Dynamic Risk Frameworks for Commodity Portfolio Optimization

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Abstract

Commodities are an important yet poorly understood asset class, but outsized losses and gains in commodities in recent years have garnered public attention. Partly in reaction to financial market crashes, the suite of risk management tools has expanded considerably since 1996 when J.P. Morgan published Value-at-Risk. In parallel, significant advancements in financial forecasting have been made since Engle's ARCH model. We compare these and other tools in extreme commodity market environments and observe that dollar-denomination effects, high volatility, and high correlation adversely affect their performance. Our results have implications for investors, commercial hedgers, and regulators tasked with reducing systemic risk.

JEL Classification: G1; G11; G17

Keywords: Commodities; Time Series; Portfolio Optimization; CVaR; VaR; Copula

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1 Introduction

While portfolio theory has been researched intensively both in academia and in industry for at least six decades, the overwhelming majority of empirical work in the field has been focused on more traditional asset classes like equities, bonds, credit, and even foreign currency (Wang, Chen, Jin, & Zhou, 2010). These analyses often overlook commodities which are – both alone and as part of a diversified portfolio – an important financial asset class for their unique return behavior, multi-decade price cycles, and correlation with inflation[1]. However, in order to realize these benefits investors must develop sound risk management tools to deal with commodities' high volatility, particularly in times of widespread market turmoil. To this end, we examine the relative and absolute performances of six commodity portfolio optimization procedures during the most extreme market environments of the last two decades and find that they depend primarily on the exogenous drivers of prevailing market conditions. These six procedures, discussed in detail below, are mean-variance, parametric Value-at-Risk, multivariate normal Conditional Value-at-Risk, *t* copula Conditional Value-at-Risk, robust forecasting Conditional Value-at-Risk, and an equally-weighted control.

Our results are relevant to commodity investors, primary producers and consumers of commodities, and regulators. Investors constantly balance the potential returns of investments against their risks and often choose to reallocate capital when the risk-return tradeoff shifts. Our work examines the evolution of risk and return over six-month time periods with weekly re-allocation and compares the performance of the most standard portfolio optimization methods in commodity markets. The profitability of primary producers and consumers of commodities depends heavily on prices of commodity futures. Both of these commercial market participants hedge all or part of their exposure to commodity price fluctuations in futures markets, but the optimal extent and timing of this hedging activity can be informed by our conclusions. Finally, one of the products of the 2007-08 financial crisis was a renewed mandate for regulators to understand and reduce systemic risk (Dodd-Frank Wall Street Reform and Consumer Protection Act, 2010). Our results demonstrate that traditional risk management tools are likely insufficient to protect against losses in commodity market crash environments. Therefore, the use of VaR-type systemic risk aggregation at the regulatory level should be used to supplement other risk metrics and only with caution as a policy tool.

[1] These features are addressed by Deaton and Laroque (1992), Radetzki (2006), and Gorton and Rouwenhorst (2005), respectively. Additionally, many studies such as Gorton and Rouwenhorst (2005) and Furlong and Ingenito (1996) link commodity prices and inflation using changes in the Consumer Price Index (CPI) as a proxy for inflation. Our analysis purposefully uses changes in the trade-weighted value of the dollar as an alternative proxy to exclude the sampling error and periodic changes in composition associated with the CPI.

While it is understood that commodity prices are negatively correlated with the real value of the dollar, we find that the relative performance of optimization methods also depends strongly on changes in the dollar's value. When the dollar's value is changing significantly, performance across optimization methods is more homogeneous than when its value is relatively static. We interpret this to suggest that when extreme commodity market conditions are driven by exogenous factors like inflation and recession, endogenous commodity market drivers like weather patterns and inventory levels become weaker and correlations among commodities increase, reducing the value of diversification. While causally linking these observations to specific exogenous factors is beyond the scope of this paper, we suspect that monetary policy often plays a significant role in strongly directional commodity markets.

We also observe that the performance of our forecasting methods in commodity portfolio optimization contexts is mixed. When high volatility or high correlations in asset returns are weak and short-lived, our forecasting methods perform well in predicting and exploiting mean-reversion. However, in some of the market environments we observe, extreme volatility and correlation behavior is persistent and profound. In these cases, our methods often prematurely predict mean-reversion and underperform as a result. Additionally, when used in conjunction with portfolio optimization methods, the volatile asset allocations suggested by our forecasting methods can generate large transaction costs. For industry applications, these costs should not be ignored.

This paper is organized as follows. Section 2 explores literature related to our research and presents the theoretical basis for our methods. Section 3 describes our data and discusses the qualitative implications of the summary statistics. Section 4 provides the methodology for each of our six optimization procedures and addresses miscellaneous issues arising from comparing the performance of these procedures. Section 5 presents the performance results of the optimization procedures in four strongly directional commodity markets from the last two decades and discusses the relationships between drivers of extreme market conditions and both relative and absolute portfolio performance. Section 6 concludes, summarizing these relationships into three categories and discussing the significance for investors, commercial hedgers, and regulators.

2 Literature Review

Markowitz (1952) postulates that for the universe of risky assets, there exists an upper limit or "efficient frontier" to the attainable returns for any given risk level. Furthermore, rational investors who maximize returns while minimizing risk should allocate their assets along the "capital market line," the set of all linear combinations between the "risk-free asset" and the tangent point to the efficient frontier, or the

"market portfolio." Finally, Markowitz describes a method called mean-variance optimization by which one can solve for the efficient frontier. However, in doing so, mean-variance optimization makes three unrealistic assumptions. Namely, Markowitz assumes returns are independently and identically distributed (iid) as multivariate normal, risk is measured by variance, and pairwise linear correlations fully capture the joint relationships of returns. The last six decades of research has focused on reconciling these assumptions with reality.

Sheikh and Qiao (2010) show that Markowitz's normality assumption is inconsistent with empirical evidence of "fat" left tails and serial correlation in financial asset returns. They find that negative returns occur more frequently and in greater magnitudes than predicted by the normal distribution; that is, financial asset returns exhibit negative skewness and leptokurtosis. Furthermore, they observe that past returns are informative of future returns. This property, called serial correlation, violates Markowitz's iid assumption which imposes that observations are temporally uncorrelated. To address the negative skewness, leptokurtosis, and serial correlation of financial asset returns, Sheikh and Qiao propose "unsmoothing" autocorrelated returns using the methodology of Fisher, Geltner, and Webb (1994) before modeling the joint return distribution with a Student's *t* copula. They argue the combination of these methods improve on Markowitz's normality and iid assumptions while also allowing the flexibility of copula dependence in the joint distribution instead of pairwise linear correlations.

The problem of measuring risk is much more difficult. Markowitz's risk proxy, variance, is simply the second moment of a random variable and describes the variable's dispersion. While this dispersion of returns is informative, investors and risk managers are often more concerned with expected losses than expected gains (Tversky & Kahneman, 1991). This phenomenon, coined "loss aversion," suggests that risk measures should focus on describing the profile of expected losses.

To this effect, J.P. Morgan published Value-at-Risk (VaR), its model for internal risk measurement (Longerstaey & Spencer, 1996). VaR estimates the expected loss of an asset or portfolio over a given time period at a given confidence level based on historical data and a distributional assumption. In this regard, the name "Value-at-Risk" is misleading as realized losses may exceed VaR; in fact, one should expect losses to exceed the daily 95%-VaR level on roughly 13 business days of every year. While VaR is a more sophisticated risk measure than variance, it is not ideal. Artzner, Delbaen, Eben, and Heath (1999) demonstrate that the undesirable mathematical properties of VaR can cause it to "[fail] to encourage a reasonable allocation of risks among agents." Namely, VaR is neither subadditive nor coherent. These facts present practical problems for the aggregation of risk and diversification in VaR frameworks.

Krohmal, Palmquist, and Uryasev (2002) demonstrate that these undesirable properties of VaR are addressed by Conditional Value-at-Risk (CVaR) or "tail VaR" which is the expected loss given that the loss exceeds VaR. In addition, Rockafellar and Uryasev (1999) show that linear programming methods make CVaR optimization problems far more computationally tractable than the large scale optimization methods needed to address the non-linearity of VaR problems.

Investors, commercial hedgers, and regulators have all struggled with the problem of quantifying risk, and variance, VaR, and CVaR represent only a few of the dozens of risk measures that have propagated and studied in response to this challenge. The task of constructing an effective risk measure includes modeling tail events (Chernobai & Rachev, 2006), satisfying a number of theoretical conditions (Szegö, 2002), and performing consistently over a range of risk levels (Alexander & Baptista, 2004), among a number of other requirements. As the search for an ideal risk measure continues, other methods have been explored to improve the suite of risk management tools available to financial market professionals. Among these methods are adaptions of forecasting techniques. Since Engle's Nobel Prize-winning contributions to volatility modeling and forecasting, the body of literature in financial time series analysis and its applications to risk management has grown significantly in volume and sophistication (Andersen, Bollerslev, Christoffersen, & Diebold, 2006). We use the most basic of these techniques in this study, and present their fundamentals below.

Forecasting financial asset returns is a multi-step process. In portfolio optimization contexts, one desires to use forecasts of asset returns in the next period to inform the allocation decision in the current period. Therefore, these forecasts must estimate the expected risk and return of every relevant asset as well as the risk and return inter-relationships within the asset universe. Generally, this forecasting is completed in three stages: univariate conditional mean estimation, univariate conditional volatility estimation, and multivariate conditional correlation estimation. Each stage of this forecasting process has been well-explored by financial econometricians.

The first standardized model for the conditional mean was propagated by Wold (1938) when he showed Yule's (1926) linear autoregressive model and Slutzky's (1937) moving average model could be combined into a joint Autoregressive Moving Average (ARMA) model for stationary time series. Box and Jenkins (1976) later popularized practical computation methods for estimating Wold's ARMA model.

Estimation of the ARMA model leaves the econometrician with a set of ARMA parameters and model "errors" or residuals. These residuals represent the variation from the conditional mean, and those of a wide variety of financial assets were found to exhibit clustering. This discovery was made by Engle (1982) and described in his Nobel Prize-winning work on Autoregressive Conditional Heteroskedasticity

(ARCH). Engle's work laid the foundation for the development of a generalized specification of the ARCH model – that is, GARCH – by Bollerslev (1986) which continues to be the cornerstone of volatility modeling today.

Bollerslev's forecasting model for conditional variance gained popularity as a simple and effective way to estimate mean-reverting volatility processes. The output of the model – a set of GARCH parameters and model residuals – created the need within portfolio optimization problems for a further model: one to describe correlation between assets. Various models were developed to describe conditional correlation including Constant Conditional Correlation, or CCC, (Bollerslev, 1990) and BEKK (Engle & Kroner, 1995) but these models were impractical for allocation problems with large numbers of assets. Finally, Engle (2002) pioneered Dynamic Conditional Correlation (DCC), a model that required parameter estimates of $O(1)$ compared to CCC's $O(k^2)$ and BEKK's $O(k^4)$.

With the development of DCC, large-scale portfolio optimization problems finally became computationally tractable. Recent literature has focused on various modifications and improvements to these basic models. For this study, we use the most basic and best-understood specifications of each of these models wherever possible. Namely, for our robust forecasting model, we use ARMA(1,1) and two-step maximum likelihood DCC(1,1)-GARCH(1,1).

3 Data

We use historical weekly logarithmic return data for the 18 most active exchange-traded front-month futures contracts across five commodity classes as summarized in Table 1 below. These classes are energy, metals, agriculture (grains), softs[2], and livestock. For the sake of qualitative discussion of our results, we also use the daily Trade Weighted "Broad" U.S. Dollar Index as compiled by the Federal Reserve Bank of St. Louis from the earliest available date, January 4, 1995, through January 25, 2013, a total of 4,352 observations.

[2] "Soft" commodities generally refer non-grain agricultural commodities like cotton, coffee, cocoa, sugar. They are so called because they are grown, not mined.

Table 1: Commodity Futures Contract Specification Summary

		Full Asset Name	Main Exchange	Symbol	Price Quote	Size	Unit	Delivery Location
Energy	WTI	Light Sweet Crude Oil	NYMEX	CL	$/bbl	1,000	barrels	Cushing, OK
	BRT	Brent Crude Oil	ICE	B	$/bbl	1,000	barrels	Sullom Voe, UK
	NG	Henry Hub Natural Gas	NYMEX	NG	$/mmBtu	10,000	mmBtu	Erath, LA
	H.O.	NY Harbor No. Heating Oil	NYMEX	HO	$/bbl	1,000	barrels	New York, NY
Metals	GLD	Physical Gold	COMEX	GC	$/tr. oz.	100	troy oz.	New York, NY
	SLV	Physical Silver	COMEX	SI	$/tr. oz.	5,000	troy oz.	New York, NY
	PLT	Physical Platinum	NYMEX	PL	$/tr. oz.	50	troy oz.	New York, NY
	PAL	Physical Palladium	NYMEX	PA	$/tr. oz.	100	troy oz.	New York, NY
	COP	Physical Copper	LME	CA	$/tonne	25	metric tons	Various
Agr.	CRN	#2 Yellow Corn	CBOT	C	¢/bu	5,000	bushels	Chicago, IL
	WHT	#2 Soft Red Winter Wheat	CBOT	W	¢/bu	5,000	bushels	Chicago, IL
	SOY	#2 Yellow Soybeans	CBOT	S	¢/bu	5,000	bushels	Chicago, IL
Softs	COT	Cotton No. 2	ICE	CT	¢/lb	50,000	pounds	Various S. U.S.
	COF	Coffee C	ICE	KC	¢/lb	37,500	pounds	Various
	CCO	Cocoa	ICE	CC	$/tonne	10	metric tons	Various E. U.S.
	SUG	Sugar No. 11	ICE	SB	¢/lb	112,000	pounds	Port of Origin
Live	CAT	Live Cattle	CME	LH	¢/lb	40,000	pounds	Various U.S.
	HOG	Lean Hogs	CME	LC	¢/lb	40,000	pounds	Various C. U.S.

Prices were taken as of the close of trading on Friday afternoon of each week, and the data range from the earliest available date for the complete series, March 30, 1990, through January 25, 2013 for a total of 1,191 weeks of returns. Summary statistics for this data are presented in Table 2 on the following page where μ is the mean, σ is the standard deviation, S is the skewness, and K is the kurtosis. The full sample linear correlation matrix is presented in Table 3.

Table 2: Summary Statistics for Historical Weekly Logarithmic Returns

		μ	σ	S	K
Energy	WTI	7.07%	37.00%	(0.79)	8.08
	BRT	8.18%	34.38%	(0.86)	7.97
	NG	2.56%	51.95%	(0.06)	4.62
	H.O.	7.73%	35.75%	(0.33)	4.98
Metals	GLD	6.61%	15.99%	(0.12)	6.15
	SLV	8.17%	29.29%	(1.19)	10.41
	PLT	5.52%	21.08%	(0.52)	5.92
	PAL	7.25%	33.55%	(0.07)	6.29
	COP	4.98%	26.56%	(0.75)	7.12
Agr.	CRN	4.56%	28.41%	(0.55)	9.59
	WHT	3.87%	30.11%	0.01	4.84
	SOY	4.61%	26.26%	(1.07)	11.88
Softs	COT	0.14%	30.00%	(0.71)	10.84
	COF	3.12%	31.23%	0.09	4.63
	CCO	1.17%	33.74%	(0.30)	4.43
	SUG	2.74%	36.87%	0.66	8.68
Live	CAT	2.00%	17.53%	(0.51)	5.69
	HOG	0.02%	36.54%	(0.09)	10.11

We observe from the summary statistics that the petroleum-based commodities and precious metals have the highest historical returns while the softs and livestock have the lowest. Natural gas is the most volatile commodity in the sample with an annualized standard deviation of 51.95% while gold and cattle were the least volatile, with annualized standard deviations of 15.99% and 17.53% respectively. We also observe that all but three commodities in our sample feature negative skewness, implying that the mode is to the right of the mean. In other words, the expected probability of extreme losses is greater than that of extreme profits. Silver and Brent crude oil have the most negative skewness, and wheat, coffee, and sugar are the only commodities with positive skewness, i.e., greater probability of extreme gains than extreme losses for long-only investors. Lastly, the returns of all commodities in our sample have kurtosis in excess of 3, indicating "fatter" tails than the normal distribution, or leptokurtosis. This property is most pronounced in soybeans, cotton, silver, and lean hogs. Together, the properties of negative skewness and excess kurtosis are indications that the normal distribution is not an appropriate model for the distribution of the sample commodities' returns.

