

# Multiparty Bargaining Strategies

Comparing Nash Bargaining Payoffs of Bilateral and Multilateral Negotiation  
Strategies during Conflict Bargaining

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## 1 Abstract

Since 2003, the United States and North Korea have been at odds over the creation and continuation of a North Korean nuclear weapons program. While North Korea lobbies strongly for these differences to be sorted out through bilateral negotiations between the two nations, the United States refuses to partake in any negotiations other than the multilateral Six-Party Talks. Seeking to determine if the bargaining framework (bilateral or multilateral) between several economic agents might grant one or more agents a strategic advantage, we developed a three-player bargaining model for both a single multilateral negotiation and for a series of bilateral negotiations involving all three players. We also included in our model “conflict coefficients” which can simulate disagreement erupting into damaging conflict between two players. Hence, our model can further simulate nations on the brink of armed conflict, companies at risk of entering a price war, or other scenarios where players might cause a decrease in each other’s initial wealth or utility. Conflict coefficients were designed in such a way that they can be removed from the model effortlessly to attain more general results. We concluded that there are indeed strategic advantages and disadvantages of multilateral and bilateral bargaining games for each player depending on their disagreement points and the surpluses being divided. In cases of conflict bargaining, expected payoffs for each player and preferred bargaining framework are further affected by their own conflict coefficient and those of the other players.

## 2 Introduction

In early 2003, several findings concerning North Korea led to an upset in global international relations. United States Intelligence Agencies announced that North Korea had an active nuclear weapons program and, shortly after, North Korea withdrew from the Nuclear Non-Proliferation Treaty and publicly announced that they were in possession of nuclear weapons.<sup>1</sup> The United States, in an effort to stymie the growth of the North Korean nuclear arsenal, knew it would have to enter negotiations with North Korea with the aim of dismantling their nuclear weapons program.

Since 2003, North Korea has sought to deal with the issues of disarmament and security guarantees through bilateral discussions with the United States alone. The United States, under President George W. Bush and Secretaries of State Collin Powell and Condoleezza Rice, have repeatedly reject the idea of holding bilateral negotiations with the North Koreans<sup>2</sup>, agreeing only to the creation of the “Six-Party Talks” – regularly scheduled multilateral negotiations between the United States, North Korea, China, South Korea, Japan, and Russia.<sup>3</sup>

Based solely on the vehemence with which each side lobbies for their form of negotiations (bilateral vs. multilateral), both nations consider the structural framework of the negotiations of noticeable importance. Principles of economics would suggest that the United States believes there are significant advantages for

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<sup>1</sup> Weldon, June 4, 2003, House of Representatives

<sup>2</sup> CBS News, Oct. 11, 2006

<sup>3</sup> CNN News, Oct. 10, 2006

its position in multilateral bargaining on this subject, whereas North Korea clearly believes it will garner greater utility if each of the Six-Party nations were to bargain bilaterally each one with the next.

There are several arguments why each nation would desire the bargaining framework they lobby for. For instance, Secretary of State Rice has often claimed that bilateral negotiations with North Korea would be subject to the commitment problem – The United States has little leverage to ensure that North Korea follows the terms of any agreement, whereas China specifically has much more leverage over its smaller, communist neighbor of North Korea.<sup>4</sup> Furthermore, State Department Officials have admitted they considered negotiating over such an important issue without including East-Asian allies a diplomatic faux pas. North Korea, on the other hand, would prefer to gain status, economic aid, and a security guarantee from the United States without having to compromise on issues with other nations such as Japan and South Korea.<sup>5</sup>

The example above of North Korean disarmament negotiations is just one example of an economic issue of more general importance: Are there strategic differences between holding multilateral negotiations between several parties and holding a series of bilateral negotiations between those same parties? In what situations would an economic agent prefer multilateral negotiations over a series of bilateral negotiations and vice-versa?

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<sup>4</sup> *ibid*

<sup>5</sup> Crowely, P. J, Senior Fellow and Director of National Defense and Homeland Security

We base our analysis of these questions on the bargaining model proposed by John Nash in 1950. Using the concepts of Nash Cooperative Bargaining, we developed a three-player bargaining model for both multilateral negotiations and a series of bilateral negotiations involving all three players. By determining their equilibrium expected payoffs using Nash Bargaining Solutions, we compared the expected payoffs under each bargaining framework (multilateral and various bilateral arrangements). In so doing, we were able to develop a system to determine when a given player would have a strategic incentive to seek multilateral bargaining over bilateral bargaining frameworks.

We expanded the applicability of our bargaining model by including an additional concept which we have termed “conflict coefficients.” Conflict coefficients allow us to simulate different players losing additional utility to conflicts in the event of a disagreement. For instance, in a simulation of North Korea, the United States, and China bargaining over nuclear disarmament and wealth, failure might lead to armed conflict between the US and North Korea. In a simulation of three competing companies, a conflict between two companies might take the form of a detrimental price war. Regardless of the situation, the conflict coefficient simulates a percentage loss of wealth (or utility) at the end of the entire bargaining process.

To maintain the ability to simulate non-conflict situations as well, setting the conflict coefficient to equal one will remove it from all equations, allowing the results to be generalized to instances without these conceivable conflicts.

We found that, given full knowledge of the bargaining game, there are indeed instances when a player gains a strategic advantage through multilateral negotiations and other instances when the player gains a strategic advantage from a series of bilateral negotiations. The specific requirements for when each player prefers a certain bargaining framework are outlined in the analysis section (5.5).

The conclusions reached in this paper have policy implications for many real-world economic agents including nations engaging in diplomacy, companies negotiating with competitors, or even individuals bargaining over monetary distributions (such as how wages should be divided). In addition to determining preferences between multilateral and bilateral negotiations based on commitment issues and rules of decorum, policy makers can also consider whether one bargaining framework offers a substantial strategic advantage in order to increase expected payoffs.

### 3 Literature Review

A large body of economic literature exists on bargaining as both a normative and positive process. However, the majority of this literature has focused on various forms of bilateral bargaining games – often making use of the Nash cooperative bargaining model or the Rubenstein strategic bargaining model to research new problems. In fact, as recently as 1994, William Zartman stated “no conceptual work addresses the vast area of multilateral negotiations... Implicitly or explicitly all negotiation theory addresses bilateral negotiations.” Although other economic researchers, such as Robert H. Mnookin, argue that Zartman’s claim is an exaggeration, it is clear that there is comparatively little literature that analyzes the effects of multilateral negotiation strategies on bargaining payoffs. However, listed below are several published papers that have sought to compare bilateral and multilateral negotiations in a given scenario.

Robert H. Mnookin (2003), in his piece *Strategic Barriers to Dispute Resolution: A Comparison of Bilateral and Multilateral Negotiations*, highlights several explanations of why real-life bargaining situations might deviate from game theory expectations. Mnookin focuses on “strategic barriers” or barriers “that can cause rational, self-interested disputants to act in a manner that proves to be both individually and collectively disadvantageous” (Mnookin, 2). His paper treats several items, both strategic and psychological, which would cause players in a bargaining game to accept sub-optimal payouts or force a bargaining game with mutually-beneficial possibilities to end in disagreement.



Again, for the purposes of this paper we elected to model all bargaining games with Nash Bargaining. In contrast to Rubenstein's bargaining model, Nash Bargaining removes the effects of bargaining procedure and seeks only equilibrium solutions that fulfill Nash's required axioms. As a result, the vast majority of Mnookin's findings are not applicable to the model derived in this paper. However, it would be improper to ignore one of Mnookin's most important arguments – that the mathematics of game theory requires very precise rules and protocols which generally do not exist in real world bargaining situations. An interesting extension to our paper would be to analyze the effects of ultimatum offers, coalitions, imperfect information of disagreement points, and holdout strategies on both the bilateral and multilateral bargaining games developed later in this paper.

Giovanni Maggi (1999) in *The Role of Institutions in International Trade Cooperation* also investigates the differences between multilateral bargaining games and a web of bilateral bargains. In some aspects, Maggi's treatment of the subject is very similar to our own, including his use of Nash Bargaining solutions to estimate equilibrium solutions and the assumption of equal bargaining power in both multilateral and bilateral bargaining games. However, Maggi's analysis differs from our own in several fundamental ways. First and most importantly, Maggi's analysis delves into the areas of enforcement issues and global efficiency. Our treatment of the subject to date assumes that agreements are perfectly enforceable by nature since distribution occurs immediately following negotiations. However, a possible future extension of our model might include enforceability and credibility concerns in contracts. Furthermore, we approach

the subject through seeking the optimal strategy for each individual player as opposed to trying to maximize the efficiency of the entire system. As a result, solutions to our model might come to conclusions that are optimal for a given player but are inefficient for the system as a whole. Since our goal is to determine when bilateral or multilateral negotiations are in the best interest of a given player, we accept that there might exist more efficient solutions that are not selected by our model.

