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On numerical modeling of the contaminant transport equations of the wetland hydrology and water quality model WETSAND

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ABSTRACT

The reliability of the MacCormack finite difference method for solving the contaminant transport equations of wetland model WETSAND is investigated. WETSAND solves the coupled advection-dispersion-reaction equations for the nitrogen cycle, total nitrogen and total phosphorus concentrations by using the implicit finite difference method. In addition to the implicit scheme, the MacCormack algorithm is implemented within WETSAND. Then, the results obtained by using the MacCormack algorithm are compared with the results obtained by using the implicit finite difference method for both synthetic examples and real data which is collected at the restored wetland site of Duke University at Sandy Creek watershed. Results show that the numerical methods are in good agreement. While the MacCormack scheme may be computationally more efficient for small velocities and dispersion coefficients (as is commonly the case for wetlands and lakes), much longer computational times are needed for the cases with high velocity and dispersion coefficient values (e.g., streams) since the magnitude of the time step has to be selected according to the CFL stability condition.

1. Introduction

Wetlands are important elements of water resources system because they stabilize water supply and clean polluted water as downstream receiving water bodies [1, 2]. The role of wetlands as one of the best management practices to decrease the surface runoff peaks and to improve surface runoff water quality has been demonstrated [3-4]. Consequently, there is a growing interest in developing accurate and efficient numerical models on wetland hydrology and wetland water quality. Kazezyılmaz-Alhan et al. [5] developed a wetland hydrology and wetland water quality model called WETland Solute TrANsport Dynamics (WETSAND), which has both surface flow and solute transport components, and accounts for surface water/groundwater interactions in both flow and mass transport. The wetland water quality model of WETSAND involves coupled advection-dispersion-reaction equations of nitrogen cycle and total nitrogen and total phosphorus. These equations take into account surface water/groundwater interactions by incorporating mass flux terms between surface water wetlands and groundwater and solved using implicit finite difference method.

There have been numerous studies published, which attempt to solve different forms of contaminant transport equations and give distribution of different pollutants either in surface or ground water system, with finite difference methods including high order schemes [6–9]. Moreover, there are a number of developments with respect to implicit unfactored and implicit

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LU-SGS schemes, including their applications to fairly complex flows [10-12]. For example, Tsoutsanis et al. [13] presented a development and implementation of WENO schemes for viscous flows on arbitrary unstructured meshes. Drikakis and Tsangaris [14] presented an application of two flux vector splitting methods to unsteady compressible Navier–Stokes equations. MacCormack is an explicit finite difference method which is first introduced to solve nonlinear fluid dynamics aeronautical problems [15]. There are various applications of the MacCormack method especially to free surface flow problems [16-20]. Chunshui and Duan [21] compared high resolution numerical schemes including MacCormack method for solving kinematic wave equation which describes overland flow. A TVD-MacCormack scheme has been adopted to numerically solve a horizontal two-dimensional model which considers the hydrodynamic, sediment transport, dissolved oxygen and nutrient cycling processes by Liang et al [22]. MacCormack finite difference method has been recently applied to fundamental forms of advection-dispersion equation [23-25]. However, a thorough review of the literature shows that the MacCormack explicit finite difference method has not been previously applied to contaminant transport equations of WETSAND. WETSAND uses an implicit numerical scheme. However, an explicit scheme has the potential to be more computationally efficient as well as being easier to implement. Therefore, the performance of the explicit MacCormack scheme in WETSAND is explored.

In this study, MacCormack finite difference method is implemented into WETSAND for the solution of contaminant transport equations. Its performance is presented via a hypothetical wetland section and a real data collected at Duke University wetland site. Its applicability is demonstrated with the experimental data. The advantages and drawbacks of the method over the implicit finite difference are discussed.

2. Governing equations of WETSAND

WETland Solute TrANsport Dynamics (WETSAND) is a wetland hydrology and water quality model developed for the simulation of distributed water velocity and water depth and distributed concentrations of phosphorus and nitrogen compounds, incorporating surface water/groundwater interaction effects [5]. WETSAND has two main components which are wetland water quantity and wetland water quality. The wetland water quantity component solves the diffusion wave equation for overland and channel flow and involves rainfall, groundwater recharge and discharge, lateral inflow, infiltration, and evapotranspiration components. The wetland water quality component solves the contaminant transport equations, which are derived for total phosphorus, total nitrogen, and each compound of the nitrogen cycle (organic nitrogen, ammonium nitrogen, and nitrate nitrogen). These equations include the effect of surface water/groundwater interactions on the concentration distribution via incoming/outgoing mass terms due to either groundwater recharge or discharge. The velocity in the advection term is one of the outputs of the wetland water quantity component and is used as input to the wetland water quality component. The equations for phosphorus and nitrogen concentrations are given as follows [5]:

