Essays in Macroeconomics and Information Theory

by

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Defense Date: March 29, 2024

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Economics in the Graduate School of Duke University
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ABSTRACT

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Abstract

This dissertation studies macroeconomics and information theory. Each essay investigates the effects of imperfect information processing on macroeconomics, international finance, and social welfare, even when the information is transmitted accurately but not processed perfectly by the recipients.

In Chapter 2, I explore the underlying reasons for the rapid spread of financial contagion across markets with weak correlations using a dynamic model of rational inattention. In my model, agents invest in two assets, whose returns are uncorrelated. Monitoring the returns on these assets entails costly information acquisition. Unlike in past work, I allow for the total amount of information that the investor acquires to endogenously respond to the change in volatility of assets' returns. By emphasizing the decomposition of attention rather than the relative signal weights of the two assets in the optimal signal, I find that increased volatility of one asset's returns heightens not only investor attention to that asset (as past research has found), but also to attention to other assets as well (counter to the conclusions of existing research), as a result of the increased subjective correlation. I validate these predictions using the text from the Financial Times during the 1997 Asian Financial Crisis. Furthermore, the theoretical results provide general mechanisms of volatility spillover in the canonical rational inattention model and the pivotal role of information structure choice in these processes.

In Chapter 3, I examine the relationship between news media credibility, information friction, and strategic complementarity, especially during uncertain times like the COVID-19 pandemic. It introduces a news media verification mechanism to the Beauty Contest model and discusses the effects of an endogenous information cost model on the reliability perception of public information and social welfare. The study highlights how increased information costs and high strategic complementarity might reduce efforts to verify public information, potentially leading to undervalued news media credibility and increased belief polarization. It suggests that, contradicting the insights of Morris and Shin (2002), improving news media trustworthiness or accuracy of public information could almost always enhance social welfare, urging the need for empirical research to further understand the news media's impact on economic outcomes and policy-making.

Contents

List of Tables

List of Figures

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1 Introduction

This dissertation contains two essays related to macroeconomics and information theory. These essays propose the possibility that, even if information providers deliver accurate information, recipients can distort it, leading to potential financial contagion and a decline in news media trust.

In Chapter 2, I primarily explore the underlying causes of financial contagion, focusing particularly on how international investors process information. Unlike many studies that have explained financial contagion through the Global Coordination game, this paper diverges by analyzing the dynamics of investors' attention reallocation in response to changing information flows, moving away from the assumption that international investors operate with a fixed information capacity. Specifically, it examines how increases in asset volatility affect international investors' focus and information processing based on the theory of rational inattention.

Specifically, the paper discusses the interaction between increases in volatility and the allocation of attention, suggesting that, when the volatility of one asset increases, it not only heightens attention towards that asset but also leads to increased information processing for other non-correlated assets. This provides new insights into how international investors allocate information under a fixed information processing cost and verifies theoretical and numerical predictions through empirical analysis using data from the 1997 Asian Financial Crisis. The study aims to deepen our understanding of the mechanisms of information processing and attention reallocation in financial markets, especially in the context of anticipated versus unanticipated financial crises.

In Chapter 3, I examine the relationship between the credibility of news media providing public information, information friction, and economic decision-making in situations with strategic complementarity, challenging traditional assumptions. By introducing a model in which agents receive different signals about the credibility of news media and can affect the accuracy of these signals by incurring endogenous information costs, the study seeks to better understand how differing beliefs about the reliability of public information impact economic welfare. I employ a modified Beauty Contest model to examine the effects of the interaction between information verification and strategic complementarity on societal outcomes.

The research investigates how the trustworthiness of news media influences economic agents' decisions, highlighting the potential negative consequences of misinformation. By integrating concepts from the Global Coordination game and rational inattention, the paper discusses the varied decision-making arising from different posterior beliefs about news credibility, ultimately suggesting that enhancing the accuracy of public information can be almost always beneficial for social welfare, particularly in contexts where news media is not fully trusted.

2 Impact of Volatility Shocks on Attention Allocation Across Uncorrelated Assets

2.1 Introduction

Understanding the fundamental causes of financial contagion is one of the most imperative topics in macroeconomics and international finance. Typically, such contagions are more globally pervasive when unexpected shocks occur (Kaminsky et al., 2003; Rigobon and Wei, 2009), and they tend to spread rapidly even among countries considered to have weak financial market linkages. For instance, the 1997 Asian Financial Crisis, which began with the devaluation of the Thai Baht on July 2, 1997, quickly spread beyond Asia to Europe, North America, and South America. Notably, Asian and Latin American markets, which usually move independently in terms of stock returns, started to show similar movements of stock returns in a negative direction with higher volatility once the contagion began.

To explain how financial contagion can spread so rapidly to markets considered less correlated in normal times, many studies have naturally begun to focus on how international investors process information. Some papers apply the Global Coordination game to explain financial contagion through the interactions of international investors (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Vives, 2005; and Ahnert and Bertsch, 2022). In contrast, Mondria (2010) and Mondria and Quintana-Domeque (2013) explain how contagion could rapidly occur without the Global Coordination game. This is through a representative rational inattentive international investor with a fixed information capacity. They argue that, when return volatility unexpectedly increases in one market (e.g., the Asian Market), it leads to increased attention to the more uncertain market and, simultaneously, less attention to an uncorrelated market (e.g., the Latin American Market). These studies use static models to explain time series data implications, with the critical assumption that, when an unexpected volatility shock occurs, international investors process only a fixed total amount of information over a given period of time and are unable to increase the total information flow.

This paper relaxes that assumption that the amount of information processed is fixed. The Financial Times' daily news data, which is a measure of public information, indicates that the total information flow fluctuates dramatically on a daily basis. Consequently, when agents can directly select total information flow, in the context of a dynamic model, I aim to investigate theoretically and empirically how the attention reallocation dynamics of international investors changes. For this research, I utilize the canonical rational inattention model proposed by Sims (2003) and Sims (2011), where an international investor seeks to achieve a pre-specified total portfolio return, which is a one-dimensional linear combination of two uncorrelated assets. I then analyze how the investor's information choices change when one asset faces a volatility shock.

More generally, this paper analyzes the interaction between volatility changes and endogenous attention allocation. While research on volatility changes and uncertainty shocks is separately well established (Bloom, 2009), and literature on endogenous attention allocation within rational inattention theory is well-known (Sims, 2011), the interaction between these two areas remains scarcely explored (see Chiang (2022) for an exception). An interesting consequence of analyzing these two mechanisms jointly is that it can generate *volatility spillover effects*. This refers to a situation in which an increase in the volatility of one exogenous process affects the amount of information processed about another exogenous process, independent of the first. This paper studies the volatility spillover effect both theoretically and empirically within the canonical rational inattention model. It also provides analytical results for multivariate dynamic rational inattention problems that are rarely available even in the steady-state.[1]

Regarding volatility spillovers, the key theoretical implications are that, when the volatility of one asset increases, (i) attention paid to that asset increases, even in a dynamic setting, (ii) agents' *subjective* posterior correlation across assets increases, and (iii) attention paid to other uncorrelated assets also increases, as a result of the increased posterior correlation. These implications extend beyond asset price applications and can be generally applied in cases involving a one-dimensional target as a linear combination of two uncorrelated fundamentals (e.g., firm price setting, investment, or consumption/saving decisions).

Especially among the general theoretical implications, understanding how posterior correlation interacts with fixed marginal information cost is essential for understanding these results. Sims (2011) notes that rational inattention naturally generates subjective correlation among initially independent sources of uncertainties, which may suggest that an increase in information processing costs should increase this subjective correlation. However, this paper clarifies this is not the case. Counterintuitively, as the fixed marginal information cost decreases towards zero, the posterior correlation does not diminish; it actually increases. This implies that volatility spillover effects arise mainly from agents' optimal choice of target variable, not from the presence of information constraints per se.

This paper also shows that, using Shannon mutual information as a measure of information, the introduced attention decomposition allows for the calculation

[1] Papers that have provided general analytical results in the canonical rational inattention model include Sims (2003), Sims (2011), Maćkowiak, Matějka, and Wiederholt (2018), Afrouzi and Yang (2021), Miao, Wu, and Young (2022), and Jurado (2023).

of individual attention for each asset and separates the part of individual attention that shares posterior correlation, providing clear insights into how agents allocate attention in multivariate rational inattention problems. Many previous studies have used the relative signal weights on the two assets in the optimal signal as an indicator of relative attention. This paper illustrates the problems with this indicator and emphasizes why the model-implied attention decomposition is more appropriate. Using this attention decomposition, this paper shows that, in the case of fixed marginal information cost, when the volatility of one asset unexpectedly increases, more attention can be allocated both to this asset and to another uncorrelated asset. This is unlike the zero-sum-game-like behavior under fixed capacity, where an increase in attention on one asset requires a reduction in attention on another.

All the analytical results discussed so far are obtained by analyzing only the steady-state of the dynamic model. To go beyond this, the paper also analyzes the transition path to the steady-state, to understand whether these steady-state results also apply in the short run. I find that the steady-state results remain qualitatively the same even in the short run. Additionally, transitional dynamics can be used to explain the dynamics of individual attention when a crisis is anticipated versus unanticipated. A decline in asset prices due to an uncertainty shock is a common outcome (Bianchi, Ilut, and Schneider, 2018). Especially, stock prices fall less when an uncertainty shock is anticipated (Mondria and Quintana-Domeque, 2013). This paper examines the difference between anticipated and unanticipated events through transitional dynamics based on the decomposition of attention. Numerical simulations indicate that in response to an anticipated change in volatility (i.e. increase in uncertainty), agents save attention by processing less information on assets before the shock period, and respond by increasing attention closer to the steady-state once the shock is realized.

Lastly, as an empirical application, I perform an empirical analysis using data on the 1997 Asian Financial Crisis to verify the analytical and numerical predictions. Analyzing the number of the Financial Times news articles, I assess the attention of international investors towards both Asian and Latin American markets. The findings confirm that, when the Asian market faced an unanticipated volatility shock, it not only drew more attention to the Asian market, but also increased attention towards the Latin American market as well, consistent with the theoretical predictions under a fixed marginal cost of information.

The structure of the paper is as follows: Section 2 deals with the information cost structure and outlines the model. Section 3 shows analytical and numerical results on the transmission of volatility shocks. Section 4 shows empirical applications, and Section 5 concludes.

2.2 Model

This section describes the canonical rational inattention tracking problem used throughout the paper and interprets it in terms of international investors seeking to achieve a pre-specified target portfolio return. There are two uncorrelated risky assets and a continuum of infinitely-lived rational inattentive agents of measure one, each with a portfolio comprising these uncorrelated assets. Given arbitrarily assigned weights to the portfolios, they select private signals regarding the stock returns of two assets at each time t.[2] Upon receiving these signals, they aim to optimally allocate their attention to each asset. The optimal attention then informs their actions, which reflect their knowledge of the exogenous target portfolio return—a linear combination of the two risky assets. The more accurately they perceive the exogenous portfolio return, the higher the benefit they receive. However, due to the presence of information costs, they are unable to perfectly achieve the optimal target return.

2.2.1 Information cost

The agents gain benefit when their actions more closely align with their target. However, striving to reach this target entails an increase in the cost of processing information that is helpful in reaching the target. In this framework, while there is no direct monetary cost for using information, it is assumed that there is an implicit cost related to subjective factors like mental effort or time spent processing additional information. In rational inattention theory, a willingness to incur higher information costs implies a desire to pay more attention or to diminish uncertainty regarding underlying fundamentals. For the purposes of this paper, this 'reduc-

[2] This process represents the initial step in deciding on the asset portfolio. While the dynamics of this attention problem can be extended to asset holding decisions, this paper is confined to the allocation of attention prior to such decisions.

tion in uncertainty' is synonymously and specifically referred to as a measure of 'attention' or 'the amount of information' acquired.

The concept of information used in this paper originates from information theory, first established by Shannon (1948) and subsequently integrated into economic models by Sims (2003). Information theory quantifies information using a fundamental metric known as entropy. This entropy measures the level of uncertainty in information and is defined as follows:

$$H(X) = -\mathbb{E} \ln p(X) = -\int p(x) \ln p(x)\, dx$$

where X is a random variable and $p(x)$ is a continuous probability density function. If $X \sim N(\bar{X}, \Sigma)$ which is a n-dimensional multivariate normal case:

$$H(X) = \frac{1}{2} \ln\left((2\pi e)^n |\Sigma|\right)$$

where $|\Sigma|$ is the determinant of the n-dimensional covariance matrix Σ. This entropy decreases when observing the realization of another random variable Y, providing a measure of uncertainty reduction. This reduction in uncertainty is called mutual information and is defined as:

$$I(X;Y) = \int\int p(x,y) \ln \frac{p(x,y)}{p(x)p(y)}\, dx\, dy = H(X) - H(X|Y).$$

Intuitively, if Y conveys information about X, the uncertainty surrounding X diminishes upon observing Y, reducing its entropy to $H(X|Y)$. Consequently, the mutual information $I(X;Y)$ quantifies this reduction in uncertainty.

Moreover, for two Gaussian random variables with a linear correlation, the mutual information can be computed as:[3]

$$I(X;Y) = H(X) + H(Y) - H(X,Y) = -\frac{1}{2} \log\left(1 - \rho^2\right).$$

[3] See Gel'fand and Yaglom (1959) and Example 9.3 in Cover and Thomas (2006)

Here, ρ represents the correlation coefficient. With this formula, I can break down the individual attention allocated to each Gaussian random variable, particularly in cases where they have a posterior correlation. I will detail this method in Section 2.3.2 *Attention decomposition*.

In the work of Sims (2003), mutual information is constrained by a capacity limit, which essentially caps the amount of information that can be processed, reflecting the human limitations in managing information. Defining the movement of this limit, however, posed a challenge. In this paper, following Sims (2011), the mutual information is included directly in the objective function as a fixed marginal information cost. It is crucial to recognize that, although agents have a finite capacity for information processing, this second approach proposes that such capacity is distributed across various domains, including economic decisions, entertainment, social interactions, etc. Consequently, by reallocating attention from these diverse and unmodeled activities, the total amount of attention on one specific economic decision, such as the predetermined target portfolio return, can be increased.

Reflecting these aspects, in this model, agents are able to decide the amount of information they want to process. If agents were dealing with multiple targets, a fixed information capacity model might be appropriate. However, given our concentration on a one-dimensional target, it seems reasonable to enable agents to instantly choose their total information flow. Notably, the influence of a fixed information capacity on volatility spillover is quite different from the effects of a fixed marginal information cost. This distinction plays a central role in the following analysis.

2.2.2 Investors' objective function

The benefits for a representative rational inattentive agent are represented by:

$$-\left(x_t - a_t|s^t\right)^2.$$

Here, x_t is the exogenous target variable which is a total portfolio return, and a_t is the agent's action based on the history of signals observed up to time t, denoted by s^t. The benefit arises as the agent's actions more closely approximate the target. The target x_t is a linear process driven by Gaussian shocks, so this quadratic objective implies that the optimal action is equal to the conditional expectation as follows:

$$a_t = \mathbb{E}(x_t|s^t).$$

The target variable x_t is constructed as a linear combination of the exogenous returns from two risky assets, θ_t and z_t, described by:

$$\theta_t = \rho_\theta \theta_{t-1} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma_\theta^2), \quad 0 \leqslant \rho_\theta < 1,$$

$$z_t = \rho_z z_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_z^2), \quad 0 \leqslant \rho_z < 1,$$

where u_t and ϵ_t represent independent noise processes with normal distributions, which remain independent over time. It is also known to the agents that u_t and ϵ_t are independent, and θ_t and z_t are not correlated at any point. Although σ_θ^2 and σ_z^2 do not have a time subscript, I later introduce an unexpected one-time shift in volatility. This allows us to explore how such a shift endogenously influences the choice of attention. The returns follow either AR(0) or AR(1) patterns. When both variables are AR(0), the dynamics of the problem simplifies to the i.i.d. case.

The signal related to x_t is denoted by s_t, with the initial signal, s^{-1}:

$$s_t = \underbrace{\begin{bmatrix} C_{11,t} \\ C_{21,t} \end{bmatrix}}_{C_{1,t}} \theta_t + \underbrace{\begin{bmatrix} C_{12,t} \\ C_{22,t} \end{bmatrix}}_{C_{2,t}} z_t + e_t, \quad e_t \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} v_{11,t}, v_{12,t} \\ v_{21,t}, v_{22,t} \end{bmatrix}}_{V_t} \right).$$

In this equation, $C_{1,t}$ and $C_{2,t}$ are the endogenous tracking coefficients for the fundamentals, θ_t and z_t, respectively, and e_t symbolizes the noise due to rational inattention. Importantly, the dimension of this signal is itself endogenous. This allows agents to receive independent information about each asset return separately or information about arbitrary linear combinations of the returns. The decision variables for the agent include the matrix $C_t = [C_{1,t}, C_{2,t}]$ and the variances of the noise, represented by V_t. The matrix C_t is unconstrained, suggesting that agents have the flexibility to identify each fundamental separately by opting for a diagonal matrix. As the signal's precision increases, so does the information cost. For instance, if the agent receives the signal $a_t = s_t = x_t = \theta_t + z_t$, then the cost of information becomes infinite. As a result, when the marginal cost of information is positive, the agent cannot select a perfect signal regarding x_t.

The total cost of the signal s_t is given by:

$$\omega \mathbb{I}((\theta^t, z^t); s^t | s^{t-1}).$$

Here, ω signifies the fixed marginal information cost, and $\mathbb{I}((\theta^t, z^t); s^t | s^{t-1})$ is the mutual information contained in the signal s^t about the history of asset returns (θ^t, z^t), conditional on the history of prior signals s^{t-1}. This allocation of attention then determines the agent's action, following the principle $a_t = \mathbb{E}(x_t | s^t)$ as previously stated.

To summarize, the representative investor faces a dynamic attention problem of the following form.

Problem 2.1. ***Dynamic Attention Problem***

$$\max_{\{C_{1,t},C_{2,t},V_t\}_{t\geqslant 0}} \mathbb{E}_{-1}\left[\sum_{t=0}^{\infty}\beta^t\{\underbrace{-0.5\left(x_t-\mathbb{E}(x_t|s^t)\right)^2}_{Benefit}-\underbrace{\omega\mathbb{I}((\theta^t,z^t);s^t|s^{t-1})}_{Information\ Cost}\}\Big|s^{-1}\right]$$

$$s.t.\quad x_t = b_1\theta_t + b_2 z_t,$$

$$s_t = \underbrace{\begin{bmatrix}C_{11,t}\\C_{21,t}\end{bmatrix}}_{C_{1,t}}\theta_t + \underbrace{\begin{bmatrix}C_{12,t}\\C_{22,t}\end{bmatrix}}_{C_{2,t}}z_t + e_t,\quad e_t\sim\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix},\underbrace{\begin{bmatrix}v_{11,t},v_{12,t}\\v_{21,t},v_{22,t}\end{bmatrix}}_{V_t}\right),$$

$$\theta_t = \rho_\theta\theta_{t-1}+u_t,\quad u_t\sim\mathcal{N}(0,\sigma_\theta^2),\quad 0\leqslant\rho_\theta<1,$$

$$z_t = \rho_z z_{t-1}+\epsilon_t,\quad \epsilon_t\sim\mathcal{N}(0,\sigma_z^2),\quad 0\leqslant\rho_z<1,$$

$$s^{-1}\ given.$$

Here, b_1 and b_2 denote the holding ratios for the two assets, respectively. For simplicity and to gain a clearer understanding of the analytical outcomes, I set both b_1 and b_2 to 1. However, the qualitative results remain consistent with arbitrary b_1 and b_2 if $b_1 \geqslant 1$ and $b_2 \geqslant 1$. It is important to realize that these parameters themselves should be interpreted as endogenous with respect to the investors' optimal asset allocation problem. To focus on the attention allocation, this analysis treats those parameters as exogenously pre-specified. This is done for simplicity but is not innocuous. For example, in response to changes in uncertainty, b_1 and b_2 may in principle respond.

The choice of signal s_t in Problem 2.1 is without loss of generality, due to the linear-quadratic-Gaussian nature of the problem. However, it is not unique without further restrictions. Nevertheless, with a target defined as a linear combination of two exogenous components, there is a distinctly unique Gaussian posterior belief that corresponds precisely to each action. Therefore, it is convenient to re-write Problem 2.1 so that the agent directly chooses a set of posterior beliefs. Given these beliefs, it is always possible to reverse engineer an appropriate signal that generates these beliefs and obtain the corresponding optimal action. Specifically, I first determine the unique posterior beliefs—namely, the posterior covariance matrix for this scenario. With this matrix, I can then trace our steps back to pinpoint the particular signals and optimal actions.[4]

Using well-known results in rational inattention theory, I can rewrite the model, which brings us to Lemma 2.1.[5] A solution to Problem 2.1 can be found by first solving this problem:

Lemma 2.1.

$$\max_{\{\Sigma_{t|t} \geq 0\}_{t \geqslant 0}} \mathbb{E}_0 \frac{1}{2} \left[\sum_{t=0}^{\infty} \beta^t \left\{ -\text{tr}\left(\Sigma_{t|t}\Omega\right) - \omega \log \left(\frac{|\Sigma_{t|t-1}|}{|\Sigma_{t|t}|} \right) \right\} \Big| \Sigma_{0|-1} \right], \tag{2.1}$$

$$s.t. \quad \Sigma_{t|t} = \begin{bmatrix} \hat{\sigma}^2_{\theta_t} & \hat{\sigma}_{\theta_t, z_t} \\ \hat{\sigma}_{\theta_t, z_t} & \hat{\sigma}^2_{z_t} \end{bmatrix}, \quad \Sigma_{t+1|t} = A\Sigma_{t|t}A' + W, \tag{2.2}$$

$$\Sigma_{t|t-1} - \Sigma_{t|t} \geq 0 \quad \textit{No-forgetting Conditions}, \tag{2.3}$$

$$0 < \Sigma_{0|-1} < \infty \textit{ given.} \tag{2.4}$$

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[4] Refer to Afrouzi and Yang (2021), Miao, Wu, and Young (2022), and Jurado (2023) for detailed discussions

[5] To gain a detailed understanding of how Problem 2.1 changes to Lemma 2.1, refer to the Afrouzi and Yang (2021).