Gilles Rotillon et. al. (1996) also present a comparison of multilateral and bilateral negotiations in their paper *Bilateral or multilateral bargaining in the face of global environmental change?* Rotillon et. al. develop a model in which nations are divided into various categories based on generally aligned characteristics. Primarily, their model discusses developed nations, as represented by the OECD countries, and developing nations. The OECD nations collectively want to reduce the level of global greenhouse emissions but for any given nation to do so on their own implies a cost to that nation. As a result, the OECD nations become the leaders or a bargaining game between themselves and the developing nations in the hopes of reducing global greenhouse emissions at minimal costs to themselves. The bargaining process is carried out as a series of offers between the OECD nations and the various developing nations in the following manner: The OECD nations offer each developing nation a certain monetary transfer in return for a given decrease in greenhouse emissions. Each developing nation can either accept or reject the offer. Those that accept sign the offer and leave. Those that reject the offer can make a counter-offer to the OECD nations. This process can carry itself out in as many iterations as necessary, however, the Rotillon

model includes a transaction cost for each round of negotiations. Rotillon et. al. seek to determine whether it is in the best of interest of the developing nations to act in concert with each other or to negotiate with the OECD separately,

Although the general comparison of multilateral and bilateral negotiations is once again similar to our own analysis, there are several key features that differentiate our analysis from that of Rotillon et. al. Firstly, the Rotillon et. al. model was derived specifically for situations in which one group (or player) is offering a transfer to all other players in order to reduce a negative neighborhood effect (greenhouse gas emissions). Our model is left much more general and would require several additional assumptions to effectively model such a system. Furthermore, our model makes no assumptions about similarities between groups nor gives them significant reason to act in concert with each other against a leading power (such as the OECD nations). Finally, our model relies on the equilibrium solutions given by Nash's axioms rather than solutions dependent on procedural aspects of the game (including the order that offers are made in).

Although the above papers have analyzed various aspects of how multilateral and a series of bilateral negotiations would differ, there are several reasons why we believe our analysis will yield novel findings.

First, by relying on Nash's model of bargaining, we hope to find bargaining solutions independent of all bargaining procedures other than the decision to hold multilateral or bilateral negotiations and in what order bilateral negotiations would occur. We assume that each bargaining game reaches its Nash Bargaining

Solution regardless of whom proposes first in each sub-game, the transaction costs of counter-offers, or other procedural aspects. We also assume that agreements are perfectly enforceable and therefore remove the ability to renege as a determinant of strategy. Since we are primarily trying to ascertain if bilateral negotiations or multilateral negotiations benefit a player in a given scenario, these assumptions are necessary for results to be derived solely from the type of bargaining and not from

Second, specifically to facilitate modeling of possible conflict bargaining situations between states, we have included the ability for “conflict” to arise between various players. In our current three player model, conflict can only arise between two of the players, however this will be generalized in the upcoming n-player model. With the addition of these “conflict multipliers” we allow for bargaining situations in which disagreement means something other than leaving with what you entered with. Disagreement can now be extended to include damaging the wealth or utilities of individual players. However, by setting the conflict multipliers to one, we leave the option of seeing the affects of bilateral and multilateral negotiations on a situation free of this special conflict case.

## 4 Mathematical Model

As stated before, we set out to determine in what situations players benefit from a series of bilateral negotiations over a single multilateral negotiation during a conflict bargaining scenario. We start by constructing the simplest model possible with three players – the United States, North Korea, and China.

The United States is unique in that it is the “central player,” and can therefore determine the bargaining framework. The United States can select to have a multilateral negotiation with all three players coming to a joint decision or can select a series of bilateral discussions instead. In the case of bilateral negotiations, the United States can either negotiate with China first or North Korea first, however, the United States must negotiate with both China and North Korea. In the case of bilateral negotiations, China and North Korea never negotiate directly with each other (this possibility could be added as a further extension in the future).

North Korea is unique in that it is the “conflict player.” In the event of a disagreement between North Korea and the United States, there is a conflict between the two players with resulting in both North Korea and the United States losing some known percentage of their final wealth.

Although there are several standard models for modeling bargaining situations, we selected to design our model as an extension of Nash Bargaining in order to minimize the effects of sub-game strategies on results. As an extension of Nash Bargaining, our solutions are required to exhibit several characteristics as outlined

by John Nash including that solutions are: (1) invariant to affine transformations, (2) Pareto optimal, (3) independent from irrelevant alternatives, and (4) symmetric.<sup>6</sup> However, in contrast to a normal Nash Bargaining problem, players now act to maximize their utility over a series of bargaining games where the resulting payoffs of each negotiation affect the setup of later negotiations.

All players have complete knowledge of the game including their own wealth, other players' wealth, the set of all possible bargaining solutions, and the percentage loss that the United States and North Korea would receive in a conflict. Hence, each player can estimate the equilibrium results of not only their current bargains, but also equilibrium results of bargains they will participate in later and also equilibrium results of bargains other players will participate in later. Furthermore, we assume that all players have equal bargaining power both in bilateral bargaining games and in multilateral bargaining games.

All objective of all three players is to maximize their final utility by maximizing their final wealth. We assume that utility is linearly increasing with final wealth, and therefore assume the utility derived from a player's wealth at during any sub-game is the amount of wealth this will generate at the end of all negotiations.

We modeled the above scenario using the following variables:

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<sup>6</sup> Nash, John F. 1950

$X_{US}$ ,  $X_{Cb}$ , and  $X_{NK}$  are used to denote the final wealth of the United States, China, and North Korea, respectively. The objective of each player is to maximize their final wealth.

$w_{US}$ ,  $w_{Cb}$ , and  $w_{NK}$  are used to denote the initial wealth of the United States, China, and North Korea, respectively.

$\pi_{Cb}$  and  $\pi_{NK}$  are used to define the set of possible bargaining solutions that exist between the United States and China and between the United States and North Korea, respectively. In the bargaining games shown hereafter, the possible set of bargaining solutions is defined as the summation of total wealth to be divided between the given players, each division resulting in a final utility for each player.

$r_{US}$  and  $r_{NK}$  give the percentage loss of total wealth experienced in the case of a conflict between Players A and C. It is not necessary for both players to incur the same percentage loss during a conflict, hence the need for a separate variable for both Player A and Player C.

$d_{US}$ ,  $d_{Cb}$ , and  $d_{NK}$  are the disagreement points for the United States, China, and North Korea, respectively, during any given bargaining situation. The disagreement values vary from player to player and further vary for each player between different bargaining games.

$Z_{US}$ ,  $Y_{US}$ , and  $w_1$  are additional variables used. Since the United States can participate in multiple bargaining games, we use  $Z_{US}$ ,  $Y_{US}$ , and  $w_1$  to give the United States' wealth at various stages of the problem. More detailed information will be given when these variables are used.

## 5 Analysis

For the three player model outlined above, a full analysis can be divided into solving three related problems. The first two problems are solving for the expected payoffs for each player during a series of bilateral negotiations. In (5.1) we will find the expected payoffs for when the US bargains with China first and then negotiates with NK last. In (5.2) we will find the expected payoffs for when the US bargains with NK first and then negotiates with China last. In (5.3) we solve for the expected payoffs of a multilateral negotiation between all three players – the US, China, and NK. Finally, in (5.4) we compare the expected payoffs given by each bargaining strategy and determine when a player would benefit from bilateral negotiations over multilateral negotiations in conflict bargaining.

For the purposes of this paper, we begin with a simplified case instead of the generalized model. The two simplifications are as follows:

1. The initial wealth of North Korea is equal to zero ( $w_{NK} = 0$ ).
2. The United States and North Korea have a non-productive relationship. Hence, no additional wealth can be generated if the United States and North Korea come to any agreement ( $s_{NK} = 0$ ). However, the loss of wealth caused by conflict in disagreement can be avoided.

Noting these simplifications, we now observe the model.



### 5.1 Bilateral Bargaining Series – Negotiate with China then North Korea

#### Result 5.1.1 –Summary of Payoffs in a Bilateral Series (China first)

In a bilateral series of negotiations in which the US first negotiates with China and then the US negotiates with NK, the final wealth (or payoffs) to each player will be as follows:

$$X_{US} = \left(\frac{1 + r_{US}}{2}\right)w_0 + \left(\frac{1 + r_{US}}{3 + r_{US}}\right)s_{ch}$$

$$X_{NK} = \left(\frac{1 - r_{US}}{2}\right)w_0 + \left(\frac{1 - r_{US}}{3 + r_{US}}\right)s_{ch}$$

$$X_{ch} = w_c + \left(\frac{1 + r_{US}}{3 + r_{US}}\right)s_{ch}$$

These payoffs assume all variable constraints outlined in section (5.4) are enforced.

Derivation:

First, let us explain this scenario as it occurs chronologically. In round one, the United States and China begin with their initial wealth  $w_0$  and  $w_{ch}$ , respectively. They negotiate over the division of  $\pi_{ch}$ . Upon conclusion of their negotiation, China has final wealth  $X_{ch}$  and the United States has some wealth  $Z_{US} = w_0 + w_1$ , where  $w_1$  is their change in wealth from negotiations with China and  $Z_{US}$  is the United States' wealth at the end of round one negotiations.

Next, the United States and North Korea enter a bargaining game. The United States begins with initial wealth  $Z_{US} = w_0 + w_1$  and North Korea begins with initial wealth  $w_{NK} = 0$ . They negotiate over the division of  $\pi_{NK}$ , which equates

to all of the possible divisions of  $Z_{US}$ . Upon conclusion of their negotiation, the United States will have some final wealth  $X_{US}$  and North Korea will have some final wealth  $X_{NK}$ . In the event of disagreement between the United States and North Korea in this bargaining game, both will take percentage losses of their final wealth given by multiplying their wealth by  $r_{US}$  and  $r_{NK}$ . Since North Korea has initial wealth of zero, in any disagreement it ends with a final wealth of zero.

