

Implications of Consumer Behavior in Retail Operations

by

Arian Aflaki

Business Administration
Duke University

Date: _____

Approved:

Robert Swinney, Supervisor

Fernando Bernstein

Kevin Shang

Santiago Balseiro

Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Business Administration
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ABSTRACT

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Abstract

Consumers have become increasingly sophisticated along multiple dimensions. For example, they may consider whether and when to purchase from a firm based on current and future prices and product availability. Notably, empirical evidence suggests that a significant fraction of consumers do not visit firms that experience frequent stock outs and may postpone their purchase with the hope of paying a lower price in the clearance period. Additionally, consumers also consider product design in their valuation for the product, and are only willing to pay premium prices for products that match design trends and retain value during the product's lifecycle. In such an environment, retailers should include elements of consumer behavior in their pricing, inventory, channel (e.g. online and offline), and fashion design decisions. In three essays, we address each of these managerial decisions with the aim of helping retailers to understand and control different elements of consumer behavior while providing a better shopping experience for their consumers. In the first essay, we focus on pricing strategies of a firm that implements multiperiod pricing and answer whether consumers necessarily benefit from strategic behavior (timing their purchase) when the firm optimally responds to this type of behavior. We find that many consumers have a lower surplus if they are strategic than if they are myopic. We then develop a model in which consumers choose to become strategic by exerting costly effort, and find that it is possible to increase firm profit, consumer, and social welfare simultaneously by increasing the cost of strategic behavior. In the second essay, considering

a product with a decreasing price path, we study how inventory and channel integration impacts whether and when consumers visit the firm by affecting product availability. We show that integration generates value by increasing regular period availability and encouraging more consumers to visit the firm, but it may also harm the firm by encouraging more consumers to delay their purchases. In the third essay, we study joint design, pricing, and inventory decisions of firms facing consumers that have a low valuation for products that are out of style and rapidly lose value during the selling season. We illustrate that firms that implement dynamic pricing should carefully consider the interdependence between their inventory and design decisions. In sum, our findings in this dissertation shed light on the importance of considering implications of consumer behavior in retail operations and provide solutions that benefit firms and consumers.

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List of Abbreviations and Symbols

Chapter 2

Superscripts m and s are used for equilibrium values under myopic and strategic behavior, respectively.

p_i	Price in period $i \in \{1, 2\}$
π^j	Firm profit
δ	Consumer discount factor
$G(\cdot); g(\cdot)$	Distribution function of consumer valuations; density function of consumer valuations
$u^j(v)$	Surplus for a consumer with valuation v
\bar{v}^j	Threshold consumer valuation
CS^j	Total consumer surplus
SW^j	Social welfare
$V(v)$	Value of strategic behavior for a consumer with valuation v
FV	Value of strategic behavior for the firm
SV	Value of strategic behavior for society
k	Cost of strategic behavior
$\hat{\eta}(v)$	Belief of a consumer with valuation v about the incremental value obtained from adopting a strategic purchasing rule
$x \wedge y$	Meet of x and y
$x \vee y$	Joint of x and y

Chapter 3

Subscript $i \in \{1, 2\}$ is used to refer to market i in a multichannel system. Subscript I refers to an integrated system. Additionally, we use symbol $(\tilde{\cdot})$ to refer to the set of beliefs.

D_i	Market size
μ_i	Mean of demand
σ_i	Standard deviation of demand
D_i^c	Consumers' posterior belief of demand conditional on their own existence in the market
h_i^c	Density function of D_i^c
ρ	Correlation between market demands
λ	Market asymmetry parameter
cv_i	Coefficient of variation in market i
$q_i; q_I$	Inventory level in market i ; total inventory in the integrated system
$\pi_i(q_i); \pi_I(q_I)$	Expected profit for a firm carrying q_i units of inventory in each market under a multichannel system; expected profit for a firm carrying q_I units of inventory under an integrated system
c	Inventory procurement cost
p	Full price
s	Clearance price
δ	Consumer discount factor
$v_l; v_h$	Lowest consumer valuation; highest consumer valuation
$G(\cdot); \bar{G}(\cdot)$	Distribution function for consumer valuations; complementary distribution function
v_i^*	Threshold consumer valuation
k	Consumer store visit cost

$\tilde{\xi}_{ij}$	Consumer belief about the probability of the product being available in period j in market i
$\tilde{\xi}_{i2}(A)$	Consumer belief about the probability of the product being available in period 2 conditional on knowing that the product has been available in the first period
$\Phi(\cdot); \phi(\cdot)$	Standard normal distribution and density functions
Δ_I	Value of integration
$\Pi(\mu, \sigma)$	Optimal newsvendor expected profit when demand is normally distributed with mean μ and standard deviation σ
\hat{z}	Standard normal z -statistic
x^+	Maximum of x and 0
$\mathbb{E}(\cdot)$	Expected value

Chapter 4

Superscript * is used to refer to the optimal values. Additionally, subscripts s and d correspond to static and dynamic pricing schemes, respectively.

T	Length of the selling season
\bar{t}	Start time of the clearance period
D	Market size
$F(\cdot); f(\cdot)$	Distribution function of the market size; density function of the market size
m	Mean of the market size
SD	Standard deviation of the market size
c	Inventory procurement cost
p	Full price
s	Clearance price
I	Inventory level
α	Level of fashionability/Product design

$\pi(\alpha, I)$	Expected firm profit when it starts the season with I units of a product with fashionability level α
W_t	Standard Wiener Process
$V_t(\alpha)$	Uncertain valuation of a product with fashionability level α at time t
$\mu(\alpha)$	Percentage drift in valuation of a product with fashionability level α
$\sigma(\alpha)$	Percentage volatility in valuation of a product with fashionability level α
$v_0(\alpha)$	Mean of the product valuation at time 0
\bar{v}_t	Expected life value of the product at time t
v_1	Expected life value of the product at time 0/Total value of the product
v_2	Expected life value of the product at time \bar{t} /Future value of the product
Δ	Additional expected value derived from purchasing in the first period rather than the second period/Current value of the product
$\alpha_{v_1}^*$	Fashionability level that maximizes the total value of the product
$\alpha_{v_2}^*$	Fashionability level that maximizes the future value of the product
α_{Δ}^*	Fashionability level that maximizes the current value of the product
$\tilde{\epsilon}$	A random variable with mean 1
$\mathbb{E}(\cdot)$	Expected value
$h'(\cdot); h''(\cdot)$	First derivative of a function h with respect to α ; second derivative of a function h with respect to α

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1

Introduction

Modern consumers are increasingly sophisticated along multiple dimensions when making purchasing decisions. For example, consumers often consider the availability risk of a product when deciding whether to visit a firm and have become aware of the intertemporal pricing strategies of firms and consider strategically delaying purchases to obtain products at a lower price. Additionally, consumers not only consider vertical characteristics of products such as price and quality, they also consider product design and fashion attributes when making their purchase decisions to express their image and identity (Le Bon 2014). In this complex environment, firms should take elements of consumer behavior into account and employ mechanisms that, first, can put them on the right course to achieve their goals, and second, that help to satisfy the ever-increasing expectations of consumers. In three sequential essays, this dissertation specifically focuses on pricing, inventory, channel, and product design decisions of retailers facing consumers that are sensitive and react to these decisions.

Chapter 2, *Choosing to Be Strategic: Implications of the Endogenous Adoption of Forward-Looking Purchasing Behavior on Multiperiod Pricing* (joint work with Pnina Feldman and Robert Swinney), answers a fundamental question: do consumers

necessarily benefit from strategic behavior? The answer to this question is important because if some consumers do not benefit from this behavior, then they are unlikely to be exogenously endowed with a particular type of behavior as is currently assumed in the literature. When the firm intertemporally sets prices, we show that only a fraction of consumers benefits from strategic behavior. Motivated by this observation, we develop a model in which consumers endogenously choose to become strategic by exerting costly effort. We then illustrate that considering this choice has a significant qualitative impact on the firm and consumers. For example, some of the most fundamental properties of the optimal prices are different when consumers endogenously choose to be strategic. Additionally, in contrast to most of the literature, under endogenous strategic behavior, price commitment can encourage more strategic waiting, and thus can reduce firm profit, by reducing the cost of strategic behavior for consumers. As such, considering the consumer choice to be strategic can have significant implications for the firm's pricing strategy.

Chapter 3, *Omnichannel Retail and Inventory Integration with Strategic Consumers* (joint work with Robert Swinney), addresses the interaction of omnichannel retail and consumer behavior: with the surge of e-commerce and technological advances, firms strive to provide a seamless purchase experience for consumers by integrating their selling channels. This challenging task enables firms to benefit from economies of scale in inventory management and can increase inventory availability for consumers, which can attract more demand to the firm. However, we show that this may also signal a high availability of the product during clearance sales and can encourage consumers to postpone their purchases until prices have been reduced. This reduces the value of integration significantly and may even lead to a negative value in the absence of any cost of integration. We study the implications of these effects and illustrate cases where integration can be used to mitigate strategic behavior or may lead to more strategic waiting.

Chapter 4, *Optimal Fashionability for Seasonal Products* (joint work with Robert Swinney), studies joint pricing, inventory, and fashion design decisions of a firm that sells seasonal products to consumers. A key characteristics of the fashion market are that it is trendy in nature, and trends are hard to forecast. Firms, long in advance of the selling season, must choose both product design and stocking quantity, while they use pricing as a more responsive tool to market realizations. A highly fashionable product design can enable firms to charge a high premium price during the regular selling period if the product is successful, though it may lose value faster, which requires deeper discounts in the clearance period. We particularly compare two pricing mechanisms: in a static pricing strategy, the firm collects leftover inventory at the end of the season without offering a discount, while in dynamic pricing, it sells excess inventory in a clearance sale. Comparing these two pricing mechanisms, we find that firms with static pricing face a much simpler task in deciding fashion design: they maximize the life value of the product and offer higher fashionability compared to dynamic pricing, because they are not dependent on clearance sales. As a result of carrying a riskier product, firm stocks less inventory to hedge against the risk of having leftover inventory at the end of the season. We further study the impact of market parameters on inventory and fashion choice of firms that implement dynamic pricing. Interestingly, while a market size with a higher mean always results in a more fashionable design, it can lead to a higher or lower optimal inventory level.

Choosing to Be Strategic: Implications of the Endogenous Adoption of Forward-Looking Purchasing Behavior on Multiperiod Pricing

2.1 Introduction

Multiperiod pricing is a central concern of firms selling physical goods, particularly those that are durable in nature. In recent years, however, an increasingly large proportion of the consumer population has become aware of and responsive to the inter-temporal pricing strategies of firms, allowing them to anticipate price changes and optimally time their purchases (Li et al. 2014b). The term “strategic consumer behavior” has become synonymous with this type of rational, forward-looking purchasing behavior (Su and Zhang 2008b; Cachon and Swinney 2009b), and understanding the extent of and optimal response to such strategic consumer behavior has become a topic of great interest to both practitioners and researchers. In the academic literature, while most work on the pricing and inventory management of physical goods had previously assumed consumers were not strategic, a new stream

of research has rapidly developed to analyze the impact of strategic consumer behavior on a firm’s optimal decisions along numerous dimensions, including pricing, inventory, and supply chain management (see, e.g., Aviv and Pazgal 2008b; Zhang and Cooper 2008; Yin et al. 2009b; Jerath et al. 2010; Levin et al. 2010; Lai et al. 2010; Osadchiy and Vulcano 2010; Cachon and Swinney 2011; Mersereau and Zhang 2012; Cachon and Feldman 2015a; Zhang and Zhang 2015; Aviv et al. 2015; and many others).

Research into this phenomenon typically classifies consumers into one of two types based on the decision rule they use when purchasing a product. Consumers are said to follow a *strategic purchasing rule* if they consider the future when making their purchasing decisions today. That is, for a product sold over two periods, a consumer follows a strategic purchasing rule if she arrives in period 1, considers her utility from delaying her purchase to period 2, and chooses to purchase in the period that maximizes her utility. Conversely, a consumer follows a *myopic purchasing rule* if she does not consider period 2 when making her purchasing decision in period 1; consequently, she purchases in period 1 if and only if her utility from a purchase is non-negative. Consumers following these rules are frequently referred to as “strategic” and “myopic” consumers, respectively, in the literature.¹ A consumer employing a strategic purchasing rule is distinct from, but related to, the phenomenon of *strategic waiting*, which occurs when a consumer intentionally decides to delay a purchase in order to obtain the product at a lower price. A consumer can only strategically wait if she follows a strategic purchasing rule; however, not all consumers who follow a strategic purchasing rule, in equilibrium, may choose to strategically wait.

Given this distinction, existing research into strategic consumer behavior is pri-

¹ While the term “strategic consumer” can also describe other types of behaviors, e.g., consumers strategizing over when to visit a congested service system (Lariviere and Van Mieghem 2004), in this paper we use the term to exclusively refer to forward-looking utility-maximizing consumers who optimally time their purchase when prices vary over time.

marily divided into theoretical studies concerning methods to reduce strategic waiting (Su and Zhang 2008b; Aviv and Pazgal 2008b; Cachon and Swinney 2009b; Yin et al. 2009b) and empirical studies estimating the prevalence of strategic waiting (Osadchiy and Bendoly 2010; Soysal and Krishnamurthi 2012; Li et al. 2014b). In both cases, it is generally assumed that consumers (or some fraction of consumers) exogenously employ a strategic purchasing rule: a particular customer is either strategic or myopic, and the extent of strategic behavior in the population is specified as a model primitive. As such, previous work has mostly ignored two key questions regarding strategic consumer behavior: first, whether consumers in fact benefit from strategic behavior, and second, whether consumers would, if given the choice, decide to be strategic by adopting a strategic purchasing rule. These are deceptively simple questions: fixing the firm's actions—in particular its prices—it is clearly true that an individual consumer can do no worse by considering the opportunity to purchase in future periods in addition to purchasing in the period in which she arrives. However, once all consumers consider the opportunity to purchase in future periods and optimally time their purchases, and once the firm responds optimally to this behavior by adjusting its prices, it is no longer obvious that strategic purchasing behavior will result in an increase in individual consumer (or, indeed, social) welfare.

These are important questions to answer for at least three reasons. First, the implicit assumption in both the popular press and in many earlier works has been that the pricing game between the firm and consumers is zero-sum in nature (Su 2007), and as such strategic consumer behavior represents a wealth transfer from the firm to consumers that unambiguously benefits the customer population. In other words, consumers are typically encouraged to be strategic, as this is in their own interests and, it would seem, is only detrimental to the firm. However, if conditions exist under which strategic behavior either harms consumers or society, then this type of consumer behavior is cast in a new light; strategic behavior by consumers

would not merely lead to a wealth transfer in a zero-sum game, but rather it would be an actively destructive force that, while possibly individually rational for consumers, leads to an equilibrium in which society is worse off. Second, in practice, consumers are unlikely to be exogenously endowed with a particular type of behavior. Strategic behavior requires consumers to exert effort and incur some associated costs, e.g., due to the hassle cost of finding the optimal time of purchase and the effort required to implement an optimal purchasing strategy by taking actions such as monitoring prices, identifying the exact time of a price reduction, and making return visits to a physical store or website. Hence, a rational consumer, aware of the net value of engaging in strategic behavior, would rationally *choose* whether to employ a strategic purchasing rule or a myopic purchasing rule. Given this, knowing whether—and which—consumers benefit from strategic behavior can help to illuminate conditions under which strategic behavior is and is not likely to be adopted by consumers, and help firms understand how to respond via their pricing policy. Third, as the magnitude of the effort costs associated with strategic behavior are likely related to the selling strategy employed by the firm, it is also probable that the population of consumers who choose to be strategic will differ under different selling strategies, which in turn may affect how beneficial different strategies are for the firm.

In this work, we explore precisely these issues. To accomplish this, we analyze a model of a firm selling a single product over two periods (§2.3). Consumers have heterogeneous valuations, and hence the firm has an incentive to set different prices in each period and segment the market. We first examine a model of exogenously specified behavior in which consumers are either myopic or strategic (i.e., they exogenously follow a myopic purchasing rule or a strategic purchasing rule), and we determine the firm’s optimal prices when they are set dynamically (i.e., established at the start of each period to maximize profit-to-go). Using this classical framework as a starting point, we examine the impact of strategic consumer behavior on all

stakeholders in the system: the firm, and, in contrast to previous work, consumers and society (§2.4). We find that all consumers do not benefit from being strategic: consumers with low valuations are indifferent to strategic behavior, consumers with moderate valuations are worse off under strategic behavior, and only consumers with high valuations are better off under strategic behavior. Moreover, social welfare is always higher under myopic consumer behavior—hence, the game between the firm and consumers is not zero-sum, and strategic consumer behavior is detrimental to society as a whole.

Motivated by this finding, in §2.5 we develop a new model of endogenous consumer purchasing behavior in which consumers choose between being “myopic” and being “strategic.” Specifically, we posit that all consumers are inherently rational, but they may choose whether to adopt a myopic purchasing rule or a strategic purchasing rule, taking into account the expected net value of the latter, which includes both a benefit (i.e., the incremental increase in utility from optimizing the purchase time) and a cost (i.e., the effort required to find and implement the optimal purchase strategy). We determine precisely which consumers do—and do not—choose to be strategic, and show that the firm’s optimal dynamic pricing policy differs qualitatively under endogenous behavior from the optimal policy under exogenous behavior: the optimal prices are not monotonic in the consumer cost of strategic behavior, and it is possible for the firm’s optimal prices in each period to be strictly lower under endogenous behavior than under myopic behavior, something that never happens if consumers are assumed to be exogenously strategic.

Our results have several further implications for firms, which we discuss in detail in §2.6. First, while making strategic behavior difficult for consumers seems like an effective strategy for firms selling to a captive customer population, it is unclear why this would be as valuable for firms selling to consumers with outside options, such as purchasing from competitors or reducing or forgoing consumption. In other words, if

strategic behavior is good for consumers, why would consumers continue to shop at firms that make it difficult to be strategic? Using our endogenous behavior model, we show that by increasing the cost of strategic behavior, the firm can not only improve profit, it can also increase consumer and social welfare. In other words, the firm, by forcing consumers to be myopic and thereby limiting consumer purchasing options, may actually make itself, consumers, and society better off. Thus, our model suggests that such firms can be successful precisely because they make strategic behavior difficult, since consumer and social welfare are also maximized when the cost of being strategic is high. Second, we show that, in contrast to conventional wisdom, selling strategies designed to mitigate strategic waiting (e.g., committing to keep prices high) may in fact decrease firm profit if they also impact the cost of being strategic. Therefore, when thinking about whether to implement a particular selling strategy, not only is it important to account for strategic behavior, it is also important to consider the costs associated with becoming strategic and the consumer choice to adopt a strategic purchasing rule, as this may have a significant effect on profit. Taken in sum, our results illustrate the importance of both considering the impact of strategic behavior on consumers and accounting for the consumer decision to become strategic.

2.2 Literature Review

While anecdotal evidence of strategic consumer behavior is pervasive, rigorous empirical evidence that consumers exhibit strategic behavior is limited (due in part to the econometric challenges associated with this problem) but growing. Li et al. (2014b) use data from the travel industry to determine that 5% to 45% of consumers are forward-looking and strategically time their purchases. Nair (2007) and Soysal and Krishnamurthi (2012) similarly demonstrate that forward-looking behavior has a significant impact on firm profits using data from the video game and apparel

industries, respectively. In a laboratory context, Osadchiy and Bendoly (2010) and Mak et al. (2014) determine that a significant fraction of subjects are strategic.

Supported by the increasing empirical evidence of this type of behavior, the literature on strategic consumers is growing rapidly, and forward-looking consumer behavior has received significant theoretical attention in the economics, marketing, and operations literatures over the last decade. Motivated by the conjecture of Coase (1972) that a monopolist attempting to “price skim” over time would be unable to prevent consumers from strategically waiting for the lowest price, Stokey (1981), Bulow (1982), and Besanko and Winston (1990) were among the first to model this dynamic as one in which consumers are forward-looking and optimally choose their purchase time. More recently, Bergemann and Välimäki (2006); Su (2007); Aviv and Pazgal (2008b); Levin et al. (2010); Mersereau and Zhang (2012); Ovchinnikov and Milner (2012); Aviv and Wei (2015); Aviv et al. (2015), and others have analyzed the multiperiod pricing problem under strategic consumer behavior in a variety of richer contexts. This work was later extended to consider the impact of strategic behavior on firm decisions beyond pricing, such as inventory (Liu and van Ryzin 2008b), supply chain design (Cachon and Swinney 2009b), advance selling (Prasad et al. 2011; Wei and Zhang 2015; Cachon and Feldman 2015b), product variety (Parlaktürk 2012), posterior price matching policies (Lai et al. 2010; Surasvadi and Vulcano 2013), assortment rotation (Bernstein and Martinez-de Albéniz 2014), and customer voting systems (Marinesi and Girotra 2013); we refer readers to Netessine and Tang (2009) for an extensive review.

Our work differs from these earlier models in two key ways. First, when consumer behavior is exogenously specified, we consider the impact of strategic behavior on consumers themselves and society (the firm and consumers) as a whole, as opposed to just the firm. This allows us to determine which consumers benefit from strategic behavior and which do not, and moreover, whether society is better off when

consumers are strategic. Second, in all earlier work on strategic consumer behavior that we are aware of, both empirical and theoretical, an implicit assumption is that consumers are *exogenously* either myopic or strategic. That is, the question of how consumers came to be strategic is not considered. In that sense, earlier work can be thought of as endogenizing *when* consumers wish to purchase, but not *whether* they are strategic in the first place. This is an important question, because while the inter-temporal consumer purchasing problem has received a great deal of attention in economics, marketing, and operations management, whether consumers benefit from engaging in strategic behavior will directly impact the results in each of these literatures. This is a key focus of our work, and we show that endogenizing the decision to be strategic can have a significant impact on the value (to the firm, consumers, and society) of the firm's pricing and selling strategy.

2.3 Model

We study a firm that sells a single product over a finite selling season. The selling season consists of two successive periods, labeled 1 and 2. The firm can charge different prices in each period: the price is p_1 in period 1 and p_2 in period 2. The firm's marginal and fixed costs are normalized to zero. The firm's objective is to maximize its total profit, π , which consists of the undiscounted sum of the profit in each period. For the majority of our analysis, we assume that the firm prices dynamically, i.e., sets the price in each period to maximize profit-to-go; in §2.6, we also discuss implications of a price commitment strategy, in which the firm credibly commits at the start of the season to prices over the entire selling horizon.

At the beginning of period 1, a deterministic mass of consumers arrives. The size of this population is normalized to one. Each consumer purchases at most once, and values the item at $v \geq 0$. Consumers have heterogeneous valuations with distribution $G(x)$, a continuous and differentiable function, and density $g(x)$. All

consumers identically discount period 2 surplus by $\delta \in (0, 1)$ (Cachon and Swinney 2011). Thus, a consumer with valuation v who purchases in period 1 receives utility $u_1(v) = v - p_1$, a consumer who purchases in period 2 receives utility $u_2(v) = \delta(v - p_2)$, and a consumer who does not purchase receives zero utility. Note that we assume that the firm does not discount period 2 profit while consumers do discount period 2 surplus, i.e., the firm is more patient than consumers; this assumption is frequently made in the literature (Landsberger and Meilijson 1985; Cachon and Swinney 2011) and may be relaxed without significantly impacting the results.

As noted in the introduction, we consider two broad models of consumer behavior. In the first, consumer behavior is exogenously specified, and all consumers are either myopic or strategic. In the second, consumers are intrinsically myopic but can endogenously choose to become strategic by exerting effort and paying a cost. We defer discussion on the latter case to §2.5; here, we discuss the exogenous behavior model. As previously indicated, we say that a consumer follows a *myopic purchasing rule* if she purchases whenever she observes a price lower than her valuation. Alternatively, a consumer follows a *strategic purchasing rule* if she anticipates the possibility of purchasing in period 2, and rationally decides whether to purchase the product in period 1 or delay until period 2. We use the terms “myopic consumer” and “strategic consumer” as short-hand to denote consumers that follow a myopic purchasing rule or a strategic purchasing rule, respectively. In addition, we say that a consumer *strategically waits* or *delays a purchase* if her utility from a purchase in period 1 is non-negative, but she delays her purchase to period 2 to obtain the product at a lower price. Given these definitions, myopic consumers purchase in period 1 if $u_1(v) \geq 0$, while strategic consumers purchase in period 1 if $u_1(v) \geq \max(u_2(v), 0)$ (note that we assume consumers that are indifferent between periods purchase in period 1).

We denote equilibrium values under myopic behavior by the superscript m , and

Table 2.1. Consumer, firm, and social welfare metrics.

Metric	Description
π^j	Firm profit
$u^j(v)$	Surplus for a consumer with valuation v
CS^j	Total consumers surplus
SW^j	Social welfare
$V(v)$	Value of strategic behavior for a consumer with valuation v
CV	Value of strategic behavior for the total consumer population
FV	Value of strategic behavior for the firm
SV	Value of strategic behavior for society

equilibrium values under strategic behavior by the superscript s . When analyzing the impact of strategic behavior on consumers and society, we will consider several metrics, summarized in Table 2.1, each defined for scenario $j \in \{m, s\}$. Most of these metrics are fairly standard definitions. For instance, $u^j(v)$ is the equilibrium surplus to a consumer with valuation v , $CS^j = \int u^j(v)g(v)dv$ is total consumer surplus, and $SW^j = CS^j + \pi^j$ is total social welfare, each for scenario $j \in \{m, s\}$. Several metrics require further explanation. First, we define the *value of strategic behavior* $V(v)$ to an individual consumer to be the difference between her optimal utility if she (and all other consumers) exhibits strategic behavior and her utility if she (and all other consumers) exhibits myopic behavior, given that the firm is aware of the type of consumer behavior and prices optimally, i.e., $V(v) = u^s(v) - u^m(v)$. The value of strategic behavior to the entire consumer population (CV) and to the firm (FV) are defined similarly, using total consumer surplus and firm profit, respectively, rather than individual consumer surplus, i.e., $CV = CS^s - CS^m$ and $FV = \pi^s - \pi^m$. Lastly, we say that the value of strategic behavior to society SV is the difference between social welfare (total consumer surplus plus firm profit) when all consumers are strategic and when all consumers are myopic, i.e., $SV = SW^s - SW^m$.

2.4 Exogenous Behavior: The Value of Strategic Behavior

We first analyze the value of strategic behavior to consumers and to society under exogenous behavior. That is, we calculate the difference between consumers' utility if they all (exogenously) behave strategically, and their utility if they all (exogenously) behave myopically. Exogenous behavior is the standard assumption in the existing pricing literature on forward-looking behavior, and as such serves as an important baseline for us to understand how strategic behavior impacts consumers and society. The sequence of events is as follows. At the start of period 1, the firm chooses the period 1 price, p_1 . Then, all consumers arrive, observe p_1 , and choose whether to purchase in period 1 or wait for period 2. Next, at the start of period 2, the firm chooses the period 2 price, p_2 . Lastly, all remaining consumers observe p_2 and choose whether to purchase or not.

We begin by formulating the firm's optimization problem with myopic consumers. Since myopic consumers purchase in period 1 if their valuations exceed the selling price, period 1 demand is $1 - G(p_1)$ and period 1 profit is $(1 - G(p_1))p_1$. Period 2 demand is thus $G(p_1) - G(p_2)$, and period 2 profit is $(G(p_1) - G(p_2))p_2$. The firm sets p_1 at the start of the horizon and p_2 dynamically in period 2 to maximize profit-to-go. Hence, the firm's period 1 optimization problem is:

$$\begin{aligned} \max_{p_1, p_2} \quad & (1 - G(p_1))p_1 + (G(p_1) - G(p_2))p_2 \\ \text{s.t.} \quad & p_2 \in \arg \max_x (G(p_1) - G(x))x \end{aligned} \tag{2.1}$$

Let p_1^m and p_2^m be the optimal prices that result from this optimization problem. If consumers' valuations follow a uniform distribution on $(0, 1)$, it is straightforward to find that the optimal prices under myopic behavior are $p_1^m = \frac{2}{3}$ and $p_2^m = \frac{1}{3}$.

When consumers are strategic, the firm and consumers play a game: the firm

chooses prices and consumers choose when (and whether) to purchase the product. We seek the subgame perfect Nash equilibrium (SPNE) to this game. The optimal actions of the firm and consumers in period 2 may be solved immediately; the first significant step in deriving the equilibrium is thus to establish the optimal action of consumers in period 1, after having observed a posted period 1 price from the firm. Lemma 1 illustrates that strategic consumers follow a threshold purchasing rule, i.e., for any p_1 chosen by the firm, there exists some \bar{v} such that consumers purchase in period 1 if and only if $v \geq \bar{v}$.

Lemma 1. *There exists a unique threshold \bar{v} such that all strategic consumers with valuation $v \geq \bar{v}$ purchase in the first period and consumers with valuation $v < \bar{v}$ delay purchasing until period 2.*

Proof. All proofs appear in the appendix. □

We thus refer to a consumer with valuation \bar{v} as the “threshold consumer.” (Note that the analogous threshold consumer under myopic behavior has valuation p_1 .) Given this, the firm’s period 1 problem can be written as follows:

$$\begin{aligned} \max_{p_1, p_2} \quad & (1 - G(\bar{v}))p_1 + (G(\bar{v}) - G(p_2))p_2 \\ \text{s.t.} \quad & \bar{v} = \{\min \hat{v}, \text{ s.t. } \hat{v} - p_1 \geq \delta(\hat{v} - p_2)^+\} \\ & p_2 \in \arg \max_x (G(\bar{v}) - G(x))x \end{aligned} \tag{2.2}$$

Let p_1^s and p_2^s be the equilibrium prices and let \bar{v}^s be the equilibrium valuation of the threshold consumer under strategic consumer behavior. These equilibrium values critically depend on δ . (For example, observe that when $\delta = 0$, strategic consumers do not have an incentive to delay their purchases and therefore behave in the same way as myopic consumers.) Therefore, we analyze the equilibrium as a function of δ . To accomplish this, we define the partial order on \mathbb{R}^n be the componentwise

order. More precisely, for some $x, y \in \mathbb{R}^n$, we say $x \geq y$ if and only if $x_i \geq y_i$ for all $i \in \{1, \dots, n\}$. We also define $x \wedge y = (\min(x_1, y_1), \dots, \min(x_n, y_n))$ and $x \vee y = (\max(x_1, y_1), \dots, \max(x_n, y_n))$ as the meet and joint of two elements of \mathbb{R}^n . A set is a lattice if it is closed under meet and joint. Lastly, we say that set A is greater than set B with respect to the strong set order (denoted by $A \geq B$) if for any $x \in A$ and $y \in B$, we have $x \vee y \in A$ and $x \wedge y \in B$. Given these preliminaries, the following lemma derives several useful properties of the optimization problem given in (2.2):

Lemma 2. *Define $\mathcal{C}(\delta) := \{(p_2^s(\delta), \bar{v}^s(\delta)) \in \mathbb{R}^2; \text{ s.t. } (p_1^s(\delta), p_2^s(\delta), \bar{v}^s(\delta)) \text{ is a solution to optimization problem given in (2.2)}\}$. Then,*

- (i) *The set $A := \{(p, \bar{v}) \in \mathbb{R}^2 : p \in \arg \max_p (G[\bar{v}] - G[p])p\}$ is a sublattice of \mathbb{R}^2 .*
- (ii) *$\mathcal{C}(\delta)$ is non-decreasing in δ respect to the strong set order. This implies that if the equilibrium is unique, $\bar{v}^s(\delta)$ and $p_2^s(\delta)$ are non-decreasing in δ .*
- (iii) *The firm's equilibrium profit is non-increasing in δ .*
- (iv) *$p_1^s(\delta) \geq p_2^s(\delta)$.*

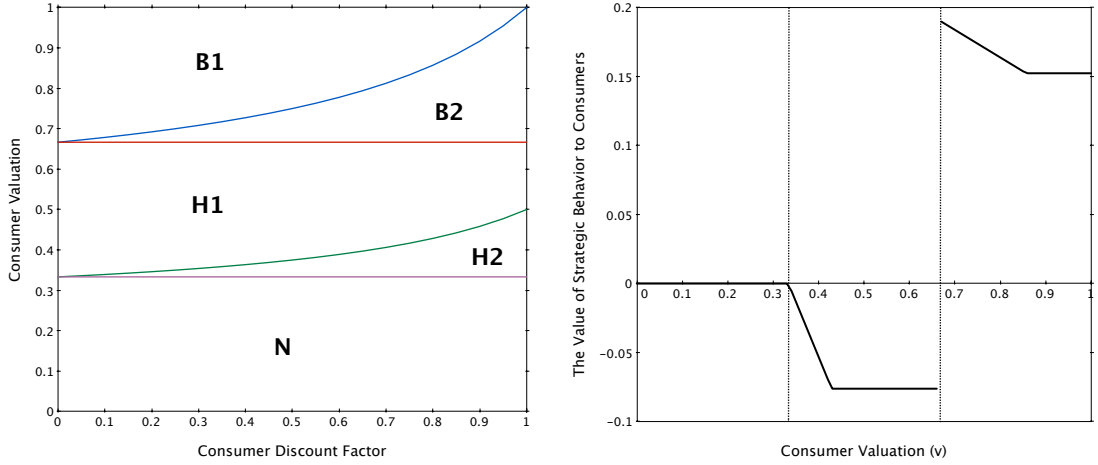
The lemma shows that as consumers value the future more and become more patient (i.e., as δ increases), more consumers wait for the second period and the firm charges a higher second period price and earns lower profit. Furthermore, in equilibrium the firm skims the market, i.e., it begins with a high price and reduces the price in the second period. This leads us to the following result:

Proposition 1. *Suppose the equilibrium prices to (2.2) are unique. Then, under any continuous and differentiable consumer valuation distribution:*

- (i) *$\bar{v}^s > p_1^m$ and $p_2^s > p_2^m$.*
- (ii) *A consumer with valuation $v \in [p_2^m, p_2^s)$ does not obtain a unit under strategic behavior, but does obtain a unit under myopic behavior.*

(iii) A consumer with valuation $v \in [p_2^s, p_1^m)$ obtains a unit under both types of behavior, but pays a higher price under strategic behavior.

