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On Population Structure and Marriage Dynamics*

Eugenio Giolito

Abstract

I develop an equilibrium, a two-sided search model of marriage with endogenous population growth, to study the interaction between fertility, the age structure of the population and the age of men and women at first marriage.

Within a simple two-period overlapping generation model, I show that given an increase of the desired number of children age at marriage is affected through two different channels. First, as population growth increases, the age structure of the population produces a thicker market for young people, inducing early marriages. The second channel comes from differential fecundity: if the desired number of children is not feasible for older women, women tend to marry younger and men older, with single men outnumbering single women in equilibrium.

Using an extended version of the model to a finite number of periods and fertility data, I show that two mechanisms described above may have acted as persistence mechanisms after the U.S. Baby Boom. Specifically, I find that the demographic transitional dynamics after the Baby Boom may account for approximately 23 percent of the increase in men's age of marriage between 1985 and 2009, although the impact on women's age is small.

KEYWORDS: Baby Boom, marriage, population structure

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1 Introduction

This paper investigates the relationship between fertility, the age structure of the population and the timing of marriage of men and women. The model developed here explains a little appreciated pattern of the U.S. Baby Boom: it is not surprising that, during that period, fertility rates were high while the age of marriage was low. However, 10 years after the Baby Boom ended, the age at first marriage remained almost at the same level as it had been at any point of the Baby Boom. The well documented rise in the at first marriage occurred a full generation later than the peak of fertility rates, as shown in Figure 1 (a).

Is this delay in the rise of the of the age of marriage linked to the Baby Boom itself? This paper argues that lagged fertility rates are causally linked to the time it takes to find a suitable mate. The basic aspects of the model formally developed below can be easily seen with the example of the Baby Boom.

Panel (b) of Figure 1 shows that the share of the younger group of people of marriageable age (those aged 15-29 among those aged 15-44) mimics fertility rate with a lag of about 20 years. Specifically, this share starts increasing in the early 1960s, when the early boomers became 15 year-olds, peaks in the mid-1970s, when they turned 30, and then decreases until the mid-1990s. Panel (c) shows the comparison between demographic structure and the median age of marriage of men and women. Notice that both measures of marriage timing start increasing approximately at the same time the population starts to become older again, around 1976. The main goal of this paper is to show that there is a causal mechanism behind those facts.

The mechanism works as follows: given an increase in the desired number of children, the age at first marriage is affected through two different channels. First, as population growth increases, the age structure of the population produces a thicker market for young people, inducing early marriages. The second channel comes from differential fecundity: if the desired number of children is not feasible for older women, they tend to marry younger and men older, with single men outnumbering single women in equilibrium. Of course, we expect the opposite effects when the biological constraint is relaxed, which may be the case after the baby-boom ended. The model then has a second prediction, that the age difference between men and women is jointly determined with the single sex ratio. Figure 1 (d) plots the evolution of the age difference and the single sex ratio, showing the strong correlation between these variables.

In the first part of the paper (Section 2), I develop a simple two-period overlapping generation model with two-sided search in which agents only differ

in gender and age and where utility is non-transferrable (NTU).¹ The only difference between genders is that women lose their fecundity relatively earlier than men (Siow, 1998).² I assume that both men and women desire to have a given number of children, which I keep as exogenous. Given desired fertility, population growth and therefore the age structure of the population are endogenous.³ This model builds on Giolito (2004), who is the first to model the age of individuals both as a state variable and as a source of agents' heterogeneity within an NTU environment.^{4,5}

The second part of the paper, starting in Section 3, analyzes the empirical implications of the basic model; it specifically studies a potential causal relationship between the demographic effects of the Baby Boom and the dynamics of marriage for several decades after the end of the boom. For that reason, I extend the basic model to a full life-cycle model with people living a finite number of yearly periods and where men and women differ not only in fecundity but also in life-expectancy.⁶

To calibrate this model, I proceed in two steps, detailed in Section 4. First, I assume that the economy is in its steady state and calibrate the model parameters to U.S. 1930 demographics. Then, since the study of the causes of the Baby Boom is outside the scope of this paper, I consider the boom as a multi-period shock in some parameter of a (not modeled) fertility choice process. Therefore, I obtain the desired fertility values from U.S. data and assume that they become constant in 1975. From that year onwards, marriage timing is exclusively determined by a persistence mechanism due to the demographic effects of the Baby Boom.

¹Following Burdett and Coles (1997), Burdett and Wright (1998) and Smith (2006), there is extensive literature in NTU equilibrium search models of marriage. Recent examples include Wong (2003), Burdett et al. (2004) and Bloch and Ryder (2000).

²The relationship between differential fecundity and the age gap in marriage was first studied by Siow (1998), whose analysis include parental investment in children, and labor markets interacting with the marriage decision and determining the gender roles. However, there is no population growth, which is a key feature in this model. For a previous paper on the topic, see Betgstrom and Bagnoli (1993).

³A related paper is Tertilt (2005), who also allows for endogenous population growth to study the effects of polygyny in certain African countries.

⁴Different from this work, Giolito (2004), assumes zero population growth.

⁵Recently, Coles and Francesconi (2007) wrote a similar model to explain the increasing number of toyboys (marriages where the wife is at least five years older than the husband). Different from Giolito (2004), the model is in continuous time, with aging represented by a constant rate of decay in fitness, and where wages are stochastic.

⁶This extended model share some aspects of its basic setup with a recent paper by Díaz-Giménez and Giolito (2010), who developed a model to account for differences in fecundity, earnings and mortality profiles between men and women. Another related paper is Caucutt, Guner and Knowles (2002), who study the roles played by wage inequality, human capital accumulation, and returns to experience for the timing of marriage and fertility. Different from this work, both of the papers mentioned focus on a steady state economy with no population growth.

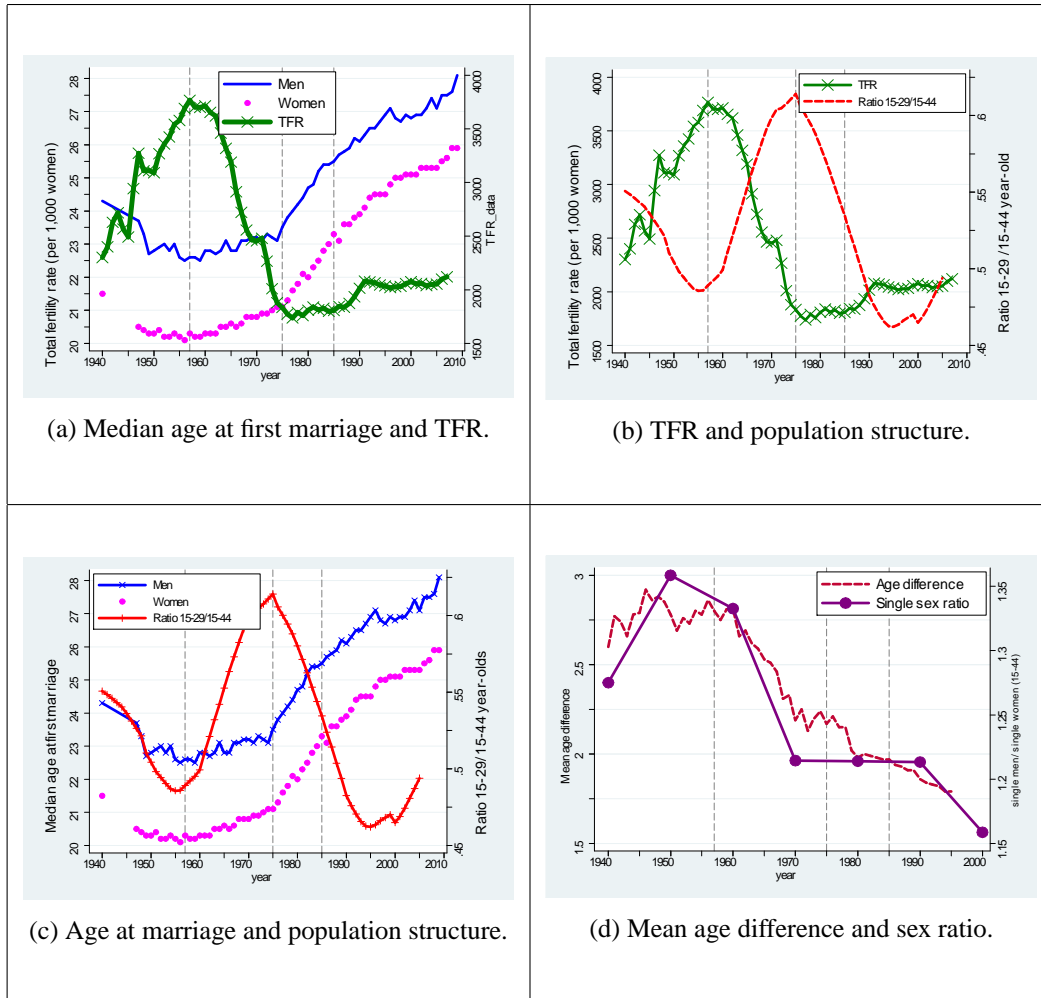


Figure 1: Marriage and fertility indicators in the U.S.

The main findings of the calibration exercise are shown in Section 5. I find that the decline in fertility in the period 1957-1976 may account for 23% of the increase in men’s median age of marriage occurring between 1985 and 2009. In the baseline model, men’s age at first marriage increases until the age structure of the population, which is a function of past fertility, reaches its steady state level (around the year 2080). The demographic structure produces two different effects. The direct effect is that matching rates increase for men and women at the beginning of the transition period, when a large share of people of marriageable age are young "Baby Boomers". The indirect effect comes through the sex ratio, increasing men’s matching rates with respect to women’s. For this same reason, the impact on women’s time of marriage is small.

In order to check the robustness of the mechanism, I perform two counterfactual experiments. In the first experiment, I assume that fertility rates converge slowly from the 1940 levels to the replacement rate, as if the Baby Boom had never occurred. In the second experiment, I assume that, independently of people's behavior, the inflow of young men and women is constant, and therefore so is the demographic structure. In other words, in this experiment all general equilibrium effects are eliminated. The findings in either case are that men's age of marriage is always higher than that of the benchmark, and only converges 50 years after fertility rates reach their steady state.

2 A two-period model of marriage

2.1 Demographics

The economy is inhabited by a continuum of men and women who live for three periods, one as children and two as adults: young (age 1) and old (age 2). Adult men and women derive utility from being married and from having children, do not discount the future, and their only economic activity is searching for a mate. In the steady state, the population grows every period at a constant rate n , which is endogenous. Therefore, in any period t , the economy is populated by N_t young and N_{t-1} old agents, where $N_t = N_{t-1}(1+n) = N_0(1+n)^t$. Since half of the agents are male and half are female, I assume for simplicity that $N_0 = 2$.

The population dynamics in this economy determines the steady state age structure of the population. The fraction of young people in this economy is given by

$$q_t = \frac{N_t}{N_t + N_{t-1}} = \frac{N_0(1+n)^t}{N_0(1+n)^{t-1}(2+n)} = \frac{1+n}{2+n}, \quad (1)$$

which is increasing in the population growth rate.

In each period t , equal measures $s_{1,t}^w$ of young single women and $s_{1,t}^m$ of young single men enter the economy, where $s_{1,t}^m = s_{1,t}^w = (1+n)^t$. Consequently, in every period there is a measure, $S_t^w = (1+n)^t + s_{2,t}^w$, of single women in the economy, of which $(1+n)^t$ are young and $s_{2,t}^w$ are old, with n and $s_{2,t}^w$ endogenous. Similarly, there is a measure of single men, $S_t^m = (1+n)^t + s_{2,t}^m$, of which $(1+n)^t$ are young and $s_{2,t}^m$ are old. Therefore, the ratio of single men to single women (henceforth, the sex ratio), $\phi_t = \frac{S_t^m}{S_t^w}$ is also endogenous.

Since men and women marry in pairs, the total number of brides, $b_{1,t}$ young plus $b_{2,t}$ old, must be equal to the total number of grooms, $h_{1,t}$ young plus $h_{2,t}$ old.

Since in the steady state the sex ratio is constant, $\phi_t = \phi$, the fact that in every period an equal number of young men and women enter the market requires the number of men who exit the marriage market, either because they marry or because they die single to be equal to the number of women who leave the market. Consequently,

$$b_{1,t} + b_{2,t} + w_t^w = h_{1,t} + h_{2,t} + w_t^m \quad (2)$$

where w_t^m and w_t^w denote the number of men and women who die unmarried, respectively. Notice that condition (2) and the fact that people marry in pairs imply that $w_t^m = w_t^w$.

2.2 Matching technology

I assume that singles of both ages meet randomly at most once every period and decide whether or not to marry. Once married, both men and women remain so until the end of their lives and people whose spouse dies are not allowed to search for a new spouse.⁷ Whenever a man and a woman meet, either one can propose marriage to the other. If they both accept, they marry. Otherwise they both stay single.

The total number of meetings between singles is represented, in the spirit of Pissarides (1990), by the following constant returns to scale matching function,

$$\eta_t = \rho (S_t^m)^\theta (S_t^w)^{1-\theta} \quad (3)$$

where $\theta \in (0, 1)$ and $\rho \in (0, \frac{1}{2}]$.⁸ As is standard in the search and matching literature, the probabilities of being matched depend on the sex ratio ϕ . Therefore, a single man meets a single woman with probability

$$\lambda^m = \frac{\eta_t}{S_t^m} = \rho (\phi)^{\theta-1} \quad (4)$$

Similarly, a single woman meets a single man with probability $\lambda^w = \rho (\phi)^\theta$. Since the sex ratio is fully endogenous in this economy, here the matching function also captures the way in which the aggregate effects of the marriage decision feed back into the individual decision problem.

⁷For a model of marriage with endogenous separations, see Brien, Lillard and Stern (2002) and Cornelius (2003). Moreover, Chiappori and Weiss (2006) examine equilibrium divorce incentives where there is a thick market externality in the remarriage market.

⁸Given that the sex ratio can be unbalanced in equilibrium, I assume that ρ is small enough to ensure that the matching probabilities are not higher than one. The interval assumed for ρ ensures the uniqueness of a symmetric equilibrium in a two-period economy.

The probability that an individual meets either a young or an old agent of the opposite sex depends on the age structure of the single population. For example, the probability that a single man meets a single young woman is $\lambda^m p_1^w$ and the probability of meeting an old single woman is given by $\lambda^m p_2^w = \lambda^m (1 - p_1^w)$, where $p_a^w = \frac{s_{a,t}^w}{S_t^w}$ is the fraction of single women of age a , which is constant in the steady state. Similarly, from a single women point of view, the probabilities of meeting a young and an old man are given by $\lambda^w p_1^m$ and $\lambda^w (1 - p_1^m)$, respectively.⁹

2.3 Payoffs

The table below describes the utility that people derive from their marital status. I assume that married people derive utility both from the match quality and from the number of children they are able to have with a given spouse, and the utility of being single is normalized to zero. I assume that men and women desire to have $k \in [\frac{1}{2}, \bar{k}]$ children during their lifetime, where each "child" is a pair of twins, one girl and one boy. I also assume that women bear all their children in the same period they marry. Finally, men and women differ in their fecundity horizons. On one hand, while young women are able to bear k children, old women are only capable of bearing at most one child. On the other hand, men of age 1 and of age 2 are able to have k children.

