On the Digital Ocean

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The language of big data has been oceanic from the beginning. In one of the earliest slide presentations to employ the phrase, "Big Data . . . and the Next Wave of InfraStress"—by the Silicon Valley computer scientist John Mashey, who may well have originated the term in the 1990's—a rudimentary stick-figure surfs ("Ok!") while another succumbs ("Uh-oh") to a giant wave labeled "InfraStress."[1] By the time the *Oxford English Dictionary* took *big data* into its fold in 2013, the accompanying message of "learn to surf or drown" had become a mainstay of contemporary culture.[2] As one of the rare truisms to bridge the chasm separating the tech sector from critical theory, headlines like "We Are Drowning in Data," "Drowning in Big Data—Finding Insight in a Digital Sea of Information," "Drowning in Data: Digging Out of the Digital Quagmire," and "World Drowning in a Rising Sea of Information" share their rhetoric of watery demise with philosophical analyses of the present from every corner of the ideological map.[3] Caught

Unless otherwise noted, all translations are my own.

1. John R. Mashey, "Big Data . . . and the Next Wave of InfraStress," slide presentation given at USENIX Annual Technical Conference, Monterey, Calif., 6–11 June 1999, p. 1, static.usenix.org/event/usenix99/invited_talks/mashey.pdf. A later slide defines "infrastress" as "bad effects of faster change in computer *subsystems & usage* . . . than in underlying *infrastructure*" (p. 3). On the possibility that Mashey originated or popularized the term *big data* in its present meaning, see Steve Lohr, "The Origins of 'Big Data': An Etymological Detective Story," *Bits*, *New York Times*, 1 Feb. 2013, bits.blogs.nytimes.com/2013/02/01/the-origins-of-big-data-an-etymological-detective-story/

2. *Oxford English Dictionary*, s.v. "big data."

3. See Charlie Warzel, "We Are Drowning in Data," *New York Times*, 7 May 2019, www.nytimes.com/2019/05/07/opinion/data-privacy.html; Josh Steimle, "Drowning in Big Data—Finding Insight in a Digital Sea of Information," *Forbes*, 25 Mar. 2015, www.forbes.com/sites/joshsteimle/2015/03/25/drowning-in-big-data-finding-insight-in-a-digital-sea-of-information/?sh=5242256a449b; John Bates, "Drowning in Data: Digging Out of the Digital Quagmire," *Business*

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in the *data deluge*, adrift on the *sea of information*, engulfed by the flood, the tsunami, the torrent, the surge of digital flows, it is as though we have (been) plunged, by virtue of our most recent computational advances, into a condition of primordial boundlessness ("and the earth was without form and void . . . and the spirit of God moved upon the face of the waters" [Gen. 1:2]), from which both analytics consultancies and theories of contemporary being aspire to save us.

Everyone on both sides of the tech versus philosophy divide agrees that the current predicament is *new*: a 2021 report by the International Data Corporation (IDC) estimates that 62.2 zettabytes of data were created in 2020 alone, and that the amount of data produced over the next five years will more than double the amount produced between the dawn of digital storage and now.[4] The oceanic comparison, however, could not possibly be any older, and therein lies a simple yet seldom formulated problem for information-age attempts to think floatation. The problem is gendered insofar as the sea is gendered, or rather *used to be*; it is mathematical insofar as the sea's gender hangs together with its traditionally presumed topology, which has the density, compactness, and connectedness of a geometrical continuum; and it is foundational insofar as this topology coincides with the "shape" historically ascribed to the primal material underpinnings of the cosmos. From the book of Genesis through the twentieth century, the oceans of the past figure the feminine matrices of divine and human creation, the wombs and tombs of the earth, the fertile yet deadly milieu of the sirens and the mermaids. They are figures of endless flux and fecund formlessness, and thus of infinitude, in the oldest known sense of *what cannot be fenced in or finished*. Terms like the Hebrew *mayim*, *tehom*, and *tohu va bohu* (the waters, the deep, without form and void), the Greek *chaos, abyssos*, and *apeiron* (chasm, the bottomless, the unlimited), and the Latin *aqua perpetua* and *aqua vitae* (permanent water, water of life) all name or describe the voluminous, undulating, indivisible substratum that lies beneath and before the created world.

Insider, 27 Sept. 2011, www.businessinsider.com/drowning-in-data-digging-out-of-the-digital-quagmire-2011-9; and Neil McIntosh, "World Drowning in a Rising Sea of Information," *Guardian*, 21 Oct. 2003, www.theguardian.com/technology/2003/nov/01/internationalnews.onlinesupplement

4. See "Worldwide Global DataSphere Forecast, 2021–2025: The World Keeps Creating More Data—Now, What Do We Do with It All?" IDC, Mar. 2021 www.idc.com/getdoc.jsp?containerId=US46410421

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This substratum can give birth or give way, depending on the creation story in question, to delimited territories and finite bodies, but it can never be remainderlessly equated with such countable, analyzable collections. Its traditional task is to engender dry land and, thereafter, to subtend all existing boundaries, binding coast to coast, polis to polis, category to category, and act to act across a vast expanse of ever-shifting, indeterminate, and undeterminable possibility.

The problem of the digital ocean, in other words, can be formulated in terms of the following question: What happens to thinking—conceived as the ability to navigate across and find orientation within this timelessly changeable foundation—when the foundation itself begins to morph? The digital ocean displays, by definition, the opposite structure from the one just described. It is composed entirely of distinct digits (0s and 1s), which is to say that it is remainderlessly divisible into discrete bits, which is to say that it is per se accessible to the logic of conceptual definition and analysis. And this situation is by no means unique to the domain of digital technologies "proper," in the sense that we could unproblematically presuppose a still continuous, still analog, still conventionally oceanic substrate for more "authentically" real, nondigital spaces. In the seventy years since Alan Turing's "Computing Machinery and Intelligence," which proposed the thought experiment now generally known as the Turing test, it has become common to defend the hypothesis that *all* thought, or even all being, takes the form of digital computation.[5] In the last ten years, models of the universe as a digital sea of quantum bits, which gives rises to the hologrammatic projection of continuous space-time via a process of computational error correction, have gained currency to the point of rendering the very notion of continuity, for many physicists, epiphenomenal if not obsolete.[6]

It is not necessary to fully comprehend or commit to such models in order to acknowledge the challenge they represent to theory. Because what this

5. For the most often cited early version of the ontological hypothesis, under the slogan "it from bit," see John Archibald Wheeler, "Information, Physics, Quantum: The Search for Links," *Proceedings III International Symposium on Foundations of Quantum Mechanics* (Tokyo, 1989), pp. 354–68. For a contemporary formulation, which proposes "it from qubit" instead—and makes use of the concept of a "qubit ocean"—see Xiao-Gang Wen, "Four Revolutions in Physics and the Second Quantum Revolution—A Unification of Force and Matter by Quantum Information," *International Journal of Modern Physics B* 32, no. 26 (2018): arxiv.org/abs/1709.03824

6. For a particularly polemical formulation of this position, see Max Tegmark, "Infinity," in *This Idea Must Die: Scientific Theories That Are Blocking Progress*, ed. John Brockman (New York, 2015), pp. 48–51. For a good basic overview of the idea that space and time are the result of quantum error-correcting code, see Natalie Wolchover, "How Space and Time Could Be a Quantum Error-Correcting Code," *Quanta Magazine*, 3 Jan. 2019, www.quantamagazine.org/how-space-and-time-could-be-a-quantum-error-correcting-code-20190103/

shift toward a digitalized ocean entails, at a minimum, is the historically unprecedented *possibility* that continuous structure is not irreducible, and this possibility alone is enough, or should be enough, to shift the terms of the debate. The history of theory offers many resources for thinking deeply about the figure of the continuum, understood as the medium, ground, or supplement from or through which all (other) forms can emerge. From the circulating mana of structuralism to the uncontainable flows and differential fluids of poststructuralism to the most recent "analog turn"—diagnosed and analyzed in this issue by Alexander Galloway—the relationship of discrete structure to continuous substrate has always been a question, indeed has often been *the* question, for theory.[7] Precisely for this reason, however, the history of theory offers relatively few resources for thinking through what it could mean to theorize in the *absence* of any kind of continuum and, therefore, in the absence of the conceptual work that the indivisible ocean in all its many manifestations—whether as substrate or as surplus—has always done.[8]

An important partial exception to this general rule of theoretical commitment can be found in the tradition of post-Lacanian feminism, whose practitioners, from Luce Irigaray to Elizabeth Grosz, seek to analyze the oceanic as the site of a specifically feminine form of subjectivity.[9] In doing so, they accept as their point of departure the two-thousand-year-old association of the oceanic with the feminine, and of both with the *un*analyzable, *in*articulable, form*less* substrate of masculine analysis, articulation, and form-bestowal. But they go on to demonstrate, unequivocally, that this ostensibly passive medium continuously and actively labors and that its purely negative characteristics perform a positive function for thought. This labor can in turn be explicated to provide an account—differentiated and delimited according to the innate laws of continuous structure—of a uniquely feminine, uniquely oceanic perspective on writability and writing. The result is a wholly other approach to the philosophical significance of the continuum, which

7. Alexander Galloway, "Golden Age of Analog," *Critical Inquiry* 48 (Winter 2022): 216.

8. On the poststructuralist passage from substrate to supplement within the context of linguistic systems theories, see Sarah M. Pourciau, *The Writing of Spirit: Soul, System, and the Roots of Language Science* (New York, 2017), pp. 206–10.