This problem becomes equivalent to the original model when I write:

$$\Omega = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, A = \begin{bmatrix} \rho_\theta & 0 \\ 0 & \rho_z \end{bmatrix}, W = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}.$$

In the model, the matrix Ω signifies the marginal benefit derived from each element of the posterior covariance matrix, $\Sigma_{t|t}$.

The second term separately represents the information cost. The condition shown in Equation (2.3) is often called the 'no-forgetting' condition, a concept that originates from the updating rules in the Kalman Filter.[6] The assumption here is that uncertainty should consistently diminish with the acquisition of new information. Though this notion is logical, it complicates the pursuit of a straightforward solution, particularly when ensuring the matrix's positive semidefiniteness. In this study, I present an exact solution for the scenario where $\rho_\theta = \rho_z$. When $\rho_\theta \neq \rho_z$, I resort to using the implicit function theorem based on steady-state optimal conditions or numerical techniques.

Lemma 2.1 allows us to deduce the optimal posterior covariance matrix, $\Sigma_{t|t}$, thus recovering the signal's nature and the consequent action. With a given Ω, A, and W, the agent selects an optimal $\Sigma_{t|t}$, which guarantees a unique action, irrespective of multiple potential signal structures. Once an optimal sequence $\{\Sigma_{t|t}\}_{t=0}^{T}$ is identified, we can map the optimal Gaussian posterior into the signal-to-noise ratio (SNR)[7], symbolized by $\Phi_t = C_t' V_t^{-1} C_t$ to recover the optimal signals.[8]

[6] See Chapter 4 of Hansen and Sargent (2008) for further reading. The linear-quadratic-Gaussian Tracking Problem has a direct connection to the Kalman Filtering framework through duality.

[7] The signal-to-noise ratio (SNR) is a commonly used metric in the fields of information and communication theory. It is used to assess the power of a signal (representing the quantity of information) against the level of noise (which represents irrelevant or extraneous data). When the SNR is high, it signifies that the signal is more pronounced relative to the noise, implying a superior quality of information transfer or processing.

[8]

$$\begin{aligned} \Phi_0 &= \Sigma_0^{-1} - \Sigma_{-1}^{-1}, \\ \Phi_t &= \Sigma_t^{-1} - \left(A_{t-1}\Sigma_{t-1}A_{t-1}' + W_{t-1}\right)^{-1} \quad \text{for } t \geqslant 1. \end{aligned} \tag{2.5}$$

2.3 Volatility spillover effect: Theory

This section presents theoretical results regarding the volatility spillover effects. These results explain the mechanisms by which volatility shocks to one asset influence the reallocation of attention across two uncorrelated assets. Intuitively, a volatility shock to one asset may result in increased attention to both: (i) the asset experiencing heightened uncertainty, and (ii) the uncorrelated asset, as the agent feels compelled to amplify the total attention due to a perceived increase in uncertainty of a linear combination target. This study aims to explore the impact of changes in the volatility of z_t on the attention allocated to θ_t. Both θ_t and z_t can follow an AR(0) or AR(1) process, with their structures being perfectly symmetric, suggesting that insights derived from one are applicable to the other. In this section, the analysis is conducted at the steady-state.

At a general level, the results presented in this section can be applied to any scenarios where there is a one-dimensional target formed by a linear combination of two-dimensional states within the canonical model, such as in firm price setting, investment, or consumption/saving decisions. Even with a one-dimensional target formed from two-dimensional states, analytical insights in a steady-state remain largely unexplored.[9] The principal challenge lies in the semidefinite constraint, which impedes the derivation of a closed-form solution. My methodology for examining the volatility spillover effects begins with analytical findings in the static case—where the persistences between the two exogenous shocks are zero—to acquire a more transparent comprehension of the model's mechanism. This will

The dimension of Φ_t determines the dimension of the signal. For details, refer to Proposition 1 in Miao, Wu, and Young (2022)

[9] Significantly, Maćkowiak, Matějka, and Wiederholt (2018) examined the univariate scenario—one-dimensional target with one-dimensional states in a dynamic setting—while Miao, Wu, and Young (2022) illuminated some aspects of the multivariate case. Jurado (2023) provided some analytical results in the frequency domain.

serve as a foundation for intuitively understanding the origins and implications of volatility spillover. Subsequently, in Section 2.3.5, *Adding persistence*, I will demonstrate that the results hold consistently through further analytical and numerical analysis.

2.3.1 Interaction between posterior correlation and fixed marginal information cost

This subsection focuses on the relationship between posterior correlation and fixed marginal information cost. As Sims (2011) indicated, canonical rational inattention theory is known for generating posterior correlation among initially independent sources of uncertainties. When the posterior correlation is zero, the two uncorrelated assets remain independent after information processing, and no volatility spillover occurs. Therefore, it is imperative to first examine why posterior correlation arises and how it responds to changes in the fixed marginal information cost, the key driver in rational inattention theory.

The following well-known result provides the foundational underpinning for volatility spillover.[10]

Proposition 2.1. *Let $\rho_\theta = \rho_z = 0$. For any non-zero value of ω, the optimal signal is one-dimensional and can be represented as:*

$$s_t = C_1\theta_t + C_2 z_t + e_t, \tag{2.6}$$

[10] Proposition 2.1 is a well-known result. Maćkowiak, Matějka, and Wiederholt (2018) proved that the agent optimally chooses a one-dimensional signal when persistences are the same each other with fixed capacity, and Miao, Wu, and Young (2022) proved that the agent optimally chooses a one-dimensional signal when persistences are the same each other with fixed marginal cost as $\beta \to 1$. Jurado (2023) generally showed when the target dimension is one with fixed capacity, agents will receive one signal no matter what dimension of state variables. Here, I showed that the agent optimally chooses a one-dimensional signal, in the case of different persistences, with fixed marginal costs by different way. See Appendix.

where $e_t \sim \mathcal{N}(0, v)$.

Proposition 2.1 says that agents, when given the option to receive two distinct signals for the stock returns of two assets, θ_t and z_t, will optimally choose a one-dimensional signal. This preference is typical in rational inattention scenarios where the target is a one-dimensional linear combination of two exogenous shocks. This means that a one-dimensional signal that is a linear combination of the stock returns from two assets is generally more beneficial for enhancing net benefit than two separate signals. From an intuitive standpoint, the optimal forecast is one-dimensional because it is always cost-effective just to buy a one-dimensional private signal which is the optimal forecast itself than to buy two signals.[11]

Receiving a one-dimensional private signal that encapsulates both shocks induces a *subjective* posterior correlation, even though it is known that the two exogenous shocks are uncorrelated. This means that the agent optimally chooses not to fully distinguish between the sources of uncertainty. If the agent had the means to deconstruct the composition of the one-dimensional signal, it would be tantamount to receiving two distinct signals, thereby allowing for more precise predictions concerning the magnitude of shocks in both assets. Hence, the presence of a non-zero posterior correlation indicates the agent's inability to know the signal's composite nature perfectly, forcing an estimation of one shock's fluctuations as in-

[11] Even when agents have the option to select two independent signals, they tend to favor a one-dimensional correlated signal with posterior correlation, as it usually enhances net benefit. In essence, receiving two independent signals could potentially increase the agent's mental cost. This phenomenon is consistent only when the targeted fundamental variable is a linear combination of two underlying fundamentals. Should the target be defined separately with benefits represented by $-(a_{1,t} - \theta_t)^2 - (a_{2,t} - z_t)^2$, agents would favor two distinct signals. However, when benefits are defined as $-(a_t - (\theta_t + z_t))^2$ and despite knowledge of the independence between θ_t and z_t, agents would opt for a one-dimensional signal encompassing a non-zero posterior correlation between the two unseen exogenous shocks. Please see more discussion in Section 2.3.7 *Interaction between target dimension and information cost* in this paper.

dicative of the other's through the one-dimensional signal. The one-dimensional nature of the optimal signal leads to the volatility spillover effect, as documented in prior research (Mondria, 2010; Miao, Wu, and Young, 2022).

Therefore, this posterior correlation acts as a precursor to volatility spillover effects, necessitating an in-depth analysis of how it interacts with the information cost structure for a thorough understanding. The following proposition shows that, when the marginal cost of information approaches zero, the steady-state posterior features a perfect negative subjective correlation between asset returns.

Proposition 2.2. *Let $\rho_\theta = \rho_z = 0$. As ω approaches to 0, the steady-state posterior correlation, $\hat{\rho} = \frac{\hat{\sigma}_{\theta,z}}{\hat{\sigma}_\theta \hat{\sigma}_z}$, converges to -1 since the steady-state posterior covariance, Σ, is given by:*

$$\Sigma = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}, \text{ where } K = \frac{\sigma_\theta^2 \sigma_z^2}{(\sigma_\theta^2 + \sigma_z^2)}. \tag{2.7}$$

Proposition 2.2 clarifies why an agent prefers a one-dimensional signal. It reveals that the best strategy does not involve obtaining complete information—two perfect signals for each stock return—even if the cost of information is vanishingly small. Intuitively, if the agent could directly observe x_t which is a linear combination of θ_t and z_t, then a one-dimensional and precise signal of the form $s_t = x_t$ would suffice for optimal decision-making, and attempting to separate θ_t and z_t would bring unnecessary costs. Indeed, it is evident that, with just a one-dimensional signal, agents can still maximize their benefit since the benefit term in the objective function still attains its upper bound of zero:

$$\lim_{\omega \to 0} -\text{tr}\left(\Sigma \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right) = \lim_{\omega \to 0} -(\hat{\sigma}_\theta^2 + \hat{\sigma}_z^2 + 2\hat{\sigma}_{\theta,z}) = 0.$$

In this situation, the posterior correlation is exactly -1, indicating a perfectly inverse relationship between the two undisclosed shocks. For instance, if the agent knows that x_t is zero and presumes θ_t to be 2, they would logically deduce that z_t must be -2. However, even in this situation, the agent cannot distinguish the source of uncertainty even though they can achieve the maximum benefit.

The question arises: when can the agent distinguish perfectly between the two shocks? This can occur only when the fixed marginal information cost, ω, is exactly zero, not as a limiting value, allowing the agent to identify each piece of information precisely without incurring any costs. Lemma 2.2 illustrates such scenarios.

Lemma 2.2. *Let $\rho_\theta = \rho_z = 0$. Given $\omega = 0$, to maximize the agent's net benefit, any values of $\hat{\sigma}_\theta^2$, $\hat{\sigma}_z^2$, and $\hat{\sigma}_{\theta,z}$ satisfies such that:*

$$-(\hat{\sigma}_\theta^2 + \hat{\sigma}_z^2 + 2\hat{\sigma}_{\theta,z}) = 0. \tag{2.8}$$

Equation (2.8) is the benefit term within the agent's objective function as $-(a_t - (\theta_t + z_t))^2$. According to Lemma 2.2, if information processing is entirely cost-free, the agent is capable of precisely interpreting the information to hit the target accurately, and can set the posterior covariance—and thus the posterior correlation—to zero. Yet, this is merely one of infinitely many solutions when $\omega = 0$. Since the agent's maximum benefit is bounded above by zero, any level of correlation is possible without information costs as long as this maximum benefit is achieved.

From the analysis provided, rational inattention theory implies that dividing the signal into two entails its own costs, making the sum of two shocks effectively equivalent to having complete information. Moreover, it becomes clear that, as the fixed marginal information cost approaches zero, the source of volatility spillover effect intensifies because the posterior correlation tends towards -1. The following proposition formalizes this result, showing that the posterior correlation increases in absolute value as the marginal cost of information falls.

Proposition 2.3. *Let $\rho_\theta = \rho_z = 0$. The square of posterior correlation, $\hat{\rho}^2$, between θ and z weakens as ω increases, i.e.,*

$$\frac{\partial \hat{\rho}^2}{\partial \omega} \leqslant 0. \tag{2.9}$$

Proposition 2.3 examines the relationship between fixed marginal information cost and the posterior correlation. It shows that, as the fixed marginal information cost rises, the posterior correlation consistently moves towards 0. This means that, with higher fixed marginal information costs, the driving force behind the volatility spillover effect weakens, and if the cost becomes too high, the correlation reduces to zero, eliminating the volatility spillover effect entirely. This tells us that fixed marginal information cost is crucial in shaping the posterior correlation and is a key factor influencing the volatility spillover effect.

On the other hand, this finding presents a novel aspect not covered by the existing literature on rational inattention and appears counterintuitive. Sims (2011) noted that rational inattention naturally generates correlation among initially independent sources of uncertainties. Based on this, it is tempting to think that this correlation arises solely due to information costs. Yet, according to Propositions 2.2 and 2.3, as the fixed marginal information cost decreases towards zero, the posterior correlation does not diminish; it actually increases. Conversely, when the fixed marginal information cost goes up, the posterior correlation disappears as it goes to zero. The importance of this behavior is particularly significant when dealing with a one-dimensional target. More details about this issue will be further explored in Section 2.3.7 *Interaction between target dimension and information cost*.

2.3.2 Attention decomposition

Before investigating the impact of increased volatility in stock returns, z_t, on how attention is reallocated for each asset, I discuss the method for decomposing total attention to individual attention on each asset. Using the properties of Shannon mutual information, the method involves measuring the level of information processing undertaken for each asset. In other words, it assesses the extent to which uncertainty is reduced. This model-consistent method of decomposing attention can be generally applied, regardless of the presence of persistence.

Lemma 2.3. ***(Attention Decomposition)*** *The amount of attention allocated to θ_t is given by:*

$$\mathbb{I}(\theta^t; s^t | s^{t-1}) = 0.5 \left(\log \frac{\hat{\sigma}^2_{\theta,t|t-1}}{\hat{\sigma}^2_{\theta,t|t}} + \log \frac{1 - \hat{\rho}^2_{t|t-1}}{1 - \hat{\rho}^2_{t|t}} \right). \tag{2.10}$$

Similarly, the amount of attention allocated to z_t is given by:

$$\mathbb{I}(z^t; s^t | s^{t-1}) = 0.5 \left(\log \frac{\hat{\sigma}^2_{z,t|t-1}}{\hat{\sigma}^2_{z,t|t}} + \log \frac{1 - \hat{\rho}^2_{t|t-1}}{1 - \hat{\rho}^2_{t|t}} \right). \tag{2.11}$$

the measure of total attention is given by:

$$\begin{aligned}\mathbb{I}((\theta^t, z^t); s^t | s^{t-1}) &= 0.5 \log \left(\frac{|\Sigma_{t|t-1}|}{|\Sigma_{t|t}|} \right) \\ &= \mathbb{I}(\theta^t; s^t | s^{t-1}) + \mathbb{I}(z^t; s^t | s^{t-1}) - 0.5 \log \frac{1 - \hat{\rho}^2_{t|t-1}}{1 - \hat{\rho}^2_{t|t}}. \end{aligned} \tag{2.12}$$

Here, $\hat{\rho}_{t|t}$ represents the posterior correlation at time t, while $\hat{\rho}_{t|t-1}$ denotes the prior correlation at time t. Also, $\hat{\sigma}^2_{\theta,t|t-1}$ is the prior variance for θ_t, while $\hat{\sigma}^2_{z,t|t-1}$ is the prior variance for z_t.

First, the expression $0.5 \log \frac{1-\hat{\rho}^2_{t|t-1}}{1-\hat{\rho}^2_{t|t}}$ quantifies the joint reduction of uncertainty in θ_t and z_t through the subjective prior and posterior correlations. Essentially, if $\mathbb{I}(\theta^t; s^t|s^{t-1}) > \mathbb{I}(z^t; s^t|s^{t-1})$, this indicates that more attention is being paid to θ_t. To the best of my knowledge, this approach has not been previously utilized in the literature, where some studies have instead focused on the relative weight of tracking coefficients C_1 and C_2 in the optimal signal structure as a proxy for relative attention. In the next subsection, I will detail why attention decomposition method is preferable over using signal weight as an indicator of attention, as the latter can result in misconceptions when evaluating which fundamental receives more attention.

2.3.3 Relationship between individual attention and subjective signal structure

In this subsection, I discuss the problems of the common practice of using the relative weights of tracking coefficients C_1 and C_2 in the optimal signal structure given by equation (2.6) as indicators of relative attention.[12] Then, I demonstrate the necessity of employing the attention decomposition method outlined in the preceding subsection. Additionally, I explain the interaction between the structure of the signal and individual attention levels.

The following proposition illustrates why it is not appropriate to use the ratio

[12] The reason why the relative weights of the tracking coefficients C_1 and C_2 in the optimal signal structure have been used as indicators of relative attention is that, although a one-dimensional optimal signal generating one specific action is generally not unique, these ratios are unique.

$\frac{C_1}{C_2}$ as a stand-in for relative attention.

Proposition 2.4. *Let $\rho_\theta = \rho_z = 0$, and suppose the relative weights of fundamentals in the steady-state optimal signal structure are represented by $\frac{C_1}{C_2}$. Then, for any variances of θ_t and z_t, the weight remains consistently equal to one, and the following relationship holds:*

$$\frac{\sigma_z^2}{\sigma_\theta^2} = \frac{\hat{\sigma}_z^2 + \hat{\sigma}_{\theta,z}}{\hat{\sigma}_\theta^2 + \hat{\sigma}_{\theta,z}}. \tag{2.13}$$

These results suggest that a rational inattentive agent prefers their posterior beliefs about fundamentals to reflect the fundamentals' relative volatility.[13] It is intuitive to conclude that the agent desires to maintain this relationship, as it implies they are aware of the actual volatility of the fundamentals. Despite information costs preventing the posterior beliefs from fully converging with complete information, the agent still strives to keep the ratio of these beliefs consistent with the ratio of the fundamentals' volatilities. To maintain this ratio, the agent adjusts the proportion of C_1 to C_2 to be equal. However, even if $C_1 = C_2$, when the volatility of z_t is greater than that of θ_t, I find $\mathbb{I}(\theta^t; s^t | s^{t-1}) < \mathbb{I}(z^t; s^t | s^{t-1})$, which means that the agent is still paying more attention to z_t than θ_t.

Furthermore, considering the example of Sims (2011) where $\rho_\theta > \rho_z$ yet $\sigma_\theta^2 < \sigma_z^2$, Miao, Wu, and Young (2022) computed the normalized steady-state signal structure, finding $C_1 : C_2 = 1.3778 : 1$, which aligns with my attention decomposition

[13] This insight is closely related to one of the optimal conditions, as illustrated by the subsequent equation:

$$(\hat{\sigma}_{\theta,-1}^2 - \hat{\sigma}_\theta^2)(\hat{\sigma}_{z,-1}^2 - \hat{\sigma}_z^2) = (\hat{\sigma}_{\theta,z,-1} - \hat{\sigma}_{\theta,z})^2.$$

analysis showing $\mathbb{I}(\theta^t; s^t|s^{t-1}) : \mathbb{I}(z^t; s^t|s^{t-1}) = 1 : 1.1813$.[14] This reveals lower attention to θ_t relative to z_t, despite a higher tracking coefficient for θ_t, attributable to its greater persistence while its smaller volatility. In an extreme case where the volatility of θ_t is zero—meaning no uncertainty exists—despite C_1 being larger than C_2, no information processing occurs on θ_t.[15]

This leads to a natural question: is the relative signal weight an appropriate measure of relative attention when the prior variances for each fundamental are the same? The answer is negative. Even with equal variances and $\rho_\theta > \rho_z$ alongside $\sigma_\theta^2 < \sigma_z^2$, z_t's information processing might still surpass that for θ_t. This is in line with Sims' finding that rational inattentive agents are slower to react to less volatile, slowly moving components of an aggregate. The same intuition applies here. A rise in ρ_θ increases uncertainty, but in the steady-state, as ρ_θ grows, the accuracy of the prior observation also improves. This diminishes the incentive for more extensive information processing for new observations for θ_t.

To study the relationship between attention allocation and the weights assigned to fundamentals in the signal structure, I consider a specific signal, s_t^*, that generates the optimal action a_t. Despite the infinite number of signals generating the same action, if the ratio between C_1 and C_2 is fixed to one, I can then derive insights about the signal's accuracy and the scale of individual attention from the subsequent proposition.

---

[14] Parameters are $\rho_\theta = 0.95, \rho_z = 0.4, \sigma_\theta^2 = 0.0975, \sigma_z^2 = 0.84, \beta = 0.9, \omega = 1$. Miao, Wu, and Young (2022) clarified that the value of σ_z^2, reported as 0.86 in Sims (2011), was identified as a typographical error upon contacting the author. Also, the value of marginal information cost is 2 in Miao, Wu, and Young (2022), but it is the same as 1 in my example.

[15] We can interpret fundamentals with large persistence but relatively small volatility as 'slowly moving component' while the 'fast-moving component' implies a low persistence and relatively high volatility in Sims (2011). The slow-moving component could represent the AR(1) aggregate Total Factor Productivity, and the fast-moving component the AR(1) idiosyncratic Total Factor Productivity. Maćkowiak and Wiederholt (2009) utilized such models to demonstrate that prices react more rapidly to idiosyncratic shocks.

Proposition 2.5. *Let $\rho_\theta = \rho_z = 0$. One specific optimal signal structure such as $s_t^* = a_t$ can be expressed as:*

$$s_t^* = C_1\theta_t + C_2 z_t + e_t, \quad e_t \sim \mathcal{N}(0, v),$$

$$C_1 = 1 - \underbrace{\frac{\hat{\sigma}_\theta^2}{\hat{\sigma}_{\theta,-1}^2}}_{\text{Direct acquisition}} - \underbrace{\frac{\hat{\sigma}_{\theta,z}}{\hat{\sigma}_{\theta,-1}^2}}_{\text{Indirect acquisition}}, \tag{2.14}$$

$$C_2 = 1 - \underbrace{\frac{\hat{\sigma}_z^2}{\hat{\sigma}_{z,-1}^2}}_{\text{Direct acquisition}} - \underbrace{\frac{\hat{\sigma}_{\theta,z}}{\hat{\sigma}_{z,-1}^2}}_{\text{Indirect acquisition}}. \tag{2.15}$$

Proposition 2.5 shows how the weight of the signal is determined by both direct acquisition of information and indirect acquisition through posterior correlation.[16] The second term in C_1, denoted by $-\frac{\hat{\sigma}_\theta^2}{\hat{\sigma}_{\theta,-1}^2}$, signifies 'direct' information processing for θ_t. This is because 'direct' attention without any posterior correlation is $0.5\log(\frac{\hat{\sigma}_{\theta,-1}^2}{\hat{\sigma}_\theta^2})$. Hence, as the direct information processing for θ_t diminishes, C_1 drifts increasingly below 1, which means it follows θ_t less closely. Yet, an upswing in the volatility of z_t can trigger a more significant increase in $-\hat{\sigma}_{\theta,z}$ may increase C_1, even if there were no changes in the uncertainty of the fundamentals of θ_t. In other words, 'indirect' information acquisition can be larger. To gain a clearer understanding, I will show main results of this paper in the next subsection.

