Optimal Stress Tests in Financial Networks

by

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Dissertation submitted in partial fulfillment of the
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ABSTRACT

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Abstract

Bank stress test has become a centerpiece of post crisis bank supervision. Current studies have thus far examined the optimal policies on stand-alone single banks, but financial systems are interconnected in practice, and disclosure about banks influences the counterparty risks of other banks. This dissertation studies the optimal stress test design in a financial network, where banks' endogenous default outcomes are determined by a fixed point payment problem that accounts for both project qualities and interbank contagion.

The first part examines the joint stress test design on all banks in the financial network. In addition to the cross-state risk sharing in models of single banks, this model highlights the novel cross-bank risk sharing that arises from the spillover effects of disclosures via interbank payments. When expected bank profitability is high or counterparty exposures are large, disclosure is non-discriminatory, and either all banks pass or all banks fail the stress tests; otherwise only less impaired banks may pass. For network structures, I find: (i) in a ring network, banks at least a specific distance away from the nearest bank with asset impairment may pass; (ii) a more connected network is not necessarily more stable under the optimal disclosure; (iii) typically more connected banks receive preferred treatment.

The second part studies a selective stress test in a financial network, where the regulator selects an optimal subset of banks for stress tests and accordingly design the optimal disclosures only on these banks. Compared with the first part, systemic risk becomes more important as the regulator is less able to fine tune beliefs about contagion and needs to contain the risks from unselected banks. For network structures, I find: (i) in a ring network, stress test is either "balanced" on banks positioned evenly and disclosure is non-discriminatory, or on "connected" banks and disclosure is truth-telling on potential shocks; (ii) in a star network, stress test is conducted on the center bank when counterparty exposure is either sufficiently small or sufficiently large.

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Finally, I would like to thank my parents, for their unconditional love and support.

Chapter 1

Introduction

Disclosing information about bank qualities is crucial in resolving a bank crisis. Stimulated by the great recession, bank supervisors around the world conduct periodic bank stress tests and publicly disclose the test results to restore market confidence regarding the banking system.[1] In contrast to direct financial support to troubled financial institutions ex post, stress testing influences market beliefs, and as a result there is a growing literature that studies the optimal informativeness of stress testing to persuade market participants to act in a way that best enhances financial stability.[2]

However, this literature thus far examines the stress test design of stand-alone single banks and remains essentially micro-prudential; it focuses on shocks to banks' own fundamentals rather than on the broader macroprudential question of how stress might be transmitted among banks. This is in sharp contrast to the 2008 financial crisis, which vividly demonstrated the significance of interbank contagion among systematically important financial institutions (SIFIs). The literature of financial network, as a collection of nodes and links to respectively represent banks and interbank connections, hence ensues. Specifically, a strand of network theories study the contagion cascades resulting from ex post shocks to specific banks, which then trigger further failures via exogenous direct links of interbank payments.

This dissertation studies the joint stress test design on the financial system as

[1]For a detailed review of the history and goals of US stress testing, see [HL15].

[2]For stress test design in an information design approach to influence market beliefs, see [GL18]; [FeCMP16]; [LW17]; [OZS18]; [IP18]; [Ino19]; and others. [PP18], on the other hand, study how the regulator should design stress scenarios to learn about banks' exposures to risk factors.

a whole, where banks are connected in an exogenous financial network. A cash-constrained regulator facing a market with low prior can commit to a disclosure policy to influence the beliefs of the financial market and thus the interim refinancing opportunities of banks to best enhance system stability.

Building information on top of financial networks is challenging. The spillover of how disclosure about one bank influences people think about the counterparty risks on the other banks is determined by a fixed point problem of interbank payments. In addition, there is a new dimension of coordinating banks by reporting the same signals. To the best of my knowledge, this is the first research to address how stress test design accounts for interbank contagion. On the other hand, it is of direct policy relevance to study the optimal information structure at the system level of a financial network; for instance, in 2017, the "Counterparty Default Component" was added by the Federal Reserve to the stress tests for eight financial institutions, which assesses the bank's financial health assuming the default of its largest counterparty.

Chapter 2 examines the case where all banks in the financial network are required for stress tests. Chapter 3 is motivated by the concern that conducting stress testing is costly in practice and studies a selective stress test. Specifically, the regulator selects an optimal subset of banks for stress tests and accordingly design the optimal disclosures only on these banks. The literature thus far takes the set of stress test banks as given, as stand-alone bank models where spillover is absent prove to be an unsuitable framework.

In addition to the cross-state risk sharing in models of single banks, this dissertation highlights the novel cross-bank risk sharing that arises from the spillover effects of disclosures via interbank payments. For the cross-state risk sharing of each bank, the regulator takes into account the spillovers that passing a bank may pull other banks over the fence. The trade off is, how passing the bank at a state enhances system stability, versus how impaired the bank is at the state, which determines the magnitude of belief deterioration. Cross-bank risk sharing, on the other hand, is passing banks together to take advantage of mutual positive spillovers. For example, by passing JP Morgan and Deusche Bank together, people think there is no counterparty risk from JP Morgan to Deusche Bank, and vice versa. On the other hand, now the regulator convince the public that the weaker bank, Deusche Bank, is healthy and thus faces a tighter constraint.

The two research chapters share similar model framework and methodology. There are three types of risk neutral agents: a regulator maximizes system stability by designing stress tests and disclosures, n banks connected via interbank liabilities which corresponds to the network, a financial market that provides banks with refinancing opportunities given public information.

There are three dates $0, 1, 2$. At the first date, given the network structure, the regulator commits to stress test and disclosure policies. The key difference between the two research chapters in terms of modeling is this part. In Chapter 2, all banks are tested and the regulator is able to influence how people think about the underlying project quality of any bank and thus finer implications of system stability. In Chapter 3, the regulator optimally selects a fixed number $m < n$ of banks for stress test, and commits to a disclosure policy only regarding these banks. The disclosure policy specifies how likely any signal is observed at different states, and thus characterizes how informative the policy is regarding each bank.

At $t = 1$, each bank's risky project realizes independently. Other banks' projects matter via repayments of interbank liabilities at $t = 2$. Then the public signal of tested banks is realized, and a bank chooses whether to sell everything for cash or not in order to avoid default at $t = 2$. Banks with high market values refinance and surely survive, whereas banks with lower market values wait in hope for the best of the underlying state. At $t = 2$, everything is revealed and banks repay liabilities. This is a fixed point, as banks' outgoing payments depend on the payments from other banks; and determines whether banks who fail the stress tests earlier survive or default.

The model is theoretically interesting because there are both cross-state risk sharing of each bank and cross-bank risk sharing about correlating banks, and they are entangled. The easiness of correlation depends on how aligned it is to design the beliefs across states of each bank; and passing together feeds back to cross-state risk sharing, because each bank is perceived to be healthier. I introduce an index to summarize the efficiency of passing any set of banks at any state. My index is the sum of cross state risk sharing price of all banks that pass. Intuitively, the regulator priorities the strategies with higher index, which is exactly where cross-state risk sharing is more valuable.

The highlight result of Chapter 2 is that when interbank complementarity is high, and contagion is important, the regulator treats all banks the same, and either passes all the banks or fails all the banks. When complementarity is smaller and contagion is nuanced, the regulator may pass only a subset of banks depending on the network structures.

To gain more intuition, Chapter 2 examines a variety of network structures. The most empirically relevant one is the core-periphery structure. I show that the core banks typically receive preferred treatment. There is strong complementarity between the core and connected peripheries and the regulator tends to pass them together. Core banks also have diversified counter-party risk from higher connectivity. Yet there is a counter force that peripheries are one distance away than connected core in face of non-local shocks. Another relevant exercise is whether more connected networks are more stable under the optimal policy. The answer is not necessarily so, for two counter forces. On the one hand, for given shocks, connectivity enhances risk sharing and regulator pass them together. On the other hand, in a less connected network, the regulator is able to design beliefs such that multiple shocks are perceived to be quarantined locally on adjacent banks. Thus other banks are perceived to be healthier. A nice example of the latter effect is a a ring network. The optimal policy features distance policy and quarantine effect. The regulator pass banks further away from the impaired banks but not the nearer banks, at states where project shocks are connected.

The problem in Chapter 3 is solved in two steps. First, for each set of selected banks, we solve for the optimal disclosure scheme. Second, we select the banks that results in the highest system stability under the corresponding optimal disclosure.

Results in Chapter 3 are different. One key insight is the emphasis on systemic importance. Compared with Chapter 3 where peripheral banks may benefit from fine tuning beliefs about contagion to be non-local and small, here the counterparty risks from unselected banks are always present. Accordingly, the incentive to contain the risks from unselected banks increases, and systemically important banks have a larger spillover effect. The other difference is the ambiguous effects when the size of interbank exposures R varies. The public knows counterparty contagion is always present, regardless of the selected banks' project risks. So cross-state risk sharing depends on R in a complex way because payoff and bank liquidity decrease state wise. In Chapter 2 on the other hand, the relative effects of R across banks is also monotone in R, as banks have different susceptibility to contagion which only arises at bad states.

Chapter 3 also examines optimal policies in specific network structures. I find that, (i) in a ring network, stress test is either "balanced" on banks positioned evenly and disclosure is non-discriminatory, or on "connected" banks and disclosure is truth-telling on potential shocks; (ii) in a star network, stress test is conducted on the center bank when counterparty exposure is either sufficiently small or sufficiently large.