Total Phosphorus (TP):

$$\frac{\partial C_{TP}}{\partial t} = -V\frac{\partial C_{TP}}{\partial x} + \frac{1}{A_x}\frac{\partial}{\partial x}\left(A_x D_x \frac{\partial C_{TP}}{\partial x}\right) + \frac{q_{Lin}}{A_x}\left(C^L_{TP} - C_{TP}\right) + \frac{q_{gwd}}{A_x}\left(C^{gw}_{TP} - C_{TP}\right) - K_{TP}C_{TP} \tag{1}$$

Total Nitrogen (TN):

$$\frac{\partial C_{TN}}{\partial t} = -V\frac{\partial C_{TN}}{\partial x} + \frac{1}{A_x}\frac{\partial}{\partial x}\left(A_x D_x \frac{\partial C_{TN}}{\partial x}\right) + \frac{q_{Lin}}{A_x}\left(C^L_{TN} - C_{TN}\right) + \frac{q_{gwd}}{A_x}\left(C^{gw}_{TN} - C_{TN}\right) - K_{TN}C_{TN} \tag{2}$$

Organic Nitrogen (ON):

$$\begin{aligned}\frac{\partial C_{ON}}{\partial t} &= -V\frac{\partial C_{ON}}{\partial x} + \frac{1}{A_x}\frac{\partial}{\partial x}\left(A_x D_x \frac{\partial C_{ON}}{\partial x}\right) + \frac{q_{Lin}}{A_x}\left(C^L_{ON} - C_{ON}\right)\\ &\quad + \frac{q_{gwd}}{A_x}\left(C^{gw}_{ON} - C_{ON}\right) - K_{ON}C_{ON} + J_{RON}\end{aligned} \tag{3}$$

Ammonium Nitrogen (AN):

$$\begin{aligned}\frac{\partial C_{AN}}{\partial t} &= -V\frac{\partial C_{AN}}{\partial x} + \frac{1}{A_x}\frac{\partial}{\partial x}\left(A_x D_x \frac{\partial C_{AN}}{\partial x}\right) + \frac{q_{Lin}}{A_x}\left(C^L_{AN} - C_{AN}\right)\\ &\quad + \frac{q_{gwd}}{A_x}\left(C^{gw}_{AN} - C_{AN}\right) + K_{ON}C_{ON} - K_{AN}C_{AN} - J_{UAN}\end{aligned} \tag{4}$$

Nitrate Nitrogen (NN):

$$\begin{aligned}\frac{\partial C_{NN}}{\partial t} &= -V\frac{\partial C_{NN}}{\partial x} + \frac{1}{A_x}\frac{\partial}{\partial x}\left(A_x D_x \frac{\partial C_{NN}}{\partial x}\right) + \frac{q_{Lin}}{A_x}\left(C^L_{NN} - C_{NN}\right)\\ &\quad + \frac{q_{gwd}}{A_x}\left(C^{gw}_{NN} - C_{NN}\right) + \psi K_{AN}C_{AN} - K_{NN}C_{NN} - J_{UNN}\end{aligned} \tag{5}$$

Where V is the velocity along x-direction (L/T) and may be a function of x and t, C is the concentration (M/L^3), C^L is the lateral concentration (M/L^3), C^{gw} is the concentration in groundwater (M/L^3), K is the first order loss rate constant ($1/T$), A_x is the cross-sectional area in x-direction (L^2), D_x is the dispersion coefficient (L^2/T), q_{Lin} is the lateral inflow (L^2/T), q_{gwd} is the groundwater

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discharge (L^2/T), J_{RON} is the release flux of organic nitrogen from biomass (M/T), J_{UAN} is the uptake flux of ammonium nitrogen absorbed by biomass (M/T), J_{UNN} is the uptake flux of nitrate nitrogen absorbed by biomass (M/T), ψ is the fraction of ammonium that is nitrified, and *TP, TN, ON, AN, NN* are the subscripts denoting total phosphorus, total nitrogen, organic nitrogen, ammonium nitrogen and nitrate nitrogen, respectively. The reaction sequence among organic, ammonium and nitrate nitrogen components is given by Kadlec and Knight [26].

In order to solve Eqs. (1)–(5), two finite difference techniques are employed, one of which is an explicit method and the other is an implicit method. The solutions obtained from application of explicit finite difference schemes are conditionally stable and the stability condition is given by the Courant–Friedrichs–Lewy (CFL) restriction [27]. Although unconditional stability is an advantage of the implicit finite difference techniques, explicit methods are easier to implement [28]. Splitting methods are introduced in order to decrease the difficulty of programming encountered with implicit methods [29]. One such splitting method is the MacCormack method which uses a forward spatial difference in its predictor step and a backward difference in its corrector step. Although the predictor and corrector steps are first order accurate, the method itself is second order accurate which makes it a true split scheme.

