From Magnitudes to Math: Developmental Precursors of Quantitative Reasoning

by

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Dissertation submitted in partial fulfillment of
the requirements for the degree of Doctor
of Philosophy in the Department of
Psychology & Neuroscience in the Graduate School
of Duke University

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ABSTRACT

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Abstract

The uniquely human mathematical mind sets us apart from all other animals. Although humans typically think about number symbolically, we also possess nonverbal representations of quantity that are present at birth and shared with many other animal species. These primitive numerical representations are thought to arise from an evolutionarily ancient system termed the Approximate Number System (ANS). The present dissertation aims to determine how these preverbal representations of quantity may serve as the foundation for more complex quantitative reasoning abilities. To this end, the five studies contained herein investigate the relations between representations of number, representations of other magnitude dimensions, and symbolic math proficiency in infants, children, and adults. The first empirical study, described in Chapter 2, investigated whether infants engage the ANS to represent the full range of natural numbers. The study presented in Chapter 3 compared infants' acuity for detecting changes in contour length to their acuity for detecting changes in number to assess whether representations of continuous quantities are primary to representations of number in infancy. The study presented in Chapter 4 compared individual differences in acuity for number, line length, and brightness in children and adults to determine how the relations between these magnitudes may change over development. Chapter 5 contains a longitudinal study investigating the relation between

preverbal number sense in infancy and symbolic math abilities in preschool-aged children. Finally, the study presented in Chapter 6 investigated the mechanisms underlying the maturation of the number sense and determined which features of the number sense are predictive of symbolic math skill. Taken together, these findings confirm that number is a salient feature of the environment for infants and young children and suggest that approximate number representations are fundamental for the acquisition of symbolic math.

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1. Introduction

1.1 Introduction to approximate number representations

Representations of quantity are critical for survival both in the wild (how many predators are charging towards me? how far away is the nearest shelter?) and in the modern world (do I have enough money to pay rent this month? how many days until it is due?), so it is not surprising that quantitative abilities are widespread throughout the animal kingdom. Although humans typically think about number symbolically, we also possess nonverbal representations of quantity that are present at birth and shared with many other animal species. However, the quantitative abilities of educated humans far surpass those of nonhuman animals. From where does the uniquely human mathematical mind originate? Does our propensity for symbolic math build off of these primitive quantity representations?

Nonsymbolic numerical representations in humans and nonhuman animals are thought to arise from an evolutionarily ancient system termed the Approximate Number System (ANS; Dehaene, 1997). The ANS represents number in an approximate, noisy fashion using mental magnitudes (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 2000). Critically, these representations are not dependent on language or formal training; both infants and nonhuman animals spontaneously attend to numerical information in the environment. ANS representations can be visualized as a series of overlapping Gaussian distributions with logarithmic compression, such that

there is less overlap between the curves representing 3 and 4 than there is between the curves representing 13 and 14 (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). As a result, ANS representations are defined by scalar variability and follow Weber's Law, according to which the discriminability of two numerosities is dependent on their ratio rather than their absolute numerical difference. This means that preverbal infants and nonhuman animals can differentiate sets of 5 from sets of 10 items more easily than they can differentiate 10 items from 15 items, despite the fact that both pairs differ by 5. Furthermore, ANS representations are too noisy to support fine-grained discriminations such as 20 versus 21 items. This ratio-dependence is a hallmark of the ANS, and ratio-dependent performance in numerical tasks is frequently interpreted as engagement of the ANS. The sharpness or acuity of the ANS can be quantified as a Weber fraction, which is an index of the precision of internal numerical representations. More precise ANS representations arise from narrower Gaussian distributions, which minimize the overlap between neighboring values, whereas wider Gaussian distributions increase the amount of overlap and result in noisier ANS representations. Importantly, the acuity of the ANS varies across the population, such that some individuals have very precise internal representations of number while others possess fuzzier, less defined representations.

A wealth of neuroimaging and animal studies have localized the neural instantiation of the ANS to the parietal cortex and specifically the intraparietal sulcus (IPS). In adults,

the IPS exhibits ratio-dependent blood-oxygen-level-dependent (BOLD) responses to numerical stimuli (Piazza et al., 2004; Piazza, Pinel, Le Bihan, & Dehaene, 2007). Furthermore, this region is engaged in numerical processing regardless of whether the stimuli are presented as nonsymbolic arrays of elements, Arabic digits, or verbal number words (Cantlon, Libertus, Pinel, Dehaene, et al., 2009a; R. Cohen Kadosh, Cohen Kadosh, Kaas, Henik, & Goebel, 2007b; Eger, Sterzer, Russ, Giraud, & Kleinschmidt, 2003; Piazza et al., 2007). Complimentary single-unit recording research in nonhuman primates have revealed neurons that are tuned to specific numerosities in the monkey ventral intraparietal area (Nieder & Miller, 2004), as well as neurons in the monkey lateral intraparietal area whose firing rates are monotonically related to numerosity (Roitman, Brannon, & Platt, 2007). Furthermore, behavioral evidence for approximate number representations have been found in a wide range of species including birds (Scarf, Hayne, & Colombo, 2011) and fish (Agrillo, Petrazzini, Tagliapietra, & Bisazza, 2012a; Agrillo, Piffer, Bisazza, & Butterworth, 2012b). These convergent findings suggest that approximate number representations in humans are rooted in an evolutionarily ancient system that is shared with our nonhuman relatives and ancestors.

1.1.1 Approximate number representations in infancy

Before human infants learn language or begin formal schooling, they are already sensitive to numerical information in the world around them. For example, just hours after birth, infants can match numerical information across sensory modalities. Izard

and colleagues (2009) familiarized newborn infants to auditory sequences of syllables with a fixed number of repetitions (e.g., "fa fa fa fa"). Infants were then presented with visual arrays containing either the same number of elements as the auditory sequence or a novel number of elements. The infants preferred to look at the images with the number of elements that matched the auditory sequence. Furthermore, their degree of preference for the matching images was modulated by the ratio of the number of elements in the test images, such that the larger the numerical ratio between the two test images, the greater infants' preference for the matching image. This ratio-dependent behavior suggests that the ANS is functional from birth.

During the first year of life, sensitivity to number increases rapidly. In the cross-modal matching paradigm described above, neonates require a 1:3 ratio difference between the test arrays to differentiate between the matching and non-matching arrays. Visual habituation, change detection, and auditory head-turn paradigms all provide converging evidence that by 6 months of age infants can differentiate between numerical values that differ by a ratio of 1:2, and by 9 months of age can differentiate between values that differ by a ratio of 2:3 (Brannon, Abbott, & Lutz, 2004; M. E. Libertus & Brannon, 2010; J. Lipton & Spelke, 2004; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005). Electrophysiological measures provide further evidence for ratio-dependent numerical discrimination infants. For example, when infants view a series of arrays containing a fixed number of dots interspersed with arrays containing deviant

numerosities, brain activity is modulated by the ratio between the standard and deviant numerosities as measured both by spectral power in the alpha frequency band and by the amplitude of the P500 component (Hyde & Spelke, 2011; M. E. Libertus, Pruitt, Woldorff, & Brannon, 2009). Consistent with the adult and animal findings, these differences in brain activity are localized to sites over the parietal cortex, suggesting that the parietal cortex exhibits sensitivity to number very early in ontological development.

In addition to numerical discrimination, the ANS also supports approximate arithmetic calculations in infants (McCrink & Wynn, 2004; 2009). In one study, McCrink and Wynn (2004) showed 9-month-old infants animations of arithmetic problems with large sets. In the displays, two arrays of shapes sequentially moved behind an occluder (addition) or one array of shapes moved behind an occluder and then a subset of the shapes moved out from behind the occluder and traveled off screen (subtraction). The infants then saw the occluder drop away to reveal either a correct or an impossible outcome (either twice or half as many items). Infants looked significantly longer at the impossible outcomes compared to the possible outcomes. This suggests that infants are sensitive to the approximate outcomes of addition and subtraction operations and that they recognize when an inappropriate number of items is present after the arithmetic transformation. The ANS also imbues infants with a sense of numerical proportions, and this ability is again ratio-dependent. For example, 10-month-old infants are sensitive to difference between arrays in which half of the dots and blue and half are yellow

compared to arrays in which three-quarters of the dots are blue and one-quarter are yellow (McCrink & Wynn, 2007). Therefore, the ANS provides not just a system for representing number, but also a system for manipulating these representations.

Interestingly, there are some situations in which infants' numerical representations appear to be limited by an upper set-size limit rather than ratio. In particular, when infants need to discriminate between small numbers of items, they seem to engage object tracking mechanisms rather than the ANS (Uller, Carey, Huntley-Fenner, & Klatt, 1999). These object tracking mechanisms, which are also active in adults and nonhuman animals, enable infants to precisely keep track of a small number of objects through space and time (see Feigenson et al., 2004 for a review). Although such mechanisms have likely not evolved for numerical purposes, they do seem to support some quantitative tasks involving up to three items. For example, in manual search and food choice tasks, infants appear to be able to track only up to three items simultaneously. In these tasks, infants watch as toys are sequentially hidden in a box (manual search) or crackers are sequentially hidden into two cups (food choice). Infants are then allowed to retrieve the toys or choose their preferred cup. The main finding from these paradigms is that infants search for the correct number of toys or choose the cup containing the larger number of crackers when tested with 1 versus 2, 1 versus 3, or 2 versus 3 items but fail when tested with contrasts of 1 or 2 versus 4 items (Feigenson & Carey, 2003; 2005; Feigenson, Carey, & Hauser, 2002a). In these studies, infants' performance clearly breaks

down when the set size exceeds 3 items but is unaffected by numerical ratio. Furthermore, in visual habitation paradigms, infants do not appear to attend to number at all when small sets are presented, and fail to discriminate between arrays with 1 and 2 or 1 and 3 dots when non-numerical variables are controlled (Xu, 2003; Xu et al., 2005). However, when these variables are allowed to covary naturally with number, infants do discriminate between these small sets (e.g., Antell & Keating, 1983; Starkey, Spelke, & Gelman, 1990). In these situations, infants may be tracking non-numerical variables such as contour length or surface area rather than numerosity. For example, when infants are habituated to arrays containing 2 or 3 squares and then tested with arrays containing either a novel number of squares with a familiar total contour length or a familiar number of squares with a novel contour length, infants dishabituate only to the arrays containing the novel contour length and not to the arrays containing the novel number of squares (Clearfield & Mix, 1999). This finding suggests that in small sets when multiple dimensions are changing, number may not be the most salient property of the array. However, attempts to replicate this finding have produced mixed results (Clearfield & Mix, 2001; Cordes & Brannon, 2009). Furthermore, in other paradigms, such as visual habituation with heterogeneous arrays, infants do attend to the number of elements rather than to the cumulative spatial extent (Feigenson, Carey, & Spelke, 2002b). In still other paradigms with small numbers, such as those involving discriminating between series containing different numbers of tones or visually tracking

multiple moving groups of elements, infants' performance again appears to be bounded by ratio rather than set size (vanMarle & Wynn, 2009; Wynn, Bloom, & Chiang, 2002). Therefore, the nature of infants' small number representations, and particularly the situations in which the ANS versus other representational strategies are engaged, are not yet well understood.

1.1.2 Malleability of the Approximate Number System

Between infancy and adulthood, the precision of nonverbal number representations improves dramatically. As mentioned previously, neonates require a 1:3 difference in order to discriminate between numerosities, and this ability improves to a 2:3 difference by 9 months of age. The average adult, however, requires just a 9:10 difference for numerical discrimination, though acuity varies widely within the population (Halberda & Feigenson, 2008; Piazza et al., 2010). How much of this improvement should be attributed to maturation, and how much to experience, culture, and education?

One avenue for exploring this question has been to study human societies with limited numerical vocabularies and without formal mathematics systems. One such society is the Mundurukú, an indigenous group in Brazil whose lexicon contains words only for the numbers one through five. Nevertheless, the Mundurukú posses an intact number sense; on a nonsymbolic numerical comparison task (choosing which of two dot arrays is more numerous), their performance is qualitatively similar to that of educated

French adults in that performance decreases as the ratio between the sets to be compared approaches one. However, their overall performance is relatively poorer for the more difficult ratios (Pica, 2004). Because of the many cultural differences between the French and Mundurukú, the implications of this difference in performance are difficult to interpret. However, there is a great deal of variation in the amount of educational exposure for individuals within the Mundurukú community, which means that the Mundurukú also provide an ideal test case for exploring the role of mathematics education on number sense acuity. Recently, Piazza and colleagues (2013) found that Mundurukú subjects with some exposure to formal mathematics schooling had significantly better numerical acuity than those without any formal schooling, even after controlling for age. A similar pattern has been found in comparisons of schooled and unschooled adults from Western cultures, with mathematics-educated adults exhibiting more precise numerical approximation skills than adults without any formal mathematics education (Nys et al., 2013). Experience with numerical symbols and calculations therefore appears to sharpen the precision of the ANS.

The anthropological studies described above suggest that culture and education can impact the precision of ANS acuity. However, other maturational forces may also exert an influence. Recent work suggests that the acuity of the ANS can also be improved through explicit training on a nonsymbolic comparison task. DeWind and Brannon (2012) trained adults on a numerical comparison task for six one-hour sessions.

After the introduction of trial-by-trial feedback, ANS acuity rapidly improved and then remained steady after the removal of feedback. Interestingly, this improvement was most evident for trials in which surface area and numerosity were incongruent. On these trials, the more numerous arrays had the smaller cumulative surface area, meaning that surface could not be used as a cue for numerosity. It seems that through training, participants became increasingly able to suppress attention to surface area and instead selectively focus on the numerosity of the arrays. It is currently unknown whether maturational forces may improve ANS acuity via the same mechanism. In theory, there are two mechanisms by which ANS acuity could improve over development. Maturational forces could improve the acuity of the ANS by enhancing the ability to attend to number and inhibit irrelevant information from other covarying dimensions, as suggested by the training study, or by enhancing the precision of the representations themselves through decreasing the width of the tuning curves. These possibilities are not necessarily mutually exclusive, and it is likely that both mechanisms may be at play during development.

1.2 The link between representations of number and other quantities

Psychophysics researchers have long realized that adults readily compare and match stimulus intensities across a wide range of magnitude dimensions including number, size, duration, brightness, and loudness. Stevens (1957) classified these dimensions, which all pertain to stimulus quantity, as prothetic dimensions and contrasted them

with metathetic dimensions, which are those that pertain to stimulus quality (e.g., hue, pitch). Because prothetic dimensions refer to an amount of some stimulus, they can be organized on an ordinal scale. Furthermore, the similarity between prothetic dimensions extends beyond the more versus less comparison – they also share a common representational format. In addition to number, other prothetic dimensions are also thought to be represented by approximate mental magnitudes with scalar variability and follow Weber's law. These structural similarities have led to proposals that representations of prothetic dimensions may arise from a common, generalized system for representing magnitude. Proponents of such a generalized magnitude system argue that number, spatial extent, time, and potentially other prothetic dimensions are represented using shared neural circuitry within the parietal cortex (Allman, Pelphrey, & Meck, 2012; Bueti & Walsh, 2009; Cantlon, Platt, & Brannon, 2009b; Gallistel & Gelman, 2000; Lourenco & Longo, 2011). Relations between magnitude dimensions that extend beyond structural similarity to neural overlap can be considered functionally overlapping, meaning that representations of one magnitude dimension automatically activate and engage with representations of other magnitude dimensions (Srinivasan & Carey, 2010).

The first explicit support for a functional overlap between magnitude dimensions came from Meck and Church's seminal study on enumeration and timing processes (1983). Meck and Church trained rats to perform a psychophysical choice procedure

using auditory signals composed of on/off cycles of tones. During training, rats learned to classify two cycle-two second stimuli as short and eight cycle-eight second stimuli as long. At test, number and duration were deconfounded by either holding the number of cycles constant and varying signal duration or by holding the signal duration constant and varying the number of cycles. With these novel stimuli, rats demonstrated accurate and equivalent performance for classifying both dimensions, indicating that rats spontaneously attended to and encoded both the total duration and total number of events. Additionally, administration of methamphetamine caused an identical shift in rats' discrimination functions for number and duration. Meck and Church argued that a single mechanism capable of representing both number and duration is the most parsimonious explanation of these results. This vision of a single mechanism for representing multiple dimensions has been recently repopularized by Walsh's A Theory of Magnitude (ATOM; Walsh, 2003). ATOM hypothesizes that number, time, and space are jointly represented in the parietal cortex as a result of the role that these dimensions play in sensorimotor transformations necessary for action. Therefore, ATOM proposes that the parietal cortex houses a representation system organized around transforming magnitudes from different dimension into common coordinates for action. According to the strongest interpretation of ATOM and other theories of general magnitude representations, the parietal cortex represents magnitude abstractly using a common neural currency that is indifferent to magnitude dimension or format. However, a

functional overlap between magnitude dimensions does not necessitate the existence of this type of abstract representation.

1.2.1 Behavioral evidence regarding general magnitude representations

Much of the support for a functional overlap between magnitude dimensions comes from the common finding that different magnitude interact and interfere with one another during the comparison process. For example, when making judgments about which of two Arabic digits is either physically or numerically larger, reaction time are faster when the physical size and numerical value are congruent compared to when they are incongruent (e.g., 2 8 compared to 2 8) (Tzelgov, Meyer, & Henik, 1992). This cross-dimensional interference effect suggests that irrelevant magnitude information is automatically processed to the extent that it interferes with the comparison process for the relevant dimension. Concordant congruity effects using similar Stroop-like paradigms have also been found for other combinations of magnitudes, which suggests that number, size, duration, and brightness may all be functionally overlapping (Casasanto & Boroditsky, 2008; R. Cohen Kadosh & Henik, 2006; V. Dormal & Pesenti, 2012; V. Dormal, Seron, & Pesenti, 2006; Xuan et al., 2007).

Interestingly, the level of interference between different dimensions is not always symmetrical. Representations of number and physical size are frequently shown to influence temporal judgments, yet number and size judgments are much less susceptible to interference from temporal information (Cappelletti, Freeman, & Cipolotti, 2009;

Casasanto & Boroditsky, 2008; Casasanto, Fotakopoulou, & Boroditsky, 2010; V. Dormal et al., 2006; V. Dormal & Pesenti, 2012; Droit-Volet, Clément, & Fayol, 2003). Dormal and Presenti (2012) investigated these asymmetries using a Stroop-like paradigm in which participants compared the numerosity, physical length, or temporal duration of two sequentially presented linear arrays of dots with each of the three dimensions manipulated independently. Consistent with previous findings, numerosity and length both influenced duration judgments, whereas duration did not influence numerosity or length judgments. In addition, though there was bidirectional influence between length and number, the influence of number on length judgments was stronger than the reverse. These instances of asymmetrical interference are inconsistent with the view that these dimensions are fully functionally overlapping, which predicts symmetrical interference between all dimensions represented in a common substrate, and instead suggests that these dimensions are only partially overlapping.

In addition to interference effects, two other predictions that arise from the functional overlap hypothesis are that individual differences in acuity should be stable across different dimensions and that training acuity in one dimension should transfer to improvements in acuity for non-trained dimensions. Currently, the support for stable individual differences in comparison performance across difference dimensions is mixed. Although some studies have found a relation between individual differences in acuity for number and line length or number and size (DeWind & Brannon, 2012;

Lourenco, Bonny, Fernandez, & Rao, 2012), other studies have not found such a relation (Agrillo, Piffer, & Adriano, 2013; Odic, Libertus, Feigenson, & Halberda, 2013). In addition, though DeWind and Brannon (2012) found a relation between participants' initial acuity for both number and line length, they found that training to improve numerical discrimination did not transfer to improvements in line length discrimination. Together, these behavioral studies suggest that although there may be some level of representational overlap between different magnitude dimensions, it is unlikely that they are fully functionally overlapping or that representations of magnitude are fully abstract.

1.2.2 Neural evidence regarding general magnitude representations

Given the common behavioral signatures seen for comparisons of numerical magnitudes and other prothetic dimensions, as well as the well-documented role of the IPS in supporting numerical representations, neuroscientists have investigated whether the IPS supports a common mechanism for representing and comparing other magnitudes. If prothetic magnitudes are represented with a generalized magnitude system, the IPS is a plausible location for the source of such representations. Although the IPS and surrounding parietal cortex could potentially contain specialized regions for each type of prothetic dimension, a more parsimonious explanation is that the IPS plays a more general role in the magnitude comparison process.

In one of the first neuroimaging studies to investigate this question, Pinel and colleagues (2004) used functional magnetic resonance imaging (fMRI) to investigate the brain regions involved in making comparative number, size, and brightness judgments. These particular magnitudes were chosen to determine if spatial dimensions such number and size may exhibit greater overlap with one another compared to a nonspatial dimension such as brightness. The fMRI results indicated that activity in bilateral parietal and occipitotemporal regions was common to all magnitude comparisons. However, the amount of overlap in the specific cortical areas recruited for each type of comparison judgment predicted the level of behavioral interference. Number and size judgments exhibited behavioral interference and elicited activity in overlapping regions of parietal and precentral cortex, whereas size and luminance exhibited behavioral interference effects and elicited activity in overlapping regions of precentral and occipitotemporal cortex. Critically, no areas of overlapping cortical activity were found for number and luminance judgments, nor was there behavioral interference between these dimensions. These findings led Pinel and colleagues to conclude that representations of number and spatial extent may be overlapping whereas representations of number and brightness may be processed independently from one another.

Many other neuroimaging studies have also found regions of the IPS or parietal cortex that are responsive to multiple magnitude dimensions (R. Cohen Kadosh et al.,

2007c; 2005; V. Dormal, Dormal, Joassin, & Pesenti, 2011; Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003; Kaufmann et al., 2005). However, there also exist studies that have found evidence for dissociations in the cortical regions involved in processing different magnitudes (Castelli, Glaser, & Butterworth, 2006; R. Cohen Kadosh et al., 2005; V. Dormal & Pesenti, 2009). Together, the behavioral and neuroimaging evidence temper the strongest interpretation of generalized magnitude representations. That is, while there is strong evidence that the IPS is involved in representations of many magnitude dimensions, there is little evidence that these magnitudes are represented using a common, abstract coding scheme or even that the representations arise from fully overlapping cortical regions. Further support for this conclusion comes from single unit recording data in monkeys. This work demonstrates that neurons tuned to numerical, temporal, and spatial magnitudes co-exist in posterior parietal cortex (Leon & Shadlen, 2003; Sawamura, Shima, & Tanji, 2002; Tudusciuc & Nieder, 2007). In addition, some of these neurons are even responsive to multiple quantity types, though these neurons exhibit dimension-specific response patterns (Tudusciuc & Nieder, 2007). Tudusciuc and Nieder (2007) trained monkeys to perform a delayed match-to-sample task using both numerical and line length stimuli while recording from neurons in the IPS. During the delay period, when the monkey needed to maintain a quantity representation in working memory, 15% of the neurons were tuned to a specific numerosity or line length. In addition, 20% of these quantity-sensitive neurons

responded to both number and length. However, none of the neurons that responded to both numerosity and length displayed an abstract magnitude preference (e.g., a preference for both larger numerosities and longer line lengths). This suggests that number and spatial extent are represented by partly overlapping populations of neurons within the IPS, but there is no abstract coding scheme for magnitude that is independent of the source dimension. Taken together, there is little persuasive evidence that the behavioral interactions between different magnitude dimensions arise solely from overlapping or abstract neural representations of magnitude. Therefore, it is possible that a general sense of magnitude may be constructed over development rather than being biologically specified.

1.2.3 Evidence for shifting relations over development

Over the past decade researchers have debated whether infants are capable of representing numerosity independent of continuous variables such as surface area and contour length (e.g., Brannon et al., 2004; Clearfield & Mix, 1999; 2001; Cordes & Brannon, 2008b; 2008a; 2009; 2011; Mix, Huttenlocher, & Levine, 2002; Xu et al., 2005; Xu & Spelke, 2000). Although these studies demonstrate that infants are capable of representing number when continuous variables are controlled, this debate presumes that representations of number and continuous variables are distinct in the infant brain (Lourenco & Longo, 2010; Mix et al., 2002). However, this may not be the case. According to the neonatal synesthesia hypothesis, infants' early perceptions are

fundamentally cross-modal and cross-dimensional, due to the superabundance of neuronal connections present throughout the infant brain (Maurer & Mondloch, 1996; 2006). As a result, in the first months of life infants may represent all magnitudes with a general ordinal scale of increasing stimulus intensity, such that all prothetic dimensions are completely functionally overlapping (Lourenco & Longo, 2010). As the infant brain matures and cross-modal connections are pruned, cross-dimensional representations of magnitude may become increasingly differentiated into their respective magnitude dimensions. Therefore, infants may be born with an undifferentiated magnitude system that only becomes differentiated into specialized spatial, temporal, and numerical systems through associational learning over ontological development (Walsh, 2003).