Since all players have full knowledge of all bargaining situations including each other players' wealth and utility function, they will act in each individual negotiation in order to maximize their total wealth over the course of all negotiations. To solve this series of bilateral negotiations, we can make use of backwards induction because of each player's ability to compute the result of future bargaining games based on the results of current and finished games.

Let us assume that the United States and China have negotiated in round one and that the United States will now begin negotiations with North Korea. The United States has some wealth  $Z_{US} = w_0 + w_1$  where  $w_1$  has some value, including possibly zero (which would mean the United States and China reached disagreement in round one). We also know the disagreement points for the negotiation, where the United States and North Korea end with their starting wealth values at the beginning of round two multiplied by their conflict values, or  $d_{US} = r_{US}(w_0 + w_1)$  and  $d_{NK} = r_{NK}w_{NK} = 0$ .

We now proceed in solving this game. As per Nash Bargaining, we are trying to maximize the product of the utility gained by each player subject to the fact that

the summation of their final wealth values do not exceed the total wealth available. Or:

$$\begin{aligned} & \max_{X_{US}, X_{NK}} ((X_{US} - d_{US})(X_{NK} - d_{NK})) \\ & \text{subject to } X_{US} + X_{NK} = \pi_{NK} = w_0 + w_1 \end{aligned}$$

Solving the above set of equations leads to the solutions:

$$\begin{aligned} X_{US} &= \frac{d_{US} + \pi_{NK} - d_{NK}}{2} \\ X_{NK} &= \frac{d_{NK} + \pi_{NK} - d_{US}}{2} \end{aligned}$$

We know  $d_{US} = r_{US}(w_0 + w_1)$  and  $d_{NK} = 0$  so by substituting these values into the functions above we find:

$$\begin{aligned} X_{US} &= \left(\frac{1 + r_{US}}{2}\right)(w_0 + w_1) \\ X_{NK} &= \left(\frac{1 - r_{US}}{2}\right)(w_0 + w_1) \end{aligned}$$

These functions describe the final payoffs for the United States and North Korea given an arbitrary  $w_i$  from the round one negotiation with China and when an agreement is reached.

Using these results for any arbitrary  $w_i$ , we can calculate the expected payoffs for the United States and China during the round one negotiations. Let us say that in round one negotiations between the United States and China, the United States has initial wealth  $w_0$  and China has initial wealth  $w_{ch}$ . In addition, there is a trade surplus of quantity  $s_{ch}$  which the two countries may negotiate over. If disagreement is reached, the trade surplus is wasted. Both players are trying to maximize their final wealth following all negotiations. For China, this simply means maximizing their payoff from this negotiation ( $X_{ch}$ ). However, for the United States, this is slightly more complicated. The United States seeks to maximize their payoffs at the end of second round negotiations with North Korea ( $X_{US}$ ) but will leave this negotiation only with the intermediate wealth  $Z_{US}$ . As a result, the United States must consider what final wealth it will achieve for any intermediate  $Z_{US}$  by observing the expected results of round two negotiations with North Korea given  $Z_{US} = w_0 + w_1$ . Hence, we seek to solve the following maximization problems as before:

$$\begin{aligned} & \max_{X_{US}, X_{ch}} ((X_{US} - d_{US})(X_{ch} - d_{ch})) \\ & \text{subject to } Z_{US} + X_{ch} = w_0 + w_{ch} + s_{ch} \end{aligned}$$

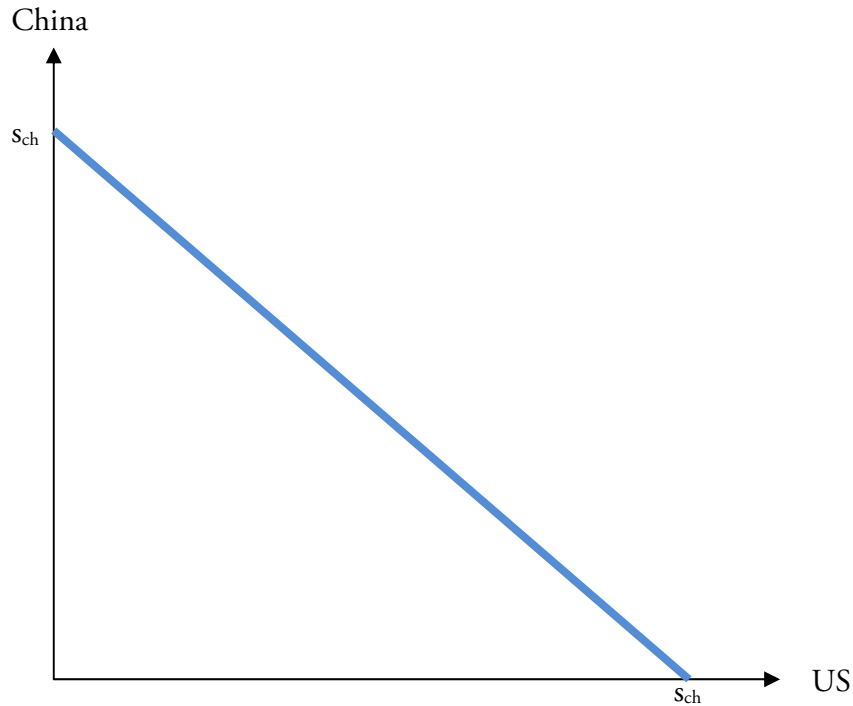
It is important to have a good understanding of each player's disagreement points in this instance. In the event of a disagreement, China will simply end negotiations with the initial wealth it began with ( $w_{ch}$ ). However, the United States' disagreement point is more complicated than simply being equal to its initial wealth. In the event of disagreement, the United States will enter round two negotiations with only their initial wealth  $w_0$ , but more importantly, its final wealth  $X_{US}$  will be equal to the value

achievable when  $Z_{US} = w_0$  (equivalently, when  $w_1 = 0$ ). Hence, using the value of  $X_{US}$  given  $w_1$  solved for above, we know the disagreement points are:

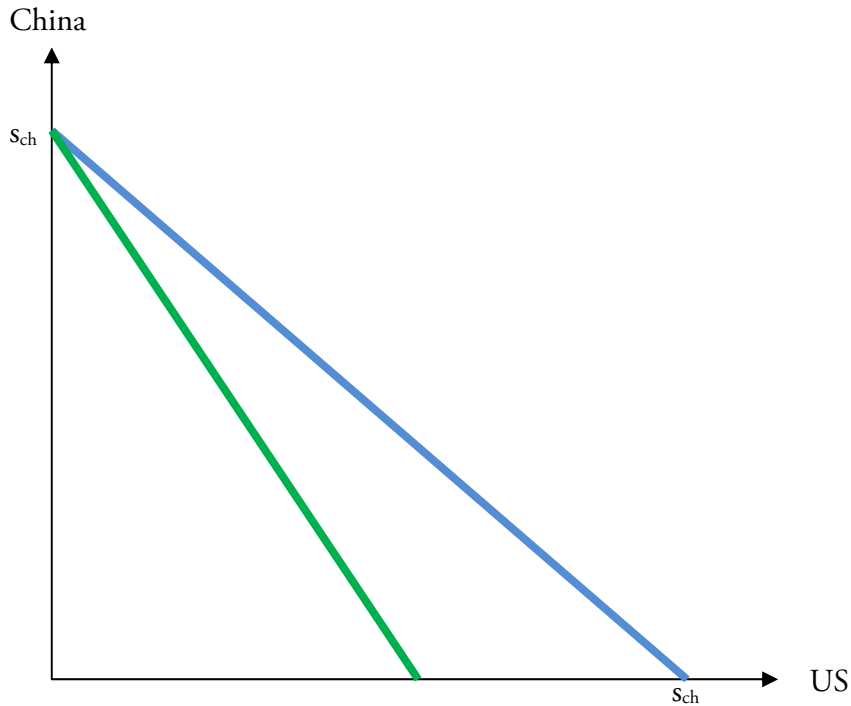
$$d_{US} = X_{US}(w_1 = 0) = \left(\frac{1 + r_{US}}{2}\right)(w_0)$$
$$d_{ch} = w_{ch}$$

Because neither party will accept any agreement in which they receive less than their disagreement point, China will require any agreement to have  $X_{ch} \geq w_{ch}$ . Similarly, the United State will require any agreement to cause  $X_{US} \geq \left(\frac{1+r_{US}}{2}\right)(w_0)$ . Since the  $X_{US}$  is strictly increasing with the value of  $Z_{US}$ , this is equivalent to requiring that  $Z_{US} \geq w_0$ . Also, the summation of wealth at the end of round one cannot exceed the total amount of wealth available to the United States and China.

