

# Scaling properties of exclusive vector meson production cross section from gluon saturation

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**Abstract** It is already known from phenomenological studies that in exclusive deep-inelastic scattering off nuclei there appears to be a scaling behavior of vector meson production cross section in both nuclear mass number,  $A$ , and photon virtuality,  $Q^2$ , which is strongly modified due to gluon saturation effects. In this work we continue those studies in a realistic setup based upon using the Monte Carlo event generator *Sartre*. We make quantitative predictions for the kinematics of the Electron-Ion Collider, focusing on this  $A$  and  $Q^2$  scaling picture, along with establishing a small region of squared momentum transfer,  $t$ , where there are signs of this scaling that may potentially be observed at the EIC. Our results are represented as pseudo-data of vector meson production diffractive cross section and/or their ratios, which are obtained by parsing data collected by the event generator through smearing functions, emulating the proposed detector resolutions for the future EIC.

## 1 Introduction

Studies of the proton partonic structure using the most precise data provided by the H1 and ZEUS experiments at HERA facility [1, 2], based upon deep inelastic scattering (DIS) measurements, have highly enriched our knowledge on small Bjorken- $x$  physics with findings of the rapid growth of gluon density at small longitudinal momentum fraction  $x$ . In particular, the HERA measurements in  $e + p$  scattering at small  $x \leq 10^{-2}$  have shown that the gluon number density seems to have an “uncontrollably” rising nature because of which the

gluonic part of the proton cross section dominates its total cross section. This strong growth of gluon density occurs in the space of high gluon occupancies of the order of  $1/\alpha_s$ , where  $\alpha_s$  is the QCD coupling constant. It results in violation of the cross section unitarity bound, which can be tamed by introducing *gluon saturation* effects into the whole high energy scattering picture.

The maximal gluon occupancy takes place for any  $x$  value at small  $x$ , for which there is a corresponding saturation scale  $Q_s(x)$ , with  $Q_s^2 \gg \Lambda_{QCD}^2$  (where  $\Lambda_{QCD}$  is the QCD intrinsic scale). The saturation effects are nonlinear QCD phenomena that can be described by the Color Glass Condensate (CGC) effective theory [3–9]. In particular, the data describing the proton structure function are well reproduced within CGC ansatz [10–12]<sup>1</sup>.

For direct studies of nonlinear QCD saturation phenomena it is necessary to have the  $e + p$  systems being collided at center-of-mass energies far exceeding those reached at HERA because the proton’s saturation scale,  $Q_{s,p}^2(x)$ , is not large enough at values of  $x$  probed at HERA energies, and consequently the evidence for gluon saturation has not been very clear so far. Nonetheless, in the case of  $e + A$  scattering one can look at higher gluon density effects, at energies by an order of magnitude lower than those used for  $e + p$  scattering at HERA. In this case the nuclear saturation momentum and gluon density will scale as  $Q_{s,A}^2(x) \sim Q_{s,p}^2(x) \times A^{1/3}$ , by which the nonlinear effects in heavy nuclei shall be amplified efficiently. Probing a nucleus with large mass number  $A$  is equivalent to probing the proton at several times larger energy. De-

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<sup>1</sup>Refs. [13–18] discuss the first hints of the onset of gluon saturation stemming from  $p(d) + A$  collisions at RHIC energies, meanwhile, there are also alternative explanations [19–21] to consider.

tailed studies like those accomplished in [22, 23] support the use of  $A^{1/3}$  dependence in various phenomenological calculations and applications. The scale  $Q_{s,A}^2(x)$  controls the nuclear dynamics at high energies.

Currently there are no available nuclear DIS data at small- $x$  region but the proposed Electron-Ion Collider (EIC) in the USA [24–27] and Large Hadron Electron Collider (LHeC) at CERN [28] will aim at directly measuring the saturation regime of large gluon densities in the upcoming high-precision EIC and LHeC era. In particular, the EIC White Paper [25] has already shown the potential of EIC to collide high energy electron and ion beams, providing unprecedented access to gluon dominated kinematic regions of nucleons and nuclei. Furthermore, strongly polarized electron and proton beams will unravel the spatial and spin structure of the proton. Ref. [26] scrutinizes the kinematic coverage of EIC and the energy dependence of key observables that are essential to assure solid and robust EIC program. The EIC Yellow Report [27] describes the program’s physics case and the resulting detector requirements/concepts.

But before these colliders come into their existence and operations, another possible way to study DIS on nuclei is provided by ultraperipheral  $A + A$  and  $p + A$  collisions (UPC), where relatively short-range strong interactions are suppressed by processes taking place in the nuclear periphery with large impact parameter. The data on *diffractive vector meson* production in such collisions [29–38] demonstrate the sensitivity of this production process on nuclear effects at small  $x$ , since the pertinent measurements are quite sensitive to gluon distributions at saturation. The reason is that in perturbative QCD (pQCD), the hard diffractive cross section at leading order is proportional to gluon density squared [39], which makes it the most sensitive probe to small- $x$  gluons, whereby the vector meson production becomes an extremely useful process to study the small- $x$  hadronic structure in general.

Phenomenological studies of Ref. [40] (see also the references therein) have already demonstrated that the onset of gluon saturation is potentially observable in exclusive vector meson production off large nuclei in high-energy  $e + A$  scattering. It is in particular shown that within CGC theory, the nuclear saturation effects significantly modify the  $A$  and  $Q^2$  scaling properties of the exclusive vector meson production cross section, if one passes from the pQCD regime ( $Q^2 > Q_{s,A}^2$ ) to the saturation regime ( $Q^2 < Q_{s,A}^2$ ). In a diffractive scattering process (see the next section for more details) an electron probe scatters off a target proton or nucleus, where the exchanged virtual photon splits into a  $q\bar{q}$  *dipole*. The dipole subsequently interacts with the target in the target’s rest frame via a color-neutral vacuum excitation,

*Pomeron*, which in pQCD is visualized as a colorless combination of two or more gluons. The parton longitudinal momentum fraction within color-neutral Pomeron (that is also transferred to the produced vector meson) is designated by  $x_{\mathbb{P}}$ , which in diffractive DIS is equivalent to the Bjorken  $x$  for exclusive processes.

Based on the aforementioned simple dipole interaction mechanism, advanced and elaborated dipole model frameworks have been developed in [23] and [41–44]. Refs. [23] and [44] have the impact parameter dependence introduced in their dipole models. The exclusive processes are also included in the dipole model of [44], which goes by the name bSat or IPSat. A linearized dipole model (to the model of [44]) called bNonSat or IPNonSat, which separates and isolates the gluon saturation effects from other small- $x$  effects, is introduced in [23]. The IPSat and IPNonSat dipole models, for both protons and nuclei, are implemented into Monte Carlo event generator *Sartre* [45, 46], the purpose of which is to simulate diffractive exclusive vector meson production and deeply virtual Compton scattering (DVCS) events in  $e + p$  and  $e + A$  scatterings at EIC and LHeC center-of-mass energies<sup>2</sup>. The current (and upcoming) work on *Sartre* includes simulations of events in  $p + A$  and  $A + A$  UPC [49], simulations of inclusive processes, as well as simulations of diffractive exclusive events after *geometrical* and *saturation scale* fluctuations of gluon spatial distributions (see [50–52] for more details) are implemented into the generator’s framework.

In this paper we partially continue the studies of Ref. [40] in a realistic EIC setup utilizing the *Sartre* generator. The content of the paper is structured as follows. In Sec. 2 we describe the basics of diffractive scattering and IPSat dipole model, outlining also the IPNonSat model and phenomenological corrections to the diffractive scattering amplitude. In this connection, Appendix A shows some details related to diffractive differential cross sections. In Sec. 3, we first focus on a realistic view of the experimental measurement of coherent and incoherent cross sections, from exclusive vector meson production processes in diffractive  $e + Au$  scattering, obtained with the integrated luminosity of  $10 \text{ fb}^{-1}/A$ . Then, we discuss the  $A$  and  $Q^2$  scaling properties of exclusive vector meson production. We make quantitative predictions for the EIC kinematics by focusing on the scaling picture as a function of  $Q^2$  at close-to-zero squared momentum transfer region of  $|t| =$

<sup>2</sup>There is also another Monte Carlo generator, *STARlight* [47], which simulates a wide variety of vector meson final states, produced in  $e + A$  scattering. The improved version of this generator, dubbed as *eSTARlight*, has been used for exclusive vector meson production studies at the EIC kinematics [48].

[0.000 – 0.001] GeV<sup>2</sup>, as well as at non-zero  $t$  regions of  $|t| = [0.003 – 0.004]$  GeV<sup>2</sup>, [0.006 – 0.007] GeV<sup>2</sup>, [0.009 – 0.010] GeV<sup>2</sup>, and [0.012 – 0.013] GeV<sup>2</sup>. Throughout the paper unless specified otherwise, instead of these  $t$  bins we will refer to their central values for simplicity: namely  $|t| = 5 \times 10^{-4}$  GeV<sup>2</sup>,  $3.5 \times 10^{-3}$  GeV<sup>2</sup>,  $6.5 \times 10^{-3}$  GeV<sup>2</sup>,  $9.5 \times 10^{-3}$  GeV<sup>2</sup>, and  $1.25 \times 10^{-2}$  GeV<sup>2</sup>. However, it should be always understood that we have performed *Sartre* simulations in the corresponding  $t$  bins, rather than at their central values. In the case of  $|t| = 5 \times 10^{-4}$  GeV<sup>2</sup>, we consider two regimes of  $Q^2$  as in [40], one for  $Q^2 > Q_{s,A}^2$  and another for  $Q^2 < Q_{s,A}^2$ .

We conclude on our results in Sec. 4 afterwards, discussing the prospects of potential future developments as well. We show our results in terms of pseudo-data on vector meson production cross sections and/or their ratios. We also calculate the uncertainties of the *Sartre*-simulated pseudo-data based upon using detector resolutions and a smearing technique outlined in the EIC Handbook [53] (see Appendix B).

## 2 IPSat dipole model in Monte Carlo event generator *Sartre*

### 2.1 Diffractive DIS picture

First let us briefly take a look at diffractive DIS kinematics of  $e + p$  scattering. The scattering process of exclusive production of a vector meson  $V$  with a momentum  $P_V$  in DIS  $e + p$  is given by

$$l(\ell) + p(P) \rightarrow l'(\ell') + p'(P') + V(P_V), \quad (1)$$

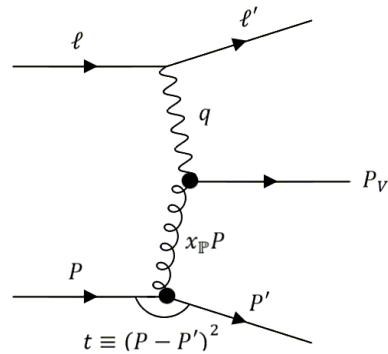
where  $\ell$  and  $\ell'$  are the electron's incoming and outgoing momenta,  $P$  and  $P'$  are the proton's incoming and outgoing momenta (see Fig. 1). The scattering process is characterized by the following Lorentz invariant quantities:

$$Q^2 \equiv -q^2 = -(\ell - \ell')^2, \quad t \equiv (P - P')^2,$$

$$x_{\mathbb{P}} \equiv \frac{(P - P') \cdot q}{P \cdot q} = \frac{M_V^2 + Q^2 - t}{W^2 + Q^2 - m_p^2}, \quad (2)$$

where  $q$  is the virtual photon momentum,  $m_p$  is the proton mass,  $M_V$  is the mass of the produced vector meson  $V$ , and  $W^2 = (P + q)^2$  is the total center-of-mass energy squared of the  $\gamma^* - p$  scattering. The scattered proton with the momentum  $P'$  can either remain intact or break up, leading to *coherent* and *incoherent* diffractive events, respectively.

The spatial distribution of gluons at small  $x$  will be studied experimentally at EIC kinematics. In order to



**Fig. 1** The “curly” line reflects a Pomeron exchange between the virtual photon and the target. The net color charge exchange with the target is zero, leading to a rapidity gap with the size equal to  $\ln(1/x_{\mathbb{P}})$  between the vector meson and the target, a gap in rapidity coverage, which is used to identify diffractive events experimentally.

obtain this distribution one should measure the diffractive cross section,  $d\sigma/dt$ , at small- $x$  region over a large range of  $t$ . Then, a Fourier transform from momentum space to coordinate space will represent the gluon source distribution as a function of the impact parameter  $\mathbf{b}_{\mathbb{T}}$ . There can be an access to  $t$  with sufficiently high precision from measurements of exclusive diffractive processes, such as exclusive vector meson production and DVCS. In this case  $t$  can be calculated from the measured  $V$  and  $\gamma$ , the scattered electron, and the known beam energies.

### 2.2 *Sartre* and IPSat dipole model basics

The purpose of the dipole model Monte Carlo event generator *Sartre* is to provide simulations of pseudo-data at kinematics in which future data are supposed to be taken at the proposed EIC [24–26] and LHeC [28] machines. With *Sartre* one can simulate diffractive exclusive vector meson production and DVCS in the following processes:

$$\begin{aligned} e + p &\rightarrow e' + V(\gamma) + p' & e + A &\rightarrow e' + V(\gamma) + A' \\ p + p &\rightarrow p' + V(\gamma) + p' & p + A &\rightarrow p' + V(\gamma) + A' \\ & & A + A &\rightarrow A' + V(\gamma) + A', \end{aligned} \quad (3)$$

where all the processes are mediated by a virtual photon ( $\gamma^*$ ) and/or a Pomeron. The vector meson  $V$  can be a  $J/\Psi$ ,  $\phi$  or  $\rho$  particle,  $\gamma$  is the DVCS real photon. However, the generator is restricted for studying these processes at  $x_{\mathbb{P}} < 10^{-2}$  and at large  $\beta \equiv x_{\mathbb{P}}/x$  because the IPSat and IPNonSat dipole models, implemented in it, are only valid for small values of  $x_{\mathbb{P}}$  and not for

too small values of  $\beta$ . If  $\beta$  becomes too small, the dipole becomes unphysically large [54]<sup>3</sup>.

As far as the IPSat dipole model is concerned, it has been very successful in describing the exclusive vector meson and photon production at HERA. In this section we represent some of the features of this dipole model [44] implemented in *Sartre* [45, 46], for simulating  $V$  and  $\gamma$ . We will be following the line of discussions and argumentations of [45–51].

### 2.2.1 IPSat framework for $e + p$ and $e + A$ scatterings

The DIS diffraction can be described in terms of states, which diagonalize the scattering matrix ( $S$ -matrix) [55]. In these states a virtual photon  $\gamma^*$  at high energies fluctuates into a  $q\bar{q}$  dipole, with a fixed dipole transverse size  $\mathbf{r}_T$  and an impact parameter  $\mathbf{b}_T$ , along with a given specific configuration of the target. Then, the scattering cross section can be obtained by averaging over multiple target configurations. One can average on the level of scattering amplitude, where the cross section is proportional to the average target density, or otherwise stated, to the average gluon density. This averaging corresponds to coherent diffraction [56], where the target remains intact. One can also average on the level of scattering cross section that includes events in which the target breaks up. This averaging corresponds to total diffraction. Subtraction of the coherent from the total cross section gives the incoherent diffractive cross section, which describes only broken up target remnants. The incoherent diffraction is proportional to the target profile's variance [57–59].