If infants enter the world with an undifferentiated sense of magnitude, then it follows that infants should spontaneously align different magnitude dimensions and exhibit cross-dimensional transfer. To test this prediction, several recent studies have examined infants' ability to recognize abstract correspondences across magnitude dimensions. In one of these studies, Lourenco and Longo (2010) habituated 9-month-old infants to magnitude color-pattern pairings (e.g., bigger objects: black with stripes; smaller objects: white with dots) and then tested them with novel magnitudes that either retained the same abstract pairing rule (e.g., greater numerosity: black with stripes) or reversed the rule (e.g., greater numerosity: white with dots). Infants looked longer at the pairings with the reversed rule, indicating that they expected the rule to transfer across

magnitudes, and this expectation held for all combinations of size, numerosity, and duration. In another study, Srinivasan and Carey (2010) habituated 9-month-old infants to pairs of lines and tones that were either congruent (e.g., long lines paired with long tones) or incongruent (e.g., long lines paired with short tones) and then presented them with both congruent and incongruent line-tone pairs at test. Only infants habituated to congruent pairings dishabituated to the novel test combinations. Furthermore, infants did not recognize congruency across all prothetic magnitudes. Srinivasan and Carey also habituated infants to a set of line-tone pairings in which the volume of the tone was varied instead of the length. This time, infants in neither the congruent nor in the incongruent condition dishabituated to the novel pairing, suggesting that infants generalize magnitude relationships between representations of length and time, but not between representations length and volume. Likewise, infants familiarized to congruent pairings of number and line length display a preference for novel congruent pairings compared to incongruent pairings at test (de Hevia & Spelke, 2010), whereas infants familiarized to congruent pairings of number and brightness exhibit no such preference (de Hevia & Spelke, 2013). These infant findings complement those from adults in suggesting that number, spatial extent, and time do exhibit some level of overlap. This overlap emerges early in postnatal life, and has recently been documented in infants just a few hours old (de Hevia, Izard, Coubart, Spelke, & Streri, 2014). At the same time, the fact that infants do not readily relate these dimensions to other dimensions such as

brightness or loudness suggests that associations with these other dimensions may not emerge until later in childhood and that there may exist a privileged relation between number, space, and time.

As a result of maturation and experience, children gain the ability to recognize and construct mappings between dimensions that pertain to a broader range of magnitudes. In this way, they are able to form connections between dimensions that are not initially overlapping. Three mechanisms that may contribute to this process are statistical learning, analogical reasoning, and shared vocabulary. In the environment, many magnitude dimensions exhibit regular correlations. For example, greater numbers of items are typically associated with greater spatial extent (e.g., three apples takes up more space than a single apple), motions occurring for greater durations are typically associated with greater spatial displacement (e.g., the longer a dog runs, the greater distance it covers), and larger items are typically associated with louder sounds (e.g., a lion's roar is louder than a cat's meow). Infants are equipped with powerful learning mechanisms that enable them to detect these types of statistical regularities (e.g., Kirkham, Slemmer, & Johnson, 2002), and these mechanisms may facilitate infants' generalization of a "more A equals more B" rule. The formation of these generalizations through statistical learning occurs implicitly, without the need for infants to consciously direct attention to statistical regularities in the environment.

Later in development, children may also use explicit analogical reasoning to form associations between previously unassociated magnitude dimensions. In particular, the process of comparison may promote the recognition of the structural similarities between prothetic magnitude dimensions (Gentner & Medina, 1998). In one study, Gentner and colleagues (1995) investigated children's ability to recognize relational similarities both within and across perceptual dimensions. Children were shown a triad of stimuli with increasing monotonic order (e.g., small-medium-large) and then presented with one of two sets of test stimuli. The sets of test stimuli were either in the same perceptual dimension (e.g., size) or in a novel perceptual dimension (e.g., hue saturation). In both cases, test stimuli consisted of one triad with the same relational similarity (e.g., increasing monotonic order) and a foil with a no relational similarity (e.g., small-large-medium or light-dark-medium). Six- and 8-year old children recognized the relational similarity for both same-dimension and cross-dimension matches, but 4-year-old children succeeded only at making same-dimension matches. However, when 4-year-old children completed the same-dimension matching task before performing the cross-dimension matching task, they succeeded in making cross-dimension matches. This suggests that experience with the relatively easier same-dimension matches aided children in recognizing the higher order relationship beyond perceptual similarity and facilitated the construction of a rule that could apply to both size and increasing saturation. Thus, children's initial representations of the relations

between items increasing in size and items increasing in saturation may be dimension-specific. However, practice with same-dimension matches may make the relatively abstract concept of monotonic increase more salient, which enables children to recognize it as a relation that is generalizable across dimensions.

Another cue that children may use to align previously unrelated magnitudes is the common vocabulary used to describe multiple magnitude dimensions (Gentner & Medina, 1998). In particular, the use of ordinal comparison words such as "more" and "less" may facilitate recognition of the polar structure shared by all prothetic dimensions. For example, though infants do not spontaneously align representations of size and loudness, children form an association between these dimensions between two and five years of age (Marks, 1978; Smith & Sera, 1992). This development occurs in parallel with children learning the words "loud" and "quiet" and recognizing that louder stimuli are "more" than quiet stimuli. When children grasp that "big" and "loud" are both words that represent "more," this realization may highlight the structural similarity between spatial extent and volume (Smith & Sera, 1992). However, aligning dimensions through analogical reasoning and polar structuring may produce an inherently different relation than that found between dimensions that exhibit overlap early in infancy. For example, even though adults can consciously form mappings between volume and spatial extent, this mapping does not occur spontaneously when not explicitly required by the task (Srinivasan & Carey, 2010). Therefore, though these

dimensions can be consciously aligned, it seems that they do not automatically engage with or activate one another.

1.3 Potential links between the ANS and math skill

Although the ANS is fully functional in educated adults, we typically interact with symbolic representations of quantity. The ability to represent and manipulate numerical quantities underlies wide-ranging facets of modern life from the paying bills to sending astronauts into outer space. Unlike other animals, educated humans are not limited to representing numbers approximately. Through the use of numerical symbols and counting routines, we can appreciate that the difference between 20 and 21 is equivalent to the difference between 1 and 2. When children learn number words and written symbols, they learn a system that supports the exact representation of numerical quantity. Numerical symbols therefore endow humans with the unique ability to represent quantities precisely, which moves our numerical representations beyond the inherent constraints of the ANS. Furthermore, these numerical symbols can be manipulated in a wide manner of complex mathematical operations that support the modern world. Nonetheless, numerical symbols are grounded in the quantities they represent. Therefore, a prominent theory is that approximate number representations form the foundation on which symbolic math is built (Dehaene, 1997; Gallistel & Gelman, 1992).

A host of recent studies have found that individual differences in ANS acuity correlate with symbolic math skill (Agrillo et al., 2013; Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; M. E. Libertus, Feigenson, & Halberda, 2011; 2013a; M. E. Libertus, Odic, & Halberda, 2012; Lourenco et al., 2012; Mazzocco, Feigenson, & Halberda, 2011; Mundy & Gilmore, 2009; Piazza, Fumarola, Chinello, & Melcher, 2011; Starr, Libertus, & Brannon, 2013b). This relation has been documented throughout the educational spectrum, from preschoolers who are just beginning formal mathematics education (M. E. Libertus et al., 2011; M. E. Libertus, Feigenson, & Halberda, 2013a; Starr, Libertus, & Brannon, 2013b) all the way through to students taking their college entrance exams (DeWind & Brannon, 2012; M. E. Libertus et al., 2012). In addition, children with dyscalculia, a learning disability specific to math, have poorer ANS acuity than their typically developing peers (Piazza et al., 2010). Other studies, however, have failed to find a relation between ANS acuity and mathematics ability (Bonny & Lourenco, 2012; Gilmore et al., 2013; Holloway & Ansari, 2009; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Sasanguie, Defever, Maertens, & Reynvoet, 2013; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2012; Soltész, Szűcs, & Szűcs, 2010; Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2010). There are many possible explanations for these divergent findings, including differences in the methods used to assess ANS acuity and mathematics ability, the size of the sample, and the age of the participants. A recent meta-analysis of studies investigating the link

between ANS acuity and math skill concluded that there is a moderate but significant association (Q. Chen & Li, 2013). Mathematics ability is a highly multifaceted construct, and many cognitive abilities are known to contribute, including working memory, executive functioning, and short-term memory (Bull & Scerif, 2001; Espy et al., 2004; Geary, 2004; Lyons & Beilock, 2011; J. F. McLean & Hitch, 1999), as well as socio-environmental factors such as income level and learning environment (N. C. Jordan, Kaplan, Ramineni, & Locuniak, 2009; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Starr, Libertus, & Brannon, 2013b; vanMarle, Chu, Li, & Geary, 2014). Therefore, ANS acuity likely one of many factors that contributes to a child's mathematical proficiency. However, because the ANS is functional very early in development, it is an attractive target for novel mathematics education interventions that can be implemented in children who are not yet proficient with numerical symbols.

A critical question, then, is the mechanism by which ANS acuity influences math ability. Given that ANS representations can be mentally manipulated in arithmetic operations, the ANS may serve as an intuitive basis for symbolic arithmetic and therefore lay the groundwork for the acquisition of formal arithmetic principles (Gilmore et al., 2010; Park & Brannon, 2013; 2014). Gilmore and colleagues found that in kindergarten, typically the start of formal schooling, children's ability to perform arithmetic with nonsymbolic quantities (arrays of dots) correlated with their mastery of symbolic mathematics curriculum. Lending further credence to this hypothesis, Park

and Brannon (2013; 2014) found that training adults to perform nonsymbolic arithmetic led to improvements in their symbolic arithmetic ability. A second possibility is that the ANS serves as an anchor for symbolic number knowledge, such that quality of the mapping between number words and their corresponding ANS values is a driving force behind the link between ANS acuity and mathematics achievement (Feigenson, Libertus, & Halberda, 2013; Piazza, 2010). In support of this hypothesis, symbol ordering proficiency has been found to mediate the link between the ANS and symbolic mathematics (Lyons & Beilock, 2011) and ANS acuity is correlated with children's understanding of the count list (Starr, Libertus, & Brannon, 2013b; vanMarle et al., 2014; Wagner & Johnson, 2011). A related finding is that ANS acuity correlates more with early-acquired symbolic mathematics concepts (e.g., number word knowledge, arithmetic using fingers or tokens) compared to later-acquired symbolic mathematics concepts (e.g., writing Arabic digits, exact multiplication and division) (M. E. Libertus, Feigenson, & Halberda, 2013b), which suggests that the ANS is most critical for the acquisition, rather than the maintenance, of symbolic mathematics skills. Yet another proposal is that the ANS serves as an online error-monitoring system during arithmetic, allowing erroneous symbolic answers to be discarded in favor of correct answers (Lourenco et al., 2012). These possibilities need not be mutually exclusive, and it is likely that the relation between the ANS and symbolic math changes over development as

children become increasingly facile with numerical symbols and learn more complex mathematical operations.

1.4 Dissertation approach

The goal of this dissertation is to determine how primitive, preverbal quantity representations facilitate the acquisition of complex symbolic quantitative reasoning abilities. To answer this question, this dissertation follows two primary lines of research. The first examines how representations of number are related to representations of other quantity dimensions in infancy, early childhood, and adulthood. The second probes the origins of the relation between approximate representations of number and symbolic math skill. To address these issues, this dissertation contains 5 empirical studies. These studies employ eye-gaze measures from infants and explicit behavioral measures from young children and adults, and they utilize both cross-sectional and longitudinal approaches. Together, these studies provide insight into which aspects of approximate number representations are most influential for symbolic math, thus laying the groundwork for identifying novel targets for mathematics education interventions.

The first empirical chapter investigates the nature of infants' small number representations to determine whether infants regard number as a salient feature of small sets. Although many studies have shown that infants can discriminate large sets on the basis of numerosity (e.g., Brannon et al., 2004; Xu & Spelke, 2000), it is unclear whether infants' spontaneously attend to the numerosity of small sets (Clearfield & Mix, 1999;

2001; Cordes & Brannon, 2008b; Feigenson, Carey, & Spelke, 2002b; Mix et al., 2002). This chapter utilizes a change detection paradigm to test 6-month-old infants' ability to discriminate small sets on the basis of numerosity when non-numerical features are controlled.

The second empirical chapter uses the change detection paradigm to assess 7-month-old infants' acuity for changes in contour length. Despite evidence suggesting that infants can discriminate large sets on the basis of numerosity (e.g., Brannon et al., 2004; Clearfield & Mix, 1999; 2001; Cordes & Brannon, 2008a; 2008b; 2009; 2011; Mix et al., 2002; Xu et al., 2005; Xu & Spelke, 2000), debate remains as to whether infants' representations of number are secondary to representations of continuous quantities that covary with number, such as surface area and contour length (Clearfield & Mix, 1999; 2001; Cordes & Brannon, 2008b; 2009; M. E. Libertus, Starr, & Brannon, 2014). An assumption has been that these continuous quantities are more immediately available for perception and are therefore a more primitive or earlier developing feature representation. In this study, acuity for contour length is compared to acuity for number and surface area to determine the likelihood that attention to non-numerical features may be driving infants' performance in purported numerical discrimination tasks.

The third empirical chapter assesses individual differences in magnitude comparison performance in 4-year-old children and adults for number, line length, and brightness to determine how the relation between these dimensions may change over development.

Although it is frequently hypothesized that representations of number, physical extent, and other continuous quantity dimensions may share common neural and cognitive resources (Cantlon, Platt, & Brannon, 2009b; Walsh, 2003), there is also research suggesting that not all magnitude dimensions share the same level of overlap, particularly early in development (de Hevia & Spelke, 2013; de Hevia, Vanderslice, & Spelke, 2012; Srinivasan & Carey, 2010). The aim of this study is to tease apart the hypotheses regarding the origins of general magnitude representations and to determine whether number and spatial extent share a privileged association that does not extend to all other prothetic dimensions.

The fourth empirical chapter uses a longitudinal design to determine whether sensitivity to number in infancy is predictive of symbolic math skill in preschool. Although many cognitive abilities contribute to math achievement, including working memory, inhibition, and other executive functions (M. L. Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Passolunghi, Mammarella, & Altoè, 2008), symbolic mathematics is commonly thought to build on a domain-specific nonverbal numerical representation (Feigenson et al., 2013). Support for this hypothesis comes from the many studies that have demonstrated that ANS acuity covaries with math ability (see Q. Chen & Li, 2013; Fazio, Bailey, Thompson, & Siegler, 2014 for reviews). With these correlational studies, however, the direction of the relation remains unclear. It is therefore necessary to investigate ANS acuity before children acquire the verbal counting system and before

exposure to mathematics education. If the ANS is truly foundational for symbolic math, then early ANS acuity should inform children's facility with written and spoken numerical symbols.

The final empirical chapter investigates the mechanism underlying the improvements in ANS acuity observed over development by determining how attention to number versus attention to other stimulus features affects numerical comparison performance in 4-year-old children, 6-year-old children, and adults. It is well established that non-numerical features influence numerical representations in both children and adults (Allik & Tuulmets, 1991; Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Defever, Reynvoet, & Gebuis, 2013; Gebuis & Gevers, 2011; Gebuis & Reynvoet, 2011; 2012a; 2012b; N. Ginsburg & Nicholls, 1988; Hurewitz, Gelman, & Schnitzer, 2006; Rousselle & Noël, 2008; Rousselle, Palmers, & Noël, 2004; Soltész et al., 2010; Sophian & Chu, 2008; D. Szűcs, Nobes, Devine, Gabriel, & Gebuis, 2013; Tibber, Greenwood, & Dakin, 2012; Tokita & Ishiguchi, 2010; 2013), but little is known about how the influence of different non-numerical features changes over development. In addition, some researchers have asserted that the ability to selectively attend to number and inhibit attention towards these non-numerical features may be driving the apparent relation between ANS acuity and symbolic math skill (Fuhs & Mcneil, 2013; Gilmore et al., 2013). This study uses a novel method for constructing numerical comparison stimuli and modeling choice behavior in order to dissociate numerical acuity from the biasing effects

of non-numerical features (DeWind, Adams, Platt, & Brannon, under review), thus enabling the assessment of how non-numerical bias changes over development and how this bias may relate to symbolic math skill.

2. Infants show ratio-dependent number discrimination regardless of set size

Evidence for approximate number system (ANS) representations in infancy is robust but has typically only been found when infants are presented with arrays of 4 or more elements. In addition, several studies have found that infants fail to discriminate between small numbers when continuous variables such as surface area and contour length are controlled. These findings suggest that under some circumstances infants fail to recruit either the ANS or object file representations for small sets. Here we used a numerical change detection paradigm to assess 6-month-old infants' ability to represent small values. In Experiment 1, infants were tested with 1 vs. 3, 1 vs. 2, and 2 vs. 3 dots. Infants successfully discriminated 1 vs. 3 and 1 vs. 2, but failed with 2 vs. 3. In Experiment 2, we tested whether infants could compare a small and large value with a 2 vs. 4 condition. Across both experiments, infants' performance exhibited ratio-dependence, the hallmark of the ANS. Our results indicate that infants can attend to the purely numerical attributes of small sets and that the numerical change detection paradigm accesses ANS representations in infancy regardless of set size. Please note that this chapter is adapted from Starr and Brannon (2013a).

2.1 Introduction

The numerical capacities of infants are frequently attributed to two core systems: an approximate number system (ANS) that represents large numbers in an imprecise

fashion and an object file system that precisely tracks small numbers of objects (Dehaene, 1997; Feigenson et al., 2004). Evidence for ANS representations in infancy comes from visual habituation and auditory head-turn behavioral studies and electrophysiological measures in which brain and behavior measures are dependent on the ratio between the sets to be compared. For example, 6-month-old infants successfully dishabituate to large sets of visual or auditory stimuli that differ by a 1:2 ratio (e.g., 8 vs. 16 dots or tones) but reliably fail to dishabituate to sets that differ by a 2:3 ratio (e.g., 8 vs. 12 dots or tones) (J. S. Lipton & Spelke, 2003; Xu, 2003; Xu et al., 2005; Xu & Spelke, 2000). A numerical change-detection paradigm modeled after a procedure used by Oakes and colleagues to test infants' working memory capacity (Ross-Sheehy, Oakes, & Luck, 2003) confirms these findings by showing that 6-month-olds look longer to the numerically changing image stream as compared to a numerically non-changing stream when both streams are presented simultaneously and the numbers in the changing stream differ by at least a 1:2 ratio (M. E. Libertus & Brannon, 2010). Moreover, infants' preference scores are graded as a function of numerical ratio providing further evidence for ratio-dependent number discrimination in infancy. Other evidence for ratio dependence in infant's numerical discriminations comes from EEG studies. In one study 7-month-old infants were habituated to a given numerosity and then presented with a novel numerical array and spectral power in the EEG alpha band was modulated by the ratio between the habituated and novel numerosities (M. E. Libertus et al., 2009). In

another study, 6-month-old infants showed a mid-latency positivity (P500) that was modulated by numerical ratio for numerically large arrays (Hyde & Spelke, 2011).

Although adults (Cordes, Gelman, Gallistel, & Whalen, 2001) and nonhuman animals (e.g., Brannon & Terrace, 1998) engage ANS representations for both large and small numerosities, ratio-dependent performance in infants often breaks down for small sets. For example, Feigenson and her colleagues found that 10- to 12-month-old infants reliably choose the larger of two food quantities when faced with 1 vs. 2 or 2 vs. 3 food items, but perform at chance when presented with 1 vs. 4 or 2 vs. 4 items (Feigenson, Carey, & Hauser, 2002a). Likewise, 12- to 14-month-old infants succeed in a manual search task with 1 vs. 2, 1 vs. 3, and 2 vs. 3, but fail to discriminate between 1 vs. 4 and 2 vs. 4 (Feigenson & Carey, 2003; 2005). These patterns of performance suggest that for small sets, infants may employ a representational system with a limited capacity that breaks down when more than 3 items have to be represented.

While search tasks with older infants suggest that they are able to use object file representations to precisely represent small sets of up to three items, studies with younger infants (≤ 7 months) suggest that they may be completely unable to discriminate between small arrays on the basis of number. Although infants at this age show precise small number discrimination when number is confounded with other perceptual variables (e.g., Antell & Keating, 1983; Feigenson, Carey, & Spelke, 2002b; Starkey et al., 1990) or when numerical information is provided in multiple modalities

(e.g., K. E. Jordan & Brannon, 2006; Wynn, 1996), discrimination often breaks down when infants must rely only on visually presented numerical information. Most studies that have investigated small number discrimination in young infants have pitted number against continuous variables such as surface area in an attempt to determine which dimension is more salient to infants (e.g., Clearfield & Mix, 2001; Feigenson, Carey, & Spelke, 2002b). For example, infants have been habituated to arrays of either 1 or 2 objects or 2 or 3 objects of a constant size and then tested with either the familiar number of objects with a novel size or a novel number of objects with the familiar size. In those studies, infants dishabituated only to changes in object size and not to changes in object number (but see Cordes & Brannon, 2009). Furthermore, when surface area was controlled during habituation, infants failed to dishabituate to the novel numerosity at test (Feigenson, Carey, & Spelke, 2002b). Similarly, with the visual habituation paradigm when continuous extent is varied throughout habituation, 6-month-old infants fail to discriminate 1 vs. 2 and 2 vs. 4 dots (Xu, 2003; Xu et al., 2005). Together, these studies provide little evidence that young infants can recruit either object files or ANS representations for visually presented small sets.

Physiological recordings provide some evidence that young infants process small and large numbers differently. When 6-month-old infants viewed arrays containing alternating large numbers of elements, a mid-latency positivity (P500) was modulated by the ratio between the number of elements in the arrays. In contrast, when viewing

arrays containing small numbers of elements, infants displayed an earlier positivity (P400) that was modulated by the absolute value of the number of elements in the array, which is suggestive of object file representations (Hyde & Spelke, 2011). In addition, although looking time measures suggest that 7-month-olds are unable to discriminate between visual arrays containing 2 and 3 objects, differences in heart rate measures of attention suggest that infants can discriminate these values, even if they are not able to explicitly demonstrate this competence (Brez & Colombo, 2012).

Thus, it is unclear how 6-month-old infants represent small numerical values. Although behavioral data provides little evidence that young infants attend to the numerical property of small sets, physiological data suggests that they may rely on object file representations for small values and ANS representations for large values, much like older infants. Here, we tested 6-month-old infants' small number discrimination using the numerical change detection paradigm to determine whether the ratio-dependent performance seen with large values would also be present for small numerical sets (M. E. Libertus & Brannon, 2010).

2.2 Experiment 1

Experiment 1 investigated 6-month-old infants' ability to discriminate small values when non-numerical continuous variables were carefully controlled. Infants were tested with 1 vs. 3 dots, 1 vs. 2 dots, or 2 vs. 3 dots.

2.2.1 Materials and Methods

2.2.1.1 Participants

Sixty 6-month-old infants participated (mean = 6 months 4 days, range: 5 months 13 days – 6 months 19 days, 33 female), with 20 infants assigned randomly to each of three conditions. Data from 13 additional infants were excluded due to fussiness (n = 8), failing to look at both screens during a trial (n = 4), or because their preference scores were more than two standard deviations away from the group mean (n = 1). Parents gave written informed consent to a protocol approved by the local Institutional Review Board.

2.2.1.2 Design

Infants were randomly assigned to one of the three conditions. In all three conditions infants were shown two streams of images, one on each of two peripheral monitors (see Figure 1). In each condition, one of the image streams contained arrays that alternated in the number of elements (changing stream) while the other image stream displayed arrays with a constant number of elements (constant stream). The conditions varied only in the number of elements presented in the changing and constant streams. In the 1 vs. 3 condition, the changing stream contained 1 and 3 elements in alternation and the constant stream contained either 1 or 3 elements. In the 1 vs. 2 condition, the changing stream contained 1 and 2 elements in alternation and the constant stream contained either 1 or 2 elements. In the 2 vs. 3 condition, the changing stream contained 2 and 3

elements in alternation and the constant stream contained either 2 or 3 elements. Half of the infants in each condition saw the larger value in the constant stream while the other half of the infants saw the smaller value (e.g., in the 1 vs. 3 condition, half of the infants saw 1 dot in the constant stream while the other half saw 3 dots).

