

Word-of-Mouth Communication, Noise-driven Volatility, and Public Disclosure*

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Abstract

This paper examines how a firm adjusts its disclosure quality in response to technological innovations that improve investors’ private information. We show that more precise private information can endogenously amplify supply shocks and, hence, increase noise-driven (or non-fundamental) price volatility. We study how the firm reacts to such changes and derive a necessary and sufficient condition under which the firm improves its disclosure quality when investors are informed with better private signals. We then apply our model to study investors’ private word-of-mouth communication. Our analysis highlights a “dark side” of word-of-mouth communication and a call for better public disclosure even if private communication is assumed to be unbiased and truthful. We provide empirical predictions regarding how price volatility, market depth, and firms’ disclosure qualities would change as technological innovations, such as social media, facilitate information sharing among investors.

Keywords: Public Disclosure; Private Information; Word-of-Mouth Communication; Price Volatility

JEL Classifications: D82; G14; M41

1 Introduction

Technological innovations, such as social media, have facilitated investors' private interpersonal communication and greatly changed the information environment in which firms operate. Shiller (2015, p.180) writes: "Word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations." As investors learn from both public disclosures and their private information channels, we ask two questions in this paper. First, how does the quality of investors' private signals affect firms' information environment (specifically, price volatility), and is the effect different from that resulting from firms' public disclosures? Second, how would a firm adjust its public disclosures in response to better informed investors? In particular, will investors' private information channels crowd out the firm's disclosures, or incentivize the firm to provide more precise public disclosures?

To answer these questions, we model an equilibrium asset market, a continuum of risk-averse investors, and a risk-averse manager who operates the firm and chooses the precision of the public disclosure. The manager (he) chooses an unobservable effort and an observable disclosure precision, and then sells his shares in a competitive market similar to Hellwig (1980) and Diamond and Verrecchia (1981). In addition to the firm's public disclosure, each investor (she) observes an idiosyncratic private signal before trading.

We first analyze the similarities and differences between public and private information in terms of their impact on price volatility. From an *ex ante* perspective, price volatility comes from two sources: (1) fundamental-driven volatility, attributed to uncertainty about the underlying firm value, and (2) noise-driven volatility, attributed to noise that is unrelated to firm value. Both private and public information increase

fundamental-driven volatility by shifting uncertainty about firm value from ex post to ex ante. The similarity between these two information channels echoes the well-known result that providing information prior to trading reduces ex post uncertainty but increases uncertainty ex ante (i.e., before the information is revealed).¹

Interestingly, public disclosure and investors' private information can have an opposite effect on the noise-driven volatility. We show that while public disclosure unambiguously mitigates the price impact of noisy supplies, more precise private signals can amplify supply shocks in the pricing process and, therefore, increase the supply shock-related volatility. The key to understanding the result is investors' attempts to learn firm value from the market price. As investors' private information becomes more precise, they trade more aggressively and, thus, their private information is better aggregated by the market-clearing price. This information aggregation process is so efficient that price informativeness increases faster than the precision of the private information per se. Therefore, investors optimally place a higher weight on the observed market price in valuing the firm. Ironically, when investors rely more heavily on price in forming their beliefs, supply shocks will be amplified in the pricing process. In contrast, improving disclosure quality reduces investors' reliance on price in valuing the firm and, therefore, mitigates the impact of supply shocks on price.

We endogenize the manager's disclosure choice and examine how the optimal disclosure quality will change as the investors' private signals become more precise. Contrary to the casual intuition that investors' private information sources crowd out firms' public disclosures, we show that the manager often provides more precise public

¹For example, Hirshleifer (1971, p. 568) states "the anticipation of public information becoming available in advance of trading adds a significant distributive risk to the underlying technological risk." This risk-shifting result is also seen in Holthausen and Verrecchia (1988) and investigated in the cost of capital literature (e.g., Christensen et al., 2010; Gao, 2010; Dutta and Nezlobin, 2017).

disclosures in response to better privately informed investors. To understand this result, recall that as investors' private signals become more precise, price informativeness increases faster than the precision of private signals (due to information aggregation). The result is that investors rely more on price in valuing the firm. Since investors' increased reliance on price amplifies supply shocks in the pricing process, it indirectly increase the *marginal benefit* of public disclosure in mitigating such shock-related price volatility. In particular, the marginal benefit of public disclosure in mitigating supply shocks is shown to depend on the relative informativeness of price in a multiplicative way. Therefore, better private information can strengthen the marginal benefit of disclosure by increasing the informativeness of price relative to private signals. We show that better private information will incentivize the manager to improve disclosure quality if and only if the variance of the noisy supply is high. Intuitively, when supply shocks are volatile, the call for a more precise disclosure to lower investors' increased reliance on price outweighs the intrinsic substitutability between public and private information in revealing firm value to investors.

We apply our model to study investors' private word-of-mouth communication, using technology from the information percolation literature (e.g., Duffie and Manso, 2007; Duffie et al., 2009). The results are relevant to recent empirical research addressing the use of social media (e.g., Twitter) and crowd-sourced content platforms (e.g., Seeking Alpha) that facilitate information sharing among investors. Existing empirical evidence suggests that inter-investor information sharing helps to amass the "wisdom of crowds" (e.g., Blankespoor et al., 2014; Bartov et al., 2017; Chen et al., 2014a; Jung et al., 2018; Da and Huang, 2020). While we show a similar wisdom-of-crowds result, our results highlight a potential *cost* of investors' information sharing in (a) exacerbating non-fundamental noise and price volatility and (b) reducing market depth. We study

how firms respond to more active word-of-mouth communication among investors, and show that a firm will improve its disclosure quality if and only if its investors' private information is of low quality. We further predict cross-sectional variations in the relative magnitude of a firm's disclosure-quality change. To the extent that retail investors are more risk averse than institutional investors, our result suggests that firms with more retail investors will improve disclosure quality more (or reduce disclosure quality less), as compared to firms with more institutional investors.

This paper is related to the literature on the relation between public and private information. Existing studies focus on how anticipated public disclosure changes private incentives to acquire information and the implications on capital market (e.g., Diamond, 1985; Demski and Feltham, 1994; Kim and Verrecchia, 1994; McNichols and Trueman, 1994). Several papers show that releasing public information can crowd out private information acquisition by reducing the rents received by informed investors (e.g., Diamond, 1985; Fischer and Stocken, 2010; Gao and Liang, 2013; Han and Yang, 2013). Fischer and Heinle (2019) study a setting in which investors are uncertain about the quality of private information before they acquire it and, therefore, use price and public information to guide their information acquisition effort across two firms. While the quality of public disclosures is generally taken as given in prior studies (for a review see Verrecchia, 2001; Goldstein and Yang, 2017), we examine how the firm revises its disclosure policy as investors' private information becomes more precise.² Our results show that better private information can actually incentivize more precise public disclosures once we take into account the information aggregation role of price.

Informational complementarity has been demonstrated in various settings. Kim

²The voluntary disclosure literature (e.g., Verrecchia, 1983; Dye, 1985) focuses on the manager's ex post information withholding choice. Stocken (2013) provides a comprehensive review of the subject.

and Verrecchia (1994) and Boot and Thakor (2001) show that public disclosure can strengthen investors' incentives to acquire private information if public and private information complement each other in understanding firm value. Demski and Feltham (1994) and McNichols and Trueman (1994) show that public disclosures can stimulate private information acquisition in a setting where the investors trade on their private information before the public announcement. Arya et al. (2017) demonstrate synergies between accounting reports and stock prices in directing firm strategies. Diamond and Verrecchia (1991) consider a setting in which only some investors have private information, and show that the firm increases its public disclosure to lower information asymmetry and its cost of capital. Goldstein and Yang (2015) show that investors' information acquisition can be complements if their information concerns different pieces of the fundamental value. Hellwig and Veldkamp (2009) show that if agents' actions are assumed to be strategic complements, then their information acquisitions are also strategic complements. Our paper offers a novel mechanism that does not require higher-order beliefs, a division between informed and uninformed investors, or investors' incentives to coordinate.

Investors' reliance on public information relative to private information is investigated in the higher-order beliefs literature. Allen et al. (2006) formalize the Keynesian-beauty-contest effect in an asset pricing model, and show that higher-order beliefs motivate investors with a short horizon to overweight public information relative to their idiosyncratic private information. Gao (2008) studies price efficiency consequences of public disclosure in such a beauty-contest stock market. He shows that, while beauty-contest effects induce investors to overweight public information and hence, potentially amplify the common noise contained in the public disclosure, better public

information always increases the overall price informativeness.³ Analyzing investors' relative reliance on different sources of information also plays an important role in this paper. Prior studies typically take the quality of public information as given. In contrast, our focus is to study how a firm adjusts its disclosure quality in response to an exogenous change of investors' private information.

Angeletos and Werning (2006) study a currency attack type of coordination game and show that less noise may increase price volatility. This noise-amplifying result also arises in our model, but the two studies rely on entirely different mechanisms. Angeletos and Werning (2006, p. 1722) write: "This novel coordinating role is crucial for our results on price multiplicity and price volatility." There is no incentive to coordinate in our model. Instead, we show that the potential cost of better private information in amplifying noise-related (or non-fundamental) price volatility is a feature intrinsic to noisy rational expectation models.

The paper proceeds as follows. Section 2 describes the model. Section 3 takes the disclosure precision as exogenous and analyzes the similarities and differences between public disclosure and private signals. Section 4 endogenizes the disclosure precision and derives the necessary and sufficient condition under which the firm improves its disclosure quality in response to better informed investors. Section 5 applies the model to study investors' private word-of-mouth communication and provides empirical predictions. Section 6 concludes.

³Chen et al. (2014b) show that public information can increase or decrease price informativeness when overlapping generations of investors have different levels of private information precision.

2 Model Setup

Our model consists of a continuum of risk-averse investors and a risk-averse manager who operates a firm. At the beginning of the game, the manager chooses an unobservable effort $a \geq 0$ at a personal cost $C(a) = a^2/2$. The manager's effort a increases the firm's value v in the following stochastic manner:

$$v = a + \phi, \tag{1}$$

where $\phi \sim N(0, \sigma_\phi^2)$ is normally distributed with mean zero and precision $\tau_\phi = 1/\sigma_\phi^2$.

Given a realization of firm value v , the firm is traded in a competitive market in which the market-clearing price p is determined. The manager owns an exogenous amount of shares that we normalize to one.⁴ In choosing his unobservable effort a at $t = 0$, the manager maximizes his expected constant absolute risk aversion (CARA) utility as follows:

$$U^M = \mathbb{E} \left[-\exp \left(-\rho (p - C(a)) \right) \right], \tag{2}$$

where ρ is the manager's constant absolute risk aversion, p is the equilibrium price at which he sells his shares, and $C(a) = a^2/2$ is his personal cost of effort.

The price p is determined in a competitive market similar to Hellwig (1980) and Diamond and Verrecchia (1981). There is a continuum of investors $i \in [0, 1]$ and a risk-free asset that serves as the numeraire. Noisy traders provide liquidity in the sense that they supply ε units of the firm's share per capita to the market, and we assume

⁴A literal interpretation is that the manager/entrepreneur initially owns one hundred percent of the firm and later sells the firm at $t = 2$. However, this normalization (hence the entrepreneur-IPO interpretation) is not important: we can assume that the manager owns $\alpha < 1$ fraction of the firm and verify that our results carry over qualitatively.

$\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is normally distributed with mean zero and precision $\tau_\varepsilon = 1/\sigma_\varepsilon^2$. Each investor is endowed with w_0 units of the risk-free asset and has the same exponential utility function:

$$U_i = -\exp(-W_i/r), \quad (3)$$

where W_i is investor i 's ending wealth and r is the common risk tolerance.

Prior to the trading stage, the firm publicly discloses a signal x that is informative about the firm's value:

$$x = v + \zeta, \quad (4)$$

with $\zeta \sim N(0, \sigma_x^2)$. The precision of the public disclosure, $\tau_x = 1/\sigma_x^2$, is publicly chosen by the manager at $t = 0$. The disclosure choice τ_x , as argued in Diamond and Verrecchia (1991) and Kanodia and Lee (1998), can be interpreted as the choice of an accounting technique or a committed policy of providing earnings guidance or other forecasts.⁵

In addition to the firm's public disclosure, each investor $i \in [0, 1]$ receives a private signal y_i about v prior to trading and we assume

$$y_i = v + \eta_i, \quad (5)$$

where $\eta_i \sim N(0, \sigma_\eta^2)$ is independent across all investors and their signal precision $\tau_\eta = 1/\sigma_\eta^2$ is the same across all investors. Figure 1 summarizes the sequence of the game.