Chapter 2

Optimal Stress Tests in Financial Networks

2.1 Introduction

Disclosing information about bank qualities is crucial in resolving a bank crisis. Stimulated by the great recession, bank supervisors around the world conduct periodic bank stress tests and publicly disclose the test results to restore market confidence regarding the banking system.[1] In contrast to direct financial support to troubled financial institutions ex post, stress testing influences market beliefs, and as a result there is a growing literature that studies the optimal informativeness of stress testing to persuade market participants to act in a way that best enhances financial stability.[2]

However, this literature thus far examines the stress test design of stand-alone single banks and remains essentially micro-prudential;[3] it focuses on shocks to banks' own fundamentals rather than on the broader macroprudential question of how stress might be transmitted among banks. This is in sharp contrast to the 2008 financial crisis, which vividly demonstrated the significance of interbank contagion among

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[3][OZS18] involve the macro-prudential recapitalization of banks that makes worse banks refinance first.

systematically important financial institutions (SIFIs).[4] The literature of financial network, as a collection of nodes and links to respectively represent banks and interbank connections, hence ensues. Specifically, a strand of network theories study the contagion cascades resulting from ex post shocks to specific banks, which then trigger further failures via exogenous direct links of interbank payments.

We study the optimal stress test disclosure that persuades the market to refinance a set of banks so as to maximize the system stability, taking into account how banks are connected with each other in an exogenous financial network. A cash-constrained regulator facing a market with low prior can commit to a disclosure policy, in order to persuade the market which set of banks are healthy and at what kinds of contingent states, such that the induced refinancing at the rate priced by the market's posterior beliefs best enhances system stability.

Building information on top of financial networks is challenging because counterparty payments are a set of fixed points. To the best of my knowledge, this is the first paper to address how stress test design accounts for interbank contagion. On the other hand, it is of direct policy relevance to study the optimal information structure at the system level of a financial network; for instance, in 2017, the "Counterparty Default Component" was added by the Federal Reserve to the stress tests for eight financial institutions,[5] which assesses the bank's financial health assuming the default of its largest counterparty.

The following two-bank example illustrates the case of no intervention, single bank stress test and system-wide stress test. Bank 1 and 2 each has a project and borrowed from each other. Projects are independent, and may either succeed or fail

[4]For a comprehensive review for the literature on the great recession, see [Bru09] and [Tha15].

[5]The eight financial institutions are Bank of America Corporation; The Bank of New York Mellon Corporation; Citigroup Inc.; The Goldman Sachs Group, Inc.; JPMorgan Chase & Co.; Morgan Stanley; State Street Corporation; and Wells Fargo & Co.

with probability 0.5. If both projects succeed, both banks survive and bank qualities are 100. If any project fails, both banks fail; a bank with failed project scores 0 whereas a bank with successful project is 20 due to contagion. The market does not observe project realizations, and only banks of at least 60 can refinance sufficiently to avoid failure. At prior, there is no refinancing, and both banks survive with probability 0.25.

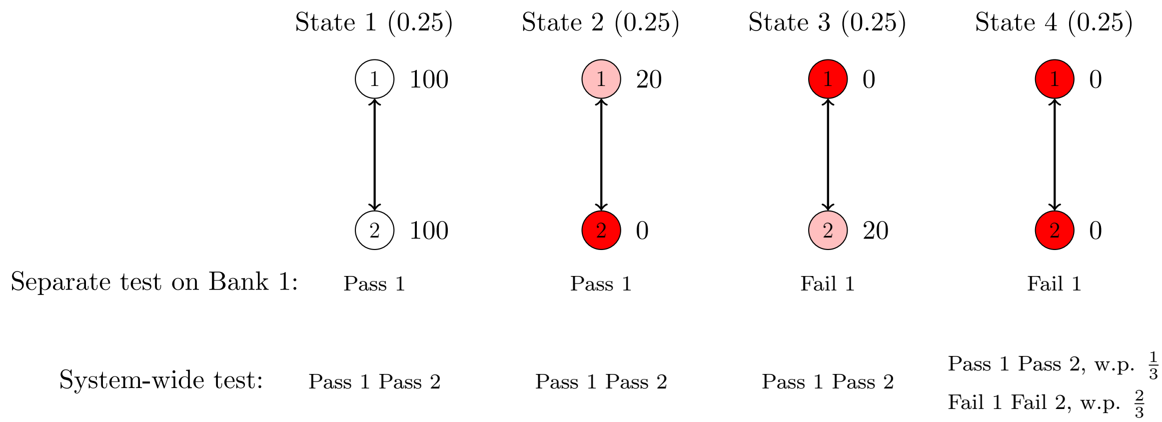

Figure 2.1: Two-Bank Illustrative Example

Suppose that the regulator conducts stress tests on banks separately. To minimize bank failure, the regulator passes the bank when its project succeeds and fails it otherwise; in this case, banks that pass scores 60 in expectation and hence is able to tap the capital market for refinancing. As a result, each bank survives with probability 0.5. This corresponds to the risk sharing across the states (of 100 and 20) studied in models of single banks, and the network structure micro-founds bank qualities that account for both project quality (100 or 0) and interbank contagion (100 or 20).

In contrast, the optimal system-wide stress test that minimizes total bank failure would pass both banks when at least one project succeed, and pass both banks with probability $\frac{1}{3}$ when both projects fail, resulting in a survival probability of $\frac{5}{6}$ per bank.

The key is the spillover effects of disclosure via interbank payments: refinancing of one bank ensures repayments to the other, whose quality then becomes 100 if its project succeeds.

We hence highlight that the system-wide stress test design involves novel cross-bank risk sharing, in addition to cross-state risk sharing. As a result, the regulator coordinates banks' stress test disclosures given the level of interbank complementarity shaped by the network. In the example, the bank with failed project is passed, which alleviates the significant contagion on the other bank. In general, the regulator trades-off allowing more banks to refinance and the involved interbank spillovers, against the constraint of convincing the public that a larger set of banks are sufficiently healthy.

In our model, there are three dates $0, 1, 2$. At the first date, given the network structure, the regulator commits to a disclosure policy which specifies how likely each bank passes or fails the test at each future contingency to maximize system stability. The disclosure policy thus determines the interpretations of test results, and in equilibrium banks who pass refinance at favorable rates. As an application of Bayesian persuasion ([KG11]) with multiple receivers, our model uses the information design framework of [BM16a, BM16b], where the payoffs are based on the fixed point problem of interbank payments introduced in [EN01].

Bank balance sheets and network structures, which are borrowed from [EN01] and [AOTS15], are exogenously given at the first date $t = 0$. Each bank has a risky loan project and interbank claims on the asset side, and has deposits and interbank liabilities on the liability side.[6] The risky loan could take binary values, and is realized independently across banks at the interim date $t = 1$. The state of nature is the collection of loan realizations across banks. At the last date $t = 2$,

[6] We take senior deposits as exogenously given in this paper; for deposits decisions that are endogenous to bank default probabilities, see [EHM17], who offer a nice framework in studying deposit demand and bank competition.

the state is revealed and liabilities are due. Deposits are senior, and the junior interbank liabilities are pari passu. As bank cash flows include payments from other banks, interbank payments are a set of fixed points such that the following rule is simultaneously satisfied for every interbank liability: a bank repays up to the face value; in the case of default, it is protected by limited liability, and repays lender banks whatever is left after deposit payments in proportion to the face values they are owed. A bank with good project may still default due to contagion, and a bank with bad project is sure to default. We assume that default on any liability entails a sufficiently large penalty on the bank.

At $t = 1$, the state is realized and not observed by banks or investors.[7] A public signal is released according to the disclosure policy. Then a bank chooses to either raise funds in a financial market against its total assets – loan plus interbank claims, or wait till $t = 2$ when the state is revealed and liabilities are due; in the former case, the market values the bank's asset side according to the Bayes rule given the public signal and bank actions. Default penalty creates incentive for risk sharing, so if the market value of the bank's asset side exceeds liabilities, it is optimal to raise funds and hence avoid default. Otherwise, the bank waits in hope for the best of the underlying state, and may still be solvent if the state turns out to be good. In this way, the financial market enables cross-state and cross-bank risk sharing.

The regulator maximizes the expected weighted number of solvent banks. Without loss of generality, the problem can be characterized by the [BM16a, BM16b] framework that imposes obedience constraints on the disclosure policy, such that the signals recommend actions to banks and banks would take the recommended actions

[7] Our analysis is robust to the case where each bank observes its own loan project realization. Whether banks know their own types, there is remaining uncertainty regarding the types of their counter-party banks. Absent public signals, banks prefer to wait for the state to reveal in hope for the best. Hence, bank actions have no signaling effects.

under the resulting posteriors. Specifically, the regulator reports a joint signal that specifies for each bank whether the value of its total assets is h (high) or l (low) in a way that the obedience constraints are satisfied; as a result, banks reported with h would raise enough funds and survive, and banks reported with l wait in hope for the best. It is worth emphasizing that first, l does not necessarily lead to bank failure but just denies the opportunity of risk sharing for self saving; second, it does not imply giving up a failed bank in practice because fiscal backstops are not modeled here.

To summarize the model timeline, at the first date the regulator commits to a disclosure policy. At the interim date, the state is realized and a public signal is released according to the disclosure policy, and the banks with signal h refinance. At the last date, the state is revealed and liabilities are due.

To solve this problem, we introduce a benefit-cost index that is a significant enrichment of the gain-to-cost ratio in [GL18]. In models of single banks, the regulator reports h at the states that he would like to allow cross-state risk sharing. The associated efficiency of each state can be characterized by the ratio of the unit incremental payoff of reporting h at this state, over the cost that captures how much pooling with this state hurts the posterior of h and thus reduces the probability of risk sharing. In the optimal disclosure, there exists a threshold ratio, such that the regulator reports h at the states with ratios above and l at the states with ratios below; at the states of the threshold ratio, the regulator randomizes between h and l. Under regularity conditions, states with smaller shocks have higher ratios, and the optimal policy corresponds to monotone partitional signaling.