9. For foundational texts, see Luce Irigaray, *Speculum of the Other Woman*, trans. Gillian C. Gill (Ithaca, N.Y., 1985) and *The Sex Which Is Not One*, trans. Catherine Porter and Carolyn Burke (Ithaca, N.Y., 1985). For contemporary approaches that use the wave-based, vibratory structure of the oceanic feminine to propose a new materialism, see Jane Bennett, *Vibrant Matter: A Political Ecology of Things* (Durham, N.C., 2010), and Elizabeth Grosz, *Chaos, Territory, Art: Deleuze and the Framing of the Earth* (New York, 2008). For an incisive critique of such wave-based approaches as a model for challenging capitalist patriarchy, see Robin James, *The Sonic Episteme: Acoustic Resonance, Neoliberalism, and Biopolitics* (Durham, N.C., 2019).

has informed every subsequent attempt to grapple theoretically with the digital/analog relation, even or especially in cases where this debt remains implicit.

Psychoanalytic feminism generally stops just short, however—as indeed it also must if it is to remain meaningfully psychoanalytic—of raising doubts about the foundational status of the discrete/continuous binary itself. The related task of analyzing the continuum all the way through to its *end*, and thus of dismantling rather than reinterpreting this binary, has historically fallen to a theory-external domain that probably could not be more opposed to the psychoanalytic feminist discourse. The discipline of mathematics, from at least the early modern period onward, has encompassed a multitude of philosophically significant attempts to delimit the unlimited and define the infinite, to acquire logical, conceptual, numerical control over the continuum, and so also, in Lacanian terms, to eradicate the (ir)rationality of the feminine. These mathematical attempts, despite their unfeminist aims and approach (which are certainly not only, or even primarily, a matter of Lacanian interpretation), have nevertheless contributed more than any other philosophical project to the comprehension of the continuum, precisely *in* its irreducible otherness. No disciplinary tradition has ever plumbed the oceanic depths more rigorously—or given more conceptual content to the claim of uncountable unfathomability or provided so many explicit characterizations of how compact, connected spaces actually *work*—than the one most intent on drying up the primordial waters, or at least on learning to mimic, with numbers, the behavior of their waves.

The history of mathematics as an agon of digital and analog, discrete and continuous modes of being, has been attracting sustained media-theoretical attention since Friedrich Kittler's late turn to ancient Greek numbers, notes, and letters; it has been attracting sustained poststructuralist attention more generally since the mathematically-inspired models of Gilles Deleuze and Alain Badiou.[10] In these contexts, theory carries on a Western tradition of math-adjacent philosophizing that runs from Plato and Aristotle through Leibniz and German idealism into the twentieth century strands of neo-Kantianism, Husserlian phenomenology and even (especially) Heideggerian ontology. One major difference, however, between contemporary theory and the history of philosophy is that theory has mostly seemed reluctant

10. For Kittler's engagement with ancient Greek mathematics, see Friedrich Kittler, *Musik und Mathematik I: Hellas 1: Aphrodite* (Munich, 2006) and *Musik und Mathematik I: Hellas 2: Eros* (Munich, 2009). In translation, see Kittler, "Number and Numeral," trans. Geoffrey Winthrop-Young, *Theory, Culture & Society* 23, nos. 7–8 (2006): 51–61 and the final six essays of *The Truth of the Technological World: Essays on the Genealogy of Presence*, trans. Erik Butler (Stanford, Calif., 2014).

to confront the problems raised in and by contemporaneous mathematics, and particularly by the computational mathematics of the digital since Turing. This reluctance is understandable. It might even be appropriate given current conditions of extreme disciplinary specialization. But the unfortunate result is that almost no serious, sustained attention has been given to the deeper philosophical stakes of what I will call here, for the sake of abbreviation, the Turing problem, which is also the problem of the digital ocean in its most contemporarily relevant manifestation.

What I mean by the Turing problem will become clearer in what follows, but a provisional formulation would go something like this: Georg Cantor's invention of set theory retains and even elevates the continuum, in digitalized form, as the ineradicable medium of human thought and natural processes; Turing's invention of computability theory eliminates the continuum—and with it the idea of an uncountable real—from the thinking/being equation. The gesture of bracketing the Turing problem, and in doing so holding *on* to the presumption of a nondenumerable substrate, is one that nearly all encounters between theory and mathematics turn out to share. Even analyses that focus specifically on the genealogy of the contemporary digital tend to follow this pattern. The most important recent work in this vein—Bernhard Siegert's *Passage des Digitalen*—effectively stops at Cantor but then extends the conclusions drawn from Cantor to a diagnosis of our contemporary situation.[11] Kittler (who treats Turing repeatedly and well) makes a structurally similar move when he juxtaposes ancient Pythagorean mathematics and quantum physics, the sea of the *alogon* and quantum randomness, in order to bypass the problem of a Turing-style digital universe.[12]

This mode of theoretical operation seems like a problem not least because it allows the various figures of a still-continuous ocean—together with its exclusively female denizens from Kittler's sirens to Bernard Stiegler's Pandora, from Deleuze's *devenir-femme* to Bruno Latour's Gaia—to keep proliferating through theory as though we still had any productive way of talking about what they are supposed to mean. My experiment here will explore, without attempting to answer, the following questions: What happens if we take seriously the idea of the digital ocean and thus also the idea that the ocean, at a particular historical moment in time, *has become* digital? What do we have to give up? What can we keep? What might we gain? What would

11. See Bernhard Siegert, *Passage des Digitalen: Zeichenpraktiken der neuzeitlichen Wissenschaften 1500–1900* (Berlin, 2003).

12. See Kittler, "The World of the Symbolic—A World of the Machine" and "There Is No Software," trans. Stefanie Harris, in *Literature, Media, Information Systems: Essays*, trans. Harris et al., ed. John Johnston (Amsterdam, 1997), pp. 130–56, and "The Artificial Intelligence of World War: Alan Turing," in *The Truth of the Technological World*, pp. 178–94.

a thinking look like that has the digital ocean for its proper medium? The ensuing discussion is not intended to provide a definitive answer to these questions but rather to prepare the ground for a serious conversation pertaining to them, and to delineate the kinds of spaces within which answers might (or might not) be sought. It will move from the digitalized ocean of Cantor's set theory to the digital machines of Turing's computation theory, laying out, in the former, what is arguably the most rigorous attempt in history to think through the structural essence of the continuum, in order to make clear in the latter what actually *disappears* when Turing argues for the structural irrelevance of this ancient ground.

1

The most important mathematical contribution to a theory of the digital ocean appeared some six or seven decades before the construction of the first digital computing machines: Cantor's *Foundations of a General Theory of Sets* (*Grundlagen einer allgemeinen Mannigfaltigkeitslehre*, 1883) laid the foundation for what would become known as modern set theory, by offering a new account of the relationship between the infinite and the finite. Cantor was well aware of the philosophical stakes of his proposal, which involved extending the numerical system into what he called the "transfinite" realm and, crucially, analyzing the geometrical continuum in terms of those numbers.[13] His argument for the legitimacy of this new method unfolds against the backdrop of an explicit engagement with the long tradition of infinite/finite *nonrelationship*—of mathematical and philosophical incommensurability—whose abandonment he was advocating. In doing so, it lays out his position with respect to the traditionally gendered categories of being and becoming, limit and unlimited, *logos* and *alogon*, which have always been inseparable from the problem of analyzing the continuum and which had returned with a vengeance to dominate much nineteenth-century mathematical thought. In the process it *also* helps to make clear why Cantor's digital ocean can no longer be ours, even if ours remains quite literally unthinkable without his.

The tradition of incommensurability against which Cantor argues is associated above all with the name of Aristotle, for whom the continuity of the cosmic substrate was the primary given of natural science. Against the idea

13. Georg Cantor, "Foundations of a General Theory of Manifolds: A Mathematico-Philosophical Investigation into the Theory of the Infinite," trans. William Ewald, in *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, ed. Ewald, 2 vols. (New York, 2005), 2:891; hereafter abbreviated "F." For a wonderful general history of set theory, see José Ferreirós, *Labyrinth of Thought: A History of Set Theory and Its Role in Modern Mathematics* (Boston, 1999).

that this continuity might actually be divisible into a collection of infinitely tiny bits, Aristotle argued that only so-called potential infinities can exist and be thought.[14] The natural counting numbers, for instance, are potentially infinite because another number can always be added to the series; a line is potentially infinite because it can be divided anywhere at any time. The actual number of numbers, however, remains at every point in time finite, and no continuous magnitude can ever be actually divided everywhere, all at once. The underlying stuff of nature, for Aristotle, is thus fundamentally undelimited (*apeiron*), despite or, indeed, rather because of the fact that formal limits (*peras*) can be limitlessly imposed upon it. Variously imagined as the pliable wax into which a *dunamis meta logou* inscribes its rational figures; as the watery mirror whose empty surface reflects the forms of being; or as the passive menstrual fluid that shelters an actively inseminating sperm, the always-feminized Aristotelian substrate remains irremediably alien to the always-masculine orders it enables to appear.[15] The substantive *hanging together* of matter, space, and time provides a womb-like, oceanic medium for the emergence of boundaries, both conceptual and bodily, which can never itself be analyzed out of existence because it admits of no common denominator and, thus also, no foundational ratio or *logos*. (The Greek *logos*, like the Latin *ratio*, designates a relationship between two quantities that share a common unit of measure.)