Table 1
Computation times of MacCormack and implicit finite difference methods for different grid sizes (sec).

	dx = 5 m		dx = 10 m		dx = 20 m	
	Implicit	MacCormack	Implicit	MacCormack	Implicit	MacCormack
dt = 1 h	0.1406	0.0469	0.1250	0.0313	0.0625	0.0156
dt = 5 h			0.0313	0.0156	0.0313	0.0156
dt = 10 h					0.0156	0.0156

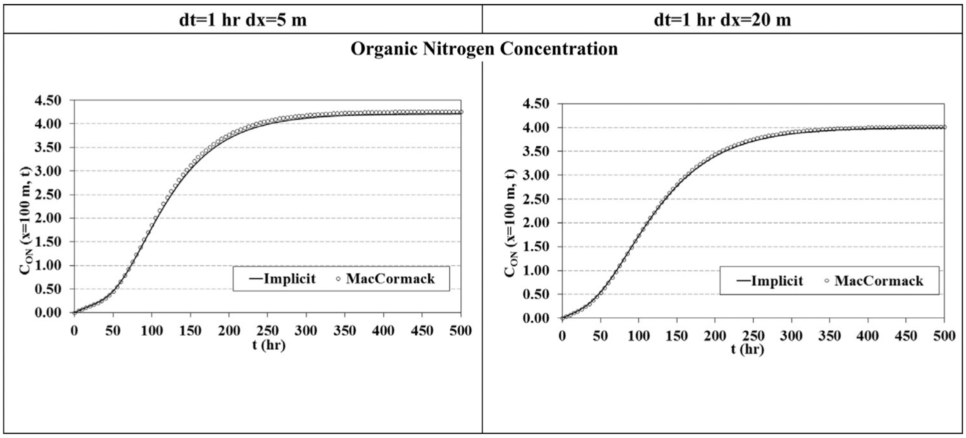

Fig. 1. Comparison of the MacCormack method with the implicit finite difference method for different grid sizes.

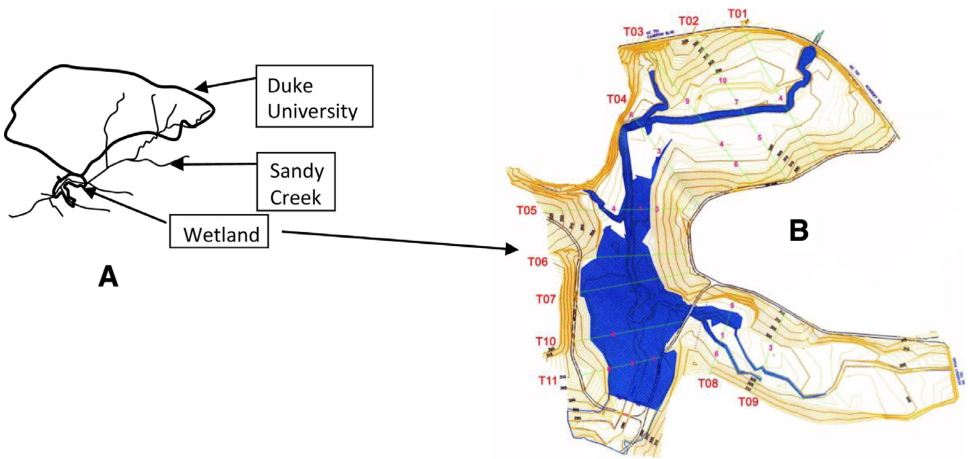

Fig. 2. (A) Duke University and wetland site watersheds, in Durham, North Carolina. (B) Restored wetland and stream site at Sandy Creek in Duke Forest, North Carolina (Duke Wetland Center) [5].

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3. Application

3.1. Synthetic example

In order to show the accuracy of the MacCormack explicit method, a synthetic example is solved for a hypothetical wetland section. The concentrations of Nitrogen compounds are calculated by using both the implicit finite difference and MacCormack