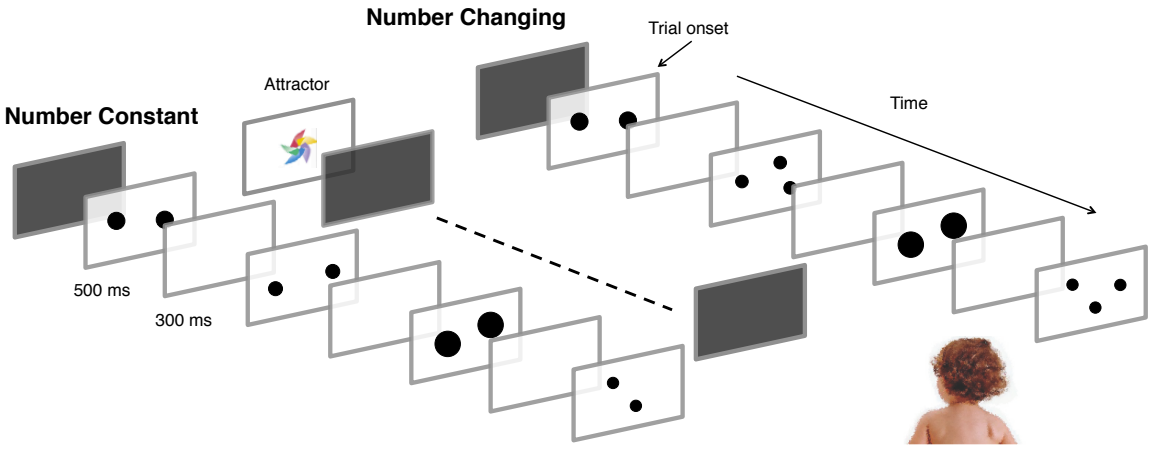

Figure 1: Schematic of the experimental design. The constant stream (left) shows images with the same number of dots each time (e.g., 2) whereas the changing stream (right) alternates between two different numerosities (e.g., 2 and 3).

2.2.1.3 Stimuli

The image streams consisted of arrays of dots presented for 500 ms followed by 300 ms of blank screen. The constant image stream always contained images with the same number of dots while the changing image stream contained images with an alternating number of dots. Every other image across both streams was identical. One-third of the images across both streams were matched for cumulative surface area, one-third were matched for cumulative contour length, and one-third were matched for individual element size. Cumulative surface area ranged from 3.14 cm^2 to 24.6 cm^2, cumulative

contour length ranged from 6.3 cm to 32 cm, and individual dot diameters ranged from 1 cm to 5.6 cm. Orthogonally, half of the images that differed in numerosity were matched on density. These manipulations ensured that the two image streams could only be differentiated on the basis of number and not based on other continuous variables. In all conditions, the images in both streams varied in the configurations of the elements such that even the images in the constant streams varied perceptually.

2.2.1.4 Procedure

Infants sat either in a highchair or on their parents' lap approximately 105 cm away from three 17" computer monitors. At the beginning of each trial, the center screen played a colorful image to orient the infant's attention towards the screens. Once the infant was looking at the center screen, the experimenter manually started the trial. Each infant was tested with four 60-second trials. The side of the changing image stream alternated between the trials, and trial order was counterbalanced across infants.

Infants' looking behavior was digitally recorded and coded off-line by an experienced observer. A second observer re-coded 25% of the participants and reliability between the observers was very high (r = .99).

2.2.1.5 Data analysis

For each infant, a preference score was calculated by subtracting the proportion of time spent looking at the constant stream from the proportion of time spent looking at the changing stream. Thus, a positive preference score indicates a preference for the

changing stream and a preference score of zero indicates equal time spent looking at the changing and constant streams.

2.2.2 Results

Infants successfully discriminated arrays of 1 vs. 3 dots and 1 vs. 2 dots, but failed to discriminate between arrays of 2 vs. 3 dots (see Figure 2). Infants' mean preference scores for the changing stream were 10.7%, 5.9%, and -1.4% for the 1 vs. 3, 1 vs. 2 and 2 vs. 3 conditions respectively[1].

One-sample t-tests were conducted to determine if the group preference score for each condition was different from zero. In the 1 vs. 3 condition, infants looked significantly longer at the changing stream compared to the constant stream ($t(19) = 3.50$, $p < .005$, $r = .63$). Infants in the 1 vs. 2 condition also looked significantly longer at the changing stream compared to the constant stream ($t(19) = 2.27$, $p < .05$, $r = .46$). Infants in the 2 vs. 3 condition, however, did not show a preference for the changing stream ($t(19) = -0.39$, $p = .35$, $r = .09$). Fourteen of the 20 infants in the 1 vs. 3 condition, 13 of the infants

[1] We calculated preference scores as in Libertus & Brannon (2010) by subtracting the proportion of time spent looking at the constant stream from the proportion of time spent looking at the changing stream. An alternate method of calculating preferences scores is to divide the amount of time spent looking at the changing stream by the total amount of time spent looking at both streams (e.g., Ross-Sheehy, Oakes, & Luck, 2003). Calculating preference scores using the latter method does not change the results. Infants demonstrate a preference for the changing stream in the 1 vs. 2 (mean preference score = .53, $t(19) = 2.29$, $p < .05$) and 1 vs. 3 conditions (mean preference score = .56, $t(19) = 3.70$, $p < .001$), whereas infants in the 2 vs. 3 condition do not show a preference for the changing stream (mean preference score = .49, $t(19) = -.39$, $p = .35$). A linear regression performed on these preference scores again demonstrates a linear trend for preference scores to increase as the ratios become easier to discriminate ($r = .38$, $p < .01$).

in the 1 vs. 2 condition, and 9 of the infants in the 2 vs. 3 condition looked longer at the changing stream.

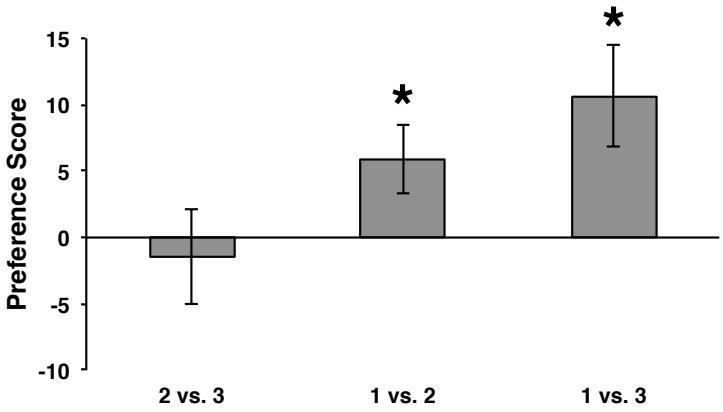

Figure 2: Mean preference scores for each of the three conditions in Experiment 1. Infants looked significantly longer at the changing stream in the 1 vs. 2 and 1 vs. 3 conditions, but not in the 2 vs. 3 condition. Error bars represent SEM.

A three-way ANOVA with condition (2:3, 1:2, 1:3), non-changing numerosity (small or large), and side of the first changing image stream (left or right) as factors and preference score as the dependent measure revealed a main effect of condition ($F(2, 48) = 4.98$, $p < .05$, $\eta^2 p = .17$). No other main effects or interactions achieved significance. Additionally, post-hoc Tukey comparisons demonstrated that infants' preference scores in the 1 vs. 3 condition were significantly higher than the preference scores in the 2 vs. 3 condition ($p < .05$), but preference scores in the 1 vs. 2 condition were not significantly different from either of the other conditions. A linear regression performed across the three ratios indicated that preference scores increased with ratio ($r = .34$, $p < .01$).

A final analysis compared data from this experiment with the large numerosity discrimination data (12 vs. 18, 8 vs. 16 and 6 vs. 18) reported in Libertus and Brannon (2010). A two-way ANOVA with ratio (2:3, 1:2, or 1:3) and set size (small or large) as factors and preference score as the dependent measure revealed a significant main effect of ratio ($F(2, 102) = 7.94$, $p = .001$, $\eta^2 p = .14$), no effect of set size, and no interaction between set size and ratio (see Figure 3).

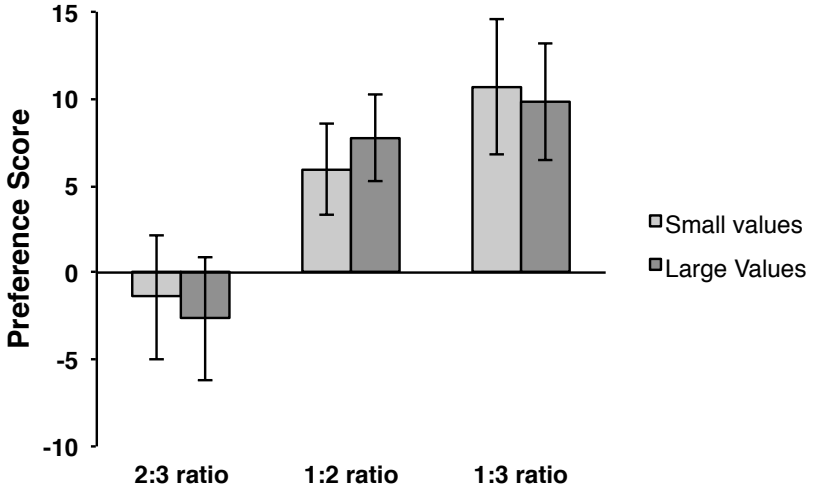

Figure 3: Infants' preference scores are modulated by ratio for both small and large values. Small value data is taken from Experiment 1 of present study and large value data from Libertus and Brannon (2010). Error bars represent SEM.

2.2.3 Discussion

The results of Experiment 1 demonstrate that 6-month-old infants are able to discriminate small arrays of elements on the basis of number. Furthermore, successful discrimination of arrays of 1 vs. 2 dots and 1 vs. 3 dots in conjunction with failure to

discriminate 2 vs. 3 dots suggests that they relied on the ANS. If infants recruited object file representations, they should have succeeded at all three comparisons including 2 vs. 3 given that all of the values were below 4. In addition, comparing infants' performance with small and large sets at the same ratios indicated that discrimination is consistently influenced by ratio and there is no effect of set size. Infants' success with 1:3 and 1:2 ratio discriminations and failure with a 2:3 ratio is also consistent with the well-established limits of 6-month-olds' numerical discrimination with large sets (e.g., Xu & Spelke, 2000). These results suggest that infants are able to represent both small and large numerosities using the ANS. Note that these results cannot be explained by constraints on infants' memory capacity because at 6 months of age infants can represent only a single item in visual short term memory (Ross-Sheehy et al., 2003). If infants were relying on short-term memory they should be unable to discriminate 1 from 2 and 1 from 3 and should not show larger preference scores for 1 and 3 compared to 1 and 2.

An alternative interpretation of these results, however, could be that infants employed set-relational representations based on a conceptual distinction between singular and plural sets rather than the ANS. Evidence for this idea comes from the change in performance somewhere between 14 and 22 months of age in food-choice and manual search tasks. Feigenson and colleagues find that 10- to 14-month-olds perform at chance when discriminating 1 from 4, despite successful performance with 1 vs. 2 or 3 in these tasks. This suggests that infants are using a system that cannot represent 4 and that

they do not even represent 4 as being more than 1 (Feigenson & Carey, 2005; Feigenson, Carey, & Hauser, 2002a). However, at 22 months infants succeed at 1 vs. 4 in a manual search paradigm, suggesting that by this age infants are able to represent 4 as a plurality (Barner, Thalwitz, Wood, Yang, & Carey, 2007; P. Li, Ogura, Barner, Yang, & Carey, 2009). Interestingly, representing the singular-plural distinction appears to come at a cost of more exact object representations, as infants in the same studies failed to differentiate between 2 and 4 objects and seemed to represent both arrays simply as pluralities. While it seems unlikely that set-relational representations could account for the results of Experiment 1 given that they have not been demonstrated in infants younger than 20- to 22-months of age, the data are consistent with this interpretation (Barner et al., 2007; P. Li et al., 2009).

Comparative research with rhesus monkeys demonstrates that task parameters, such as simultaneous versus successive item presentation, can influence whether object file or set-relational representations are engaged when discriminating between small sets of objects. Monkeys who watch food items placed sequentially into containers exhibited behavior indicative of object file representations whereas monkeys who watched the food items placed all at once into the containers exhibited behavior consistent with set-relational representations (Barner, Wood, Hauser, & Carey, 2008). A remote possibility is that the change detection paradigm, like the unified set condition in Barner et al. (2008), encourages infants to employ set-relational representations and inhibits object file or

ANS representations. If this is the case, then infants may have succeeded in the 1 vs. 2 and 1 vs. 3 conditions because they successfully represented both 2 and 3 as plural sets larger than 1. In contrast, the 2 vs. 3 condition would fail to elicit a significant preference score if both 2 and 3 were represented as plural sets (note, however, that this explanation cannot explain why infants' discrimination showed a linear trend of increasing preference scores with ratio).

2.3 Experiment 2

In Experiment 2, we tested infants with a 2 vs. 4 condition to determine whether infants' performance in Experiment 1 resulted from ANS or set-relational representations. If infants recruit ANS representations during the change detection task with small numbers, then they should succeed at discriminating 2 vs. 4 because they are capable of discriminating both small and large sets that differ by a 1:2 ratio. On the other hand, if infants recruit set-relational representations, they should fail to discriminate 2 vs. 4 because both values are represented as plural sets.

2.3.1 Materials and Methods

2.3.1.1 Participants

Twenty six-month-old infants participated (mean age = 6 months 0 days, range: 5 months 14 days – 6 months 15 days, 9 females). An additional six infants were excluded due to not looking at both screens during a trial (n = 4), failure to complete the experiment (n = 1), or because their preference scores were more than two standard

deviations away from the group mean (n = 1). Parents gave written informed consent to a protocol approved by the local Institutional Review Board.

2.3.1.2 Design

The design, stimuli, and procedure were identical to Experiment 1 with the exception that the stimuli consisted of arrays of 2 and 4 dots. Cumulative surface area ranged from 6.28 cm^2 to 25.12 cm^2, cumulative contour length ranged from 9.42 cm to 42.7 cm, and individual dot diameters ranged from 0.8 cm to 3.4 cm. A second observer re-coded 25% of the participants and reliability between the observers was very high (r = .99).

2.3.1.3 Results and Discussion

Infants' mean preference score for the changing stream was 8.14%. Infants looked significantly longer at the changing stream compared to the constant stream ($t(19) = 2.11$, $p < .05$, $r = .44$; 12 out of 20 infants showed this pattern)[2]. A two-way ANOVA with non-changing numerosity (small or large) and side of the first changing image stream (left or right) as factors and preference scores as the dependent measure revealed no significant main effects or interactions.

To compare infants' performance on 2 vs. 4 with their performance on exclusively small or large values at the same 1:2 ratio, we compared this data with data from the 1

[2] If we calculate the preference score as the amount of time spent looking at the changing stream divided by the total amount of looking time, the preference score is .54, which is significantly above chance ($t(19) = 2.11$, $p < .05$).

vs. 2 condition of Experiment 1 and the 8 vs. 16 data from Libertus & Brannon (2010) in a one-way ANOVA with set size (small, medium, or large) as the factor and preference score as the dependent measure. There was no effect of set size ($F(2,53) = .147$, $p = .975$, $\eta^2 p = .006$), indicating that infants' discrimination of sets that differ by a ratio of 1:2 is not influenced by numerical magnitude (see Figure 4).

Infants' successful discrimination of 2 vs. 4 supports the conclusion that infants recruit ANS representations throughout the range of small and large values and provides no evidence for object file or set-relational representations. The similarities in infants' preference scores for 1 vs. 2, 2 vs. 4, and 8 vs. 16 suggests that a single representational system, the ANS, is recruited for both small and large sets, as well as for sets that cross the small-large divide.

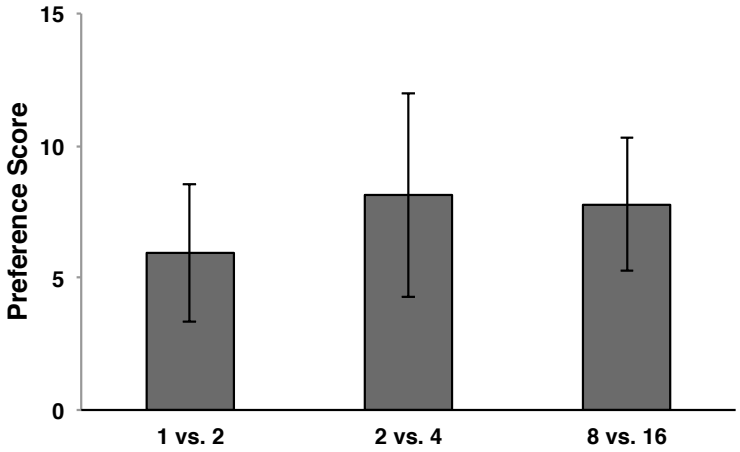

Figure 4: Infants can successfully discriminate arrays that differ by a ratio of 1:2 with small (Experiment 1), medium (Experiment 2), and large (Libertus & Brannon, 2010) numerical magnitudes. Error bars represent SEM.

2.4 General Discussion

The results of the present experiments demonstrate that 6-month-old infants are capable of making purely numerical discriminations for visually presented small sets and that discrimination is ratio-dependent. Furthermore, analyses that included previously published data with the same procedure (M. E. Libertus & Brannon, 2010) revealed that preference scores increased linearly with the ratio between the two values in the changing stream regardless of the magnitude of the values. This convergent pattern of results for small and large set sizes suggests that infants can extend ANS representations to small values in the first year of life.

While the pattern observed in Experiment 1 is consistent with the ratio-dependent performance that is the hallmark of ANS representations, we reasoned that the results could also be consistent with the idea that infants used set-relational representations to distinguish singular sets from plural sets (Barner et al., 2007; Feigenson & Carey, 2005). To tease apart these possible explanations, we tested infants of the same age in a 2 vs. 4 condition. Infants successfully discriminated 2 dots from 4 dots, looking longer at the changing stream regardless of the magnitude of the constant stream (2 or 4). Taken together, these results are best explained by ANS representations rather than object file or set-relational representations. If infants were recruiting object files, their performance should have been dictated by a maximum set size, allowing them to succeed at discriminating 2 from 3 but not 2 from 4. If infants were recruiting set-relational

representations, they should have failed at discriminating 2 from 3 and 2 from 4 because all of those values should be represented as pluralities. However, infants failed at discriminating 2 from 3 and succeeded at discriminating 2 from 4. As predicted by ANS recruitment, infants' performance was influenced by the ratio between the sets rather than set size or a singular-plural distinction.

Why have previous studies found young infants unable to discriminate small numbers while we found success? A critical difference between the present study and prior studies investigating infants' small number representations is the method used to assess numerical discrimination. We used a numerical change detection procedure that does not involve habituation or familiarization to a single numerical value but instead requires infants to detect numerical change in one visual stream. Why would this procedure be more likely to invoke the ANS than the habituation paradigm? One possibility is that there are different working memory demands in the two conditions. Hyde and Wood (2011) found that adults' electrophysiological signatures of processing small numerical arrays were associated with both the ANS and object file representations depending on the working memory load of the task. When the working memory load was low, adults exhibited neural correlates of object file representations, whereas when the working memory load was high they exhibited neural correlates of the ANS. The change detection paradigm, which was originally developed to assess infants' working memory (Ross-Sheehy et al., 2003), may load infants' memory and

prohibit the recruitment of object files. Because the change detection paradigm presents infants with two rapidly changing streams of information, it is also much more dynamic than the habituation paradigm which involves a single stream of images and longer presentation times for each image. In addition, in order to notice which stream is changing and which stream is constant, infants must maintain a representation of the previously displayed values in working memory and compare them to the currently displayed numbers. Hyde and Spelke (Hyde & Spelke, 2011) similarly propose that differences in stimulus complexity may predict whether the ANS is invoked for small values or represented by object files. The ANS may only be recruited for small values when the stimuli are complex and prohibit parallel individuation. Further support for these hypotheses comes from the fact that infants can successfully discriminate between 2 and 4 moving groups of elements (Wynn et al., 2002) whereas infants tested with static displays failed to discriminate 2 from 4 (Xu, 2003). Movement of the elements in visual displays may increase the complexity of the displays and the working memory load, leading to the recruitment of the ANS.

In conclusion, the present experiments demonstrate that like adult humans and non-human animals, infants in the first year of life possess an approximate number system that is ratio-dependent and operates over a seemingly unrestricted range of quantities. Other paradigms that involve active searching and tracking food items indicate that infants fail to recruit the ANS under some circumstances (Barner et al., 2007; Feigenson

& Carey, 2003; 2005; Feigenson, Carey, & Hauser, 2002a; P. Li et al., 2009), and further research is needed to determine the conditions that elicit different types of numerical representations throughout development. Our results are consistent with the idea that while there may be factors that differentially inhibit the ANS in infants, small and large numbers are nevertheless represented along the same numerical continuum in infancy.

3. Infants exhibit less sensitivity to changes in contour compared to numerosity

Over the past decades, abundant evidence has amassed that demonstrates infants' sensitivity to changes in number. Nonetheless, a prevalent view is that infants are more sensitive to continuous properties of stimulus arrays such as surface area and contour length than they are to numerosity. Very little research, however, has directly addressed infants' sensitivity to contour. Here we used a change detection paradigm to assess infants' acuity for the cumulative contour length of an array when the array's surface area and number was held constant. Seven-month-old infants detected a threefold change in contour length but failed to detect a twofold change. These results, in conjunction with previously published data on numerosity discrimination using the same experimental paradigm, suggest that infants' exhibit greater sensitivity for changes in number compared to changes in contour length. Consequently, these findings undermine the claim that attention towards contour length is the primary driver of numerical discrimination in infancy.

3.1 Introduction

It is well established that infants are sensitive to numerical information in their environment, beginning just hours after birth (e.g., Brannon et al., 2004; Izard et al., 2009; M. E. Libertus & Brannon, 2010; J. Lipton & Spelke, 2004; Xu et al., 2005; Xu & Spelke, 2000). Despite these findings, however, debate remains as to whether infants' representations of number are secondary to representations of continuous quantities that

covary with number, such as surface area and contour length (Cantrell & Smith, 2013; Clearfield, 2005; Clearfield & Mix, 1999; 2001; Cordes & Brannon, 2009; M. E. Libertus et al., 2014; Mix et al., 2002; Piaget, 1952). An assumption has been that continuous quantities are more immediately available for perception and are therefore more primitive or earlier developing feature representations. Although infants' can discriminate arrays on the basis of number when other dimensions are controlled, a related question is whether number is less salient compared to continuous properties of an array. When presented with an array of objects, do infants preferentially attend to number, or is number a last resort that infants' rely on only when other dimensions cannot be used?

Although a few studies indicate that infants may preferentially attend to surface area rather than number when tracking small numbers of objects (Clearfield & Mix, 2001; Feigenson, Carey, & Spelke, 2002b), other research suggests that infants may be more adept at tracking number compared to surface area, particularly when presented with large numbers of objects (Cordes & Brannon, 2011; M. E. Libertus et al., 2014). Cordes and Brannon (2011) found that 7-month-old infants required a fourfold difference to detect a change in the cumulative surface area of an array containing multiple elements, whereas only a twofold change in number resulted in dishabituation. Likewise, Libertus and colleagues (2014) found that when changes in surface area and number were directly pitted against one another in two dynamic image streams, 7-month-old infants'

preferred to look at the stream that was changing in number rather than the stream that was changing in surface area. This preference for the numerical change was so strong that a threefold change in number had to be paired with a tenfold change in surface area in order for infants to find the two streams equally engaging. Collectively, these findings suggest that number is actually easier to extract than surface area from arrays containing multiple elements.

A second variable of interest that covaries with number but has received relatively little attention is contour length (sometimes referred to as perimeter). Some researchers have proposed that in studies purporting to show number discrimination, infants are actually responding to changes in contour length (Clearfield & Mix, 1999; 2001; Mix et al., 2002). Indeed, contour length has been shown to influence numerical discrimination performance even in adults (DeWind et al., under review). A recent review of infant numerical discrimination studies argued that changes in contour length may explain infants' looking behavior in many studies that ostensibly controlled for continuous variables in order to isolate numerical discrimination (Cantrell & Smith, 2013). Despite the clear evidence that infants are sensitive to changes in visual edges and contour length (Karmel, 1969; McCall & Melson, 1970), few studies have directly investigated infants' acuity for contour discrimination (Clearfield & Mix, 1999; 2001; Cordes & Brannon, 2009). In one study, infants were habituated to arrays containing either two or three squares and then tested with arrays that contained either a familiar number of

squares with a novel contour length or a novel number of squares with a familiar contour length (Clearfield & Mix, 1999). Infants dishabituated only to the arrays with the novel contour length, suggesting that the contour length of the arrays was more salient than the number of elements. However, in an attempt to replicate this finding, another research group found that infants dishabituated to both changes in number and to changes in contour length (Cordes & Brannon, 2009). This makes it unclear whether contour length is actually more salient than number. A point to note is that these studies used a paradigm in which infants were habituated to arrays that were constant in both contour length and number and then tested with arrays in which one variable was held constant and the other was varied. Although this method may inform us as to which of two covarying dimensions infants spontaneously attend, it does not provide a metric of acuity for either dimension.