[16] If both components follow an AR(1) process but with different persistence levels, the forms may become more intricate, yet the underlying intuitions remain consistent.

2.3.4 Volatility shocks and attention reallocation

In this subsection, I present the main findings on how volatility spillover effects operate and then explain how these results are closely linked to changes in posterior correlation.

Proposition 2.6. ***(Fixed Marginal Information Cost)*** *Let $\rho_\theta = \rho_z = 0$. As the volatility of one component increases, the attention to both components increases. i.e.,*

$$\frac{\partial \mathbb{I}(\theta^t; s^t | s^{t-1})}{\partial \sigma_z^2} \geqslant 0, \quad \frac{\partial \mathbb{I}(z^t; s^t | s^{t-1})}{\partial \sigma_z^2} \geqslant 0.$$

Proposition 2.7. ***(Fixed Marginal Information Cost)*** *Let $\rho_\theta = \rho_z = 0$. As the volatility of one component increases, the attention allocated to that component increases more than to the other if and only if $\omega \leqslant 2\sigma_z^2$.* [17] *i.e.,*

$$\frac{\partial \mathbb{I}(z^t; s^t | s^{t-1})}{\partial \sigma_z^2} \geqslant \frac{\partial \mathbb{I}(\theta^t; s^t | s^{t-1})}{\partial \sigma_z^2} \text{ iff } \omega \leqslant 2\sigma_z^2. \tag{2.16}$$

Figure 2.1 visually shows the main results. The analysis shows that the amount of attention to θ_t consistently increases when the volatility of z_t rises, but the relative attention to θ_t decreases.[18] The principal observation here is the intensified attention on θ_t, which strengthens even without any inherent fundamental changes in θ_t.

[17] Even with an increase of one standard deviation of z_t, this condition can be readily met since the agent will not process any information if $\omega > \sigma_\theta^2 + \sigma_z^2$.

[18] Propositions 2.6 and 2.7 state that, as the volatility of z_t grows, the information concerning z_t also increases in a consistent manner. This finding is in line with the general conclusion from the static model of rational inattention by Maćkowiak, Matějka, and Wiederholt (2023), which asserts that more volatile variables receive more attention. These propositions suggest that this pattern holds true in dynamic models as well.

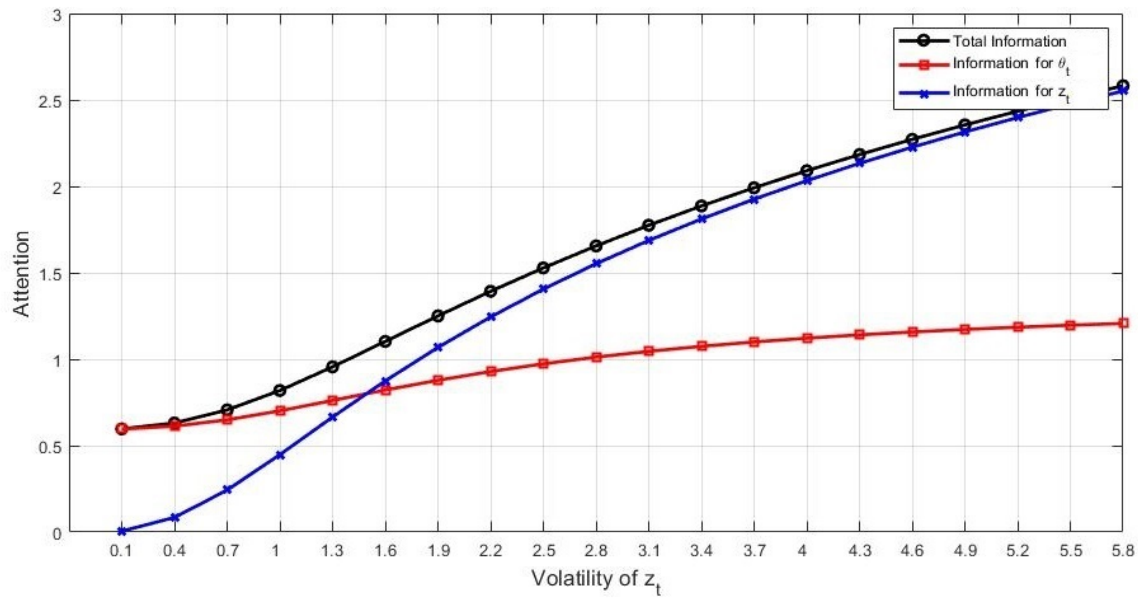

FIGURE 2.1: Attention decomposition: Parameter values are $\beta = 0.9, \rho_\theta = 0.8, \rho_z = 0, \sigma_\theta^2 = 0.8$, and $\omega = 1$

One might question the fundamental reason behind this pattern. The explanation is closely related to Proposition 2.3. According to this proposition, agents are inclined to increase the square of the posterior correlation when the fixed marginal information cost, ω, decreases in comparison to other variables. As ω nears zero, every element of the posterior covariance matrix tends to equalize, reflecting the agent's preference to maximize the square of posterior correlation. A rise in the volatility of z_t implies that agents are dealing with a relatively smaller information cost compared to the degree of overall uncertainty, which leads them to a higher posterior correlation. This dynamic consistently dominates any tendency to reduce information processing regarding θ_t, as confirmed by Proposition 2.6.

Should the marginal information cost increase, we would see a reduced posterior correlation, leading to an outcome that differs from Proposition 2.6. This reflects the scenario with a fixed capacity as analyzed by Mondria (2010). Within

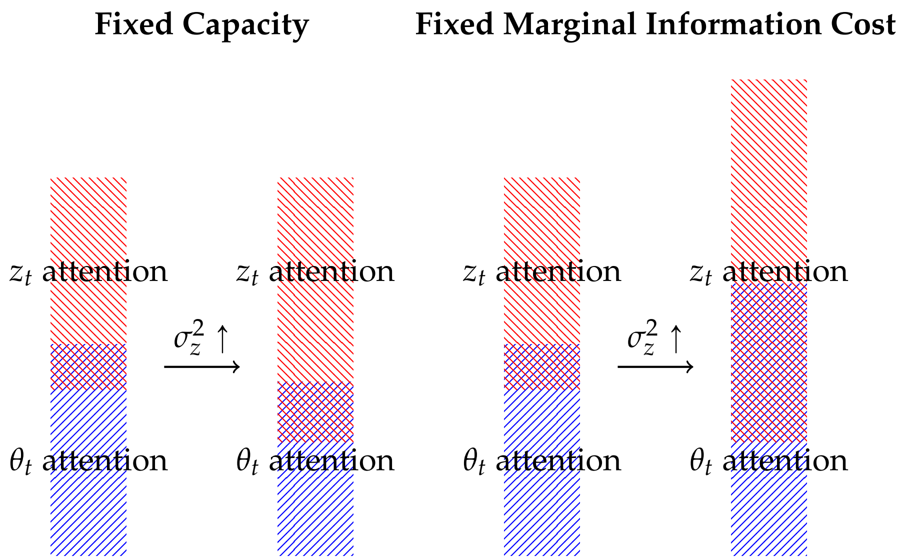

FIGURE 2.2: Comparison of information processing between fixed capacity and fixed marginal information cost

such a fixed capacity setting, even if the total level of uncertainty increases, the quantity of processed information remains unchanged. Consequently, the marginal information cost, often referred to as the 'shadow price', is likely to be higher than before the volatility shock. Therefore, understanding this phenomenon requires consideration of both the one-dimensional signal configuration and the posterior correlation dynamics.

Furthermore, Proposition 2.7 outlines another aspect where this finding significantly departs from the observations in Mondria (2010). It underscores that relative attention naturally shifts towards more uncertain elements, a behavior consistent even within a fixed capacity context. When the total inflow of information is constant, agents prioritize the more uncertain elements, despite a general tendency to increase attention on all variables.

In the fixed capacity scenario, the relative focus on z_t is anticipated to rise, while the overall information influx remains at a fixed upper boundary. In Figure 2.2, the diagram on the left side illustrates this concept: the red lines moving northwest indicate the processing of information related to z_t, and the blue lines moving northeast represent the flow of information about θ_t. The intersecting area of these patterns indicates the information processing due to prior correlation. Initially, it is assumed that both variables are equally uncertain and persistent. After a volatility shock affects z_t, the relative attention to z_t grows, though the total information inflow does not change, suggesting a decrease in the individual attention to θ_t. By contrast, in the fixed marginal information cost framework of this study, the agent aims to boost posterior correlation. In turn, this increases the individual attention to θ_t. This dynamic is believed to be at the heart of the volatility spillover effect.

In the fixed capacity model, even though the posterior correlation is assumed to be restricted to zero, attention to θ_t is expected to decrease, while attention to z_t increases due to the zero-sum nature. Therefore, subjective correlation is not a dominant factor in fixed capacity scenarios, and as a result, volatility spillover effects do not play a significant role in terms of qualitative results. In essence, this means that, when agents freely choose the signal to create an intentional subjective posterior correlation, attention to θ_t decreases less compared to cases where the posterior correlation is zero, due to volatility spillover effects.

2.3.5 Adding persistence

The situation regarding volatility spillover effects when the exogenous processes are persistent is similar to the previous case. First, all qualitative results are still consistent when persistences are the same to each other.[19] For AR(1) processes with different persistences, deriving analytical results becomes more diffi-

[19] Analytical results are in the Appendix.

cult. However, numerical evidence indicates that the underlying intuition remains the same. A comprehensive numerical analysis was undertaken over the parameters: $\beta \in (0,1)$, $\rho_\theta \in (0,1)$, $\rho_z \in (0,1)$, $\omega \in (0,50)$, $\sigma_\theta \in (0,50)$, and $\sigma_z \in (0,50)$, with increments of 0.1 for $\beta, \rho_\theta, \rho_z$ and 1 for ω, σ_θ, and σ_z. For each parameter combination, the change in $\Delta I(\theta^t; s^t|s^{t-1})$ was assessed as σ_z increased from 0.001 to 49.901. The findings reveal an increase in information processing on θ_t with an increase in the volatility of z_t.

2.3.6 Transitional dynamics

In this subsection, I examine the attention allocation during unanticipated and anticipated financial crises within the canonical rational inattention model, focusing on transitional dynamics. A financial crisis generally spreads faster when it is unexpected than when it is predicted. For example, studies by Kaminsky, Reinhart, and Végh (2003) and Rigobon and Wei (2009) have suggested that the global impact of unanticipated financial crises—such as those in Mexico in 1994, Asia in 1997, and Russia in 1998—could be more significant than crises that were anticipated, like those in Brazil in 1999, Turkey in 2000, and Argentina in 2002. Specifically, Rigobon and Wei (2009) used the frequency of newspaper articles as a measure of anticipation and pointed out that for the earlier events, there was no change in news frequency before the episodes, indicating that the effects of unexpected crises could be greater. This was particularly true for the 1997 Asian Financial Crisis; despite Thailand experiencing a speculative attack, news frequency remained unchanged, with the frequency of articles only rising after the event.

I perform numerical analyses for scenarios with one and two assets and conduct comparative analyses. While specific parameters are set, it shows that similar qualitative outcomes appear across various parameters. For anticipated volatility

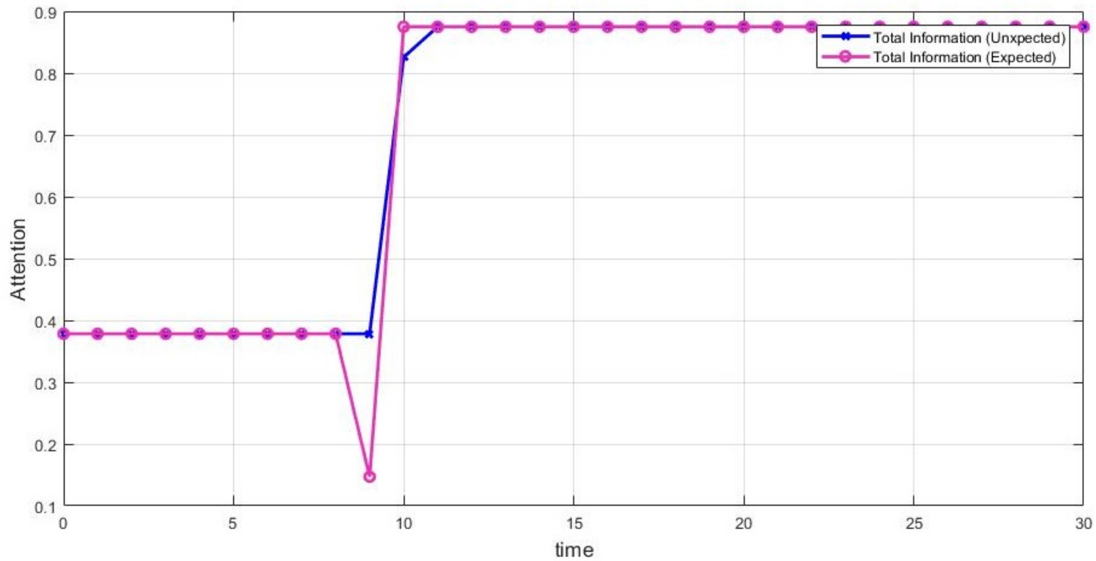

FIGURE 2.3: Transitional dynamics of attention (one shock case, AR(1)): Parameters are $\beta = 0.9, \rho_z = 0.9, \omega = 2$, and $\sigma_z = 1 \rightarrow 2$

shocks, I assume $z_t = \rho_z z_{t-1} + \epsilon_t$, with agents knowing at period 0 that a shock is due at period 10, thus determining an optimal path for attention in advance.

Figure 2.3 illustrates how attention changes for agents unaware and aware of a volatility shock at period 10 with just one asset. In the situation of unexpected shock at period 10, agents maintain a steady-state until period 9, heighten attention at period 10, and return to a steady-state in period 11. Agents anticipating the shock increase their posterior variance from period 9, reducing their attention in that period and increasing attention at the shock in period 10, entering steady-state instantly. The model, due to its AR(1) structure, does not react before period 9.

Figure 2.4 presents the scenario with a one-dimensional target with two assets. In the case of an unexpected shock, total attention rises gradually from period 10, with a return to a steady-state from period 12. When agents anticipate the shock, they start to increase their posterior variances from period 9, lowering total

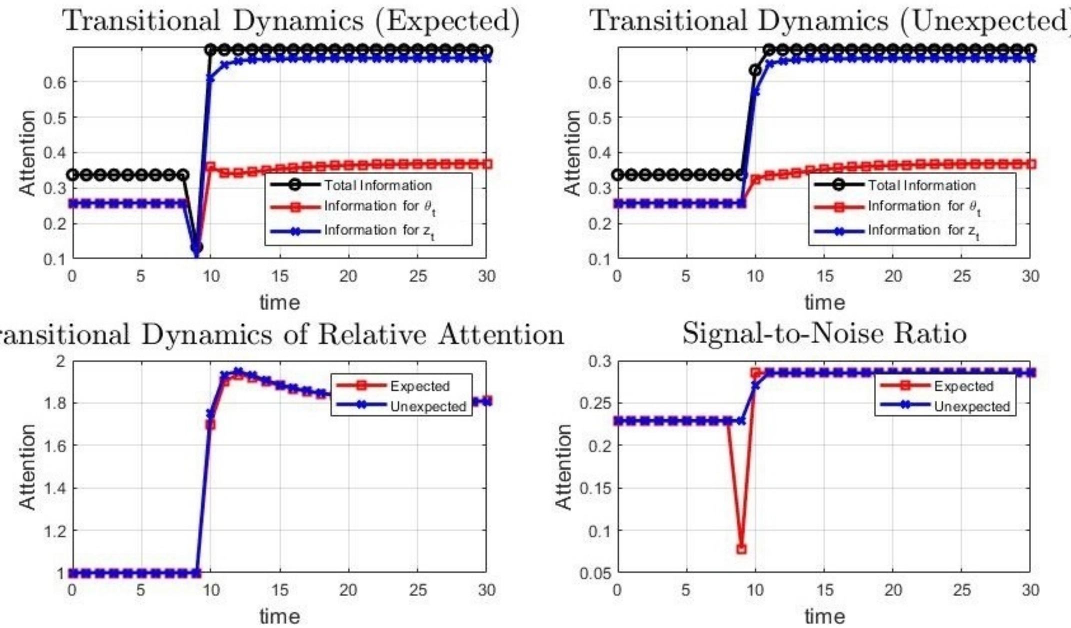

FIGURE 2.4: Transitional dynamics of attention (two shock case, AR(1)): Parameters are $\beta = 0.9, \rho_\theta = 0.9, \rho_z = 0.6, \omega = 3, \sigma_\theta = 0.8$, and $\sigma_z = 0.8 \rightarrow 1.8$

attention in that period and raising it at the shock period 10, quickly entering the steady-state.

The presence of one or two assets does not significantly alter the qualitative characteristics of the overall attention. Without anticipation, attention is gradually heightened until reaching a steady-state. With anticipation, attention is decreased before the shock, favoring an increase in posterior variance. This trend may arise from the model's assumption that efforts to reduce volatility can only be transmitted through persistence, implying that today's attention cannot diminish tomorrow's volatility. Regardless of foreknowledge of a shock, the benefit of today's attention is optimized for today's target x_t, with future volatility considered separately. Thus, without persistence, the anticipation of a shock does not play any role; the benefits of current attention are transferred solely through non-zero persistence. Intuitively, if agents pay more attention today, it only enhances prior observations' accuracy for tomorrow via persistence, and volatility concerns can only be resolved on the shock day. Therefore it compels agents to conserve attention before the shock period and then apply it intensively to mitigate forthcoming volatility. This resembles saving current income to invest in future expenditures. In this context, agents want to save mental income for a more accurate signal upon tomorrow's shock.

A distinctive difference noted is in individual attention when there are two shocks but one linear combination target. In the unexpected case, even as total attention reaches a steady-state, attention towards the unchanged fundamental θ_t slowly increases, hinting at a slow adjustment in posterior correlation. Conversely, in the expected case, attention towards θ_t also shifts more swiftly during the shock period. This implies that, without prior anticipation of a shock, agents fail to reserve attention, thereby finding it advantageous to promptly acquire direct attention towards z_t, which is more uncertain, rather than investing in posterior correlation.

2.3.7 Interaction between target dimension and information cost

This subsection clarifies when the volatility spillover effect is observable and when it is not. In addition, it explains why posterior correlation emerges as an interaction between a linear combination target dimension and fixed marginal information cost in the framework of rational inattention.

Using Lemma 2.3, with a one-dimensional linear combination target, I can reformulate the objective function in the static model as follows:

$$U(\hat{\sigma}_{\theta}^2, \hat{\sigma}_z^2, \hat{\sigma}_{\theta,z}) = -(\hat{\sigma}_{\theta}^2 + \hat{\sigma}_z^2 + 2\hat{\sigma}_{\theta,z}) - \omega \left(\log \frac{\sigma_{\theta}^2}{\hat{\sigma}_{\theta}^2} + \log \frac{\sigma_z^2}{\hat{\sigma}_z^2} - \log(1 - \hat{\rho}^2) \right).$$

Within this equation, we can see the posterior covariance in the benefit term and the posterior correlation in the information cost. This form allows us to understand the situation more intuitively, and it highlights why attention decomposition is useful. To begin, purchasing more of the square of posterior correlation translates into an increase in information cost. The fixed marginal information cost, ω, can be understood as the price of acquiring information. As ω decreases, the agent desires to purchase more of the square of posterior correlation. Even in the case of fixed marginal information cost, if the prior uncertainty levels, represented by σ_{θ}^2 and σ_z^2, increase, it implies that the cost of acquiring information, ω, has relatively decreased, prompting the purchase of more of the square of the posterior correlation.

Why is buying more of the square of posterior correlation meaningful? It is because it lowers the posterior covariance, $\hat{\sigma}_{\theta,z}$, in the benefit term, leading to an increase in benefit.[20] In other words, the agent simplifies the understanding of one side's fluctuations as indicative of the other's, allowing the action to be closer to the target.

However, if I assume there are two targets, the situation could change completely. Given dual targets, specifically:

$$-\left(\theta_t - \mathbb{E}[\theta^t | s^t]\right)^2 - \left(z_t - \mathbb{E}[z^t | s^t]\right)^2.$$

[20] Posterior covariance is always negative since the target is a sum of two fundamentals. Therefore, lowering posterior covariance increases the benefit of the agent.

Then, I derive as follows:

$$U(\hat{\sigma}_\theta^2, \hat{\sigma}_z^2, \hat{\sigma}_{\theta,z}) = -\hat{\sigma}_\theta^2 - \hat{\sigma}_z^2 - \omega \left(\log \frac{\sigma_\theta^2}{\hat{\sigma}_\theta^2} + \log \frac{\sigma_z^2}{\hat{\sigma}_z^2} - \log(1 - \hat{\rho}^2) \right).$$

In this scenario, purchasing the square of the posterior correlation has no impact on the benefit. The optimal posterior correlation becomes zero, and no volatility spillover effect occurs.

In conclusion, the existence of posterior correlation across independent sources of uncertainty must be understood as a result of the interaction between a one-dimensional linear combination target and rational inattention. More specifically, with the previous analysis of interaction between the fixed marginal information cost and posterior correlation, this implies that volatility spillover effects arise mainly from agents' optimal choice of target variable, not from the presence of information constraints per se. This understanding enables us to apply rational inattention theory more aptly to each appropriate context.

I conclude with the theoretical implications and will move on to the empirical application in the next section.

2.4 Empirical application

This section presents empirical findings on attention allocation shifts during the 1997 Asian Financial Crisis. The crisis is chosen for validation due to its representation of minimally connected markets, as explained by Mondria and Quintana-Domeque (2013). They showed how a crisis starting in Asian markets quickly impacted Latin American markets due to a shift in investor attention towards Asia, and away from Latin America. In particular, they noted a reduction in attention to the Latin American market after constructing a structural model with fixed capacity. My research, however, provides contrasting implications, focusing solely on how individual attention direction unfolded in each market.

