

# THE SHORT-RUN STABILITY OF THE FOREIGN EXCHANGE MARKET

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IN his frequently quoted attack on the free exchange market, Ragnar Nurkse claimed that: "Experience has shown that, apart from exchange control, the only effective means to prevent the disturbing exchange movements . . . is direct stabilization of the exchange market. . . ." <sup>1</sup> Free exchange markets, in his opinion, are inherently unstable.

Anticipatory purchases of foreign exchange tend to produce or at any rate to hasten the anticipated fall in the exchange value of the national currency, and the actual fall may set up or strengthen expectations of a further fall. . . . Exchange rates in such circumstances are bound to be highly unstable, and the influence of psychological factors may at times be overwhelming. <sup>2</sup>

Milton Friedman does not accept Nurkse's conclusion.

Despite the prevailing opinion to the contrary, I am very dubious that in fact speculation in foreign exchange would be destabilizing. . . . The widespread belief that speculation is likely to be destabilizing is doubtless a major factor accounting for the rejection of a system of flexible exchange rates in the immediate postwar period. Yet this belief does not seem to be founded on any systematic analysis of the available empirical evidence. <sup>3</sup>

A systematic analysis of the available empirical evidence must be conducted within the framework of a theory of short-run exchange rate determination. Using the short-run model developed by Stein, <sup>4</sup> we shall examine the

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<sup>1</sup> Ragnar Nurkse, *International Currency Experience* (League of Nations, 1944), 121.

<sup>2</sup> *Ibid.*, 118.

<sup>3</sup> Milton Friedman, "The Case for Flexible Exchange Rates," *Essays in Positive Economics* (Chicago, 1953), 175.

<sup>4</sup> Jerome L. Stein, "International Short-term Capital Movements," *American Economic Review*, LV (March 1965). A large scale econometric model designed to cope with this problem was developed by Rudolf R. Rhomberg, in "Canada's Foreign Exchange Market," *International Monetary Fund Staff Papers* (1960) and "A Model of the Canadian Economy," *Journal of Political Economy*, LXXII (1964). Rhomberg's model uses quarterly data and seeks to explain income, as well as exchange rates. One orientation is to explain weekly exchange rates and forward rates.

sterling-U.S. dollar market and the Canadian dollar-U.S. dollar market during the periods January 2, 1959 to April 26, 1962 and May 3, 1962 to October 1964. Our objective is to determine whether, in the absence of government stabilization operations, exchange markets tend to be unstable. Section I contains our analytical framework and section II contains our statistical analysis. Our main empirical conclusions are that: (1) The evidence strongly supports Friedman's view rather than Nurkse's view. Speculation does not tend to destabilize an otherwise stable market in the instances examined. (2) Interest rate policy is often a statistically significant means of defending an exchange rate. Section III contains several statistical experiments designed to predict the exchange rate in the short run. The maximum average absolute error was one-third of a cent, and the maximum standard deviation of error was two-fifths of a cent. These results are less impressive than they look, for they mainly reflect the correlation of the current price with a lagged price.

## I Analytical Framework

### A Short-Run Exchange Rate Determination

Only a brief summary of our analytical framework will be presented here. The interested reader may consult Stein <sup>5</sup> for a more detailed description of this model. For short-run equilibrium in the foreign exchange market, we require that the supply equal the demand for spot dollars and that the supply equal the demand for forward dollars. We consider the determination of the dollar price of foreign exchange, be it the pound or the Canadian dollar.

The supply of dollars is the stock of dollars existing at the end of the previous period  $S(t-1)$ , plus the basic balance deficit (or minus the basic balance surplus) of the United

<sup>5</sup> Jerome L. Stein, *op. cit.* However, the explicit solution of that dynamic model is one of the contributions of the present article.

States during the current period plus foreign official sales ( $T$ ) (or minus the purchases) of dollars during the period.<sup>6</sup> Let the basic balance surplus (+) or deficit (-) of the United States be a function of the dollar price of foreign exchange ( $p$ ) and the disturbance term ( $w$ ). The latter contains all the influences upon the basic balance not contained in  $p$ , and  $w$  is exogenous to our model.<sup>7</sup> If  $X$  is the basic balance of the United States, then

$$X = c_1 p + w. \quad (1)$$

Assume that, if the exchange rate is not at its stabilization limits,  $T$  is an exogenous variable. On the other hand, if the exchange rate is at one of the stabilization limits, the price of foreign exchange is given and  $T$  adjusts to absorb the excess supply or demand for dollars of the private sector.

The supply of dollars at period  $t$  is  $S(t)$ .

$$S(t) = S(t-1) + T - c_1 p - w. \quad (2)$$

There are two components of the demand for the stock<sup>8</sup> of dollars. First, there is the stock that is hedged in the forward market. Let  $x_1$  be the net covered yield on dollar assets. There are two components of  $x_1$ : The United States minus the foreign interest rate differential  $x_3$  and the forward premium (+) or discount (-) on the foreign currency  $x_2$ . The hedged demand for dollar assets is an increasing function of  $x_1 \equiv x_3 - x_2$ . Second, there is an unhedged demand for a stock of dollars which depends upon the expected appreciation of the dollar ( $p - p^*$ ) and the United States minus the foreign interest rate ( $x_3$ ). The expected appreciation (+) or depreciation (-) of the United States dollar is the difference between the current price  $p$  and the expected price  $p^*$  of foreign exchange. A substantive part of this paper will be concerned with the determination of  $p^*$ .

The demand for dollar assets  $D(t)$  is

$$D(t) = h(x_3 - x_2) + a_1(p - p^*) + a_3 x_3. \quad (3)$$

Each coefficient is positive. The term with the  $h$  coefficient refers to hedgers, and the terms with the  $a_i$  coefficients refer to non-hedgers.

<sup>6</sup> Assume that United States officials are passive and official foreign institutions buy and sell dollars to stabilize the exchange rate. A realistic function explaining  $T$  is introduced in subsection D of section I.

<sup>7</sup> The specification of the  $w$  function is discussed below.

<sup>8</sup> A very similar analysis results if  $S$  and  $D$  are flows. See Jerome L. Stein, *op. cit.*

Equilibrium in the spot market requires that  $S(t) = D(t)$ , and is equation (4).

$$(a_1 + c_1)p - h x_2 = W + a_1 p^* - (a_3 + h)x_3, \quad (4)$$

where  $W \equiv S(t-1) + T - w$  and  $x_3$  is exogenous.

Full equilibrium in the short run requires that the forward market also be in equilibrium. The stock of forward dollars supplied (+) or demanded (-) by hedgers is  $h(x_3 - x_2)$ . On the opposite side of the market will be "speculators" who take positions in the forward market for the sake of an expected profit. Their position depends upon the difference between the expected price of foreign exchange and the forward price of foreign exchange,  $p^* - p(1 + x_2)$ . If this term is positive, speculators desire to be short forward dollars, and if it is negative, they desire to be long forward dollars. Let  $G$  be the desired short (-) or long (+) positions of speculators in forward dollars.<sup>9</sup>

$$G = g_1(p - p^*) + g_2 x_2. \quad (5)$$

Equilibrium in the forward market required that the sum of all positions by speculators and hedgers combined must be zero.<sup>10</sup> Hence,

$$g_1 p + (g_2 + h)x_2 = h x_3 + g_1 p^* \quad (6)$$

is the equilibrium condition in the forward market.