The present study systematically assessed infants' sensitivity to changes in contour length using the change detection paradigm to provide a metric of acuity (M. E. Libertus & Brannon, 2010; Ross-Sheehy et al., 2003; Starr, Libertus, & Brannon, 2013a). In addition, we constructed irregularly shaped stimuli to overcome the inherent surface area confounds present in homogenous dot arrays. Due to the geometry of circles, relatively small changes in contour length are necessarily coupled with large changes in surface area. For example, a threefold change in the contour length of a circle is accompanied by a tenfold change in surface area. The use of irregularly shaped stimuli

enabled us to manipulate contour length while holding surface area constant. In Conditions 1A and 1B, 7-month-old infants were tested with a twofold change in contour length. In Condition 2, 7-month-old infants were tested with a threefold change in contour length. These results were compared to previously collected data using the same paradigm that assessed sensitivity to changes in number (M. E. Libertus & Brannon, 2010) and surface area (M. E. Libertus et al., 2014) to determine the dimension for which infants are most sensitive to change.

3.2 Materials and Methods

3.2.1 Participants

Forty-two 7-month-old infants participated in this study (mean age 6 months 30 days; range 6 months 15 days to 7 months 15 days; 12 female). Twenty-six infants were tested in Condition 1 (16 in Condition 1A and 10 in Condition 1B). Sixteen infants were tested in Condition 2. Data from 8 additional infants were excluded due to parent interference (n = 2), excessive fussiness (n = 3), failure to complete the experiment (n = 2), or video equipment malfunction (n = 1). Parents of all infants gave written informed consent to a protocol approved by the local Institutional Review Board.

3.2.2 Design

Infants were seated in front of two peripheral monitors. During each trial, one monitor displayed a stream of images that was constant in contour length while the other monitor displayed arrays that alternated twofold or threefold in contour length.

3.2.3 Stimuli

We used a custom Matlab (Mathworks) program that utilized a random walk technique to create shapes that varied in contour length while holding surface area constant. In Condition 1A, the images in both streams contained arrays of 6 black irregularly shaped items on a white background (see Figure 5A). The items in each array were constructed with an area of 6 square pixels and a contour length of 12 pixels or an area of 9 square pixels and a contour length of 24 pixels. These shapes were then scaled up to a mean diameter of 2 cm. Note than in Condition 1A, the twofold change in contour length was accompanied by a 1.5-fold change in surface area. Prior research indicates that this change in surface area is below the discrimination threshold for 7-month-old infants (Brannon, Lutz, & Cordes, 2006; Cordes & Brannon, 2008b; 2011; M. E. Libertus et al., 2014), though it is possible that there could be an additive effect in combination with contour length changes. This concurrent change in surface area was eliminated in Condition 1B and Condition 2. In Condition 1B, there were 10 (rather than 6) items in each array, and the items in each array were larger in size and red in color to increase their saliency. The items in each array had a constant area of 60 square pixels and contour lengths of either 60 or 120 pixels (see Figure 5B). The shapes were then scaled up to a mean diameter of 2.5 cm. In Condition 2, the images contained arrays of 10 irregularly shaped red items on a white background. The shapes were constructed with a constant area of 60 square pixels and had contour lengths of either 40 or 120

pixels (see Figure 5C). The shapes were then scaled up to a mean diameter of 1.3 cm. In all conditions, each of the items in a given array had the same contour length but a unique shape. Accordingly, both the contour length of the individual elements and the cumulative contour length of the array varied by a factor of two (Conditions 1A and 1B) or three (Condition 2) while surface area remained constant (Conditions 1B and 2) or varied by a factor of 1.5 (Condition 1A).

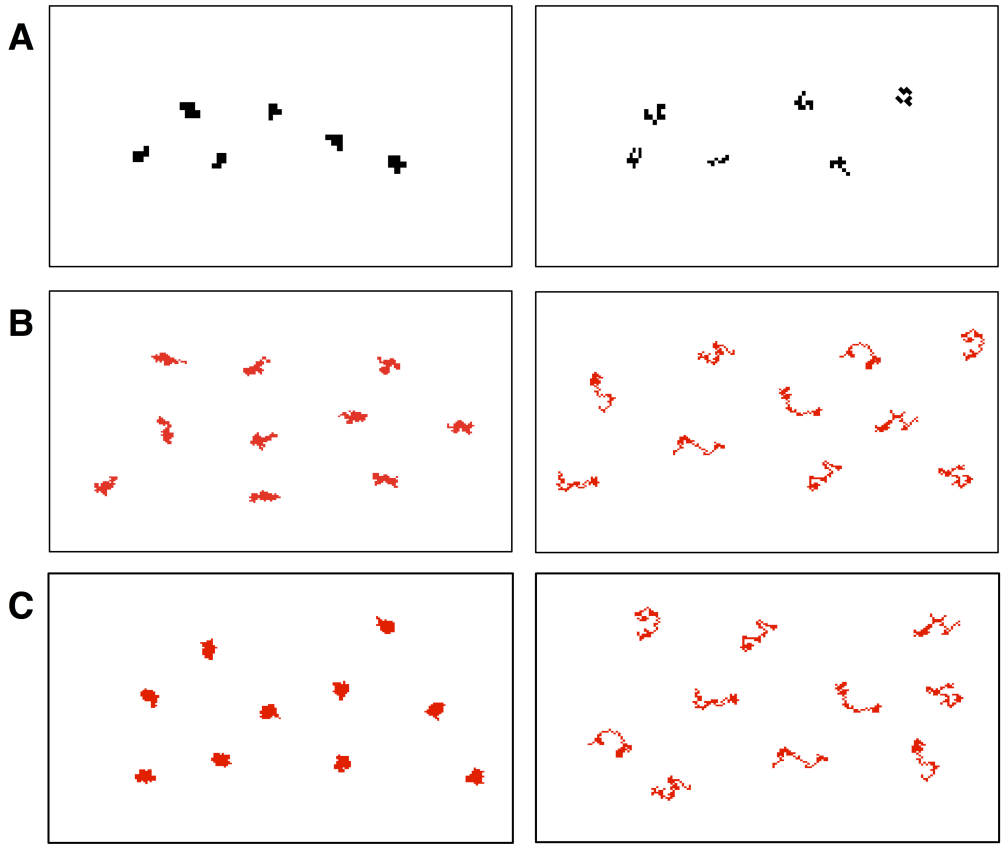

Figure 5: Example stimuli. Smaller contour lengths are displayed on the left and larger contour lengths are displayed on the right. (A) Condition 1 (twofold change in contour with a 1.5-fold change in area. (B) Condition 2 (twofold change in contour with a constant area). (C) Condition 3 (threefold change in contour with a constant area).

3.2.4 Procedure

Infants sat in a high chair or on a parent's lap approximately 105 cm away from three 17-inch monitors. Before each trial, the central screen displayed a colorful attractor video to orient infants' attention directly ahead. As soon as the infant looked at the attractor stimulus, an experimenter manually started each trial. Each stimulus array was presented for 500 ms followed by 300 ms of black screen. Each stream consisted of four

different alternating arrays. Infants were tested with four 60-second trials. The constant and changing streams alternated sides across the four trials for each infant, and the constant stream appeared on the left side first for 50% of the infants. Half of the infants viewed a constant contour stream with arrays that had the longer contour length and the other half of the infants viewed arrays in which the shorter contour length were presented in the constant stream. Infants' looking behavior was digitally recorded and analyzed offline. An experienced coder blind to the experimental condition coded looking behavior using a custom RealBasic program (K. Libertus, 2008). A second coder, also blind to the conditions, re-coded 25% of the data and reliability was extremely high across all conditions (r = .97).

3.2.5 Data analysis

Looking time toward each of the visual streams was measured as a percentage of each infant's total stimulus-directed looking time. Preference scores were calculated by subtracting the proportion of time spent looking at the constant stream from the proportion of time spent looking at the changing stream. A positive preference score therefore indicates a preference for the changing stream, whereas a preference score near zero indicates equal time spent looking at the constant and changing streams.

3.3 Results

Preference scores in each condition were analyzed using one-sample t-tests comparing the observed preference score to a chance expectation of zero. Infants did not

exhibit a preference for the changing contour stream in either Condition 1A (mean preference score = -.08, t(15) = -.02, p = .98) or Condition 1B (mean preference score = -2.70, t(9) = -1.09, p = .30). Furthermore, infants' preference scores did not differ in Conditions 1A and 1B (t(24) = .51, p = .61). Thus infants failed to detect a twofold change in cumulative surface area in Condition 1. In Condition 2, however, where contour length in the changing stream differed by a 1:3 ratio, infants exhibited a clear preference for the changing stream (mean preference score = 4.15, t(15) = 2.41, $p < .05$) (Figure 2). Nonparametric analyses confirm these results. In Conditions 1A & 1B, only 7 out of 16 and 5 out of 10 infants, respectively, looked longer at the changing contour stream (binomial *ps* > .5). By contrast, in Condition 2, 12 out of 16 infants looked longer at the changing contour stream in comparison to the constant contour stream (binomial $p < .05$).

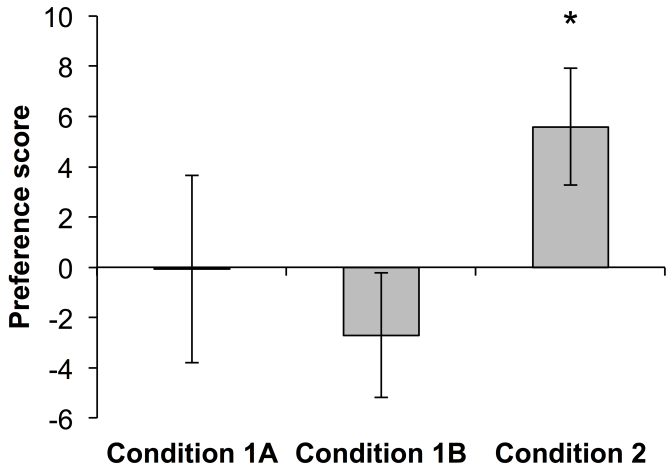

Figure 6: Infants did not exhibit a significant preference for a twofold change in contour length (Conditions 1A and 1B) but did exhibit a significant preference when the change in contour length was threefold (Condition 2). Error bars represent SEM.

We next compared infants' change detection preference scores for a threefold change in cumulative contour (Condition 2) to preference scores from previously published data in which infants of the same age were tested in the same paradigm with threefold changes in cumulative area (M. E. Libertus et al., 2014) and number (M. E. Libertus & Brannon, 2010). We found a significant linear trend for increasing preference scores, with the lowest scores for cumulative area, intermediate scores for contour length, and the highest scores for numerosity change detection ($F(1,47) = 4.484$, $p < .05$) (Figure 7). This finding is consistent with previous studies indicating that 7-month-old infants do not detect a threefold change in cumulative area (Cordes & Brannon, 2011; M. E. Libertus et al., 2014) but are sensitive to a twofold change in number (Brannon et al., 2004; M. E. Libertus & Brannon, 2010; Xu & Spelke, 2000).

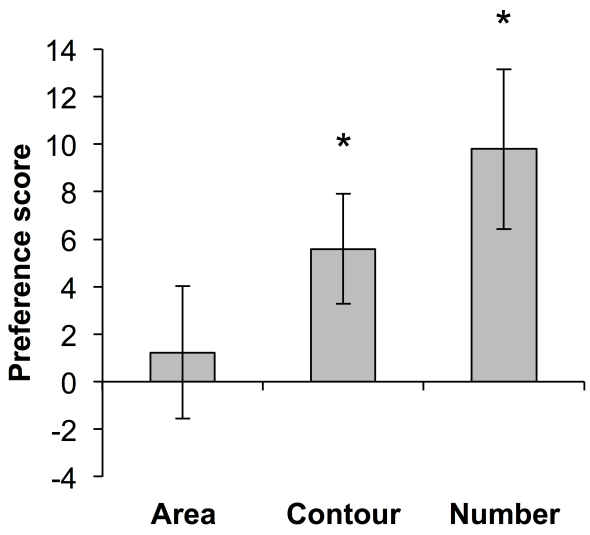

Figure 7: Comparison of preference scores at a 1:3 change for cumulative area, cumulative contour, and number (data for area from Libertus, Starr, & Brannon 2014; data for number from Libertus & Brannon, 2010). Error bars represent SEM.

3.4 Discussion

In the present study we sought to identify the limits of 7-month-old infants' ability to detect changes in contour length. To overcome the inherent confound between contour length and surface area present in dot arrays, we created a novel stimulus set that enabled us to vary contour length while holding surface area constant. We then employed these shapes in a change detection paradigm (M. E. Libertus & Brannon, 2010; Oakes, Ross-Sheehy, & Luck, 2006; Ross-Sheehy et al., 2003). The main result was that infants successfully detected a threefold change in cumulative contour length but were unable to detect a twofold change. Because 7-month-old infants readily detect twofold changes in number (Brannon et al., 2004; M. E. Libertus & Brannon, 2010; Xu & Spelke, 2000), this suggests that infants' acuity for the cumulative contour length in large arrays

is actually worse than their acuity for number. However, because infants' require a fourfold change in cumulative surface area in order to recognize change (Cordes & Brannon, 2011; M. E. Libertus et al., 2014), it appears that infants are more sensitive to changes in contour length than they are to changes in surface area.

In a few previous studies, it has been reported that infants can detect smaller changes in cumulative contour length, such as 1.5-fold changes, in arrays with two or three items (Clearfield & Mix, 1999; 2001; Cordes & Brannon, 2009). However, there are two critical differences between these prior findings and the results reported here. First, prior studies manipulated contour length in arrays made up of square elements, such that changes in contour length were concurrent with large changes in area. In the present study, we used irregular shapes that enabled us to manipulate contour length while holding surface area constant. It is possible that contour length and surface area have additive effects that facilitate representation and discrimination. If this true, then concurrent changes in both area and contour may lead to more precise representations compared to when each dimension is manipulated independently (c.f. Cantrell & Smith, 2013). Note however that in Condition 1A, in which a twofold change in contour length was paired with a 1.5-fold change in surface area, we did not find evidence for these additive effects. A second difference is that the prior studies employed arrays of two or three elements in habituation paradigms, which may encourage the use of object tracking mechanisms rather than approximate number or approximate magnitude

representations (see Feigenson et al., 2004 for a review). Therefore, although object tracking mechanisms may enable more precise representations of contour length, it appears that when many items are present, acuity for contour length, like acuity for surface area, is not as precise as that of number (Cordes & Brannon, 2011; M. E. Libertus et al., 2014).

An additional point to consider is that in Conditions 1B and 2, both the cumulative contour length and the contour length of each individual element varied twofold or threefold in the changing stream. Therefore, in theory infants could have attended to either the change in the contour length of an individual element or to the change in the cumulative contour length of the entire array. Prior research, however, suggests that infants do not readily attend to continuous variables of individual elements when presented with large arrays. For example, Cordes and Brannon found that 7-month-old infants required a fourfold change in cumulative surface area of large arrays to detect a change despite readily detecting a twofold change in surface area when presented with a single element (Cordes & Brannon, 2011).

In the present study, changes in contour length were unavoidably accompanied by a change in shape due to the constraint of holding surface area constant. As a consequence, infants could have attended to changes in either contour length or overall shape of the individual elements. However, we found in two separate samples (Conditions 1A and 1B) that infants failed to detect a twofold change in contour length

when changes in contour length covaried with element shape. Therefore, it seems unparsimonious to argue that that shape changes accounted for discrimination with a threefold change in contour length in Condition 2 when they were not sufficient to yield discrimination with a twofold change in contour length in Condition 1.

In sum, our results are difficult to reconcile with the idea that numerical information is more difficult for infants to extract compared to information regarding contour length. Our results call into question the proposal that small changes in contour length present in previous studies of infant numerical cognition drive discrimination performance. Instead, it appears that infants are less sensitive to contour changes than they are to changes in numerosity, which lends credence to the argument that number is indeed a salient dimension for infants.

4. Developmental continuity in the link between sensitivity to numerosity and physical size

Converging evidence suggests that representations of number, space, and other dimensions depend on a general representation of magnitude. However, it is unclear whether there may be a privileged relation between certain magnitude dimensions or if all continuous magnitudes are equivalently related. Four-year-old children and adults were tested with three magnitude comparison tasks – nonsymbolic number, line length, and luminance – to determine whether individual differences in sensitivity are stable across dimensions. A Weber fraction (w) was calculated for each participant with each stimulus dimension. For both children and adults, accuracy and w values for number and line length comparison were significantly correlated, whereas neither accuracy nor w was correlated for number and luminance comparison. However, although line length and luminance comparison performance were not correlated in children, there was a significant relation in adults. These results suggest that there is a privileged relation between number and line length that emerges early in development and that relations between other magnitude dimensions may be constructed over the course of development.

4.1 Introduction

The ability to represent and reason about magnitude emerges early in human development and is widespread throughout the animal kingdom. Quantities such as

number, physical extent, and time can be expressed on a continuum of increasing amount, and representations of these quantities follow Weber's Law, suggesting that they share an analog magnitude format. Beyond sharing a common format, it has also been suggested that representations of different physical magnitudes may arise from a generalized magnitude system that is dependent on shared circuitry in the parietal cortex (Bueti & Walsh, 2009; Cantlon, Platt, & Brannon, 2009b; R. Cohen Kadosh, Lammertyn, & Izard, 2008; Walsh, 2003). Magnitude dimensions that are structurally similar and automatically aligned with one another can be said to be functionally overlapping (Srinivasan & Carey, 2010). However, it is unclear whether this functional overlap reflects a privileged relation between number, physical extent, and duration or whether it extends to all continuous dimensions (Bueti & Walsh, 2009; Cantlon, Platt, & Brannon, 2009b; Lourenco & Longo, 2011; Walsh, 2003).

Early evidence for a functional overlap between magnitude dimensions comes from Meck and Church's (1983) seminal study demonstrating that rats spontaneously and simultaneously encode numerical and temporal information using a common mechanism. Additional supporting evidence comes from studies in humans demonstrating that irrelevant magnitude dimensions cause interference during magnitude comparison tasks. For example, judgments regarding the physical size of two Arabic digits are faster when the larger physical digit represents the larger numerical magnitude compared to when the physical and numerical magnitudes are incongruent

(Tzelgov et al., 1992). This cross-dimensional interference suggests that not only are both numerical and physical magnitudes processed automatically, even when they are task irrelevant, but also that the irrelevant information interferes with the comparison process in the relevant dimension. Similar instances of interference also occur for judgments of number, size, duration, and luminance when the irrelevant dimensions are varied orthogonally to the target dimensions (Casasanto & Boroditsky, 2008; R. Cohen Kadosh & Henik, 2006; V. Dormal & Pesenti, 2012; V. Dormal, Seron, & Pesenti, 2006; Xuan, Zhang, He, & Chen, 2007) . For instance, when judging the relative duration of two stimuli, larger stimuli (e.g., physically larger, more numerous, or more luminous) are judged to have longer durations than smaller stimuli (e.g., physically smaller, less numerous, or less luminous) (Xuan et al., 2007).

The functional overlap between number, physical extent, and duration appears to be present early in human development. Infants, like adults, spontaneously align representations of number, physical extent, and duration. For example, infants habituated to congruent pairs of lines and tones (e.g., long lines presented simultaneously with long tones) or lines and numerosities (e.g., long lines presented simultaneously with arrays containing large numbers of dots) look longer at incongruent pairs (e.g., long lines paired with short tones or small numerosities) compared to congruent pairs at test (de Hevia & Spelke, 2010; Srinivasan & Carey, 2010). Likewise, infants habituated to arbitrary color-magnitude pairings (e.g., large objects:

black with stripes; small objects: white with dots) expect the color-pattern pairs to hold across the dimensions of size, number, and time (e.g., more numerous arrays and longer durations: black with stripes; less numerous arrays and shorter durations: white with dots), and look longer when the pattern is violated (e.g., less numerous arrays and shorter durations: black with stripes; more numerous arrays and longer durations: white with dots) (Lourenco & Longo, 2010). These results suggest that infants are sensitive to the relational congruence between large numerosities, large objects, and long durations in comparison to small numerosities, small objects, and short durations. Furthermore, these associations have recently been found in neonates just hours after birth (de Hevia et al., 2014), suggesting that they are present even before infants have had a chance to experience correlations between these dimensions in the external world.

If representations of magnitude are functionally overlapping, then it follows that individual differences in magnitude comparison acuity should correlate across different dimensions. Recent studies probing this question, however, have provided conflicting answers. Some studies with adults have found correlations between acuity for number and line length or between number and the cumulative area of dot arrays (DeWind & Brannon, 2012; Lourenco et al., 2012), whereas other studies have failed to find a correlation in comparison performance between magnitude dimensions (Agrillo et al., 2013; Odic et al., 2013). Furthermore, DeWind and Brannon (2012) found that while numerical comparison training resulted in an improvement in Weber fraction for the

same numerosity task, the improvement did not transfer to a line length comparison task. These prior studies thus provide contradictory evidence as to whether representations of number and length or area draw on common or independent cognitive resources.

There is also research suggesting that not all magnitude dimensions share the same level of functional overlap, particularly early in development. As described above, infants recognize congruity between numerosity and line length or numerosity and duration. However, infants do not readily form associations between line length and auditory volume when tested in the same paradigms (Srinivasan & Carey, 2010) or between numerosity and luminance (de Hevia & Spelke, 2013) when tested in the same paradigms. Likewise, preschool children appear to be better at mapping number to line length compared to mapping number to brightness or line length to brightness (de Hevia et al., 2012). Furthermore, adults do not spontaneously align auditory volume and line length when the task does not explicitly promote such a mapping (Srinivasan & Carey, 2010). These findings raise the question of whether the level of functional overlap found between representations of number, physical extent, and duration may be greater than the overlap exhibited by other magnitude dimensions.

The present study was designed to assess the relation between different magnitude representations over development. Although the infant studies reviewed above suggest that the relation between some dimensions may be privileged, studies with children and

adults that have parametrically measured individual differences in acuity have only assessed acuity for spatial magnitudes (e.g., number and physical extent) and have not included nonspatial dimensions such as loudness or luminance. Given the infant data, we predicted that acuity for line length and numerosity would be more closely linked than for other magnitudes. To test this hypothesis we tested four-year-old children and adults with three magnitude comparison tasks: nonsymbolic number, line length, and luminance.

4.2 Materials and Methods

4.2.1 Participants

Data from 85 participants were included in the final analyses: 53 children (mean age: 3 years 11 months; range: 3 years 6 months to 4 years 5 months; 28 females) and 32 adults (mean age: 19 years 3 months, range 18 years 0 months to 21 years 7 months; 21 females). Seventeen additional children were excluded from all analyses due to failure to complete two or more of the three tasks and two additional adults were excluded due to chance-level performance on all tasks. Twelve of the 85 participants included in the final sample had incomplete data sets. Specifically, data from 3 children were excluded from analysis of the number task because they touched the stimulus with the largest individual dots exclusively, and data from 1 adult was excluded due to chance-level performance. Data from 2 children were excluded from analysis of the line length task due to computer error. Data from 5 children were excluded from analysis of the

luminance task due to computer error and data from 1 adult was excluded due to chance-level performance. Adult participants and parents of child participants gave written informed consent to a protocol approved by the local Institutional Review Board.

4.2.2 Stimuli and Design

Stimulus presentation and response collection were controlled using Psychophysics Toolbox Version 3 for Matlab (Mathworks). In the number task, stimuli consisted of two arrays of colored dots surrounding a central cartoon image on a black background (Figure 8A). The number of dots in each array ranged from 6 to 18. Five ratios were tested (larger number/smaller number): 3.0, 2.0, 1.5, 1.2, and 1.14. To control for non-numerical properties of the arrays, the average item size was equated in half of the trials, and cumulative surface area was equated in the other half of the trials. Therefore, in half of the trials the numerically larger had equally sized dots to the numerically smaller array, and in the other half the numerically larger array had smaller sized dots than the numerically smaller array. Dot arrays were drawn in real time before each trial such that each trial featured novel arrays. In the luminance task, stimuli consisted of two gray squares surrounding a central cartoon image on a black background (Figure 8B). The luminance of the squares ranged from 50 to 150 cd/m^2. The same five ratios were tested: 3.0, 2.0, 1.5, 1.2, and 1.14. A Spyder4Elite calibration system (Datacolor) was used to perform gamma correction and determine accurate luminance levels. In the line length

task, stimuli consisted of two colored vertical lines surrounding a central cartoon image on a black background (Figure 8C). The lines lengths ranged in size from 108 to 216 pixels with a constant width of 20 pixels. The vertical position of the lines was jittered such that the distance between the endpoints of the line and the frame of the monitor could not be used as a reference. Five ratios were tested: 1.5, 1.2, 1.14, 1.11, and 1.1. Note that while the number and luminance tasks use the same ratios, more difficult ratios were used for the line length task after pilot testing indicated that some children performed at ceiling with the original ratios.

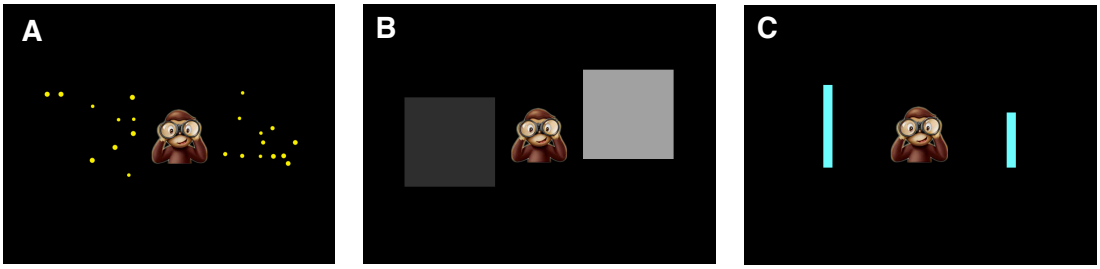

Figure 8: Example stimuli for the nonsymbolic number task (A), luminance task (B), and line length task (C).