The utility that a woman derives from being married to a specific man also depends on the man's type, assumed to be a random variable with cumulative distribution $G^m(y)$, support in $[0, \bar{y}]$, and mean μ_y . Similarly, I assume that the utility that a man derives from being married to a specific woman depends on the woman's type, which is a random variable with cumulative distribution $G^w(x)$, support in $[0, \bar{x}]$, and mean μ_x . I further assume that both cumulative distributions are continuous and differentiable, and that their probability density functions are $g_m(y)$ and $g_w(x)$, respectively. Since I assume that singles are homogeneous in their quality, the distributions G^m and G^w are identical across singles, and the realizations that people draw at each meeting are independent.¹⁰

For example, if a young woman marries a young man, their utilities are $2ky$ and $2kx$, because the marriage will last for two periods. If a young woman marries an old man, they will also receive ky and kx but only for one period. On the other hand, if a young man marries an old woman, their utilities are only $\min\{k, 1\}x$ and $\min\{k, 1\}y$ for one period, respectively. Since old women are less fecund and only

⁹Given that the matching technology is CRS and that $s_{1,t}^m = s_{1,t}^w$, it is straightforward that $\lambda^w p_1^m = \frac{\eta}{S_t^w} \frac{(1+n)^t}{S_t^m} = \lambda^m \cdot p_1^w$

¹⁰The assumption about the two separate distributions for men and women, as in Burdett and Coles (1997) is made exclusively for analytical tractability and does not affect the results.

able to bear one child, they receive the same utility regardless of the age of their spouse. Furthermore, since in the model economy people who do not marry miss out on the joys of both companionship and childbearing, I normalize the utility of being single to zero.

| Payoffs for Men and Women | | |
|---------------------------|---------------------|-------------------|
| | <i>Young Spouse</i> | <i>Old Spouse</i> |
| <i>Women</i> | | |
| Marry young | $2ky$ | ky |
| Marry old | $\min\{k, 1\}y$ | $\min\{k, 1\}y$ |
| Never marry | 0 | 0 |
| <i>Men</i> | | |
| Marry young | $2kx$ | $\min\{k, 1\}x$ |
| Marry old | kx | $\min\{k, 1\}x$ |
| Never marry | 0 | 0 |

In order to ensure existence of an equilibrium, I assume that the maximum value of k is such that the utility of marrying an average spouse and having k children with them is not higher than the joy of finding a perfect match and having only one child. That is, $k\mu_x \leq \bar{x}$ and $k\mu_y \leq \bar{y}$, which implies

$$\bar{k} = \min \left\{ \frac{\bar{x}}{\mu_x}, \frac{\bar{y}}{\mu_y} \right\}. \tag{5}$$

2.4 The decision problem of singles

Since people match randomly in our economy, the single men's problem is to choose reservation values for young and old single women. Let $R_{a,b}^m$ denote the steady state reservation value that single men of age a set for single women of age b , and, similarly, let $R_{b,a}^w$ be the reservation values that age b single women set for age a single men. Since I assume that people meet only once per period, and that they derive zero utility from being single, the reservation values of old single men for both young and old single women is zero, trivially. That is, $R_{2,1}^m = R_{2,2}^m = 0$. Similarly, the reservation values of old single women for both old and young single men are also zero, that is, $R_{2,1}^w = R_{2,2}^w = 0$.¹¹

¹¹This rules out the trivial equilibrium where all women reject all marriages because they know that all men will reject them and vice versa.

2.4.1 Expected utility of marriage for old single people

Denote as $\gamma_{a,b}^j$ the steady state probability that a single agent of age a and gender $j \in \{m, w\}$ marries someone of age b . This probability depends first, on the probability of meeting someone of the opposite sex and age b , $\lambda^j p_b^{-j}$. Given the meeting probability, the probability of marriage depends on mutual acceptance. That is, for example for men,

$$\gamma_{a,b}^m = \lambda^m p_b^w [1 - G^m(R_{b,a}^w)] [1 - G^w(R_{a,b}^m)] \quad (6)$$

Let us observe the case of old single people. Since I assume that old people who do not marry obtain zero utility, they accept any marriage offer. For example, if an old single man (woman) meets a young single woman (man) they will marry provided the youngest agent accepts. Therefore,

$$\gamma_{2,1}^m = \lambda^m p_1^w [1 - G^m(R_{1,2}^w)] \quad \text{and} \quad \gamma_{2,1}^w = \lambda^w p_1^m [1 - G^w(R_{1,2}^m)] \quad (7)$$

On the other hand, if an old single man meets an old single woman, they marry with certainty. Without loss of generality, I assume that everybody eventually marries at some point in life. Consequently, those old men and women that were either unmatched or rejected by younger counterparts will face a frictionless technology that automatically matches them among themselves.¹² Therefore $\gamma_{2,2}^m = [1 - \gamma_{2,1}^m]$ and $\gamma_{2,2}^w = [1 - \gamma_{2,1}^w]$.

Given the payoffs described above, the expected utility of marriage for old single men is

$$\begin{aligned} U_2^m &= \gamma_{2,1}^m k \mu_x + (1 - \gamma_{2,1}^m) \min\{k, 1\} \mu_x \\ &= \lambda^m p_1^w [1 - G^m(R_{1,2}^w)] k \mu_x + (1 - \lambda^m p_1^w [1 - G^m(R_{1,2}^w)]) \min\{k, 1\} \mu_x \end{aligned} \quad (8)$$

For old single women, since they will enjoy the same utility regardless of whom they marry, we have,

$$\begin{aligned} U_2^w &= \gamma_{2,1}^w \min\{k, 1\} \mu_y + (1 - \gamma_{2,1}^w) \min\{k, 1\} \mu_y \\ &= \min\{k, 1\} \mu_y. \end{aligned} \quad (9)$$

2.4.2 Expected utility of marriage for young single people

Since young single men and women have a chance to marry in the future, they choose their reservation values taking into account the next period's prospects. For

¹²This assumption is for convenience, and will be relaxed in the next section, when I study the transitional dynamics of the model. Recall that by condition (2), steady state implies that an equal number of old men and women would remain unmatched after period 2.

example, provided that a single young man meets a single young woman (with probability $\lambda^m p_1^w = \lambda^w p_1^m$), they will marry if both of them find each other mutually acceptable. A single young woman will accept a single young man with probability $[1 - G^m(R_{1,1}^w)]$, and he will accept her with probability $[1 - G^w(R_{1,1}^m)]$. Therefore, the probability that a young man marries a young woman (or vice-versa) is

$$\gamma_{1,1}^m = \gamma_{1,1}^w = \lambda^m p_1^w [1 - G^w(R_{1,1}^m)] [1 - G^m(R_{1,2}^w)] \quad (10)$$

On the other hand, if a young man or woman meets an old single agent of the opposite sex (with probability $\lambda^m p_2^w$ or $\lambda^w p_2^m$, respectively), the acceptance of the young agent will be sufficient for marriage to occur. Consequently,

$$\gamma_{1,2}^m = \lambda^m p_2^w [1 - G^w(R_{1,2}^m)] \quad (11)$$

and

$$\gamma_{1,2}^w = \lambda^w p_2^m [1 - G^m(R_{1,2}^w)], \quad (12)$$

for men and women, respectively.

The expected utility of marriage for young single men, U_1^m , is then,

$$U_1^m = \gamma_{1,1}^m 2kE [x | x \geq R_{1,1}^m] + \gamma_{1,2}^m \min\{k, 1\} E [x | x \geq R_{1,2}^m], \quad (13)$$

and the one for women, U_1^w ,

$$U_1^w = \gamma_{1,1}^w 2kE [y | y \geq R_{1,1}^w] + \gamma_{1,2}^w kE [y | y \geq R_{1,2}^w] \quad (14)$$

2.4.3 The young single people's optimization problem

Young single people of gender $j \in \{m, w\}$ choose the reservation values for young and old single agents of the opposite sex, $R_{1,1}^j$ and $R_{1,2}^j$, which maximize their expected lifetime utility. In order to do this, they solve the following problem:

$$V_1^j = \max_{R_{1,1}^j, R_{1,2}^j} [U_1^j + (1 - \Gamma^j) U_2^j] \quad (15)$$

subject to

$$\Gamma^j = \gamma_{1,1}^j + \gamma_{1,2}^j \quad (16)$$

where Γ_1^j stands for the steady state probability that a single individual of gender j marries at age 1.

The solutions to this problem require a young single agent to be indifferent between marrying when young or resampling again in the following period. If a

young single agent meets an old individual of the opposite sex he/she must compare the utility of a one period marriage with the value of marrying when old. In the case of a young man who meets an old woman, this type of marriage becomes less valuable if their desired number of children is greater than the one that an old woman is able to conceive ($k > 1$). Therefore,

$$\begin{aligned} \min\{k, 1\}R_{1,2}^m &= U_2^m \\ R_{1,2}^m &= \frac{U_2^m}{\min\{k, 1\}} \end{aligned} \quad (17)$$

The case of a young woman who meets an old man is different. Since men of all ages are able to have k children, she must compare the utility of marrying young with the value of waiting until the next period and facing the limited fecundity which will be binding in the case where $k > 1$. Then we have

$$kR_{1,2}^w = U_2^w = \min\{k, 1\}\mu_y,$$

so

$$R_{1,2}^w = \frac{\min\{k, 1\}\mu_y}{k}. \quad (18)$$

Now we focus on the reservation values that young singles set for people of their same cohort. Given that a young single man meets a young single woman (or vice-versa), they have to compare between the value of waiting and the utility of a two-period marriage with k children. Therefore,

$$2kR_{1,1}^j = U_2^j,$$

or

$$R_{1,1}^j = \frac{U_2^j}{2k} \quad (19)$$

for $j \in \{m, w\}$.

To obtain the reservation values chosen by young single men, substituting expression (8) into Equations (17) and (19), respectively, we get

$$R_{1,2}^m = \mu_x \left\{ 1 + \lambda^m p_1^w [k - \min\{k, 1\}] [1 - G^m(R_{1,2}^w)] \right\}, \quad (20)$$

and

$$\begin{aligned} R_{1,1}^m &= \frac{\mu_x}{2k} \left\{ \min\{k, 1\} + \lambda^m p_1^w [k - \min\{k, 1\}] [1 - G^m(R_{1,2}^w)] \right\} \\ &= \frac{\min\{k, 1\}}{2k} R_{1,2}^m. \end{aligned} \quad (21)$$

Notice that the young single men's reservation values depend positively on the average match quality of single women, μ_x , and on the difference between their desired number of children and the number that old women are capable of bearing ($k - \min\{k, 1\}$). When the desired number of children is not biologically binding for old women ($k \leq 1$), we have $R_{1,1}^m = \frac{\mu_x}{2}$ and $R_{1,2}^m = \mu_x$. On the other hand, when $k > 1$, we have

$$R_{1,2}^m = \mu_x \{1 + \lambda^m p_1^w [k - 1] [1 - G^m(R_{1,2}^w)]\} \quad (22)$$

and

$$R_{1,1}^m = \frac{\mu_x}{2k} \{1 + \lambda^m p_1^w [k - 1] [1 - G^m(R_{1,2}^w)]\}, \quad (23)$$

which are increasing in the probability of meeting a single young woman, $\lambda^m p_1^w$, and decreasing in $R_{1,2}^w$, i.e., the reservation value of young single women for old single men. Naturally, when $R_{1,2}^w$ decreases, that is, when it becomes more likely that young single women accept the proposals of old single men, young men are more willing to wait and therefore become choosier. Finally, notice that while $R_{1,1}^m$ is decreasing in the desired number of children, k , $R_{1,2}^m$ is increasing in k . That is, as the desired number of children increases, young men become choosier with respect to old women and less choosy with young women.

On the other hand, reservation values chosen by young single women are simply,

$$R_{1,1}^w = \frac{\min\{k, 1\}}{2k} \mu_y, \quad (24)$$

and

$$R_{1,2}^w = \frac{\min\{k, 1\}}{k} \mu_y. \quad (25)$$

Notice that, in this case, when fecundity is binding ($k > 1$), the reservation values that young women set for all men become decreasing functions of their desired number of children.¹³

2.5 The number of single men and women

This section presents the endogenous number of single people in the steady state economy as a function of the probabilities of marriage at age 1, Γ_i^j for $j \in \{m, w\}$ and the rate of population growth, n .

¹³Notice that the reservation values of men and women are all *continuous* in the value of the parameter k , which allows us to find an equilibrium for each value of k .

Since the population of old single men and women is composed of the unmarried young singles of the previous period, and given that the population of young singles is $s_{1,t}^m = s_{1,t}^w = (1+n)^t$, we have

$$s_{2,t}^j = s_{1,t-1}^j (1 - \Gamma^j) = (1+n)^{t-1} (1 - \Gamma^j), \quad (26)$$

Therefore, the total number of singles is

$$S_t^j = s_{1,t}^j + s_{2,t}^j = (1+n)^{t-1} (2+n - \Gamma^j) \quad (27)$$

The age distributions of singles—that is the fraction of young single people of gender $j \in \{m, w\}$, is

$$p_1^j = \frac{s_{1,t}^j}{S_t^j} = \frac{(1+n)}{2+n - \Gamma^j}, \quad (28)$$

Finally, the sex ratio is

$$\phi = \frac{S_t^m}{S_t^w} = \frac{2+n - \Gamma^m}{2+n - \Gamma^w}. \quad (29)$$

Note that in the steady state the sex ratio and the fractions of young single people are constant. Moreover, the fractions of young singles depend positively on the population growth rate.

2.6 Fertility and population growth

To obtain the rate of population growth, recall that a woman that marries at age 1 is capable of having k pairs of twins, and that a woman who does so at age 2 is able to bear only one pair of twins. Given those assumptions, the number of young men and women who enter the market each period is

$$s_{1,t}^m = s_{1,t}^w = k s_{1,t-1}^w \Gamma^w + \min\{k, 1\} s_{2,t-1}^w. \quad (30)$$

Given that $s_{1,t}^w = (1+n)^t$, substituting (26) in (30), and rearranging we have

$$(1+n)^2 = k(1+n)\Gamma^w + \min\{k, 1\}(1 - \Gamma^w). \quad (31)$$

Solving the quadratic equation (31) and taking the positive root, the rate of population growth in this economy is

$$n = \frac{k\Gamma^w}{2} - 1 + \frac{\sqrt{(k\Gamma^w)^2 + 4\min\{k, 1\}(1 - \Gamma^w)}}{2}. \quad (32)$$

Notice that women's fecundity is normalized to obtain $n = 0$ when $k = 1$. The population growth rate reaches its minimum when $k = 0.5$ and $\Gamma^w = 1$ ($n = -0.5$), and its maximum when $k = \bar{k}$ and $\Gamma^w = 1$ ($n = \bar{k} - 1$).

2.7 Marriage market clearing

Since men and women marry in pairs, the total number of brides must be equal to the total number of grooms in each type of marriage. Therefore, for men of age a and women of age b , for $a, b \in \{1, 2\}$, we have

$$s_{a,t}^m \gamma_{a,b}^m = s_{b,t}^w \gamma_{b,a}^w. \quad (33)$$

People marrying within their cohort. The condition above is trivially satisfied when $a = b = 1$, since $s_{1,t}^m = s_{1,t}^w = (1+n)^t$ and, by (10), $\gamma_{1,1}^m = \gamma_{1,1}^w$.