*$e + p$  scattering.* Let us now discuss this entire picture in more technical terms, starting with the amplitude for diffractively producing an exclusive vector meson  $V$  (or a DVCS  $\gamma$ ) in an interaction between the virtual photon  $\gamma^*$  and proton  $p$  [44, 47–51]:

$$\begin{aligned} \mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(x_P, Q^2, \mathbf{\Delta}) &= \\ &= i \int d^2 \mathbf{r}_T \int d^2 \mathbf{b}_T \int \frac{dz}{4\pi} \left[ (\Psi \Psi_V^*)_{T,L}(Q^2, r_T, z) \right] \times \\ &\quad \times e^{-i[\mathbf{b}_T - (1-z)\mathbf{r}_T] \cdot \mathbf{\Delta}} \frac{d\sigma_{q\bar{q}}^p}{d^2 \mathbf{b}_T}(x_P, r_T, b_T), \end{aligned} \quad (4)$$

where the subscripts  $T/L$  refer to the transversely and longitudinally polarized  $\gamma^*$ ;  $Q^2$  is the photon virtuality;  $\mathbf{\Delta} = (P' - P)_T$  is the momentum transfer in

<sup>3</sup>In any case the program has an imposed cut-off on the dipole size,  $r$ , against its any unphysical increase. For nuclei it is  $r < 3R_N$ , where  $R_N$  is the nuclear radius given in the Woods-Saxon parametrization. For the proton it is  $r < 3\text{fm}$ . This cut-off does not show any changes in final simulated cross sections, though it can be altered in a broad kinematic range.

the scattering process assumed to be  $|\mathbf{\Delta}| \approx \sqrt{-t}$ ;  $z$  and  $1 - z$  are the longitudinal momentum fractions of the dipole taken by the quark and antiquark, respectively<sup>4</sup>;  $(\Psi^* \Psi_V)_{T,L}(Q^2, r_T, z)$  is the wave-function overlap between the incoming  $\gamma^*$  and outgoing  $V$  or  $\gamma$ ;  $\sigma_{q\bar{q}}^p(x_P, r_T, b_T)$  is the  $q\bar{q}$ - $p$  cross section describing the dipole scattering off the target proton<sup>5</sup>, as given by [23]:

$$\begin{aligned} \frac{d\sigma_{q\bar{q}}^p}{d^2 \mathbf{b}_T}(x_P, r_T, b_T) &= \\ &= 2\mathcal{N}^p(x_P, r_T, b_T) = 2[1 - \Re(S)], \end{aligned} \quad (5)$$

where  $\mathcal{N}^p$  is the scattering amplitude (with  $r_T = |\mathbf{r}_T|$  and  $b_T = |\mathbf{b}_T|$ ), and  $\Re(S)$  is the real part of the  $S$ -matrix. The amplitude in turn is given by

$$\mathcal{N}^p(x_P, r_T, b_T) = 1 - \exp(-\mathbf{r}_T^2 F(x_P, \mathbf{r}_T^2) T_p(b_T)), \quad (6)$$

where  $T_p(b_T)$  is the proton transverse density profile function assumed to be Gaussian:

$$T_p(b_T) = \frac{1}{2\pi B_p} e^{-\mathbf{b}_T^2/(2B_p)}. \quad (7)$$

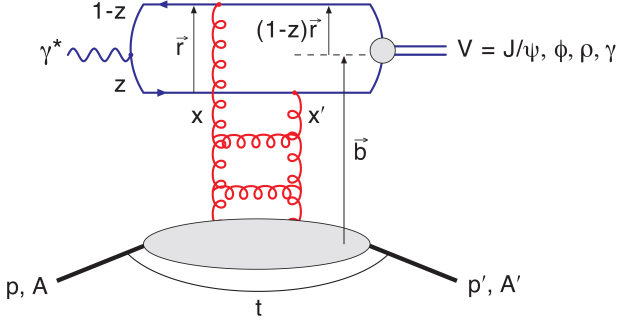
The parameter  $B_p$  is called proton width. The function  $F(x_P, \mathbf{r}_T^2)$  in Eq. (6) is proportional to gluon distribution, which undergoes DGLAP evolution [43]:

$$F(x_P, \mathbf{r}_T^2) = \frac{\pi^2}{2N_c} \alpha_s(\mu^2) x_P g(x_P, \mu^2). \quad (8)$$

For initial gluon density one uses a parametrization  $x_P g(x_P, \mu^2) = A_g x_P^{-\lambda_g} (1 - x_P)^6$ , where  $\mu^2 = (4/\mathbf{r}_T^2) + \mu_0^2$  with  $\mu_0^2$  being a cut-off scale in the gluon DGLAP evolution. The model parameters  $A_g$  and  $\lambda_g$  are determined by fitting the IPSat and IPNonSat models to HERA DIS data [11].  $B_p = 4\text{GeV}^{-2}$  and  $\mu_0^2 = 1.1\text{GeV}^2$  are fixed in the fitting process. For  $A_g$  and  $\lambda_g$  as well as for the used quark masses (treated as parameters in the model) we refer to Ref. [11]. The QCD running coupling  $\alpha_s$  takes into account some next-to-leading log effects but generally IPSat is a multiple two-gluon exchange model at leading log.

<sup>4</sup>The phase factor  $\exp[i(1-z)\mathbf{r}_T \cdot \mathbf{\Delta}]$  standing in Eq. (4) stems from the difference between forward and non-forward wavefunctions. Although used in many phenomenological applications, the validity of the exponent  $(1-z)\mathbf{r}_T \cdot \mathbf{\Delta}$  is challenged in [60], nevertheless, we keep it in our framework because it is anyway expected to have a negligible effect at small- $x$  region [40].

<sup>5</sup>This cross section is Fourier transformed into momentum space with  $\mathbf{\Delta}$ , which is the Fourier conjugate to the center-of-mass of the dipole,  $\mathbf{b}_T - (1-z)\mathbf{r}_T$ , relative to the proton's center (see Fig. 2 for details).



**Fig. 2** (Color online) A schematic view of the dipole-target scattering and its variables in  $e + p$  and  $e + A$  scatterings. In the figure  $\vec{r} \equiv \mathbf{r}_T$  and  $\vec{b} \equiv \mathbf{b}_T$ . This figure is from [45], and is a more detailed version of Fig. 1.

The total diffractive  $\gamma^* - p$  cross section is given by averaging the absolute square of the scattering amplitude:

$$\begin{aligned} \left( \frac{d\sigma_{T,L}^{\gamma^* p \rightarrow Vp}}{dt} \right)_{\text{tot. diff.}} &= \\ &= \frac{1}{16\pi} \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* p \rightarrow Vp}(x_P, Q^2, t) \right|^2 \right\rangle. \end{aligned} \quad (9)$$

The coherent diffractive  $\gamma^* - p$  cross section is given by averaging the amplitude and taking the absolute square of it:

$$\begin{aligned} \left( \frac{d\sigma_{T,L}^{\gamma^* p \rightarrow Vp}}{dt} \right)_{\text{coh. diff.}} &= \\ &= \frac{1}{16\pi} \left| \left\langle \mathcal{A}_{T,L}^{\gamma^* p \rightarrow Vp}(x_P, Q^2, t) \right\rangle \right|^2. \end{aligned} \quad (10)$$

Then, the incoherent diffractive  $\gamma^* - p$  cross section can be written as the following variance [57–59] (as it was already mentioned before):

$$\begin{aligned} \left( \frac{d\sigma_{T,L}^{\gamma^* p \rightarrow Vp}}{dt} \right)_{\text{incoh. diff.}} &= \\ &= \left( \frac{d\sigma_{T,L}^{\gamma^* p \rightarrow Vp}}{dt} \right)_{\text{tot. diff.}} - \left( \frac{d\sigma_{T,L}^{\gamma^* p \rightarrow Vp}}{dt} \right)_{\text{coh. diff.}} \Rightarrow \\ &\Rightarrow \frac{1}{16\pi} \left( \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* p \rightarrow Vp}(x_P, Q^2, t) \right|^2 \right\rangle - \left| \left\langle \mathcal{A}_{T,L}^{\gamma^* p \rightarrow Vp}(x_P, Q^2, t) \right\rangle \right|^2 \right). \end{aligned} \quad (11)$$

Thereby, calculating the incoherent and coherent diffractive cross sections becomes a matter of finding the second and first moments of the scattering amplitude, respectively.

*Extension to  $e + A$  scattering.* The explicit  $\mathbf{b}_T$  dependence of IPSat makes it possible to consider a nucleus as a collection of nucleons, and model it based upon a

given nuclear transverse density distribution. At small- $x$  values the dipole has a large life-time such that it passes through the entire longitudinal scope of the nucleus, where the nucleus is considered to be a two-dimensional object in the transverse plane. The exact position of each nucleon within the nucleus, nevertheless, is not an observable quantity. That is why in order to calculate the total diffractive  $\gamma^* - A$  cross section correctly, one has to average the squared amplitude over all possible states of nucleon configurations (designated by  $\Omega$ ):

$$\begin{aligned} \left( \frac{d\sigma_{T,L}^{\gamma^* A \rightarrow VA}}{dt} \right)_{\text{tot. diff.}} &= \\ &= \frac{1}{16\pi} \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right|^2 \right\rangle_{\Omega}. \end{aligned} \quad (12)$$

Analogously to Eq. (10), the coherent diffractive  $\gamma^* - A$  cross section will be given by

$$\begin{aligned} \left( \frac{d\sigma_{T,L}^{\gamma^* A \rightarrow VA}}{dt} \right)_{\text{coh. diff.}} &= \\ &= \frac{1}{16\pi} \left| \left\langle \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right\rangle_{\Omega} \right|^2, \end{aligned} \quad (13)$$

and analogously to Eq. (11), the incoherent diffractive  $\gamma^* - A$  cross section will be represented as a dipole-nucleus variance given by

$$\begin{aligned} \left( \frac{d\sigma_{T,L}^{\gamma^* A \rightarrow VA}}{dt} \right)_{\text{incoh. diff.}} &= \\ &= \left( \frac{d\sigma_{T,L}^{\gamma^* A \rightarrow VA}}{dt} \right)_{\text{tot. diff.}} - \left( \frac{d\sigma_{T,L}^{\gamma^* A \rightarrow VA}}{dt} \right)_{\text{coh. diff.}} \Rightarrow \\ &\Rightarrow \frac{1}{16\pi} \left( \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right|^2 \right\rangle_{\Omega} - \left| \left\langle \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right\rangle_{\Omega} \right|^2 \right). \end{aligned} \quad (14)$$

For the  $q\bar{q} - p$  cross section we have the scattering amplitude  $\mathcal{N}^p$  from Eq. (6). For the  $q\bar{q} - A$  cross section one should use the following approximation to construct the nuclear scattering amplitude from that of the proton:

$$\begin{aligned} \mathcal{N}^A(x_P, r_T, b_T) &= \\ &= 1 - \prod_{i=1}^A (1 - \mathcal{N}^p(x_P, r_T, |\mathbf{b}_T - \mathbf{b}_{T_i}|)), \end{aligned} \quad (15)$$

where  $\mathbf{b}_{T_i}$  is the position of each nucleon within the nuclear transverse plane. The nucleon positions are treated according to projections of a three-dimensional Woods-Saxon function onto the transverse plane. Making use of Eqs. (5), (6), (8) and (15) gives us the  $q\bar{q} - A$  cross

section, which is expressed by

$$\begin{aligned} \frac{d\sigma_{q\bar{q}}^A}{d^2\mathbf{b}_T}(x_P, r_T, b_T, \Omega_j) = \\ = 2 \left[ 1 - \exp\left(-\frac{\pi^2}{2N_c} \mathbf{r}_T^2 \alpha_s(\mu^2) x_P g(x_P, \mu^2) \times \right. \right. \\ \left. \left. \times \sum_{i=1}^A T(|\mathbf{b}_T - \mathbf{b}_{T_i}|) \right) \right], \end{aligned} \quad (16)$$

where  $\Omega_j = \{|\mathbf{b}_{T1}, \mathbf{b}_{T2}, \dots, \mathbf{b}_{TA}|\}$  represents a specific Woods-Saxon nucleon configuration. And here is a formula for the dipole cross-section average [23], which should be used in calculations of the first moment of the amplitude:

$$\left\langle \frac{d\sigma_{q\bar{q}}^A}{d^2\mathbf{b}_T} \right\rangle_{\Omega} = 2 \left[ 1 - \left( 1 - \frac{T_A(b_T)}{2} \sigma_{q\bar{q}}^p \right)^A \right], \quad (17)$$

where  $\sigma_{q\bar{q}}^p$  is integrated over  $\mathbf{b}_T$ , and  $T_A(b_T)$  is the nuclear transverse density profile function, which is taken to be the Woods-Saxon potential in the transverse plane. We continue the discussion of this section in Appendix A, where we generalize and show how *Sartre* calculates (simulates) the  $\gamma^*$ - $A$  diffractive differential cross sections.

Along with the IPSat framework, *Sartre* also has the IPNonSat dipole model implemented in it. The purpose of this model is to separate the gluon saturation effects from other small- $x$  effects. Eq. (6) has an exponential term through which the gluon saturation is introduced in IPSat. Its non-saturation version is constructed if the dipole-target cross section is linearized. This means that one should keep the first term in the expansion of the exponent in the IPSat dipole-target cross section, in order to obtain that for IPNonSat. It results in gluon density becoming unsaturated for small  $x_P$  as well as when the ratio  $\beta = x_P/x$  is also large. In IPSat the rise of the cross section at large  $\mathbf{r}_T$  is under control, whereas in IPNonSat there is no taming for such a rise. For more details on the IPNonSat framework, we refer to Refs. [23, 45].

### 2.2.2 Phenomenological corrections to dipole-target cross sections

*Real part of the diffractive amplitude.* In derivation of the  $\gamma^*$ - $p$  ( $\gamma^*$ - $A$  in general) diffractive scattering amplitude shown in Eq (4), the assumption is that the amplitude is imaginary. Meanwhile,  $\mathcal{N}^p$  in Eq. (5), where only  $\Re(S)$  is included, is purely real. Nonetheless, the real part of the diffractive scattering amplitude can be taken into account in a dipole-target final calculated cross section, if the cross section is multiplied by a coefficient  $(1 + B^2)$ , where  $B$  is the ratio of the real and

imaginary parts of the scattering amplitude [44]. The resulting expression for  $B$  is given by

$$B = \tan\left(\frac{\pi\lambda}{2}\right), \quad \text{with } \lambda = \frac{\partial \ln\left(\mathcal{A}_{T,L}^{\gamma^*p(A) \rightarrow Vp(A)}\right)}{\partial \ln(1/x_P)}. \quad (18)$$

*Skewness correction.* In the IPSat model, the diagonal (*collinear factorization*) gluon distribution,  $x_P g(x_P, \mu^2)$ , should be corrected to correspond to the off-diagonal (*skewed*) gluon distribution, which depends on longitudinal momentum fractions  $x_1$  and  $x_2$  of two gluons in a two-gluon exchange at lowest order in a dipole-target (proton or nucleus) scattering<sup>6</sup>, where  $x_1$  and  $x_2$  satisfy the condition of  $x_1 - x_2 = x_P$ . In the high energy limit, the  $x_1$  gluon exchange brings the  $q\bar{q}$  dipole mass close to  $M_V$  because the dominant contribution of the diffractive scattering amplitude is obtained when the intermediate propagators are close to the mass shell. In this case the second gluon is left with a significantly smaller  $x_2$ , and the dominant kinematic regime will be at  $x_2 \ll x_1 \approx x_P$  [61–63].

Thereby, in order to account for scenarios, where the gluons in the two-gluon exchange carry different longitudinal momentum fractions, the so-called skewness correction should be applied to a dipole-target final calculated cross section, by multiplying it with a coefficient  $R_g$  [44], represented by

$$\begin{aligned} R_g = \frac{2^{2\lambda_g+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda_g + 5/2)}{\Gamma(\lambda_g + 4)}, \\ \text{with } \lambda_g = \frac{\partial \ln(x_P g(x_P, \mu^2))}{\partial \ln(1/x_P)}. \end{aligned} \quad (19)$$

Both corrections discussed in this section grow drastically in the large- $x_P$  range outside the validity of theoretical models, where  $x_P > 0.01$ . For this reason, the upper limit imposed on  $x_P$  in *Sartre* is set to 0.01.

## 3 The cross section scaling in production of exclusive diffractive vector mesons in $e + A$ scatterings at EIC kinematics

In this section, we exhibit our results represented as pseudo-data from *Sartre*-made vector meson production events. We show the data uncertainties obtained from a combined framework of using proposed detector resolutions and a smearing technique [53]. We refer to Appendix B for more details on the error analysis and producing pseudo-data.

<sup>6</sup>In a dipole-target scattering there is no exchange of color charge, which is already mentioned in the introduction.

### 3.1 Coherent and incoherent diffractive cross section distributions

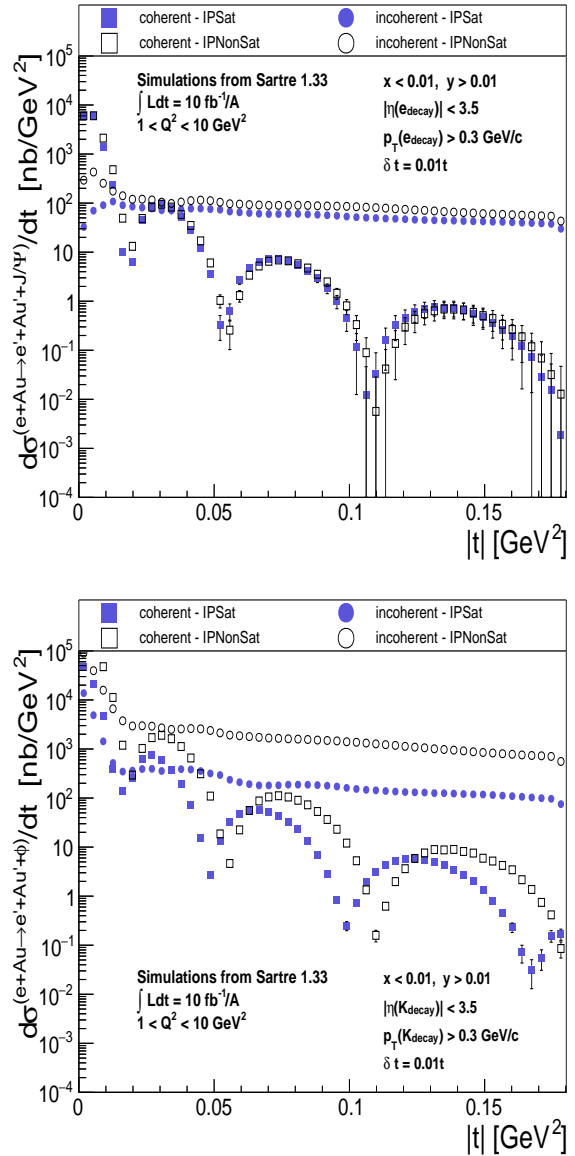
Before discussing how the nuclear saturation effects modify the  $A$  and  $Q^2$  scaling properties of the exclusive vector meson production cross section [40] in an EIC-like kinematical setup, it is relevant to show several plots on  $d\sigma/dt$  coherent and incoherent distributions for exclusive  $J/\Psi$  and  $\phi$  vector meson production in diffractive  $e + Au$  scattering. This production is the experimentally cleanest process in such processes, due to a small particle number in the final state, by which one can systematically investigate the saturation physics.