In contrast, our model is a two-dimensional problem in which the regulator chooses to report which banks are h at each states. First, we show that the efficiency of reporting each joint test results at each future contingency (signal-state pair) could still be summarized by an index, although the endogenous cross-state structures of signals with multiple obedience constraints may depend on the allocations of other signals.[8] Second, at any stage where a corner is reached, i.e. reporting a signal at a state with probability one, we adjust for the option of switching the original corner signal to another that reports more banks are h. This is new from models of single banks with only state choice, where upon corners the regulator moves to include the next best state for cross-state risk sharing under h. We hence provide an algorithm for the optimal policy in which the regulator reports the joint signal across banks at the state(s) associated with the highest index. Whenever a corner is reached, the regulator moves to a next stage, adjusts the remaining indices and reports the signal-state pair of resulting highest signal, until all risk sharing opportunities with the good states have been exhausted.

Our index also has a price-approach interpretation that relates to the shadow values in the dual problem. If the optimal policy is an interior solution, for a given joint signal the highest index across states is proportional to the sum of the obedience constraint multipliers, or equivalently the cross-state borrowing price, of banks that are reported with h. If instead corners are reached in the algorithm, the representation has an extra part of the effects on the assigned signals, because the marginal signal that exhausts the risk sharing opportunities reflects the price of risk sharing in general. This idea is similar to the virtual valuation in auction theory, which adjusts for the surplus of the types above the marginal type.

Based on the algorithm, we first discuss properties of the optimal disclosure in general networks. Policy is non-discriminatory across banks when 1) the expected bank profitability is high, because cross-state risk sharing is less constrained; or 2)

[8]For example, in a network of two symmetric banks, naturally hh is reported at states where only one bank has asset shock with equal probability.

when counterparty exposure is large and thus cross-bank risk sharing effect is large. In both cases, there exists a threshold index such that the regulator reports all banks are h at states with an index above the threshold, reports all banks are l at states with an index below and mixes at the states of the threshold index. Otherwise, when both bank profitability and interbank exposure are small, the regulator either separates a subset of less harmed banks with h from the other banks (report l), or report all banks are l, in a similar threshold policy monotone in the sequence of states ranked by the indices. Again, we emphasize that the all-l signal does not necessarily lead to system failure, but instead all banks are denied the opportunity of cross-state risk sharing and wait in hope for the best. One example of this signal is that in the worst time of 2008, the Federal Reserve required all key banks to participate in TARP instead of self saving.

Whether the optimal disclosure is monotone in the partially ranked states, i.e. prioritizing cross-state risk sharing at the states with less asset shocks, depends on the system stability in the benchmark absent disclosure policy. Benchmark stability sets the hurdle of risk sharing benefit, whereas the cost of risk sharing – banks' shortage of payments that captures the magnitude of belief deterioration, is always higher at the worse states with additional asset shocks.

We then apply the algorithm to solve the optimal policies in specific networks. In a complete network, each bank borrows equally from the rest. When counterparty exposure is sufficiently large, the system is fragile as banks with good projects also fail due to contagion, and the optimal disclosure is monotone across states. Within this parameter range of a fragile system, whether disclosure is discriminatory across banks depends on the size of interbank exposure: when it is larger, the regulator either reports all banks are h or all banks are l; when it is smaller, the regulator either separates the banks of good loans with h to allow for risk sharing, or reports all banks are l. Otherwise, there is transition in benchmark system stability: at states with only a few shocks the majority of banks with good loans are solvent, whereas at the states with more shocks the whole system fail. Hence the regulator cares more about the worse states, which are however costlier for cross-state risk sharing. If good loan payoff is small, the cost disadvantage is dampened because the extra good loans at better states accumulate less liquidity comparatively.[9] Then the optimal policy is non-monotone across states and prioritizes risk sharing at worse states. The stability transition relates to "robust-yet-fragile" nature of interconnectedness in the network literature, which provides strong incentive for non-monotone policy.

In a ring network where a bank only borrows from the next neighboring bank, the regulator reports h on banks at least a specific distance away from the nearest asset shock, to allow them for risk sharing at states where the asset shocks are connected. Reporting h on a bank removes the contagion effect on its lender bank, and a distance strategy fully exploits this directed cross-bank spillover. As disconnected asset shocks cut the cross-bank risk sharing and recounts the distance to shock, the same distance strategy is most efficient when asset shocks are connected. The endogenous threshold distance depends on the magnitude of cross-bank risk sharing, so the optimal policy is less discriminatory and allows more banks for risk sharing when interbank exposure is larger.

With only three banks, the complete network is more stable than the ring network under the optimal policy. The comparison is ambiguous when there are more than four banks. On the one hand, efficient sharing of cash flows in the complete network both decreases an average bank's shortage of payments and increases interbank complementarity for more efficient cross-bank risk sharing. On the other hand, in the

[9] To the extreme of a small project payoff that just covers senior deposits, system liquidity is similar across states; to the other extreme of large project payoff, reporting h at better states does not deteriorate the belief of h.

complete network multiple asset shocks are also shared, whereas in the ring network, they are concentrated locally under the optimal policy and do not accumulate due to limited liability.

In a core-periphery network where banks have asymmetric centrality, typically the core banks receive preferred treatment. Due to better risk sharing with counterparties, a core bank is rarely faced with a large contagion shock. In contrast, a periphery bank is subject to serious contagion whenever its connected core bank has a bad loan. Under general parameter range, the regulator reports h on core banks with weakly higher probability. One exception is when bank profitability is small and counterparty exposure falls into an intermediate range, where the regulator reports h on a few periphery banks but l on the connected core bank. This arises because the periphery banks are one distance away than their connected core bank from outside asset shocks, and interbank complementarity is weak under the parameter range, which denies the core bank the opportunity of cross-state risk sharing.

Literature

Our paper is related to several strands of literature. The first strand is the literature on stress test. [GS14] and [Lei14] give overall discussions on the benefits and costs of regulatory disclosure on banks. Several papers study the optimal disclosures of test results, as applications of Bayesian persuasion ([KG11]) and information design ([BM16a]). In [GL18], the optimal disclosure stochastically reports bad banks to prevent market breakdown, and pools the rest of banks for cross-state risk sharing. [FeCMP16] argue that the optimal disclosure policy depends on fiscal capacity because of the costly backstops that ensue. [Wil17] examines the effects of disclosure on a bank's ex ante asset choice. [LW17] discuss whether to disclose stress test models to banks, and argue that model secrecy may lead to socially undesirable investments by banks. [OZS18] consider the design of macro-prudential stress tests with capital requirements on banks to avoid future default. A dynamic disclosure which forces weak banks to raise capital first leads to efficient recapitalization of stronger banks. [IP18] study the information design with multiple receivers in the global games framework of regime change.[10] The optimal policy removes strategic uncertainty, whereas the structural uncertainty of disagreement on fundamentals persists. [Ino19] incorporates rollover risks, and the optimal policy first discloses the banks with good assets, and additionally test the banks with bad assets on liquidity positions with contingent recapitalization requirements. All these papers examine stand-alone single banks, and to the best of our best knowledge, this is the first stress test paper to account for interbank contagion in a financial network. [FHK17] provide empirical evidence that stress testing provides information about both the tested banks and the banking industry in general, but there is no evidence on a reduction of private information production.

Our paper is also related to the more general literature of information disclosure on financial institutions. The classic concern about perfect disclosure is the Hirshleifer effect ([Hir71]) that reduces risk sharing opportunities. [DGHO17] explain why banks should be opaque, which is consistent with our implication of coordinating regulatory disclosures across banks. Other related contributions include [BCM15], [SS15], [HM16].

Our paper is an application of Bayesian persuasion with multiple receivers, and adopts the framework of information design.[11] This literature traces to [Mye86] who argues that the designer can restrict to action recommendations to the agents, in a

[10] [GH16] also consider a global game model of regime change.

[11] See [Kam18], [BM19] for surveys of this literature.

general class of multi-stage games of incomplete information. [KG11] study the optimal persuasion between a sender and a single receiver. Other contributions on the single-sender, single receiver persuasion are [BC07], [RS10], [GK14]. [DM19] present a price-theoretic approach to Bayesian persuasion and characterize the conditions for monotone partitional signaling, when payoffs depend only on the mean of posterior. [EFK15], [Ely17], [Xan16] and [DE16] study persuasions in dynamic settings, and [GK16] allow for multiple senders. [BM16a, BM16b, BM19] present the information design framework with multiple receivers that unifies communication in games and Bayesian persuasion. We build on the notion of Bayes-correlated equilibrium (BCE) in [BM16a, BM16b], who argue that the set of Bayes-Nash equlibria that can arise correspond to the BCEs that are obedient – the agents take the recommended actions under the resulting posteriors. [Tan15], [MPT17], [AC16a, AC16b], [BHM15] and [BG18] also study the information design with multiple receivers. [GP18] formalize the dual problem of the [BM16a] information design framework. [GP19] study persuasion on a social network where a designer can communicate with only a limited number of agents, who then share the information with neighbors.[12]

The micro-foundation of the payoffs in our paper relates to the financial network literature.[13] In the seminal work of [AG00], interbank lending networks allow banks in different regions to share liquidity risk (a la [DD83]), and a complete network provides the most efficient risk sharing. [EN01] introduce a basic framework to study financial contagion in exogenous networks determined by interbank liabilities, and show that the set of fixed points of interbank repayments exists and is generically unique.[14] [AOTS15] study the extent of financial contagion under different network

[12][ES19] study how the optimal level of propaganda depends on the social network structure.

[13]For surveys see [AB09], [GY16] and [Sum13].

[14]This departure from the possible multiple equilibrium that arise from coordination on payments requires positive net cash flows in a subset of the network, or as in [AOTS15], the existence of

structures. We introduce uncertainty, and study the information design to maximize system stability as determined by the interbank payments. Other contributions on financial contagion and system stability in exogenous network structures include [Das04], [CS13], [EGJ14], [GY15] and [GPY15]. Recent work on network interventions include [AB15], [GGG17], [Kan18], [Ram19], [BCS17] and [Ero18]. The closest to our paper is [AB15], who discuss whether mandatory disclosure can improve welfare as opposed to voluntary disclosures by banks who have strategic substitutability or complementarity in equity levels due to contagion. Other papers that study the role networks play in strategic behaviors include [BCAZ06], [GGJ+10], [BKD14], [CGG17], [BK18]. [BK18] micro-found a [Kyl89] model in connected intermediaries and analyze how decentralization affect information diffusion. There is also a growing literature that considers endogenous interbank linkages: [GVR05], [Far17], [EO17], [Wan16], [CM18] and others.