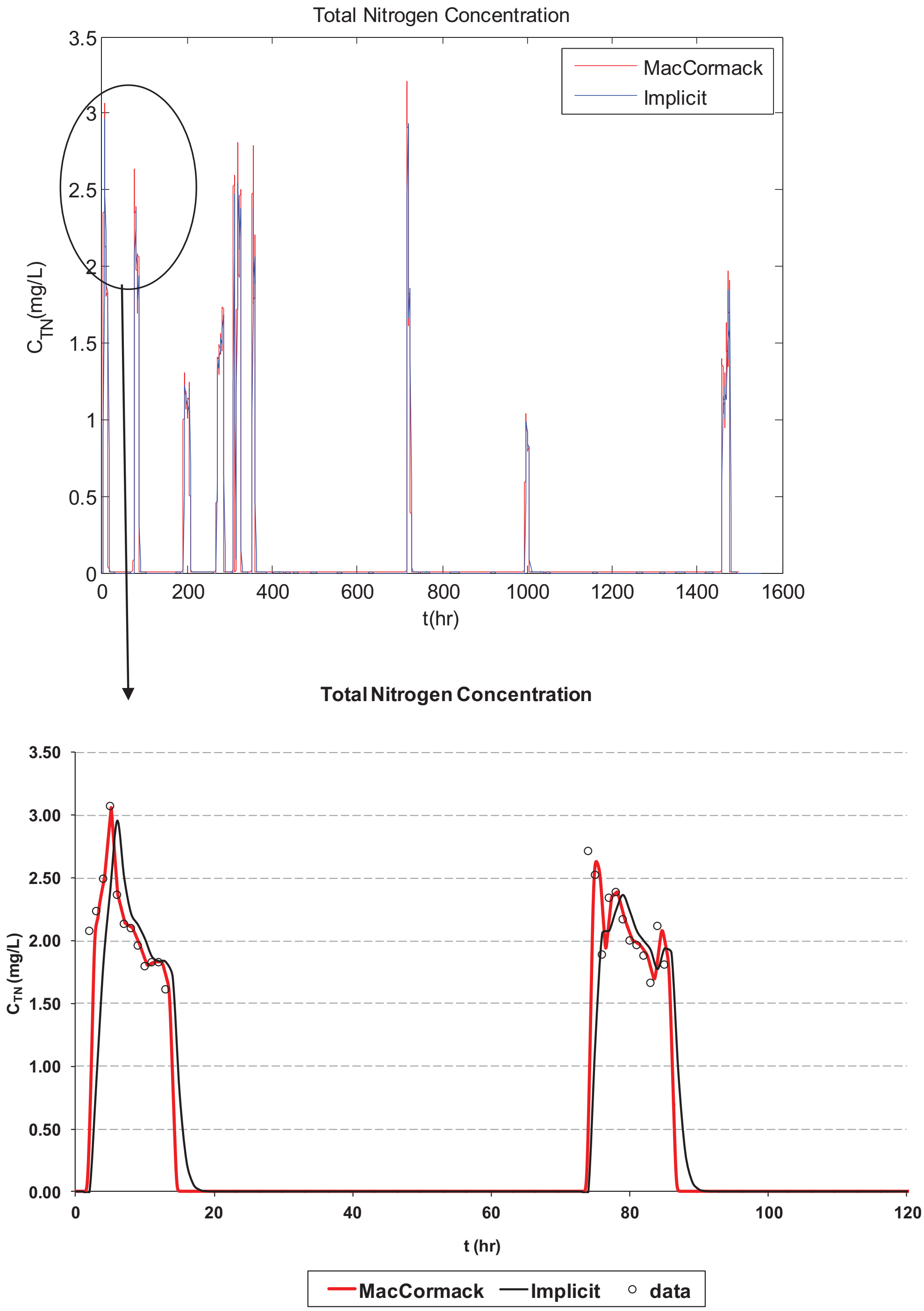

Fig. 3. Comparison of the MacCormack method with the implicit finite difference method for Total Nitrogen at the outlet of the stream of the Duke University wetland site.

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methods which are incorporated into WETSAND, and results are compared. The selected wetland section is a square with dimensions of 100 m × 100 m. The water quantity parameters are chosen as follows: the water velocity $V = 0.5$ m/h, diffusion coefficient $D_x = 10$ m^2/h, lateral inflow $q_{Lin} = 0$, water depth $y = 0.02$ m and groundwater discharge $q_{gwd} = 0.02$ m^2/h. In addition to these, typical values for water quality parameters are chosen [26] as follows: the background concentrations for Organic Nitrogen (ON), Ammonium Nitrogen(AN) and Nitrate Nitrogen(NN) are selected as $C_{gw}{}^{ON} = 1.5$ mg/L, $C_{gw}{}^{AN} = 0.1$ mg/L and $C_{gw}{}^{NN} = 0$mg/L, respectively; the decay rates for ON, AN and NN are selected as $K_{ON} = 17$ yr^{-1}, $K_{AN} = 18$ yr^{-1} and $K_{NN} = 35$ yr^{-1}, respectively;