The coherent distribution depends on a target shape, by which one can study the nuclear spatial gluon distributions. The incoherent distribution provides crucial information on geometric fluctuations of a target [50–52]. Experimentally, the clue to the spatial gluon distribution and fluctuations is thereby the measurements of the  $d\sigma/dt$  distributions. Figure 54 of the EIC White Paper [25] shows Sartre-simulated  $d\sigma/dt$  distributions for exclusive  $J/\Psi$  and  $\phi$  production, in both coherent and incoherent<sup>7</sup> events, in diffractive  $e + Au$  scattering. The simulations were restricted to  $1 \text{ GeV}^2 < Q^2 < 10 \text{ GeV}^2$  and  $x < 0.01$ . Besides, the produced events were passed through an experimental filter, and scaled for mimicking the integrated luminosity of  $10 \text{ fb}^{-1}/A$ . Also, a simple 5% smearing in  $t$  was performed on the simulated data. We replicate the two plots in that Figure 54, and make several other plots with smearing in  $t$ , which all are shown in Figs. 3-7 (the replica plots are actually shown in Fig. 4). The basic experimental cuts in all these figures are listed in their legends, similar to those in Figure 54 of [25] or in Figure 7.83 of [27], except for the  $p_{T(e_{\text{decay}})}$  cut. This cut removes quite a lot of events at low- $|t|$  region, instead of which we now use  $p_{T(K_{\text{decay}})}$ .

Our generated pseudo-data are obtained with the integrated luminosity of  $10 \text{ fb}^{-1}/A$ . Generating these figures with pseudo-data captures a realistic glance at the experimental measurement of the diffractive cross section. The sensitivity of such a measurement relies heavily on the resolution of  $t$ , as can be seen by the gradual smearing of the diffractive peak for increasing relative  $t$  uncertainty. These figures stress the importance of a high-resolution far-forward detector (like the Roman Pots) design for EIC. The pseudo-data are generated with the  $e + Au$  scattering energies<sup>8</sup> of  $10 \times 110 \text{ GeV}$ , taken from Table 10.3 of the Yellow Report [27]. The top plots of Figs. 3-5, and 7 show sufficient

<sup>7</sup>It is shown in [64] how the incoherent cross section is affected by saturation effects that is not negligible for  $J/\Psi$  production.

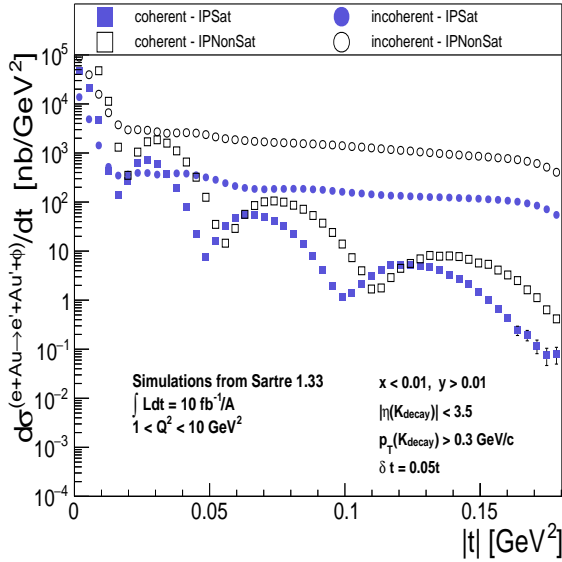
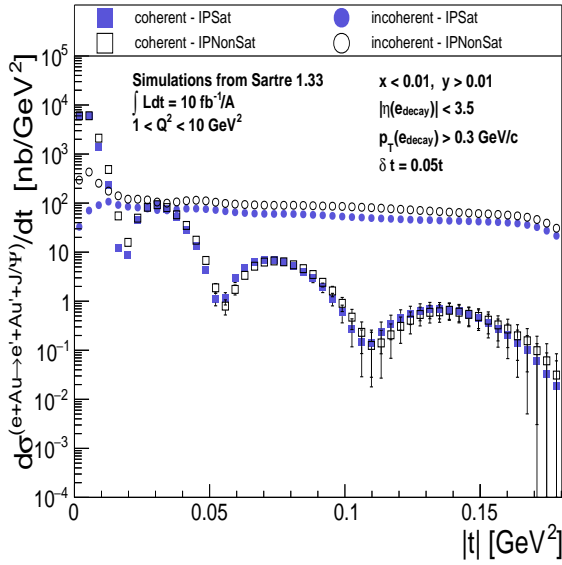
<sup>8</sup>The ion beam energy here represents that of per nucleon.



**Fig. 3** (Color online) Coherent and incoherent cross sections, from exclusive  $J/\Psi$  (top) and  $\phi$  (bottom) production processes in diffractive  $e + Au$  scattering, by having outputted the cross section pseudo-data from Sartre and passed it through detector resolutions described in the EIC Handbook [53] with smearing  $\delta t = 0.01t$ , within the validity range of the saturation (IPSat) and non-saturation (IPNonSat) models. The  $\eta$  cut helps determine which direction a particle will move towards, and the  $p_T$  cut helps find out if the particle will even reach the detectors. These cut numbers are taken from [27] and [53].

statistics in the first three minima of the coherent cross section of the produced  $J/\Psi$  events for the simulation's integrated luminosity of  $10 \text{ fb}^{-1}/A$ . Thereby, one can think of the first three minima as being experimentally measurable in terms of statistics. However, the largest problem here is related to the separation of coherent and incoherent processes. The coherent produc-

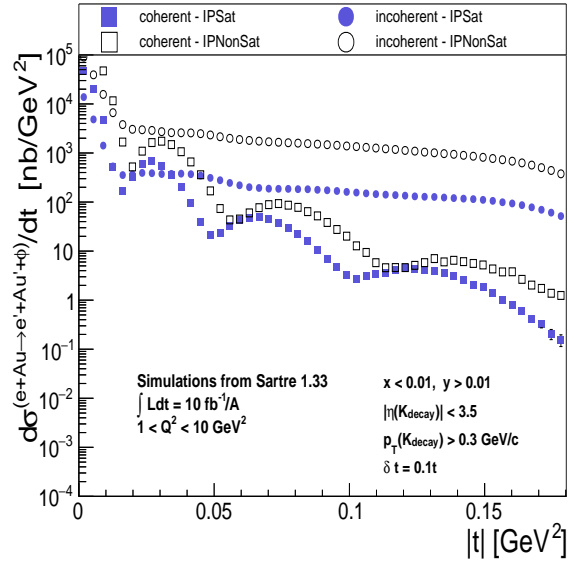
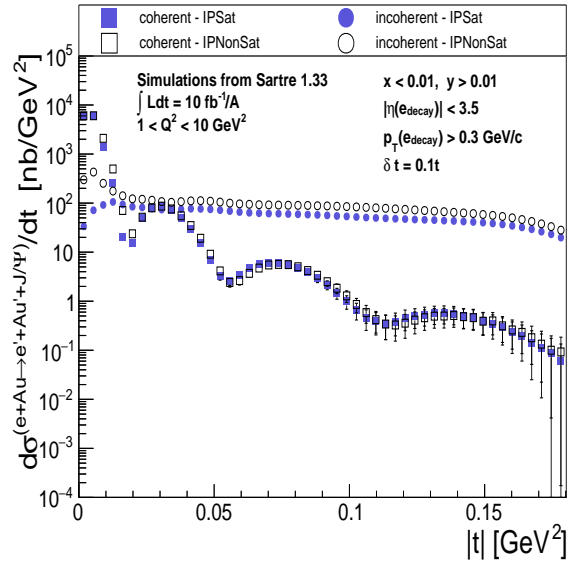




**Fig. 4** (Color online) Similar cross section distributions as in Fig. 3 but simulated with smearing  $\delta t = 0.05t$ .

tion dominates at low  $t$ , whereas the incoherent production starts to take over at  $t \gtrsim 0.02 \text{ GeV}^2$ . The current experimental cuts are not yet sufficient to suppress the incoherent contribution to make the diffractive pattern of the coherent contribution measurable at the required level. Nonetheless, there is a prospect of improvement for such a separation (see the end of Sec. 8.4.6 in [27]).

In addition to what is stated above, because of the  $J/\Psi$  heavy mass (and therefore small size of its wavefunction) we see little difference between the IPSat and IPNonSat models for exclusive coherent and incoherent  $J/\Psi$  electroproduction at moderate  $Q^2$ . The bottom plots of the same Figs. 3-5, and 7 show more of a favorable situation for  $\phi$ , with a clear separation between the

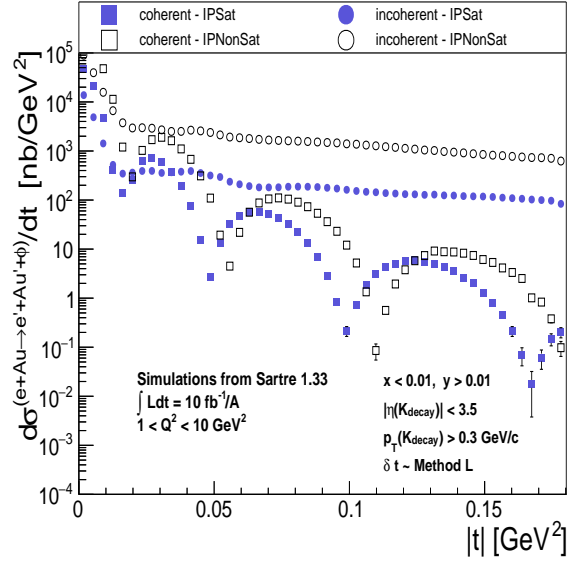
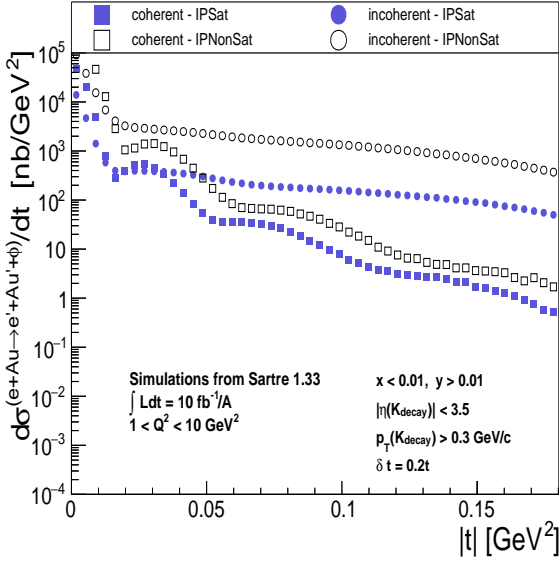
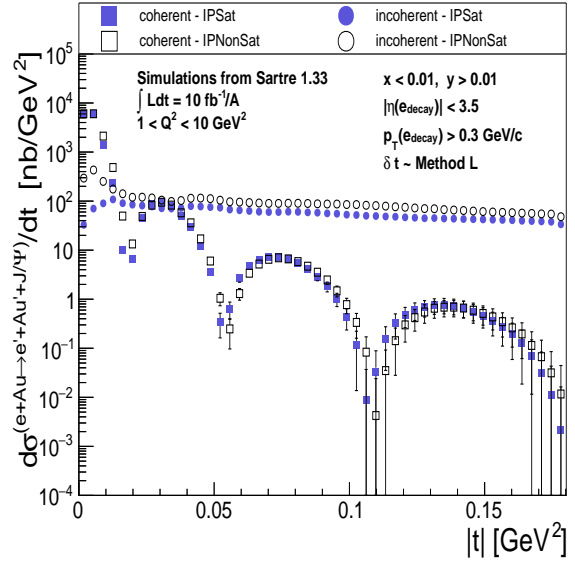
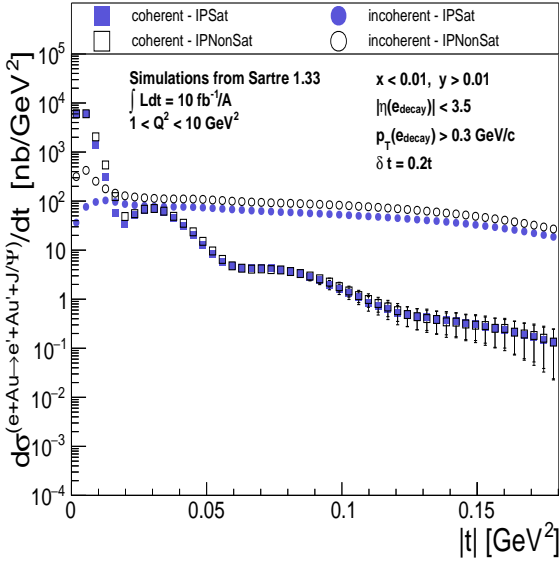


**Fig. 5** (Color online) Similar cross section distributions as in Fig. 3 but simulated with  $\delta t = 0.1t$ .

events coming from the saturation and non-saturation scenarios for exclusive  $\phi$  electroproduction. Thus,  $J/\Psi$  is less sensitive to non-linear saturation effects than the much lighter  $\phi$  meson, the contribution of which is enhanced in the saturation region relative to that of the heavier  $J/\Psi$ . In Fig. 6, all minima are already flattened out for both  $J/\Psi$  and  $\phi$  cross sections at  $\delta t = 0.2t$ , though the first two dips are still discernible. The decay channel specifications for producing the  $\phi$  and  $J/\Psi$  mesons are shown at the beginning of Sec. 3.2.1.

In Figs. 3-6, we progressively increase the relative uncertainty in the event  $t$  reconstruction. For these figures, the true event  $t$  is smeared with a Gaussian whose width is some factor (written explicitly in the plot la-





**Fig. 6** (Color online) Similar cross section distributions as in Fig. 3 but simulated with  $\delta t = 0.2t$ .

**Fig. 7** (Color online) Similar cross section distributions as in Fig. 3 but simulated with  $\delta t$  from the “Method L”.

bels) multiplied by  $t$ . In Fig. 7, the  $t$ -multiplying factor for obtaining the Gaussian width changes depending on the value of  $t$  itself. Table 8.10 in [27], produced for the  $Q^2$  region of  $1 < Q^2 < 10 \text{ GeV}^2$ , gives six different ranges of  $t$ , placed sequentially from  $0.00 \text{ GeV}^2$  to  $0.18 \text{ GeV}^2$ , in which one can find six values describing the effect of beam momentum spread and beam divergence on  $t$ -resolution,  $\sigma_t/t$ . The procedure whereby those values have been obtained is titled “Method L”<sup>9</sup>,

which can also be used to improve the impact of momentum resolution effects on the  $t$ -resolution. Thus, for Fig. 7, depending on which range the true event  $t$  falls within, the respective multiplicative factor is pulled from that Table 8.10 and used to calculate the width for smearing, based upon the “Method L” that is currently the most promising procedure to improve the  $t$ -resolution as small  $t$  values, such as  $|t| = [0.000 - 0.001] \text{ GeV}^2$ . The  $t$ -resolution numbers coming from the “Method L” has been obtained for  $J/\Psi$  production. Nevertheless, we assume the same numbers for  $\phi$  production, and since this method relies on the reconstruction of a final-state decay, we plan to look into that thoroughly for the kaon decay mode in another paper.

<sup>9</sup>The procedure is bottomed on an assumption leading to using a constraint when the invariant mass of the outgoing nucleus should be taken as  $M_{A'}^2$ , to find the incoming electron’s longitudinal momentum, instead of just assuming the nominal electron beam momentum (see Sec. 8.4.6 of [27]).

### 3.2 $A$ and $Q^2$ scaling picture in the regime of high $Q^2 > Q_{s,A}^2$

At EIC kinematics, the would-be measurable coherent  $t$ -spectrum, for example in Fig. 4, may be used to model-independently obtain the nuclear spatial gluon distribution, from exclusive  $J/\Psi$  and  $\phi$  production in  $e+Au$  scattering, by measuring that distribution in impact parameter space,  $F(b)$ , through a two-dimensional Fourier transform of the square root of the coherent elastic cross section. Also, by measuring  $F(b)$  for both  $J/\Psi$  and  $\phi$  mesons<sup>10</sup>, we will be in a position to extract valuable information on how sensitive the corresponding measurement can be to saturation effects. Consequently, the studies of the exclusive vector meson production coherent cross section are quite important, so that in this section we will focus on its scaling properties, in terms of the nuclear mass number  $A$  and virtuality  $Q^2$ , as has been done in Ref. [40]. However, now we will reproduce its main results in a realistic EIC setup by using *Sartre*, in order to have better insight on whether the strong modification of the  $A$  and  $Q^2$  scaling of the coherent cross section, stemming from the gluon saturation in high-energy exclusive DIS off nuclei, can potentially be observed at EIC<sup>11</sup>. Such an observation shall be anchored upon measurements, which will allow to conspicuously identify different systematics in vector meson production, in the presence and absence of the gluon saturation in nuclei.