The theorem demonstrates that, for *any* continuous and differentiable valuation distribution, there always exists a nonempty set of consumers that are harmed by strategic behavior, i.e., they enjoy lower utility if consumers are strategic than they would if all consumers were myopic. There are two distinct mechanisms by which consumers can be harmed by strategic behavior. First, in case (ii), consumers with $v \in [p_2^m, p_2^s)$ do not obtain a unit under strategic behavior. These consumers, who would have purchased (and obtained positive utility) under myopic behavior, are priced out of the market under strategic behavior. The reason for this is that strategic behavior results in higher valuation consumers purchasing in period 2 than under myopic behavior and this, in turn, leads the firm to raise the period 2 price to accommodate the higher valuations of period 2 customers. Hence, low valuation consumers are excluded from the market, specifically those with valuations between the myopic and the strategic period 2 prices. Second, in case (iii), consumers with $v \in [p_2^s, p_1^m)$ always obtain a unit in period 2 under either type of behavior, but are forced to pay a higher price under strategic behavior due to the presence of higher valuation customers that caused the firm to raise the period 2 price. Taken in sum, cases (ii) and (iii) of the theorem provide our first results that indicate some consumers are actively harmed by strategic behavior. Bazhanov et al. (2015) find a similar result in a very different setting—consumers may be harmed by being more forward-looking, although in their model this is due to the destructive effects of competition between firms. The fact that consumers may be better off if they are myopic also echoes observations by Zhou et al. (2015) that in an oligopoly, firms may be better off if they are “non-strategic,” i.e., do not react to the actions of their competitors.



(a) Consumer segments as a function of δ . Segments B1 and B2 benefit from strategic behavior; H1 and H2 are harmed by strategic behavior; and N feels no impact from strategic behavior. (b) The value of strategic behavior to individual consumers ($V(v)$) when $\delta = 0.8$.

FIGURE 2.1. The impact of strategic behavior on consumers.

To facilitate our analysis and gain further insights, we assume throughout the remainder of the paper that consumer valuations are uniform on the interval $(0, 1)$. It is straightforward to determine that with uniform valuations, there exists a unique SPNE under strategic behavior, and in this equilibrium $\bar{v}^s = \frac{2-\delta}{3-2\delta}$, $p_1^s = \frac{(2-\delta)^2}{6-4\delta}$, and $p_2^s = \frac{2-\delta}{6-4\delta}$. Comparing the resulting prices under each type of behavior, we note that $p_2^m < p_2^s < p_1^s < p_1^m$, i.e., strategic behavior results in a lower period 1 price and a higher period 2 price than myopic behavior. The impact of strategic behavior on consumers in this case is illustrated graphically in Figure 2.1(a), which shows that consumers are, in fact, impacted by strategic behavior in five different ways.

Consumers with the lowest valuations—specifically, those with valuations in the bottom third of the distribution—never purchase a unit under either type of behavior; hence, these consumers always earn zero utility, and they effectively feel no impact of strategic behavior (segment N in the figure). On the other hand, consumers with moderate valuations—that is, those with valuations in the middle third of the

distribution, segments H1 and H2 in the figure—are harmed by strategic behavior similarly to the results in Theorem 1. Consumers with $v \in [\frac{1}{3}, \frac{2-\delta}{6-4\delta})$ do not obtain a unit under strategic behavior (segment H2) whereas consumers with $v \in [\frac{2-\delta}{6-4\delta}, \frac{2}{3})$ always obtain a unit in period 2 under either type of behavior, but pay a higher price under strategic behavior (segment H1). Note that the set of consumers who pay a higher price under strategic behavior includes some who, in equilibrium, strategically delay a purchase themselves, i.e., some consumers with valuations in the interval $[p_1^s, p_1^m)$. Despite the fact that these consumers engage in strategic waiting, they would have been better off had all consumers been myopic; in that case, they still would have purchased in period 2 (because the period 1 price would have been higher than their valuations) but, due to the absence of even higher valuation period 2 customers strategically delaying from period 1, the firm would set a lower period 2 price and increase the utility of these consumers. Lastly, consumers with the highest valuations benefit from strategic behavior, again in two ways. Consumers with $v \in [\frac{2}{3}, \frac{2-\delta}{3-2\delta})$ purchase in period 1 under myopic behavior, but strategically delay and purchase in period 2 under strategic behavior (and hence obtain the product at a lower price); this is segment B2 in the figure. Consumers with $v \in [\frac{2-\delta}{3-2\delta}, 1]$ purchase in period 1 under both types of behavior, but do so at a lower price under strategic behavior (segment B1 in the figure). Hence, these consumers—who have the highest valuations of all—never strategically wait themselves, but benefit from the strategic behavior of their fellow consumers.

In addition, observe that the overall sizes of the segments that are indifferent to, harmed by, and benefitted by strategic behavior is insensitive to δ , the consumer discount factor. This occurs because these sets are defined by the *myopic* prices, which are independent of δ ; specifically, consumers with valuations less than p_2^m are indifferent to strategic behavior (segment N), consumers with valuations between p_2^m

and p_1^m are harmed by strategic behavior (the union of segments H1 and H2), and consumers with valuations greater than p_1^m benefit from strategic behavior (the union of segments B1 and B2). However, within each of these terciles, the way in which consumers are harmed by or benefitted by strategic behavior shifts as δ increases: for instance, a fraction $\frac{\delta}{18-12\delta}$ of consumers are deprived of the item under strategic behavior, while a fraction $\frac{6-5\delta}{18-12\delta}$ are forced to pay a higher price under strategic behavior, hence as δ increases, a larger fraction of this set is harmed due to being priced entirely out of the market and a smaller fraction is harmed due to paying a higher price.

As part (b) of the figure shows, under a uniform valuation distribution, a majority of customers *do not* benefit from strategic behavior: for a third of the population (those with the lowest valuations) the value of strategic behavior is zero, while for another third of the population (those with moderate valuations) the value of strategic behavior is negative. Only consumers with valuations in the highest third of the distribution benefit from strategic behavior. Hence, strategic behavior is beneficial to a minority of the consumer population, and indeed those that benefit are precisely the consumers with the highest valuations. We note here that while the specific sizes of the segments that benefit from and are harmed by strategic behavior will, naturally, depend on the distribution of consumer valuations, it is true that under *any* continuously differentiable valuation distribution, a nonempty set of consumers is worse off under strategic behavior. Hence, the result that strategic behavior can be detrimental to some consumers is not sensitive to the choice of the consumer valuation distribution.²

² In contrast, this result is sensitive to the assumption that the firm correctly recognizes that its consumers are strategic and optimally accounts for their behavior by adjusting prices. If the firm does not do this, e.g., because it is unaware that consumers are strategic or incorrectly adjusts for strategic behavior (as laboratory experiments suggest may be plausible; see Kremer et al. 2015), consumers may benefit more from strategic behavior.

Having derived the impact of strategic behavior on individual consumers, we may now determine the impact on the entire consumer population, the firm, and society as a whole:

Proposition 2. *When consumer valuations are uniform on $(0, 1)$,*

(i) The value of strategic behavior to the entire consumer population is positive ($CV > 0$).

(ii) The value of strategic behavior to the firm is negative ($FV < 0$).

(iii) The value of strategic behavior to society is negative ($SV < 0$).

Part (i) of Theorem 2 shows that despite the fact that strategic behavior harms *some* consumers, total consumer surplus is higher when consumers are strategic. Thus, the gain in high valuation consumer surplus more than outweighs the loss in low and moderate valuation consumer surplus due to strategic behavior, which can be seen in Figure 2.1(b). The value of strategic behavior to the firm is, as expected, negative; interestingly, though, the value of strategic behavior to society as a whole is negative, meaning strategic behavior *reduces* social welfare. This shows that even though some consumers (specifically high valuation consumers) are better off under strategic behavior, this does not make up for the combined reduction in moderate valuation consumer surplus and firm profit. Figure 2.2 graphically illustrates these results. Observe that, in Figure 2.2(c), the gap between social welfare under myopic and strategic settings is increasing in δ . In other words, as strategic consumers become more patient, they harm social welfare more, compared to the myopic case. This is consistent with Figure 2.1(a), which showed that as δ increases, more consumers are harmed by being excluded from the market altogether (rather than simply paying a higher price) due to strategic behavior.

Taken as a whole, these results illustrate that strategic consumer behavior is neither beneficial to all consumers nor to society. This behavior does not represent

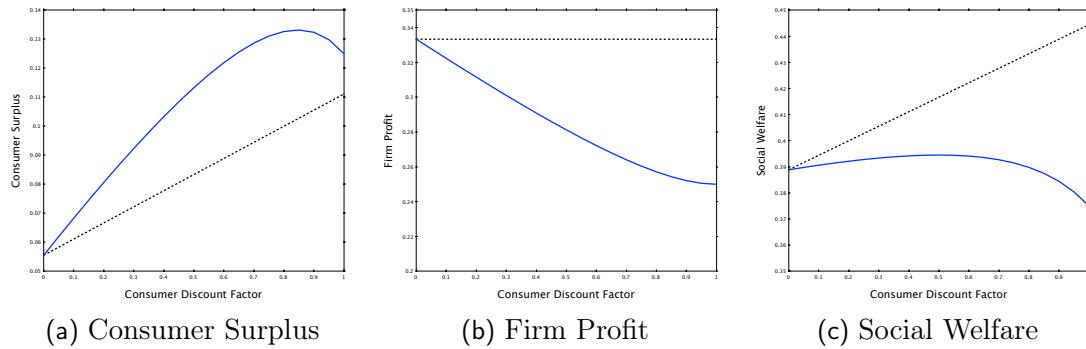


FIGURE 2.2. The impact of strategic behavior on society. The solid line represents equilibrium values when all consumers are strategic, and the dashed line represents values when all consumers are myopic.

consumers simply taking surplus from the firm in a zero-sum game: it reduces firm profit, causes some consumers to be excluded from the market or pay a higher price, and only benefits consumers with the highest valuations. As a result, social welfare is lower under strategic behavior than under myopic behavior. This fact motivates us to examine precisely how consumers may choose to be strategic, and what the firm can do to influence that choice, in the following section.

2.5 Endogenous Behavior: Choosing to be Strategic

In the literature on strategic consumer behavior and multiperiod pricing, an almost universal assumption is that consumer behavior is exogenous—that is, each individual consumer is assumed to follow either a myopic or strategic purchasing rule, but does not choose between these two types of purchasing rules. However, in practice, whether or not a consumer considers a strategic delay when making her initial purchasing decision is unlikely to be a completely exogenous trait; rather, it is plausible that rational consumers, aware of the costs and benefits of employing a strategic purchasing rule and subsequently executing the optimal purchasing strategy, decide which purchasing rule to adopt. In this section, we consider precisely this dynamic by endogenizing the consumer choice of purchasing rule.

Specifically, we consider a model in which all consumers are initially myopic—that is, they employ a myopic purchasing rule by default—but they may choose to follow a strategic purchasing rule if they anticipate that this will increase their utility. Aside from any potential benefits (i.e., the ability to obtain the item at a lower price), adopting a strategic purchasing rule and implementing an optimal purchasing strategy also comes with a cost. For example, a customer that chooses to adopt a strategic purchasing rule will have to calculate her optimal purchase period (in our stylized model, period 1 or 2), which will require costly effort, and, if she further decides to strategically delay her purchase, she must actively monitor the firm to obtain the item as soon as a markdown occurs (the precise timing of which may be unpredictable) and make a return visit to the physical store or website, both of which incur some additional costs.³ While, in reality, consumers may exert varying degrees of effort and, as a result, may end up possessing varying effort costs and degrees of strategic behavior, we abstract away from such details and assume that consumers either exert zero effort, meaning they remain “myopic,” or precisely enough effort to become fully “strategic.” In the latter case, the effort required to achieve this comes with a positive cost $k \geq 0$, which is the same for all consumers regardless of their valuation.

When choosing whether to adopt a strategic purchasing rule, each consumer has a belief $\hat{\eta}(v)$ about the incremental value they would obtain relative to the myopic purchasing rule, i.e., they believe that if they adopt a strategic purchasing rule, their surplus will increase by $\hat{\eta}(v)$. Following the rational expectations framework (Su and Zhang 2008b; Cachon and Swinney 2009b; Cachon and Feldman 2015a), we assume that these beliefs are correct in equilibrium. Consumers might develop these “rational expectations” of the value of strategic behavior from their past shopping experience

³ Recent empirical estimates by Moon et al. (2015) using data from an online retailer place the dollar value of similar “consumer monitoring costs” between \$2 and \$25 per visit to the store’s website; for a brick-and-mortar retailer, costs are likely to be even greater.

with the firm or with other similar firms; based on this experience, each consumer knows how much her net utility changes if she adopts a strategic purchasing rule and she chooses to become strategic only if her net change is positive. Note $\hat{\eta}(v)$ may depend on the individual consumer’s valuation. We emphasize that while consumers have *ex ante* rational expectations about the net value of adopting a strategic purchasing rule, actually implementing that rule requires exerting effort and incurring the effort cost k ; hence, there is a distinction between knowing the potential value of strategic behavior and being able to realize that value by implementing the optimal purchasing strategy. Thus, in our model each individual consumer begins the game as myopic but *rational*, and chooses to remain myopic or adopt a strategic purchasing rule to maximize her own utility. We say that a consumer who chooses to adopt a strategic purchasing rule chooses to “become strategic,” i.e., forward-looking in the sense of the previous literature on inter-temporal consumer purchasing behavior (Su and Zhang 2008b; Aviv and Pazgal 2008b). After this decision, consumers who choose to become strategic exert effort to find and implement the optimal purchasing strategy.⁴

The utility earned by a myopic consumer is $u_1(v)$ if she purchases in period 1 (i.e., if $u_1(v) \geq 0$), $u_2(v)$ if she purchases in period 2 (i.e., if $u_1(v) < 0$ and $u_2(v) \geq 0$), and zero otherwise; the utility earned by a strategic consumer is $\max(u_1(v), u_2(v), 0) - k$.

The rational expectations assumption implies that a consumer chooses to be strategic

⁴ This model of “endogenous strategic behavior” is also related to broader theories of bounded rationality (Simon 1978) and, in particular, satisficing (Tyson 2008). A consumer in our model can be thought of as engaging in something analogous to satisficing behavior because she remains myopic (and does not strategically wait) if her gain from becoming strategic is less than k , i.e., if being myopic leads to a payoff within k of her optimal payoff. The key difference is that in our model consumers actually pay the cost k if they become strategic, while under satisficing k is not a real cost; this feature impacts the equilibrium total consumer surplus and social welfare calculations but not the equilibrium consumer decisions, leading to similar insights. Also related is the concept of “rational ignorance,” in which consumers may rationally choose to obtain less than perfect information if the cost of obtaining information is non-zero (Downs 1957; Martinelli 2006; Hu et al. 2015); our model may be thought of as “rational myopia” in a similar spirit to this line of work.

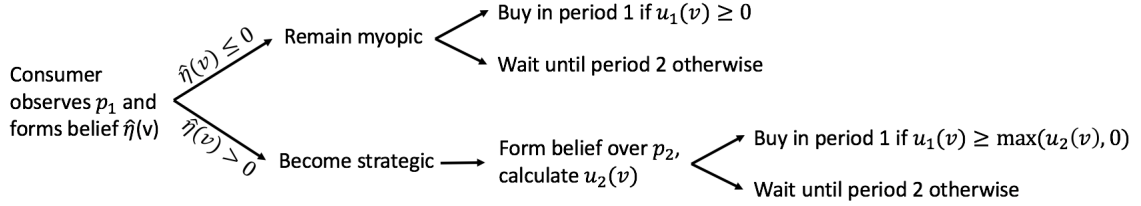


FIGURE 2.3. Consumer decision tree in the endogenous behavior model.

if her utility from behaving in this manner is greater than her utility from remaining myopic, i.e., if $\hat{\eta}(v) > 0$, where $\hat{\eta}(v)$ is the equilibrium incremental value of a strategic purchasing rule. The rational expectations assumption also implies that, in deciding between a myopic and a strategic purchasing rule, a consumer is only required to have correct expectations about $\hat{\eta}(v)$ given the first period price, i.e., she does not need to perfectly anticipate every parameter that enters her utility from a strategic purchasing rule separately (such as the second period price or the cost of being strategic) nor is she required to be able to calculate the utility $\max(u_1(v), u_2(v), 0) - k$ precisely and find the optimal purchasing period. Only a consumer who decides to follow a strategic purchasing rule pays the cost k and determines $u_2(v)$. The cost of being strategic, k , is assumed to be sunk once paid, and thus after choosing to be strategic, consumers ignore this cost when optimally timing their purchases. Figure 2.3 depicts the consumer decision process in the first period.

We denote the endogenous behavior model by a superscript k . Note that this is indexed by the cost of becoming strategic; in the special case where $k = 0$, it is trivially true the model reduces to the exogenous behavior model with strategic consumers. Conversely, if k is sufficiently large (under uniform $(0, 1)$ valuations, greater than 1) then no consumer will ever choose to be strategic, and the model reduces to the exogenous behavior model with myopic consumers. Thus, the endogenous behavior model can be thought of as a generalization of both exogenous behavior models.

Our first result with this model provides the equilibrium prices and consumer actions when consumers may individually choose whether to adopt a strategic purchasing rule:

Lemma 3. *With endogenous behavior and uniform valuations on $(0, 1)$, there exists a unique SPNE in which all consumers with valuations greater than \bar{v}^k purchase in period 1 and all consumers with valuations less than \bar{v}^k wait until period 2. Furthermore:*

- (i) *If $k \geq \frac{\delta}{3}$, all consumers choose to be myopic, and $p_1^k = \frac{2}{3}$, $p_2^k = \frac{1}{3}$, and $\bar{v}^k = \frac{2}{3}$.*
- (ii) *If $\frac{2\delta - \delta^2}{6 - 2\delta} \leq k < \frac{\delta}{3}$, all consumers choose to be myopic, and $p_1^k = \frac{2k}{\delta}$, $p_2^k = \frac{k}{\delta}$, and $\bar{v}^k = \frac{2k}{\delta}$.*
- (iii) *If $0 \leq k < \frac{2\delta - \delta^2}{6 - 2\delta}$, consumers with $v \in [p_1^k, \bar{v}^k)$ choose to be strategic, and $p_1^k = \frac{(2-\delta)^2 + 2(1-\delta)k}{6-4\delta}$, $p_2^k = \frac{2-\delta-2k}{6-4\delta}$, and $\bar{v}^k = \frac{2-\delta-2k}{3-2\delta}$.*

In part (i), the cost of executing a strategic purchasing rule is sufficiently high that no consumer would ever choose strategic behavior regardless of the prices set by the firm; hence, the myopic outcome is replicated. In part (ii), it is also true that no consumer chooses to be strategic in equilibrium; however, this outcome is achieved because the firm's prices have induced such behavior. In particular, as k decreases from $\delta/3$, the firm sets a lower period 1 price to induce consumers to be myopic. In part (iii), the cost of executing a strategic purchasing rule is sufficiently low that the firm cannot profitably "price out" strategic behavior, as doing so would require a significant price reduction in period 1; hence, in equilibrium, the firm prices higher than the level that eliminates strategic behavior, and some consumers choose to be strategic (and, in addition, strategically wait for period 2). Specifically, these are consumers with valuations in the interval $[p_1^k, \bar{v}^k)$. Interestingly, while many theoretical models that incorporate both myopic and strategic consumers assume that consumer behavior is independent of consumer valuations, part (iii) of the lemma

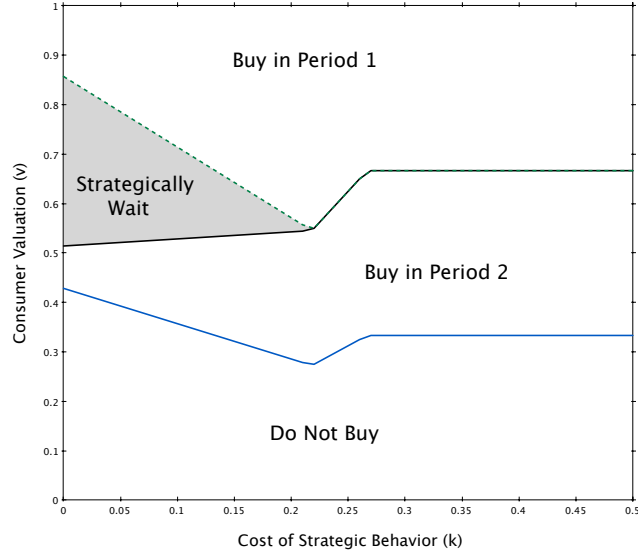


FIGURE 2.4. Equilibrium consumer behavior under endogenous behavior for $\delta = 0.8$. The dashed line is the threshold consumer valuation (\bar{v}^k), the upper solid line is the first period price, the lower solid line is the second period price, and the gray shaded region corresponds to consumers who choose to be strategic and, in addition, strategically delay a purchase.

suggests that only moderate valuation consumers will choose to become strategic, calling into question this common assumption from the literature.

Note that the firm's equilibrium prices and the threshold consumer valuation are not necessarily monotonic in k . Specifically, p_1^k is always (weakly) increasing in k , but the other two equilibrium quantities (p_2^k and \bar{v}^k) are non-monotonic: in case (i) these values are independent of k , in case (ii) they are increasing in k , and in case (iii) they are decreasing in k . These patterns are depicted graphically in Figure 2.4. The reason for this non-monotonicity is the aforementioned way in which the firm uses the period 1 price to eliminate strategic behavior: initially as k decreases from $\delta/3$, the firm reduces the period 1 price rapidly to induce consumers to be myopic, which means that \bar{v}^k initially decreases as k decreases. However, as k continues to fall, this becomes too expensive for the firm and as a result the firm “gives up” on eliminating strategic behavior. After this point, \bar{v}^k grows as k decreases, i.e., as being strategic becomes less costly, more consumers choose to be strategic. Because the period 2

price is simply $\bar{v}^k/2$ under uniform valuations, this value exhibits the same behavior as \bar{v}^k , leading to a non-monotonic period 2 price.

Comparing the equilibrium derived in the lemma to the equilibrium under exogenous behavior, observe that under endogenous behavior, the firm should set a first period price between the two extreme cases (i.e., between the myopic and strategic optimal prices, found at $k = 0$ and $k \geq \frac{\delta}{3}$, respectively), but the optimal second period price may actually be lower than under either of the exogenous behavior models. The reason for this is that under endogenous behavior, the firm may (at intermediate k) intentionally set a very low period 1 price to eliminate strategic behavior; in turn, this implies very low valuations of period 2 customers, and hence a low optimal period 2 price. As a result, both the period 1 and period 2 prices under endogenous behavior may be lower than the prices under purely myopic behavior; this is in contrast to the exogenous strategic behavior model, which recommends firms lower the period 1 price but raise the period 2 price compared to the myopic optimal levels. This shows that the firm's optimal dynamic pricing policy is qualitatively different when strategic behavior is endogenously determined than when it is exogenous, and setting a strictly lower price path may be an optimal response to endogenous strategic behavior. In other words, it is possible that one of the most basic managerial insights about the optimal response to strategic behavior—that firms should raise the final period price in response to strategic consumers—may no longer be true when consumers can endogenously choose to be strategic, and in fact a very low period 2 price may be optimal.

Also note that under a uniform valuation distribution, a minority of the consumer population will choose to be strategic. Specifically, the maximum proportion of consumers who choose to be strategic is 50%; the maximum occurs when $k \rightarrow 0$ and $\delta \rightarrow 1$. These proportions are quite sensitive to δ , and as δ decreases they fall rapidly; for instance, when $\delta = 0.8$, 34% of consumers choose to be strategic;

this is remarkably close to estimates of the prevalence of strategic behavior found in the empirical literature (e.g., Li et al. 2014b). In addition, despite the fact that consumers may voluntarily choose whether to be strategic, it remains true that not all consumers benefit from strategic behavior, as the following theorem shows:

Proposition 3. *Let $\bar{k} = \frac{2\delta - \delta^2}{6 - 2\delta}$. Then, under endogenous behavior and uniform valuations on $(0, 1)$,*

(i) If $0 \leq k \leq \frac{\delta}{6}$, all consumers with $v \in (\frac{1}{3}, \frac{2}{3})$ are strictly worse-off under strategic behavior than if all consumers were myopic. Furthermore, in this set, consumers with $v \in [\frac{(2-\delta)^2 + 2(1-\delta)k}{6-4\delta}, \frac{2}{3})$ choose to become strategic.

(ii) If $\frac{\delta}{6} < k \leq \bar{k}$, all consumers with $v \in [\frac{(2-\delta)^2 + 2(1-\delta)k}{6-4\delta}, \frac{2}{3})$ are strictly worse-off under strategic behavior than if all consumers were myopic. Furthermore, in this set, consumers with $v \in [\frac{(2-\delta)^2 + 2(1-\delta)k}{6-4\delta}, \frac{2-\delta-2k}{3-2\delta})$ choose to become strategic.

(iii) If $\bar{k} < k < \frac{\delta}{3}$, all consumers with $v \in [\frac{2k}{\delta}, \frac{2}{3})$ are strictly worse-off under strategic behavior than if all consumers were myopic. Furthermore, no consumer chooses to become strategic.

Interestingly, even when consumers may freely choose to become strategic or remain myopic, there is always a nonempty set of consumers who are worse off than if the entire population were forced to be myopic. This will clearly be true for those consumers who are priced out of the market due to strategic behavior and those consumers who always buy in period 2 due to a period 1 price higher than their valuations. However, in cases (i) and (ii) of the theorem, some consumers that are harmed by strategic behavior choose, themselves, to adopt a strategic purchasing rule, yet are worse off than if they had not had the option of becoming strategic. In case (iii), no consumer chooses to be strategic, but the threat of strategic behavior causes the firm to react by changing its prices in such a way that some consumers are worse off (in particular, those who would buy in period 2, and obtain a large

surplus, under myopic behavior, but end up purchasing in period 1, at a smaller surplus, under endogenous behavior).

Comparing the segment sizes under the endogenous case to those under the exogenous behavior case with uniform valuations (discussed in §2.4), it is clear that in case (i) of the theorem, the same consumers are harmed by strategic behavior as in the exogenous model, while in cases (ii) and (iii), fewer consumers are harmed by strategic behavior in the endogenous behavior case than in the exogenous behavior case; this is due to the fact that endogenous behavior reduces the number of consumer who are strategic in equilibrium, thereby reducing, but never eliminating (unless $k > \delta/3$), the negative impact on some individual consumers compared to an exogenous model in which all consumers are strategic.

2.6 Implications

Having derived the equilibrium prices and consumer actions under endogenous behavior in the previous section, we now discuss two important implications of these results: first, the impact of increasing the cost of strategic behavior on the firm, consumers, and society, and second, the impact of endogenous behavior on how the firm values commitment strategies designed to mitigate strategic waiting.

2.6.1 Increasing the Cost of Strategic Behavior

Lemma 3 illustrates that in order to reduce strategic behavior amongst its customers, the firm could make being strategic more “costly” for them, i.e., it could attempt to increase k . How might firms go about increasing the cost of strategic behavior? One way to accomplish this might be through offering equivalent but more complicated pricing schemes to consumers that require more effort to calculate and thus to compare the value of purchasing in different periods. For example, the firm can offer stacking discount schemes (e.g., 40% + an additional 10% discount) instead of single

price discounts. Another way to increase the cost of executing a strategic purchasing rule is to make markdowns less frequent and more random in nature (Moon et al. 2015), necessitating consumers monitor the store more frequently or otherwise exert more effort to learn about pricing patterns and identify precisely when a price reductions occurs.

In fact, several of the most successful apparel retailers in the world, like Zara and H&M, have built their business strategy around precisely this approach, “training” their customers to be myopic by, among other strategies, infrequent and unpredictable price reductions (Ghemawat et al. 2003; Cachon and Swinney 2011). While mitigating strategic behavior with a captive customer base seems beneficial, in reality, customers have outside options, such as purchasing from competitors or reducing or forgoing consumption, and faced with a decrease in their own utility due to firm efforts to minimize strategic behavior, in practice consumers may choose to abandon the firm. This presents the following riddle: if strategic behavior is good for consumers, why do so many consumers shop at companies that make it difficult to be strategic? How can these companies be so successful in the marketplace? On one hand, it is possible that companies such as Zara and H&M succeed in spite of the fact that they make strategic behavior difficult for consumers, e.g., because they offer other benefits to consumers like better design, or because the benefits of mitigating strategic behavior for consumers who continue to shop at the firm outweigh any demand loss that results from reducing consumer utility. On the other hand, our model indicates that a different explanation may be possible as well, which the following theorem illustrates:

Proposition 4. *Let $\bar{k} = \frac{2\delta - \delta^2}{6 - 2\delta}$. Under endogenous behavior and uniform valuations on $(0, 1)$:*

(i) Firm profit is increasing in k .

(ii) Total consumer surplus achieves its maximum at either $k = 0$ or $k = \bar{k}$. Specifically, let x^* be the first root of $x^3 - 12x^2 + 24x - 12 = 0$. Then, $x^* \approx 0.8$, and if $\delta > x^*$, consumer surplus is maximized at $k = \bar{k}$. Otherwise, consumer surplus is maximized at $k = 0$.

(iii) Social welfare is quasi-concave in k and maximized at $k = \bar{k}$.

