is not optimal in this case and hence the unique maximum is achieved by the interior solution. Notice that the interior solution is guaranteed to exist because by the assumption of Case 3,  $P(\frac{\sigma_z^2}{\exp(2\kappa)}) > 0$  and as derived previously  $P(\sigma_e^2) < 0$ . Since the polynomial is continuous, there exists a  $\sigma^2 \in [\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_e^2]$  for which  $P(\sigma^2) = 0$  and the interior solution is  $\sigma_h^2 = \sigma^2$ . Again, this means that the maximum is unique in this case as well.

This concludes the proof of Proposition 1. It was shown that the information acquisition problem has a unique solution such that  $\sigma_h^2 < \sigma_f^2$ , i.e. the agent always acquires more home information than foreign information.  $\square$

### B.2.2 Proposition 2:

*Proof.* The second derivatives of the utility function  $U$  in terms of  $\ln(\sigma_h^2)$  and  $\ln(\sigma_f^2)$  are:

$$\frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2} = B \frac{\sigma_h^2(\sigma_e^2 - \sigma_h^2)}{(\sigma_h^2 + \sigma_e^2)^3}$$

$$\frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2} = A \frac{\sigma_f^2(\sigma_e^2 - \sigma_f^2)}{(\sigma_f^2 + \sigma_e^2)^3}$$

Since  $A < 0, B < 0$  it follows that  $\frac{\partial^2 U}{\partial \ln(\sigma_i^2)^2} \geq 0$  whenever  $\sigma_i^2 \geq \sigma_e^2$ , and  $\frac{\partial^2 U}{\partial \ln(\sigma_i^2)^2} < 0$  otherwise, for  $i \in \{h, f\}$ . Hence utility is convex in both signals' precisions whenever the size of learnable uncertainty,  $\sigma_i^2$ , is greater than the size of non-learnable uncertainty,  $\sigma_e^2$ . Thus, acquiring information about the forecastable fundamentals (both home and foreign) exhibits increasing returns whenever  $\sigma_i^2 \geq \sigma_e^2$  and decreasing returns otherwise.  $\square$

### B.2.3 Proposition 3:

*Proof.* [(i)]

As detailed in the Proof of Proposition 1, the corner allocation where  $\sigma_h^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$  is

the solution to the information acquisition problem whenever  $\frac{\partial U}{\partial \ln(\sigma_h^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2) \leq \frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2)$ . Using the analytical expression for the derivatives, this condition amounts to

$$A \frac{\sigma_z^2}{e^{2\kappa}(\frac{\sigma_z^2}{e^{2\kappa}} + \sigma_e^2)^2} \leq B \frac{\sigma_z^2}{(\sigma_z^2 + \sigma_e^2)^2}$$

In particular, it is true that when  $\kappa = 0$  the above inequality is strict. Moreover, as proved in Proposition 2, the function  $f(x) = \frac{x}{(x+\sigma_e^2)^2}$  is increasing for  $x \geq \sigma_e^2$  and decreasing otherwise. Hence, the above expression can hold with an equality for one and only one value of  $\kappa$ . Call this particular value of  $\kappa$ ,  $\bar{\kappa}$  and notice that it is a function of all parameters of the model and in particular  $\gamma, \delta, \sigma_e^2$  and thus we can write  $\bar{\kappa}(\gamma, \delta, \sigma_e^2)$  to make this dependence explicit. And since the left hand side of the above is always strictly less than the right hand side when  $\kappa < \bar{\kappa}(\gamma, \delta, \sigma_e^2)$  it follows from the proof of Proposition 1 that in this case the optimum is achieved at the corner solution  $\sigma_h^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$ ,  $\sigma_f^2 = \sigma_z^2$ . Thus, for  $\kappa \leq \bar{\kappa}(\gamma, \delta, \sigma_e^2)$ :

$$\begin{aligned} \Lambda &= \frac{1}{2}(\ln(\sigma_f^2) - \ln(\sigma_h^2)) \\ &= \frac{1}{2}(\ln(\sigma_z^2) - \ln(\sigma_z^2) + 2\kappa) \\ &= \kappa \end{aligned}$$

Clearly,  $\Lambda$  is an increasing function of  $\kappa$  for  $\kappa \leq \bar{\kappa}(\gamma, \delta, \sigma_e^2)$ . On the other hand, whenever  $\kappa > \bar{\kappa}(\gamma, \delta, \sigma_e^2)$  we have  $\frac{\partial U}{\partial \ln(\sigma_h^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2) > \frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2)$  and the maximum is described by the unique interior allocation which makes the two partial derivatives equal and exhausts the whole information capacity. Let  $\bar{\kappa} \leq \kappa_1 < \kappa_2$  and

consider the respective optimal allocations  $\{\sigma_h^2, \sigma_f^2\}$  and  $\{\sigma_h'^2, \sigma_f'^2\}$ . I will show that the corresponding information asymmetry is smaller in the allocation corresponding to  $\kappa_2$  as compared to the allocation under  $\kappa_1$ . There are two cases to analyze.

Case 1: Let  $\sigma_f^2 \geq \sigma_e^2$ . Recall that the interior solution  $\sigma_h^2$  is the unique positive root of the following polynomial:

$$(-B\sigma_z^4 + \exp(2\kappa)A\sigma_e^4)\sigma_h^4 + 2\sigma_z^2\sigma_e^2(A - B)\sigma_h^2 + \sigma_z^4(A\sigma_z^4 \exp(-2\kappa) - B\sigma_e^4)$$

Call the polynomial  $P_\kappa(\sigma_h^2)$  where this time the notation makes it explicit that the polynomial depends on the parameter  $\kappa$ . Given that the optimal solution for  $\kappa_1$  is  $\{\sigma_h^2, \sigma_f^2\}$ , it must be the case that  $P_{\kappa_1}(\sigma_h^2) = 0$ . The proof will proceed by showing that  $P_{\kappa_2}(\sigma_h^2) \geq 0$ , which will be enough because from the proof of Proposition 1 we know that  $P_{\kappa_2}(\sigma_e^2) < 0$ . Consider the derivative of the polynomial in terms of  $\kappa$ :

$$\frac{\partial P_{\kappa_1}(\sigma_h^2)}{\partial \kappa} = 2e^{2\kappa}A\sigma_e^4\sigma_h^4 - 2A\sigma_z^8e^{-2\kappa} \quad (\text{B.2})$$

$$= 2e^{2\kappa}A\sigma_e^4\left(\sigma_h^4 - \frac{\sigma_z^8}{e^{2\kappa}\sigma_e^4}\right) \quad (\text{B.3})$$

From the binding entropy constraint, we can get the following relationship:  $\sigma_f^2 = \frac{\sigma_z^4}{e^{2\kappa}\sigma_h^2}$ . And since we have assumed that  $\sigma_f^2 \geq \sigma_e^2$ , it follows that  $\frac{\sigma_z^4}{e^{2\kappa}\sigma_h^2} \geq \sigma_e^2$  and hence  $\frac{\sigma_z^4}{e^{2\kappa}\sigma_e^2} \geq \sigma_h^2$ . Substituting this in equation (B.3) above, we obtain that

$$\frac{\partial P_{\kappa_1}(\sigma_h^2)}{\partial \kappa} = 2e^{2\kappa}A\sigma_e^4\left(\sigma_h^4 - \frac{\sigma_z^8}{e^{2\kappa}\sigma_e^4}\right) \geq 0$$

and thus,  $P_{\kappa_2}(\sigma_h^2) \geq 0$ . From the proof of Proposition 1 we also know that  $P(\sigma_e^2) < 0$  and therefore, the unique  $\sigma_h'^2$  such that  $P_{\kappa_2}(\sigma_h'^2) = 0$  is in the interval  $[\sigma_h^2, \sigma_e^2)$ .

Moreover, since the information acquisition constraint must be binding, it is the case that  $\sigma_f'^2 \leq \sigma_f^2$  (otherwise the constraint will be lax), so we have that

$$\sigma_h^2 \leq \sigma_h'^2$$

$$\sigma_f^2 \geq \sigma_f'^2$$

and only one of the inequalities can hold with an equality. Therefore, we can conclude that

$$\frac{1}{2}(\ln(\sigma_f^2) - \ln(\sigma_h^2)) > \frac{1}{2}(\ln(\sigma_f'^2) - \ln(\sigma_h'^2))$$

which proves that  $\Lambda$  is a decreasing function of  $\kappa$  for  $\kappa \geq \bar{\kappa}$ .