Assuming international investors have stakes in both the Asian and Latin American markets, they strive to maximize the accuracy of their subjective signal about the combined payoff from both markets. Consistent with the previous model, θ_t denotes the Latin American market returns, and z_t symbolizes the Asian market returns. The investors' goal is to narrow the difference between their action a_t and the total return x_t, which is a linear combination of returns from both markets. The 1997 Asian Financial Crisis is typically attributed to a volatility shock in the Thai market. The aim here is to examine how this shock influenced attention reallocation in both the Thai and Latin American markets. Specifically, if fixed capacity assumption is more appropriate, attention to the Latin American market would decline. Under the fixed marginal information cost assumption, this leads to a heightened posterior correlation and an increase in attention to both markets.

To create data akin to that of Mondria and Quintana-Domeque (2013), I emulate their strategy. The review period spans from January 1, 1997, to June 30, 1998, following the timeline of the 1997 Asian Financial Crisis outlined by Nouriel Roubini. The section's aim is to confirm three key predictions:

Prediction 1 (from Proposition 2.6 and 2.7): Increased volatility in the Asian market heightens posterior correlation between the two markets.

Prediction 2 (from Proposition 2.6 and 2.7): Increased volatility in the Asian market heightens individual attention to this market.

Prediction 3 (from Proposition 2.6): As attention to the Asian market intensifies, there is a simultaneous increase in attention to the Latin American market.

2.4.1 Data: Price volatility and attention

This study begins by examining Thailand, Argentina, Brazil, and Chile. As per Mondria and Quintana-Domeque (2013), the latter three are economically pivotal nations in Latin America. Thailand is chosen to represent the Asian market due to its significant role in the commencement of the 1997 Asian Financial Crisis with the devaluation of its currency on July 2, 1997. To assess price volatility and attention allocation for these countries, data from the Datastream Global Index and the Lexis Nexis Database are utilized.

From the Datastream Global Index, a stock market price index for each country is compiled, with daily entries from January 1, 1997, to June 30, 1998. Price volatility is measured in various methodologies to conduct a thorough robustness check, serving as a proxy for the volatility of θ_t and z_t.

Additionally, news article frequency from the Financial Times (FT) in the Lexis Nexis Database is analyzed for the corresponding period. This frequency, counting mentions of a country in article titles or lead paragraphs, is taken as a measure of investor interest. An increase in article count is interpreted as a rise in attention towards the market. This method is consistent with that used in the established literature, such as Veldkamp (2006), which considers article frequency as an indicator of attention.

A key assumption is that FT's coverage is an accurate proxy for the attention paid to these countries. As Mondria and Quintana-Domeque (2013) state, FT had a readership exceeding 1.8 million across over 140 countries in 2012, believed to reflect the attention of international investors. Furthermore, FT was recognized as the most influential business read by the world's leading financial institutions in the 2011 Global Capital Markets Survey.[21] Therefore, FT's news coverage is

[21] https://aboutus.ft.com/press_release/financial-times-named-the-most-important-business-

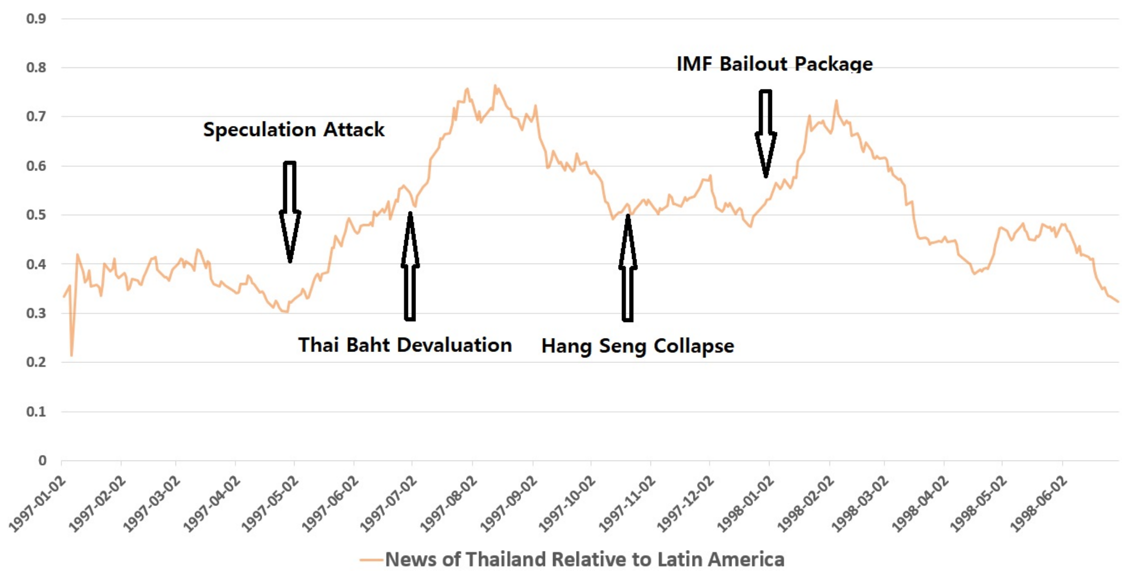

FIGURE 2.5: Relative attention based on the frequency of news articles about Thailand compared to Latin America: 30-day moving average of daily news article count

considered an appropriate proxy for the attention of international investors. In line with Nimark (2014), I assume that news editors supply content that aligns with readers' interests, indicating that the news reflects the attention of readers.

Before I proceed with the econometric analysis, I first examine the trend in news article frequency for each country as depicted in Figure 2.5. Here, I have marked significant events to contextualize the relative attention. We can observe that interest in Thailand gradually surpasses that in Latin America following these events. A look at news frequency prior to June 1997 suggests that the volatility shock in Thailand was unexpected. The reason why the relative attention during the Hang Seng Collapse did not escalate as much as during other events might be due to Hong Kong's market being more globally related in 1997.

Figure 2.6 illustrates the pattern of individual attention. After the speculation

read-by-the-worlds-largest-financial-institutions

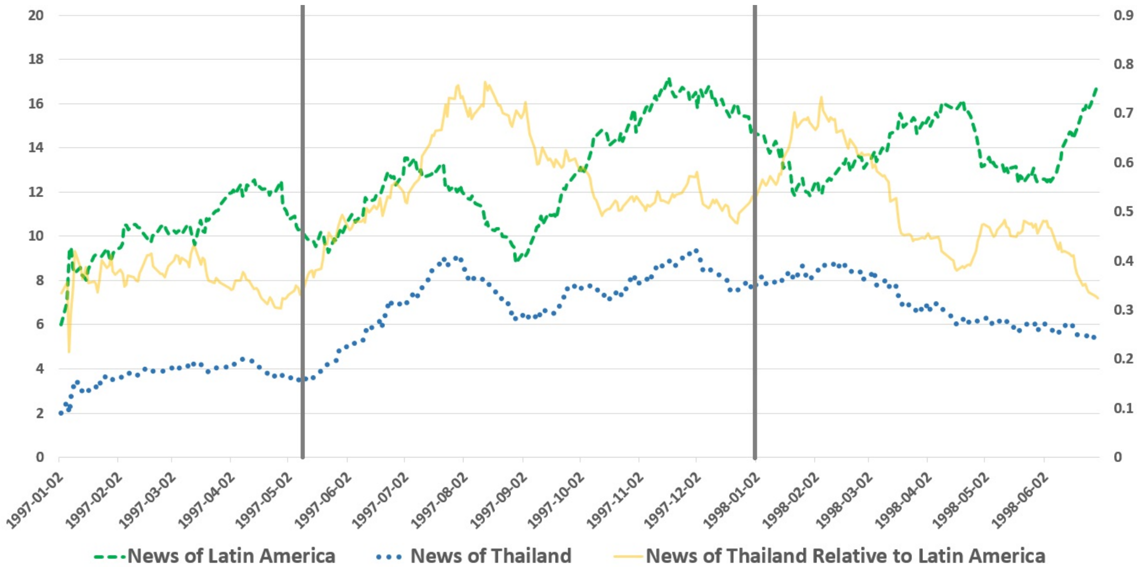

FIGURE 2.6: Relative vs. individual market attention: 30-day moving average of daily news article count

attack in Thailand in May, the frequency of news about Thailand began to climb, with the number of articles about Latin American countries also rising. This trend continued until the announcement of the IMF Bailout Package, after which the attention towards each region began to diverge. Notably, before May 1997, Thailand scarcely appeared in the news. Starting in May 1997, there was a noticeable trend where article counts for Thailand and Latin American countries rise and fall together until December 1997. This pattern prompts us to consider whether the attention of international investors is indeed fixed.

Adding to this observation, Table 2.1 presents the correlation coefficients, revealing a positive correlation in news frequencies. This indicates that, as the volume of news about Thailand increases, so does the volume of news about Latin American countries. With these observations in hand, I will now advance to a detailed econometric analysis in the following subsection.

Table 2.1: Correlations between Asian and Latin American news frequencies

	Argentina	Brazil	Chile	Thailand
Argentina	1.0000			
Brazil	0.5343***	1.0000		
	(0.0000)			
Chile	0.4940***	0.3434***	1.0000	
	(0.0000)	(0.0000)		
Thailand	0.2912***	0.4438***	0.2035***	1.0000
	(0.0000)	(0.0000)	(0.0001)	

*** p<0.01, ** p<0.05, * p<0.1

2.4.2 Econometric analysis

I begin by assuming the following linear relationship:

$$A_t^{Asia} = \alpha^{Asia} + \gamma^{Asia} Vol_{t-1}^{Asia} + \delta^{Asia} Vol_{t-1}^{Latin} + \mathbf{U}^{\mathbf{Asia}'}\mathbf{X_t} + \epsilon_t^{Asia}$$

$$A_t^{Latin} = \alpha^{Latin} + \beta^{Latin} A_t^{Asia} + \gamma^{Latin} Vol_{t-1}^{Asia} + \delta^{Latin} Vol_{t-1}^{Latin} + \mathbf{U}^{\mathbf{Latin}'}\mathbf{X_t} + \epsilon_t^{Latin}.$$

In this model, $\mathbf{X_t}$ represents a set of control variables that include an annual indicator, a quadratic trend for each day, indicators for the days of the week, and United States interest rates (specifically, daily rates for three-month, six-month, five-year, and ten-year US Treasury Bills). The variable A_t quantifies the level of individual attention, which is determined by the frequency of news articles for each country. The first model equation assesses the impact of the Asian market's price volatility on the volume of news articles about the Asian market. The second model equation gauges the influence of the Latin American market's price volatility on its news coverage, while also considering the effect of the volume of news about the Asian market on the volume of news about the Latin American market. This approach aims to understand the shifts in attention through posterior correlation, building on the insights from the previous theoretical discussion.

The exclusion of the Latin American attention variable in the Asia-focused equation is rationalized by the time difference across the globe, which ensures the temporal ordering in the analysis. Specifically, there is a time difference of 10 hours between Thailand and Brazil or Argentina and 11 hours between Thailand and Chile. Hence, when the Thai stock market ends its trading day, the Latin American market commences after the closure in Thailand, even if on the same calendar day. Since London's time is between these markets, articles on Thailand begin to emerge as its market closes, while the Latin American market is still open. As a result, news about the Latin American market tends to be published after those about Thailand, despite bearing the same date. In essence, reports on Thailand typically come first in this sequence.

2.4.3 Empirical findings: Posterior correlation and attention comovement

Table 2.2 presents the initial key empirical findings of this study. It examines the correlation of attention before the speculation attack in Thailand in May 1997, from May to December 1997 as shock periods, and from the announcement of the IMF Bailout Package to June 30, 1998. These timeframes showcase the movement of posterior correlation, aligning with my theoretical framework.

The results are consistent with the theoretical implications. Prior to the volatility shock, the correlation is negligible and not statistically significant. During the shock period, the correlation increases to 0.50 and is statistically significant. As the shock diminishes from January 1998 onwards, the correlation decreases to 0.35.

Table 2.3 outlines the regression analysis concerning attention, presenting the second and third major empirical results of this paper. Initially, the previous day's price volatility in the Asian market increases individual attention to Thailand. This amplified attention to Thailand is associated with increased attention to the Latin American market. Specifically, a three-article increase on Thailand corresponds with a one-article increase on Latin America. The prior day's price volatility in Argentina, Brazil, and Chile does not significantly sway attention towards Latin America, indicating the Asian market as the shock's origin. Multicollinearity among price volatilities could be present but usually reduces t-values, so the significant result that news on Thailand affects news on Latin America supports the second forecast robustly.

In conclusion, the data indicate a swift increase in the total information flow, suggesting a fixed marginal information cost analysis is apt for the 1997 Asian Financial Crisis. Given the relatively stable daily number of news articles in the paper in 1997 (before the prevalence of internet news), as focus on each stock market heightens, coverage of other topics, unrelated to the 1997 Asian Financial Crisis, appears to diminish. This pattern implies that a fixed marginal information cost assumption is more appropriate when focusing on a one-dimensional target. However, if multiple targets are considered alongside the combined return from two markets, a fixed capacity assumption may be more relevant, particularly since the total news volume is largely fixed.

Table 2.2: Correlations between Asian and Latin American news frequencies

January – April 1997

	Latin America	Thailand
Latin America	1.00	
Thailand	0.17	1.00
	(0.12)	

May – December 1997

	Latin America	Thailand
Latin America	1.00	
Thailand	0.50***	1.00
	(0.00)	

January – June 1998

	Latin America	Thailand
Latin America	1.00	
Thailand	0.35***	1.00
	(0.00)	

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 2.3: Regressions of individual attention measures on the price volatilities

News Articles	Thailand	Latin America
Thailand		0.328***
		(0.0922)
Volatility Argentina (t-1)	0.683	2.002
	(0.697)	(1.228)
Volatility Brazil (t-1)	-0.158	-0.0695
	(0.573)	(1.008)
Volatility Chile (t-1)	1.320	-6.744
	(2.515)	(4.428)
Volatility Asia (t-1)	0.419***	0.152
	(0.146)	(0.259)
Observations	379	379
R-squared	0.346	0.398

*** p<0.01, ** p<0.05, * p<0.1

2.5 Conclusion

This paper explores the attention reallocation dynamics of international investors, to understand fundamental causes of financial contagion, focusing on a total return which is a linear combination of two uncorrelated assets' returns, especially when one of two assets faces a volatility shock. The 1997 Asian Financial Crisis serves as a prime example, illustrating how a crisis can quickly propagate from its epicenter in Asia to markets across the globe.

The paper's key theoretical implications, from canonical rational inattention model with a fixed marginal information cost, include the finding that increased volatility in one asset heightens attention on that asset, even within a dynamic framework. Additionally, it demonstrates that agents strive to increase the subjective posterior correlation for uncorrelated assets as the overall uncertainty escalates, resulting in heightened attention towards another uncorrelated asset in the steady-state—contrasting the zero-sum nature of fixed capacity models. It also establishes that these steady-state findings hold true in dynamic short-term situations in transitional dynamics. These results are broadly applicable (e.g., firm price setting, investment, or consumption/saving decisions).

In an empirical application, this paper uses data from the 1997 Asian Financial Crisis, analyzing the Financial Times news articles to gauge the attention of international investors on both the Asian and Latin American markets. The results confirm that the unanticipated volatility shock in the Asian market not only garnered increased attention but also led to a concurrent rise in attention towards the Latin American market through amplified posterior correlation.

This application focuses only on the implications for attention, without connecting these to equilibrium asset returns. This is done because the analysis in the paper takes the optimal portfolio weights (and so the equilibrium returns) as exogenous when analyzing volatility spillover effects. Therefore, it would be inappropriate to make inferences about equilibrium returns without further modification to make the portfolio weights endogenous as well. That is left for future work.

3 Beauty Contest under Uncertain Media Credibility

3.1 Introduction

During the COVID-19 pandemic, we experienced various forms of belief polarization (Allcott, Boxell, Conway, Gentzkow, Thaler, and Yang, 2020; Coibion, Gorodnichenko, and Weber, 2020; and Ternullo, 2022). Notably, in times of uncertainty, even public media failed to provide accurate information, leading to greater belief polarization. In such situations, there was a consensus on the need to enhance the credibility and accuracy of public information.

However, it has been known since Morris and Shin (2002) that in scenarios with strategic complementarity among economic agents, increasing the accuracy of public information does not always lead to better social welfare. This seems to contradict our experience during COVID-19, where we sought more accurate information in uncertain times, believing it would lead to greater welfare. However, Morris and Shin (2002) assumed that (i) agents had equal beliefs about the credibility of public information, (ii) the nature of the information was exogenous, with agents having no influence over the quality of information, and (iii) there was no information cost.

In this paper, I aim to relax these three assumptions. Specifically, in situations with strategic complementarity, I propose a more realistic yet simple assumption that (i) agents can receive different signals about the credibility of public information *before receiving it*, and (ii) they can choose to increase the accuracy of the signal before receiving it by conducting costly additional research. More generally, this paper explores the interaction between the credibility of public information and strategic complementarity, in line with the Global Coordination game and rational inattention (Sims 2003, 2011). This research integrates the role of information friction and news media into economic models, investigating how the reliability of information plays a crucial role in the decision-making processes of agents, particularly in how different beliefs about news media information can impact economic activity and welfare. Through this analysis, I seek to broaden our understanding of the quality of public information and its effects on the economy.

Initially, I discuss how survey data and existing literature indicate that economic agents have divergent opinions on the trustworthiness of public news, making it a public issue. Particularly, Gentzkow and Shapiro (2006) strongly suggests that public information covering the same phenomena can provide biased information or different signals, interpreting the same events in almost entirely different ways. Additionally, according to a 2019 survey by the Pew Research Center, there are news media that are trusted or distrusted based on political leanings, indicating a stark division along ideological lines. These observations form the basis for the model in this paper.

The model focuses on the verification process by information receivers while all agents receive the same public information from news media. It assumes Bayesian agents with equal prior beliefs about the credibility of news media. For simplicity, I consider a single common news media source not fully trusted by viewers. Economic agents receive signals about credibility of public information (news media), but these signals have a probability of being erroneous. Although all agents have the same prior probabilities and thus measure the reliability of the signals they receive in exactly the same way, they end up with different posterior beliefs about the credibility of news media information after receiving different signals. This is the source of varied decision-making emphasized in this paper.

To apply this situation effectively, I use a modified version of the Beauty Contest model by Morris and Shin (2002). Similar to Morris and Shin's model but with a significant difference: the signals from news media, which provide public information to agents, are assumed to have a probability of error. Initially, I explore the implications when this error probability is fixed exogenously, showing that verifying public information to reduce error probability results in higher welfare than not verifying it at all, especially in the case of news media is truly reliable.

Furthermore, I present an endogenous cost model allowing for the reduction of signal error probability, showing that increasing the accuracy of public information can lead to higher welfare than not verifying the signals at all. Overall, the analysis suggests that, when agents do not fully trust news media providing public information, it negatively affects the accuracy of that information. This, in turn, seems to reduce the domain in which Morris and Shin's argument, in which the accuracy of news media can have a negative effect on welfare, applies.

The paper is structured as follows. Section 2 introduces related studies in both macroeconomic and microeconomic contexts. Section 3 discusses some facts about mass media through survey data. Section 4 introduces a model featuring Bayesian agents who do not fully trust public information. This model extends the Beauty Contest framework by Morris and Shin (2002), resulting in a divided society characterized by varied posterior beliefs. Furthermore, I examine the implications of integrating endogenous information costs into the model in terms of social welfare. Section 5 then highlights the contributions of this study and discusses plans for future research.

3.2 Related literature

This paper is not the first to examine news media and information friction in economic models. In macroeconomic literature, Nimark (2014) demonstrated that agents' posterior uncertainty could increase as news media sought to report notable events. In his paper, the incentive for news media is explained by introducing the Man-bites-dog signal, which includes more unusual tail events than frequent events around the mean of the true distribution. As a result, agents updating their beliefs with this signal can end up with a broader distribution than the actual distribution, explaining significant output fluctuations in economic cycles. Their findings share a spirit with my work, claiming a stronger response in average action when the Man-bites-dog signal is available. Similarly, in my model, the responsiveness of average action is enhanced when public information is accessible. However, my work diverges from Nimark (2014) as I focus on how agents receive and update their beliefs upon receiving public information.

Chahrour, Nimark, and Pitschner (2021) attempt to explain business cycle fluctuations using an exogenous editorial function that mimics the gatekeeping role of journalism. They posit that newspapers prefer to report unusual events, negative outcomes, or incidents in sectors deemed more critical, such as finance, insurance, and real estate. This study illustrates the significant impact news media can have on business cycle fluctuations in a multi-sector economy, with their model replicating aspects of the 2009 Great Recession.

Both aforementioned papers presuppose that information is reported to agents in the economy exogenously by information providers, addressing significant macroeconomic issues with news media. My work is distinct from Nimark (2014) and Chahrour, Nimark, and Pitschner (2021) as, in their analyses, all agents interpret a single piece of information identically. In contrast, my analysis posits that each agent may hold different posterior beliefs about the credibility of public information.

On information friction without interaction between news media and economic agents, there is notable research to consider. Angeletos and Lian (2022) propose a mechanism for how business cycles can be driven by shifts in aggregate demand primarily through information friction. They detail a scenario where some agents are informed about current events while others only have knowledge of past events. This discrepancy leads households with information friction to misunderstand whether income shocks are aggregate or idiosyncratic, causing misperceptions about permanent income and leading to decreased spending and confidence. This feedback loop helps explain the propagation of demand shocks throughout the macroeconomy.

Additionally, Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017) introduce the 'Uncertainty Trap,' illustrating endogenous information acquisition where more risk-averse entrepreneurs refrain from market entry during recessions, causing a cascading effect of prolonged belief updating due to missing production signals from other firms. This sequence underscores the reinforcing nature of uncertainty, where the absence of firms in the market begets further market abstention due to escalating uncertainty.

Collectively, Angeletos and Lian (2022) and Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017) highlight the endogenous interactions among economic agents under information friction, albeit without the presence of an information provider such as news media in my paper.