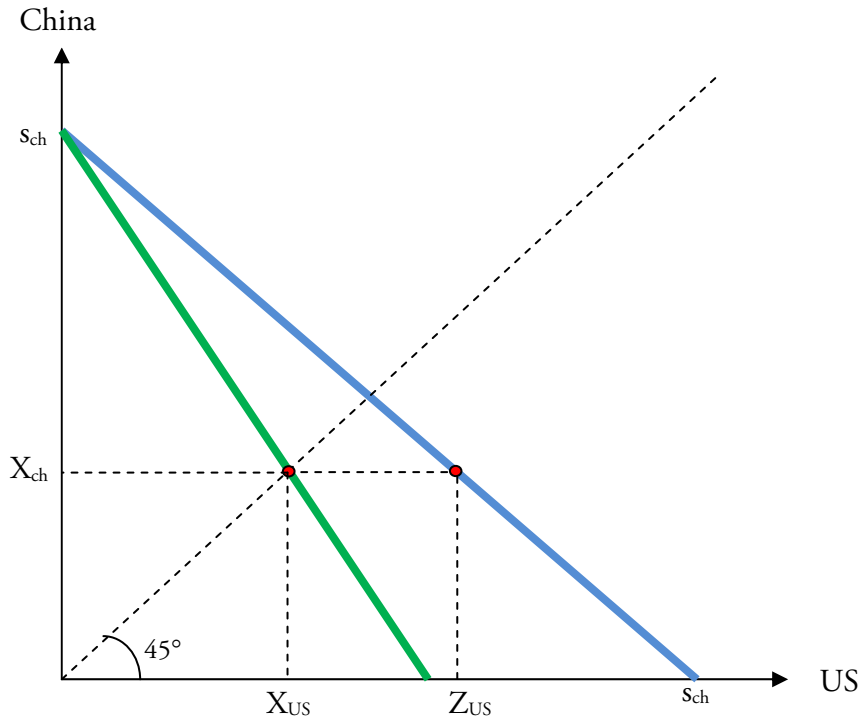
Hence, we know that in any agreement, since China will demand at least  $w_{ch}$  and the United States will demand at least  $w_0$ , the only wealth that can still be divided amongst the two is  $s_{ch}$ . The possible divisions of  $s_{ch}$  between China and the US can be represented by the blue line in the graph below.



It is important to keep in mind, however, that any wealth the United States gains from this negotiation is then subject to change in its round two negotiation with North Korea. Hence, although the China's utility is equivalent to amount of wealth it has at the end of this negotiation, the United State's final utility is subject to a transformation in the round two negotiation with North Korea. As solved for before, the final wealth of the United States after all rounds of negotiation is equal to  $X_{US} = \left(\frac{1+r_{US}}{2}\right)(w_0 + w_1)$  when it first negotiates with China and then with North Korea. Hence, any wealth the United States has at the end of round two will be multiplied by a factor of  $\left(\frac{1+r_{US}}{2}\right)$ . As a result, although the possible divisions of wealth are represented by the blue line in the graph, the final utility division between the United States and China are shown as the green line:



The next step is to determine which utility division the United States and China will both agree to and, as a result, what division of wealth round one will result in. As a characteristic of our Nash Bargaining game, we have required symmetric payouts. Hence, the United States and China will agree only to divisions of wealth in which they receive at least half of the surplus utility. Since the only possible solution to both players receiving “at least half” of the surplus is for both players to receive *exactly* half of the surplus utility, we can solve for equilibrium division of wealth using the following figure:



At the designated point on the green line, both the United States and China are receiving exactly half of the surplus utility. Using this, we can derive the wealth for the United States and China at the end of round one:

$$Z_{US} = w_0 + \left(\frac{2}{3 + r_{US}}\right) s_{ch}$$

$$X_{ch} = w_c + \left(\frac{1 + r_{US}}{3 + r_{US}}\right) s_{ch}$$

Now that we have a meaningful value for  $Z_{US}$  above we can use it to solve for the actual expected results of the round two negotiation between the United States and North Korea. As before, we solve



$$\begin{aligned} & \max_{X_{US}, X_{NK}} ((X_{US} - d_{US})(X_{NK} - d_{NK})) \\ & \text{subject to } X_{US} + X_{NK} = \pi_{NK} = Z_{US} = w_0 + \left(\frac{2}{3+r_{US}}\right) s_{ch} \end{aligned}$$

And our results are:

$$\begin{aligned} X_{US} &= \left(\frac{1+r_{US}}{2}\right) (Z_{US}) = \left(\frac{1+r_{US}}{2}\right) w_0 + \left(\frac{1+r_{US}}{3+r_{US}}\right) s_{ch} \\ X_{NK} &= \left(\frac{1-r_{US}}{2}\right) (Z_{US}) = \left(\frac{1-r_{US}}{2}\right) w_0 + \left(\frac{1-r_{US}}{3+r_{US}}\right) s_{ch} \end{aligned}$$

And from before:

$$X_{ch} = w_c + \left(\frac{1+r_{US}}{3+r_{US}}\right) s_{ch}$$

## 5.2 Bilateral Bargaining Series – Negotiate with North Korea then China

### Result 5.2.1 – Summary of Payoffs in a Bilateral Series (North Korea negotiates first)

In a bilateral series of negotiations in which Player A first negotiates with Player C and then Player A negotiates with Player B, the final wealth (or payoff) to each player will be as follows:

$$X_{US} = \left(\frac{1 + r_{US}}{2}\right) w_0 + \left(\frac{1}{4}\right) \left(\frac{3r_{US} + 1}{r_{US} + 1}\right) s_{ch}$$

$$X_{ch} = w_{ch} + \frac{s_{ch}}{2}$$

$$X_{NK} = \left(\frac{1 - r_{US}}{2}\right) w_0 + \frac{1}{4} \left(\frac{1 - r_{US}}{r_{US} + 1}\right) s_{ch}$$

These payoffs assume all variable constraints outlined in section (5.4) are enforced.

Derivation:

Still assuming a series of bilateral negotiations involving all three players, the United States could also chose to negotiate first with North Korea and then negotiate with China afterwards. We can solve this scenario in much the same way as we solve section (5.1) above – through backwards induction.

As before, to solve the round one negotiations between the United States and North Korea, we must determine the result of the round two negotiation given any specific outcome for the round one negotiation. We will begin our analysis for what would happen in round two if the United States and North Korea were to reach

disagreement in the first round. Recall that in the event of disagreement between the United States and North Korea, both players will lose some percentage of their wealth at the end of both rounds of negotiations. The amount lost is determined separately for each by  $r_{US}$  and  $r_{NK}$ . Since North Korea is assumed in this simplified case to have initial wealth of zero, in a disagreement their final wealth is equal to zero.

Because we assumed that disagreement was reached in round one negotiations, the United States enters the second round negotiation with only their initial wealth  $w_0$ . China enters the negotiation with wealth  $w_{ch}$ . The players negotiate over the wealth they will hold at the end of the round, which we will call  $Y_{US}$  and  $X_{ch}$ . Furthermore, both the United States and China are aware that  $X_{US}$ , or the United States' final wealth, will be only  $X_{US} = r_{us}Y_{US}$  as the United States has its wealth diminished by their conflict with North Korea following both rounds of negotiations. Hence, we are trying to solve the set of equations:

$$\begin{aligned} & \max_{Y_{US}, X_{ch}} ((r_{US}Y_{US} - d_{US})(X_{ch} - d_{ch})) \\ & \text{subject to } Y_{US} + X_{ch} = w_0 + w_{ch} + s_{ch} \end{aligned}$$

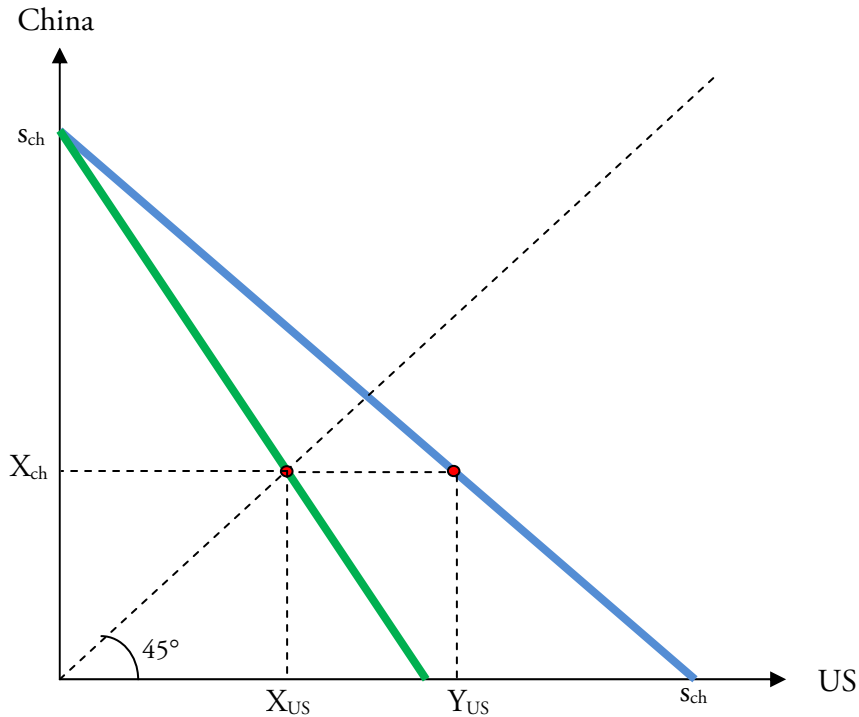
Disagreement points for China and the United States are as follows:

$$\begin{aligned} d_{US} &= r_{US}w_0 \\ d_{ch} &= w_{ch} \end{aligned}$$

In the division of their wealth, the United States will accept no division that leaves them with less than their disagreement point. Hence,  $Y_{US} \geq w_0$ . China will also only

accept divisions that grant them greater wealth than their disagreement point. Hence,  $X_{ch} \geq w_{ch}$ . Similar to the negotiation between the United States and China in section (5.1), this leaves only  $s_{ch}$  as the wealth that can be divided freely amongst the two players.