People marrying across cohorts

1. Young men with old women

$$\begin{aligned} s_{1,t}^m \gamma_{1,2}^m &= s_{2,t}^w \gamma_{2,1}^w & (34) \\ s_{1,t}^m \lambda_2^m [1 - G^w(R_{1,2}^m)] &= s_{2,t}^w \lambda_1^w p_1^m [1 - G^w(R_{1,2}^m)] \\ s_{1,t}^m \frac{\eta_t}{S_t^m} \frac{s_{2,t}^w}{S_t^w} &= s_{2,t}^w \frac{\eta_t}{S_t^w} \frac{s_{1,t}^m}{S_t^m}. \end{aligned}$$

2. Old men with young women.

$$\begin{aligned} s_{2,t}^m \gamma_{2,1}^m &= s_{1,t}^w \gamma_{1,2}^w & (35) \\ s_{2,t}^m \lambda_1^m p_1^w [1 - G^m(R_{1,2}^w)] &= s_{1,t}^w \lambda_2^w p_1^m [1 - G^m(R_{1,2}^w)] \\ s_{2,t}^m \frac{\eta_t}{S_t^m} \frac{s_{1,t}^w}{S_t^w} &= s_{1,t}^w \frac{\eta_t}{S_t^w} \frac{s_{2,t}^m}{S_t^m}. \end{aligned}$$

2.8 Equilibrium definition

Given a desired number of children, k , a steady state equilibrium for this economy is a vector of reservation values for young single men, $R_1^m = (R_{1,1}^m, R_{1,2}^m)$, a vector of reservation values for young single women, $R_1^w = (R_{1,1}^w, R_{1,2}^w)$, a probability of marriage for young single men, $\Gamma^m = \gamma_{1,1}^m + \gamma_{1,2}^m$, a probability of marriage for young single women, $\Gamma^w = \gamma_{1,1}^w + \gamma_{1,2}^w$, and a population growth rate n , such that:

- (i) Given R_1^w , Γ^m , Γ^w and n , R_1^m solves the young single men's decision problem described in expression (15), and is determined by equations (21) and (20).
- (ii) Given R_1^m , Γ^m , Γ^w and n , R_1^w solves the young single women's decision problem described in expression (15), and is determined by equations (24) and (25).

- (iii) The young men's probability of marriage, Γ^m , determined by (10) and (11), is consistent with the reservation values chosen by both young single men and young single women, the probability of marriage of young single women and the population growth rate.
- (iv) The young women's probability of marriage, Γ^w , determined by (10) and (12), is consistent with the reservation values chosen by both young single men and young single women, the probability of marriage of young single women and the population growth rate.
- (v) The population growth rate, n , determined by (32), is consistent with the reservation values and marriage probabilities of single young men and women.¹⁴
- (vi) Marriage market clears, by (34) and (35).

2.9 Equilibrium: existence

Here I show the existence of a steady state equilibrium for this economy. Notice that, by expression (21), $R_{1,1}^m$ is uniquely determined by $R_{1,2}^m$. In addition, the reservation values of women are completely determined by k and μ_y (equations (24) and (25)). Therefore, I have to show the existence of an interior solution of a system of four equations in four unknowns: $R_{1,2}^m$, Γ^m , Γ^w and n .

Theorem 1 (Existence) *A steady state equilibrium exists for this economy.*

Proof. To prove existence I use Brouwer's Fixed Point Theorem. See Appendix A.1 ■

The following result rules out the existence of corner solutions and establishes that in equilibrium marriages across cohorts occur with positive probability.

Corollary 2 *The equilibrium is interior,*

$$\begin{aligned} R_{1,1}^m, R_{1,2}^m &\in (0, \bar{x}), \\ \Gamma^m &\in (0, 1), \\ \Gamma^w &\in (0, 1), \end{aligned}$$

¹⁴Notice that, strictly speaking, the reservation values for old single men, $R_2^m = (R_{2,1}^m, R_{2,2}^m)$, and old single women, $R_2^w = (R_{2,1}^w, R_{2,2}^w)$, are also part of the steady state equilibrium of this economy. However, as shown in Section 2.4, they are trivially zero, and I have omitted them from the definition of the equilibrium for the sake of simplicity.

and

$$n \in (-0.5, \bar{k} - 1).$$

Furthermore, in equilibrium, all types of marriages occur with positive probability,

$$\begin{aligned} \gamma_{1,1}^m &= \gamma_{1,1}^w > 0 \\ \gamma_{1,2}^m, \gamma_{2,1}^w &> 0 \\ \gamma_{1,2}^w, \gamma_{2,1}^m &> 0. \end{aligned}$$

Proof. See Appendix A.2 ■

2.10 Equilibrium: symmetric case

Consider the case where people desire to have $k \in [0.5, 1]$ children.¹⁵ Consequently, differential fecundity plays no role because the decision problems faced by single men and single women are identical. Moreover, in this case the reservation values of single men and single women are identical, and so are the numbers of single men and single women in the economy. Notice that in this case the reservation values of men and women are independent of their desired fertility.

Assumption 1 $G^m = G^w = G$ with mean μ and density g .

Assumption 2 The probability density function g is log-concave.

Under Assumption 1, by (20), (21), (24) and (25) we trivially have

$$R_{1,1}^m = R_{1,1}^w = \frac{\mu}{2}, \tag{36}$$

and

$$R_{1,2}^m = R_{1,2}^w = \mu. \tag{37}$$

Proposition 3 Under Assumption 1, if people only want to have a number of children that is biologically feasible for old women, $k \in [\frac{1}{2}, 1]$, there is always a steady state equilibrium where men and women marry at age 1 with equal probability, $\Gamma^m = \Gamma^w = \Gamma^s$, and where population is not increasing over time, $n \leq 0$. This implies

¹⁵Note that the results depend exclusively on symmetry and not on the negative population growth rates implied by the assumption of $k \leq 1$. This assumption is made merely for expositional reasons and will be relaxed in the next section.

that the number of single men is equal to the number of single women, $S_t^m = S_t^w$, and therefore the sex ratio is equal to one, $\phi = 1$. Furthermore, the economy is mostly populated by old agents, $q \leq \frac{1}{2}$.

Proof. See Appendix B.1 ■

Proposition 4 Under Assumptions 1 and 2, there is a unique steady state equilibrium for $k \in [\frac{1}{2}, 1]$.

Proof. See Appendix B.2 ■

Given that men and women face identical problems the existence of a unique equilibrium where men and women marry at the same age seems trivial. However, the interesting feature of the symmetric case comes from comparative statics. As we show below, when men and women are identical, an increase in the desired number of children leads to men and women marrying earlier as population growth increases, despite the constant reservation rules derived above.

Proposition 5 Under Assumptions 1 and 2, if $k \in [\frac{1}{2}, 1)$, given an increase in the desired number of children:

1. Population growth rate increases: $\frac{\partial n}{\partial k} > 0$. This implies that the fraction of young people in the economy increases: $\frac{q}{\partial k} > 0$.
2. Men and women marry at age 1 with higher probability: $\frac{\partial \Gamma^s}{\partial k} > 0$.

Proof. See Appendix B.3 ■

The result above comes from the interaction between young people's preferences for marrying within their cohort and the effect of desired fertility on the age structure of the population. When k increases, population growth increases and therefore the age structure of the economy is affected, with a larger share of young people

$$q = \frac{1+n}{2+n}.$$

Therefore, there is a higher probability that young people meet people of the same cohort. Since young agents obtain higher utility from marrying other young people (and therefore set lower reservation values for them), they tend to marry younger as the structure of the population is also becoming younger.

2.11 Equilibrium: differential fecundity

Now consider the case where people desire to have $k \in (1, \bar{k}]$ children. If $k > 1$, old women are not able to bear their desired number of children. As shown in Section 2.4.1, if men wait until age 2 they still can marry a young woman and have k children. Recall from (8) that the expected utility that a man obtains from marrying at age 2 is $U_2^m = \gamma_{2,1}^m k \mu + (1 - \gamma_{2,1}^m) \mu$. On the other hand, if women wait until age 2, they will get the same utility regardless of whom they marry, $U_2^w = \mu$, (9). Since in equilibrium $\gamma_{2,1}^m > 0$ (from Theorem 1), this implies that differential fecundity increases the value of waiting for men. Therefore, men become relatively choosier than women.

Proposition 6 *Under Assumption 1, if people want to have more children than old women are able to bear $k \in (1, \bar{k}]$, there is always a steady state equilibrium where young men are choosier than young women, that is $R_{1,1}^m > R_{1,1}^w$ and $R_{1,2}^m > R_{1,2}^w$.*

Proof. See Appendix C.1 ■

From simple observation of (24) and (25) it is clear that women's reservation values both for young and old men are decreasing in k . From Proposition 6 we know that if differential fecundity plays a role, men are choosier than women. The previous result does not imply that as desired fertility increases men's reservation values will necessarily increase. In fact, as I show below, if desired fertility increases men also reduce their reservation values for young women, but to a lesser extent than women do. However, young men will increase their reservation values for older women, which is the main driving force behind the difference in the timing of marriage.

Proposition 7 *Under Assumptions 1 and 2, then in the neighborhood of the unique steady state equilibrium where $k = 1$, given a small increase in the desired number of children:*

1. *Young men become choosier with old women and less choosy with young women: $\frac{\partial R_{1,2}^m}{\partial k} > 0$ and $\frac{\partial R_{1,1}^m}{\partial k} < 0$. In the latter case, the decrease in the reservation value is lower than the decrease in the young women's reservation value: $\left| \frac{\partial R_{1,1}^m}{\partial k} \right| < \left| \frac{\partial R_{1,1}^w}{\partial k} \right|$.*
2. *Women marry young with higher probability: $\frac{\partial \Gamma^w}{\partial k} > 0$. In addition, the difference between the probability of marriage at age 1 of women and men, $\Gamma^w - \Gamma^m$, is positive and increasing in k .*

3. *The ratio of single men to single women, ϕ , is greater than 1 and increasing in k .*
4. *Similarly to Proposition 5, population growth increases, therefore the fraction of young people in the economy, q increases.*

Proof. See Appendix C.2 ■

Proposition 7 establishes that when people desire to have more children than a woman who marries at age 2 is able to bear, differential fecundity plays a role and single men become choosier than single women. Consequently, single women tend to marry younger than men, and the age gap is an increasing function of the desired number of children. Moreover, it establishes that, as the desired number of children increases, single men increasingly outnumber single women.

The mechanism behind Proposition 7 is the following. If the desired number of offspring is feasible for women that marry at age 2, the biological differences between men and women play no role in the marriage market. However, biological gender asymmetries do appear if society want to have more children than women of age 2 are able to bear ($k > 1$). Therefore, young men are less willing than young women to accept an older spouse and the value of waiting until age 2 increases for men relative to women. Notice that men also decrease their reservation values for young women (albeit in a lesser extent) because if they wait they may not be able to marry a young woman when old.

The increase in the sex ratio is the necessary counterpart of the increase in the age gap. As young women are more likely to marry old men, in equilibrium more young men have to wait until period 2 in order to marry. Therefore, it must be the case that single men outnumber single women. This result has implications in light of the empirical literature on marriage timing. It is a common practice in this literature to use the single sex ratio as a measure of relative availability of men and women, ignoring the endogeneity problem caused by the relationship between the sex ratio and the age gap.¹⁶

Notice that while an increase in k makes women unambiguously marry younger, it has ambiguous effects on men's timing of marriage, subject to two countervailing effects. While differential fecundity induces men to marry older as k increases, the decrease of frictions due to changes in the demographic structure makes them marry younger (Proposition 5). These two effects will be analyzed in more detail in the next section when we use the model to study marriage dynamics in the U.S.

¹⁶See, for example, Fitzgerald (1991), McLaughlin et al. (1993), and Parrado and Centeno (2002).

3 A generalized model

In this section I extend the model developed in Section 2 to a full life cycle model with a finite number of yearly periods. The objective of this extension is to study the empirical implications of the model developed above, specifically the dynamics of marriage behavior in the U.S. after the Baby Boom.

Since in this stylized model the fertility choice process is taken as exogenous, we can consider the boom as a multi-period temporary shock in some parameter of a (not modeled) fertility choice process, subsumed in the variable that represents desired fertility, k_t . Recall that main focus of interest in of this paper is not the Baby Boom itself, but mainly the marriage behavior of the generations born during and after the Baby Boom, affected by the demographic changes caused by this large temporary increase in fertility rates.

In this section, I study a model economy populated by a continuum of men and a continuum of women. Men and women live for at most J years, which we denote with subindex $j = 15, 16, \dots, J$. Men and women in this model economy differ in their fecundity, and in their longevity. And they derive utility from being married and having children. From now on I relax the assumption of the constant population growth rate maintained thorough Section 2.

3.1 Mortality

In our model economy each period every person faces an exogenous probability of dying until the following period, which is gender, age and time dependent. We denote these probabilities by $\delta_{a,t}^i$, for $a \in \{1, 2, \dots, J\}$ and $i \in \{m, w\}$,¹⁷ with $\delta_{J,t}^m = \delta_{J,t}^w = 1$ for any period $t = 1, 2, \dots$. These assumptions also imply that the probability that a person of gender i who is a years old in period t survives until age b is

$$\Psi^i(a, b, t) = \prod_{j=a}^{b-1} \left(1 - \delta_{,j,t+j-a}^i\right) \quad (38)$$

3.2 Matching technology

Costly double sided search for spouses is a distinguishing feature of our model economies. The probabilities of being matched depend on an exogenous parameter that measures the search frictions, and on the ratio of available singles. Let $s_{j,t}^i$

¹⁷When necessary, I will also use the traditional numerical gender notation used by the U.S. Census: $i \in \{1, 2\}$, where 1 represents men and 2 represent women.

denote the number of j year-old singles of gender i in period t and let $S_t^i = \sum_{j=15}^J s_{j,t}^i$ denote the total number of singles of gender i . Therefore, the probability that a single man meets a single b year-old woman is

$$\lambda_{b,t}^m = \rho \left[\min \left(\frac{S_t^w}{S_t^m}, 1 \right) \right] \frac{s_{b,t}^w}{S_t^w}, \quad (39)$$

where parameter $0 < \rho \leq 1$ measures the search frictions.¹⁸

In this model economy, the decision problems of single men and women are exactly identical. For notational convenience, I describe the problem and variables that pertain to single men only. To obtain the corresponding variables for single women simply substitute the m 's with w 's.

3.3 Fecundity

The expected number of children depends on an exogenous, time varying parameter of desired fertility k_t and biological constraints based on men's and women's age. (determined at the time of marriage). Even though this is not an easy matter, I will make the following assumptions, which are roughly consistent with the literature on the subject.^{19,20} Define as $c_t^m(a, b)$ the number of children that a man of age a expects to have if he marries a woman of age b at time t . Then,

$$c_t^m(a, b) = \begin{cases} k_t & \text{if } a \leq 55 \text{ and } b \leq 30 \\ \min [k_t, 3] & \text{if } a \leq 55 \text{ and } 30 < b \leq 35 \\ \min [k_t, 2.5] & \text{if } a \leq 55 \text{ and } 35 < b \leq 38 \\ \min [k_t, 2] & \text{if } a \leq 55 \text{ and } 38 < b \leq 40 \\ \min [k_t, 1] & \text{if } a \leq 55 \text{ and } 41 < b \leq 44 \\ \min [k_t, 0.5] & \text{if } a \leq 55 \text{ and } 44 < b \leq 49 \\ 0 & \text{if } a > 55 \text{ or } b > 49. \end{cases} \quad (40)$$

¹⁸Notice that this matching function is different from the one in expression (3). While the model of Section 2 required differentiability in the matching function, keeping the same function would add one more free parameter to the calibration exercise. This change does not affect the results at all.

¹⁹According to Wood and Weinstein (1988), women's probability of any conception changes rapidly after age 40, as a result of changes in the ovarian function. Therefore, any reduction in fecundity between ages 25 and 40 is attributable to an elevation in intra-uterine mortality. However, even accounting for that, the pattern of effective fecundability remains fairly flat between ages 20 and 35.

²⁰According to Hassan and Killick (2003), the effect of men's age on fecundity remains uncertain. They study male infecundity by comparing the time to pregnancy for men of different age groups. They found that male aging leads to a significant increase in the time to pregnancy, especially after ages 45 to 50.

In a similar way $c_t^w(a, b)$ is defined.²¹ Therefore, I assume that men are fecund until age 55 and women married after age 30 do not have enough time to have more than three children.²²

3.4 Utility of marriage

The period utility of marriage that a single man a obtains from marrying someone of age b in period t with a given marriage quality is $zc_t^m(a, b)$, where z is an independent and identically distributed realization from $G(z)$, the same for men and women (Assumption 1). This utility is determined at the time of marriage and remains constant until one of the spouses dies, so the surviving spouse receives zero utility for the rest his/her life. Since life expectancy is stochastic, the expected value of this marriage is

$$u_t^m(a, b, z) = zc_t^m(a, b) \sum_{j=0}^D \beta^j \Psi^m(a, a+j, t+j) \Psi^w(b, b+j, t+j). \quad (41)$$

where $D = \min\{J - a - 14, J - b - 14\}$, and $\beta \in (0, 1)$ is the discount factor.