4.2.3 Procedure

Participants performed the number, line length, and luminance tasks in a counterbalanced order within a single experimental session lasting approximately 20 minutes. Participants were instructed to choose the array containing the "most dots", the "longest line", or the "brightest square." All ratios were presented in a randomized order.

In the child version of the experiment, each task was preceded by three practice trials to ensure that children understood how to play the game. Each task consisted of 50 test trials. On each trial, the stimuli appeared on the screen until the child made a response by touching one of the two stimuli. Immediately following the response, children received differential auditory feedback (a high tone for correct responses and a low tone for incorrect responses) and were given a small sticker for each correct response to maintain motivation throughout the session.

The adult version of the experiment consisted of 150 test trials for each task. On each trial, the stimuli were displayed for 500 ms followed by a 2500 ms response window. Adults pressed a button (F or J) to indicate their response. Because adults immediately understood the task and did not require external motivation to maintain focus, practice trials and feedback were not included. Cronbach's alpha was calculated for each task separately for children and adults, and reliability was high (.79, .80, and .67 for number, line length, and luminance respectively in children; .52, .76, and .85 in adults).

4.2.4 Modeling

We used a psychophysical modeling technique to calculate a Weber fraction (w) that served as an estimate for each participant's acuity in each of the three magnitude dimensions (e.g., Halberda & Feigenson, 2008). We modeled the error rate in the tasks as $\frac{1}{2} erfc\left(\frac{n_1 - n_2}{\sqrt{2}w\sqrt{n_1^2 + n_2^2}}\right)$, where n_1 is the larger value, n_2 is the smaller value, w is a

measure of variance in the internal representation, and *erfc* is the complementary error function. The resulting value of w represents the noise in each participant's internal magnitude representations, such that lower values of w correspond to less noise (i.e., higher acuity). Due to highly variable responses and performance at or below chance, the model was not always able to settle on a fit and in those cases data from those participants for those conditions were excluded from further analyses involving w. This was the case for 15 children in the numerical task, 1 adult in the line length task, and 1 child in the luminance task.

4.3 Results

For each task we analyzed accuracy using a repeated-measures analysis of variance (ANOVA) with ratio as a within-subjects factor and age group as a between-subjects factor (see Table 1 for descriptive statistics). In the nonsymbolic number task, there was a main effect of age group ($F(1,77) = 32.68$, $p < .001$, $\eta^2p = .30$), indicating that adults were more accurate than children (Figure 9A). There was also a main effect of ratio ($F(4,308) = 37.78$, $p < .001$, $\eta^2p = .33$) . In addition, there was a significant age group by ratio interaction ($F(4,308) = 6.14$, $p < .005$, $\eta^2p = .07$) indicating that the ratio effect was stronger in adults compared to children. In the line length task there was a marginal effect of age group ($F(1,79) = 3.46$ $p = .07$, $\eta^2p = .04$), reflecting higher performance for adults compared to children (Figure 9B). There was also a main effect of ratio ($F(4,316) = 50.53$, $p < .001$, $\eta^2p = .39$). The age group by ratio interaction was not significant ($p = .18$).

Finally, in the luminance task there was a main effect of age group, again reflecting better performance for adults ($F(1,78) = 6.69$, $p < .05$, $\eta^2p = .08$), and a main effect of ratio ($F(4,312) = 86.08$, $p < .001$, $\eta^2p = .53$) (Figure 9C). The age group by ratio interaction was not significant ($p > .8$).

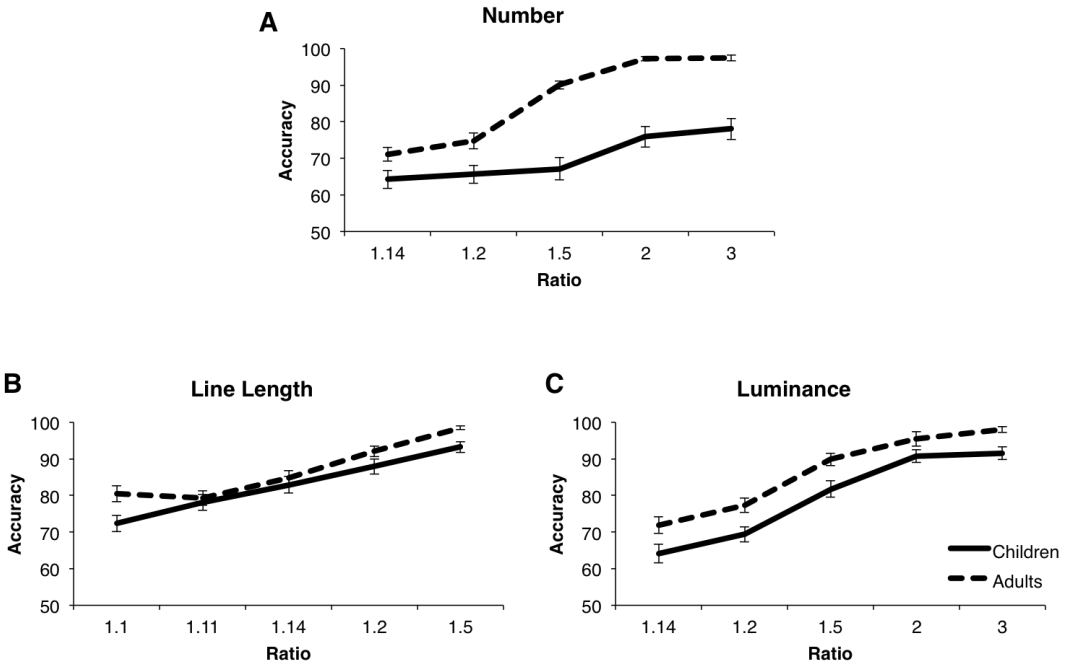

Figure 9: Mean performance in the nonsymbolic number task (A), line length task (B), and luminance task (C). Error bars represent SEM.

Table 1: Descriptive statistics for each task.

	Number		**Line Length**		**Luminance**	
	Accuracy (SEM)	**w (SEM)**	**Accuracy (SEM)**	**w (SEM)**	**Accuracy (SEM)**	**w (SEM)**
Children	70.18 (2.73)	.50 (.06)	82.88 (2.03)	.12 (.01)	79.53 (2.06)	.31 (.03)
Adults	85.93 (1.64)	.20 (.02)	89.28 (1.66)	.08 (.01)	87.94 (1.59)	.18 (.02)

To determine the relation between acuity for different magnitudes, we performed correlational analyses between participants' accuracy in each of the three tasks (Figures 10 and 11). The main finding was that number and line length were significantly correlated in both children ($r(47) = .58$, $p < .001$) and in adults ($r(31) = .46$, $p < .01$). In contrast, neither children nor adults exhibited a significant correlation in performance for number and luminance (children: $r(47) = .26$, $p = .07$; adults: $r(30) = .20$, $p = .30$). Finally, performance in the line length and luminance tasks was not correlated in children ($r(51) = .21$, $p = .12$), but was significantly correlated in adults ($r(31) = .60$, $p < .001$).

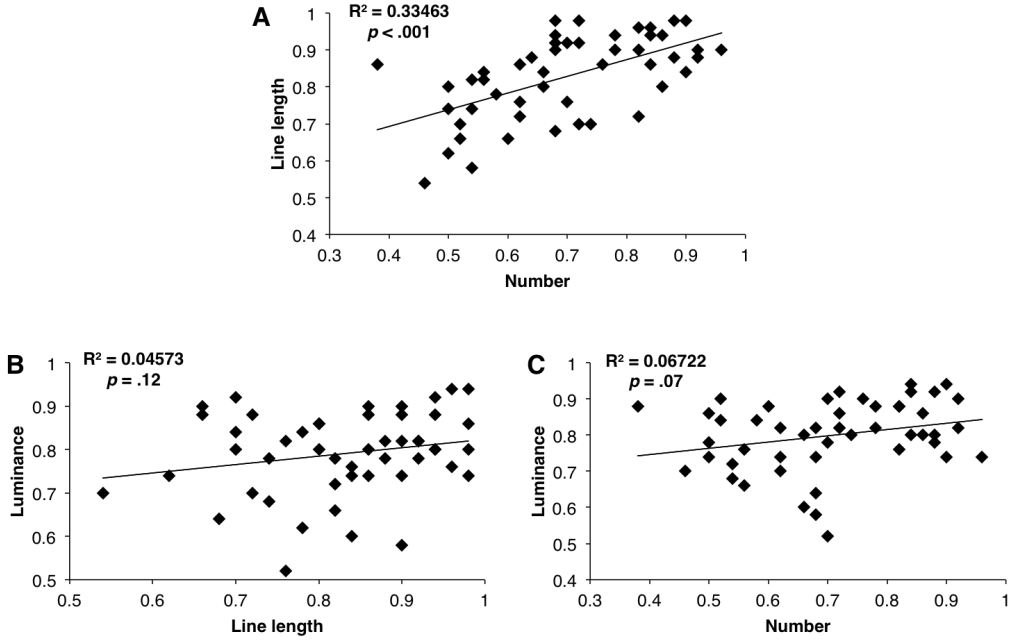

Figure 10: Correlational analyses illustrating the relation between children's performance in the different magnitude comparison tasks. Number and line length (A) are significantly correlated whereas neither line length and luminance (B) nor number and luminance (C) exhibit a significant correlation.

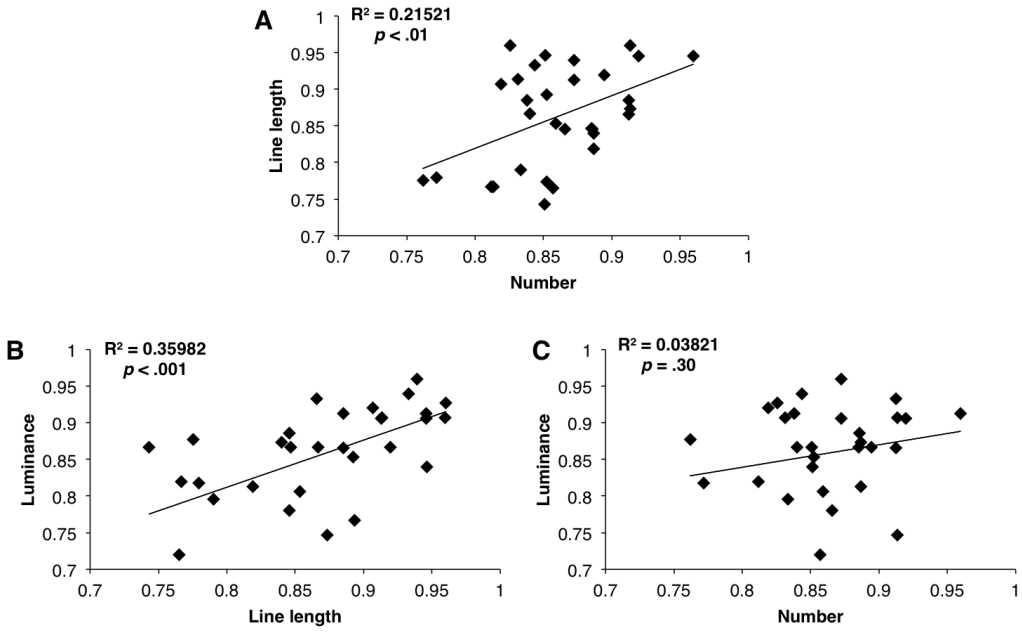

Figure 11: Correlational analyses illustrating the relation between adults' performance in the different magnitude comparison tasks. Number and line length (A) and line length and luminance (B) are significantly correlated whereas number and luminance (C) do not exhibit a significant correlation.

The same analyses were repeated using Weber fractions (w) rather than accuracy for each participant. The w values for one child in the number task, two children in the line length task, and one child and one adult in the luminance task differed from the group means by more than three standard deviations, and these values were excluded from further analyses. The analyses based on w revealed the same pattern as those based on accuracy. The w values for number and line length were significantly correlated in children ($r(34) = .34$, $p < .05$) and in adults ($r(30) = .47$, $p < .01$). Number and luminance did not exhibit a significant correlation in either children ($r(31) = .19$, $p = .30$) or adults ($r(29) = -.07$, $p = .73$). There was again no correlation between line length and luminance

for children (r(42) = -.19, p = .21) , but the correlation was significant in adults (r (29)= .40, p < .05).

To compare the relative strengths of the correlations between task pairs, we used Lee and Preacher's method for comparing correlation strengths between dependent correlations (I. A. Lee & Preacher, 2013). This method involves converting the correlation coefficients into z-scores and then computing asymptotic covariance of the estimates (Steiger, 1980) for use in an asymptotic z-test to account for the samples being dependent rather than independent. Given that accuracy and w revealed the same pattern of correlations and accuracy allowed use of the full sample, we conducted the remaining analyses on accuracy. In children, the relation between number and line length was significantly greater than the relations between number and luminance or line length and luminance (zs > 2.03, ps < .05). In adults, there was a trend for the correlation between number and line length correlation to be greater than that between number and luminance (z = 1.68 p = .09), and the correlation between line length and luminance was significantly greater than that between number and luminance (z = 2.45, p < .05). However, the correlation between number and line length was not significantly different than that between line length and luminance (z = .77, p = .45).

A final analysis indicated that there was no statistical difference between the strength of the significant correlation between number and line length between children and adults (z = .68, p = .50), suggesting that the relation between these dimensions

remains stable between early childhood and adulthood. However, the correlation between line length and luminance was significantly stronger in adults than in children ($z = 2.02$, $p < .05$).

4.4 Discussion

The primary goal of this research was to investigate the functional overlap between representations of different magnitude dimensions by assessing the relations between acuity for three types of magnitudes: number, line length, and luminance. The study was designed to ask whether there is a privileged relation between number and line length compared to luminance and whether these relations change over development. Our first finding was that both children and adults showed a positive relationship between acuity for number and line length whereas number and luminance acuity were not significantly correlated for either age group. In contrast, we found age-related differences in the relationship between line length and luminance acuity, in that acuity for these dimensions was correlated in adults but not in children. These results generally support the hypothesis that number has a privileged relation with space that does not extend to all other magnitude dimensions. They are also consistent with research in adults demonstrating a correlation between numerical acuity and acuity for physical extent (DeWind & Brannon, 2012; Lourenco et al., 2012), as well as with previous work demonstrating that infants and preschool children spontaneously form associations between representations of number and physical extent but fail to do so when relating

these dimensions to luminance or volume (de Hevia et al., 2012; de Hevia & Spelke, 2013; Srinivasan & Carey, 2010).

Notably, while the relation between number and line length was comparable in four-year-old children and adults, the relation between line length and luminance appears to change over development. Whereas four-year-old children's acuity for line length and luminance were not correlated, by adulthood the correlation was significant. By one theory of generalized magnitude representations, infants enter the world with an undifferentiated sense of magnitude and come to segregate their representations over development (Walsh, 2003). This theory predicts that acuity for number, line length, and luminance would be more strongly correlated in young children and that the correlations would dissipate over development. Our results, however, reveal a different pattern and are more consistent with the idea that new relations between magnitudes can be constructed over the course of development.

Research measuring looking-time in infants has shown that in the first year of life infants are already sensitive to congruent pairings of number and physical size (de Hevia et al., 2014; de Hevia & Spelke, 2010; Lourenco & Longo, 2010) but do not recognize congruency for pairings of number and luminance (de Hevia & Spelke, 2013). Likewise, preschool children can explicitly form mappings between number and line length yet are unable to do so for number and luminance (de Hevia et al., 2012). Although adults can readily learn or form meaningful mappings across any ordered

continuum (e.g., Stevens, 1957), the finding that this ability is not present early in development suggests that the relation between number and size may be primary to relations with nonspatial dimensions such as luminance.

If not all magnitude dimensions are related to one another in infancy, when and how do adults develop the ability to make vivid cross-dimensional associations? Future studies could examine this developmentally to assess when these relations becomes robust. However, two factors that may promote the formation of cross-dimension relations are analogical reasoning and the common language used across magnitude dimensions (e.g., more, less). In particular, the process of comparison may promote the recognition of the structural similarities between prothetic magnitude dimensions (Gentner & Medina, 1998). Older children, for example, readily match relational similarity (e.g., increasing monotonic order) both within and across perceptual dimensions, whereas younger children succeed only at same-dimension matches (Kotovsky & Gentner, 1996). When younger children are given practice with same-dimension matches before being asked to perform across-dimension matches, they then succeed at making cross-dimension matches, suggesting that experience with same-dimension matches facilitates children's recognition of abstract relational rules that can be applied across perceptual dimensions. Practice therefore appears to encourage children to compare relations within and across dimensions, helping them to align representations of previously independent magnitude dimensions.

Common vocabulary used for different magnitude dimensions may also promote the formation of associations between previously independent dimensions (Gentner & Medina, 1998). For example, although infants do not spontaneously align representations of physical extent and loudness, children come to form associations between these dimensions by two to five years of age (Marks, 1978; Smith & Sera, 1992). One possibility is that the acquisition of words such as "loud," "quiet," and "more" underlies this developmental shift. When children grasp that "big" and "loud" are both words that represent "more," they may become able to recognize the structural similarity between physical extent and volume and form a congruent alignment between these dimensions (Smith & Sera, 1992). Other research suggests that even though adults can consciously form mappings between volume and physical extent, this mapping does not always occur spontaneously (Srinivasan & Carey, 2010), which is consistent with our conclusion that associations between magnitude dimensions formed later in childhood may differ qualitatively from the cross-dimensional associations that exist early in development. As such, the relation between number and physical size may reflect an early-developing, privileged relation between these dimensions whereas the relation between size and luminance found in adults may reflect a relation that has been constructed through experience. Further work with children is needed to investigate the emergence of the relation between size and luminance and the possible contributions of analogy and language.

It is also likely that there are different neural substrates that support relations between different magnitude types. The role of the parietal cortex in numerical representations has been well documented (see Nieder & Dehaene, 2009 for review). Parietal cortex, however, is also involved in representations of other continuous magnitudes. Comparison judgments for number, time, size, and luminance have all been found to engage neighboring or overlapping regions of parietal cortex, and cross-dimensional interference between these dimensions modulates parietal activity (R. Cohen Kadosh et al., 2005; R. Cohen Kadosh, Cohen Kadosh, & Henik, 2007a; Fias et al., 2003; Pinel et al., 2004). Furthermore, single unit recording data from macaques indicates that individual neurons tuned to numerical, temporal, and spatial magnitudes co-exist in posterior parietal cortex (Leon & Shadlen, 2003; Onoe et al., 2001; Sawamura et al., 2002; Tudusciuc & Nieder, 2007). Together, these results suggest that the parietal lobe is a locus of magnitude representation and magnitude comparison. However, there is also evidence suggesting a role for occipito-temporal cortex in magnitude representations, particularly with regards to the dimensions of size and luminance (Pinel et al., 2004). Consistent with the present results, an fMRI study investigating comparative judgments for number, size, and luminance found cross-dimensional interference effects for number and line length and for line length and luminance, but no interference between number and luminance (Pinel et al., 2004). Critically, these patterns of behavioral interference predicted the amount of overlap in the brain areas engaged by each of the

comparison judgments. Number and size judgments recruited overlapping areas of the parietal cortex whereas number and line length judgments recruited overlapping areas of occipito-temporal cortex. No areas of overlap were found between number and luminance judgments. This suggests the existence of both distributed and overlapping neural populations that contribute to magnitude judgment, with interference effects arising in multiple cortical areas.

4.4.1 Conclusions

We investigated the relation between acuity for three magnitude dimensions in four-year-old children and adults to shed light on the question of whether number and size share a privileged relation. We found that in both young children and adults, numerical acuity predicted line length acuity (or vice versa). Furthermore, line length acuity was related to luminance acuity in adults but not in children. No relations between numerical and luminance acuity were found. Therefore, the present results support the idea that number and physical extent exhibit signs of a functional overlap early in human development and maintain a privileged relation. Other dimensions such as luminance may become integrated with a general sense of magnitude later in childhood but may never reach the same level of functional overlap.

5. Number sense in infancy predicts mathematical abilities in childhood

Human infants in the first year of life possess an intuitive sense of number. This preverbal number sense may serve as a developmental building block for the uniquely human capacity for mathematics. In support of this idea, several studies have demonstrated that nonverbal number sense is correlated with mathematical abilities in children and adults. However, there has been no direct evidence that infant numerical abilities are related to mathematical abilities later in childhood. Here, we provide evidence that preverbal number sense in infancy predicts mathematical abilities in preschool-aged children. Numerical preference scores at 6 months of age correlated with both standardized math test scores and nonsymbolic number comparison scores at 3.5 years of age, suggesting that preverbal number sense facilitates the acquisition of numerical symbols and mathematical abilities. This relationship held even after controlling for general intelligence, indicating that preverbal number sense imparts a unique contribution to mathematical ability. These results validate the many prior studies purporting to show number sense in infancy and support the hypothesis that mathematics is built upon an intuitive sense of number that predates language. Please note that this chapter is adapted from Starr, Libertus, and Brannon (2013b).

5.1 Introduction

Where does the uniquely human capacity for abstract mathematical concepts come from? What are the ontological building blocks that scaffold our ability for representing

number symbolically and performing exact arithmetic? One hypothesis is that the preverbal, nonsymbolic numerical capacities exhibited by human infants in the first year of life serve as a conceptual basis for learning to count and acquiring symbolic mathematical knowledge (Dehaene, 1997; Gallistel & Gelman, 1992). Although many cognitive abilities contribute to math achievement, including working memory, inhibition, and other executive functions (M. L. Meyer et al., 2010; Passolunghi et al., 2008), symbolic mathematics is commonly thought to build on a domain-specific nonverbal numerical representation (Feigenson et al., 2013). This system, the approximate number system (ANS), is an evolutionarily and ontogenetically ancient system that allows approximate representation of number without the need to count or rely on numerical symbols (Dehaene, 1997; Feigenson et al., 2004).

Support for this hypothesis comes from a handful of studies that have shown a correlation between math ability and individual differences in ANS acuity. For example, ANS acuity in adolescence retroactively predicts math ability in elementary school (Halberda et al., 2008) and ANS acuity in preschool-aged children correlates with their current and future math performance (Gilmore et al., 2010; M. E. Libertus et al., 2011; M. E. Libertus, Feigenson, & Halberda, 2013a; Mazzocco et al., 2011; Piazza et al., 2010). In addition, children with dyscalculia, a severe deficit specific to math, have poorer ANS acuity than their typically developing peers (Mazzocco et al., 2011; Piazza et al., 2010). These findings demonstrate that ANS acuity covaries with math ability, but the direction

of this relationship remains unclear. One possibility is that ANS acuity guides the acquisition of the verbal counting system and symbolic math knowledge, such that children with greater ANS acuity learn to count earlier and have enhanced facility with spoken and written numerical symbols (Piazza, 2010). Alternatively, learning the verbal counting system and early symbolic math concepts may refine ANS acuity, such that children with greater symbolic number knowledge end up with greater ANS acuity (Piazza, 2010). To differentiate between these possibilities, it is necessary to investigate ANS acuity before children acquire the verbal counting system and before exposure to mathematics education. If the ANS is truly foundational for symbolic math, then early ANS acuity should inform children's facility with written and spoken numerical symbols.

Can the acuity of the preverbal number sense in the first year of life, well before an infant can count and understand verbal or written numerical symbols, predict later developing math abilities? A major stumbling block for answering this question has been the lack of parametric measures available for studying infant cognition in the first year of life. Our understanding of number sense in preverbal infants comes primarily from studies showing that groups of infants repeatedly shown pictures of the same number of objects (e.g., 8 dots) look longer when arrays with a new numerical value are presented (e.g., 16 dots) (Brannon et al., 2004; M. E. Libertus & Brannon, 2010; Xu, 2003; Xu et al., 2005; Xu & Spelke, 2000). These studies provide binary information (i.e.,

success vs. failure to discriminate between different numbers) about behavior at the group level but do not provide parametric scores of numerical sensitivity at the individual level. The development of a numerical change detection procedure that yields preference scores modulated by the ratio between numerical values in a changing visual stream provides a solution to this disconnect between infant and adult measures of numerical discrimination (M. E. Libertus & Brannon, 2010; Starr, Libertus, & Brannon, 2013a). In the numerical change detection paradigm, infants observe two streams of visual arrays, one of which alternates between two numerical values while the other stays numerically constant and only changes in dot size and arrangement. Infants' preference for the numerically changing stream, as indexed by longer looking times, is thought to reflect the acuity of their ANS, such that infants with better ANS acuity will show a greater preference for the numerically changing stream. At the group level, preference scores are parametrically related to the ratio between the numerical values in the changing numerical stream. However, a particularly promising aspect of this dependent measure is that it is sensitive to individual differences between infants and can be used to track these differences longitudinally. Previous work has demonstrated that individual differences in ANS acuity remain stable during infancy: numerical preference scores at 6 months of age predict numerical preference scores at 9 months of age (M. E. Libertus & Brannon, 2010).