Part (i) of the theorem confirms our intuition that the firm benefits from a higher cost of strategic behavior. Interestingly, part (ii) of the theorem shows that consumers may also benefit from higher k : when consumers are relatively patient ($\delta \gtrsim 0.8$), total consumer surplus is maximized at $k = \bar{k}$. This is due to the fact that when consumers are patient and k is small, “too many” consumers will adopt a strategic purchasing rule, which causes the firm to respond by adjusting its prices in a way that is detrimental to total consumer welfare. In this case, the consumer population would be better off with a larger k , in which fewer consumers will choose to be strategic but the firm’s prices are more attractive. Note that \bar{k} is exactly the minimum k at which no consumers, in equilibrium, choose to be strategic. Figure 2.5(a) illustrates this case. Part (iii) shows that social welfare is maximized at an intermediate k ; this is the same point that maximizes consumer welfare. Thus, not only are consumers potentially better off with a higher cost of strategic behavior, but the firm and society are as well. These effects are depicted in Figure 2.5(b). The reason for this is that, when the cost of being strategic is low, strategic behavior has the flavor of a Prisoner’s Dilemma: many consumers, individually, want to be strategic, but the result of these individual decisions is that consumers reduce social welfare and, as we saw in the exogenous behavior model, the individual utility of moderate valuation consumers. Hence, consumers “over-strategize” when the cost of engaging in this behavior is low, to the detriment of the firm and society as a whole. This illustrates that an opportunity exists for a Pareto-improving outcome in which

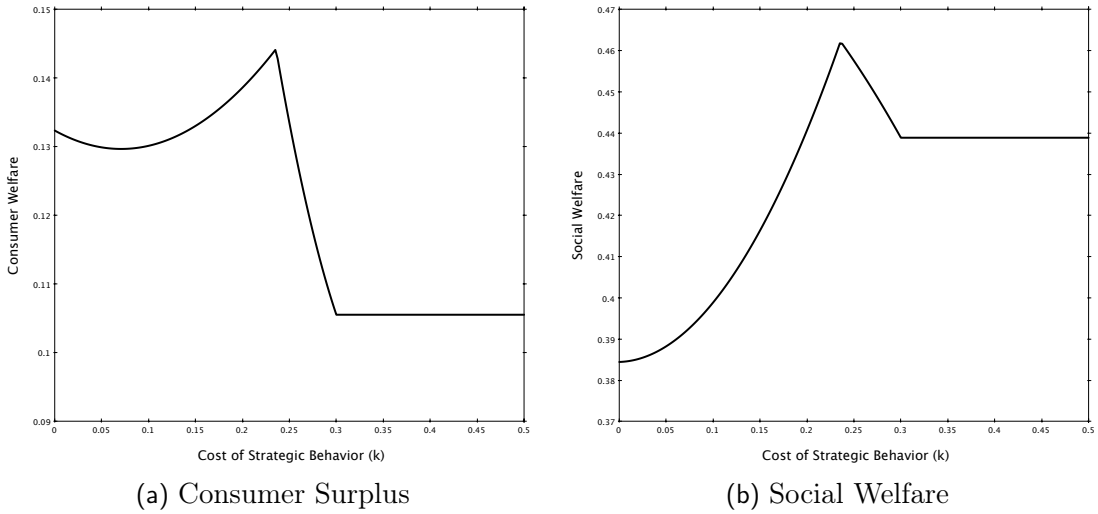


FIGURE 2.5. Consumer and social welfare under endogenous behavior for $\delta = 0.9$.

the firm makes it more difficult for consumers to be strategic, and both consumers and the firm benefit as a result.

These results suggest that successful apparel companies such as Zara and H&M may not succeed *in spite of* the fact that they make it hard for consumers to be strategic; rather, they may succeed precisely *because of* this fact. Theorem 4 shows that if consumers are sufficiently patient, making strategic behavior more difficult—to the point that all consumers, in equilibrium, choose to be myopic—increases not only the firm’s profit, but also consumer and social welfare. In other words, these firms make consumers act myopically, and consumers and the firm are both better off because of it. While we do not explicitly model competition or long term industry dynamics, we posit that this, in turn, may attract more demand over time and cause these firms to grow their customer base, leading to the observed phenomenon that a number of the most successful apparel retailers also do the most to minimize strategic behavior.

2.6.2 Price Commitment and Endogenous Behavior

Making it costlier to be strategic is not the only approach a firm could take to mitigate strategic consumer behavior. Indeed, previous work on strategic consumer behavior has identified several methods that a firm can employ to reduce strategic waiting and increase profits. Perhaps the most important and well-studied such strategy is *price commitment*, where the firm pre-announces and commits to a series of prices at the beginning of the selling season rather than setting them dynamically (Aviv and Pazgal 2008b; Elmaghraby et al. 2008; Su and Zhang 2008b; Elmaghraby et al. 2009; Mersereau and Zhang 2012). Although previous studies consider different settings and assumptions, such as the number of price changes or the consumer valuation distribution, they all find that committing to prices can reduce the negative impact of strategic waiting to the firm by credibly raising the price in later periods, giving forward-looking consumers less incentive to strategically delay a purchase.⁵ Price commitment, rather than increasing the cost of strategic behavior (as discussed in the preceding section), works by reducing the “value” of a strategic purchasing delay. However, while price commitment has demonstrated benefits when consumer behavior is exogenously specified, its effects on the endogenous behavior model have not been considered; hence, in this section we analyze the performance of price commitment under endogenous behavior.⁶ We use the superscript *c* to refer to price commitment with endogenous behavior.

⁵ An exception is Cachon and Swinney (2009b), who find dynamic pricing can perform better than price commitment; however, in their model this result is driven by the ability of dynamic pricing to react to fluctuations in uncertain demand, a feature not present in our model of deterministic demand.

⁶ Similar to price commitment, *inventory commitment* allows the firm to commit to a pre-specified amount of inventory in future periods. This strategy can generate value by creating availability risk which encourages consumers to purchase earlier (Su 2007; Liu and van Ryzin 2008b; Zhang and Cooper 2008; Levin et al. 2010). In our model, because demand is deterministic, price and inventory commitment are in fact equivalent. Thus, in what follows, although we state our results in terms of price commitment, all results also apply to inventory commitment.

To begin our analysis, Lemma 4 replicates the result of Lemma 3 under a price commitment strategy:

Lemma 4. *With price commitment, endogenous behavior, and uniform valuations on $(0, 1)$, there exists a unique SPNE in which all consumers with valuations greater than \bar{v}^c purchase in period 1 and all consumers with valuations less than \bar{v}^c wait until period 2. Furthermore:*

- (i) *If $k \geq \frac{\delta}{3}$, all consumers choose to be myopic, and $p_1^c = \frac{2}{3}$, $p_2^c = \frac{1}{3}$, and $\bar{v}^c = \frac{2}{3}$.*
- (ii) *If $\frac{\delta - \delta^2}{3 - \delta} \leq k < \frac{\delta}{3}$, all consumers choose to be myopic, and $p_1^c = \frac{\delta + k}{2\delta}$, $p_2^c = \frac{\delta - k}{2\delta}$, and $\bar{v}^c = \frac{\delta + k}{2\delta}$.*
- (iii) *If $0 \leq k < \frac{\delta - \delta^2}{3 - \delta}$, consumers with $v \in [p_1^c, \bar{v}^c)$ choose to be strategic, and $p_1^c = \frac{2 + k}{\delta + 3}$, $p_2^c = \frac{\delta - k + 1}{\delta + 3}$, and $\bar{v}^c = \frac{2 - \delta^2 - \delta - 2k}{3 - 2\delta - \delta^2}$.*

In many ways, the equilibrium under price commitment is similar to the equilibrium under dynamic pricing: in the first two cases, no consumer chooses to be strategic, while in the last case consumers with valuations slightly higher than the first period price choose to be strategic in equilibrium. However, observe that equilibrium prices are always monotonic (and increasing) in k under price commitment; because of this, the prices with endogenous behavior lie between the equilibrium prices under the two exogenous behavior models (i.e., corresponding to $k = 0$ for the exogenous strategic model and $k = 1$ for the exogenous myopic model). Importantly, it is easy to see that fewer consumers choose to adopt a strategic purchasing rule under price commitment than under dynamic pricing for any fixed k . Therefore, keeping the cost of being strategic, k , equal, the firm benefits from price commitment as it reduces consumer incentives to be strategic.

However, this argument ignores a critical detail: the cost of being strategic, k , likely depends on the pricing strategy that the firm chooses. In particular, it is reasonable that the cost of being strategic is lower under price commitment than

under dynamic pricing: by credibly announcing a second period price at the start of the selling season, price commitment makes it relatively easy for consumers to be strategic, as they no longer need to compute an expected second period price. This presents an interesting tension: on one hand, keeping the cost k equal, price commitment is beneficial as it mitigates consumer incentives to be strategic (consistent with previous works such as Aviv and Pazgal 2008b). On the other hand, adopting a pricing strategy that makes it “easier” to be strategic may lower k and encourage more consumers to adopt a strategic purchasing rule. Given these opposite forces, it is not immediately clear which pricing strategy is most valuable to the firm. The following theorem formalizes this argument:

Proposition 5. *Let k_c and k_d be the costs of strategic behavior under price commitment and dynamic pricing, respectively. Then, under endogenous behavior and uniform valuations on $(0, 1)$:*

(i) *If $k_c = k_d$, firm profit is weakly greater under price commitment than under dynamic pricing. Furthermore, both pricing mechanisms yield the same profit if and only if $k_c \geq \frac{\delta}{3}$ and $k_d \geq \frac{\delta}{3}$.*

(ii) *For every discount factor δ and every $k_c < \frac{\delta}{3}$, there exists a unique threshold $\bar{k}_d(k_c, \delta) \in (k_c, \frac{\delta}{3})$, such that dynamic pricing results in strictly greater firm profit than price commitment if and only if $k_d > \bar{k}_d(k_c, \delta)$.*

The theorem shows that while price commitment dominates dynamic pricing when the costs of strategic behavior are equal (part (i)), dynamic pricing dominates if the costs of strategic behavior are sufficiently different (part (ii)). Figure 2.6 illustrates this tension by plotting the firm’s optimal pricing strategy as a function of k_c and k_d ; previous research, with exogenously specified strategic behavior, essentially focuses solely on the origin of the graph ($k_c = k_d = 0$). As the figure shows, for most of the parameter space, dynamic pricing is weakly optimal. Indeed, it is

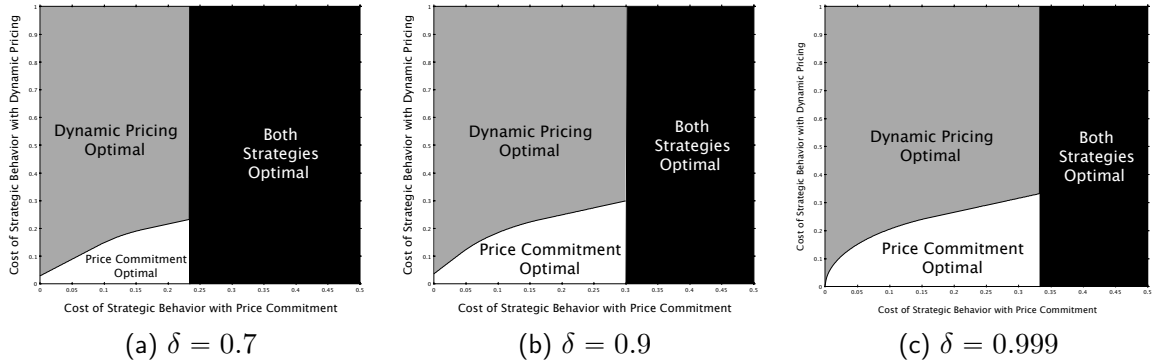


FIGURE 2.6. Firm preference between dynamic pricing and price commitment as a function of the cost of strategic behavior (k_d , vertical axis, and k_c , horizontal axis).

reasonable that the cost of strategic behavior is negligible with price commitment (i.e., $k_c \rightarrow 0$). In that case, dynamic pricing is optimal even for a small non-zero cost to strategic behavior ($k_d \gtrsim 0.04$ when $\delta = 0.9$, for example). Thus, in contrast to numerous earlier works that demonstrate the (often significant) value of price commitment under exogenous strategic behavior, under endogenous behavior, price commitment can actually reduce firm profit if it results in a reduction of the cost of being strategic for consumers. This finding may provide a solution to a common puzzle found in the pricing literature: if price commitment is so effective at mitigating strategic waiting, why do most firms seem to use dynamic pricing instead? Our model suggests that one possible answer to this question is that price commitment is less effective at mitigating strategic behavior than previously believed because it makes it easier (and less costly) for consumers to be strategic. Combined with the benefits of dynamic pricing to match supply with stochastic demand, this fact may mean that, in practice, firms find little value in committing to future prices. Indeed, perhaps the most notorious practitioner of price commitment in practice, Filene’s Basement department store—famous for automatically marking down inventory by set percentages at regular intervals—experienced poor performance throughout the early 2000s and closed all stores in 2011; the firm eventually re-opened as an exclu-

sively online retailer in 2015, no longer employing automatic markdowns (Radsken 2015).

More broadly, this discussion illustrates that it is important to consider how an alternative selling strategy designed to mitigate strategic waiting impacts consumer incentives to adopt a strategic purchasing rule in the first place. Some mitigation strategies, such as price commitment, seem likely to reduce the cost of strategic behavior; others, such as limiting inventory displays to obscure availability information (Yin et al. 2009b) or rotating product assortment more frequently (Bernstein and Martinez-de Albéniz 2014) may increase the cost of strategic behavior, giving these strategies additional value beyond that which has been previously understood.

2.7 Conclusion

While strategic consumer behavior has received significant research and practical attention in recent years, a key question has, until now, gone unanswered: should consumers be strategic? In this paper, we have shown that the answer to this question is not as straightforward as it might seem. Although, in the absence of any costs, a strategic purchasing rule is clearly optimal for each individual consumer, in equilibrium many consumers (and society) are worse off than they would have been had all consumers followed a myopic purchasing rule. Motivated by this finding, we have investigated a model of endogenous consumer behavior in which individual consumers must choose whether to exert costly effort to adopt a strategic purchasing rule, with the goal of understanding how this choice impacts firm decisions and equilibrium consumer behavior.

Our analysis has several important implications. First, we have shown that when consumer behavior is endogenous, in contrast to the case when consumer behavior is exogenously specified, consumer (and social) welfare may be maximized when the cost of strategic behavior is strictly positive: if the cost is too low, too many con-

sumers choose to be strategic, leading to a smaller market and higher prices. This suggests that firms may not only benefit *themselves* by increasing the cost of strategic behavior (e.g., by offering complicated sales schemes or making markdowns less predictable); they may also benefit consumers and society as a whole, highlighting why making it harder for consumers to be strategic may actually attract more consumers. Further research into this issue might study specific strategies to increase the cost of strategic behavior in greater detail. For instance, although we have abstracted away from the details of dynamic consumer learning to isolate our main research question, future work may attempt to investigate more detailed models of information gathering by consumers to explore precisely how firms can make learning more difficult and, as a result, strategic behavior more costly to execute.

Second, our results show that strategies to mitigate strategic behavior that are believed to perform well may in fact decrease profit if they impact the cost of being strategic. Using this logic, we have shown that committing to a pre-specified price path, long believed to be more effective than dynamic pricing at mitigating strategic behavior, can in fact backfire and reduce firm profit if it also reduces the cost of strategic behavior for consumers. This may help to explain why dynamic pricing is far more prevalent in practice than commitment; in addition to the inherent ability of dynamic pricing to react to stochastic demand, in practice it may be the case that price commitment reduces the cost of strategic behavior enough that there is little or no behavioral benefit from this strategy.

Third, we have developed a novel model of endogenous strategic consumer behavior, and shown that considering the consumer choice to be strategic can significantly impact both firm and consumer optimal decisions. For the firm, this impact extends to both tactical decisions (i.e., the firm's optimal dynamic pricing policy is non-monotonic in the cost of strategic behavior and may consist of lower prices in both periods than the myopic optimal prices) and strategic decisions (i.e., the firm may

find value in raising the cost of strategic behavior and in pricing dynamically instead of committing to a price path). Indeed, our model makes empirically testable predictions about these decisions that run counter to previous theoretical research that assumes exogenous behavior that may be interesting avenues to explore in future research. In addition, future theoretical work may employ our endogenous behavior model to investigate the impact of the consumer choice to be strategic on other firm actions, such as product assortment or supply chain design.

Lastly, while we have focused on a particular type of strategic behavior, i.e., inter-temporal purchase timing, many other types of more broadly defined “strategic” consumer actions have been identified in the literature (e.g., choosing when to arrive to a service system, Lariviere and Van Mieghem 2004, or choosing whether to shop given limited inventory availability or price information, Dana and Petruzzi 2001b; Cachon and Feldman 2015a). An intriguing issue is whether strategic behavior in other contexts can be detrimental to consumers in the same way that strategic behavior in our model is, which could have serious consequences for the way the firm manages its marketing efforts and operations, and the way consumers are advised to behave.

Omnichannel Retail and Inventory Integration with Strategic Consumers

3.1 Introduction

In the past decade, retailers have increasingly moved to integrate previously independent selling channels to create a unified shopping experience for consumers, transitioning from a *multichannel* paradigm—in which multiple channels, e.g., online and offline, are managed independently—to an *omnichannel* paradigm in which these channels are managed jointly (Brynjolfsson et al. 2013; Bell et al. 2014; Forrester 2014). While there are many elements to an omnichannel retail strategy, including marketing and sales efforts, we focus on a critical operational component: *inventory integration*, which refers to the virtual pooling of inventory across different channels and locations to form a single stock that can be used to satisfy consumer demand from any channel, allowing consumers to, for example, purchase a product online and pick it up in a physical store, or have an item that is stocked out in a physical store delivered to the customer via the online channel. Integration creates a consis-

tent customer experience across channels, preventing, for example, one channel (such as an offline store) from being out-of-stock while another channel (the online store) has inventory in-stock. While such capabilities were at first rare, many consumers now expect firms to follow an inventory integration strategy; Forrester Research reports that cross-channel fulfillment programs are a top strategic priority for retailers, and moreover, 62% of retailers invest in omnichannel initiatives mainly because of customer expectations (Forrester 2014).

Inventory integration is perceived to be valuable to retailers for two reasons. First, it increases inventory availability and reduces lost sales; if one channel stocks out, the other can fulfill its demand, resulting in statistical economies of scale that reduce a retailer's inventory costs and increase profit (in the classic sense of inventory pooling, e.g., Eppen 1979). Second, by providing greater inventory availability, customer service is improved, encouraging existing customers to shop more frequently and attracting new customers to the firm (Dana and Petruzzi 2001a; Clifford 2010). Consumers are increasingly intolerant of stock-outs and frequently do not waste their time with a store visit when they anticipate low inventory availability; Forrester reports that more than 30% of consumers that are likely or very likely to visit a store when the perceived availability is high are, at the same time, not likely to visit if perceived availability is low (Forrester 2014). As a result, ensuring high inventory availability to satisfy customers is critical, and many retailers, such as the department store Nordstrom and the Jones Apparel Group, as well as third-party e-commerce and logistics providers, have begun to integrate inventory systems and fulfillment capabilities across offline and online channels (Clifford 2010; Fowler and Dodes 2010).

Concurrent with these advances, and in many of the same industries, consumers have become more sophisticated and strategic in their shopping behavior. Particularly in industries characterized by predictable seasonality and markdown patterns

such as apparel, consumers have become accustomed to end-of-season clearance sales and have learned to strategically wait for discounts before purchasing (Rozhon 2005; Soysal and Krishnamurthi 2012; Li et al. 2014a). Inter-temporal consumer purchasing incentives are directly tied to inventory availability, and, generally speaking, greater availability encourages more consumers to strategically wait for discounts, hurting firm profit in the process (Liu and van Ryzin 2008a; Su and Zhang 2008a; Cachon and Swinney 2009a, 2011). Thus, while inventory integration is perceived to be valuable via its ability to encourage more consumers to visit the firm, how it influences inter-temporal purchase timing, and whether integration ultimately benefits the firm when considering both effects, remains an open question.

In this paper we explore precisely these issues by analyzing a model of inventory integration for a firm selling to strategic consumers. The product is seasonal in nature and sold over two periods; in the first period, the firm charges a high (“full”) price, while in the second period it charges a low (“clearance”) price. Consumers in our model are strategic along two dimensions: they decide *whether* to visit the firm (taking into account anticipated product availability and their own cost of visiting the firm) and also *when* to purchase the product (taking into account the expected price path and inventory availability over both periods). The firm sells the product in two distinct markets, which may be separate channels, e.g. online and offline markets, or two distinct geographic locations, and chooses between two selling strategies: a multichannel strategy—in which inventory for each market is managed independently and there is no transfer of inventory between markets—and an integrated strategy in which inventory for both markets is managed jointly and consumer demands from either market may be fulfilled by either stock of inventory.

We derive the firm’s optimal inventory decisions and the equilibrium consumer actions under each of these strategies, and show that inventory integration possesses two sources of value to the firm: operational (i.e., the value associated with

statistical economies of scale resulting from pooling inventory across channels) and behavioral. While the operational value is always positive, it is decreasing in the consumer visit cost and patience level (i.e., their willingness to wait for a clearance sale). Conversely, the behavioral value, which derives from the impact of integration on inventory availability and hence consumer purchasing incentives, is more ambiguous: while integration always increases inventory availability in the first (full price) period, it may increase or decrease availability in the second (clearance price) period. As a result, integration will generally encourage more consumers to visit the firm, but it may mitigate or aggravate strategic waiting until the second period. These two effects can push firm profit in opposite directions, and as a result, the behavioral value of integration may be positive or negative, depending on the characteristics of consumers. Specifically, integration is most likely to mitigate strategic waiting (and thus yield significant positive behavioral value to the firm) when consumers have a high visit cost and are very patient, or if the markets are negatively correlated or symmetric in size. On the other hand, integration is mostly likely to aggravate strategic waiting (and thus yield low or negative behavioral value) when consumers have a low visit cost and are very patient, or if the markets are positively correlated or asymmetric in size. Overall, we find that the total value of integration (operational plus behavioral) is lowest when consumers have a high visit cost and are very patient, suggesting the decrease in the operational value outweighs the potentially high behavioral value in these circumstances. Because of this, inventory integration tends to be less valuable (in terms of the absolute change in firm profit) in the presence of strategic consumers, and indeed can backfire and reduce firm profit even without considering any fixed cost to implementing an integrated system. In particular, this shows that firms should take care to consider the characteristics of their market—including consumer characteristics like visiting costs and patience level—in addition to more traditional operational measures like market correlation

when choosing whether to implement inventory integration.

The remainder of this paper is organized as follows. In §3.2, we review the related literature. In §3.3 and §3.4, we introduce and derive the equilibrium to our model, respectively. In §3.5 we determine the components of the value of integration, and in §3.6 we analyze that value in three special cases: with non-strategic consumers, with consumers that strategize only over whether to visit the store, and with consumers that strategize only over when to purchase the product. In §3.7 we return to the general case in which consumers strategize simultaneously over whether and when to purchase the product. In §3.8 we generalize our results to the case of asymmetric component markets, and §4.5 concludes the paper. Proofs and supporting results are included in the appendix.

3.2 Related Literature

Our work contributes to several streams of literature. The first concerns inventory pooling—the practice of consolidating multiple inventory locations—which has received significant attention following the seminal work of Eppen (1979), who analyzes the value of pooling with normally distributed demand. Subsequent work in this area, including Eppen and Schrage (1981), Federgruen and Zipkin (1984), Gerchak and Mossman (1992), Gerchak and He (2003), and Corbett and Rajaram (2006), extended these early results to inventory systems with positive lead times and to more general demand distributions. More recently, Bimpikis and Markakis (2015) show that some common intuition regarding inventory pooling may not hold under heavy-tailed demand distributions. We focus specifically on the value of inventory integration on under strategic consumer behavior, a topic that has not, to the best of our knowledge, been addressed in the pooling literature. Indeed, the vast majority of the literature ignores consumer behavior altogether; an exception to this is Anupindi and Bassok (1999), who show that pooling may hurt a manufacturer supplying two

retailers when customers are sufficiently willing to search for inventory themselves. However, they do not consider the same set of consumer decisions that we do, i.e., consumers that choose whether to visit the firm and when to purchase the product.

Our research is also related to the literature on newsvendor models with availability-dependent market size. Early examples include Baker and Urban (1988), Gerchak and Wang (1994), and Urban and Baker (1997). In these works, however, demand exogenously depends on the product availability and is not driven from an explicit model of consumer choice. An exception is Dana and Petruzzi (2001a), who study a model in which consumers choose between visiting the firm and consuming an exogenous outside option. They show that, given such dynamics, the firm should maintain a higher fill rate to attract more customers. However, they consider neither inventory integration nor the consumer decision to strategically delay a purchase. A number of recent papers in operations have addressed the latter point—strategic purchase timing by forward-looking consumers—including, to name a few, Aviv and Pazgal (2008a), Su and Zhang (2008a), Liu and van Ryzin (2008a), Cachon and Swinney (2009a), and Aviv et al. (2015); however, none of these works have considered inventory integration, as we do.

Finally, our model is also related to the emerging body of research on omnichannel retail. Chen et al. (2008) consider service-dependent demand when a manufacturer manages online and offline retail channels. In their model, these two channels compete with one another and are hence substitutes, rather than complements, for both customers and the firm. Gallino and Moreno (2014) empirically study the impact of buy-online-pick-up-in-store programs on sales and consumer behavior. Bell et al. (2015) use data to examine the impact of showrooms—offline locations that do not sell goods or hold inventory, but rather only provide information for consumers—on online sales. Gao and Su (2016) consider the consumer choice to gather information and buy the product either online or offline, when both the value and availability

of the product are uncertain. In contrast to existing papers in this stream of literature, we analyze the impact of inventory integration on whether and when consumers choose to purchase the product.

3.3 Model

A firm sells a single product in two distinct markets (i.e., channels or physical locations), labeled 1 and 2, in a selling season with two successive periods. The size of the consumer population in each market is stochastic and denoted by the normal random variable D_i , $i \in \{1, 2\}$, with mean μ_i and standard deviation σ_i . The correlation between market demands is $\rho \in (-1, 1)$. Initially, we assume that market demands are identically distributed, i.e., $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$; in §3.8 we relax this assumption and consider the impact of market asymmetry. The firm may operate in one of two systems. In a *multichannel* system, denoted by subscript m , inventory is committed to an individual market prior to the resolution of demand uncertainty, and once inventory has been committed to one market it cannot be used to satisfy demand in the other market (i.e., there is no transshipment or consumer search). In a multichannel system, the amount of inventory committed to market i is q_i , and expected firm profit in market i is $\pi_i(q_i)$, $i = 1, 2$. In an *integrated* system, denoted by subscript I , demand streams from both markets pull from a centralized stock of inventory; this may represent a literal centralized inventory repository or it may represent virtual centralization with transshipment. We assume that the fixed and variable costs of integration are zero; this is consistent with much of the literature (e.g., Eppen 1979) and represents a best case scenario for the value of integration.¹ In an integrated system, total inventory is q_I and firm profit is $\pi_I(q_I)$.

In both systems, the dynamics of the selling season follow the classical newsven-

¹ Non-zero fixed costs are trivial to incorporate to our model, but non-zero variable costs, i.e., transshipment costs, can become quite complicated even in the absence of consumer behavior effects; see Rudi et al. (2001) and Zhao et al. (2008).

dor model with salvaging. Before the season starts, the firm procures inventory at unit cost c . During the first (full price) period the firm charges price p , and in the second (clearance price) period charges price s , where $s < c < p$. We assume that any amount of unsold inventory at the end of the second period can be salvaged to a discount store (or any other selling channel) at price s to serve a distinct set of consumers which we refer to as bargain hunters; this assumption is made to ensure that all inventory can be cleared at the end of the season, which will enable us to derive closed form expressions for firm inventory and profit (see also Su and Zhang 2008a and Cachon and Swinney 2011). Consistent with the literature on inventory pooling and channel management (e.g. Corbett and Rajaram 2006; Netessine and Rudi 2006; Gao and Su 2016), prices in our model are exogenously given and the same for both systems. In practice, this holds for products where price is primarily determined by firm strategy and not by the inventory integration decision; for example, Gao and Su (2016) note that Warby Parker eyewear is sold at a fixed price that did not change even when the firm introduced an offline channel to complement the pre-existing online channel. Indeed, a key aspect of omnichannel retail is setting consistent prices in different channels even if inventory is not integrated, and moreover there is little evidence that apparel retailers such as Nordstrom have changed their pricing tactics after adopting inventory integration (Clifford 2010), e.g., a shirt that sells for \$99 before integration is likely sold at \$99 after integration, although, importantly, integration may impact the likelihood of and amount of inventory remaining for a clearance sale.

Consumers have heterogeneous valuations distributed according to the continuous function $G(\cdot)$, where $\bar{G}(x) = 1 - G(x)$, with support on the interval (v_l, v_h) . Consumers exert costly effort, $k \geq 0$, each time they visit the store. This could be the cost of traveling to a physical store or the hassle of visiting a website for an online purchase (Moon et al. 2016). We assume that this cost is the same in both

markets, which is a reasonable assumption when markets represent different physical locations. In the case of distinct channels, e.g. online and offline, this assumption may not hold (although, as Moon et al. 2016 empirically demonstrate, the “visit cost” for online purchasing can be extremely high, as much as \$2-\$25 per click), although we expect that incorporating different costs for different markets would not qualitatively impact our results. Given this visit cost, consumers decide whether to visit the store (i.e., whether to pay k and attempt to purchase a unit) and when to purchase the product (in period 1 or period 2). Consumers also anticipate the availability of the product in both periods when deciding whether—and when—to visit the firm. In period 1, they discount the future value of the product by $\delta \in [0, 1]$. Thus, a strategic consumer that values the product at v in the first period has valuation δv for the product in the second period. Consequently, δ represents the patience level of consumers, where higher values of δ represent very patient consumers that are more willing to delay a purchase for a future discount.²

All consumers are thus strategic along two dimensions, determining whether to visit the store and when to purchase in order to maximize their expected utility, trading off price, the depreciation in value of the product, and the availability of the product in each period. Formally, at the beginning of the season, consumers form a belief about the probability of the product being available in each period, namely $\tilde{\xi}_{ij}$ in market $i \in \{1, 2, I\}$ and period $j \in \{1, 2\}$, where the (\cdot) symbol denotes a belief. The precise nature of these beliefs are discussed in the following section. For now, we merely state that all consumers possess common beliefs. Each consumer then has two options: visit the store in period 1, or wait until period 2. If they visit the store in the first period and the firm is out-of-stock the game ends (i.e., there is no reason for the consumer to strategically delay). If the firm is in-stock, then the consumer

² An alternative interpretation is that δ is a measure of the perishability (or fashionability) of the product; we focus on the interpretation that δ represents consumer patience throughout the paper, although the perishability interpretation is equally valid.

updates her belief about the availability of the product in the second period, $\tilde{\xi}_{i2}(A)$, where A stands for the event of observe the product is available in the first period, and can further choose between a certain purchase at a high price (in period 1) and a delayed, but uncertain, purchase at the lower clearance price (in period 2) with the additional hassle of a return visit to the store.

Consequently, there are three possible ways that a consumer can purchase a unit. If a consumer purchases the product in the first period she receives surplus $v - p - k$. If she visits only in the second period and purchases the product she receives surplus $\delta v - s - k$. If she visits in both periods and purchases in the second period, she receives surplus $\delta v - s - 2k$. If the consumer decides not to visit the firm, then she receives zero surplus. Given this, a strategic consumer visits the firm in the first period if

$$\tilde{\xi}_{i1} \times \max \left(v - p, \tilde{\xi}_{i2}(A)(\delta v - s) - k \right) - k \geq \max \left(\tilde{\xi}_{i2}(\delta v - s) - k, 0 \right).$$

Conditional on a visit in the first period, the consumer would purchase in the first period if $v - p \geq \tilde{\xi}_{i2}(A)(\delta v - s) - k$. To maintain interesting solutions, we assume $v_h > p + k$ (otherwise, no consumers would ever purchase at the full price), and for technical purposes we assume the consumer valuation distribution has no mass on the endpoints ($\bar{G}(v_h) = G(v_l) = 0$). Without loss of generality we also assume that $s = 0$.³ The following proposition provides our first result concerning the purchasing behavior of the strategic consumer segment:

Proposition 6. *In equilibrium, each consumer visits the firm at most once. Fur-*

³ Note that consumers in our model are paying the cost k to visit the firm. Therefore, a salvage value equal to 0 does not mean that consumers in the second period are purchasing the product at a cost, to them, of zero. Thus, we assume $s = 0$ to eliminate one variable from the notation, while keeping the insights the same.

thermore, there exists a critical consumer valuation in market i

$$v_i^* = \min \left(v_h, \max \left(\frac{\tilde{\xi}_{i1} p}{\tilde{\xi}_{i1} - \tilde{\xi}_{i2} \delta}, p + \frac{k}{\tilde{\xi}_{i1}} \right) \right), \quad (3.1)$$

$i \in \{1, 2, I\}$, such that a consumer visits the store and purchases in period 1 if and only if $v \geq v_i^*$.