Table 3: Correlation of Historical Weekly Returns

		Energy				Metals					Agriculture			Softs				Live	
		WTI	BRT	NG	H.O.	GLD	SLV	PLT	PAL	COP	CRN	WHT	SOY	COT	COF	CCO	SUG	CAT	HOG
Energy	WTI	1.00																	
	BRT	0.87	1.00																
	NG	0.24	0.25	1.00															
	H.O.	0.75	0.80	0.32	1.00														
Metals	GLD	0.24	0.25	0.11	0.20	1.00													
	SLV	0.26	0.28	0.08	0.22	0.71	1.00												
	PLT	0.22	0.24	0.13	0.20	0.50	0.51	1.00											
	PAL	0.12	0.14	0.07	0.10	0.34	0.41	0.57	1.00										
	COP	0.22	0.24	0.07	0.19	0.27	0.34	0.34	0.32	1.00									
Agriculture	CRN	0.14	0.16	0.09	0.11	0.18	0.20	0.16	0.15	0.15	1.00								
	WHT	0.14	0.16	0.06	0.12	0.14	0.16	0.13	0.10	0.17	0.59	1.00							
	SOY	0.13	0.15	0.10	0.14	0.09	0.12	0.18	0.13	0.21	0.53	0.32	1.00						
Softs	COT	0.12	0.12	0.02	0.07	0.09	0.11	0.14	0.13	0.15	0.17	0.15	0.21	1.00					
	COF	0.10	0.11	0.06	0.09	0.20	0.22	0.19	0.13	0.15	0.17	0.11	0.12	0.19	1.00				
	CCO	0.15	0.17	0.09	0.14	0.11	0.16	0.14	0.13	0.18	0.16	0.18	0.18	0.12	0.12	1.00			
	SUG	0.08	0.08	0.02	0.06	0.11	0.18	0.17	0.11	0.14	0.16	0.12	0.10	0.10	0.16	0.14	1.00		
Live	CAT	0.05	0.07	0.06	0.07	0.00	0.02	0.06	0.09	0.07	0.05	0.01	0.06	0.05	0.04	0.07	0.02	1.00	
	HOG	0.02	0.03	(0.03)	0.01	0.02	0.03	0.04	0.05	0.00	(0.03)	0.02	0.01	(0.02)	0.01	(0.01)	0.03	0.10	1.00

The summary of historical correlations in Table 3 captures many fundamental relationships between the benchmark commodities. For example, the petroleum products' correlations are all in excess of 75%. The primary differences between these assets' returns are related to their quality, location of delivery, and degree of refinement, but several driving factors like overall economic strength and petroleum inventories are fundamental to all of them. These common drivers explain the high correlation across petroleum commodities' returns. Conversely, grain commodities are sometimes driven by common factors like weather conditions, industrial demand, and fertilizer costs but because they each compete for planting acreage, their prices can be negatively correlated for extended periods of time. As a result, the average correlations among grains over are significantly lower than those of petroleum products. Soft commodities and livestock are each driven by mostly unique sets of price determinants and therefore have low absolute correlations with one another and other commodities. In our sample, most commodities are positively correlated with one another due to long term, large-scale drivers like dollar[3]-denomination, inflation, and population growth.

From the summary statistics and correlations of our commodities, we conclude that a static Gaussian model of our commodities returns in a portfolio context is likely to be insufficient for robust risk management. In fact, the relatively high probability of extreme returns – especially losses – should be modeled when estimating risk and optimizing portfolio returns.

4 Methodology

4.1 Geometric Returns

In order to capture the effects of compounding, we use geometric returns to calculate the cumulative performance of portfolios during the sample period. If T is the number of out-of-sample testing weeks and R_i is the realized portfolio return for the ith week, then R_{geo}, the cumulative geometric portfolio return is defined as

$$R_{geo} = \prod_{i=1}^{T}(1 + R_i) - 1 \tag{1}$$

4.2 Mean-Variance Optimization

The most basic portfolio optimization method is Markowitz's (1952) mean-variance (MV) optimization. Using the normality and iid assumptions and considering the sample mean return vector, $\boldsymbol{r}_t$ and sample

[3]Throughout this paper, we use "dollar" strictly as "U.S. dollar"

asset covariance structure, $\boldsymbol{\sigma}_t$ to be representative of expected returns at time t, we calculate the decision vector $\boldsymbol{w}_t$ that solves the following optimization problem for k assets

$$\min_{w_1,\dots,w_k} \sigma_t^2 = \sum_{i=1}^{k} w_{i,t}^2\, \sigma_{i,t}^2 + 2 \sum_{i=1}^{k} \sum_{j>1}^{k} w_{i,t} w_{j,t} \sigma_{ij,t} \tag{2}$$

$$\text{where } \sigma_{ij,t} = \sigma_{i,t}\sigma_{j,t}\rho_{ij,t} \tag{3}$$

$$\text{subject to } 0 \leq w_1, \dots, w_k \leq 1 \tag{4}$$

$$\sum_{i=1}^{k} w_i = 1 \tag{5}$$

That is, we minimize[4] the conditional portfolio variance at time t given long-only and unlevered investment constraints.

$$X_t = \sum_{i=1}^{k} w_{i,t} r_{i,t} \tag{6}$$

The portfolio return X_t is the sum-product of the conditional mean return vector and the decision vector, $\boldsymbol{w}_t$.

4.3 Value-at-Risk Optimization

Value-at-Risk (VaR) is defined as the dollar value associated with the quantile of an asset or portfolio's empirical cumulative distribution of returns. Artzner *et al.* (1999) express this as

$$\mathrm{VaR}_\alpha(L) = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\} \tag{7}$$

$$\text{for } \alpha \in (0,1) \tag{8}$$

where α is the chosen confidence level, l is the VaR associated with the α-quantile, L is the portfolio loss, and $F_L(l)$ is the empirical cumulative distribution of returns.

VaR optimization entails a similar problem to mean-variance optimization in Section 4.2. Namely, we seek to solve

[4] In practice, some optimization algorithms, including ours, maximize returns for given risk levels. This discrepancy does not affect the result.

$$\min_{w_1,\ldots,w_k} \mathrm{VaR}_{\alpha,t}(\boldsymbol{w}_t, \boldsymbol{r}_t) \tag{9}$$

$$X_t = \sum_{i=1}^{k} w_{i,t} r_{i,t} \tag{10}$$

subject to constraints (4) and (5), and where X_t, $\boldsymbol{w}_t$, and $\boldsymbol{r}_t$ are defined as in Section 4.2. Generally, this problem is non-linear due to the convexity of some financial asset returns like those of options and bonds, but for our purposes, we are able to optimize VaR parametrically without compromising our results. The parametric (or delta-normal) method expresses VaR as

$$\mathrm{VaR}_{\alpha,t}(L) = \sqrt{\boldsymbol{w}_t{}^T \boldsymbol{Q} \boldsymbol{w}_t} \tag{11}$$

$$\boldsymbol{Q} = z_\alpha^2 \boldsymbol{Q}^* \tag{12}$$

$$\boldsymbol{r} \sim N(0, \boldsymbol{Q}^*) \tag{13}$$

where z_α is the value of the standard normal evaluated at α, and $\boldsymbol{Q}^*$ is the sample covariance matrix (Mausser & Rosen, 1999).

4.4 Multivariate Normal Conditional Value-at-Risk Optimization

CVaR is the expected loss given that the loss amount exceeds VaR. Most generally, this is written as

$$\mathrm{CVaR}_\alpha(\boldsymbol{w}, \boldsymbol{r}) = \frac{1}{1-\alpha} \int_{f(\boldsymbol{w},\boldsymbol{r}) \geq \mathrm{VaR}_\alpha(L)} f(\boldsymbol{w}, \boldsymbol{r}) p(\boldsymbol{r}) dy \tag{14}$$

where $f(\boldsymbol{w},\boldsymbol{r})$ is the loss function associated with the decision vector, $\boldsymbol{w}$, and the expected return vector, $\boldsymbol{r}$. The probability density function (pdf) of the return vector is given by $p(\boldsymbol{r})$ (Rockafellar & Uryasev, 2000).

Unlike the parametric method we use to calculate and optimize VaR, we use a simulation-based approach to optimize all of the CVaR-based models (i.e., multivariate normal CVaR, *t* copula CVaR, and robust forecasting CVaR). That is, instead of analytically solving for $p(\boldsymbol{r})$, we sample a given distribution 10,000 times for observations of $\boldsymbol{r}$ and calculate CVaR from the resultant empirical distribution. In the case of multivariate normal CVaR (MVN CVaR), the distribution we assume is the *k*-dimensional multivariate normal distribution which has a pdf

$$f_r(\boldsymbol{r}) = \frac{1}{(2\pi)^{k/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\boldsymbol{r}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{r}-\boldsymbol{\mu})\right) \tag{15}$$

where $\boldsymbol{\mu}$ is the mean return vector, $\boldsymbol{\Sigma}$ is the sample covariance matrix, and $|\boldsymbol{\Sigma}|$ is its determinant.

With this specification of MVN CVaR, we proceed as in Section 3.2 and 3.3. That is, we solve for the decision vector $\boldsymbol{w}_t$ that satisfies

$$\min_{w_1,\dots,w_k} \mathrm{CVaR}_{\alpha,t}(\boldsymbol{w}_t, \boldsymbol{r}_t) \tag{16}$$

$$X_t = \sum_{i=1}^{n} w_{i,t} r_{i,t} \tag{17}$$

subject to constraints (4) and (5).

4.5 *t* Copula Conditional Value-at-Risk Optimization

The procedure for *t* copula CVaR (tCVaR) differs from that of MVN CVaR only in the distributional assumption imposed on $p(\boldsymbol{r})$. Where we previously assumed $p(\boldsymbol{r})$ was distributed as multivariate normal, here we sample from a *t* copula for the empirical distribution of $p(\boldsymbol{r})$.

The copula, C_Θ, can be written as

$$C_\Theta(u_t) = t_{\nu,\Sigma}\left(t_\nu^{-1}(u_{1,t}), \dots, t_\nu^{-1}(u_{n,t})\right) \tag{18}$$

$$\Theta = \{(\nu, \Sigma): \nu \in (1, \infty), \Sigma \in \mathbb{R}^{m\times m}\} \tag{19}$$

where, t_ν is the univariate Student's *t* distribution with ν degrees of freedom and $t_{\nu,\Sigma}$ is the multivariate Student's *t* distribution with correlation matrix Σ and ν degrees of freedom (Demarta & McNeil, 2005). The *t*-copula does not require specification of univariate densities as it is simply the dependence structure between densities that we are interested in modeling with this copula method, but for consistency with the distributional assumptions embedded in the robust forecasting model, we impose that the copula's univariate densities be drawn from the Student's *t* distribution which has a pdf

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \tag{20}$$

We complete the optimization by solving for the decision vector $\boldsymbol{w}_t$ that satisfies (16) and (17) subject to constraints (4) and (5).

4.6 Robust Forecasting Conditional Value-at-Risk Optimization

For our final optimization method, robust forecasting CVaR (RFCVaR), we first use well-documented forecasting techniques to estimate parameters which are used to simulate 10,000 return scenarios based on historical data. We then solve for the decision vector $\boldsymbol{w}_t$ that satisfies the CVaR optimization problem given in (16) and (17) subject to constraints (4) and (5).

To accomplish this, we begin by filtering the return vector. We individually test for single-lag serial correlation in each commodity's return series using the Ljung-Box test where the test statistic, Q_i, is distributed as χ^2 with one degree of freedom under the null and is defined as

$$Q_i = T(T+2)\frac{\rho(1)^2}{T-1} \tag{21}$$

where T is the sample size, and $\rho(1)$ is the first-lag sample autocorrelation. Where we cannot reject of the null of no autocorrelation at the 5% significance level, we filter the return series simply by subtracting the mean from every observation. Where we reject the null, we filter the return series using an ARMA(1,1), model for the conditional return, $r_{i,t}$, of each commodity of the form

$$r_{i,t} = c_{i,1} + \varphi_i r_{i,t-1} + \theta_i \varepsilon_{i,t-1} + \varepsilon_{i,t} \tag{22}$$

where $c_{i,1}$ is some constant, φ_i is the ith commodity's AR parameter, $r_{i,t-1}$ is the return observation at time t-1, θ_i is the ith commodity's MA parameter, and $\varepsilon_{i,t-1}$ is the innovation observation at time t-1. We estimate this ARMA model by quasi-maximum likelihood estimation where

$$\varepsilon_{i,t} \sim t(\nu) \tag{23}$$

and ν is degrees of freedom.

Next, we estimate Bollerslev's (1986) GARCH(1, 1) model on $\varepsilon_{i,t}$ for each commodity where

$$\varepsilon_{i,t} = \sigma_{i,t}\eta_{i,t} \tag{24}$$

$$\sigma_{i,t}^2 = c_{i,2} + \alpha_i \sigma_{i,t-1}^2 + \beta_i \varepsilon_{i,t-1}^2 \tag{25}$$

$$\eta_{i,t} \sim t(\nu) \tag{26}$$

That is, the forecast innovation for time t, $\varepsilon_{i,t}$, is the product of a Student's t innovation, $\eta_{i,t}$, with degrees of freedom ν, and the square root of the conditional variance, $\sigma_{i,t}^2$. The conditional variance itself is fully defined by some constant, $c_{i,2}$, the autoregressive conditional variance coefficient, α_i, the conditional variance at time t-1, $\sigma_{i,t-1}^2$, the innovation coefficient, β_i, and squared innovation at time t-1, $\varepsilon_{i,t-1}^2$.

Next, we estimate Engle's (2002) DCC model on $\varepsilon_{i,t}$ for each commodity where:

$$\varepsilon_{i,t} \sim \mathrm{F}(0, H_t) \tag{27}$$

$$H_t = D_t R_t D_t \tag{28}$$

$$D_t = \mathrm{diag}\{\sqrt{h_{i,t}}\} \tag{29}$$

$$R_t = \frac{Q_t}{\sqrt{\mathrm{diag}\{Q_t\}\mathrm{diag}\{Q_t\}'}} \tag{30}$$

$$Q_t = S(1 - \alpha - \beta) + \alpha(\varepsilon_{t-1}\varepsilon_{t-1}') + \beta Q_{t-1} \tag{31}$$

and $h_{i,t}$ are the expressions for the univariate GARCH models estimated in the previous step. In other words, the mean-zero conditional distribution from which $\varepsilon_{i,t}$ is drawn is fully specified by the conditional correlation matrix H_t which is related to the time-varying correlation matrix, R_t, by (28). D_t is a matrix containing the square root of the individual GARCH residuals along its diagonal and R_t is simply a normalization of the conditional covariance matrix, Q_t. Q_t is estimated with a GARCH-like mean-reverting specification where Q_t is fully defined by two DCC parameters, α and β, the unconditional sample covariance, S, the lagged, squared innovation matrix, ε_{t-1}, and the lagged conditional covariance matrix, Q_{t-1}.

Having estimated models that can be combined to forecast returns with serial correlation and a GARCH-like covariance structure, we fit a dependence structure for $\varepsilon_{i,t}$, the correlated residuals. For this purpose,

we use a parameterized Student's t copula model where the extreme 5% masses of the pdf are estimated by the generalized Pareto distribution (GPD) given by

$$f\left(\varepsilon_{i,t}\middle|k_{i,t},\sigma_{i,t},\theta_{i,t}\right)=\left(\frac{1}{\sigma_{i,t}}\right)\left(1+k_{i,t}\frac{\varepsilon_{i,t}-\theta_{i,t}}{\sigma}\right)^{-1-\frac{1}{k_{i,t}}} \tag{32}$$

$$\text{for } \theta_{i,t}<\varepsilon_{i,t}, \text{when } k_{i,t}>0, \text{or for } \theta_{i,t}<\varepsilon_{i,t}<-\frac{\sigma_{i,t}}{k_{i,t}} \text{ when } k_{i,t}<0 \tag{33}$$

The GPD is commonly used in extreme value theory where there is a lack of support for specifying a more rigid distributional assumption (McNeil, 1997). The inner 90% of distributional mass is smoothed using a Gaussian specification of the Rosenblatt-Parzen method (Parzen, 1962). This kernel density estimation method is a standard signal-processing technique used to estimate the shape of the unknown density f of sample data drawn from some distribution. The use of the Gaussian specification here is primarily for computational convenience and does not significantly affect the macroscopic distribution of returns.

To prepare our data for copula fitting, we first transform the data to the unit hyperspace using its estimated parameterized cumulative distribution function (cdf) as follows

$$\boldsymbol{u}_t=\int_{-\infty}^{\varepsilon_{i,t}} f(x)dx \;\; \forall\, i \tag{34}$$

After transforming each $\varepsilon_{i,t}$, we are left with a unit hyperspace vector $\boldsymbol{u}_t$ to which the Student's t copula described in (18)-(20) can be fit using maximum-likelihood estimation. Next, we sample the copula 10,000 times for $\boldsymbol{u}_t$ and then transform the result back into the return space using the GPD inverse cdf, i.e., by reversing (34). The resulting correlated $\varepsilon_{i,t}$ can then be reconstituted into return estimates $r_{i,t}$ by reversing the ARMA(1, 1) process in (22). These $r_{i,t}$ represent 10,000 correlated, simulated estimates of the individual returns for each commodity at time t and constitute the empirical distribution $p(\boldsymbol{r})$ needed to solve (16) and (17) subject to constraints (4) and (5) as in Section 4.5.

4.7 Comparing Optimization Methods

Comparing the performance of the various optimization methods under consideration is complicated for several reasons. First, the risk measures are disparate. That is, each risk measure has no relevance in the others' frameworks. Second, we must be careful to avoid data snooping when selecting the out-of-sample periods over which we compare the various optimization methods' performances. Finally, we simplify the

optimization problem by using a number of assumptions with regard to data usage, transaction costs, and the universe of assets.

The disparity of the various risk frameworks poses a fundamental problem. Because the optimization procedures each produce their own efficient frontier, the hypothetical investor must select a risk level in each framework in order to determine the optimal allocation of assets. That risk level, however, is measured differently in each framework. That is, variance cannot be compared to VaR which cannot be compared to CVaR. Furthermore, once each portfolio has been selected, there is a question as to how the portfolio's performance should be measured and compared to the others'. Any Sharpe-like metric which considers risk-adjusted performance by taking the ratio of return to risk cannot be reconciled across frameworks because it requires arbitrary preference for one risk proxy or another. This preference will bias the results.