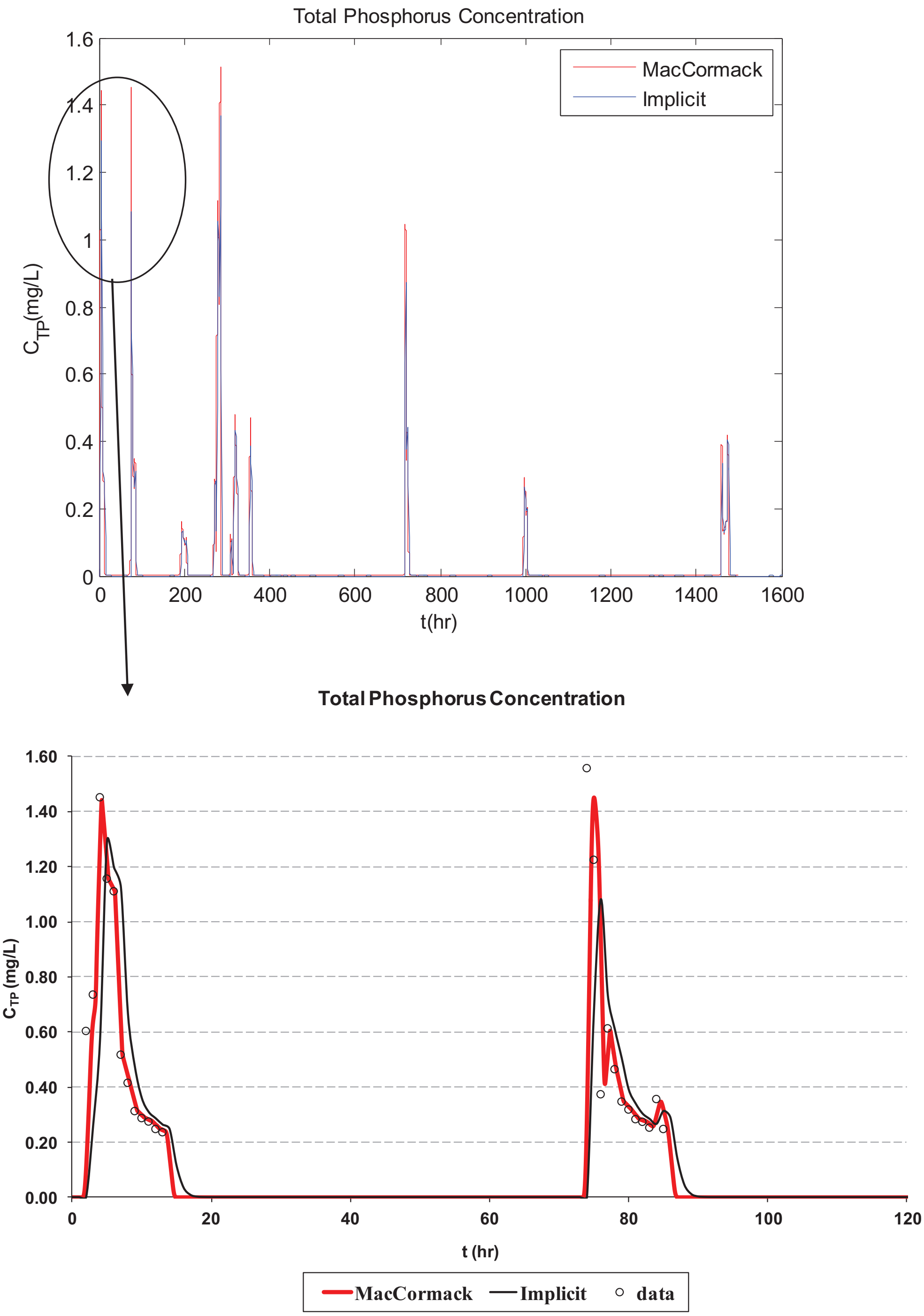

Fig. 4. Comparison of the MacCormack method with the implicit finite difference method for Total Phosphorus at the outlet of the stream of Duke University wetland site.

and the fraction of ammonium that is nitrified is selected as $\psi = 1$. A continuous type of boundary condition is tested for the Nitrogen cycle and for this purpose, 10 mg/L of concentration of each Nitrogen compound is assigned at the inlet of the wetland site.

The simulations are done for several time steps and grid sizes as given in Table 1 in order to see the effects of grid sizes on the accuracy and efficiency of the finite difference methods. Fig. 1 shows the change of Organic Nitrogen concentrations at the outlet of the wetland site obtained by using both implicit finite difference method and MacCormack method for different grid sizes. As seen from these graphs that the results for Organic Nitrogen are in good agreement and the results do not differ significantly for different grid sizes. The results for Ammonium and Nitrate Nitrogen show similar behavior. Moreover, the computation time required by the MacCormack scheme is less than the computation time required by the implicit method as provided in Table 1. As it can be seen from the table, the computational time reaches the minimum value for MacCormack method for a grid size of $\Delta t = 1$ hand $\Delta x = 20$ m. Thus, the MacCormack method is preferable over the implicit method if computational time is an issue.