The rest of the paper is organized as follows. Model setting is presented in Section 2. Section 3 solves the problem and provides general properties of the optimal policy. Section 4 discusses symmetric networks and the effect of connectivity. Section 5 discusses asymmetric networks. Section 6 concludes.

2.2 Model Setup

The economy lasts for three dates $t = 0,\ 1,\ 2$ without discounting, and is populated by three types of risk-neutral agents: banks, a regulator and investors. There are n banks indexed by $i = 1,\ 2, \cdots,\ n$, who are connected by interbank liabilities, which correspond to the network structure. At $t = 0$ the regulator commits to a disclosure policy before the state is realized at $t = 1$, to maximize the system stability at $t = 2$

senior liabilities.

when banks meet debt obligations. At $t = 1$, the state realizes and a public signal is released according to the committed disclosure policy. Given the public signal, a bank can raise funds from a continuum of deep-pocketed investors at the market value of its total assets, which enables risk sharing across states and across banks. We provide a notation table for all variables in Appendix A.

Banks and Financial Network. We introduce uncertainty and information design to the financial network model of [EN01] and [AOTS15], who focus on interbank payments at contingent states. At the beginning of $t = 0$, bank balance sheet is exogenously given as:

Bank i's Balance Sheet (Book Value)

Assets:	**Liabilities:**
Risky loan $\tilde{A}_i$	Deposit D_i
Interbank claims $\sum_{j \neq i} R_{ij}$	Interbank debts $\sum_{j \neq i} R_{ji}$
	Equity

Figure 2.2: A Typical Bank's Balance Sheet (Book Value)

On the asset side, each bank i has a risky loan asset that delivers binary random payoff $\tilde{A}_i \in \{A_i > 0,\ 0\}$ at the beginning of $t = 1$, and

$$\mathbb{P}(\tilde{A}_i = A_i) = p_i,\ \mathbb{P}(\tilde{A}_i = 0) = 1 - p_i$$

is the common prior. $\tilde{A}_i$ realizes independently across banks, and state of nature $\theta \in \Theta$ is the collection of loan asset realizations at $t = 1$:

$$\Theta \equiv \left\{ \theta = (\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n) | \tilde{A}_i(\theta) \in \{A_i, 0\},\ \forall i = 1, \cdots, n \right\}.$$

For example, with two symmetric banks ($A_i = A$), there are four states $\{AA, A0, 0A, 00\}$. Bank i also has interbank claims on borrower banks, $\sum_{j\neq i} R_{ij}$, where R_{ij} is the face value lent to bank j, which is due at $t = 2$. Throughout this paper, for any directed interbank lending, the first subscript represents the lender bank and the second subscript represents the borrower bank.

On the liability side, all liability obligations are due at $t = 2$. Each bank i has senior liabilities – deposits of face value D_i, and pari passu junior interbank liabilities, $\sum_{j\neq i} R_{ji}$, where R_{ji} is the amount borrowed by bank i from lender bank j. At $t = 2$, bank i is protected by limited liability, and repays deposits first before any payments to other banks. As a bank's cash flows depend on payments from other banks, interbank payments are a set of fixed point which we will later characterize. Last, bank equity is the residual value.

We assume that bank i is faced with a sufficiently large penalty if it defaults on any liability, and hence for simplicity its utility at $t = 2$ is characterized as:

$$u_i = \begin{cases} 0, & \text{solvent,} \\ -K < 0, & \text{default.} \end{cases} \tag{2.1}$$

We emphasize that the network structure corresponds to the collection of interbank liabilities $\{R_{ij}\}$.

Information Design. At $t = 0$, the regulator commits to an information structure $\{\hat{\mathbf{S}}, \boldsymbol{\pi}\}$, which consists of the signal space $\hat{\mathbf{S}}$ and probability distribution $\boldsymbol{\pi}$. Specifically, $\hat{\mathbf{S}} = \hat{\mathbf{S}}_\mathbf{1} \times \cdots \times \hat{\mathbf{S}}_\mathbf{n}$ is the collection of available signals, where $\hat{\mathbf{S}}_\mathbf{i}$ is a finite set of available signals for an individual bank i. A typical signal $s \in \hat{\mathbf{S}}$ is an n-element vector $s = s_1 \times \cdots \times s_n$. $\boldsymbol{\pi} : \Theta \to \triangle\hat{\mathbf{S}}$ is an $|\hat{\mathbf{S}}| \times |\Theta|$ matrix that specifies the conditional probability of any $s \in \hat{\mathbf{S}}$ at any contingent state $\theta \in \Theta$. i.e., a typical element

$$\pi(s,\theta) \equiv \mathbb{P}(s|\theta) \in [0,1], \text{ and } \sum_{s \in \hat{\mathbf{S}}} \pi(s,\theta) = 1, \ \forall \theta \in \Theta.$$

At the beginning of $t = 1$, state $\theta = (\tilde{A}_1, \cdots, \tilde{A}_n)$ realizes, which is not observed by banks or investors.[15] Then a public signal s is released according to $\boldsymbol{\pi}$. Given the signal realization, each bank i chooses an action x_i from a binary set

$$x_i \in X_i = \{\text{raise funds}, \ \text{wait}\}.$$

A bank can either "raise funds" in the financial market against its total assets – loan plus interbank claims receivable,[16] or "wait" till the next date $t = 2$ when the state is revealed and liabilities are due. Let $X \equiv X_1 \times \cdots \times X_n$ and $x \in X$ is a collection of bank actions. Competitive investors value bank assets at Bayes' posterior $m : \ \hat{\mathbf{S}} \times \boldsymbol{\pi} \times X \to \mathbb{R}^n$ given the public signal s and bank actions x. Let $y_{ij}(x,\theta)$ be the contingent payment from bank j to i. Then if bank i raises funds, it receives cash of m_i,

$$m_i = \mathbb{E}[\tilde{A}_i(\theta)|s,x] + \sum_{j \neq i} \mathbb{E}[y_{ij}(x,\theta)|s,x], \tag{2.2}$$

with which to repay liabilities at $t = 2$; the investors receive the claim on bank i's

[15] Our model is robust to the case where each bank i privately observes its own asset realization $\tilde{A}_i$. Whether banks know their types, there is remaining uncertainty about the actual interbank payments they receive, which depend on the other banks' types. Absent public signals, they prefer to wait in hope for the best of the underlying state to be revealed. In contrast, there is signaling on private information in [GL18], where the reservation values of trading in the financial market are heterogeneous across bank types.

To relate to [BM16a]'s framework, in our application the obedience constraints and thus belief updating are degenerate in bank types. One interpretation is that the regulator knows more than banks on the stress test models, changes in future monetary policies and etc.

[16] This could be interpreted as derivative contracts that creates super-seniority for the investors, or directly selling all the assets as in [GL18].

loan and interbank payments receivable.[17] Below is the interim market value balance sheet:

Bank i's Balance Sheet (Market Value)

Assets: total value m_i	**Liabilities:**
Risky loan $\mathbb{E}[\tilde{A}_i(\theta)\vert s,x]$	Deposit D_i
Interbank claims $\sum_{j\neq i}\mathbb{E}[y_{ij}(x,\theta)\vert s,x]$	Interbank debts $\sum_{j\neq i} R_{ji}$
	Equity

(y_{ij} is the actual payment from bank j)

Figure 2.3: A Typical Bank's Balance Sheet (Interim Market Value)

Default penalty (see Eq (3.1)) at $t = 2$ creates the incentive for risk sharing. Then at $t = 1$, if the resulting m_i of cash (see Eq (2.2)) is enough to repay total liabilities, bank i raises funds and hence avoid default, and otherwise waits till $t = 2$ in hope for the best of the underlying state:

$$x_i = \begin{cases} \text{raise funds}, & \text{if } m_i \geq D_i + \sum_{j\neq i} R_{ji}, \\ \text{wait}, & \text{if } m_i < D_i + \sum_{j\neq i} R_{ji}, \end{cases} . \tag{2.3}$$

Note that a bank who waits does not necessarily default, because the underlying state may turn out to be favorable (e.g., the state where all banks have good loan assets), despite the low interim m_i.

The role of investors and financial market that enables risk sharing in our model is thus similar to that in [GL18], where a bank with value above a threshold receives extra payoff and has risk sharing incentive due to ex post idiosyncratic asset shocks.

[17] Suppose there are infinite many investors and each gets a fraction of the asset claims. Hence, we do not allow netting of interbank liabilities on the part of investors. See [DP17] for why banks do not net gross liabilities.

The major difference here is the additional cross-bank risk sharing because raising funds also guarantees full interbank payments. In contrast, the financial market in [OZS18] serves as the means of costly liquidation, where the regulator requires banks to sell part of the risky asset for cash buffers against future loss.