To address the relationship between infants' number sense and their later developing math abilities, we investigated the symbolic and nonsymbolic numerical abilities of 48 3.5-year-old children who had previously been tested in a numerical change detection study at 6 months of age. At 3.5 years of age, we assessed the children's numerical and mathematical understanding, as well as their general intelligence, using four widely used, age-appropriate measures. First, we tested children's ANS acuity using a nonsymbolic numerical comparison task in which they were asked to choose the numerically larger of two dots arrays. Based on performance on this task, we calculated a Weber fraction for each child (*w*, a measure of ANS acuity) using standard psychophysical modeling (DeWind & Brannon, 2012; Halberda et al., 2008; Halberda & Feigenson, 2008; M. E. Libertus et al., 2011; 2012; Piazza et al., 2010). The value of *w* reflects the amount of noise in internal ANS representations, such that lower values of *w* correspond to more precise nonsymbolic numerical representations. Second, we assessed children's math ability using the Test of Early Mathematics Ability (TEMA-3) (H. P. Ginsburg & Baroody, 2003), which is a standardized math test designed for children as young as three years of age that consists of a series of verbally administered questions to assess counting ability, number-comparison facility, numeral literacy, and basic calculation skills. Third, children's knowledge of the verbal counting system was assessed using a variant of the Give-a-Number task (Wynn, 1992) to determine the largest number word whose meaning the child understands exactly.

Finally, children's general intelligence was assessed with the Reynolds Intellectual Assessment Scales (RIAS) (Reynolds & Kamphaus, 2003), a standardized test that assesses both verbal and nonverbal intelligence with a series of verbally administered questions.

5.2 Materials and Methods

5.2.1 Subjects

Sixty-six infants who participated in a cross-sectional numerical change detection experiment (M. E. Libertus & Brannon, 2010) at 6 months of age (mean age 6 months 2 days) were re-recruited at 3.5 years of age. Seven children were excluded because they were unable to complete the nonsymbolic numerical comparison task at 3.5 years of age. Eleven children were excluded because w could not be modeled effectively (Halberda et al., 2008; M. E. Libertus et al., 2011; Piazza et al., 2010). The final sample contained 48 children (mean age 3.6 years, ranging from 3.5-3.9 years; 22 females).

5.2.2 Procedure

At 6 months of age, infants came into the lab for a single session during which the infant completed the numerical change detection task as part of one of five different cross-sectional studies. Half of the infants also completed a non-numerical change detection task, and the order of the two tasks was counterbalanced. Results from some of these cross-sectional studies have been previously reported (M. E. Libertus & Brannon, 2010).

At 3.5 years of age, children came into the lab for two visits, each lasting approximately one hour. Children were tested individually in a quiet room and were given small stickers throughout the session to maintain interest. During the first visit, children completed the Test of Early Mathematics Ability (TEMA-3) (H. P. Ginsburg & Baroody, 2003) and one session of the nonsymbolic number comparison task. During the second visit, children completed the verbal and non-verbal components of the Reynolds Intellectual Assessment Scales (RIAS) (Reynolds & Kamphaus, 2003), the counting knowledge task, and a second session of the nonsymbolic number comparison task. The order of the tasks within each session was counterbalanced across participants. See Table 2 for a complete list of all tasks administered at each time point.

At each visit, parents gave written consent to a protocol approved by the local Institutional Review Board and were compensated monetarily and with a small gift for the child. After completing all three visits, families were given at $50 bonus.

Table 2: Experimental design of the longitudinal study.

6 months			3.5 years		
Construct	**Test**	**Measure**	**Construct**	**Test**	**Measure**
ANS acuity	Numerical change detection task	Numerical preference score	ANS acuity	Non-symbolic number comparison task	Weber fraction (w)
Perceptual discrimination ability	Non-numerical change detection task (color or size)	Non-numerical preference score	Math ability	Test of Early Mathematics Ability (TEMA-3)	Standardized math score
			Counting knowledge	Give-A-Number task	Number knowledge (dichotomous)
			General intelligence (IQ)	Reynolds Intellectual Assessment Scales (RIAS)	IQ Score

5.2.2.1 Numerical change detection task

Infants were shown two streams of images, one on each of two peripheral monitors. One of the image streams contained arrays that alternated in the number of elements (number changing stream) while the other image stream contained arrays with a constant number of elements (number constant stream). A schematic of this task is shown in Figure 12. The cross-sectional studies varied in the number of elements presented in the changing and constant streams: 6 vs. 24 (n = 2), 5 vs. 15 (n = 18), 6 vs. 18 (n = 13), 8 vs. 16 (n = 2), or 10 vs. 20 (n = 13). In all conditions, element size, cumulative contour length, cumulative surface area, and density were controlled across the two streams. For each infant, a numerical preference score was calculated by subtracting the proportion of time spent looking at the numerically constant stream from the proportion of time spent looking at the numerically changing stream. A positive preference score

therefore indicates a preference for the changing stream, whereas a preference score of zero indicates no preference. To enable comparison of preference scores across numerical conditions, preference scores were normalized by dividing each score by the highest score in its respective condition.

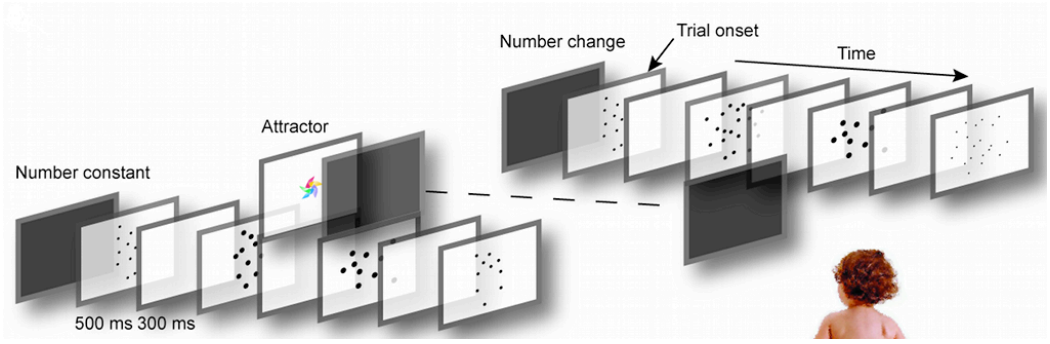

Figure 12: Schematic of the numerical change detection paradigm used to assess ANS acuity in infancy. The right side shows a numerically changing stream, which alternates between images containing 10 or 20 dots, while the left side shows a numerically constant stream, which in this example always contains 10 dots with variable dot sizes and arrangements.

5.2.2.2 Non-numerical change detection task

Half of the infants performed either a color or size change detection task using the same procedure described above (size: n = 17; color: n = 7). In the color version, the constant stream displayed a single square with a constant color while the changing stream displayed a square that randomly changed between eight different colors. In the size version, the constant screen displayed a single Elmo face with a constant size, while the changing stream alternated between two Elmo faces that differed in size by a factor of three. In these tasks, a non-numerical preference score was calculated by subtracting

the proportion of time spent looking at the constant stream from the proportion of time spent looking at the changing stream.

5.2.2.3 Nonsymbolic numerical comparison task

On each trial, children were presented with two boxes (8 x 9.5 cm) on a touchscreen computer containing arrays of dots and were asked to touch the numerically larger array. Arrays contained between 4 and 14 elements, and the numerical ratio between the arrays was 1:2, 2:3, 3:4, or 6:7. To control for non-numerical perceptual cues, the parameters of the arrays varied such that the smaller and larger numerical array each had the larger cumulative surface area on 50% of trials. All of the dots within a single array were homogenous in element size and color, and the color of each array varied randomly from trial to trial. Differential audio-visual feedback was provided after each trial, and children received a small sticker for each correct response to keep them engaged. Children performed practice trials until they made three consecutive correct responses or a maximum of ten trials. Children performed 60 test trials in each test session for a total of 120 trials. Four children only have data from a single session due to a computer error (n = 2) or participating in only one session (n = 2).

To estimate each child's ANS acuity, we used a psychophysical modeling technique that has been used previously in the literature (DeWind & Brannon, 2012; Halberda et al., 2008; Halberda & Feigenson, 2008; M. E. Libertus et al., 2011; 2012; Piazza et al., 2010) to calculate Weber fractions (w) based on the performance in the

nonsymbolic numerical comparison task. We modeled the error rate as $\frac{1}{2}erfc\left(\frac{n_1 - n_2}{\sqrt{2}w\sqrt{n_1^2 + n_2^2}}\right)$, where n_1 is the numerosity of the larger set, n_2 is the numerosity of the smaller set, w is a measure of variance in the internal representation, and *erfc* is the complementary error function. The resulting value of w represents the noise in each participant's internal ANS representations, such that lower values of w correspond to less noise (i.e., higher ANS acuity). The model was unable to fit the performance of five children and settled on a very poor fit to the data ($r^2 < .2$) for six additional children. As in previous studies (Halberda et al., 2008; M. E. Libertus et al., 2011; Piazza et al., 2010), these children were excluded from further analyses.

5.2.2.4 Counting knowledge task

This task was modeled after the Give-a-Number task (Wynn, 1992). The experimenter introduced the child to a dinosaur puppet and asked the child to give the dinosaur a certain number of fish. On the first trial, the experimenter asked the child to give the dinosaur one fish. If the child successfully produced one fish, the child was asked to give the dinosaur three fish. If the child failed to produce one fish, the child was asked to give the dinosaur two fish. If the child provided the correct number of fish, the trials progressed in the order 1-3-5-6-6. If the child provided an incorrect number of fish for any number, the child was asked for N-1 fish. The trials proceeded until the child answered correctly at least twice for N and failed at least twice for N+1, or until the child successfully provided six fish twice. Children were grouped into two categories: those

who understood the exact meaning of the number words "one" to "six" and those who did not. Data from 8 of the 48 children are missing because they were not administered the task.

5.2.2.5 Standardized tests

Children's mathematical ability was assessed with the Test of Early Mathematics Ability (TEMA-3) (H. P. Ginsburg & Baroody, 2003), which consists of a series of verbally administered questions that assess age-appropriate counting ability, number-comparison facility, numeral literacy, and basic calculation skills. To assess general intelligence, children completed two verbal and two nonverbal subtests of the Reynolds Intellectual Assessment Scales (RIAS) (Reynolds & Kamphaus, 2003), and a composite IQ score was calculated for each child. RIAS scores from 3 of the 48 children are missing because they refused to complete all subtests.

5.3 Results

Our main finding was that numerical preference scores from the numerical change detection task administered at 6 months of age significantly predicted both ANS acuity as measured by w (r = -.29, p = .02) and standardized math scores (r = .28, p = .03) measured at 3.5 years of age (Figure 13)[1]. Note that because lower w scores are

[1] Several of the children who returned for the follow-up visits at 3.5 years of age had negative numerical preference scores at 6 months of age despite the fact that the mean score for each of the conditions from which they were drawn exhibited significant positive preference scores.

associated with greater ANS acuity, the negative correlation between numerical preference scores and *w* indicates that greater ANS acuity in infancy is associated with greater ANS acuity at 3.5 years. Critically, these relationships held even after controlling for general intelligence (*w*: r_p = -.30, *p* = .03; math: r_p = .35, *p* = .01). In addition, we performed a multiple regression analysis to assess the contributions of numerical preference scores at 6 months of age and IQ at 3.5 years of age to standardized math scores at 3.5 years of age. This model captured a significant amount of variance in children's math achievement (R^2 = 0.28, $F(2,42) = 8.18$, *p* = .001) and both factors were unique predictors (numerical preference score: beta = .31, *p* = .02; IQ: beta = .45, *p* = .001). A second model assessing the contributions of numerical preference scores and IQ to ANS acuity at 3.5 years was marginally significant (R^2 = .13, $F(2,42) = 3.06$, *p* =.06) and numerical preference scores were a significant predictor (beta = -.30, *p* = .04) whereas IQ was not (beta = -.21, *p* = .16). These analyses suggest that the link between ANS acuity in infancy and math ability cannot be attributed solely to differences in general intelligence.

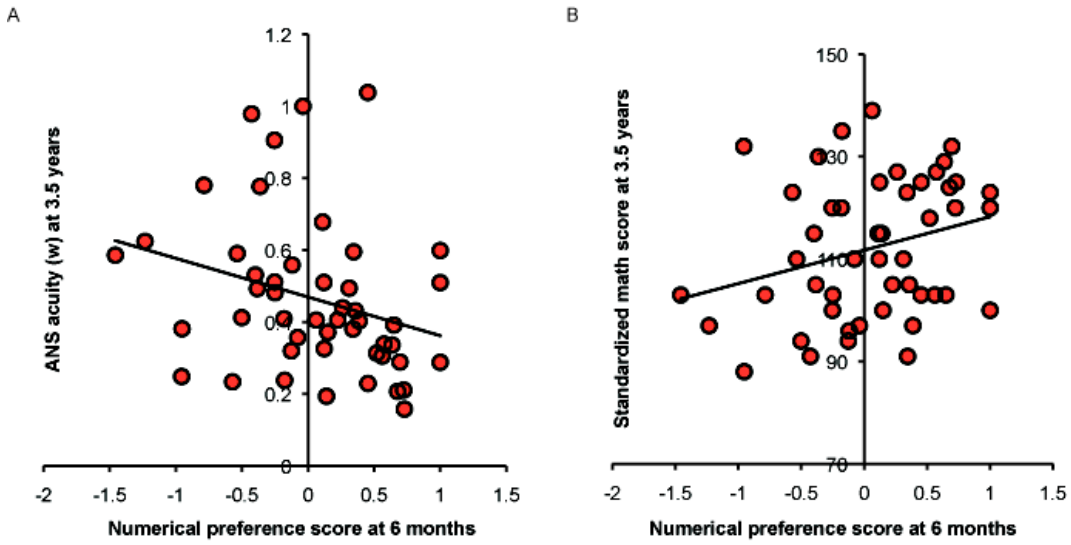

Figure 13: Numerical preference scores in the numerical change detection task at 6 months of age are significantly correlated with ANS acuity as indexed by Weber fractions (w) (A) and with math ability (standardized math scores) (B) at 3.5 years of age.

A median split based on math achievement scores also revealed that children with high math achievement scores (TEMA > 110) had significantly higher numerical preference scores than children with low math achievement scores (TEMA < 111) (p = .02). Children with high math achievement scores had numerical preference scores that were significantly greater than zero ($t(22) = 2.06$, p = .05) whereas children with low math achievement scores had preference scores that were not different from zero ($t(24) = -0.87$, p = .39).

To assess whether memory or general perceptual discrimination abilities in infancy could account for our results, we additionally examined the relationship between infant non-numerical change detection scores and both w and standardized math scores at 3.5 years of age. Half of the 6-month-old infants had also been tested on a non-numerical

version of the change detection task in which the color or size of a single shape varied in one visual stream and remained constant in the other. Preference scores in this task are thought to reflect short-term memory capacity (Ross-Sheehy et al., 2003) or perceptual discrimination ability and are independent of numerical preference scores (M. E. Libertus & Brannon, 2010). In this reduced sample, numerical preference scores, but not non-numerical preference scores, were correlated with ANS acuity at 3.5 years (numerical preference scores: r = -.42, $p < .02$; non-numerical preference scores: r = .14, $p = .25$), and the correlation between numerical preference scores and ANS acuity was significantly greater than the correlation between non-numerical preference scores and ANS acuity (z = -1.91, $p < .03$). However, there was no significant difference between the correlations for numerical versus non-numerical preference scores and standardized math scores at 3.5 years of age ($p > .2$), and neither correlation reached significance (numerical preference scores: r = .16, $p = .22$; non-numerical preference score: r = -.02, $p = .47$). An important caveat is that in this reduced sample we may lack the statistical power to detect a true correlation between preference scores and math achievement. Given the significant correlation between numerical preference scores and childhood *w*, the lack of a correlation between numerical preference scores and childhood IQ, and the finding that numerical and non-numerical preference scores are uncorrelated with each other both in the present sample and in previously published reports (M. E. Libertus & Brannon, 2010), it does not seem likely that the observed correlation between numerical

preference scores and math achievement is driven by domain-general factors in infancy. Therefore, we believe that the most parsimonious conclusion is that ANS acuity in infancy, rather than domain-general memory or perceptual abilities, uniquely predicts later emerging math abilities and ANS acuity.

Numerical preference scores in infancy also predicted children's mastery of verbal counting principles. Children who understood the exact meaning of the number words "one" to "six" at 3.5 years of age had significantly higher numerical preference scores in infancy than children who only understood a subset of those count words ($t(38) = -2.70$, $p = .01$), even though the groups did not differ in general intelligence ($p > .2$).

Finally, consistent with previous findings, we found a significant concurrent link between w and standardized math scores ($r = -0.42$, $p = .002$), which extends prior findings of this correlation to 3.5-year-old children who have not yet begun formal math education. The relationship between children's math abilities and ANS acuity both measured at 3.5 years of age remained significant even after controlling for general intelligence ($r_p = -.35$, $p = .01$).

5.4 Discussion

A fundamental question for researchers and educators alike is to understand the cognitive bases of uniquely human mathematical abilities. This longitudinal study was designed to probe the relationship between infants' nascent numerical representations and the acquisition of symbolic math knowledge in early childhood. Our results

demonstrate that ANS acuity at 6 months of age is predictive of math achievement, number word knowledge, and ANS acuity at 3.5 years of age. Critically, these relationships hold after controlling for general intelligence in childhood. Although previous studies have provided evidence for a relationship between ANS acuity and math achievement (Halberda et al., 2008; M. E. Libertus et al., 2011; 2012; Piazza et al., 2010), they have been unable to address the origins of this relationship. By demonstrating that ANS acuity in infancy, long before the acquisition of number words or exposure to formal math instruction, predicts future math achievement, the present results suggest that symbolic arithmetic builds upon more primitive numerical representations.

Several possible explanations have been proposed to explain the mechanism underlying the relationship between ANS acuity and math performance. One hypothesis that has received some support from cross-cultural studies (Pica, 2004) is that exposure to and proficiency with symbolic number presentations sharpens ANS representations and improves acuity. An alternate hypothesis is that the ANS interfaces with arithmetic operations as a form of online error detection (Lourenco et al., 2012), such that sharper ANS acuity allows detection of erroneous symbolic answers. A third hypothesis is that greater ANS precision leads to improved facility with acquiring the meaning of number words and symbols and learning symbolic arithmetic (Dehaene, 1997; Feigenson et al., 2013). Children with a sharper sense of quantity may be at an advantage for mapping

numerical symbols to approximate magnitudes and for acquiring basic mathematical concepts. In support of this idea, recent studies have shown that experience with approximate arithmetic using nonsymbolic arrays leads to improvements in symbolic math performance (Hyde, Khanum, & Spelke, 2014; Park & Brannon, 2013; 2014). In addition, the automaticity of ordering numerical symbols may mediate the relationship between ANS acuity and symbolic math performance in adults (Lyons & Beilock, 2011). Thus, heightened ANS acuity may aid children in the mapping of number words onto ANS representations and this may lead to earlier proficiency with numerical symbols and provide a jumpstart for math achievement.

Given that the correlation between ANS acuity and math achievement holds into adulthood (DeWind & Brannon, 2012; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; M. E. Libertus et al., 2012), and given that adults in math literate societies have on average slightly higher ANS acuity than adults from cultures without systematic count lists (Pica, 2004), it seems likely that there are bidirectional influences between ANS acuity and math achievement that continue throughout the lifespan. Nevertheless, the present finding that ANS acuity in infancy predicts number word knowledge and math scores in early childhood suggests that the influence of ANS acuity on math achievement precedes exposure to verbal counting and math education. This finding is therefore most consistent with the hypothesis that ANS acuity has a causal influence on math achievement.

However, despite the studies referenced above that find a relationship between ANS acuity and symbolic math, a number of other studies have found that ANS acuity either does not correlate with symbolic math achievement (Holloway & Ansari, 2009; Nosworthy et al., 2013; Sasanguie et al., 2013), that the relationship is mediated by executive function (Fuhs & Mcneil, 2013), or that the relationship holds only for children with low math ability (Bonny & Lourenco, 2012). The presence of these discrepant findings indicates that the relationship between ANS acuity and symbolic math is not clear-cut, and further research is needed to elucidate how the ANS may serve as a scaffold for symbolic math skills. Given that the present findings concern children who are just beginning to acquire numerical symbols and symbolic arithmetic, one possibility is that it is at this point that the relationship between ANS acuity and burgeoning math knowledge is strongest (Mussolin, Nys, Leybaert, & Content, 2012).

It is also important to note that while we found that numerical sensitivity in infancy was a unique predictor of later math achievement, this relationship explained only a small proportion of the total variance. In fact, in many prior studies where individual differences in ANS acuity has been found to correlate with variance in math knowledge, ANS acuity was neither the only nor the strongest determiner of a child's math achievement (Fuhs & Mcneil, 2013; Gilmore et al., 2010; Halberda et al., 2008; M. E. Libertus, Feigenson, & Halberda, 2013a). Therefore, future research will need to investigate how ANS acuity relates to other factors that influence math achievement,

and researchers will need to consider many potentially mediating variables when investigating the relationship between ANS and symbolic math

This longitudinal study provides evidence that preverbal number sense in infancy is predictive of both nonsymbolic number sense and symbolic mathematical ability in early childhood. Our uniquely human mathematical abilities appear to be fundamentally linked to an ontogenetically and evolutionarily ancient number sense that emerges in the first days of human life (Izard et al., 2009) and is ubiquitous throughout the animal kingdom (Brannon & Terrace, 1998; Gallistel & Gelman, 1992; Meck & Church, 1983). Although there may be bidirectional influences of ANS acuity and math ability over development, these data implicate a developmentally primary role for the preverbal number sense. This work may open the door for educational interventions to improve children's number sense even before they learn to count.

6. The role of non-numerical features in the development of the number sense

Thirty-nine 4-year-old children, 45 6-year-old children, and 30 adults were tested with a nonsymbolic numerical comparison task, a symbolic math assessment, and an inhibitory control task to determine how the influence of non-numerical stimulus features changes over development and whether bias towards these features is related to symbolic math skill. The influence of non-numerical features decreased significantly over development, particularly with respect to field area. In addition, numerical acuity was a stronger predictor of math achievement than either non-numerical bias or inhibitory control in children. These results suggest that that the ability to selectively attend to number contributes to the maturation of the number sense and that numerical acuity, independent of inhibitory control, contributes to math achievement in early childhood.

6.1 Introduction

Beginning with Piaget, there has been great controversy regarding how children make decisions about number. In his classic number conservation studies, Piaget observed that although children correctly viewed two lines that were composed of the same number of objects as equally numerous when the spacing of the objects was equal, they erroneously judged one line to become more numerous when the objects were spaced further apart (Piaget, 1952). As a result, Piaget concluded that children are

unable to disentangle representations of number and space until they reach the concrete operations stage around seven years of age. More recent work, however, suggests that even infants can indeed selectively attend to both number and size, particularly with large set sizes (see Cantrell & Smith, 2013; Mou & vanMarle, 2013 for reviews). At the same time, it is clear that numerical representations, even in adulthood, are influenced by non-numerical properties such as element size, field area, and density (Allik & Tuulmets, 1991; Dakin et al., 2011; Defever et al., 2013; Gebuis & Gevers, 2011; Gebuis & Reynvoet, 2011; 2012a; 2012b; N. Ginsburg & Nicholls, 1988; Hurewitz et al., 2006; Rousselle et al., 2004; Rousselle & Noël, 2008; Soltész et al., 2010; Sophian & Chu, 2008; D. Szűcs et al., 2013; Tibber et al., 2012; Tokita & Ishiguchi, 2010; 2013). In general, numerical decisions are more accurate when these non-numerical features co-vary with number compared to when they do not.

In most assessments of nonsymbolic numerical discrimination, the stimuli are constructed such that in half of the trials the more numerous array contains larger individual elements and a larger total surface area than the less numerous array, meaning that numerosity and surface area are congruent. In the other half of trials, numerosity and area are incongruent such that the more numerous array has smaller elements and a smaller total surface area (e.g., DeWind & Brannon, 2012; Halberda & Feigenson, 2008; Piazza et al., 2010). With this design, performance is typically better for congruent trials compared to incongruent trials, and the effects of congruency are

strongest in young children and attenuate with age (Defever, Sasanguie, Gebuis, & Reynvoet, 2011; Rousselle et al., 2004; Rousselle & Noël, 2008; Soltész et al., 2010). These studies demonstrate that non-numerical features influence numerical discrimination, but they do not provide information as to which non-numerical features are most salient. Furthermore, they do not enable us to quantify how the influence may change over development. Does reliance on all non-numerical cues decrease in parallel or do the trajectories vary by feature? More generally, does a decrease in the reliance on non-numerical cues underlie the improvement in number sense acuity that occurs over development?