3.2. Duke University wetland site

In order to show the applicability of the MacCormack method, a real wetland site is selected and collected water quality data at the field site is used in order to calculate the concentration distribution. The selected site is a restored wetland located within the Duke University territory in the Sandy Creek watershed and has a surface area of 554 ha. A great portion of the storm water runoff generated over the Duke University Campus and City of Durham, NC flows into this area. Sandy Creek is one the important tributaries which reaches the drinking water supplies of North Carolina. A stream restoration project was applied to Sandy Creek in order to enhance water flow over the floodplain via meanders and aid in the removal of nutrients and sediments. Fig. 2 shows the location and the topography of the restored wetland site, new restored stream and the flooded area which was formed after the construction of an earthen dam. Water quality measurements for major nutrients (N, P and cations) are taken monthly at the wetland, restored stream and lake site [30].

In this example, total nitrogen and total phosphorus data measured during August–October 2002 is used as boundary conditions upstream of Sandy Creek and the concentration levels at downstream point of Sandy Creek and at the outlet of the lake are calculated by using both the implicit finite difference method and the MacCormack method incorporated into the WETSAND model. Typical parameters are used in the analyses and are as follows: diffusion coefficient $D_x = 1000$ m^2/h for stream and $D_x = 3$ m^2/h for the lake, the decay rates $K_{TN} = 20$ yr^{-1}for total nitrogen and $K_{TP} = 11.5$ yr^{-1} for total phosphorus. Figs. 3 and 4 show the simulation results for total nitrogen and total phosphorus concentrations at downstream of Sandy Creek, and Figs. 5 and 6 show the simulation results for total nitrogen and total phosphorus concentrations at the outlet of the lake. The total simulation times for lake and stream are 3320 h and 1500 h, respectively. Since the velocity in the lake is much smaller than in stream, we observe advection process in a greater period of time. As seen from these graphs that the results for both stream and lake concentrations are in general good agreement. The MacCormack method catches the peaks slightly smaller than the implicit method in the lake, whereas a better match is observed in the stream. The grid sizes for stream and lake are choosen

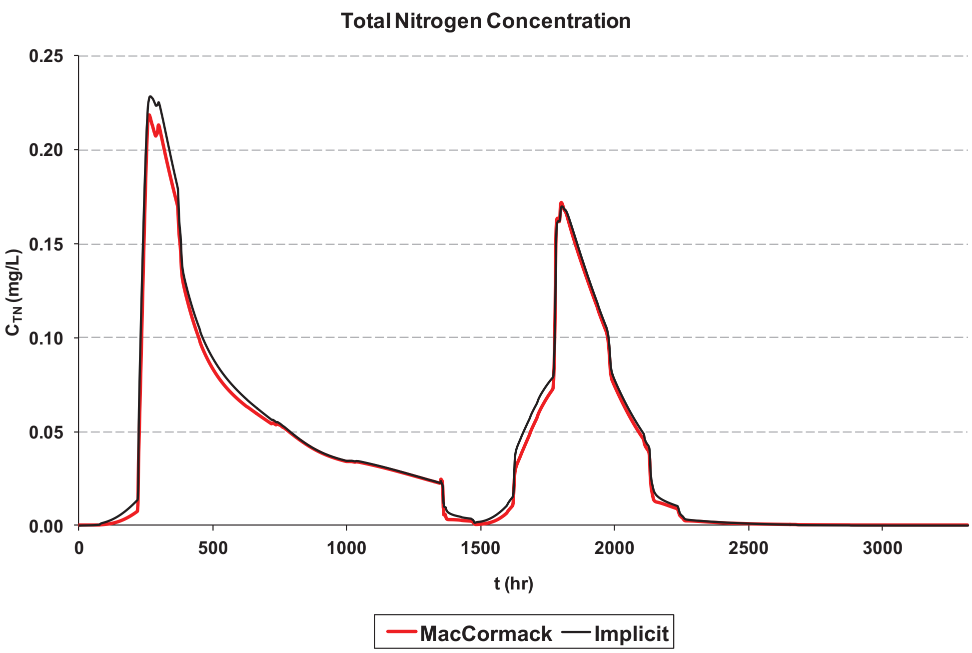

Fig. 5. Comparison of the MacCormack method with the implicit finite difference method for Total Nitrogen at the outlet of the lake of the Duke University wetland site.