Our model of short-run exchange rate determination consists of equations (4) and (6). The *independent* variables are the expected price ( $p^*$ ), the interest rate differential ( $x_3$ ), and a disturbance term  $W$  which contains official purchases or sales of dollars and those factors, other than the exchange rate, which affect the basic balance. The *dependent* variables are the dollar price of foreign exchange  $p(t)$  and the forward premium (+) or discount (-) on foreign exchange  $x_2(t)$ . Solving for  $p(t)$  and  $x(t)$ , we obtain:

$$p(t) = B_1 p^* + B_2 W - B_3 x_3 \quad (7)$$

$$x_2(t) = A_1 p^* - A_2 W + A_3 x_3, \quad (8)$$

where each coefficient is positive. In particular,  $B_1$  is less than unity.<sup>11</sup>

<sup>9</sup> Alternatively, the desired long or short position of speculators in forward foreign exchange is the dual of  $G$ .

<sup>10</sup> Whether an institution is classified as a hedger or a speculator is dependent on whether its behavior is described by the function  $h(x_3 - x_2)$  or  $a_1(p - p^*) + a_3 x_3$ .

<sup>11</sup>  $B_1 = \frac{[a_1(g_2 + h) + g_1 h]}{[(a_1 + c_1)(g_2 + h) + g_1 h]} < 1$ .

B *The Determinants of the Expected Price*

We consider several functions which may explain the determination of the speculators' price expectations ( $p^*$ ) and we examine their implications concerning the short-run stability of the foreign exchange market.<sup>12</sup> The empirical validity of the price expectation functions is examined in section II.

1) Constant coefficients of expectations

One simple theory of price expectations is that the expected price,  $p^*$ , is equal to a multiple of last period's price  $\lambda_1 p(t-1)$  plus a constant  $\lambda$  multiplied by the change in price that has occurred,  $\Delta p(t) = p(t) - p(t-1)$ . Thus,

$$p^* = \lambda_1 p(t-1) + \lambda \Delta p(t) \tag{9}$$

$$p^* = \lambda p(t) + \mu p(t-1), \text{ where } \mu \equiv \lambda_1 - \lambda. \tag{9'}$$

Equation (9) is formally equivalent to saying that the expected price is a linear combination of the current and lagged prices.

Equation (9') may also be derived from the assumption that the expected price is a linear combination of the current price and the change in price that has occurred:

$$p^* = \lambda_1 p(t) + \lambda_2 \Delta p(t).$$

We then derive

$$p^* = (\lambda_1 + \lambda_2) p(t) - \lambda_2 p(t-1).$$

An exponentially weighted expectations function, of past prices, yields similar results.

If (9') is substituted into (4) and (6), we obtain (10) and (11), respectively.

$$[a_1(1-\lambda) + c_1]p - hx_2 = W - (a_3 + h)x_3 + a_1\mu p(t-1). \tag{10}$$

$$g_1(1-\lambda)p + (g_2 + h)x_2 = hx_3 + g_1\mu p(t-1). \tag{11}$$

Solving for  $p(t)$ , the dollar price of foreign exchange at any time  $t$ , we obtain

$$p(t) = B_1' p(t-1) + B_2' W(t) - B_3' x_3(t), \tag{12}$$

<sup>12</sup> We shall not concern ourselves with hypothetical questions of the following sort: "If speculators followed rule R, what would have been their profits or losses?" Instead, we construct hypotheses concerning the functioning of exchange markets in the real world, and then subject these hypotheses to statistical tests. The difference between Nurkse and Friedman concerns the actual functioning of exchange markets and can only be resolved by explaining how the real world functions.

where

$$B_1' \equiv \frac{[a_1(g_2 + h) + hg_1]\mu}{(1-\lambda)[a_1(g_2 + h) + hg_1] + c_1(g_2 + h)}. \tag{13}$$

Coefficient  $B_1'$  will determine whether the dynamic system is stable. Coefficients  $B_2'$  and  $B_3'$  have the same numerators as  $B_2$  and  $B_3$  in (7), but they have the same denominator as  $B_1'$  in (13).

The forward rate at any time  $t$  is determined in a similar manner.

$$x_2(t) = A_1' p(t-1) - A_2' W + A_3' x_3(t), \tag{14}$$

where

$$A_1' = \frac{c_1 g_1}{J} \mu, \tag{15}$$

where  $J$  is the denominator in equation (13).

2) Intrinsic premium or discount

Some authors<sup>13</sup> have conjectured that there are two components of price expectations. First, there are constant coefficients of expectations as in equation (9). Second, the deviation of the forward rate from its interest parity  $x_1 \equiv x_3 - x_2$  is also a determinant of exchange rate expectations. When  $x_1$  is positive (i.e., the dollar is at an intrinsic premium), then speculators expect the dollar to appreciate. On the other hand, if  $x_1$  is negative (i.e., the dollar is at an intrinsic discount), then the dollar is expected to depreciate.

Formally, this hypothesis states that:

$$p^* = \lambda_1 p(t-1) + \lambda \Delta p(t) + s(x_2 - x_3), \tag{16}$$

where  $s(x_2 - x_3)$  is the effect of an intrinsic premium (+) or discount (-) on the forward foreign exchange upon exchange rate expectations.<sup>14</sup>

The resulting reduced form equations for  $p(t)$  and  $x_2(t)$  are very similar to (12) and (14), because the independent variables in the model are still  $p(t-1)$ ,  $x_3(t)$  and  $W$ . Although  $(x_3 - x_2)$  influences price expectations,

<sup>13</sup> In Rhomberg's early study (1960), he used the intrinsic premium or discount ( $x_1$ ) as a measure of speculative expectations. He nevertheless entertained doubts of the validity of this measure (p. 455, footnote 19). Whereas he treated  $x_1 \equiv x_3 - x_2$  as a pre-determined variable, we treat it as an endogenous variable. In his later study (1964),  $x_2$  was omitted from consideration.

<sup>14</sup>  $x_2 - x_3 \equiv$  intrinsic premium or net covered yield on the forward foreign exchange and  $x_3 - x_2 \equiv$  intrinsic premium on the forward dollar.

$x_2$  is a dependent variable which does not appear in the reduced form equations.

### C The Stability of the Exchange Market

Nurkse claimed that, in the absence of government purchases and sales of foreign exchange to stabilize the price, or of a gold standard, the exchange market is inherently unstable. In terms of our model he would claim that: if the interest rate differential  $x_3(t)$ , government purchases and sales of foreign exchange,  $T$ , and the export supply and import demand curves described by  $w$  are given, then the exchange rate  $p(t)$  would not converge smoothly and quickly to an equilibrium value. On the contrary, it would be subject to explosive forces: "Exchange rates . . . are bound to be highly unstable, and the influence of psychological factors may at times be overwhelming."