For our study, we address the problem of risk-level selection by optimizing 21 discrete portfolios and choosing the 11th portfolio in the risk-ordered set to represent the "middle" risk level for each optimization method. While technically an efficient frontier is a continuous boundary, practically, we solve for a discrete number of portfolios that lie on the frontier and linearly interpolate between them to construct the frontier. If the geometries of the efficient frontiers derived from each optimization method are similar – that is, they are scaled or translated versions of one another – this idealization is effective in comparing like portfolios. However, if the geometries of the efficient frontiers vary significantly, the comparison loses validity. For our purposes, the geometries of the frontiers are similar enough for comparison.

To address the second issue, we compare performance on a total return basis. The methods which have the highest total returns at the end of each out-of-sample period are deemed to be the best performing. For the sake of comparison, the variability of portfolio returns throughout the out-of-sample period is considered to be irrelevant in determining the best portfolio, but we discuss the qualitative importance of this variation.

We avoid data snooping by using a rules-based methodology for selecting the out-of-sample windows. First, we choose an arbitrary length of time – 24 weeks, or roughly 6 months – as the window length. Given this window length, we calculate the 24 week rolling average return of an equally-weighted portfolio of assets (the control) for every observation date (beginning with the 24th observation). From these rolling averages, we select the single worst average returning period (Jun. 13, 2008 – Nov. 28, 2008) to represent the bear scenario and the single best average returning period (Sep. 7, 2007 – Feb. 22, 2008) to represent the bull scenario. We also consider the second worst (May 4, 2001 – Oct. 19, 2001) and

second best (Aug. 20, 2010 – Feb. 4, 2011) non-overlapping periods for further comparison. Henceforth, we will refer to the best (worst) returning period as "strong bull (bear)" and the second best (worst) returning period as "weak bull (bear)."

Finally, in order to maintain consistency across simulation periods, we truncate the historical data such that the initial return observation set is equal for each simulation. Because May 4, 2001 is the earliest simulation period, the 579 weeks in its initial data set are the limiting factor for the other simulations. We use a rolling window for the historical data considered by each optimization method so the historical data set for every period in each simulation has length $n = 579$. We also assume transaction costs are negligible, the universe of assets is strictly limited to the 18 front-month commodity future contracts listed in Table 1, and that the hypothetical investor uses no leverage, is long-only, and is fully allocated.

5 Results and Discussion

In this section, we present our results and discuss their implications. For each market environment, we present and discuss the time plot of cumulative returns by optimization method and a table of total returns by optimization method.

As a matter of nomenclature, we refer to MV, VaR, MVN CVaR, tCVaR, and RFCVaR as "active" portfolios because their allocations can vary over time. The control, which is an equally-weighted portfolio of the 18 commodities, is the only passive portfolio.

5.1 Summary Results

Table 4 summarizes descriptive information about each market environment including their start and end dates, relative and absolute portfolio performance statistics, and the 24 week change in the "Broad" Index, a trade-weighted index calculated by the Federal Reserve of St. Louis which is a proxy for the strength of the dollar. The portfolio performance statistics include the highest and lowest portfolio returns as of the end date of each market environment, the difference between the highest and lowest portfolio returns, and the standard deviation of the six portfolio returns as of the end date of the market environment. The dollar index change is calculated as the logarithmic difference between the beginning and end of each market environment. This change is then presented as the percentile rank within the ordered set of all 24 week logarithmic differences in the dollar index from the beginning of the data set until that point in history. Higher percentiles correspond to larger (positive) changes in value.

Table 4: Performance Summary by Market Environment

	Start Date	End Date	Best Return	Worst Return	Best-Worst	Return Std Dev	Dollar Index %ile
Strong Bull	09/07/07	02/22/08	46.76%	27.57%	19.19%	8.07%	7.80%
Weak Bull	08/20/10	02/04/11	40.11%	18.28%	21.84%	9.96%	7.00%
Strong Bear	06/13/08	11/28/08	(25.40%)	(35.69%)	10.29%	3.70%	99.40%
Weak Bear	05/04/01	10/19/01	(16.42%)	(28.67%)	12.25%	4.37%	23.30%

We address the contents of this table in greater detail in the following sections, but it is immediately obvious that both the absolute difference in the best and worst portfolios and the standard deviation of final portfolio returns is significantly lower for the bear markets than for the bull markets. Additionally, change in the strength of the dollar seems to be related to the strength and direction of the commodity market. We explore potential explanations for these phenomena in the following sub-sections, and simply observe here that the drivers of extreme market conditions may be determinants of both relative and absolute portfolio performance.

5.2 Strong Bull (Sep. 7, 2007 – Feb. 22, 2008)

The 2007-08 commodity bull market is characterized by strong cross-commodity class performance. The top three performing commodities, in descending order, are platinum, natural gas, and soybeans. The fact that each of these commodities comes from a different commodity class with generally disparate return drivers and low historical correlation (ρ_{avg} = 13.48%, see Table 3) is an indication that common factors outside individual commodity markets were significant during the 2007-08 bull market.

This observation is consistent with history. Between September 2007 and February 2008, the dollar lost 4.51% of its value according to the Fed's dollar index, a change which is in the 8th percentile of 24 week changes as shown in Table 4. Because the commodities in our sample are dollar-denominated, an exogenous change in the value of the dollar without any endogenous changes in the fundamental value of the commodities will result in an equal, opposite, and perfectly correlated change in the prices of the commodities. Likewise, excluding term structure effects, if there is net dollar-denominated demand for the sample commodities, an exogenous real change in this demand will result in an opposite (but likely unequal) change in the value of the dollar. Therefore, the strong, highly-correlated performance of commodities during this period can in part be linked to the extreme weakening of the dollar but the direction or even existence of causality are beyond the scope of this discussion and rigorously explored by Krytsou and Labys (2006).

As shown in Figure 1, MVN CVaR performs the best during the strong bull market, returning 46.76% over the 24 week period. Table 5 shows that MVN CVaR benefits most from a large allocation to

platinum which, alone, delivered a 25.57% return to the portfolio. Conversely, VaR and RFCVaR suffer from under-allocation to platinum and over-allocation to the underperforming livestock commodities which deliver the only negative returns during the sample period. Overall, the worst performing portfolio is RFCVaR which had extremely weak performances in energy and metals and relatively strong performances in agriculture and livestock.

Figure 1 shows RFCVaR to be an outlier in its performance pattern. Until February 2008, each of the other portfolios has returns within a 10% band of one another while RFCVaR underperformed the next weakest portfolio (VaR) by up to 10%. We hypothesize this performance is the result of the mean-reverting features embedded in RFCVaR. Because the outperformance of platinum and natural gas are persistent and pronounced during this period, an allocation method that rewarded recent outperformance rather than penalizing it would have been preferable. Cross-commodity class correlation is similarly persistent and pronounced during this bull period; in this regard, the mean-reverting features embedded in RFCVaR's DCC model are also likely detrimental to RFCVaR's performance.

RFCVaR's temporal allocation of assets differs significantly from the other portfolios as well. Across all four market environments, the standard deviation of RFCVaR's allocation to the average commodity is 7.51%[5] compared to 3.59% for tCVaR, the next highest, and 1.70% for variance, the lowest. This is likely the byproduct of the single-lag features embedded in RFCVaR's forecasting procedure. Whereas each of the other active portfolios treat each observation of $\boldsymbol{r}$ equally, a result of RFCVaR's forecasting procedure is bifurcation between "old" data (i.e., $\boldsymbol{r}_{t-579},\ldots,\boldsymbol{r}_{t-2}$) and the newest data (i.e., $\boldsymbol{r}_{t-1}$). The old data is used to estimate parameters that describe the mean, variance, and correlation processes that drive $\boldsymbol{r}$. The last observation of $\boldsymbol{r}$ is then transformed by the estimated parameters to forecast $\boldsymbol{r}_t$. A side effect of this bifurcation between data is volatile portfolio allocation due to noise in $\boldsymbol{r}_{t-1}$. We do not penalize volatile portfolio allocation but, in reality, the effect of transaction costs on portfolio performance should not be overlooked.

[5] We obtain these values for each portfolio by first calculating the 18 standard deviations associated with commodity-level allocations over 24 weeks. The number we report for each portfolio is the average of these 18 standard deviations.

Table 5: Performance by Optimization Method (Strong Bull)[6]

1-BULL ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	0.30%	0.25%	(0.07%)	1.05%	0.21%	1.57%
	BRT	2.23%	1.31%	3.17%	0.74%	0.00%	1.58%
	NG	0.37%	0.36%	0.57%	2.17%	0.27%	3.22%
	H.O.	0.15%	0.12%	0.16%	0.00%	0.26%	1.60%
Metals	GLD	5.40%	6.79%	3.93%	5.48%	0.14%	1.87%
	SLV	0.48%	0.00%	1.64%	1.55%	1.37%	2.31%
	PLT	19.05%	7.78%	25.57%	20.75%	5.50%	3.45%
	PAL	0.02%	0.10%	0.46%	0.26%	5.71%	2.92%
	COP	1.79%	1.03%	1.94%	1.83%	(0.64%)	1.06%
Agr.	CRN	0.00%	0.00%	0.00%	0.00%	2.84%	2.85%
	WHT	2.84%	2.04%	2.97%	4.47%	4.02%	1.41%
	SOY	2.29%	2.73%	1.37%	0.44%	1.62%	2.96%
Softs	COT	0.02%	1.62%	0.00%	0.51%	(1.52%)	1.70%
	COF	1.25%	1.73%	0.69%	1.87%	1.20%	2.04%
	CCO	0.00%	0.60%	0.00%	0.01%	1.26%	2.51%
	SUG	0.07%	1.13%	0.00%	(0.09%)	1.80%	2.22%
Live	CAT	(0.90%)	(1.07%)	(0.65%)	(0.53%)	(0.79%)	(0.27%)
	HOG	(0.00%)	0.28%	0.00%	(0.11%)	2.12%	(0.12%)

1-BULL ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	3.07%	2.05%	3.85%	4.01%	0.75%	8.20%
Metals	28.36%	16.40%	35.85%	32.04%	12.48%	12.14%
Agr.	5.19%	4.82%	4.38%	4.93%	8.71%	7.40%
Softs	1.34%	5.18%	0.69%	2.31%	2.74%	8.75%
Live	(0.90%)	(0.79%)	(0.65%)	(0.63%)	1.32%	(0.40%)

1-BULL ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(5.13%)	(6.14%)	(4.35%)	(4.19%)	(7.45%)	0.00%
Metals	16.22%	4.26%	23.71%	19.90%	0.35%	0.00%
Agr.	(2.21%)	(2.57%)	(3.02%)	(2.47%)	1.31%	0.00%
Softs	(7.41%)	(3.57%)	(8.06%)	(6.45%)	(6.01%)	0.00%
Live	(0.51%)	(0.40%)	(0.26%)	(0.24%)	1.71%	0.00%

[6] The total return table in each section is divided into three parts. In the first, we tabulate percentage contributions to each portfolio's cumulative return by commodity (grouped by commodity-class). In the second, we present the geometric returns of each commodity class as defined in (1). Finally, we present the geometric returns of each class *relative* to the control portfolio. This final sub-table represents class-level under- and over-performance relative to the control by portfolio.

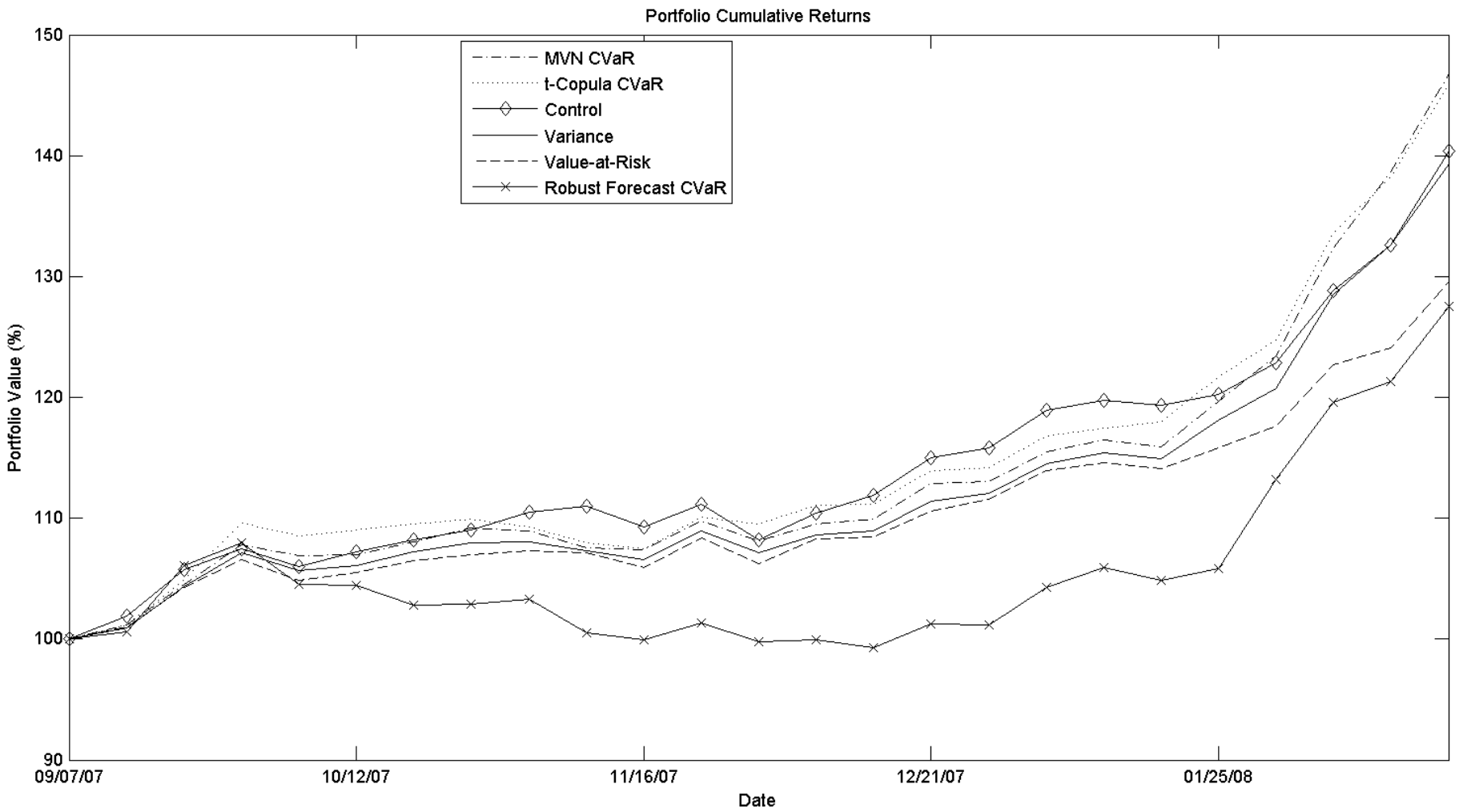

Figure 1: Strong Bull Cumulative Returns[7]

[7] The legend for the cumulative return plot in each section ranks portfolio performance from best to worst.

5.3 Weak Bull (Aug. 20, 2010 – Feb. 4, 2011)

Similar to the strong bull market, the weak bull market is also characterized by a weakening dollar as the dollar index was fell 4.96%, a change in the 7th percentile of the to-date data set as shown in Table 4. Notwithstanding this fact, the factors which drove commodity returns during the 2010-11 weak bull market seem to be more diverse than those in 2007-08. For example, the average commodity class return in 2007-08 is 2.21% (σ = 1.03%) while the average in 2010-11 is 1.78% (σ = 1.10%). In other words, the weak bull market returns are less pronounced and also less evenly distributed across commodity classes.

The average historical correlations between the top three performing commodities in the period – cotton, palladium, and corn – are also marginally higher than in 2007-08 (ρ_{avg} = 15.12%, see Table 3). Consistent with these apparent differences between the two bull markets, the performance of portfolios is almost completely reversed in 2010-11. The best performing portfolio is RFCVaR, and the worst-performing portfolio is tCVaR, the strong bull market's second-best performing portfolio.

Additionally, Table 4 shows that the performance across portfolios is more homogeneous in the strong bull market than in the weak bull market. The difference between the best and worst portfolios in 2010-11 is 21.84% compared to 19.19% in 2007-08 and the standard deviation in portfolio returns in 2010-11 is 9.96% compared to 8.07% in 2007-08. The graph of cumulative portfolio returns in Figure 2 depicts these observations.

Analysis of the commodity-level returns in Table 6 shows that RFCVaR's exposure to copper, wheat, and soybeans are the main driver of its strong performance. The control largely benefits from broad exposure, particularly to historically lower-returning soft commodities like cotton (4.25%) and sugar (2.95%). These results suggest that the control and RFCVaR tend to outperform other methods during bull markets in historically lower-returning commodities, and when mean and covariance trends are less persistent and pronounced.

We also observe that large portfolio allocations to gold, despite their positive returns, cause MV, tCVaR, and MVN CVaR to underperform. In fact, we see that in each of the post-2007 market environments (chronologically, strong bull, strong bear, and weak bull), gold accounts for a disproportionately large and volatile percentage of each of the active portfolio's allocations with the exception of RFCVaR. The average allocation to gold by ex-RFCVaR active portfolios in post-2007 market environments is 28.36%, over five times the control allocation. Furthermore, the temporal standard deviation associated with these allocations is 7.51%, over three times the average for the respective portfolios. By contrast, RFCVaR's average allocation to gold is 14.62%, 2.63 times the control allocation, and its temporal allocation standard deviation is 13.68%, 1.82 times the portfolio average. These observations suggest that the

behavior of gold returns in post-2007 markets is not only anomalous relative to other commodities, but also systematically disrupts the diversification features of all of the active portfolios although the extent is reduced for RFCVaR.