In the rest of Sec. 3 the figures are made for an integrated luminosity of  $10 \text{ fb}^{-1}/A$  with beam energies  $10 \times 110 \text{ GeV}$  in diffractive  $e+Au$  and  $e+Ca$  scatterings, as well as for  $100 \text{ fb}^{-1}/A$  with  $10 \times 100 \text{ GeV}$  in diffractive  $e+p$ . In this Sec. 3.2, we discuss exclusive vector  $J/\Psi$  and  $\phi$  meson production in the virtuality region  $Q^2 > 1 \text{ GeV}^2$ , which is the only relevant scale because the mass difference between the two mesons is less appropriate in this case. The differential cross section tables included in *Sartre* 1.33 reach a maximum  $Q^2$  range of  $20 \text{ GeV}^2$  for  $A > 1$  nuclear targets, with which we cannot address the whole extent of the  $A$  and  $Q^2$  scaling properties discussed in [40]. Because of this reason we will use the expression ‘‘scaling onset’’, meaning that a lower  $Q^2$  range is considered for showing the scaling trends of the cross sections and their ratios.

<sup>10</sup>The  $\rho$  meson might be more sensitive to saturation effects at moderate  $Q^2$  than  $\phi$ , nevertheless, there are large theoretical uncertainties in the knowledge of its wavefunction, making perturbative calculations less reliable at smaller  $Q^2$  values. Perhaps it also somewhat applies to  $\phi$ 's wavefunction but we prefer to work with  $\phi$  and  $J/\Psi$  following [25] and [46].

<sup>11</sup>The major motivation, which the EIC and LHeC proposals are based upon, is to search for gluon many-body non-linear dynamics, and the gluon saturation in particular.

#### 3.2.1 The scaling onset at $|t| = 5 \times 10^{-4} \text{ GeV}^2$

For producing the scaling-related figures, truth information (see Appendix B) from the event generator is used to separate out the longitudinally and transversely polarized virtual photon events. Experimentally, separating out the individual polarization cross sections for these events involves a delicate technique known as Rosenbluth separation [65]. The ramifications of this technique are yet to be analyzed using the pseudo-data generated, thus making it a crucial focus in future work. In this Sec. 3, we analyze the  $\phi$  production cross section with the  $e+A(p) \rightarrow \phi+A(p) \rightarrow K^+K^-$  decay channel, which is both statistically and experimentally practical. The  $K^+K^-$  branching ratio for the  $\phi$  meson, experimentally measured to be  $48.9\% \pm 0.5\%$  [66], is the largest of its decay modes. Additionally, exhaustive detector R&D for the future EIC has made identification of light final-state mesons, such as kaons and pions, a vital focus. For the heavier  $J/\Psi$  production, the vast list of hadronic decays overwhelms its simple, yet less frequent leptonic decay channels ( $e^+e^-$  and  $\mu^+\mu^-$ ). For simplicity, we choose the  $e^+e^-$  decay mode to the analysis, which has a branching ratio measured to be  $5.94\% \pm 0.06\%$  [67]. Future studies may consider doubling the statistics by combining the contribution from the equally likely  $\mu^+\mu^-$  decay channel.

Below are given the cross sections and number of events simulated for the three scattering types under consideration with the given branching ratios:

- $e+Au | \phi \rightarrow K^+K^-$ , total cross section =  $80.20 \text{ nb}$ ,  $N_{\text{events}} = 4.07 \times 10^6$ ;
- $e+Ca | \phi \rightarrow K^+K^-$ , total cross section =  $6.21 \text{ nb}$ ,  $N_{\text{events}} = 1.55 \times 10^6$ ;
- $e+p | \phi \rightarrow K^+K^-$ , total cross section =  $6.8 \times 10^{-3} \text{ nb}$ ,  $N_{\text{events}} = 6.80 \times 10^5$ ;
- $e+Au | J/\Psi \rightarrow e^+e^-$ , total cross section =  $0.99 \text{ nb}$ ,  $N_{\text{events}} = 5.04 \times 10^4$ ;
- $e+Ca | J/\Psi \rightarrow e^+e^-$ , total cross section =  $0.053 \text{ nb}$ ,  $N_{\text{events}} = 1.32 \times 10^4$ ;
- $e+p | J/\Psi \rightarrow e^+e^-$ , total cross section =  $37.2 \times 10^{-6} \text{ nb}$ ,  $N_{\text{events}} = 3.72 \times 10^3$ .

First, we consider the forward limit at  $t \rightarrow 0$ , by going back to and starting with Eq. (4) but considering it for the case of  $\gamma^*A \rightarrow VA$ . However, in the limit of  $Q^2 \gg Q_{s,A}^2$ , instead of the  $\gamma^*-p$  amplitude (that can be used from the GBW model [41]) one should use the  $\gamma^*-A$  amplitude by making the substitution  $Q_{s,p}^2 \rightarrow Q_{s,A}^2 \sim A^{1/3}Q_{s,p}^2$ :

$$\frac{d\sigma_{q\bar{q}}^A}{d^2\mathbf{b}_T} = 2 \left[ 1 - (1 - \mathbf{r}_T^2 Q_{s,A}^2) \right], \quad (20)$$

which is a simplified version of Eq. (17). Consequently, the diffractive  $\gamma^*A \rightarrow VA$  scattering amplitude will become

$$\mathcal{A}_{T,L}^{\gamma^*A \rightarrow VA} \sim \sim i \int d^2\mathbf{r}_T d^2\mathbf{b}_T (\Psi_{\gamma^* \rightarrow q\bar{q}} \Psi_{q\bar{q} \rightarrow V}^*)_{T,L} \mathbf{r}_T^2 Q_{s,A}^2. \quad (21)$$

The ‘‘Boosted Gaussian’’ parametrization is used for the vector meson wavefunction [44]. At this point let us follow some of the argumentation of [40] for addressing its principal scaling results.

*A<sup>2</sup> scaling:* The  $\mathbf{b}_T$  integration of Eq. (21) gives the cross sectional area factor of  $A^{2/3}$ , resulting in

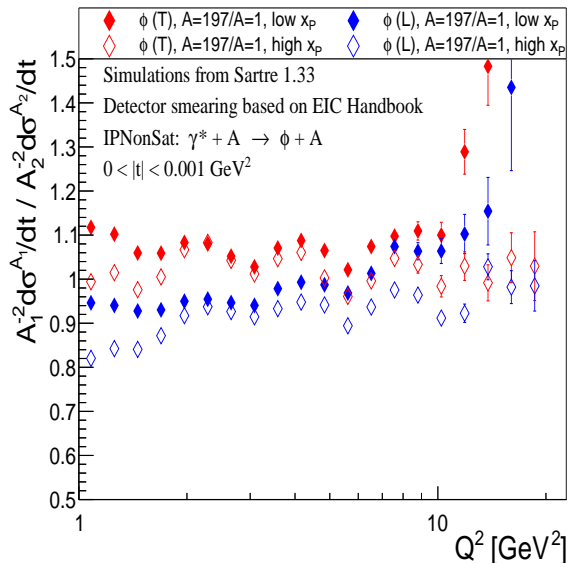
$$\mathcal{A}_{T,L}^{\gamma^*A \rightarrow VA} \sim \sim iA \int d^2\mathbf{r}_T (\Psi_{\gamma^* \rightarrow q\bar{q}} \Psi_{q\bar{q} \rightarrow V}^*)_{T,L} \mathbf{r}_T^2 Q_{s,p}^2, \quad (22)$$

which according to Eq. (13) leads to the coherent diffractive cross section, such as

$$\left( \frac{d\sigma_{T,L}^{\gamma^*A \rightarrow VA}}{dt} \Big|_{t=0} \right)_{\text{coh. diff.}} \equiv \frac{d\sigma_{T,L}^{\gamma^*A \rightarrow VA}}{dt} \Big|_{t=0} \sim A^2. \quad (23)$$

This asymptotic  $A^2$  is used in the normalization of the exclusive coherent  $\phi$  and  $J/\Psi$  cross section ratios for Gold over Calcium and Gold over proton at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  in Figs. 8 and 9. As it is mentioned at the end of the introduction, in *Sartre* simulations one cannot feasibly generate events at specific  $t$  values, nevertheless, we are able to generate events in given  $t$  bins: in this case, in the bin of  $0 < |t| < 0.001 \text{ GeV}^2$ , and by just referring to its central value instead of the bin. The same convention is adopted for the other  $t$  bins discussed in the next Sec. 3.2.2.

Fig. 8 shows pseudo-data of some normalized ratios in the IPNonSat model, which are consistent with a horizontal line that is expected to take place in the absence of gluon saturation effects. Fig. 9 shows pseudo-data of normalized ratios in the IPSat model, where one can see a substantial suppression due to gluon saturation at low- $Q^2$  region. It is obvious that the suppression is larger for transversely polarized photon case. For simulations, the version 1.33 of *Sartre* is used as in the previous figures. The cuts shown in Figs. 3-7, are used for making these two figures as well (and the subsequent figures). The pseudo-data exhibited in Figs. 8 and 9 are normalized by the scaling factor from Eq. (23). More details are presented in the corresponding captions. An analysis at a fixed  $|t|$  value, although ideal, is impractical statistically for the actual experiment. The specific



**Fig. 8** (Color online) The  $A^2$ -ratio at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  for the exclusive coherent  $\phi$  electroproduction cross section in the IPNonSat model.  $T$  and  $L$  refer to transverse and longitudinal polarization, respectively. The vertical axis shows the  $A^{-2}$  normalized ratio of the cross sections for the Gold over proton. The high  $x_p$  shows the range of pseudo-data obtained in  $x_p = [0.005 - 0.009]$ , the low  $x_p$  shows the pseudo-data range of  $x_p = [0.001 - 0.005]$ .

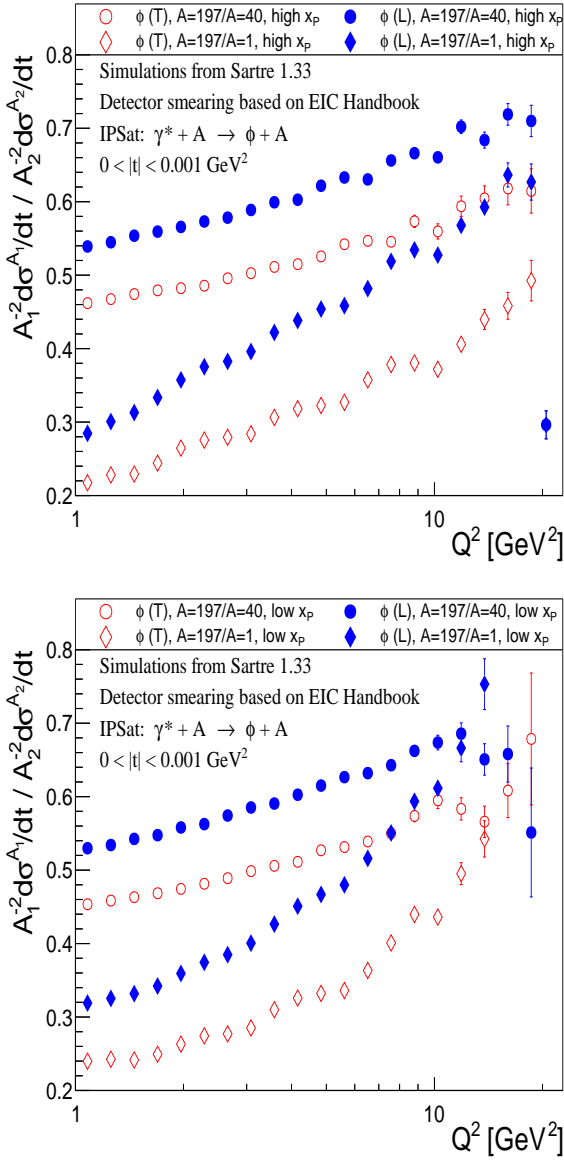
$|t| \sim 0$  range we selected provides us with ample experimental statistics to distinguish the growth of the cross section ratio to within statistical uncertainties. We argue that this range accurately reflects the theoretical behavior of the cross section at  $|t| = 0$ , upon direct comparison with the figures from [40].

*A<sup>4/3</sup> scaling:* Fig. 10 shows pseudo-data of the normalized ratios, for the total exclusive coherent cross section in the IPSat model, obtained after  $t$ -integration of Eq. (23):

$$\sigma_{T,L}^{\gamma^*A \rightarrow VA} \sim A^2 A^{-2/3} \sim A^{4/3}, \quad (24)$$

where  $A^{-2/3}$  comes from the width of the coherent peak. In Fig. 9 it is expected that the normalized cross section ratios go to unity at asymptotically large  $Q^2$ , as shown in [40]<sup>12</sup>. Meanwhile, the ratios in Fig. 10 do not approach unity at large  $Q^2$  because of oversimplification of the  $t$ -integration, based on the coherent peak assumption, which kind of changes the shape of the pseudo-data curves. But the ratio Gold over Calcium

<sup>12</sup>Currently, the look-up amplitude tables in *Sartre* have maximum reach of  $Q^2$  as  $20 \text{ GeV}^2$  for  $e + Au$  and  $e + Ca$ , and  $200 \text{ GeV}^2$  for  $e + p$ . We plan to update those tables to have a higher  $Q^2$  reach, along with making others for various  $e + A$  beams that can be found in Table 10.3 of [27].

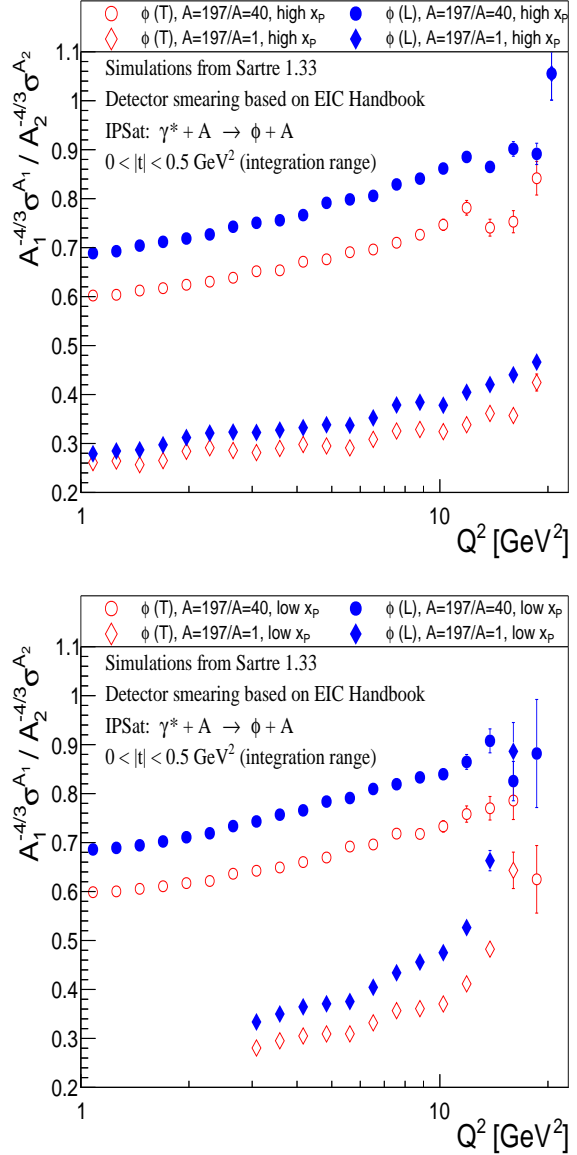


**Fig. 9** (Color online) The onset of the  $A^2$  scaling at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  for the exclusive coherent  $\phi$  electroproduction cross section in the IPSat model. The vertical axis shows the  $A^{-2}$  normalized ratio of the cross sections for the Gold over Calcium and the Gold over proton. The nomenclature is the same as in Fig. 8. The top plot shows the pseudo-data at high  $x_p$ , the bottom plot shows the pseudo-data at low  $x_p$ .

still shows suppression though not in a correct way<sup>13</sup>. On the other hand, the result in Eq. (24) is valid at asymptotically large  $A$ . Nonetheless, by looking also at the ratio Gold over Calcium and over proton in Fig. 9 and Fig. 10, one can conclude that generally, in normalized ratios of arbitrary  $A_1/A_2$  and  $A_1/A_3$  with three

<sup>13</sup>It should be emphasized that a better calculation gives an  $A$ -dependent parameter, standing in front of  $A^{4/3}$  in the r.h.s of Eq. (24), which e.g., for the Gold nucleus is  $C[197] \approx 1/2$  [11].

heavy nuclei  $A_1 > A_2 > A_3$ , the suppression would be stronger for  $A_1/A_3$  rather than for  $A_1/A_2$ . Note that a similar integrated cross section ratio is reported in [27, 48] too.