On the other hand, Ilut and Valchev (2023) shift focus to the information receivers' side similar to my paper, dealing with an exogenous information structure. They theorize economic agents with limited cognitive perception regarding their optimal policy functions, where updating beliefs incurs reasoning costs. Hence, interpreting information becomes a costly endeavor, leading agents to adjust their interpretation based on the familiarity of observed states. This mechanism explains macroeconomic phenomena not captured by standard models, such as high marginal propensity to consume (MPC) and wealth inequality. While their research shares my focus on information reception under common public information, my paper distinguishes itself by examining the implications of strategic complementarity in depth.

The microeconomic literature, too, has seen extensive studies, albeit with little focus on the interplay between news media credibility, strategic complementarity, and belief polarization, along with its welfare implications, which my research seeks to address.

For instance, Nimark and Sundaresan (2019) analyzes how rational Bayesian agents respond to the same public information, exploring why belief polarization occurs among them. They found that this polarization stems from agents choosing signal structures more likely to reinforce their prior beliefs (confirmation effect) and opting for less informative signals when their prior beliefs are more precise (complacency effect). In their paper, the key insight is that attention is a scarce resource that agents allocate, in line with rational inattention.

Further bridging the gap, Nimark and Pitschner (2019) conceptualized a news selection function based on the notion that many economic agents outsource their information gathering to news specialists who have direct access to real-world states or events. They posited that news providers aim to maximize readers' utility, thus favoring reports aligned with what their audience prefers to hear. These works lay a solid groundwork for understanding the dynamics between news media and economic agents.

However, their research diverges from mine as it does not study the nuances of the interaction between strategic complementarity and information cost.

On the empirical front, there have been studies on how news media or social networking services (SNS) influence decisions. For example, Martin and Yurukoglu (2017) empirically demonstrated how biased news can politically sway its audience, estimating that Fox News could potentially persuade viewers to vote Republican by a margin of 0.3 percentage points for every additional 2.5 minutes watched per week. In another context, Bianchi, Kind, and Kung (2023) analyzed the impact of President Trump's Twitter activity on expectations regarding monetary policy, concluding that markets perceive the Federal Reserve as susceptible to political pressure. These findings underscore the significant influence of media on public opinion and policy perceptions, offering valuable insights into the interplay between media exposure and political and economic decision-making processes.

However, while these macroeconomic and microeconomic studies provide a foundation, they have not studied the nuances of the accuracy of news media, strategic complementarity, and the resulting belief polarization and its welfare effects, areas my research aims to explore. This paper seeks to offer a comprehensive analysis of how news media credibility and strategic complementarity among agents shape economic outcomes and social welfare.

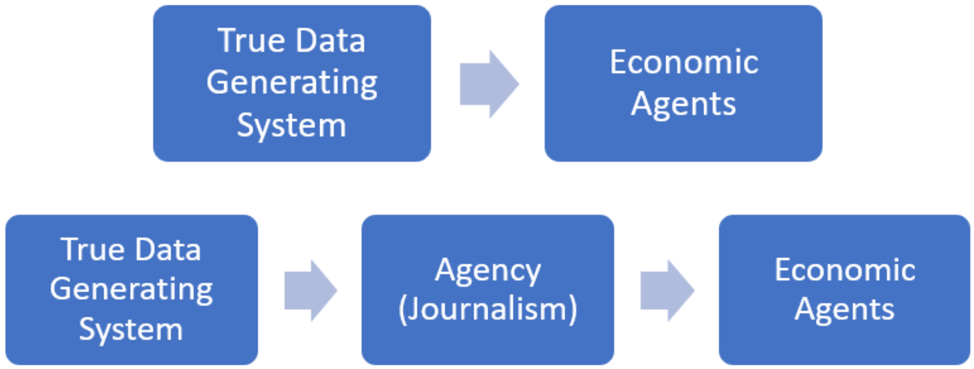

FIGURE 3.1: Information receiving process

3.3 News media perspectives and process for receiving information

There are two distinct processes for receiving information: one directly from the source, the true data generating system, and another mediated by news outlets, which is far more common and potentially introduces significant biases, especially if trust levels in these outlets vary widely. Figure 3.1 briefly illustrates this phenomenon through a diagram.

Many studies have traditionally assumed economic agents directly receive information, sometimes tainted by noise. However, in some previous studies, economic agents primarily get their news through news media channels (Blinder and Krueger, 2004; Nimark, 2014; and Chahrour, Nimark, and Pitschner, 2021), and it has been pointed out that this can significantly impact political and economic outcomes.

In Figure 3.2, the 2020 survey by the Pew Research Center[1] unveils a varied

[1] https://www.pewresearch.org/journalism/2020/08/31/americans-see-skepticism-of-news-media-as-healthy-say-public-trust-in-the-institution-can-improve/

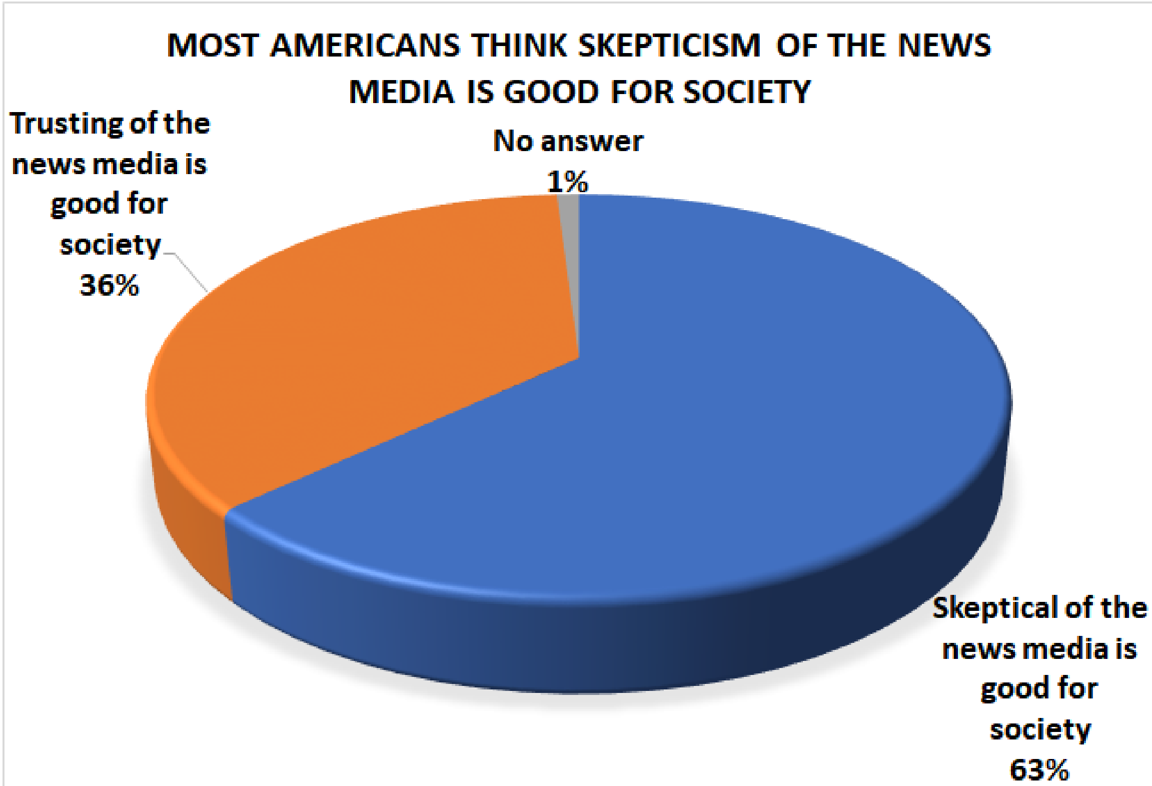

FIGURE 3.2: American perceptions of trust in news media and its societal impact
Source: Survey of U.S. adults conducted Feb 18 – March 2, 2020, Pew Research Center

landscape of trust in news media among economic agents in the United States. This challenges the assumption of earlier research (Morris and Shin, 2002; Nimark, 2014; and Chahrour, Nimark, and Pitschner, 2021) that modeled economic agents as uniformly processing and accepting public information in their decision-making processes. As illustrated in the figure, only 36 percent of American adults expressed trust in the news media, whereas a significantly higher percentage, 63 percent, responded that they are pessimistic about the news media's impact on society. In essence, this survey clearly demonstrates that there is a significant di-

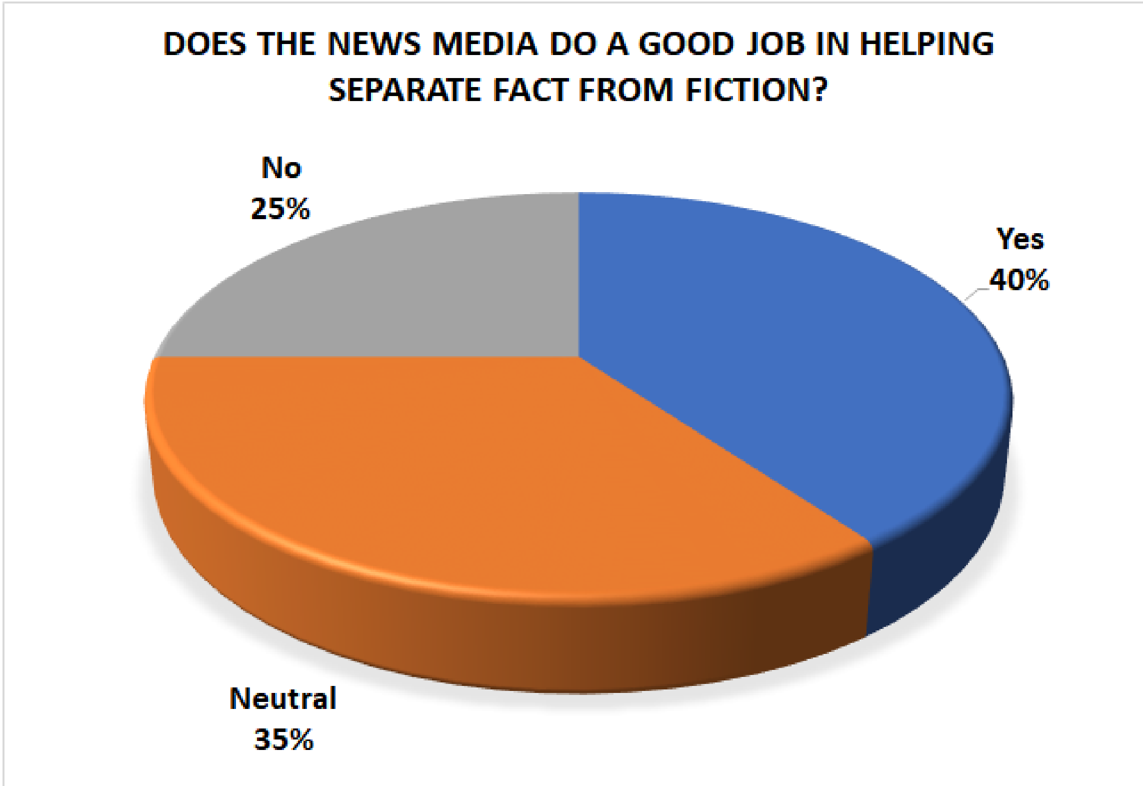

FIGURE 3.3: Public trust in news media: evaluating its role in fact-checking throughout nine nations (United States, UK, Ireland, Spain, Germany, Denmark, Australia, France, and Greece)
Source: Newman and Fletcher (2017)

versity in individuals' trust in the news.

Additionally, in Figure 3.3, Newman and Fletcher (2017) conducted a study through the Reuters Institute, surveying perceptions of public trust in news media across nine countries (United States, UK, Ireland, Spain, Germany, Denmark, Australia, France, and Greece). In this survey, 40 percent responded that they are doing well in evaluating its role in fact-checking, while a significant proportion, 25 percent, believed they were not. This survey demonstrates that belief polarization regarding the trustworthiness of news media is not confined to the United States

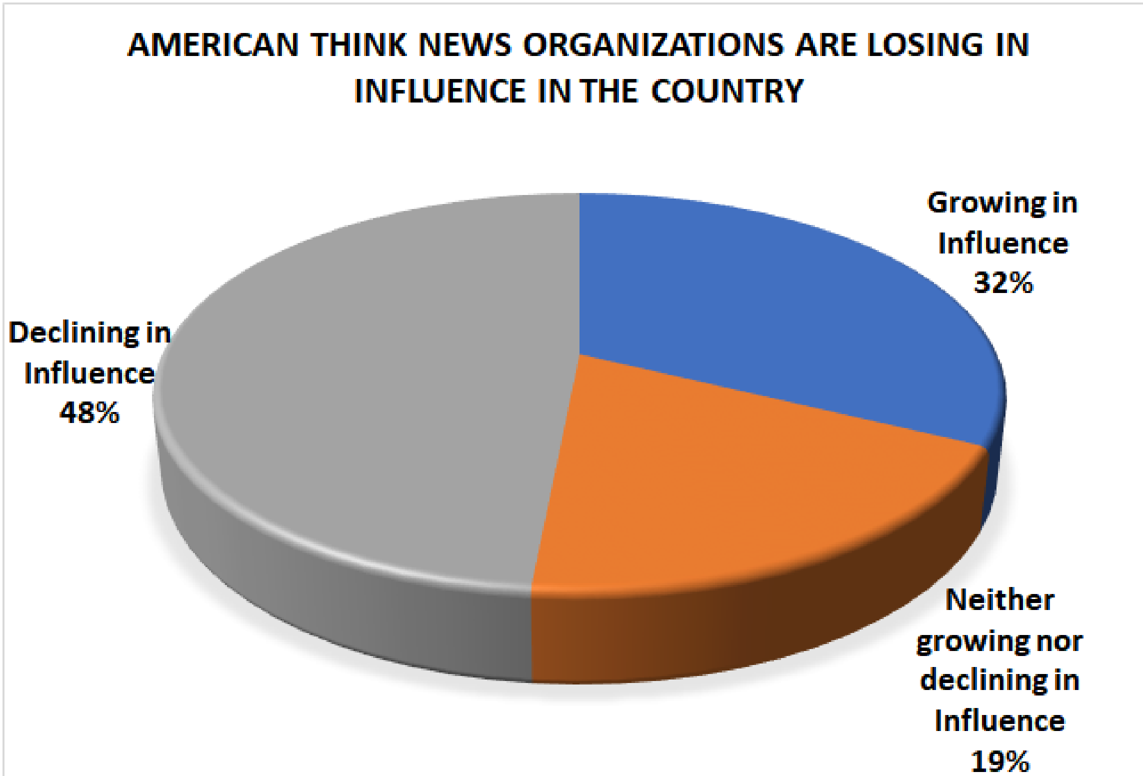

FIGURE 3.4: The influence of news media
Source: Survey of U.S. adults conducted Feb 18 – March 2, 2020, Pew Research Center

alone.

In Figure 3.4, the survey by the Pew Research Center goes beyond the issue of trust, showing that the influence of news media has been gradually weakening recently. These results could indeed be seen as stemming from a decline in trust in news media and a weakening of strategic complementarity due to belief polarization. In other words, the current situation is not so much that the accuracy of public information has become too high, causing a negative effect on welfare, but rather that opinions on public information are extremely divided or there is a notable lack of trust, presenting a basis for the new modeling discussed in this

Table 3.1: Trust of 30 media and polarization in the United States

	Trust	**Neutral**	**Distrust**
Republicans	22	0	8
Democrats	7	3	20

Source: Survey of US adults conducted Oct 29 – Nov 11, 2019. US Media Polarization and the 2020 Election: A Nation Divided by Pew Research Center.

paper.

Especially, the phenomenon of belief polarization is closely associated with issues of political ideology. The current landscape of news media consumption indicates that viewers may exhibit more partisan tendencies than previously thought when receiving public information. For instance, Table 3.1 highlights the potential for extreme polarization, showing how Americans' trust varies across 30 major US news media outlets.[2] According to a 2019 Pew Research Center survey, the political leanings of viewers or readers significantly influence their trust in news outlets, with Republicans and Democrats showing stark differences in whom they trust.

This polarization suggests that if economic agents come to trust only specific news sources, the issue could extend to economic perceptions and decisions. Even when presented with identical information, differing views on its relevance or accuracy can lead to varied interpretations of its economic implications. For example, the perceived threat of North Korean missile tests to the market might alarm some groups more than others. An announcement by North Korea of intent to

[2] The 30 news outlets include ABC News, BBC, Breitbart, Business Insider, Buzzfeed, CBS News, CNN, Daily Caller, Fox News, HuffPost, MSNBC, NBC News, New York Post, The New York Times, Newsweek, NPR, PBS, Politico, Rush Limbaugh Show (radio), Sean Hannity Show (radio), The Guardian, The Hill, Time, Univision, USA Today, Vice, Vox, The Wall Street Journal, Washington Examiner, and The Washington Post.

"turn Seoul into a sea of fire"[3] could cause significant market reactions among those who see it as a serious issue, while others might disregard it. This variability in response could depend on factors such as age, gender, occupation, and political beliefs, illustrating the widespread nature of differing perceptions based on the same information.

Moreover, even when different news media outlets report the same event, their narratives can vary dramatically. Gentzkow and Shapiro (2006) illustrate this with an example where Fox News, The New York Times, and Al Jazeera cover the same incident in Samarra, Iraq, but with markedly different tones and perspectives.[4] This divergence in reporting could lead to significantly different perceptions among viewers who exclusively follow one of these sources.

To summarize the above-described phenomena into two questions, it would be: (i) what are the reasons economic agents interpret their *same public information* differently, and (ii) what are the economic implications of such cases? Figure 3.5 briefly depicts this scenario, prompting a deeper investigation into how diverse interpretations of news media information among viewers can affect overall economic behavior and decisions. To explore this, the next section applies these concepts to the extended Beauty Contest model of Morris and Shin (2002)

[3] https://www.cnn.com/2011/11/24/world/asia/north-korea-sea-of-fire/index.html

[4] Gentzkow and Shapiro (2006) mentioned as follows:

"On December 2, 2003, American troops fought a battle in the Iraqi city of Samarra.
(1) Fox News began its coverage of the event with the following paragraph*: In one of the deadliest reported firefights in Iraq since the fall of Saddam Hussein's regime, US forces killed at least 54 Iraqis and captured eight others while fending off simultaneous convoy ambushes Sunday in the northern city of Samarra.*
(2) The New York Times article on the same event began*: American commanders vowed Monday that the killing of as many as 54 insurgents in this central Iraqi town would serve as a lesson to those fighting the United States, but Iraqis disputed the death toll and said anger against America would only rise.*
(3) And the English-language website of the satellite network Al Jazeera began*: The US military has vowed to continue aggressive tactics after saying it killed 54 Iraqis following an ambush, but commanders admitted they had no proof to back up their claims. The only corpses at Samarra's hospital were those of civilians, including two elderly Iranian visitors and a child."*

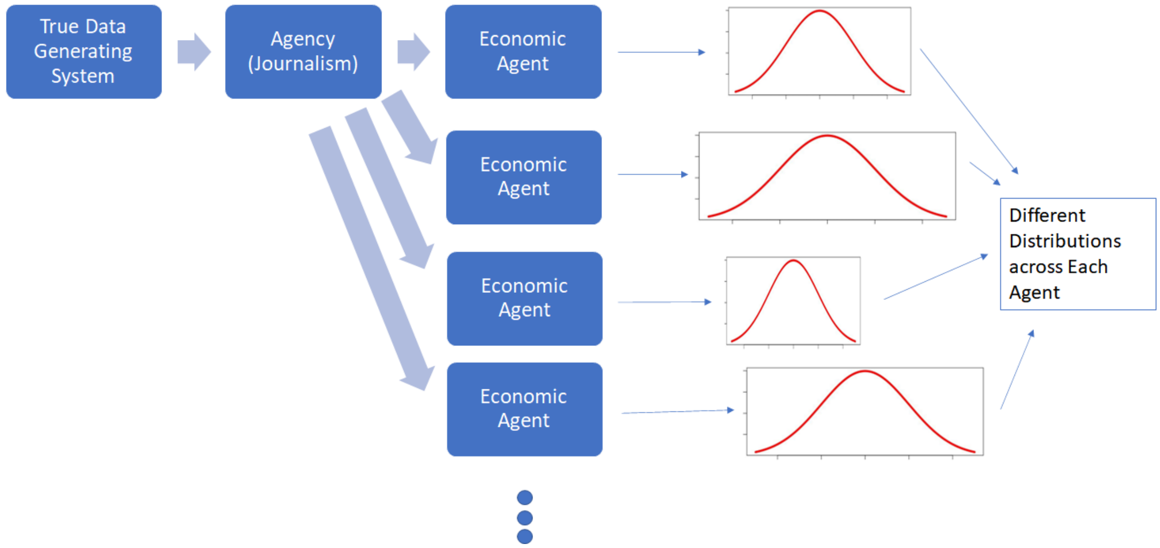

FIGURE 3.5: The process of interpreting the same information differently among agents

3.4 Social value of information with verification process for public information

As discussed earlier, this model predicts that society will fragment into two groups holding different posterior beliefs about public information following the process of verifying media credibility. Specifically, in this simplified model, agents diverge in their perceptions of the variance of public information after Bayesian updating. It is assumed that agents start with neutral priors regarding media credibility but encounter some errors during the verification process.

I outline the model setup and explore the linear rational equilibrium in the subsequent subsection. Following that, I examine how each agent's decisions are influenced by their access to public and private information. Initially, the error probability of news media credibility is treated as exogenous in this section, but it will later be extended to include an endogenous cost model within the same section.

Lastly, I discuss the impact on social welfare. The model primarily draws upon Morris and Shin (2002), which in turn, is inspired by the Beauty Contest example presented in Keynes (1936). Essentially, these models explore the dynamics of agent interaction under the influence of strategic complementarity.

3.4.1 Model

In this subsection, I will discuss the key assumptions of this model. First of all, there is a continuum of players as follows:

- **Players**
  - There is a continuum of (ex-ante) homogeneous players $i \in [0, 1]$.

They are all homogeneous before the verification process with errors. The exact verification process and private information structure that lead to differing post-hoc beliefs will be introduced later.