It is without loss of generality to focus on binary signal space for each individual bank, because bank action is binary and joint signals allow for coordination among banks. Specifically,

$$\mathbf{S} = \{s = s_1 \times \cdots \times s_n | s_i \in \{h, l\},\ \forall i\}, \tag{2.4}$$

under which each bank is assigned with one of two signals, h or l. In addition, without loss of generality we can represent the information design problem by the [BM16a] framework $\{\mathbf{S}, \boldsymbol{\pi}, \psi\}$ with obedience constraints on $\boldsymbol{\pi}$: the signal space $\mathbf{S}$ (2.4) in which signals are action recommendations, h for "raise funds" and l for "wait"; the signal distribution $\boldsymbol{\pi} : \Theta \to \triangle\mathbf{S}$; and the decision rule $\psi : \mathbf{S} \to X$ such that each bank takes the recommended action under the resulting posteriors,

$$x_i = \begin{cases} \text{raise funds}, & \text{if } s_i = h, \\ \text{wait}, & \text{if } s_i = l. \end{cases} \tag{2.5}$$

Hence, the obedience constraints say that the regulator can report $s_i = h$ only when bank i prefers raising funds to waiting, which under the binary utility of banks translates to a larger market value m_i (see Eq (2.2)) than total liabilities; otherwise the regulator should report $s_i = l$. i.e.,

$$m_i(s, \boldsymbol{\pi}, \psi(s)) \geq D_i + \sum_{j \neq i} R_{ji} \text{ if } s_i = h,$$
$$m_i(s, \boldsymbol{\pi}, \psi(s)) < D_i + \sum_{j \neq i} R_{ji} \text{ if } s_i = l.$$

We will fully characterize the obedience constraints after introducing the interbank payments. If $pA_i + \sum_{j\neq i} R_{ij} < D_i + \sum_{j \neq i} R_{ji}$ for some i, an example of violation is to always report all banks are h,

$$s_h: \ s_i = h, \ \forall i; \ \pi(s_h, \theta) = 1, \ \forall \theta, \tag{2.6}$$

because investors then value bank assets at prior and no bank is willing to raise funds.

Interbank Payments. At $t = 2$, the state is revealed and liabilities are due. Let $y_{ij}(s, \theta)$ denote the actual payment from bank j to bank i at $t = 2$ given the signal s and the revealed state θ,[18] and then

$$y_{ij}(s,\theta) = \begin{cases} R_{ij}, & s_j = h, \\ \left\{ \min \left[R_{ij}, \frac{R_{ij}}{\sum_{k\neq j} R_{kj}} \left(\tilde{A}_j(\theta) + \sum_{k \neq j} y_{jk}(s,\theta) - D_j \right) \right] \right\}^+, & s_j = l. \end{cases} \tag{2.7}$$

If $s_j = h$, bank j raised enough cash and makes full payment at $t = 2$. If instead $s_j = l$, it repays junior creditor bank i up to the face value of R_{ij}; when it defaults, it pays out whatever is left after deposit payments, $\tilde{A}_j(\theta) + \sum_{k\neq j} y_{jk}(s,\theta) - D_j$, to junior

[18] We used $y_{ij}(x, \theta)$ for actual interbank payments before we introduce [BHM15]'s framework. Here we abuse notation a little to use $y_{ij}(s, \theta)$ under the decision rule (3.3) that maps signal to bank actions.

creditors in proportion to the face values, and is protected by limited liability. As a bank's cash flows depend on payments from borrower banks, interbank payments are a set of fixed point in which the above payment rule (3.5) is simultaneously satisfied for every interbank liability.

Note that interbank payments here are consistent with [EN01] and [AOTS15], where the $s_j = h$ case is absent. $s_i = h$ is as if assets were sold with enough cash in bank i, so that it always make full payments under their payment rule.

Regulator's Payoff. The regulator commits to an information structure $\{\mathbf{S}, \boldsymbol{\pi}\}$ at $t = 0$ to maximize the total weighted number of banks that survive. Let w_i be the exogenous weight of bank i, $v(s, \theta)$ the contingent payoff at $t = 2$, and $V(\boldsymbol{\pi})$ the regulator's payoff at $t = 0$. Then

$$v(s,\theta) = \sum_i w_i \mathbf{1}_{\{\sum_{j\neq i} y_{ji}(s,\theta)=\sum_{j\neq i} R_{ji}\}}, \tag{2.8}$$

$$V(\boldsymbol{\pi}) = \mathbb{E}\left(v(s,\theta)\right) = \sum_\theta \mathbb{P}(\theta) \sum_{s\in\mathbf{S}} \pi(s,\theta) v(s,\theta). \tag{2.9}$$

Timeline. The model timeline is summarized as follows:

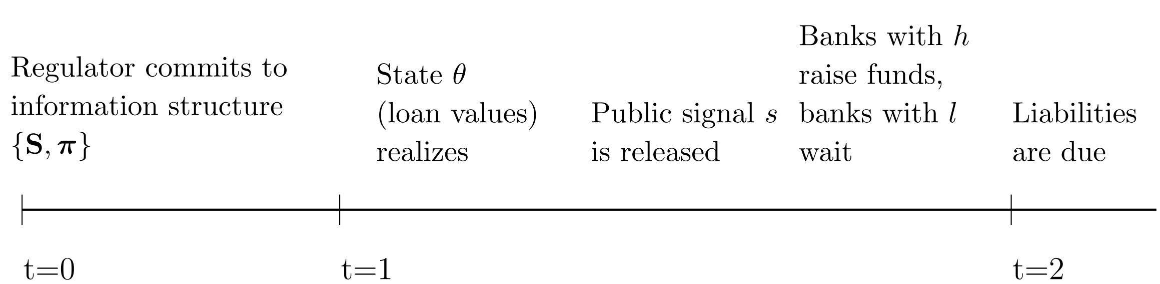

Figure 2.4: Model Timeline

2.3 General Case

In this section, first we characterize the information design problem, and show that the solution exists and is generically unique. Then we propose a general method to solve the problem analytically. Last, we present a few general properties of the optimal policy for all connected networks.

2.3.1 Benchmark Case: Autarky

This part characterizes the benchmark case absent information design. The following assumption restricts common priors to rule out uninteresting cases.

Assumption 2.3.1 (Low Prior)**.**

$$pA_i + \sum_{j \neq i} R_{ij} < D_i + \sum_{j \neq i} R_{ji}, \ \forall i = 1, \cdots, n. \tag{2.10}$$

Assumption 3.3.1 says, absent information design, no bank can raise enough cash to repay liabilities even if bank coordination is perfect, so all banks wait in hope for the best of the underlying state.[19] It is also a sufficient condition to rule out the babbling policy of always reporting all banks are h (3.4).

Let $y^0_{ij}(\theta)$ br the payment from bank j to i at state θ in benchmark. Under Assumption 3.3.1,

$$y^0_{ij}(\theta) = \left\{ \min \left[R_{ij}, \frac{R_{ij}}{\sum_{k \neq j} R_{kj}} \left(\tilde{A}_j(\theta) + \sum_{k \neq j} y^0_{jk}(\theta) - D_j \right) \right] \right\}^+, \ \forall i \text{ and } j \neq i.$$

[19] Then the information structure with singleton signal space $\{(l, \cdots, l)\}$ corresponds to the benchmark case. If Assumption 3.3.1 is relaxed such that there is multiple equilibrium absent information design, null information structures with singleton signal space ($|\hat{\mathbf{S}}| = 1$) are not identical as they additionally serve to coordinate banks on financial market participation.

Let $v_0(\theta)$ and V_0 denote respectively the $t = 2$ contingent and $t = 0$ expected payoffs in benchmark:

$$v_0(\theta) = \sum_i w_i \mathbf{1}_{\{\sum_{j\neq i} y_{ij}^0(\theta) = \sum_{j\neq i} R_{ji}\}},$$
$$V_0 = \mathbb{E}\left(v_0(\theta)\right) = \sum_\theta \mathbb{P}(\theta) v_0(\theta).$$

2.3.2 Regulator's Problem and Equilibrium

Obedience Constraints. Let $L_i(s, \theta)$ denote bank i's inflows net of deposits (or net assets, or "liquidity"):

$$L_i(s, \theta) = \tilde{A}_i(\theta) + \sum_{j\neq i} y_{ij}(s, \theta) - D_i. \tag{2.11}$$

The following are the obedience constraints in [BM16a], which guarantee that bank i raises funds when $s_i = h$ and waits when $s_i = l$:

$$\sum_\theta \left[\frac{\mathbb{P}(\theta)\pi(s,\theta)}{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)} \cdot L_i(s_{-i}, s_i = l, \theta)\right] \geq \sum_{j\neq i} R_{ji}, \qquad (\forall s, \forall i \text{ with } s_i = h \text{ in } s)$$
$$\sum_\theta \left[\frac{\mathbb{P}(\theta)\pi(s,\theta)}{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)} \cdot L_i(s, \theta)\right] < \sum_{j\neq i} R_{ji}. \qquad (\forall s, \forall i \text{ with } s_i = l \text{ in } s)$$

The first set of inequalities say, given s and the fact that the other banks follow the recommended actions, the market value of net assets L_i of any bank i with $s_i = h$ is no less than its interbank liabilities $\sum_{j\neq i} R_{ji}$, so bank i would indeed raise funds. Similarly, the second set of inequalities say any bank i with $s_i = l$ prefers to wait in hope for the best of the state to be revealed.

One important thing is, for any bank i with $s_i = h$, the correct representation of its contingent state inflow value flips $s_i = h$ to $s_i = l$, i.e., $L_i(s_{-i}, s_i = l, \theta)$. This is because, bank i's inflow value should only depend on the loan asset $\tilde{A}_i(\theta)$ and the payments from other banks $\sum_{j\neq i} y_{ij}(s_{-i}, s_i = l, \theta)$ without that assuming bank i pays in full as the result of the obedience constraint, and the flipping of s_i cuts the effect on the rest of the network. Otherwise, part of bank i's value is double counted because outflows may eventually become inflows via interbank payments.[20]

Regulator's Problem. The regulator's problem $(\hat{\mathcal{P}})$ at $t = 0$ is

$$V = \max_{\boldsymbol{\pi}} \sum_{\theta} \mathbb{P}(\theta) \sum_{s\in\mathbf{S}} \pi(s,\theta) \sum_{i} w_i \mathbf{1}_{\{\sum_{j\neq i} y_{ji} = \sum_{j\neq i} R_{ji}\}}$$

$$\text{(Obedience: } h\text{)} \qquad s.t. \quad \sum_{\theta} \left[\frac{\mathbb{P}(\theta)\pi(s,\theta)}{\sum_{\theta} \mathbb{P}(\theta)\pi(s,\theta)} \cdot L_i(s_{-i}, s_i = l, \theta) \right] \geq \sum_{j\neq i} R_{ji}, \ (\forall s_i = h) \tag{2.12}$$

$$\text{(Obedience: } l\text{)} \qquad \sum_{\theta} \left[\frac{\mathbb{P}(\theta)\pi(s,\theta)}{\sum_{\theta} \mathbb{P}(\theta)\pi(s,\theta)} \cdot L_i(s,\theta) \right] < \sum_{j\neq i} R_{ji}, \ (\forall s_i = l) \tag{2.13}$$

$$\text{(Prior Consistency)} \qquad \sum_{s} \pi(s,\theta) = 1, \ (\forall \theta) \tag{2.14}$$

$$\text{(Probability)} \qquad 0 \leq \pi(s,\theta) \leq 1. \ (\forall \theta, \mathbf{s}) \tag{2.15}$$

The regulator commits to the optimal information structure at $t = 0$ that maximizes the weighted number of banks that survive $V(\boldsymbol{\pi})$, subject to the obedience constraints for $s_i = h$ and $s_i = l$ respectively, the prior consistency constraints that at each state the conditional probabilities of reporting all possible signals add up to 1, and the probability constraints that restricts $\pi(s,\theta)$ in between 0 and 1.