⁵This assumption is also standard in finance literature (e.g., Admati and Pfleiderer, 2000; Kurlat and Veldkamp, 2015).

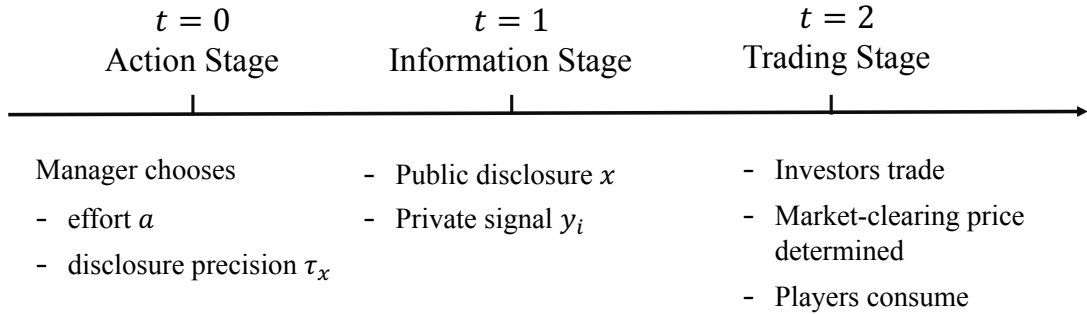


Figure 1: Timeline

3 Analysis with Exogenous Disclosure Precision

In this section, we take the precision of the public disclosure as given and solve for the manager’s equilibrium effort, as well as the subsequent trading game. We also demonstrate how a firm’s public disclosure and investors’ private signals can have qualitatively different effects on investors’ inferences from price and on price volatility.

3.1 Equilibrium

Our trading subgame is built on Diamond and Verrecchia (1981) and incorporates a public disclosure as well as an unobservable effort. The equilibrium is solved in three steps. We first reason from the investors’ perspective and solve for the linear pricing function that clears the market, while taking the investors’ conjecture \hat{a} about the manager’s effort as given. In particular, we guess and verify the following linear pricing function:

$$p(\hat{a}) = \hat{\alpha}_0 + \hat{\alpha}_v v + \hat{\alpha}_x \zeta - \hat{\alpha}_\varepsilon \varepsilon, \tag{6}$$

where the coefficients can depend on the conjectured effort \hat{a} but not on the actual a that is unobservable by assumption.

In the second step, we reason from the manager's perspective. The manager, taking the market conjecture \hat{a} and the pricing function (6) as given, chooses a to maximize his payoff (2). Given the CARA-normal setup, this is equivalent to maximizing the following certainty equivalent:

$$\max_a \mathbb{E}[p|a, \hat{a}, \tau_x] - C(a) - \frac{\rho}{2} \text{var}(p|a, \hat{a}, \tau_x),$$

where $\mathbb{E}[p|a, \hat{a}, \tau_x] = \hat{\alpha}_0 + \hat{\alpha}_v a$ and $\text{var}(p|a, \hat{a}, \tau_x)$ are derived from (6). The first-order condition yields the manager's best response:

$$a^*(\hat{a}, \tau_x) = \hat{\alpha}_v. \tag{7}$$

In the third step, we impose rational expectations to determine the equilibrium. That is, the conjectured effort equals the actual one in equilibrium (i.e., $a^* = \hat{a}$) and the conjectured linear pricing function coincides with the actual market-clearing price. We summarize the equilibrium in Proposition 1 and defer the details to the Appendix.⁶

Proposition 1 (Equilibrium with Exogenous Precision) *Fixing the disclosure precision τ_x , there exists a unique linear pricing function:*

$$p = \alpha_0 + \alpha_v v + \alpha_x \zeta - \alpha_\varepsilon \varepsilon, \tag{8}$$

⁶We show in the proof that the coefficient $\hat{\alpha}_v$ in (6) is independent of the investor's conjecture \hat{a} and only depends on the primitives in the model that are commonly known. Therefore, equation (7) suggests that the manager has a dominant strategy a^* in the sense that it is independent of the market's belief \hat{a} . Such a dominant strategy response rules out potential multiple equilibria. We thank Phillip Stocken for helping us with the uniqueness argument.

where $\alpha_0 = \frac{\tau_\phi}{\tau_\phi + \tau_x + \tau_\eta + \tau_p} a^*$, $\alpha_v = \frac{\tau_x + \tau_\eta + \tau_p}{\tau_\phi + \tau_x + \tau_\eta + \tau_p}$, $\alpha_x = \frac{\tau_x}{\tau_\phi + \tau_x + \tau_\eta + \tau_p}$, $\alpha_\varepsilon = \frac{1}{r\tau_\eta} \frac{\tau_\eta + \tau_p}{\tau_\phi + \tau_x + \tau_\eta + \tau_p}$, and $\tau_p = (\tau_\eta r)^2 \tau_\varepsilon$ is the precision of price used as an independent signal of v . The manager's equilibrium effort choice is

$$a^* = \alpha_v = 1 - \frac{\tau_\phi}{\tau_\Sigma}, \quad (9)$$

where $\tau_\Sigma = \text{var}^{-1}(v|\mathcal{F}_i) = \tau_\phi + \tau_x + \tau_\eta + \tau_p$ is an individual investor's posterior precision given her information set $\mathcal{F}_i = \{x, y_i, p\}$.

Proof. All proofs are in the Appendix.

The analysis with exogenous disclosure quality is standard in noisy rational expectation equilibrium models.⁷ All else equal, the market-clearing price (8) will be higher if the firm's fundamental v is higher, the asset supply ε is lower, or the common noise ζ contained in the public disclosure x is higher. The idiosyncratic noises η_i contained in investors' private signals y_i do not affect the price because they are aggregated away by the law of large numbers. In equilibrium, observing price p is informationally equivalent to observing

$$q \doteq (p - \alpha_x x - \alpha_0) / (\alpha_v - \alpha_x) = v - \frac{\varepsilon}{r\tau_\eta}, \quad (10)$$

which is a normally distributed signal of firm value v with a precision $\tau_p = (\tau_\eta r)^2 \tau_\varepsilon$.

To understand the manager's equilibrium effort choice, $a^* = \alpha_v$ in (9), note that while the market price satisfies $\mathbf{E}[p] = \mathbf{E}[v]$ in equilibrium, it is formed in a process that is only partially responsive to its fundamental value v (and hence effort a) in the sense that $\frac{d}{dv} \mathbf{E}[p|v] = \alpha_v < 1$. This partial responsiveness arises because, when assessing

⁷Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981) are among the first to characterize the noisy rational expectation equilibrium, which has become a standard framework to study asset pricing in competitive markets. See, for example, Gao (2008) Lemma 1 for a similar price function.

firm value, investors always attach some weight to the conjectured effort level \hat{a} that the manager takes as given and cannot change (see Holmström and Tirole, 1993; Edmans and Manso, 2011, for a similar argument). The manager's moral hazard problem arises because the rate at which his effort increases the market price, $\frac{d\mathbb{E}[p|a]}{da} = \alpha_v$, is lower than the rate at which it increases the firm's value, $\frac{d\mathbb{E}[v|a]}{da} = 1$. The coefficient α_v measures the manager's perceived marginal benefit from exerting effort, which explains his effort choice, $a^* = \alpha_v$, in equilibrium.

Given the manager's equilibrium effort, $a^* = 1 - \frac{\tau_\phi}{\tau_\phi + \tau_x + \tau_\eta + \tau_p}$, it is clear that improving the precision of either public or private information (i.e., τ_x or τ_η) will incentivize the manager to exert more effort. This, however, does not mean that the manager always benefits from a more precise information environment. The reason is that revealing information to investors prior to trading can also make the price p more volatile from an ex ante perspective, which exposes the manager to greater risk and lowers his certainty equivalent, $CE = \mathbb{E}[p|a^*] - C(a^*) - \frac{\rho}{2}\text{var}(p)$.

3.2 Price Volatility and Investors' Inferences From Price

To analyze how the two types of information affect price volatility $\text{var}(p)$ (and, hence, the manager's certainty equivalent), we decompose it as follows:

$$\text{var}(p) = \underbrace{\text{var}[\mathbb{E}(p|v)]}_{\text{Fundamental-driven volatility: } V_F} + \underbrace{\mathbb{E}[\text{var}(p|v)]}_{\text{Noise-driven volatility: } V_N}. \quad (11)$$

The Fundamental-driven volatility, $V_F \doteq \text{var}[\mathbb{E}(p|v)] = \frac{\alpha_v^2}{\tau_\phi}$, is caused by the volatility of the underlying firm value v , and the Noise-driven volatility, $V_N \doteq \mathbb{E}[\text{var}(p|v)] = \frac{\alpha_x^2}{\tau_x} + \frac{\alpha_\varepsilon^2}{\tau_\varepsilon}$, is driven by noise terms that are unrelated to firm value.

Because the noise-driven volatility V_N is unrelated to firm value, it is detrimental in the sense that the risk V_N is imposed on the manager without any incentive benefit. In comparison, the fundamental-driven volatility V_F is intrinsically tied to the power of the manager's incentives: both $V_F = \alpha_v^2/\tau_\phi$ and the equilibrium effort $a^* = \alpha_v$ are increasing functions of α_v . That is, while a more responsive pricing function (i.e., a higher α_v) motivates the manager to exert a higher effort, it also exposes the manager more to the volatility $1/\tau_\phi$ of the underlying firm value. To see the similarity between public and private information in increasing the fundamental-driven volatility V_F , we use Proposition 1 to express V_F as

$$V_F \doteq \text{var} [\mathbb{E}(p|v)] = \alpha_v^2/\tau_\phi = \left(1 - \frac{\tau_\phi}{\tau_\Sigma}\right)^2 / \tau_\phi. \quad (12)$$

Equation (12) shows that any reduction in ex post uncertainty (i.e., a higher posterior precision $\tau_\Sigma = \tau_\phi + \tau_x + \tau_\eta + \tau_p$) due to new information – be it public or private – is accompanied by an increase in ex ante fundamental-driven volatility V_F . That is, releasing information prior to trading redistributes uncertainty from ex post to ex ante. This risk-shifting result dates back to Hirshleifer (1971) and Ross (1989), and is formally investigated in the cost of capital literature (e.g., Christensen et al., 2010; Gao, 2010; Dutta and Nezlobin, 2017).

While both public and private information increase the fundamental-driven volatility V_F , they affect the noise-drive volatility V_N differently. A key difference is how public and private information affect the $\alpha_\varepsilon^2/\tau_\varepsilon$ part of V_N , that is, the price volatility tied to supply shocks. Investors' inferences from market price are important in understanding

the differences. To illustrate, we use the market-clearing condition to express price as

$$p = \underbrace{\int_i \mathbb{E}(v|\mathcal{F}_i) di}_{\text{Aggregate Belief}} - \underbrace{\frac{\text{var}(v|\mathcal{F})}{r}}_{\text{Risk Premium}} \varepsilon, \quad (13)$$

where $\mathcal{F}_i = \{\hat{a}, x, y_i, p\}$ is investor i 's information set.⁸ That is, the market-clearing price equals the aggregate belief among investors minus a risk premium that investors demand to absorb the supply shock ε . It follows from Bayes' rule that investor i 's posterior assessment $\mathbb{E}(v|\mathcal{F}_i)$ is a precision-weighted average of signals in her information set as follows:

$$\mathbb{E}(v|\mathcal{F}_i) = w_0 \hat{a} + w_x x + w_y y_i + w_p q, \quad (14)$$

with $w_0 = \frac{\tau_\phi}{\tau_\Sigma}$, $w_x = \frac{\tau_x}{\tau_\Sigma}$, $w_y = \frac{\tau_\eta}{\tau_\Sigma}$, $w_p = \frac{\tau_p}{\tau_\Sigma}$, and $\tau_\Sigma = \tau_\phi + \tau_x + \tau_\eta + \tau_p$. (Recall from (10) that $q = v - \frac{\varepsilon}{r\tau_\eta}$ is informationally equivalent to price p .)