Second, the utility function and actions of each player are as follows:

- **Utility function**

$$u_i(a_i|\mathbf{a_{-i}}, x) \equiv -\underbrace{(1-r)(a_i - x)^2}_{\text{standard quadratic loss}} - \underbrace{r(L_i - \bar{L})}_{\text{beauty contest}} \tag{3.1}$$

where $a_i \in \mathbb{R}^{[0,1]}$, and r is a constant with $0 < r < 1$ and

$$L_i \equiv \int_0^1 (a_j - a_i)^2 dj,$$

$$\bar{L} \equiv \int_0^1 L_j dj.$$

$u_i(a_i|\mathbf{a_{-i}}, x)$ is a utility function of each player. $\mathbf{a_{-i}}$ is a vector including every action of all the players except for a_i. x can be interpreted as fundamental of the economy or underlying state. The first term in equation (3.1) measures standard quadratic loss as the distance between one's action and the underlying state. It can be interpreted as one's own interest in the game. In other words, each person wants to get close to the underlying state.

The 'beauty contest' component, as described by Morris and Shin (2002) within equation (3.1), introduces a motive for second-guessing, implying that players must consider the actions of others to maximize their utility. This element becomes zero when the actions of all players align perfectly.

However, this beauty contest term would not become zero in the end, and ultimately, agents should conjecture that any deviation in actions is quantified by the aggregate of the distances between each player's choices. For instance, if a player i selects action a_i, they must also account for L_i, representing the distance from every other player's choices. The collective deviation, $\bar{L}$, encompasses these individual discrepancies.

Additionally, the parameter r assigns importance to this second-guessing aspect. A value of r equal to zero indicates a lack of incentive for coordination among players, whereas an r value of one signifies that players prioritize minimizing the distance in their actions over the actual state x.

Third, I express social welfare as follows:

- **Social welfare**

$$W(\mathbf{a}, x) \equiv \frac{1}{1-r}\int_0^1 u_i(a_i|\mathbf{a_{-i}}, x)di$$
$$= -\int_0^1 (a_i - x)^2 di. \tag{3.2}$$

Social welfare is the normalized average of each player's utility as in Morris and Shin (2002). If there is a social planner, each a_i should be controlled going to close to the underlying state x.

Each player's action a_i is determined by the first order condition of each player as follows:

- **First order condition**

$$a_i = (1-r)E_i(x|I_i, s_i = k) + rE_i(\bar{a}|I_i, s_i = k) \tag{3.3}$$

where $\bar{a} \equiv \int_0^1 a_j dj$. and $k \in \{0, 1\}$.

The average action across all players is denoted by $\bar{a}$, while $E_i(x|I_i, s_i = k)$ represents player i's expected value of the underlying state x, based on their information set I_i, which includes both public and private signals, and the credibility signal s_i of the public information.

In a scenario where players have access to complete information, a rational decision-making process would lead them to choose an action $a_i = x$, aligning the social optimum with the equilibrium derived from individual actions, without the need for a social planner's intervention.

However, the presence of information friction, stemming from individual players facing distinct private signals and developing divergent posterior beliefs after evaluating the credibility of public information, can deviate outcomes from the socially optimal equilibrium.

Finally, I present the public and private information structure as follows:

- **Information structure**
  - Each player $i \in [0,1]$ observes a **private information**:

$$x_i = x + \xi_i \tag{3.4}$$

    where $\xi_i \perp x$, $\xi_i \sim N(0, \sigma_\xi^2)$, and $E(\xi_i \xi_j) = 0$ for $i \neq j$.
  - In addition, the players commonly observe a **public information**:

$$y = x + \eta \tag{3.5}$$

    where $\eta \perp x$, and $\eta \sim N(0, (\theta + \gamma(1-\theta))\sigma_\eta^2)$ where $\gamma > 1$ and $\theta \in \{0, 1\}$. ($\theta = 1$: News media is reliable, $\theta = 0$: News media is unreliable)
  - Each agent's prior belief for the variance of the public information is 0.5 for σ_η^2 and 0.5 for $\gamma\sigma_\eta^2$. In other words, $\Pr(\theta = 1) = \Pr(\theta = 0) = 0.5$ for all $i \in [0,1]$.
  - The precision of the public and the private information with prior beliefs:

$$\alpha = \frac{1}{(0.5 + 0.5\gamma)\sigma_\eta^2}, \quad \beta = \frac{1}{\sigma_\xi^2}.$$

  - The information set for i is as follows:

$$I_i = (y, x_i) \tag{3.6}$$

– Player i's expected value of x with prior beliefs is as follows:

$$E_i(x) = \frac{\alpha y + \beta x_i}{\alpha + \beta}. \tag{3.7}$$

Each player i bases their action on the information set I_i, which allows us to express one's action as $a_i(I_i, s_k)$. The actual state of affairs, denoted by x, a real number, is established before any actions are taken, yet it is shrouded in uncertainty. Although each player receives a private signal x_i, the precise value of x remains unknown to them. Therefore, they also turn to an information agency, news media, for an additional source of information, which provides a common public information y.

However, this public information y is also subject to noise, represented by η, as illustrated above. A more challenging issue arises from the players' uncertainty regarding the credibility of this information. Initially, it is assumed that players are somewhat skeptical about this credibility, holding half priors as previously mentioned. Therefore, they go through a stage of investigating whether the news media, providing the public information, they will receive is trustworthy. The next subsection will detail a process for verifying the credibility of public information more thoroughly.

3.4.2 Verification process for the public information

First of all, nature determines the state of the news media $\theta \in \{0, 1\}$. An agent cannot directly observe this state of the news media and their prior is $\pi = p(\theta = 1) = p(\theta = 0) = 0.5$. Instead of observing the real state, they receive one signal about news media credibility without any costs. I refer to the information structure defined by the error probability ϵ.

Table 3.2: Signals about news media and its errors

	$s = 1$	$s = 0$
$\theta = 1$	$1 - \epsilon$	ϵ
$\theta = 0$	ϵ	$1 - \epsilon$

$\theta = 1$: News media is reliable, $\theta = 0$: News media is unreliable

Definition 3.1. *A channel $S \in (0,1) \times (0,1)$ is defined by the error probabilities $\epsilon = p(s = 1|\theta = 0) = p(s = 0|\theta = 1)$ where $\epsilon \in [0, 0.5]$*

Table 3.2 shows how agents will have different signals by Definition 3.1. For the simplicity of the model, I assume every agent will have signals concerning media credibility *before receiving public information y*. With this information structure in Table 3.2, agents update their beliefs before taking action.

Lemma 3.1. *When a prior belief is $\pi = 0.5$, agents update their belief as follows by Bayesian updating:*

$$p(\theta = 1|s = 1) = p(\theta = 0|s = 0) = 1 - \epsilon \tag{3.8}$$

Then, I consider two distinct scenarios. The first involves a group of agents who receive the signal $s = 1$, indicating that the news media is considered reliable. Conversely, the second group is presented with the signal $s = 0$, suggesting skepticism towards media reliability. The following lemma delineates these two cases in detail.

Lemma 3.2. *Consider two cases for an agent i based on the received signal s_i as follows: (1) When an agent i receives $s_i = 1$,*

$$E_i(x|I_i, s_i = 1) = \frac{\alpha_1 y + \beta x_i}{\alpha_1 + \beta}$$

where $\alpha_1 = \dfrac{1}{[(1-\epsilon) + \gamma\epsilon]\sigma_\eta^2}$

(2) When an agent i receives $s_i = 0$,

$$E_i(x|I_i, s_i = 0) = \frac{\alpha_0 y + \beta x_i}{\alpha_0 + \beta}$$

where $\alpha_0 = \dfrac{1}{[\epsilon + \gamma(1-\epsilon)]\sigma_\eta^2}$

For an agent i to calculate $E_i(\bar{a}|I_i, s_i)$ in his first order condition to maximize his utility, he should conjecture how many agents will have same signals and how many agents will have different signals given his own signal.

Lemma 3.3. *Each agent conjectures as follows:*

$$p(s_j = 1|s_i = 1) = p(s_j = 0|s_i = 0) = 1 - f(\epsilon)$$
$$p(s_j = 0|s_i = 1) = p(s_j = 1|s_i = 0) = f(\epsilon)$$

for $i \neq j$ where $f(\epsilon) = -2\epsilon^2 + 2\epsilon$ and s_i for an agent i's signals.

With Lemma 3.3, an agent i can calculate $E_i(\bar{a}|I_i, s_i)$. In the next subsection, I solve for a unique equilibrium of the model showing how $E_i(\bar{a}|I_i, s_i)$ can be different by signals.

3.4.3 Linear equilibrium

In this subsection, I solve for a unique equilibrium of the model. I focus on a linear strategy of the form:

$$a_i(I_i, s_i = k) = \kappa_k x_i + (1 - \kappa_k) y \tag{3.9}$$

where $k \in \{0, 1\}$. It means that players who received $s = 1$ choose the strategy κ_1 while players who received $s = 0$ choose the strategy κ_0. In order to get κ_1 and κ_0, I need to solve $E_i(\bar{a}|I_i, s_i = k)$ for $k \in \{0, 1\}$.

Lemma 3.4. *Consider two cases for an agent i based on the received signal s_i:*
(1) When an agent i received $s_i = 1$,

$$E_i(\bar{a}|I_i, s_i = 1) = \kappa_1(1-f)\frac{\alpha_1 y + \beta x_i}{\alpha_1 + \beta} + \kappa_0 f \frac{\alpha_0 y + \beta x_i}{\alpha_0 + \beta} + (1-\kappa_1)(1-f)y + (1-\kappa_0) f y \tag{3.10}$$

(2) When an agent i received $s_i = 0$,

$$E_i(\bar{a}|I_i, s_i = 0) = \kappa_1 f \frac{\alpha_1 y + \beta x_i}{\alpha_1 + \beta} + \kappa_0(1-f)\frac{\alpha_0 y + \beta x_i}{\alpha_0 + \beta} + (1-\kappa_1) f y + (1-\kappa_0)(1-f) y \tag{3.11}$$

With Lemma 3.2 and Lemma 3.4, I can solve for an agent i's optimal action.

Proposition 3.1. *The linear equilibrium of action a_i, for $k \in \{0, 1\}$, is given by*

$$a_i(I_i, s_i = k) = \kappa_k x_i + (1 - \kappa_k) y \tag{3.12}$$

where $\kappa_1 = \dfrac{\beta(1-r) + rf\beta\frac{\alpha_1+\beta}{\alpha_0+\beta}\frac{\beta(1-r)}{\beta(1-r)+\alpha_0+rf\beta}}{\beta(1-r) + \alpha_1 + rf\beta - \frac{(rf\beta)^2}{\beta(1-r)+\alpha_0+rf\beta}}$ *if $s_i = 1$, and*

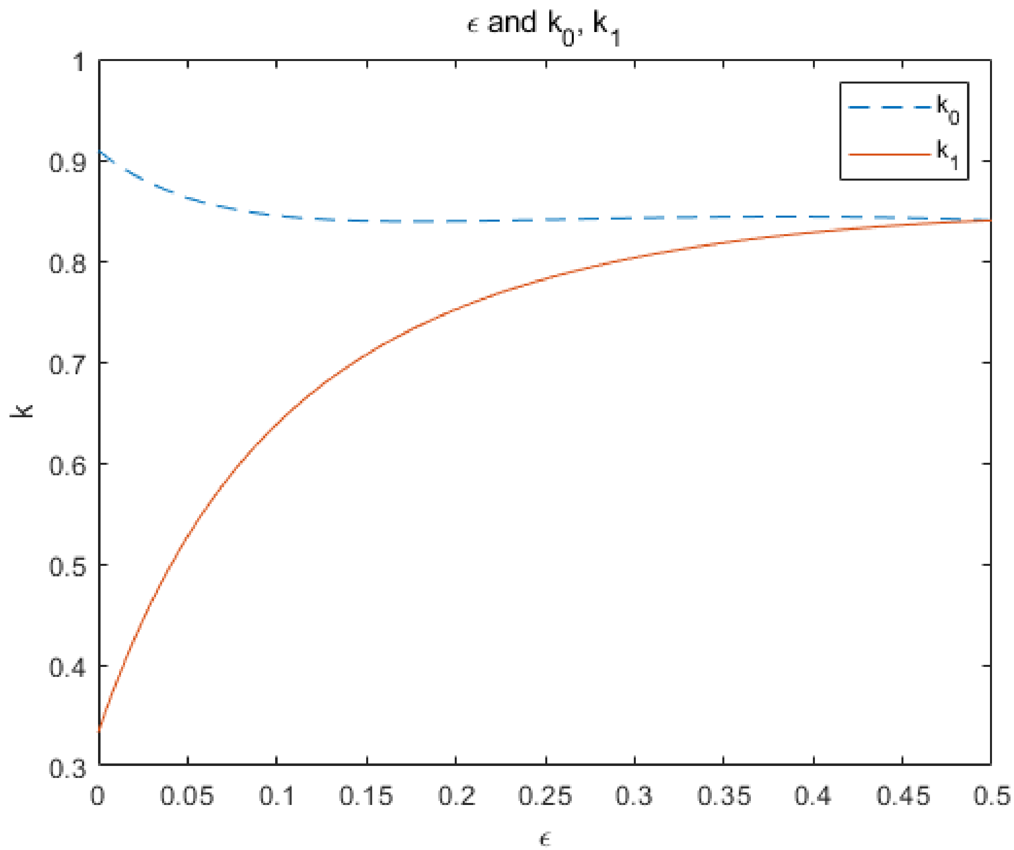

FIGURE 3.6: Responses of k_1 and k_0 as the change of error probability ϵ: Parameters are $\gamma = 20$, $\sigma_\eta = 1$, $\sigma_\epsilon = 1$, and r = 0.5

$$\text{where } \kappa_0 = \frac{\beta(1-r) + rf\beta\frac{\alpha_0+\beta}{\alpha_1+\beta}\frac{\beta(1-r)}{\beta(1-r)+\alpha_1+rf\beta}}{\beta(1-r)+\alpha_0+rf\beta-\frac{(rf\beta)^2}{\beta(1-r)+\alpha_1+rf\beta}} \text{ if } s_i = 0.$$

The implication of Proposition 3.1 is straightforward. The next proposition shows its implication.

Proposition 3.2. *Agents who receive the signal that public information is reliable assign more weight to the public information. That is,* $\kappa_1 \leqslant \kappa_0$.

Furthermore, numerical simulations reveal that, as the error probability increases, agents who receive the signal $s = 1$ (indicating that public information is reliable) tend to rely more on their private information. Conversely, for agents receiving $s = 0$, there appears to be a clash between the incentive to focus on private signals and the incentive to concentrate on public signals for coordination, leading to fluctuating tendencies. This outcome aligns with intuitions: (i) when faced with higher error probabilities, agents informed that public information is reliable are inclined to trust it less, as illustrated in Figure 3.6 and (ii) regardless of the signal received, as the error probability approaches 0.5, κ_0 and κ_1 become equivalent. This reflects a scenario where no verification of news media is performed, and regardless of the signal received, the probability becomes one-half, thus rendering the signals themselves inconsequential.

3.4.4 Welfare effects of public information and verification process

In this subsection, I study the welfare implications of public information and the verification process. To begin with, a comparison between the ex-ante expected welfare, absent the verification process for news media, and the social planner's perspective, incorporating the verification process, seems prudent.

The welfare of the ex-ante expected welfare is as follows:

$$E_p[W(\mathbf{a}, x)|x] = -\left[(1 - \kappa_{prior})^2 E(\eta^2) + \kappa_{prior}^2 E(\xi_i^2)\right]$$

Considering that the social planner, like any other agent in this economy, lacks precise knowledge of the true value of x, it is assumed that the social planner also has a prior belief, with a probability of 0.5, regarding the state of the news media.

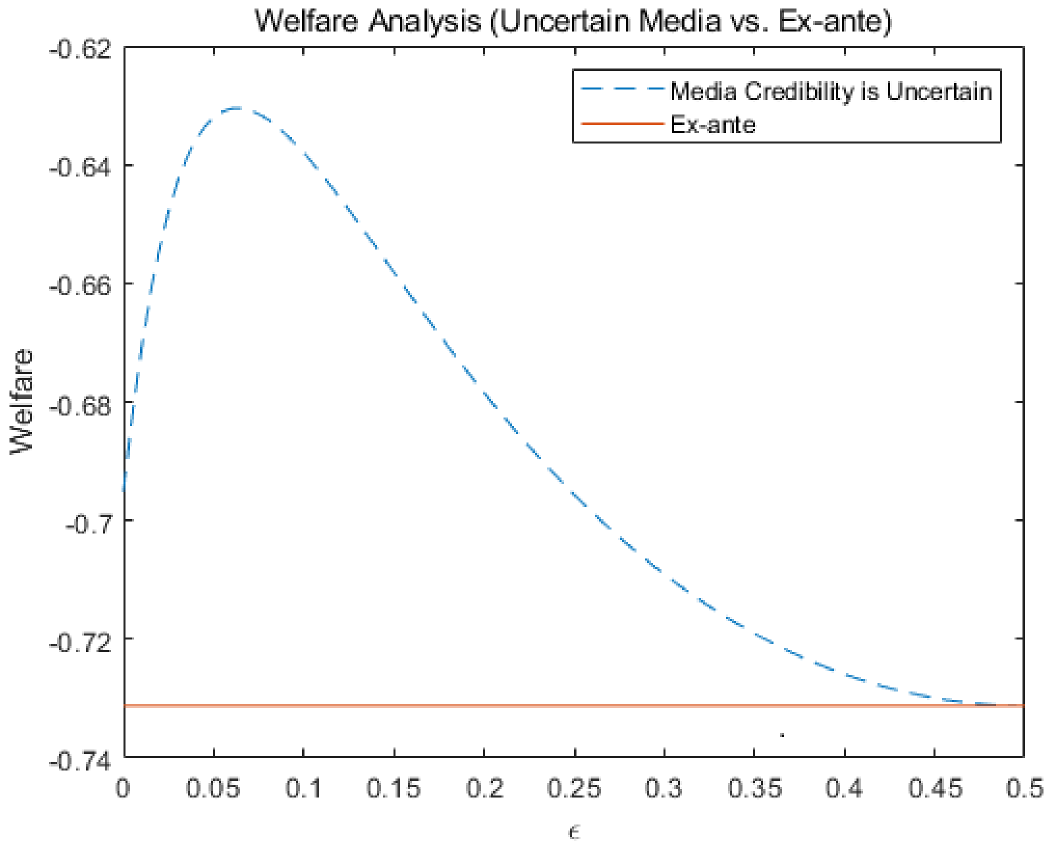

FIGURE 3.7: Welfare analysis when “the social planner does not exactly know the state of news media” vs. “ex-ante expected welfare”: Parameters are $\gamma = 20$, $\sigma_\eta = 1$, $\sigma_\xi = 1$, r = 0.5, and prior belief of the social planner is 0.5

It is given by:

$$E_p[W(\mathbf{a}, x)|x] = -0.5\left[(1-\kappa_1)^2 E(\eta^2) + \kappa_1^2 E(\xi_i^2)\right] - 0.5\left[(1-\kappa_0)^2 E(\eta^2) + \kappa_0^2 E(\xi_i^2)\right]$$

When the parameter of unreliable news media γ is small, the social planner’s expected welfare decreases as the divergence between κ_1 and κ_0 increases. Therefore, in such circumstances, the planner prefers scenarios where there is no verification process for public information. However, when γ is sufficiently large, the verification process consistently benefits society compared to the ex-ante expected

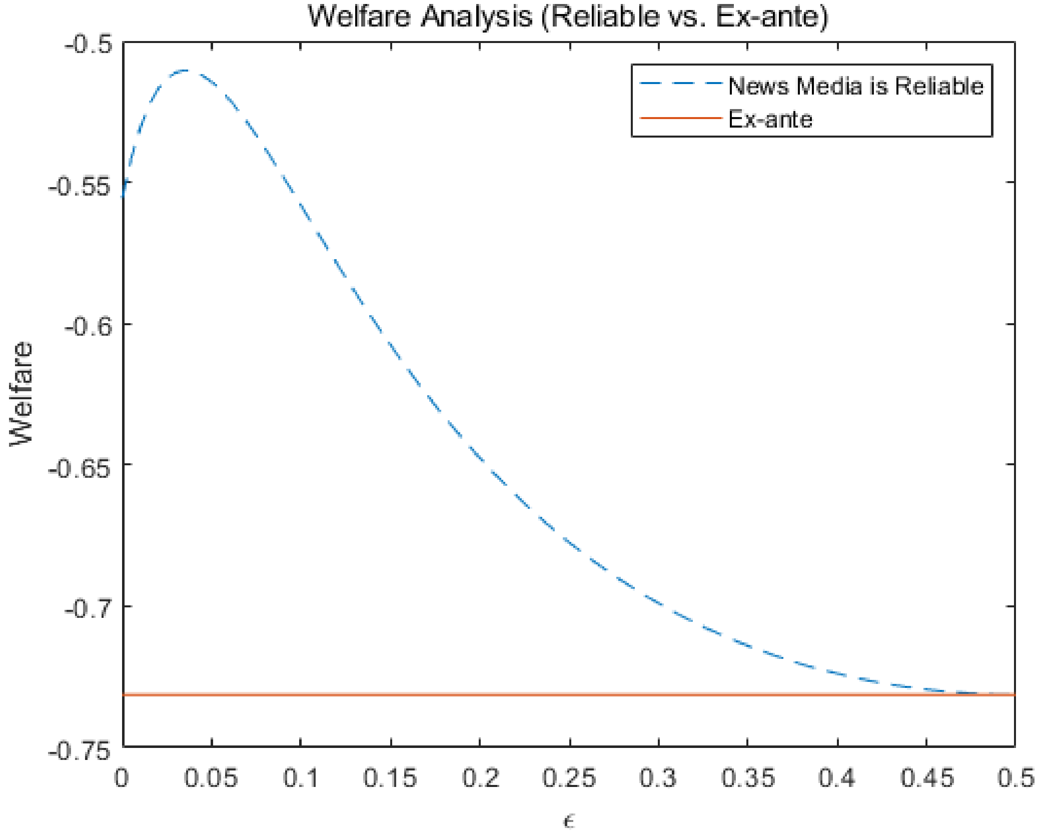

FIGURE 3.8: Welfare analysis when "the social planner knows the state of news media (reliable)" vs. "ex-ante expected welfare": Parameters are $\gamma = 20$, $\sigma_\eta = 1$, $\sigma_\xi = 1$, and r = 0.5

welfare. This is depicted in Figure 3.7, where the blue dotted line represents scenarios with the verification process, and the red solid line represents scenarios without it.