The precision with which numerical comparisons are made, referred to here as numerical acuity, undergoes rapid development in infancy and early childhood and continues to increase into adulthood (Halberda et al., 2012; Halberda & Feigenson, 2008; Piazza et al., 2010). This improvement may reflect an increase in the precision of the internal numerical representations themselves or, alternatively, an increase in the ability to selectively attend to number and inhibit attention towards other stimulus features. Although these explanations are not necessarily mutually exclusive, there is some evidence supporting the latter. When adults repeatedly practiced a nonsymbolic numerical comparison task with feedback for a total of six one-hour sessions, performance improved with practice (DeWind & Brannon, 2012). Notably, in conjunction with the increase in performance subjects exhibited a decrease in the

congruency effect, and particularly a diminished reliance on surface area. This suggests that training nonsymbolic numerical comparison improves the ability to specifically attend to number. One possibility then is that the improvements in numerical acuity observed over development are due to a diminished reliance on non-numerical cues that typically covary with number.

The development of selective attention to number may also have implications for the acquisition of symbolic math skills. Several studies have documented a relation between performance on numerical comparison tasks and symbolic math performance (DeWind & Brannon, 2012; Gilmore et al., 2010; Halberda et al., 2008; 2012; M. E. Libertus et al., 2011; 2012; Mazzocco et al., 2011; Mussolin et al., 2012; Starr, Libertus, & Brannon, 2013b). However, a growing number of studies have observed no relation (Holloway & Ansari, 2009; Nosworthy et al., 2013; Sasanguie et al., 2013) or a relation that exists only in young children or children with low math ability (Bonny & Lourenco, 2012; Inglis, Attridge, Batchelor, & Gilmore, 2011; Sasanguie, De Smedt, Defever, & Reynvoet, 2011). Two recent meta-analyses on this topic concluded that there is a significant yet small relation between the symbolic math and non-symbolic numerical acuity that is strongest in young children (Q. Chen & Li, 2013; Fazio et al., 2014), lending credence to the hypothesis that approximate number representations may be foundational for the acquisition of symbolic math (Dehaene, 1997; Gallistel & Gelman, 1992). An alternative theory, however, is that the apparent relation between numerical acuity and math ability

actually reflects a relation between inhibitory control and math ability. It is well documented that inhibitory control is predictive of math achievement (Bull & Lee, 2014; Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006), and some researchers have proposed that the inhibitory demands of numerical comparison tasks (i.e., the ability to attend to number and ignore irrelevant non-numerical features) create a spurious correlation between number sense and math performance (Fuhs & Mcneil, 2013; Gilmore et al., 2013). According to this view, children (and adults) with better inhibitory control perform better on nonsymbolic numerical comparison tasks because they are better able to inhibit attention towards non-numerical stimulus features, and this better inhibitory control also leads to higher math performance. Therefore, higher numerical acuity itself may not directly contribute to math achievement.

The present study was designed to investigate how children's attention to numerical and non-numerical stimulus features shifts over development and the degree to which attention to these features is related to math performance. We tested 4- and 6-year-old children, as well as adults, on a nonsymbolic numerical comparison task, a symbolic math assessment, and an inhibitory control task. The nonsymbolic numerical comparison task and analysis meth was adapted from DeWind et al. (under review) and differed from previously published studies in that multiple non-numerical features were each manipulated partially independently of numerical ratio and varied within the same range as the numerical ratios. By fitting a generalized linear model to each participant's

data, we were able to determine the relative weight given to number and non-numerical features when choosing which array was ostensibly numerically greater. This method and model therefore enabled us to quantify the influence that non-numerical features exert on numerical representations and to examine how this influence may change over development. Further, because this method provides an objective measurement of non-numerical bias, we could explicitly test the hypothesis that the ability to inhibit attention towards non-numerical features may drive the relation between numerical acuity and symbolic math skill.

6.2 Materials and Methods

6.2.1 Participants

Thirty-nine 4-year-old children (mean age 4.64 years, range: 4.45-4.96 years, 17 females), 45 6-year-old children (mean age 6.63 years, range: 6.29-6.96 years, 19 females), and 30 adults (mean age 21.9 years, range 18.5-54.4 years, 20 females) participated in the study. An additional 11 4-year-olds were excluded due to failure to cooperate (n = 10), or inability to speak English (n = 1), and one 6-year-old was excluded due to failure to cooperate. All participants or their parents consented to a protocol approved by the local IRB. Children were given a small gift and parents were compensated monetarily at each visit. Adults were compensated either monetarily or with course credit.

6.2.2 Measures

6.2.2.1 Nonsymbolic numerical comparison task

This task was modified from DeWind et al. (under review). Construction of the dot arrays was based on the principle that although there are many non-numerical features that covary with number within dot arrays, these arrays have only three degrees of freedom: the number of dots, the parameters that describe the size of the dots, and the parameters that describe the spacing of the dots. Furthermore, the size and spacing of the dots are independent of one another and can each be manipulated orthogonally to the numerosity of the array. This creates a three-dimensional stimulus space in which any given array of dots is defined by a single point that reflects the number, size, and spacing of the array (Figure 14). Within this three-dimensional space, we constructed a set of stimuli that divided numerosity, size, and spacing into 13 levels that were evenly spaced on a logarithmic scale. On each trial, the experimental program randomly chose one of the 4 different numerical ratios (closest whole numbers to $1:2^{1/6}$,$1:2^{1/3}$,$1:2^{1/2}$, or 1:4 ratios), one of the 13 size ratios, and one of the 13 spacing ratios. In order to sample the stimulus space evenly along a logarithmic scale, all values were rounded to the nearest whole number. See the Modeling section below for additional description of the mathematical relation between number, size, and spacing.

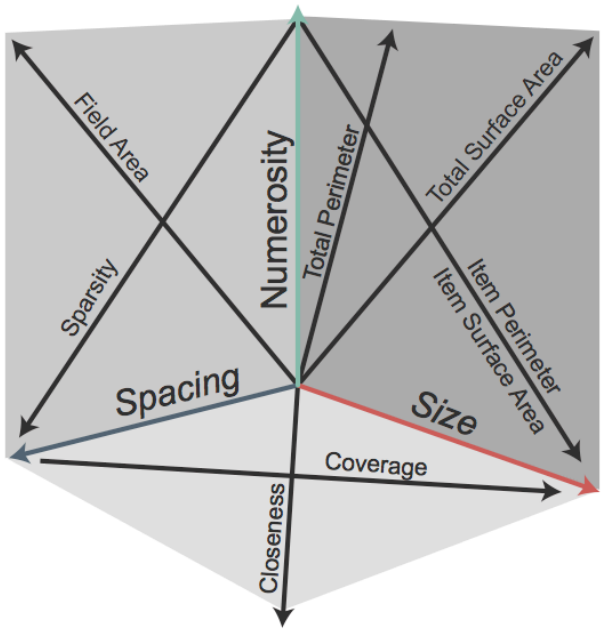

Figure 14: Illustration of 3-D stimulus space demonstrating the relations between the number, size, and spacing dimensions.

On each trial, participants were presented with two arrays of dots and asked to indicate which side had the greater number of dots. Each trial began with a white fixation cross in the center of the screen. After 500 ms, the dot arrays were displayed on either side of the fixation cross. The dots were visible for a set presentation time that varied as a function of age (1000 ms for 4-year-olds, 750 ms for 6-year-olds, and 250 ms for adults), after which participants were prompted to provide a response using the keyboard. These variations in presentation time were implemented in order for accuracy in each age group to be approximately 75%, which is optimal for fitting our choice behavior model. No feedback was given during test trials. To ensure that all participants understood the task, 4- and 6-year-olds began with four practice trials and adults began

with eight practice trials. Practice trials included feedback, had a slower presentation time, and always presented a 1:4 numerical ratio. If children did not answer at least three out of four practice trials correctly, the practice was repeated. Four-year-old children completed 60 test trials per session at each of two sessions (120 trials total), 6-year-old children completed 150 trials per session at each of two sessions (300 trials total), and adults performed 500 trials in a single session.

6.2.2.2 Number Sense Screener

The Number Sense Screener (NSS; Glutting & Jordan, 2012) was used to assess math skill in 4-year-old children. The NSS is orally administered and assesses counting skill, number recognition, numerical magnitude comparison, and arithmetic. Standardized scores in the NSS are based on level of schooling. None of the 4-year-old participants had begun kindergarten, but scores were standardized based on the norms for the first semester of kindergarten because this was the lowest level of schooling for which standardized scores were available.

6.2.2.3 Wide Range Achievement Test – 4th Edition, Calculation subtest

The Wide Range Achievement Test-4th Edition (WRAT-4) Calculation test (Wilkinson & Robertson, 2006) was used to assess math skill in 6-year-old children and adults. The WRAT-4 consists of orally-administered number knowledge and arithmetic questions (6-year-old children only) and a written portion in which participants have 15 minutes

to complete arithmetic problems that sequentially increase in difficulty. Scores were standardized based on participants' age.

6.2.2.4 Day/Night task

The Day/Night task (Gerstadt, Hong, & Diamond, 1994) was used to assess inhibitory control in 4- and 6-year-old children. In the warm-up version, children were shown a card containing 16 sun and moon pictures in a pseudo-random order and instructed to say "day" for the sun pictures and "night" for the moon pictures as quickly as possible. Next, children were told they were going to play a silly version of the game that required saying the opposite picture names ("day" for the moon picture and "night" for the sun picture). They were then shown a new card with 16 sun and moon pictures and instructed to say the opposite picture names as quickly as possible. The total time and number of errors were combined into a single efficiency score (number of correct responses divided by total time).

6.2.2.5 Flanker task

The Flanker task was used to assess inhibitory control in adults. The task consisted of a row of five fish, and participants were instructed to push a button indicating whether the center fish was facing to the left or to the right. The center fish could either be facing the same way as the surrounding fish (congruent) or the opposite way (incongruent). Participants were instructed to respond as quickly as possible. On each trial, the fish array appeared on the screen for 300 ms. After the fish disappeared, a

green fixation cross remained on the screen until a response was made. After the response, the fixation cross turned white and there was a 2 second ITI before the start of the next trial. Participants performed four practice trials with feedback followed by 144 test trials without feedback. A version of this task was also administered to child participants but was excluded from further analyses because as a group children did not exhibit the classic Flanker effect. Six-year-old children tended to perform very slowly and deliberately, such that there was no performance difference between congruent and incongruent trials, whereas 4-year-old children struggled to perform at chance level on incongruent trials.

6.2.3 Procedure

Child participants completed two laboratory visits. At the first visit, children performed the nonsymbolic numerical comparison task, Day/Night, and the appropriate symbolic math test. At the second visit, children performed the nonsymbolic numerical comparison task and the Flanker task. Adults were tested in a single visit comprised of the nonsymbolic numerical comparison task, the Flanker task, and the WRAT-4 Calculation test.

6.2.4 Modeling choice behavior in the nonsymbolic numerical comparison task

For each subject, we fit a generalized linear model to their choice data with regressors for the log of the ratio of numerosity, size, and spacing for the dot arrays

appearing on the right and left sides of the screen (DeWind et al., under review). The model equation is as follows:

$$p(ChooseRight) =$$

$$(1-\gamma)\left(\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{\beta_{side}+\beta_{num}log_2(r_{num})+\beta_{Size}log_2(r_{Size})+\beta_{Spacing}log_2(r_{Spacing})}{\sqrt{2}}\right)\right]-\frac{1}{2}\right)+\frac{1}{2}$$

We included a guessing term in our model, γ, to account for trials in which participants guessed randomly. This allows the choice curves to asymptote below 100% (Halberda & Feigenson, 2008; Pica, 2004). The guess parameter was set based on each participant's mean accuracy on the easiest numerical ratio (1:4).

The model enabled us to calculate the weight that participants place on the stimulus features that make up the variables of size and spacing. In addition to number, the other two parameters that define any given numerical array are size and spacing. Size is defined by the product of item surface area and total surface area. Therefore, the size term captures the effect of changes to the size of the dots on choice behavior. If the size coefficient equals zero, then the participant's numerical decision-making is indifferent to changes in dot size. A positive size coefficient indicates a tendency to view larger dots as more numerous, whereas a negative size coefficient indicates a tendency to regard smaller dots as more numerous. Spacing is defined by the product of sparsity (the inverse of density) and field area. The spacing coefficient therefore captures the effects of changes to the spacing of the dots on choice behavior. If the spacing coefficient equals zero, then the participant's numerical decision-making is indifferent to changes in dot

spacing. A positive size coefficient indicates a tendency to view more spaced out arrays as more numerous, whereas a negative size coefficient indicates a tendency to regard denser arrays as more numerous. The advantage of this modeling approach is therefore that it enabled us to calculate the independent effects of numerical ratio, size, and spacing on nonsymbolic numerical comparison performance.

The effects of number, size, and spacing can also be combined into a single composite vector that acts as a summary statistic of overall comparison performance. The magnitude of this vector (i.e., the Euclidean distance from the origin to the point specified by the number, size, and spacing coefficients) reflects a participant's overall discrimination acuity. The angle of this vector (i.e., the angle between the number-axis and the vector) reflects the participant's overall deviation from purely numerically based discrimination. Therefore, larger vector magnitudes reflect greater acuity, whereas larger vector angles reflect greater bias towards non-numerical stimulus features. If a participant has no non-numerical bias and is deciding purely based on number, than this composite vector will be equal to the value of the number coefficient. However, if the size or spacing coefficients are nonzero, than the composite vector will deviate from the number coefficient. Further details and mathematical justification of this modeling approach are provided in DeWind et al. (under review).

6.3 Results

6.3.1 Developmental change in the nonsymbolic numerical comparison task

In our first set of analyses we investigated how bias towards non-numerical cues in the nonsymbolic number comparison task changes over development. All age groups performed the task above chance and at approximately the same level (4-year-olds: 72.6%; 6-year-olds: 77.5%; adults: 78.5%), indicating that our attempt to equalize performance by manipulating stimulus presentation time was relatively successful. Next we fit each participant's choice data using our generalized linear model to calculate the relative weights placed on number, size, and spacing of the array items. We compared these coefficient weights across the three age groups. For all features there was a main effect of age (number: $F(2,111) = 10.44$, $p < .001$, n2p = .158; size: $F(2,111) = 3.59$; $p < .05$, n2p = .06; spacing: $F(2,111) = 10.53$, $p < .001$, n2p = .42). In addition, all age groups exhibited number coefficients significantly greater than zero (all $ts > 16$, $ps < .001$), as well as spacing coefficients significantly greater than zero (all $ts > 10$, $ps < .001$). However, none of the age groups exhibited size coefficients that were greater than zero (all $ts > 1.3$, $ps > .09$). This indicates that number was a significant predictor of choice behavior for all age groups, yet there was also a significant biasing effect of spacing (Figure 15). Follow-up comparisons revealed that 4-year-olds had significantly lower number coefficient weights compared to 6-year-olds and adults ($ps \leq .005$), and 6-year-olds had significantly lower number coefficient weights than adults ($p < .05$). Four-year-

olds had also significantly higher coefficient weights for size and spacing compared to 6-year-olds and adults (*ps* < .05), whereas 6-year-olds and adults did not differ (*ps* > .4). These results suggest that the biasing effects of size and spacing variables reach adult-like levels between 4 and 6 years of age, whereas numerical acuity continues to improve between the ages of 6 and adulthood.

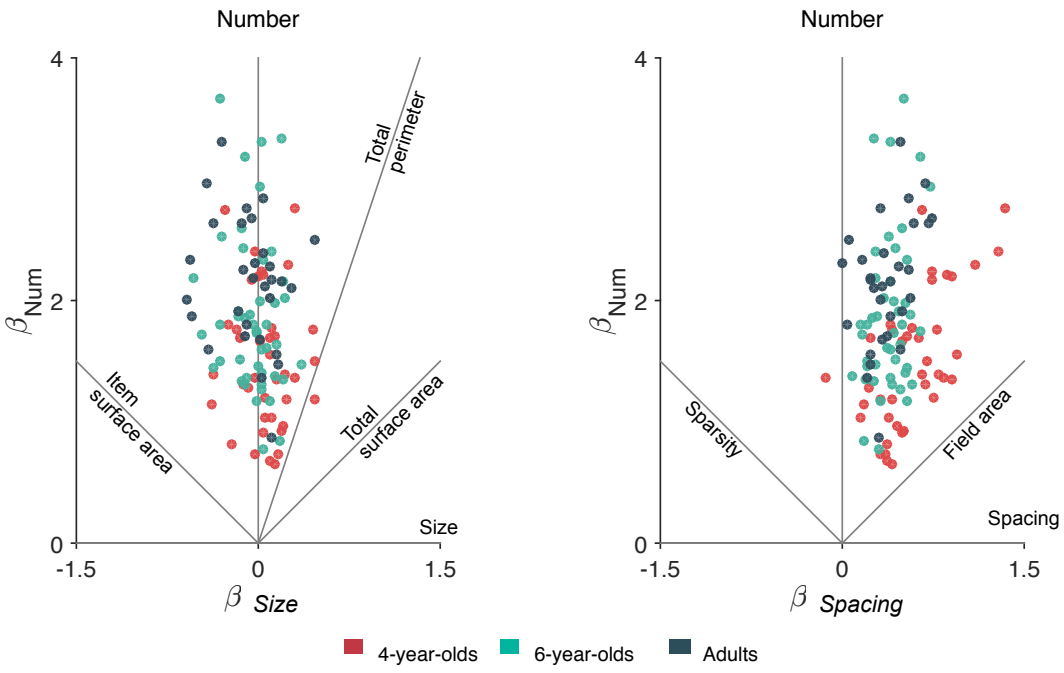

Figure 15: Scatterplots of the coefficient weights after fitting the choice model to the nonsymbolic numerical comparison data. (A) Number vs. size dimensions. (B) Number vs. spacing dimensions. Each dot represents a single participant.

To further explore how numerical decision-making changes over development, we next investigated age-related differences in the bias angle and vector magnitude of the composite choice vectors, which represent participants' bias towards non-numerical features and overall discrimination performance. To reiterate from above, the bias angle

reflects the influence of non-numerical features, and the vector magnitude reflects discrimination acuity. There was a main effect of age on both bias angle ($F(2,111) = .39.49$, $p < .001$, n2p = .42) and vector magnitude ($F(2,111) = 7.42$, $p = .001$, n2p = .118). Furthermore, both bias angle and vector magnitude were significantly greater than zero in all age groups (all $ts > 13$, $ps < .001$). Follow-up comparisons revealed that 4-year-olds had significantly larger bias angles and smaller vector magnitudes in comparison to 6-year-olds and adults ($ps < .05$). Six-year-olds and adults exhibited no significant differences in bias angle ($p = .14$), but adults exhibited larger vector magnitudes ($p = .05$) (Figure 16). These analyses support the previous analyses in suggesting that the influences of non-numerical cues decreases between the ages of 4 and 6 and then remains relatively stable yet significant, whereas discrimination acuity has a more protracted developmental trajectory.

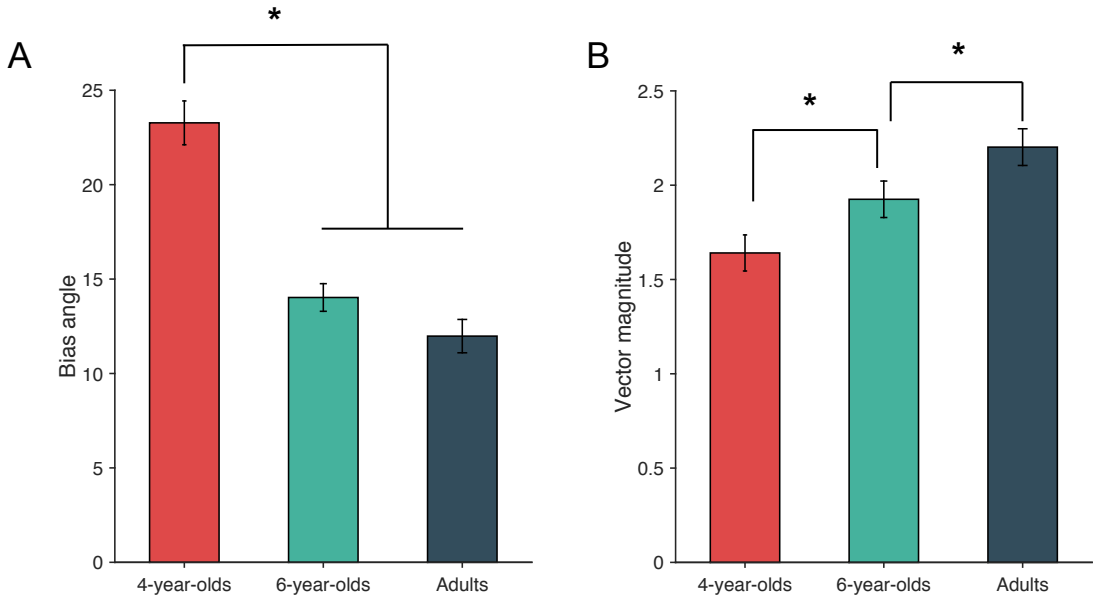

Figure 16: Group averages for the bias angle (A) and composite vector magnitude (B). Four-year-olds have larger bias angles in comparison to 6-year-olds and adults. Vector magnitude continues to increase between 4 and 6 years of age and between 6 years and adulthood. Error bars represent SEM.

Our next series of analyses compared the coefficients for number relative to the full list of stimulus features accounted for by our model. Although these other stimulus features (individual element size, total surface area, total perimeter, field area, and density) are not explicitly included in the model, their collinearity with number, size, and spacing enables us to calculate coefficient weights that reflect their influence on numerical comparison performance (DeWind et al., under review). We conducted paired t-tests comparing coefficient weights for number and each of the other stimulus features in each of our age groups. In 6-year-olds and adults, the relative weight placed on number was greater than the weight placed on any of the other stimulus features (all

ts > 8.1, all *ps* < .001). In 4-year-olds, however, number and field area were weighted approximately equally (t(38) = .99, *p* = .33). Therefore, while 6-year-olds and adults successfully focused their attention on number, 4-year-olds exhibited a strong field area bias and used number and field area information equally when purportedly making decisions about number. Furthermore, although it was not weighted significantly more than number, both 6-year-olds and adults weighted perimeter more strongly than any other non-numerical features (all ts > 6.7, all *ps* < .001). Figure 17 illustrates how the influence of field area on numerical discrimination decreases rapidly between 4 and 6 years of age, whereas the influence of perimeter remains relatively stable across the three age groups.

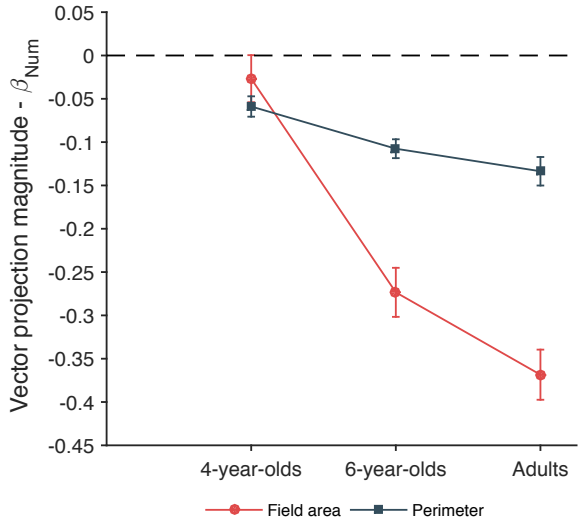

Figure 17: Level of explanatory power that field area and perimeter have on choice behavior relative to number. Values closer to 0 indicate greater explanatory power, and more negative values indicate less explanatory power. The influence of perimeter remains relatively stable, whereas the influence of field area drops off sharply between 4 and 6 years of age. Error bars represent SEM.

6.3.2 Assessing the relation between symbolic math skill, non-numerical bias, and inhibitory control

The aim of our final set of analyses was to investigate the role of inhibitory control in mediating the relation between numerical acuity and math achievement. Therefore, we conducted linear regression analyses to determine which factors best predict symbolic math performance. The regression model included vector magnitude, bias angle, and Day/Night performance as possible predictors of math performance (Table 3). In both 4- and 6-year-old children, only vector magnitude was a significant predictor of math ability (*ps* ≤ .05). These results suggest that numerical acuity contributes significant variance to math achievement in young children. We did not find evidence that this relation is mediated by inhibitory control, and bias angle was unrelated to Day/Night performance (4-year-olds: r(37) = .02, *p* = .92; 6-year-olds: r(43) = -.14, *p* = .37). In adults, the model was unable to make any significant predictors regarding math performance. This suggests that neither numerical acuity nor inhibitory control exerted significant influence on symbolic math performance in adults.