Within the history of mathematics, this Aristotelian dualism takes the shape of a recurring *continuum problem*, which precedes Aristotle himself and goes on to dog Cantor's successors. The problem's earliest manifestation—the discipline's first "foundational crisis"—has often been identified with the discovery of incommensurable magnitudes, which dashed the Pythagorean dream of a perfectly rational because perfectly relational cosmos, composed solely of harmonious whole-number intervals or *logoi*. In a much-cited passage from Pappus's commentary on Euclid, for instance, one finds the following account, which consigns the bearer of bad news to drown in the same boundless depths from which his *un*knowledge about the irrational $\sqrt{2}$ derives:

> Indeed the sect (or school) of Pythagoras was so affected by its reverence for these things that a saying became current in it, namely, that

14. For a collection of the most important Aristotelian passages on mathematics and the infinite, together with extensive commentary, see Thomas L. Heath, *Mathematics in Aristotle* (New York, 1949). On Aristotle's account of infinity, see Jonathan Lear, "Aristotelian Infinity," in *Proceedings of the Aristotelian Society* 80 (June 1980): 187–210.

15. For an insightful, Irigaray-inspired analysis of the role of the feminine-as-*apeiron* in Aristotelian philosophy, particularly with respect to the Aristotelian account of biological reproduction and *energeia*, see Emanuela Bianchi, *The Feminine Symptom: Aleatory Matter in the Aristotelian Cosmos* (New York, 2014).

> he who first disclosed the knowledge of surds or irrationals and spread it abroad among the common herd, perished by drowning: which is most probably a parable by which they sought to express their conviction . . . that the soul which by error or heedlessness discovers or reveals anything of this nature . . . wanders [thereafter] hither and thither on the sea of non-identity . . . immersed in the stream of the coming-to-be and the passing-away, where there is no standard of measurement.[16]

Aristotle, who studied Euclid and takes the incommensurability of the substrate as his starting point, is more sanguine than his supposed Pythagorean forbears. In place of their denial and retributive justice, he proposes a strict distribution of mental labor intended to keep the *apeiron* down below where it belongs. He insists (1) that arithmetic and geometry must remain entirely separate because, as the sciences of discrete and continuous quantities, respectively, they are themselves incommensurable and (2) that the potential infinity of the counting numbers will suffice to solve every *relevant* arithmetic problem.

The overwhelming influence of Aristotelian thought on scholastic philosophy ensured that this dualist approach to mathematics—as well as to the *homonymy* of material and formal, feminine and masculine modes of being—remained dominant, or at least widespread, well into the baroque period. Cantor cites the ubiquitous pronouncement of *infinitum actu non datur* (the actual infinite does not exist) as evidence of *The Philosopher*'s long, repressive shadow, and he takes the time to criticize the arguments of *Metaphysics*, book 11 in particular—where Aristotle discusses counting—as falling prey to a vicious circle (see "F," p. 889).[17] The actual precipitating situation to which his polemic responds, however, is neither ancient and Aristotelian nor medieval and scholastic but rather modern and, in Cantor's own words, "criticist" ("F," p. 892; trans. mod.).[18]

By the beginning of the nineteenth century, a new "crisis of foundations" (*Grundlagenkrise*) had engulfed the world of (European) mathematics, which threatened to lay waste to the towering achievements of a new baroque method of infinitist analysis. The invention of the calculus, whose etymology ties it back to the *calculi*, or "little pebbles," of Pythagorean arithmetic, had made it

16. *The Commentary of Pappus on Book X of Euclid's Elements*, trans. William Thomson, ed. Gustav Junge and Thomson (Cambridge, 1930), p. 64. Historically, this commentary has often been attributed to Proclus.

17. See Aristotle, *Metaphysics* 11.10.

18. The translation in the Ewald edition effaces the fact that Cantor is here referring not to general criticism of the Spinozistic and Leibnizian positions but to the specifically Kantian doctrine of criticism (*Kritizismus*), which claims to determine the limits of what can be legitimately thought.

possible to calculate over the whole breadth and depth of the continuum with the help of a new, more expansive concept of ratio or *logos*. This new concept made room for infinitesimally small intervals called differentials, which could be written in ratio form as *dy/dx*, set into mathematically definable relationships with conventional numerical ratios and physically interpreted as rates of continuous change. The result was a quasi-Pythagorean solution to the bona fide Pythagorean problem of a pancosmic rationality: the new differential *logoi* could be employed in contexts where "regular" numerical *logoi* could not, in order to account for phenomena that had traditionally escaped the grasp of number and thus also of *logos*. This perspective finds an emblem in the famous Leibnizian image of an infinite ocean of becoming that is *actually* analyzable everywhere at once, if only for God: "The present is great with the future; the future could be read in the past; the distant is expressed in the near. . . . Each soul knows the infinite, knows everything, but confusedly. Thus when I walk along the seashore and hear the great noise of the sea, I hear the separate sounds of each wave but do not distinguish them . . . Only God has a distinct knowledge of everything, for he is the source of everything."[19]

The problem with this perspective, from a nineteenth-century standpoint, was that the new, more general understanding of numerical relation exposed new rifts in the concept of number itself. By the time of Augustin-Louis Cauchy's *Cours d'analyse* (1821), it had become clear that the mysterious, infinitesimal nonthings of the calculus could be reinterpreted in terms of finite operations on real numbers. But, by the same point in the century, it had *also* become clear that the concept of real number itself needed clarification or, more precisely, creation. The phrase *real number* refers, intuitively, to the totality of rational and irrational numbers that correspond to the points on a continuous number line. But while all rational numbers can be unproblematically defined with reference to the natural counting numbers whose ratios they are, and while many irrational numbers can be precisely defined with reference to more general, algebraic ratios like quadratic equations, there remain infinitely many irrational numbers that cannot be thus identified, solved for, calculated, constructed, or defined. There is consequently no general algorithmic rule or procedure for establishing the kinds of relationships that count when determining what does and does not fall under the category of number, and, in the absence of such a rule, mathematicians have traditionally relied on their geometrical intuition of the numerical continuum as a unity

19. Gottfried Wilhelm Leibniz, "The Principles of Nature and of Grace, Based on Reason, 1714," in *Philosophical Papers and Letters*, trans. and ed. Leroy E. Loemker, 2 vols. (Boston, 1989), 2:640. For a treatment of the Leibnizian waves in the context of the cybernetically relevant genealogy of noise or *Rauschen*, see Rüdiger Campe, "The *Rauschen* of the Waves: On the Margins of Literature," *SubStance* 19, no. 1 (1990): 21–38.

that incontrovertibly, if somewhat mysteriously, *hangs together*. The nineteenth-century discovery of multiple ways in which this "incontrovertible" intuition can fail—from functions that are everywhere continuous but nowhere differentiable to functions that are continuous nowhere to functions that are continuous at all irrational values and discontinuous at all rational ones—put paid to this traditional relationship of dependency and called forth several discipline-transforming attempts to confront the "foundational crisis" of number.

Much of the nineteenth-century mathematical establishment argued that the end of the crisis, and the salvation of the baroque edifice of infinitesimal analysis, should be sought in a Kantian-style gesture of critical retreat: a drawing of fixed limits around the island of what can be clearly defined and legitimately deduced, a return to the solid ground of the natural counting numbers and their ratios, a purging of "transcendent" infinities from the discipline, a reduction of analysis to arithmetic so as to avoid the antinomies of the continuum.[20] Cantor, together with a handful of his most significant contemporaries, vehemently disagreed with this proposal to place the oceanic infinite (back) out of bounds. And it is against this modern, Kantian form of the traditional Aristotelian dualism, associated above all with the name of Cantor's teacher and nemesis, Leopold Kronecker, that the "Foundations" stakes out its unapologetically infinitist position. Broadly speaking, Cantor argues that what is needed is not a regression to the thinglike solidity of the counting numbers but a progression toward the conceptual solidity of his *manifolds*, which turns out to entail a more general account of what it means *both* to count and to be (a thing). "In place of the Aristotelian-scholastic proposition discussed in §4 [*infinitum actu non datur*] I accordingly set another: Omnia seu finita seu infinita *definita* sunt et excepto Deo ab intellectu determinari possunt [All things finite or infinite are *definite* and, God excepted, can be determined by the intellect]" ("F," p. 891).