Intuitively, high valuation consumers do not wish to risk waiting for a markdown and attempt to purchase at a high price, while low valuation consumers (with less potential loss if there is a stock-out) are more willing to strategically delay a purchase. Proposition 6 implies that demand from strategic consumers in period 1 is given by $\bar{G}(v_i^*)D_i$. In the second period, only those strategic consumers would demand the product that (i) have not purchased in the first period, i.e., their valuation is less than v_i^* , and (ii) find the expected utility of visiting in the second period positive, i.e., $\tilde{\xi}_{i2}\delta v - k \geq 0$. As such, the fraction of strategic consumers who visit the firm in the second period is given by $[G(v_i^*) - G(\frac{k}{\tilde{\xi}_{i2}\delta})]^+ D_i$, where $(x)^+ = \max(x, 0)$.⁴

Consumers are not the only entities possessing beliefs in our model; the firm possesses a belief concerning the critical consumer valuation in market i (and hence the total period 1 demand in each market), which we label \tilde{v}_i . Given these consumer characteristics and firm beliefs, we may now write the expression for expected firm profit in market i as a function of inventory given a particular belief about consumer purchasing behavior:

$$\pi_i(q_i) = \mathbb{E}_{D_i} \left[p \min(q_i, \bar{G}(\tilde{v}_i)D_i) - cq_i \right]. \quad (3.2)$$

Lastly, we define a useful quantity

$$\Pi(\mu, \sigma) = (p - c)\mu - p\sigma L(\hat{z}) - c\hat{z}\sigma,$$

⁴ Note that $G(v_i^*) - G(\frac{k}{\tilde{\xi}_{i2}\delta}) < 0$ can only occur when $v_i^* = \min(v_h, p + \frac{k}{\tilde{\xi}_{i1}})$; otherwise, there exists a customer who is not purchasing the product that could purchase in the first period and receive a positive surplus.

where \hat{z} is the standard normal z -statistic corresponding to an in-stock probability of $\frac{p-c}{p}$, $L(\hat{z}) = \int_{\hat{z}}^{\infty} (t - \hat{z}) \phi(t) dt$ is the standard normal loss function, and $\phi(t)$ is the standard normal density function. Hence, $\Pi(\mu, \sigma)$ is the optimal newsvendor expected profit when demand is normally distributed with mean μ and standard deviation σ ; this will represent an upper bound on firm profit in our model.

3.4 Equilibrium to the Firm-Consumer Game

Having defined the underlying characteristics of both the firm and the consumer population, we may now derive the equilibrium to the game between the firm and consumers. We begin by discussing beliefs in the game, starting with consumer beliefs about inventory availability. Strategic consumers make their purchasing decisions based on the anticipated probability of obtaining a unit in each period, $\tilde{\xi}_{ij}$, $i \in \{1, 2, I\}$ and $j \in \{1, 2\}$. We assume that consumers do not directly observe and react to the inventory level of the firm when making their purchasing decisions. In other words, the firm does not act as a sequential leader in an inventory game; rather, we assume that consumers possess a common, fixed belief $\tilde{\xi}_{ij}$ of the inventory availability, a belief which we will require to be correct in equilibrium (i.e., a rational expectations assumption, see Su and Zhang 2008a; Cachon and Swinney 2009a, 2011).⁵ We make this assumption due to the fact that, in practice, it is difficult for individual consumers to accurately observe the inventory level of a firm (e.g., because inventory is held in many locations such as retail shelves, back rooms, warehouses and distribution centers, and throughout the supply chain) and moreover a firm does not necessarily have incentives to reveal this information to consumers (Yin et al. 2009a), implying that even if consumers could directly observe inventory they may

⁵ Consumers might form such expectations via repeated interaction with the firm in many single shot games, learning about the average clearance sale availability on similar products over time. Su and Zhang (2009) demonstrate how consumer beliefs about availability can converge to the one-shot fixed expectation equilibrium in such a setting.

not believe that this information is credible.⁶ We assume that inventory in each period is randomly allocated to consumers (Liu and van Ryzin 2008a). In other words, the probability of obtaining a unit for a customer in a period is given by the period’s *fill rate*, i.e., the fraction of demand that is fulfilled. As such, we generically refer to the first period and second period availability as first and second period fill rates, respectively.

Consequently, consumers and the firm play a simultaneous game: the firm chooses an inventory level subject to some belief about consumer purchasing behavior (i.e., what fraction of consumers will purchase at the full price) while consumers choose whether to visit and when to purchase given their belief about the availability of the product in the first and second period. We call this game the *firm-consumer game*, and an equilibrium to this game is a set of actions (inventory level and critical consumer valuation) in which both consumers and the firm choose optimal actions in response to their beliefs, which are consistent with the equilibrium outcome. Thus, we define the equilibrium as follows:

Definition 1. *An equilibrium to the firm-consumer game satisfies the following conditions, for $i = 1, 2$ in the multichannel system and $i = I$ in the integrated system:*

1. *The firm maximizes expected profit subject to beliefs about consumer behavior:*

$$q_i^* = \arg \max_{q_i} \pi_i(q_i, \tilde{v}_i).$$
2. *Consumers purchase in the period that maximizes their expected surplus subject*

⁶ Some firms do announce online whether a product is in-stock or not, both for online and offline purchases (Forrester 2014; Gallino and Moreno 2014). In this case, consumers may know whether the product is available before visiting the store, but not the precise inventory level. In our model, if the firm marks the product as unavailable, then game between consumers and the firm ends. However, if the product is marked available in the first period, strategic consumers should update their beliefs about the availability of the product in the second period, given that it is available in the first period, before deciding whether to visit the store in either period. Therefore, a model with credible availability announcements can be treated as a special case of our model with an effective visit cost of $k' = 0$ and an effective price of $p' = p + k$. We return to this point in the conclusion of the paper.

to beliefs about product availability in the clearance period:

$$v_i^* = \min \left(v_h, \max \left(\frac{\tilde{\xi}_{i1} p}{\tilde{\xi}_{i1} - \tilde{\xi}_{i2} \delta}, p + \frac{k}{\tilde{\xi}_{i1}} \right) \right).$$

3. Firm beliefs are rational: $\tilde{v}_i = v_i^*$.

4. Consumer beliefs are rational: $\tilde{\xi}_{ij} = \xi_{ij}(q_i^*, v_i^*)$.

In the above definition, $\xi_{ij}(q_i^*, v_i^*)$ represents the actual probability of obtaining a unit in each period given an inventory level q_i^* and critical consumer valuation v_i^* . First period demand is given by $\bar{G}(v_i^*)D_i$ and second period demand is given by $(G(v_i^*) - G(k/\xi_{i2}(q_i^*, v_i^*)\delta))^+ D_i$. Sales are equal to the minimum of demand and the available inventory. Similar to Dana and Petruzzi (2001a), we assume that consumers condition their beliefs about demand on their own existence in the market. In other words, they learn about demand as they find that they are a consumer. Let D_i^c be the consumers' posterior belief of demand, conditioning on their own existence. Then, D_i^c is distributed according to the density function

$$h_i^c(x) = \begin{cases} \frac{x h_i(x)}{\mathbb{E}[D_i | D_i > 0] \Pr(D_i > 0)} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad (3.3)$$

where $h_i(x)$ is the density function of D_i . Thus, the first period fill rate—as perceived by a consumer conditioning on her own existence—is

$$\xi_{i1}(q_i^*, v_i^*) = \mathbb{E}_{D_i^c} \left[\frac{\text{Period 1 Sales}}{\text{Period 1 Demand}} \right] = \mathbb{E}_{D_i^c} \left[\min \left(1, \frac{q_i^*}{\bar{G}(v_i^*)D_i^c} \right) \right],$$

while the second period fill rate is

$$\begin{aligned} \xi_{i2}(q_i^*, v_i^*) &= \mathbb{E}_{D_i^c} \left[\frac{\text{Period 2 Sales}}{\text{Period 2 Demand}} \right] \\ &= \mathbb{E}_{D_i^c} \left[\min \left(1, \frac{(q_i^* - \bar{G}(v_i^*)D_i^c)^+}{(G(v_i^*) - G(k/\xi_{i2}(q_i^*, v_i^*)\delta))^+ D_i^c} \right) \right]. \end{aligned} \quad (3.4)$$

The first period fill rate, $\xi_{i1}(q_i, v_i)$, is a well defined continuous function of v_i and q_i . However, note that $\xi_{i2}(q_i, v_i)$ implicitly depends on itself, which suggests that $\xi_{i2}(q_i, v_i)$ may not exist, and even if it does, it may not be continuous in v_i . However, Lemmas 7 and 8 in Appendix B.1 establish that the second period fill rate $\xi_{i2}(q_i, v_i)$ is indeed well defined, and moreover there exists a unique and continuous explicit function $\gamma_i(q_i, v_i)$, such that $\gamma_i(q_i, v_i) = \xi_{i2}(q_i, v_i)$; as such, from now on we use the function $\gamma_i(q_i, v_i)$ whenever we need to emphasize the explicit form of the second period fill rate. Next, we derive the firm's best response function (i.e., the optimal inventory decision given a particular belief about consumer purchasing behavior) for both the multichannel and integrated systems:

Proposition 7. (i) *In the multichannel system, the firm's best response inventory level in each market is $q_i^*(\tilde{v}_i) = \bar{G}(\tilde{v}_i) (\mu + \hat{z}\sigma)$, and the expected profit of the firm in market i is $\pi_i^*(\tilde{v}_i) = \bar{G}(\tilde{v}_i)\Pi(\mu, \sigma)$.*

(ii) *In the integrated system, the firm's best response inventory level is $q_I^*(\tilde{v}_I) = \bar{G}(\tilde{v}_I) \left(2\mu + \hat{z}\sigma\sqrt{2(1+\rho)} \right)$, and the expected profit of the firm is $\pi_I^*(\tilde{v}_I) = \bar{G}(\tilde{v}_I)\Pi \left(2\mu, \sqrt{2(1+\rho)}\sigma \right)$.*

With the firm's best response functions in-hand, we may now demonstrate the existence and uniqueness of an equilibrium to the firm-consumer game:

Proposition 8. *In both the multichannel and integrated systems, an equilibrium to the firm-consumer game exists and is unique. In the multichannel system, the equilibrium is symmetric across markets ($v_1^* = v_2^* \equiv v_m^*$ and $q_1^* = q_2^* \equiv q_m^*$). In addition, the first period fill rate, $\xi_{i1}(q_i^*, v_i^*)$, is independent of the threshold consumer valuation and is given by $\xi_{m1}(q_m^*, v_m^*) = \mathbb{E}_{D_m^c} \left[\min \left(1, \frac{\mu + \hat{z}\sigma}{D_m^c} \right) \right] \equiv \xi_{m1}$ in the multichannel system and $\xi_{I1}(q_I^*, v_I^*) = \mathbb{E}_{D_I^c} \left[\min \left(1, \frac{2\mu + \hat{z}\sqrt{2(1+\rho)}\sigma}{D_I^c} \right) \right] \equiv \xi_{I1}$ in the integrated system.*

Interestingly, the first period fill rate only depends on the distribution of demand in each market and is independent of the threshold consumer valuation, which is a function of δ and k . In other words, from the perspective of a consumer, although strategic behavior changes the optimal *inventory level*, it does not change the equilibrium *inventory availability* in the first period; this is because the firm always adjusts its inventory level to meet a constant critical fractile of demand in the first period.

3.5 The Value of Integration

Having established that a unique equilibrium to the firm-consumer game exists, we may now analyze how that equilibrium is influenced by integration. We first characterize the equilibrium *value of integration*, which we define to be the difference between total expected firm profit in the integrated and multichannel systems. From Propositions 6-8, the absolute value of integration is:

$$\Delta_I \equiv \pi_I^* - (\pi_1^* + \pi_2^*) = \bar{G}(v_m^*)\Delta + (G(v_m^*) - G(v_I^*))\Pi\left(2\mu, \sqrt{2(1+\rho)}\sigma\right), \quad (3.5)$$

where $\Delta \equiv (pL(\hat{z}) + c\hat{z})\left(2 - \sqrt{2(1+\rho)}\right)\sigma$. Thus, the value of integration can be represented as the sum of two distinct terms. The first term in (3.5) is the product of the total fraction of demand that buys in the first period in the multichannel system, $\bar{G}(v_m^*)$, and Δ , an expression that captures the change in the optimal expected newsvendor costs resulting from integrating demand. We refer to this term as the *operational value* of integration, because this value is non-zero even when consumers are not strategic (i.e., when $\delta = k = 0$). We label the second term in equation (3.5) the *behavioral value of integration* due to the fact that it is non-zero only if $v_m^* \neq v_I^*$, i.e., only if integration results in a change in equilibrium consumer behavior. In what follows, first we analyze the operational value of integration, then we consider the behavioral value.

3.5.1 The Operational Value of Integration

The operational value of integration (in particular, the Δ term) is the source of value that has been explored by the vast majority of the operations literature on integration, starting with Eppen (1979), and represents the statistical economies of scale that occur when combining multiple stochastic demand streams to pull from a single stock of inventory. The presence of strategic consumers impacts the magnitude of the operational value of integration, as demonstrated in the following proposition:

Proposition 9. *The operational value of integration is positive and is decreasing in ρ , δ , and k . In addition, in both the multichannel and integrated systems, as consumers become more patient (δ increases), the firm carries less inventory and the second period fill rate $\xi_{i2}(q_i^*, v_i^*)$ decreases.*

It is not surprising that the operational value is decreasing in the correlation of the market demands (ρ), in line with classical results such as Eppen (1979); however, as we will see in the next section, the same may not be true of the behavioral value of integration. In addition to confirming this result, the proposition also illustrates how consumer behavior factors impact the operational value of integration. First, the proposition shows that while the operational value of integration is always positive with strategic consumers, its magnitude is smaller because of their presence; observe that since the operational value is decreasing in δ and k , this value is highest if $\delta = k = 0$, i.e., if consumers are non-strategic. This occurs because demand in the first period reduces under strategic behavior for two reasons: first, fewer consumers visit the firm because they consider availability risk and their visit cost k when deciding whether to shop; and second, some consumers that do choose to shop at the firm may strategically delay a purchase until the second period.

Note that it is not obvious that as δ increases more consumers will strategically wait until the second period in equilibrium. Indeed, according to Proposition 9, as

δ increases, the firm stocks less inventory and the second period fill rate $\xi_{i2}(q_m^*, v_m^*)$ decreases; all else equal, this will discourage consumers from postponing a purchase. However, as δ increases, strategic consumers also enjoy more value from a delayed purchase, i.e., δv increases as well. Proposition 9 proves that the increase in value dominates the decrease in availability, and hence more consumers strategically delay as δ increases, which in turn causes the firm's profit to decrease in δ . In addition, the proposition shows that as it becomes more costly for consumers to visit the firm (k increases), the operational value of integration decreases. Note that this result is not *a priori* obvious either, because there are counteracting forces in play: first, as k increases, fewer consumers are willing to risk a visit to the firm (all else equal) and therefore the firm serves a smaller market in the first period, which reduces the operational value of integration. However, as k increases, consumers that are willing to risk a visit to the store have higher valuations, meaning they are less likely to strategically delay a purchase until period 2; this effect increases the proportion of consumers that purchase in the first period, and hence increases the operational value of integration. Proposition 9 shows that the latter force is dominated by the former.

3.5.2 The Behavioral Value of Integration

The second term in (3.5) represents the behavioral value of integration. This value is proportional to two key factors: the incremental change in period 1 demand resulting from integration, $(G(v_m^*) - G(v_I^*))$, and the newsvendor optimal expected profit in the integrated system, $\Pi(2\mu, \sqrt{2(1+\rho)}\sigma)$. In other words, the behavioral value represents the incremental change in profit resulting from changing consumer behavior by influencing the critical consumer valuation. If $v_m^* > v_I^*$, the behavioral value of integration is positive, while if $v_m^* < v_I^*$ it is negative; consequently, a key determinant of the behavioral impact is whether integration increases or decreases the

equilibrium critical consumer valuation. Combining the equilibrium conditions from Definition 1, it follows that the critical consumer valuation in either the multichannel or pooled systems is determined by the solution to

$$v_i^* = \min \left(v_h, \max \left(\frac{\xi_{i1}(q_i^*, v_i^*)p}{\xi_{i1}(q_i^*, v_i^*) - \gamma(q_i^*, v_i^*)\delta}, p + \frac{k}{\xi_{i1}(q_i^*, v_i^*)} \right) \right).$$

Both the first period fill rate ($\xi_{i1}(q_i^*, v_i^*)$) and the second period fill rate ($\gamma(q_i^*, v_i^*)$) appear in the above implicit function; thus, to understand the behavioral value of integration, it is critical to understand how integration changes the first and second period fill rates. Note that if D_i has positive support, then from (3.3), D_i^c is distributed according to density function $h_i^c(x) = \frac{xh_i(x)}{\mu_i}$. It then follows that

$$\mathbb{E}_{D_i^c} \left[\frac{\text{Sales}}{\text{Demand}} \right] = \frac{\mathbb{E}_{D_i} [\text{Sales}]}{\mathbb{E}_{D_i} [\text{Demand}]}.$$

In our model, however, demand is assumed to be normally distributed, and its support includes negative values, implying

$$\begin{aligned} \mathbb{E}_{D_i^c} \left[\frac{\text{Sales}}{\text{Demand}} \right] &= \int_0^\infty \frac{\min(x, q_i^*)}{x} \frac{xh_i(x)}{\mathbb{E}[D_i|D_i > 0]Pr(D_i > 0)} dx \\ &= \frac{\int_0^\infty \min(x, q_i^*)h_i(x)dx}{\mathbb{E}[D_i|D_i > 0]Pr(D_i > 0)}, \end{aligned}$$

which is not necessarily equal to $\frac{\mathbb{E}_{D_i}[\text{Sales}]}{\mathbb{E}_{D_i}[\text{Demand}]}$. However, since demand in reality is non-negative, it is implicitly assumed (both in our model and in related inventory integration models such as Eppen 1979) that $Pr(D_i < 0)$ is very small. In this case, $\mathbb{E}_{D_i^c} \left[\frac{\text{Sales}}{\text{Demand}} \right] \approx \frac{\mathbb{E}_{D_i}[\text{Sales}]}{\mathbb{E}_{D_i}[\text{Demand}]}$. Consequently, we assume that consumers use the latter expression as the first period fill rate. In this case,

$$\xi_{m1} = \frac{\mathbb{E}_{D_m} [\min(\mu + \hat{z}\sigma, D_m)]}{\mu},$$

and

$$\xi_{I1} = \frac{\mathbb{E}_{D_I} \left[\min(2\mu + \hat{z}\sqrt{2(1+\rho)}\sigma, D_I) \right]}{2\mu}.$$

Similarly, we use $\frac{\mathbb{E}_{D_i} [\text{Second Period Sales}]}{\mathbb{E}_{D_i} [\text{Second Period Demand}]}$ for the second period fill rate instead of $\mathbb{E}_{D_i^c} \left[\frac{\text{Second Period Sales}}{\text{Second Period Demand}} \right]$. With this background, we may consider the impact of integration on the first period fill rate:

Proposition 10. *The first period fill rate in the integrated system, ξ_{I1} , is decreasing in ρ . Furthermore, $\xi_{I1} \geq \xi_{m1}$.*

This result shows that in the integrated system, inventory availability is higher in the first period than in the multichannel system; this, in turn, makes shopping in the first period more appealing to strategic consumers. Interestingly, despite the fact that when \hat{z} is positive firm carries more inventory as the market correlation increases, the first period fill rate is always decreasing in ρ . This is because as ρ increases, demand under the integrated system becomes more uncertain, and the increase in inventory does not offset the increase in uncertainty. However, an increase in the first period fill rate due to integration does not necessarily translate to higher demand from consumers in the first period, as demand in period 1 also depends on the period 2 fill rate.

In fact, integration impacts availability in period 2 in three different ways. First, an increase in the chance of obtaining a unit may increase the total number of consumers that consider visiting the firm, which intensifies the competition for inventory among consumers, i.e., increases the magnitude of demand in period 2. We refer to this as the *demand endogeneity effect*. Second, because period 2 demand is a multiple of the total market size, and integration decreases the variability of the aggregate market size, integration can decrease the variability of demand in period 2. We call this the *demand variability effect*. Third, integration may decrease the total amount

of inventory remaining in period 2. To illustrate, note that total expected leftover inventory (after period 1) in the multichannel system is $2\bar{G}(v_i^*)\sigma(\hat{z} + L(\hat{z}))$, while in the integrated system is $\sqrt{2(1+\rho)}\bar{G}(v_i^*)\sigma(\hat{z} + L(\hat{z}))$. Thus, holding v_i^* constant, integration reduces the total amount of inventory available to sell during the clearance period. In other words, for a particular critical consumer valuation, integration enables the firm to better match supply and demand, which reduces inventory sold during the clearance period. We label this the *inventory effect*.

In sum, these three effects imply that integration affects the fill rate in period 2 through multiple mechanisms: the magnitude of demand in period 2 (the demand endogeneity effect), the variability of demand in period 2 (the demand variability effect), and the amount of inventory in period 2 (the inventory effect). The interplay of these three effects determines the change in the second period fill rate and, as a result, the behavioral value of integration.

3.6 Special Cases

Due to the complex nature of the interactions governing the behavioral value of integration, their effects are challenging to analytically characterize. Thus, to gain insight into how inventory integration is impacted by strategic consumer behavior, we analyze three special cases. First, we consider the case when consumers are completely non-strategic, i.e., they do not consider the future ($\delta = 0$) and they have zero store visit cost ($k = 0$) meaning they do not need to consider inventory availability when deciding whether to purchase from the firm. In this case, consumers purchase in period 1 if and only if their valuations exceed the selling price, p , in both systems. Thus, $v_i^* = p$ in both the multichannel and integrated systems, and moreover:

Proposition 11. *When consumers are non-strategic, ($\delta = k = 0$), the behavioral*

value of integration is zero. Thus, the total value of integration is positive and decreasing in ρ .

This corresponds to a traditional model of inventory integration (e.g., Eppen 1979); in this case, integration has purely operational value that is decreasing in market correlation, as one would expect. Having established this, in §3.6.1, we assume that strategic consumers are impatient, i.e., $\delta = 0$, and hence always prefer to purchase in period 1, but have some non-zero visit cost $k > 0$. This allows us to isolate the behavioral value of integration that derives from encouraging strategic consumers to visit the store in period 1. Then, in §3.6.2, we assume that $k = 0$ (store visits are costless) but allow $\delta > 0$ to focus on the impact of integration on the inter-temporal purchasing incentives of consumers. In §3.7, we return to the general case and consider strategic consumers that are both patient ($\delta > 0$) and experience some costs associated with visiting the store ($k > 0$).

3.6.1 Impatient Strategic Consumers: $\delta = 0$

Next, we analyze the situation in which strategic consumers are impatient—and hence always prefer to purchase in period 1 over period 2—but still consider the availability of the product when deciding whether to visit the firm. The following proposition analyzes the behavioral value of integration in this case:

Proposition 12. *When strategic consumers are impatient ($\delta = 0$),*

(i) *The behavioral value of integration is positive.*

(ii) *The behavioral value of integration is increasing in k if and only if $\frac{g(p+k/\xi_{m1})}{g(p+k/\xi_{I1})} \geq \frac{\xi_{m1}}{\xi_{I1}}$.*

(iii) *The behavioral value of integration is decreasing in ρ . Consequently, the total value of integration is decreasing in ρ .*

The proposition shows that the behavioral value of integration is always posi-

tive when strategic consumers are impatient, i.e., when $\delta = 0$. This follows from Proposition 10: integration provides higher availability for consumers in the first period and therefore attracts more demand, and when consumers do not also consider strategically delaying their purchases, this implies a positive behavioral value.

Interestingly, though, part (ii) shows that the behavioral value of integration is not necessarily monotonic in k . Specifically, the behavioral value of integration is increasing in the visit cost if and only if the ratio of the marginal decrease in demand between the integrated and multichannel systems is higher than the ratio of their corresponding fill rates.⁷ Note that the threshold consumer valuation is less sensitive to an increase in k under the integrated system than the multichannel system, because the risk of not obtaining a unit upon visiting is smaller under the integrated system. Thus, increasing k causes a smaller change in v_I^* than it does in v_m^* . However, since the threshold consumer valuation under the integrated system might be located at a point where the valuation distribution is more dense, even a small change in v_I^* can exclude a large fraction of consumers, and hence the behavioral value of integration is not necessarily increasing in k . This depends heavily on the shape of the valuation distribution. In particular, if $G(\cdot)$ is convex on its support, then the condition in part (ii) is satisfied at all p and k , and the behavioral value of integration is always increasing in k . This includes, for example, the uniform valuation distribution; thus, if consumer valuations are uniformly distributed, the behavioral value of integration is increasing in the visit cost k .

Part (iii) of the proposition shows that, intuitively, when consumers are impatient the behavioral value and operational value are both decreasing in ρ . This is due to the fact that the improvement in period 1 availability in the integrated system is smaller if demands are more highly correlated; as a result, integration becomes less

⁷ Note that, as stated in Proposition 8, ξ_{i1} only depends on distribution of D_i and is independent from k and δ .

effective at encouraging customers to shop as the underlying demands become more correlated. In other words, both sources of value—behavioral and operational—are tightly related to the availability of the product in the first period, and since a higher ρ translates to a lower first period fill rate, there is less value—behaviorally and operationally—in integrating the system as markets become more correlated.

3.6.2 Free Visits: $k = 0$

We now allow strategic consumers to exhibit patience ($\delta > 0$), but we eliminate consumer visiting costs by setting $k = 0$. In effect, this eliminates the demand endogeneity effect from the behavioral value of integration, allowing us to focus solely on the trade-off between the inventory and demand variability effects. In this case, from Proposition 6,

$$v_i^* = \min \left(v_h, \frac{\xi_{i1} p}{\xi_{i1} - \xi_{i2} \delta} \right) = \min \left(v_h, \frac{p}{1 - \xi_{i2}(A) \delta} \right), \quad (3.6)$$

where $\xi_{i2}(A) = \frac{\xi_{i2}}{\xi_{i1}}$ is the probability that the product is available in the second period conditional on its availability in period 1. From Lemma 8 in Appendix B.1, for each belief \tilde{v} , the second period fill rate (and consequently the second period conditional fill rate) exists. Therefore, with some manipulation one can combine equations 3.4 and 3.6 to find the following:

$$\xi_{i2}(A) = \frac{\mathbb{E}_X \left[\min \left(B(\xi_{i2}(A)) (\hat{z} - X)^+, X + \frac{1}{cv} \right) \right]}{\mathbb{E}_X \left[\min(\hat{z}, X) + \frac{1}{cv} \right]}, \quad (3.7)$$

where $B(\xi_{i2}(A)) \equiv \frac{\bar{G} \left(\min \left(v_h, \frac{p}{1 - \xi_{i2}(A) \delta} \right) \right)}{G \left(\min \left(v_h, \frac{p}{1 - \xi_{i2}(A) \delta} \right) \right)}$ is the first period to second period demand ratio, cv is the coefficient of variation, and $X \sim N(0, 1)$. Observe that the second period conditional fill rate, and therefore the threshold consumer valuation, is not directly a function of the demand mean (μ) or standard deviation (σ); rather, it

solely depends on the coefficient of variation of the underlying distribution, a result that also holds in the general case, i.e., $\delta \geq 0$ and $k \geq 0$. With this background, the following proposition sheds light on the behavior of $\xi_{P2}(A)$ —and consequently the behavioral value of integration—as a function of market correlation:

Proposition 13. *When $k = 0$,*

(i) The conditional second period fill rate in the integrated system, $\xi_{I2}(A)$, is quasi-concave in ρ .

(ii) There exists a unique threshold $\bar{\rho}$ (possibly $\bar{\rho} = 1$) such that the behavioral value of integration is positive if and only if $\rho \leq \bar{\rho}$.

Proposition 13 illustrates that if the underlying markets are sufficiently negatively correlated, integration possesses *positive* behavioral value. This is because, in this case, integration substantially reduces supply-demand mismatch, which results in a decrease in the second period conditional fill rate, which discourages strategic waiting. Thus, the firm can utilize integration as a new method (beyond those already suggested in the literature such as price commitment and inventory rationing) to mitigate strategic waiting. In this case, since the operational value of integration is also positive, integration results in a “win-win” scenario, possessing positive operational *and* behavioral value, and will unequivocally increase firm profit as a result. This occurs, e.g., when overall firm demand is relatively predictable but precisely which channel consumers will utilize is less certain, leading to low or even negative correlation between the component demands.

Conversely, Proposition 13 also illustrates that when demands in the individual markets are sufficiently correlated, integration *increases* second period inventory availability and induces more forward-looking consumers to strategically delay a purchase until the clearance sale. This can happen, for example, for fashion products in which the overall success of a new product for the firm is uncertain, but that success

is relatively correlated in across channels or locations. In this case, integration leads to more strategic waiting because of an increase second period availability, which leads to a negative behavioral value that reduces, and may potentially even make negative, the total value of integration to the firm.

The impact of the consumer discount factor δ on the behavioral value of integration is less straightforward to characterize. Indeed, we have numerically observed that many different patterns are possible, including behavioral value that is monotonic and non-monotonic in δ . Recalling that the behavioral value of integration is proportional to $G(v_m^*) - G(v_I^*)$, observe that as δ increases, more consumers strategically delay a purchase until the clearance period, which means both v_I^* and v_m^* increase. However, depending on the shape of $G(\cdot)$ and the rate at which v_I^* and v_m^* increase, the behavioral value may increase or decrease as a result. We thus defer further study of the impact of δ on the behavioral value of integration to the next section.

Overall, examination of these special cases (in which either $\delta = 0$ or $k = 0$) shows that although inventory integration always benefits the firm operationally, it may not possess positive behavioral value. Integration is beneficial in encouraging consumers to visit the store when such visits are costly (Proposition 12), and this value typically increases in the store visit cost k . However, integration can also induce more consumers to strategically wait for discounts (Proposition 13), particularly if demand is sufficiently positively correlated between channels.

3.7 General Case: The Principles of Integration

Having isolated the impact of each type of strategic behavior by analyzing several special cases, in this section, we return to the full model (in which, potentially, both $\delta > 0$ and $k > 0$) to understand the value of integration when strategic consumers *simultaneously* decide whether to visit the store and in which period to purchase.

Table 3.1. Parameters used in numerical study.

Parameter	Values
p	10
c	{3, 5, 7, 9}
μ	150
σ	{50, 70, 90}
ρ	{-0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75}
δ	{0, 0.4, 0.8}
k	{0, 0.5, 1}
v_l	{4, 6, 8}
v_h	{12, 14, 16}
m	$\{v_h, \frac{v_l+v_h}{2}, v_l\}$

Given the complexity of the equilibrium conditions and the resulting firm profit expressions, analytical treatment of the full model is challenging; thus, in this section, we employ an extensive numerical study with the goal of answering the following questions. First, how often and under what conditions does integration possess positive or negative behavioral value? Second, how significant is the behavioral value of integration compared to the operational value, and is it ever the case that the total value of integration (operational plus behavioral) is negative? And third, how do the market and consumer characteristics impact the total value of integration?

To answer these questions we analyzed 20,412 instances comprised of every combination of the parameter values in Table 3.1. The parameters are chosen to represent a wide range of realistic scenarios. Consumer valuations are distributed according to a triangular distribution with support (v_l, v_h) and mode m . When $m = v_h$, the density function is increasing, meaning that consumers are concentrated at the high end of the valuation distribution; when $m = \frac{v_l+v_h}{2}$, consumers are concentrated at the midpoint of the distribution; and when $m = v_l$, consumers are concentrated at the low end of the distribution. Price and cost parameters are chosen such that critical ratios range from 0.1 to 0.7, while demand parameters are chosen such that the coefficient of variation ranges from 0.33 to 0.6, both of which are reasonable for

industries where omnichannel retail is common, e.g., fashion apparel. To eliminate pathological cases, we removed instances with both a very low critical ratio and high demand variability (as these could leave to low or negative inventory and profit), and in addition, we removed the instances with the total value of integration in the highest and lowest 0.01 percentile of the sample as outliers. This left us with 16,922 suitable instances. For each parameter combination in the sample, we calculated the equilibrium expected firm profit in both the multichannel and integrated systems, and determined the value of integration.

To aide in our analysis and maintain a consistent baseline for comparing the value of integration across the instances in our sample, we define the *relative value of integration* to be the change in profit in the integrated system normalized by the total profit in the multichannel system, i.e.:

$$\frac{\Delta_I}{\pi_1^* + \pi_2^*} = \frac{\Delta}{2\Pi(\mu, \sigma)} + \frac{(G(v_m^*) - G(v_I^*))}{\bar{G}(v_m^*)} \times \frac{\Pi(2\mu, \sqrt{2(1+\rho)}\sigma)}{2\Pi(\mu, \sigma)}. \quad (3.8)$$

Observe that the relative value also consists of an operational component (the first term) and a behavioral component (the second term). Given these preliminaries, we now describe several key principles of integration in omnichannel retail with strategic consumer behavior. Our first insight concerns when integration is most likely to result in non-trivial, positive behavioral value for the firm. To answer this question, we examined the likelihood (percentage of cases) under which integration had a relative total value greater than 1% of the the total multichannel system profit.⁸ These

⁸ For example, we calculate $\text{Prob}(\text{Relative Behavioral Value} > 0.01 | \delta = 0.4, k = 1)$ to measure how likely it is to observe a relative value larger than 1% when $\delta = 0.5$ and $k = 1$. Note that since we have eliminated some inappropriate instances from our initial sample, we may be biased in estimating the percentage of instances that had a positive total value with a specific parameter. For example, we may find that it is more likely to observe a positive total value when $c = 5$, than $c = 7$, simply because more instances with $c = 7$, were eliminated from the original sample. To fix this bias, we scale the probabilities relative to the size of the instances that were kept in our data. In other word, instead of focusing on the number of observations, we calculate the conditional probabilities.