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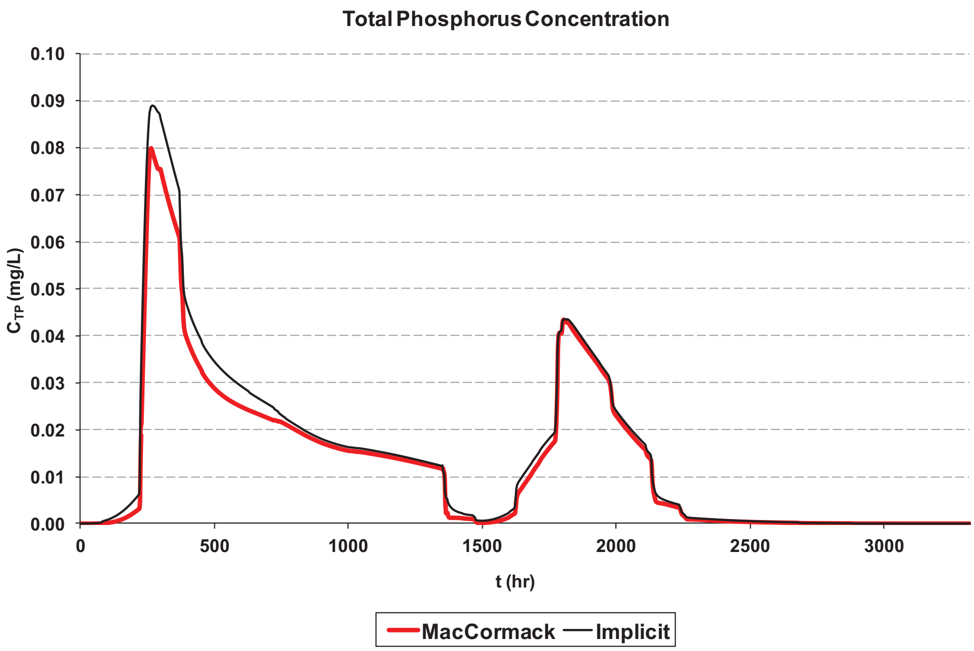

Fig. 6. Comparison of the MacCormack method with the implicit finite difference method for Total Phosphorus at the outlet of the lake of the Duke University wetland site.

as $\Delta x = 9.3$ m and $\Delta x = 15.1$ m, respectively. Although a time step $\Delta t = 1$ h was enough for routing through lake, a time step of $\Delta t = 0.005$ h has to be selected for routing through the stream in order to satisfy the CFL condition since both the velocity and dispersion coefficient of stream is much higher than the ones of the lake. For this reason, the analyses with MacCormack method took much longer time than the analyses with the implicit finite difference method. While the computation times for the MacCormack and implicit methods for the lake simulation are 0.35 s and 0.9 s, respectively, the computation times of the MacCormack and implicit methods for the stream simulation are 1.5×10^4 s and 1.6 s, respectively.

4. Conclusions

In this study, the MacCormack finite difference method to approximate the contaminant transport equations of WETSAND is derived. The water quality component of WETSAND involves advection-dispersion-reaction equations of the Nitrogen cycle, which are coupled equations, and Total Nitrogen and Total Phosphorus. Surface water/ground water interactions are also incorporated into these equations. The MacCormack method is implemented in the WETSAND model and examples, for both a hypothetical wetland section and a real site with measured data, solved in order to show the applicability of the MacCormack scheme. These results are compared to implicit finite difference solutions. The results show that the MacCormack scheme provides accurate results for both the hypothetical and real cases. While it is also computationally more efficient than the implicit method for the case with small velocity and dispersion coefficient (as is the case for wetlands and lakes), much longer computational times are needed for the cases with high velocity and dispersion coefficient values (e.g., in streams).

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References

[1] C.J. Richardson, Wetlands ecology, Encycl. Env. Biology 3 (1995) 535–550.
[2] W.J. Mitsch, J.G. Gosselink, Wetlands, Third Edition, Published by John Wiley, New York, 2000.
[3] C.J. Richardson, S. Qian, C.B. Craft, R.G. Qualls, Predictive models for phosphorus retention in wetlands, Wetlands Ecol. Manag. 4 (1997) 159–175.
[4] G.F. Mitchell, C.L. Hunt, Y.M. Su, Mitigating highway runoff constituents via a wetland, Soil Mechanics 2002 Transportation Research Record 2002 (1808) 127–133.
[5] C.M. Kazezyılmaz-Alhan, M.A. Medina Jr, C.J. Richardson, A wetland hydrology and water quality model incorporating surface water/groundwater interactions, Water Resources Res. 43 (4) (2007) W04434.