Substituting $\mathbb{E}(v|\mathcal{F}_i)$ into (13) and using the fact $\int_i y_i di = v$, we obtain $p = \left[w_0 \hat{a} + w_x x + w_y v + w_p \left(v - \frac{\varepsilon}{r\tau_\eta} \right) \right] - \frac{\text{var}(v|\mathcal{F})}{r} \varepsilon$. We can therefore derive the price impact of a supply shock to be

$$\begin{aligned} \alpha_\varepsilon &= \left| \frac{dp}{d\varepsilon} \right| = \frac{\text{var}(v|\mathcal{F})}{r} + \left| \frac{d}{d\varepsilon} \int_i \mathbb{E}(v|\mathcal{F}_i) di \right| \\ &= \frac{1}{r\tau_\Sigma} + \frac{w_p}{r\tau_\eta}. \end{aligned} \quad (15)$$

Equation (15) shows that a supply shock ε moves the market price in two ways. First, it affects the risk premium, $\frac{1}{r\tau_\Sigma}$, required to compensate investors for clearing the supply shock. Intuitively, investors demand a higher premium when they face more uncertainty (i.e., lower τ_Σ) or are less tolerant to risk (i.e., lower r). Second, ε also affects investors'

⁸We drop the subscript of \mathcal{F}_i in $\text{var}(v|\mathcal{F})$ because the posterior variance is the same for all i .

aggregate assessment of firm value: its magnitude, $\left| \frac{d}{d\varepsilon} \int_i \mathbb{E}(v|\mathcal{F}_i) di \right| = \frac{w_p}{r\tau_\eta}$, depends on (i) the weight w_p investors place on price, and (ii) the amount of supply noise contained in the price in the first place. The thinking behind $\frac{1}{r\tau_\eta}$ can be illustrated by noting that price p is informationally equivalent to $v - \frac{\varepsilon}{r\tau_\eta}$, as shown in (10). That is, for every unit of supply noise ε , $\frac{1}{r\tau_\eta}$ enters the investors' information sets under the camouflage of price p . It is clear from (15) that the investor's reliance on price w_p has the potential to amplify supply shocks in the pricing process. Lemma 1 shows that public and private information have opposite influences on investors' reliance on price.

Lemma 1 (Different Effects on Inferences) *Investors will rely more on price p in assessing firm value when their private signals are more precise. In contrast, they rely less on price when public disclosure becomes more precise. That is,*

$$\frac{d}{d\tau_\eta} w_p > 0, \quad \frac{d}{d\tau_x} w_p < 0, \quad (16)$$

where $w_p = \frac{\tau_p}{\tau_\Sigma} = \frac{\tau_p}{\tau_\phi + \tau_x + \tau_\eta + \tau_p}$ is derived according to Bayes' rule in (14).

The thinking behind Lemma 1 can be illustrated by examining investor i 's equilibrium demand function below (after substituting p and $\mathbb{E}(v|\mathcal{F}_i)$ from Equations (13) and (14)):

$$D_i = \frac{r \left[\mathbb{E}(v|\mathcal{F}_i) - p \right]}{\text{var}(v|\mathcal{F}_i)} = r \tau_\eta (y_i - v) + \varepsilon. \quad (17)$$

As private information becomes more precise (i.e., higher τ_η), investors trade more aggressively on their private signal y_i . As a result, the market price better aggregates investors' idiosyncratic private information and, hence, becomes a more informative signal about firm value (i.e., higher τ_p). Note that price informativeness τ_p grows *faster* than the precision of private signals per se so that even the relative precision $\frac{\tau_p}{\tau_\eta} = r^2 \tau_\eta \tau_\varepsilon$

increases in τ_η . The anticipation of an increasingly informative τ_p (relative to private signals) motivates investors rationally to rely more on the market price in valuing the firm, i.e., $\frac{dw_p}{d\tau_\eta} > 0$. In contrast, a similar increase in reliance on price does *not* occur when we improve public disclosure precision τ_x . This is because public disclosures are observed by everyone and will *not* affect investors' trading strategies (17), leaving the price informativeness τ_p unchanged. Without affecting price informativeness, we know from Bayes' rule that a higher τ_x induces investors to rely more on the public disclosure x and lowers the weights given to other signals in their information sets. Our next result follows from Lemma 1.

Proposition 2 (Supply Shock Mitigation versus Amplification) *More precise public disclosure mitigates supply shock related volatility: $\frac{d(\alpha_\varepsilon^2/\tau_\varepsilon)}{d\tau_x} < 0$. In comparison, more precise private information amplifies such volatility when $\tau_\varepsilon \in (\underline{\tau}_\varepsilon, \bar{\tau}_\varepsilon)$.*

We specify the exogenous boundaries $\underline{\tau}_\varepsilon$ and $\bar{\tau}_\varepsilon$ in the Appendix. It is easy to see why improving disclosure quality τ_x lowers $\alpha_\varepsilon = \frac{1}{r\tau_\Sigma} + \frac{w_p}{r\tau_\eta}$: a higher τ_x reduces both the risk premium $\frac{1}{r\tau_\Sigma}$ (by increasing τ_Σ) and the sensitivity of aggregate belief to supply shock (recall $\frac{dw_p}{d\tau_x} < 0$). Improving private signal precision τ_η has countervailing effects. In particular, while a higher τ_η also reduces the risk-premium $\frac{1}{r\tau_\Sigma}$, it motivates investors to rely more on the price (recall $\frac{dw_p}{d\tau_\eta} > 0$) and, therefore, can increase α_ε . Proposition 2 shows that the effect of a higher τ_η in inducing more reliance on price is the dominant effect if the precision of supply noise is not too extreme. The intuition can be illustrated by analyzing the limiting case of $\tau_\varepsilon \rightarrow 0$ or $\tau_\varepsilon \rightarrow \infty$: price p will be either completely uninformative (for $\tau_\varepsilon \rightarrow 0$) or perfectly revealing (for $\tau_\varepsilon \rightarrow \infty$). In both cases, however, the marginal effect of private information τ_η on investors' inference w_p diminishes in the sense that $\frac{dw_p}{d\tau_\eta} \rightarrow 0$. Without a meaningful influence on the investors' reliance on

price w_p , we know from (15) that providing more precise information will lower the price impact of the noisy supply.

4 Endogenous Disclosure and Complementarity

In this section, we endogenize the optimal disclosure quality τ_x^* and investigate how the manager would adjust τ_x^* in response to better informed investors.

4.1 Optimal Disclosure Quality

The manager takes the pricing function p and his equilibrium effort a^* in Proposition 1 as given and chooses an optimal disclosure quality τ_x^* to maximize his certainty equivalent $\mathbb{E}[p|a^*] - C(a^*) - \frac{\rho}{2}\text{var}(p)$. We can simplify the certainty equivalent as follows after substituting p from Proposition 1 and $\text{var}(p) = V_F + V_N$ from (11):

$$CE = a^* - \frac{(a^*)^2}{2} - \frac{\rho}{2}(V_F + V_N). \quad (18)$$

Differentiating the manager's certainty equivalent (18), we obtain the first-order condition that characterizes the optimal disclosure choice τ_x^* as follows:

$$\begin{aligned} FOC = \frac{dCE}{d\tau_x} = & \underbrace{\frac{d[a^* - (a^*)^2/2]}{d\tau_x}}_{\tau_\phi^2 \tau_\Sigma^{-3}} - \underbrace{\frac{\rho dV_F}{2 d\tau_x}}_{\rho (\tau_\Sigma - \tau_\phi) \tau_\Sigma^{-3}} \\ & + \underbrace{\rho \left(\tau_x - \frac{1}{2} \tau_\Sigma \right) \tau_\Sigma^{-3}}_{-\frac{\rho}{2} \frac{d(\alpha_x^2/\tau_x)}{d\tau_x}} + \underbrace{\rho \frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2 \tau_\varepsilon} \tau_\Sigma^{-3}}_{-\frac{\rho}{2} \frac{d(\alpha_\varepsilon^2/\tau_\varepsilon)}{d\tau_x}} - \frac{\rho dV_N}{2 d\tau_x} = 0, \end{aligned} \quad (19)$$

where $\tau_\Sigma = \tau_\phi + \tau_x + \tau_\eta + \tau_p$ is the investors' total posterior precision in Proposition 1.

The first term in (19) is the marginal benefit of improving disclosure quality τ_x in motivating a higher equilibrium effort a^* net of the cost of effort $(a^*)^2/2$. The second term is the marginal cost of τ_x in driving up the fundamental-driven volatility V_F . The marginal effect of disclosure on both a^* and V_F is tied to the fact that improving disclosure quality τ_x makes the equilibrium price more responsive to firm value, i.e., a higher α_v in the pricing function. The first two terms of the *FOC* formalize our earlier discussion: while a more responsive equilibrium price motivates the manager to exert a higher effort $a^* = \alpha_v = 1 - \frac{\tau_\phi}{\tau_\Sigma}$ (in Proposition 1), it also exposes the manager more to the volatility of the underlying firm value $V_F = \alpha_v^2/\tau_\phi$, as shown in (12).⁹

The remaining two terms in (19) capture the marginal effect of a higher τ_x on the noise-driven volatility $V_N = \alpha_x^2/\tau_x + \alpha_\varepsilon^2/\tau_\varepsilon$, which has two components because V_N is tied to both supply shock ($\alpha_\varepsilon^2/\tau_\varepsilon$) and the common noise contained in the public disclosure (α_x^2/τ_x). A higher τ_x affects the two parts of V_N differently, and investors' reliance on different sources of information is important to understand the difference. We show in Proposition 2 that a higher τ_x always reduces supply shock related volatility $\alpha_\varepsilon^2/\tau_\varepsilon$, in part because better disclosures lower investors' reliance on price in valuing the firm. In comparison, a higher τ_x can increase or decrease volatility α_x^2/τ_x tied to the common noise in the public disclosure, which may be surprising because a higher τ_x directly reduces disclosure noise. The counterforce is that a higher τ_x also motivates investors to rely more on the public disclosure in valuing the firm, amplifying any given noise in the disclosure. This counterforce can be seen from the fact that the pricing coefficient $\alpha_x = \frac{\tau_x}{\tau_\Sigma}$ increases in τ_x , and is the same as the weight w_x that investors place on public

⁹It is not surprising that tradeoffs between a higher a^* and a higher V_F in (19) depend on the total precision τ_Σ , for both $a^* = 1 - \frac{\tau_\phi}{\tau_\Sigma}$ and $V_F = \left(1 - \frac{\tau_\phi}{\tau_\Sigma}\right)^2 / \tau_\phi$ are functions of τ_Σ , and $\frac{d\tau_\Sigma^{-1}}{d\tau_x} = -\tau_\Sigma^{-2}$.

disclosure in valuing the firm in (14). Solving the first-order condition (19) yields a unique optimal precision τ_x^* . We summarize the equilibrium below.

Proposition 3 (Equilibrium) *When the precision of public disclosure is endogenous, the game has a unique linear equilibrium in which the disclosure precision is*

$$\tau_x^* = \max\left\{0, 2\frac{1}{r^2\tau_\varepsilon} + \frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \tau_\eta - (\tau_\eta r)^2 \tau_\varepsilon\right\}, \quad (20)$$

with $\tau_x^* > 0$ if and only if $\tau_\varepsilon < \frac{4}{r^2 \left[\sqrt{\left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \tau_\eta\right)^2 + 8\tau_\eta^2} - \left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \tau_\eta\right) \right]}$. Substituting the value of τ_x^* into Proposition 1 fully characterizes the equilibrium.