I will also assume that never married people receives a constant endowment of A per period.²³

3.5 The decision problem of singles

3.5.1 Probability of marriage

The probability that an a year-old bachelor marries a b year-old single woman is

$$\gamma_t^m(a, b) = \lambda_{b,t}^m \{1 - G[R_t^m(a, b)]\} \{1 - G[R_t^w(b, a)]\}, \quad (42)$$

where $R_t^m(a, b)$ denotes the reservation value that a single man who is a years-old in period t set for b year-old single women, and $R_t^w(a, b)$ is the reservation value that b year-old single women set for a year-old bachelors.

²¹Here "2.5 children" should be interpreted as "two children and a 50% probability of a third child".

²²This restriction will be binding only during the peak of the Baby Boom.

²³In the two-period model of Section 2, this endowment was normalized to zero for simplicity.

Consequently, the probability that a single a year-old bachelor marries in period t is,

$$\begin{aligned}\Gamma_{a,t}^m &= \sum_{b=15}^J \lambda_{b,t}^m \{1 - G[R_t^m(a, b)]\} \{1 - G[R_t^w(b, a)]\} \\ &= \sum_{b=15}^J \gamma_t^m(a, b),\end{aligned}\quad (43)$$

which naturally depends on the reservation values of both men and women.

3.5.2 Expected value of marrying and remaining single in period t

The value that an a year-old single of gender $i \in \{m, w\}$ expects to obtain from marrying in the current period t before any matches have taken place is

$$U_{a,t}^i = \sum_{b=15}^J \gamma_t^i(a, b) E u_t^i[a, b, z \mid z \geq R_t^i(a, b)] \quad (44)$$

Now we are able to calculate the expected value that an a year-old single attaches to his/her marital status at the beginning of period t . That is,

$$V_{a,t}^i = U_{a,t}^i + \sum_{\ell=1}^{J-a-14} E_t \left\{ \prod_{j=a}^{a+\ell-1} (1 - \Gamma_{j,t+j-a}^i) (1 - \delta_{j,t+j-a}^i) \beta^\ell [A + U_{a+\ell,t+\ell}^i] \right\}, \quad (45)$$

or equivalently,

$$V_{a,t}^i = U_{a,t}^i + (1 - \Gamma_{a,t}^i) \left\{ A + (1 - \delta_{a,t}^i) \beta E [V_{a+1,t+1}^i/t] \right\} \quad (46)$$

3.5.3 Reservation values

The optimal reservation values that a year-old men and b year-old women set for each other in each period can be found solving the system of $2J^2 \times NT$ equations in $2J^2 \times NT$ unknowns that result from equating the expressions (41) and (46) for the men and the corresponding J^2 women's equations for periods $t = 1, 2, 3 \dots NT$. Here NT is the period in which all the variables in the economy reach their steady state value. Formally, the $\{R_t^i(a, b)\}$ are the values of z that solve

$$u_t^m(a, b, R_t^m(a, b)) = \left\{ A + (1 - \delta_{a,t}^m) \beta E [V_{a+1,t+1}^m/t] \right\} \quad (47)$$

and

$$u_t^w(a, b, R_t^w(a, b)) = \left\{ A + (1 - \delta_{a,t}^w) \beta E [V_{a+1,t+1}^w/t] \right\}. \quad (48)$$

3.6 Fertility and population dynamics

The number of children born in any period t is a function of desired fertility at that point in time and of the number of women married at different ages. Recall that, independently of the desired number of children, the age distribution of newlyweds is crucial because biological constraints of women in their thirties and forties may become binding, especially in times when fertility is high. Therefore,

$$N_{0,t}^i = \varpi_t^i \left\{ \sum_{a=1}^J s_{a,t}^w \sum_{b=15}^J \gamma_t^w(a,b) c_t^w(a,b) \right\}, \quad (49)$$

where ϖ_t^i is the fraction of children of gender $i \in \{m, w\}$ born in period t .²⁴

Every period, a number of 15 year-old single men and women, $s_{15,t}^m = N_{15,t}^m$ and $s_{15,t}^w = N_{15,t}^w$, enter the market. Since now agents live 15 years as children, the population entering the market at period t is born at period $t - 15$. That is,

$$N_{15,t}^i = \varpi_{t-15}^i \Psi^i(0, 15, t - 15) N_{0,t-15}^i, \quad (50)$$

where $\Psi_{t-15}^i(0, 15)$ is their survival probability up to age 15. Similarly, the stock of agents of older ages $\{N_{j,t}^i\}$ is given by

$$N_{a,t}^i = \left(1 - \delta_{a-1,t-1}^i\right) N_{a-1,t-1}^i \text{ for } 15 < a \leq J. \quad (51)$$

3.7 Equilibrium

Given a vector of desired number of children over time, k_t , an equilibrium for this economy is a vector of measures of 15 year-old men and women, $\{N_{15,t}^i\}$, a matrix of measures of single people, $\{s_{a,t}^i\}$ for gender $i \in \{m, w\}$, and matrices of reservation values that single males set for single females and vice versa, $\{R_t^m(a, b), R_t^w(a, b)\}$, for ages $a, b \in \{15, 16, \dots, J\}$ and period $t \in \{1, \dots\}$ such that,

- (i) Given $R_t^i(a, b)$ and $s_{a,t}^i$, measures $\{N_{15,t}^i\}$ satisfy expressions (49) and (50) and are consistent with the reservation values and the measures of singles.

²⁴For simplicity's sake, Equation (49) implies that all children are born in the first year of marriage. Given that this assumption may affect results significantly in periods of rapidly changing fertility, when solving the problem I actually restricted women to having one child per period in up to four consecutive periods.

(ii) Given $R_t^i(a, b)$ and $N_{15,t}^i$, measures $\{s_{a,t}^i\}$ satisfy

$$s_{15,t}^i = N_{15,t}^i \text{ and} \tag{52}$$

$$s_{a+j,t+j}^i = s_{a,t}^i \left(1 - \delta_{a+j-1,t+j-1}^i\right) \left(1 - \Gamma_{a+j-1,t+j-1}^i\right) \text{ for } 15 < j \leq (J - a - 14),$$

where $\Gamma_{a,t}^i$ is defined in expression (43), and is also consistent with the reservation values and the measures of 15 year-old people.

(iii) Given $N_{15,t}^i$ and $s_{j,t}^i$, the reservation values $\{R_t^m(a, b), R_t^w(a, b)\}$ solve the decision problems of singles described in expressions (47) and (48) and are consistent with the measures of singles and 15 year-old people.

(iv) Marriage market clears every period t , that is,

$$s_{a,t}^m \gamma_t^m(a, b) = s_{b,t}^w \gamma_t^w(b, a), \tag{53}$$

where $\gamma_t^m(a, b)$ is defined in expression (42).

3.8 Expectations formation

As stated above, in this model the Baby Boom is considered as a multi-period temporary shock in desired fertility k_t . In the meantime, several cohorts of agents have to make their marriage decisions under uncertainty about the nature and extension of the shock. Therefore, there are two issues that have to be resolved. The first one is to determine the mechanism through which the agents adjust their expectations as they receive new information about the shock (i.e. the observed value of k_t). The second issue is to specify the way the agents predict their future environment, assuming they have no information about how the shock will evolve in the future.

To answer the first question, I assume that the learning process can be represented by a weakly increasing function of time $\pi(t) \in [0, 1]$, which determines the transition from zero information to perfect foresight. Therefore, the last term of equation (46), can be expressed as

$$E[V_{a+j,t+j}^i/t] = \pi(t) [V_{a+j,t+j}^i] + (1 - \pi(t)) E[V_{a+j,t+j}^i/k_{t+j} = k_t], \tag{54}$$

where and $j \in \{1, 2, \dots, (J - a - 14)\}$. I assume further that the learning function takes the following form,

$$\pi(t) = \begin{cases} 0 & \text{if } t < X \\ \exp \left[\frac{t-T}{t-X+1} \right] & \text{if } X \leq t \leq T \\ 1 & \text{if } t > T, \end{cases} \quad (55)$$

where X and T represent the start and the end of the learning process, respectively, and $\xi > 0$ is a parameter. Notice that the first term of expression (54), together with (55), implies that the agents have perfect foresight from period T onwards.

Now suppose that the agents have no information about the shock's characteristics. However, they are able to use several available pieces of information in order to predict their future environment. For example, they observe the conditions faced by older people in the current period. In addition, they also know the future demographic composition of the marriage market with extreme accuracy for the following fifteen years, just by observing the current population of children aged 0-14. The term $E \left[V_{a+j,t+j}^i / k_{t+j} = k_t \right]$ in expression (54) represents the agent's expectations about the future assuming all the exogenous variables in the economy will remain constant in the future, that is, $k_{t+j} = k_t$, $\delta_{a,t+j}^i = \delta_{a,t}^i$ and $\varpi_{t+j}^i = \varpi_t^i$.²⁵ Therefore, in order to calculate those expected values it is necessary to solve for the transitional dynamics of the model for all combinations $\{a, t\}$, with $a \in \{1, \dots, J-1\}$ and $t \in \{1, \dots, T-1\}$.

4 Calibration

To calibrate this model I proceed in two steps. First, I assume the economy to be in its steady state and calibrate the model parameters to the demographics of the initial period. Then, to analyze the model dynamics, I introduce a vector of desired fertility values constructed using U.S. time series data and assume the variable become constant in some period T .²⁶ Therefore, to calibrate this model economy I have to choose the duration of the model period, an initial period, a functional form for the distribution of match values, $G(z)$, and a value for every parameter in the model economy. These parameters are the maximum life-time, J ; the fecundity profiles; the mortality probabilities, $\delta_{j,t}^i$; the fraction of people born of gender i , ϖ_t^i ; the search friction parameter ρ ; the beginning and the end of the learning process, X

²⁵The assumptions about mortality and the sex ratio at birth are for consistency, because their quantitative impact is negligible.

²⁶Notice that period T , where people's fertility preferences become constant, is the same period chosen for the end of the learning process in expression (55).

and T ; the learning parameter, ξ ; the discount factor, β ; and the parameters that characterize the payoffs, the desired number of children per period, k_t , and the period utility of remaining single, A .

The model period. I assume the period in the model is yearly. The model starts in 1930 but the first period where young men and women entering the market are actually an outcome by the model is 1945. For the interval between 1930 and 1945 the initial stocks of men and women are calculated using fertility and mortality data.

The time discount factor. I choose $\beta = 0.96$, as is standard in the literature.

The distribution of the match values. I assume that the distribution of match values, $G(z)$, is the logarithm of a normally distributed function with mean 0 and standard deviation σ .

The maximum life-time. I assume that $J = 75$.

The mortality probabilities. The mortality probabilities are death rates taken from the Human Mortality Database for the years 1933-2006.^{27,28}

The fraction of people born by gender. I use the sex ratios at birth for 1940-2002 from Mathews and Hamilton (2005).

The fecundity profiles. The assumptions about fecundity profiles are detailed in expression (40). These values are chosen so that the fecundity profiles in the model economy are roughly consistent with the findings of the literature on the subject.²⁹

Steady state free parameters (1930). To complete the calibration of the baseline model economy we are left with three free parameters for the steady state, a vector of the number of desired children per period, k_t , and the parameters of the learning function $\pi(t)$, defined in (55). I will proceed in two steps. First, to obtain an initial state for this economy, I have to choose the numerical values of the standard deviation of the distribution of match values, σ ; the matching function parameter,

²⁷The Human Mortality Database is compiled by the University of California, Berkeley (USA) and the Max Planck Institute for Demographic Research (Germany). This dataset is available at www.mortality.org.

²⁸I use the 2006 mortality rates for 2007 onwards.

²⁹For example Hassan and Killick (2003) and Wood and Weinstein (1988), discussed above.

Table 1: The Initial (steady state) Economy and the 1930 U.S. Census.

| | <i>1930 Census</i> | <i>Model</i> |
|-----------------------------------------------|--------------------|--------------|
| Median age at first marriage for men | 24.27 | 24.33 |
| Median age at first marriage for women | 21.41 | 22.57 |
| Mean Age difference at first marriage (years) | 2.86 | 2.81 |
| Fraction ever married men * | 0.66 | 0.66 |
| Fraction ever married women * | 0.73 | 0.72 |
| Sex ratio for never married people | 1.32 | 1.23 |
| 15-29 / 15-44 year-old Ratio * | 0.54 | 0.54 |

*Calibration targets.

ρ and the period utility of remaining single, A , using three targets from the 1930 U.S. census. These targets are the fraction of ever married men, the fraction of ever married women and 15-29 / 15-44 year-old ratio. For this initial state, I choose $k_1 = 3.1$, the average total fertility rate during the period 1920-29. Therefore, I choose the values of σ , ρ , and A which minimize the sum of the squared differences between outcomes in the model economy and our United States targets. The values of the parameters that deliver this result are $\sigma = 0.495$, $\rho = 0.38$, and $A = 1.31$. Observe in Table 1, some indicators that were not included as model targets, as the median age at first marriage for men and women, the mean of age differences and the sex ratio for never married people are very close to the U.S. data.^{30,31} One exception is the women's median age at first marriage, which is around one year higher than in the data. This difference will remain when we compare the model dynamics with the time series.

The desired number of children. Once the parameters for the steady state are obtained, the next step is to construct the vector of desired number of children per period, k_t in order to analyze the dynamic of agents' behavior. As stated before, I use the Total Fertility Rates (TFR) for the period 1940- 2000, and for crude birth rates from 1920-1939 all provided by the NCHS.³² Specifically I run the following

³⁰The U.S. Census Bureau has published the estimated median age at first marriage annually since 1947, using an indirect method based on the proportion of people who were ever married within the single age cohort. This methodology prevents the age of marriage from being affected mechanically by population structure. Since the mean age of marriage is conditional on marriage, this measure is subject to those mechanical effects. However, I use the mean of the age at first marriage in order to calculate the age gap.

³¹Other indicators are obtained for several years of the U.S. Census, available through IPUMS.

³²The total fertility rate computed by the National Center of Health Statistics is the sum of the birth rates of mothers in 5-year age groups multiplied by five. The birth rates are the num-

OLS regression, for the period 1930-2000,

$$TFR_t = \mu_0 + \sum_{i=1}^6 \mu_i(t)^i + \varepsilon_t, \quad (56)$$

where μ 's are the estimated parameters of the regression. Therefore, using the previously chosen parameters obtains the values of k_t . In addition, I assume that the economy eventually reaches a steady state where population growth is equal to zero. Therefore, from period $T = 1975$ onwards, I impose the value of k which generates an equilibrium steady state replacement rate, $k_T = 2.36$.³³ Since the fertility rate in the model depends on marriage probabilities, I replace the value μ_0 in equation (56) with α_0 , which becomes a fourth free parameter chosen to minimize the sum square differences between the TFR in the data and those produced by the model. The values of k_t over time are shown in Figure 2 (a).

The learning parameters. I assume that the agents start learning about the temporary nature of the shock in period $X = 1958$ (just after TFR reaches its maximum) and that the learning process continue until $T = 1975$, when they become fully informed.