A systematic analysis of the available empirical evidence, insofar as it relates to Nurkse's contention, can be conducted within the framework of our dynamic model. The stability of the short-run foreign exchange market can be determined by solving equation (12) for  $p(t)$  and examining whether it converges to its equilibrium value. In effect, equation (12) implies that  $p(t)$  is determined by a second order difference equation. Using equation (12) to solve for  $p(t+1)$  and  $p(t)$ , we obtain<sup>15</sup> equation (17a). Primes will be dropped when no confusion is likely to occur.

$$p(t+1) - p(t) = B_1 p(t) - B_1 p(t-1) + B_2 [S(t) - S(t-1)], \quad (17a)$$

given  $x_3$  and  $(T-w)$ . The value of  $S(t) - S(t-1)$  is determined from equation (2). Making this substitution, we derive a second order difference equation (17b).

$$p(t+1) + [c_1 B_2 - (1 + B_1)] p(t) + B_1 p(t-1) = B_2 (T-w). \quad (17b)$$

The equilibrium value of  $p(t)$ , call it  $\bar{p}$ , is given by equation (18).

<sup>15</sup> From equation (12),  
 $p(t) = B_1 p(t-1) + B_2 S(t-1) + B_2 (T-w) - B_3 x_3$   
 and  
 $p(t+1) = B_1 p(t) + B_2 S(t) + B_2 (T-w) - B_3 x_3$   
 hence  
 $p(t+1) - p(t) = B_1 p(t) - B_1 p(t-1) + B_2 [S(t) - S(t-1)],$   
 assuming  $(T-w)$  and  $x_3$  are given.

$$\bar{p} = \frac{T-w}{c_1} = \frac{W'}{c_1}, \quad W' \equiv T-w. \quad (18)$$

At the equilibrium rate of exchange, there will be no change in the stock of dollars, as defined in equation (2). The basic balance minus official net sales of dollars will be zero. The interest rate differential does not affect the equilibrium rate of exchange, if it is not a determinant of  $(T-w)$ .<sup>16</sup> It does, however, affect the equilibrium forward rate  $\bar{x}_2$ .<sup>17</sup>

$$\bar{x}_2 = \frac{h}{h+g_2} x_3 - \frac{g_1}{h+g_2} (1-\mu-\lambda) \bar{p}. \quad (18a)$$

The equilibrium forward rate  $\bar{x}_2$  will not change by as much as the interest parity if  $g_2/h$  is positive, i.e., if the hedging function is not perfectly elastic with respect to the net covered yield<sup>18</sup> or the speculative function is not perfectly inelastic with respect to expected profits from futures speculation.<sup>19</sup>

Whether or not  $p(t)$  converges to its equilibrium value  $\bar{p}$  depends upon the roots of the characteristic equation (17c), derived from equation (17b).

$$x^2 + [c_1 B_2 - (1 + B_1)]x + B_1 = 0. \quad (17c)$$

If the roots of equation (17c) are complex conjugates, then the exchange rate will fluctuate in damped or anti-damped cycles. The cycles will be damped, i.e., the exchange rate  $p(t)$  will converge to its equilibrium value,  $\bar{p}$ , if, and only if, the value of  $B_1$  is less than unity.<sup>20</sup> If the roots are both positive, then  $p(t)$  will converge to  $\bar{p}$  if, and only if, the (absolute) value of  $B_1$  is less than unity and  $B_2$  is positive.<sup>21</sup> The latter condition,  $B_2 > 0$ ,

<sup>16</sup> This paradoxical result suggests that the equilibrium rate of exchange equilibrates the supply of and demand for the flow of foreign exchange. The interest rate differential on the other hand seems to affect the demand for a stock. See equation (19a) below for the general case where  $x_3$  affects  $w$  and thereby  $\bar{p}$ .

<sup>17</sup> In equation (11), let  $p(t) = p(t-1) = \bar{p}$  and solve for  $x_2$ .

<sup>18</sup> From equation (3),  $\partial D / \partial (x_3 - x_2) = h$ .

<sup>19</sup> From equation (5),  $\partial G / \partial (p - p^*) = g_1$ .

<sup>20</sup> See W. Baumol, *Economic Dynamics* (New York, 1959), 220. Empirically, there is no doubt that  $B_1$  is positive.

<sup>21</sup> Since  $B_1$  is positive, the product of the roots is positive. When there are two positive real roots, then stability will occur if the coefficient of  $x$  in the characteristic equation exceeds  $-(1+B_1)$ . That is  $c_1 B_2 - (1+B_1) > -(1+B_1)$  or  $c_1 B_2 > 0$ . Since  $c_1 > 0$ ,  $B_2$  must be positive. W. Baumol, *op. cit.*, 223. Since the sum of the two positive roots is less than unity, their product  $B_1$  must also be less than unity.

must be satisfied since  $B_2$  is the effect of an autonomous increase in the demand for foreign exchange upon the price of foreign exchange.

We considered two cases: either the exchange rate fluctuates or it exhibits monotonic movements.<sup>22</sup> Within our model, a value of  $B_1$  less than unity (in absolute value) is a necessary and sufficient condition for the stability of the foreign exchange market.

We shall postulate a function to explain  $T - w \equiv W'$  in equations (12) and (18). Variable  $W'(t)$  is negatively related to the basic balance of the United States. A tendency for imports to rise relative to exports, given the exchange rate, is associated with a rise in  $W'(t)$ . We hypothesize that  $W'(t)$  is a function of the ratio of the money stock in the United States to the money stock in the other country,  $M(t)$ , and a random term  $u'$ . Suppose that  $M(t)$  rose, i.e., the United States money supply rose at a faster rate than the "foreign" money supply. Then United States aggregate demand would have increased relative to the other country's aggregate demand. Regardless of the relative importance of price increases to output increases, United States imports would tend to rise faster than its exports — given the exchange rate. Either the supply of United States exports is reduced as a result of rises in domestic costs and prices, or the demand for United States imports is increased as a result of higher real incomes and rises in United States prices relative to foreign prices. Equation (19) can be accepted by either a Keynesian or a Quantity theorist.

$$W'(t) = mM(t) + u'. \quad (19)$$

Substitute (19) into (12) to obtain the predicted value of the dollar price of foreign exchange, equation (20). Notice that both the relative money stocks  $M(t)$  and the interest rate differential  $x_3(t)$  affect the exchange rate at any moment in time. An increase in aggregate demand which is not accompanied by

<sup>22</sup> There is a third case which is possible when  $B_1$ , the product of the roots, is positive — two negative real roots. We exclude this case from consideration since an explosive movement of  $p(t)$  would require that  $p(t)$  be below the equilibrium  $\bar{p}$  in one period and above it in the next period. These one period oscillations differ from fluctuations, and are not economically realistic. By concentrating on two cases, fluctuations and monotonic movements, we think that we have covered the economically relevant range of phenomena.

monetary expansion can be expected to raise the interest rate differential. It would tend to produce short-run appreciation of the dollar in this model.<sup>23</sup>

$$p(t) = B_1'p(t-1) + B_2''M(t) - B_3'x_3(t) + u, \quad (20)$$

where  $u$  is a randomly distributed disturbance term plus a function of last periods' stock of dollars.<sup>24</sup> The stability condition is unchanged, that is, the absolute value of  $B_1'$  must be less than unity for  $p(t)$  to converge to  $\bar{p}$ .