**Fig. 10** The onset of the  $A^{4/3}$  scaling for the total exclusive coherent  $\phi$  electroproduction cross section in the IPSat model. The vertical axis shows the  $A^{-4/3}$  normalized ratio of the cross sections for the Gold over Calcium and over proton. The nomenclature is the same as in Fig. 8. The top plot shows the pseudo-data at high  $x_p$ , the bottom plot shows the pseudo-data at low  $x_p$ . Contrary to Fig. 8 and Fig. 9, the range of  $|t|$  here is the cross-section integration region. The integration used in Eq. (24) is valid in this range because we generate events only in  $0 < |t| < 0.5 \text{ GeV}^2$ .

In Figs. 8, 9 and 10, for making pseudo-data we use the  $Q^2$  interval  $1 < Q^2 < 20 \text{ GeV}^2$  instead of

$1 < Q^2 < 10 \text{ GeV}^2$ . First, we have checked out all these figures with the exact same smearing ‘‘Method L’’ (as in Fig. 7), the model numbers of which, made for  $1 < Q^2 < 10 \text{ GeV}^2$  in totally six ranges of  $t$ , are shown in Table 8.10 of [27]. Then, our further cross-check has shown that the figures made in both  $Q^2$  intervals are quite similar with each other quantitatively and qualitatively. Therefore, based upon this observation we assume that the current L model numbers are well applicable for making the pseudo-data in the range of  $1 < Q^2 < 20 \text{ GeV}^2$ . We also expand the last cell of Table 8.10, assuming all events with  $|t| > 0.18 \text{ GeV}^2$  have a  $t$ -resolution of  $\sigma_t/t = 0.005$ .

It is also relevant to emphasize that though the cross sections we are analyzing for all the figures in Sec. 3.2 are beam energy-independent via the division of the virtual photon flux factor (see Appendix A and Appendix B), the kinematic phase space of the final-state particles are not. For the scattered electron, which is used to reconstruct the event kinematics such as  $Q^2$  and  $y$ , it is favorable for its kinematic phase space to be as identical as possible when taking cross section ratios of  $e + A$  and  $e + p$  scatterings. For the beam energies of  $e + Au$  and  $e + Ca$  at  $10 \times 110 \text{ GeV}$ , also  $e + p$  at  $10 \times 100 \text{ GeV}$ , the reconstruction quality of the event’s scattered electron is practically identical. In turn, we eliminate the need to consider different constraints (or difficulties) in event reconstruction after smearing at these beam energies. For Figs. 3-7, which are produced with the  $e + Au$  beam at  $10 \times 110 \text{ GeV}$ , the cross section is not beam-independent and should be expected to alter depending on the energies. For these plots of Sec. 3.1, which do not intend to show comparisons between nuclear targets, we only place a cut to remove events with  $y < 0.01$ . This inelasticity cut, discussed frequently in [27], alleviates the issue of large relative uncertainty in  $y$  and  $Q^2$  for high-energetic and low-angle scattered electrons. Without a beam energy difference, there is no need to increase the  $y$  cut. But the sheer variability of the final-state electron’s phase space can muddle our cross-section ratio figures, if we do not widen the  $y$  cut. At small event  $y$  below 0.05, the event reconstruction quality after smearing at low  $Q^2 \sim 1 \text{ GeV}^2$ , and even at moderate  $Q^2 \sim 20 \text{ GeV}^2$ , varies considerably between the  $e + p$  and  $e + A$  scatterings. To account for this, all the scaling figures (the ones shown later as well) contain an event cut, where any event with  $y < 0.05$  is discarded. After this cut is placed, Figs. 8, 9 and 10 exhibit a clearer rise as a function of  $Q^2$ , qualitatively matching onto theoretical figures made in [40].

It is also necessary to mention that in the lower plot of Fig. 10 made for the low- $x_{\mathbb{P}}$  range, the cross-section ratio of  $\phi(\text{T})$  and  $\phi(\text{L})$  vector mesons has a cut-

off at  $Q^2 \sim 3 \text{ GeV}^2$ . The reason for doing so is that there is an artificial bump seen below  $Q^2 < 3 \text{ GeV}^2$ . We have found that, due to the difference in final-state particle kinematics between  $e + p$  at  $10 \times 100 \text{ GeV}$  and  $e + Au$  at  $10 \times 110 \text{ GeV}$ , the final-state lepton detection efficiencies (scattered electron and decay pair) for both  $e + p$  and  $e + Au$  differ noticeably enough within this specific phase space to create an imbalance in the cross-section ratio<sup>14</sup>. In particular, the placement of the  $p_T > 0.3 \text{ GeV}$  cut on all final-state leptons play a significant role in producing this imbalance. Regardless, our study shows that the scaling behavior at larger  $Q^2$  can still be extracted without a careful consideration of the slight difference in the final-state kinematics between the EIC’s  $e + p$  and  $e + Au$  energies. On the other hand, there is an overall caveat related to both top and bottom plots of Fig. 10, based on the coherent peak assumption in the  $t$ -integration of Eq. (23) leading to  $A^{4/3}$  (as discussed below of Eq. (24)).

$A^2 Q^{-6}$  scaling: The vector meson wavefunction has an overlap with the longitudinally polarized photon wavefunction, given by the following functional form:

$$(\Psi_{\gamma^* \rightarrow q\bar{q}} \Psi_{q\bar{q} \rightarrow V}^*)_L \sim z(1-z) Q K_0(\varepsilon r_T) \phi_L(r_T, z), \quad (25)$$

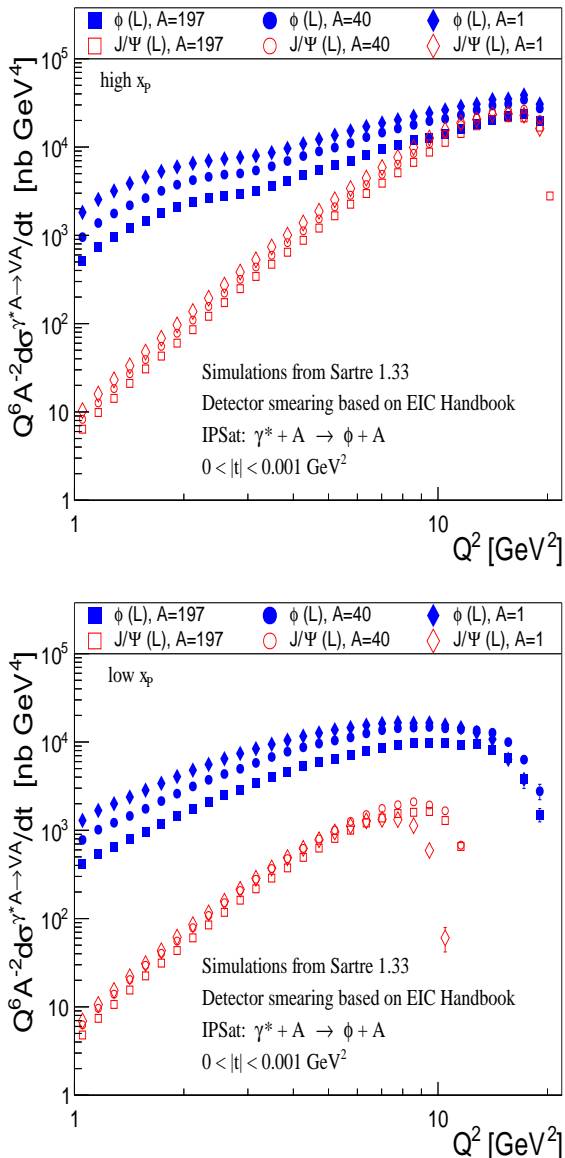
where  $\varepsilon = \sqrt{Q^2 z(1-z) + m_q^2} \approx Q$  at  $Q^2 \gg Q_s^2$ , and  $K_0(\varepsilon r_T)$  is the modified Bessel function of the second kind of the 0th order. The scalar part of the vector meson wavefunction,  $\phi_L(r_T, z) \sim z(1-z)e^{-r_{\mathbb{T}}^2 M_V^2}$ , restricts contributions from dipoles with the sizes larger than  $1/M_V$ . Thereby, the diffractive longitudinal scattering amplitude reads as

$$\begin{aligned} \mathcal{A}_L^{\gamma^* A \rightarrow VA} &\sim i \int d\mathbf{r}_{\mathbb{T}} \mathbf{r}_{\mathbb{T}}^3 Q K_0(Q r_T) \sim \frac{1}{Q^3}, \\ \text{leading to } \frac{d\sigma_L^{\gamma^* A \rightarrow VA}}{dt} \Big|_{t=0} &\sim \frac{1}{Q^6}. \end{aligned} \quad (26)$$

Fig. 11 shows pseudo-data on exclusive coherent  $\phi$  and  $J/\Psi$  longitudinal electroproduction cross section at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  in the IPSat model. For this specific case, the numerical calculations in [40] show that the cross section becomes flattish in the region  $Q^2 \gtrsim 10^2 \text{ GeV}^2$  at  $x_{\mathbb{P}} = 0.01$ . In our case, the top plot of Fig. 11 implies that the proton curves describing the vector meson production tend to have a flattish structure above  $10^2 \text{ GeV}^2$ , for the events simulated in the range of high  $x_{\mathbb{P}} = [0.005 - 0.009]$ . Besides, since the Gold and Calcium curves go along with the proton curves at low- $Q^2$ , then one could expect that their  $Q^6$  scaling trend would likewise continue above

<sup>14</sup>This effect is absent in the lower plot of Fig. 9 as the events are analyzed within a much narrower range of  $|t|$ .

$\sim 10^2 \text{ GeV}^2$ . The  $A^2$  scaling is also visible in the simulated  $Q^2$  ranges. This scaling is approximate for  $\phi$  production given by the Gold and Calcium curves, however, it is better for  $J/\Psi$  production.



**Fig. 11** The onset of the  $A^2 Q^{-6}$  scaling at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  for the exclusive coherent  $\phi$  and  $J/\Psi$  longitudinal electroproduction cross section in the IPSat model for the Gold, Calcium and proton, multiplied by the scaling factor of  $Q^6 A^{-2}$ . The nomenclature is the same as in Fig. 8. The top plot shows the pseudo-data at high  $x_{\mathbb{P}}$ , the bottom plot shows the pseudo-data at low  $x_{\mathbb{P}}$ .

As regards the bottom plot of Fig. 11, the curves decreasing at their largest  $Q^2$  reach have one very plausible reason. That is the physical phase space of an event within low  $x_{\mathbb{P}} = [0.001 - 0.005]$ , which is also limited in  $Q^2$ . For example, if  $Q^2 = 2 \text{ GeV}^2$  or so, then the

event  $x_{\mathbb{P}}$  is kinematically allowed to fall within the entire low- $x_{\mathbb{P}}$  range. However, as  $Q^2$  increases, the kinematically allowed  $x_{\mathbb{P}}$  range begins to shrink. This leads to a shrinking cross section at larger  $Q^2$  since we are averaging over the entire low- $x_{\mathbb{P}}$  range. For the high- $x_{\mathbb{P}}$  range this effect is not noticeable, however, it occurs in a lower- $x_{\mathbb{P}}$  one.

One can see non-physical dips in the top plot of Fig. 11 for the high- $x_{\mathbb{P}}$   $\phi$  cross sections, appearing at low  $Q^2 \sim 1 \text{ GeV}^2$ . These features of the  $\phi$  cross section are also visible in Figs. 12, 17 and 18. The dips, which are absent when recreating the figures using unsmearred event generator data, arise in regions of particularly poor event reconstruction. When the beam electron scatters at very negative pseudorapidities ( $\eta < -3.0$ ), the equation for the event inelasticity,  $y$ , approaches the form of  $y = 1 - E'/E$ . In this limit, the  $y$  reconstruction is ultra-sensitive to events with  $E' \sim E$  (at  $y \ll 1$ ), in comparison to events with  $E' \neq E$ . The photon flux factors that construct our figures' cross sections are functions of  $y$  (see Appendix A), so that when they are calculated with pseudo-data, unreliable results are produced. The future Electron Ion Collider's hermetic detector system will precisely measure event kinematics in this unreliable regime with alternative techniques such as the Jacquet-Blondel method or Double Angle method [53]. An analysis of our vector meson production events with these techniques is an area of future work.

$A^2 Q^{-8}$  scaling: For transversely polarized photons, one can perform a similar estimation as shown for the longitudinal polarization case in Eq. (25), bringing up

$$\begin{aligned} (\Psi_{\gamma^* \rightarrow q\bar{q}} \Psi_{q\bar{q} \rightarrow V}^*)_T &\sim \\ &\sim \frac{1}{z(1-z)} \varepsilon K_1(\varepsilon r_T) \partial_r(\phi_T(r_T, z)), \end{aligned} \quad (27)$$

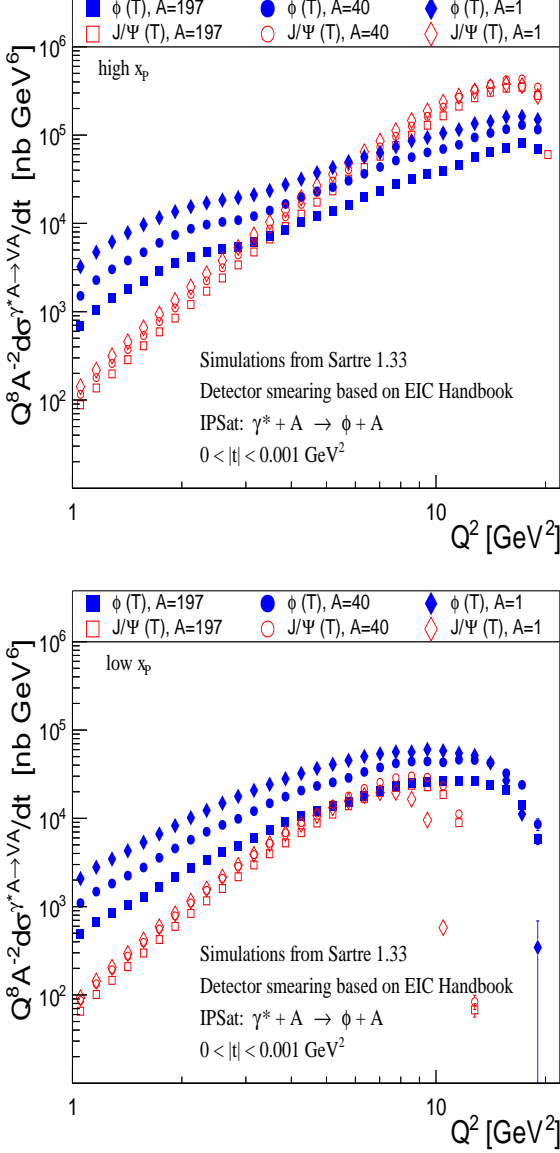
where the scalar part of the vector meson wavefunction now is  $\phi_T(r_T, z) \sim z^2(1-z)^2 e^{-r_{\mathbb{T}}^2 M_V^2}$ . Then, the diffractive transverse scattering amplitude will be given by

$$\begin{aligned} \mathcal{A}_T^{\gamma^* A \rightarrow VA} &\sim i \int d\mathbf{r}_{\mathbb{T}} r_{\mathbb{T}}^4 Q K_1(Q r_T) \sim \frac{1}{Q^4}, \\ \text{leading to } \left. \frac{d\sigma_T^{\gamma^* A \rightarrow VA}}{dt} \right|_{t=0} &\sim \frac{1}{Q^8}. \end{aligned} \quad (28)$$

Fig. 12 shows pseudo-data on exclusive coherent  $\phi$  and  $J/\Psi$  transverse electroproduction cross section at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  in the IPSat model. Some features observed in the two plots of this figure by explanation are similar to those seen in Fig. 11. Strong suppression is present in both figures, nevertheless, the  $Q^8$  transverse scaling is less accurate than the  $Q^6$  longitudinal scaling



because the transversely polarized photon contribution from large dipoles is not suppressed by high  $Q^2$ , because of strong dependence of the wavefunction overlap on the  $z \rightarrow 0, 1$  limits.



**Fig. 12** The onset of the  $A^2 Q^{-8}$  scaling at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  for the exclusive coherent  $\phi$  and  $J/\Psi$  transverse electroproduction cross section in the IPSat model for the Gold, Calcium and proton, multiplied by the scaling factor of  $Q^8 A^{-2}$ . The nomenclature is the same as in Fig. 8. The top plot shows the pseudo-data at high  $x_p$ , the bottom plot shows the pseudo-data at low  $x_p$ .