Table 6: Performance by Optimization Method (Weak Bull)

2-BULL ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	0.00%	0.00%	0.00%	0.04%	1.75%	1.16%
	BRT	0.00%	0.00%	0.00%	0.02%	(0.22%)	1.90%
	NG	0.00%	0.16%	0.00%	0.03%	0.67%	0.32%
	H.O.	0.66%	0.00%	0.74%	1.28%	0.08%	2.06%
Metals	GLD	3.92%	1.80%	4.39%	3.39%	1.11%	0.51%
	SLV	0.00%	0.00%	0.00%	0.00%	0.03%	2.93%
	PLT	1.28%	0.00%	1.41%	(0.14%)	0.73%	1.24%
	PAL	0.00%	0.52%	0.00%	0.00%	3.31%	3.39%
	COP	2.40%	0.00%	3.18%	2.04%	11.17%	2.15%
Agr.	CRN	0.00%	0.08%	0.27%	0.32%	1.03%	2.99%
	WHT	0.94%	1.01%	0.81%	0.88%	6.83%	1.51%
	SOY	1.91%	1.84%	1.64%	0.23%	5.87%	2.25%
Softs	COT	2.99%	3.94%	3.09%	1.36%	1.65%	4.25%
	COF	0.50%	0.05%	0.87%	(0.06%)	0.02%	1.06%
	CCO	4.24%	0.39%	4.61%	6.71%	(0.10%)	2.95%
	SUG	0.17%	2.59%	0.02%	0.26%	0.34%	2.13%
Live	CAT	1.24%	4.03%	0.68%	0.77%	1.29%	0.58%
	HOG	0.33%	1.24%	(0.03%)	0.33%	0.01%	0.75%

2-BULL ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	0.66%	0.16%	0.74%	1.37%	2.29%	5.55%
Metals	7.78%	2.33%	9.23%	5.35%	17.01%	10.62%
Agr.	2.88%	2.95%	2.75%	1.43%	14.27%	6.90%
Softs	8.08%	7.10%	8.81%	8.39%	1.91%	10.78%
Live	1.58%	5.32%	0.64%	1.10%	1.30%	1.34%

2-BULL ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(4.89%)	(5.39%)	(4.81%)	(4.18%)	(3.26%)	0.00%
Metals	(2.84%)	(8.29%)	(1.39%)	(5.27%)	6.38%	0.00%
Agr.	(4.02%)	(3.94%)	(4.15%)	(5.47%)	7.37%	0.00%
Softs	(2.70%)	(3.68%)	(1.97%)	(2.39%)	(8.87%)	0.00%
Live	0.24%	3.98%	(0.69%)	(0.24%)	(0.04%)	0.00%

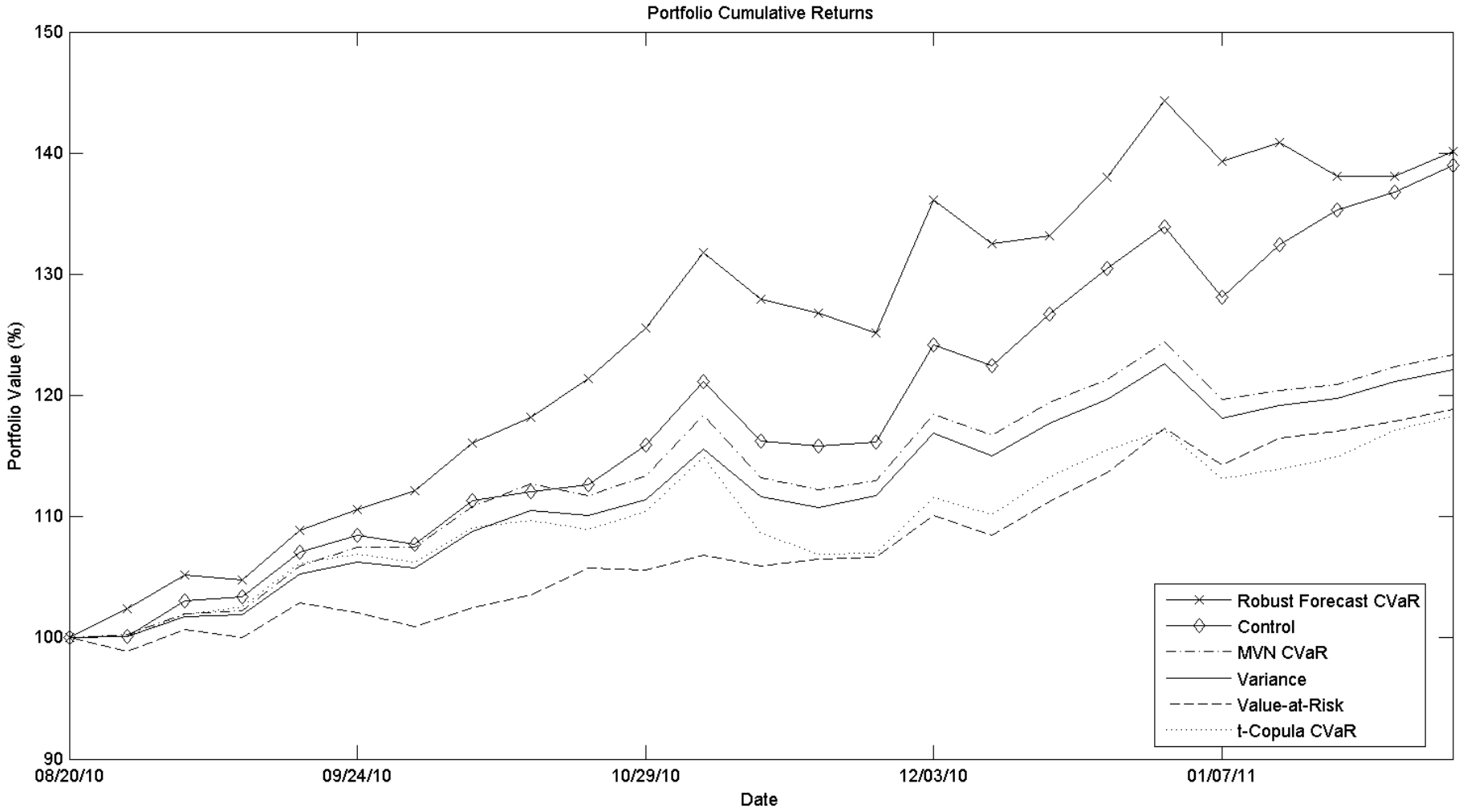

Figure 2: Weak Bull Cumulative Returns

5.4 Strong Bear (Jun. 13, 2008 – Nov. 28, 2008)

Both bear markets are characterized by similar performances across portfolio optimization methods. While the average difference between the final cumulative returns of the best and worst portfolios in the bull markets is 20.51%, the comparable difference for the bear market portfolios is only 11.27% as seen in Table 4. Similarly, the average standard deviation in final performance between portfolios in the two bear markets is 4.03% compared to 9.02% in bull markets. This is only in part due to the effects of compounding. More importantly, the well-documented tendency for asset correlations to increase in market crash scenarios is likely the salient factor in this characteristic. That is, the value added by the more sophisticated of our optimization methods is generally smaller[8] in bear markets because asset performances are more uniformly poor than those in bull markets are uniformly strong.

The best performing portfolio in the strong bear market is VaR which limited losses to 25.40%. Table 7 shows that VaR's performance benefitted from relatively larger allocations to the less poorly-performing softs (-0.81%) and livestock (-0.57%) while other portfolios suffered largely from their allocations to energy (-3.70%) and metals (-2.82%) – in particular Brent and platinum. Interestingly, all of the active portfolios except for RFCVaR completely avoid investments in palladium which is the worst performing asset of the period. If not for the control's investment in palladium, it would not have been the worst performing portfolio. The poor and relatively uniform portfolio performances for the strong bear market are shown in Figure 3.

The strong bear market losses are at least in part due to the 13.39% appreciation of the dollar, which represents a change in the 99th percentile of all 24 week changes as shown in Table 4. This result is analogous to the observation that dollar weakness is an important driver of strong bull market returns. The extremity of the change in the dollar's value during the strong bear period is another reason for the performance homogeneity across portfolios.

As discussed above, causally linking specific qualitative drivers of extreme market conditions to the relative and absolute performances of our optimization methods is not the focus of this paper. Instead, we simply observe that certain market environments are correlated with distinct performance characteristics. However, we will briefly explore the extreme changes in the dollar's value during the strong bear period in the context of monetary policy. Between September 2007 and December 2008, the Federal Reserve cut the policy federal funds rate over 500bp from 5.25% to 0-0.25% in reaction to the

[8] Importantly, this is not to say that more sophisticated optimization techniques should not be sought. On the contrary, the fact that traditional robust risk management tools are less effective in commodity markets is motivation for researchers to develop forecasting and optimization methods specific to commodity markets.

2007-08 financial crisis as its balance sheet swelled 156% to $2.24 trillion from $877 billion. These unprecedented levels of monetary easing had many side effects including plummeting nominal commodity prices.

It is somewhat counterintuitive that expansion of the money supply can be associated with deflation and dollar strength, but the experience of the financial crisis shows us that it is well within the realm of plausibility (Ugai, 2007). For this reason, traditional monetary policy tools like open market operations targeting lending rates may be insufficient to address economic crises if the velocity of money is also decreasing. That is, when financial institutions and corporations hoard marginal cash instead of lending and spending it, respectively, monetary policy has little to no effect. Monetary policy's failure to boost consumer prices (as a result of falling velocity of money) and simultaneous slumping real demand for commodities are likely the primary drivers of the unique performance characteristics of our optimization methods in the strong bear market.

Table 7: Performance by Optimization Method (Strong Bear)

1-BEAR ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	(0.26%)	(0.31%)	(1.48%)	0.28%	(4.23%)	(3.87%)
	BRT	(8.02%)	(4.16%)	(8.87%)	(6.13%)	0.00%	(3.95%)
	NG	(0.04%)	(0.09%)	(0.43%)	0.25%	0.31%	(3.39%)
	H.O.	(3.83%)	(2.97%)	(4.74%)	(2.38%)	(5.08%)	(3.58%)
Metals	GLD	(2.39%)	(2.61%)	(3.09%)	(4.64%)	(4.35%)	(0.31%)
	SLV	(0.09%)	0.00%	0.27%	0.81%	(2.36%)	(2.30%)
	PLT	(12.17%)	(6.43%)	(13.89%)	(9.03%)	(1.60%)	(3.98%)
	PAL	0.00%	0.00%	0.00%	0.00%	(3.82%)	(4.16%)
	COP	(5.39%)	(4.60%)	(5.89%)	(3.91%)	(5.01%)	(3.33%)
Agr.	CRN	(0.17%)	(0.04%)	(0.34%)	(5.89%)	(0.54%)	(3.51%)
	WHT	(1.31%)	(3.14%)	(0.76%)	(1.93%)	(0.58%)	(2.30%)
	SOY	(0.28%)	(1.70%)	(0.04%)	(0.13%)	0.25%	(2.74%)
Softs	COT	0.00%	(0.58%)	0.00%	0.00%	(0.73%)	(1.99%)
	COF	0.38%	(0.26%)	0.16%	(0.31%)	(0.50%)	(1.24%)
	CCO	0.00%	0.67%	0.00%	0.01%	(5.45%)	0.72%
	SUG	0.00%	0.09%	0.00%	0.02%	(1.95%)	(0.74%)
Live	CAT	(1.22%)	(1.69%)	(0.83%)	(1.22%)	(2.03%)	(0.31%)
	HOG	0.10%	(0.13%)	0.05%	0.31%	0.07%	(0.83%)

1-BEAR ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(11.80%)	(7.38%)	(14.83%)	(7.88%)	(8.81%)	(13.99%)
Metals	(18.97%)	(13.07%)	(21.25%)	(15.98%)	(16.04%)	(13.36%)
Agr.	(1.76%)	(4.83%)	(1.13%)	(7.82%)	(0.87%)	(8.31%)
Softs	0.38%	(0.09%)	0.16%	(0.28%)	(8.43%)	(3.23%)
Live	(1.12%)	(1.82%)	(0.78%)	(0.91%)	(1.96%)	(1.14%)

1-BEAR ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	2.19%	6.61%	(0.84%)	6.11%	5.18%	0.00%
Metals	(5.61%)	0.29%	(7.89%)	(2.62%)	(2.69%)	0.00%
Agr.	6.55%	3.48%	7.18%	0.49%	7.44%	0.00%
Softs	3.61%	3.14%	3.39%	2.95%	(5.20%)	0.00%
Live	0.02%	(0.68%)	0.35%	0.23%	(0.82%)	0.00%

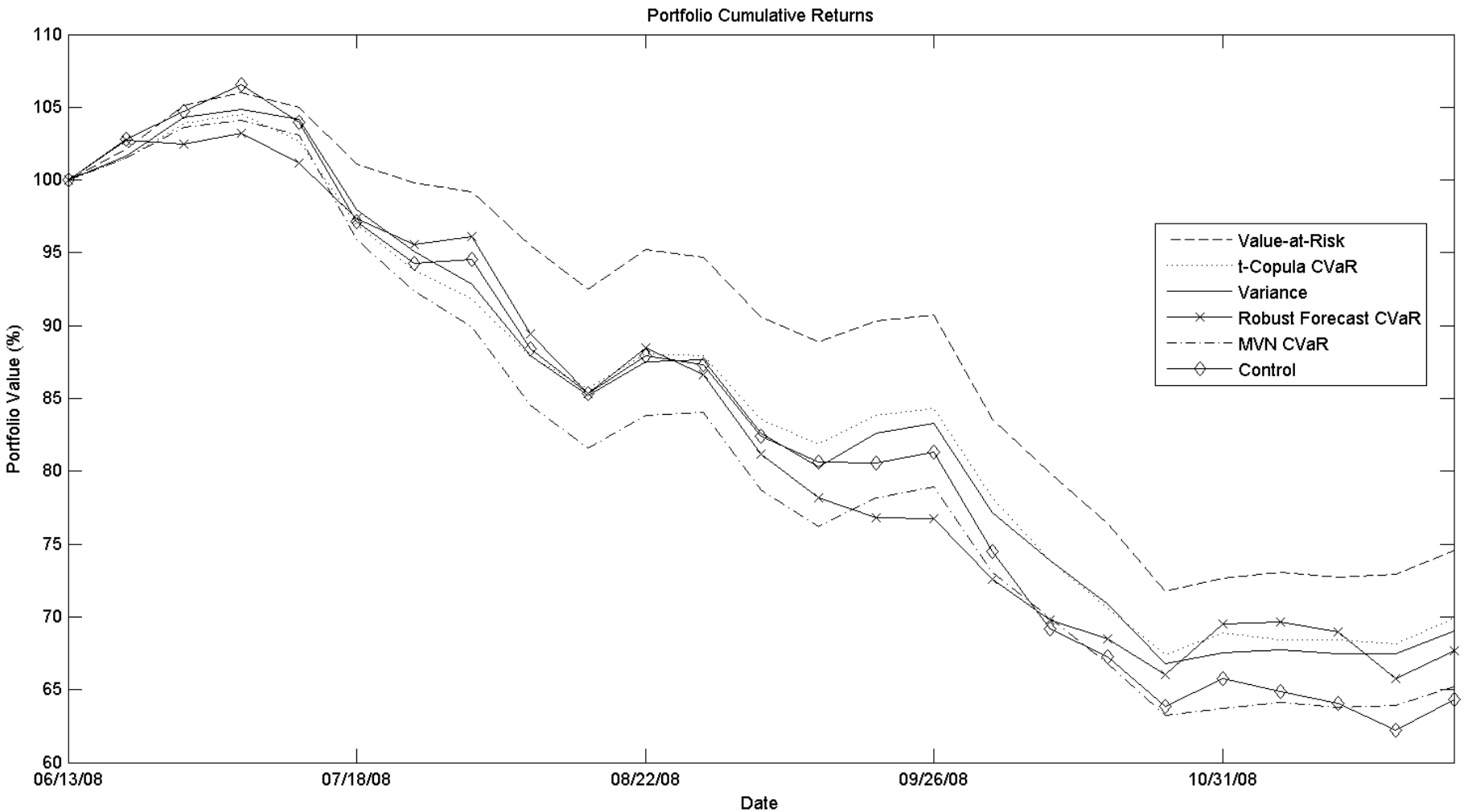

Figure 3: Strong Bear Cumulative Returns

5.5 Weak Bear (May 4, 2001 – Oct. 19, 2001)

Similar to the pattern of lower portfolio performance dispersion in more pronounced bull markets, the weak bear market exhibits marginally greater cumulative return variation than the strong bear market. Table 4 shows the difference between the best (tCVaR) and worst (VaR) portfolios is 12.25% compared to 10.29% in the strong bear market. Likewise, the final cumulative return standard deviation is 4.37% for the weak bear versus 3.70% for the strong bear. Also analogous to the strong-weak dynamic in bull markets is the near reversal in portfolio performance. Whereas VaR is the best portfolio in the strong bear market, it is the worst in the weak bear market. The worst performing portfolio in the strong bear market, the control, is the second-best performing portfolio in the weak bear market. These facts suggest the drivers each of the two bear markets (like the two bull markets) are much different.

History lends support to this hypothesis. In mid to late 2001, the U.S. economy was entering a recession. In May 2001, many economic indicators were still strong but deteriorating. According to the Labor Department, from May 2001 through October 2001, a net of 1.47 million American jobs were lost as the seasonally-adjusted unemployment rate climbed steadily from 4.3% to 5.3%. Consumer sentiment as reported by the University of Michigan fell 10.11% from 92.0 to 82.7. To accommodate the slowing economy, the Federal Reserve cut policy rates 200bp[9] from 4.50% to 2.50% between May and October. To compound the U.S.'s recession, the 9/11 attacks claimed thousands of American lives and resulted in the worst single-day point loss in the Dow Jones Industrial Average until that point in history, destroying hundreds of billions of dollars of wealth.