The difference between Nurkse and Friedman can be expressed precisely in terms of our model. Nurkse would claim that the absolute value of  $B_1'$  exceeds unity. This means that deviations of the exchange rate from its equilibrium value would increase in magnitude over time. To support this view, he adduced the history of the French franc from 1924 to 1926. Friedman, on the other hand, would claim that the absolute value of  $B_1'$  is less than unity. If the equilibrium rate of exchange,  $\bar{p}(t)$ , were constant, then  $p(t)$  would converge smoothly and quickly to it. However, the pursuit of inflationary finance is likely to produce an explosive movement of  $\bar{p}(t)$ , the equilibrium rate of exchange.<sup>25</sup> It is quite possible that the French foreign exchange market was stable, in the sense that the absolute value of  $B_1$  was less than unity. However, as a result of the monetization of the *bons de la défense nationale*,  $\bar{p}(t)$  was changing constantly.<sup>26</sup> Hence  $p(t)$

<sup>23</sup> Rhomberg found this to be the case for United States-Canadian case. See his 1964 article cited above, where he uses a 19-equation model.

The function  $W' \equiv T - w$  could also be assumed to be  $W'(t) = m_1M(t) - m_2x_3(t) + u'$ , (19a) to indicate the effect of interest rates upon income. Then the equilibrium rate of exchange would be

$$\bar{p} = [m_1M(t) - m_2x_3(t) + u'] / c_1. \quad (19b)$$

Equation (19) is a special case of equation (19a) with  $m_2 = 0$ .

<sup>24</sup> Equation (20) implies a second order difference equation, because of the inclusion of the  $S(t-1)$  term, with the same characteristic equation as (17b). We will use equation (20) for statistical estimation. Statistically both  $M(t)$  and  $S(t-1)$  should have trends. Since variable  $M(t)$ , and not  $S(t-1)$ , is contained in the general solution of our dynamic model, we decided to include  $M(t)$  and suppress  $S(t-1)$  from our statistical estimating equation.

<sup>25</sup> See equation (19b) in footnote 23.

<sup>26</sup> The French experience was quite complicated. The monetary authorities pegged some interest rates and stood ready to monetize the *bons de la défense nationale*. Speculative anticipations of the depreciation of the franc, in excess of the *bons* yield, induced Frenchmen not to renew the ma-

could have exhibited explosive behavior although the exchange market was stable.

The stability or instability of the short-run exchange market refers to the deviation between the actual price and the short-run equilibrium price  $\bar{p}(t)$ , as defined in equations (18) or (19b) above. If the market is stable, the deviation converges to zero. If the market is unstable, the deviation widens over time. What must be realized is that the stability or instability of the short-run exchange market cannot be inferred simply from the variance of the exchange rate during a given period of time. The crucial question, concerning the stability of the equilibrium rate in the absence of government intervention, is the absolute value of  $B_1$  in equation (20). If the absolute value of  $B_1$  is less than unity, we conclude that the market is stable. Any explosive movements that occur in the exchange rate result from movements in the short-run *equilibrium* rate of exchange (as defined in equations 18 or 19b). If  $B_1$  exceeds unity in absolute value, however, we conclude that the exchange market is unstable. In such a case, if the exchange rate does not exhibit explosive movements then the government purchases and sales of foreign exchange or its monetary-fiscal policy must have offset the other shocks.

*D Official Intervention to Smooth Out Exchange Rate Fluctuations*

Let us relax the assumption that  $T$ , foreign official sales (+) or purchases (-) of the United States dollars, is exogenous within the stabilization limits. Instead, assume that official purchases and sales (within the stabilization limits, if they exist) are related to the rate at which the exchange rate is changing. That is, the exchange authorities intervene to smooth out exchange rate fluctuations. For example, equation (21) may be a description of the official intervention:

turing *bons*. The monetary authorities were unable or unwilling to offset the maturing issues (and redemptions) with the sale of unpegged securities, since the required market rates of interest were higher than they were willing to pay. Hence the monetary authorities had to print new money. As Nurkse points out, the exchange rate and the money supply influenced each other. The chain of causation ran both ways. Mr. A. Grissa is currently writing a Ph.D. thesis at Brown University re-examining the stability of the French exchange market in the 1920's.

$$T = k[p(t - 1) - p(t)], \quad k > 0. \quad (21)$$

The official sales (purchases) of United States dollars may be proportional to the rate at which the United States dollar is appreciating (depreciating) relative to the foreign currency. When the dollar appreciates, the expression in brackets is positive, hence  $T$  is positive. On the other hand, when the dollar is depreciating the expression in brackets is negative and purchases of dollars occur ( $T < 0$ ).

Substitute equation (21) into equations (10) and (11) to derive equations (10a) and (11a) respectively.

$$[a_1(1 - \lambda) + c_1 + k]p(t) - hx_2 = (k + a_1\mu)p(t - 1) - (a_3 + h)x_3 + W \quad (10a)$$

$$g_1(1 - \lambda)p(t) + (g_2 + h)x_2 = g_1\mu p(t - 1) + hx_3, \quad (11a)$$

where  $W \equiv S(t - 1) - w$ .

Solving for  $p(t)$ , the dollar price of foreign exchange, we obtain

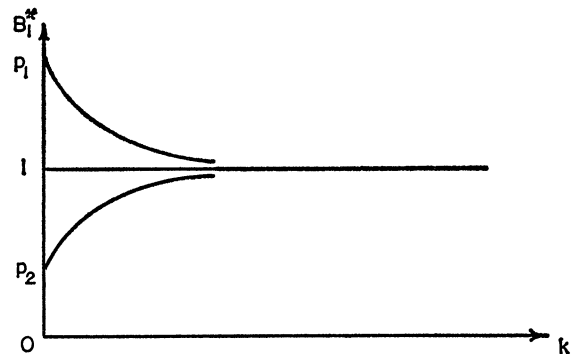
$$p(t) = B_1^*p(t - 1) + B_2^*W(t) - B_3^*x_3(t). \quad (12a)$$

The short-run stability of the foreign exchange market requires that the absolute value of  $B_1^*$ , given by equation (13a), be less than unity:

$$B_1^* = \frac{\mu[a_1(g_2 + h) + g_1h] + k(g_2 + h)}{(1 - \lambda)[a_1(g_2 + h) + g_1h] + c_1(g_2 + h) + k(g_2 + h)}. \quad (13a)$$

Coefficient  $B_1^*$  can now be considered a function of  $k$ , the sensitivity of government intervention with respect to exchange rate changes. This function is graphed in figure 1. The limit of  $B_1^*$ , as  $k$  grows indefinitely is unity and the value of  $B_1^*$  approaches this

FIGURE 1. — THE RELATION BETWEEN  $B_1^*$  AND THE MAGNITUDE OF THE STABILIZATION POLICY



limit monotonically and smoothly.<sup>27</sup> Intuitively, this result is sensible. If  $k$  were infinite, then official purchases would always tend to keep the current rate equal to the previous period's rate.