[20] An example for better understanding is disclosing information on only one bank i of a connected network. Then investors' belief under babbling (always reporting $s = h$) should be the prior, which corresponds to the specification of $L_i(s_i = l, \theta)$.

Equilibrium. An equilibrium is the collection of $(\{\mathbf{S}, \boldsymbol{\pi}\}, \{y_{ij}(s,\theta)\})$ such that:

1) Given (s,θ), $\{y_{ij}(s,\theta)\}$ is a set of interbank payment equilibrium such that the payment rule (3.5) is simultaneously satisfied for every i and every $j \neq i$.

2). $\{\mathbf{S}, \boldsymbol{\pi}\}$ solves the regulator's problem $\hat{\mathcal{P}}$.

The following lemma characterizes the interbank payments when all banks default.

Lemma 2.3.1. *If all banks default on junior liabilities, generically some bank defaults on senior liability.*

Lemma 2.3.2 allows us to remove the obedience constraints of $s_i = l$ (3.9). In our model, because both the action choice for single banks is monotone in signals and the interbank payments receivable weakly increase in others banks' signals, (3.9) is slack in equilibrium and can be omitted.

Lemma 2.3.2. *The following relaxed problem $\mathcal{P}$ has the same solution as the original problem $\hat{\mathcal{P}}$.*

$$V = \max_{\boldsymbol{\pi}} \sum_{\theta} \mathbb{P}(\theta) \sum_{s \in \mathbf{S}} \pi(s,\theta) \sum_{i} w_i \mathbf{1}_{\{\sum_{j\neq i} y_{ji} = \sum_{j \neq i} R_{ji}\}}$$

$$(\textit{Obedience: } h) \qquad s.t. \quad \frac{\sum_{\theta} \mathbb{P}(\theta)\pi(s,\theta) L_i(s_{-i}, s_i = l, \theta)}{\sum_{\theta} \mathbb{P}(\theta)\pi(s,\theta)} \geq \sum_{j \neq i} R_{ji}, \ (\forall s_i = h)$$

$$(\textit{Prior Consistency}) \qquad \sum_{s} \pi(s,\theta) = 1, \ (\forall \theta)$$

$$(\textit{Probability}) \qquad 0 \leq \pi(s,\theta) \leq 1. \ (\forall \theta, \mathbf{s})$$

2.3.3 Existence and Uniqueness

To prove existence, first we borrow from [EN01] and [AOTS15] to show that given any signal s and state θ, the set of interbank payments exists and is generically unique.

The case of $s_i = h$ is robust because these banks' assets were sold with enough cash for full payments. Then we argue that the optimal information structure exists by the standard extreme value theorem.

Proposition 2.3.1 (Existence)**.** *For any (s, θ), the interbank payment equilibrium $\{y_{ij}(s, \theta)\}$ exists and is generically unique.*[21] *The optimal information structure $\boldsymbol{\pi}^*$ exists.*

$\mathcal{P}$ is a linear programming problem, and the optimal solution $\boldsymbol{\pi}^*$ lies on the vertex of the feasible set polyhedron. Multiple solutions arise only when some binding constraint through $\boldsymbol{\pi}^*$ is accidentally parallel to the objective function. For example, multiplicity may arise in networks with symmetry, and unless otherwise stated, we focus on the symmetric solutions. Note that although a bank's participation in the financial market depends on the other banks, in the equilibria where all banks are obedient,[22] the set of bank actions is unique given s.

Proposition 2.3.2 (Uniqueness)**.** *The optimal information structure $\boldsymbol{\pi}^*$ is generically unique.*

2.3.4 Solution Structure

In this section, first we discuss the trade-offs in choosing which of the banks to allow for risk sharing at what states, and then present some regularity results and the other parameter assumptions.

---

[21] In the non-generic case, there may exist a continuum of payment equilibria. For example, in a system of two symmetric banks, $R_{12} = R_{21} = R$, $D_1 = D_2 = 0$, $\theta : \tilde{A}_1 = \tilde{A}_2 = 0$, then any set of payments with $y_{12} = y_{21} \in [0, R]$ is an equilibrium. The key for generic uniqueness is the outside-network flows. Eg., $y_{12} = y_{21} > 0$ cannot be an equilibrium if $D_1 > 0$, which should be paid first from the y_{12} received.

[22] Otherwise, multiplicity could arise when counter party banks do not follow signal instructions. [BM16a] assume that the designer could coordinate the participants on one specific equilibrium.

Marginal Efficiencies. In this part, suppose that bank profitability is extremely low ($p_i \to 0$ and/or $A_i \to D_i^+$), and we discuss the marginal efficiency of (s, θ). Intuitively, the unit payoff of reporting h on some banks at state θ is the incremental total solvency[23] across all banks under s, from the total solvency at θ in autarky. The associated cost comes from the small inflows or large shortage of payments of the banks reported with h, which reduces the posterior of s, and thus the probability and total payoff of reporting s. From the price-approach perspective, a larger shortage increases the shadow price of the obedience constraint (3.8), which corresponds to costlier cross-state borrowing.

This trade-off is also reflected in other stress test studies (see [GL18], [OZS18] and etc.). The major difference here is that signals may report h on multiple banks. First, the signals that report more banks are h have larger unit payoffs regardless of marginal efficiencies. We elaborate the effects in the next part, where we assume large bank profitability and corner solutions arise. Second, the multiple-h signals have endogenous cross-state structure due to multiple obedience constraints and the distributions of other signals.[24] We address the sufficiency of efficiency measures in the next subsection. In this subsection, we discuss the efficiencies absent the influence of other signals.

The first dimension of choice is which state(s) to send a high signal. At states where the initial asset impairments are weakly larger, banks have larger shortage and thus reporting high signals are costlier. On the other hand, in autarky the whole sys-

[23] A bank i could be saved from default in autarky either because of a direct $s_i = h$ report, or although $s_i = l$, the large enough indirect spillover effects of counter-party banks who are reported with h. Typically, the incremental survival consists of only the first case, because the regulator would report $s_i = h$ on the banks in the later whenever possible.

[24] For example, with two symmetric banks, the efficiency of (hh, gb) (sending $s_1 = s_2 = h$ at state $gb : \tilde{A}_1 > 0, \tilde{A}_2 = 0$) depends on how the regulator reports other signals. If $\mathbb{P}(hl|gb) = \mathbb{P}(lh|bg) = 0$, hh is reported at gb and bg with equal probability. However, if for example $\mathbb{P}(lh|bg) = 1$, hh could only be reported at gb.

tem may fail at these states due to contagion, which makes them more important for the regulator. [DM19] characterize the conditions for monotone partitional signals in persuasion when the payoffs depends only on the posterior mean, which is reflected in the stress test studies on single banks. In contrast, here the regulator cares about the system survival instead of individual bank survival, and system stability is endogenous in the network structure. We present an example of non-monotonic disclosure in Section 5, where the complete network is stable at states with a few asset impairments and fragile at states with more asset impairments, which makes the worse states more important.

The second dimension of choice is to report high signals on which banks. If only one bank is reported with h, there is only cross-state risk sharing, and this corresponds to [GL18] with the state space micro-founded by the interbank payments in a financial network.[25] Under multiple-h signals, there is also cross-bank subsidy risk sharing. Intuitively, signals that report more banks are h result in higher system solvency, reduce bank's shortage via interbank spillovers, but may be costlier to send because the regulator needs to convince the public that the worse banks who are involved can also repay in full. As the regulator maximizes system survival, the endogenous cross-state structure of a multiple-h signal compromises the optimal structures of the single-h signals that report only each of the involved banks is h. The incentive to report h on multiple banks depend on their complementarities.

Bank Profitability/ System Liquidity. If, instead, bank profitability (large A_i and/or p_i) is high enough, the regulator is able to report $s \neq s_l \equiv (l, \cdots, l)$ at some the bad state with probability 1 (also reported at good state for cross-state

[25] Specifically, the asset realization of the bank reported with h is its type, and the asset realizations of other banks are its ex post idiosyncratic risk.

risk sharing). For the rest of this paper, we refer this as corner solutions. Bank profitability matters because at a given state, the signals that report more banks are h have larger unit payoffs, or payoff upper-bounds that are reached when reported with probability 1, regardless of marginal efficiencies. In models of single banks, at a given state, the unit payoffs of high signals are the same, and marginal efficiency comparison is sufficient.

When corner solution is reached, in models of single banks, the regulator simply saves the next best state. In our paper, the regulator can also randomize with another signal that reports more $s_i = h$ with the original signal at the same state where corner solution is reached. This new choice thus involves scaling back of the original signal, and we adjust for this when measuring the efficiencies of (s, θ) in the next subsection. When bank profitability is even larger, the regulator reports the signals with more $s_i = h$, regardless of marginal efficiencies. This is because the regulator is able to report the signals with probability high enough such that the associated payoffs approach the upperbounds.