As before, the blue line in the following graph represents all possible divisions of wealth between China and the United States. Once again, the utility China derives is exactly equal to the amount of wealth it has at the end of the negotiation. However, the wealth the United States has ( $Y_{US}$ ) will be transformed by being multiplied by  $r_{US}$ . Hence, the green line in the graph below represents all possible utility divisions between China and the United States. Neither player will accept any agreement which grants them less than half of the available surplus utility, hence the final utilities will be at the point indicated on the green utility line.



Solving this system of equations leaves us with the following:

$$Y_{US} = w_0 + \left( \frac{1}{1 + r_{US}} \right) s_{ch}$$

$$X_{US} = r_{US} Y_{US} = r_{US} w_0 + \frac{1}{\left( 1 + \frac{1}{r_{US}} \right)} s_{ch}$$

$$X_{ch} = w_{ch} + \left( \frac{r_{US}}{1 + r_{US}} \right) s_{ch}$$

Since  $X_{US} = r_{US} w_0 + \frac{1}{\left( 1 + \frac{1}{r_{US}} \right)} s_{ch}$  will be the final wealth of the United States any time they reach round one disagreement with North Korea, we can input this as their disagreement point in round one with North Korea. By inputting this true value for

$d_{US}$  we are able to solve for the true value of the United State's wealth following round one negotiations with North Korea.

If, however, an agreement was made in round one between the United States and North Korea, then the two players will not have their final wealth decreased by their conflict coefficients. As a result, the round two negotiations between the United States and China will be very different. Let us call the wealth of the United States following round one negotiations with North Korea  $Z_{US} = w_0 + w_1$  where, once again,  $w_0$  is the United State's initial wealth and  $w_1$  is their change in wealth from round one (recall,  $w_1$  can be negative or positive). In this case, round two negotiations between the United States and China will be determined through the following:

$$\begin{aligned} & \max_{X_{US}, X_{ch}} ((X_{US} - d_{US})(X_{ch} - d_{ch})) \\ & \text{subject to } X_{US} + X_{ch} = Z_{US} + w_{ch} + s_{ch} \end{aligned}$$

In this instance, the disagreement points of the United States and China are simply the level of wealth they enter round two negotiations with:

$$\begin{aligned} d_{US} &= Z_{US} = w_0 + w_1 \\ d_{ch} &= w_{ch} \end{aligned}$$

In this case, the players final payoffs will be the following:

$$X_{US} = Z_{US} + \frac{s_{ch}}{2}$$

$$X_{ch} = w_{ch} + \frac{S_{ch}}{2}$$

In this case, both China and the United States split their trade surplus evenly in addition to keeping the wealth they began the round two negotiations with.

Using the above results for the round two negotiations given any possible round one payoffs (including both disagreement and agreement between the United States and North Korea), we can determine the equilibrium payoffs of round one negotiations between the United States and North Korea. Let us continue to refer to the wealth of United States after round one negotiations as  $Z_{US}$ . We now solve for the results of round one negotiations between the United States and North Korea given the known results of disagreement in round one for both players. We solve the set of equations:

$$\begin{aligned} & \max_{X_{US}, X_{NK}} ((X_{US} - d_{US})(X_{NK} - d_{NK})) \\ & \text{subject to } Z_{US} + X_{NK} = \pi_{NK} = w_0 \end{aligned}$$

Inputting the following known disagreement values:

$$\begin{aligned} d_{US} &= X_{US}(w_1 = 0, \text{disagreement}) \\ d_{US} &= r_{US}w_0 + \frac{1}{\left(1 + \frac{1}{r_{US}}\right)} S_{ch} \\ d_{NK} &= r_{NK}w_{NK} = 0 \end{aligned}$$

This is further complicated by the fact that North Korea will not simply bargain over a portion of the United State's current wealth,  $w_0$ . North Korea can predict the upcoming results of round two negotiations with China and see that the United States will be gaining an additional wealth of  $\frac{s_{ch}}{2}$  from the trade surplus with China if agreement is reached in round one. Hence, North Korea will also demand a portion of that future wealth – a demand the United States will have to agree to or risk losing a percentage of all of their wealth including both  $w_0$  and  $s_{ch}$ .

Hence, the United States will not accept any agreement with North Korea in which they cannot achieve a wealth equal to or greater than  $r_{US}w_0 + \frac{1}{\left(1+\frac{1}{r_{US}}\right)}s_{ch}$  in round two with China. In order to achieve a wealth of at least  $r_{US}w_0 + \frac{1}{\left(1+\frac{1}{r_{US}}\right)}s_{ch}$  in round two, the United States will need  $Z_{US} \geq \left(r_{US}w_0 + \frac{1}{\left(1+\frac{1}{r_{US}}\right)}s_{ch}\right) - \frac{s_{ch}}{2} = r_{US}w_0 + \left(\frac{r_{US}-1}{2r_{US}+2}\right)s_{ch}$ . North Korea simply will not accept any agreement in which it receives negative wealth. Since the United States will demand at least  $r_{US}w_0 + \left(\frac{r_{US}-1}{2r_{US}+2}\right)s_{ch}$  and North Korea will demand at least zero, the two will negotiate over the remaining wealth, which is  $(1 - r_{US})w_0 + \left(\frac{1-r_{US}}{2r_{US}+2}\right)s_{ch}$ .

Due to our assumption of symmetric payoffs, neither the United States nor North Korea will accept less than half of the excess utility. As a result, they will split this wealth of  $(1 - r_{US})w_0 + \left(\frac{1-r_{US}}{2r_{US}+2}\right)s_{ch}$  in two parts with half going to North Korea and the other half remaining with the United States. This gives:



$$X_{NK} = \left(\frac{1 - r_{US}}{2}\right)w_0 + \frac{1}{4}\left(\frac{1 - r_{US}}{r_{US} + 1}\right)s_{ch}$$
$$Z_{US} = \left(\frac{1 + r_{US}}{2}\right)w_0 + \frac{1}{4}\left(\frac{r_{US} - 1}{r_{US} + 1}\right)s_{ch}$$

Hence, whenever an agreement is reached between the United States and North Korea in round one negotiations, they will be described by the equations above. We can use the new, proper value of  $Z_{US}$  to solve for the subsequent results of round two negotiations with China. We assume that the wealth of the United States at the beginning of round two negotiations is new value of  $Z_{US}$ . We now solve the functions:

$$\max_{X_{US}, X_{ch}} ((X_{US} - d_{US})(X_{ch} - d_{ch}))$$

subject to  $X_{US} + X_{ch} = Z_{US} + w_{ch} + s_{ch}$

With known disagreement points:

$$d_{US} = Z_{US} = \left(\frac{1 + r_{US}}{2}\right)w_0 + \frac{1}{4}\left(\frac{r_{US} - 1}{r_{US} + 1}\right)s_{ch}$$
$$d_{ch} = w_{ch}$$

And solution:

$$X_{US} = \left(\frac{1 + r_{US}}{2}\right)w_0 + \left(\frac{1}{4}\right)\left(\frac{3r_{US} + 1}{r_{US} + 1}\right)s_{ch}$$
$$X_{ch} = w_{ch} + \frac{s_{ch}}{2}$$

And from before:

$$X_{NK} = \left(\frac{1 - r_{US}}{2}\right) w_0 + \frac{1}{4} \left(\frac{1 - r_{US}}{r_{US} + 1}\right) s_{ch}$$

Hence, any time both rounds of negotiation are completed successfully without disagreements, the equations above will describe the final wealth of all players.

### 5.3 Multilateral Bargaining – Single Negotiation Involving All Three Players

#### Result 5.3.1 – Summary of Payoffs in a Multilateral Negotiation

In a multilateral bargaining scenario between the US, China, and NK during which all three players must unanimously agree to a division of the combined surpluses, the final wealth (or payoff) to each player will be as follows:

$$X_{US} = \frac{(2r_{US} + 1)w_0}{3} + \frac{s_{ch}}{3}$$

$$X_{ch} = w_{ch} + \frac{(1 - r_{US})w_0}{3} + \frac{s_{ch}}{3}$$

$$X_{NK} = \frac{(1 - r_{US})w_0}{3} + \frac{s_{ch}}{3}$$

These payoffs assume all variable constraints outlined in section (5.4) are enforced.

Derivation:

In place of the two methods of bilateral negotiations outlined in (5.1) and (5.2), the United States might choose instead to host a single multilateral negotiation between all players. In this case, all players enter the negotiation with their respective initial wealth and bargain over the distribution of the combined surpluses  $\pi_{ch} + \pi_{NK} = w_0 + w_{ch} + s_{ch}$ . All three players must unanimously accept the terms of the agreement or else disagreement is reached. In case of a disagreement, China leaves with its initial wealth  $w_{ch}$ . However, the United States and North Korea have a conflict following disagreement and leave with  $r_{US}w_0$  and  $r_{NK}w_{NK} = 0$ , respectively.