Table 3.2. Percentage of cases with behavioral value greater than +1%.

	$\delta = 0$	$\delta = 0.4$	$\delta = 0.8$
$k = 0$	0%	48%	57%
$k = 0.5$	69%	79%	80%
$k = 1$	85%	87%	91%

results are summarized in Table 3.2, and lead us to make the following observation:

Principle 1. *Integration is most likely to generate significant positive behavioral value when consumers are very patient and the store visit cost is high.*

In 91% of cases in which the store visit cost is high ($k = 1$) and consumers are highly patient ($\delta = 0.8$), the behavioral value of integration was significant and positive. Thus, integration has a significant impact on influencing consumer behavior—both by inducing strategic consumers to visit the firm, and by encouraging them to purchase in the first period rather than strategically delay until the second period—under these conditions. This is an intuitive result: integration is most likely to have significant positive behavioral value precisely when consumers are most prone to behavioral effects, i.e., when the store visit cost and discount factor are both high. However, when we consider instances in which integration has a significant negative value (defined to be less than -1% of the total multichannel profit), an interesting picture emerges, as illustrated in Table 3.3 and the following principle:

Principle 2. *Integration is most likely to generate significant negative behavioral value when consumers are very patient and the visit cost is small.*

When consumers can costlessly visit the firm ($k = 0$), there is no benefit from integration in encouraging more consumers to shop; consequently, when consumers are highly patient, integration is much more likely to backfire in these instances.

Table 3.3. Percentage of cases with behavioral value less than -1%.

	$\delta = 0$	$\delta = 0.4$	$\delta = 0.8$
$k = 0$	0%	20%	23%
$k = 0.5$	0%	2.4%	7.2%
$k = 1$	0%	0.18%	0.76%

Table 3.4. Percentage of the total value of integration deriving from behavioral value.

	$\delta = 0$	$\delta = 0.4$	$\delta = 0.8$
$k = 0$	0%	10%	14%
$k = 0.5$	15%	23%	25%
$k = 1$	28%	30%	35%

Combining the insights from Principles 1 and 2, we see that integration is most likely to have a significant positive *or negative* behavioral impact with highly patient consumers. Thus, while there is significant potential for positive behavioral value with patient consumers, there is also significant risk, and particularly if the visit cost is low, integration could encourage more consumers to strategically wait for discounts and thus yield negative behavioral value. This risk reduces as the visit cost increases, making it more likely that the behavioral value of integration is positive at high k ; this is due to both the positive value of encouraging more consumers to visit the store and the fact that the negative impact of inducing more consumers to delay a purchase is mitigated at higher visit costs. The latter effect occurs because, when k is high, fewer consumers can justify a visit to the store during the clearance period, and, if the consumers are willing to purchase at all, they are more likely to do so in the first period when they value the product more; thus, a potential increase in clearance period availability due to integration is less likely to encourage strategic waiting when k is high, and consequently integration is less likely to “backfire” and result in negative behavioral value. This illustrates that integration, strategic store visits, and strategic waiting interact in a non-trivial manner: strategic stores visits moderate the potential negative consequence of integration encouraging more consumers to strategically delay a purchase.

Next, we consider how significant the behavioral value of integration is relative to the operational value; to accomplish this, we determine the average percentage of the total value of integration (operational and behavioral) coming from behavioral sources (Table 3.4), leading us to make the following observation:

Principle 3. *A significant fraction of the total value of integration comes from influencing consumer behavior—on average 19%—and this fraction is highest when consumers are patient and the store visit cost is high.*

Thus, the behavioral value of integration, when it is largest at high k and δ , is roughly the same order of magnitude as the operational value, illustrating that behavioral factors can be of equal importance to operational factors when determining the value of an integration strategy. This occurs for two reasons: one, the behavioral value increases as k and δ increase, and two, the operational value decreases under the same conditions (because fewer consumers visit the store and purchase in the first period). The net effect is that a larger and larger fraction of the total value of integration comes from behavioral sources as k and δ increase.

Given this, we next consider when integration has low or negative total value. In our sample, there are 61 instances in which the total value of integration is negative. In these cases, firm profit in the integrated system is, on average, 27% lower than its profit in the multichannel system (on average over the entire sample, integration increases firm profit by 42%). Thus, interestingly, integration can reduce firm profit (due to behavioral factors) to a significant degree even without considering any costs to integration, such as fixed costs associated with implementing an integrated IT system or variable costs associated with transshipping inventory. This leads us to conclude the following principle:

Principle 4. *Integration can have negative total value that can be significant relative to firm profit, even in the absence of any fixed or variable costs of integration.*

Below (in Principles 6 and 7) we further examine when integration is most likely to have low or high total value. Before we consider this, we examine how correlation between the markets impacts the behavioral value of integration. Figure 3.1 illustrates this impact, which leads us to propose the next principle of integration:

Principle 5. *Integration is more likely to produce a significant negative behav-*

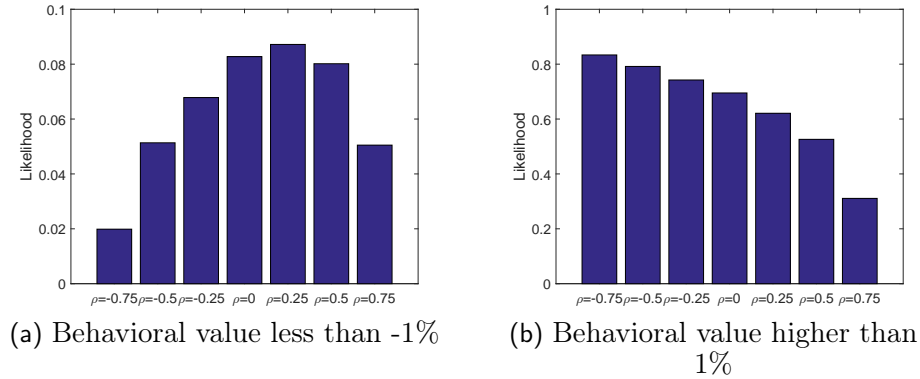


FIGURE 3.1. Likelihood of observing a behavioral value less than -1% or a behavioral value higher than 1% as a function of demand correlation ρ .

ioral value when the markets are moderately positively correlated. It is more likely to generate a significant positive behavioral value when the markets are strongly negatively correlated.

When the markets have a strong negative correlation, behavioral value is the highest: in this case, the firm can both increase availability in the first period and decrease availability in the second period because integration results in a drastic decrease in demand variability. However, the opposite is not true: the behavioral value is not most likely to be significantly negative when markets are perfectly correlated, but rather when they are mildly positively correlated. This occurs because of the opposing variability and inventory effects: when demand is highly correlated, this increase in variability in clearance period demand lowers availability in that period, which tempers the negative behavioral effects of integration. Consequently, integration is most likely to backfire (behaviorally) when markets have a slight positive correlation. Recall that Proposition 13 showed, when $k = 0$, that the second period fill rate is quasiconcave in ρ ; this means that strategic waiting is most exacerbated not at high ρ but at some intermediate ρ , which aligns with our numerical observation that integration leads to negative behavioral value in these circumstances.

Lastly, we consider how the full set of parameters impacts the absolute value of

Table 3.5. Log-linear model of the absolute value of integration.

Predictor	Estimate	SE	tStat
Intercept	-0.94646	0.029747	-31.817
c	-0.08558	0.0016462	-51.985
σ	0.011127	0.00020948	53.115
ρ	-1.5514	0.0065926	-235.32
δ	-0.022001	0.010118	-2.1745
$v_h - v_l$	0.031171	0.0014218	21.923
$\frac{v_l + m + v_h}{3}$	0.42173	0.0020903	201.76
k	-0.31716	0.0081436	-38.946

Number of observations: 16861, Error degrees of freedom: 16853

Root Mean Squared Error: 0.428

R-squared: 0.858, Adjusted R-Squared 0.858

F-statistic vs. constant model: 1.46e+04

Predictors and the model are significant at 0.05 level

integration (i.e., Δ_I) as well as the relative value of integration ($\Delta_I/(\pi_1^* + \pi_2^*)$). Due to the non-monotonocities illustrated in the preceding analysis, there is no simple answer to this question; however, we can analyze the average impact of changes in market correlation, variability, etc. on the value of integration. To accomplish this, we log-linearly regressed the absolute and relative total value of integration on all of the problem parameters.⁹ Rather than regress directly on the lower and upper bound of the consumer valuation distribution, we used the range ($v_h - v_l$) and mean ($(v_l + m + v_h)/3$) of the distribution. Table 3.5 summarizes the results of this regression when the dependent variable is the *absolute* total value of integration. All coefficients are significant even at very small levels; this is not surprising, as we numerically generated the results from these predictors and we would expect that the coefficients are non-zero. In addition, the model provides a good fit (86% adjusted R-squared). Examination of the results in Table 3.5 leads us to our next principle:

Principle 6. *On average, the absolute total value of integration increases in σ , the mean consumer valuation, and range of valuations, while it decreases in c , ρ , δ , and k .*

⁹ We also tested for other forms, such as pure linear instead of log-linear, and found that the log-linear model provided the best fit. To handle negative total value of integration as a dependent variable, we treated these instances as missing data; these values comprised only 0.3% of our sample.

In other words, integration is most valuable (in absolute terms) if σ is high and ρ is low; this is consistent with the existing literature that considers only the operational value of integration.¹⁰ Recall from Proposition 9 that the absolute operational value of integration is decreasing in k , because the operational benefits of integration are enjoyed on a smaller market as k increases. While the behavioral value of integration can be increasing in k (e.g., when $\delta = 0$ and consumer valuations are uniform; see Proposition 12), our numerical analysis shows that this does not make up for the loss in operational value when consumers have higher visit costs. Hence, the *total* value of integration is decreasing in k . The same holds for the consumer discount factor δ : on average in our sample, the absolute value of integration decreases as consumers become more patient, despite any benefits that may occur from reducing clearance period availability and discouraging strategic waiting (Principles 1-3). This leads us to conclude that, while integration possesses behavioral value that can be of significant benefit to the firm, the reduction in operational value due to strategic behavior more than outweighs behavioral benefits, leading integration to have lower (absolute) value under strategic consumer behavior.

Although the absolute value of integration is a critical metric that, in practice, would be used to justify the fixed costs associated with integrating the channels, the relative value (percentage increase in firm profit relative to the multichannel system) may also be an important consideration for firms. We analyze this value by repeating the previous regression analysis with the relative total value of integration as the dependent variable; results are reported in Table 3.6. This leads to our final principle:

Principle 7. *On average, the relative total value of integration increases in c , σ , δ , and k , and decreases in ρ , the mean consumer valuation, and the range of*

¹⁰ It is important to note that in the presence of behavioral considerations, this result is only true *on average* in our sample and may be violated for specific instances, as illustrated by the results in §§3.5-3.6.

Table 3.6. Log-linear model of the relative value of integration.

Predictor	Estimate	SE	tStat
Intercept	-4.5605	0.023659	-192.76
c	0.35425	0.0013094	270.55
σ	0.020152	0.00016661	120.95
ρ	-1.5513	0.0052435	-295.86
δ	0.19772	0.0080475	24.569
$v_h - v_l$	-0.029853	0.0011309	-26.398
$\frac{v_l + m + v_h}{3}$	-0.021897	0.0016625	-13.171
k	0.26866	0.0064771	41.478

Number of observations: 16861, Error degrees of freedom: 16853

Root Mean Squared Error: 0.34

R-squared: 0.909, Adjusted R-Squared 0.909

F-statistic vs. constant model: 2.39e+04

Predictors and the model are significant at 0.00001 level

valuations.

While the behavior as a function of ρ and σ is consistent with the absolute value (Principle 6), interestingly, as δ and k increase, the relative total value increases as well. Note that the *absolute* total value of integration exhibits precisely the opposite behavior: it tends to be lowest when consumers have a high visit cost and discount factor. Thus, integration results in the greatest relative profit boost (but the smallest absolute profit boost) when δ and k are high. Interestingly, in much of the extant literature on integration the focus has been on the impact of market correlation on the value of integration. As can be seen from the last two principles, the relative and absolute value of integration are both decreasing in ρ . Hence, whether the firm is more interested in the relative value or absolute value of integration, the qualitative impact of ρ is the same. However, the behavioral elements of our model—specifically δ and k —impact these two values in opposite ways, and thus firms should carefully consider the most important way to gauge the value of integration before determining whether it is a desirable strategy, given the behavioral characteristics of their consumers.

3.8 Asymmetric Markets

An important practical feature faced by firms considering an inventory integration strategy is that the component markets may not be symmetric. For example, the online store may receive much higher demand than the offline channel (as is the case with Warby Parker and other online retailers that began online and later added offline stores; Bell et al. 2014), or vice versa (e.g., as is the case for many traditional brick-and-mortar retailers that expand into the online space). Thus, in this section, we consider the impact of market asymmetry on the value of integration. Specifically, to maintain tractability and facilitate our analysis, we assume that both markets have the same coefficient of variation of demand, but their demands may be “larger” or “smaller” than one another, holding their combined demands constant, i.e., we let $D_1 \sim \lambda D$ and $D_2 \sim (2 - \lambda)D$, where $D \sim N(\mu, \sigma)$ is an identical (but possibly correlated) random variable in each market, and $1 \leq \lambda \leq 2$. With this model, the mean of the combined demand from both markets is always 2μ , and the model studied in the previous sections (with symmetric markets) corresponds to $\lambda = 1$.

Since λ does not change the coefficient of variation under the multichannel system, under this model the threshold consumer valuation and the fill rates will be the same in both markets; moreover, they are independent of λ . However, under the integrated system the coefficient of variation is $cv_I = \frac{\sqrt{\lambda^2 + (2-\lambda)^2 + 2\rho\lambda(2-\lambda)\sigma}}{2\mu}$, which is increasing in λ . In other words, as the markets become more asymmetric, the coefficient of variation in the integrated system increases, even though the coefficient of variation in each individual market remains constant; this is because, as one market comes to dominate the other, integration becomes less effective at reducing aggregate variability. Because of this, the threshold consumer valuation and fill rates depend on λ in the integrated system.

Similar to Propositions 7 and 8 one can show that a unique equilibrium ex-

ists under both systems, and $q_1^* = \bar{G}(v_m^*)\lambda(\mu + \hat{z}\sigma)$, $\pi_1 = \bar{G}(v_m^*)\lambda\Pi(\mu, \sigma)$, $q_2^* = \bar{G}(v_m^*)(2 - \lambda)(\mu + \hat{z}\sigma)$, and $\pi_2 = \bar{G}(v_m^*)(2 - \lambda)\Pi(\mu, \sigma)$. Therefore, $\pi_1^* + \pi_2^* = 2\bar{G}(v_m^*)\Pi(\mu, \sigma)$. Similarly, $q_I^* = \bar{G}(v_I^*(\lambda))(2\mu + \hat{z}\sqrt{\lambda^2 + (2 - \lambda)^2 + 2\rho\lambda(2 - \lambda)}\sigma)$ and $\pi_I^* = \bar{G}(v_I^*(\lambda))\Pi(2\mu, \sqrt{\lambda^2 + (2 - \lambda)^2 + 2\rho\lambda(2 - \lambda)}\sigma)$, which may increase or decrease in λ depending on the behavior of $v_I^*(\lambda)$. The value of integration is thus

$$\Delta_I = \bar{G}(v_m^*)\Delta(\lambda) + (G(v_m^*) - G(v_I^*(\lambda)))\Pi\left(2\mu, \sqrt{\lambda^2 + (2 - \lambda)^2 + 2\rho\lambda(2 - \lambda)}\sigma\right),$$

where $\Delta(\lambda) = (pL(\hat{z}) + c\hat{z})\left(2 - \sqrt{2(2 - \lambda)(2 - \lambda)(1 - \rho)}\right)\sigma$. From this, it is straightforward to verify that the results in §§3.5-3.6 hold for all values of λ . Furthermore, notice that the problem with $\lambda > 1$ and correlation ρ is equivalent to the problem with $\lambda = 1$ and correlation $\rho' = \lambda^2(1 - \rho) - 2\lambda(1 - \rho) + 1$, which is an increasing function of λ . Therefore, in essence, asymmetric markets are equivalent to symmetric markets with an adjusted (higher) coefficient of variation. As such, we may assert the following proposition:

Proposition 14. *The operational value of integration is decreasing in λ , and*

$$\Delta(\lambda) \leq (pL(\hat{z}) + c\hat{z})\left(2 - \sqrt{2(1 + \rho)}\right)\sigma.$$

In addition, for $k = 0$, $\xi_{I2}(A)$ is quasiconcave in λ and $\xi_{I2}(A)|_{\lambda=2} = \xi_{m2}(A)$. Consequently, if the behavioral value of integration is negative for $\lambda = 1$, then it is negative for all values of λ . Moreover, there exists a unique threshold $\bar{\lambda}$ (possibly $\bar{\lambda} = 2$) such that the behavioral value of integration is negative if and only if $\lambda \geq \bar{\lambda}$.

Intuitively, as the coefficient of variation in the integrated system increases (λ increases), integration operationally becomes less valuable. Thus, the operational value of integration when the markets are asymmetric is always less than when they are symmetric. The other parts of the proposition can be interpreted similar to

interpreting the impact of ρ , as any asymmetric problem has a one-to-one correspondence with a symmetric problem with a higher market correlation. In particular, this illustrates that integration is most likely to generate a negative behavioral value when the component markets are asymmetric.

3.9 Conclusion

In this paper, we have studied the behavioral consequences of influencing product availability via inventory and channel integration. While the existing literature on operational implications of omnichannel retail (including the literature on inventory pooling) typically assumes that demand is independent from any changes in product availability resulting from integration, we focus precisely on how such changes impact consumer behavior and, consequently, firm profit. Our results show that integration does not influence the availability of the product—and therefore consumer behavior—in a trivial manner. While integration always increases first period availability of the product, which encourages more consumers to visit the firm, it can also increase or decrease the second period availability of the product, and as a result can either encourage or discourage consumers to strategically delay their purchases.

We thus identified two sources of value due to integration: operational and behavioral. The operational value, which is always positive, results from the classical “statistical economies of scale” that occur due to integration. The behavioral value, on the other hand, can be negative or positive depending on the change in second period availability due to integration. The behavioral value can be significant compared to the operational value, and can even outweigh the operational value and lead to a negative total value, illustrating the importance of considering behavioral issues when considering inventory integration across channels. Integration is most likely to backfire and result in negative behavioral value when consumers have a low visit cost and are very patient, or if the markets are highly correlated or asymmetric in

magnitude; in these cases, strategic waiting is a serious issue and there is little value in encouraging consumers to visit the store by increasing inventory availability. On the other hand, integration is also most likely to have a positive behavioral value when consumers have a high visit cost and are very patient, or if the markets are negatively correlated or symmetric; in these conditions, integration is beneficial in both encouraging consumers to visit the store and discouraging strategic waiting by consumers. Despite this, the operational benefits of integration are decreasing in both the consumer visit cost and patience level, and this decrease tends to outweigh any behavioral benefits, meaning integration is, on average, less valuable if consumers are strategic (in an absolute sense).

For firms considering an omnichannel strategy, our results have several important implications. First, they show that behavioral factors are a key determinant of when a firm should—and should not—pursue an omnichannel approach that incorporates inventory integration. Integration is most beneficial when consumers are not very strategic, either because they have a low visit cost or a low discount factor. This adds to the operations literature on the value of inventory integration (Eppen 1979; Corbett and Rajaram 2006) by illustrating the role that behavioral factors play in determining the efficacy of this strategy. Second, as noted in the introduction, many firms feel compelled to implement inventory integration because consumers have come to expect an omnichannel approach (Forrester 2014); in these cases, our results illustrate that firms can make integration more profitable taking steps to decrease the consumer visit cost or discount factor, given that their customer base demands that integration be implemented. Firms could, for instance, make visit costs lower by credibly announcing inventory availability online (Gallino and Moreno 2014), or make consumers less patient by selling more “fashionable” products that consumers value more at the beginning of the season relative to the end of the season (Ghemawat and Nueno 2003). Third, our results show that integration can be an effective tool to

mitigate strategic waiting by consumers, in addition to other well-known mechanisms like price commitment, inventory rationing, and quick response production (Aviv and Pazgal 2008a; Liu and van Ryzin 2008a; Cachon and Swinney 2009a). It is most effective in this regard when markets are negatively correlated and the customer visit cost is high. However, integration can backfire, and encourage *more* strategic waiting when the customer visit cost is small; hence, while integration can mitigate strategic waiting, it must be used with caution for this purpose. Taken in sum, our results illustrate that strategic consumer behavior has a significant impact on firm decisions and profit due to inventory integration in an omnichannel retail strategy.

Optimal Fashionability for Seasonal Products

4.1 Introduction

Fashion is characterized by its trendy nature in a wide range of industries such as apparel and jewellery. Manufacturers and shoppers use fashion to express their image and identity, which adds value to products beyond their functionality (Le Bon 2014). In spite the desire of manufacturers to produce fashionable products (products that match current trends), it is a challenging task. First, trends are hard to forecast given long design and production lead times that manufacturers face. A design that is fashionable and appealing to customers at the time of design could be dull after being launched in stores. Second, in addition to high unpredictability, fashion trends have immensely short cycles. The concept of a “season” has completely changed in the past few decades. Zara, a leader in fashion apparel industry, updates design of its products as frequently as twice a week (Hansen 2012), reflecting the rapid changes in market trends. As such, even a design that had been correctly forecasted may appeal to customers for only a few weeks. Third, in a world with ever growing access to information, customers have become quite savvy in their shopping behavior: they

want the right product at the right price. In other words, customers are not willing to pay high prices for outdated products, while they may be interested in those products at a discounted price because of the functionality of the product, or simply because they are willing to wait until the product becomes trendy again.

The many challenges of matching trends have forced successful manufacturers to take a strategic approach to their supply chain and fashion design management. Among these efforts, quick response and postponement of product differentiation have been cited as strong supply chain strategies to cut the forecast lead times (Cachon and Terwiesch 2009; Lee and Tang 1998). For example, Zara, by internalizing its production and using expensive freight transportation, has been able to cut its forecast horizon significantly (Ferdows et al. 2004). As another example, Benetton, an international manufacturer based in Italy, improved their forecast accuracy by first knitting bleached yarns into different styles and then dyeing them in different colors (Signorelli and Heskett 1984).

Although all these supply chain capabilities are helpful to an extent in improving forecast accuracy, for any given fixed supply chain capability, firms should take a strategic position regarding the functionality and fashionability of their brands. A recent example from Banana Republic reported by Racked clarifies the importance of this strategic position: with the hope of speeding up slow sales growth, after a new creative director joined in 2015 with the motto of “Don’t be so dull”, Banana Republic had a major shift in its design approach by producing more fashionable products (Adams 2016). This transition however, was not perceived well by their customers, and caused a 15% loss in store sales just a month later, and 9% in total annual sales (Gap Inc. 2015; 2016). The brand’s president later explained the reason as “customer confusion” and having “a really hard time outfitting for the occasions that she’s used to shopping for” at their stores. To address the issue, the creative director was later removed, and the firm brought back more traditional styles to

brick and mortar stores, while keeping more fashionable items online and in safe zones with higher price points, Racked reports.

Banana Republic's story sheds light on the challenging task of managing optimal product fashionability. While a more fashionable design can potentially provide an upside for firms by providing a high value to customers, which in turn may allow firms to charge premium prices, it comes with a higher success risk. In Banana Republic's case, customers simply were not "used to" very fashionable design of the products at the stores. Instead, they wanted products with high functionality at low price points, which is exactly what the creative director referred to as "dull." This transition in design therefore, caused low sales during the season and high leftover inventory of unsuccessful products. To deal with the risk of having leftover inventory at the end of the season, firms usually adjust their clearance prices to reflect the value of unsuccessful products. However, highly trendy products in addition to having higher success risk, also do not retain value and consequently do not provide a high salvage price, which further dampened Banana Republic's revenues.

In this work, we precisely address these challenges by modeling a firm that sells a trendy product over a selling season with two successive periods. Before the start of the selling season, firm decides the level of fashionability of the product and a stocking quantity based on its forecast about demand and customers' valuation for that level of fashionability. We model the fluctuations in valuation of the product as a Geometric Brownian Motion that evolves over a selling season. While a product with a higher level of fashionability can provide a higher initial value for customers, it suffers a higher obsolesce rate during the season, and therefore provides a lower salvage value. At the beginning of the selling season, the firm learns about consumer valuations of the product and uses prices as a tool to respond to realized valuations. We specifically consider two pricing mechanisms: under a "dynamic pricing" model, the firm updates the salvage value based on the realization of product value at the

beginning of the clearance period, while under a “static pricing” the firm does not discount the product and disposes the remaining inventory.

By analyzing this model, we find that under static pricing, firm maximizes the total value of the product during the selling season, and sets a higher fashionability and price, but stocks less inventory than the dynamic model. This is because firms highly dependent on their clearance sales, should offer less fashionable products that better retain value. As such, firms such as Zara, with fewer discounts, can offer more trendy designs. In addition, in contrast to a static pricing scheme where optimal fashionability is independent from inventory, procurement cost, and demand parameters, under dynamic pricing, the firm should carefully consider its pricing, inventory, and fashion management jointly. Particularly, we find that a higher procurement cost results in a higher fashionability and a lower stocking quantity under a dynamic pricing scheme. Furthermore, as the market size becomes more variable, the firm moves toward a less risky option by offering a less fashionable product. Interestingly, however, as mean demand increases, the firm may stock more or less inventory due to the concurrent change in optimal fashionability.

The rest of the paper is organized as follows: §4.2 reviews the related literature. §4.3 explains our model setup. §4.4 analyses the model and reveals the main findings. §4.5 concludes.

4.2 Related Literature

Economists have long been interested in determining optimal durability of products under different market assumptions. Kleiman and Ophir (1966) considering the trade-off between value added to the product because of an increase in product durability and the resulting increase in production cost, argues that a monopolist offers less durability than the one in competitive market. Swan (1970) later challenged this result in a framework where the product retains the same value in its life cycle,

and becomes obsolete at a given threshold. In this setting, new and old products are perfect substitutes for customers. In contrast, Waldman (1996) and Hendel and Lizzeri (1999) considering products that deteriorate in quality over time, show that the firm may intentionally reduce durability in order to create competitive advantage for introduction of new products. For further discussion on this literature we refer the reader to Waldman (2003).

We are also related to the literature on quality management, where demand characteristics depend on quality of the product. Mussa and Rosen (1978) studies joint pricing and quality management of a firm facing heterogeneous tastes, and finds that a monopolist offers a lower quality than that of under competition. Krishnan and Zhu (2006) study quality management of products with high development cost in presence of customers that have heterogeneous willingness to pay based on the quality of the product. By studying products that have multiple quality dimensions, they find that offering low quality to low end customers is not necessarily optimal. In fact, by offering products with over-lapping quality, firm can sufficiently segment the market. Heese and Swaminathan (2006) study a manufacturer who produces two products that can potentially use common components. They find that a manufacturer that jointly manages quality and effort to reduce production costs, may earn a higher revenue by offering higher quality for the products that have commonality. They argue that ignoring all these elements can result in excessively differentiated product line with low quality. Netessine and Taylor (2007), also considering supply schedule that involves a fixed and variable cost, find that a more expensive technology can result in products with higher quality and lower price, with lower segment differentiation. Jerath et al. (2017), in a newsvendor setting, illustrates that considering endogenous inventory can result in a higher quality in a decentralized supply chain than a centralized channel.

In the operations literature, a stream of work mainly focuses on optimal schedules

to introduce a new design to market. The key trade-off in this stream is that introducing a new design is costly, time consuming, and not all technologies are available at a given time. If the firm introduces a new technology to market, it can boost sales because of the new product's superior technology, however, it will slow down the sales of the previous product. As such, introducing technology too frequently is very costly and introducing very slowly cannot generate enough revenue (Cohen et al. 1996; Mahajan and Muller 1996; Krankel et al. 2006; Sunar et al. 2015; Lobel et al. 2015). This framework, best suits introduction of technology products such as cell phones and computers. Similar to this stream of work, in more of a fashion context, Fujita (2008) considers the problem where a retailer sustains its brand value by introducing new designs. It assumes that market trends evolve according to a Geometric Brownian Motion with jumps upon the introduction of new products. They find that a fashion retailer should introduce the products frequently if the market as a whole is less uncertain and if sales of the previous products are not impacted significantly by new introductions.

Despite fundamental differences in these streams of literature, they all share similar assumptions: first products are vertically differentiated. In other words, new designs (more durable and higher technology or quality) are preferable to old designs. Second, offering a superior design is more costly. As such, the key driver of results is the trade-off between the cost of introducing the superior product and resulting value added. In our work however, more appropriate in fashion context, products are not vertically differentiated, i.e., a product with higher fashionability initially offers a high value to customers, but loses value faster. We also assume that different fashion designs does not necessarily result in a higher production cost. Therefore, the key trade-off in our model is inventory risk versus a two dimensional price risk.

4.3 Model

There is a firm selling a fashion product over two consecutive periods, namely 1 and 2, during a selling season with length T . Time is indexed forward starting from 0 denoted by t . There is a cut-off point in time, \bar{t} , that divides the periods. Under a static pricing scheme, the firm sets price p at the beginning of the season (time $t = 0$) to be charged during the season for both periods, and collects unsold products at the end of the season (time T). Under a dynamic pricing scheme, however, in the first period, i.e., $t \in [0, \bar{t})$, the firm charges a regular price p for the product, while at the beginning of second period, i.e., $t = \bar{t}$, adjusts its price to s . At the beginning of the season there is a potential set of customers that are willing to buy the product if they receive a non-negative surplus. The size of this population, D , is unknown to the seller, however, firm has knowledge over its distribution. Let $F(\cdot)$ and $f(\cdot)$ denote the market size distribution and density functions, respectively, with mean m and standard deviation SD. At time \bar{t} , an infinite number of bargain-hunters enter the market. Before the season starts, firm strategically decides the design (fashionability) of the product, denoted by $\alpha \geq 0$, as well as the stocking quantity I , which is procured at unit cost c . At any given point in time during the selling season, customers value the product at $V_t(\alpha)$, which depends on design of the product. An important characteristic of fashion products is that they are trendy in nature, i.e., value of the product at any given point in time is uncertain, however, they are expected to lose value over time. We model this aspect of fashion products by assuming that $V_t(\alpha)$ is random in nature and evolves according to a Geometric Brownian Motion (GBM), with percentage drift $-\mu(\alpha)$ and percentage volatility $\sigma(\alpha)$, where $\mu(\alpha)$ and $\sigma(\alpha)$ are non-negative functions of α . As such, we assume that customer valuations evolve over time according to the following stochastic differential

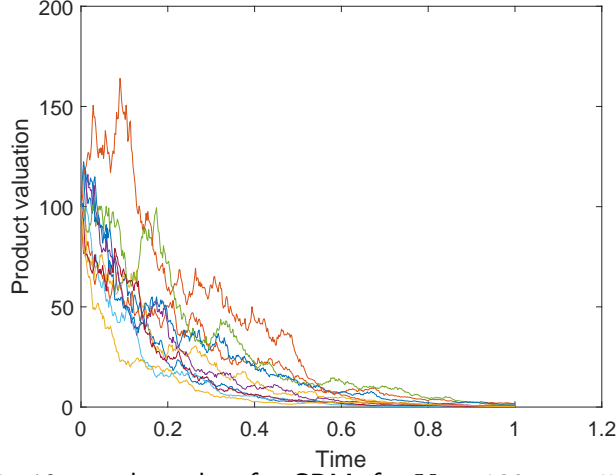


FIGURE 4.1. 10 sample paths of a GBM, for $V_0 = 100$, $\mu = 5$, and $\sigma = 1$.

equation:

$$dV_t(\alpha) = -\mu(\alpha)V_t(\alpha)dt + \sigma(\alpha)V_t(\alpha)dW_t,$$

where W_t is a standard Wiener process. Knowing the initial value of the product, $V_0(\alpha)$, one can solve this stochastic differential equation to find the following closed form:

$$V_t(\alpha) = V_0(\alpha)e^{((-\mu(\alpha)-\sigma(\alpha)^2/2)t+\sigma(\alpha)W_t)}. \quad (4.1)$$

Figure 4.1, plots 10 sample paths of this stochastic process for $V_0 = 100$, $\mu = 5$, and $\sigma = 1$ to illustrate how the valuations evolve over time.