The value of parameter ξ in expression (55), determines the pace of the learning process. I choose $\xi = 0.8$, implying that the learning function has the shape shown in Panel (b) of Figure 2. With this choices, I am assuming that the learning process started very slowly between 1958 and 1961 (when TFR stays around the maximum level (see Figure 1 (a)), but making progress at faster rates until around 1968, to finally slow down from the late 1960's to the mid 1970's.³⁴

5 Findings

As stated above, the extended model in Section 3 was developed to study a potential cause-effect relationship from demographic changes to marital behavior. Specifically, the main empirical question is whether the demographic changes originated by the Baby Boom had any impact in the marriage timing of the generations born during and after the boom. Therefore, the starting point is the evolution of fertility

bers of live births per 1,000 women in a given age group. TFR data for 1940-2000 is available at www.cdc.gov/nchs/data/statab/t991x07.pdf. Data on birth rates for the period 1909-2000 at <http://www.cdc.gov/nchs/data/statab/t001x01.pdf>

³³To ensure that desired fertility converges smoothly to their steady state value, I adjust the values of k_t using the following formula: $k_t = k_T + (k_{1965} - k_T) e^{-0.35t}$, for $1965 \leq t \leq 1975$.

³⁴The results are robust to either a more concave (lower ξ , rapid learning at the beginning and then slowing down) or a more convex learning function between 1958 and 1975.

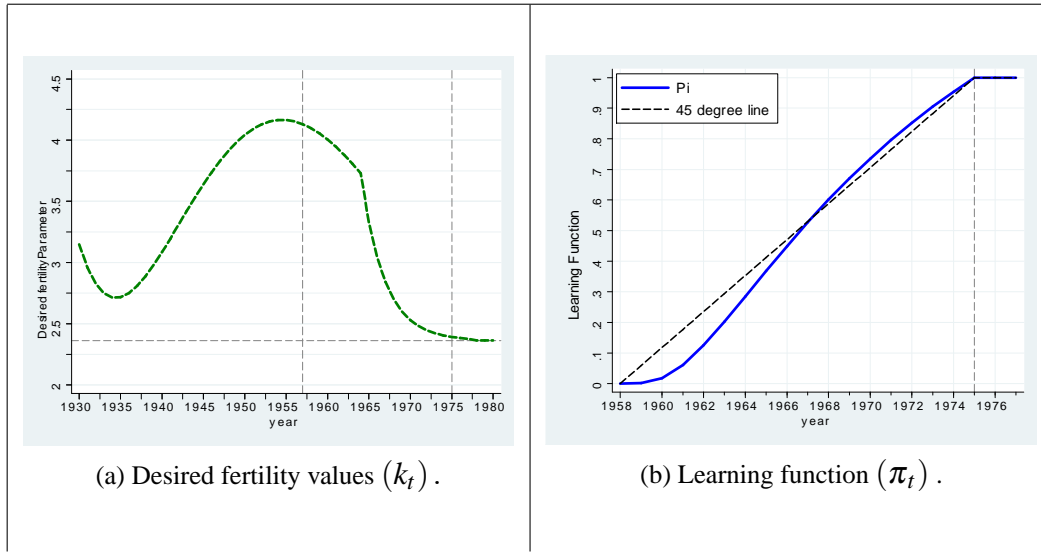


Figure 2: Desired fertility and Learning Parameters

in the model, which is shown in Panel (a) of Figure 3 and is very close to data. Two vertical dashed lines are shown in the figure: the first one shows the peak on the total fertility rate, in 1957, while the second is located in 1985, which will be the starting point of the analysis. I choose 1985 as a "conservative" starting point (remember that as of 1975 the fertility parameters are constant and agents have perfect foresight) to ensure that any change on variables is exclusively due to the transitional dynamics of the model.

A key force affecting model dynamics is the age structure of the population, which is mostly a function of past fertility.³⁵ As a measure of the age structure I choose the fraction of 15-29 over 15-44 years-old (from now on "the fraction of young people"), that is

$$q_t = \frac{\sum_{i=1}^2 \sum_{a=15}^{29} N_{a,t}^i}{\sum_{i=1}^2 \sum_{a=15}^{44} N_{a,t}^i}, \quad (57)$$

where in this case, $i \in \{1, 2\}$ represents gender.³⁶ Notice in Panel (b) of Figure 3 that this demographic measure reaches its maximum in 1975, when the first cohort of Baby Boomers, born starting in 1946, are about to turn 30 years old. Therefore, while the numerator of q_{1975} includes only Baby Boomers, the denominator

³⁵By comparing both panels of Figure 3, it does not appear that immigration has played an important role on the age structure of the population so far.

³⁶This choice was made assuming the "marriage market" includes people aged 15-44, and I divide this range in two "periods" to be consistent with the two-period model of Section 2. Any other measure of population structure would have delivered the same results.

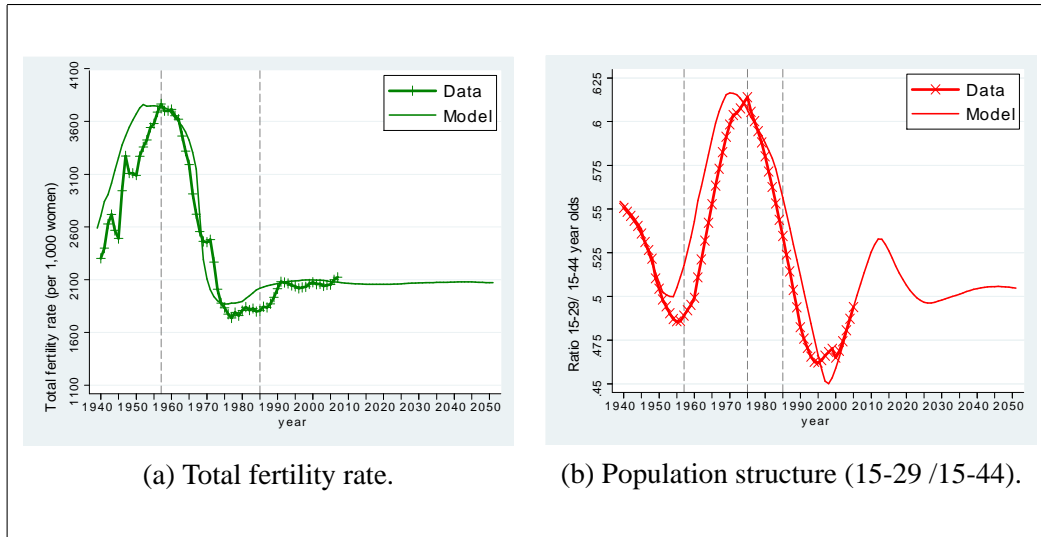


Figure 3: TFR and age structure of the population.

also include people from previous, smaller cohorts (i.e., those born in 1931-1945). Then, as the youngest singles entering the market belong to the smaller cohorts born after the Baby Boom (often called "Generation X"), q_t decreases until reaching a global minimum in 1995, just before the children of Baby Boomers, ("Generation Y"), start entering the market. This generation includes cohorts born after 1980, naturally larger than those from Generation X, which explains the structure of the population becoming "younger". Notice that q_t shows another peak in the model around 2015, when the cohorts born in the mid-1980's, children of those born during the peak of the Baby Boom (mid to late 1950's), will turn 30.³⁷

5.1 Median age at first marriage, age gap and sex ratio

Table 2 and Panels (a) and (b) of Figure 4 show the evolution of the median age at first marriage in the model and U.S. data, for men and women respectively. Panel (a) shows that men's age in the model closely follows the evolution of the data from 1940 to the early 1980's. Then, while in the data men's age continues increasing up to around 28.1 years in 2009, in our benchmark economy it increases at a slower pace. As shown in Column 1 of Table 2, men's median age at first marriage increased by 2.6 years between 1985 and 2009, while, during the same period the model shows an increase of 0.6 years, 23% of the variation in the data. Notice that

³⁷Data of the age structure of the population is taken from the Interim State Population Projections from the U.S. Census Bureau.

Table 2: Median age at first marriage: Benchmark and U.S. data

| <i>Year</i> | <i>Men</i> | | <i>Women</i> | |
|---------------------------|------------------|--------------|------------------|--------------|
| | (1) U.S. Data | (2) Model | (3) U.S. Data | (4) Model |
| <i>1957</i> | 22.6 | 23.3 | 20.3 | 21.5 |
| <i>1985</i> | 25.5 | 25.0 | 23.3 | 24.2 |
| <i>2009</i> | 28.1 | 25.6 | 25.9 | 24.1 |
| <i>S.S.</i> | | 26.1 | | 24.4 |
| Δ <i>1957-1985</i> | 2.9 | 1.7 | 3.0 | 2.7 |
| Δ <i>1985-2009</i> | 2.6 | 0.6 | 2.6 | -0.1 |
| Δ <i>2009-</i> | | 0.5 | | 0.3 |

men's age only reaches its steady state level around 2080 (with an additional increase of 0.5 years), so the transitional dynamics of the model accounts for a 1.1 years increase in men's median age of marriage since 1985. The comparison for women is shown in Panel (b) of Figure 4 and Column 2 of Table 2. Note that women in the model are more than one year older than in the data in 1930 (see Table 1 above), and that initial divergence makes the evolution of women's age at first marriage in the model follow the data less closely than in the case of men. Moreover, the increase in the period 1985-2009 is very small and even negative (-0.07), and only accounts for an increase of 0.3 years from 1985 to the steady state.

The last two panels of Figure 4 display comparisons of important variables that merit being shown despite their data limitations. The evolution of the average age gap at first marriage is shown in Panel (c) from 1940 to 1995, while Panel (d) displays the evolution of the single men/ single women ratio (only decennial data is available until 2000).^{38,39} Note that both model variables decrease along with the data from the late 1950's to the mid 1980's, to then recover in their transition to the new steady state level. At least between 1990 and 2000, decennial data on the sex ratio does not show signs of recovery.

³⁸In order to calculate the average age gap, we need data on actual marriages. Although highly correlated, the age gap is not a measure equivalent to the difference in median age at marriage (taken from the fraction of ever married population from either census data or the Current Population Survey.), which is a relative measure of marriage timing within a cohort.

³⁹Unfortunately, there is no consistent series on age at first marriage in order to calculate the age gap. I use census data for the period 1940-1970 (this question stopped being asked in 1980), and data from the Marriage Data Files for the period 1968-95. From 1995 to date, there is no individual information available about the age at first marriage.

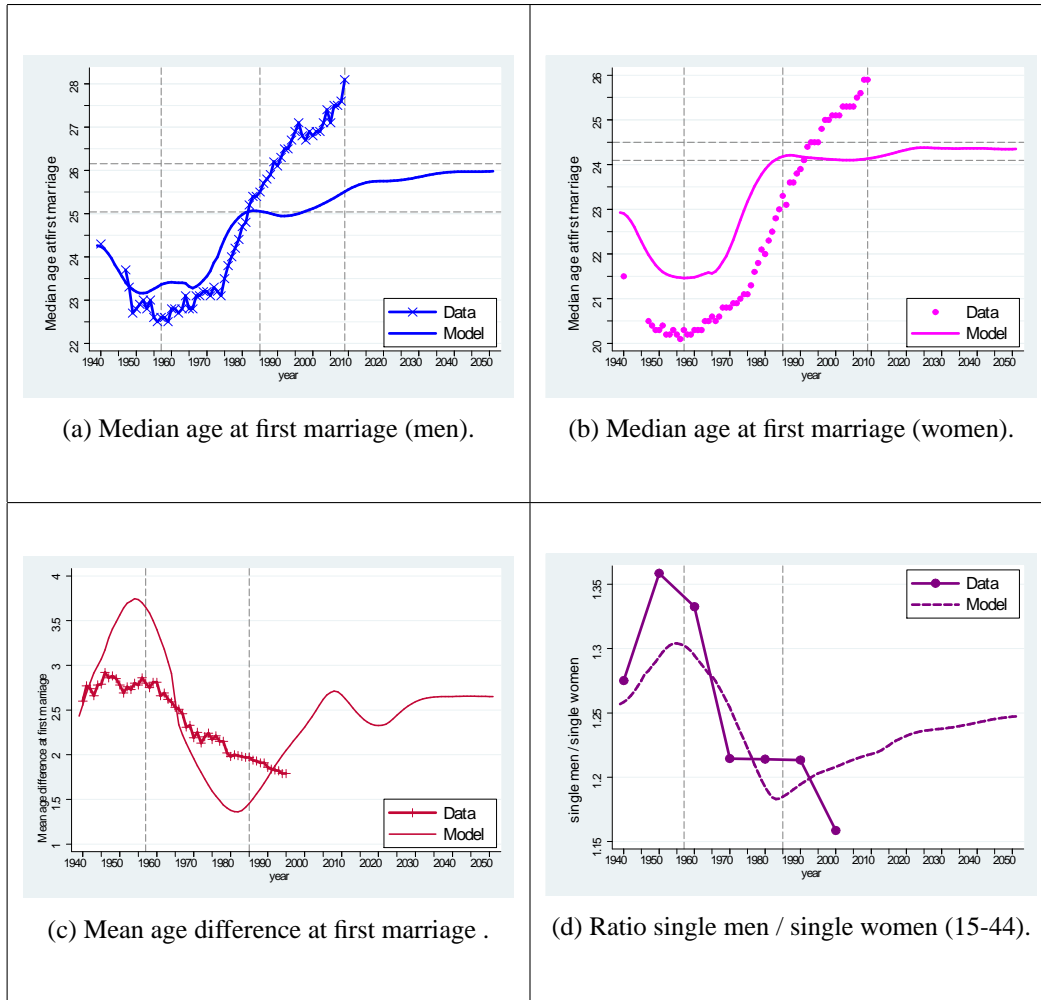


Figure 4: The Benchmark Economy and U.S. data.

5.2 The benchmark model economy

In this section I explore the benchmark economy in more detail, to get a better understanding of the previous findings. First of all, I would like to focus briefly on what happened during the Baby Boom. The first four Panels of Figure 5 are a clear example of Propositions 5 and 7: the population becomes younger (Panels (a) and (b)), men and women marry younger (Panel (c)) with the sex ratio and the age gap increasing (Panel (d)). However, different from the model in Section 2, here the changes in fertility and in the age structure of the population are long-lasting but only temporary.

Giolito: On Population Structure and Marriage Dynamics

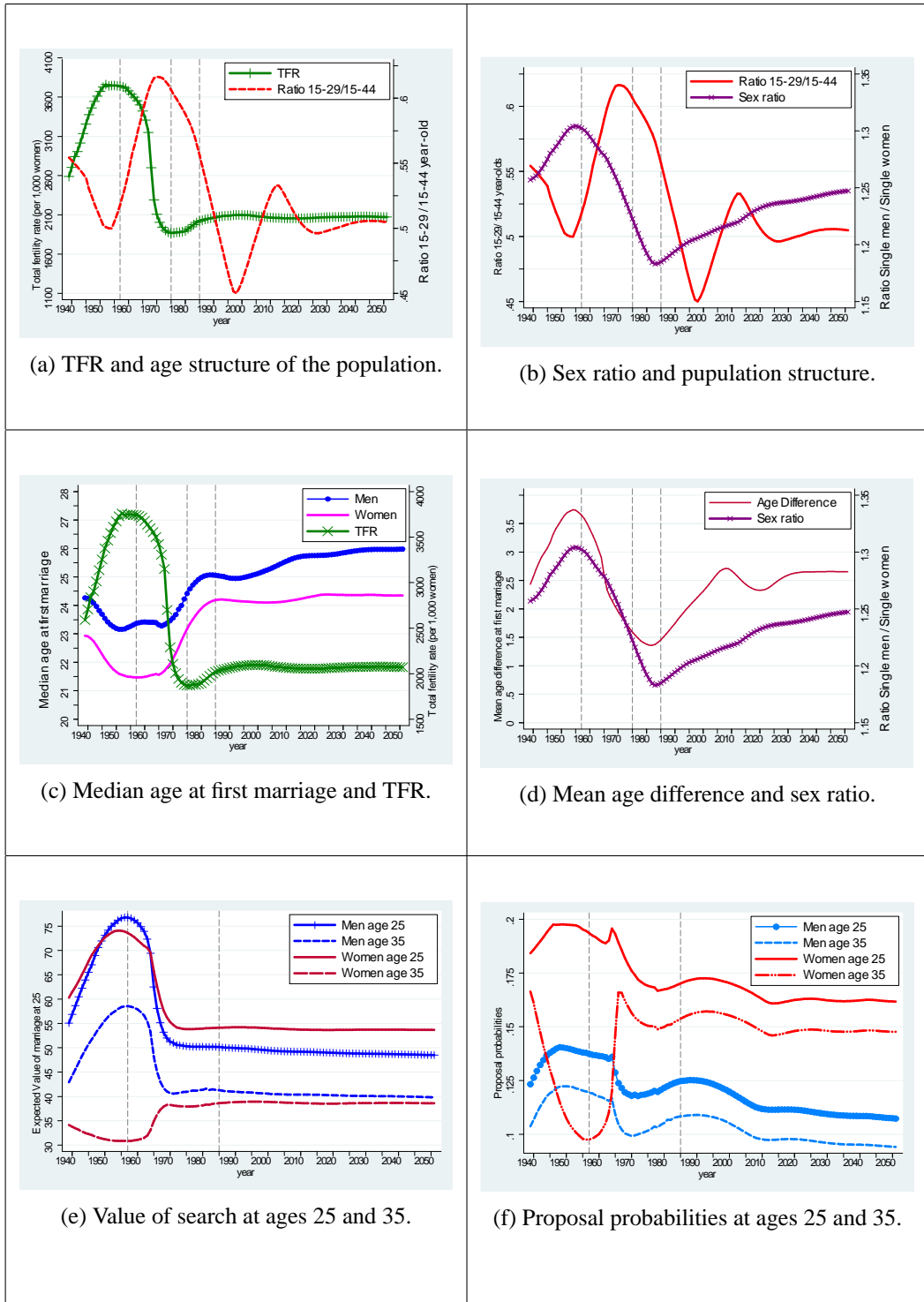


Figure 5: The Benchmark Economy.