[6] G. Gurarslan, H. Karahan, D. Alkaya, M. Sari, Y. Mutlu, Numerical solution of advection-diffusion equation using a sixth-order compact finite difference method, Math. Problems Eng. 2013 (2013) 7 pages, Article ID 672936.
[7] M. Meenal, T.I. Eldho, Two-dimensional contaminant transport modeling using meshfree point collocation method(PCM), Eng. Anal. Boundary Elem. 36 (2012) 551–561.
[8] Q. Zhu, Q. Wang, J. Fu, Z. Zhang, New second-order finite difference scheme for the problem of contaminant in groundwater flow, J. Appl. Math. 2012 (2012) 13 pages Article ID 575493.
[9] V.A. Titarev, D. Drikakis, Uniformly high-order schemes on arbitrary unstructured meshes for advection-diffusion equations, Comput. Fluids 46 (2011) 467–471.
[10] V. Titarev, P. Tsoutsanis, D. Drikakis, WENO schemes for mixed-element unstructured meshes, Commun. Comput. Phys. 8 (3) (2010) 585–609.
[11] P. Tsoutsanis, V. Titarev, D. Drikakis, WENO schemes on arbitrary mixed-element unstructured meshes in three space dimensions, J. Comput. Phys. 230 (4) (2011) 1585–1601.
[12] D. Drikakis, M. Hahn, A. Mosedale, B. Thornber, Large eddy simulation using high-resolution and high-order methods, Philos. Trans. R. Soc. A, Math. Phys. Eng. Sci. 367 (2009) (1899) 2985–2997.
[13] P. Tsoutsanis, A.F. Antoniadis, D. Drikakis, WENO schemes on arbitrary unstructured meshes for laminar, transitional and turbulent flows, J. Comput. Phy. 256 (2014) 254–276.
[14] D. Drikakis, S. Tsangaris, On the solution of the compressible Navier–Stokes equations using improved flux vector splitting methods, Appl. Math. Model. 17 (1993) 282–297.
[15] R.W. MacCormack, Numerical solution of the interaction of a shock wave with a laminar boundary layer, Lecture Notes In Physics, 8, Springer-Verlag, 1971, pp. 151–163.
[16] D. Pantelakis, Th. Zissis, E. Anastasiadou-Partheniou, E. Baltas, Numerical models for the simulation of overland flow in fields within surface irrigation systems, Water Resour Manage 26 (2012) 1217–1229.
[17] C.M. Kazezyılmaz-Alhan, M.A. Medina Jr, Kinematic and diffusion waves: analytical and numerical solutions to overland and channel flow, J. *Hydr. Eng. ASCE* 133 (2) (2007) 217–228.
[18] D. Liang, R.A. Falconer, B. Lin, Comparison between TVD-MacCormack and ADI-type solvers of the shallow water equations, Adv. Water Res. 29 (2006) 1833–1845.
[19] D. Liang, R.A. Falconer, B. Lin, Coupling surface and subsurface flows in a depth averaged flood wave model, J. Hydrol. 337 (2007) 147–158.
[20] P. Porizkova, K. Kozel, J. Horacek, Numerical solution of compressible and incompressible unsteady flows in channel inspired by vocal tract, J. Comput. Appl. Math 270 (2014) 323–329.
[21] C. Yu, J.G. Duan, High resolution numerical schemes for solving kinematic wave equation, J. Hydrol. 519 (2014) 823–832.
[22] D. Liang, X. Wang, B.N. Bockelmann-Evans, R.A. Falconer, Study on nutrient distribution and interaction with sediments in a macro-tidal estuary, Adv. Water Res. 52 (2013) 207–220.
[23] G. Li, C.R. Jackson, Simple, Accurate, and efficient revisions to MacCormack and saulyev schemes: high peclet numbers, Appl. Math. Comput. 186 (2007) 610–622.
[24] D. Liang, X. Wang, R.A. Falconer, B.N. Bockelmann-Evans, Solving the depth-integrated solute transport equation with a TVD-MacCormack scheme, Environ. Model. Software 25 (2010) 1619–1629.
[25] N. Pochai, Numerical treatment of a modified MacCormack scheme in a nondimensional form of the water quality models in a nonuniform flow stream, J. Appl. Math. (2014) 16 pages, Article ID 274263.
[26] R.H Kadlec, R.L. Knight, Treatment Wetlands, CRC Press, Inc, 1996.
[27] M.H Chaudhry, Open-Channel Flow, Prentice Hall, Inc, Englewood Cliffs, New Jersey 07632, 1993.
[28] G.D. Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods, Oxford University Press, 1985.
[29] J.H. Ferziger, Numerical Methods for Engineering Application, John Wiley & Sons, Inc, 1998.
[30] C.J. Richardson, R. Elting, J. Pahl, G. Katul, Restoration of hydrologic and biogeochemical functions in bottomland hardwoods, in: in Hydrology and Management of Forested Wetlands: Proceedings of the International Conference. American Society of Agricultural and Biological Engineers, St. Joseph, Michigan, USA, 2006, pp. 50–57.

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