Suppose that  $B_1'$  were initially greater than unity, i.e., equal to  $OP_1$ . The introduction of government intervention, to smooth out exchange rate variations, will reduce the value of  $B_1^*$  below  $OP_1$ . However, it will never result in a value of  $B_1^*$  below unity. If the short-run exchange market were unstable, in the absence of government intervention, we would obtain an estimate of  $B_1'$  greater than unity. The introduction of official smoothing operations, as described by equation (21), will change the value of  $B_1'$  to  $B_1^*$ . Nevertheless, the estimate of  $B_1^*$  will still exceed unity. We conclude that if the private foreign exchange market is unstable in the absence of government intervention the coefficient  $B_1^*$  of  $p(t-1)$  will exceed unity in absolute value.

Similarly, if  $B_1'$  were less than unity ( $OP_2$ ) in the absence of government intervention to smooth out exchange rate fluctuations,  $B_1^*$  would also be less than unity. Our statistical tests of the stability of the exchange market remain valid even when there is official intervention to smooth out exchange rate movements.

## II Statistical Analysis

We examine the stability of the exchange rate in a free exchange market and in a stabilized exchange market, by estimating the value of  $B_1$  in equation (20). Part of the variation in the exchange rate results from changes in the *equilibrium* rate of exchange. This source of variation will be reflected in  $B_2M(t) - B_3x(t)$ . The effects of the exchange authorities' purchases and sales, and of random forces, are subsumed under variables  $u$  and  $B_1$ . Nurkse's view will be confirmed if  $B_1$  exceeds unity; Friedman's view will be confirmed if  $B_1$

$$\frac{\partial B_1^*}{\partial k} \text{ is positive if } B_1' < 1 \text{ and negative if } B_1' > 1.$$

Since the limit of  $B_1^*$  is unity, as  $k$  goes to infinity, we have the function graphed in figure 1. It is easy to see why  $\lim B_1^* = 1$ . Divide both the numerator and denominator of (13a) by  $k$  and let  $k \rightarrow \infty$ . We assume that  $B_1^*$  is positive on the basis of our statistical analysis. Moreover, assume that the denominator in (13a) is positive.

is less than unity. It appears that Nurkse's view is contradicted by the data.

### A *The Free United States Dollar — Canadian Dollar Market*

During the period January 2, 1959 through April 26, 1962, the Canadian dollar was not stabilized with respect to any other currency. Weekly data for  $p(t)$ ,  $p(t-1)$ , and  $x_3(t)$  were used,<sup>28</sup> and monthly data were employed to obtain the ratio of the United States to the Canadian money supply.<sup>29</sup> The resulting regression, describing the free exchange market, is equation (22). It is based on 173 observations.

$$\begin{aligned} \hat{p}(t) = & 0.9515p(t-1) + 0.1184M(t) \\ & (0.0182) \qquad (0.0507) \\ & - 0.0014x_3(t) + 0.0204, \qquad (22) \\ & (0.0005) \end{aligned}$$

with an  $R^2 = 0.99$  and a standard error of estimate of 0.0036. Each regression coefficient has the correct sign and is significantly different from zero at the one per cent level. The higher the ratio of the United States to the Canadian money supply, the higher the United States dollar price of the Canadian dollar.<sup>30</sup> The higher the United States interest rate relative to the Canadian interest rate, the lower the United States dollar price of the Canadian dollar.<sup>31</sup> A problem of multi-collinearity exists among the variables. The zero order correlation between the ratio of the United States to the Canadian money supply ( $M$ ) and the United States minus the Canadian interest rate differential ( $x_3$ ) is  $-0.45$ . However, this negative correlation is what is expected on theoretical grounds. All that multi-collinearity means, for our purposes is that we

<sup>28</sup> Exchange rates were taken from the *Bank and Quotations Record* and the interest rate differentials were taken from Samuel I. Katz, "Yield Differentials in Treasury Bills, 1959-64," *Federal Reserve Bulletin* (Oct. 1964). Forward rates also came from Katz's paper. Forward rates and interest rates are measured in percentages per annum.

<sup>29</sup> The data came from the International Monetary Fund, *International Financial Statistics*. A monthly observation is associated with each of its component weeks. The United States money supply is measured in billions of United States dollars, the Canadian money supply is measured in ten millions of Canadian dollars and the United Kingdom money supply is measured in ten millions of pounds.

<sup>30</sup> The zero order correlation between  $p(t)$  and  $M(t)$  was 0.89.

<sup>31</sup> The zero order correlation between  $p(t)$  and  $x_3(t)$  was  $-0.45$ .

really do not know how much of the variation in  $\bar{p}(t)$  is due to  $M(t)$  and how much is due to  $x_3(t)$ . Since both  $M(t)$  and  $x_3(t)$  may be contained in our short-run equilibrium rate of exchange,<sup>32</sup>  $\bar{p}(t)$ , the multi-collinearity problem does not prevent us from distinguishing between the forces which affect the short-run equilibrium rate and the forces affecting the deviation of the actual from the equilibrium rate.

Let us turn to our main question: is  $B_1$  significantly different from unity? The estimate of  $B_1$  is 0.9515. Hence  $t = \frac{1 - 0.9515}{0.0182} = 2.66$ . We may reject the hypothesis that  $B_1$  is unity, at the one per cent level. Nurkse's view is inconsistent with the data, whereas Friedman's view is consistent with the experience of the free Canadian exchange market during 1959 to 1962.

Obviously, the current price of foreign exchange is highly correlated with its lagged price. Consequently, the results of equation (22) do not reveal whether our price expectations function (equation 9') is valid. To test the validity of our expectations function we must consider equation (14), which claims that the forward rate is related to the lagged price of foreign exchange. That such a relation exists is not obvious on statistical grounds.<sup>33</sup> Let  $x_2(t)$  = the premium (+) or discount (-) on the Canadian dollar, measured in percentage per annum. Initially  $x_2(t)$  was regressed on  $\bar{p}(t-1)$ ,  $x_3(t)$ , and  $M(t)$ . Since the coefficient of  $M(t)$  was not statistically significant, it was eliminated, and  $x_2(t)$  was regressed on  $\bar{p}(t-1)$  and  $x_3(t)$ . The results are given in equation (23).

$$x_2(t) = \begin{matrix} -5.4233\bar{p}(t-1) & + & 0.7138x_3(t) \\ (0.9239) & & (0.0457) \end{matrix} + 5.4809 \quad (23)$$

with  $R^2 = 0.72$ . The lagged price and the interest parity explain more than 70 per cent of the variance in the forward rate.<sup>34</sup> What is of

<sup>32</sup> Equation (19b) says  $\bar{p}(t) = \frac{m_1M(t) - m_2x_3(t) + u}{c_1}$ .