Case 2: Let  $\sigma_f^2 < \sigma_e^2$ . Under this condition, it is immediate to see that

$$\frac{\partial P_{\kappa_1}(\sigma_h^2)}{\partial \kappa} = 2e^{2\kappa} A \sigma_e^4 (\sigma_h^4 - \frac{\sigma_z^8}{e^{2\kappa} \sigma_e^4}) < 0$$

which implies that  $P_{\kappa_2}(\sigma_h^2) < 0$ . On the other hand, from the proof of Proposition 1 we know that there exists a unique interior solution and that  $P_{\kappa_2}(\sigma_e^2) < 0$  as well, which lets us conclude that the solution  $\sigma_h'^2 < \sigma_h^2$ . Having determined that, we can use Proposition 2 to show that it must also be the case that  $\sigma_f'^2 < \sigma_f^2$ , or otherwise the partial derivatives would not be equal, which is a condition of the interior solution. Hence, we have shown that whenever  $\sigma_f^2 < \sigma_e^2$ , increasing  $\kappa$  leads to increasing both home and foreign information (i.e., decreasing  $\sigma_h^2$  and  $\sigma_f^2$  both). This means that we can rewrite  $\{\sigma_h'^2, \sigma_f'^2\}$  as

$$\sigma_h'^2 = \sigma_h^2 e^{-\nu_h}$$

$$\sigma_f^{2\nu_f} = \sigma_f^2 e^{-\nu_f}$$

where  $\nu_h + \nu_f = \kappa_2 - \kappa_1$  and  $\nu_h, \nu_f > 0$ . One can think of the  $\nu$ 's as the extra home and foreign information, respectively, the agent will acquire if we were to increase his capacity from  $\kappa_1$  to  $\kappa_2$ . I will show that  $\nu_h < \nu_f$  by the way of contradiction, so to this end assume the opposite:  $\nu_h \geq \nu_f$ . Since both  $\{\sigma_h^2, \sigma_f^2\}$  and  $\{\sigma_h'^2, \sigma_f'^2\}$  characterize interior solutions, it is the case that

$$\frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma_h^2, \sigma_f^2) = \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h^2, \sigma_f^2)$$

$$\frac{\partial U}{\partial \ln(\sigma_h'^2)}(\sigma_h'^2, \sigma_f'^2) = \frac{\partial U}{\partial \ln(\sigma_f'^2)}(\sigma_h'^2, \sigma_f'^2)$$

Subtract the two lines from one another, move all terms on the left side and rearrange:

$$\begin{aligned} & \left( \frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma_h^2, \sigma_f^2) - \frac{\partial U}{\partial \ln(\sigma_h'^2)}(\sigma_h'^2, \sigma_f'^2) \right) - \left( \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h^2, \sigma_f^2) - \frac{\partial U}{\partial \ln(\sigma_f'^2)}(\sigma_h'^2, \sigma_f'^2) \right) = \\ & \left( \frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma_h^2, \sigma_f^2) - \frac{\partial U}{\partial \ln(\sigma_h'^2)}(\sigma_h'^2, \sigma_f'^2) \right) - \left( \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h^2, \sigma_f^2) - \frac{\partial U}{\partial \ln(\sigma_f'^2)}(\sigma_h'^2, \sigma_f'^2) \right) = \\ & \int_{\sigma_h'^2}^{\sigma_h^2} \frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(x, \sigma_f^2) d \ln(x) - \int_{\sigma_f'^2}^{\sigma_f^2} \frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, x) d \ln(x) = \\ & \int_{\ln(\sigma_h'^2)}^{\ln(\sigma_h^2)} \frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(e^y, \sigma_f^2) dy - \int_{\ln(\sigma_f'^2)}^{\ln(\sigma_f^2)} \frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, e^y) dy = \\ & \int_{\ln(\sigma_h^2) - \nu_h}^{\ln(\sigma_h^2)} \frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(e^y, \sigma_f^2) dy - \int_{\ln(\sigma_f^2) - \nu_f}^{\ln(\sigma_f^2)} \frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, e^y) dy = \\ & \int_0^{\nu_h} \frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(e^{\ln(\sigma_h^2) - \nu}, \sigma_f^2) d\nu - \int_0^{\nu_f} \frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, e^{\ln(\sigma_f^2) - \nu}) d\nu = \\ & \int_0^{\nu_f} \frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(e^{\ln(\sigma_h^2) - \nu}, \sigma_f^2) d\nu + \int_{\nu_f}^{\nu_h} \frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(e^{\ln(\sigma_h^2) - \nu}, \sigma_f^2) d\nu - \int_0^{\nu_f} \frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, e^{\ln(\sigma_f^2) - \nu}) d\nu \end{aligned}$$

The first equality follows from  $\frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma^2, x) = \frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma^2, y)$  for all  $x, y > 0$  as the cross partials are 0.

Computing the second derivative expressions yields:

$$\frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(\sigma_h^2, \sigma_f^2) = A \frac{\sigma_e^2 - \sigma_h^2}{(\sigma_h^2 + \sigma_e^2)^3}$$

$$\frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, \sigma_f^2) = B \frac{\sigma_e^2 - \sigma_f^2}{(\sigma_f^2 + \sigma_e^2)^3}$$

and since  $\sigma_h^2 < \sigma_f^2 < \sigma_e^2$  it follows that

$$\frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(\sigma_h^2, \sigma_f^2) < \frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, \sigma_f^2)$$

Then, we have

$$\int_0^{\nu_f} \frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2}(e^{\ln(\sigma_h^2)-\nu}, \sigma_f^2) d\nu - \int_0^{\nu_f} \frac{\partial^2 U}{\partial \ln(\sigma_f^2)^2}(\sigma_h^2, e^{\ln(\sigma_f^2)-\nu}) d\nu < 0$$

and since  $\frac{\partial^2 U}{\partial \ln(\sigma_h^2)^2} < 0$  it follows that

$$\left( \frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma_h^2, \sigma_f^2) - \frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma_h'^2, \sigma_f'^2) \right) - \left( \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h^2, \sigma_f^2) - \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_h'^2, \sigma_f'^2) \right) < 0$$

which is a contradiction. Thus, we must have  $\nu_f > \nu_h$ , which implies that

$$\begin{aligned}\frac{1}{2}(\ln(\sigma_f'^2) - \ln(\sigma_h'^2)) &= \frac{1}{2}(\ln(\sigma_f^2) - \ln(\sigma_h^2) - (\nu_f - \nu_h)) \\ &< \frac{1}{2}(\ln(\sigma_f^2) - \ln(\sigma_h^2))\end{aligned}$$

and thus information asymmetry,  $\Lambda$ , decreases with  $\kappa$ .

2. This follows directly from the Proof of Proposition 1. For  $\kappa \leq \bar{\kappa}$  the solution is characterized by the corner allocation where  $\sigma_h^2 = \frac{\sigma_z^2}{\exp(2\kappa)}$  and  $\sigma_f^2 = \sigma_z^2$ . In the case of  $\kappa > \bar{\kappa}$  we have an interior solution, and from the Proof of Proposition 1 we know that this only happens when  $\frac{\partial U}{\partial \ln(\sigma_h^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2) > \frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2)$ . This condition implies that  $P(\frac{\sigma_z^2}{\exp(2\kappa)}) > 0$  and hence the solution is such that  $\sigma_h^2 > \frac{\sigma_z^2}{\exp(2\kappa)}$ . Since the information acquisition constraint is always binding, this implies that  $\sigma_f^2 < \sigma_z^2$  and we can conclude that for  $\kappa > \bar{\kappa}$  the agent pays a positive amount of attention to foreign information.
3. First, I will establish that  $\sigma_h^2 \rightarrow 0$ ,  $\sigma_f^2 \rightarrow 0$  as  $\kappa \rightarrow \infty$ . Since the information acquisition constraint always holds, it must be the case that at least one of the two variances goes to zero - we just need to prove that both do. To do so, assume to the contrary that one of the variances converges to a number strictly greater than 0, and without loss of generality assume that it is  $\sigma_f^2 \rightarrow \sigma^2 > 0$ . In order to formalize things, let  $\{\kappa_n\}$  be a sequence of real numbers that diverges to infinity and  $\{\sigma_{h,n}^2, \sigma_{f,n}^2\}$  be the corresponding sequence of optimal information allocations. When  $\kappa_n$  is big, we obtain the interior solution hence

$$A \frac{\sigma_{h,n}^2}{(\sigma_{h,n}^2 + \sigma_e^2)^2} = B \frac{\sigma_{f,n}^2}{(\sigma_{f,n}^2 + \sigma_e^2)^2}$$

As  $n \rightarrow \infty$ , the left hand side converges to 0, while the right hand side converges to

$B \frac{\sigma^2}{\sigma_e^4} < 0$ . Hence there exists an integer  $N$  such that for all  $n \geq N$ ,

$$A \frac{\sigma_{h,n}^2}{(\sigma_{h,n}^2 + \sigma_e^2)^2} > B \frac{\sigma_{f,n}^2}{(\sigma_{f,n}^2 + \sigma_e^2)^2}$$

which means that the sequence  $\{\sigma_{h,n}^2, \sigma_{f,n}^2\}$  does not characterize solutions to the information problem and we have reached a contradiction - it must be the case that both variances converge to 0. Having established this, consider again the fact that the two partial derivatives must be equal:

$$\begin{aligned} A \frac{\sigma_{h,n}^2}{(\sigma_{h,n}^2 + \sigma_e^2)^2} &= B \frac{\sigma_{f,n}^2}{(\sigma_{f,n}^2 + \sigma_e^2)^2} \\ \Rightarrow \frac{A (\sigma_{f,n}^2 + \sigma_e^2)^2}{B (\sigma_{h,n}^2 + \sigma_e^2)^2} &= \frac{\sigma_{f,n}^2}{\sigma_{h,n}^2} \end{aligned}$$

As  $n \rightarrow \infty$  the left hand side converges to  $\frac{A}{B}$  hence we get that

$$\frac{\sigma_{f,n}^2}{\sigma_{h,n}^2} \rightarrow \frac{A}{B}$$

And since natural log is a continuous function we get that

$$\Lambda = \frac{1}{2} (\ln(\sigma_f^2) - \ln(\sigma_h^2)) \rightarrow \frac{1}{2} \ln\left(\frac{A}{B}\right)$$