### 3.2.2 The scaling onset as a function of small $|t|$

In Eq. (4), for the factor  $\exp(-i[\mathbf{b}_T - (1-z)\mathbf{r}_T] \cdot \Delta)$  we can also use an approximation based on applying

$|\mathbf{b}_T| \gg |\mathbf{r}_T|$ . However, one should note that in Sartre this phase factor in the same formula actually has the form of  $J_0([1-z]r_T \Delta) e^{-i\mathbf{b}_T \cdot \Delta}$  [45, 46]. Thereby, one may find out whether the scaling picture discussed in the previous section exists as a function of small values of  $t$ . This can be done if we derive the  $t$ -dependent  $A^2$  scaling. In this case the diffractive amplitude for  $\gamma^* A \rightarrow VA$  in a simplified version will be given by

$$\mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA} \sim i \int d^2 \mathbf{r}_T d^2 \mathbf{b}_T (\Psi_{\gamma^* \rightarrow q\bar{q}} \Psi_{q\bar{q} \rightarrow V}^*)_{T,L} \times J_0([1-z]r_T \Delta) e^{-i\mathbf{b}_T \cdot \Delta} \mathbf{r}_T^2 Q_{s,A}^2, \quad (29)$$

which can also be written as

$$\mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA} \approx \text{const} \times i A^{1/3} \int \mathbf{b}_T e^{-i\mathbf{b}_T \cdot \Delta} d\mathbf{b}_T \times \int d^2 \mathbf{r}_T (\Psi_{\gamma^* \rightarrow q\bar{q}} \Psi_{q\bar{q} \rightarrow V}^*)_{T,L} J_0([1-z]r_T \Delta) \mathbf{r}_T^2. \quad (30)$$

After the  $\mathbf{b}_T$  integration from 0 to  $b_{0,j} A^{1/3}$  we will have

$$\mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA} \approx \text{const} \times i A^{1/3} \frac{e^{b_{0,j} A^{1/3} (-i\Delta_k)} [1 + b_{0,j} A^{1/3} (i\Delta_k)] - 1}{\Delta_k^2} \times \int d^2 \mathbf{r}_T (\Psi_{\gamma^* \rightarrow q\bar{q}} \Psi_{q\bar{q} \rightarrow V}^*)_{T,L} J_0([1-z]r_T \Delta) \mathbf{r}_T^2, \quad (31)$$

where

(i) for  $b_{0,j}$  we take

$$b_{0,Au} = 1.2 \text{ fm} \rightarrow 6.082 \text{ GeV}^{-1}, b_{0,Ca} = 1.017 \text{ fm} \rightarrow 5.155 \text{ GeV}^{-1} [68] \text{ and } b_{0,p} = 0.831 \text{ fm} \rightarrow 4.212 \text{ GeV}^{-1} [69];$$

(ii) for  $\Delta_k$  we take the values of  $t_k$  at  $|t_0| = 5 \times 10^{-4} \text{ GeV}^2$ ,  $|t_1| = 3.5 \times 10^{-3} \text{ GeV}^2$ ,  $|t_2| = 6.5 \times 10^{-3} \text{ GeV}^2$ ,  $|t_3| = 9.5 \times 10^{-3} \text{ GeV}^2$  and  $|t_4| = 12.5 \times 10^{-3} \text{ GeV}^2$  (all already shown in the introduction).

Thus, the final result for the amplitude is the following:

$$\mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA} \sim i A^{1/3} \frac{e^{b_{0,j} A^{1/3} (-i\Delta_k)} [1 + b_{0,j} A^{1/3} (i\Delta_k)] - 1}{\Delta_k^2}, \quad (32)$$

and the coherent diffractive cross section is given by

$$\left. \frac{d\sigma_{T,L}^{\gamma^* A \rightarrow VA}}{dt} \right|_{t=t_k} \sim A^{2/3} \left| \frac{e^{b_{0,j} A^{1/3} (-i\Delta_k)} [1 + b_{0,j} A^{1/3} (i\Delta_k)] - 1}{\Delta_k^2} \right|^2, \quad (33)$$

which in the limit of  $t \equiv t_0 \rightarrow 0$  is proportional to  $A^2 b_{0,j}^4$ .

Let us designate the functional form in Eq. (33) divided by  $b_{0,j}^4$  to be  $G(A, t)$ , which will be the normalization scaling factor for the cross section ratio or cross section at the above  $t_k$  values. Therefore, the scaling

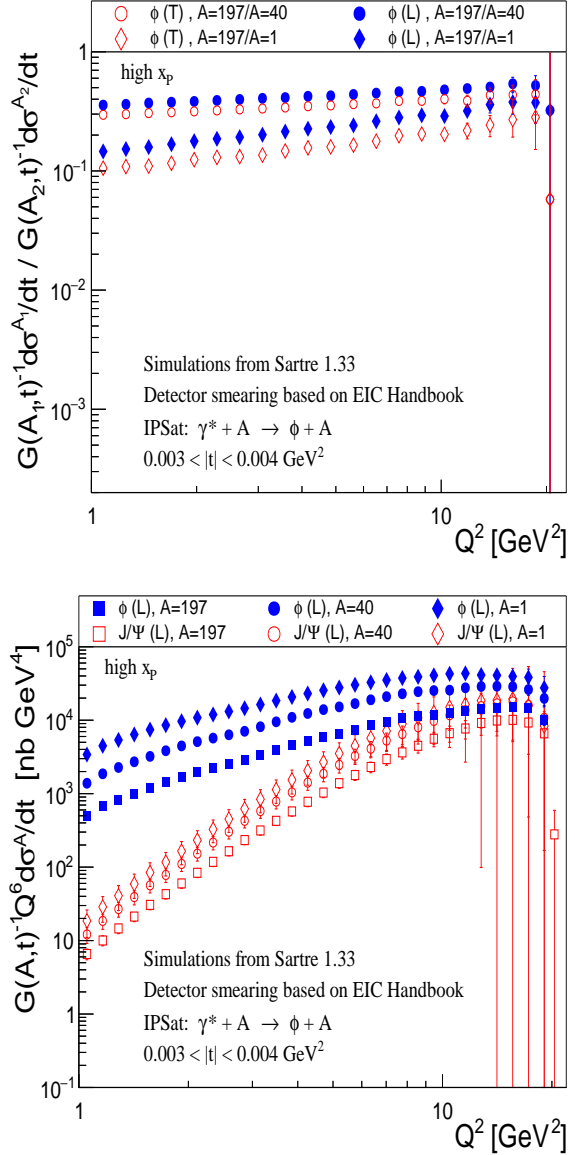


factor in this formula is also a function of small  $|t|$ . In this case we can show, for example, the reproduction of the normalized cross section ratio of Fig. 9, and the normalized cross section of Fig. 11, at the given four values of  $t_1$ ,  $t_2$ ,  $t_3$  and  $t_4$ . Thereby, we use the cross section asymptotic scaling factor defined as  $G(A, t)$  that at  $t_0 \rightarrow 0$  gives  $A^2$ , which is the same as the factor in Eq. (23). Thus, one can see how the behavior of the scaling patterns of the cross section ratio and the longitudinal cross section change as a function of small values of  $|t|$  shown correspondingly in Figs. 9 and 11 as well as in Figs. 13-16. Contrary to the high- $x_{\mathbb{P}}$  and low- $x_{\mathbb{P}}$  ranges used in the previous section, we employ the entire range of  $x_{\mathbb{P}} = [0.001 - 0.008]$  in producing these figures. In particular, in the cross section ratio figures of the Gold over Calcium and the Gold over proton, we see the suppression in magnitude of the pseudo-data curves. Meanwhile, the shapes of the ratios, which have rising behaviors as a function of  $Q^2$ , survive up to the case of  $|t| = 12.5 \times 10^{-3} \text{ GeV}^2$ . Consequently, within the approximations used we may assume that the saturation pattern, which is observable from the pseudo-data in Fig. 9 also occur as a function of small values of  $|t|$ , at least, in Figs. 13-16.

At the end of this section one should stress that in order to accurately reflect an experimental binning of  $t$  in a given range, e.g., for the cross-section ratio in Fig. 9, we allow the event generator to produce events in a  $t$  range larger than what is binned. For all the figures binned in  $|t| = [0.000 - 0.001] \text{ GeV}^2$ , we select our simulated  $t$  range to extend to  $|t| = [0.000 - 0.002] \text{ GeV}^2$ . In doing so, we account for the realistic possibility that an event simulated outside the binning range will become smeared into the very same range. For instance, an event simulated with  $|t| \sim 0.0011 \text{ GeV}^2$  can be reconstructed as  $|t| \sim 0.0009 \text{ GeV}^2$ , which places it into the  $|t| = [0.000 - 0.001] \text{ GeV}^2$  binning range. Meanwhile, for the cross-section ratio in Fig. 13, we select the minimum and maximum  $t$  to be separated by  $\pm 0.0005 \text{ GeV}^2$  on each side of that figure's  $t$  binning range. Thereby, in this case we select our simulated  $t$  range to be  $|t| = [0.0025 - 0.0045] \text{ GeV}^2$ . Then, an outside  $t$  event, say, at  $|t| \sim 0.0041 \text{ GeV}^2$ , can be reconstructed as  $|t| \sim 0.0039 \text{ GeV}^2$ , placing it into the  $|t| = [0.003 - 0.004] \text{ GeV}^2$  binning range. This procedure is also the case for the other figures (with other  $t$  bins) displayed in this section.

### 3.3 $A$ and $Q^2$ scaling picture in the regime of low $Q^2 < Q_{s,A}^2$

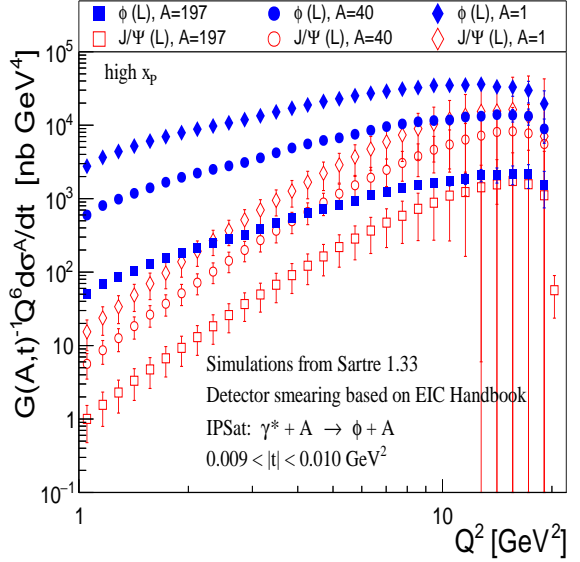
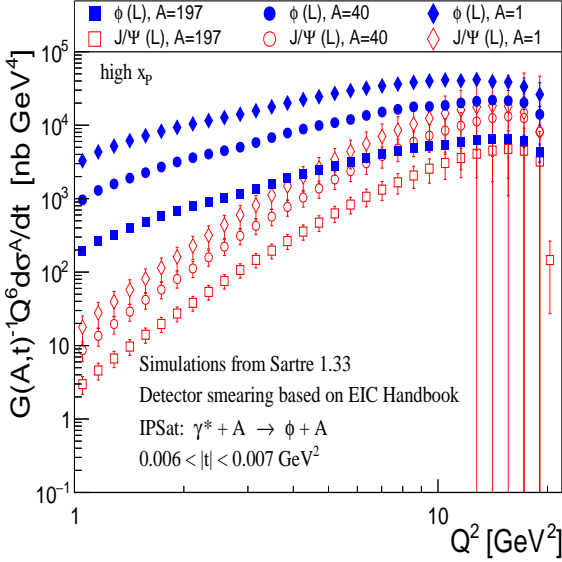
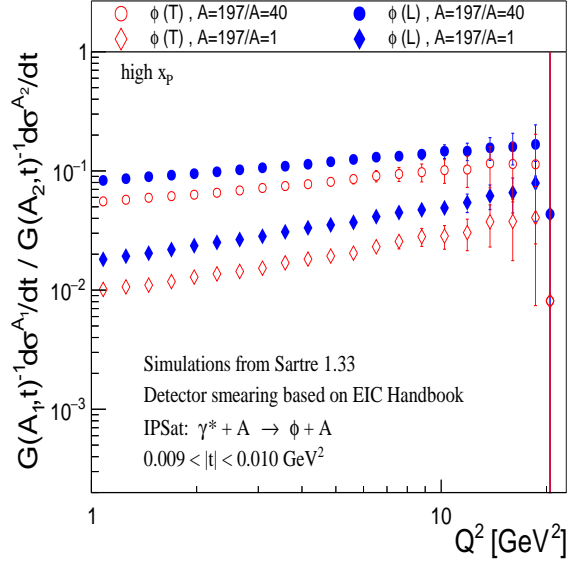
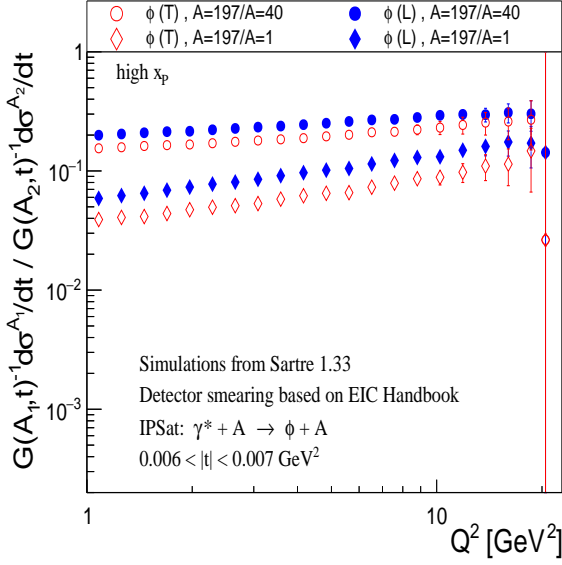
Here we discuss the asymptotics in which one can approximate  $\mathbf{r}_{\mathbf{T}}^2 Q_{s,A}^2$  in the limit of  $Q^2 \ll Q_{s,A}^2$ . In this



**Fig. 13** (Color online) The top plot shows the onset of the  $G(A, t)$  scaling at  $|t| = 3.5 \times 10^{-3} \text{ GeV}^2$  for the exclusive coherent  $\phi$  electroproduction cross section pseudo-data at high  $x_{\mathbb{P}}$  in the IPSat model. The vertical axis shows the  $G(A, t)^{-1}$  normalized ratio of the cross sections for the Gold over Calcium and the Gold over proton. The nomenclature is the same as in Fig. 9. The bottom plot shows the onset of the  $G(A, t)Q^{-6}$  scaling at  $|t| = 3.5 \times 10^{-3} \text{ GeV}^2$  for the exclusive coherent  $\phi$  and  $J/\Psi$  longitudinal electroproduction cross section pseudo-data at high  $x_{\mathbb{P}}$  in the IPSat model for the Gold, Calcium and proton, multiplied by the scaling factor of  $Q^6 G(A, t)^{-1}$ . The nomenclature is the same as in Fig. 11.

limit the  $\gamma^* - A$  cross section is represented by

$$\frac{d\sigma_{q\bar{q}}^A}{d^2\mathbf{b}_{\mathbf{T}}} = 2 [1 - \exp(-\mathbf{r}_{\mathbf{T}}^2 Q_{s,A}^2)] \rightarrow 2, \quad (34)$$



**Fig. 14** (Color online) Similar figure as in Fig. 13 but at  $|t| = 6.5 \times 10^{-3} \text{ GeV}^2$ .

**Fig. 15** (Color online) Similar figure as in Fig. 13 but at  $|t| = 9.5 \times 10^{-3} \text{ GeV}^2$ .

and at  $t = 0$  the diffractive scattering amplitude from Eq. (4) becomes

$$\mathcal{A}_{T,L}^{\gamma^* A \rightarrow V A} \sim \sim i \int d^2 \mathbf{r}_T d^2 \mathbf{b}_T (\Psi_{\gamma^* \rightarrow q \bar{q}} \Psi_{q \bar{q} \rightarrow V}^*)_{T,L} \times 2. \quad (35)$$

In Eq. (35) the  $Q^2$  and  $A$  dependence is determined by the dipole radius scale, and how it affects the wavefunction overlap. Let us again follow Ref. [40] for looking into two scaling behaviors deep in the saturation domain.