The following result that says the excess liquidity in the good state is exhausted.

Lemma 2.3.3. *Under Assumption 3.3.1, for every $s \neq s_l = (l, \cdots, l)$, the obedience constraint (3.8) binds for at least one bank. i.e.,*

$$\forall s \neq s_l, \exists i \text{ with } s_i = h \text{ in } s, \; \frac{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)L_i(s_{-i}, s_i = l, \theta)}{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)} = \sum_{j \neq i} R_{ji},$$

and at the best state $\theta_1 : \; \tilde{A}_i(\theta) = A_i > 0$, the planner reports at least one bank is h,

$$\sum_{\forall s \neq s_l} \pi(s, \theta_1) = 1.$$

The following assumption says that there is interbank contagion effect, and banks with bad asset realization repays 0.

Assumption 2.3.2.

1) (Contagion)

$$0 < A_i - D_i < \sum_{j \neq i} R_{ji},$$

2) (Senior Default with Bad Asset)

$$0 < \sum_{j \neq i} R_{ij} < D_i.$$

Specifically, a good asset realization generates positive cash flow but is not enough to repay junior debts, and a bad asset realization makes the bank default on senior debts even if it receives full payments from other banks.

For specific network examples, we make the following assumption that liabilities scale with loan asset:

Assumption 2.3.3 (Capital Structure)**.** *There exists constants r_1, r_2, d such that*

$$R_i \equiv \sum_{j \neq i} R_{ji} = r_1 A_i, \quad \sum_{j \neq i} R_{ij} = r_2 A_i, \quad D_i = d A_i.$$

2.3.5 Efficiency Measure: Benefit-Cost Index

Signals that report multiple $s_i = h$ complicate our analysis in three ways, as compared with models of single banks. First, the cross-state structure of multiple-h signals may depend on the distributions of other signals (see footnote 24 for an example). Second, the efficiency of reporting multiple-h signals depend on their cross-state structures. Third, when corner solution is reached, the regulator could switch the original signal to another that reports additional $s_i = h$.

In this part, first we argue that the cross-state structure of a specific signal has a degree of independence from other signals, such that an efficiency measure could be introduced. Then we propose the cost-benefit index for each (s, θ) that takes into account the cross-state structures of multiple-h signals and the potential signal switching at corner solutions. Last, we show the algorithm to get the optimal solution based on the index, and present a few general properties of the optimal information structure.

Cross-state Structure. First, notice that the cross-state structure of any signal should be optimal in its own sub-game with restricted signal space and probability space. The restricted signal space is binary and includes only this signal and a reservation signal $s_l : \; s_i = l, \; \forall i$.[26] The restricted probability space represents a residual space from the assigned distribution of other signals $s \neq s_l$ in the original game. Let $I_h(s)$ denote the set of banks that receive $s_i = h$ under s:

$$I_h(s) \equiv \{i | s_i = h \text{ in } s\}, \tag{2.16}$$

and $\theta_G(s)$ an excess liquidity state where all banks reported h have higher inflows than their liabilities:

$$\theta_G \in \Theta_G(s) \equiv \{\theta | L_i(s_{-i}, \theta | s_i = l) \geq \sum_{k \neq i} R_{ki}, \forall i \in I_h(s)\}. \tag{2.17}$$

Then $\Theta_G(s)$ is the collection of states from which the banks reported with h borrow

[26]Note that s_l is not necessarily the only reservation signal, but this setting is without loss of generality. For example, if some banks are solvent in the benchmark at state θ, another reservation signal at θ reports $s_i = h$ on these banks and $s_i = l$ on other banks. However, the second reservation signal could be nested in the setting that we introduce, by introducing another sub-game of the second reservation signal.

liquidity under s. Formally, a signal subgame is introduced for each signal $s \neq s_l$ and each $\theta_G(s) \in \Theta_G(s)$ with restricted signal space $\{s, s_l\}$ and restricted probability space.

Sub-game optimality allows us to rule out the influence from the other signals that are unreasonable. One result is that we can assign the (s, θ) in the sequence of their efficiencies, which allows for efficiency measures of each (s, θ). In the next part, we show that all obedience constraints are binding, as a result of the independent asset shocks and thus switching healthiness of banks across states.

Benefit Cost Index and Dual Problem. In this part, we introduce an efficiency measure, the benefit cost index, for each (s, θ) element. [GL18] introduce a similar gain-to-cost ratio for each state in a model of single banks, and argue that the optimal disclosure is monotone in the array of states sorted by the ratio. In a network system with joint signals, our index adjusts for the multi-dimension of multiple-h signals, and the additional option of switching to signals that report more $s_i = h$ at corner solutions. We also relate the index to the shadow values in dual problem, and show that it is the total cross-state borrowing price of banks with $s_i = h$, and in corner solutions, plus an extra term of the effects on the assigned signals. Last, we present the algorithm for optimal solution based on index ranking, and a few properties of the optimal information structure for all networks.

First, we follow [GP18] to characterize the dual problem. We introduce

$$\chi(s, \theta) \equiv \mathbb{P}(\theta)\pi(s, \theta) = \mathbb{P}(\theta)\mathbb{P}(s|\theta).$$

Let $\lambda_i(s)$, $q(\theta)$, $\nu(s, \theta)$ respectively denote the Lagrangian multipliers of the obedience constraints for banks with $s_i = h$, the prior consistency constraint at θ, and the probability constraint. Then

Primal

$$
\begin{aligned}
\mathcal{P} = \max_{\chi \in \mathbb{R}^{\mathbf{S}\times\Theta}} & \sum_{\theta\in\Theta, s\in\mathbf{S}} v(s,\theta)\chi(s,\theta) \\
(O) \quad & \sum_{\theta\in\Theta, s_{-i}\in\mathbf{S}_{-i}} [L_i(s_{-i}, s_i = l, \theta) - \sum_{j\neq i} R_{ji}]\chi(s,\theta) \geq 0, \ (\forall s, \forall i \in I_h(s)) \quad && \lambda_i(s_i, s_{-i}) \\
(PC) \quad & \sum_{s\in\mathbf{S}} \chi(s,\theta) = \mathbb{P}(\theta), \ (\forall\theta) && q(\theta) \\
(P) \quad & \chi(s,\theta) \geq 0, \ (\forall s, \theta) && \nu(s,\theta)
\end{aligned}
$$

Dual

$$
\begin{aligned}
\mathcal{P}^* &= \min_{q\in\mathbb{R}^\Theta} \sum_\theta \mathbb{P}(\theta) q(\theta) \\
& q(\theta) \geq v(s,\theta) + \sum_{i\in I_h(s)} \lambda_i(s_i, s_{-i})[L_i(s_{-i}, s_i = l, \theta) - \sum_{j\neq i} R_{ji}], \ (\forall s, \theta) \quad \chi(s,\theta) \\
& \lambda_i(s) \geq 0, \ q(\theta) \geq 0.
\end{aligned}
$$

where $v(s,\theta)$ is the contingent system solvency, and $I_h(s)$ is the banks with $s_i = h$ under s (see 2.16). The dual says, the optimal information structure minimizes the expected cost of sending signals across states, $\sum_\theta \mathbb{P}(\theta)q(\theta)$. At θ, the cost of sending signals $q(\theta)$, must be no less than the value of any signal, which has two parts: the contingent state payoff $v(s,\theta)$, and the value of cross-state borrowing

$$
\lambda_i(s_i = h, s_{-i})[L_i(s_{-i}, s_i = l, \theta) - \sum_{j\neq i} R_{ji}].
$$

The first term $\lambda_i(s)$ is the multiplier of the obedience constraint, and captures the cross-state borrowing price of bank i under s. The second term, bank i's shortage of payments, captures the quantity of cross-state borrowing. When $L_i(s_{-i}, s_i = l, \theta) \geq \sum_{j \neq i} R_{ji}$ (good state), sending s at θ has the additional value of enhancing its posterior, which enables the regulator to report s at other states. Otherwise, a negative second term captures the cost of belief deterioration of reporting s. At each state, the signals associated with the highest value are reported ($\chi(s, \theta) > 0$) and reflect the cost of reporting signal $q(\theta)$.

Now we introduce the cost benefit index. Whenever corner solution is reached and there is residual excess liquidity, we readjust the index for the option of switching the original signal to another that reports more $s_i = h$, and call this a new round. Although our problem is static, we abuse a little of the dynamic language, such as previous and later rounds to refer the sequence of signal assignments.

We introduce a collection of potential states to report s as

$$\Theta'(s, \theta_G) \equiv \{\theta_G \in \Theta_G(s)\} \cup \{\theta | \theta \notin \Theta_G(s)\}, \tag{2.18}$$

where $\theta_G(s)$ (see (2.17)) is the only source of excess-liquidity.

Definition 2.3.1 (Benefit Cost Index $k = 1$). Under Assumption 3.3.1 and 2.3.2, for any signal $s \neq s_l$, any good state with excess liquidity $\theta_G \in \Theta_G(s)$ and any bad state $\theta \notin \Theta_G(s)$ that consumes liquidity, let $\xi^{(k)}(s, \theta; \theta_G)$ denote the k-th round benefit-cost index of (s, θ). For $k = 1$,

$$\xi^{(1)}(s, \theta; \theta_G) = \frac{1}{\epsilon} \max_{\hat{\Theta}(s,\theta;\theta_G) \subset \Theta'} \left[\sum_{\theta \in \hat{\Theta}} \mathbb{P}(\theta) \pi(s, \theta) v(s, \theta) - V_0 \right] \tag{2.19}$$

$$\text{(Obedience)} \qquad s.t. \ \frac{\sum_{\theta \in \hat{\Theta}} \mathbb{P}(\theta) \pi(s, \theta) L_i(s_{-i}, s_i = l, \theta)}{\sum_{\theta \in \hat{\Theta}} \mathbb{P}(\theta) \pi(s, \theta)} = \sum_{j \neq i} R_{ji}, \ (\forall s_i = h) \tag{2.20}$$

(Marginal Liquidity)	$\pi(s, \theta_G) = \epsilon > 0,$
(Probability)	$\pi(s, \theta) \in [0, 1], \text{ for } \theta \in \Theta'.$

The above problem asks, if the amount of excess liquidity at an endogenous good state $\theta_G(s)$ is a very small, and the regulator reports s at both θ_G and a bad state θ, what is the associated payoff. From linearity, this corresponds to the case where the optimal solution is interior. Here, given the scarce excess liquidity at $\theta_G(s)$, independent asset shocks across banks, the obedience constraints of assigning a single signal s are binding.[27] The following Lemma 2.3.4 presents the result of $\xi^{(1)}(s, \theta; \theta_G)$.