In the remainder of this paper, I will argue that the change in demographic structure caused by the Baby Boom acts as a persistence mechanism that generates a slow adjustment in men's age of marriage after the boom is over.

When comparing the single sex ratio, which is a key determinant of meeting rates (see (39)), with the fraction of young people, we can observe a negative correlation between the two variables, especially after the peak of the fertility rates in 1957 and until the early 1980's. After the imbalance produced in the sex ratio during the Baby Boom (through the increase of the age gap), the arrival of larger cohorts of younger people to the marriage market helped to balance the sex ratio relatively quickly (late 1960's and 1970's). Beginning in the early 1980's, the single sex ratio recovers, mainly during the period where the fraction of young people was also below its steady state level.

In Panel (c) I compare the median age at first marriage with the TFR, while Panel (d) shows that, despite the data limitations, the high correlation between the age difference and the single sex ratio seems to be consistent with the evolution shown in Panel (e) of Figure 1. Notice that, after reaching a minimum in 1957 (the peak of fertility rates), the median age at first marriage of both men and women started to recover, with women's recovery beginning earlier and at a much more rapid pace than men's. Observe also that if we either look at the differences in timing (median ages at first marriage Panel (c)) or at the mean of the age gap at marriage directly (Panel (d)), the gap seems to shrink from the peak of the Baby Boom to the early 1980's and then to increase again, to reach a steady state that is lower than the initial state. Moreover, Panel (d) shows that the evolution of the single sex ratio and that of the age gap seem to mirror each other.⁴⁰

The last two panels of Figure 5 show the evolution of two model variables that may be helpful in order to understand the mechanism behind the results. Panel (e) shows the expected value of being in the marriage market at ages 25 and 36, which is $V_{a,t}^i$ from expression (46), for $i \in \{m, w\}$ and $a \in \{25, 35\}$. Not surprisingly, the value of marriage increases for both 25 year-old men and women during the Baby Boom (reaching a peak in 1957) and then stabilizes in the early 1990's. Notice that the case of 35 year-old is different: while for men it replicates the pattern of 25 year-olds at a lower level, the search value for 35 year-old women sharply decreases during the Baby Boom and then recovers during the 1970's, as fertility decreases. The narrowing of the gap between the value of search of 25 and 35 year-old women can be seen as a relative increase in the value of waiting and helps to explain why women's median age at first marriage recovers more rapidly than men's.

Finally, in Panel (f) of Figure 5, I show the evolution of the probability of receiving a proposal at ages 25 and 35 for men and women. We define the

⁴⁰See the interpretation of Proposition 7 on page 18.

probability that a man of age a receives a marriage proposal from a woman of age b as the probability of meeting such a woman ($\lambda_{b,t}^m$ from expression (39)) times the probability that she is willing to marry a man of age a , $\{1 - G[R_t^w(b, a)]\}$. Therefore the probability that a man of age a receives a marriage proposal is

$$\Lambda_{a,t}^m = \sum_{b=15}^J \lambda_{b,t}^m \{1 - G[R_t^w(b, 25)]\} \quad (58)$$

Since proposals are determined by the other gender's reservation value, here we can see the asymmetric market faced by men and women during and after the Baby Boom. Notice again that during the Baby Boom, 35 year-old women are less likely to receive a marriage proposal than men of their same cohort. At the end of the Baby Boom, proposals for older women increase sharply, explaining the pattern of the value of search in Panel (e).

So far I have presented arguments to justify why women's age at marriage increased more rapidly than men's after the Baby Boom. However, the reasons behind the slow adjustment in men's age remains to be explained, which I will try to do in the next section by doing two counterfactual experiments.

6 Counterfactual experiments

6.1 Experiment I: "No Baby Boom"

In the first counterfactual experiment, I assume that there is not such an increase in desired fertility from the mid 1940's to the 1960's, as if the Baby Boom had never occurred. Therefore, here k_t converges slowly from the initial level (before 1940) to the steady state (replacement rate). The results of this experiment are shown in Figure 6 and in columns (2) and (5) of Table 3. Panel (a) of Figure 6 shows the nature of the experiment. Notice that fertility rates fall slowly from 1940 to reach their steady state values in 1990. By "erasing" the Baby Boom, the structure of the population is only slightly affected and approaches its steady state value by the year 2000, with the population structure being slightly older than in the initial state. Panel (b) shows the evolution of the median age at first marriage for men and women. As shown in the figure, if the Baby Boom had not occurred, the age at first marriage for men and women would have slowly risen to a new steady state level.⁴¹

⁴¹If we look at these findings in light of Proposition 7, as a result of a permanent decrease in desired fertility, we should expect women marrying unambiguously older, with the impact on men's age at marriage being ambiguous. There are two countervailing effects on men's age at marriage: search frictions will decrease because men will be more eager to marry older women, and the "aging"

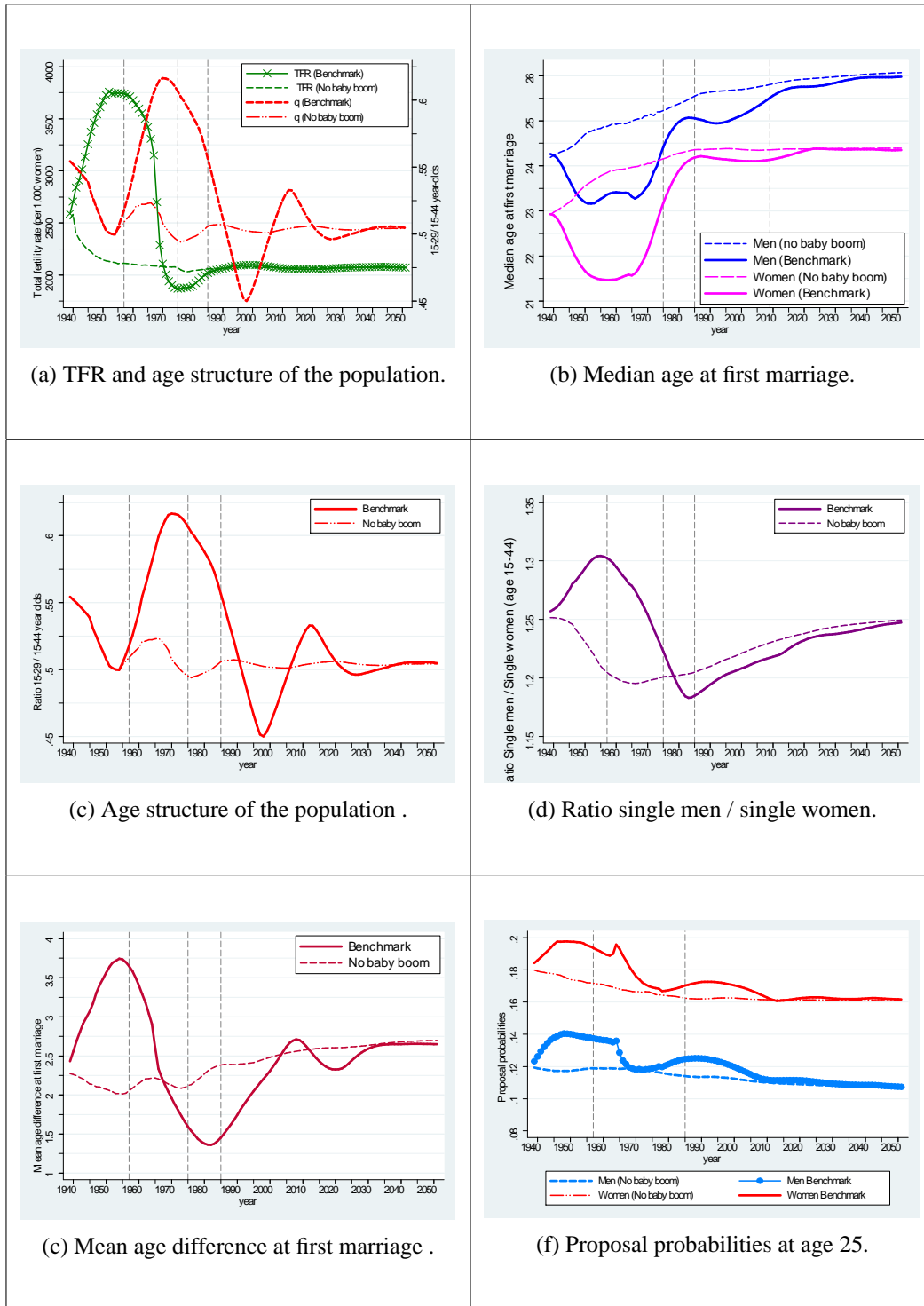


Figure 6: Counterfactual Economy I (No baby boom).

Table 3: Median age at first marriage: Benchmark and Counterfactual Experiments

| <i>Year</i> | <i>Men</i> | | | <i>Women</i> | | |
|-----------------------|-----------------|----------------|-----------------|-----------------|----------------|-----------------|
| | (1) Baseline | (2) No boom | (3) Constant | (4) Baseline | (5) No boom | (6) Constant |
| <i>1957</i> | 23.3 | 24.9 | 23.3 | 21.5 | 23.8 | 21.5 |
| <i>1985</i> | 25.0 | 25.6 | 26.0 | 24.2 | 24.4 | 24.1 |
| <i>2009</i> | 25.6 | 25.8 | 26.0 | 24.1 | 24.4 | 24.3 |
| <i>S.S.</i> | 26.1 | 26.1 | 26.1 | 24.4 | 24.4 | 24.4 |
| \triangle 1957-1985 | 1.7 | 0.7 | 2.7 | 2.7 | 0.5 | 2.6 |
| \triangle 1985-2009 | 0.6 | 0.2 | 0.0 | 0.0 | 0.0 | 0.2 |
| \triangle 2009- | 0.5 | 0.3 | 0.2 | 0.3 | 0.0 | 0.1 |

Notice, however, that the convergence in marriage timing with the baseline model occurs much later than the convergence in fertility rates (around 1990). As shown in Table 3, in 2009, for example, men's median age at marriage was still 0.2 years higher than in the baseline model, while women's age was 0.3 years higher.

As fertility rates decrease from the 1940's to the early 1980's, we observe an older age structure of the population (Panel (c)), along with a joint decrease in the sex ratio and in the age gap (Panels (d) and (e)). However, when fertility rates stabilize, the sex ratio is below its steady state level, as in the baseline model. Therefore, the age gap starts recovering as of the mid-1980's, and so does the sex ratio, to reach their new steady state. Observe also in Panel (f) that proposals received for both men and women are below those in the benchmark model.

The connection between the slow convergence in the age at marriage and demographic structure is suggested by the evolution of marriage proposals shown in Panel (f). Notice that if the Baby Boom had never occurred, the period probability of receiving a marriage proposal would have been lower than in the benchmark model for both men and women in the post Baby Boom period, to converge as the demographic structure is getting closer to its steady state value.

In order to further investigate further the connection between the demographic structure and the age of marriage, in the next experiment the economy will be like the one in the baseline model during the Baby Boom period. However, I will assume that the Baby Boom does not affect the age structure of the population, in order to isolate the general equilibrium effect caused by the demographic structure.

structure of the population will induce them to marry older (see the interpretation of the analytical results on pp. 18). In this case it appears that the second factor plays a more important role.

6.2 Experiment II: Constant inflow of young men and women

In this experiment, I assume that, in every period, the inflow rate of 15 year-old people to the economy is constant, everything else being equal. As shown in Panel (a) of Figure 7, in this case, the evolution of fertility rates mimics those in the baseline model, but the age structure of the population is constant from 1970 onwards. As a consequence, men's age at first marriage recovers much more rapidly than in the benchmark economy, starting to increase in 1957 and reaching its steady state in the early 1980's (see Panel (b)). For example, Table 3 shows that in 1985 the age of marriage of men is one year higher than the baseline if population inflow is constant. In the case of women, the evolution is much closer to the one in the baseline economy, but there is also a slow adjustment to the steady state, albeit small.

If we look at the evolution of the sex ratio on Panel (d), we see that in both the benchmark and the counterfactual economy, the sex ratio, together with the age gap (Panel (e)), starts rising with the Baby Boom because of the incentives driven by differential fecundity analyzed in Section 2. In the partial equilibrium case the sex ratio continues increasing until 1970, because, as desired fertility remains high during the early 1960's, the sex ratio would mechanically continue growing, because it is already unbalanced (due to the initial shock) and any new marriage takes one man and one woman out of the market. However, in the baseline economy, the sex ratio starts decreasing in the late 1950's, falling below its steady state levels around the mid-1980's and then recovering to reach the steady state only sometime after the year 2050. The evolution of the age gap, shown in Panel (e), is a mirror of what happens with the sex ratio. While in the partial equilibrium experiment the age difference decreases smoothly towards the steady state once the Baby Boom is over, in the benchmark economy it decreases sharply below the steady state level and recovers to reach the steady state only forty years later.