The solution to this problem as a Nash Bargaining problem can be solved from the following set of equations:

$$\begin{aligned} & \max_{X_{US}, X_{NK}, X_{ch}} ((X_{US} - d_{US})(X_{ch} - d_{ch})(X_{NK} - d_{NK})) \\ & \text{subject to } X_{US} + X_{ch} + X_{NK} = \pi_{NK} + \pi_{ch} = w_0 + w_{ch} + s_{ch} \end{aligned}$$

Giving solutions:

$$\begin{aligned} X_{US} &= \frac{\pi_{ch} + \pi_{NK} + 2d_{US} - d_{ch} - d_{NK}}{3} \\ X_{ch} &= \frac{\pi_{ch} + \pi_{NK} + 2d_{ch} - d_{US} - d_{NK}}{3} \\ X_{NK} &= \frac{\pi_{ch} + \pi_{NK} + 2d_{NK} - d_{ch} - d_{US}}{3} \end{aligned}$$

Since the disagreement points for each players are known to be:

$$\begin{aligned} d_{US} &= r_{US}w_0 \\ d_{ch} &= w_{ch} \\ d_{NK} &= 0 \end{aligned}$$

We can input these values to solve for the final wealth of each player from the multilateral negotiation:

$$\begin{aligned} X_{US} &= \frac{(2r_{US} + 1)w_0}{3} + \frac{s_{ch}}{3} \\ X_{ch} &= w_{ch} + \frac{(1 - r_{US})w_0}{3} + \frac{s_{ch}}{3} \end{aligned}$$

$$X_{NK} = \frac{(1 - r_{US})w_0}{3} + \frac{s_{ch}}{3}$$

The above equations give the payouts for each player from the single multilateral negotiation so long as disagreement is avoided. The conditions for disagreement to be avoided are outlined in the following section (5.4).

#### 5.4 Variable Constraints

It is important to remember that the final wealths solved for in (5.1), (5.2), and (5.3) are only achieved in the event that no disagreement is reached in any negotiation. In this section, we seek to understand in what instances, if any, one or more rounds of negotiation will end in disagreement. We will now outline fully the assumptions used in the above model.

First, we require that all players have finite initial wealth greater than or equal to zero. Since we have defined North Korea's initial wealth to be zero in this simplified case, this amounts to:

$$\begin{aligned}w_0 &\geq 0 \\w_{ch} &\geq 0 \\w_{NK} &= 0\end{aligned}$$

We also require that the trade surplus between China and the United States is truly a surplus. The trade surplus between the United States and North Korea has been defined as zero. Hence:

$$\begin{aligned}s_{ch} &\geq 0 \\s_{NK} &= 0\end{aligned}$$

Furthermore, we have require the conflict coefficients to take on values ranging from zero to one, or:

$$\begin{aligned}0 &\leq r_{US} \leq 1 \\0 &\leq r_{NK} \leq 1\end{aligned}$$

Although there might be a situation in which disagreement between players caused a percentage increase in their wealth, we were willing to limit our own model to describe only situations in which disagreement caused a percentage loss of wealth between 0% and 100%.

The constraints on initial wealth and conflict coefficients are self-imposed to increase intuitive understanding of the model. However, the most important constraints are imposed by the rules of bargaining themselves. Primarily, no utility maximizing agent will accept in any agreement which would place them worse off than their disagreement point. As a result, the following inequalities holding is both necessary and sufficient for agreement to be reached in a bilateral negotiation:

$$\begin{aligned}d_{US} + d_{ch} &\leq \pi_{ch} \\d_{US} + d_{NK} &\leq \pi_{NK}\end{aligned}$$

And in the multilateral bargaining situation we require:

$$d_{US} + d_{NK} + d_{ch} \leq \pi_{ch} + \pi_{NK}$$

Using the manners in which we have defined all these variables in the model, these three constraints will always be met. As a result, there is no instance in this model in which rational actors would end in disagreement. There is always an agreement that leads to equal or greater utility for all players. Hence, the final payoffs found in (5.1), (5.2), and (5.3) will always be true for this simplified case and the findings derived in the following sections will always be true as a result.

## 5.5 Comparing Bilateral and Multilateral Payoffs for the United States

### Result 5.5.1– The United State’s Preferences

We find that –between the two forms of bilateral series negotiations – the United States always prefers to negotiate first with China and then second with North Korea.

The United States always prefers this bilateral series of negotiations with China first over a single multilateral negotiation (except in the case where  $r_{US} = 1$ , when the US is indifferent).

Hence, in this simplified case, the United States has a dominant bargaining structure which is to use a bilateral series of negotiations in which they negotiate first with China and then with North Korea.

In addition to solving for the final payoffs for each player in various bargaining scenarios, the original intent of this paper was to determine in what situations a player would prefer multilateral bargaining and in what situations they would prefer a series of bilateral negotiations.

We begin by seeking which form of bargaining would be most beneficial to the United States. We do so by first determining which form of bilateral negotiations (China first or North Korea first) would result in the highest final wealth of the United States. The United States would prefer negotiating with China first rather than negotiating with North Korea first whenever the following inequality holds:

$$X_{US}(\text{bilateral, China first}) \geq X_{US}(\text{bilateral, NK first})$$



$$\left(\frac{1+r_{US}}{2}\right)w_0 + \left(\frac{1+r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{1+r_{US}}{2}\right)w_0 + \left(\frac{1}{4}\right)\left(\frac{3r_{US}+1}{r_{US}+1}\right)s_{ch}$$

Which reduces to:

$$r_{US}^2 - 2r_{US} + 1 \geq 0$$

Considering  $0 \leq r_{US} \leq 1$ , the above inequality is always true. Hence, in this simplified case, the United States will always prefer to negotiate first with China and then with North Korea if it makes use of a series of bilateral negotiations (except when  $r_{US} = 1$ , at which point the United States is indifferent between the two forms of bilateral series).

We now pause to consider whether or not this finding makes sense intuitively. We find that it does. When the United States negotiates first with China and then with North Korea, the United States knows that in round two it will be forced to give a certain percentage of its wealth to North Korea – including whatever portion of  $s_{ch}$  it received in round one. When the United States is negotiating first with China and then with North Korea, it can use this fact as a bargaining chip to force China to give them more than half of  $s_{ch}$ . However, if the United States negotiates with North Korea first, North Korea still demands a portion of  $s_{ch}$  that the United States will gain in round two. However, China feels no need to give the United States more than half of  $s_{ch}$  because the United States could not reasonably reject a deal in which they receive exactly half of the surplus. As a result, the US captures a greater portion of  $s_{ch}$  whenever it negotiates first with China.

Next, we must determine when the United States will prefer a bilateral series of negotiations (starting with China first) and when it will instead prefer a single

multilateral negotiation involving all three players. The United States will prefer a bilateral negotiation (starting with China first) whenever the following inequality holds:

$$X_{US}(\text{bilateral, China first}) \geq X_{US}(\text{multilateral})$$
$$\left(\frac{1+r_{US}}{2}\right)w_0 + \left(\frac{1+r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{2r_{US}+1}{3}\right)w_0 + \left(\frac{s_{ch}}{3}\right)$$

This reduces to:

$$(1-r_{US})w_0 + \left(\frac{4r_{US}}{3+r_{US}}\right)s_{ch} \geq 0$$

Since  $s_{ch}$  and  $w_0$  are defined to be equal to or greater than zero, and since  $0 \leq r_{US} \leq 1$ , we find that this inequality is always true. Hence, the United States will always prefer a bilateral series of negotiations starting with China over a single multilateral negotiation (except when  $r_{US} = 1$ , at which point the United States is indifferent between the two).

Again, we pause to consider whether or not this finding makes any sense. We observe the following: in a bilateral series, regardless of the value of  $r_{US}$ , the United States always preserves a greater portion of its initial wealth,  $w_0$ , in the bilateral series. Furthermore, in the multilateral negotiation, the United States always receives exactly one third of  $s_{ch}$ . In the bilateral series, the US can receive anywhere from one third to one half of  $s_{ch}$ . Hence, the mathematics make sense on an intuitive level.

The mathematics describes the following facts: in the bilateral series, North Korea has no direct ability to overrule the United States and China's division of their

trade surplus,  $s_{ch}$ . Hence, they can receive only whatever portion of  $s_{ch}$  they can coerce from the United States – a value that never exceeds one third of  $s_{ch}$ . However, in the multilateral case, North Korea can always directly reject any proposal in which they receive less than one third of this surplus. Hence, in this regard, the United States prefers the bilateral series. Also, in both the bilateral series and the multilateral case, the United States has a certain percentage of its initial wealth at risk ( $r_{USW_0}$ ). In the bilateral series, it must only split this wealth-at-risk between itself and North Korea with each getting one half. However, in the multilateral case, China also demands an equal portion, causing each player to receive one third of this wealth-at-risk. Again, the bilateral series is preferable over the multilateral negotiation for the United States.

## 5.6 Comparing Bilateral and Multilateral Payoffs for China

### Result 5.6.1– China’s Preferences

We find that –between the two forms of bilateral series negotiations – China always prefers the United States to negotiate first with North Korea and then second with China (except when  $r_{US} = 1$ , in which case China is indifferent between the two forms of bilateral series negotiations).