Moreover, consider a second scenario where the planner can observe the true state of the public information. This scenario further divides into two distinct cases

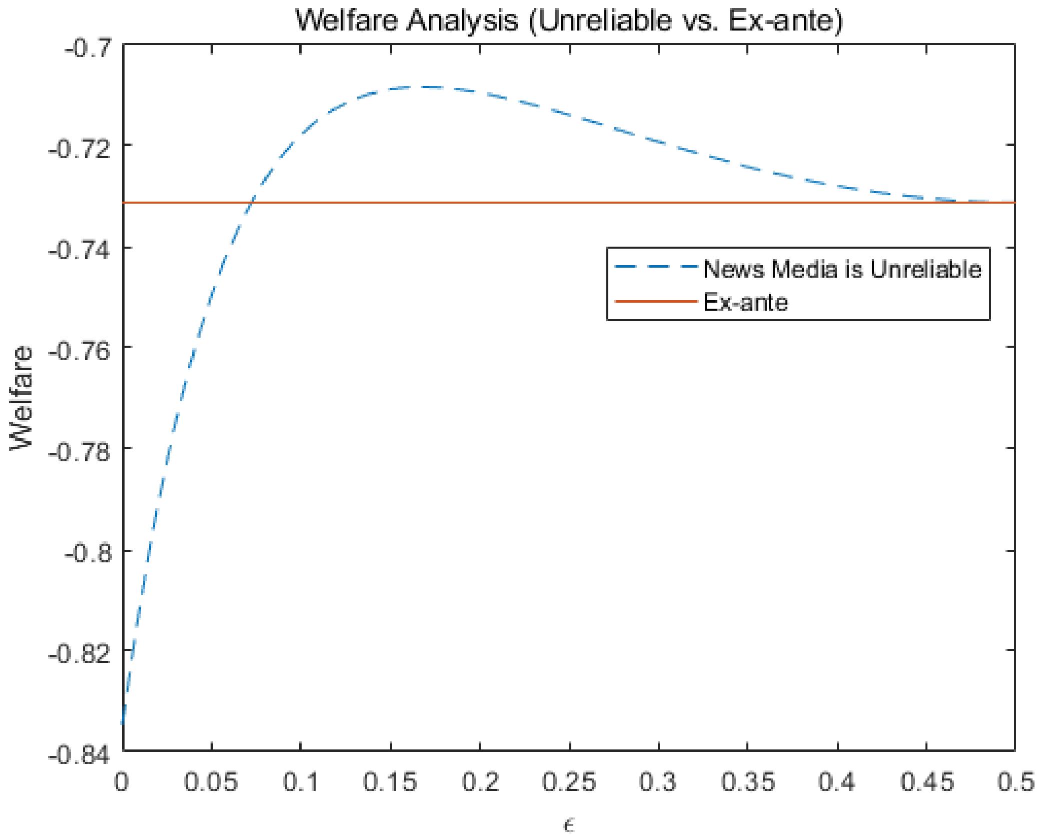

FIGURE 3.9: Welfare analysis when "the social planner knows the state of news media (unreliable)" vs. "ex-ante expected welfare": Parameters are $\gamma = 20$, $\sigma_\eta = 1$, $\sigma_\xi = 1$, and r = 0.5

as follows:

$$
\begin{aligned}
E_p[W(\mathbf{a}, x)|x, \theta = 1] = &-(1-\epsilon)\left[(1-\kappa_1)^2 E(\eta^2) + \kappa_1^2 E(\xi_i^2)\right] \\
&- \epsilon\left[(1-\kappa_0)^2 E(\eta^2) + \kappa_0^2 E(\xi_i^2)\right] \\
E_p[W(\mathbf{a}, x)|x, \theta = 0] = &-\epsilon\left[(1-\kappa_1)^2 E(\eta^2) + \kappa_1^2 E(\xi_i^2)\right] \\
&- (1-\epsilon)\left[(1-\kappa_0)^2 E(\eta^2) + \kappa_0^2 E(\xi_i^2)\right]
\end{aligned}
$$

If the news media is indeed reliable ($\theta = 1$) and the social planner is aware of this, the verification process consistently results in higher welfare compared to the ex-ante model, as shown in Figure 3.8. In this case, investigating the news media in advance always results in better welfare than not investigating at all.

Conversely, if the news media is unreliable ($\theta = 0$) and the social planner is aware of this, the *meticulous* verification process can detrimentally affect welfare because excessive coordination drives agents' actions away from the fundamentals, as illustrated in Figure 3.9. This region corresponds with the scenario described by Morris and Shin (2002) where increasing the accuracy of public information can impair welfare. However, if there is a significant information cost associated with the agent's verification process, moving away from an error probability ϵ of zero, this area decreases.

In other words, in cases where there is a verification process and belief polarization regarding the reliability of news media providing public information, it can be concluded that generally, moving towards increasing the accuracy of public information tends to improve welfare.

3.4.5 Adding endogenous information cost

In this subsection, I add an endogenous cost to the model presented in the previous section and examine its implications. The steps of the game remain the same as in the previous model. That is, in the first step, an investigation into the credibility of the news media is conducted, where more detailed information about whether the news media is reliable or not is obtained. However, unlike the previous model, the cost of information must be paid for this investigation, and paying more for information costs can reduce the error probability of the signal. Then, in the second step, as before, an action is chosen.

In this scenario, the utility function is modified as follows:s:

$$u_i(a_i|\mathbf{a_{-i}}, x, \epsilon) \equiv - \underbrace{(1-r)(a_i - x)^2}_{\text{standard quadratic loss}} - \underbrace{r(L_i - \bar{L})}_{\text{beauty contest}} + \underbrace{\lambda \log(2\epsilon)}_{\text{information cost}} \tag{3.13}$$

where $\lambda > 0$ and $\epsilon \in [0, 0.5]$. In this formulation, the utility function indicates that, when the error probability ϵ remains at the prior of 0.5, the information cost is 0, and as ϵ approaches 0, the cost becomes infinitely large, implying that it is impossible to reduce the error probability to zero completely.

The form of the solution for a_i remains similar to the model discussed earlier because, in the second step, ϵ is considered given for calculation. The only difference here is the possibility of reducing ϵ in the first step.

Given that we already know the solution for a_i, I can insert the solution into the above equation and derive the first-order condition for ϵ. The challenge arises when determining the policy function for ϵ as it results in the form of an implicit function, making it impossible to find an explicit solution. Also, it is impossible to analytically determine the direction of ϵ as r changes because of the complexity of the system.

However, by comparing the values of ϵ at $r = 0$ and $r = 1$ or through numerical simulations, we can obtain some idea of the situation. Figure 3.10 presents the numerical simulation results for a specific scenario graphically.

In Figure 3.10, we can observe that, as the strategic complementarity term r increases, the error rate in news media credibility gradually increases. In an extreme case, when $r = 1$, agents leave ϵ at 0.5. From several simulation results, as γ decreases, this monotonically increasing behavior disappears, and areas of increase and decrease appear non-linearly. However, generally, an increasing tendency is observed as γ increases.[5] Furthermore, the following proposition provides us with

[5] It does not necessarily increase monotonically.

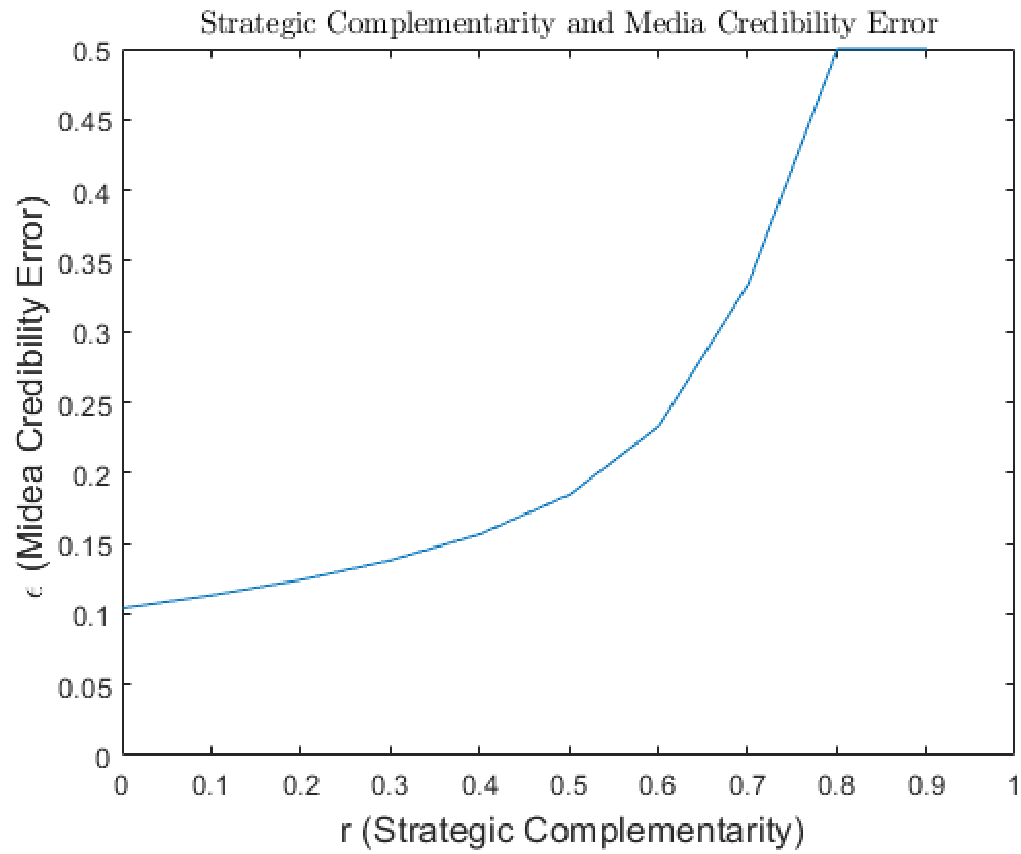

FIGURE 3.10: The optimal error, ϵ, to the strategic complementary term, r: Parameters are $\gamma = 20$, $\sigma_\eta = 1$, $\sigma_\xi = 1$, and $\lambda = 0.1$

greater insight.

Proposition 3.3. *When agents place a weight of 1 on the strategic complementarity term, r, they do not investigate the credibility of news media at all. That is,* $\epsilon(r = 1) = 0.5$.

The proof of Proposition 3.3 is straightforward. Since agents do not concern themselves with the fundamental x but only with the average action of others, they simply adopt the public information y as it is, even though this public information may be highly uncertain. In other words, high strategic complementarity undermines the benefit of enhancing accuracy by investigating the news media. Conversely, when r is not equal to 1, that is, when moving closer to the fundamental provides some utility, they become incentivized to investigate the news media.

These results effectively moderate the main findings of Morris and Shin (2002). Their principal conclusion was that, as the term of strategic complementarity increases, the precision of public information could have a negative impact on welfare. In this analysis, instead of choosing to increase the accuracy of public information by paying an information cost, agents opt to forego accuracy, maintaining ϵ at 0.5 and seeking coordination among themselves. This implies that agents do not trust any signal they receive, meaning this approach generally leads to a decrease in welfare due to overly inaccurate public information, as compared to the welfare analysis in the previous subsection.

Additionally, even when assuming that the information from the news media is completely accurate, $\theta = 1$, endogenous information costs lead agents to choose a significantly larger ϵ than zero. This is because an ϵ of zero would incur infinite information costs. Hence, there is an incentive for agents to endogenously make news media more uncertain, which slightly modifies the results of Morris and Shin (2002).

The key implication of this analysis is that, in cases of significant strategic complementarity, individual agents may not have an incentive to investigate the news media, but government intervention trying to increase the accuracy of public information can increase welfare. Therefore, in this scenario, enhancing the accuracy of public information can still be beneficial to society.

In conclusion, while empirical research is needed to determine where we currently stand, if belief polarization is indeed significant, policymakers should work to increase the accuracy of public information.

3.5 Conclusion

This paper explores the intricate interplay between the credibility of news media, information friction, and strategic complementarity in an economic context. By incorporating a news media verification process into the Beauty Contest model and analyzing based on an endogenous information cost model, it reveals how economic agents perceive the reliability of public information, the resulting decision-making process, and its impact on social welfare.

This research begins with the exploration of belief polarization phenomena, prominently observed during periods of uncertainty, such as the COVID-19 pandemic. If we indeed live in an era of significant uncertainty and pronounced belief polarization, this paper suggests a possibility that contradicts the insights of Morris and Shin (2002). It proposes that enhancing the accuracy of public information could almost always improve social welfare, by reducing the region in which public information decreases welfare.

The addition of the endogenous cost model brings a nuanced layer to the narrative. This illustrates that, in the absence of strategic complementarity, agents who were initially motivated to uncover the truth might collectively opt for less investigative actions due to the information costs associated with strategic complementarity. This phenomenon can lead to a societal equilibrium where the credibility of news media, providing public information, is undervalued, undermining the potential welfare gains from more accurate information dissemination and exacerbating belief polarization.

Exploring the welfare implications of these dynamics reveals the crucial role of social planners in mitigating the negative effects of information friction and strategic behaviors. The analysis suggests that, in specific scenarios, governmental measures to boost the trustworthiness of news media can notably improve social welfare. This highlights the importance of securing information reliability in the modern era of uncertain media credibility, presenting new challenges and opportunities for economic policy and research. This research lays the groundwork for a deeper understanding of the role of news media and the quality of information on economic outcomes.

However, this paper does not fully reveal whether we are currently at a point where enhancing the accuracy of public information is necessary. Determining where we stand requires thorough empirical research, which remains a task for future work.

4 Conclusion

This dissertation contains two essays related to macroeconomics and information theory. Both essays share a common focus on the recipients of information, rather than the providers. That is, despite efforts by providers to accurately convey information, distortions arise due to the costs associated with analyzing information on the part of the recipients.

In Chapter 2, I investigate the dynamics of attention reallocation among international investors to understand the fundamental causes of financial contagion, using the 1997 Asian Financial Crisis as a primary example of how a crisis can globally propagate. It utilizes a theoretical framework based on the canonical rational inattention model, demonstrating that increased volatility in one asset not only heightens attention to that asset but also to an uncorrelated asset, due to an increased subjective posterior correlation, challenging the assumptions of fixed capacity models.

Through an analysis of Financial Times news articles during the 1997 Asian Financial Crisis, the study finds that a volatility shock in the Asian market not only heightened investor attention towards the Asian markets but also towards uncorrelated markets such as Latin America. This shift in attention is significant; however, the paper refrains from directly linking these shifts to equilibrium asset returns, pointing out the necessity for further research to include endogenous portfolio weights for a comprehensive understanding of volatility spillover effects and equilibrium returns. However, more importantly, theoretical results of the volatility spillover effects can be broadly applicable (e.g., firm price setting, investment, or consumption/saving decisions).

In Chapter 3, I study how the credibility of news media, information friction, and strategic complementarity interact within an economic framework, particularly highlighting the impact of a news media verification process on agents' perceptions of information reliability and social welfare. It posits that in times of uncertainty, such as the COVID-19 pandemic, improving the accuracy of public information might counteract belief polarization and enhance social welfare. The study introduces an endogenous information cost model to explain why agents might reduce their efforts to seek truth due to strategic complementarity, potentially leading to a societal undervaluation of news media credibility and worsening belief polarization.

The research underscores the importance of governmental intervention in enhancing news media trustworthiness or accuracy of public information to improve social welfare in an era marked by uncertain news media credibility. However, it also acknowledges the need for further empirical research to conclusively determine our stance, setting a direction for future investigations into the interplay between news media credibility and economic outcomes.

Appendix A. (Appendix for Chapter 2)

A.1 Closed-form solutions to Problem 2.1 for $\rho_\theta = \rho_z \in [0, 1)$

From Lemma 2.1, first order conditions are stated as follows:

$$
\begin{aligned}
&-1-\frac{\beta\omega\rho_\theta^2\left(\rho_z^2\hat{\sigma}_z^2+\sigma_z^2\right)}{\left(\left(\rho_\theta^2\hat{\sigma}_\theta^2+\sigma_\theta^2\right)\left(\rho_z^2\hat{\sigma}_z^2+\sigma_z^2\right)-\rho_\theta^2\rho_z^2\hat{\sigma}_{\theta,z}^2\right)}+\frac{\omega\hat{\sigma}_z^2}{\hat{\sigma}_\theta^2\hat{\sigma}_z^2-\hat{\sigma}_{\theta,z}^2}\\
&\quad+\lambda\left(\beta\rho_\theta^2-1\right)\left(\rho_z^2\hat{\sigma}_z^2+\sigma_z^2-\hat{\sigma}_z^2\right)=0,\\
&-1-\frac{\beta\omega\rho_z^2\left(\rho_\theta^2\hat{\sigma}_\theta^2+\sigma_\theta^2\right)}{\left(\left(\rho_\theta^2\hat{\sigma}_\theta^2+\sigma_\theta^2\right)\left(\rho_z^2\hat{\sigma}_z^2+\sigma_z^2\right)-\rho_\theta^2\rho_z^2\hat{\sigma}_{\theta,z}^2\right)}+\frac{\omega\hat{\sigma}_\theta^2}{\hat{\sigma}_\theta^2\hat{\sigma}_z^2-\hat{\sigma}_{\theta,z}^2}\\
&\quad+\lambda\left(\beta\rho_z^2-1\right)\left(\rho_\theta^2\hat{\sigma}_\theta^2+\sigma_\theta^2-\hat{\sigma}_\theta^2\right)=0,\\
&-1+\frac{\beta\omega\rho_\theta^2\rho_z^2\hat{\sigma}_{\theta,z}}{\left(\left(\rho_\theta^2\hat{\sigma}_\theta^2+\sigma_\theta^2\right)\left(\rho_z^2\hat{\sigma}_z^2+\sigma_z^2\right)-\rho_\theta^2\rho_z^2\hat{\sigma}_{\theta,z}^2\right)}-\frac{\omega\hat{\sigma}_{\theta,z}}{\hat{\sigma}_\theta^2\hat{\sigma}_z^2-\hat{\sigma}_{\theta,z}^2}\\
&\quad+\lambda\left(-\beta\rho_\theta^2\rho_z^2+\beta\rho_\theta\rho_z+\rho_\theta\rho_z-1\right)\hat{\sigma}_{\theta,z}=0,\\
&\quad\left(\rho_\theta^2\hat{\sigma}_\theta^2+\sigma_\theta^2-\hat{\sigma}_\theta^2\right)\left(\rho_z^2\hat{\sigma}_z^2+\sigma_z^2-\hat{\sigma}_z^2\right)-\left(\rho_\theta\rho_z-1\right)^2\hat{\sigma}_{\theta,z}^2=0.
\end{aligned}
$$

When $\rho_\theta = \rho_z \neq 0$,

$$
\hat{\sigma}_\theta^2=-\frac{\sigma_\theta^2\left(2\sigma_z^4\rho_\theta^2+\sigma_\theta^2(-1+3\rho_\theta^2)\sigma_z^2+(-1+\rho_\theta^2)\sigma_\theta^2\left((-1+\beta)\rho_\theta^2\omega+\sigma_\theta^2+\sqrt{\sigma_\theta^4+(-1+\beta)^2\rho_\theta^4\omega^2+2(1+\beta)\rho_\theta^2\omega\sigma_z^2+\sigma_z^4+2\sigma_\theta^2((1+\beta)\rho_\theta^2\omega+\sigma_z^2)}\right)\right)}{2\rho_\theta^2(-1+\rho_\theta^2)(\sigma_\theta^2+\sigma_z^2)^2},
$$

$$
\hat{\sigma}_z^2=-\frac{\sigma_z^2\left(2\sigma_\theta^4\rho_\theta^2+\sigma_\theta^2(-1+3\rho_\theta^2)\sigma_z^2+(-1+\rho_\theta^2)\sigma_z^2\left((-1+\beta)\rho_\theta^2\omega+\sigma_z^2+\sqrt{\sigma_\theta^4+(-1+\beta)^2\rho_\theta^4\omega^2+2(1+\beta)\rho_\theta^2\omega\sigma_z^2+\sigma_z^4+2\sigma_\theta^2((1+\beta)\rho_\theta^2\omega+\sigma_z^2)}\right)\right)}{2\rho_\theta^2(-1+\rho_\theta^2)(\sigma_\theta^2+\sigma_z^2)^2},
$$

$$
\hat{\sigma}_{\theta,z}=\frac{\sigma_\theta^2\sigma_z^2\left(\sigma_\theta^2(1+\rho_\theta^2)+(1+\rho_\theta^2)\sigma_z^2-(-1+\rho_\theta^2)\left((-1+\beta)\rho_\theta^2\omega-\sqrt{\sigma_\theta^4+(-1+\beta)^2\rho_\theta^4\omega^2+2(1+\beta)\rho_\theta^2\omega\sigma_z^2+\sigma_z^4+2\sigma_\theta^2((1+\beta)\rho_\theta^2\omega+\sigma_z^2)}\right)\right)}{2\rho_\theta^2(-1+\rho_\theta^2)(\sigma_\theta^2+\sigma_z^2)^2}.
$$

When $\rho_\theta = \rho_z = 0$,

$$\hat{\sigma}_\theta^2 = \frac{\sigma_\theta^2(\sigma_\theta^2\omega + \sigma_\theta^2\sigma_z^2 + \sigma_z^4)}{(\sigma_\theta^2 + \sigma_z^2)^2},$$

$$\hat{\sigma}_z^2 = \frac{\sigma_z^2(\sigma_\theta^4 + \sigma_\theta^2\sigma_z^2 + \omega\sigma_z^2)}{(\sigma_\theta^2 + \sigma_z^2)^2},$$

$$\hat{\sigma}_{\theta,z}^2 = -\frac{\sigma_\theta^2\sigma_z^2(\sigma_\theta^2 - \omega + \sigma_z^2)}{(\sigma_\theta^2 + \sigma_z^2)^2}.$$