The factor behind the slower adjustment patterns in the baseline model with respect to this partial equilibrium experiment is the entrance of Baby Boomers during the 1960's. Even though the asymmetries in the incentives about marriage timing between men and women remain high until the mid 1960's, the fact that larger, sex-balanced cohorts enter the economy prevents the sex ratio from continuing to grow. Therefore, when the Baby Boom is over, the sex ratio and the age gap will decrease sharply, not only because women's biological constraints become less binding, but also because the demographic structure is temporarily "younger" due to the Baby Boomers cohorts. As the temporary change in the age structure of the population delays the adjustment of the sex ratio to the steady state, it also alters the matching process and therefore the timing of marriage. As shown in Panel (f), although both men and women receive more proposals than in the steady state because the larger cohorts of young people create "a thicker market externality",

Giolito: On Population Structure and Marriage Dynamics

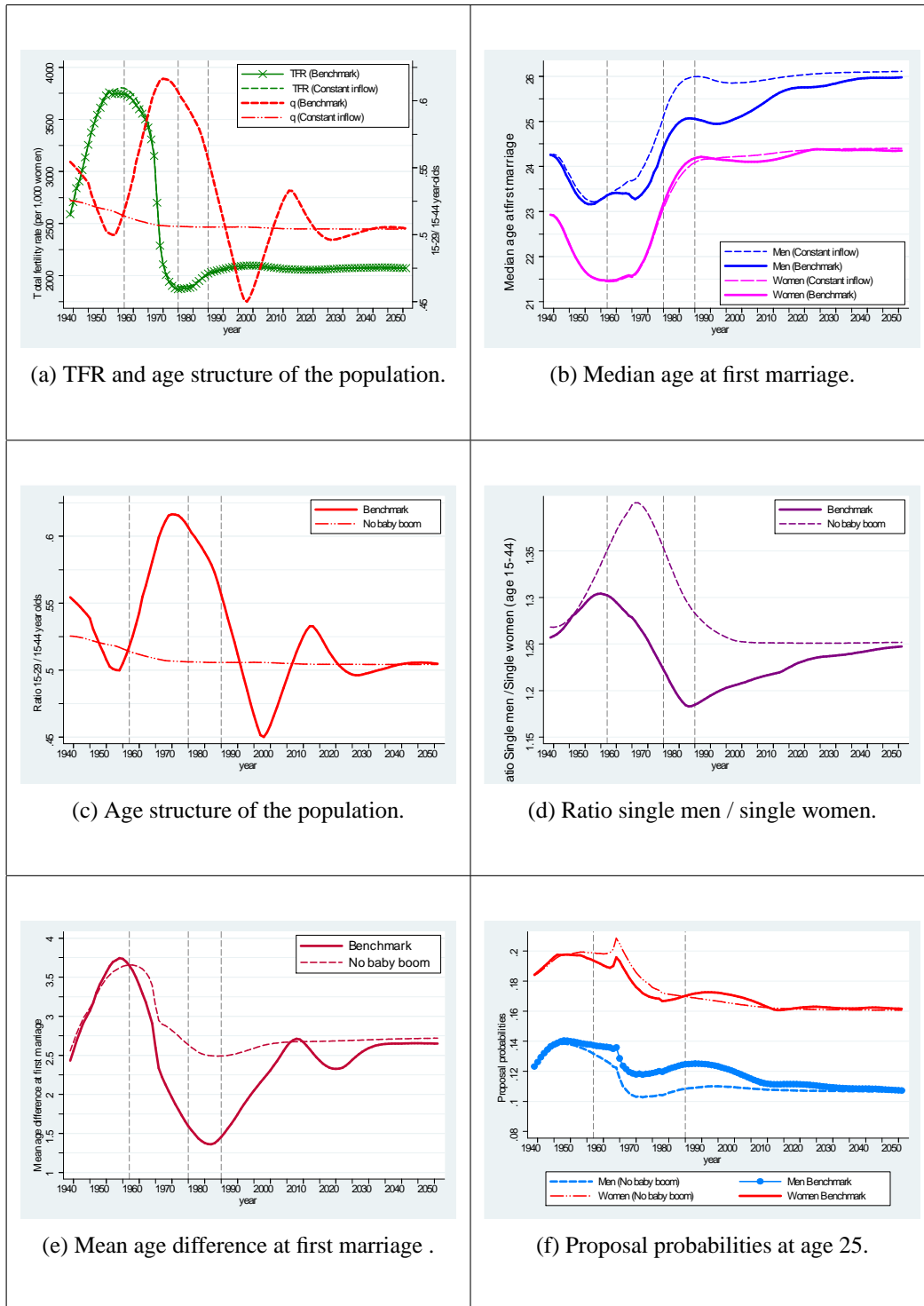


Figure 7: Counterfactual Economy II (Constant population inflow).

the impact is sizable for men, since they face relatively higher meeting rates than women. As men keep receiving relatively more proposals due to the impact of age structure of the population on the sex ratio, their median age at first marriage will slowly adjust toward its steady state as the demographic structure finds its own steady-state level.

7 Conclusion

In this paper, I develop an equilibrium, two-sided search model of marriage with endogenous population growth to study the interaction between fertility, the age structure of the population and the age at first marriage of men and women. I show that, given an increase in the desired number of children (which is an exogenous parameter in this model), age at marriage is affected through two different channels. First, as population growth, the age structure of the population acts as a "thicker market" externality, inducing early marriages. The second channel comes from differential fecundity: if the desired number of children is not feasible for older women, young women become relatively less choosy than young men. In equilibrium, women are more likely to marry older men and single men outnumber single women.

The results above have dynamic implications that are consistent with patterns observed in U.S. data, specifically after the Baby Boom period, which in the context of the model can be seen as a temporary increase of desired fertility. The main finding that the change in the demographic structure caused by the entrance of Baby Boomers in the marriage markets acts as a persistence mechanism, affecting marriage timing. Specifically, I found that the change in demographic structure may have delayed the adjustment in the sex ratio after the Baby Boom, increasing matching rates of men relative to those of women.

Even though that this stylized model is not able to account for all the factors that may have influenced the continuous increase in the age of marriage as of the mid-1970's, it provides a link, demographic composition, to reconcile the evolution of fertility rates with time series of the age of marriage. However, as fertility rates reached the replacement rates in the U.S. decades ago, the biological time frame for women to have two children in their lifetime is so wide that additional structure would be necessary to study the causes of the changes in marriage timing further.

The marriage market model developed in this article can be easily extended to a richer environment that includes human capital accumulation, labor supply and fertility choice in order to make a quantitative analysis of the dynamics of marriage in the last few decades. The results above suggest that the Baby Boom may have affected the marriage behavior of Baby Boomers themselves through identifiable

persistence factors, whose study requires going beyond standard, comparative static analysis.

Technical appendix

A Existence

A.1 Proof of theorem 1

To proof existence, I will make use of Brouwer's Fixed Point Theorem to show that the following functional relationship is a fixed point

$$\Psi(R_{1,2}^m, \Gamma^m, \Gamma^w, n) \longrightarrow \begin{pmatrix} R_{1,2}^m = \Psi_1(R_{1,2}^m, \Gamma^m, \Gamma^w, n) \\ \Gamma^m = \Psi_2(R_{1,2}^m, \Gamma^m, \Gamma^w, n) \\ \Gamma^w = \Psi_3(R_{1,2}^m, \Gamma^m, \Gamma^w, n) \\ n = \Psi_4(R_{1,2}^m, \Gamma^m, \Gamma^w, n) \end{pmatrix} \quad (59)$$

where

$$\Psi_1 = (1 + \mu_x) \left\{ \rho(\phi)^{\theta-1} \left(\frac{1+n}{2+n-\Gamma^w} \right) \left[1 - G^m \left(\frac{\min\{k, 1\}}{k} \mu_y \right) \right] [k - \min\{k, 1\}] \right\} \quad (60)$$

$$\begin{aligned} \Psi_2 = \rho(\phi)^{\theta-1} & \left\{ p_1^w \left[1 - G^w \left(\frac{\min\{k, 1\} R_{1,2}^m}{2k} \right) \right] \left[1 - G^m \left(\frac{\min\{k, 1\} \mu_y}{2k} \right) \right] \right. \\ & \left. + (1 - p_1^w) [1 - G^w(R_{1,2}^m)] \right\} \quad (61) \end{aligned}$$

$$\Psi_3 = \rho(\phi)^\theta \left\{ p_1^m \left[1 - G^w \left(\frac{\min\{k, 1\} R_{1,2}^m}{2k} \right) \right] \left[1 - G^m \left(\frac{\min\{k, 1\} \mu_y}{2k} \right) \right] \right. \\ \left. + (1 - p_1^m) \left[1 - G^m \left(\frac{\min\{k, 1\} \mu_y}{k} \right) \right] \right\} \quad (62)$$

$$\Psi_4 = \frac{k\Gamma^w}{2} - 1 + \frac{\sqrt{(k\Gamma^w)^2 + 4\min\{k, 1\}(1 - \Gamma^w)}}{2} \quad (63)$$

Recall that, from (28) and (29),

$$p_1^j = \frac{(1+n)}{2+n-\Gamma^j}, \text{ for } j \in \{1, 2\} \text{ and } \phi = \left(\frac{2+n-\Gamma^m}{2+n-\Gamma^w} \right).$$

Define a set $\Omega = [0, \bar{x}] \times [0, 1] \times [0, 1] \times [-0.5, (\bar{k} - 1)]$. To prove that $\Psi(\cdot)$ is a fixed point, I have to show that the system $\Psi(\cdot)$ is continuous and maps from Ω to itself.

The continuity of $\Psi(\cdot)$ is ensured because of the continuity of $G^w(\cdot)$ and $G^m(\cdot)$, and also because the matching probabilities $\rho(\phi)^{\theta-1}$ and $\rho(\phi)^\theta$ are bounded from above and below.⁴²

The next step is to show that $\Psi(\cdot)$ maps from Ω to itself. In order to show this, I pick a point $\chi_0 = (R_0^m, \Gamma_0^m, \Gamma_0^w, n_0)$, where $R_0^m \in [0, \bar{x}]$, $\Gamma_0^m \in [0, 1]$, $\Gamma_0^w \in [0, 1]$ and $n_0 \in [-0.5, (\bar{k} - 1)]$. I have to show that $\Psi_1(\chi_0) \in [0, \bar{x}]$, $\Psi_2(\chi_0) \in [0, 1]$, $\Psi_3(\chi_0) \in [0, 1]$ and $\Psi_4(\chi_0) \in [-0.5, (\bar{k} - 1)]$.

Consider first the case of Ψ_1 . By simple observation of equation (60), we know that $\Psi_1(\chi_0) \geq \mu_x$, so we only have to look for its upper bound, which occurs at $k = \bar{k}$. By definition,

$$\lambda_1^m(\chi_0) = \rho \left(\frac{2+n_0-\Gamma_0^m}{2+n_0-\Gamma_0^w} \right)^{\theta-1} \left(\frac{1+n_0}{2+n-\Gamma_0^w} \right) \leq 1 \text{ and } G^m \left(\frac{\mu_y}{\bar{k}} \right) \in (0, 1).$$

Therefore, evaluating Ψ_1 at $\bar{k} = \frac{\bar{x}}{\mu_x}$ (by condition (5)), we have

$$\Psi_1(\chi_0) = \mu_x + \left\{ \lambda_1^m(\chi_0) \left[1 - G^m \left(\frac{\mu_y}{\bar{k}} \right) \right] [\bar{k} - 1] \right\} \mu_x \quad (64) \\ = \mu_x + \left\{ \lambda_1^m(\chi_0) \left[1 - G^m \left(\frac{\mu_y}{\bar{k}} \right) \right] [\bar{x} - \mu_x] \right\} < \bar{x},$$

which implies that $\Psi_1 = R_{1,2}^m \in [\mu_x, \bar{x}]$.

⁴²See footnote 8 on page 5

Consider now Ψ_2 . Define

$$\phi_0 = \left(\frac{2 + n_0 - \Gamma_0^m}{2 + n_0 - \Gamma_0^w} \right), p_0^w = \left(\frac{1 + n_0}{2 + n_0 - \Gamma_0^w} \right) \text{ and } p_0^m = \left(\frac{1 + n_0}{2 + n_0 - \Gamma_0^m} \right).$$

Given that $\Gamma^w \in [0, 1]$ and $n_0 \in [-0.5, (k-1)]$, we know that p_0^w and $p_0^m \in (0, 1]$. We also know that, by definition $\rho(\phi)^{\theta-1} \in (0, 1]$, and that, since the reservation values of women are interior,

$$G^m(R_{1,2}^w) = G^m\left(\frac{\min\{k, 1\}\mu_y}{2k}\right) \in (0, 1) \text{ and } G^m\left(\frac{\min\{k, 1\}\mu_y}{2k}\right) \in (0, 1)$$

Therefore we have,

$$\begin{aligned} \Psi_2(\chi_0) = \rho(\phi_0)^{\theta-1} & \left\{ p_0^w \left[1 - G^w\left(\frac{\min\{k, 1\}R_0^m}{2k}\right) \right] [1 - G^m(R_{1,2}^w)] \right. \\ & \left. + (1 - p_0^w) [1 - G^w(R_0^m)] \right\} \in (0, 1), \end{aligned} \quad (65)$$

so

$$\Psi_2 = \Gamma^m \in (0, 1).$$

A similar argument can be used to show that

$$\Psi_3 = \Gamma^w \in (0, 1)$$

Finally, in Section 2.6 (page 12) I have already shown that

$$\Psi_4 = n \in [-0.5, \bar{k} - 1]$$

I have shown that the system $\Psi(\cdot)$ maps from Ω to itself. Therefore, a steady state equilibrium exists for this economy.

A.2 Proof of corollary 2

From the above equations, it is easy to show that the equilibrium of this economy will always have an interior solution. From expression (64) we know that $R_{1,2}^m \in [\mu_x, \bar{x}]$. We also know that both Γ^m and $\Gamma^w \in (0, 1)$. Given the interior solution of Γ^w we have that $\Psi_4 \in (-0.5, \bar{k} - 1)$. Therefore any solutions of this system will be interior.

Recall from expression (16) that for $j \in \{m, w\}$.

$$\Gamma^j = \gamma_{1,1}^j + \gamma_{1,2}^j$$

In expression (65) above, the first line represents $\gamma_{1,1}^w$ and the second line $\gamma_{1,2}^w$. Notice that the fact that both Γ^m and Γ^w are both greater than zero and less than one

implies that p_0^w and p_0^m are also between zero and one. We also know that $R_{1,2}^m < \bar{x}$, as shown above. Therefore since $(1 - p_0^w) > 0$ and $(1 - p_0^m) > 0$, we have that $\gamma_{1,2}^m > 0$ and $\gamma_{1,2}^w > 0$, which, by (34) and (35), imply $\gamma_{2,1}^w > 0$ and $\gamma_{2,1}^m > 0$. This trivially implies $\gamma_{1,1}^m = \gamma_{1,1}^w > 0$.

B Symmetry

B.1 Proof of proposition 3 (Equilibrium)

I have to show that when $G^m = G^w = G$ and $k \in [\frac{1}{2}, 1)$, the point $\chi_s = (\mu, \Gamma^s, \Gamma^s, n^s)$ is a solution of $\Psi(\cdot)$.⁴³ Plugging χ_s into $\Psi(\cdot)$, making $k < 1$ and manipulating we get

$$\begin{aligned} \Psi_2(\chi_s) &= \rho(\phi)^{\theta-1} \left\{ \left(\frac{1+n^s}{2+n^s-\Gamma^s} \right) \left[1 - G\left(\frac{\mu}{2}\right) \right]^2 \right. \\ &\quad \left. + \left(\frac{1-\Gamma^s}{2+n^s-\Gamma^s} \right) [1 - G(\mu)] \right\} \end{aligned} \quad (66)$$

$$\begin{aligned} &= \rho \left(\frac{1+n^s}{2+n^s-\Gamma^s} \right) \left[1 - G\left(\frac{\mu}{2}\right) \right]^2 \\ &\quad + \left(\frac{1-\Gamma^s}{2+n^s-\Gamma^s} \right) [1 - G(\mu)] \end{aligned}$$

$$\Psi_2(\chi_s) = \Psi_3(\chi_s) \quad (67)$$

$$\Psi_4(\chi_s) = \frac{k\Gamma^s}{2} - 1 + \sqrt{\frac{(k\Gamma^s)^2}{4} + k(1-\Gamma^s)}. \quad (68)$$

Therefore χ_s is an equilibrium of $\Psi(\cdot)$ and satisfies

$$\begin{aligned} \Gamma^m &= \Gamma^w \in (0, 1) \\ n &\in (-0.5, 0). \end{aligned}$$

Given that $\Gamma^m = \Gamma^w$, it is straightforward that the number of single old men equal the number of old single women, $s_{2,t}^m = s_{2,t}^w = (1+n)^{t-1}(1-\Gamma^s)$, by (26), which makes $S_t^m = S_t^w$ and $\phi = 1$. Furthermore, since $n < 0$, the fraction of young people in the population, $q_1 = \frac{1+n}{2+n} < \frac{1}{2}$, by (1).