To see why τ_x^* is zero for sufficiently high τ_ε , we consider an extreme case of $\tau_\varepsilon \rightarrow \infty$ in which the market price is fully revealing, i.e., $p = v$. A fully-revealing price takes away the demand for any disclosure because, even without disclosures, the manager's equilibrium effort is already at its first-best level $a^* = \alpha_v = 1$ and the noise-driven price volatility $V_N = \alpha_x^2/\tau_x + \alpha_\varepsilon^2/\tau_\varepsilon = 0$ is already at its lowest level.¹⁰

4.2 Response to Changes in Private Information

How would the manager adjust the optimal disclosure precision τ_x^* in response to more precise private information? We are particularly interested in understanding why more precise private signals can motivate the manager to provide more precise disclosures. To answer these questions, we apply the implicit function theorem to the manager's *FOC* (19) and obtain $\frac{d\tau_x^*}{d\tau_\eta} = -\frac{dFOC}{d\tau_\eta} / \frac{dFOC}{d\tau_x}$, where the denominator $\frac{dFOC}{d\tau_x} = -\frac{\rho}{2}\tau_\Sigma^{-3} < 0$ is the

¹⁰Mathematically, the marginal benefit in (19) converges to zero faster than the marginal cost as $\tau_\varepsilon \rightarrow \infty$. That is, the marginal benefit of τ_x is negligible compared to its marginal cost for $\tau_\varepsilon \rightarrow \infty$.

second-order condition of the manager's maximization problem. It follows that

$$\frac{d\tau_x^*}{d\tau_\eta} = \left(\underbrace{\frac{\partial FOC}{\partial \tau_\Sigma} \frac{\partial \tau_\Sigma}{\partial \tau_\eta}}_{\text{Total Precision Effect}} + \underbrace{\frac{\partial FOC}{\partial \frac{\tau_p}{\tau_\eta}} \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta}}_{\text{Relative Precision Effect}} \right) / \left(\frac{\rho}{2} \tau_\Sigma^{-3} \right). \quad (21)$$

Equation (21) shows that the way private information quality τ_η influences the manager's disclosure choice τ_x^* can be summarized into two effects. First, more precise private information increases the *total* amount of information revealed to investors, captured by $\tau_\Sigma = \tau_\phi + \tau_x + \tau_\eta + \tau_p$. Second, better private information also changes the *relative importance* of price in an investor's information set, captured by the ratio $\frac{\tau_p}{\tau_\eta}$. Examining the manager's *FOC* (19) reveals that the "Relative Precision Effect" arises exclusively from the marginal benefit of public disclosures in reducing the $\alpha_\varepsilon^2/\tau_\varepsilon$ part of the noise-driven volatility V_N . That is,¹¹

$$\begin{aligned} \frac{\partial FOC}{\partial \frac{\tau_p}{\tau_\eta}} \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta} &= \frac{\partial}{\partial \frac{\tau_p}{\tau_\eta}} \left(-\frac{\rho}{2} \frac{dV_N}{d\tau_x} \right) \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta} \\ &= \frac{\partial}{\partial \frac{\tau_p}{\tau_\eta}} \left(-\frac{\rho}{2} \frac{d\alpha_\varepsilon^2/\tau_\varepsilon}{d\tau_x} \right) \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta} = 2\rho \left(1 + \frac{\tau_p}{\tau_\eta} \right) \tau_\Sigma^{-3}. \end{aligned} \quad (22)$$

In comparison, a higher τ_η affects all other parts of the *FOC* only through improving the total information τ_Σ revealed in equilibrium, regardless of whether an investor learns that information directly from her private signal or indirectly from an informative price. For example, it is clear from the *FOC* that private information quality τ_η influences the manager's tradeoffs between a higher effort a^* and a higher V_F only through the total

¹¹The $-\frac{\rho}{2} \frac{d\alpha_\varepsilon^2/\tau_\varepsilon}{d\tau_x}$ in (22) is shown to be $\rho \frac{(1+\frac{\tau_p}{\tau_\eta})^2}{r^2\tau_\varepsilon} \tau_\Sigma^{-3}$ in the *FOC* (19) and will be discussed in (25).

precision τ_Σ . We show in the Appendix that

$$\frac{\partial FOC}{\partial \tau_\Sigma} \frac{\partial \tau_\Sigma}{\partial \tau_\eta} = -\frac{3}{2}\rho(1 + 2r^2\tau_\eta\tau_\varepsilon)\tau_\Sigma^{-3}. \quad (23)$$

Substituting Equations (23) and (22) into (21), we obtain

$$\begin{aligned} \frac{d\tau_x^*}{d\tau_\eta} &= \left(\frac{\partial FOC}{\partial \tau_\Sigma} \frac{\partial \tau_\Sigma}{\partial \tau_\eta} + \frac{\partial FOC}{\partial \tau_\eta} \frac{\partial \tau_\Sigma}{\partial \tau_\eta} \right) / \frac{\rho}{2}\tau_\Sigma^{-3} \\ &= \underbrace{-3(1 + 2r^2\tau_\eta\tau_\varepsilon)}_{\text{Total Precision Effect (-)}} + \underbrace{4(1 + r^2\tau_\eta\tau_\varepsilon)}_{\text{Relative Precision Effect (+)}}. \end{aligned} \quad (24)$$

The two effects in (24) nicely separate the countervailing forces that private information precision τ_η has on the manager's disclosure choice τ_x^* . The negative “Total Precision Effect” suggests that more precise private information would unambiguously reduce the manager's disclosure choice τ_x^* if he only cared about the total information τ_Σ revealed to the investors (rather than where the information is learned). The intuition can be understood by noting that public and private information are *additively separable* in improving the total precision $\tau_\Sigma = \tau_\phi + \tau_x + \tau_\eta + \tau_p$. Therefore, if what matters to the manager is only the total precision τ_Σ , he will reduce the disclosure quality in response to more precise private signals in order to maintain τ_Σ at the level that initially balances its marginal benefit and marginal cost.

The positive “Relative Precision Effect” can be illustrated by examining the marginal benefit of disclosure in reducing the $\alpha_\varepsilon^2/\tau_\varepsilon$ part of noise-driven volatility V_N . Equation (15) shows $\alpha_\varepsilon = \frac{1}{r\tau_\Sigma} + \frac{w_p}{r\tau_\eta}$, where $\frac{w_p}{r\tau_\eta}$ captures how much a unit of supply noise moves the investors' aggregate assessment of firm value, i.e., $|\frac{d}{d\varepsilon} \int_i \mathbb{E}(v|\mathcal{F}_i) di|$. It is the interaction between public and private information in affecting $|\frac{d}{d\varepsilon} \int_i \mathbb{E}(v|\mathcal{F}_i) di|$ that gives rise to the relative precision effect in (24). To see this, we substitute investors' reliance on

price, $w_p = \frac{\tau_p}{\tau_\Sigma}$, from Lemma 1 to rewrite $\alpha_\varepsilon = \frac{1}{r\tau_\Sigma}(1 + \frac{\tau_p}{\tau_\eta})$ as a function of the total precision τ_Σ and the relative precision $\frac{\tau_p}{\tau_\eta}$. Since $\frac{\tau_p}{\tau_\eta}$ is independent of τ_x , it is easy to see the marginal benefit of τ_x in reducing the $\alpha_\varepsilon^2/\tau_\varepsilon$ part of V_N as

$$-\frac{\rho}{2} \frac{d(\alpha_\varepsilon^2/\tau_\varepsilon)}{d\tau_x} = \frac{\rho\tau_\Sigma^{-3}}{r^2\tau_\varepsilon} \times (1 + \frac{\tau_p}{\tau_\eta})^2. \quad (25)$$

Note that the relative precision $\frac{\tau_p}{\tau_\eta}$ enters the marginal benefit above *multiplicatively*. Therefore, a better private information (i.e., a higher τ_η) can strengthen the marginal benefit in (25) by increasing $\frac{\tau_p}{\tau_\eta} = r^2\tau_\eta\tau_\varepsilon$. Intuitively, as investors' private information improves, price becomes more informative relative to private signals (i.e., a higher $\frac{\tau_p}{\tau_\eta}$) thanks to its information aggregation role. A higher $\frac{\tau_p}{\tau_\eta}$ incentivizes investors to rely more on price in assessing firm value, which amplifies the price impact of a supply shock $\alpha_\varepsilon = \frac{1}{r\tau_\Sigma}(1 + \frac{\tau_p}{\tau_\eta})$. This, in turn, can increase the marginal benefit of public disclosure τ_x in mitigating such supply-shock related volatility, as shown in (25).

The net effect of the two countervailing forces in (24) determines whether the manager increases or decreases the optimal disclosure quality τ_x^* in response to better privately informed investors. Proposition 4 shows that the positive Relative Precision Effect dominates the negative Total Precision Effect when the noisy supply is expected to be volatile.

Proposition 4 (Stimulating More Disclosure) *As investors' private signals become more precise, the manager responds by improving the firm's disclosure quality if the variance of the noisy supply is large. That is,*

$$\frac{d}{d\tau_\eta}\tau_x^* \geq 0 \text{ if and only if } \tau_\varepsilon < \frac{1}{2\tau_\eta r^2}, \quad (26)$$

and the above inequality is strict (i.e., $\frac{d}{d\tau_\eta}\tau_x^* > 0$) for any $\tau_x^* > 0$.

To gain some intuition about the condition $\tau_\varepsilon < \frac{1}{2\tau_\eta r^2}$, note that the two countervailing forces shown in (24) are both stronger for a higher τ_ε . The precision τ_ε of noisy supplies matters because it affects price informativeness: all else equal, a higher τ_ε improves price informativeness τ_p . The fact that Relative Precision Effect increases in τ_ε is intuitive. Recall from Equation (25) that the marginal benefit of disclosure in mitigating $\alpha_\varepsilon^2/\tau_\varepsilon$ depends on the relative precision $\frac{\tau_p}{\tau_\eta}$ multiplicatively. Therefore, a higher τ_ε strengthens the effect by increasing τ_p and, hence, $\frac{\tau_p}{\tau_\eta}$.

To see why the Total Precision Effect is also stronger for a higher τ_ε , we derive the comparative advantage of private information (over public disclosures) in revealing firm value to investors as

$$\frac{d\tau_\Sigma}{d\tau_\eta} / \frac{d\tau_\Sigma}{d\tau_x} = 1 + \frac{d\tau_p}{d\tau_\eta} = 1 + 2r^2\tau_\eta\tau_\varepsilon. \quad (27)$$

The comparative advantage comes from the information aggregation role of price and is captured by $\frac{d\tau_p}{d\tau_\eta} = 2r^2\tau_\eta\tau_\varepsilon$ in (27). Another interpretation of (27) is that, for every unit increase in private information quality τ_η , the manager has to lower his disclosure choice τ_x by $1 + 2r^2\tau_\eta\tau_\varepsilon$ units to keep the total precision τ_Σ unchanged. A higher τ_ε clearly strengthens the degree of substitution by making price more informative. Furthermore, it follows from Equation (24) that a higher τ_ε increases the Total Precision Effect faster than it increases the Relative Precision Effect. We therefore expect that the positive Relative Precision Effect dominates (and, hence, $\frac{d\tau_x^*}{d\tau_\eta} > 0$) for a low τ_ε .

5 An Application to Studying Investors’ Word-of-Mouth Communication

Our model can be used to study investors’ private word-of-mouth communication that has been increasingly relevant in light of technological innovations. The New York Stock Exchange (NYSE) recently noted that “social media has become a crucial source of information for the financial services community.”¹² A natural question is how firms’ public disclosure would react in response to the change of information environment.

We use the technology developed by Duffie and Manso (2007) and Duffie et al. (2009) to model investors’ private word-of-mouth communication. In particular, each investor meets other investors (e.g., family members or friends) at a sequence with Poisson arrival time with a mean arrival rate $\lambda \geq 0$ that is exogenous and common across all investors. When two investors meet, they exchange their posterior beliefs about firm value v . Given the joint-normal information structure, it is sufficient for the purpose of updating investors’ beliefs about v that each investor i tells her counterpart j , at each meeting, her current conditional mean $\tilde{\mu}_i$ and the total number of signals N_i from which $\tilde{\mu}_i$ is derived. The number N_i is initially one at $t = 1$ (i.e., the endowed private signal), and then increases at each meeting by the number of signals N_j gathered by her counterpart j prior to the meeting. The word-of-mouth communication takes place continuously prior to the trading date $t = 2$. According to Andrei and Cujean (2017, Proposition 1),

¹²“NYSE Technologies and SMA to Distribute Social Media Analysis Data via SFTI”, published on NYSE Technologies.

the cross-sectional distribution of the number of signals N_i at the trading date is:¹³

$$\pi(n) = \begin{cases} e^{-\lambda} & \text{if } n = 1, \\ e^{-(n-1)\lambda}(e^\lambda - 1)^{n-2}(1 - e^{-\lambda}) & \text{if } n = 2, 3, 4, \dots \end{cases} \quad (28)$$

A higher Poisson arrival rate λ corresponds to more active word-of-mouth communication ($\lambda = 0$ means that no one shares information, as in the main model). As is clear from (28), modeling word-of-mouth communication inevitably results in asymmetrically informed investors in terms of heterogeneous signal precision. This added information asymmetry complicates the analysis in our main model. Nonetheless, Lemma 2 shows that our previous results are qualitatively unaffected.