□

#### B.2.4 Proposition 4:

*Proof.* By the proof of Proposition 3, we know that  $\bar{\kappa}$  is the value of  $\kappa$  such that



$$\frac{\partial U}{\partial \ln(\sigma_h^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right) = \frac{\partial U}{\partial \ln(\sigma_f^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right)$$

which reduces to

$$A \frac{\sigma_z^2}{e^{2\kappa} \left(\frac{\sigma_z^2}{e^{2\kappa}} + \sigma_e^2\right)^2} = B \frac{\sigma_z^2}{\sigma_z^2 + \sigma_e^2} \quad (\text{B.4})$$

First, let's analyze the effect of  $\delta$  on the value of  $\bar{\kappa}$ . The expression for  $A$  and  $B$  are given in the proof of Proposition 1, and we can see from them that only  $A$  is a function of  $\delta$ . In particular, we can show that  $A$  is a decreasing function of  $\delta$ , hence if we have  $\delta_1 < \delta_2$  it follows that  $A(\delta_1) > A(\delta_2)$ . Let  $\bar{\kappa}_1$  and  $\bar{\kappa}_2$  be the values of  $\kappa$  which make (B.4) hold with an equality, when  $A$  is calculated under  $\delta_1$  and  $\delta_2$  respectively. The right hand side will be the same under both  $\delta_1$  and  $\delta_2$ , but since  $A(\delta_1) > A(\delta_2)$ , it follows that

$$B \frac{\sigma_z^2}{(\sigma_z^2 + \sigma_e^2)^2} = A(\delta_1) \frac{\sigma_z^2}{e^{2\bar{\kappa}_1} \left(\frac{\sigma_z^2}{e^{2\bar{\kappa}_1}} + \sigma_e^2\right)^2} > A(\delta_2) \frac{\sigma_z^2}{e^{2\bar{\kappa}_2} \left(\frac{\sigma_z^2}{e^{2\bar{\kappa}_2}} + \sigma_e^2\right)^2}$$

Using Proposition 2, it is straightforward to show that  $A \frac{\sigma^2}{(\sigma^2 + \sigma_e^2)^2} = B \frac{\sigma_z^2}{(\sigma_z^2 + \sigma_e^2)^2}$  if and only if  $\sigma^2 < \sigma_e^2$ . Moreover, when  $\sigma^2 < \sigma_e^2$ , the left hand side of the equality is decreasing in  $\sigma^2$ . Hence, if  $x, y$  are such that

$$A(\delta_1) \frac{x}{(x + \sigma_e^2)^2} = A(\delta_2) \frac{y}{(y + \sigma_e^2)^2}, x < \sigma_e^2, y < \sigma_e^2$$

it must be the case that  $y < x$ . Therefore, since

$$B \frac{\sigma_z^2}{(\sigma_z^2 + \sigma_e^2)^2} = A(\delta_1) \frac{\sigma_z^2}{e^{2\bar{\kappa}_1} \left(\frac{\sigma_z^2}{e^{2\bar{\kappa}_1}} + \sigma_e^2\right)^2}$$

and

$$B \frac{\sigma_z^2}{(\sigma_z^2 + \sigma_e^2)^2} = A(\delta_2) \frac{\sigma_z^2}{e^{2\bar{\kappa}_2} (\frac{\sigma_z^2}{e^{2\bar{\kappa}_2}} + \sigma_e^2)^2}$$

it must be the case that  $\frac{\sigma_z^2}{e^{2\bar{\kappa}_1}} > \frac{\sigma_z^2}{e^{2\bar{\kappa}_2}}$  which means that  $\bar{\kappa}_1 < \bar{\kappa}_2$  and thus we conclude that  $\bar{\kappa}(\delta, \gamma, \alpha)$  is increasing in  $\delta$ .

Next I analyze the relationship between  $\bar{\kappa}$  and  $\gamma$ .  $A$  is a decreasing function of  $\gamma$  as well and  $B$  is still not affected by changes in  $\gamma$ , hence using the exact same arguments as above, we can conclude that  $\bar{\kappa}(\gamma, \delta, \alpha)$  is increasing in  $\gamma$ .

Lastly, let us consider how changes in  $\alpha$  affect the value of  $\bar{\kappa}$ . By definition  $\alpha$  is the fraction of the total variance of the financial assets which is forecastable, i.e.  $\sigma_z^2 = \alpha\sigma_y^2$ . First, notice that  $\bar{\kappa}$  exists only when  $\alpha > \frac{1}{2}$ , as otherwise the information acquisition problem is strictly concave, the solution is always interior and the agent always acquires both home and foreign information. A situation in which the agent acquires only home information and ignores foreign information for  $\kappa \leq \bar{\kappa}$  can only occur if  $\alpha > \frac{1}{2}$  because this would imply a measure of (eventually disappearing) increasing returns to information acquisition.

Next, rewrite ( B.4) in terms of  $\alpha$  by using the expressions  $\sigma_z^2 = \alpha\sigma_y^2$  and  $\sigma_e^2 = (1-\alpha)\sigma_y^2$ :

$$\frac{A}{(\frac{\alpha}{e^\kappa} + (1-\alpha)e^\kappa)^2} = B$$

Expanding the expression in parentheses and combining terms I get:

$$(1-\alpha)e^{2\kappa} - (\frac{A}{B})^{\frac{1}{2}}e^\kappa + \alpha = 0$$

The left-hand side of the above expression is a second order polynomial in  $\exp(\kappa)$  which roots are given by

$$e^\kappa = \frac{\left(\frac{A}{B}\right)^{\frac{1}{2}} \pm \left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}}{2(1-\alpha)}$$

Since  $A < B$ , we know that  $\frac{\partial U}{\partial \ln(\sigma_h^2)}(\sigma_z^2, \sigma_z^2) < \frac{\partial U}{\partial \ln(\sigma_f^2)}(\sigma_z^2, \sigma_z^2)$ . Moreover, by Proposition 2  $\frac{\partial U}{\partial \ln(\sigma_h^2)}\left(\frac{\sigma_z^2}{\exp(2\kappa)}, \sigma_z^2\right)$  is at first a decreasing and then an increasing function of  $\kappa \geq 0$ . Therefore, there can be only one  $\kappa \geq 0$  such that (B.4) is satisfied. With this in mind, it is immediate that the appropriate root of the above polynomial is then

$$e^\kappa = \frac{\left(\frac{A}{B}\right)^{\frac{1}{2}} + \left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}}{2(1-\alpha)}$$

This is the case because the other root is strictly lower than this one, hence if the other one was to be obtained at  $\kappa \geq 0$  then there would be two positive solutions to equation (B.4) which will be a contradiction. Thus the above equation gives an implicit function for  $\bar{\kappa}$ , and its partial derivative in respect to  $\alpha$  will help determine the relationship between  $\alpha$  and  $\bar{\kappa}$ :

$$\frac{\partial e^\kappa}{\partial \alpha} = \frac{\left(\frac{\frac{\partial |A|}{\partial \alpha}}{(|AB|^{\frac{1}{2}})^2} + \frac{\frac{\partial |A|}{\partial \alpha} - 4(1-\alpha) + 4\alpha}{2\left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}}\right)2(1-\alpha) + \left(\left(\frac{A}{B}\right)^{\frac{1}{2}} + \left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}\right)2}{4(1-\alpha)^2}$$

The sign of the above is determined by the sign of its numerator:

$$\begin{aligned}
& \left( \frac{\frac{\partial |A|}{\partial \alpha}}{(|AB|^{\frac{1}{2}})2} + \frac{\frac{\frac{\partial |A|}{\partial \alpha}}{|B|} - 4(1-\alpha) + 4\alpha}{2(\frac{A}{B} - 4\alpha(1-\alpha))^{\frac{1}{2}}} \right) 2(1-\alpha) + 2\left(\left(\frac{A}{B}\right)^{\frac{1}{2}} + \left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}\right) = \\
& \frac{A-B}{(|AB|^{\frac{1}{2}})} + \frac{\frac{A-B}{|B|} - 4(1-2\alpha)(1-\alpha)}{\left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}} + 2\left(\frac{A}{B}\right)^{\frac{1}{2}} + 2\frac{\frac{A}{B} - 4\alpha(1-\alpha)}{\left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}} > \\
& \left(-\left(\frac{A}{B}\right)^{\frac{1}{2}} + 2\left(\frac{A}{B}\right)^{\frac{1}{2}}\right) + \frac{2\frac{A}{B} - \frac{A}{B} + 1 - 4 + 12\alpha - 8\alpha^2 - 8\alpha + 8\alpha^2}{\left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}} > \\
& \frac{2(2\alpha - 1)}{\left(\frac{A}{B} - 4\alpha(1-\alpha)\right)^{\frac{1}{2}}} >
\end{aligned}$$

0

The first equality follows from

$$\begin{aligned}
\frac{\partial |A|}{\alpha}(1-\alpha) &= -\gamma(1-\delta)(\mu - pR)\sigma_\varepsilon^2 - \gamma^2(1-\delta)^2\frac{\sigma_\varepsilon^4}{2} \\
&= A - B
\end{aligned}$$

the first inequality follows from the fact that  $-B > 0$ , the second from  $\frac{A}{B} > 1$  and the last inequality is due to  $\alpha > \frac{1}{2}$ . These derivations let us conclude that

$$\frac{\partial e^{\bar{\kappa}}}{\partial \alpha} > 0$$

which implies  $\bar{\kappa}$  is increasing in  $\alpha$ .