By comparison, the economic distress of the nation was more severe during the strong bear period. Unemployment rose by 1.2 percentage points from June through November 2008 as the economy shed a net of 1.96 million jobs. Industrial production as reported by the Federal Reserve Bank of St. Louis slumped 6.49% from 98.56 to 92.16 between June and November 2008 compared to a 2.16% decline over the weak bear period. Real GDP plummeted an annualized 3.21% over the strong bear period compared to a marginal rise of 0.07% over the weak bear period.

Additionally, the dollar appreciation in the weak bear period was a mere 0.62% (23rd percentile) compared to the strong bear period's 13.39% (99th percentile) as shown in Table 4. We have already observed that the effects of dollar-denomination are significant for relative and absolute commodity portfolio performance, and therefore it is unsurprising that the markedly different performance of the dollar in 2001 produced qualitative differences in the portfolios' performances.

[9] 1 basis point, or bp, is 0.01%

Quantitative comparison between the return data in the strong and weak bear markets reflect these historical events. Aside from the two weeks following 9/11, the control has only 2 weeks of portfolio returns in the 10th or less percentile of the entire data set during the weak bear market compared to 13 for the strong bear market. The control has no weekly portfolio returns in the 90th or greater percentile in the weak bear market while it has 4 such weeks during the strong bear market, including two in the 99th percentile. Additionally, the weekly standard deviation of control portfolio returns in the strong bear market, 4.88%, is 3.43 times that of the weak bear market. While it is clear from these facts that volatility was significantly greater in the 2008 commodity market crash, it is also apparent that the driving factors of a particular market movement and the resultant features of the asset returns are the primary determinants of which optimization methods are preferable. The near reversal of portfolio performance rank between the strong (Figure 3) and weak (Figure 4) bear markets along with the contrast between portfolio performance dispersion between the two (Table 4) are evidence of this performance bifurcation.

As shown in Table 8, the underperformance of nearly all of the active portfolios (with the exception of tCVaR) can be explained by their allocations to palladium, the worst performing asset by nearly a full percentage point (-3.73%). For metals as a commodity class, every active portfolio underperformed the control by double-digits. This underperformance is somewhat ameliorated by relatively strong performances across the active portfolios in energy, the worst performing commodity class, and softs. Finally, the average pairwise correlation of the worst three performing commodities in the weak bear market is significantly lower than that of the strong bear market (0.07 versus 0.31, respectively). The disparity between the strong and weak bear average worst cross-asset correlation is also in stark contrast to those of relatively consistent average best cross-asset correlations for the strong and weak bull markets (0.13 and 0.15 respectively). Such a discrepancy emphasizes the quantitative differences in correlation and volatility in the two bear markets and, again, suggests disparate qualitative drivers.

Table 8: Performance by Optimization Method (Weak Bear)

2-BEAR ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	(0.08%)	0.16%	(0.16%)	0.23%	(0.78%)	(1.31%)
	BRT	(0.86%)	(0.50%)	(0.99%)	(0.64%)	1.57%	(1.40%)
	NG	(1.57%)	(2.12%)	(1.91%)	(1.34%)	0.70%	(2.80%)
	H.O.	0.20%	0.37%	0.39%	(1.19%)	(0.58%)	(1.02%)
Metals	GLD	(0.01%)	0.00%	0.00%	(0.44%)	0.31%	0.26%
	SLV	(0.00%)	0.00%	(0.09%)	(0.06%)	(4.22%)	(0.04%)
	PLT	(0.38%)	(0.14%)	(0.16%)	(0.78%)	(5.49%)	(1.79%)
	PAL	(19.90%)	(29.31%)	(22.82%)	(17.59%)	(8.48%)	(3.73%)
	COP	0.00%	0.00%	0.00%	(0.14%)	0.10%	(1.11%)
Agr.	CRN	0.01%	0.00%	(0.04%)	(0.21%)	1.38%	0.07%
	WHT	1.35%	1.26%	1.27%	0.76%	(0.21%)	0.39%
	SOY	0.48%	0.43%	(0.04%)	0.69%	0.82%	(0.17%)
Softs	COT	0.00%	0.00%	0.00%	(0.02%)	(1.71%)	(1.92%)
	COF	1.01%	1.07%	1.06%	4.72%	(1.34%)	0.02%
	CCO	0.02%	0.00%	0.01%	(0.03%)	0.23%	(1.45%)
	SUG	(0.06%)	0.01%	0.02%	(0.40%)	(1.12%)	(1.90%)
Live	CAT	(1.34%)	0.27%	(1.13%)	0.05%	(1.79%)	(0.29%)
	HOG	0.13%	0.08%	0.23%	0.16%	(2.85%)	(1.42%)

2-BEAR ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(2.31%)	(2.09%)	(2.66%)	(2.91%)	0.89%	(6.38%)
Metals	(20.21%)	(29.41%)	(23.02%)	(18.75%)	(16.81%)	(6.30%)
Agr.	1.85%	1.69%	1.19%	1.24%	2.00%	0.29%
Softs	0.96%	1.09%	1.10%	4.25%	(3.89%)	(5.16%)
Live	(1.22%)	0.35%	(0.90%)	0.21%	(4.58%)	(1.70%)

2-BEAR ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	4.07%	4.29%	3.72%	3.47%	7.27%	0.00%
Metals	(13.91%)	(23.11%)	(16.72%)	(12.45%)	(10.51%)	0.00%
Agr.	1.55%	1.40%	0.90%	0.95%	1.71%	0.00%
Softs	6.12%	6.25%	6.26%	9.41%	1.27%	0.00%
Live	0.48%	2.05%	0.80%	1.91%	(2.88%)	0.00%

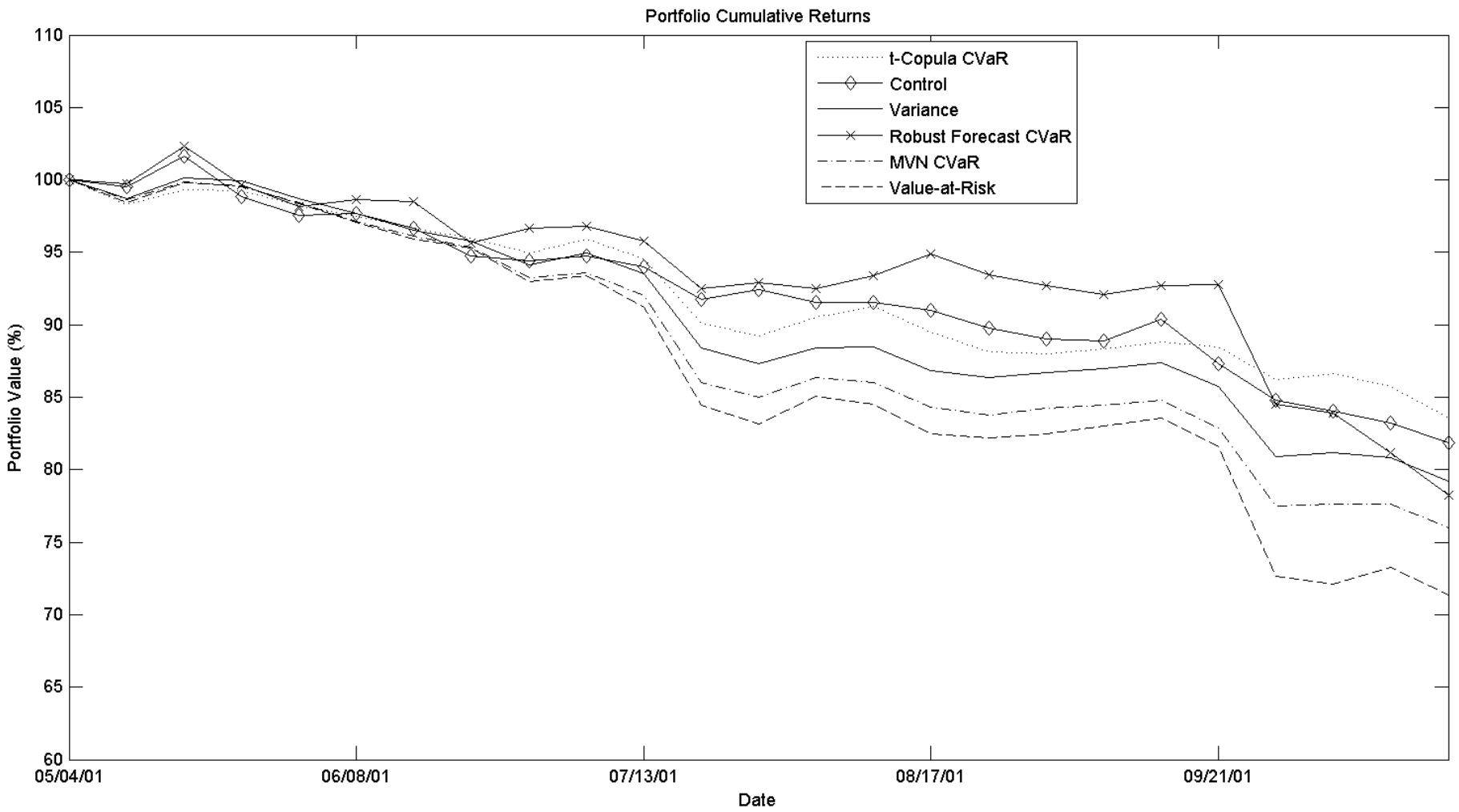

Figure 4: Weak Bear Cumulative Returns

6 Conclusion

We implement six portfolio optimization methods including Markowitz's mean-variance optimization, VaR, three specifications of CVaR, and an equally weighted control. Our comparison between these methods attempts to assess the empirical differences in performance provided by various distributional assumptions, risk measures, and forecasting methods. We observe a number of interesting relationships between asset return features and the characteristics of performance across optimization methods, but do not find any single method to be strictly better. Nonetheless, the phenomena we observe are significant for future work on commodity portfolio optimization. Specifically, these phenomena can be grouped into three categories: dollar-denomination effects, value-add of diversification, and the persistence and profundity of asset return features.

The effects of dollar-denomination on commodity portfolio behavior are significant. The relationship between commodity prices and inflation are well-understood (Furlong & Ingenito, 1996), but we also find evidence that the magnitude of change in the dollar's value is negatively related with the dispersion of portfolio returns and may inform investors', commercial hedgers', and regulators' preferences among risk frameworks. As a corollary, we find that the value-add of diversification is eroded in market environments driven by dollar-denomination effects. In other words, more sophisticated optimization and forecasting techniques provide less marginal benefit during strongly directional markets where exogenous effects drive commodity return behavior.

We observe that the persistence and profundity of asset return features affect the performance of our forecasting methods. Where asset return features like high volatility and high correlation are persistent and profound, forecasting methods are poor at predicting future return scenarios. We believe this underperformance can be attributed to the mean-reverting features of ARMA, GARCH, and DCC. Commodity returns periodically exhibit serial correlation, high volatility, and high correlation. Other research suggests this behavior may be related to long-term structural factors related to certain commodity markets like storage and transportation costs, seasonal factors like weather patterns (Pindyck, 1993), and other political, macroeconomic, and monetary factors like embargoes and interest rates (Hua, 1998). These unique characteristics serve as the reason commodities are an important asset class for investors seeking diversification and as the reason traditional forecasting techniques perform poorly in some commodity market environments.

The results of this thesis have implications for investors, commercial hedgers, and regulators. The objectives of investors vary, but generally, investors are interested in maximizing their returns while

minimizing their risk. Our results show that, while the theoretical basis for certain distributional assumptions, risk measures, or forecasting techniques may be preferable, no single optimization procedure was strictly better. While the out-of-sample performance of many of these techniques have been demonstrated in the context of other financial asset markets such as in Khindanova, Rachev, and Schwartz (2001) and Wang, Chen, Jin, and Zhou (2010), the unique features of commodity returns necessitate entirely new methods or commodity-specific adaptations to established techniques. As distinct from investors, commercial hedgers, while also motivated by profit, are primarily concerned with reducing their exposure to commodity price fluctuations. Our results are significant to these market participants as they demonstrate that drivers exogenous to commodity markets and beyond the expertise of commercial hedgers can have dire implications for the performance of optimization strategies. This is less relevant to producers or consumers of single commodities, but for commercial participants that have exposure to many different commodities, extreme joint scenarios can be devastating. Finally, regulators should be cognizant of the relatively poor and uniform performance of each of the optimization methods during market crashes. Reducing systemic risk is an important objective for regulators, and our results demonstrate that even robust optimization techniques are insufficient to protect against large losses. The use of VaR-type systemic risk aggregators at the regulatory level is particularly concerning.

Future work should focus on adapting and comparing commodity-specific innovations to traditional risk management and forecasting techniques across a variety of market conditions. Understanding of the commodity market is still relatively elementary, but given its importance as a financial asset class, resources should be committed to improving the robustness of the suite of risk management tools available to commodity investors, commercial hedgers, and regulators. Finally, the importance of dollar-denomination, interest rates, and monetary policy in the discussion of commodity return behavior should not be understated. It would be particularly interesting to directly and rigorously study the effects the recent, unprecedented monetary measures have on commodity returns measured at both low and high frequencies.

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8 Appendix

This appendix includes a full set of tables and figures for each market environment. The first table in each market environment set tabulates the average of the commodity-level decision vector $\boldsymbol{w}_t$ for each optimization method. The first figure depicts the class-level decision vector $\boldsymbol{w}_t$ for each optimization method. The second table is identical to the cumulative return tables presented in Section 5. The second figure plots the class-level cumulative returns for each method over time. The last table in the appendix, Table 17, is a complete set of final portfolio values, weekly Sharpe ratios for each portfolio, and relative rankings for each of these sets of values.

8.1 Strong Bull (Sep. 7, 2007 – Feb. 22, 2008) Tables and Figures

Table 9: Allocation by Optimization Method (Strong Bull)

1-BULL AVERAGE ASSET ALLOCATION							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	0.81%	0.87%	1.03%	1.53%	0.28%	5.56%
	BRT	7.57%	4.36%	9.02%	5.87%	0.02%	5.56%
	NG	0.46%	0.52%	0.53%	1.45%	2.13%	5.56%
	H.O.	0.52%	0.60%	0.81%	2.47%	0.77%	5.56%
Metals	GLD	15.99%	20.26%	10.05%	13.52%	0.30%	5.56%
	SLV	0.63%	0.00%	2.77%	1.31%	12.35%	5.56%
	PLT	26.84%	15.49%	34.34%	28.15%	9.70%	5.56%
	PAL	0.03%	0.21%	0.25%	0.89%	9.89%	5.56%
	COP	12.51%	8.81%	15.22%	12.62%	8.81%	5.56%
Agr.	CRN	0.00%	0.00%	0.00%	0.00%	4.39%	5.56%
	WHT	9.37%	7.76%	9.89%	12.62%	4.64%	5.56%
	SOY	4.04%	5.06%	2.36%	1.03%	3.76%	5.56%
Softs	COT	0.01%	4.05%	0.00%	0.38%	5.52%	5.56%
	COF	3.33%	4.60%	1.74%	2.64%	10.76%	5.56%
	CCO	0.00%	0.69%	0.00%	0.01%	9.60%	5.56%
	SUG	0.51%	2.66%	0.00%	0.76%	4.05%	5.56%
Live	CAT	17.37%	22.16%	12.00%	14.16%	9.88%	5.56%
	HOG	0.01%	1.91%	0.00%	0.59%	3.15%	5.56%

1-BULL AVERAGE CLASS ALLOCATION (ABS. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	9.36%	6.34%	11.38%	11.32%	3.21%	22.22%
Metals	56.00%	44.76%	62.63%	56.49%	41.04%	27.78%
Agr.	13.40%	12.82%	12.25%	13.65%	12.80%	16.67%
Softs	3.86%	12.00%	1.74%	3.79%	29.93%	22.22%
Live	17.38%	24.07%	12.00%	14.75%	13.03%	11.11%

1-BULL AVERAGE CLASS ALLOCATION (CONTROL REL. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(12.87%)	(15.88%)	(10.84%)	(10.91%)	(19.02%)	0.00%
Metals	28.22%	16.99%	34.85%	28.72%	13.26%	0.00%
Agr.	(3.26%)	(3.85%)	(4.42%)	(3.02%)	(3.87%)	0.00%
Softs	(18.37%)	(10.22%)	(20.49%)	(18.43%)	7.70%	0.00%
Live	6.27%	12.96%	0.89%	3.64%	1.92%	0.00%