Lemma 2 (Equilibrium with Word-of-Mouth Communication) *All results derived in the main model are preserved once we replace the private signal precision τ_η with the cross-sectional average precision $\bar{N}\tau_\eta$ after word-of-mouth communication. The constant \bar{N} is the cross-sectional average number of signals that investors accumulate by the time of trading:*

$$\bar{N} = \sum_{n=1,2,3,\dots} n \pi(n) = e^\lambda. \quad (29)$$

Lemma 2 shows that more active word-of-mouth communication (i.e., higher λ) affects our previous equilibrium analysis by increasing the average precision of investors' private signals. This average precision only feature has been shown in the literature (e.g., Kim and Verrecchia, 1991; Lambert et al., 2011) to be a standard feature of the classical noisy rational expectation models. Given the monotonic relation between word-of-mouth communication λ and the cross-sectional average precision $\bar{N}\tau_\eta$, we can apply

¹³Andrei and Cujean (2017) derive the distribution of the number of *incremental* signals, while (28) is the distribution of the total number of signals, including each investor's signal endowment y_i .

Proposition 4 to obtain the following result.¹⁴

Corollary 1 *More active word-of-mouth communication will lead to more public disclosure if and only if the variance of the noisy supply is large. That is, for any $\tau_x^* > 0$, $\frac{d}{d\lambda}\tau_x^* > 0$ if and only if $\tau_\varepsilon < \frac{1}{2\mathbf{N}\tau_\eta r^2}$.*

Corollary 1 addresses the concern that, by helping investors share private information, the rise of social media and similar developments will make firms' public disclosures less relevant. Can the social-media induced improvement in private communication among investors imply a need for more precise public disclosure in a fully rational market? Corollary 1 suggests the answer is yes. We show that, even in the absence of misleading rumors or other forms of biased communication, investors' word-of-mouth communication can drive the price ex post away from its fundamental value and, hence, introduce excess price volatility. That is, we show that a cost of word-of-mouth communication (and a call for better public disclosure) arises even under the benevolent assumption that investors' private communication is unbiased and truthful. We acknowledge that because we assume unbiased and truthful word-of-mouth communication, our model cannot study possible mispricing caused by biased communication.¹⁵ It is conceivable that biased communication would exacerbate the potential mispricing and excess price volatility shown in our model. If that is the case, the policy remedy might be even more public disclosure to "correct" any ex post mispricing, or perhaps, a regulatory mechanism to preempt biased communication in

¹⁴Upon discussing the word-of-mouth application, we confine attention to the case in which the disclosure precision is $\tau_x^* > 0$. Proposition 3 provides the necessary and sufficient condition.

¹⁵We also acknowledge anecdotal evidence of the existence of this type of biased communication. For example, in 2017 Centrex sued Richard Pearson and others for disseminating false and libelous statements through the Seeking Alpha website to drive Centrex's stock price down, while having a short position to gain a financial windfall. The suit claimed damages of \$170 million. (<https://www.businesswire.com/news/home/20170306006247/en/Centrex-Files-170-Million-Lawsuit-Richard-Pearson>.) We thank the Editor for directing our attention to this litigation.

the first place.

The model also yields the following predictions about the implications of investors' word-of-mouth communication.

Proposition 5 (Empirical Predictions) *As investors' private word-of-mouth communication becomes more active,*

- (i) *price is more informative, i.e., a higher τ_p ,*
- (ii) *price volatility increases, i.e., a higher $\text{var}(p)$,*
- (iii) *market depth decreases, i.e., a lower α_ε^{-1} ,*
- (iv) *a firm will improve its disclosure quality if its investors' private information is of low quality (i.e., $\tau_\eta < \frac{1}{2N\tau_\varepsilon r^2}$); otherwise, it will reduce its disclosure quality,*
- (v) *firms with more risk-averse investors improve disclosure quality more (or reduce disclosure quality less), i.e., $\frac{d^2\tau_a^*}{d\lambda dr^{-1}} > 0$.*

The predictions are relevant to the growing literature on the use of social media (e.g., Twitter) and crowd-sourced content platforms (e.g., Seeking Alpha) that facilitate information sharing among investors. Empirical research documents that investor opinions communicated through social media predict future stock returns and earnings surprises (e.g., Bartov et al., 2017; Chen et al., 2014a; Da and Huang, 2020). A consensus in this literature is that inter-investor communication is beneficial as they help to obtain the “wisdom of crowds.” Part (i) of Proposition 5 shows a similar wisdom-of-crowds result. In particular, we show that information sharing among investors enables price to aggregate investors' private information more effectively.¹⁶

¹⁶Following Han et al. (2016), we use τ_p to measure price informativeness: it captures how much extra information the price conveys regarding firm value v beyond the public disclosure and private signals of investors. We also prove Part (i) of Proposition 5 using $\text{var}^{-1}(v|p)$ as an alternative measure price informativeness.

In contrast, Parts (ii) and (iii) of Proposition 5 highlight a potential cost of investors' information sharing in (a) amplifying non-fundamental noise and, hence, price volatility and (b) decreasing market depth, which is a liquidity measure in our paper.¹⁷ To our best knowledge, these cost considerations are under-explored in the empirical literature studying inter-investor communication using social media.

Parts (iv) and (v) of Proposition 5 predict how a firm adjusts its disclosure quality when innovations such as social media facilitate information sharing among investors. Part (iv) predicts the sign of the adjustment (i.e., increase or decrease), and the result follows from the necessary and sufficient condition $\tau_\varepsilon < \frac{1}{2N\tau_\eta r^2}$ under which word-of-mouth communication leads to more precise public disclosures. Investors' private information is likely to be of low quality when the firm's information environment is opaque (e.g., a firm with low media and analyst coverage). If this is the case, our result suggests that firms with opaque information environments are likely to improve (rather than lower) their disclosure quality in response to investors' word-of-mouth communication.¹⁸ Part (v) further predicts how the relative magnitude of a firm's disclosure-quality change varies with its investors' risk preference. Compared to institutional investors, retail investors are likely to be more risk averse to the extent their portfolios are less diversified. This implies that firms with more retail investors will improve disclosure quality more (or reduce disclosure quality less), as compared to firms with more institutional investors.

Note that Part (iii) of Proposition 5 can be compared to a well-known result that more precise private information can lower market liquidity if the private information

¹⁷We use the inverse of coefficient α_ε in the pricing function (8) to measure market depth (as in, for example, Vives, 2010; Han and Yang, 2013). The idea behind the measure is that a change in noise trading by one unit moves prices by α_ε ; a market is deep if a noise trader shock is absorbed without moving prices much.

¹⁸See Balakrishnan et al. (2014) for related evidence.

exacerbates information asymmetry among investors (e.g., Verrecchia, 2001).¹⁹ The mechanism behind our liquidity-reducing result is different because it does not require information asymmetry. We can eliminate information asymmetry by letting *all* investors' private signal precision be τ_η (as in our main model) and then derive the monotonic decrease in market depth, i.e., $\frac{d}{d\tau_\eta}\alpha_\varepsilon^{-1} < 0$. We obtain an unambiguous negative correlation because our model endogenizes the firm's disclosure quality τ_x^* . We believe that allowing the manager to adjust the firm's disclosure quality is empirically descriptive for many firms.

6 Conclusion

Technological innovations such as social media greatly facilitate individual investors' private information discoveries and communication. In this paper, we examine firms' responses to such changes. Contrary to the casual intuition that more precise private information competes with and crowds out firms' public disclosures, we show that firms often commit to more precise public disclosures when investors' private information becomes more precise. This complementarity arises because, as investors' private information becomes more precise, the information aggregation role of price ensures that investors choose to rely more heavily on price in valuing the firm. This increased reliance on price has a side effect of amplifying the price impact of noisy supplies, which in turn strengthens the value of public disclosures because more precise disclosure helps mitigate such price impact and its resulting price volatility. We show that when the asset supply is expected to be volatile, the call for more precise public disclosures to lower

¹⁹Kim and Verrecchia (1994) show conditions under which disclosure reduces liquidity by affecting information asymmetry. Caskey et al. (2015) study the effect on the bid-ask spread of information dissemination in networks in a sequential trade model à la Glosten and Milgrom (1985).

the otherwise exacerbated shock-driven volatility outweighs the intrinsic substitutability between the public and private signals in revealing information to the investors.

We apply our model to study investors' private word-of-mouth communication. The analysis highlights a cost of such inter-investor information sharing in (a) amplifying non-fundamental noise and price volatility and (b) reducing market depth, which is a liquidity measure in our model. We also predict how a firm adjusts its disclosure quality in response to more active word-of-mouth communication, including both the sign of a firm's disclosure-quality change and cross-sectional variation in the relative magnitude of the change. These empirical predictions are relevant in light of the use of social media and crowd-sourced content platforms (such as Twitter and Seeking Alpha) that facilitate information sharing among investors.

The manager's risk aversion plays an important role in our analysis (in particular, his disutility when facing a more volatile price). In a way, our emphasis on the manager's utility is in line with Beyer et al. (2010, p. 305), who state in their review of the disclosure literature that “[i]t is management and not the ‘firm’ that makes disclosure decisions. As a result, the costs and benefits of disclosure that explain disclosure decisions reflect management's utility and disutility from making a disclosure.” According to Beyer et al. (2010), “[m]ost models assume that the managers attempt to maximize share price.” Our model complements previous studies by also considering the manager's disutility that is associated with price volatility. While we acknowledge that some managers may prefer a more volatile price, the risk-aversion assumption and, in our opinion, the incentives to avoid price volatility are empirically descriptive in many cases. For future research, it seems to be an interesting avenue to study the interactions between firms' disclosures and investors' private communication in a dynamic setting.

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A Appendix: Proofs

Proof of Proposition 1: We organize the proof in two steps. In the first step, we reason from the investors' perspective: we take the investors' conjecture \hat{a} about the manager's effort as given, and solve for the linear pricing function that clears the market (given the conjecture \hat{a}). We then reason from the manager's perspective in the second step: we take the linear pricing function derived in the first step as given and solve for the manager's optimal effort choice a^* . The rationality condition ensures that $\hat{a} = a^*$ in equilibrium.

Step 1: We guess and verify the following linear pricing equilibrium:

$$p = \hat{\alpha}_0 + \hat{\alpha}_v v + \hat{\alpha}_x \zeta - \hat{\alpha}_\varepsilon \varepsilon, \quad (\text{A.1})$$

where the coefficients can depend on the investors' conjecture \hat{a} (among other primitives of the model) but not on the manager's actual effort a , which is unobservable by assumption.

Consider the demand of the risky asset from any investor i who observes (i) the public signal x , (ii) the market price p , and (iii) an independent private signal $y_i = v + \eta_i$ prior to trading. The market price p is informationally equivalent to

$$q \doteq \frac{p - \hat{\alpha}_x x - \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} = \frac{p - \hat{\alpha}_x(v + \zeta) - \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} = v - \frac{\hat{\alpha}_\varepsilon}{\hat{\alpha}_v - \hat{\alpha}_x} \varepsilon, \quad (\text{A.2})$$

which is a noisy signal of firm value v with precision $\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon$. Note that q is easier to work with because its mean is v . We can express investor i 's information set as $\mathcal{F}_i = \{y_i, x, q, \hat{a}\}$, where \hat{a} is the investors' conjecture of the manager's effort. The joint

normality implies that:

$$\text{var} (v|\mathcal{F}_i) = \frac{1}{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta}, \quad (\text{A.3})$$

$$\mathbb{E} (v|\mathcal{F}_i) = \frac{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon q + \tau_x x + \tau_\phi \hat{a} + \tau_\eta y_i}{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta}. \quad (\text{A.4})$$

Therefore, investor i 's demand for the risky asset is

$$\begin{aligned} D_i &= \frac{r \left(\mathbb{E} (v|\mathcal{F}_i) - p \right)}{\text{var} (v|\mathcal{F}_i)} \\ &= r \left[\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon} \right)^2 \tau_\varepsilon \frac{p - \hat{\alpha}_x x - \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} + \tau_x x + \tau_\phi \hat{a} + \tau_\eta y_i \right. \\ &\quad \left. - p \left(\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon} \right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta \right) \right]. \end{aligned} \quad (\text{A.5})$$

Integrating D_i over the continuum of investors and making use of the market-clearing condition $\int_i D_i di = \varepsilon$, we can show the following:

$$\begin{aligned} &r \left[\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon} \right)^2 \tau_\varepsilon \frac{p - \hat{\alpha}_x x - \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} + \tau_x x + \tau_\phi \hat{a} + \int_i \tau_\eta y_i di \right. \\ &\quad \left. - p \left(\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon} \right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta \right) \right] = \varepsilon, \end{aligned}$$

$$\begin{aligned} \Leftrightarrow &r \left[\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon} \right)^2 \tau_\varepsilon \frac{p - \hat{\alpha}_x (v + \zeta) - \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} + \tau_x (v + \zeta) + \tau_\phi \hat{a} + \tau_\eta v \right. \\ &\quad \left. - p \left(\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon} \right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta \right) \right] = \varepsilon, \end{aligned}$$

from which we know the market-clearing price is

$$p = \frac{-\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{\hat{\alpha}_x \zeta + \hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} + \tau_x \zeta + \tau_\phi \hat{a} + \left(\tau_x + \tau_\eta - \left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{\hat{\alpha}_x}{\hat{\alpha}_v - \hat{\alpha}_x}\right) v - \frac{\varepsilon}{r}}{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta - \left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{1}{\hat{\alpha}_v - \hat{\alpha}_x}}. \quad (\text{A.6})$$

To determine the coefficients, we impose the rational condition that the conjectured pricing function (A.1) coincides with the true market-clearing price (A.6) in equilibrium.