According to Cantor, this position is fundamentally Platonic and, by extension, Pythagorean. The first line of his paper, which contains the phrase "theory of manifolds" (*Mannigfaltigkeitslehre*), sends the reader to the following note:

> In general, by a "manifold" or "set" I understand every Many [*jedes Viele*] which can be thought of as a One [*als Eines*], i.e. every aggregate [*Inbegriff*] of definite elements which can be bound up into a whole by some law. I believe that in this I am defining something akin to the

20. The most influential proponent of this position was the Berlin mathematician Leopold Kronecker; see Leopold Kronecker, "On the Concept of Number" (1887), trans. Ewald, in *From Kant to Hilbert*, 2:947–955.

> Platonic *eidos* or *idea*, as well as to that which Plato called *mikton* in his dialogue "Philebus or the Highest Good." He contrasts this to the *apeiron* (i.e. the unbounded, undetermined, which I call the inauthentic infinite [*uneigentliche Unendlichkeit*]) as well as to the *peras*, i.e. the boundary; and he explains it as an ordered "mixture" of both. Plato himself indicates that these concepts are of Pythagorean origin; see A. Boeckh, *Philolaos des Pythagoreers Lehren*, Berlin 1819. ["F," pp. 882, 916; trans. mod.]

To make sense of this comparison, and so also of Cantor's version of the digital ocean, it will be necessary to expend some energy uncovering the significance he attaches to these Platonic-Pythagorean concepts, which are here cast as the ancestors of his own revolutionary collections. Cantor's primary concern in the "Foundations" is with the sovereign *reach* of mathematical thought: Does the mathematical *logos* have the power (and thus also the right) to range freely over the totality of all conceivable mathematical objects, including infinite ones, or must it restrict itself to permutations of finite multiplicities lest it fall into the "abyss of the 'transcendent'—where, it is said with fear and wholesome alarm, 'anything is possible'" ("F," p. 889)? Everything depends, of course, on what we mean by *conceivable mathematical objects*, and the point of Cantor's Plato comparison is to signal his affinity with the Platonic rather than the Aristotelian interpretation of *conceivable*.

To conceive means "to grasp as a concept," and *to grasp as a concept* has traditionally meant "to grasp in accordance with a rule for determining what belongs"—in ancient Greek terms, "to think a many under the aegis of a one." The theory of concept formation is thus always also a theory of collecting and collections. The word *logos* derives etymologically from the verb *legein*, "to collect," "to gather," and the passage from Plato's *Philebus* to which Cantor refers can be read as a rumination on the precise nature of the relationship between what is gathered and the principle of gathering. Plato there proposes that true knowledge consists in discovering the "definite number" of foundational units, or proper divisions (*peras*), for an undelimited domain (*apeiron*), in terms of which a continuous spectrum of "more and less" becomes comprehensible as a structured whole of proportional relations (*mikton*). Once this definite number "between unity and infinity" has been found, via the back and forth of dialectical analysis, "then, and not till then, we may rest from division" and allow the other, irrelevant differences "to drop into the *apeiron*."[21] Plato's privileged examples for such a procedure are the arts of music and

21. Plato, *Philebus* 18c, 24b; hereafter abbreviated *P*. On the significance of *Philebus* for Cantor's concept of set, see Kai Hauser, "Cantor's Concept of Set in Light of Plato's *Philebus*," *The Review of Metaphysics* 63 (June 2010): 783–805.

grammar, which analyze the continuum of perceptible sound—the boundless flux and flow of acoustic material—into harmonious, orderly collections of notes and letters.

Nothing about Cantor's first footnote, or indeed the rest of the paper, makes clear *why* Plato's account of collection in the *Philebus* should be more congenial than Aristotle's to the Cantorian theory of manifolds. The main thrust of the latter, after all, is the anti-Aristotelian argument that actual infinity can be conceptualized. And certainly, no matter how one chooses to read the story of concept formation provided by the *Philebus*, the point cannot be that Plato shares Cantor's belief in the rationality of actually infinite collections. The relationship between Plato's "definite number," which is most definitely only finite, and Cantor's "definite infinite," which is most definitely not, goes entirely unexplained in the "Foundations" (as well as everywhere else in Cantor's oeuvre) ("F," p. 882). The paper does contain some hints, however, about the direction in which a more precise articulation of the Cantor-Plato relation can be found. These hints are historical-contextual in the sense that they point outward toward a specifically nineteenth-century tradition of Platonic-Pythagorean interpretation with which Cantor sees his sets as interacting. But the historical-contextual is here substantive and philosophical in the sense that it makes visible the deeper, extra-mathematical stakes of Cantor's work on infinite sets. It is Cantor's implicit but radical reinterpretation of the structure-substrate, form-matter, *logos-alogon* interaction—and thus also of idealisms both ancient and modern—that makes the set theoretical perspective so central to the development of twentieth- and twenty-first-century thought. It is not possible to fully understand what happens in Turing's model without a clear sense of the problem *and* solution it requires us to give up.

In section 5 of the "Foundations," which is devoted to a philosophical history of the infinite, Cantor refers to the opposition between mechanical and organic approaches to nature. He alludes to the "one-sidedness and insufficiency" of purely mechanical approaches so "strikingly exposed by Kant" in his *Critique of Judgment*. He claims that "not even the start of an *organic* explanation of nature, equipped with the same mathematical rigor but transcending the mechanistic one has been developed" ("F," p. 892; trans. mod.). And he insists, finally, that any such organicist account would need to return to the precritical, inherently infinitistic approaches of Leibniz and Spinoza. Even if Cantor had not elsewhere, and repeatedly, cast his own theory of manifolds as the foundation of this new organicist account, it would be obvious from the "Foundations" that his argument tends in this direction.[22]

22. See Cantor, letter to Magnus Gustaf Mittag-Leffler, 22 Sept. 1884, in *Briefe*, ed. Herbert Meschkowski and Winfried Nelson (New York, 1991), p. 202. See also Cantor, letter to

What is less obvious is how this new organicism works and why we should care: What does the post-Kantian problem of accounting for life and life's freedom—the organism's apparent ability to transcend the confines of mechanist causality—have to do with Cantor's theory of infinite collections, and so also with the figure of the digital ocean?

The answer runs through the German-idealist mobilization of Plato's *Philebus*, to which Cantor here implicitly refers when he polemically insists that a viable organicist account of nature does not yet exist. The goal of German-idealist organicism—as encapsulated perhaps most emphatically in Friedrich Schelling's speculative nature philosophy—is to transcend the boundary that Kant, like Aristotle, had erected between organic and inorganic, mental and material, teleological and mechanical modes of being and explanation. This boundary comes very close to coinciding, due to the traditional equation of the rational and the calculable, with the boundary between the potential infinity of arithmetic and the actual infinity of the continuum. And the point of turning to Plato's numerical account of how beings emerge and are known, as Schelling does throughout his nature-philosophical writings, is its profoundly *un*mechanical monism, which comes very close to coinciding—due to Plato's emphasis on the generative "mixture" of the finite and the infinite—with a monism of the rational and the real. The Platonic account holds for all beings, whether living or not: "Whatever is said to be consists of one and many, having in its nature limit and unlimitedness" (*P* 16c–d). But it also holds eminently and paradigmatically of the kinds of rational, purposive, *autonomous* structures associated with life forms. (The task of the dialectic, as Plato says elsewhere, is to "cut along the natural joints" of the cosmos, which is itself envisioned as a giant animal animated by a rational world soul or *nous* [*P* 265e].) The consequence of this position, according to Schelling, is that inorganic multiplicities need to be understood on the model of organic ones rather than the reverse, which means, in turn, that multiplicity itself—the formless profusion of Plato's uncollected *many*—must be reconceived to make *room* for an inherent proclivity toward form. Schelling envisions this specifically oceanic *energeia* as a kind of irregular, arbitrary oscillation between quasi-electromagnetic poles, which is to say, as waves. The dyad of *peras* and *apeiron* reappears in his nature-philosophical writings as a cosmic love affair along traditionally gendered lines, between masculine seed and feminine womb waters, with the monumental difference that Schelling's waters

Mittag-Leffler, 16 Nov. 1884; letter to Mittag-Leffler, 18 Nov. 1884; letter to Elie Blanc, 22 May 1887; letter to Aloys Schmid, 5 Aug. 1887, in *Briefe*, pp. 224–5, 228–9, 292–3, 298–9. For a more extended treatment of the nature-philosophical and natural-scientific backdrop to Cantor's set theoretical reflections, see Ferreirós, "The Motives behind Cantor's Set Theory—Physical, Biological, and Philosophical Questions," *Science in Context* 17 (June 2004): 49–83.

are no longer inert and receptive, as they are for Aristotle, but rather active and procreative. The specifically organic capacity to move *spontaneously and freely*, which can also mean *arbitrarily*, and which is associated by both Kant and Aristotle exclusively with the masculine realm of purposive form bestowal, has here become a defining feature of the inorganic, feminine realm of the substrate (matter, time, *apeiron*, nature) per se.[23]