Lastly, lets analyze how  $\Lambda$  behaves given changes in  $\gamma$  or  $\delta$  - imagine again we move from  $\gamma_1$  to  $\gamma_2$  or from  $\delta_1$  to  $\delta_2$ , where  $\gamma_1 < \gamma_2$  and  $\delta_1 < \delta_2$ . Let  $\bar{\kappa}_1$  correspond to  $\gamma_1$  (or  $\delta_1$ ) and  $\bar{\kappa}_2$  correspond to  $\gamma_2$  (or  $\delta_2$ ). Then, if  $\kappa < \bar{\kappa}_1$ ,  $\Lambda = \kappa$  and there is no change when moving from  $\gamma_1$  to  $\gamma_2$  (or from  $\delta_1$  to  $\delta_2$ ). When  $\bar{\kappa}_1 \leq \kappa \leq \bar{\kappa}_2$ , then clearly  $\Lambda$  increases as

under the new parameter values  $\Lambda = \kappa$ , but  $\Lambda < \kappa$  under  $\gamma_1$  or  $\delta_1$ . Lastly, consider  $\kappa > \bar{\kappa}_2$ . In this case, the solution must be the unique positive root of the polynomial  $P(\sigma_h^2)$  defined in the Proof of Proposition 1. Notice that

$$\frac{\partial P(\sigma_h^2)}{\partial \gamma} = \frac{\partial A}{\partial \gamma} (e^{2\kappa} \sigma_\varepsilon^4 \sigma_h^4 + 2\sigma_z^2 \sigma_\varepsilon^2 \sigma_h^2 + \sigma_z^8 e^{-2\kappa}) < 0$$

and

$$\frac{\partial P(\sigma_h^2)}{\partial \delta} = \frac{\partial A}{\partial \delta} (e^{2\kappa} \sigma_\varepsilon^4 \sigma_h^4 + 2\sigma_z^2 \sigma_\varepsilon^2 \sigma_h^2 + \sigma_z^8 e^{-2\kappa}) < 0$$

where the inequalities follow from  $\frac{\partial A}{\partial \gamma} < 0$  and  $\frac{\partial A}{\partial \delta} < 0$ . The above two results, combined with  $P(\sigma_\varepsilon^2) < 0$  and the fact that the unique positive solution must be strictly less than  $\sigma_\varepsilon^2$  imply that the solution  $\sigma_{h2}^2$ , under  $\gamma_2$  (or respectively  $\delta_2$ ), must be strictly lower than  $\sigma_h^2$ , the solution under  $\gamma_1$  (or respectively  $\delta_1$ ). Therefore, we conclude that  $\Lambda$  is increasing in  $\gamma$  and  $\delta$  for any given  $\kappa$ .  $\square$

### B.2.5 Proposition 5

*Proof.* First, I will address the case  $\sigma_e^2 = 0$ . Under this condition, the objective function becomes:

$$U(\sigma_h^2, \sigma_f^2) = \frac{1}{2} \left( -2 + \frac{\sigma_z^2}{\sigma_h^2} \left( 1 + \frac{(\mu - pR)^2}{\sigma_z^2} \right) + \frac{\sigma_z^2}{\sigma_f^2} \left( 1 + \frac{(\mu - pR)^2}{\sigma_z^2} \right) \right)$$

and the information capacity constraint is:

$$\frac{1}{2} (\ln(\text{Var}(z_h|s)) - \ln(\sigma_h^2) + \ln(\sigma_z^2) - \ln(\sigma_f^2)) \leq \kappa$$

All of the notation is the same as the above, except for  $\sigma_h^2$  which is now defined as  $\sigma_h^2 = \text{Var}(z_h|s, \eta_h)$  - the posterior variance of  $z_h$  after observing both the exogenous, free

signal  $s$  and the endogenously chosen signal  $\eta_h$ . The change in the information capacity constraint comes from the assumption that the agents receive the exogenous home signal  $s$  for free. All uncertainty reduction delivered by  $s$  does not count against the information capacity that constrains the choice of the endogenous signals  $\eta_h$  and  $\eta_f$ . The agent is restrained in the amount of uncertainty he can reduce, over and above the uncertainty reduction delivered by the exogenous home signal  $s$ .

Notice that the constraint can also be rewritten as:

$$\frac{\text{Var}(z_h|s)\sigma_z^2}{\sigma_h^2\sigma_f^2} \leq \exp(2\kappa)$$

Hence the agent's problem is to maximize a sum, under a product constraint and the no forgetting constraints. The solution to this type of problem (as also derived by Van Nieuwerburgh and Veldkamp (2010)) is the corner solution where the agent spends his whole constraint on the term with the highest linear weight. A quick inspection of the objective function shows that the linear weight on the home signal term ( $\frac{\text{Var}(z_h|s)}{\sigma_h^2}$ ) is

$$\frac{\sigma_z^2}{\text{Var}(z_h|s)} \left(1 + \frac{(\mu - pR)^2}{\sigma_z^2}\right)$$

and the linear weight on the foreign signal ( $\frac{\sigma_z^2}{\sigma_f^2}$ ) is:

$$\left(1 + \frac{(\mu - pR)^2}{\sigma_z^2}\right)$$

Because of the exogenous information advantage over the home asset, we have that  $\frac{\sigma_z^2}{\text{Var}(z_h|s)} > 1$  and since the home linear weight is bigger. The unique solution of this problem is the corner solution where the agent allocates all of his information capacity to the home asset and  $\Lambda = \kappa$ . This the same result as in Van Nieuwerburgh and Veldkamp (2010).

On the other hand, if  $\sigma_\varepsilon^2 > 0$  the agent solves the following problem:

$$\max_{\sigma_h^2, \sigma_f^2} U(\sigma_h^2, \sigma_f^2) = \frac{1}{2} \left( -2 + \frac{\sigma_z^2 + \sigma_e^2}{\sigma_h^2 + \sigma_e^2} \left( 1 + \frac{(\mu - pR)^2}{\sigma_z^2 + \sigma_e^2} \right) + \frac{\sigma_z^2 + \sigma_e^2}{\sigma_f^2 + \sigma_e^2} \left( 1 + \frac{(\mu - pR)^2}{\sigma_z^2 + \sigma_e^2} \right) \right)$$

s.t.

$$\frac{1}{2} (\ln(\text{Var}(z_h|s)) - \ln(\sigma_h^2) + \ln(\sigma_z^2) - \ln(\sigma_f^2)) \leq \kappa$$

$$-\infty \leq \ln(\sigma_h^2) \leq \ln(\sigma_z^2), \quad -\infty \leq \ln(\sigma_f^2) \leq \ln(\sigma_z^2)$$

This problem does not share the same convexity properties as in the case of  $\sigma_e^2 = 0$  and the corner solution is not always optimal. In particular, I will show that for  $\kappa < \bar{\kappa}$  the problem obtains the corner solution, for  $\kappa = \bar{\kappa}$  the agent is indifferent between any attention allocation (multiplicity of solutions), and for  $\kappa > \bar{\kappa}$  the interior solution is the unique solution to the problem. Where the  $\bar{\kappa}$  is defined as

$$\bar{\kappa} = \frac{1}{2} (\ln(\text{Var}(z_h|s)) - \ln(\sigma_e^2) + \ln(\sigma_z^2) - \ln(\sigma_e^2))$$

Intuitively, this is just the information capacity which allows the agent to set both the home and foreign asset posterior variances equal to  $\sigma_e^2$  (the size of unlearnable uncertainty). To prove the statements above, start by analyzing the Karush-Kuhn-Tucker conditions, which imply that there are three possible optimal allocations - an interior solution at which the partial derivatives are equal, and two corner solutions - one where the agent allocates all of the capacity to the home signal, and the opposite one where all of the capacity is allocated to the foreign signal. Consider first  $\kappa < \bar{\kappa}$  and note that for such  $\kappa$  it is impossible to set both  $\sigma_h^2 < \sigma_e^2$  and  $\sigma_f^2 < \sigma_e^2$ . On the other hand, an interior solution must satisfy

$$\frac{\sigma_h^2}{(\sigma_h^2 + \sigma_e^2)^2} = \frac{\sigma_f^2}{(\sigma_f^2 + \sigma_e^2)^2}$$

and by the proof of Proposition 2 both the LHS and the RHS are decreasing whenever  $\sigma_h^2 \geq \sigma_e^2$  or respectively,  $\sigma_f^2 \geq \sigma_e^2$ . Without loss of generality, assume that  $\sigma_h^2 \geq \sigma_e^2$ . Then, it is straightforward to show (using the same arguments as in the Proof of Proposition 1) that a small deviation from any interior point to  $\{\frac{\sigma_h^2}{e^\varepsilon}, e^\varepsilon \sigma_f^2\}$  would achieve a strictly higher utility. Intuitively, this is because in this situation the marginal benefit of increasing the precision of any of the signals is increasing, and with a convex function like this the interior solution cannot be optimal. Thus, the only thing we need to determine is which of the two corner solutions achieves higher utility:

$$\begin{aligned} U\left(\frac{\text{Var}(z_h|s)}{e^{2\kappa}}, \sigma_z^2\right) - U\left(\sigma_z^2, \frac{\sigma_z^2}{e^{2\kappa}}\right) &= (\sigma_z^2 + \sigma_e^2)\left(1 + \frac{(\mu - pR)^2}{\sigma_z^2 + \sigma_e^2}\right)\left(\frac{1}{\frac{\text{Var}(z_h|s)}{e^{2\kappa}} + \sigma_e^2} + \frac{1}{\sigma_z^2 + \sigma_e^2} - \frac{1}{\text{Var}(z_h|s) + \sigma_e^2} - \frac{1}{\frac{\sigma_z^2}{e^{2\kappa}} + \sigma_e^2}\right) \\ &= C(\sigma_z^2 - \text{Var}(z_h|s))\left(\frac{e^{2\kappa}}{(\text{Var}(z_h|s) + e^{2\kappa}\sigma_e^2)(\sigma_z^2 + e^{2\kappa}\sigma_e^2)} - \frac{1}{(\sigma_z^2 + \sigma_e^2)(\text{Var}(z_h|s) + \sigma_e^2)}\right) \end{aligned}$$

where  $C = (\sigma_z^2 + \sigma_e^2)\left(1 + \frac{(\mu - pR)^2}{\sigma_z^2 + \sigma_e^2}\right)$ . To determine the sign of the last term in parentheses, notice that

$$\begin{aligned} e^{2\kappa}(\sigma_z^2 + \sigma_e^2)(\text{Var}(z_h|s) + \sigma_e^2) - (\text{Var}(z_h|s) + e^{2\kappa}\sigma_e^2)(\sigma_z^2 + e^{2\kappa}\sigma_e^2) &= (e^{2\kappa} - 1)(\text{Var}(z_h|s)\sigma_z^2 - e^{2\kappa}\sigma_e^4) \\ &> (e^{2\kappa} - 1)(\text{Var}(z_h|s)\sigma_z^2 - e^{2\kappa}\sigma_e^4) \\ &= 0 \end{aligned}$$



where the inequality follows from  $\kappa < \bar{\kappa}$ , and the last equality from  $e^{2\bar{\kappa}} = \frac{\text{Var}(z_h|s)\sigma_z^2}{\sigma_e^4}$ .

The last result lets us conclude that  $U(\frac{\text{Var}(z_h|s)}{e^{2\kappa}}, \sigma_z^2) - U(\sigma_z^2, \frac{\sigma_z^2}{e^{2\kappa}}) > 0$  and hence the corner solution where the agent allocates all of his information capacity to learning about the home asset is optimal whenever  $\kappa < \bar{\kappa}$ . Consequently, for  $\kappa < \bar{\kappa}$  we have  $\Lambda = \kappa$ .

Now consider  $\kappa = \bar{\kappa}$ . First, using the argument of the previous paragraph we see that the two mirror corner solutions achieve the same level of utility. On the other hand, using the fact that the information constraint is always binding ( $\sigma_f^2 = \frac{\text{Var}(z_h|s)\sigma_z^2}{\sigma_h^2 e^{2\kappa}}$ ) we can derive that any interior solution is a positive root of the following quadratic equation (derivations are same as in the Proof of Proposition 1):

$$P(\sigma_h^2) = \sigma_h^4(-\text{Var}(z_h|s)\sigma_z^2 + e^{2\kappa}\sigma_e^4) + (\frac{\text{Var}(z_h|s)^2\sigma_z^4}{e^{2\kappa}} - \text{Var}(z_h|s)\sigma_z^2\sigma_e^4)$$

But at  $\kappa = \bar{\kappa}$  the RHS of the above reduces to 0, hence *any*  $\sigma_h^2$  is an interior solution. This, plus the fact that the partial derivatives are equal at all interior solutions is enough to conclude that the agent is indifferent between any information allocation.

Lastly, consider  $\kappa > \bar{\kappa}$ . First, note that  $\frac{\frac{\sigma^2}{e^{2\kappa}}}{(\frac{\sigma^2}{e^{2\kappa}} + \sigma_e^2)^2}$  is increasing in  $\frac{\sigma^2}{e^{2\kappa}}$  and thus decreasing in  $\kappa$ . Therefore, it is straightforward to show that  $|\frac{\partial U}{\partial \ln(\sigma_h^2)}(\frac{\text{Var}(z_h|s)}{e^{2\kappa}}, \sigma_z^2)| < |\frac{\partial U}{\partial \ln(\sigma_f^2)}(\frac{\text{Var}(z_h|s)}{e^{2\kappa}}, \sigma_z^2)|$  and that  $|\frac{\partial U}{\partial \ln(\sigma_h^2)}(\text{Var}(z_h|s), \frac{\sigma_z^2}{e^{2\kappa}})| > |\frac{\partial U}{\partial \ln(\sigma_f^2)}(\text{Var}(z_h|s), \frac{\sigma_z^2}{e^{2\kappa}})|$ , which intuitively means that at each corner solution the marginal benefit of acquiring information for the *other* fundamental is bigger than the marginal benefit of pushing the corner even further out. Then, by a similar argument as in the Proof of Proposition 1, it follows that the corner solutions are dominated by the interior solution.

Moreover, the interior solution is unique, again by an argument in the same spirit as the ones presented in the Proof of Proposition 1, and it is given by the condition  $\sigma_h^2 = \sigma_f^2$ .

Thus, we have  $\frac{\sigma_h^2}{\sigma_f^2} = 1$ , and

$$\begin{aligned}\Lambda &= \frac{1}{2}[(\ln(\text{Var}(z_h|s)) - \ln(\sigma_h^2)) - (\ln(\sigma_z^2) - \ln(\sigma_f^2))] \\ &= \frac{1}{2}(\ln(\text{Var}(z_h|s)) - \ln(\sigma_z^2)) < 0\end{aligned}$$

And finally, consider how  $\bar{\kappa} = \frac{\text{Var}(z_h|s)\sigma_z^2}{\sigma_e^4}$  varies with  $\sigma_u^2$  (the variance of the error in the exogenous signal  $s$ ) and  $\alpha$ , where  $\alpha$  is defined as above -  $\sigma_z^2 = \alpha\sigma_y^2$ . Using the standard expression for posterior variance  $\text{Var}(z_h|s) = \frac{\sigma_z^2\sigma_u^2}{\sigma_u^2 + \sigma_z^2}$  I obtain:

$$\frac{\partial \bar{\kappa}}{\partial \sigma_u^2} = \frac{\sigma_z^6 \sigma_e^4}{((\sigma_u^2 + \sigma_z^2)\sigma_e^4)^2} > 0$$

and

$$\frac{\partial \bar{\kappa}}{\partial \alpha} = \frac{\alpha^2(1-\alpha)^2\sigma_u^2\sigma_y^2 + 2\alpha(1-\alpha)^2\sigma_u^4 + 2\alpha^2(1-\alpha)\sigma_u^2(\alpha\sigma_y^2 + \sigma_u^2)}{((1-\alpha)^2(\alpha\sigma_y^2 + \sigma_u^2))^2} > 0$$

Thus,  $\bar{\kappa}$  is increasing with  $\sigma_u^2$  and  $\alpha$ . □

## B.3 Empirical Results Appendix

### *B.3.1 The list of countries in the dataset*

The countries covered by the dataset are: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Israel, Italy, Japan, Korea, Latvia, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Romania, Slovak Republic, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.

### *B.3.2 Data Description*

All variables are on an annual basis and have been collected for the time period 2001-2008. The measure of home bias I use is the previously defined EHB index, which is standard in the literature. I follow standard practice and calculate the EHB index for each country-year pair combining portfolio data from the IMF's Coordinated Portfolio Investment Survey (CPIS) with domestic stock market capitalization data from the World Bank.

The data on Labor Income comes from the OECD database. I use the series Total Labor Cost, which is Total Employee Compensation adjusted for self-employment income and the amount of self-employed people in the economy. The Financial Wealth proxies are constructed from CPIS and World Bank data. The Total Equity Holdings measure comes directly from CPIS and World Bank data series and is constructed as follows:

$$\text{Total Equity Holdings} = \text{Domestic Portf Equity Assets} + \text{Foreign Portf Equity Assets}$$

where Domestic Portfolio Equity Assets is computed as is standard in the literature:

$$\text{Domestic Portfolio Equity Assets} = \text{Domestic Market Cap} - \text{Foreign Portfolio Equity Liabilities}$$

The data on Domestic Market Capitalization is from the World Bank, and data

on Foreign Portfolio Equity Assets and Liabilities is from the CPIS.

Constructing Total Financial Assets takes into consideration the holdings of portfolio debt securities as well, but is a little harder because there is no data on the total Domestic Market Capitalization of Debt Securities. Thus, I cannot compute a measure of Domestic Portfolio Debt Assets from directly observable data. Instead, I assume that the share of the total market value of domestic debt securities that is held by foreigners is the same as the share of the stock market that is held by foreigners. With that assumption in hand, we can express the total market value of domestic debt securities as:

$$\text{Market Value of Domestic Debt Securities} = \frac{\text{Foreign Portfolio Debt Liabilities}}{1 - \text{DomesticShare}}$$

Then, we can compute the Domestic Portfolio Debt Assets using the imputed value of the Market Value of Domestic Debt Securities, and finally arrive at Total Financial Assets:

$$\text{Total Financial Assets} = \text{Equity Holdings} + \text{Foreign Portf Debt Assets} + \text{Domestic Portf Debt Assets}$$

Finally, I turn these variable into per person measures by dividing by the number of adults, i.e. people over the age of 15, which data I obtain from the World Bank. The Appendix shows that the results are virtually unchanged if instead I divide by total population. However, the number of adults is theoretically more appealing because these are the people that earn income and make financial decisions.