That is, the coefficients satisfy:

$$\hat{\alpha}_0 = \frac{-\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{\hat{\alpha}_0}{\hat{\alpha}_v - \hat{\alpha}_x} + \tau_\phi \hat{a}}{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta - \left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{1}{\hat{\alpha}_v - \hat{\alpha}_x}}, \quad (\text{A.7})$$

$$\hat{\alpha}_v = \frac{\tau_x + \tau_\eta - \left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{\hat{\alpha}_x}{\hat{\alpha}_v - \hat{\alpha}_x}}{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta - \left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{1}{\hat{\alpha}_v - \hat{\alpha}_x}}, \quad (\text{A.8})$$

$$\hat{\alpha}_x = \frac{-\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{\hat{\alpha}_x}{\hat{\alpha}_v - \hat{\alpha}_x} + \tau_x}{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta - \left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{1}{\hat{\alpha}_v - \hat{\alpha}_x}}, \quad (\text{A.9})$$

$$\hat{\alpha}_\varepsilon = \frac{\frac{1}{r}}{\left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon + \tau_x + \tau_\phi + \tau_\eta - \left(\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon}\right)^2 \tau_\varepsilon \frac{1}{\hat{\alpha}_v - \hat{\alpha}_x}}. \quad (\text{A.10})$$

It is easy to verify that $\frac{\hat{\alpha}_v - \hat{\alpha}_x}{\hat{\alpha}_\varepsilon} = \tau_\eta r$. We can then simplify (A.2) as $q = v - \frac{\varepsilon}{r\tau_\eta}$ and verify that $\tau_p = (\tau_\eta r)^2 \tau_\varepsilon$ is the precision of market price p , which is informationally equivalent

to q . The system of linear equations shown above determines the pricing coefficients:

$$\hat{\alpha}_0 = \frac{\tau_\phi}{\tau_\phi + \tau_x + \tau_\eta + \tau_p} \hat{a} = \frac{\tau_\phi}{\tau_\Sigma} \hat{a}, \quad (\text{A.11})$$

$$\hat{\alpha}_v = \frac{\tau_x + \tau_\eta + \tau_p}{\tau_\phi + \tau_x + \tau_\eta + \tau_p} = 1 - \frac{\tau_\phi}{\tau_\Sigma}, \quad (\text{A.12})$$

$$\hat{\alpha}_x = \frac{\tau_x}{\tau_\phi + \tau_x + \tau_\eta + \tau_p} = \frac{\tau_x}{\tau_\Sigma}, \quad (\text{A.13})$$

$$\hat{\alpha}_\varepsilon = \frac{1}{r\tau_\eta} \frac{\tau_\eta + \tau_p}{\tau_\phi + \tau_x + \tau_\eta + \tau_p} = \frac{\tau_\eta + \tau_p}{r\tau_\eta\tau_\Sigma}, \quad (\text{A.14})$$

where $\tau_\Sigma = \text{var}(v|\mathcal{F})^{-1} = \tau_\phi + \tau_x + \tau_\eta + \tau_p$ is investor i 's posterior precision about v given her information set $\mathcal{F}_i = \{y_i, x, p, \hat{a}\}$. (We drop the subscript i in $\text{var}(v|\mathcal{F})$ because the posterior variance is the same for all investors.)

Step 2: We next solve for the manager's equilibrium effort choice. In particular, the manager takes the linear pricing function characterized above as given and chooses a to maximize his certainty equivalent:

$$\max_a \mathbb{E}[p|a, \hat{a}] - C(a) - \frac{\rho}{2} \text{var}(p|a, \hat{a}), \quad (\text{A.15})$$

where $\mathbb{E}[p|a, \hat{a}] = \hat{\alpha}_0 + \hat{\alpha}_v \times a$ and $\text{var}(p|a, \hat{a}) = (\hat{\alpha}_v)^2 / \tau_\phi + (\hat{\alpha}_x)^2 / \tau_x + (\hat{\alpha}_\varepsilon)^2 / \tau_\varepsilon$ follow from the linear pricing function characterized in Step 1. Inspecting the first-order condition yields the manager's best response as follows:

$$a^*(\tau_x) = \hat{\alpha}_v = 1 - \frac{\tau_\phi}{\tau_\Sigma}. \quad (\text{A.16})$$

Since the manager's best response is independent of the investors' conjecture \hat{a} , \hat{a} must equal $a^*(\tau_x) = 1 - \frac{\tau_\phi}{\tau_\Sigma}$ to be correct in equilibrium. Finally, replacing the conjectured effort \hat{a} in (A.11) with the equilibrium effort a^* yields the equilibrium linear pricing

coefficients $(\alpha_0, \alpha_v, \alpha_x, \alpha_\varepsilon)$ shown in the proposition. \blacksquare

Proof of Lemma 1: Denote by $\mathcal{F}_i = \{y_i, x, p, \hat{a}\}$ investor i 's information set, where p is informationally equivalent to $q = \frac{p - \alpha_x x - \alpha_0}{\alpha_v - \alpha_x} = v - \frac{\varepsilon}{r\tau_\eta}$. It follows from Bayes rule that

$$\mathbb{E}(v|\mathcal{F}_i) = \frac{\tau_p q + \tau_x x + \tau_\phi \hat{a} + \tau_\eta y_i}{\tau_\phi + \tau_x + \tau_\eta + \tau_p}, \quad (\text{A.17})$$

which can be denoted as $\mathbb{E}(v|\mathcal{F}_i) = w_0 \hat{a} + w_p q + w_x x + w_y y_i$, with $w_0 = \frac{\tau_\phi}{\tau_\Sigma}$, $w_x = \frac{\tau_x}{\tau_\Sigma}$, $w_y = \frac{\tau_\eta}{\tau_\Sigma}$, $w_p = \frac{\tau_p}{\tau_\Sigma}$, and $\tau_\Sigma = \tau_\phi + \tau_x + \tau_\eta + \tau_p$. It is easy to verify that

$$\frac{dw_p}{d\tau_x} = -\frac{\tau_p}{\tau_\Sigma^2} < 0, \quad (\text{A.18})$$

and

$$\begin{aligned} \frac{dw_p}{d\tau_\eta} &= \frac{\tau_\Sigma \frac{d\tau_p}{d\tau_\eta} - \tau_p \frac{d\tau_\Sigma}{d\tau_\eta}}{\tau_\Sigma^2} \\ &= \frac{2\frac{\tau_p}{\tau_\eta}(\tau_\phi + \tau_x) + \tau_p}{\tau_\Sigma^2} > 0, \end{aligned} \quad (\text{A.19})$$

where we use $\frac{d\tau_p}{d\tau_\eta} = \frac{d(\tau_\eta^2 r^2 \tau_\varepsilon)}{d\tau_\eta} = 2r^2 \tau_\eta \tau_\varepsilon = 2\frac{\tau_p}{\tau_\eta}$ and $\frac{d\tau_\Sigma}{d\tau_\eta} = 1 + \frac{d\tau_p}{d\tau_\eta}$. \blacksquare

Proof of Proposition 2: Since τ_ε is independent of τ_x and τ_η , we prove the claim by studying the sign of $\frac{d\alpha_\varepsilon}{d\tau_x}$ and $\frac{d\alpha_\varepsilon}{d\tau_\eta}$. Using the market-clearing condition $\int_i D_i di = \varepsilon$ and the investor i 's demand function $D_i = \frac{r(\mathbb{E}(v|\mathcal{F}_i) - p)}{\text{var}(v|\mathcal{F}_i)}$, we rewrite the pricing function as

$$p = \int_i \mathbb{E}(v|\mathcal{F}_i) di - \frac{\text{var}(v|\mathcal{F})}{r} \varepsilon, \quad (\text{A.20})$$

where $\text{var}(v|\mathcal{F}) = \tau_\Sigma^{-1} = \frac{1}{\tau_\phi + \tau_x + \tau_\eta + \tau_p}$ is investors' conditional variance. Substituting $\mathbb{E}(v|\mathcal{F}_i)$ from (A.17) into (A.20) and using the fact $\int_i y_i di = v$, we simplify (A.20) to

$p = \left[w_0 \hat{a} + w_x x + w_y v + w_p \left(v - \frac{\varepsilon}{r\tau_\eta} \right) \right] - \frac{\text{var}(v|\mathcal{F})}{r} \varepsilon$, which allows us to show

$$\begin{aligned} \alpha_\varepsilon &= \left| \frac{dp}{d\varepsilon} \right| = \frac{\text{var}(v|\mathcal{F})}{r} + \left| \frac{d}{d\varepsilon} \int_i \mathbb{E}(v|\mathcal{F}_i) di \right| \\ &= \frac{1}{r\tau_\Sigma} + \frac{w_p}{r\tau_\eta}. \end{aligned} \tag{A.21}$$

It is straightforward to verify that

$$\begin{aligned} \frac{d}{d\tau_x} \alpha_\varepsilon &= -\frac{1}{r\tau_\Sigma^2} \frac{d\tau_\Sigma}{d\tau_x} + \frac{1}{r\tau_\eta} \frac{dw_p}{d\tau_x} \\ &= -\frac{1 + \frac{\tau_p}{\tau_\eta}}{r\tau_\Sigma^2} < 0. \end{aligned} \tag{A.22}$$

Similarly, we show

$$\begin{aligned} \frac{d}{d\tau_\eta} \alpha_\varepsilon &= -\frac{1}{r\tau_\Sigma^2} \frac{d\tau_\Sigma}{d\tau_\eta} + \frac{1}{r} \frac{d}{d\tau_\eta} \left(\frac{w_p}{\tau_\eta} \right) \\ &= -\frac{(r^2\tau_\eta\tau_\varepsilon)^2 - r^2\tau_\varepsilon(\tau_\phi + \tau_x - 2\tau_\eta) + 1}{r\tau_\Sigma^2}, \end{aligned} \tag{A.23}$$

which is positive if and only if $\tau_\varepsilon \in (\underline{\tau}_\varepsilon, \bar{\tau}_\varepsilon)$, where

$$\underline{\tau}_\varepsilon \doteq \frac{(\tau_\phi + \tau_x - 2\tau_\eta) - \sqrt{(\tau_\phi + \tau_x)(\tau_\phi + \tau_x - 4\tau_\eta)}}{r^2\tau_\eta^2}, \tag{A.24}$$

$$\bar{\tau}_\varepsilon \doteq \frac{(\tau_\phi + \tau_x - 2\tau_\eta) + \sqrt{(\tau_\phi + \tau_x)(\tau_\phi + \tau_x - 4\tau_\eta)}}{r^2\tau_\eta^2}. \tag{A.25}$$

The set $(\underline{\tau}_\varepsilon, \bar{\tau}_\varepsilon)$ is not empty if and only if $\tau_\phi + \tau_x > 4\tau_\eta$. Collecting the conditions proves the result. \blacksquare