Since the product is designed long before the actual trend realizes, we assume that the initial valuation is also random. We specifically assume that $V_0(\alpha) = \tilde{\epsilon}v_0(\alpha)$, for some deterministic function $v_0(\alpha)$ and random variable $\tilde{\epsilon}$ with $\mathbb{E}[\tilde{\epsilon}] = 1$. Therefore, from Equation 4.1, it follows that $V_t(\alpha)$ has mean $\mathbb{E}[V_t(\alpha)] = v_0(\alpha)e^{-\mu(\alpha)t}$.

As mentioned earlier, more fashionable items may match trends and provide a higher initial value, however, they also lose value faster. Therefore, we assume that $\mu(\alpha)$ and $v_0(\alpha)$ are increasing functions of α . From now on, whenever there is no confusion, we remove argument α from the relevant functions. For example, we write

$\mathbb{E}[V_t] = v_0 e^{-\mu t}$, keeping in mind that V_t , v_0 , and μ are all functions of α .¹

At the beginning of time t , both customers and the firm learn actual value of V_t . Hence, a customer that purchases at time t , receives expected utility

$$\bar{v}_t = \mathbb{E} \left[\int_t^T \hat{v}_t e^{((-\mu - \sigma^2/2)x + \sigma W_x)} dx \right], \quad (4.2)$$

from the product, where \hat{v}_t is the realization of V_t at time t . After observing \hat{v}_0 , the firm sets price p to maximize its profit. At time \bar{t} , the firm updates its price upon learning $\hat{v}_{\bar{t}}$ (under dynamic pricing). Since the firm adjusts prices upon learning the initial valuation at a given time, it is optimal for the firm to set prices exactly equal to the expected remaining value of the item, i.e., $p = \bar{v}_0$ and $s = \bar{v}_{\bar{t}}$. The assumption that the firm sets prices after learning about the initial valuation aligns with real practice where prices are usually used as a more responsive lever to demand trends than inventory and design of the product, due to long lead times. However, one should bear in mind that before the start of the selling season, these prices are unknown to the firm.

Since the main population of customers are present at the beginning of the season, and the firm extracts full surplus from customers, and since the product on average loses value over time, all main customers would attempt to buy the product at time 0. At time \bar{t} , an infinite set of bargain-hunters would clear the inventory as they receive 0 surplus.² Figure 4.2 illustrates the sequence of events.

We also define the following useful functions:

¹ We also use notation $h'(\cdot)$ and $h''(\cdot)$ to refer to the first and second derivatives of an arbitrary function h respect to α .

² We assume that $\frac{T-\bar{t}}{T}$ is sufficiently small such that probability of product having a higher valuation in the second period than the first period is very small. This is why bargain-hunters would only attempt to buy the product in the second period.

product should have higher expected current to future valuation ratio. We establish this property in the next lemma, which will also help us in proof of some of the upcoming results.

Lemma 5. *The ratio $A \equiv \frac{\Delta}{v_2}$, is increasing in α .*

Proof. All proofs are provided in Appendix B.2. \square

This shows that as α increases, the product indeed becomes more trendy in nature, independent from the actual valuations of the product in each period. Next, we adopt the following assumption for the rest of our analysis.

Assumption 1. v_2 and Δ are strongly unimodal and attain their maximum in an interior solution. Additionally, v_1 attains its maximum.

Assumption 1 insures that, the current and future values of the product, initially increase as the product becomes more fashionable, however, after a threshold the product becomes “too fashionable” and loses value. Let $\alpha_{v_1}^* = \arg \max_{\alpha} v_1$, $\alpha_{\Delta}^* = \arg \max_{\alpha} \Delta$, and $\alpha_{v_2}^* = \arg \max_{\alpha} v_2$, to be used later. The following lemma provides a simple condition in terms of v_0 and μ for Assumption 1 to hold.

Lemma 6. *Assume that v_0 and $-\mu$ are concave in α with $\lim_{\alpha \rightarrow 0 \text{ or } \infty} \frac{v_0}{\mu} = 0$.³ Then, Assumption 1 holds.*

Lemma 6 requires that at higher levels of fashionability, initial value of the product increases with a slower rate, while the product loses value faster. Additionally, when the product has “too little” or “too much” fashionability, the product either has a very small initial valuation or loses value very fast.

As is common in practice, many firms (e.g. Banana Republic and Gap) heavily rely on end of season sales to clear their inventory and help with their revenues. In

³ One may consider a bounded interval for the domain of fashionability. In this case, we replace interval $[0, \infty)$ by $[\alpha_{min}, \alpha_{max}]$.

contrast, some companies such as Zara, mostly follow a static pricing scheme. We study these two practices of pricing separately to shed light on their differences in terms of fashion, inventory, and price management.

4.4.1 Static Pricing

In this section, we assume that firm collects (disposes) remaining inventory at the end of the season, instead of clearing it by offering a discount. Without loss of generality, assume that there is no cost or profit involved in disposing the products.⁴ As such, before the start of the selling season, the firm solves optimization Problem 4.6 to determine optimal fashionability and inventory level of the product.

$$\begin{aligned} \max_{\alpha, I} \pi(\alpha, I) = \mathbb{E}_{p, D} [p \min(I, D)] - cI \\ v_1 \mathbb{E}_D [\min(I, D)] - cI \end{aligned} \tag{4.6}$$

Without loss of generality, assume that there exists some $\hat{\alpha}$ such that $v_1(\hat{\alpha}) > c$; otherwise, the firm does not have any incentive to remain in the market and stock any inventory. Let α_s^* and I_s^* be the optimal fashionability and inventory levels, respectively, under static pricing. Our first result characterizes the optimal solution to optimization problem 4.6.

Proposition 15. *Under static pricing, a unique solution to firm's problem exists. Furthermore, $\alpha_s^* = \alpha_{v_1}^*$ and I_s^* is the unique solution to $\bar{F}(I) = \frac{c}{v_1(\alpha_{v_1}^*)}$.*

As expected, under static pricing the firm does not consider the future value of the product and, therefore, solely maximizes the expected total value, as it allows charging the highest price during the season. Furthermore, Proposition 15 illustrates that under static pricing, the optimal design is independent from the optimal inventory, procurement cost, and market size parameters. Therefore, it is easier for firms

⁴ We can similarly incorporate a negative or positive salvage value by changing the induced price and marginal cost.

without end of season sales to determine their optimal design: they just offer what customers are willing to pay the highest price for at the start of the season. As we will see, this is not the case for firms with dynamic pricing.

4.4.2 *Dynamic Pricing*

Under dynamic pricing, the firm adjusts the prices at the beginning of each period. Since prices are not a priori known to the firm, one can obtain that $\mathbb{E}[p] = v_1$ and $\mathbb{E}[s] = v_2$. Hence, the firm's problem can be written as

$$\begin{aligned} \max_{\alpha, I} \pi(\alpha, I) &= \mathbb{E}_{p,s,D} [p \min(I, D) + s \max(I - D, 0) - cI] \\ &= \Delta \mathbb{E}_D [\min(I, D)] - (c - v_2)I \end{aligned} \tag{4.7}$$

Although we can show our results (with minor modifications) for cost functions that are increasing and convex in α , we intentionally are interested in the cases where c is constant, because in contrast to quality, fashion design is unlikely to significantly influence the procurement cost.⁵ By doing this, we further focus on the trade-off between current and future values of the product. In addition, we also assume that for all values of α , $c > v_2(\alpha)$. If this assumption is violated for some $\hat{\alpha}$, then the firm can earn infinite revenue by setting $\alpha = \hat{\alpha}$ and stocking infinite inventory. This assumption can further be justified by noting that when length of the discount period ($T - \bar{t}$) is short, or when the market is very trendy and products lose value fast, firms usually need to clear the market by offering very deep discounts. Let α_d^* and I_d^* be the optimal fashionability and inventory level, respectively, under dynamic pricing (if exist). If multiple solutions arise we assume that the firm chooses the one with minimum fashionability. We have the following proposition.

⁵ Fashion design can potentially change the fixed cost if the firm uses a different set of designers. However, same designers usually work hourly or per project basis and will not charge differently for different designs.

Proposition 16. *Current value of the product achieves its maximum at a higher fashionability level than the future value, i.e., $\alpha_{v_2}^* \leq \alpha_{\Delta}^*$. Furthermore, under dynamic pricing, if an optimal solution to the firm's problem exists, then it should be such that $\alpha_{v_2}^* \leq \alpha_d^* \leq \alpha_{\Delta}^*$.*

Intuitively, higher fashionability hurts the future value more through the resulting cumulative increase in the decay rate of valuations (μ), while current value is more impacted by an immediate increase in the initial value (v_0). Hence, a smaller fashionability level would maximize the future value compared to the current value. Additionally, Proposition 16 illustrates that the optimal fashionability lies in between optimizers of current and future values of the product. In other words, the firm should not set too high fashionability that harms the future value significantly, and it should not be so basic that it does not provide an appropriate current value. As an important consequence, we only need to search for optimal fashionability in a compact region where Δ is increasing and at the same time v_2 is decreasing in α . More specifically, at the optimal fashionability level, we will increase the current value if we make the product a little more fashionable, while future value would be reduced. This reduction, will outweigh the increase in current value, and hence, it is suboptimal to set that level of fashionability. We then have the following proposition.

Proposition 17. *An interior solution to optimization problem 4.7 exists and satisfies*

$$\begin{cases} \frac{\partial \Delta}{\partial \alpha} \times \mathbb{E}[\min(I, D)] + \frac{\partial v_2}{\partial \alpha} \times I = 0 & \alpha \in [\alpha_{v_2}^*, \alpha_{\Delta}^*] \\ \bar{F}(I) = \frac{c-v_2}{\Delta} \end{cases} \quad (4.8)$$

In addition to the existence of an interior solution, Proposition 17 also suggests that one can sequentially solve the problem by first solving for the unique optimal inventory level for each fixed level of α , and then replace it in Problem 4.7 and solve for a simple one dimensional optimization problem on compact interval $[\alpha_{v_2}^*, \alpha_{\Delta}^*]$.

Equivalently, one can solve for the roots of system of equations 4.8. If the solution is unique, Proposition 17 guarantees that it is a maximizer of the firm's problem.

One might be interested in conditions that guarantee the uniqueness of the solution to system of equations 4.8. The following proposition provides a sufficient condition for that.

Proposition 18. *Assume that v_2 and Δ^2 are concave on domain $[\alpha_{v_2}^*, \alpha_\Delta^*]$. Furthermore, assume that $\int_0^y xf(x)dx \leq y^2 f(y)$ for all values of y in the support of f . Then, a unique solution to system of equations 4.8 exists.*

Clearly, if $xf(x)$ is increasing on its domain, the assumption $\int_0^y xf(x)dx \leq y^2 f(y)$ would hold. Therefore, if $xf(x)$ is increasing, and v_2 and Δ are sufficiently concave on domain $[\alpha_{v_2}^*, \alpha_\Delta^*]$, the uniqueness of the solution is guaranteed.

Next we study how changing different parameters of the problem would impact the optimal inventory and fashionability level.

Proposition 19. *As the product becomes more costly to procure, the firm sets a higher fashionability level and stocks less inventory. Consequently, the product starts the season with a higher average price followed by a deeper average discount.*

Proposition 19 suggests that firms facing a higher procurement cost should design more fashionable products, while providing more scarcity. These products therefore face a trendy market: initially, they receive a lot of attention from customers, and as a result, firm charges a high price during the regular season; however, these products lose value fast and the firm should discount them deeply to clear the inventory. Note that this result is not a priori intuitive: a firm with a higher procurement cost also faces a higher value at risk. In other words, unsold units become more costly. Therefore, the firm may attempt to offset this cost by offering a higher salvage value, i.e., a product that retains more value. If the firm can successfully do this, then it

may not reduce its inventory level. On the other hand, a firm that faces a higher marginal cost may also need to offer a higher price to make up for the resulted margin loss. Additionally, the firm can reduce the overstocking risk by reducing inventory in the system. Proposition 19 illustrates that the latter approach benefits the firm more than the former. Next, we study the impact of market size parameters.

Proposition 20. *Under a dynamic pricing model,*

(i) As the mean of the market size increases, the firm sets a higher fashionability level and charges a higher regular price. However, it may stock more or less inventory.

(ii) As the standard deviation of the market size increases, the firm sets a lower fashionability level and charges a lower regular price. However, it may stock more or less inventory.

Proposition 20 illustrates that impact of demand variability on fashionability is unambiguous: as demand mean (standard deviation) increases (decreases), the resulting reduction in variability makes room for firm to go toward riskier options, i.e., offering a more fashionable design. However, the interplay of inventory and fashionability is not straightforward: a system with lower inventory and higher fashionability can yield the same cost and risk as a system with lower fashionability and higher inventory. From one hand, an increase in mean demand translates to a bigger market size, which requires stocking more inventory. On the other hand, since the firm offers a more fashionable and riskier product, it may need to reduce its stocking quantity to hedge against the overstocking cost. The interplay of these two forces then would determine the behavior of optimal inventory as a function of mean demand.

It is not hard to find examples where optimal inventory decreases in standard deviation. In fact, it is enough to consider a normally distributed demand and valuations such that $v_1 + v_2 \leq 2c$ for all values of α . This will result in a negative critical fractile which in turn concludes that inventory is decreasing in the standard

deviation. However, what is more interesting is that increasing mean demand can also result in a lower inventory. To see this, consider an example where $v_0(\alpha) = 1/2\sqrt{\alpha}$, $\mu(\alpha) = 1/2\alpha^2$, $c = 1$, $T = 10$ and $\bar{t} = 5$. Figure 4.3c plots the resulting valuations (v_1 , v_2 , and Δ) as a function of α . In this example, the product has very little value for α close to 0 and more than 5. The total value of product, v_1 , is maximized in $\alpha \simeq 0.33$, while the future value, v_2 , and current value, Δ , attain their maximum at $\alpha \simeq 0.26$ and $\alpha \simeq 0.47$, respectively. We consider a Log-normally distributed demand with standard deviation 300 and vary mean demand from 200 to 600, i.e., from a high coefficient of variation (1.5) to a smaller value (0.5). Log-normal distribution is an especially appropriate distribution to estimate the market size in fashion retail because it lies in a positive support and allows for high variability in the system. Figure 4.3 plots optimal fashionability and inventory as a function of mean demand.

As illustrated by Proposition 20, optimal fashionability always increases as a function of mean demand. When there is relatively high variability in the system (mean is small) the firm sets a fashionability close to the one that maximizes the future value (close to 0.26). However, as the system becomes less variable, the firm stocks a design that is closer to maximizing the current and total values. Interestingly, as Figure 4.3b illustrates, inventory may go down as mean demand increases for “medium” values of variability. This is because initially, when demand variability is too high, the system is too risky and any unit reduction in variability will greatly help the firm to stock more. In the other end, when there is very little variability in the system, the risk of overstocking is small and therefore the firm increases its inventory level as mean demand increases. However, for medium values of variability, a reduction in variability will increase the optimal fashionability, which contributes to increase in the risk of having too much inventory to the extent that can outweigh the positive impact of increase in mean demand, and can result in a decrease in inventory.

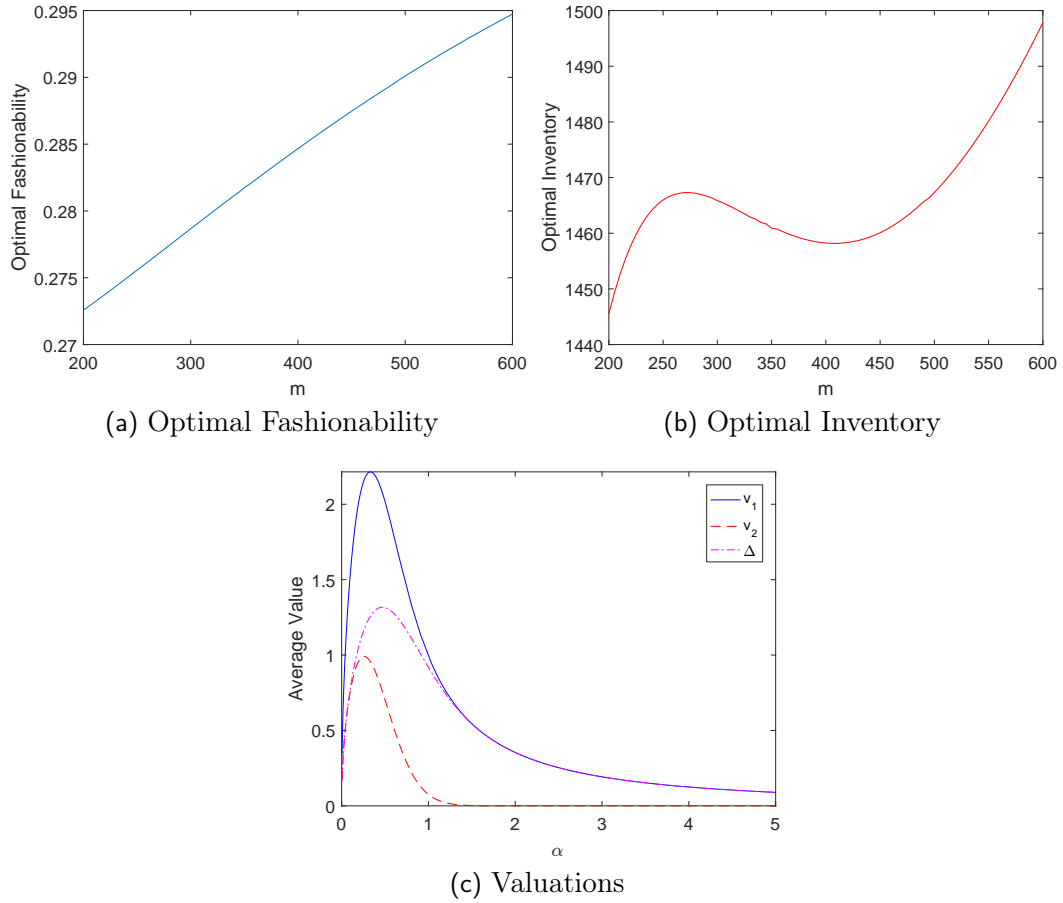


FIGURE 4.3. Example with $v_0(\alpha) = 1/2\sqrt{\alpha}$, $\mu(\alpha) = 1/2\alpha^2$, $T = 10$, $\bar{t} = 5$, $c = 1$, and Log-normally distributed demand with standard deviation 300 and varying mean between 200 and 600. Although optimal fashionability is increasing in mean demand, optimal inventory may increase or decrease.

Another important factor in determining the firm's strategy is the initial valuation of the product, i.e., v_0 . The next proposition studies this impact.

Proposition 21. *Consider a new market with initial value $v_{0\delta}(\alpha) = \delta v_0(\alpha)$ for some constant $\delta > 1$. Then, in the new market, the firm offers a less fashionable product and stocks more inventory.*

Not surprisingly, the impact of an increase in the initial valuation is exactly the opposite of the impact of an increase in procurement cost. As the initial value increases, the firm faces a higher under-stocking cost and as a result stocks more. At

the same time, the firm has relatively good margins in the first period, and therefore, can afford offering less trendy products with lower prices to further hedge against the risk of having leftover inventory in the clearance period.

As the results in this section illustrate, firms that implement dynamic pricing face a much more complicated problem designing their products than those with static pricing strategy. Neglecting interconnections between pricing, inventory, and design management can lead to designs that do not reflect important market characteristics such as demand parameters and procurement cost. In the next proposition, we compare the optimal outcome of the problem under dynamic and static pricing strategies.

Proposition 22. *Under static pricing, the firm sets higher fashionability, stocks less inventory, and charges a higher regular price.*

Intuitively, under static pricing, since the firm is not concerned about the future value, it can manage to set a higher fashionability during the season and consequently charge a higher price. The relation between inventory levels is less intuitive: under static pricing, firm charges a higher price and therefore under-stocking is more costly, which encourages stocking more inventory. However, the absence of a salvage period increases the overstocking cost and pushes the firm to be more conservative and stock less. Proposition 22 illustrates that the latter effect outweighs the former, and firm stocks less under static pricing.

Our results shed light on the Banana Republic story discussed in the introduction: initially the management team relied heavily on clearance sales, correctly (given the pricing strategy) recognizing the need for producing products that were less trendy to retain more value. As a fashion oriented manufacturer, this contributed to their slow sales in 2014. The new creative director, recognizing this problem, decided to design products that were less “dull”, i.e., more fashionable and trendy. However,

given their pricing mechanism and customer base that were used to low prices and clearance sales, they lost even further share by producing products that were not retaining value and had higher price points. In the end, recognizing this mistake, they brought less trendy products back to stores. However, they also launched a more fashionable and pricey collection in collaboration with Timo Weiland in selected stores and online (Adams, 2016), which resembled the static pricing strategy.

4.5 Conclusion

In this paper, we have studied joint price, inventory, and fashion management of a firm that faces uncertain demand and trends. Too fashionable products face a high success risk and do not provide a high salvage value for firms. In contrast, not sufficiently fashionable products do not excite customers during the regular season and command a lower price, but can retain value for the salvage period. While the previous literature has focused on operational capabilities that enable firms to react to market changes quickly, we focus on optimal design of products for any fixed level of operational capability, to reduce inventory costs and maximize profit.

We built a tractable model that can capture these trade-offs, and established sufficient conditions that guarantee existence and uniqueness of the optimal solution. Our analysis provide interesting insights: first, we find that while under static pricing, optimal fashion design simply translates to maximizing the total life value of the product, under a dynamic pricing model, pricing and inventory decisions should be jointly considered with fashion design. This suggests that firms that offer fewer discounts, face less complexity deciding about their fashion choice. Additionally, we find that firms that highly depend on their clearance sales, inevitably, should produce less fashionable products to retain value. In contrast, firms with very limited discounts, such as Zara, can afford offering more fashionable products to their customers.

We also find that an increase in the procurement cost results in higher optimal

fashionability through its impact on inventory. Furthermore, in a system with highly variable market size, the firm should offer a lower fashionability to provide a higher salvage value. Interestingly, when the firm manages its inventory, price, and fashion design jointly, an increase in mean demand can result in a lower inventory. This is because the firm can afford more risk by offering a more trendy product and can charge a higher regular price; however, as the product becomes trendier and loses value faster, the firm may need to limit its stocking quantity. Finally, as customers initially value the product more (at any fixed fashionability level), the firm has better margins in the first period, which justifies stocking more inventory. With higher inventory in the system, the firm produces products that retain value to reduce the risk of having left over inventory. Taken in sum, we emphasize the importance of considering price, inventory, and design management jointly.

4.6 Future Directions

This paper also opens interesting avenues for future research. First, in this work, we assumed that both the firm and consumers are risk neutral. The risk neutral assumption is reasonable for big firms such as Zara and Banana Republic (even though we observed some level of caution when Banana Republic decided to offer their riskier products only in safe zones). However, for smaller designers and consumers, it is natural to observe some level of risk aversion. Since both inventory and design impact value-at-risk in a non-trivial manner, it is interesting to consider a risk-averse firm and/or consumers to see whether the firm would necessarily offer lower fashionability/inventory.

Second, fashionability is a manifestation of consumer behavior, and therefore, interplays greatly with known behavioral factors such as forward-looking behavior, where customers time their purchase to maximize their surplus by trading-off the product value, price, and availability over time. Since adding design management to

the firm problem will impact all these parameters, it is then interesting to study the impact of forward-looking behavior on the firm problem and vice versa. On one hand, when customers anticipate future purchasing opportunities, the firm should offer a more fashionable product to reduce the value of future purchases. However, with more stylish products, the firm may also stock more which can make the product less scarce and encourage strategic waiting. Therefore, it is important to understand which one of these managerial decisions can be used as a more effective lever in mitigating forward-looking behavior.

Finally, although in this paper we considered a single product, one should interpret it as the firm's general approach toward fashionability. A higher α then can be interpreted as a higher tendency toward fashion. However, it is still interesting to consider fashion management of a firm that offers multiple products. This way, the firm might be able to benefit from pooling products together. It is not a priori clear whether the firm would bundle products with high and low fashionability or simply would choose only fashionable/stable assortments.

Appendix A

Proofs and Supplementary Materials for Chapter 2

Proof of Lemma 1. For any pair of prices \hat{p}_1 and \hat{p}_2 , a consumer would purchase in the first period if and only if $v - \hat{p}_1 \geq \max(\delta(v - \hat{p}_2), 0)$. The result follows from noting that $v - \hat{p}_1 - \max(\delta(v - \hat{p}_2), 0)$ is an increasing function of v . \square

Proof of Lemma 2. (i) One can see that $\frac{\partial(G[\bar{v}] - G[p])p}{\partial \bar{v} \partial p} = g[\bar{v}] \geq 0$ and since \mathbb{R} is a lattice, from Corollary 2 of Milgrom and Shannon (1994), $\arg \max_p (G[\bar{v}] - G[p])p$ is a sublattice of \mathbb{R} . We need to prove that A is a sublattice of \mathbb{R}^2 . Let $v_2 \geq v_1$; if $(v_1, p_1) \in A$ and $(v_2, p_2) \in A$, then $(v_1, p_1) \wedge (v_2, p_2) = (v_1, \min(p_1, p_2))$ and $(v_1, p_1) \vee (v_2, p_2) = (v_2, \max(p_1, p_2))$. From Theorem 4 of Milgrom and Shannon (1994), we have $\arg \max_p (G[v_2] - G[p])p \geq \arg \max_p (G[v_1] - G[p])p$ with respect to the strong set order. Hence, if $p_1 \in \arg \max_p (G[v_1] - G[p])p$ and $p_2 \in \arg \max_p (G[v_2] - G[p])p$, then we have $\min(p_1, p_2) \in \arg \max_p (G[v_1] - G[p])p$ and $(v_1, \min(p_1, p_2)) \in A$. Similarly, one can see that $(v_2, \max(p_1, p_2)) \in A$. As such, A is a lattice.

(ii) First of all note that since $\lim_{p \rightarrow +\infty} (G[\bar{v}] - G[p])p \leq 0$ and $\lim_{p \rightarrow 0} (G[\bar{v}] - G[p])p = 0$, then if $G[\bar{v}] > 0$, the maximizer of $(G[\bar{v}] - G[p])p$ will be attained in an interior solution. Therefore, $\arg \max_p (G[\bar{v}] - G[p])p \subseteq \{p : (pG[p])' = G[\bar{v}]\}$. Thus,

in equilibrium, $G[\bar{v}] = G[p_2] + p_2g[p_2]$ which concludes that $G[\bar{v}] \geq G[p_2]$. We claim that in equilibrium $\bar{v} \geq p_2$. If $G[\bar{v}] > G[p_2]$, then the result easily follows from noting that $G(x)$ is an increasing function of x . Also in equilibrium, if $G[\bar{v}] = G[p_2] \neq 0$, the firm receives a zero revenue in the second period. However, there exists some $p > 0$ such that $G[\bar{v}] > G[p]$ and yields a positive revenue in the second period, which is a contradiction with optimality of p_2 for the second period. If $G[\bar{v}] = G[p_2] = 0$, then the firm receives total revenue p_1 . In this case, $\bar{v} = \sup\{v : G(v) = 0\}$, because otherwise one can increase p_1 and receive a higher revenue. Therefore, we have $\bar{v} \geq p_2$. Consequently, one can replace constraint $\bar{v} = \{\min \hat{v}, \text{ s.t. } \hat{v} - p_1 \geq \delta(\hat{v} - p_2)^+\}$ by $\bar{v} - p_1 = \delta(\bar{v} - p_2)$. From this equality, we replace p_1 to obtain the following equivalent optimization problem:

$$\begin{aligned} \text{Max}_{p_2, \bar{v}} \quad & \pi(\bar{v}, p_2, \delta) = (1 - G[\bar{v}])(1 - \delta)\bar{v} + \delta p_2 + (G[\bar{v}] - G[p_2])p_2 \\ \text{s.t.} \quad & p_2 \in \arg \max_p (G[\bar{v}] - G[p])p \end{aligned} \tag{A.1}$$

For simplicity of notation we use v instead of \bar{v} and p instead of p_2 for the remainder of the proof. From Part (i), the feasible set is a sublattice of \mathbb{R}^2 . Also one can see that $\frac{\partial \pi(v, p, \delta)}{\partial p \partial v} = (1 - \delta)g[v] \geq 0$. In addition, if (\hat{v}, \hat{p}) and (\tilde{v}, \tilde{p}) be two elements of the feasible set such that $\hat{v} \geq \tilde{v}$, then from Part (i), $(\hat{v}, \hat{p}) \vee (\tilde{v}, \tilde{p}) = (\hat{v}, \hat{p})$ and $(\hat{v}, \hat{p}) \wedge (\tilde{v}, \tilde{p}) = (\tilde{v}, \tilde{p})$. Hence, $\pi(v, p, \delta)$ is supermodular in (v, p) and consequently is a quasisupermodular function. To be able to use Theorem 4 of Milgrom and Shannon (1994), one should prove that $\pi(v, p, \delta)$ has the single crossing property in (v, p, δ) . A function $f : X \times T \rightarrow \mathbb{R}$ is said to have a single crossing property in $(x; t)$, if the following two conditions hold when $x' > x''$ and $t' > t''$: 1) $f(x', t'') \geq f(x'', t'') \Rightarrow f(x', t') \geq f(x'', t')$, 2) $f(x', t') > f(x'', t') \Rightarrow f(x', t'') > f(x'', t'')$. Let $p' \geq p''$, $v' \geq v''$, and $\delta' \geq \delta''$. To check the first condition assume that $\pi(v', p', \delta'') \geq \pi(v'', p'', \delta'')$, i.e.,

$$\begin{aligned}
& (1 - G[v']) \left((1 - \delta'')v' + \delta''p' \right) + (G[v'] - G[p'])p' \\
& \geq (1 - G[v'']) \left((1 - \delta'')v'' + \delta''p'' \right) + (G[v''] - G[p''])p''.
\end{aligned}$$

Equivalently,

$$\begin{aligned}
& \delta'' \left((p' - v')(1 - G[v']) - (p'' - v'')(1 - G[v'']) \right) \\
& + v'(1 - G[v']) - v''(1 - G[v'']) \tag{A.2} \\
& + (G[v'] - G[p'])p' - (G[v''] - G[p''])p'' \geq 0.
\end{aligned}$$

Note that only the first part of this inequality depends on δ'' . Therefore, if $(p' - v')(1 - G[v']) - (p'' - v'')(1 - G[v'']) \geq 0$, then we can replace δ'' by δ' without changing the direction of the Inequality A.2, which concludes that $\pi(v', p', \delta') \geq \pi(v'', p'', \delta')$. Next, consider the case in which $(p' - v')(1 - G[v']) - (p'' - v'')(1 - G[v'']) < 0$. In this case, the left side of Inequality A.2 attains its minimum value when $\delta'' = 1$. Consequently, it is sufficient to show that for $\delta'' = 1$, the left side of the inequality remains nonnegative. Replacing $\delta'' = 1$ and with some manipulation we obtain

$$\begin{aligned}
& 1 \times \left((p' - v')(1 - G[v']) - (p'' - v'')(1 - G[v'']) \right) + v'(1 - G[v']) \\
& - v''(1 - G[v'']) + (G[v'] - G[p'])p' - (G[v''] - G[p''])p'' \\
& = p'(1 - G[v']) - p''(1 - G[v'']) + p'(G[v'] - G[p']) - p''(G[v''] - G[p'']) \\
& = p'(1 - G[p']) - p''(1 - G[p'']).
\end{aligned}$$

To show that this equation is positive, it is sufficient to show that $p(1 - G[p])$ is increasing in p or equivalently $(pG[p])' \leq 1$. However, note that as argued, the solution to $\max_p (G[\bar{v}] - G[p])p$ is an interior solution and satisfies $(pG[p])' = G[\bar{v}] \leq 1$, which is what we wanted. To check for the second condition of the single crossing property, we replace inequality sign \geq with strict inequality $>$ in Inequality A.2. Note

that again if $(p' - v')(1 - G[v']) - (p'' - v'')(1 - G[v'']) \geq 0$, then the left side of the inequality is increasing in δ'' , and therefore, we can replace δ'' by δ' without changing the direction of the strict inequality. For $(p' - v')(1 - G[v']) - (p'' - v'')(1 - G[v'']) < 0$, then the left side of the inequality is strictly decreasing in δ and for $\delta = 1$ is non-negative. But note that δ is strictly less than 1, hence, the left side would be strictly positive. As such, since the feasible set is a lattice, the objective function is quasisupermodular and has the single crossing property, we can utilize Theorem 4 of Milgrom and Shannon (1994) to obtain the result.