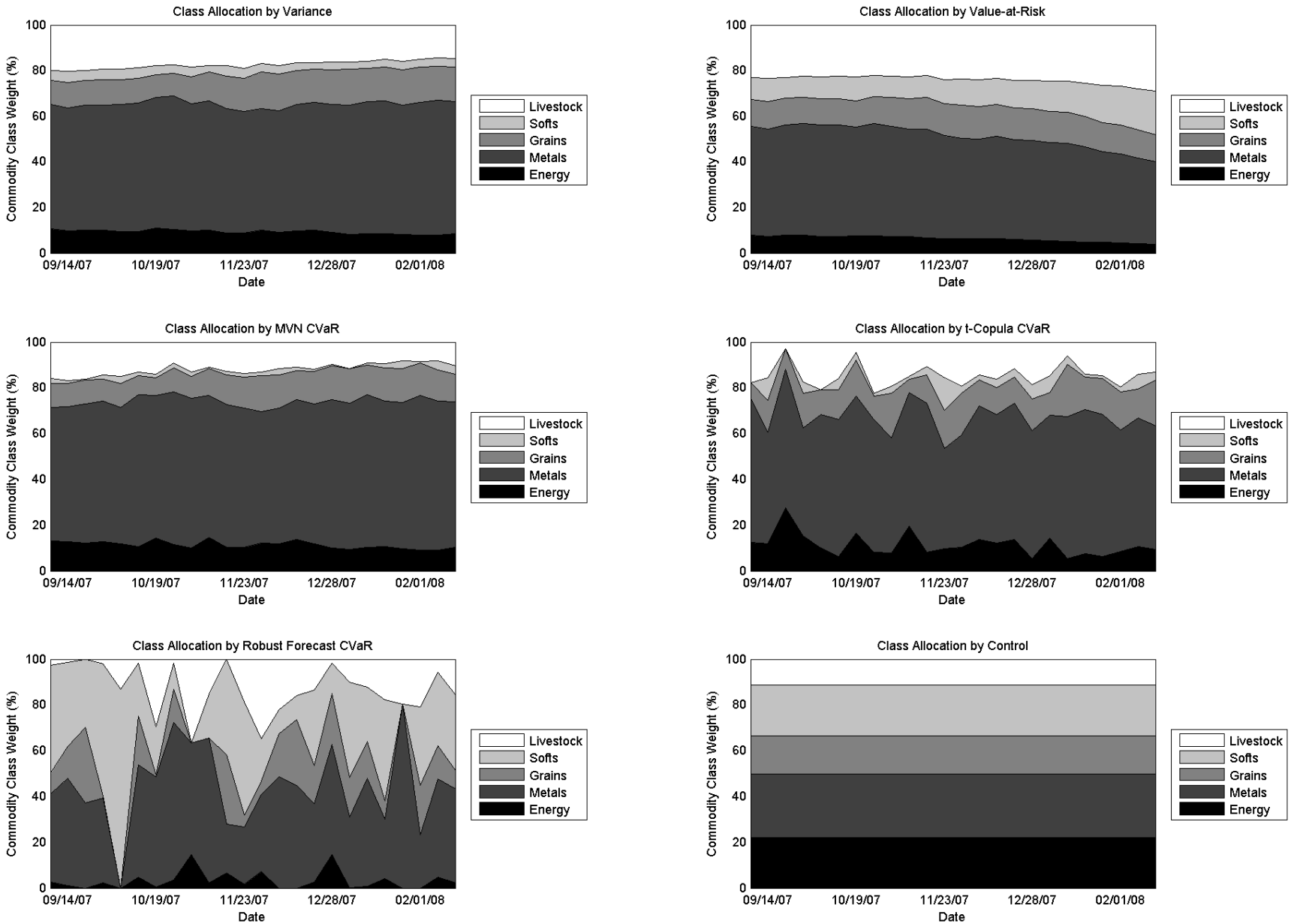

Figure 5: Class-Level Allocation by Optimization Method (Strong Bull)

Table 10: Performance by Optimization Method (Strong Bull)

1-BULL ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	0.30%	0.25%	(0.07%)	1.05%	0.21%	1.57%
	BRT	2.23%	1.31%	3.17%	0.74%	0.00%	1.58%
	NG	0.37%	0.36%	0.57%	2.17%	0.27%	3.22%
	H.O.	0.15%	0.12%	0.16%	0.00%	0.26%	1.60%
Metals	GLD	5.40%	6.79%	3.93%	5.48%	0.14%	1.87%
	SLV	0.48%	0.00%	1.64%	1.55%	1.37%	2.31%
	PLT	19.05%	7.78%	25.57%	20.75%	5.50%	3.45%
	PAL	0.02%	0.10%	0.46%	0.26%	5.71%	2.92%
	COP	1.79%	1.03%	1.94%	1.83%	(0.64%)	1.06%
Agr.	CRN	0.00%	0.00%	0.00%	0.00%	2.84%	2.85%
	WHT	2.84%	2.04%	2.97%	4.47%	4.02%	1.41%
	SOY	2.29%	2.73%	1.37%	0.44%	1.62%	2.96%
Softs	COT	0.02%	1.62%	0.00%	0.51%	(1.52%)	1.70%
	COF	1.25%	1.73%	0.69%	1.87%	1.20%	2.04%
	CCO	0.00%	0.60%	0.00%	0.01%	1.26%	2.51%
	SUG	0.07%	1.13%	0.00%	(0.09%)	1.80%	2.22%
Live	CAT	(0.90%)	(1.07%)	(0.65%)	(0.53%)	(0.79%)	(0.27%)
	HOG	(0.00%)	0.28%	0.00%	(0.11%)	2.12%	(0.12%)

1-BULL ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	3.07%	2.05%	3.85%	4.01%	0.75%	8.20%
Metals	28.36%	16.40%	35.85%	32.04%	12.48%	12.14%
Agr.	5.19%	4.82%	4.38%	4.93%	8.71%	7.40%
Softs	1.34%	5.18%	0.69%	2.31%	2.74%	8.75%
Live	(0.90%)	(0.79%)	(0.65%)	(0.63%)	1.32%	(0.40%)

1-BULL ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(5.13%)	(6.14%)	(4.35%)	(4.19%)	(7.45%)	0.00%
Metals	16.22%	4.26%	23.71%	19.90%	0.35%	0.00%
Agr.	(2.21%)	(2.57%)	(3.02%)	(2.47%)	1.31%	0.00%
Softs	(7.41%)	(3.57%)	(8.06%)	(6.45%)	(6.01%)	0.00%
Live	(0.51%)	(0.40%)	(0.26%)	(0.24%)	1.71%	0.00%

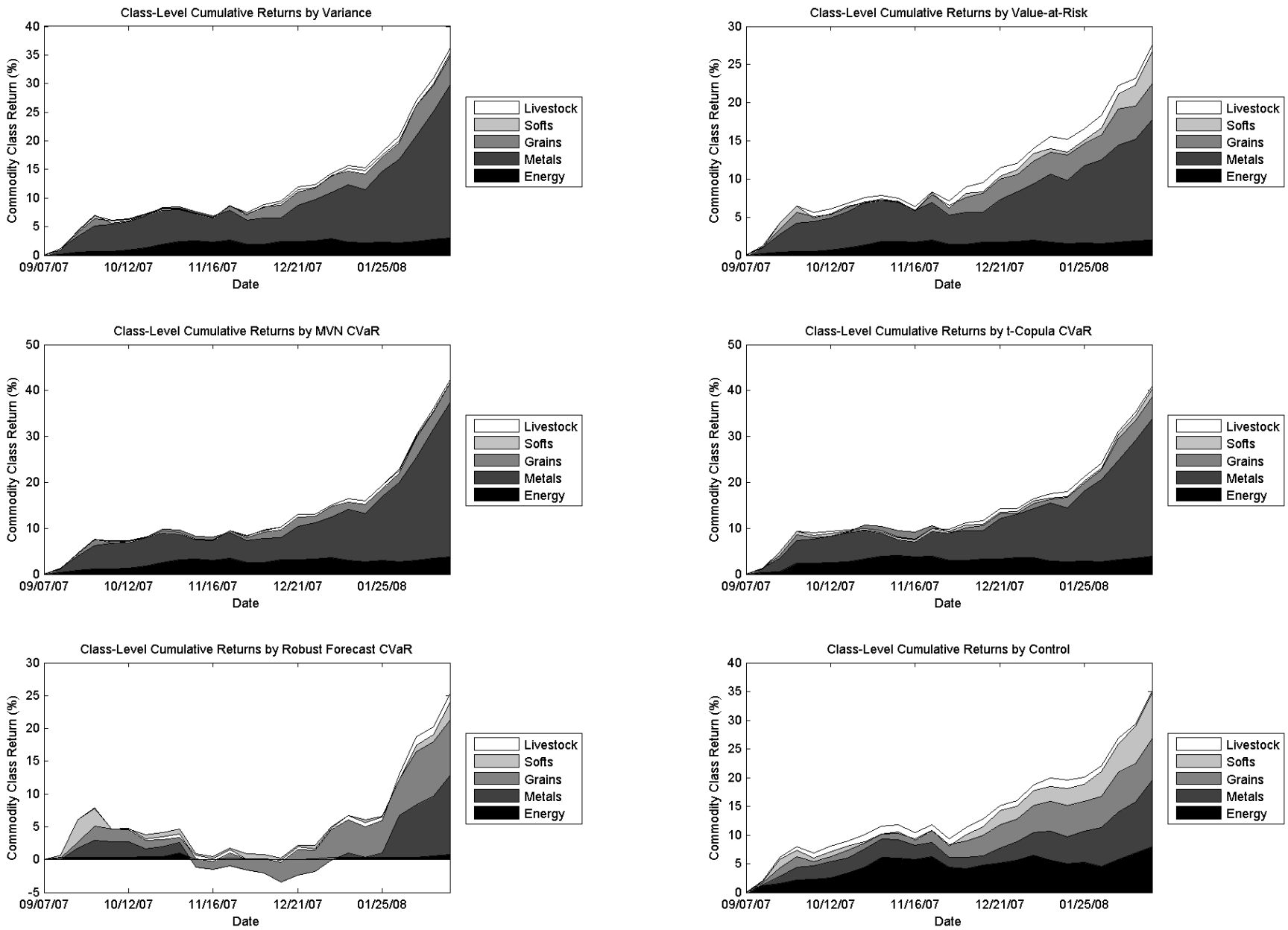

Figure 6: Class-Level Performance by Optimization Method (Strong Bull)

8.2 Weak Bull (Aug. 20, 2010 – Feb. 4, 2011) Tables and Figures

Table 11: Allocation by Optimization Method (Weak Bull)

2-BULL AVERAGE ASSET ALLOCATION							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	0.00%	0.00%	0.00%	0.10%	7.79%	5.56%
	BRT	0.00%	0.00%	0.00%	0.11%	0.36%	5.56%
	NG	0.00%	2.97%	0.00%	0.14%	0.56%	5.56%
	H.O.	1.67%	0.00%	1.87%	3.43%	0.60%	5.56%
Metals	GLD	45.66%	17.96%	50.02%	41.98%	29.57%	5.56%
	SLV	0.00%	0.00%	0.00%	0.00%	0.03%	5.56%
	PLT	5.52%	0.00%	7.13%	4.36%	12.20%	5.56%
	PAL	0.00%	1.18%	0.00%	0.00%	5.52%	5.56%
	COP	6.33%	0.00%	8.29%	6.10%	12.59%	5.56%
Agr.	CRN	0.00%	0.08%	0.15%	0.85%	0.71%	5.56%
	WHT	3.18%	4.67%	2.10%	6.88%	4.39%	5.56%
	SOY	4.85%	4.93%	4.08%	0.64%	6.69%	5.56%
Softs	COT	3.40%	5.86%	2.86%	4.01%	1.52%	5.56%
	COF	1.51%	1.72%	1.40%	2.83%	2.18%	5.56%
	CCO	8.05%	0.55%	9.47%	10.10%	1.88%	5.56%
	SUG	0.39%	7.30%	0.22%	0.47%	1.75%	5.56%
Live	CAT	15.35%	42.82%	9.34%	12.70%	10.61%	5.56%
	HOG	4.09%	9.96%	3.05%	5.31%	1.07%	5.56%

2-BULL AVERAGE CLASS ALLOCATION (ABS. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	1.67%	2.97%	1.87%	3.77%	9.31%	22.22%
Metals	57.51%	19.14%	65.45%	52.44%	59.90%	27.78%
Agr.	8.04%	9.68%	6.34%	8.37%	11.79%	16.67%
Softs	13.35%	15.42%	13.95%	17.41%	7.32%	22.22%
Live	19.43%	52.78%	12.39%	18.01%	11.68%	11.11%

2-BULL AVERAGE CLASS ALLOCATION (CONTROL REL. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(20.55%)	(19.25%)	(20.35%)	(18.45%)	(12.91%)	0.00%
Metals	29.74%	(8.64%)	37.67%	24.66%	32.13%	0.00%
Agr.	(8.63%)	(6.98%)	(10.33%)	(8.30%)	(4.88%)	0.00%
Softs	(8.88%)	(6.80%)	(8.27%)	(4.81%)	(14.90%)	0.00%
Live	8.32%	41.67%	1.28%	6.90%	0.57%	0.00%

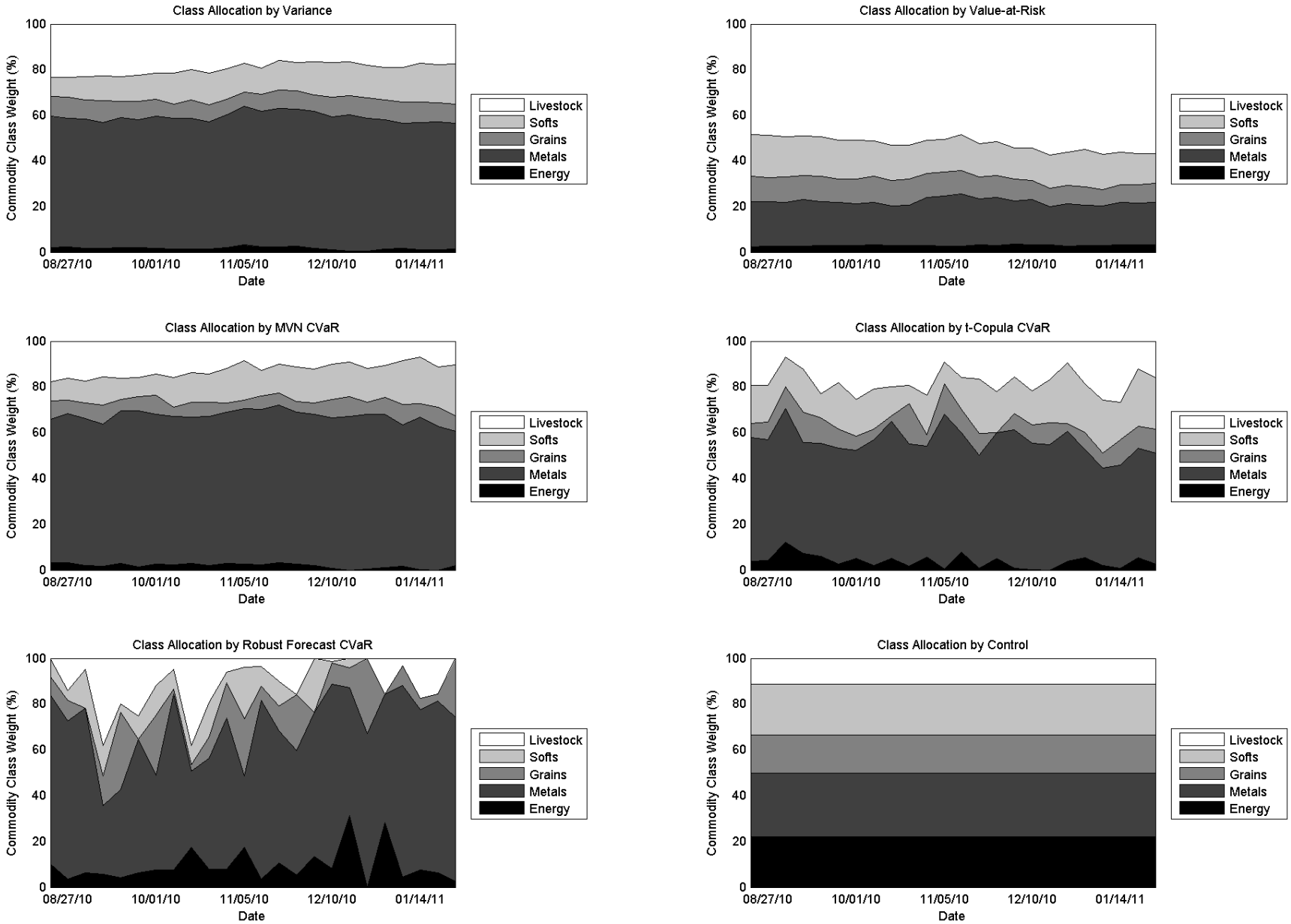

Figure 7: Class-Level Allocation by Optimization Method (Weak Bull)

Table 12: Performance by Optimization Method (Weak Bull)

2-BULL ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	0.00%	0.00%	0.00%	0.04%	1.75%	1.16%
	BRT	0.00%	0.00%	0.00%	0.02%	(0.22%)	1.90%
	NG	0.00%	0.16%	0.00%	0.03%	0.67%	0.32%
	H.O.	0.66%	0.00%	0.74%	1.28%	0.08%	2.06%
Metals	GLD	3.92%	1.80%	4.39%	3.39%	1.11%	0.51%
	SLV	0.00%	0.00%	0.00%	0.00%	0.03%	2.93%
	PLT	1.28%	0.00%	1.41%	(0.14%)	0.73%	1.24%
	PAL	0.00%	0.52%	0.00%	0.00%	3.31%	3.39%
	COP	2.40%	0.00%	3.18%	2.04%	11.17%	2.15%
Agr.	CRN	0.00%	0.08%	0.27%	0.32%	1.03%	2.99%
	WHT	0.94%	1.01%	0.81%	0.88%	6.83%	1.51%
	SOY	1.91%	1.84%	1.64%	0.23%	5.87%	2.25%
Softs	COT	2.99%	3.94%	3.09%	1.36%	1.65%	4.25%
	COF	0.50%	0.05%	0.87%	(0.06%)	0.02%	1.06%
	CCO	4.24%	0.39%	4.61%	6.71%	(0.10%)	2.95%
	SUG	0.17%	2.59%	0.02%	0.26%	0.34%	2.13%
Live	CAT	1.24%	4.03%	0.68%	0.77%	1.29%	0.58%
	HOG	0.33%	1.24%	(0.03%)	0.33%	0.01%	0.75%