The Schellingian account of a freely oscillating nature would almost certainly have remained utterly irrelevant to Cantor had it not been for the work of the French physicist Joseph Fourier and the use to which this work was put by nineteenth-century German scientists of the psyche.[24] In the course of studying the mathematics of heat waves, Fourier had discovered that any periodic function at all, no matter how apparently arbitrary or irregular, could be analyzed into a *definite number* of simple wave functions, or sine curves.[25] This method of decomposition made it possible to rigorously relate every conceivable kind of oscillatory operation to every other across the *common measure* of the sine function, which rendered even the most seemingly irrational vibrations commensurable. The technique was therefore almost immediately exploited by scientists concerned with mental phenomena, first in acoustics and then in psychology, in order to argue for the inherent commensurability of physical and mental oscillations. The approach culminated, for those willing to leave Kant entirely behind, in the image of an undulating all on the model of the Leibnizian ocean, where every entity, from rocks to animals to musical chords to mental concepts, possessed the analyzable structure of a compound wave system.[26] In this context, Fourier's mathematics of functional transformation performed the task that Leibniz himself had reserved for God, by mediating numerically between chaotic depths and readable surface, infinitesimal perceptions and visible wave patterns, continuous noise (*Rauschen*) and navigable information. The dynamism of a freely operating, self-propelling human *logos* could now be scientifically traced back

23. For Schelling's most extensive and influential nature-philosophical deployment of Plato, see F. W. J. Schelling, *On the World Soul*, trans. pub., in *Geo/Philosophy*, vol. 6 of *Collapse*, ed. R. Mackay (Bristol, 2010), pp. 58–95.

24. Scientists of the psyche for whom Fourier's law played a crucial role include Hermann von Helmholtz, Gustav Fechner, and Wilhelm Wundt. Cantor corresponded with Wundt about the physical ramifications of his manifolds; see Cantor, letter to Wilhelm Wundt, 5 Oct. 1883, in *Briefe*, pp. 136–40.

25. The phrase *definite number*, which draws attention to the perceived link between Fourier's discovery and the *mikton* of Plato's *Philebus*, appears in an influential paraphrase of Fourier's Law by Hermann von Helmholtz; see Hermann von Helmholtz, "On the Physiological Causes of Harmony in Music," *Science and Culture: Popular and Philosophical Essays*, trans. pub., ed. David Cahan (Chicago, 1995), p. 62.

26. For a more extensive treatment of this vibratory model of logos-cosmos relation, see Pourciau, *The Writing of Spirit*, pp. 160–188.

or *deduced*—as envisioned but not performed by Schelling's transempirical nature philosophy—from its profoundly un-Kantian, because nonhuman, conditions of possibility.

Cantor, to make a long and complicated story very short, believes that this direction of reasoning is right but that the existing picture of the substrate is not. A coherent account of organic and mathematical freedom requires a coherent account of the continuous ground of being as fruitfully, actively, and actually infinite, which means also, as fully analyzed: human cognition can only range without restriction over the real once the real itself has been rendered cogniz*able*. The doctrine of universal Fourier transformability, and with it of arbitrary operational power, thus requires a correspondingly universal and arbitrary doctrine of collection—that is, a theory of manifolds—to raise the operational domain—that is, the continuum of all reals—to the level of a self-consistent concept.[27] Such a doctrine, however, according to Cantor, is precisely what nineteenth-century philosophy does not have. The post-Kantian work of rethinking the subject-substrate relationship had been performed almost exclusively in and as a rethinking of the category of becoming, or time. Cantor does not think much of this approach or, indeed, of time itself: "I must explain that in my opinion to bring in *the concept of time* or the *intuition of time* in discussing the much more fundamental and more general concept of the continuum is *not* the correct way to proceed. . . . Such a thing as *objective* or *absolute time* exists nowhere in nature" ("F," p. 904; trans. mod.). Nor does Cantor believe that the philosophical tradition has any persuasive alternative candidates to offer. (He dispenses with the intuition of continuous space, along with the related history of "aesthetic contemplation," "philosophical sharp-wittedness," and "inaccurate comparisons," in a single sentence.) And he therefore concludes that it will be up to "sober and exact mathematical investigations"—in other words, up to him—to produce the first-ever, rigorous analysis of freedom's foundations: "Thus I am left with no choice but to attempt, with the help of the concept of real number as defined in Section 9, as general as possible a definition of a purely arithmetical concept of a point-continuum" ("F," p. 904; trans. mod.).

This, then, is the sense in which the theory of manifolds is related to but also transcends the Platonic theory of eidetic forms, together with its later German-idealist revivals from Leibniz to Schelling to Wundt. Cantor's theory,

27. The earliest set theoretical approaches employed by both Cantor and his contemporary Richard Dedekind occur in the context of their investigations into the behavior of trigonometric functions. These investigations were inspired by Fourier's work on heat waves and by the further generalizations of the implications of that work undertaken in the 1850s by Lejeune Dirichlet and Bernhard Riemann.

like Plato's, offers an account of how coherent collections or concepts can be generated and analyzed. Cantor's theory, like Plato's, does so in terms of a rule for establishing the *definite number* of elements that make up such a collection. Cantor's theory, however, unlike Plato's, includes the entire substrate of numerical thinking, and thus also of all possible numerical concept formation within its purview. Cantor accomplishes the seemingly paradoxical feat of *collecting* the one domain that, by definition, has never been thinkable *as one*, by generalizing the concept of conceptual collection itself: where Plato had required a symmetrical relation among relations, or ratio among ratios, along the lines of the Pythagorean *analogos* (written in modern notation as x/y = a/b); where Leibniz had expanded this rule of proportional harmony to include ratios between continuously changing ratios, as encapsulated by the modern notion of function; and where Fourier had further expanded this model to include ratios between potentially infinite *series* of functions, Cantor drops all requirements from the notion of relational *rule* other than the principle *that there be a relation among relations*. His technique of establishing a one-to-one correspondence or representational map between sets (the German *Abbildung* means "image" or "copy"), which are themselves nothing more than collections of relations—and his willingness to apply this general rule of rule-governedness without restriction—allows him to accomplish the historically impossible: namely, to introduce distinction, and with it definiteness, into the realm of the infinite. Cantor conclusively demonstrates, via the principle of the one-to-one map, that the infinity of the natural counting numbers is not the same size as the infinity of all reals and that, by extension, both infinities must *have* a size, which is to say a *definite* (but not finite) *number* of elements. Cantor calls this number the *cardinality* of the manifold. He uses this conclusion to position the cardinality of the continuum in a series of other lesser and greater infinities (or at least to posit such a positioning) and also to hypothesize that the elements of the continuum can be sequentially ordered.[28]

The result is a new, maximally general theory of analogical control over the analog: a "dialectical generation of concepts" that subsumes Plato's own, and in the process *sublates* Plato's *apeiron* into the starry constellations of Cantor's infinite point sets.[29] If this "purely arithmetical concept of a point-continuum" still has anything at all in common with the traditional domain

28. The hypothesis that the cardinality of the continuum is the next greatest cardinality after the cardinality of the natural numbers is (one possible formulation of) Cantor's Continuum Hypothesis. The Continuum Hypothesis was the first problem on David Hilbert's subsequent, equally famous list of mathematical problems for the future. The question still has not been resolved, although we now know a great deal more about what an answer would entail.

29. Cantor, "Ueber unendliche, lineare Punktmannichfaltigkeiten," *Mathematische Annalen* 20 (Mar. 1882): 114.

of the *alogon*—and thus also with the materiality, irrationality, fluidity, and femininity of the Aristotelian substrate—then it can only be on the model of a Pythagorean purification or *distillation*, like the dynamic that binds Plato's chthonic sirens to his heavenly ones. Those sexless, cosmic beings from the end of the *Republic*—who dwell with the planets, know past, present, and future, and sing the harmonies of the Pythagorean tetrakys—find in Cantor's comprehensively crystalline universe their first truly appropriate medium.[30]

2

I have spent a great deal of time on Cantor's conception, even though our digital ocean is no longer the one he discovered—or created—because Cantor's conception provides the foundation for everything Alan Turing later does, both methodologically and philosophically, to reconfigure this domain. Methodologically, this is because the cut that Cantor introduces into the *apeiron*, his limitation of the traditionally unlimited, is the small (infinitely thin) slice of terra firma from which Turing sets out. Turing uses the same ingenious mapping technique that Cantor invented, known as diagonalization, to demonstrate that the set of all computable numbers has the same *size* as the set of the natural counting numbers, which—given the demonstrably astronomical difference between the cardinalities of the counting numbers and the reals—means that nearly all real numbers are *not* computable and, by extension, *not* accessible by any known or knowable rule. Philosophically, this insight will then provide Turing with a basis for implicitly severing his relationship with the continuum of the reals, together with everything the continuum meant for Cantor and still means, in many contexts, for us. The result is a model that has only grown in power and prevalence, in contexts from big tech to information theory to cosmology to pop culture, even as it has failed to attract much attention, for continuum-related reasons, from the humanities. This model does not have an associated philosophy, and it (consequently?) often sounds arid, even banal, and certainly un-European to theory-trained ears. But it may also, at some presumably relevant level, be fundamentally right. The question I want to pose, therefore—the question whose urgency I am hoping here at least to make plausible—is whether

30. See Plato, *Republic* 10. The corresponding Pythagorean formulation, which is attributed to Pythagoras himself and reported among the so-called *acusmata*, is: "What is the oracle at Delphi? The *tetraktys*, which is the harmony in which the Sirens are" (Iamblichus, *On the Pythagorean Life*, trans. John Dillon and Jackson Hershbell [Atlanta, Ga., 1991], p. 82). On the relationship between the chthonic and the heavenly sirens, see Carine Van Liefferinge, "Les Sirènes: du chant mortel à la musique des sphères. Lectures homériques et interprétations platoniciennes" [The Sirens: From the Deadly Song to the Music of the Spheres. Homeric Readings and Platonic Interpretations], *Revue de l'histoire des religions* 229 (Dec. 2012): 479–501.

anything recognizably philosophical can actually be done with such a figure of finite, discontinuous, computable foundations (other than run from it) and, if so, what that *doing* could look like.