<sup>33</sup> If  $\bar{p}(t-1)$  is close to  $\bar{p}$  and  $x_2(t)$  is close to  $\bar{x}_2$ , then equation (23) may be an approximation of equation (18a). The fact that the coefficient of  $\bar{p}(t-1)$  is negative suggests that  $(1 - \mu - \lambda)$  is positive.

<sup>34</sup> For a detailed critique of the interest parity theory see Jerome L. Stein, "The Forward Rate and The Interest Parity," *Review of Economic Studies* (1965).

interest to us, is that the lagged price is a significant variable in equation 23. This means that there is a presumption that our price expectations function (9') is valid and that the statistically significant coefficient of the lagged price is not just a statistical phenomenon.<sup>35</sup>

### B *The Stabilized United States Dollar — Canadian Dollar Market*

From May, 1962, the price of the Canadian dollar was stabilized relative to the United States dollar. Our problem is to determine whether the exchange market tended to be unstable in the absence of official counter-measures. It is theoretically possible that  $B_1$  exceeded unity, i.e., the exchange rate tends to move away from its short-run equilibrium value in the manner described by Nurkse, but the Monetary Authorities intervened by (a) purchasing or selling foreign exchange and (b) engaging in open market operations to affect interest rates and the money supply. In this manner, the Monetary Authorities acted to prevent an inherently explosive market from exploding. To answer our question, whether the market was inherently unstable, we repeat the statistical analysis of section IIA on the stabilized Canadian market, May 3, 1962 to October 9, 1964. There were 127 observations in this series.

We regressed the United States dollar price of the Canadian dollar  $p(t)$  on the lagged price  $p(t-1)$ , the ratio of the United States to the Canadian money stocks  $M(t)$  and the United States minus the Canadian Treasury bill differential. The problem of multi-collinearity between  $M(t)$  and  $x_3(t)$  is quite strong during this period. The zero order correlation between  $M(t)$  and  $x_3(t)$  was  $-0.70$ . In fact, the regression coefficient of  $M(t)$  has the wrong sign. We therefore dropped  $M(t)$  and regressed  $p(t)$  upon  $p(t-1)$  and  $x_3(t)$  in equation (24).

<sup>35</sup> There appear to be several possibilities for the relation between  $x_2(t)$  and  $\bar{p}(t-1)$ . First, there are equations (14) and (15) which claim that the coefficient of  $\bar{p}(t-1)$  is  $c_1g_1\mu/J$ , which should be positive. Second, there is equation (18a) which claims that

$$\bar{x}_2 = \frac{h}{h+g_2}x_3 - \frac{g_1}{h+g_2}(1-\lambda-\mu)\bar{p}.$$

Hence if  $\bar{p}(t-1)$  is close to  $\bar{p}$  and  $x_2(t)$  is close to  $\bar{x}_2$ , the relation between  $x_2(t)$  and  $\bar{p}(t-1)$  will be given by equation (18a). The sign of the coefficient of  $\bar{p}(t-1)$  would depend on the sign of  $(1 - \mu - \lambda)$ .



$$\begin{aligned}
 p(t) = & 0.8754p(t-1) - 0.0004x_3(t) \\
 & (0.0416) \quad (0.0001) \\
 & + 0.1152 \quad (24)
 \end{aligned}$$

with  $R^2 = 0.79$ .

The interest rate differential is statistically significant. A rise in the interest rate differential in favor of the United States tends to lower the United States dollar price of the Canadian dollar.

We now ask whether our estimate of  $B_1 = 0.8754$  is significantly less than unity. The difference between 0.8754 and unity is three times the standard error.

$$t = \frac{1 - 0.8754}{0.0416} = 3.$$

We therefore reject the Nurkse hypothesis that the exchange market is inherently unstable.

Next, we examine whether the lagged price is associated with the forward rate. According to our expectations function, the reduced form equation for the forward rate should contain the lagged price as well as the interest rate differential. We therefore regressed the forward premium (+) or discount (-) on the Canadian dollar,  $x_2$ , on the lagged United States dollar price of the Canadian dollar,  $p(t-1)$ , the ratio of the United States to the Canadian money stocks,  $M(t)$ , and the United States minus the Canadian Treasury bill differential,  $x_3(t)$ . As we mentioned earlier, the zero order correlation between  $M(t)$  and  $x_3(t)$  was  $-0.70$ , hence there was no clear way of distinguishing between the influence of  $M(t)$  and the influence of  $x_3(t)$ . The regression coefficient of  $M(t)$  was not significant, so we eliminated  $M(t)$  and regressed  $x_2(t)$  on  $p(t-1)$  and  $x_3(t)$  in equation (25).

$$\begin{aligned}
 x_2(t) = & -22.6916p(t-1) + 0.9056x_3(t) \\
 & (5.8084) \quad (0.0199) \\
 & + 21.1448 \quad (25)
 \end{aligned}$$

with  $R^2 = 0.94$ .

Both the lagged price and the interest differential are statistically significant at the one per cent level. Again, our expectations function seems to be consistent with the data.

### C The Dollar-Pound Market: 1959 to 1964

Was the stabilized dollar-pound market inherently unstable? If the United Kingdom monetary authorities did not stabilize the exchange market or engage in an interest rate

policy designed to keep the pound within the \$2.78 to \$2.82 limits, would the market have been unstable for a given level of aggregate demand? The answer depends upon the estimated value of  $B_1$ .

#### 1) 1959 to 1962:

Regress  $p(t)$  upon  $p(t-1)$ ,  $M(t)$  which is the ratio of the United States to the United Kingdom money stocks, and  $x_3(t)$  which is the United States minus the United Kingdom Treasury bill differential. The resulting regression is equation (26). The dependent variable is the dollar price of the pound. There were 173 observations in this series.

$$\begin{aligned}
 p(t) = & 0.8589p(t-1) + 0.2631M(t) \\
 & (0.0279) \quad (0.0496) \\
 & - 0.0009x_3(t) + 0.3559, \quad (26) \\
 & (0.0001)
 \end{aligned}$$

with  $R^2 = 0.94$ . Each coefficient was significantly different from zero at the one per cent level. There was a positive correlation between  $M(t)$  and  $x_3(t)$  which could conceivably be interpreted as follows: As the United Kingdom money supply rose (relative to that of the United States), the value of the pound fell. This induced the monetary authorities to raise interest rates to defend the pound. But regardless of the interpretation of the positive correlation between  $M(t)$  and  $x_3(t)$ , the fact remains that a rise in the United States/United Kingdom money supply raises the dollar price of the pound, and a rise on the United States minus the United Kingdom Treasury bill rate lowers the dollar price of the pound.