(iii) As argued in the previous part, $\bar{v} \geq p_2$. Additionally, $\frac{\partial \pi(v, p, \delta)}{\partial \delta} = (1 - G[v])(p - v) \leq 0$ and the feasible set is independent from δ . Therefore, if $\delta_2 \geq \delta_1$, for all feasible solutions (v, p) we have

$$\pi(v, p, \delta_2) \leq \pi(v, p, \delta_1) \leq \pi(v^*(\delta_1), p^*(\delta_1), \delta_1).$$

The result then follows from replacing (v, p) with $(v^*(\delta_2), p^*(\delta_2))$.

(iv) Follows from $v = \frac{p_1 - \delta p_2}{(1 - \delta)} \geq p_2$. \square

Proof of Proposition 1. (i) Note that consumers being myopic is equivalent to $\delta = 0$ in optimization problem (2.2). Then from Lemma 2, $\bar{v}^s \geq \bar{v}^m = p_1^m$ and $p_2^s \geq p_2^m$. First we claim that $\bar{v}^s > p_1^m$. By contradiction assume otherwise, i.e., $\bar{v}^s = p_1^m$. Then from the optimality of the second period prices we have $p_2^s = p_2^m$. Let $L^m(p_1, p_2, \lambda)$ and $L^s(\bar{v}, p_2, \lambda)$ be the Lagrange functions of optimization problems 2.1 and A.1, respectively, where λ is the Lagrange multiplier of the corresponding constraints. Furthermore, let λ^j be the Lagrange multiplier in the optimal solution in problem $j \in \{m, s\}$. Writing the KKT conditions for the problem with myopic consumers we have

$$\begin{aligned} \frac{\partial L^m(p_1, p_2, \lambda)}{\partial p_1} &= \bar{G}(p_1) - g(p_1)p_1 + \frac{\partial (G(p_1) - G(p_2)) p_2}{\partial p_1} \\ &\quad - \lambda \frac{\partial (G(p_1) - G(p_2) - g(p_2)p_2)}{\partial p_1} = 0, \end{aligned} \quad (\text{A.3})$$

$$\frac{\partial L^m(p_1, p_2, \lambda)}{\partial p_2} = \frac{\partial ((G(p_1) - G(p_2)) p_2 - \lambda (G(p_1) - G(p_2) - g(p_2)p_2))}{\partial p_2} = 0, \quad (\text{A.4})$$

$$\frac{\partial L^m(p_1, p_2, \lambda)}{\partial \lambda} = G(p_1) - G(p_2) - g(p_2)p_2 = 0. \quad (\text{A.5})$$

From Equations A.5 and A.4 we observe that

$$\lambda^m \frac{\partial (G(p_1) - G(p_2) - g(p_2)p_2)}{\partial p_2} \Big|_{(p_1, p_2) = (p_1^m, p_2^m)} = 0.$$

Therefore, either $\lambda^m = 0$ or $\frac{\partial (G(p_1) - G(p_2) - g(p_2)p_2)}{\partial p_2} \Big|_{(p_1, p_2) = (p_1^m, p_2^m)} = 0$. Additionally, if $\lambda^m = 0$, then from Equation A.3, we obtain that

$$(\bar{G}(p_1) - g(p_1)p_1 + G(p_1)p_2) \Big|_{(p_1, p_2) = (p_1^m, p_2^m)} = 0. \quad (\text{A.6})$$

Similarly, for the problem with strategic consumers, using optimization problem A.1, we have the following necessary conditions:

$$\begin{aligned} \frac{\partial L^s(\bar{v}, p_2, \lambda)}{\partial \bar{v}} &= (1 - \delta) (\bar{G}(\bar{v}) - g(\bar{v})\bar{v}) - \delta p_2 g(\bar{v}) \\ &\quad + \frac{\partial ((G(\bar{v}) - G(p_2)) p_2 - \lambda (G(\bar{v}) - G(p_2) - g(p_2)p_2))}{\partial \bar{v}} \\ &= (1 - \delta) (\bar{G}(\bar{v}) - g(\bar{v})\bar{v} + g(\bar{v})p_2) - \lambda g(\bar{v}) = 0, \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \frac{\partial L^s(\bar{v}, p_2, \lambda)}{\partial p_2} &= \delta \bar{G}(\bar{v}) \\ &\quad + \frac{\partial ((G(\bar{v}) - G(p_2)) p_2 - \lambda (G(\bar{v}) - G(p_2) - g(p_2)p_2))}{\partial p_2} = 0, \end{aligned} \quad (\text{A.8})$$

$$\frac{\partial L^s(\bar{v}, p_2, \lambda)}{\partial \lambda} = G(\bar{v}) - G(p_2) - g(p_2)p_2 = 0. \quad (\text{A.9})$$

If $(\bar{v}, p_2) = (p_1^m, p_2^m)$ is optimal for optimization problem A.6, then by simplifying Equation A.8 using Equation A.9 we have

$$\left(\delta \bar{G}(\bar{v}) - \lambda \frac{\partial (G(\bar{v}) - G(p_2) - g(p_2)p_2)}{\partial p_2} \right) \Big|_{(\bar{v}, p_2) = (p_1^m, p_2^m)} = 0. \quad (\text{A.10})$$

As argued, if $\lambda^m \neq 0$, then $\frac{\partial (G(\bar{v}) - G(p_2) - g(p_2)p_2)}{\partial p_2} \Big|_{(\bar{v}, p_2) = (p_1^m, p_2^m)} = 0$, and therefore from Equation A.10, $\bar{G}(p_1^m) = 0$. This is a contradiction with the optimality of (p_1^m, p_2^m) for the problem with myopic consumers.

If $\lambda^m = 0$, then from Equations A.6 and A.7 we have $\lambda^s = 0$. Consequently, from Equation A.8, we conclude that $\bar{G}(p_1^m) = 0$, which again is a contradiction. Therefore, $\bar{v}^s > p_1^m$.

Next we prove that $p_2^s > p_2^m$. If $\bar{v}^s > p_1^m$ and $G[\bar{v}^s] = G[p_1^m]$, then facing with myopic consumers, one can change p_1^m to \bar{v}^s and earn a higher revenue which is a contradiction with optimality of p_1^m . If $\bar{v}^s > p_1^m$ and $G[\bar{v}^s] > G[p_1^m]$, then since the solution in the second period's problem is an interior solution and $\frac{\partial (G[\bar{v}^s] - G[p_2])p_2}{\partial p_2} > \frac{\partial (G[p_1^m] - G[p_2])p_2}{\partial p_2}$, then $p_2^s > p_2^m$. This completes the proof of the first part.

(ii), (iii) Consider a consumer with valuation $v \in [p_2^m, p_1^m)$. In a myopic environment this consumer would purchase in the second period and her utility is given by $\delta(v - p_2^m)$. From Lemma 2 we know that $\bar{v}^s \geq p_1^m$. Hence, this consumer would wait for the second period when consumers are strategic and receives utility $\delta \max(v - p_2^s, 0) \leq \delta(v - p_2^m)$. Furthermore, if $v < p_2^s$, then this consumer cannot afford the item when consumers are strategic. \square

Proof of Proposition 2. Using myopic equilibrium prices, one can find that $\pi^m = \frac{1}{3}$, $CS^m = \int_{\frac{1}{3}}^{\frac{2}{3}} \delta(v - \frac{1}{3})dv + \int_{\frac{2}{3}}^1 v - \frac{2}{3}dv = \frac{1+\delta}{18}$, and $SW^m = \frac{1+\delta}{18} + \frac{1}{3} = \frac{7+\delta}{18}$. Similarly, using the equilibrium outcome $\bar{v}^s = \frac{2-\delta}{3-2\delta}$, $p_1^s = \frac{(2-\delta)^2}{6-4\delta}$, and $p_2^s = \frac{2-\delta}{6-4\delta}$, we have $\pi^s = \frac{(2-\delta)^2}{12-8\delta}$, $CS^s = \int_{\frac{2-\delta}{6-4\delta}}^{\frac{2-\delta}{3-2\delta}} \delta(v - \frac{2-\delta}{6-4\delta})dv + \int_{\frac{2-\delta}{3-2\delta}}^1 (v - \frac{(2-\delta)^2}{6-4\delta})dv = \frac{(\delta-2)\delta(5\delta-2)+4}{8(3-2\delta)^2}$, and

$SW^s = CS^s + \frac{(2-\delta)^2}{12-8\delta} = \frac{\delta(\delta(\delta+10)-36)+28}{8(3-2\delta)^2}$. It is then not hard to check that $CS^s > CS^m$, $\pi^s < \pi^m$, and $SW^s < SW^m$. \square

Proof of Lemma 3. First, note that for any pair of prices, a consumer with valuation v would exert effort if and only if

$$\max(v - p_1, \delta(v - p_2)^+) - k > \begin{cases} v - p_1 & v \geq p_1 \\ \delta(v - p_2)^+ & \text{otherwise} \end{cases}.$$

This holds if and only if $\delta(v - p_2) - k > v - p_1 \geq 0$. Therefore, the last consumer who purchases in the first period is given by $\bar{v} = \max(p_1, \frac{p_1 - \delta p_2 - k}{1 - \delta})$. In the dynamic model, note that demand and profit in the second period are given by $(\bar{v} - p_2)$ and $(\bar{v} - p_2)p_2$, respectively. Therefore, the optimal price in the second period is $p_2 = \frac{\bar{v}}{2}$. Consequently, \bar{v} is the solution to $\bar{v} = \max(p_1, \frac{p_1 - \delta \frac{\bar{v}}{2} - k}{1 - \delta})$. Solving this equation, we obtain that if $k \geq \frac{p_1 \delta}{2}$, then $\bar{v} = p_1$; otherwise, $\bar{v} = \frac{2(p_1 - k)}{2 - \delta}$. As such, one needs to solve the following optimization problem for the first period to find the equilibrium outcome:

$$\begin{aligned} & \text{Max}_{p_1, \bar{v}} (1 - \bar{v})p_1 + \bar{v}^2/4 \\ & \text{s.t.} \quad \bar{v} = \begin{cases} p_1 & k \geq \frac{p_1 \delta}{2} \\ \frac{2(p_1 - k)}{2 - \delta} & \text{otherwise} \end{cases} \end{aligned}$$

Solving this yields

$$p_1^k = \begin{cases} \frac{2}{3} & k \geq \frac{\delta}{3} \\ \frac{2k}{\delta} & \frac{\delta^2 - 2\delta}{2\delta - 6} \leq k < \frac{\delta}{3}, \\ \frac{(\delta - 2)^2 - 2(\delta - 1)k}{6 - 4\delta} & 0 \leq k < \frac{\delta^2 - 2\delta}{2\delta - 6} \end{cases}, \quad p_2^k = \begin{cases} \frac{1}{3} & k \geq \frac{\delta}{3} \\ \frac{k}{\delta} & \frac{\delta^2 - 2\delta}{2\delta - 6} \leq k < \frac{\delta}{3}, \\ \frac{2 - \delta - 2k}{6 - 4\delta} & 0 \leq k < \frac{\delta^2 - 2\delta}{2\delta - 6} \end{cases},$$

$$\text{and } \bar{v}^k = \begin{cases} \frac{2}{3} & k \geq \frac{\delta}{3} \\ \frac{2k}{\delta} & \frac{\delta^2 - 2\delta}{2\delta - 6} \leq k < \frac{\delta}{3}. \\ \frac{\delta + 2k - 2}{2\delta - 3} & 0 \leq k < \frac{\delta^2 - 2\delta}{2\delta - 6} \end{cases}.$$

This completes the proof. \square

Proof of Proposition 3. Under strategic behavior, the utility of a consumer with valuation v is given by

$$u^k(v, k, \delta) = \begin{cases} v - \frac{2}{3} & v \geq \frac{2}{3} \wedge k \geq \frac{\delta}{3} \\ \delta \left(v - \frac{1}{3} \right) & \frac{2}{3} > v \geq \frac{1}{3} \wedge k \geq \frac{\delta}{3} \\ v - \frac{2k}{\delta} & \frac{\delta v}{2} \geq k \wedge \frac{(\delta-2)\delta}{2(\delta-3)} \leq k < \frac{\delta}{3} \\ \delta v - k & \frac{\delta^2-2\delta}{2\delta-6} \leq k < \frac{\delta}{3} \wedge \frac{k}{\delta} \leq v < \frac{2k}{\delta} \\ v - \frac{(\delta-2)^2-2(\delta-1)k}{6-4\delta} & k < \frac{\delta^2-2\delta}{2\delta-6} \wedge v \geq \frac{\delta+2k-2}{2\delta-3} \\ \delta \left(v - \frac{2-\delta-2k}{6-4\delta} \right) - k & 0 \leq k < \frac{\delta^2-2\delta}{2\delta-6} \wedge \frac{-\delta^2+4\delta+2\delta k-2k-4}{4\delta-6} \leq v < \frac{\delta+2k-2}{2\delta-3} \\ \delta \left(v - \frac{2-\delta-2k}{6-4\delta} \right) & 0 \leq k < \frac{\delta^2-2\delta}{2\delta-6} \wedge \frac{\delta+2k-2}{4\delta-6} \leq v < \frac{-\delta^2+4\delta+2\delta k-2k-4}{4\delta-6} \end{cases}$$

The result then follows from comparing this utility function with the utility function of a consumer who is myopic under the exogenous model. \square

Proof of Proposition 4. (i) Under dynamic pricing and endogenous behavior,

$$\pi^k = \begin{cases} \frac{1}{3} & k \geq \frac{\delta}{3} \\ \frac{k(2\delta-3k)}{\delta^2} & \frac{\delta^2-2\delta}{2\delta-6} \leq k < \frac{\delta}{3} \\ -\frac{\delta^2+4(k^2+k+1)-4\delta(k+1)}{8\delta-12} & 0 \leq k < \frac{\delta^2-2\delta}{2\delta-6} \end{cases}$$

The result follows from performing piece-wise analysis.

(ii) We have

$$CS^k = \begin{cases} \frac{\delta+1}{18} & k \geq \frac{\delta}{3} \\ \frac{\delta^2+4k^2+\delta(k-4)k}{2\delta^2} & \frac{\delta^2-2\delta}{2\delta-6} \leq k < \frac{\delta}{3} \\ \frac{(2-\delta)((2-5\delta)\delta+4(5-4\delta)k^2+8\delta^2k)+8k-4}{8(3-2\delta)^2} & 0 \leq k < \frac{\delta^2-2\delta}{2\delta-6} \end{cases}$$

For $\frac{\delta^2-2\delta}{2\delta-6} \leq k < \frac{\delta}{3}$, $CS^k = \frac{\delta^2+4k^2+\delta(k-4)k}{2\delta^2}$ and attains its maximum in $k = \frac{2\delta-\delta^2}{6-2\delta}$ with

the value $\frac{\delta(2-\delta)^2+4}{8(3-\delta)^2} \geq \frac{1+\delta}{3}$. For $0 \leq k < \frac{\delta^2-2\delta}{2\delta-6}$, $CS^k = \frac{(2-\delta)((2-5\delta)\delta+4(5-4\delta)k^2+8\delta^2k)+8k-4}{8(3-2\delta)^2}$.

Let x^* be the first root of $x^3 - 12x^2 + 24x - 12 = 0$. Then, CS^k attains its maximum

in this region at $k = \begin{cases} \frac{\delta^2-2\delta}{2\delta-6} & \delta > x^* \\ 0 & \text{otherwise} \end{cases}$ with the value $\begin{cases} \frac{\delta(2-\delta)^2+4}{8(3-\delta)^2} & \delta > x^* \\ \frac{\delta(2-\delta)(2-5\delta)+4}{8(3-2\delta)^2} & \text{otherwise} \end{cases}$.

One can also check that $\frac{\delta(2-\delta)(2-5\delta)+4}{8(3-2\delta)^2} \geq \frac{\delta(2-\delta)^2+4}{8(3-\delta)^2}$ for $\delta \leq x^*$. As such CS^k is

maximized in $k = \begin{cases} \frac{2\delta-\delta^2}{6-2\delta} & \delta > x^* \\ 0 & \delta \leq x^* \end{cases}$.

(iii) Combining parts (i) and (ii), we have

$$SW^k = \begin{cases} \frac{7+\delta}{18} & k \geq \frac{\delta}{3} \\ \frac{1}{2} \left(\frac{(\delta-2)k^2}{\delta^2} + 1 \right) & \frac{\delta^2-2\delta}{2\delta-6} \leq k < \frac{\delta}{3} \\ \frac{\delta(\delta(\delta+10)-36)+4(\delta(4\delta-17)+16)k^2-8(\delta-2)(\delta-1)^2k+28}{8(3-2\delta)^2} & 0 \leq k < \frac{\delta^2-2\delta}{2\delta-6} \end{cases}$$

For $0 \leq k < \frac{\delta^2-2\delta}{2\delta-6}$, the social welfare is non-decreasing in k because $\frac{dSW^k}{dk} = \frac{-\delta(\delta^2-4\delta(k+1)+17k+5)+16k+2}{(3-2\delta)^2} \geq 0$. Also, for $\frac{\delta^2-2\delta}{2\delta-6} \leq k < \frac{\delta}{3}$, SW^k is concave in k and attains its maximum at $k = \frac{\delta(2-\delta)}{2(3-\delta)}$. Additionally, SW^k is constant in k for $k \geq \frac{\delta}{3}$.

The result then follows by noting that SW^k is continuous in k . \square

Proof of Lemma 4. Similar to the proof of Lemma 3, one needs to solve the following optimization problem:

$$\begin{aligned} & \text{Max}_{p_1, p_2, \bar{v}} (1 - \bar{v})p_1 + (\bar{v} - p_2)p_2 \\ \text{s.t.} \quad & \bar{v} = \max\left(p_1, \frac{p_1 - \delta p_2 - k}{1 - \delta}\right) \end{aligned}$$

to obtain

$$p_1^c = \begin{cases} \frac{2}{3} & k \geq \frac{\delta}{3} \\ \frac{\delta+k}{2\delta} & \frac{\delta^2-\delta}{\delta-3} \leq k < \frac{\delta}{3} \\ \frac{k+2}{\delta+3} & 0 \leq k < \frac{\delta^2-\delta}{\delta-3} \end{cases}, \quad p_2^c = \begin{cases} \frac{1}{3} & k \geq \frac{\delta}{3} \\ \frac{\delta-k}{2\delta} & \frac{\delta^2-\delta}{\delta-3} \leq k < \frac{\delta}{3} \\ \frac{\delta-k+1}{\delta+3} & 0 \leq k < \frac{\delta^2-\delta}{\delta-3} \end{cases},$$

$$\text{and } \bar{v}^c = \begin{cases} \frac{2}{3} & k \geq \frac{\delta}{3} \\ \frac{\delta+k}{2\delta} & \frac{\delta^2-\delta}{\delta-3} \leq k < \frac{\delta}{3} \\ \frac{\delta^2+\delta+2k-2}{\delta^2+2\delta-3} & 0 \leq k < \frac{\delta^2-\delta}{\delta-3} \end{cases}.$$

This completes the proof. \square

Proof of Proposition 5. Plugging the equilibrium outcomes under dynamic pricing and price commitment strategies in the corresponding objective functions we have

$$\pi^k = \begin{cases} \frac{1}{3} & k_d \geq \frac{\delta}{3} \\ \frac{k_d(2\delta-3k_d)}{\delta^2} & \frac{\delta^2-2\delta}{2\delta-6} \leq k_d < \frac{\delta}{3} \\ \frac{\delta^2+4(k_d^2+k_d+1)-4\delta(k_d+1)}{12-8\delta} & 0 \leq k_d < \frac{\delta^2-2\delta}{2\delta-6} \end{cases}, \text{ and}$$

$$\pi^c = \begin{cases} \frac{1}{3} & k_c \geq \frac{\delta}{3} \\ \frac{(\delta-k_c)(\delta+3k_c)}{4\delta^2} & \frac{\delta^2-\delta}{\delta-3} \leq k_c < \frac{\delta}{3} \\ \frac{\delta(k_c+1)-k_c(k_c+1)-1}{(\delta-1)(\delta+3)} & 0 \leq k_c < \frac{\delta^2-\delta}{\delta-3} \end{cases}.$$

(i) If $k_c = k_d = k$, then

$$\pi^c - \pi^k = \begin{cases} 0 & \frac{\delta}{3} \leq k \\ \frac{(\delta-3k)^2}{4\delta^2} & \frac{2\delta-d^2}{6-2\delta} \leq k < \frac{\delta}{3} \\ \frac{\delta^2+4(k^2+k+1)-4\delta(k+1)}{\delta^2(\delta-2k-1)^2} + \frac{(\delta-k)(\delta+3k)}{4\delta^2} & \frac{\delta-\delta^2}{3-\delta} \leq k < \frac{\delta}{3} \\ \frac{8\delta-12}{4(\delta-1)(\delta+3)(2\delta-3)} & \text{otherwise} \end{cases}.$$

One can then perform piece-wise analysis to find the result.

(ii) Observe that profit functions are strictly increasing and continuous in $k_i \in [0, \frac{\delta}{3})$ for $i \in \{s, c\}$. Let $\pi^i(\delta, k)$ be the firm's profit function under the pricing mechanism $i \in \{s, c\}$ when consumer discount factor and cost of strategic behavior are δ and k , respectively. Note that from Part (i), for any fixed $k_c < \frac{\delta}{3}$, we have $\pi^s(\delta, k_c) < \pi^c(\delta, k_c)$. In addition, $\pi^s(\delta, \frac{\delta}{3}) = \pi^c(\delta, \frac{\delta}{3}) > \pi^c(\delta, k_c)$. Therefore, from Intermediate Value Theorem, there exists some $\bar{k}_d(k_c, \delta) \in (k_c, \frac{\delta}{3})$ such that

$\pi^s(\delta, \bar{k}_d(k_c, \delta)) = \pi^c(\delta, k_c)$. Since $\pi^s(\delta, k)$ is increasing in k , for $k_d \geq \bar{k}_d(k_c, \delta)$ we have $\pi^s(\delta, k_d) \geq \pi^s(\delta, \bar{k}_d(k_c, \delta)) = \pi^c(\delta, k_c)$. The uniqueness of $\bar{k}_d(k_c, \delta)$ follows from monotonicity of $\pi^s(\delta, k)$ in k . \square

Appendix B

Proofs and Supplementary Materials for Chapter 3

B.1 Supporting Results

Lemma 7. *Let Y be a metric space and X be a compact metric space.¹ In addition, let $f : X \times Y \rightarrow X$ be a continuous function such that there exists a unique map $\gamma(y) : Y \rightarrow X$ that satisfies $f(\gamma(y), y) = \gamma(y)$. Then $\gamma(y)$ is continuous in y .²*

Proof of Lemma 7. From the Closed Graph Theorem, we know that map $\gamma(y)$ is continuous if and only if its graph is closed (since X is compact and Y is metric). Graph of $\gamma(y)$ is given by the following set: $Gr(\gamma) = \{(y, x) \in X \times Y | x = \gamma(y)\}$. But if the fixed point to $f(\cdot)$ is unique, then $x = \gamma(y)$ if and only if $f(x, y) = x$. Therefore, $Gr(\gamma) = \{(y, x) \in X \times Y | f(x, y) = x\}$. Define function $h(x, y) := f(x, y) - x$, then since $f(\cdot)$ is continuous, $h(\cdot)$ is also continuous and $Gr(\gamma) = h^{-1}(\{0\})$. But $\{0\}$ is a closed set and h is continuous; therefore $h^{-1}(\{0\})$ is closed, which concludes that $\gamma(y)$ is continuous. \square

¹ This result could be generalized to Hausdorff spaces.

² Szymik (2015) brings a counter example to show that the result does not hold in general. However, in his example the solution to the fixed point is not unique and Szymik (2015) does not show that the result holds if we restrict attention to the functions with a unique fixed point.

Lemma 8. *There exists a unique and continuous explicit function $\gamma_i(q_i, v_i)$ such that $\gamma_i(q_i, v_i) = \xi_{i2}(q_i, v_i)$.*

Proof of Lemma 8. Define $f(x, v_i, q_i) := \mathbb{E}_{D_i^c} \left[\min\left(1, \frac{(q_i - \bar{G}(v_i)D_i^c)^+}{(G(v_i) - \bar{G}(\frac{k}{x\delta}))^+ D_i^c}\right) \right]$. Since $f(x, v_i, q_i)$ is a continuous function from $[0, 1]$ to itself, from Brouwer's Fixed Point Theorem, at least one fixed point to this equation exists for all q_i and v_i . The uniqueness follows from noting that $f(x, v_i, q_i)$ is decreasing in x . The continuity then follows from Lemma 7. \square

B.2 Proofs from Main Text

Proof of Proposition 6. Clearly, it is not optimal for a consumer to buy the product in the first period and give a redundant visit in the second period. Therefore, if a consumer visits twice, she should buy the product in the second period. In other words, $\tilde{\xi}_{i2}(A)\delta v - k > v - p$, where the consumer receives ex ante utility $\tilde{\xi}_{i1} \times (\tilde{\xi}_{i2}(A) \times \delta v - k) - k$. If the consumer only visits in the second period, her ex ante expected utility would be $\tilde{\xi}_{i2} \times \delta v - k = \tilde{\xi}_{i1} \times \tilde{\xi}_{i2}(A)\delta v - k > \tilde{\xi}_{i1} \times (\tilde{\xi}_{i2}(A)\delta v - k) - k$, which proves the first part. To see the second part, note that a consumer visits in the first period if and only if $\tilde{\xi}_{i1}(v - p) - k \geq \tilde{\xi}_{i2}\delta v - k$ and $\tilde{\xi}_{i1}(v - p) - k \geq 0$. The result then follows from noting that $\tilde{\xi}_{i1} \in (0, 1]$ and $\tilde{\xi}_{ij} \in [0, 1]$. \square

Proof of Proposition 7. From Equation (3.2), following the newsvendor solution, the optimal inventory level in market $i = 1, 2$ is $q_i^*(\tilde{v}_i) = \bar{G}(\tilde{v}_i)F_i^{-1}\left(\frac{p-c}{p}\right)$, where $F_i^{-1}(x)$ is the inverse cumulative distribution function in market i . Due to the normal demand assumption, this may be written as $q_i^*(\tilde{v}_i) = \bar{G}(\tilde{v}_i)(\mu + \hat{z}\sigma)$ in the multichannel system and $q_I^*(\tilde{v}_I) = \bar{G}(\tilde{v}_I)\left(2\mu + \hat{z}\sigma\sqrt{2(1+\rho)}\right)$ in the integrated system, where \hat{z} is the standard normal z -statistic corresponding to an in-stock probability of $\frac{p-c}{p}$. The expressions for expected profit follow from the newsvendor profit

function evaluated at the quantities above. \square

Proof of Proposition 8. (i) *The multichannel System.* From Proposition 7 and Definition 1, the equilibrium in the multichannel system must satisfy the following conditions in each market: (1) the firm chooses the optimal inventory level, $q_i^* = \bar{G}(\tilde{v}_i)(\mu + \hat{z}\sigma)$, (2) consumers purchase in the period that maximizes their utility, $v_i^* = \min\left(v_h, \max\left(\frac{\tilde{\xi}_{i1}p}{\tilde{\xi}_{i1} - \tilde{\xi}_{i2}\delta}, p + \frac{k}{\tilde{\xi}_{i,1}}\right)\right)$, and expectations are rational, (3) $\tilde{v}_i = v_i^*$, and (4) $\tilde{\xi}_{i1} = \xi_{i1}(q_i^*, v_i^*)$ and $\tilde{\xi}_{i2} = \gamma(q_i^*, v_i^*)$. Combining conditions (1) with (3) and (2) with (4) yields $q_i^* = \bar{G}(v_i^*)(\mu + \hat{z}\sigma)$, and $v_i^* = \min\left(v_h, \max\left(\frac{\xi_{i1}(q_i^*, v_i^*)p}{\xi_{i1}(q_i^*, v_i^*) - \gamma(q_i^*, v_i^*)\delta}, p + \frac{k}{\xi_{i1}(q_i^*, v_i^*)}\right)\right)$. Thus, a simultaneous solution to these two equations will provide the equilibrium. To derive this equilibrium we must provide a functional form of $\xi_{ij}(q_i^*, v_i^*)$, the actual probability that (in equilibrium) a consumer will obtain a unit if the consumer visits the store in period j . Inserting the expression for the optimal inventory level of the firm and rearranging terms one finds that

$$\xi_{i1}(v_i^*) = \xi_{i1} = \mathbb{E}_{D_i} \left[\min\left(1, \frac{\mu + \hat{z}\sigma}{D_i^c}\right) \right],$$

and $\gamma(v_i^*)$ would be the solution to

$$x = \mathbb{E}_{\hat{D}} \left[\min\left(1, \frac{\bar{G}(v_i^*)}{(G(v_i^*) - G(\frac{k}{x\delta}))^+} \hat{D}\right) \right], \quad (\text{B.1})$$

where $\hat{D} = \frac{(\mu + \hat{z}\sigma - D_i^c)^+}{D_i^c}$, with density function $h(\cdot)$. Since the individual market demands are identically distributed, any equilibria must be identical in the both markets and we replace i with m . Define

$$\Omega_m(v) := \min\left(v_h, \max\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma(\bar{G}(v)(\mu + \hat{z}\sigma), v)\delta}, p + \frac{k}{\xi_{i1}}\right)\right).$$

To prove the existence of the equilibrium, note that $p + k \leq \Omega_m(v) \leq v_h$ and \bar{G} is continuous. Also from Lemma 8, γ is continuous; therefore $\Omega_m(v)$ is continuous, implying Brouwer's Fixed Point Theorem applies. To prove the uniqueness it is enough to show that $\Omega_m(v)$ is decreasing in v . Since ξ_{i1} is independent from v , it is sufficient to show that $\gamma(\bar{G}(v)(\mu + \hat{z}\sigma), v)$ is decreasing in v . We know that $\gamma(\bar{G}(v)(\mu + \hat{z}\sigma), v)$ is the solution to B.1. If $0 \geq G(v_i^*) - G(\frac{k}{x\delta})$, then there is nothing to prove. If $0 < G(v_i^*) - G(\frac{k}{x\delta})$, then since \hat{D} is supported on $[0, \infty)$ we have

$$\mathbb{E}_{\hat{D}} \left[\min\left(1, \frac{\bar{G}(v)}{G(v) - G(\frac{k}{x\delta})} \hat{D}\right) \right] = \int_0^{\frac{G(v) - G(\frac{k}{x\delta})}{\bar{G}(v)}} \frac{\bar{G}(v)t}{G(v) - G(\frac{k}{x\delta})} h(t) dt + \int_{\frac{G(v) - G(\frac{k}{x\delta})}{\bar{G}(v)}}^{\infty} h(t) dt.$$

From the implicit function theorem,

$$\frac{\partial x}{\partial v} = - \frac{\frac{\partial(\mathbb{E}_{\hat{D}} \left[\min\left(1, \frac{\bar{G}(v)}{G(v) - G(\frac{k}{x\delta})} \hat{D}\right) \right] - x)}{\partial v}}{\frac{\partial(\mathbb{E}_{\hat{D}} \left[\min\left(1, \frac{\bar{G}(v)}{G(v) - G(\frac{k}{x\delta})} \hat{D}\right) \right] - x)}{\partial x}} = - \frac{\int_0^{\frac{G(v) - G(\frac{k}{x\delta})}{\bar{G}(v)}} \frac{G'(v)(1 - G(\frac{k}{x\delta}))th(t)}{(G(v) - G(\frac{k}{x\delta}))^2} dt}{\int_0^{\frac{G(v) - G(\frac{k}{x\delta})}{\bar{G}(v)}} - \frac{\bar{G}(v)G'(\frac{k}{x\delta})kth(t)}{\delta x^2 (G(v) - G(\frac{k}{x\delta}))^2} dt - 1} \leq 0.$$

Therefore, $\gamma(\bar{G}(v)(\mu + \hat{z}\sigma), v)$ is decreasing in v and consequently $\Omega_m(v)$ is also decreasing in v , which proves the uniqueness of the fixed point.