At an explicit level, the question of the continuum barely arises for Turing in "On Computable Numbers, with an Application to the Entscheidungs-problem" (1937). His opening gambit, which will turn out to be a Turingism, is to translate a large, amorphous philosophical concept with no consensually agreed-upon definition—in this case, the concept of computation—into a precisely bounded thought experiment involving machines, with the aim of establishing what belongs inside these hypothetically constructed limits. At the outset of the introduction, he stipulates the kind of translation he has in mind: "The 'computable' numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. . . . According to my definition, a number is computable if its decimal can be written down by a machine."[31] And in the beginning of section 1, he explains what he takes this stipulation to mean:

> We may compare a man in the process of computing a real number to a machine which is only capable of a finite number of conditions q_1, q_2, . . . , q_R which will be called "*m*-configurations." The machine is supplied with a "tape" (the analogue of paper) running through it, and divided into sections (called "squares") each capable of bearing a "symbol." At any moment there is just one square, say the *r*-th, bearing the symbol $\mathfrak{S}(r)$ which is "in the machine." We may call this square the "scanned square." The symbol on the scanned square may be called the "scanned symbol." The "scanned symbol" is the only one of which the machine is, so to speak, "directly aware." However, by altering its *m*-configuration the machine can effectively remember some of the symbols which it has "seen" (scanned) previously. The possible behavior of the machine at any moment is determined by the *m*-configuration q_n and the scanned symbol $\mathfrak{S}(r)$. This pair q_n, $\mathfrak{S}(r)$ will be called the "configuration": thus the configuration determines the possible behavior of the machine. ["O," p. 231]

Turing continues by describing a process that will allow the machine to "take notes" (to write down symbols that do not belong to the decimal expansion of the number it is computing), and he then concludes: "It is my contention that these operations include all those which are used in the computation of a number" ("O," p. 232).

31. A. M. Turing, "On Computable Numbers, with an Application to the Entscheidungs-problem," *Proceedings of the London Mathematical Society* 42 (Nov. 1937): 230; hereafter abbreviated "O."

It makes sense to take a moment, here, to appreciate up front how far away we already are from Cantor's fully analyzed ocean of mathematical thought. In place of the celestial medium of a dynamically vibrating, punctiform ether, which extends infinitely in every direction to place precisely zero restrictions on the possibilities of deep mathematical analysis, we have the medium of a one-dimensional tape, which is "the analogue of paper" but also of the *thinnest* imaginable conception of mathematical-computational time.[32] This paper-time moves backward and forward in fits and starts, one square or discrete mental state after another, in order to serve as the substrate for the equally discrete inscriptions of a purely symbolic, purely syntactic, purely nonreferential mode of analysis. The result is an explicitly deterministic process ("the possible behavior of the machine at any moment is determined by the *m*-configuration q_n and the scanned symbol $\mathfrak{S}[r]$"), which, according to Turing, coincides with the only mode of operation we need in order to range—if not freely then at least sovereignly—over the whole domain of finitely articulable mathematical operations. ("Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth" ["O," p. 230].)

Turing does not (yet) intend his explosive refiguring of mathematical power and possibility as a general statement about what can be mathematically conceived. He intends it, rather, as a general statement about what can be mathematically computed. But the reason he is interested in the question of computation in the first place has everything to do with the problems that had afflicted the Cantorian model of mathematical concept formation almost from its inception, and it is in this sense that the problem of the continuum hovers constantly in the background of his paper. As the second half of the title, "with an application to the Entscheidungsproblem," makes clear, Turing considers the primary contribution of his foray into the nature of computability to be his solution to David Hilbert's famous question about mathematical decidability. Hilbert's question about decidability, in turn, grew out of his project for axiomatically *re*founding, and thereby rescuing, the Cantorian theory of manifolds from a new battery of antinomies—the

32. The elements of Cantor's point sets presumably do not vibrate. But this picture does seem to be quite close to the structure he actually attributed to matter, on the basis of his mathematical analyses. Cantor was a proponent, at the physical level, of the punctiform ether theories advanced by the eighteenth- and nineteenth-century founders of field theory (he names Boscovich, Ampere, and Cauchy) over against *both* the void-plus-atoms theories of the Newtonian mechanists *and* the quasi-mystical plenum-of-waves theories of the Fourier-inspired vitalists. See, for example, Cantor, letter to Mittag-Leffler, 16 Nov. 1884 and 18 Nov. 1884, in *Briefe*, pp. 224–5, 228–9.

so-called set theoretical paradoxes—which threatened to demote Cantor's uncountable point set back into an *apeiron*.

One way of understanding what the set theoretical paradoxes reveal is that Cantor's utterly general understanding of conceptual definition is *too* general: his "rule" for what constitutes a manifold, which studiously avoids placing any restrictions whatsoever on the *kinds* of procedures used to think a many as a one, makes room for paradoxical, inconsistent manifolds like the set of all sets that are not members of themselves. As Cantor himself had shown, however, it is precisely this kind of absolutely unbounded generality which is, in fact, required to encompass the full range of operational potential represented by the continuum of the reals. Hilbert's idea was therefore, in essence, to approach this full, Cantorian generality in a different way. He would simply *take* what was needed in order to construct the entirety of the numerical system via a series of axioms, and then he would work backwards from there in order to demonstrate, with purely finite, logical means, that the axioms were (1) *independent* of one another, (2) *consistent* with one another, (3) *complete* relative to the domain they were describing, and (4) that all statements they made possible were *decidable* within the syntax of the system.

The meaning of *decidable*, which Hilbert considered the most mathematically central of the four criteria, was later clarified by Hilbert's assistant, Heinrich Behmann, in the following manner: "It is of fundamental importance for the character of this problem that *only mechanical calculations* according to given instructions, without any thought activity in the stricter sense, are admitted as tools for the proof. One could, if one wanted to, speak of *mechanical* or *machinelike* thought. (Perhaps one could later let the procedure be carried out by a machine.)"[33] At the time of Behmann's pronouncement, there did not yet exist a precise, generally accepted definition of the kind of "mechanical calculations" he was describing. Nineteenth-century and early twentieth-century mathematicians, to whom the distinction between "machinelike thought" and "thought activity in the stricter sense" was nonetheless of paramount importance, had to content themselves with circumlocutions of various kinds, including determinate law, writable function, and finite rule. Such a determinate, writable, finite, and mechanical mode of reasoning will not suffice, as Cantor and others had already made clear, to give us the domain of all reals; but Hilbert hoped that it *would* be able to guarantee the integrity of a numerical continuum that had already been axiomatically, rather than operationally, grasped.

33. Quoted in Paolo Mancosu, "Between Russell and Hilbert: Behmann on the Foundations of Mathematics," *The Bulletin of Symbolic Logic* 5 (Sept. 1999): 321.

Many of Hilbert's contemporaries, both inside and outside of mathematics, celebrated him explicitly for rescuing the highest achievements of modern thought, by which they meant the rigorous set theoretical investigation of the continuum. They compared axiomatics with a ship and Hilbert himself with the ancient Greek mathematician Eudoxus, originator of the method of analogical proportion, which had made it possible for Greek geometers to operate over the abyss of the numerically incommensurable.[34] In one sense, these jubilant contemporaries were certainly right: a version of Hilbert's axiomatic systematization still lies at the foundation of every distinct mathematical domain, from arithmetic to geometry to set theory. But in another deeper sense, the party was premature because Hilbert's system did not survive in anything similar to the form that he envisioned, namely as a definitive rewriting of the Cantorian continuum with purely finitistic and syntactic symbolic means. In 1931, Kurt Gödel published his famous Incompleteness Theorem in "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems."[35] The paper used the Cantorian technique of diagonalization to show that no consistent system of axioms about the natural numbers, whose theorems can be listed by a mechanical procedure, is capable of proving all truths about the arithmetic of the natural numbers; a second related result shows that such a system cannot demonstrate its own consistency. And in 1936, no fewer than *four* different, rigorous definitions of finite computability—all of which were later shown to be extensionally equivalent—yielded two different proofs that Hilbert's decidability criterion, too, had to go.[36]

The question was what these failures should be taken to *mean*. Gödel credits Turing, alone among the four new definers of *computability*, with having brought the true philosophical stakes of Hilbert's project to the fore by explicitly identifying the syntactic formalization of a system with the activity of a finite machine.[37] He vehemently disagrees, however, with Turing's claim that such machines can adequately represent the activity of a human computer and, by extension, human thought more generally: "What Turing disregards

34. See, for example, Helmut Hasse and Heinrich Scholz, "Die Grundlagenkrisis der griechischen Mathematik," *Kant-Studien* 33, nos. 1–2 (1928): 13.