The Chinn-Ito index of financial openness is taken directly from Chinn and Ito (2006). It is the first principal component of the four binary measures of capital controls that are tracked by the IMF in their Annual Report on Exchange Arrangements

and Exchange Restrictions. As a robustness check, the Appendix also presents results when instead I use the Lane-Milesi-Ferretti Index of Financial Openness, which is the sum of foreign assets and liabilities over total GDP. The results are unchanged. Lastly, the real GDP per capita data comes from the World Bank.

All income variables are reported in constant Purchasing Power Parity terms (PPP). The results are virtually unchanged if instead the income variables are measured in constant US dollars and a Table with estimates of these regressions is included in this Appendix.

### B.3.3 Robustness Checks

#### *Robustness Checks on the Population, Income and Financial Openness Measures*

Table B.2: Controls Robustness Checks

	Per Capita		Constant 2000 USD		Lane-Milesi-Ferretti	
Log Internet Users	-0.122** (0.052)	-0.118** (0.052)	-0.122** (0.049)	-0.129** (0.051)	-0.092** (0.038)	-0.076* (0.043)
Log Labor Income	0.258*** (0.091)	0.222* (0.117)	0.309*** (0.103)	0.334*** (0.118)	0.183 (0.125)	0.053 (0.139)
Log Total Fin Assets	-0.024 (0.015)		-0.026 (0.018)		0.021* (0.012)	
Log Equity Assets		0.011 (0.031)		-0.014 (0.044)		0.073** (0.035)
Financial Openness	-0.068*** (0.016)	-0.059*** (0.017)	-0.064*** (0.018)	-0.059*** (0.016)	-0.033*** (0.011)	-0.035*** (0.009)
Log RGDP <sub>2001</sub> p.c.	-0.307*** (0.080)	-0.351*** (0.085)	-0.320*** (0.106)	-0.358*** (0.099)	-0.332*** (0.124)	-0.328** (0.141)
N	35	35	35	35	35	35
R <sup>2</sup>	0.685	0.676	0.674	0.663	0.756	0.775

OLS point estimates with heteroskedasticity robust standard errors in parentheses.  
 \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% respectively.

Table B.2 provides regression results when income variables are standardized by total population, instead of number of adults (columns (1) and (2)), when all monetary variables are expressed in 2000 US dollars (columns (3) and (4)) and when

the Lane-Milesi-Ferretti Index of financial openness is used instead of the Chinn-Ito index. The results are robust to these alternative specifications. The coefficient on Internet users per capita is negative, statistically significant and of the same magnitude as the specifications reported in the main body of the paper.

On the other hand, in columns (1) through (4) the coefficient of Labor Income is positive, significant and of the same magnitude as in the main body of the paper and the coefficient on financial wealth is again insignificant. Using the Lane-Milesi-Ferretti Index of financial openness produces slightly different results. In that case the coefficient on labor income is insignificant, but the coefficient on Financial Assets is found to be positive and marginally significant in column 5 and significant at the 5% level in column 6. It appears that in this case the estimated effect of the hedging motive dominates the estimated effect of the information acquisition motive, as the relative size of labor income seems to have a negative relationship with the home bias. Thus, under the Lane-Milesi-Ferretti Financial Openness index the second empirical restriction is not rejected and hence the evidence of relative labor income is consistent with both models. Nevertheless, we can still reject the first restriction as the coefficient on Internet users is found to be negative and the regressions still rule in favor of the endogenous information asymmetry model.

#### *Regressing on the Ratio of Labor and Financial Incomes*

Table B.3 presents two new sets of results. First, it includes estimates when I impose the restriction  $\beta_l = \beta_f$ , i.e. use the log of the ratio of Labor Income over Financial Wealth as a single regressor, and second it presents results when I use the standard measure of labor income share of GDP. In both cases there are no significant changes to the results. The coefficient on log Internet users is found to be negative, significant and of the same magnitude as before, and the coefficient on the income composition

Table B.3: Restricted Labor Income and Financial Holdings Coefficient

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Ln(Int. Users)	-0.236*** (0.030)	-0.219*** (0.035)	-0.229*** (0.031)	-0.132*** (0.042)	-0.161*** (0.037)	-0.078* (0.048)	-0.077* (0.045)
Ln( $\frac{\text{Labor Income}}{\text{Total Assets}}$ )		0.036 (0.029)		0.044** (0.019)		0.024 (0.018)	
Labor Share			-0.110 (0.377)		0.089 (0.274)		0.323 (0.218)
Chinn-Ito				-0.076*** (0.022)	-0.073*** (0.023)	-0.056** (0.023)	-0.0506*** (0.017)
Ln(RGDP <sub>2001</sub> ).						-0.118*** (0.040)	-0.165*** (0.035)
N	35	35	35	35	35	35	
R <sup>2</sup>	0.475	0.503	0.513	0.628	0.61	0.661	0.653

OLS point estimates with heteroskedasticity robust standard errors in parentheses.

\*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% respectively.

ratio is generally found to be positive but insignificant (although it is significant at the 5% level in Column (4)). Again, the results are virtually the same as in the main body of the paper and support the conclusions reached there.

#### *Including A Measure of Non-Tradable Consumption*

Table B.4 presents regression results when I also control for the relative size of non-tradables in consumption expenditures. This regressor is motivated by the theoretical work of Tesar (1993) and I follow the empirical work of Lewis (1996) and Pesenti and Van Wincoop (2002) when I construct the empirical measure of non-tradable consumption. The data I use is from the OECD and consists of aggregate consumption expenditures broken down into different types such as Food, Health Services, Housing Expenses etc. Identifying non-tradable consumption in the data is tricky and no one approach is perfect, hence the table contains results for two different proxies of non-tradable consumption. The first two columns present results when I use the disaggregated data to arrive at a figure for non-tradable consumption and then I subtract this from total consumption to obtain tradable consumption. The second pair of columns does the opposite - I use the disaggregated data to compute

Table B.4: Results when controlling for non-tradable consumption

	Computing Non-Tradables		Computing Tradables	
Log Internet Users	-0.179*** (0.049)	-0.173*** (0.049)	-0.163*** (0.044)	-0.157*** (0.043)
Log Labor Income	0.394* (0.224)	0.250 (0.287)	-0.001 (0.240)	-0.042 (0.298)
Log Total Fin Assets	0.003 (0.016)		0.032 (0.027)	
Log Equity Assets		0.047 (0.036)		0.053 (0.036)
Log $\frac{\text{Non-tradable Con}}{\text{Tradable Con}}$	0.106 (0.204)	0.136 (0.215)	0.431** (0.200)	0.367** (0.166)
Chinn-Ito Index	-0.060*** (0.016)	-0.059*** (0.015)	-0.038* (0.021)	-0.049*** (0.017)
Log RGDP <sub>2001</sub> p.c.	-0.627*** (0.202)	-0.601*** (0.199)	-0.380* (0.222)	-0.369 (0.233)
N	24	24	24	24
R <sup>2</sup>	0.755	0.764	0.795	0.796

OLS point estimates with heteroskedasticity robust standard errors in parentheses.

\*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% respectively.

tradable consumption and then impute the non-tradable part. I follow Lewis (1996) and Pesenti and Van Wincoop (2002) as closely as possible in determining whether each type of disaggregated consumption is tradable or not. For example, I categorize Food, Beverages, Furniture and Vehicle purchases as tradable consumption and Health care, Financial and Restaurant services as non-tradable and so on.<sup>9</sup>

*Using a proxy for  $\kappa$  in the spirit of Mondria and Wu (2010)*

Mondria and Wu (2010) also study the empirical relationship between  $\kappa$  and the Equity Home Bias but use a slightly different proxy for  $\kappa$ . Instead of using a proxy for information technology level normalized by total population (like Internet Users per 1000 ppl) they further normalize this figure by real GDP per capita. Hence,

<sup>9</sup> The two approaches yield different results because the disaggregated consumption categories do not sum up to total consumption expenditures. The OECD only collects disaggregated data on certain types of consumption expenditures.



they use Proxy for  $\kappa = \frac{\text{Information Tech per 1000 ppl}}{\text{Real GDP p.c.}}$  which they argue captures the average amount of information capacity per \$1000 of economic activity. This extra normalization is appealing because the theoretical models studied here and in Mondria and Wu (2010) analyze only financial decisions and not the full array of decisions an individual must make in the real world. Thus,  $\kappa$  measures the information capacity that the agent allocates to financial decisions, and not necessarily his whole ability to process information. In light of this, an ideal empirical model would also attempt to control for how much of the total information capacity of the agent is allocated to financial decisions. Mondria and Wu (2010) attempt to do this by dividing by real GDP per capita, which they argue is a good proxy for how many other economic activities the agent needs to pay attention to. However, normalizing by real GDP per capita is not likely to capture the desired effect because it also proxies for a number of other important considerations. For example, real GDP per capita is an additional proxy for information capacity itself because a simple measure like Internet Users per capita cannot possibly characterize the whole information technology infrastructure in a given country. Furthermore, richer agents with large financial portfolios would likely prefer to apportion more (and not less) of their fixed information capacity to financial decisions as they have more at stake.