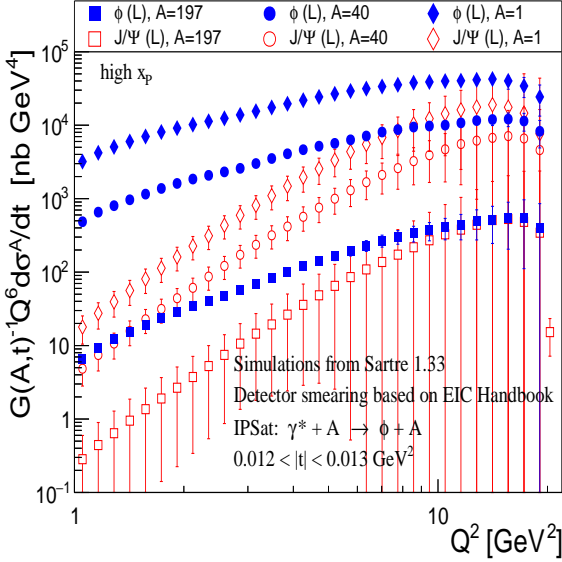
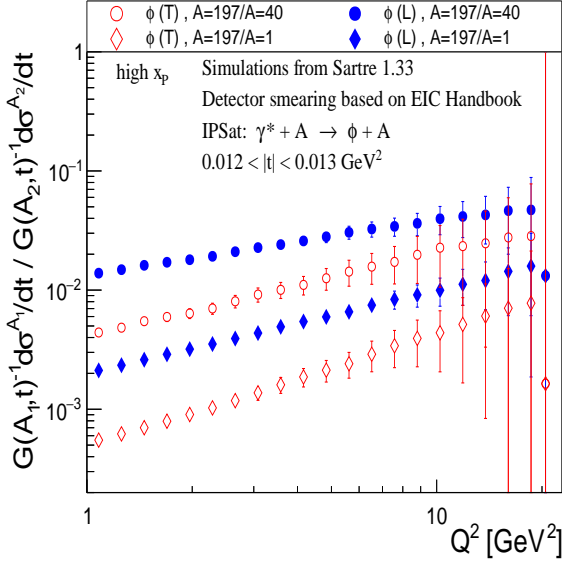
$A^{4/3} Q^2$  scaling: Eq. (25) shows the overlap of the vector meson wavefunction with the longitudinally polar-

ized photon wavefunction. One can use it in the saturation region but with  $\varepsilon = \sqrt{Q^2 z(1-z) + m_q^2} \approx m_q$  for  $Q \ll m_q$ . Then, the diffractive longitudinal scattering amplitude reduces to

$$\mathcal{A}_L^{\gamma^* A \rightarrow V A} \sim iQ \int_{1/Q_{s,A}}^{1/m_q} d\mathbf{r}_T \mathbf{r}_T K_0(m_q r_T). \quad (36)$$

The ultimate result in the limit of low- $Q^2$  is given by

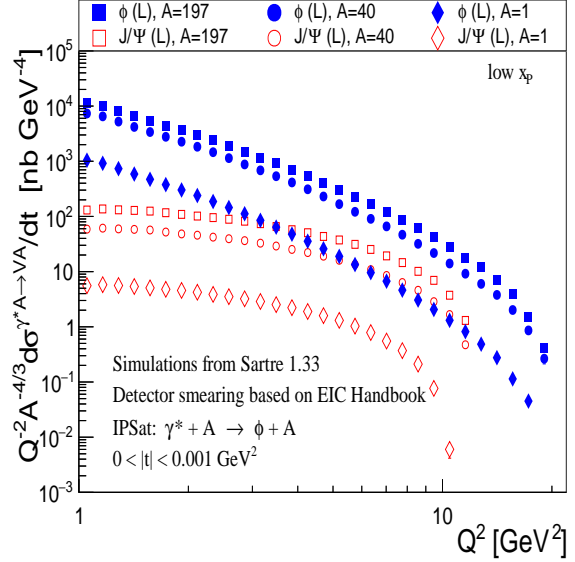
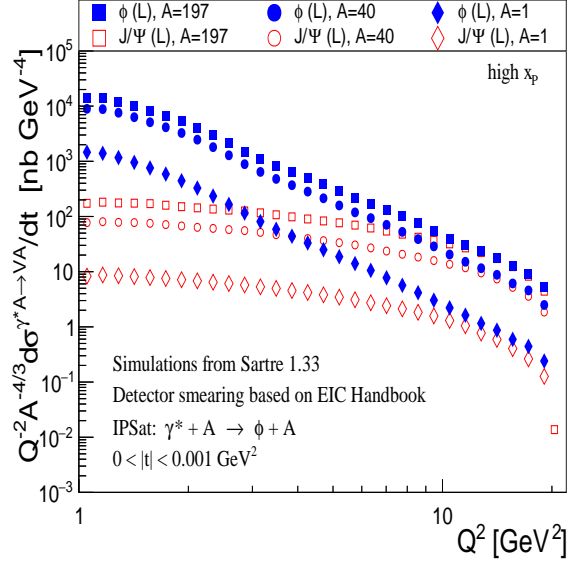
$$\begin{aligned} \mathcal{A}_L^{\gamma^* A \rightarrow V A} &\sim iQ \left( \text{const} + \mathcal{O} \left( \frac{m_q^2}{Q_{s,A}^2}, \frac{M_V^2}{Q_{s,A}^2}, \frac{Q^2}{Q_{s,A}^2} \right) \right) \\ \Rightarrow \left. \frac{d\sigma_L^{\gamma^* A \rightarrow V A}}{dt} \right|_{t=0} &\sim Q^2. \end{aligned} \quad (37)$$



**Fig. 16** (Color online) Similar figure as in Fig. 13 but at  $|t| = 12.5 \times 10^{-3} \text{ GeV}^2$ .

Fig. 17 shows pseudo-data of the exclusive coherent  $\phi$  and  $J/\Psi$  longitudinal electroproduction cross section at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  in the IPSat model, where contrary to Fig. 11 the cross section is now multiplied by the scaling factor  $Q^{-2}$ , and is also scaled in  $A$  by the asymptotic analytical expectation  $A^{4/3}$ . For  $J/\Psi$  production, the cross sections become flatter at low- $Q^2$  when scaled by  $Q^2$ . For  $\phi$  production, this trend could be obtained at asymptotically small values of  $Q^2$  (not shown here), which is beyond applicability of the approximations used.

$A^{4/3}Q^0$  scaling: Here is the case of transversely polarized photons, where one can make use of Eq. (27) with



**Fig. 17** The cross sections for the Gold, Calcium and proton at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  for the exclusive coherent  $\phi$  and  $J/\Psi$  longitudinal electroproduction in the IPSat model, multiplied by the factor  $Q^{-2}A^{-4/3}$ . The nomenclature is the same as in Fig. 8. The top plot shows the pseudo-data at high  $x_p$ , the bottom plot shows the pseudo-data at low  $x_p$ .

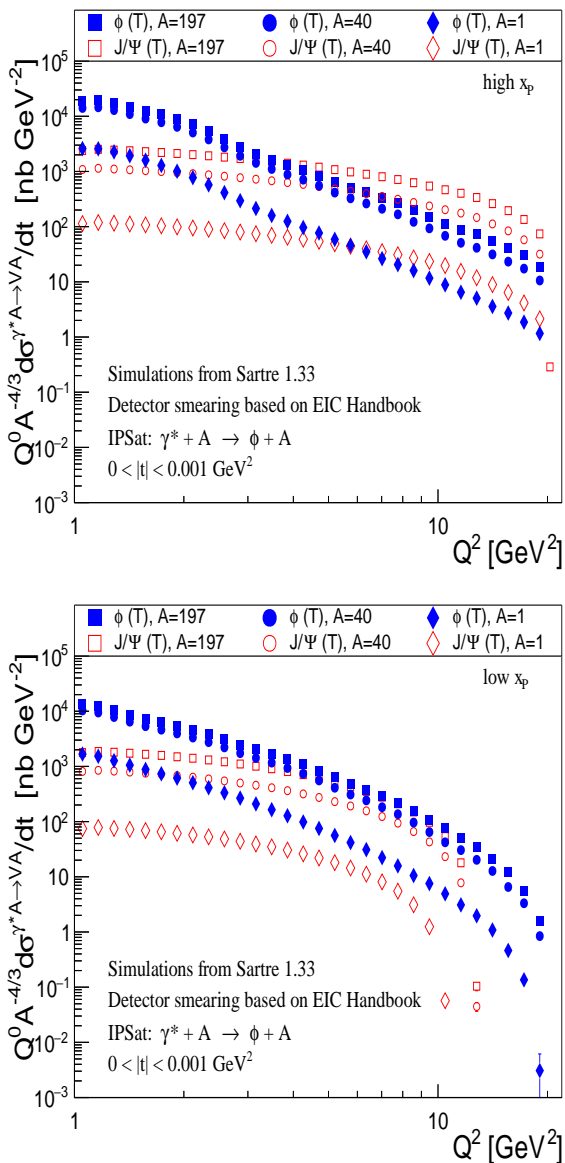
the above approximation  $\varepsilon \approx m_q$ .

$$\mathcal{A}_T^{\gamma^* A \to VA} \sim im_q \int_{1/Q_{s,A}}^{1/m_q} d\mathbf{r}_T r_T^2 K_1(m_q r_T). \quad (38)$$

The ultimate result in the limit of low- $Q^2$  is given by

$$\begin{aligned} \mathcal{A}_T^{\gamma^* A \to VA} &\sim iQ^0 \left( \text{const} + \mathcal{O} \left( \frac{m_q^2}{Q_{s,A}^2}, \frac{M_V^2}{Q_{s,A}^2}, \frac{Q^2}{Q_{s,A}^2} \right) \right) \\ \Rightarrow \left. \frac{d\sigma_T^{\gamma^* A \to VA}}{dt} \right|_{t=0} &\sim Q^0. \end{aligned} \quad (39)$$

Fig. 18 shows pseudo-data of the exclusive coherent  $\phi$  and  $J/\Psi$  transverse electroproduction cross section at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  in the IPSat model, where contrary to Fig. 12 the cross section is now  $Q^2$ -independent, and is scaled in  $A$  by the asymptotic analytical expectation  $A^{4/3}$ .



**Fig. 18** The cross sections for the Gold, Calcium and proton at  $|t| = 5 \times 10^{-4} \text{ GeV}^2$  for the exclusive coherent  $\phi$  and  $J/\Psi$  transverse electroproduction in the IPSat model, multiplied by the factor  $Q^0 A^{-4/3}$ . The explanation in Fig. 17 is applied here as well, except for that the cross section is  $Q^2$ -independent. The nomenclature is the same as in Fig. 8. The top plot shows the pseudo-data at high  $x_p$ , the bottom plot shows the pseudo-data at low  $x_p$ .

## 4 Conclusions and outlook

The perturbative QCD cross section in exclusive vector meson production processes in high-energy DIS of electrons scattered off nuclei is proportional to the squared nuclear gluon distribution according to [39], by which the measurements of exclusively produced vector mesons might be sensitive to gluon distributions at the regime of saturation (since these distributions grow rapidly at small  $x$ ). Consequently, the systematics that could have been determined from such measurements, based upon the presence of saturation, might be conspicuously different from the systematics of those measurements for which the saturation would be absent. With calculations performed to leading logarithmic accuracy, Ref. [40] has demonstrated that the  $A$  and  $Q^2$  scaling properties of the cross section in exclusive vector meson production is substantially modified by saturation effects, taking place in the crossover region between the perturbative QCD and saturation regimes.

The results demonstrated in our paper seem to confirm this scaling picture, which one would expect from a realistic “EIC setup” based upon the Monte Carlo event generator *Sartre*. But before addressing the scaling problem, we first generated and analyzed pseudo-data to extract the coherent and incoherent cross section distributions of exclusive  $J/\Psi$  and  $\phi$  production in diffractive  $e + Au$  scattering with an integrated luminosity of  $10 \text{ fb}^{-1}/A$  and the scattering energies  $10 \times 110 \text{ GeV}$ , by gradually smearing the diffractive peak for increasing relative  $t$  uncertainty. These results are given in Figs. 3-7. For obtaining all the pseudo-data along with the projected uncertainties, we have used detector resolutions and smearing functions from the EIC Yellow Report [27] and the EIC Handbook [53].

Afterwards, based upon the same framework, we reproduced the cross section and cross section ratio results of [40] in terms of exclusive  $J/\Psi$  and  $\phi$  production pseudo-data produced for an integrated luminosity of  $10 \text{ fb}^{-1}/A$  with beam energies  $10 \times 110 \text{ GeV}$  in diffractive  $e + Au/e + Ca$  scatterings, and for  $100 \text{ fb}^{-1}/A$  with  $10 \times 100 \text{ GeV}$  in diffractive  $e + p$  scattering. We used somewhat different kinematics than [40], as well as studied the  $\phi$  production instead of the  $\rho$  production. Our results, though based upon restricted kinematics, in principle substantiate the main conclusions of [40] within projected uncertainties. The scaling relations are visualized in Figs. 8-12, as well as in Figs. 17 and 18. In particular, the pseudo-data in Fig. 9 shows a sign of gluon saturation, in terms of a low- $Q^2$  part of the asymptotic  $A^2$  scaling at  $|t_0| = 5 \times 10^{-4} \text{ GeV}^2$  in the IPSat model. While if we integrate over  $t$ , we will have a low- $Q^2$  part of the  $A^{4/3}$  scaling that is shown in

Fig. 10. The entire  $Q^2$  coverage for both scaling cases cannot be shown because of the lack of pseudo-data above  $Q^2 \sim 20 \text{ GeV}^2$ . In the absence of the saturation effects (in the IPNonSat model), the  $A^2$  scaling should look like what is demonstrated in Fig. 8, where the trend is expected to be similar at high  $Q^2$  too. In addition, we discussed the scaling onset at values of  $|t|$  equal to  $|t_1| = 3.5 \times 10^{-3} \text{ GeV}^2$ ,  $|t_2| = 6.5 \times 10^{-3} \text{ GeV}^2$ ,  $|t_3| = 9.5 \times 10^{-3} \text{ GeV}^2$ , and  $|t_4| = 12.5 \times 10^{-3} \text{ GeV}^2$ , where we see the extent to which the suppression pattern seen due to gluon saturation survives as a function of small values of  $|t|$  within all the approximations used (see Figs. 13-16). Also, the  $\phi$  and  $J/\Psi$  production cross sections seem to have different  $Q^2$  scaling behavior, which is to be expected as they probe different dipole sizes, which are directly dependent on  $Q^2$ . The  $J/\Psi$  particle probes very small dipoles at all  $Q^2$ , so the cross section is not very dependent on  $Q^2$  as shown in Figs. 17 and 18. In these two figures the trend is that the pseudo-data curves become flatter at low- $Q^2$ , which means that the expected scaling is observed. However, we emphasize that due to small masses of the light vector mesons (like  $\phi$ ), the results in the low- $Q^2$  region are at the edge of applicability of the employed weak-coupling framework [40].

The next studies may include more elaborate simulations/calculations of the scaling as a function of  $|t|$  discussed in Sec. 3.2.2, by including also pseudo-data on vector meson production for  $e+d$ ,  $e+{}^4\text{He}$ ,  $e+C$ ,  $e+Cu$  collision species, in addition to  $e+Au$ ,  $e+Ca$ , and  $e+p$ , considered in this paper. This means that new look-up IPSat and IPNonSat tables (see Appendix A) should be added to *Sartre*, along with adding an IPNonSat look-up tables for  $e+Ca$  that currently are absent in *Sartre*. In new studies, it will also be important to incorporate the energy mode of  $10 \times 41 \text{ GeV}$ , which is common for all those collision species according to Table 10.3 in [27], thereby, the energy difference of the proton and nuclear beams will be zero. For the purpose of more precise simulations, we should obtain new  $t$ -resolution values from the ‘‘Method L’’ discussed at the end of Sec. 3.1, for the  $Q^2$  interval of  $1 < Q^2 < 20 \text{ GeV}^2$  (or even larger). This latter particular study should include extraction of those resolution values for both  $J/\Psi$  and  $\phi$  production. Besides, the  $Q^2$  scaling itself is interesting and worth of additional detailed investigation, since it will be a unique feature at EIC. Note that LHC and RHIC produce vector mesons in UPC at  $Q^2 = 0$ .

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## Appendix A: Calculating diffractive differential cross sections

An extremely complicated task is to calculate and generate total cross-sections, for which one has to evaluate complex four-dimensional integrals at each phase-space point. But *Sartre* uses an approach based on computing the first and second moments of the scattering amplitudes separately, and then stores the results in three-dimensional look-up tables, in terms of  $Q^2$ ,  $W^2$  and  $t$  independent variables. The ultimate outcome is a set of four look-up tables for each nuclear species, each final-state vector meson (and DVCS photon), each polarization, and each dipole model (either IPSat or IPNonSat).

$$\begin{aligned} &\langle \mathcal{A}_T(Q^2, W^2, t) \rangle_\Omega, & \langle \mathcal{A}_L(Q^2, W^2, t) \rangle_\Omega, \\ &\langle |\mathcal{A}_T(Q^2, W^2, t)|^2 \rangle_\Omega, & \langle |\mathcal{A}_L(Q^2, W^2, t)|^2 \rangle_\Omega, \end{aligned} \quad (\text{A.1})$$

These look-up tables contain all the physics information from both dipole models. The program also provides tables for calculating the phenomenological corrections described in Sec. 2.2.2.