35. See Kurt Gödel, "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I," in *Collected Works*, ed. Solomon Feferman et al., 5 vols. (New York, 1986), 1:144–95.

36. See Alonzo Church, "A Note on the Entscheidungsproblem," *The Journal of Symbolic Logic* 1 (Mar. 1936): 40–41; S. C. Kleene, "General Recursive Functions of Natural Numbers," *Mathematische Annalen* 112 (Dec. 1936): 727–42; Emil L. Post, "Finite Combinatory Processes—Formulation 1," *The Journal of Symbolic Logic* 1 (Sept. 1936): 103–5; and Turing, "On Computable Numbers." The applications to the decidability question are in Church and Turing.

37. See Oron Shagrir, "Gödel on Turing on Computability," in *Church's Thesis after 70 Years*, ed. Adam Olszewski, Jan Woleński, and Robert Janusz (Piscataway, N.J., 2006), p. 393.

completely is the fact that *mind, in its use, is not static, but constantly developing.* . . . Therefore, although at each stage the number and precision of the abstract terms at our disposal may be *finite*, both (and, therefore, also Turing's number of *distinguishable states of mind*) may *converge toward infinity* in the course of the application of the procedure."[38] Gödel, in other words, acknowledges that human mental capacity must be physically finite, but he believes that this mental capacity has a special, *non*mechanical relationship to the *continuous* passage of time, which allows it to nonetheless transcend the limits of a purely mechanical finitude (by, say, formalizing new techniques and/or giving itself new problems). His interpretation of the meaning of Turing's decidability results—and of his own incompleteness results—is thus that they "do not establish any bounds for the powers of human reason, but rather for the potentialities of pure formalism in mathematics."[39] The discrepancies they highlight occur when we try to use discrete, finite, formal, *mechanical* means to grasp, ground, or otherwise confine the arc of human thought with its nonnegotiable, if necessarily indeterminate, ties to continuity, infinity, intuition, and organic purpose.[40]

What Gödel only implicitly admits, however, is that even according to his own insistently organicist account of cognition, Turing's analysis has turned the tables because nearly everything that used to belong eminently to the order of the organic—including order itself—has now been relegated to the order of the machine. Turing proves that the set of all Turing-machine-computable numbers, and by extension all Turing-machine-computable functions, is countable rather than uncountable (in other words, both sets can be put into one-to-one correspondence with the natural counting numbers). But Turing also shows that these collections include every kind of numerical relation that we might ever want to describe, from infinitesimal calculus to Fourier transformations and beyond, all without requiring any illicit recursions to the (much "bigger") resources of the continuum. And what this means, in turn, is that the *only* thing on the far side of Gödel's mechanical-organic boundary, the only thing uniquely associated with the supposedly triumphal transcendence of human reason, is the *dis*order of total

38. Gödel, "Some Remarks on the Undecidability Results," in *Collected Works*, 2:306. See also Shagrir, "Gödel on Turing on Computability."

39. Quoted in Feferman, Robert M. Solovay, and Judson C. Webb, "Introductory Note to *1972a*," in Gödel, in *Collected Works*, 2:292.

40. For a contemporary version of this argument, see Roger Penrose, *The Emperor's New Mind: Concerning Computers, Minds and the Laws of Physics* (New York, 1989). The mainstream philosophical position is that Gödel's incompleteness results cannot, in fact, be used to mount a cogent critique against computational theories of mind. See, for example, Stewart Shapiro, "Mechanism, Truth, and Penrose's New Argument," *Journal of Philosophical Logic* 32 (Feb. 2003): 19–42.

randomness.[41] Turing's machines, in their slow, plodding, binary way, are perfectly capable of establishing every possible "definite number" as envisioned by Plato's *Philebus*, but what they allow to "drop into the *apeiron*" when they halt is Plato's *nous*—together with Cantor's set theoretical notion of human freedom. Such freedom reappears, here, still indexed to infinity, as the essentially superfluous capacity to range unbounded over the entirety of the meaningless and arbitrary. Insofar as we humans choose to align ourselves with this supplemental remainder, as Gödel proposes, we can perhaps succeed in subverting or deconstructing the patient, orderly activity of the machines—by stopping them from halting—but we do so only at the expense of abandoning our own claims to order, and thus also to the one-over-many of "rational" selfhood.

Gödel himself, of course, never puts the predicament of human reason in so many words, but it is clear that his poststructuralist readers have learned, through him, Turing's lesson in both the mathematical and linguistic domains. Thinkers who are reluctant to relinquish the uncountable infinitude of the real(s), because they see in it the nonnegotiable condition for a thought that could transcend computation, have had no choice, since the middle of the last century, but to prioritize the *alogon* as the antiessentialist essence of *logos*.[42] Linguistically speaking, this implicitly Gödelian perspective finds a powerful articulation in the early work of Jacques Derrida and Deleuze, who explore the outer limits of structuralism's binary sign systems in order to argue that an unsystematizable *outside* (still) exists. The point of such analyses is not to revert to a prestructuralist understanding of the continuum as originary given, or ground, but rather to establish the uncomputable as the ineradicable *endpoint* of thought, and so also to beat the new digital models of language at their own digitalizing game—by demonstrating, formally and syntactically, that a fully formalized, syntactical thought cannot *decide* on its own formal boundaries. "Undecidability," writes Derrida in relation to the word *hymen*, "is not caused here by some enigmatic equivocality, some inexhaustible

41. The only numbers whose decimal expansions cannot be written down by a machine are those that never repeat. These decimal expansions are fully random in the sense of being governed by no finite writable law whatsoever. All noncomputable numbers are random in this sense.

42. A contemporary mathematical variation on this poststructuralist theme can be found in Gregory Chaitin's quest for the numerical endpoint of mathematical thought—the so-called omega number—which encapsulates the probability that any randomly constructed Turing machine will halt and, in doing so, marks the uncomputable boundary of computation. See Gregory Chaitin, *Meta Math! The Quest for Omega* (New York, 2005). The uncomputable domain of "infinitely dark blackness" is coded as fertile and feminine and associated with the source of Chaitin's own mathematical creativity throughout the book (p. 140).

ambivalence of a word in a 'natural' language. . . . What counts here is not the lexical richness, the semantic infiniteness of a word or concept, its depth or breadth. . . . What counts here is the formal or syntactical *praxis* that composes and decomposes it."[43] It is on this basis that Derrida can speak with such confidence of the "analogy" between Gödelian undecidability and "the formal or syntactical praxis" of *différance*. Derridean deconstruction relates to the traditional philosophical domain of the *alogon*, and thus also of sexual difference, like Gödel's undecidable boundary to the traditional mathematical continuum.[44] The unfathomable depths of the infinite, feminine ur-substance, whose penetrable limit the hymen traditionally represents, have collapsed here into a dimensionless, formal alterity that can be called upon to destabilize the computational state of things even if it can no longer engender new orders. When Deleuze later proposes, in partnership with Félix Guattari, that thought should seek to *transgress* this deconstructive boundary—following lines of flight that culminate in the deterritorialized continuity of "the sea, the archetype of smooth space"—he therefore does so by asking us to *become woman* rather than attempting to possess her.[45] In the wake of Turing's computability thesis, the philosophical project of rationally mastering the irrational, of inseminating chaos, of penetrating the hymen, has been rendered officially obsolete. In the process, however, impenetrability itself—the ostensible inviolability of a real conceived as fundamentally irrational, random, and chaotic—has emerged as a *reservoir* of Gödelian hope because it suggests that the infinite openness of the continuum might still exist, albeit in a post-Cantorian, computationally superfluous form. The feminine and oceanic figures that haunt the margins of so many contemporary attempts to think information age technology, ecology, political eventhood, and general being are emblems of that hope, as well as ciphers of the loss to which it is tethered.