Proof of Proposition 3: Using the pricing function in Proposition 1, we express the

manager's objective function U^M as

$$\begin{aligned}
U^M &= \mathbb{E}[p|a^*] - C(a^*) - \frac{\rho}{2} \text{var}(p) \\
&= a^* - \frac{(a^*)^2}{2} - \frac{\rho}{2} (V_F + V_N) \\
&= \frac{1}{2} - \frac{\tau_\phi^2}{2\tau_\Sigma^2} - \frac{\rho}{2} \left(\text{var} [\mathbb{E}(p|v)] + \mathbb{E} [\text{var} (p|v)] \right), \tag{A.26}
\end{aligned}$$

where the last equality uses the result $a^* = \alpha_v = 1 - \frac{\tau_\phi}{\tau_\Sigma}$ and the mathematical observation of $\text{var}(p) = \text{var} [\mathbb{E}(p|v)] + \mathbb{E} [\text{var} (p|v)]$. Using the pricing function, we can rewrite the *Fundamental-driven volatility* V_F as

$$V_F \doteq \text{var} [\mathbb{E}(p|v)] = \alpha_v^2 / \tau_\phi = \left(1 - \frac{\tau_\phi}{\tau_\Sigma}\right)^2 \frac{1}{\tau_\phi}, \tag{A.27}$$

and the *Noise-driven volatility* V_N as

$$V_N \doteq \mathbb{E} [\text{var} (p|v)] = \alpha_x^2 / \tau_x + \alpha_\varepsilon^2 / \tau_\varepsilon = \frac{1}{\tau_\Sigma^2} \left(\tau_x + \frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2 \tau_\varepsilon} \right). \tag{A.28}$$

Differentiating the manager's certainty equivalent (A.26) with respect to τ_x , we obtain the first-order condition as

$$\begin{aligned}
FOC = \frac{dU^M}{d\tau_x} &= \underbrace{\frac{d[a^* - (a^*)^2/2]}{d\tau_x}}_{\tau_\phi^2 \tau_\Sigma^{-3}} - \underbrace{\frac{-\rho}{2} \frac{dV_F}{d\tau_x}}_{\rho (\tau_\Sigma - \tau_\phi) \tau_\Sigma^{-3}} \\
&\quad + \underbrace{\rho \left(\tau_x - \frac{1}{2} \tau_\Sigma \right) \tau_\Sigma^{-3} + \rho \frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2 \tau_\varepsilon} \tau_\Sigma^{-3}}_{-\frac{\rho}{2} \frac{dV_N}{d\tau_x}}. \tag{A.29}
\end{aligned}$$

Substituting $\tau_\Sigma = \tau_\phi + \tau_x + \tau_\eta + \tau_p$ and $\tau_p = (\tau_\eta r)^2 \tau_\varepsilon$ into (A.29), we set $FOC = 0$

and solve for the optimal precision τ_x^* :

$$\tau_x^* = 2\frac{1}{r^2\tau_\varepsilon} + \frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \tau_\eta - (\tau_\eta r)^2 \tau_\varepsilon, \quad (\text{A.30})$$

with $\tau_x^* > 0$ if and only if

$$\tau_\varepsilon < \bar{\tau}_I(\tau_\eta) \doteq \frac{\left[\sqrt{\left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \tau_\eta\right)^2 + 8\tau_\eta^2} + \left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \tau_\eta\right) \right]}{2r^2\tau_\eta^2}. \quad (\text{A.31})$$

To complete the proof, we verify below that the second-order condition of the maximization problem is negative.

$$\begin{aligned} \frac{d^2U^M}{d\tau_x^2} \Big|_{\tau_x=\tau_x^*} &= \frac{dFOC}{d\tau_x} \Big|_{\tau_x=\tau_x^*} \\ &= \frac{\partial \left[\tau_\phi^2 - \rho(\tau_\Sigma - \tau_\phi) + \rho\left(\tau_x - \frac{1}{2}\tau_\Sigma\right) + \rho\frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2\tau_\varepsilon} \right]}{\partial \tau_x} \cdot \tau_\Sigma^{-3} \\ &\quad + \left[\tau_\phi^2 - \rho(\tau_\Sigma - \tau_\phi) + \rho\left(\tau_x - \frac{1}{2}\tau_\Sigma\right) + \rho\frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2\tau_\varepsilon} \right] \cdot \frac{\partial \tau_\Sigma^{-3}}{\partial \tau_x} \\ &= \left(-\frac{\rho}{2}\right) \cdot \tau_\Sigma^{-3} + 0 \cdot \frac{\partial \tau_\Sigma^{-3}}{\partial \tau_x} = -\frac{\rho}{2}\tau_\Sigma^{-3} < 0, \end{aligned} \quad (\text{A.32})$$

where the second equality above uses the fact that FOC in (A.29) can be expressed as $[\tau_\phi^2 - \rho(\tau_\Sigma - \tau_\phi) + \rho(\tau_x - \frac{1}{2}\tau_\Sigma) + \rho\frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2\tau_\varepsilon}] \cdot \tau_\Sigma^{-3}$; the third equality uses $FOC = 0$ at $\tau_x = \tau_x^*$, which, given $\tau_\Sigma^{-3} > 0$, implies the following at $\tau_x = \tau_x^*$:

$$\tau_\phi^2 - \rho(\tau_\Sigma - \tau_\phi) + \rho\left(\tau_x - \frac{1}{2}\tau_\Sigma\right) + \rho\frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2\tau_\varepsilon} = 0. \quad (\text{A.33})$$

■

Proof of Proposition 4: We present our proof in two parts. In Part I, we first apply the implicit function theorem to analyze $\frac{\partial \tau_x^*}{\partial \tau_\eta}$, taking as given that the precision choice is interior, i.e., $\tau_x^* > 0$. In Part II, we remove the restriction of $\tau_x^* > 0$ and take into account that the value of τ_η affects whether τ_x^* is strictly positive, as shown in Proposition 3.

Part I (Taking $\tau_x^* > 0$ as given): We apply the implicit function theorem to obtain

$$\frac{d\tau_x^*}{d\tau_\eta} = -\frac{\frac{dFOC}{d\tau_\eta}}{\frac{dFOC}{d\tau_x}} \Big|_{\tau_x=\tau_x^*} = -\frac{\frac{\partial FOC}{\partial \tau_\Sigma} \frac{\partial \tau_\Sigma}{\partial \tau_\eta} + \frac{\partial FOC}{\partial \tau_\eta} \frac{\partial \tau_\eta}{\partial \tau_\eta}}{\frac{dFOC}{d\tau_x}}, \quad (\text{A.34})$$

where we have shown in (A.32) that $\frac{dFOC}{d\tau_x} = -\frac{\rho}{2\tau_\Sigma^3} < 0$.

First, the optimality of τ_x^* allows us to simplify $\frac{\partial FOC}{\partial \tau_\Sigma}$ as:

$$\begin{aligned} \frac{\partial FOC}{\partial \tau_\Sigma} \Big|_{\tau_x=\tau_x^*} &= \frac{\partial \left[\tau_\phi^2 - \rho(\tau_\Sigma - \tau_\phi) + \rho\left(\tau_x - \frac{1}{2}\tau_\Sigma\right) + \rho \frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2\tau_\varepsilon} \right]}{\partial \tau_\Sigma} \cdot \tau_\Sigma^{-3} \\ &\quad + \left[\tau_\phi^2 - \rho(\tau_\Sigma - \tau_\phi) + \rho\left(\tau_x - \frac{1}{2}\tau_\Sigma\right) + \rho \frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2\tau_\varepsilon} \right] \cdot \frac{\partial \tau_\Sigma^{-3}}{\partial \tau_\Sigma} \\ &= \left(-\frac{3\rho}{2}\right) \cdot \tau_\Sigma^{-3} + 0 \cdot \frac{\partial \tau_\Sigma^{-3}}{\partial \tau_\Sigma} = -\frac{3\rho}{2} \tau_\Sigma^{-3} < 0, \end{aligned} \quad (\text{A.35})$$

where the second equality follows from $FOC = 0$ at $\tau_x = \tau_x^*$ (see Equation (A.33)).

Combining the equality above with $\frac{d\tau_\Sigma}{d\tau_\eta} = \frac{d\tau_\eta}{d\tau_\eta} + \frac{d\tau_p}{d\tau_\eta} = 1 + 2r^2\tau_\eta\tau_\varepsilon$, we obtain

$$\frac{\partial FOC}{\partial \tau_\Sigma} \frac{\partial \tau_\Sigma}{\partial \tau_\eta} = -\frac{3\rho}{2} (1 + 2r^2\tau_\eta\tau_\varepsilon) \cdot \tau_\Sigma^{-3}. \quad (\text{A.36})$$

Second, it follows from (A.29) and (A.33) that

$$\begin{aligned}
\frac{\partial FOC}{\partial \frac{\tau_p}{\tau_\eta}} \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta} &= \frac{\partial}{\partial \frac{\tau_p}{\tau_\eta}} \left(-\frac{\rho}{2} \frac{dV_N}{d\tau_x} \right) \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta} = \frac{\partial}{\partial \frac{\tau_p}{\tau_\eta}} \left(-\frac{\rho}{2} \frac{d\alpha_\varepsilon^2/\tau_\varepsilon}{d\tau_x} \right) \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta} \\
&= \frac{\partial \left(\rho \frac{\left(1 + \frac{\tau_p}{\tau_\eta}\right)^2}{r^2 \tau_\varepsilon} \cdot \tau_\Sigma^{-3} \right)}{\partial \frac{\tau_p}{\tau_\eta}} \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta} \\
&= 2\rho \left(1 + \frac{\tau_p}{\tau_\eta}\right) \tau_\Sigma^{-3} = 2\rho \left(1 + r^2 \tau_\eta \tau_\varepsilon\right) \cdot \tau_\Sigma^{-3}, \tag{A.37}
\end{aligned}$$

where the third equality uses $\frac{d(-\frac{\rho}{2}\alpha_\varepsilon^2/\tau_\varepsilon)}{d\tau_x} = \rho \frac{(1+\frac{\tau_p}{\tau_\eta})^2}{r^2 \tau_\varepsilon} \tau_\Sigma^{-3}$ as shown in the *FOC* (A.29).

Substituting (A.36) and (A.37) into (A.34), we obtain

$$\begin{aligned}
\frac{d\tau_x^*}{d\tau_\eta} &= -\frac{\frac{dFOC}{d\tau_\eta}}{\frac{dFOC}{d\tau_x}} = -\frac{\frac{\partial FOC}{\partial \tau_\Sigma} \frac{\partial \tau_\Sigma}{\partial \tau_\eta} + \frac{\partial FOC}{\partial \frac{\tau_p}{\tau_\eta}} \frac{\partial \frac{\tau_p}{\tau_\eta}}{\partial \tau_\eta}}{\frac{dFOC}{d\tau_x}} \\
&= \underbrace{-3(1 + 2r^2 \tau_\eta \tau_\varepsilon)}_{\text{Total Precision Effect (-)}} + \underbrace{4(1 + r^2 \tau_\eta \tau_\varepsilon)}_{\text{Relative Precision Effect (+)}} \\
&= 1 - 2r^2 \tau_\eta \tau_\varepsilon. \tag{A.38}
\end{aligned}$$

It follows from (A.38) that $\frac{d\tau_x^*}{d\tau_\eta} > 0$ if and only if $r^2 \tau_\eta \tau_\varepsilon < \frac{1}{2}$, which is equivalent to

$$\tau_\varepsilon < \bar{\tau}_C(\tau_\eta) \doteq \frac{1}{2r^2 \tau_\eta}, \tag{A.39}$$

as claimed in the proposition.

Part II (Allowing for any $\tau_x^* \geq 0$): The proof in Part I assumes the solution is interior, i.e., $\tau^* > 0$. This implicit assumption is satisfied if the condition $\tau_\varepsilon < \bar{\tau}_C(\tau_\eta)$ in (A.39) ensures the condition $\tau_\varepsilon < \bar{\tau}_I(\tau_\eta)$ for an interior solution in (A.31). One can show that

$\tau_\varepsilon < \bar{\tau}_C(\tau_\eta)$ ensures $\tau_\varepsilon < \bar{\tau}_I(\tau_\eta)$ and, hence, an interior $\tau_x^* > 0$ if and only if

$$\bar{\tau}_C(\tau_\eta) \leq \bar{\tau}_I(\tau_\eta). \quad (\text{A.40})$$

The inequality (A.40) is equivalent to $\tau_\eta \geq \frac{2}{9} \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)$, which holds easily if $\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \leq 0$, or equivalently, $\rho \leq 2\tau_\phi$. That is, whenever $\rho \leq 2\tau_\phi$ holds, the condition $\tau_\varepsilon < \bar{\tau}_C(\tau_\eta)$ ensures $\tau_x^* > 0$ and, hence, is a necessary and sufficient condition for $\frac{d}{d\tau_\eta} \tau_x^* > 0$.