The master equation of *Sartre* is the total diffractive differential cross section, which for electron-nucleus scattering has the following form:

$$\begin{aligned} &\left( \frac{d^3 \sigma^{\gamma^* A \rightarrow VA}}{dQ^2 dW^2 dt} \right)_{\text{tot. diff.}} = \\ &= \sum_{T,L} \frac{R_g^2(1+B^2)}{16\pi} \frac{d\Gamma_{T,L}}{dQ^2 dW^2} \times \\ &\times \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_F, Q^2, t, \Omega) \right|^2 \right\rangle_\Omega, \end{aligned} \quad (\text{A.2})$$

where  $B$  is given in Eq. (18), and  $R_g$  in Eq. (19). The quantity  $\Gamma_{T,L}$  is the flux of transversely  $T$  and longitudinally  $L$  polarized virtual photons [70–74], given by

$$\Gamma_T = \frac{\alpha}{2\pi} \left( \frac{1+(1-y)^2}{yQ^2} \right), \quad \Gamma_L = \frac{\alpha}{2\pi} \left( \frac{2(1-y)}{yQ^2} \right), \quad (\text{A.3})$$

where  $y$  is the inelasticity defined as the fraction of the electron's energy lost in the nucleon rest frame. The averaging over configurations  $\Omega$  is defined as

$$\begin{aligned} \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right|^2 \right\rangle_{\Omega} &= \\ &= \frac{1}{N_{\max}} \sum_{j=1}^{N_{\max}} \left| \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega_j) \right|^2, \quad (\text{A.4}) \end{aligned}$$

where  $N_{\max}$  is the number of configurations. If it is large enough, then the sum in Eq. (A.4) converges to a true average. It is already shown in [46] that  $2 \times 500$  configurations, 500 for  $T$  polarized  $\gamma^*$  and 500 for  $L$  polarized  $\gamma^*$ , give a good convergence. Such that there are 1000 such integrals for each  $(Q^2, W^2, t)$  phase-space point.

The photon flux may emanate from electrons, as in the case of  $e + p$  and  $e + A$  scatterings, however, it may be also radiated from protons or nuclei [75], as in the case of  $A + A$  or  $p + A$  UPC. A phase-space point together with a given beam energy fully determines the final state of a produced vector meson, except for its azimuthal angle, which is distributed uniformly.

The second moment of the amplitude in Eq. (A.4) for the nucleon configurations  $\Omega_j$  can be calculated based upon [45, 46]:

$$\begin{aligned} \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right|^2 \right\rangle_{\Omega} &= \\ &= \sum_{j=1}^{N_{\max}} \left| \int d^2\mathbf{r}_{\mathbf{T}} \int d^2\mathbf{b}_{\mathbf{T}} \int \frac{dz}{4\pi} \times \right. \\ &\quad \times \left[ (\Psi^* \Psi_V)_{T,L}(Q^2, \mathbf{r}_{\mathbf{T}}, z) \right] J_0([1-z]r_T \Delta) \times \\ &\quad \times \left. e^{-i\mathbf{b}_{\mathbf{T}} \cdot \Delta} \frac{d\sigma_{q\bar{q}}^A}{d^2\mathbf{b}_{\mathbf{T}}}(x_P, \mathbf{r}_{\mathbf{T}}, \mathbf{b}_{\mathbf{T}}, \Omega_j) \right|^2, \quad (\text{A.5}) \end{aligned}$$

where the last term  $d\sigma_{q\bar{q}}^A/d^2\mathbf{b}_{\mathbf{T}}$  is defined in Eq. (16).

Thus, Eqs. (A.2) and (A.5) determine the total diffractive differential cross section. Its coherent part is given by

$$\begin{aligned} \left( \frac{d^3\sigma^{\gamma^* A \rightarrow VA}}{dQ^2 dW^2 dt} \right)_{\text{coh. diff.}} &= \\ &= \sum_{T,L} \frac{R_g^2(1+B^2)}{16\pi} \frac{d\Gamma_{T,L}}{dQ^2 dW^2} \times \\ &\quad \times \left| \left\langle \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right\rangle_{\Omega} \right|^2, \quad (\text{A.6}) \end{aligned}$$

For the first moment of the amplitude, the integral to calculate will be

$$\begin{aligned} \left| \left\langle \mathcal{A}_{T,L}^{\gamma^* A \rightarrow VA}(x_P, Q^2, t, \Omega) \right\rangle_{\Omega} \right|^2 &= \left| \int d^2\mathbf{r}_{\mathbf{T}} \int d^2\mathbf{b}_{\mathbf{T}} \times \right. \\ &\quad \times \int \frac{dz}{4\pi} \left[ (\Psi^* \Psi_V)_{T,L}(Q^2, \mathbf{r}_{\mathbf{T}}, z) \right] J_0([1-z]r_T \Delta) \times \\ &\quad \times \left. J_0(b_T \Delta) \left\langle \frac{d\sigma_{q\bar{q}}^A}{d^2\mathbf{b}_{\mathbf{T}}}(x_P, \mathbf{r}_{\mathbf{T}}, \mathbf{b}_{\mathbf{T}}, \Omega) \right\rangle_{\Omega} \right|^2, \quad (\text{A.7}) \end{aligned}$$

where the average  $\langle d\sigma_{q\bar{q}}^A/d^2\mathbf{b}_{\mathbf{T}} \rangle_{\Omega}$  in the last term is defined in Eq. (17).

Thereby, as in Eq. (14), the incoherent part of the total diffractive differential cross section is taken to be the difference between the total and coherent cross sections:

$$\begin{aligned} \left( \frac{d^3\sigma^{\gamma^* A \rightarrow VA}}{dQ^2 dW^2 dt} \right)_{\text{incoh. diff.}} &= \left( \frac{d^3\sigma^{\gamma^* A \rightarrow VA}}{dQ^2 dW^2 dt} \right)_{\text{tot. diff.}} - \\ &\quad - \left( \frac{d^3\sigma^{\gamma^* A \rightarrow VA}}{dQ^2 dW^2 dt} \right)_{\text{coh. diff.}} \quad (\text{A.8}) \end{aligned}$$

The incoherent part directly gives the probability for the nuclear breakup.

## Appendix B: Error analysis of the pseudo-data using the EIC Handbook

We use *Sartre* 1.33, an exclusive event generator, to produce vector meson production data. A user-edited runcard is called at the initialization of a simulation to set beam energies, decay modes, and ranges on event kinematics such as  $Q^2$  and  $|t|$ . The generator outputs the kinematics of final-state particles, such as their momentum and pseudorapidity. Additionally, important event information such as  $Q^2$ ,  $x_P$ , and  $y$  are recorded. We refer to the event generator output as *truth* data. Truth data, while unobtainable in a physical experiment, allows us to perform perfect event identification and to create pseudo-data. Using detector resolutions outlined in the EIC Handbook [53], true particle kinematics are smeared to create pseudo-data. For instance, according to the handbook, the barrel ( $|\eta| < 1$ ) tracking resolution for electrons is  $\sigma_p/p = 0.05\%p + 0.5\%$ . By having such resolutions written out explicitly for the relevant final-state particles from vector meson production simulations allows us to calculate pseudo-data based on the handbook's projections. The smearing functions immediately produce smeared kinematics, such as momentum and energy. Event-by-event, we manually smear the final-state kinematics according to these functions. Subsequently, the pseudo-data reconstruction of the scattered electron is used to determine event kinematics such as  $Q^2$ . The event  $|t|$  is the only

quantity which we smear independently. One of the immediate effects of generating pseudo-data is the reduction of statistics. Final-state particles, which exit at pseudorapidities beyond the coverage of the detector system, render the exclusive event unrecoverable. Systematic errors introduced through particle/event misidentification are not accounted for when analyzing the pseudo-data. Instead, we opt to use true, event generator information to identify the scattered electron and decay particles. Lastly, we use truth information to separate coherent vector meson production from the incoherent one, as well as events with transversely polarized virtual photons from longitudinally polarized virtual photons.

After the pseudo-data is generated, we are left with the reconstructed particle and event kinematics for a pile of production events. Then, the data is stored event-by-event in a *ROOT TTree*, which is read and analyzed to produce plots. From the *TTree*, the analyzed data is used to fill *ROOT* histogram objects. These histogram objects, initially storing the number of events per bin, are scaled in a variety of ways. This includes scaling to the correct decay mode branching ratios, scaling to the desired luminosity, and scaling by bin size (to create differential cross sections). What follows is a detailed look at how the pseudo-data is analyzed, how this analysis is piped into histograms, how they are scaled, and how the error is propagated.

*Pseudo-data and error propagation for Figs. 3-7.* After the pseudo-data is generated, the following cuts are placed event-by-event:

- Reconstructed  $x_{\mathbb{P}} < 0.01$ .
- Reconstructed  $1.0 < Q^2 < 10.0 \text{ GeV}^2$ .
- True event coherent vector meson production.
- Reconstructed final-state particle pseudorapidity between  $-3.5 < \eta < 3.5$ <sup>15</sup>.
- Reconstructed final-state particle momentum greater than 1 GeV.

Then, using true event  $|t|$  information outputted by *Sartre*, we generate the reconstructed  $|t|$  using the *Gaus()* function from the *ROOT TRandom* class. For each event, we select a random variable from a Gaussian centered at true  $|t|$  with spread equal to true  $|t| \times tsmear$ . This random variable represents the reconstructed  $|t|$  of the event. The quantity *tsmear* is changed depending on our desired resolution in  $|t|$ . It ranges from 0 all the way to  $0.3 \text{ GeV}^2$ , or is parameterized by the “Method L” in

<sup>15</sup>In the original Figure 54 of the EIC White Paper [25], the final-state particle pseudorapidity is restricted between  $-4.0 < \eta < 4.0$ . In our case, the Handbook smearing functions from [53] are implemented to cover tracking up to  $-3.5 < \eta < 3.5$ .

[27]. Once all of the events have been parsed through and either added or discarded, a *TGraphErrors* object is created. This object will pick up information from the filled histogram and produce a plot.

Below are the individual modifications we make to the entries of the filled histogram, given in order. Beforehand, we define  $N_{\text{entries}}$  to be the number of total entries in the histogram, and  $w$  to be the amount of entries in an arbitrary bin.

1. First, we multiply each bin of the histogram by the quantity  $\sigma \cdot L/A$ , where  $\sigma$  is the total event cross section (in nanobarns) outputted by *Sartre*, and  $L$  is the integrated luminosity. We also divide by the atomic number  $A$  of a nucleus to isolate the scattering cross section of a single nucleon for a given nucleus beam.
2. Then, we multiply each bin by the branching ratio (BR) of the truth decay mode.
3. Next, we divide each bin by  $N_{\text{entries}}$ . And now for each bin, we are left with *the number of events expected in an experiment with integrated luminosity  $L$ , within the reconstructed  $|t|$  bin range:*

$$N_{\text{expected}} \equiv w' = \frac{\text{BR} \times L \times \sigma \times w}{N_{\text{entries}} \times A}. \quad (\text{B.9})$$

4. We further calculate the statistical uncertainty for each bin as the square root of its entries,  $\sigma_{w'} = \sqrt{w'}$ .
5. Afterwards, we divide each bin quantity  $w$  by the branching ratio, the bin width  $\Delta t$ , and integrated luminosity. We repeat this for  $\sigma_w$  as well.

$$\begin{aligned} \frac{d\sigma}{dt}[\text{pseudo-data}] &= \frac{A}{\text{BR} \times \Delta t \times L} N_{\text{expected}}, \\ \frac{d\sigma}{dt}[\text{pseudo-data statistical uncertainty}] &= \\ &= \frac{A}{\text{BR} \times \Delta t \times L} \sqrt{N_{\text{expected}}}. \end{aligned} \quad (\text{B.10})$$

This step completes the pseudo-data and error analysis for Figs. 3-7.

*Pseudo-data and error propagation for Figs. 8-18.* The analysis of these figures differ from the above method in subtle ways<sup>16</sup>. For each collision type ( $e + p$  and/or  $e + A$ ), vector meson produced ( $J/\Psi$  or  $\phi$ ), and dipole model used (IPSat or IPNonSat), we generate  $N = 10^8$  exclusive events using *Sartre*. At the beginning of the simulation, we save the total cross section of the simulation’s phase space. Using this cross section, the projected EIC luminosities ( $100 \text{ fb}^{-1}/A$  for  $e + p$  at

<sup>16</sup>The bin size in these figures changes as a function of  $Q^2$ . When the pseudo-data are used to fill those asymmetrically-sized bins, the error bars in each bin are calculated accordingly.



$10 \times 100 \text{ GeV}$ ,  $10 \text{ fb}^{-1}/A$  for  $e+A$  at  $10 \times 110 \text{ GeV}$ ), and the branching ratios of the studied vector mesons' decay modes, we calculate the number of expected events produced at the EIC. This number, we call  $n$ , can be divided by  $N$  to tell us how much we should scale our simulation size to reflect the specific luminosity. We call this parameter  $\text{Scale} = n/N$ .

Event by event, we use the true event generator output to determine if the event was coherent or incoherent. Skipping incoherent events, we then store the virtual photon polarization for an event, also given by *Sartre*. We place a hard cut on the true event  $y$  to minimize the impact of poor smearing in that kinematic regime. By using detector uncertainties projected in [53], we smear the energy, momentum, and polar angle of the final-state electron and decay products. We skip events in which any one of these three-final state particles' true pseudorapidity falls outside the reach of the detector system's coverage. After the smearing is completed, we use the true beam energy of the incoming electron and nuclear/proton target to calculate event kinematics such as  $Q^2$ ,  $y$ , and  $x_{\mathbb{P}}$ . We smear the event's  $t$  in a way described in Sec. 3, skipping events with  $y < 0.05$ .

When filling our histograms, we weigh the fill by the factor

$$\text{Scale} \times \frac{dx_{\mathbb{P}}}{dy} \frac{Q^m}{\Gamma_{T,L}},$$

where  $m$  is determined by the specific  $Q^2$  scaling being analyzed, the photon flux factor  $\Gamma_{T,L}$  depends on the polarization of the event, and the derivative is given by

$$\frac{dx_{\mathbb{P}}}{dy} = \frac{s \times x_{\mathbb{P}}}{M_V^2 + Q^2 - t}. \quad (\text{B.11})$$

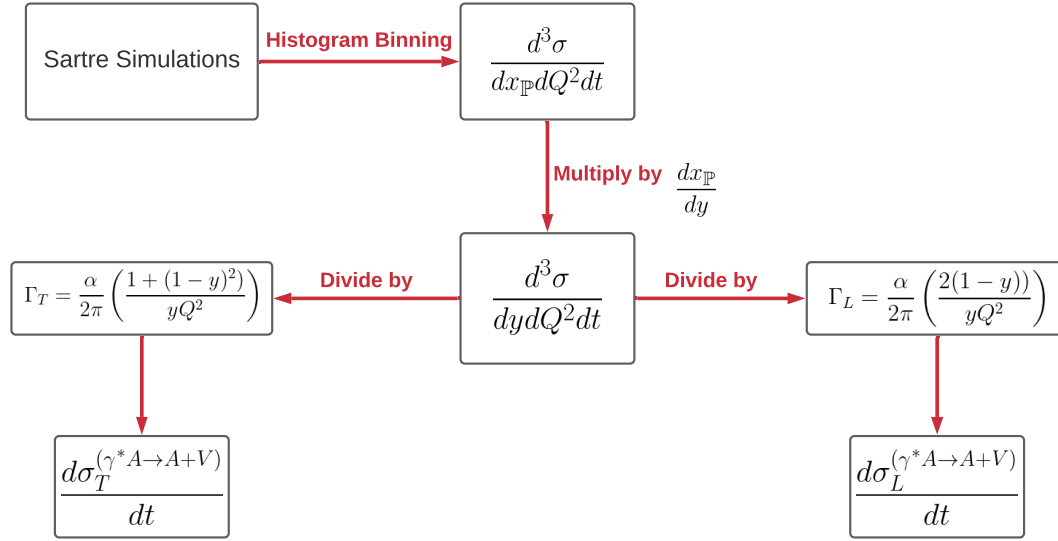
The  $dx_{\mathbb{P}}/dy$  and  $\Gamma_{T,L}$  factor effectively divide out the virtual photon flux element of the event cross section, leaving us with (see Fig. 19) a beam-independent cross section  $\sigma(\gamma^* + p/A \rightarrow V + p'/A')$ . The scale factor is included to reflect the number of events expected, given the experimental luminosity. If we had run *Sartre* and generated exactly the number of events expected in a given luminosity, then the  $\text{Scale}$  would equal 1. Each time a histogram's bin is filled, ROOT updates the statistical error of the bin to be the summation in quadrature of all the bin's individual weights.

After all events have been simulated, we divide each bin of each histogram by the factor  $\text{BR} \times L/A$ . We then divide out by the  $Q^2$  bin width,  $x_{\mathbb{P}}$  range, and proper nuclear size  $A^k$  scaling, which leaves the bin content being equivalent to  $A^{-k} Q^m \sigma^{T,L}$ . To obtain the differential cross section, we finally divide by the  $t$  binning,

which is equal 0.001. The proper nuclear size  $A$  scaling and is applied to each histogram depending on  $x$ . For the non-zero  $|t|$  scaling plots in Figs. 13-16, we additionally divide the vertical axis by  $G(A, t)$  (see Eq. (33)).

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**Fig. 19** Cross section conversion scheme for obtaining the scaling plots in Figs. 8-18 based upon Sartre 1.33 simulations.

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