The stability of the exchange market depends upon the value of  $B_1$ . Our estimate of  $B_1 = 0.8589$  is significantly less than unity.

$$t = \frac{1 - 0.8589}{0.0279} = 5.03.$$

We may reject the Nurkse hypothesis that the exchange market was unstable. Given the short-run equilibrium rate of exchange  $\bar{p}(t)$ , the actual rate would converge to it.

As a further test of the validity of our expectations hypothesis, we regressed the forward premium (+) or discount (-) on the pound on the lagged dollar price of the pound  $p(t-1)$ , the United States/United Kingdom money stock  $M(t)$  and the United States minus the United Kingdom Treasury bill rate  $x_3(t)$ .

The money stock variable was not statistically significant, so it was eliminated and  $x_2(t)$  was regressed on  $p(t-1)$  and  $x_3(t)$  in equation (27).

$$x_2(t) = 22.4140p(t-1) + 0.8685x_3(t) - 62.8702. \quad (27)$$

(6.6224)                      (0.0299)

The coefficient of the lagged dollar price of the pound is statistically significant, but it has a positive sign instead of the negative sign found for Canada.<sup>36</sup>

## 2) 1962 to 1964

There are three noteworthy features of the United States-United Kingdom experience from May 1962 to October 1964. First, the zero order correlation between the ratio of the United States to the United Kingdom money supply and the United States minus the United Kingdom Treasury bill differential was 0.22. This positive relation was also observed during the 1959-1962 period. Second, in a regression of the price of the pound on  $p(t-1)$ ,  $M(t)$ , and  $x_3(t)$ , the interest rate variable was not significant during the 1962-1964 period although it was highly significant during the 1959-1962 period. Possibly the small variation in the interest rate variable during the later period contributed to the failure to observe the negative relationship between  $p(t)$  and  $x_3(t)$ . This lack of significance should not be interpreted as an indication that monetary policy is ineffective in affecting the exchange rate. The forward rate was significantly related to the interest rate differential. (See equation 28.) Consequently, there is reason to believe that the interest rate differential affects capital flows.

$$x_2(t) = 32.666 - 13.8686p(t-1) + 42.4553M(t) + 0.5098x_3(t). \quad (28)$$

(3.7298)                      (9.1997)                      (0.0601)

Third, in contrast to the other three cases examined, the estimate of  $B_1$  was not significantly less than unity. Next,  $p(t)$  was regressed on  $p(t-1)$  and  $M(t)$  to obtain equation (29). The interest rate variable was

excluded from this regression because it was not statistically significant.

$$p(t) = 0.0601 + 0.9735p(t-1) + 0.0974M(t) \quad (29)$$

(0.0198)                      (0.0466)

with an  $R^2 = 0.97$  and a standard error of 0.0011. Each coefficient has the correct sign and is significantly different from zero at the five per cent level. The estimate of  $B_1$  is 0.9735. Although it is less than unity, it is not significantly below unity at the five per cent level. The value of

$$t = \frac{1 - 0.9735}{0.0198} = 1.3.$$

The data tend to support Friedman's view, but Nurkse's view is not contradicted by the data at the five per cent level.

## E Summary of the Statistical Analysis

1) Our quest was to determine the stability of the short-run foreign exchange market. Theoretically, the rate would be stable if  $B_1$  were less than unity, and it would be unstable if it exceeded unity. To estimate  $B_1$  we regressed the exchange rate upon its lagged value, the ratio of money stocks in the two countries and the Treasury bill differential. The regression coefficient of the lagged price was our estimate of  $B_1$ . In three out of four cases we were able to reject the hypothesis that the exchange market tended to be unstable. Nurkse's influential view was contradicted by the data. In the fourth case, we found  $B_1$  to be less than unity, but could not reject Nurkse's hypothesis.

2) It should be noted that the introduction of the lagged price term  $p(t-1)$  into equations (22), (24), (26), and (29), reduced the auto-correlation of the residuals. The Durbin-Watson statistics were 1.53, 1.87, 1.48, and 2.15 respectively. Hence we do not think that the measured standard errors of  $B_1$  are serious underestimates of the true standard errors.

3) To be sure, the Canadian foreign exchange policy was not uniform during the free exchange market period January 2, 1959 to April 26, 1962. The Ministry of Finance announced, on June 20, 1961, that there would be large and continued use of gold and dollar re-

<sup>36</sup> Consider a situation which is not close to equilibrium. Then the coefficient of  $p(t-1)$  is  $\frac{c_1 g_1}{J} \mu$  which is positive. See equations (14) and (15).

serves to manipulate the exchange rate. It seems reasonable to expect large scale speculation as a result of the official attempt to establish a new equilibrium rate  $\bar{p}$ .<sup>37</sup> Did the attempt to establish a new equilibrium rate, via official intervention, tend to destabilize the exchange market?

In connection with the prediction experiments (described in section III), we examined the sub-period September 1960 to May 1962, wherein there was official manipulation of the exchange rate. The regression with the best fit contained the lagged price and the net covered yield as independent variables.<sup>38</sup> In that regression, the coefficient of  $B_1$  was significantly below unity. We never obtained a value of  $B_1$  equal to, or greater than, unity in any of our regressions. These results lead us to conclude that Nurkse's view is not supported by the data.<sup>39</sup>

4) To assess the relative importance of  $p(t-1)$ ,  $M(t)$ , and  $x_3(t)$  in explaining  $p(t)$ , in the reduced form equation (20), we present the beta coefficients of these variables. These coefficients estimate the change in the exchange rate from its mean measured in standard deviations, that would occur as a result of a change in the other variable from its mean, by one standard deviation. The beta coefficients are the regression coefficients of the normalized variables. As a result of the correlation between  $M(t)$  and  $x_3(t)$ , it is not possible to determine with any precision the importance of  $M(t)$  relative to  $x_3(t)$ . However, our interest is in comparing the beta coefficient of  $M(t)$  or  $x_3(t)$  with that of  $p(t-1)$ . This is done in table 1.

Clearly, the lagged price is the most significant variable in explaining the exchange rate. However, the interest rate differential does exert a powerful effect. For a one standard deviation change in the Treasury bill differential (from its mean), the exchange rate changes by one-fifth of a standard deviation

<sup>37</sup> From equation (18),  $\bar{p} = \frac{T-w}{c_1}$  where  $T$  is official sales (+) or purchases of United States dollars.

<sup>38</sup> During this period, we did not include  $M(t)$  as an independent variable, because of our interest in predicting weekly changes in the exchange rate.

<sup>39</sup> When  $\Delta p(t)$  is regressed on  $\Delta p(t-1)$ ,  $\Delta x_3(t)$ , and  $\Delta M(t)$ , the coefficient of  $\Delta p(t-1)$  is very significantly below unity in every period examined.