The analysis is more involved for $\rho > 2\tau_\phi$, in which case τ_x^* may not be interior given the condition $\tau_\varepsilon < \bar{\tau}_C(\tau_\eta)$ in (A.39). We need to compare τ_ε with $\bar{\tau}_I(\tau_\eta)$ (see (A.31)) to determine whether τ_x^* is interior. Differentiating $\bar{\tau}_I(\tau_\eta)$ with respect to τ_η yields

$$\frac{d\bar{\tau}_I(\tau_\eta)}{d\tau_\eta} = -\bar{\tau}_I^2(\tau_\eta) \frac{r^2}{4} \left[\frac{\left(\frac{2}{\rho} \tau_\phi^2 - \tau_\phi + \tau_\eta \right) + 8\tau_\eta}{\sqrt{\left(\frac{2}{\rho} \tau_\phi^2 - \tau_\phi + \tau_\eta \right)^2 + 8\tau_\eta^2}} - 1 \right], \quad (\text{A.41})$$

which is positive (negative) if τ_η is smaller (bigger) than $\frac{2}{9} \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)$, and $\frac{2}{9} \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)$ is positive given $\rho > 2\tau_\phi$. That is, $\bar{\tau}_I$ first increases and then decreases with τ_η (i.e., $\bar{\tau}_I(\tau_\eta)$ is inverse U-shaped), and it achieves its maximum at $\bar{\tau}_I(\tau_\eta) \big|_{\tau_\eta = \frac{2}{9} \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)} = \frac{9}{4r^2 \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)} = \bar{\tau}_C(\tau_\eta) \big|_{\tau_\eta = \frac{2}{9} \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)}$. We complete the proof by analyzing two scenarios:

1. If $\tau_\varepsilon \geq \frac{9}{4r^2 \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)}$, we know $\tau_\varepsilon \geq \bar{\tau}_I(\tau_\eta)$ holds for $\forall \tau_\eta > 0$. In this case, $\tau_x^* = 0$ for all $\tau_\eta > 0$ and, hence, $\frac{d}{d\tau_\eta} \tau_x^* = 0$.
2. If $\tau_\varepsilon < \frac{9}{4r^2 \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)}$, the optimal τ_x^* can be interior. Specifically, $\tau_\varepsilon < \bar{\tau}_I(\tau_\eta)$ (hence, $\tau_x^* > 0$) if and only if $\tau_\eta \in (\tau_{\eta 1}(\tau_\varepsilon), \tau_{\eta 2}(\tau_\varepsilon))$, where $\tau_{\eta 1}(\tau_\varepsilon)$ and $\tau_{\eta 2}(\tau_\varepsilon)$ are two real roots to equation $\bar{\tau}_I(\tau_\eta) = \tau_\varepsilon$ such that $0 < \tau_{\eta 1}(\tau_\varepsilon) < \frac{1}{2r^2 \tau_\varepsilon} < \tau_{\eta 2}(\tau_\varepsilon)$.²⁰

²⁰Note that given the condition $\tau_\varepsilon < \frac{9}{4r^2 \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)}$, we obtain $\frac{1}{2r^2 \tau_\varepsilon} > \frac{2}{9} \left(\tau_\phi - \frac{2}{\rho} \tau_\phi^2 \right)$ so

To verify $\frac{d}{d\tau_\eta}\tau_x^* \geq 0$ for $\tau_\eta < \frac{1}{2r^2\tau_\varepsilon}$, we analyze how the optimal τ_x^* changes as τ_η increases continuously from zero to $\frac{1}{2r^2\tau_\varepsilon}$. In particular, the optimal τ_x^* remains at $\tau_x^* \equiv 0$ for $0 < \tau_\eta \leq \tau_{\eta 1}(\tau_\varepsilon)$; and becomes positive and monotonically increases in τ_η for $\tau_\eta \in \left(\tau_{\eta 1}(\tau_\varepsilon), \frac{1}{2r^2\tau_\varepsilon}\right)$. The last part holds because both $\tau_\varepsilon < \bar{\tau}_I(\tau_\eta)$ and $\tau_\varepsilon < \bar{\tau}_C(\tau_\eta)$ are satisfied for $\tau_\eta \in \left(\tau_{\eta 1}(\tau_\varepsilon), \frac{1}{2r^2\tau_\varepsilon}\right)$.

Inspecting the conditions in Part II, one can verify that $\tau_\eta < \bar{\tau}_C(\tau_\eta) = \frac{1}{2r^2\tau_\varepsilon}$ is a sufficient condition for $\frac{d}{d\tau_\eta}\tau_x^* \geq 0$, which completes the proof. \blacksquare

Proof of Lemma 2: Consider an investor i who observes (i) the public signal x , (ii) the market price p , and (iii) has accumulated N_i independent private signals $\{y_k = v + \eta_k\}_{k=1}^{N_i}$ via word-of-mouth communication prior to trading. Note that observing N_i independently normally distributed signals is informationally equivalent to observing one signal $y_i = v + \varepsilon_i$, where $\varepsilon_i = \frac{\sum_{k=1}^{N_i}\eta_k}{N_i}$ and $\varepsilon_i \sim N\left(0, \frac{1}{N_i\tau_\eta}\right)$. Following the steps in the proof of Proposition 1, we can derive the equilibrium price and the equilibrium effort a^* . The result is the same as Proposition 1 except that we replace τ_η with $\bar{\mathbf{N}}\tau_\eta$, which is the mean precision of investors' private information. The amount of information investors learn from equilibrium price is now $\tau_p = (\bar{\mathbf{N}}\tau_\eta r)^2 \tau_\varepsilon$. It remains to show that $\bar{\mathbf{N}} = e^\lambda$. We know from Andrei and Cujean (2017, Proposition 1) that the average of the number of *incremental* signals collected by the investors prior to the trading is $e^\lambda - 1$. Therefore, $\bar{\mathbf{N}} = (e^\lambda - 1) + 1 = e^\lambda$ follows by adding the one private signal y_i each investor is endowed with.

Introducing private word-of-mouth communication will result in asymmetrically informed investors in terms of their private signal precisions. Therefore, different investors may face different levels of residual uncertainty and rely on price to a different

$$\bar{\tau}_I(\tau_\eta) \Big|_{\tau_\eta = \frac{1}{2r^2\tau_\varepsilon}} > \bar{\tau}_C(\tau_\eta) \Big|_{\tau_\eta = \frac{1}{2r^2\tau_\varepsilon}} = \tau_\varepsilon.$$

extent. Nonetheless, we can construct a “representative investor” R whose private information is as precise as the cross-sectional average precision among all the investors $\bar{\mathbf{N}}\tau_\eta$ (see, for example, Kim and Verrecchia, 1991). We denote by $\mathcal{F}_R = \{p, x, y_R\}$ the information set of the representative investor R , where p is the market price, x is the firm’s public disclosure, and y_R is the private signal with a precision of $\bar{\mathbf{N}}\tau_\eta$. We can derive R ’s posterior precision $\tau_\Sigma = \text{var}(v|\mathcal{F}_R)^{-1} = \tau_\phi + \tau_x + \tau_p + \bar{\mathbf{N}}\tau_\eta$ and R ’s reliance w_p^R on the market price the same way as we did in the main model. All derivations in Propositions 2, 3, and 4 then follow. \blacksquare

Proof of Corollary 1: By replacing τ_η with $\bar{\mathbf{N}}\tau_\eta$, we derive the optimal precision of public disclosure as follows:

$$\tau_x^* = 2\frac{1}{r^2\tau_\varepsilon} + \frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta - (\bar{\mathbf{N}}\tau_\eta r)^2 \tau_\varepsilon, \quad (\text{A.42})$$

with $\tau_x^* > 0$ if and only if $\tau_\varepsilon < \frac{\left[\sqrt{\left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta\right)^2 + 8(\bar{\mathbf{N}}\tau_\eta)^2} + \left(\frac{2}{\rho}\tau_\phi^2 - \tau_\phi + \bar{\mathbf{N}}\tau_\eta\right)\right]}{2r^2(\bar{\mathbf{N}}\tau_\eta)^2}$.

It is easy to show:

$$\frac{d\tau_x^*}{d\lambda} = \frac{d\tau_x^*}{d\bar{\mathbf{N}}} \frac{d\bar{\mathbf{N}}}{d\lambda} = \left(\tau_\eta - 2\bar{\mathbf{N}}(\tau_\eta r)^2 \tau_\varepsilon\right) e^\lambda, \quad (\text{A.43})$$

which is positive if and only if $\tau_\varepsilon < \frac{1}{2\bar{\mathbf{N}}\tau_\eta r^2}$. \blacksquare

Proof of Proposition 5: For Part (i), note that $\tau_p = (r\bar{\mathbf{N}}\tau_\eta)^2 \tau_\varepsilon$ follows from Proposition 1 and Lemma 2. It is easy to show

$$\frac{d}{d\lambda}\tau_p = \frac{d\tau_p}{d\bar{\mathbf{N}}} \frac{d\bar{\mathbf{N}}}{d\lambda} = 2\bar{\mathbf{N}}(r\tau_\eta)^2 \tau_\varepsilon e^\lambda > 0. \quad (\text{A.44})$$

For Part (ii), we substitute V_F and V_N from (A.27) and (A.28) and obtain

$$\begin{aligned}\text{var}(p) &= V_F + V_N = \left(1 - \frac{\tau_\phi}{\tau_\Sigma}\right)^2 \frac{1}{\tau_\phi} + \frac{1}{\tau_\Sigma^2} \left(\tau_x^* + \frac{\left(1 + \frac{\tau_p}{\bar{\mathbf{N}}\tau_\eta}\right)^2}{r^2\tau_\varepsilon} \right) \\ &= \frac{1}{\tau_\phi} - \frac{\frac{2}{\rho}\tau_\phi^2 + \frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta}{4\left(\frac{1}{\rho}\tau_\phi^2 + \frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right)^2},\end{aligned}\tag{A.45}$$

where τ_x^* is derived in Proposition 3 (with $\tau_\Sigma = \tau_\phi + \tau_x + \tau_p + \bar{\mathbf{N}}\tau_\eta$ and $\tau_p = (r\bar{\mathbf{N}}\tau_\eta)^2 \tau_\varepsilon$).

Straightforward calculation shows

$$\frac{d}{d\lambda}\text{var}(p) = \frac{d\text{var}(p)}{d\bar{\mathbf{N}}} \frac{d\bar{\mathbf{N}}}{d\lambda} = \frac{\tau_\eta \left(\frac{3}{\rho}\tau_\phi^2 + \frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta \right)}{4\left(\frac{1}{\rho}\tau_\phi^2 + \frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right)^3} e^\lambda > 0.\tag{A.46}$$

To prove Part (iii), we substitute τ_x^* from Proposition 3 and $\tau_p = (r\bar{\mathbf{N}}\tau_\eta)^2 \tau_\varepsilon$ into the linear pricing function, and rewrite the coefficient α_ε as

$$\alpha_\varepsilon = \frac{1}{r\bar{\mathbf{N}}\tau_\eta \tau_\phi + \tau_x^* + \bar{\mathbf{N}}\tau_\eta + \tau_p} = \frac{\frac{1}{r} + r\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon}{2\left(\frac{1}{r^2\tau_\varepsilon} + \bar{\mathbf{N}}\tau_\eta\right) + \frac{2}{\rho}\tau_\phi^2}.\tag{A.47}$$

One can verify

$$\frac{d}{d\lambda}\alpha_\varepsilon^{-1} = \frac{d\alpha_\varepsilon^{-1}}{d\bar{\mathbf{N}}} \frac{d\bar{\mathbf{N}}}{d\lambda} = -\frac{\frac{2}{\rho}r\tau_\eta\tau_\varepsilon\tau_\phi^2}{\left(\frac{1}{r} + r\bar{\mathbf{N}}\tau_\eta\tau_\varepsilon\right)^2} e^\lambda < 0.\tag{A.48}$$

Part (iv) follows from Corollary 1, in which we show the necessary and sufficient condition $\tau_\varepsilon < \frac{1}{2\bar{\mathbf{N}}\tau_\eta r^2}$ under which a higher λ leads to more precise public disclosures. To prove Part (v), recall from Corollary 1 $\frac{d\tau_x^*}{d\lambda} = \left(\tau_\eta - 2\bar{\mathbf{N}}(\tau_\eta r)^2 \tau_\varepsilon\right) e^\lambda$. It is easy to verify $\frac{d^2\tau_x^*}{d\lambda dr^{-1}} > 0$, where r^{-1} is investors' risk aversion parameter. \blacksquare