In general, it is not clear why Information Technology per capita would be a worse proxy for  $\kappa$  than Information technology per \$1000 of real GDP. On the other hand, Information Technology per capita is an intuitive and readily available measure and this paper opts to include it in its raw form rather than apply any transformations to it. Instead, I include initial period real GDP per capita as a separate regressor to control for the initial differences between countries. This leads to an empirical framework that is a bit more flexible, and still controls for all important effects.

In the sake of completeness, however, Tables B.5 and B.6 present estimation

results when the proxy for  $\kappa$  is defined as in Mondria and Wu (2010) –

$\frac{\text{Internet Users per 100 ppl}}{\text{Real GDP p.c.}}$ . Table B.5 uses all 35 countries in my data set, while Table B.6 has only the 19 countries used by Mondria and Wu (2010). The conclusions of the main body of the paper are unchanged - the relationship between the home bias and information capacity is found to be negative and statistically significant. In fact, the size of the coefficient  $\beta_\kappa$  is estimated to be about 4-5 times bigger (in absolute value) than the one found in the specifications in the main body of the paper. The only case in which I find a positive and significant  $\beta_\kappa$  is when I regress Home Bias only on the proxy for  $\kappa$  and the Chinn-Ito Financial Openness index. However, if I control for initial conditions (or add measures of labor and financial income) the sign flips back to being negative.

Table B.5: Results with Mondria and Wu (2010) normalization for  $\kappa$

	(1)	(2)	(3)	(4)	(5)	(6)
Log $\frac{\text{Internet Users}}{\text{Real GDP p.c.}}$	0.108*** (0.0223)	-0.570** (0.287)	-0.039 (0.164)	0.006 (0.164)	-0.560** (0.284)	-0.5553** (0.271)
Log Labor Income			-0.062 (0.126)	-0.064 (0.127)	0.181 (0.130)	0.183 (0.149)
Log Total Fin Assets			-0.039* (0.021)		-0.0300* (0.018)	
Log Equity Assets				-0.012 (0.044)		0.002 (0.040)
Chinn-Ito Index	-0.072*** (0.013)	-0.061*** (0.022)	-0.078*** (0.019)	-0.071*** (0.018)	-0.070*** (0.021)	-0.063*** (0.019)
Log RGDP <sub>2001</sub> p.c.		-0.563** (0.234)			-0.689*** (0.253)	-0.736*** (0.240)
N	35	35	35	35	35	35
R <sup>2</sup>	0.570	0.645	0.602	0.575	0.676	0.659

OLS point estimates with heteroskedasticity robust standard errors in parentheses. \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% respectively.

This is an interesting finding because Mondria and Wu (2010) find the opposite - a positive and significant  $\beta_\kappa$ . There are three possibilities for the discrepancy in the results. First, the datasets and data periods are quite different. Mondria and

Wu (2010) use portfolio data from the IMF’s Balance of Payments Statistics and International Financial Statistics (IFS) databases over the span of 1988-2004, while I use data from the IMF’s Coordinated Portfolio Investment Survey for the years 2001-2008. There is only a little overlap in the available years of data. And the data itself is significantly different. The portfolio data in the IFS and the BPS is imputed based on reported capital flows between countries and is not based on direct observations of actual portfolio holdings. The CPIS, on the other hand, was commissioned by the IMF to specifically provide high quality portfolio data based on direct observations of portfolio holdings. In principle, it does not suffer from a lot of statistical measurement errors that could plague the IFS and BPS imputed numbers and is now the principal source of International Portfolio data for the literature (e.g. Coeurdacier and Rey (2013b), Sercu and Vanpée (2008)). Thus, the different time span of the data and the different methodologies under which it was collected and constructed could explain the discrepancies in the results.

Table B.6: Mondria and Wu (2010) normalization for  $\kappa$  (19 countries sample)

	(1)	(2)	(3)	(4)	(5)	(6)
Log $\frac{\text{Internet Users}}{\text{Real GDP p.c.}}$	-0.013 (0.112)	-1.39* (0.806)	-1.30* (0.711)	-1.45* (0.821)	-1.54* (0.850)	-1.81** (0.916)
Log Labor Income			-0.912 (0.578)	-1.23* (0.655)	-0.644 (0.523)	-0.891 (0.644)
Log Total Fin Assets			-0.074* (0.039)		-0.073* (0.038)	
Log Equity Assets				0.058 (0.062)		0.070 (0.066)
Chinn-Ito Index	-0.086* (0.047)	-0.086* (0.045)	-0.210*** (0.051)	-0.118* (0.068)	-0.197*** (0.062)	-0.100 (0.063)
Log RGDP <sub>2001</sub> p.c.		-1.24* (0.710)			-0.517 (0.595)	-0.733 (0.584)
N	19	19	19	19	19	19
$R^2$	0.095	0.253	0.364	0.308	0.379	0.337

OLS point estimates with heteroskedasticity robust standard errors in parentheses.  
 \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% respectively.

Secondly, Mondria and Wu (2010) do not use labor and financial income data, nor control for initial GDP per capita in their specifications. Table B.5 and B.6 show that such considerations are important, as controlling for these effects has important implications for the sign and significance of  $\beta_\kappa$ . Thirdly, Mondria and Wu (2010) run their regressions on a panel data set of 17 years and 19 countries, while I first time average the data and then use cross-sectional regressions. Theoretically, the cross-sectional regressions are the correct specification for testing the models considered in both this paper and Mondria and Wu (2010) because both are static models. At this point, the literature has not considered a dynamic model of endogenous information acquisition and portfolio choice and it is not clear what are the testable implications of such models. In any case, when I re-estimate Tables B.5 and B.6 on the the panel dimensions of my dataset I find once again that  $\beta_\kappa < 0$  and statistically significant.

Lastly, the standard OLS based fixed effects estimator that relies on cross sectional variation for identification is known to suffer potentially significant biases in panels where the number of time periods available is of the same order as the available cross-sections and some of the variables are near unit root (like the Home bias for example). Perhaps an econometric issue could also help explain some part of the differences in the reported estimates.<sup>10</sup>

#### *Evidence for a Non-Monotonic Relationship Between the Home Bias and $\kappa$*

Proposition 3 implies that the home bias should have a non-monotonic relationship with  $\kappa$ , which is increasing for small  $\kappa$  and decreasing for all  $\kappa$  larger than a tipping point  $\bar{\kappa}$ . However, all regression specifications reported up to this point allow only for a monotonic (either positive or negative) relationship. The primary reason is the

<sup>10</sup> On the other hand, the panel regressions I consider on the 2001-2008 dataset at hand are less likely to suffer from such problems for two reasons. First, I use GMM estimators that do not have asymptotic biases and secondly, the time periods (8) is much smaller than the number of cross sections (35).

Table B.7: Allowing for a non-monotonic  $\kappa$  relationship

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Ln(Int Users)	-0.018 (0.483)	0.194 (0.409)	0.420 (0.464)	0.705* (0.43)	0.816* (0.489)	0.831*** (0.320)	1.094*** (0.346)
Ln(Int Users) <sup>2</sup>	-0.031 (0.070)	-0.042 (0.062)	-0.072 (0.068)	-0.111* (0.062)	-0.126* (0.069)	-0.131*** (0.048)	-0.167*** (0.050)
Ln(Lab. Inc.)		-0.139** (0.063)	-0.234** (0.118)	-0.036 (0.057)	-0.113 (0.121)	0.283*** (0.090)	0.212** (0.108)
Ln(Fin. Assets)		-0.017 (0.026)		-0.032* (0.019)		-0.012 (0.014)	
Ln(Equity)			0.025 (0.040)		0.005 (0.040)		0.042 (0.033)
Chinn-Ito				-0.077*** (0.020)	-0.0689*** (0.024)	-0.074*** (0.012)	-0.067*** (0.013)
Ln(RGDP <sub>2001</sub> )						-0.359*** (0.086)	-0.421*** (0.099)
N	35	35	35	35	35	35	35
R <sup>2</sup>	0.4765	0.5719	0.5692	0.661	0.642	0.715	0.720

OLS point estimates with heteroskedasticity robust standard errors in parentheses.  
 \*\*\*, \*\* and \* denote significance at the 1%, 5% and 10% respectively.

relatively small sample size (35 observations). Allowing for a more flexible, possibly non-monotonic, relationship with  $\kappa$  would put a higher identification burden on the regression specification and one could run into problems of overfitting the available data. On the other hand, finding a negative  $\beta_k$  in the main specification which only allows for a linear effect is enough to conclude that the overall relationship between  $\kappa$  and home bias is negative. And an overall negative relationship is enough to conclude that the data is at odds with the implications of a standard exogenous information advantage model. Moreover, a priori it seems unlikely that any significant portion of OECD countries in the 21st century would be informationally constrained, i.e. have  $\kappa < \bar{\kappa}$ . Thus, for the purposes of differentiating the exogenous information advantage model from the endogenous information asymmetry model, the main specification reported in the paper appears to be enough.