China prefers this form of bilateral negotiations over a single multilateral negotiation whenever the following inequality holds:

$$\left(\frac{1}{2}\right) s_{ch} \geq (1 - r_{US})w_0$$

We now follow the same process for China to determine which series of bilateral negotiations China prefers most. In this case, when the following inequality is true, China prefers to negotiate with the United States first and then have the United States negotiate with North Korea second:

$$w_{ch} + \left(\frac{1 + r_{US}}{3 + r_{US}}\right) s_{ch} \geq w_{ch} + \left(\frac{1}{2}\right) s_{ch}$$

This reduces to:

$$r_{US} - 1 \geq 0$$

Since  $0 \leq r_{US} \leq 1$ , this inequality is never true. China will always prefer a bilateral series in which the United States negotiates first with North Korea over a bilateral

series in which the United States negotiates first with China (except in the case of  $r_{US} = 1$ , in which China is indifferent between the two).

Next, we must compare China's preferred bilateral bargaining strategy with the multilateral strategy. Since China always prefers being in the second negotiation with the United States (or, when  $r_{US} = 1$ , is indifferent) we use the bargaining series with North Korea and the United States negotiating first as the preferred bilateral bargaining strategy. China prefers this bilateral series over a single multilateral negotiation whenever the following inequality holds:

$$w_{ch} + \left(\frac{1}{2}\right) s_{ch} \geq w_{ch} + \left(\frac{1 - r_{US}}{3}\right) w_0 + \left(\frac{1}{3}\right) s_{ch}$$

This reduces to:

$$\left(\frac{1}{2}\right) s_{ch} \geq (1 - r_{US})w_0$$

Given the constraints on the variables, this inequality can be either true or false. When true, China's preferred bargaining strategy will be a bilateral series of negotiations in which the United States negotiates first with North Korea and then negotiates second with China. If it is false, then China prefers a single multilateral negotiation. China never prefers a bilateral series of negotiations in which the United States first negotiates with China and then negotiates with North Korea second.

## 5.7 Comparing Bilateral and Multilateral Payoffs for North Korea

### Result 5.7.1– North Korean Preferences

We find that –between the two forms of bilateral series negotiations – North Korea always prefers the United States to negotiate first with China and then second with North Korea.

North Korea prefers this form of bilateral negotiations over a single multilateral negotiation whenever the following inequality holds:

$$\left(\frac{1 - r_{US}}{2}\right) w_0 \geq \left(\frac{4r_{US}}{3 + r_{US}}\right) s_{ch}$$

Finally, we investigate which bargaining structure is most preferred by North Korea in this instance. We begin by determining which form of bilateral series North Korea prefers most. When the following inequality holds, North Korea prefers a bilateral series in which the United States negotiates first with China and then second with North Korea:

$$\left(\frac{1 - r_{US}}{2}\right) w_0 + \left(\frac{1 - r_{US}}{3 + r_{US}}\right) s_{ch} \geq \left(\frac{1 - r_{US}}{2}\right) w_0 + \left(\frac{1}{4}\right) \left(\frac{1 - r_{US}}{r_{US} + 1}\right) s_{ch}$$

This reduces to:

$$-2r_{US}^2 + 2 + 1 \geq 0$$

Since  $0 \leq r_{US} \leq 1$ , this inequality is always true. Hence, North Korea will always prefer a bilateral series in which the first negotiation is between the United States and

China over a bilateral series in which the first negotiation is between the United States and North Korea.

Next, we determine whether North Korea would prefer a bilateral negotiation with China first over a single multilateral negotiation. North Korea would prefer the bilateral series with China first whenever the following inequality holds:

$$\left(\frac{1-r_{US}}{2}\right)w_0 + \left(\frac{1-r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{1-r_{US}}{3}\right)w_0 + \left(\frac{s_{ch}}{3}\right)$$

This reduces to:

$$\left(\frac{1-r_{US}}{2}\right)w_0 \geq \left(\frac{4r_{US}}{3+r_{US}}\right)s_{ch}$$

Given the constraints on the variables, this inequality can be either true or false. When true, North Korea's preferred bargaining strategy will be a bilateral series of negotiations in which the United States negotiates first with China and then negotiates second with North Korea. If it is false, then North Korea prefers a single multilateral negotiation. North Korea never prefers a bilateral series of negotiations in which the United States first negotiates with North Korea and then negotiates with China.

## 6 Conclusion

As stated earlier in this paper, since 2003, the United States has refused to meet bilaterally with North Korea, choosing instead to pursue a series of multilateral negotiations in the form of the Six Party Talks. However, contrary to this preference, our findings suggest that the United States would achieve higher utility if it instead allowed bilateral discussions with North Korea to occur. What could be the reasons for this misalignment between theory and practice?

Firstly, it is possible that our simplified model is missing important characteristics of the actual negotiations. For instance, all of the agreements in our model are assumed to be perfectly enforceable. However, in practice, it might be seen that contracts are difficult to enforce. This has been shown to be particularly true in the case of North Korean disarmament talks, during which North Korea has at several points reversed their decision during implementation. As put by Bloomberg News, “Kim [Jong Il]’s negotiating style is given to sudden reversals, brinkmanship and even disappearing acts.”<sup>7</sup> It is possible that multilateral negotiations have a significant advantages when it comes to enforceability since China, which is possibly North Korea’s closest ally and most important trading partner, would also be aligned. Also, there may be political ramifications for the United States administration if they are seen as being too lenient to North Korea by partaking in bilateral negotiations with them. Finally, the United States might suffer negative consequences from their Asian allies (Japan, South Korea, etc) if they are seen as handling such an important issue as North Korean denuclearization without allowing others to participate multilaterally.

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<sup>7</sup> Bloomberg News, December 8<sup>th</sup>, 2008, North Korea Told Weapons Balk Won’t Work With Obama



Also, it might be possible that our model's findings are correct and that it is in the United State's best interest to have bilateral talks with North Korea. In this case, the current Administration of the United States is pursuing a sub-optimal strategy due to either lack of knowledge or political gamesmanship. The fact that President Barack Obama has publically stated that his administration will consider having bilateral negotiations with the North Koreans in addition to negotiations with other nations might be seen as verification of this.<sup>8</sup>

Our greatest hopes for future expansion of this model would be to model a more general case in which we remove constraints on North Korea's initial wealth and the trade surplus between North Korea and the United States. Though these changes would grant the model greater generality, this extension would be much more difficult mathematically and would, as a result, be much less intuitive.

Also, as the bilateral series currently stands, two players (China and North Korea) have no ability to negotiate with each other. A possible, though difficult, extension would be to alter bilateral negotiations to allow all three players to negotiate with each other in term, therein effectively removing the central player's uniqueness in being able to negotiate with both other players during series of bilateral negotiations.

Lastly, we look forward to receiving comments and suggestions from our instructor and peers for areas to change or clarify for future versions of this paper.

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<sup>8</sup> Bloomberg News, December 8<sup>th</sup>, 2008, North Korea Told Weapons Balk Won't Work With Obama

## 7 Appendix – Omitted Calculations

The following is some more detailed derivations of some of the findings reported in the previous sections. The paper was designed to be understandable without viewing these calculations, but interested readers seeking to find the derivation of results in section 5 will find them listed by subsection hereafter.

### 5.1 Bilateral Series, China first

Note that for the purposes of the following derivations,  $s = s_{ch}$  and  $r = r_{US}$ . Also,  $Y_{US}$  and  $Y_{ch}$  are used to denote the portion of  $s_{ch}$  that each player will receive, not their wealth at the end of the round. Hence,  $Z_{US} = w_0 + Y_{US}$  and  $X_{ch} = w_c + Y_{ch}$ .

$$\begin{aligned} Y_{US} &= s - \left(\frac{2}{1+r}\right)Y_{US} \\ \left(1 + \frac{2}{1+r}\right)Y_{US} &= s \\ \left(\frac{3+r}{1+r}\right)Y_{US} &= s \\ Y_{US} &= \left(\frac{1+r}{3+r}\right)s \end{aligned}$$

Solving for  $Z_{US}$  using  $Y_{US}$ :

$$\begin{aligned} Z_{US} &= \left(\frac{2}{1+r}\right)Y_{US} \\ Z_{US} &= \left(\frac{2}{1+r}\right)\left(\frac{1+r}{3+r}\right)s \\ Z_{US} &= \left(\frac{2}{3+r}\right)s \end{aligned}$$

Solving for  $Y_{ch}$  using  $Y_{US}$ :

$$\begin{aligned} Y_{ch} &= s - Y_{US} = s - \left(\frac{2}{3+r}\right)s \\ Y_{ch} &= \left(\frac{1+r}{3+r}\right)s \end{aligned}$$

Solving for  $X_{US}$  given  $Z_{US}$ :

$$\begin{aligned}
X_{US} &= \left(\frac{1+r}{2}\right) Z_{US} \\
&= \left(\frac{1+r}{2}\right) \left(w_0 + \frac{2}{3+r} s\right) \\
&= \left(\frac{1+r}{2}\right) w_0 + \left(\frac{1+r}{3+r}\right) s
\end{aligned}$$

Solving for  $X_{NK}$  given  $Z_{US}$ :

$$\begin{aligned}
X_{NK} &= \left(\frac{1-r}{2}\right) (Z_{US}) \\
&= \left(\frac{1-r}{2}\right) \left(w_0 + \frac{2}{3+r} s\right) \\
&= \left(\frac{1-r}{2}\right) w_0 + \left(\frac{1-r}{3+r}\right) s
\end{aligned}$$