What does Turing himself think about all this, and can we learn anything important from following his interpretive trajectory rather than Gödel's? Turing thinks, first and foremost, that the concept of thinking is very amorphous, and he proposes that we translate it into a "closely related" thought

43. Jacques Derrida, "The Double Session," *Dissemination*, trans. Barbara Johnson (Chicago, 1981), p. 220.

44. Ibid., p. 219.

45. Gilles Deleuze and Félix Guattari, *A Thousand Plateaus*, trans. Brian Massumi, vol. 2 of *Capitalism and Schizophrenia*, trans. Hurley et al. (Minneapolis, 1987), p. 480. For a seminal feminist reading of the concept of *le devenir-femme*, see Alice Jardine, "Woman in Limbo: Deleuze and His Br(others)," *SubStance* 13, nos. 44–45 (1984): 46–60. On Deleuze's large but also largely implicit debt to Gödel, see Daniel W. Smith, "Axiomatics and Problematics as Two Modes of Formalization: Deleuze's Epistemology of Mathematics," in *Virtual Mathematics: The Logic of Difference*, ed. Simon B. Duffy (Manchester, 2006), pp. 145–68.

experiment—one involving the relationship between humans and machines—which will allow us to analyze its operational contents.[46]

> The new form of the problem can be described in terms of a game which we call the "imitation game." It is played with three people, a man (A), a woman (B), and an interrogator (C) who may be of either sex. The interrogator stays in a room apart front the other two. The object of the game for the interrogator is to determine which of the other two is the man and which is the woman. He knows them by labels X and Y, and at the end of the game he says either "X is A and Y is B" or "X is B and Y is A." . . . We now ask the question, "What will happen when a machine takes the part of A in this game?" Will the interrogator decide wrongly as often when the game is played like this as he does when the game is played between a man and a woman? These questions replace our original, "Can machines think?" ["C," pp. 433–34]

This thought experiment of Turing's, from "Computing Machinery and Intelligence" (1950), has received at least as much attention and is at least as well known as the earlier thought experiment involving Turing machines from "On Computable Numbers." Unlike his earlier experiment, however, which translates the concept of computation, this translation of *thinking* is almost always subjected by commentators to a *second* translation back *out* of the "relatively unambiguous words" used by Turing himself. And what gets programmatically lost in such second-degree translations is the gender component.[47]

On the surface, this translational impulse makes perfect sense because it is not at all obvious, reading through Turing's description, what the convoluted

46. Turing, "Computing Machinery and Intelligence," *Mind* 59 (Oct. 1950): 433; hereafter abbreviated "C." "I propose to consider the question, 'Can machines think?' This should begin with definitions of the meaning of the terms 'machine' and 'think.' . . . Instead of attempting such a definition I shall replace the question by another, which is closely related to it and is expressed in relatively unambiguous words" (p. 433).

47. Among treatments of the Turing test that do pay attention to the original setup, it is common to point out that it is a test of *passing* and thus of gender identity as performance, a fact which may well have had additional biographical significance for Turing as a homosexual man in a profoundly homophobic society; see Homay King, *Virtual Memory: Time-Based Art and the Dream of Digitality* (Durham, N.C., 2015), pp. 18–46, esp. pp. 42–46. Another important exception to this rule of gender erasure is Katherine Hayles's discussion in the prologue to *How We Became Posthuman*, where Turing's setup is used to highlight the *un*gendered or rather spliced nature of the posthuman subject. This account cannot make sense of the significance of gender for Turing's concept of thinking, but it does make good sense of the *splicing effect* of the test understood as a social exercise, which is Hayles's primary concern. See N. Katherine Hayles, prologue to *How We Became Posthuman: Virtual Bodies in Cybernetics, Literature, and Informatics* (Chicago, 1999), pp. xi–xiv.

setup of three humans, one computer, and a game of deception could possibly have to do with the question "Can machines think?" Once we generalize gender out of the equation, on the other hand, and construe the test as posing the question "Can a machine, pretending to be a human, fool a human interlocutor?" then the connection to thinking becomes clear. The problem is, first, that Turing does not frame his test this way, which means that this more general formulation does not describe a/the Turing test. And the problem is, second, that the gender-free formulation effaces the philosophical stakes of the formulation he *does* propose. It is not actually all that hard to make sense of the gender dimension here if we keep the historico-philosophical backdrop of the *apeiron* and its inherent femininity firmly in mind. The giveaway is in the game's asymmetry: Turing identifies thinking with what he calls the imitation game, and this game is played *with* three people, but it is only played *by* one of them, namely the man, A. Turing stipulates that A's task is to *imitate* the woman, B, to the point of indistinguishability for the interlocutor, C: "It is A's object in the game to try and cause C to make the wrong identification" ("C," p. 434). The task of the woman, on the other hand, is not to imitate (or to think) but to *be* womanly, and nothing (else) that she says or does within the scope of the game will make the slightest difference: "The object of the game for the third player (B) is to help the interrogator. The best strategy for her is probably to give truthful answers. She can add such things as 'I am the woman, don't listen to him!' to her answers, but it will avail nothing as the man can make similar remarks" ("C," p. 434). In the second phase of the test, the computer takes the place of the man. In both scenarios, so the implication, we will be able to gauge the extent of A's capacities to think by the effectiveness with which A uses its/his discrete, finite means to imitate—which means, also, to represent, in the strong, mathematical sense of an *Abbildung* or functional map—the necessarily unarticulated, undifferentiated being of B.

Thinking, for Turing, is thus still very much a matter of capturing or picturing or mimicking the continuous being of a feminine-encoded *apeiron*, with the admittedly radical difference that the behavior of the *apeiron* itself has become entirely irrelevant to the constellation.[48] Turing knows, based on his definition of computable numbers, that we no longer need the continuum in order to construct our representations, which is to say, to think. And he can therefore approach the longstanding science-fictional trope of a machinic woman come to efface or replace the real—a trope that runs from E. T. A. Hoffmann's "The Sandman" (1816) to Fritz Lang's *Metropolis* (1927)

48. It is therefore *also* irrelevant whether Turing conceives of his woman as a finitistically calculating human in her own right or as an infinitistic, continuous entity with her own unique model of (non)thought. The point of thought for him, and thus also of the imitation game, is that it can imitate without needing to concern itself with this question.

to the *Blade Runner* franchise (1982/2017)—with something closer to glee than anxiety. The "real" woman in his proposed test is nothing but a placeholder. And by 1939 Turing had already envisioned a technique for subordinating even this placeholder activity to the demands of machinic computation via the integration of his so-called "oracle machines."[49] The task of such machines is to act like total randomness generators and *at the same time* to provide prophetic hints, which can effectively shorten the computational process. The Platonic-Pythagorean oracle of Delphi—that "harmony in which the Sirens sing"—reappears here, in other words, as a purely heuristic tool in the deflated manner of Kant's organicist guiding thread and at the same time as the emblem of Turing's triumph over infinitist conceptions of nonmachinic time. Destiny and randomness, as the two faces of the unknowable, irrelevant temporality that lies beyond the boundaries of machinic determinism, are consigned by Turing to dwell together, perhaps even to merge together—does it actually matter which?—inside the black box of his ultimate feminine machine.

The point, here, is not that Turing can dispense with real-life women but rather that his definition of thinking does not require the reality of Woman. If Gödel's post-Turing reinterpretation of the *apeiron* points forward toward the sirens (and *khôras*, Pandoras, Gaias) of contemporary theory, then Turing's own approach to the outside of thought leads just as naturally in the direction of Siri, Apple's ubiquitous digital secretary, who helps us navigate the informational sea without ever gesturing toward an uncomputable beyond. Siri does not concern herself with the primordial waters of deep time and infinite possibility, but she is also not a *she* except in the most superficial and casually sexist of ways. It is therefore perfectly appropriate to align Turing's legacy, as Kittler does, with the demise of the female secretary (and by extension the bourgeois letter-teaching mother and the wartime woman computer).[50] What is less clear is whether we also need to mourn this loss or to correct for it by returning—in Kittler's case, bodily as well as textually—to the ancient Greek origins of a "properly" oceanic femininity.[51] Turing's extremely un-Heideggerian exploration of what thinking means (*Was heißt Denken?*) offers at least a hint

49. Turing, "Systems of Logic Based on Ordinals," *Proceedings of the London Mathematical Society* 45 (Jan. 1939): 173.

50. See Kittler, "Computeranalphabetismus," in *Literatur im Informationszeitalter*, ed. Kittler and Dirk Matejovski (Frankfurt, 1996), pp. 237–51, esp. p. 239.

51. In 2004, Kittler travelled to the Li Galli islands in order to perform an acoustic analysis of the precise physical environment that ostensibly produced the sirens' song; see Winthrop-Young, "Siren Recursions," in *Kittler Now: Current Perspectives in Kittler Studies*, ed. Stephen Sale and Laura Salisbury (Malden, Mass., 2015), pp. 71–94.

of an alternative approach, when he responds to traditional humanist concerns about creativity, unpredictability, and the productivity of error with what amounts to a rhetorical shrug. It may well be the case, he admits in "Computing Machinery and Intelligence," that machinic thought is in principle predictable, error-free, and devoid of spontaneous leaps; but machines still manage to surprise *him* all the time, errors in thought are best avoided, and a truly spontaneous creativity would be indistinguishable from arbitrariness (see "C," pp. 448–51). Such a refusal to hand-wring about the consequences of a finite, digital medium for thought, whether conceived as a web of Turing tapes or as quantum entanglement, may not amount to a philosophical position. But it does entail one, or rather, it should. Turing's implicit thesis is that the countable set of computables is big enough—as big as an ocean, even, albeit not an *apeiron*—and the question we need to ask and answer in response is: Big enough for what? We will have a better chance of making headway on this problem, I think, if we stop calling on the remainder of Woman to do the work for us.