2-BULL ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	0.66%	0.16%	0.74%	1.37%	2.29%	5.55%
Metals	7.78%	2.33%	9.23%	5.35%	17.01%	10.62%
Agr.	2.88%	2.95%	2.75%	1.43%	14.27%	6.90%
Softs	8.08%	7.10%	8.81%	8.39%	1.91%	10.78%
Live	1.58%	5.32%	0.64%	1.10%	1.30%	1.34%

2-BULL ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(4.89%)	(5.39%)	(4.81%)	(4.18%)	(3.26%)	0.00%
Metals	(2.84%)	(8.29%)	(1.39%)	(5.27%)	6.38%	0.00%
Agr.	(4.02%)	(3.94%)	(4.15%)	(5.47%)	7.37%	0.00%
Softs	(2.70%)	(3.68%)	(1.97%)	(2.39%)	(8.87%)	0.00%
Live	0.24%	3.98%	(0.69%)	(0.24%)	(0.04%)	0.00%

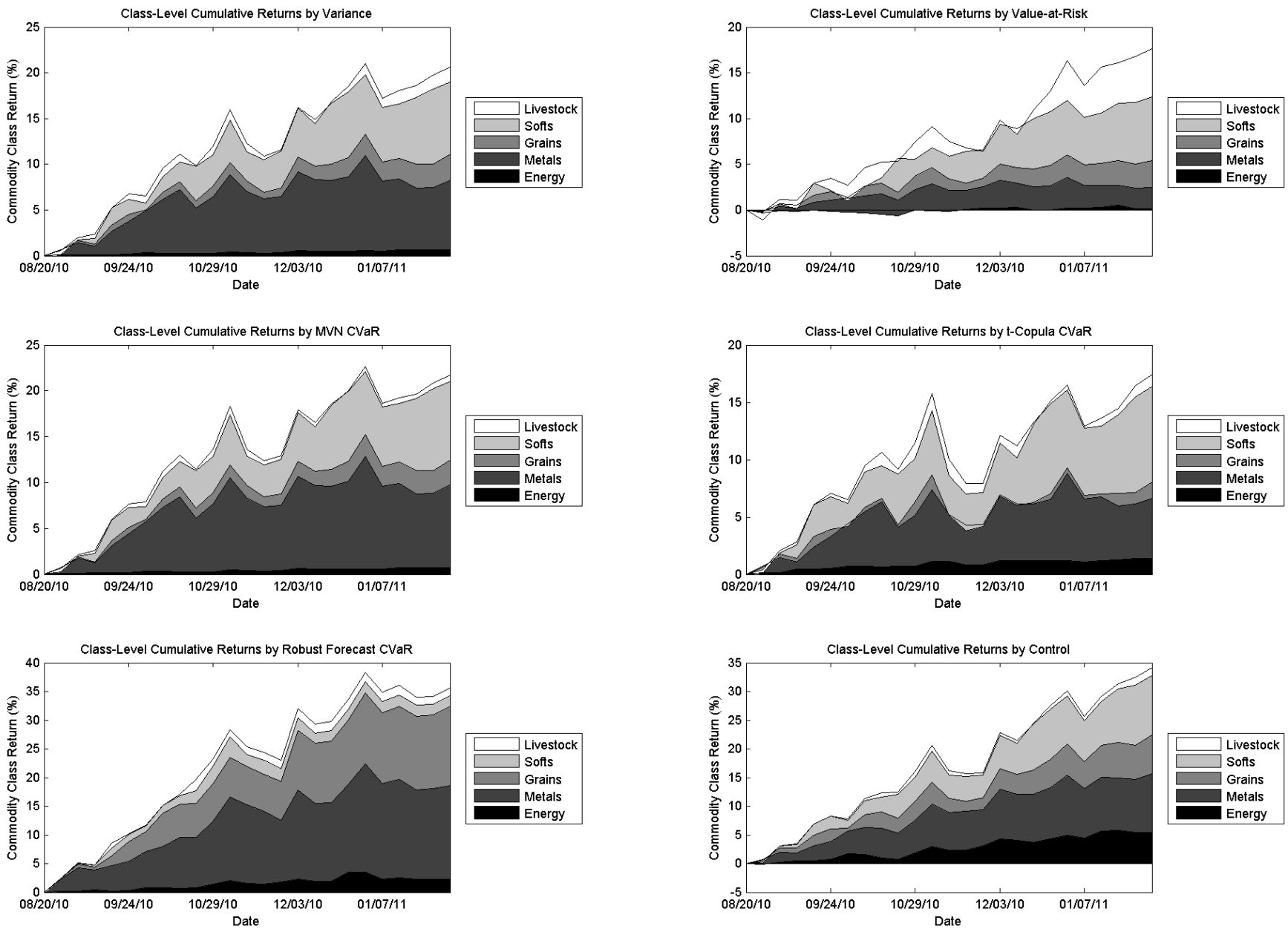

Figure 8: Class-Level Performance by Optimization Method (Weak Bull)

8.3 Strong Bear (Jun. 13, 2008 – Nov. 28, 2008) Tables and Figures

Table 13: Allocation by Optimization Method (Strong Bear)

1-BEAR AVERAGE ASSET ALLOCATION							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	0.41%	0.51%	1.32%	2.59%	3.23%	5.56%
	BRT	11.67%	4.58%	13.96%	6.26%	0.00%	5.56%
	NG	0.30%	0.35%	0.53%	1.87%	1.66%	5.56%
	H.O.	5.42%	3.90%	7.50%	6.62%	6.30%	5.56%
Metals	GLD	31.63%	33.12%	31.80%	28.33%	13.98%	5.56%
	SLV	0.18%	0.00%	0.72%	0.74%	6.48%	5.56%
	PLT	15.72%	8.10%	18.01%	13.55%	2.69%	5.56%
	PAL	1.40%	1.70%	1.64%	1.38%	3.00%	5.56%
	COP	8.83%	5.76%	10.16%	8.19%	13.03%	5.56%
Agr.	CRN	0.90%	0.12%	1.23%	3.88%	3.01%	5.56%
	WHT	5.56%	6.23%	3.62%	7.81%	4.44%	5.56%
	SOY	0.99%	2.90%	0.60%	0.25%	2.11%	5.56%
Softs	COT	0.00%	2.31%	0.00%	0.00%	1.85%	5.56%
	COF	0.48%	2.10%	0.42%	3.09%	4.19%	5.56%
	CCO	0.00%	0.57%	0.00%	0.12%	4.79%	5.56%
	SUG	0.04%	1.13%	0.05%	0.04%	6.24%	5.56%
Live	CAT	16.16%	24.51%	8.19%	14.55%	22.79%	5.56%
	HOG	0.31%	2.10%	0.26%	0.75%	0.20%	5.56%

1-BEAR AVERAGE CLASS ALLOCATION (ABS. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	17.80%	9.35%	23.31%	17.34%	11.19%	22.22%
Metals	57.76%	48.68%	62.33%	52.19%	39.18%	27.78%
Agr.	7.45%	9.26%	5.45%	11.94%	9.56%	16.67%
Softs	0.52%	6.11%	0.47%	3.25%	17.07%	22.22%
Live	16.48%	26.61%	8.45%	15.30%	22.99%	11.11%

1-BEAR AVERAGE CLASS ALLOCATION (CONTROL REL. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(4.42%)	(12.87%)	1.08%	(4.89%)	(11.03%)	0.00%
Metals	29.98%	20.90%	34.55%	24.41%	11.40%	0.00%
Agr.	(9.22%)	(7.41%)	(11.22%)	(4.73%)	(7.10%)	0.00%
Softs	(21.70%)	(16.12%)	(21.76%)	(18.98%)	(5.15%)	0.00%
Live	5.37%	15.50%	(2.66%)	4.19%	11.88%	0.00%

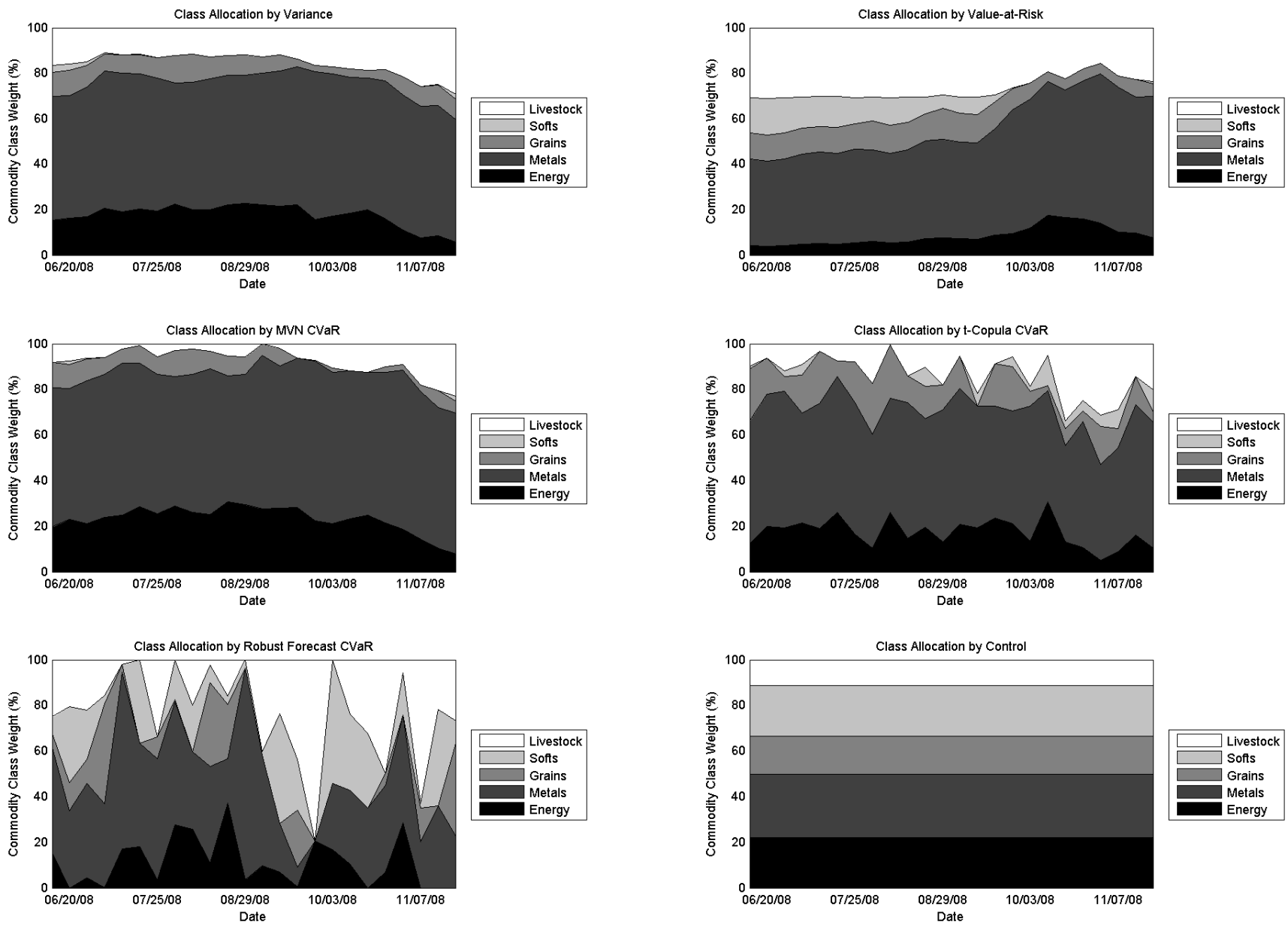

Figure 9: Class-Level Allocation by Optimization Method (Strong Bear)

Table 14: Performance by Optimization Method (Strong Bear)

1-BEAR ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	(0.26%)	(0.31%)	(1.48%)	0.28%	(4.23%)	(3.87%)
	BRT	(8.02%)	(4.16%)	(8.87%)	(6.13%)	0.00%	(3.95%)
	NG	(0.04%)	(0.09%)	(0.43%)	0.25%	0.31%	(3.39%)
	H.O.	(3.83%)	(2.97%)	(4.74%)	(2.38%)	(5.08%)	(3.58%)
Metals	GLD	(2.39%)	(2.61%)	(3.09%)	(4.64%)	(4.35%)	(0.31%)
	SLV	(0.09%)	0.00%	0.27%	0.81%	(2.36%)	(2.30%)
	PLT	(12.17%)	(6.43%)	(13.89%)	(9.03%)	(1.60%)	(3.98%)
	PAL	0.00%	0.00%	0.00%	0.00%	(3.82%)	(4.16%)
	COP	(5.39%)	(4.60%)	(5.89%)	(3.91%)	(5.01%)	(3.33%)
Agr.	CRN	(0.17%)	(0.04%)	(0.34%)	(5.89%)	(0.54%)	(3.51%)
	WHT	(1.31%)	(3.14%)	(0.76%)	(1.93%)	(0.58%)	(2.30%)
	SOY	(0.28%)	(1.70%)	(0.04%)	(0.13%)	0.25%	(2.74%)
Softs	COT	0.00%	(0.58%)	0.00%	0.00%	(0.73%)	(1.99%)
	COF	0.38%	(0.26%)	0.16%	(0.31%)	(0.50%)	(1.24%)
	CCO	0.00%	0.67%	0.00%	0.01%	(5.45%)	0.72%
	SUG	0.00%	0.09%	0.00%	0.02%	(1.95%)	(0.74%)
Live	CAT	(1.22%)	(1.69%)	(0.83%)	(1.22%)	(2.03%)	(0.31%)
	HOG	0.10%	(0.13%)	0.05%	0.31%	0.07%	(0.83%)

1-BEAR ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(11.80%)	(7.38%)	(14.83%)	(7.88%)	(8.81%)	(13.99%)
Metals	(18.97%)	(13.07%)	(21.25%)	(15.98%)	(16.04%)	(13.36%)
Agr.	(1.76%)	(4.83%)	(1.13%)	(7.82%)	(0.87%)	(8.31%)
Softs	0.38%	(0.09%)	0.16%	(0.28%)	(8.43%)	(3.23%)
Live	(1.12%)	(1.82%)	(0.78%)	(0.91%)	(1.96%)	(1.14%)

1-BEAR ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	2.19%	6.61%	(0.84%)	6.11%	5.18%	0.00%
Metals	(5.61%)	0.29%	(7.89%)	(2.62%)	(2.69%)	0.00%
Agr.	6.55%	3.48%	7.18%	0.49%	7.44%	0.00%
Softs	3.61%	3.14%	3.39%	2.95%	(5.20%)	0.00%
Live	0.02%	(0.68%)	0.35%	0.23%	(0.82%)	0.00%

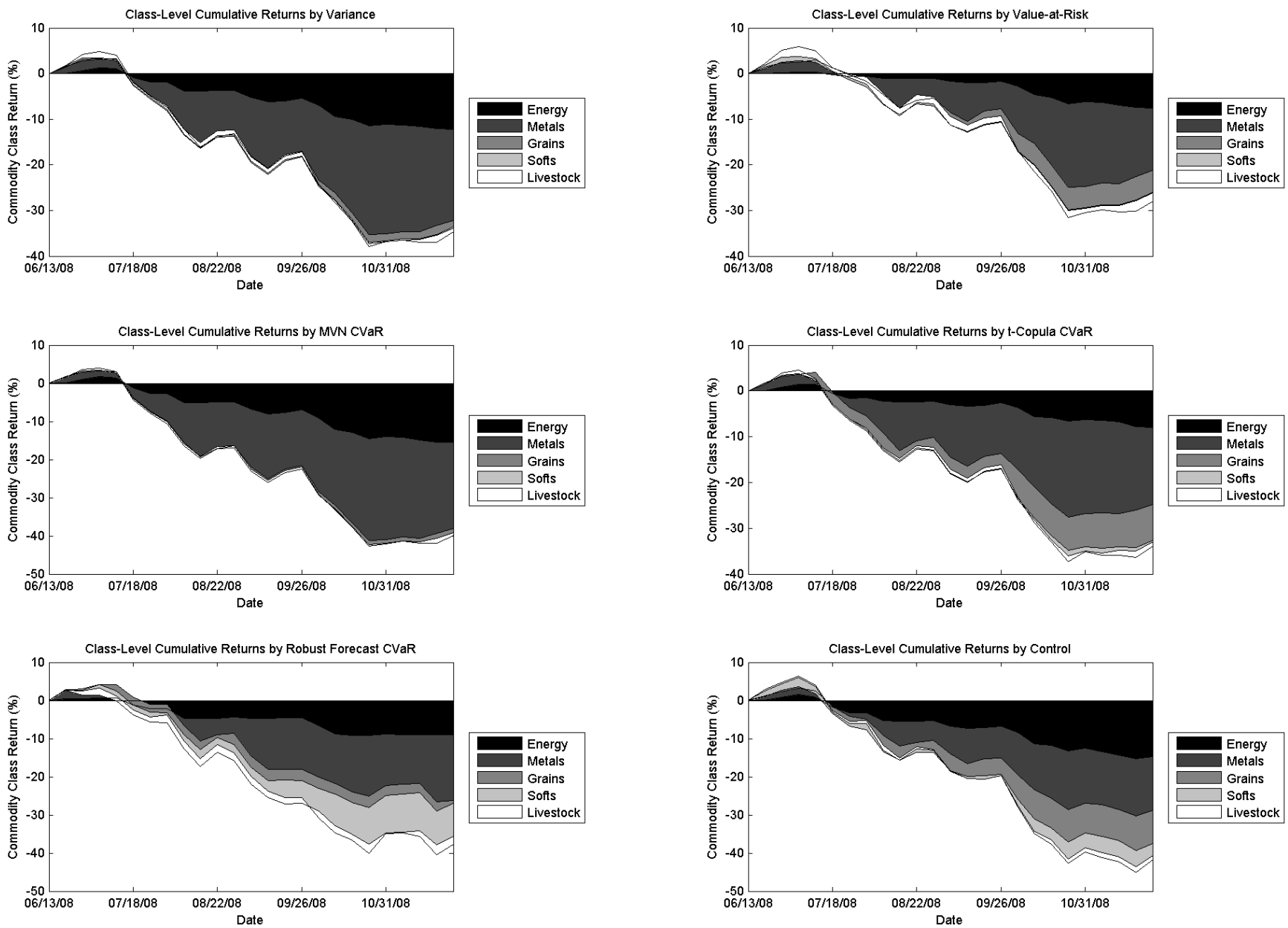

Figure 10: Class-Level Performance by Optimization Method (Strong Bear)