### 5.3 Multilateral Bargaining

Deriving payoff functions in detail:

$$\begin{aligned}
X_{US} &= \frac{\pi_{ch} + \pi_{NK} + 2d_{US} - d_{ch} - d_{NK}}{3} = \frac{w_0 + w_{ch} + s_{ch} + 2r_{US}w_0 - w_{ch}}{3} \\
&= \frac{(2r_{US} + 1)}{3} w_0 + \frac{s_{ch}}{3} \\
X_{ch} &= \frac{\pi_{ch} + \pi_{NK} + 2d_{ch} - d_{US} - d_{NK}}{3} = \frac{w_0 + w_{ch} + s_{ch} + 2w_{ch} - r_{US}w_0}{3} \\
&= w_{ch} + \frac{(1 - r_{US})}{3} w_0 + \frac{s_{ch}}{3} \\
X_{NK} &= \frac{\pi_{ch} + \pi_{NK} + 2d_{NK} - d_{ch} - d_{US}}{3} = \frac{w_0 + w_{ch} + s_{ch} - w_{ch} - r_{US}w_0}{3} \\
&= \frac{(1 - r_{US})}{3} w_0 + \frac{s_{ch}}{3}
\end{aligned}$$

### 5.5 United States Preferences

Determining whether the United States prefers a bilateral series with NK first or bilateral series with China first:

$$X_{US}(\text{bilateral, China first}) \geq X_{US}(\text{bilateral, NK first})$$

$$\left(\frac{1+r_{US}}{2}\right)w_0 + \left(\frac{1+r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{1+r_{US}}{2}\right)w_0 + \left(\frac{1}{4}\right)\left(\frac{3r_{US}+1}{r_{US}+1}\right)s_{ch}$$

$$\left(\frac{1+r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{1}{4}\right)\left(\frac{3r_{US}+1}{r_{US}+1}\right)s_{ch}$$

$$\left(\frac{1+r_{US}}{3+r_{US}}\right) \geq \left(\frac{1}{4}\right)\left(\frac{3r_{US}+1}{r_{US}+1}\right)$$

$$4(r_{US}+1)(1+r_{US}) \geq (3+r_{US})(3r_{US}+1)$$

$$4r_{US}^2 + 8r_{US} + 4 \geq 3r_{US}^2 + 10r_{US} + 3$$

$$r_{US}^2 - 2r_{US} + 1 \geq 0$$

Determining the United States' preference between a bilateral series with China first or a single multilateral negotiation:

$$X_{US}(\text{bilateral, China first}) \geq X_{US}(\text{multilateral})$$

$$\left(\frac{1+r_{US}}{2}\right)w_0 + \left(\frac{1+r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{2r_{US}+1}{3}\right)w_0 + \left(\frac{s_{ch}}{3}\right)$$

$$\left(\frac{3+3r_{US}-4r_{US}-2}{6}\right)w_0 \geq \left(\frac{3+r_{US}-3-3r_{US}}{3(3+r_{US})}\right)s_{ch}$$

$$\left(\frac{1-r_{US}}{6}\right)w_0 \geq \left(\frac{-2r_{US}}{3(3+r_{US})}\right)s_{ch}$$

$$(1-r_{US})w_0 \geq \left(\frac{-4r_{US}}{3+r_{US}}\right)s_{ch}$$

$$(1-r_{US})w_0 + \left(\frac{4r_{US}}{3+r_{US}}\right)s_{ch} \geq 0$$

## 5.6 China Preferences

Determining whether China prefers a bilateral series with China first or bilateral series with North Korea first:

$$X_{ch}(\text{bilateral}, \text{China first}) \geq X_{ch}(\text{bilateral}, \text{NK first})$$

$$w_{ch} + \left(\frac{1 + r_{US}}{3 + r_{US}}\right) s_{ch} \geq w_{ch} + \left(\frac{1}{2}\right) s_{ch}$$

$$\left(\frac{1 + r_{US}}{3 + r_{US}}\right) \geq \left(\frac{1}{2}\right)$$

$$2(1 + r_{US}) \geq (3 + r_{US})$$

$$r_{US} - 1 \geq 0$$

Determining if China prefers a bilateral series with North Korea first or a single multilateral negotiation:

$$X_{ch}(\text{bilateral}, \text{NK first}) \geq X_{ch}(\text{multilateral})$$

$$w_{ch} + \left(\frac{1}{2}\right) s_{ch} \geq w_{ch} + \left(\frac{1 - r_{US}}{3}\right) w_0 + \left(\frac{1}{3}\right) s_{ch}$$

$$\left(\frac{1}{6}\right) s_{ch} \geq \left(\frac{1 - r_{US}}{3}\right) w_0$$

$$\left(\frac{1}{2}\right) s_{ch} \geq (1 - r_{US}) w_0$$

## 5.7 North Korea Preferences

Determining whether North Korea prefers a bilateral series with China first or bilateral series with North Korea first:

$$X_{NK}(\text{bilateral}, \text{China first}) \geq X_{NK}(\text{bilateral}, \text{NK first})$$

$$\left(\frac{1-r_{US}}{2}\right)w_0 + \left(\frac{1-r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{1-r_{US}}{2}\right)w_0 + \left(\frac{1}{4}\right)\left(\frac{1-r_{US}}{r_{US}+1}\right)s_{ch}$$

$$\left[\left(\frac{1-r_{US}}{3+r_{US}}\right) - \left(\frac{1}{4}\right)\left(\frac{1-r_{US}}{1+r_{US}}\right)\right]s_{ch} \geq 0$$

$$\left[\left(\frac{4-4r_{US}}{4(3+r_{US})}\right) - \left(\frac{1-r_{US}}{4(1+r_{US})}\right)\right]s_{ch} \geq 0$$

$$\left[\frac{(4-4r_{US})(1+r_{US}) - (3+r_{US})(1-r_{US})}{4(3+r_{US})(1+r_{US})}\right]s_{ch} \geq 0$$

$$\left[\frac{-2r_{US}^2 + 2 + 1}{4(3+r_{US})(1+r_{US})}\right]s_{ch} \geq 0$$

$$\left[\frac{-2r_{US}^2 + 2 + 1}{4(3+r_{US})(1+r_{US})}\right]s_{ch} \geq 0$$

$$-2r_{US}^2 + 2 + 1 \geq 0$$

Determining if North Korea prefers a bilateral series with China first or a single multilateral negotiation:

$$X_{NK}(\text{bilateral, China first}) \geq X_{NK}(\text{multilateral})$$

$$\left(\frac{1-r_{US}}{2}\right)w_0 + \left(\frac{1-r_{US}}{3+r_{US}}\right)s_{ch} \geq \left(\frac{1-r_{US}}{3}\right)w_0 + \left(\frac{s_{ch}}{3}\right)$$

$$\left[\left(\frac{1-r_{US}}{2}\right) - \left(\frac{1-r_{US}}{3}\right)\right]w_0 \geq \left[\left(\frac{1}{3}\right) - \left(\frac{1-r_{US}}{3+r_{US}}\right)\right]s_{ch}$$

$$\left(\frac{1-r_{US}}{6}\right)w_0 \geq \left(\frac{4r_{US}}{3(3+r_{US})}\right)S_{ch}$$

$$\left(\frac{1-r_{US}}{2}\right)w_0 \geq \left(\frac{4r_{US}}{3+r_{US}}\right)S_{ch}$$

## 8 References

CBS News, Bush Talks Tought on North Korea, Oct. 11, 2006.

<http://www.cbsnews.com/stories/2006/10/10/world/main2079519.shtml>

CNN News, Rice: Bilateral talks with North Korea won't work, Oct. 10, 2006.

<http://www.cnn.com/2006/WORLD/asiapcf/10/10/rice.korea/index.html>.

Crowely, P. J, Senior Fellow and Director of National Defense and Homeland Security, Center for American Progress, CampusProgress Interview,

<http://www.campusprogress.org/tools/101/ask-the-expert-north-korea>.

Giovanni Maggi, "The Role of Multilateral Institutions in International Trade Cooperation," *The American Economic Review*, Vol. 89, No. 1 (Mar., 1999), pp. 190-214. Published by: American Economic Association

Krishna, Vijay and Roberto Serrano, "Multilateral Bargaining", *The Review of Economic Studies*, Vol. 63, No. 1 (Jan., 1996), pp. 61-80, Published by: The Review of Economic Studies Ltd.

Mnookin, Robert H., "Strategic Barriers to Dispute Resolution: A Comparison of Bilateral and Multilateral Negotiations," 8 *Harv. Negot. L. Rev.* 1 (2003).

Nash, John F. Jr. "The Bargaining Problem", *Econometrica*, Vol. 18, No. 2 (1950), pp. 155-162.

Rotillon, Gilles, Tarik Tazdait, and Sylvain Zeghni, "Bilateral or multilateral bargaining in the face of global environmental change?" *Ecological Economics*, Vol. 18 (1996), pg 177-187.

Mr. Weldon of Pennsylvania addresses the House of Representatives, June 4<sup>th</sup>, 2003, Congressional Record, reproduced by the Federatiuon of American Scientists, <http://www.fas.org/nuke/guide/dprk/nuke/codel0603.html>.