A.2 Proofs

Proof of Proposition 2.1. (1) I shall extend the proof to a more general scenario, where $\rho_\theta \in [0,1)$ and $\rho_z \in [0,1)$, as opposed to the specific instance where $\rho_\theta = \rho_z = 0$. This approach allows for a broader examination of the underlying principles. (2) According to Proposition 1 by Miao, Wu, and Young (2022), the dimensionality of $C'V^{-1}C$ is indicative of the dimensionality of the optimal signal structure. Then, from Lemma 2.1:

$$A = \begin{bmatrix} \rho_\theta & 0 \\ 0 & \rho_z \end{bmatrix}, W = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}.$$

Then, without loss of generality, C and V can be normalized as:

$$C = \begin{bmatrix} 1 & c_1 \\ 1 & c_2 \end{bmatrix}, V = \begin{bmatrix} v_1 & v_2 \\ v_2 & v_3 \end{bmatrix}.$$

Then,

$$
\begin{aligned}
C'V^{-1}C &= \Sigma^{-1} - (A\Sigma A' + W)^{-1} \\
&= \frac{1}{v_1 v_3 - v_2^2} \begin{bmatrix} v_1 - 2v_2 + v_3 & v_3 c_1 - v_2 c_1 + v_1 c_2 - v_2 c_2 \\ v_3 c_1 - v_2 c_1 + v_1 c_2 - v_2 c_2 & v_3 c_1^2 - 2v_2 c_1 c_2 + v_1 c_2^2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{\hat{\sigma}_z^2}{\hat{\sigma}_\theta^2 \hat{\sigma}_z^2 - \hat{\sigma}_{\theta,z}^2} - \frac{\rho_z^2 \hat{\sigma}_z^2 + \sigma_z^2}{(\rho_\theta^2 \hat{\sigma}_\theta^2 + \sigma_\theta^2)(\rho_z^2 \hat{\sigma}_z^2 + \sigma_z^2) - \rho_\theta^2 \rho_z^2 \hat{\sigma}_{\theta,z}^2} & \frac{-\hat{\sigma}_{\theta,z}}{\hat{\sigma}_\theta^2 \hat{\sigma}_z^2 - \hat{\sigma}_{\theta,z}^2} + \frac{\rho_\theta \rho_z \hat{\sigma}_{\theta,z}}{(\rho_\theta^2 \hat{\sigma}_\theta^2 + \sigma_\theta^2)(\rho_z^2 \hat{\sigma}_z^2 + \sigma_z^2) - \rho_\theta^2 \rho_z^2 \hat{\sigma}_{\theta,z}^2} \\ \frac{-\hat{\sigma}_{\theta,z}}{\hat{\sigma}_\theta^2 \hat{\sigma}_z^2 - \hat{\sigma}_{\theta,z}^2} + \frac{\rho_\theta \rho_z \hat{\sigma}_{\theta,z}}{(\rho_\theta^2 \hat{\sigma}_\theta^2 + \sigma_\theta^2)(\rho_z^2 \hat{\sigma}_z^2 + \sigma_z^2) - \rho_\theta^2 \rho_z^2 \hat{\sigma}_{\theta,z}^2} & \frac{\hat{\sigma}_\theta^2}{\hat{\sigma}_\theta^2 \hat{\sigma}_z^2 - \hat{\sigma}_{\theta,z}^2} - \frac{\rho_\theta^2 \hat{\sigma}_\theta^2 + \sigma_\theta^2}{(\rho_\theta^2 \hat{\sigma}_\theta^2 + \sigma_\theta^2)(\rho_z^2 \hat{\sigma}_z^2 + \sigma_z^2) - \rho_\theta^2 \rho_z^2 \hat{\sigma}_{\theta,z}^2} \end{bmatrix}.
\end{aligned}
$$

In the given context, if $\frac{v_1 - 2v_2 + v_3}{v_3 c_1 - v_2 c_1 + v_1 c_2 - v_2 c_2} = \frac{v_3 c_1 - v_2 c_1 + v_1 c_2 - v_2 c_2}{v_3 c_1^2 - 2v_2 c_1 c_2 + v_1 c_2^2}$, it follows algebraically that $c_1 = c_2$ and, consequently, the signal dimension is one. To show this, it is sufficient to show that the dimension of $\Sigma^{-1} - (A\Sigma A' + W)^{-1}$ is one. That is, if the determinant of $\Sigma^{-1} - (A\Sigma A' + W)^{-1}$ is zero, then the proof is complete. The numerator of the determinant is given by:

$$(\rho_\theta^2 \hat{\sigma}_\theta^2 + \sigma_\theta^2 - \hat{\sigma}_\theta^2)(\rho_z^2 \hat{\sigma}_z^2 + \sigma_z^2 - \hat{\sigma}_z^2) - (\rho_\theta \rho_z - 1)^2 \hat{\sigma}_{\theta,z}^2.$$

This equation, being one of the first order conditions from Appendix A.1, should equal zero. □

Proof of Proposition 2.2. Considering the Riccati equation for the full information case about the target variable:

$$\Sigma_{-1} = A\Sigma_{-1}A' + W - A\Sigma_{-1}H'(H\Sigma_{-1}H')^{-1}H\Sigma_{-1}A',$$

we can derive the optimal steady-state posterior covariance matrix, Σ. □

Proof of Proposition 2.3. By the Implicit Function Theorem and Cramer's rule from steady-state optimal conditions, I can show

$$\frac{\partial \hat{\sigma}_\theta^2}{\partial \omega} \geqslant 0, \frac{\partial \hat{\sigma}_z^2}{\partial \omega} \geqslant 0, \text{and } \frac{\partial \hat{\sigma}_{\theta,z}^2}{\partial \omega} \leqslant 0.$$

These three inequalities directly show the proposition. $\square$

Proof of Lemma 2.3. We begin the proof with an algebraic assertion that holds generally:

$$\begin{aligned}\mathbb{I}((\theta^t, z^t); s^t | s^{t-1}) &= 0.5 \log \left(\frac{|\Sigma_{t|t-1}|}{|\Sigma_{t|t}|} \right) \\ &= 0.5 \left(\log \frac{\hat{\sigma}^2_{\theta,t|t-1}}{\hat{\sigma}^2_{\theta,t|t}} + \log \frac{\hat{\sigma}^2_{z,t|t-1}}{\hat{\sigma}^2_{z,t|t}} + \log \frac{1 - \hat{\rho}^2_{t|t-1}}{1 - \hat{\rho}^2_{t|t}} \right).\end{aligned}$$

Applying the chain rule of mutual information[1] yields:

$$\mathbb{I}((\theta^t, z^t); s^t | s^{t-1}) = \mathbb{I}(\theta^t; s^t | s^{t-1}) + \mathbb{I}(z^t; s^t | \theta^t, s^{t-1}).$$

Consequently, we deduce:

$$\mathbb{I}(\theta^t; s^t | s^{t-1}) = \mathbb{I}((\theta^t, z^t); s^t | s^{t-1}) - \mathbb{I}(z^t; s^t | \theta^t, s^{t-1}).$$

The last term exclusively represents the *direct* reduction in uncertainty about z^t upon observing s^t. Hence:

$$\begin{aligned}\mathbb{I}(\theta^t; s^t | s^{t-1}) &= 0.5 \left(\log \frac{\hat{\sigma}^2_{\theta,t|t-1}}{\hat{\sigma}^2_{\theta,t|t}} + \log \frac{\hat{\sigma}^2_{z,t|t-1}}{\hat{\sigma}^2_{z,t|t}} + \log \frac{1 - \hat{\rho}^2_{t|t-1}}{1 - \hat{\rho}^2_{t|t}} \right) - 0.5 \log \frac{\hat{\sigma}^2_{z,t|t-1}}{\hat{\sigma}^2_{z,t|t}} \\ &= 0.5 \left(\log \frac{\hat{\sigma}^2_{\theta,t|t-1}}{\hat{\sigma}^2_{\theta,t|t}} + \log \frac{1 - \hat{\rho}^2_{t|t-1}}{1 - \hat{\rho}^2_{t|t}} \right).\end{aligned}$$

This proves Lemma 2.3. $\square$

[1] See Theorem 2.8.1 in Cover and Thomas (2006)

Proof of Proposition 2.4 and 2.5. Consider the system where Y_t represents the loading of optimal signals on the state at time t. The dynamics are described by the following relationship:

$$a_t - \mathbb{E}[a_t|s^{t-1}] = Y_t'(x_t - \mathbb{E}[x_t|s^{t-1}]) + e_t, \quad \forall t.$$

Kalman Filtering provides the update for the covariance as:

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}Y_t(Y_t'\Sigma_{t|t-1}Y_t + \Sigma_{e,t})^{-1}Y_t'\Sigma_{t|t-1}, \quad \forall t.$$

In the steady-state, we arrive at the following equilibrium:

$$\begin{aligned}
\Sigma &= \Sigma_{-1} - \Sigma_{-1}Y(Y'\Sigma_{-1}Y)^{-1}Y'\Sigma_{-1}, \\
\Sigma(\Sigma_{-1})^{-1} &= I - \Sigma_{-1}Y(Y'\Sigma_{-1}Y)^{-1}Y', \\
H'\Sigma_{-1}Y(Y'\Sigma_{-1}Y)^{-1}Y' &= H'(I - \Sigma(\Sigma_{-1})^{-1}), \text{where } H = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
Y(\Sigma_{-1}Y(Y'\Sigma_{-1}Y)^{-1})'H &= (I - \Sigma(\Sigma_{-1})^{-1})'H, \\
Y &= (I - \Sigma(\Sigma_{-1})^{-1})'H, \\
Y &= \begin{bmatrix} 1 - \frac{\hat{\sigma}_\theta^2}{\hat{\sigma}_{\theta,-1}^2} - \frac{\hat{\sigma}_{\theta,z}}{\hat{\sigma}_{\theta,-1}^2} \\ 1 - \frac{\hat{\sigma}_z^2}{\hat{\sigma}_{z,-1}^2} - \frac{\hat{\sigma}_{\theta,z}}{\hat{\sigma}_{z,-1}^2} \end{bmatrix}.
\end{aligned}$$

This proves Proposition 2.5.

Subsequently, the ratio of the elements of Y is shown to be 1 when the closed solutions for the posterior variances and covariances are substituted under the condition $\rho_\theta = \rho_z$:

$$\frac{1 - \frac{\hat{\sigma}_\theta^2}{\hat{\sigma}_{\theta,-1}^2} - \frac{\hat{\sigma}_{\theta,z}}{\hat{\sigma}_{\theta,-1}^2}}{1 - \frac{\hat{\sigma}_z^2}{\hat{\sigma}_{z,-1}^2} - \frac{\hat{\sigma}_{\theta,z}}{\hat{\sigma}_{z,-1}^2}} = 1.$$

Specifically, in the case where $\rho_\theta = \rho_z = 0$, the variances reduce to $\hat{\sigma}^2_{\theta,-1} = \sigma^2_\theta$ and $\hat{\sigma}^2_{z,-1} = \sigma^2_z$, leading to:

$$\frac{\sigma_z^2}{\sigma_\theta^2} = \frac{\hat{\sigma}_z^2 + \hat{\sigma}_{\theta,z}}{\hat{\sigma}_\theta^2 + \hat{\sigma}_{\theta,z}}.$$

This proves Proposition 2.4. □

Proof of Proposition 2.6 and 2.7. Utilizing the closed-form solutions from Appendix A.1 and the individual attention formula articulated in Lemma 2.3, we derive the respective results. □

A.3 Fixed capacity

Proposition A.1. ***(Fixed Capacity)*** *Let $\rho_\theta = \rho_z = 0$. As the volatility of one component increases, the attention given to that component increases, while it decreases for the other. i.e.,*

$$\frac{\partial \mathbb{I}(\theta^t; s^t|s^{t-1})}{\partial \sigma_z^2} \leqslant 0, \quad \frac{\partial \mathbb{I}(z^t; s^t|s^{t-1})}{\partial \sigma_z^2} \geqslant 0.$$

Proposition A.2. ***(Fixed Capacity)*** *Let $\rho_\theta = \rho_z = 0$. As the volatility of one component increases, the attention allocated to that component increases more than to the other. i.e.,*

$$\frac{\partial \mathbb{I}(z^t; s^t|s^{t-1})}{\partial \sigma_z^2} \geqslant \frac{\partial \mathbb{I}(\theta^t; s^t|s^{t-1})}{\partial \sigma_z^2}.$$

A.4 Dynamic cases

Proposition A.3. *As the marginal information cost, ω, approaches to zero, the steady-state posterior covariance, Σ, is given by:*

$$\Sigma = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix},$$

$$\textit{where } K = \frac{1}{2\rho_\theta(\rho_\theta - \rho_z)^2\rho_z}\Big(-\rho_\theta\rho_z\sigma_\theta^2 + \rho_\theta\rho_z^3\sigma_\theta^2 - \rho_\theta\rho_z\sigma_z^2 + \rho_\theta^3\rho_z\sigma_z^2$$
$$+ \sqrt{\rho_\theta^2\rho_z^2\left[4(\rho_\theta - \rho_z)^2\sigma_\theta^2\sigma_z^2 + \left((-1+\rho_z^2)\sigma_\theta^2 + (-1+\rho_\theta^2)\sigma_z^2\right)^2\right]}\Big),$$

if $\rho_\theta \neq \rho_z$,

$$K = \frac{\sigma_\theta^2\sigma_z^2}{(1-\rho^2)(\sigma_\theta^2 + \sigma_z^2)},$$

if $\rho_\theta = \rho_z = \rho$.

Proof. Considering the Riccati equation for the full information case about the target variable:

$$\Sigma_{-1} = A\Sigma_{-1}A' + W - A\Sigma_{-1}H'(H\Sigma_{-1}H')^{-1}H\Sigma_{-1}A',$$

we can derive the optimal steady-state posterior covariance matrix, Σ. $\square$

Proposition A.4. *Let $\rho_\theta = \rho_z \in [0,1)$. The square of posterior correlation, $\hat{\rho}^2$, between θ and z weakens as ω increases, i.e.,*

$$\frac{\partial\hat{\rho}^2}{\partial\omega} \leqslant 0.$$

Proof. By the Implicit Function Theorem and Cramer's rule from steady-state optimal conditions, I can show

$$\frac{\partial\hat{\sigma}_\theta^2}{\partial\omega} \geqslant 0, \frac{\partial\hat{\sigma}_z^2}{\partial\omega} \geqslant 0, \text{and } \frac{\partial\hat{\sigma}_{\theta,z}^2}{\partial\omega} \leqslant 0.$$

These three inequalities directly show the proposition. □

Proposition A.5. ***(Fixed Marginal Information Cost)*** *Let $\rho_\theta = \rho_z \in (0,1)$. As the volatility of one component increases, attention to both components increases. i.e.,*

$$\frac{\partial \mathbb{I}(\theta^t; s^t | s^{t-1})}{\partial \sigma_z^2} \geqslant 0, \quad \frac{\partial \mathbb{I}(z^t; s^t | s^{t-1})}{\partial \sigma_z^2} \geqslant 0.$$

Appendix B. (Appendix for Chapter 3)

B.1 Proofs

Proof of Lemma 3.1.

$$
\begin{aligned}
p(\theta = 1|s = 1) &= \frac{p(s = 1|\theta = 1)p(\theta = 1)}{p(s = 1|\theta = 1)p(\theta = 1) + p(s = 1|\theta = 0)p(\theta = 0)} \\
&= \frac{(1-\epsilon) \times 0.5}{(1-\epsilon) \times 0.5 + \epsilon \times 0.5} \\
&= 1 - \epsilon \\
p(\theta = 0|s = 0) &= \frac{p(s = 0|\theta = 0)p(\theta = 0)}{p(s = 0|\theta = 1)p(\theta = 1) + p(s = 0|\theta = 0)p(\theta = 0)} \\
&= \frac{(1-\epsilon) \times 0.5}{(1-\epsilon) \times 0.5 + \epsilon \times 0.5} \\
&= 1 - \epsilon
\end{aligned}
$$

This proves Lemma 3.1. □

Proof of Lemma 3.3. Each agent conjectures as follows:

$$
\begin{aligned}
p(s_j = 1|s_i = 1) = p(s_j = 0|s_i = 0) = (1-\epsilon)^2 + \epsilon^2 = 1 - f(\epsilon) \\
p(s_j = 0|s_i = 1) = p(s_j = 1|s_i = 0) = 1 - (1-\epsilon)^2 - \epsilon^2 = 1 - f(\epsilon)
\end{aligned}
$$

for $i \neq j$ where $f(\epsilon) = -2\epsilon^2 + 2\epsilon$ and s_i for an agent i's signals. □

Proof of Proposition 3.1. In this proof, I show the derivation of κ_1 and κ_0. First of all, Note that, for $k \in \{0, 1\}$,

$$a_i(I_i, s_i = k) = (1-r)E_i(x|s_i = k) + rE_i(\bar{a}|s_i = k)$$

Combining Lemma 3.2 and Lemma 3.4 with the above equations, I can get two equations as follows.

First, for $s_i = 1$,

$$a_i(I_i, s_i = 1) = x_i \left[(1-r)\frac{\beta}{\alpha_1+\beta} + r\{\kappa_1(1-f)\frac{\beta}{\alpha_1+\beta} + \kappa_0 f\frac{\beta}{\alpha_0+\beta}\}\right]$$

$$+ y\left[(1-r)\frac{\alpha_1}{\alpha_1+\beta} + r\{\kappa_1(1-f)\frac{\alpha_1}{\alpha_1+\beta} + \kappa_0 f\frac{\alpha_0}{\alpha_0+\beta} + (1-\kappa_1)(1-f) + (1-\kappa_0)f\}\right]$$

Second, for $s_i = 0$,

$$a_i(I_i, s_i = 0) = x_i \left[(1-r)\frac{\beta}{\alpha_0+\beta} + r\{\kappa_1 f\frac{\beta}{\alpha_1+\beta} + \kappa_0(1-f)\frac{\beta}{\alpha_0+\beta}\}\right]$$

$$+ y\left[(1-r)\frac{\alpha_0}{\alpha_0+\beta} + r\{\kappa_1 f\frac{\alpha_1}{\alpha_1+\beta} + \kappa_0(1-f)\frac{\alpha_0}{\alpha_0+\beta} + (1-\kappa_1)f + (1-\kappa_0)(1-f)\}\right]$$

Comparing coefficients in 3.12 and the above two equations, I have two variables with two simultaneous equations.

$$\kappa_1 = \frac{(1-r)\beta + rf\beta\kappa_0\frac{\alpha_1+\beta}{\alpha_0+\beta}}{(1-r)\beta + \alpha_1 + rf\beta}$$

$$\kappa_0 = \frac{(1-r)\beta + rf\beta\kappa_1\frac{\alpha_0+\beta}{\alpha_1+\beta}}{(1-r)\beta + \alpha_0 + rf\beta}$$

From these equations, I can algebraically get κ_1 and κ_0 as follows:

$$\kappa_1 = \frac{\beta(1-r) + rf\beta\frac{\alpha_1+\beta}{\alpha_0+\beta}\frac{\beta(1-r)}{\beta(1-r)+\alpha_0+rf\beta}}{\beta(1-r) + \alpha_1 + rf\beta - \frac{(rf\beta)^2}{\beta(1-r)+\alpha_0+rf\beta}}$$

$$\kappa_0 = \frac{\beta(1-r) + rf\beta\frac{\alpha_0+\beta}{\alpha_1+\beta}\frac{\beta(1-r)}{\beta(1-r)+\alpha_1+rf\beta}}{\beta(1-r) + \alpha_0 + rf\beta - \frac{(rf\beta)^2}{\beta(1-r)+\alpha_1+rf\beta}}.$$

□

Proof of Proposition 3.2. Using the solution in Proposition 3.1, calculate $\kappa_0 - \kappa_1$ algebraically as follows:

$$\begin{aligned}
\kappa_0 - \kappa_1 &= \frac{(1-r)\beta + rf\beta\kappa_1\frac{\alpha_0+\beta}{\alpha_1+\beta}}{(1-r)\beta + \alpha_0 + rf\beta} - \frac{(1-r)\beta + rf\beta\kappa_0\frac{\alpha_1+\beta}{\alpha_0+\beta}}{(1-r)\beta + \alpha_1 + rf\beta} \\
&= \frac{\frac{\beta(\alpha_1-\alpha_0)(1-r)\{\beta(\alpha_1+\beta-\alpha_1 fr-2\beta fr)+\alpha_0(\alpha_1+\beta-\beta fr)\}}{(\alpha_0+\beta)(\alpha_1+\beta)}}{\alpha_0(\alpha_1+\beta+\beta(-1+f)r)+\beta\{\alpha_1+\alpha_1(-1+f)r-\beta(-1+r)(1+(-1+2f)r)\}}
\end{aligned}$$

From Lemma 3.3 $f = -2\epsilon^2 + 2\epsilon$, we find that $\epsilon = \frac{1}{2}$ maximizes f at $\frac{1}{2}$. Therefore, the denominator is clearly positive since $0 \leqslant r \leqslant 1$ and $0 \leqslant f \leqslant \frac{1}{2}$. Also, in the numerator, apart from the term $\{\beta(\alpha_1 + \beta - \alpha_1 fr - 2\beta fr) + \alpha_0(\alpha_1 + \beta - \beta fr)\}$, all other terms are clearly positive because $\alpha_1 > \alpha_0$.

Since the maximum value of r is 1 and the maximum value of f is $\frac{1}{2}$, the minimum value of $-fr$ is $-\frac{1}{2}$. Substituting this into the term $\{\beta(\alpha_1 + \beta - \alpha_1 fr - 2\beta fr) + \alpha_0(\alpha_1 + \beta - \beta fr)\}$,

$$\alpha_0\alpha_1 + \frac{1}{2}\beta(\alpha_0 + \alpha_1) \geqslant 0$$

Hence, it follows that $\kappa_0 - \kappa_1 \geqslant 0$. □

Proof of Proposition 3.3. Consider the case where $r = 1$. Under this condition, the utility function for agent i simplifies to:

$$u_i(a_i|\mathbf{a_{-i}}, x, \epsilon) \equiv -r(L_i - \bar{L}) + \lambda \log(2\epsilon)$$

This simplification implies that the term encouraging agents to align their actions with the fundamental value x vanishes. Hence, $a_i = y$.

More specifically, in the second step, each agent aims to synchronize their actions based solely on the given public information y, disregarding their private information. Under these circumstances, the precision of the public information y

is irrelevant to their utility, prompting them to select $\epsilon = \frac{1}{2}$ in the first step. Consequently, this choice equates to opting out of media verification. $\square$

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