8.4 Weak Bear (May 4, 2001 – Oct. 19, 2001) Tables and Figures

Table 15: Allocation by Optimization Method (Weak Bear)

2-BEAR AVERAGE ASSET ALLOCATION							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	4.08%	4.76%	3.94%	4.63%	4.44%	5.56%
	BRT	6.11%	7.06%	6.83%	3.06%	5.47%	5.56%
	NG	3.93%	6.25%	4.59%	3.70%	0.88%	5.56%
	H.O.	2.05%	1.53%	2.12%	3.86%	4.09%	5.56%
Metals	GLD	0.19%	0.00%	0.00%	2.92%	0.34%	5.56%
	SLV	2.37%	0.02%	1.56%	1.40%	21.61%	5.56%
	PLT	0.85%	0.18%	0.25%	1.86%	11.05%	5.56%
	PAL	32.68%	49.19%	38.44%	30.51%	5.66%	5.56%
	COP	0.00%	0.00%	0.00%	0.73%	2.92%	5.56%
Agr.	CRN	0.17%	0.00%	0.17%	1.84%	6.28%	5.56%
	WHT	7.38%	6.22%	7.45%	4.54%	1.28%	5.56%
	SOY	5.33%	2.42%	3.52%	4.53%	9.27%	5.56%
Softs	COT	0.00%	0.00%	0.00%	0.33%	2.23%	5.56%
	COF	5.24%	2.07%	4.09%	8.63%	5.64%	5.56%
	CCO	0.13%	0.00%	0.02%	1.00%	2.93%	5.56%
	SUG	0.27%	0.11%	0.09%	0.67%	2.29%	5.56%
Live	CAT	28.24%	19.85%	26.30%	23.84%	8.19%	5.56%
	HOG	0.97%	0.34%	0.63%	1.94%	5.42%	5.56%

2-BEAR AVERAGE CLASS ALLOCATION (ABS. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	16.16%	19.60%	17.48%	15.25%	14.89%	22.22%
Metals	36.09%	49.39%	40.25%	37.42%	41.58%	27.78%
Agr.	12.89%	8.65%	11.14%	10.91%	16.83%	16.67%
Softs	5.65%	2.17%	4.20%	10.64%	13.09%	22.22%
Live	29.21%	20.19%	26.93%	25.78%	13.61%	11.11%

2-BEAR AVERAGE CLASS ALLOCATION (CONTROL REL. %)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(6.06%)	(2.62%)	(4.74%)	(6.97%)	(7.33%)	0.00%
Metals	8.31%	21.61%	12.47%	9.65%	13.80%	0.00%
Agr.	(3.78%)	(8.02%)	(5.52%)	(5.76%)	0.17%	0.00%
Softs	(16.57%)	(20.05%)	(18.02%)	(11.58%)	(9.13%)	0.00%
Live	18.10%	9.08%	15.82%	14.67%	2.50%	0.00%

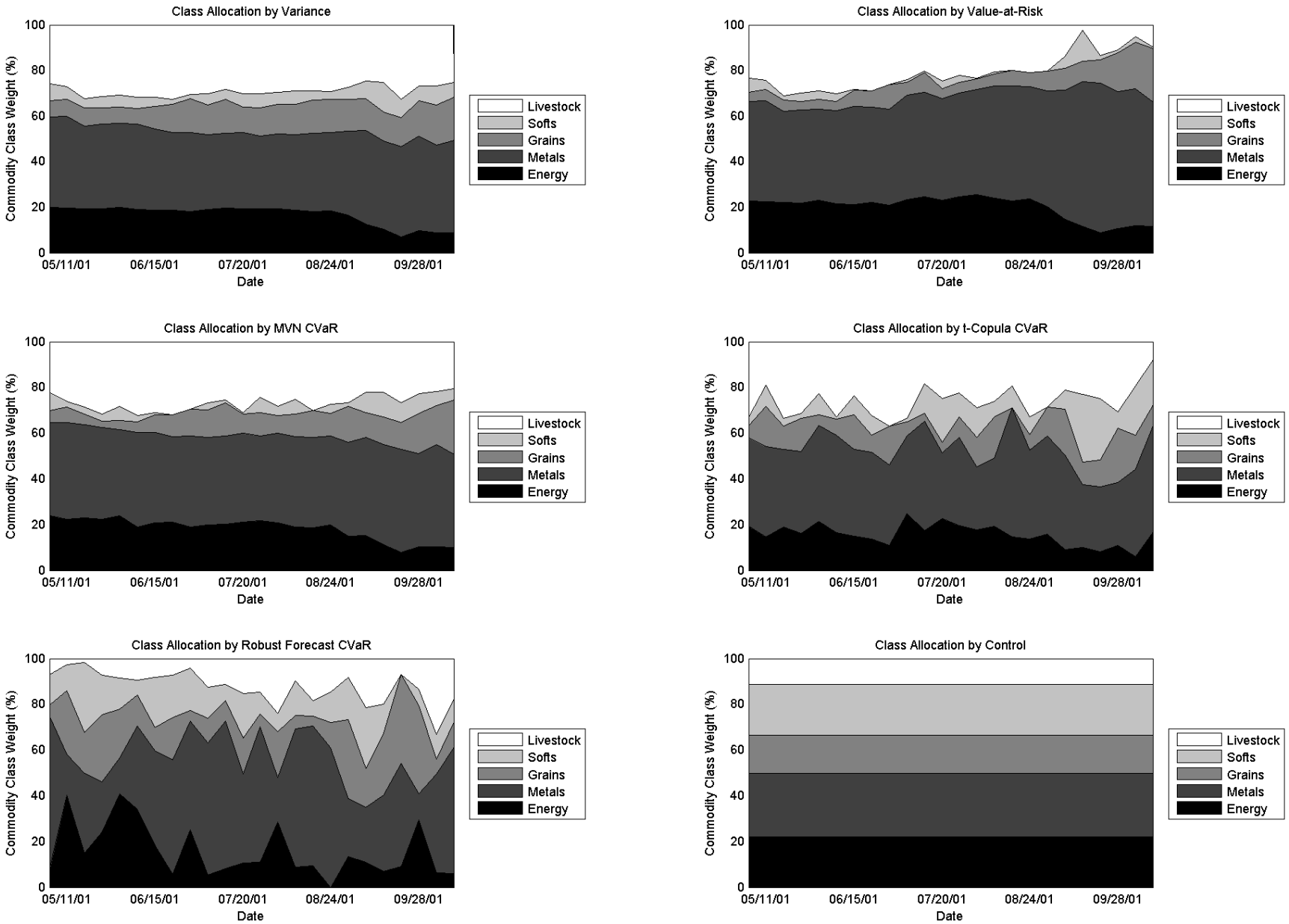

Figure 11: Class-Level Allocation by Optimization Method (Weak Bear)

Table 16: Performance by Optimization Method (Weak Bear)

2-BEAR ASSET PERFORMANCE							
		Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	WTI	(0.08%)	0.16%	(0.16%)	0.23%	(0.78%)	(1.31%)
	BRT	(0.86%)	(0.50%)	(0.99%)	(0.64%)	1.57%	(1.40%)
	NG	(1.57%)	(2.12%)	(1.91%)	(1.34%)	0.70%	(2.80%)
	H.O.	0.20%	0.37%	0.39%	(1.19%)	(0.58%)	(1.02%)
Metals	GLD	(0.01%)	0.00%	0.00%	(0.44%)	0.31%	0.26%
	SLV	(0.00%)	0.00%	(0.09%)	(0.06%)	(4.22%)	(0.04%)
	PLT	(0.38%)	(0.14%)	(0.16%)	(0.78%)	(5.49%)	(1.79%)
	PAL	(19.90%)	(29.31%)	(22.82%)	(17.59%)	(8.48%)	(3.73%)
	COP	0.00%	0.00%	0.00%	(0.14%)	0.10%	(1.11%)
Agr.	CRN	0.01%	0.00%	(0.04%)	(0.21%)	1.38%	0.07%
	WHT	1.35%	1.26%	1.27%	0.76%	(0.21%)	0.39%
	SOY	0.48%	0.43%	(0.04%)	0.69%	0.82%	(0.17%)
Softs	COT	0.00%	0.00%	0.00%	(0.02%)	(1.71%)	(1.92%)
	COF	1.01%	1.07%	1.06%	4.72%	(1.34%)	0.02%
	CCO	0.02%	0.00%	0.01%	(0.03%)	0.23%	(1.45%)
	SUG	(0.06%)	0.01%	0.02%	(0.40%)	(1.12%)	(1.90%)
Live	CAT	(1.34%)	0.27%	(1.13%)	0.05%	(1.79%)	(0.29%)
	HOG	0.13%	0.08%	0.23%	0.16%	(2.85%)	(1.42%)

2-BEAR ASSET CLASS PERFORMANCE (ABS. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	(2.31%)	(2.09%)	(2.66%)	(2.91%)	0.89%	(6.38%)
Metals	(20.21%)	(29.41%)	(23.02%)	(18.75%)	(16.81%)	(6.30%)
Agr.	1.85%	1.69%	1.19%	1.24%	2.00%	0.29%
Softs	0.96%	1.09%	1.10%	4.25%	(3.89%)	(5.16%)
Live	(1.22%)	0.35%	(0.90%)	0.21%	(4.58%)	(1.70%)

2-BEAR ASSET CLASS PERFORMANCE (CONTROL REL. % RTN)						
	Variance	5%-VaR	MVN CVaR	t-copula CVaR	Forecast CVaR	Control
Energy	4.07%	4.29%	3.72%	3.47%	7.27%	0.00%
Metals	(13.91%)	(23.11%)	(16.72%)	(12.45%)	(10.51%)	0.00%
Agr.	1.55%	1.40%	0.90%	0.95%	1.71%	0.00%
Softs	6.12%	6.25%	6.26%	9.41%	1.27%	0.00%
Live	0.48%	2.05%	0.80%	1.91%	(2.88%)	0.00%

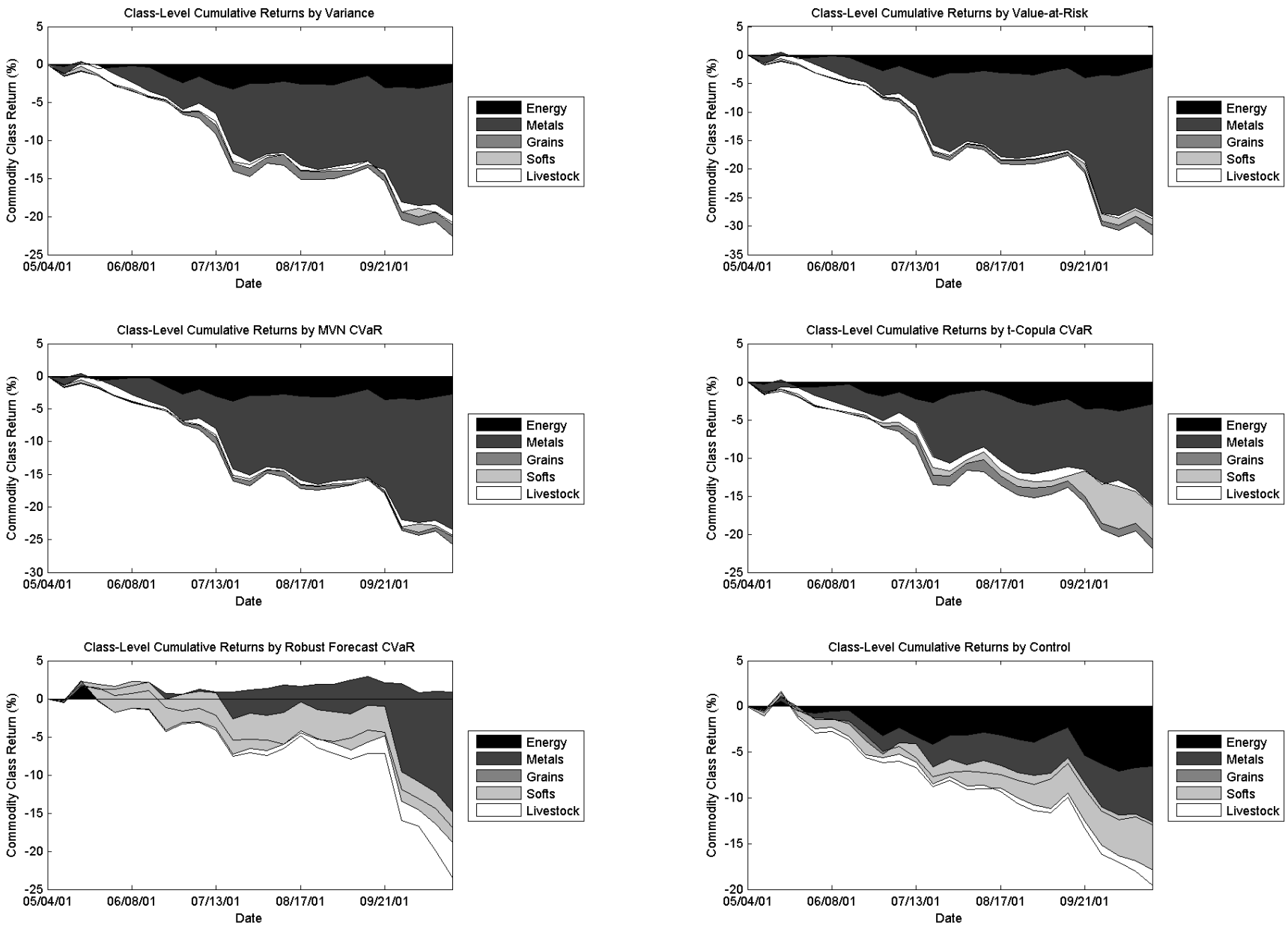

Figure 12: Class-Level Performance by Optimization Method (Weak Bear)

8.5 Performance Summary

Table 17: Absolute and Risk-Adjusted Portfolio Performance Summary

Final Portfolio Value	1-bull	2-bull	1-bear	2-bear	Bull Avg	Bear Avg	All Avg
Variance	139.30%	122.15%	69.06%	79.20%	130.72%	74.13%	102.43%
5%-VaR	129.58%	118.84%	74.60%	71.33%	124.21%	72.96%	98.59%
MVN 5%-CVaR	146.76%	123.35%	65.25%	75.98%	135.05%	70.62%	102.83%
t-copula 5%-CVaR	145.90%	118.28%	69.94%	83.58%	132.09%	76.76%	104.42%
Forecast 5%-CVaR	127.57%	140.11%	67.65%	78.24%	133.84%	72.95%	103.39%
Control	140.42%	139.01%	64.31%	81.88%	139.71%	73.09%	106.40%

Ranked Final Portfolio Value	1-bull	2-bull	1-bear	2-bear	Bull Avg	Bear Avg	All Avg
Variance	4	4	3	3	5	2	5
5%-VaR	5	5	1	6	6	4	6
MVN 5%-CVaR	1	3	5	5	2	6	4
t-copula 5%-CVaR	2	6	2	1	4	1	2
Forecast 5%-CVaR	6	1	4	4	3	5	3
Control	3	2	6	2	1	3	1

Weekly Sharpe Ratio	1-bull	2-bull	1-bear	2-bear	Bull Avg	Bear Avg	All Avg
Variance	0.70	0.41	(0.55)	(0.47)	0.56	(0.49)	0.04
5%-VaR	0.67	0.45	(0.50)	(0.40)	0.57	(0.45)	(0.02)
MVN 5%-CVaR	0.70	0.38	(0.58)	(0.52)	0.55	(0.53)	0.04
t-copula 5%-CVaR	0.72	0.31	(0.53)	(0.48)	0.51	(0.47)	0.07
Forecast 5%-CVaR	0.40	0.46	(0.42)	(0.51)	0.43	(0.47)	0.05
Control	0.73	0.53	(0.61)	(0.49)	0.62	(0.49)	0.09

Ranked Weekly Sharpe Ratio	1-bull	2-bull	1-bear	2-bear	Bull Avg	Bear Avg	All Avg
Variance	4	4	4	2	3	4	5
5%-VaR	5	3	2	1	2	1	6
MVN 5%-CVaR	3	5	5	6	4	6	4
t-copula 5%-CVaR	2	6	3	3	5	3	2
Forecast 5%-CVaR	6	2	1	5	6	2	3
Control	1	1	6	4	1	5	1