For the sake of robustness, however, it is interesting to also examine the possibility of a non-monotonic relationship. An attempt in this direction is given by Table B.7

which reports results for a regression where I also include a squared Internet Users per 100 ppl term. The estimates show that there is indeed evidence of nonmonotonicity - home bias is increasing in  $\kappa$  for low values and decreasing for high values. For example, the point estimates from the main specification (column 6) imply that the home bias is increasing in Internet Users if a country has less than roughly 24 Internet users per 100 ppl and decreasing for any values above that. For the sample at hand, this implies that Brazil, Bulgaria, Mexico, Romania, and Turkey are the five OECD countries which fall into the information constrained set where  $\kappa < \bar{\kappa}$ .

A visually appealing way to examine the potential non-monotonicity is to graphically look at the relationship between  $\kappa$  and the home bias after controlling for other effects. In particular, I fit a 3rd order polynomial of Log Internet Users per 100 people on the residual from a regression of the Home bias on all other variables. The results are presented in Figure B.3.3. The left panel shows the residual of the following regression

$$\overline{EHB}_i = const + \beta_l \ln(\overline{LabIncome}) + \beta_f \ln(\overline{FinWealth}) + \beta' \overline{X}_i + \varepsilon_i$$

plotted against Log Internet Users per 100 people and also draws the 3rd order polynomial fitted line. The second plot estimates the main regression specification of the model:

$$\overline{EHB}_i = const + \beta_k \ln(\overline{InetU}_i) + \beta_l \ln(\overline{LabIncome}) + \beta_f \ln(\overline{FinWealth}) + \beta' \overline{X}_i + \varepsilon_i$$

and then looks at the residual Home Bias after I control for all variables except for the Internet Users per 100 people:

$$e_r = \overline{EHB}_i - const - \beta_l \ln(\overline{LabIncome}) - \beta_f \ln(\overline{FinWealth}) - \beta' \overline{X}_i$$

The results from both exercises are pretty similar. The 3rd order polynomial has a well pronounced hump, where it is increasing for low values of Internet Users and decreasing for high values. In particular, in both figures the polynomial is increasing for the same five countries found above (Brazil, Bulgaria, Mexico, Romania, and Turkey), and decreasing for all others.

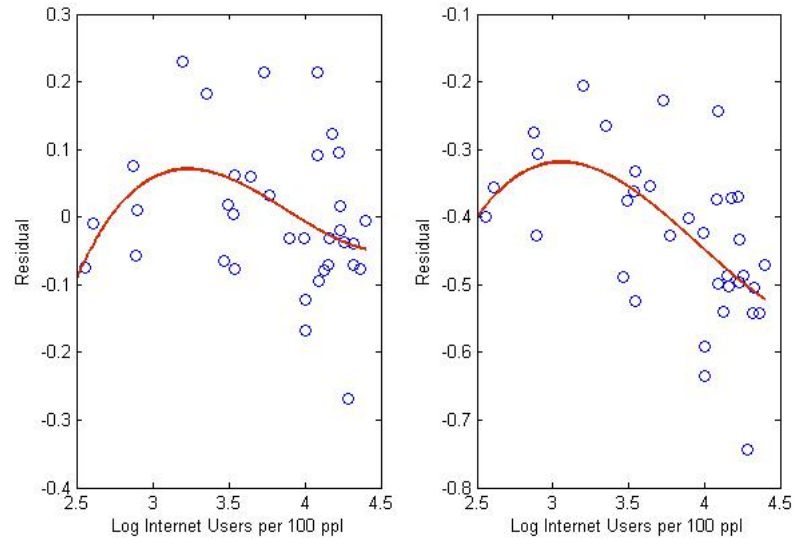


FIGURE B.2: Home Bias and  $\kappa$

# Appendix C

Appendix for Chapter 3



Table C.1: A.1: Countries and Data Time Span

Country	Years of Data
Argentina	1950 – 2007
Australia	1950 – 2007
Austria	1950 – 2007
Belgium	1950 – 2007
Canada	1950 – 2007
Switzerland	1950 – 2007
Chile	1951 – 2007
Denmark	1950 – 2007
Spain	1950 – 2007
Finland	1950 – 2007
France	1950 – 2007
UK	1950 – 2007
USA	1950 – 2007
Germany	1970 – 2007
Greece	1951 – 2007
Hong Kong	1960 – 2007
Ireland	1950 – 2007
Israel	1950 – 2007
Italy	1950 – 2007
Iceland	1950 – 2007
Japan	1950 – 2007
Korea	1953 – 2007
Luxembourg	1950 – 2007
Netherlands	1950 – 2007
New Zealand	1950 – 2007
Norway	1950 – 2007
Poland	1970 – 2007
Portugal	1950 – 2007
Singapore	1960 – 2007
Sweden	1950 – 2007
Uruguay	1950 – 2007

Table C.2: A.2: Country By Country VAR Variance Decompositions

Country	$Var(r_t)$ due to:			$Var(tot_t)$ due to:			$Var(gdp_t)$ due to:		
	$u^r$	$u^{tot}$	$u^{gdp}$	$u^r$	$u^{tot}$	$u^{gdp}$	$u^r$	$u^{tot}$	$u^{gdp}$
Argentina	99.02	0.58	0.41	1.75	96.99	1.26	20.72	5.98	73.30
Australia	86.42	4.18	9.40	4.41	90.34	5.25	7.84	0.60	91.56
Austria	95.82	0.61	3.57	3.38	89.39	7.23	17.34	2.50	80.16
Belgium	97.96	0.34	1.70	15.68	84.16	0.16	6.07	24.82	69.11
Canada	94.75	2.68	2.57	1.12	98.19	0.69	15.58	2.65	81.77
Switzerland	76.21	23.16	0.63	3.05	95.58	1.38	1.78	10.26	87.96
Chile	98.33	0.08	1.58	7.42	91.09	1.49	0.41	8.10	91.49
Denmark	83.53	16.29	0.18	8.26	90.83	0.91	14.33	4.39	81.28
Spain	65.25	32.64	2.11	5.27	92.67	2.06	0.26	8.34	91.39
Finland	98.69	0.07	1.24	5.09	94.50	0.40	7.09	16.60	76.31
France	83.78	16.06	0.16	1.15	98.46	0.39	6.71	3.21	90.07
UK	95.27	1.10	3.63	1.93	85.50	12.57	4.85	0.40	94.75
USA	74.52	23.25	2.24	9.03	90.36	0.61	4.12	3.75	92.12
Germany	74.58	23.28	2.14	0.93	91.14	7.93	5.21	19.04	75.75
Greece	91.73	1.08	7.18	1.88	96.83	1.28	4.31	4.83	90.85
Hong Kong	97.68	0.29	2.03	4.17	95.45	0.38	3.25	1.78	94.97
Ireland	89.47	10.30	0.23	6.86	90.87	2.27	15.18	6.53	78.29
Israel	97.74	1.75	0.51	0.94	98.69	0.37	0.49	9.21	90.29
Italy	91.47	2.76	5.77	4.16	94.81	1.02	1.08	2.32	96.60
Iceland	74.40	15.93	9.67	9.85	87.74	2.40	0.98	5.70	93.32
Japan	68.18	29.89	1.93	6.18	93.28	0.54	3.36	0.62	96.02
Korea	97.85	2.12	0.02	0.33	99.11	0.56	0.31	13.22	86.48
Luxembourg	98.61	0.79	0.61	1.34	87.19	11.47	0.41	1.05	98.54
Netherlands	78.08	17.60	4.32	13.64	84.27	2.08	14.19	3.90	81.91
New Zealand	96.56	2.86	0.57	6.54	85.45	8.00	5.49	13.35	81.17
Norway	93.76	5.16	1.08	0.56	90.10	9.35	0.21	5.40	94.39
Poland	63.86	4.75	31.39	15.68	80.07	4.26	0.06	8.05	91.88
Portugal	93.54	3.29	3.17	5.86	88.17	5.98	1.46	16.87	81.67
Singapore	98.15	0.57	1.28	3.04	83.91	13.05	7.59	10.77	81.65
Sweden	99.93	0.05	0.02	4.68	93.51	1.82	1.99	8.79	89.22
Uruguay	96.18	0.57	3.26	11.42	86.35	2.23	18.22	4.72	77.06
M. Share < 15%	90.36	6.29	3.36	5.29	91.11	3.60	6.54	7.53	85.94
M. Share < 10%	90.92	5.61	3.47	5.38	90.93	3.69	6.73	7.42	85.86
M. Share < 5%	87.41	6.91	5.67	6.61	89.75	3.64	6.04	7.44	86.52
M. Share < 15%, Gr	91.02	5.59	3.38	5.68	91.71	2.61	7.36	7.77	84.87
M. Share < 10%, Gr	91.02	5.59	3.38	5.68	91.71	2.61	7.36	7.77	84.87
M. Share < 5%, Gr	88.71	6.01	5.27	6.29	89.95	3.76	6.55	7.62	85.84

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# Biography

Rosen Valchev was born on September 14, 1986 in Bourgas, Bulgaria on the Black Sea coast. At the age of six, his family moved to Bulgaria's capital Sofia where Rosen graduated from high school. He attended Duquesne University in Pittsburgh, Pennsylvania from where he graduated in May 2009 as the Valedictorian of his class. He then enrolled in the PhD program at Duke University, where he earned his Master's degree in 2010 and is submitting his PhD thesis. In the Fall of 2015, Rosen will join Boston College as an Assistant Professor of Economics.