⁴³Recall that, when $k \leq 1$, $R_{1,2}^m = R_{1,2}^w = \mu$, by (36). Similarly, $R_{1,1}^m = R_{1,1}^w = \frac{\mu}{2}$.

B.2 Proof of proposition 4 (Uniqueness)

Now I have to show that the equilibrium found in Proposition 3 is unique for each value of $k \in [\frac{1}{2}, 1]$. Define

$$F(R_{1,1}^m, \Gamma^m, \Gamma^w, n) = \begin{pmatrix} F_1(R_{1,1}^m, \Gamma^m, \Gamma^w, n) \\ F_2(R_{1,1}^m, \Gamma^m, \Gamma^w, n) \\ F_3(R_{1,1}^m, \Gamma^m, \Gamma^w, n) \\ F_4(R_{1,1}^m, \Gamma^m, \Gamma^w, n) \end{pmatrix} \quad (69)$$

where $F_1 = [\frac{\min\{k,1\}}{2k}\Psi_1 - R_{1,1}^m]$, $F_2 = [\Psi_2 - \Gamma^m]$, $F_3 = [\Psi_3 - \Gamma^w]$ and F_4 is given by equation (31).⁴⁴ In Proposition 3 we found that the point $\chi_s = (\Gamma^s, \Gamma^s, n^s)$ is a solution of $\Psi(\cdot)$. To show the uniqueness of χ_s using the Implicit Function Theorem, we need continuity in all the partial derivatives of $F(\cdot)$. We also need that Jacobian determinant of $F(\cdot)$ with respect to the endogenous variables is nonzero when evaluated at χ_s . We know that the system is continuous and differentiable for $k \in [\frac{1}{2}, 1]$, being their partial derivatives also continuous because g is differentiable.⁴⁵

Given that under symmetry $R_{1,1}^m = \frac{\mu}{2}$ (by (36)), we can eliminate one equation (F_1) and one unknown ($R_{1,1}^m$) from the system $F(\cdot)$. Therefore, evaluating our system at point χ_s we have,

$$\begin{pmatrix} F_2(\Gamma^s, \Gamma^s, n^s) \\ F_3(\Gamma^s, \Gamma^s, n^s) \\ F_4(\Gamma^s, \Gamma^s, n^s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (70)$$

and we need that,

$$|J| = \text{Det} \begin{bmatrix} F_{22} & F_{23} & F_{24} \\ F_{32} & F_{32} & F_{33} \\ F_{42} & F_{43} & F_{44} \end{bmatrix} \neq 0. \quad (71)$$

⁴⁴Notice that, in order to make the exposition of the following results easier, now the system is expressed in terms of $R_{1,1}^m$.

⁴⁵The system $F(\cdot)$ is not differentiable at $k = 1$, that is, the right hand side and the left hand side derivatives with respect to k are not equal. However, $F(\cdot)$ is continuous in k and differentiable for the intervals $k \in [\frac{1}{2}, 1)$ and $k \in (1, \bar{k}]$.

Calculating the partial derivatives of $F(\cdot)$ and manipulating, we get

$$\begin{aligned}
 |J| &= \frac{1}{(2+n^s-\Gamma^s)^3} (2+n^s-\rho[1-G(\mu)]) \\
 &\times \left\{ (2+n^s-\Gamma^s)^2 (2+2n-k\Gamma^s) \right. \\
 &+ (2+n^s(4+k+2n)-k(1+2n)\Gamma^s) \\
 &\left. \times \rho \left(\left[2-G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) + G(\mu) \right) \right\}
 \end{aligned} \tag{72}$$

Given that by assumption $\rho \leq \frac{1}{2}$ and that always $n^s \geq -\frac{1}{2}$, we have that $(2+n^s-\rho[1-G(\mu)]) > 0$. Therefore $|J|$ will be positive if the expression under curly brackets, $\{x_1+x_2*x_3\} > 0$, where $x_1 = (2+n^s-\Gamma^s)^2(2+2n-k\Gamma^s)$, $x_2 = (2+n^s(4+k+2n)-k(1+2n)\Gamma^s)\rho$ and $x_3 = \left(\left[2-G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) + G(\mu) \right)$. We clearly have that $x_1 > 0$, $x_2 > 0$. In the case of x_3 , the log-concavity of g implies

$$\left[2-G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) \leq G(\mu) \tag{73}$$

and therefore $x_3 \leq 0$ and $x_2*x_3 \leq 0$.⁴⁶ However, given that $|x_3| \leq G(\mu)$, it is sufficient that $\rho \leq \frac{1}{2}$ to have $|x_1| > |x_2|*|x_3|$. Therefore

$$|J| > 0$$

which implies that the equilibrium is unique.

B.3 Proof of proposition 5 (comparative statics).

In Proposition 4 we found that $|J| > 0$, which allows us to define Γ^s and n^s as implicit functions of k in a neighborhood of any point χ_s that is a solution of the system $F(\cdot)$ defined in (69). Given that $F(\cdot)$ has a unique solution for each value of $k \in [\frac{1}{2}, 1)$, the following results hold for the whole interval.

Now I turn to comparative statics. Applying Cramer's rule, the derivatives of the endogenous variables in $F(\cdot)$ with respect to k are

$$\frac{\partial \Gamma^m}{\partial k} = \frac{\partial \Gamma^w}{\partial k} = \frac{|J_2|}{|J|}, \text{ and } \frac{\partial n}{\partial k} = \frac{|J_4|}{|J|},$$

⁴⁶Log-concave probability densities have the "New is better than used" (NBU) property: $[1-F(x+y)] \leq [1-F(x)][1-F(y)]$. Making $x=y=\frac{\mu}{2}$, this implies that $\left(\left[2-F\left(\frac{\mu}{2}\right) \right] F\left(\frac{\mu}{2}\right) \right) \leq F(\mu)$. See An (2003).

where $|J|$ is defined in (71) and $|J_2|$ and $|J_4|$ are obtained by replacing $-F_{ik} = -\frac{\partial F_i}{\partial k}$ for $i = \{2, 3, 4\}$ in the respective column of $|J|$. Therefore we have,

$$|J_2| = -\frac{(1 - \Gamma^s)}{(2 + n^s - \Gamma^s)^3} (1 + n\Gamma^s) (2 + n^s - \rho [1 - G(\mu)]) \times \rho \left(\left[2 - G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) + G(\mu) \right) \quad (74)$$

Given that, by (73),

$$\left(\left[2 - G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) + G(\mu) \right) \leq 0,$$

we have that $|J_2| > 0$. Therefore,

$$\frac{\partial \Gamma^s}{\partial k} > 0, \text{ for } k \in \left[\frac{1}{2}, 1 \right).$$

Similarly,

$$|J_4| = \frac{(2 + n^s - \rho [1 - G(\mu)])}{(2 + n^s - \Gamma^s)^3} \left\{ (2 + n^s - \Gamma^s)^2 + (1 + n^s) \rho \left(\left[2 - G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) + G(\mu) \right) \right\} \quad (75)$$

We know that $(2 + n^s - \Gamma^s) > 1 + n^s \geq 0.5$ (because of $\Gamma^s < 1$ and the boundaries of n) and that $\left[2 - G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) + G(\mu) \leq G(\mu)$. Given that $\rho \leq \frac{1}{2}$, it must follow that $|J_4| > 0$. Therefore

$$\frac{\partial n}{\partial k} > 0, \text{ for } k \in \left[\frac{1}{2}, 1 \right).$$

Finally, since $q_1 = \frac{1+n}{2+n}$, it must follow that $\frac{\partial q_1}{\partial k} > 0$.

C Differential fecundity

C.1 Proof of Proposition 6

When $k > 1$ we have from (24) and (25) that $R_{1,1}^w = \frac{\mu}{2k}$ and $R_{1,2}^w = \frac{\mu}{k}$. Substituting $R_{1,2}^w$ in (21) and (20) we have,

$$R_{1,1}^m = \frac{\mu}{2k} \left\{ 1 + \lambda^m p_1^w [k - 1] \left[1 - G^m\left(\frac{\mu}{k}\right) \right] \right\} > R_{1,1}^w,$$

and

$$R_{1,2}^m = \mu \left\{ 1 + \lambda^m p_1^w [k - 1] \left[1 - G^m\left(\frac{\mu}{k}\right) \right] \right\} > R_{1,2}^w,$$

since $\lambda^m p_1^w > 0$.

C.2 Proof of proposition 7 (comparative statics).

To show the comparative statics with differential fecundity we use the system defined in (69). Given that now $k \in (1, \bar{k}]$, F_1 is defined by (21) and we work with the full system of four equations in four unknowns. For tractability I pick a point s' defined by the unique solution of the system at $k = 1$. Since the right-hand side derivatives of $F(\cdot)$ are defined and are continuous for $k = 1$, we are able to make local comparative statics for the neighborhood of χ'_s (for $k = 1 + \varepsilon$, with ε small). First of all, we have to show that the Jacobian of the full system, $|H|$ is nonzero when evaluated at χ'_s . Calculating the partial derivatives of $F(\cdot)$ and manipulating, we get

$$|H| = \frac{(2 - \Gamma^s - \rho [1 - G(\mu)])}{(2 - \Gamma^s)^2} \times \left\{ (2 - \Gamma^s)^2 + \rho \left(\left[2 - G\left(\frac{\mu}{2}\right) \right] G\left(\frac{\mu}{2}\right) + G(\mu) \right) \right\} > 0 \quad (76)$$

Therefore, we can define $R_{1,1}^m$, Γ^m , Γ^w and n as implicit functions of k in a neighborhood of χ'_s . Applying Cramer's rule, the derivatives of the endogenous variables in $F(\cdot)$ with respect to k are $\frac{\partial R_{1,1}^m}{\partial k} = \frac{|H_1|}{|H|}$, $\frac{\partial \Gamma^m}{\partial k} = \frac{|H_2|}{|H|}$, $\frac{\partial \Gamma^w}{\partial k} = \frac{|H_3|}{|H|}$ and $\frac{\partial n}{\partial k} = \frac{|H_4|}{|H|}$, where $|H_1|$, $|H_2|$, $|H_3|$ and $|H_4|$ are obtained by replacing $-F_{ik} = -\frac{\partial F_i}{\partial k}$ for $i = \{1, 2, 3, 4\}$ in the respective column of $|H|$.

Now I will show that in a neighborhood of χ'_s we have that, given an increase in k :

1. *Young men are less choosy with young women and choosier with old women:*

$$\frac{\partial R_{11}^m}{\partial k} < 0 \text{ and } \frac{\partial R_{12}^m}{\partial k} > 0.$$

To show that men's reservation values of men for young women are decreasing in k we need that,

$$\frac{|H_1|}{|H|} = -\frac{\mu (2 - \Gamma^s - \rho [1 - G(\mu)])}{2 (2 - \Gamma^s)} < 0.$$

In addition, since

$$\frac{(2 - \Gamma^s - \rho [1 - G(\mu)])}{(2 - \Gamma^s)} < 1,$$

we also have that

$$\left| \frac{\partial R_{11}^m}{\partial k} \right|_{k=1} < \left| \frac{\partial R_{11}^w}{\partial k} \right|_{k=1} = \frac{\mu}{2}$$

On the other hand, we have that young men become choosier with respect to old women. By (20), $R_{12}^m = 2kR_{11}^m$ when $k \geq 1$. Therefore,

$$\begin{aligned} \frac{\partial R_{12}^m}{\partial k} &= 2 \left[R_{1,1}^m + \frac{\partial R_{11}^m}{\partial k} \right] \\ &= 2 \left[\frac{\mu}{2} \left(1 - \frac{(2 - \Gamma^s - \rho [1 - G(\mu)])}{(2 - \Gamma^s)} \right) \right] > 0 \end{aligned} \tag{77}$$

2. *Women marry young with higher probability. In addition, the difference between the probability of marriage at age 1 of women and men, $\Gamma^w - \Gamma^m$, is positive and increasing in k :*

$$\frac{\partial \Gamma^w}{\partial k} > \frac{\partial \Gamma^m}{\partial k}$$

To prove that Γ^w is increasing in k is enough to show that $|H_3| > 0$. Therefore,

$$\begin{aligned} |H_3| &= \left\{ - (2 - \Gamma^s - \rho [1 - G(\mu)]) (1 - \Gamma^s) \frac{([2 - G(\frac{\mu}{2})] G(\frac{\mu}{2}) - G(\mu))}{(2 - \Gamma^s)^3} \right. \\ &\quad + \frac{\mu [1 - G(\frac{\mu}{2})]}{(2 - \Gamma^s)} (2 - \Gamma^s - \rho [1 - G(\mu)]) (4 - 2\Gamma^s - \rho [1 - G(\mu)]) g\left(\frac{\mu}{2}\right) \\ &\quad + \frac{2\mu f(\mu)}{(2 - \Gamma^s)^2} \left\{ [(2 - \Gamma^s)^2 [2 - \Gamma^s - (1 - \theta)\rho + \rho^2\theta] \right. \\ &\quad \left. - (1 + \theta(3 - 2\Gamma^s))\rho^2 \right\} \\ &\quad + \left[(2 - \Gamma^s)(1 - \theta)\rho F(\mu) + \rho^2 G(\mu)^2 (1 + \theta(1 - \Gamma^s)) \right] \\ &\quad \left. + (2 - \Gamma^s - \rho [1 - G(\mu)]) \left[2 - G\left(\frac{\mu}{2}\right) \right] (1 - \theta) G\left(\frac{\mu}{2}\right) \right\} \end{aligned}$$

We know by (73) that the first line is positive. Recalling that $\rho \leq \frac{1}{2}$, it is also clear that the second, fifth and sixth lines are also positive. In the third line we have $[2 - \Gamma^s - (1 - \theta)\rho + \rho^2\theta] \geq 1$, so it is also positive. In the fourth line we have $-1 < (1 + \theta(3 - 2\Gamma^s))\rho^2 < 0$. However, at $k = 1$ we have $\phi = 1$ which implies $\Gamma^s \leq \rho$. Therefore, the fourth line is also positive and $|H_3| > 0$, which implies $\frac{\partial \Gamma^w}{\partial k} > 0$ given that $|H| > 0$.

To show that the difference between the probability of marriage at age 1 of women and men, $\Gamma^w - \Gamma^m$, is positive and increasing in k we need that,

$$\frac{|H_3| - |H_2|}{|H|} = \frac{(1 - \Gamma^s)\rho\mu(2 - \Gamma^s + \rho[1 - G(\mu)])g(\mu)}{(2 - \Gamma^s)2 - \Gamma^s - \rho[1 - G(\mu)]} > 0$$

3. *The ratio of single men to single women, ϕ , is greater than 1 and increasing in k .*

From (26) we know that $s_{2,t}^m = s_{1,t-1}^m (1 - \Gamma^m)$ and $s_{2,t}^w = s_{1,t-1}^w (1 - \Gamma^w)$. Since $s_{1,t}^m = s_{1,t}^w$, $s_{2,t}^m > s_{2,t}^w$ because $\Gamma^w > \Gamma^m$. Therefore $S_t^m > S_t^w$ and $\phi > 1$. Furthermore, since

$$\frac{\partial \Gamma^w}{\partial k} > \frac{\partial \Gamma^m}{\partial k}, \text{ we have } \frac{\partial \phi}{\partial k} > 0.$$

4. *Population growth and the fraction of young people in the economy increases.*

In this case we need

$$\frac{|H_4|}{|H|} = \frac{\Gamma^s}{(2 - \Gamma^s)} > 0.$$

It is straightforward that here we also have $\frac{\partial q_1}{\partial k} > 0$, as in Proposition 5.

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