(ii) *The Integrated System.* The proof follows analogously to Part (i), replacing the individual market demand mean and standard deviation (μ and σ , respectively) with the integrated market mean and standard deviation (2μ and $\sigma\sqrt{2(1+\rho)}$). \square

Proof of Proposition 9. Since $L(\hat{z}) = \phi(\hat{z}) - \hat{z}(1 - \Phi(\hat{z}))$ (see Porteus 2002), $L(\hat{z}) + \hat{z} = \phi(\hat{z}) + \hat{z}\Phi(\hat{z}) \geq 0$ implies $pL(\hat{z}) + c\hat{z} \geq 0$. Hence, operational value of integration is positive. Additionally, $\frac{d\Delta}{d\rho} = -\frac{(pL(\hat{z}) + c\hat{z})\sigma}{\sqrt{2(1+\rho)}} \leq 0$, and coefficient $\bar{G}(v_m^*)$ is independent from ρ , which concludes that operational value is decreasing in ρ . To show that the operational value is decreasing in δ , it is enough to show that v_m^* is non-decreasing in δ (since \bar{G} is a decreasing function and only depends on δ through v^*). From Propositions 6 & 7, $v_m^* = \min\left(v_h, \max\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma(\bar{G}(v_m^*)(\mu + \hat{z}\sigma), v_m^*)\delta}, p + \frac{k}{\xi_{i,1}}\right)\right)$.

If $v_m^* = v_h$ or $p + \frac{k}{\xi_{i1}}$, then it is straightforward to see the results. For simplicity of the notation let $\gamma = \gamma(q^*, v_m^*)$. Suppose $v_m^* = \frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}$, then γ would be the solution to

$$\gamma = \mathbb{E}_{\hat{D}} \left[\min\left(1, \frac{\bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right)}{G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)} \hat{D}\right) \right],$$

where $\hat{D} = \frac{(\mu + \hat{z}\sigma - D_i^c)^+}{D_i^c}$, with density function $h(\cdot)$. Then,

$$\gamma = \int_0^{\frac{G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)}{\bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right)}} \frac{\bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right)t}{G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)} h(t) dt + \int_{\frac{G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)}{\bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right)}}^{\infty} h(t) dt,$$

and

$$\frac{\partial\gamma}{\partial\delta} = \frac{\int_0^{\frac{G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)}{\bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right)}} - \frac{\left((\xi_{i1} - \gamma\delta)^2 k \bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) G'\left(\frac{k}{\gamma\delta}\right) + \bar{G}'\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) \left(G\left(\frac{k}{\gamma\delta}\right) - 1\right) p \xi_{i1} \gamma^2 \delta^2\right) th(t)}{(\xi_{i1} - \gamma\delta)^2 \left(G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)\right)^2 \gamma \delta^2} dt}{1 - \int_0^{\frac{G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)}{\bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right)}} - \frac{\left((\xi_{i1} - \gamma\delta)^2 k \bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) \bar{G}'\left(\frac{k}{\gamma\delta}\right) + \bar{G}'\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) \left(G\left(\frac{k}{\gamma\delta}\right) - 1\right) p \xi_{i1} \gamma^2 \delta^2\right) th(t)}{(\xi_{i1} - \gamma\delta)^2 \left(G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)\right)^2 \gamma^2 \delta} dt} \leq 0,$$

which proves the last part of the proposition. One can also see that

$$\frac{\partial(\gamma \times \delta)}{\partial\delta} = \delta \frac{\partial\gamma}{\partial\delta} + \gamma = \frac{\gamma}{1 - \int_0^{\frac{G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)}{\bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right)}} - \frac{\left((\xi_{i1} - \gamma\delta)^2 k \bar{G}\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) G'\left(\frac{k}{\gamma\delta}\right) + \bar{G}'\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) \left(G\left(\frac{k}{\gamma\delta}\right) - 1\right) p \xi_{i1} \gamma^2 \delta^2\right) th(t)}{(\xi_{i1} - \gamma\delta)^2 \left(G\left(\frac{\xi_{i1}p}{\xi_{i1} - \gamma\delta}\right) - G\left(\frac{k}{\gamma\delta}\right)\right)^2 \gamma^2 \delta} dt} \geq 0.$$

Therefore, $\gamma \times \delta$ is non-decreasing in δ , and hence, v^* is non-decreasing, which completes this part of the proof. The fact that the firm carries less inventory as consumers become more patient is the direct conclusion of non-decreasing behavior of v^* in δ .

Lastly, we consider the operational value as a function of k . Particularly, we consider the behavior of $\bar{G}(v_m^*)$ in k . We have

$$\frac{\partial \gamma}{\partial k} = \frac{\int_0^{\frac{G(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})-G(\frac{k}{\gamma\delta})}{\bar{G}(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})}} \frac{\bar{G}(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})G'(\frac{k}{\gamma\delta})}{\gamma\delta\left(G(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})-G(\frac{k}{\gamma\delta})\right)^2} th(t) dt}{1 - \int_0^{\frac{G(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})-G(\frac{k}{\gamma\delta})}{\bar{G}(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})}} \frac{-p\gamma^2\delta^2\xi_{i1}\bar{G}(\frac{k}{\gamma\delta})G'(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})-(\xi_{i1}-\gamma\delta)^2\bar{G}(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})G'(\frac{k}{\gamma\delta})k}{\gamma^2\delta\left(G(\frac{\xi_{i1}p}{\xi_{i1}-\gamma\delta})-G(\frac{k}{\gamma\delta})\right)^2(\xi_{i1}-\gamma\delta)^2} th(t) dt} \leq 0,$$

which completes the proof of this part. \square

Proof of Proposition 10. In equilibrium

$$\xi_{I1} = \frac{\mathbb{E}_{D_P} \left[\min(2\mu + \hat{z}\sqrt{2(1+\rho)}\sigma, D_I) \right]}{2\mu} = \frac{2\mu + \sqrt{2(1+\rho)}\sigma\mathbb{E}_X [\min(\hat{z}, X)]}{2\mu},$$

where $X \sim N(0, 1)$. The first part follows from noting that $\mathbb{E}_X [\min(\hat{z}, X)] < 0$. The second part, follows from the fact that for $\rho = 1$ we have $\xi_{I1}(q_I^*, v_I^*) = \xi_{m1}(q_m^*, v_m^*)$.

\square

Proof of Proposition 11. Omitted; follows immediately from the preceding results. \square

Proof of Proposition 12. To study the behavioral value of integration, it is sufficient to study $G(v_m^*) - G(v_I^*)$. Since $\delta = 0$, then $v_m^* = \min(v_h, p + \frac{k}{\xi_{m1}})$ and $v_I^* = \min(v_h, p + \frac{k}{\xi_{I1}})$. From Proposition 10, $p + \frac{k}{\xi_{m1}} \geq p + \frac{k}{\xi_{I1}}$. Therefore, $v_m^* \geq v_I^*$, which proves the first part of the proposition. To prove the second part, if $k \geq (v_h - p)\xi_{m1}$, then $v_m^* = v_h$ and the behavioral value is always decreasing in k and the result follows from noting that $g(p + \frac{k}{\xi_{m1}}) = 0$. If $k < (v_h - p)\xi_{m1}$, then $v_m^* = p + \frac{k}{\xi_{m1}}$ and $v_I^* = p + \frac{k}{\xi_{I1}}$. Then

$$\frac{\partial(G(v_m^*) - G(v_I^*))}{\partial k} = \frac{g(p + \frac{k}{\xi_{m1}})}{\xi_{m1}} - \frac{g(p + \frac{k}{\xi_{I1}})}{\xi_{I1}}.$$

The behavioral value is then increasing in k if and only if this value is non-negative, which proves Part (ii). Lastly to see Part (iii), observe that v_m^* is independent from ρ while, using Proposition 10, $v_I^* = \min(v_h, p + \frac{k}{\xi_{I1}})$ is increasing in ρ . Therefore, $G(v_m^*) - G(v_I^*)$ is decreasing in ρ . Additionally, $\Pi\left(2\mu, \sqrt{2(1+\rho)}\sigma\right)$ is decreasing in ρ . The result hence follows from noting that $G(v_m^*) - G(v_I^*)$ and $\Pi\left(2\mu, \sqrt{2(1+\rho)}\sigma\right)$ are both non-negative. \square

Proof of Proposition 13. First let $v^* = \frac{p}{1-\xi_{I2}(A)\delta}$ and define $B := \frac{\bar{G}(\frac{p}{1-\xi_{I2}(A)\delta})}{G(\frac{p}{1-\xi_{I2}(A)\delta})}$. During the proof, one should keep in mind that B is a function of $\xi_{I2}(A)$ and ρ . Note that

$$\xi_{I2}(A) = \frac{\xi_{I2}}{\xi_{I1}} = \frac{\mathbb{E}\left[\min\left(\frac{2\mu}{\sqrt{2(1+\rho)}\sigma} + X, B(\hat{z} - X)^+\right)\right]}{\frac{2\mu}{\sqrt{2(1+\rho)}\sigma} + \mathbb{E}[\min(X, \hat{z})]},$$

where $X \sim N(0, 1)$. For simplicity of the notation let $C := \frac{2\mu}{\sqrt{2(1+\rho)}\sigma}$. Expanding the expectations and rearranging the terms we have

$$\begin{aligned} \xi_{I2}(A) \times \left(C + \int_{-\infty}^{\hat{z}} x\phi(x)dx + \int_{\hat{z}}^{\infty} z\phi(x)dx\right) \\ = \int_{-\infty}^{\frac{-C+\hat{z}B}{1+B}} (x+C)\phi(x)dx + \int_{\frac{-C+\hat{z}B}{1+B}}^{\hat{z}} B(z-x)\phi(x)dx. \end{aligned}$$

To show that $\xi_{I2}(A)$ is quasiconcave in ρ , we should show that if $\frac{\partial \xi_{I2}(A)}{\partial \rho} = 0$, then $\frac{\partial^2 \xi_{I2}(A)}{\partial \rho^2} < 0$. From implicit differentiation (again note the dependence of B , C , and $\xi_{I2}(A)$ on ρ) and simplification we have,

$$\frac{\partial \xi_{I2}(A)}{\partial \rho} = \frac{-\left(\Phi\left(\frac{-C+\hat{z}B}{1+B}\right) - \xi_{I2}(A)\right) \times \frac{\mu}{\sqrt{(2)(1+\rho)}\sigma}}{C + \int_{-\infty}^{\hat{z}} x\phi(x)dx + \int_{\hat{z}}^{\infty} z\phi(x)dx - \int_{\frac{-C+\hat{z}B}{1+B}}^{\hat{z}} (z-x)B'\phi(x)dx}. \quad (\text{B.2})$$

This equation is 0 if and only if $\Phi\left(\frac{-C+\hat{z}B}{1+B}\right) - \xi_{I2}(A) = 0$. Note that since \bar{G} is a

decreasing function, $\frac{\bar{G}(\frac{p}{1-x\delta})}{G(\frac{p}{1-x\delta})}$ is decreasing in x and therefore $B' = \frac{\partial B}{\partial x} \leq 0$. Consequently, $\int_{\frac{-C+\hat{z}B}{1+B}}^z (z-x)B'\phi(x)dx \leq 0$. Also note that

$$C + \int_{-\infty}^{\hat{z}} x\phi(x)dx + \int_{\hat{z}}^{\infty} z\phi(x)dx = \frac{\mathbb{E} \left[\min \left(D_I, 2\mu + \hat{z}\sqrt{2(1+\rho)}\sigma \right) \right]}{\sqrt{2(1+\rho)}\sigma} = \xi_{I1}C > 0.$$

Therefore, the denominator is positive. Let $h(\rho)$ and $u(\rho)$ be the numerator and denominator of Equation B.2 and let ρ^* be any solution to $\frac{\partial \xi_{I2}(A)}{\partial \rho} = 0$. Then,

$$\frac{\partial^2 \xi_{I2}(A)}{\partial \rho^2} \Big|_{\rho=\rho^*} = \frac{h'(\rho)u(\rho) - u'(\rho)h(\rho)}{u(\rho)^2} \Big|_{\rho=\rho^*} = \frac{h'(\rho)u(\rho)}{u(\rho)^2} \Big|_{\rho=\rho^*},$$

where the last equality follows from noting that $h(\rho^*) = 0$. Since we showed that $u(\rho) > 0$, we only need to show that $h'(\rho)|_{\rho=\rho^*} < 0$. We have,

$$\begin{aligned} h'(\rho) \Big|_{\rho=\rho^*} &= \\ & \left(- \left(\Phi \left(\frac{-C + \hat{z}B}{1+B} \right) - \xi_{P2}(A) \right)' \times \frac{\mu}{\sqrt{(2)(1+\rho)^{\frac{3}{2}}}\sigma} + \right. \\ & \left. \left(\Phi \left(\frac{-C + \hat{z}B}{1+B} \right) - \xi_{P2}(A) \right) \times \left(\frac{\mu}{\sqrt{(2)(1+\rho)^{\frac{3}{2}}}\sigma} \right)' \right) \Big|_{\rho=\rho^*} \\ &= - \left(\Phi \left(\frac{-C + \hat{z}B}{1+B} \right)' \times \frac{\mu}{\sqrt{(2)(1+\rho)^{\frac{3}{2}}}\sigma} \right) \Big|_{\rho=\rho^*}. \end{aligned}$$

The last equality followed from noting that $\left(\Phi \left(\frac{-C + \hat{z}B}{1+B} \right) - \xi_{P2}(A) \right) = 0$ and $\xi'_{I2}(A) = 0$ in $\rho = \rho^*$. As such it is sufficient to show that $\Phi \left(\frac{-C + \hat{z}B}{1+B} \right)' > 0$. Since $\Phi(\cdot)$ is increasing, we only need to show that $\left(\frac{-C + \hat{z}B}{1+B} \right)' > 0$. We have

$$\frac{\partial \left(\frac{-C + \hat{z}B}{1+B} \right)}{\partial \rho} = \frac{-(1+B)\frac{\partial C}{\partial \rho} + (c + \hat{z})\xi'_{I2}(A)B'}{(1+B)^2} \Big|_{\rho=\rho^*} = \frac{-(1+B)\frac{\partial C}{\partial \rho}}{(1+B)^2} > 0.$$

This shows that $\xi_{I2}(A)$ is quasiconcave in ρ . For the last part of the proposition note that for $\rho = 1$, $\xi_{I2}(A) = \xi_{m2}(A)$. Also when $\rho = -1$ demand is deterministic

in the integrated system and therefore $\xi_{I_2}(A) = 0$. These two observations show that $v_m^*|_{\rho=1} = v_I^*|_{\rho=1}$ and $v_m^*|_{\rho=-1} \geq p = v_I^*|_{\rho=-1}$. And since we showed that $\xi_{I_2}(A)$ is quasiconcave, v_I^* is also quasiconcave, which implies the result. The proof for $v^* = \min(v_h \frac{p}{1-\xi_{I_2}(A)\delta})$ follows from the fact that v^* is increasing in $\xi_{I_2}(A)$ and by replacing the minimum function by its continuous approximation in all the steps above. \square

Proof of Proposition 14. To see that the operational value of integration is positive and decreasing in λ note that $\left(2 - \sqrt{2(2 - \lambda(2 - \lambda)(1 - \rho))}\right) \geq 0$ for $\lambda \in [1, 2]$. In addition $\frac{\partial\left(2 - \sqrt{2(2 - \lambda(2 - \lambda)(1 - \rho))}\right)}{\partial\lambda} = -\frac{\sqrt{2}(\lambda-1)(1-\rho)}{\sqrt{2+(1-\rho)\lambda(\lambda-2)}} \leq 0$. The result then follows from noting that v_m^* is independent from λ . Consequently, $\Delta(\lambda) \leq \Delta(\lambda)|_{\lambda=2} = (pL(\hat{z}) + c\hat{z}) \left(2 - \sqrt{2(1 + \rho)}\right) \sigma$. To see the other parts it is enough to consider the problem's equivalent symmetric form with a higher correlation and use the results in Proposition 13. \square

Appendix C

Proofs and Supplementary Materials for Chapter 4

Proof of Lemma 5. We have $A = \frac{1-e^{-\mu\bar{t}}}{e^{-\mu\bar{t}}-e^{-\mu T}}$. Then,

$$\frac{\partial A}{\partial \alpha} = \frac{\mu' e^{(\bar{t}+T)\mu} (\bar{t}(e^{\mu T} - 1) - T(e^{\mu\bar{t}} - 1))}{(e^{\mu\bar{t}} - e^{\mu T})^2}.$$

Since μ is increasing in α , we only need to show that $B \equiv (\bar{t}(e^{\mu T} - 1) - T(e^{\mu\bar{t}} - 1)) \geq 0$. We have

$$\frac{\partial B}{\partial \mu} = \bar{t}T(e^{\mu T} - e^{\mu\bar{t}}) \geq 0.$$

As such, it is enough to show that $B|_{\mu=0} \geq 0$. But $B|_{\mu=0} = 0$, which completes the proof. \square

Proof of Lemma 6. To show that v_2 is unimodal, first we note that $\frac{x}{1-e^{-xt}}$ is convex and increasing in x for all values of t because

$$\frac{d(\frac{x}{1-e^{-xt}})}{dx} = \frac{e^{xt}(e^{xt} - xt - 1)}{(1 - e^{xt})^2} \geq 0,$$

and

$$\frac{d^2(\frac{x}{1-e^{-xt}})}{dx^2} = \frac{te^{xt}(xt(e^{xt} + 1) - 2(e^{xt} - 1))}{(e^{xt} - 1)^3} > 0,$$

for $x, t > 0$. Also, one can easily check that $\frac{1}{e^{-xt}}$ is convex and increasing in x for all values of t . Therefore, $\frac{x}{e^{-xt}-e^{-xT}} = \frac{1}{e^{-xt}} \cdot \frac{x}{1-e^{-x(T-t)}}$ is convex and increasing in x (product of two increasing and convex functions). Hence, its composition with μ is a convex function of α . In other words, $\frac{\mu}{e^{-\mu t}-e^{-\mu T}}$ is a convex function of α . We have

$$v_2 = \frac{v_0}{\frac{\mu}{e^{-\mu t}-e^{-\mu T}}}.$$

Since v_0 is concave and non negative and denominator is strictly positive and convex, v_2 is strictly quasiconcave. To obtain the result for Δ , we replace \bar{t} and T with 0 and \bar{t} , respectively.

To show that maximum would be achieved in an interior solution, note that $e^{-\mu \bar{t}} - e^{-\mu T}$ is bounded, and therefore $\lim_{\alpha \rightarrow 0 \text{ or } \infty} \Delta = \lim_{\alpha \rightarrow 0 \text{ or } \infty} v_2 = 0$. Since Δ and v_2 are non negative functions, maximum should be achieved in an interior solution. \square

Proof of Proposition 15. We first solve for the optimal fashionability level. Clearly, at any given fixed inventory level, the firm designs a product that maximizes v_1 . Replacing the maximum value of v_1 , the firm's inventory management problem will be converted to a simple newsvendor model with price $p = v_1$, $s = 0$, and marginal cost c . \square

Proof of Proposition 16. We have $\Delta = Av_2$. Therefore, $\frac{\partial \Delta}{\partial \alpha} = A \frac{\partial v_2}{\partial \alpha} + \frac{\partial A}{\partial \alpha} v_2$. Since $\frac{\partial A}{\partial \alpha} v_2 \geq 0$ and $A \geq 0$, then whenever $\frac{\partial v_2}{\partial \alpha} \geq 0$, necessarily $\frac{\partial \Delta}{\partial \alpha} \geq 0$. Since Δ and v_2 are unimodal in α , essentially whenever v_2 increases Δ would also increase. This concludes the first part of the proposition.

For the second part, note that the firm's profit is increasing in Δ and v_2 . Since Δ and v_2 are unimodal with an interior maximizer, the firm would always achieve a higher profit by increasing α in the regions that both Δ and v_2 are increasing. Given that $\alpha_{\Delta}^* \geq \alpha_{v_2}^*$, we have $\alpha_d^* \geq \alpha_{v_2}^*$. Additionally, for $\alpha \geq \alpha_{\Delta}^*$, both Δ and v_2 are decreasing in α , which suggests that the firm's profit is a decreasing function of α in this region. As such $\alpha_d^* \leq \alpha_{\Delta}^*$. This completes the proof. \square

Proof of Proposition 17. Using Proposition 16, we only need to consider domain $\alpha \in [\alpha_{v_2}^*, \alpha_{\Delta}^*]$. We first differentiate the function respect to the inventory level for a fixed value of α . We have

$$\frac{\partial \pi(\alpha, I)}{\partial I} = \bar{F}(I)\Delta - (c - v_2). \quad (\text{C.1})$$

Also, $\frac{\partial^2 \pi(\alpha, I)}{\partial I^2} = -f(I) \leq 0$. Therefore, equating Equation C.1 to 0 would provide us with optimal inventory level for a fixed value of α . We replace this value in the profit function and solve the resulting single dimensional optimization problem in α . We use notation $'$ to refer to the derivative with respect to α . We have

$$\begin{aligned} \pi'(\alpha, I(\alpha)) \equiv \pi'(\alpha) &= \Delta' \mathbb{E}[\min(I, D)] + \Delta I' \bar{F}(I) + v_2' I + I' v_2 - c I' = \\ & \Delta' \mathbb{E}[\min(I, D)] + v_2' I + I'(\Delta \bar{F}(I) + v_2 - c) = \\ & \Delta' \mathbb{E}[\min(I, D)] + v_2' I. \end{aligned} \quad (\text{C.2})$$

It is enough to show that equating this equation to 0 has a solution in $\alpha \in [\alpha_{v_2}^*, \alpha_{\Delta}^*]$ and this solution is indeed a maximizer for $\pi(\alpha)$. We have $\pi'(\alpha_{\Delta}^*) = 0 + v_2' I < 0$ and $\pi'(\alpha_{v_2}^*) = \Delta' \mathbb{E}[\min(I, D)] + 0 > 0$. Since $\pi'(\alpha)$ is continuous in α , using intermediate value theorem, $\pi'(\alpha) = 0$ has at least one solution. But since $\pi(\alpha)$ is increasing in $\alpha = \alpha_{v_2}^*$ and decreasing in $\alpha = \alpha_{\Delta}^*$, maximizer of the problem would be an interior solution. \square

Proof of Proposition 18. Similar to the proof of Proposition 17, we first optimize over the inventory level and plug the value in the objective function. Using Equation C.2 we have

$$\pi''(\alpha) = \Delta'' \mathbb{E}[\min(I, D)] + v_2'' I + I'(\bar{F}(I)\Delta' + v_2'). \quad (\text{C.3})$$

It is enough to show that whenever $\pi' = 0$, we have $\pi'' < 0$. Equating Equation C.1 to 0 we obtain that

$$I' = \frac{-v_2' \Delta - \Delta'(c - v_2)}{-\Delta^2 f(I)} = \frac{(v_2' + \Delta' \bar{F}(I))}{\Delta f(I)}. \quad (\text{C.4})$$

Also, expanding $\mathbb{E}[\min(I, D)]$ in Equation C.2 we have

$$\pi' = \Delta' \int_0^I xf(x)dx + I(v'_2 + \Delta' \bar{F}(I)),$$

and therefore equating it to 0 will give

$$v'_2 + \Delta' \bar{F}(I) = \frac{-\Delta' \int_0^I xf(x)dx}{I}. \quad (\text{C.5})$$

Using this value and Equations C.3 & C.4 we obtain

$$\begin{aligned} \pi'' &= \Delta'' \left(\int_0^I xf(x)dx + I\bar{F}(I) \right) + v''_2 I + \frac{(\Delta')^2}{\Delta} \cdot \frac{\int_0^I xf(x)dx}{I^2 f(I)} \cdot \int_0^I xf(x)dx \leq \\ & \quad \left(\Delta'' + \frac{(\Delta')^2}{\Delta} \right) \int_0^I xf(x)dx + (\Delta'' I \bar{F}(I) + v''_2 I). \end{aligned}$$

Therefore, it is sufficient to have $(\Delta'' + \frac{(\Delta')^2}{\Delta}) \leq 0$, or equivalently $(\Delta^2)'' < 0$. \square

Proof of Proposition 19. Using Equations 4.8, first we find the value of $I(\alpha)$ for each fixed fashionability level and plug it in the derivative with respect to α . We have

$$\frac{\partial \alpha^*}{\partial c} = -\frac{\frac{\partial \pi'}{\partial c}}{\pi''(\alpha^*)}.$$

Since the optimal solution is an interior point, $\pi''(\alpha^*) \leq 0$. Also $\frac{\partial \pi'}{\partial c} = \frac{\partial I}{\partial c} \cdot (\Delta' \bar{F}(I) + v'_2)$. From Equation C.5 we have

$$\frac{\partial \pi'}{\partial c} = \frac{\partial I}{\partial c} \cdot (\Delta' \bar{F}(I) + v'_2) = \frac{-1}{\Delta f(I)} \cdot \frac{-\Delta' \int_0^I xf(x)dx}{I} \geq 0,$$

where the last inequality has used the fact that Δ is increasing on domain $[\alpha_{v_2}^*, \alpha_{\Delta}^*]$.

As such $\frac{\partial \alpha^*}{\partial c} \geq 0$.

To get the result for the inventory level we have

$$\begin{aligned} \frac{\partial I^*}{\partial c} &= \frac{\Delta - \frac{\partial \alpha}{\partial c} (v'_2 \Delta + \Delta' (c - v_2))}{-\Delta^2 f(I)} = \\ & \left(-\frac{1}{\Delta f(I^*)} + \frac{\partial \alpha}{\partial c} \cdot \frac{(v'_2 + \Delta' \bar{F}(I^*))}{\Delta f(I^*)} \right) \Big|_{\alpha = \alpha^*} \leq 0, \end{aligned}$$

which resulted from using $\frac{\partial \alpha^*}{\partial c} \geq 0$ and $(v_2' + \Delta' \bar{F}(I^*)) = \frac{-\Delta' \int_0^I x f(x) dx}{I} \leq 0$. \square

Proof of Proposition 20. Let Z be the standard form of the demand, D , with distribution function $G(\cdot)$. In other words, $D \sim m + \text{SD}Z$. One can re-write the firm's problem as follows:

$$\max_{\alpha, \hat{I}} \pi(\alpha, \hat{I}) = \text{SD} \left(\Delta \mathbb{E} \left[\min(\hat{I}, Z) \right] - (c - v_2) \hat{I} \right) + m(v_1 - c). \quad (\text{C.6})$$

Then, $I^* = \text{SD} \cdot \hat{I}^* + m$, where $\hat{I}^* = \bar{G}^{-1} \left(\frac{c - v_2}{\Delta} \right)$ for any fixed level of fashionability.

Also optimal fashionability is the solution to $\pi'(\alpha, \hat{I}^*) = 0$. Therefore,

$$\frac{\partial \alpha^*}{\partial m} = -\frac{\frac{\partial \pi'}{\partial m}}{\pi''} = -\frac{v_1'}{\pi''}.$$

But since α^* is an interior solution we have $\pi'' \leq 0$. In addition, using Propositions 15 & 22 one can see that $\alpha^* \leq \alpha_{v_1}^*$, and therefore, in the optimal solution $v_1' \geq 0$. As such $\frac{\partial \alpha^*}{\partial m} \geq 0$.

For the second part of the proof note that

$$\frac{\partial \alpha^*}{\partial \text{SD}} = -\frac{\frac{\partial \pi'}{\partial \text{SD}}}{\pi''} = -\frac{\Delta' \mathbb{E} \left[\min(\hat{I}, D) \right] + v_2' \hat{I}}{\pi''}.$$

In the optimal solution we have

$$\Delta' \mathbb{E} \left[\min(\hat{I}, D) \right] + v_2' \hat{I} = -\frac{m \cdot v_1'}{\text{SD}}.$$

Since $v_1' \geq 0$, one can conclude the result. \square

Proof of Proposition 21. We add subindex δ to the previous notation to refer to the new system. For example, $v_{2\delta}$ is the future value in the new system. Note that we will have $v_{2\delta} = \delta v_2$ and $\Delta_\delta = \delta \Delta$. Similarly, $v_{2\delta}' = \delta v_2'$ and $\Delta_\delta' = \delta \Delta'$. Therefore the optimal solution should satisfy

$$\bar{F}(I) = \frac{c - \delta v_2}{\delta \Delta} = \frac{c/\delta - v_2}{\Delta},$$

and

$$\delta \Delta' \mathbb{E} [\min(I, D)] + \delta v'_2 I = 0 \iff \Delta' \mathbb{E} [\min(I, D)] + v'_2 I = 0.$$

As such, problem is equivalent to having the same initial value as in the old system with a lower marginal cost. The result is then followed from the proof of Proposition 19. \square

Proof of Proposition 22. Define $v_\delta := \delta v_2$ and $\Delta_\delta := v_1 - \delta v_2$. Static pricing model is corresponding to a dynamic pricing model with $v_2 = 0 \times v_2$ and $\Delta = v_1 - 0 \times v_2$. Hence, it is sufficient to show that under the dynamic pricing model with parameters Δ_δ and v_δ , the firm sets a lower fashionability, higher inventory, and lower price as δ increases. The optimal solution should satisfy

$$\begin{cases} \frac{\partial \Delta_\delta}{\partial \alpha} \times \mathbb{E} [\min(I, D)] + \frac{\partial v_\delta}{\partial \alpha} \times I = 0 & \alpha \in [\alpha_{v_2}^*, \alpha_\Delta^*] \\ \bar{F}(I) = \frac{c-v_\delta}{\Delta_\delta} \end{cases}. \quad (\text{C.7})$$

First, we solve the problem for the optimal inventory level and plug its value in the derivative with respect to α .

$$\frac{\partial \alpha^*}{\partial \delta} = -\frac{\frac{\partial \pi'}{\partial \delta}}{\pi''(\alpha^*)}.$$

Since the optimal solution is an interior point, $\pi''(\alpha^*) \leq 0$. We have

$$\begin{aligned} \frac{\partial \pi'}{\partial \delta} &= -v'_2 \mathbb{E} [\min(I, D)] + \frac{\partial I}{\partial \delta} \bar{F}(I)(v'_1 - \delta v'_2) + v'_2 I + \frac{\partial I}{\partial \delta} \delta v'_2 = \\ &v'_2 (I - \mathbb{E} [\min(I, D)]) + \frac{\partial I}{\partial \delta} \cdot (\bar{F}(I) \Delta'_\delta + \delta v'_2) = \\ &v'_2 (I - \mathbb{E} [\min(I, D)]) + \frac{\partial I}{\partial \delta} \cdot \frac{-\Delta'_\delta \int_0^I x f(x) dx}{I} \leq 0. \end{aligned}$$

Where the last inequality is resulted from noting that $\frac{\partial I}{\partial \delta} = \frac{v_2(v_1-c)}{(v_1-\delta v_2)^2 f(I)} \geq 0$, $\Delta' > 0$, and $v'_2 < 0$. As such $\frac{\partial \alpha^*}{\partial \delta} \leq 0$.

For the inventory level, this time we first solve for α and then we plug its value in the derivative with respect to I . We have

$$\frac{\partial I^*}{\partial \delta} = \frac{(-v_2 - \delta \frac{\partial v_2}{\partial \delta})(v_1 - \delta v_2) - (\frac{\partial v_1}{\partial \delta} - v_2 - \frac{\partial v_2}{\partial \delta} \delta)(c - \delta v_2)}{-(v_1 - \delta v_2)^2 f(I)} =$$

$$-\frac{1}{\Delta_\delta f(I)} \left(\frac{v_2(c - v_1)}{\Delta_\delta} - \frac{\partial \alpha}{\partial \delta} \cdot (\delta v_2' + \Delta_\delta' \bar{F}(I)) \right) \geq 0,$$

where the last inequality followed from knowing that $v_1 > c$, $\frac{\partial \alpha^*}{\partial \delta} \leq 0$, and $(\bar{F}(I)\Delta_\delta' + \delta v_2') = \frac{-\Delta_\delta' \int_0^I x f(x) dx}{I} \leq 0$. To see the result for the optimal prices note that $v_1(\alpha) \geq \Delta(\alpha)$ for all values of α and $p_s = \max_\alpha v_1$. \square

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Biography

Arian Aflaki is a Ph.D. Candidate in Operations Management at The Fuqua School of Business at Duke University. His research contributes to two specific areas of Operations Management: Retail Operations and Humanitarian Operations. In retail, he studies the impact of consumer behavior on firm pricing, inventory, channel, and product design decisions; in humanitarian operations, he focuses on the influence of contributor behavior on the funding strategies of humanitarian organizations. Arian was born on June 8th, 1988, in Tehran, Iran. Prior to joining Duke, he obtained his Master of Science in Applied Mathematics and Bachelor of Science in Industrial Mathematics from Sharif University of Technology. Starting July 2017, he will be an Assistant Professor of Business Analytics and Operations at the Joseph M. Katz Graduate School of Business at the University of Pittsburgh.