TABLE 1. — BETA COEFFICIENTS OF LAGGED PRICE, RELATIVE MONEY STOCKS, AND TREASURY BILL DIFFERENTIAL: UNITED STATES, CANADA, AND UNITED KINGDOM, 1959-1964

	$\beta p(t-1)$	$\beta M(t)$	$\beta x_3(t)$
1) United States-Canada, free exchange market	0.9433 <sup>c</sup>	0.0426 <sup>a</sup>	-0.0288 <sup>a</sup>
2) United States-Canada, stabilized market	0.8678 <sup>c</sup>	-0.1682 <sup>b</sup>	-0.2243 <sup>a</sup>
3) United States-United Kingdom, 1959-1962	0.8524 <sup>c</sup>	0.1945 <sup>a</sup>	-0.1996 <sup>a</sup>
4) United States-United Kingdom, 1962-1964	0.9606 <sup>a</sup>	0.0409 <sup>a</sup>	not significant

<sup>a</sup> Significantly different from zero at the five per cent level.

<sup>b</sup> Wrong sign.

<sup>c</sup> Significantly different from unity at the five per cent level (non-normalized variable).

(from its mean), in lines 2 and 3 of table 1. There is good reason to believe that interest rate policy is effective in defending an exchange rate.

### III Exchange Rate Prediction

What is the best way to predict *weekly* changes in the exchange rate? According to equation (20), the independent variables should be the lagged price  $p(t-1)$ , the Treasury bill differential,  $x_3(t)$ , and the relative money supplies,  $M(t)$ . For our prediction experiments, we modified our regression equation. First, we eliminated the relative money stocks since the data are obtained monthly and they do not exhibit much variation from week to week. Second, we included the net covered yield  $x_1' \equiv x_2 - x_3$  as an independent variable.<sup>40</sup> Theoretically,  $x_1'$  is an endogenous variable which should not be contained in the reduced form equation. However, we were curious to discover whether  $x_1'$  was a significant variable in explaining  $p(t)$ . Third, we added two price expectations variables: weighted percentage changes in price from  $t-2$  to  $t-1$  and from  $t-3$  to  $t-2$ . In the case of the free Canadian market the weights were unity. In the case of the stabilized Canadian or United Kingdom market, the weights were the (positive) difference between the more recent price and the upper (lower) stabilization limit if the price were rising (falling). Formally, these last variables were:<sup>41</sup>

<sup>40</sup>  $x_1'$  is the intrinsic premium (+) or discount (-) on the foreign currency. It is the negative of  $x_1$ .

<sup>41</sup> The  $10^4$  was introduced for computational convenience.

$$E(I) = \frac{p(I) - p(I-1)}{p(I-1)} \cdot 10^4 \cdot |p(I) - L|.$$

where  $L$  = lower United States dollar price stabilization limit if  $p(I)$  is less than  $p(I-1)$ ,  $L$  = upper United States dollar price stabilization limit if  $p(I)$  exceeds  $p(I-1)$ , and  $I = t-1, t-2$ .

The resulting regression equation is equation (30).

$$p(t) = b_0 + b_1p(t-1) + b_2x_1'(t) + b_3x_3(t) + b_4E(t-1) + b_5E(t-2). \quad (30)$$

We consider three series: (a) The first 85 observations of the period of the free Canadian exchange market, January 2, 1959 through September 1, 1960; (b) the first 62 observations of the period of the stabilized Canadian market beginning May 24, 1962; and (c) The United States-United Kingdom exchange market, January 1959 to May 1962. Table 2 contains the results of the regressions, and we indicate which coefficients were significant and which were not.

The lagged price  $p(t-1)$  and the Treasury bill differential  $x_3(t)$  are always statistically significant at the one per cent level. The net covered yield  $x_1'$  is never significant. It has no place in a reduced form equation predicting the exchange rate on theoretical grounds. The percentage change in price from  $t-2$  to  $t-1$  was significant in two out of the three cases and had the correct sign. But the per cent change in price from  $t-3$  to  $t-2$  was never significant.

TABLE 2.—RESULTS OF "FIRST PERIOD" REGRESSIONS

	United States-United Kingdom 1959-1962	Free United States Canada First 88 Observations	Stabilized United States Canada First 65 Observations
$p(t-1)$ , lagged price $x_3(t)$ , United States minus foreign Treasury bill differential	0.9730 <sup>a</sup>	0.9378 <sup>a</sup>	0.8692 <sup>a</sup>
$x_1(t)$ net covered yield (+) favor foreign	0.0001	-0.0005	0.0005
$E(t-1)$ per cent change in price ( $t-2$ ) to ( $t-1$ )	0.0031 <sup>a</sup>	0.0022 <sup>a</sup>	-0.0017
$E(t-2)$ per cent change in price ( $t-3$ ) to ( $t-2$ )	-0.0004	-0.0014	0.0005

<sup>a</sup> Significantly different from zero at the five per cent level.

The exchange rate was then regressed on the *significant* (five per cent) variables in each column in table 2.

For the United Kingdom-United States experience January 1959 to May 1962, we obtained regression equation (31).

$$p(t) = 0.0677 - 0.0003x_3(t) + 0.9758p(t-1) + 0.0030E(t-1). \quad (31)$$

This regression equation, and the actual values of  $x_3(t)$ ,  $p(t-1)$ , and  $E(t-1)$ , were used to predict the exchange rates for the period May 1962 to October 1964.

For the free Canadian market, January 1959 to September 1960, we obtained regression equation (32).

$$p(t) = 0.0553 - 0.0004x_3(t) + 0.9456p(t-1) + 0.0019E(t-1). \quad (32)$$

This regression equation, and the actual values of  $x_3(t)$ ,  $p(t-1)$ , and  $E(t-1)$  were used to predict the exchange rates for the second half of the free exchange market period — September 1960 to May 1962.

Similarly, for the stabilized Canadian market, May 4, 1962 to August 2, 1963, we obtained regression equation (33).

$$p(t) = 0.1356 + 0.8532p(t-1) - 0.0005x_3(t). \quad (33)$$

This regression equation, and the actual values of  $x_3(t)$  and  $p(t-1)$ , were used to predict the exchange rate from August 1963 to October 1964.

Table 3 presents the results of these predictions.

TABLE 3.—RESULTS OF PREDICTION EXPERIMENTS

	United States-United Kingdom, May 1962-October 1964	Free Canadian second part of period	Stabilized Canadian second part of period
average error <sup>a</sup>	-\$0.0003	-\$0.0029	\$0.0003
average absolute error	0.0007	0.0035	0.0006
standard deviation of error	0.0010	0.0042	0.0007

<sup>a</sup> Error = actual minus expected. The maximum average absolute error was one-third of a cent and the maximum standard deviation of error was two-fifths of a cent.

Separate inspection of these three periods reveals that it is the correlation of  $p(t)$  with  $p(t-1)$ , rather than with the other variables, which enabled us to predict price. In the case of the United Kingdom, we know from equation (20) that the relative money stocks

constituted a significant explanatory variable. But  $M(t)$  was excluded from our prediction experiments. With respect to Canada, the interest rate differential was a significant variable in the first parts of the free and the stabilized market periods (see table 2). However, in the second parts of these periods, it either was not significant or had the wrong sign.