Table 3: Linear regression analyses predicting math ability from vector magnitude, bias angle, and inhibitory control.

	4-year-olds		6-year-olds		Adults	
R^2	.207		.193		.08	
F-stat	3.041		3.264		.635	
p-stat	.042		.031		.60	
	β	*p*	β	*p*	β	*p*
Vector magnitude	.309	**.053***	.379	**.039***	.226	.293
Bias angle	.177	.253	-.047	.787	.154	.464
Inhibitory control	.266	.09	.264	.089	-.169	.425

6.4 Discussion

The aims of the study were threefold. First, we sought to characterize the changing influence of non-numerical stimulus features on numerical acuity over development. Second, we investigated whether the biasing effects of these non-numerical features contributes to the relation between numerical acuity and symbolic math skill. Third, we tested the hypothesis that inhibitory control may mediate the link between numerical acuity and symbolic math skill. To this end, we used a nonsymbolic comparison task that allowed us to quantify the relative weight that participants' placed on numerical and non-numerical stimulus features when making decisions about number. This method enabled us to objectively quantify the biasing effects of non-numerical stimulus

features and to dissociate numerical acuity from attention towards non-numerical features.

6.4.1 Developmental changes in the influence of non-numerical stimulus features

Consistent with previous results (Allik & Tuulmets, 1991; Dakin et al., 2011; Defever et al., 2013; DeWind et al., under review; Gebuis & Gevers, 2011; Gebuis & Reynvoet, 2011; 2012a; 2012b; N. Ginsburg & Nicholls, 1988; Hurewitz et al., 2006; Rousselle et al., 2004; Rousselle & Noël, 2008; Soltész et al., 2010; Sophian & Chu, 2008; D. Szűcs et al., 2013; Tibber et al., 2012; Tokita & Ishiguchi, 2010; 2013), we found that numerical decision making in all age groups was influenced by non-numerical features. When participants were attempting to make decisions based on the numerosity of the arrays, even adults were unable to entirely ignore non-numerical stimulus features. However, the biasing effects of non-numerical features decreased significantly with age. Four-year-old children's numerical judgments were most influenced by non-numerical features, and they weighed numerosity and field area equally when deciding which of two arrays contained more dots. Six-year-old children, on the other hand, exhibited strikingly adult-like performance. The similarities in the weighting of non-numerical cues by 6-year-olds and adults, in contrast to the larger biases observed in 4-year-olds, suggests that numerical decision-making undergoes substantial change between 4 and 6 years of age. The 4-year-olds in the present study behaved much like the children in Piaget's classic number conservation task: as the dots became more spread out over a larger field

area, the arrays were viewed as being more numerous (Piaget, 1952). The biasing effects of field area have been noted elsewhere (Allik & Tuulmets, 1991; Dakin et al., 2011; Kramer, Di Bono, & Zorzi, 2011; Tokita & Ishiguchi, 2013), and suggest that field area is a dimension that requires greater consideration in future studies. However, the influence of field area and other features was markedly decreased in 6-year-olds. This overall decrease in influence from non-numerical features between 4 and 6 years of age suggests that the ability to selectively attend to number may contribute to the developmental increase in numerical acuity. At the same time, the fact that we observed continued improvements in numerical acuity between 6 years of age and adulthood, a time period in which bias towards non-numerical features remained relatively stable, suggests that selective attention to number may be just one of multiple mechanisms that promotes refinement of the number sense.

In comparison to the strong biasing effects of field area for 4-year-olds, the influence of size-related features (individual dot size, cumulative surface area, and cumulative perimeter) was less pronounced. However, cumulative perimeter did exert a strong influence in all of the age groups tested, including adults. DeWind et al. (under review) also found that cumulative perimeter biased adults' numerical choices, and a small percentage of adults in that study were actually better characterized as basing their choices on cumulative perimeter rather than number. Therefore, perimeter appears to

play a persistent role in numerical representation over development, and this role requires further characterization.

An important caveat for interpreting our results is that stimulus presentation time varied across the age groups, such that presentation time decreased with increasing age. Although 6-year-olds demonstrated choice behavior that was remarkably similar to that seen in adults, their behavior may have looked dissimilar had they been tested with the shorter presentation time used for adults. Indeed, previous work has demonstrated that as presentation times increases, numerical representations become more precise (Inglis & Gilmore, 2013). However, we chose to vary presentation time as a function of age to equate performance and maximize model fitting in order to best characterize numerical decision-making strategies. Thus we can conclude that at a given level of performance, 4-year-olds exhibit much stronger spacing biases than 6-year-olds, and 6-year-olds exhibit non-numerical biases that are equivalent to those seen in adults. The effect of presentation time on the biasing effects of non-numerical stimulus features is another interesting avenue for future research.

6.4.2 Relations between numerical acuity, non-numerical bias, and math ability

The second focus of the study concerned the role that attention towards non-numerical stimulus features may play in mediating the relation between numerical acuity and symbolic math skill. In two recent studies, inhibitory control was found to mediate the relation between children's numerical acuity and math skill (Fuhs & Mcneil,

2013; Gilmore et al., 2013). In these studies, the relation between math skill and numerical acuity was significant only for the incongruent trials in a nonsymbolic numerical comparison task, in which the more numerous dot arrays had smaller dots than the less numerous arrays. In these trials, participants need to inhibit attention towards dot size, which may be a salient feature, and instead focus on the potentially less-salient feature of numerosity. Furthermore, these studies found that the relation between math skill and numerical acuity did not hold once a domain-general measure of inhibitory control was accounted for. The methodology of the present study, however, allowed for a more precise measure of numerical acuity and non-numerical bias as compared to simply assessing performance differences for congruent versus incongruent trials. In 4- and 6-year-old children, we found that children's discrimination acuity predicted math performance, whereas their overall bias towards non-numerical features did not. Although inhibitory control likely contributes to math performance (Bull & Lee, 2014; e.g., Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006), we did not find support for the hypothesis that it mediates the relation between numerical acuity and math skill in children. In addition, we did not find a relation between children's non-numerical bias and their Day/Night performance, and it is not presently clear whether the ability to selectively attend to number in a nonsymbolic numerical comparison task is directly related to domain-general measures of inhibitory control.

Therefore, it appears that both numerical acuity and inhibitory control make independent contributions to children's math achievement.

We did not find any evidence for a relation between numerical acuity and math achievement in adults. This result is consistent with the idea that approximate number representations may be most critical when children are first acquiring symbolic number knowledge and arithmetic, and the link between nonsymbolic number representations and formal math may therefore decrease with increasing age and math experience (Fazio et al., 2014; Mussolin et al., 2012). In particular, the precision of nonsymbolic number representations may influence the acquisition of numerical symbols, and indirectly influence math performance through facility with these symbols (Starr, Libertus, & Brannon, 2013b; vanMarle et al., 2014). Therefore, children who have greater precision in their internal number representations, regardless of their bias towards non-numerical features, may be more adept at learning number words and digits in comparison to their peers who exhibit poorer nonsymbolic numerical precision and this may provide an advantage for performing basic arithmetic operations.

6.4.3 Conclusions

Our study demonstrates that the ability to selectively attend to number and inhibit attention towards non-numerical stimulus features is a critical component of the development of the number sense. The ability to selectively attend to number appears to undergo rapid maturation in early childhood, and reaches adult-like levels by the age of

6. However, given that numerical acuity continued to increase beyond 6 years of age, this finding suggests that refinement of the number sense occurs both through decreasing bias towards non-numerical cues and through improvements in the precision of the internal numerical representations themselves. Our study also revealed that numerical acuity was a unique predictor of symbolic math achievement in 4- and 6-year-old children, above and beyond the contributions of non-numerical bias and inhibitory control. Taken together, these findings shed light on the mechanisms that support the fine-tuning of the number sense over development and provide insight into how these evolutionarily ancient representations may promote the acquisition of symbolic math.

7. General discussion

Mathematics is a uniquely human invention. However, humans share with all animals a system for representing number nonsymbolically. This system, the Approximate Number System (ANS), may serve as a foundation for the construction symbolic mathematics principles. To address this claim, the studies contained within this dissertation focused on two primary lines of research. The first was to determine how the ANS interfaces with representations of other magnitude dimensions. The second was to investigate how the ANS may support the acquisition of symbolic math. Together, these studies provide new insight into how primitive quantity representations, which are present at birth and widespread throughout the animal kingdom, give rise to distinctively human abilities.

7.1 Chapter summaries

The study described in Chapter 2 demonstrated that infants can engage the ANS to represent small numerosities. This finding compliments previous findings in adults and nonhuman animals (e.g., Brannon & Terrace, 1998; Cordes et al., 2001) in showing that the ANS in infants is functional over the full range of natural numbers. At first blush, it may seem difficult to reconcile this result with the many previous studies in which infants do not seem to engage the ANS when representing small sets (Clearfield & Mix, 1999; 2001; Feigenson & Carey, 2003; Feigenson, Carey, & Hauser, 2002a; Hyde & Spelke, 2011; Xu, 2003). In most of these studies, infants appeared to engage object-tracking

mechanisms rather than the ANS, while in other studies it seemed that infants may not have attended to number at all. However, whereas the previous studies used visual habituation or sequential presentation paradigms, the present study implemented a change detection paradigm. This paradigm involves two constantly changing arrays, which increases both stimulus complexity as well as working memory demands. Recent work with adults suggests that small numbers are more likely to be represented by the ANS when stimulus complexity is high (Hyde & Wood, 2011), and the present study suggests that this may also hold for infants. Therefore, in addition to demonstrating that infants' can represent the full continuum of numerical values using a single representational format, this study suggests that infants' representational strategies are flexible and depend on the specifics of the current situation.

The second empirical study, described in Chapter 3, explored infants' acuity for detecting changes in contour length. Recently it has been proposed that infants' sensitivity to contour may be far greater than their sensitivity to number, which raises the possibility that in previous studies purporting to show numerical discrimination, infants may actually have been responding to small changes in contour length (Cantrell & Smith, 2013). Although contour is known to be a salient dimension for infants (Karmel, 1969; McCall & Melson, 1970), no prior studies have explicitly tested infants' sensitivity to changes in contour length. In this study, it was found that infants required a threefold change in contour length in order to detect change, whereas only a twofold

change is needed for number (e.g., Brannon et al., 2004; M. E. Libertus & Brannon, 2010; Xu & Spelke, 2000). Therefore, infants' acuity for number is actually greater than their acuity for contour length, making it unlikely that infants preferentially attend to contour when numerical information is available. A more parsimonious explanation is that number is a salient dimensions for infants and not a last-resort strategy to be used only when information from non-numerical features is unavailable.

In the study described in Chapter 4, magnitude comparison performance for number, line length, and brightness in 4-year-old children and adults was assessed to determine whether individual differences in acuity are stable across magnitude dimensions. It was hypothesized that if infants' possess a general representation of magnitude that becomes differentiated over development (Walsh, 2003), then individual differences in magnitude comparison performance should exhibit stronger correlations in children compared to adults. Contrary to this prediction, performance for number and line length comparisons was correlated in both children and adults, with no apparent change in the strength of this relation. Further, line length and brightness acuity were correlated in adults but not in children, suggesting that a link between these magnitudes is constructed between the ages of four and adulthood. Together, these findings suggest that there may exist a privileged relation between number and line length early in ontological development. In addition, relations between other dimensions not privy to this initial association, such as brightness, can be constructed

later in development. These later developing associations between magnitudes may contribute to a general sense of magnitude without necessitating a biologically determined explanation.

The main finding from the study described in Chapter 5 is that sensitivity to number in 6-month-old infants, as measured by their numerical change detection scores, was predictive of their symbolic math knowledge at age 3.5. In addition, infants with higher numerical acuity were more likely to have mastered the cardinal principle of counting three years later than infants with poor numerical acuity. This is the first demonstration the numerical acuity, prior to exposure to any formal mathematics schooling, is related to later mathematics achievement. This finding lends support to the hypothesis that the ANS is foundational for the acquisition of symbolic math principles (Dehaene, 1997; Gallistel & Gelman, 1992) and suggests a causal direction for the correlation between ANS acuity and math performance. In particular, it suggests that children who have greater precision in their internal number representations may be at an advantage when it comes to learning numerical symbols and arithmetic principles.

The final empirical study, described in Chapter 6, explored the factors that contribute to the maturation of the number sense and the relation between number sense and symbolic math. This study employed a novel analytic technique adapted from DeWind et al. (under review) to objectively quantify the biasing effects of non-numerical features on children and adults' nonsymbolic numerical comparison performance. In 4-

year-old children, numerical decision-making was strongly biased by field area, such that children consistently viewed spaced out dot arrays as being more numerous than their actual numerical value. By 6 years of age, however, this field area bias was attenuated and children had developed numerical decision-making strategies that were indistinguishable from adults, albeit with poorer numerical acuity. This suggests that the ability to selectively attend to number over other non-numerical features may be one mechanism that underlies improvements in ANS acuity during early childhood. The second focus of this study was to assess the relation between attention to number, inhibitory control, and symbolic math skill. Previous studies have suggested that inhibitory control may mediate the relation between ANS acuity and math skill (Fuhs & Mcneil, 2013; Gilmore et al., 2013). However, this hypothesis was not supported by the present data. Numerical discrimination acuity, not non-numerical bias, was the strongest predictor of math achievement in 4- and 6-year-old children and this relation was not mediated by inhibitory control. This study therefore provides further support for the theory that symbolic math builds off of approximate number representations and specifically suggests that it is the precision of the ANS that is driving this relation. However, no relation between numerical acuity and math performance was found in adults, which suggests that the link between the ANS and symbolic math may be strongest when children are first learning numerical symbols and arithmetic principles.

7.2 Synthesis and future directions

Humans are predisposed to care about number, even before learning to count or beginning mathematics schooling. The studies presented in Chapters 2 and 3 contribute to a large body of research demonstrating that the tendency to spontaneously attend to number is present in infancy (see Feigenson et al., 2004 for a review). This tendency is not unique to humans; monkeys also spontaneously attend to number and, for example, will preferentially match arrays on the basis of number rather that physical size (Cantlon & Brannon, 2007). Numbers, it seems, are inherently special. However, the ubiquity of numbers in modern human societies extends beyond this evolutionarily privileged status. Modern cultures reinforce the emphasis on numbers through the use of numerical symbols. As children learn number words and begin formal math education, they may learn that number is a dimension to which attention should be specifically directed. In addition, children's increasing facility with numerical symbols improves the automaticity with which these symbols are processed, which results in more efficient symbol use and manipulation. However, this increase in automaticity may also make it increasingly difficult to suppress processing numerical symbols, even when it is advantageous to do so. In general, the ability to selectively attend to a feature of interest and inhibit attention towards other features improves over development (e.g., Gebuis, Cohen Kadosh, De Haan, & Henik, 2009a, see also Chapter 6 of the current dissertation). However, the interference effect of irrelevant numerical information on physical

judgments actually increases over development for Arabic digits (Gebuis, Herfs, Kenemans, & van der Smagt, 2009b). Cultural emphasis therefore appears to heighten the relative salience of number with respect to other magnitude dimensions.

Number may be a highly salient dimension, but it is still just one of many magnitude types that is represented with approximate mental magnitudes. After decades of research positioning numerical processing as a core domain of cognition (Dehaene, 1997; Hauser & Spelke, 2004; Kinzler & Spelke, 2007), there is renewed interest in the idea that the capacity for numerical abstraction may be just one aspect of a domain-general system for representing magnitudes (Bueti & Walsh, 2009; Cantlon, Platt, & Brannon, 2009b; R. Cohen Kadosh et al., 2008; Lourenco & Longo, 2011; Walsh, 2003). Currently, it is unclear to what extent abstract representations of magnitude arise from such a system. As demonstrated by the study in Chapter 4, not all associations between magnitude dimensions are necessarily biologically constrained. Just as cultural factors may emphasize one magnitude dimension over another, they may also influence the relations between different dimensions. Beyond shared ordinal vocabulary for describing magnitudes, metaphors that explicitly describe one dimension in terms of another are universal. Across languages and cultures, the most common of these are spatiotemporal metaphors. In particular, spatial vocabulary is commonly co-opted to refer to time (e.g., "she went on a *long* vacation," "she tried to put the past *behind* her"), whereas temporal terms are rarely used to refer to space (Lakoff & Johnson, 1980). Interestingly, the

asymmetry in these metaphors is consistent with the finding that spatial information biases temporal judgments more so than temporal information biases spatial judgments (Boroditsky, 2000; Bottini & Casasanto, 2010; Casasanto et al., 2010; Casasanto & Boroditsky, 2008; V. Dormal & Pesenti, 2012). Although it may be tempting to conclude that asymmetry in language can induce asymmetry in the representations themselves, it seems that spatiotemporal metaphors are a marker of, rather than the driving force behind, the asymmetrical relation. In the Greek language, spatiotemporal metaphors based on length are much less common than in English, and temporal durations are frequently described in terms of quantity (e.g., "a *big* meeting" versus "a *long* meeting"). Greek-speaking children, however, show the same asymmetrical effect of irrelevant length information biasing temporal judgments that is found in English-speaking children and adults (Casasanto, Fotakopoulou, & Boroditsky, 2010). This suggests that experience with length-based spatiotemporal metaphors is not necessary to create an asymmetry in the mapping between spatial length and temporal durations. However, experience with these metaphors does appear influence which types of magnitudes are likely to cause interference. Greek-speaking adults, who have a lifetime of experience using quantity-temporal metaphors, do not exhibit interference from length information and are instead swayed by irrelevant quantity information (Casasanto, 2008). Therefore, it seems that the language we use does influence magnitude processing. Consistent with the idea that facility with numerical symbols increases the automaticity with which they

are processed, the specific language used in spatiotemporal metaphors may likewise increase the saliency of those dimensions and increase their processing automaticity. Additional work is needed to determine both how the connections between magnitude dimensions may reorganize over development and across cultures, as well as to identify the mechanisms responsible for these changes. Although many functional neuroimaging studies have investigated the relation between different magnitude dimensions in adults, very few of these studies have included children (Kaufmann et al., 2006; Kaufmann, Vogel, Wood, & Kremser, 2008; G. Wood, Ischebeck, Koppelstaetter, Gotwald, & Kaufmann, 2009). These types of studies are necessary for determining the origins of the apparent overlap between magnitude dimensions, particularly for dimensions such as number and brightness that do not appear to be connected early in development.

Additional research will also be critical for understanding the basis of the relation between the ANS and symbolic math. If the ANS is truly foundational, as suggested by the study in Chapter 5 and the many studies documenting a correlational relation (see Q. Chen & Li, 2013 for a review), then improvements in ANS acuity should transfer to improvements in mathematics ability. To date, only a few studies have directly tested this tantalizing hypothesis. In one series of studies, college students were trained on an approximate arithmetic task that involved addition and subtraction problems with arrays of dots (Park & Brannon, 2013). This approximate arithmetic training led not just

to improvement on the task itself, but also to improvements in symbolic addition and subtraction as assessed by arithmetic tests administered before the first training session and immediately after the final training session. In contrast, subjects who received training on tasks involving other factors known to correlate with math performance (e.g., numerical comparison, working memory, and symbol ordering) did not exhibit improvement in their symbolic arithmetic performance (Park & Brannon, 2014). In a complimentary study, children's symbolic arithmetic performance was enhanced following practice with nonsymbolic arithmetic problems, whereas practice with adding and subtracting line lengths led to no such enhancement (Hyde et al., 2014). Together, these studies suggest that improving or engaging the nonverbal number sense boosts some aspects of symbolic mathematics ability. These studies also provide the strongest evidence to date that the ANS supports the acquisition of symbolic mathematics skills.

The possibility that there is a causal relationship between ANS acuity and symbolic mathematics skills brings about the exciting possibilities for developing novel mathematics education strategies. In particular, it suggests that interventions targeting the ANS may be particularly beneficial for young children who have not yet learned to count or begun formal arithmetic instruction. Improving children's nonsymbolic numerical competence may improve the foundation that symbolic mathematics builds upon, thus facilitating the acquisition of symbolic number and arithmetic principles. The question then becomes, which aspect of the ANS should be targeted for maximum

benefit? Although a link between ANS precision and mathematics ability has been documented throughout the lifespan (Halberda et al., 2012), it is possible that the nature of the relation may change with development and experience. Indeed, the association between ANS acuity and math achievement appears to be strongest in young children under the age of 6 (Fazio et al., 2014) and for informal math concepts (M. E. Libertus, Feigenson, & Halberda, 2013b). Consistent with these findings, the studies described in Chapters 5 and 6 found a relation between numerical acuity and math performance in 3-, 4-, and 6-year-old children, but no such relation in adults. Perhaps in children who have not yet begun formal schooling, the precision of the ANS is critical for learning the meaning of number words, such that children with sharper ANS acuity are better positioned to form mappings between approximate number representations and the counting words that they are just beginning to understand. However, for children with fuzzier internal representations of number, the difficulty with discriminating neighboring quantities may impede mapping number words onto those quantities. For these children, it may be beneficial to improve the precision of the ANS, so as to facilitate mastery of the count list and Arabic digits. Later, in older children and adults who are performing arithmetic operations, the fact that the ANS supports quantity manipulations may be the critical factor. At this stage, it may be manipulability of the representations, rather than their precision, that influences math achievement. This may explain why Park and Brannon (2014) found that nonsymbolic arithmetic training

translated to improvements in symbolic arithmetic performance, whereas nonsymbolic comparison training did not produce a transfer effect. For older children then, the most effective intervention strategies may focus on approximate arithmetic, which gives children the opportunity to become familiar with arithmetic principles in the absence of numerical symbols. Much more work is needed to explore these possibilities, as well as to explore possible mediating factors. In the study described in Chapter 6, it was found that numerical acuity and inhibitory control each contributed unique variance to children's mathematical skill. However, mathematics is a multi-faceted concept, and many factors including working memory, executive functions, socio-economic status are known to impact mathematics performance (e.g., Bull & Scerif, 2001; Espy et al., 2004; Geary, 2004; N. C. Jordan et al., 2009; Klibanoff et al., 2006). Determining how these factors interact with the ANS is an important avenue for future research.

A related open question concerns the neural changes that are induced by ANS-focused interventions. Neuroimaging studies with both adults and children have found that recovery of the BOLD signal in the IPS after adaptation to a specific numerosity is dependent on the ratio between the novel and adapted numerosity (Cantlon, Brannon, Carter, & Pelphrey, 2006; Piazza et al., 2004; 2007). A plausible hypothesis then is that improving ANS acuity may result in improved sensitivity to number within the IPS. Individual differences in sensitivity to symbolic numerosities in the IPS have previously been correlated with arithmetic competence in school-aged children (Bugden, Price,

McLean, & Ansari, 2012), suggesting that the numerical sensitivity of the IPS is one source of variance in math skill. If improving the acuity of the ANS serves to enhance IPS sensitivity, this would provide a biological explanation for the observed link between ANS acuity and symbolic math skill. However, symbolic math draws on a large network of brain regions in addition to the IPS, including prefrontal cortex, angular gyrus, and occipto-temporal cortex (e.g., Dehaene, 1999; Menon, Rivera, White, Glover, & Reiss, 2000; Rivera, Reiss, Eckert, & Menon, 2005). Furthermore, functional connectivity between the IPS and prefrontal regions has also been shown to correlate with children's math performance (Emerson & Cantlon, 2012; Park, Li, & Brannon, 2014). Therefore, it will be critical to assess how ANS-focused interventions may influence responsivity both within and outside of the IPS, as well as connectivity patterns between these regions.

7.3 Conclusions

The studies described in this dissertation provide new insights into the nature of quantity representations over the course of ontological development. This work provides important information about the situations in which infants engage the ANS, the relation between the ANS and representations of other magnitude dimensions, and the aspects of the ANS that support the acquisition of symbolic mathematics. Infants are born with a natural tendency to attend to number. When children learn numerical symbols and map them to their internal approximate number representations, they are

able to harness the power of the ANS for a new representational system. At the same time, numerical symbols enable children to move beyond the inherent constraints of the ANS in order to represent and manipulate exact quantities. This reciprocation is a critical factor behind the development of the human mathematical mind. Although many questions remain regarding the specific mechanisms, this dissertation provides broad support for the significance of approximate number representations in the acquisition of symbolic math principles.

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Biography

Ariel Starr was born on November 18, 1985, in Seattle, Washington. She received her Bachelor's degree in 2007 from Wesleyan University where she double-majored in Neuroscience & Behavior and Psychology, and was awarded high honors in Psychology. Ariel spent the next three years working as a research assistant at the University of Southern California and the University of California, Los Angeles. In the fall of 2010, Ariel joined the Department of Psychology & Neuroscience at Duke University to pursue a doctoral degree under the guidance of Dr. Elizabeth Brannon. Her doctoral research was supported by a National Science Foundation Graduate Research Fellowship and a Society for Research in Child Development Student and Early Career Council Dissertation Funding Award. Ariel's work on the development of quantity representations has been published in journals including *Proceedings of the National Academy of Sciences*, *Infancy*, and *Cognitive Development*.