Lemma 2.3.4 (**Benefit Cost Index** $k = 1$)**.** *Given s is reported at θ_G, θ, let $\Theta^{*(1)}(s, \theta; \theta_G)$ denote the collection of states where s is reported excluding θ_G:*

$$\Theta^{*(1)}(s, \theta; \theta_G) = \arg \max_{\hat{\Theta}(s,\theta;\theta_G) \subset \Theta'} \left[\sum_{\theta \in \hat{\Theta}} \mathbb{P}(\theta) \pi(s, \theta) v(s, \theta) - V_0 \right] \setminus \{\theta_G\}. \tag{2.21}$$

Let $\Delta \boldsymbol{v}_{|\Theta^*| \times 1}$, $\Delta \boldsymbol{L}(s; \theta_G)_{|I_h(s)| \times 1}$, and $-\Delta \boldsymbol{\mathcal{L}(s)}_{|I_h(s)| \times |\Theta^*|}$ be the vector and matrix presentations of respectively the incremental payoff $v(s, \theta) - v_0$, the excess-liquidity at θ_G – $L_i(s_{-i}, s_i = l, \theta_G) - \sum_{j \neq i} R_{ji}$, and the shortage $\sum_{j \neq i} R_{ji} - L_i(s_{-i}, s_i = l, \theta)$, where $i \in I_h(s) : s_i = h$. Under Assumption 3.3.1 and 2.3.2,

$$\xi^{(1)}(s, \theta; \theta_G) = \underbrace{\mathbb{P}(\theta_G) \Delta \boldsymbol{L}(s; \theta_G)}_{\textit{excess liquidity}} \cdot \underbrace{\left[(\Delta \boldsymbol{\mathcal{L}(s)})' \right]^{-1} \Delta \boldsymbol{v}}_{\textit{benefit-to-cost}}. \tag{2.22}$$

If the optimal solution is indeed interior, the maximum index of each s is proportional to the sum of size-adjusted cross-state borrowing price of the banks reported with

[27] Otherwise, for s that reports multiple $s_i = h$ with a slack constraint on bank i, the regulator could increase the probability of s at the state where only bank i has bad asset, to exploit the 0-cost cross-state borrowing of bank i.

$s_i = h$:

$$\max_{\theta \notin \Theta_G} \xi^{(1)}(s, \theta; \theta_1) = c \cdot \left[\sum_{i \in I_h(s)} \lambda_i(s) \mu_i \right],$$

where c is a constant and $\mu_i \propto A_i - D_i + \sum_{j \neq i} R_{ij}$ adjusts for the size of excess liquidity.

The above lemma presents both the quantity-approach and the price-approach presentations of $\xi^{(1)}(s, \theta; \theta_G)$. In the quantity-approach, the index consists of first, the amount of excess liquidity at the good state that scales the unit efficiency, and second the unit efficiency characterized by a benefit-to-cost ratio: the unit payoff of incremental system solvency, over the shortage which captures the rate at which posterior of s deteriorates when reported at $\theta \in \Theta^{*(1)}$ and restricts the probability of reporting s. If the optimal solution is indeed interior, the price-approach presentation of each signal is the sum of the size-adjusted cross-state borrowing price of all banks reported with $s_i = h$, which are the shadow values of the obedience constraints. Intuitively, the regulator reports s at θ associated with the highest value of cross-state borrowing.

For a single-h signal s that reports $s_i = h$ and $s_j = l$ for all $j \neq i$,

$$\xi^{(1)}(s, \theta; \theta_G) = \underbrace{\mathbb{P}(\theta_G)[L_i(s_{-i}, s_i = l, \theta_G) - \sum_{j \neq i} R_{ji}]}_{\text{excess liquidity}} \cdot \underbrace{\frac{\overbrace{v(s, \theta) - v_0(\theta)}^{\text{benefit}}}{\sum_{j \neq i} R_{ji} - L_i(s_{-i}, s_i = l, \theta)}}_{\text{cost}}, \tag{2.23}$$

where the unit efficiency part is the same in essence as the gain-to-cost ratio in [GL18]. Specifically, their ratio is the probability of a type failing the target (potential gain) over the type's distance to the target (the cost of posterior deterioration).

For s that reports multiple $s_i = h$, we use a two-symmetric-bank example for illustration, and naturally $w_1 = w_2 = 1$. Suppose whenever there is asset shock, both banks default in autarky:

$$v_0(gg) = 2,\ v_0(gb) = v_0(bg) = v_0(bb) = 0.$$

Thus, the only state with excess liquidity is gg. For (hh, gb), first we solve for the optimal cross-state structure of hh that maximizes the regulator's payoff, given hh is reported at gg and gb. As a result, hh is reported at the one-shock states with equal probability,

$$\Theta^{*(1)}(hh, gb; gg) = \{gb, bg\}, \quad \pi(hh, gb) = \pi(hh, bg).$$

In the index, both the benefit and cost are summed over gb and bg:

$$\xi^{(1)}(hh, gb; gg) = \underbrace{\mathbb{P}(gg)(A - D)}_{\text{excess liquidity}} \cdot \underbrace{\frac{\sum_{\theta=gb,bg} [v(hh,\theta) - v_0(\theta)]}{2D - A}}_{\text{benefit-to-cost}}.$$

Specifically, the unit benefit is 4, as both banks are solvent at both gb and bg. Per unit report of hh at gb enhances the posterior of bank 1 at a rate of $A - D$, and at bg deteriorates the posterior at a rate of D, totaling $2D - A$, for both banks due to symmetry.

As we have argued, the index for rounds $k > 1$ is a function of the previous-round corner solution, because of the option to switch an assigned signal to another that reports more $s_i = h$. Let

$$(s^{*(k)}, \theta^{*(k)}, \theta_G^{*(k)}) \equiv \arg\max \xi^{(k)}(s, \theta; \theta_G),$$

and $\Delta\pi^{(k)}(s,\theta)$ respectively record the combination with the highest index and signal allocation in round k. Then

$$\eta(s,\theta;\theta_G) \equiv \frac{\Delta\pi^{(k)}(s,\theta)}{\Delta\pi^{(k)}(s,\theta_G)},\ \forall\theta\in\Theta^*(s,\theta;\theta_G)$$

characterizes the cross-state structure of the allocation, and similarly $\eta^{*(k)}(\theta)$ for the structure of $(s^{*(k)},\theta^{*(k)})$. If $\eta^{*(1)}(\theta)>1$ for some $\theta\in\Theta^{*(1)}$, corner solution is reached and $k=2$ is entered.

Definition 2.3.2 (Benefit Cost Index $k>1$). $\xi^{(k)}(s,\theta;\theta_G(s))$ is scaled such that the $k-$th round signal allocation of (s,θ) reaches the next corner solution, and rebates the payoff of assigned signals when signal switching is involved. Specifically, let $\Theta_C^{*(k)} \equiv \{\theta|\sum_i^{k-1}\pi^{*(i)}(s^{*(i)},\theta)=1\}$ denote the states that have reached corner solution at the k-th round. Under Assumption 3.3.1 and 2.3.2,

1). (No signal switching) If $\Theta^{*(k)}(s,\theta;\theta_G)\cap\Theta_C^{(k-1)}=\emptyset$,

$$\begin{aligned}
&\xi^{(k)}(s,\theta;\theta_G) = \max_{\hat\Theta(s,\theta;\theta_G)\subset\Theta'}\left[\sum_{\theta\in\hat\Theta}\mathbb{P}(\theta)\pi(s,\theta)v(s,\theta)-V_0\right]\\
&\text{(Obedience)}\quad s.t.\quad \frac{\sum_{\theta\in\hat\Theta}\mathbb{P}(\theta)\pi(s,\theta)L_i(s_{-i},s_i=l,\theta)}{\sum_{\theta\in\hat\Theta}\mathbb{P}(\theta)\pi(s,\theta)}=\sum_{j\neq i}R_{ji},\ (\forall i\in I_h(s))\\
&\text{(Scaling)}\quad \pi(s,\tilde\theta)=1-\sum_i^{k-1}\pi^{*(i)}(s^{*(i)},\tilde\theta), \qquad (2.24)\\
&\quad\text{where } \tilde\theta=\arg\min_{\theta'\in\Theta^*(s,\theta;\theta_G)\cup\{\theta_G\}}\frac{1-\sum_i^{k-1}\pi^{*(i)}(s^{*(i)},\theta')}{\eta(s,\theta';\theta_G)},\\
&\text{(Probability)}\quad \pi(s,\theta)\in[0,1],\ \text{for } \theta\in\Theta'.
\end{aligned}$$

2) (Signal switching) If $\Theta^{*(k)}(s,\theta;\theta_G)\cap\Theta_C^{(k-1)}\neq\emptyset$, there exists $\pi(\hat s,\hat\theta)=1$[28] for some

[28] This is the result of binding obedience constraints and linearity. Otherwise, the regulator scales up the signal reported at $\hat\theta$ with higher efficiency.

$\hat{\theta} \in \Theta^*(s,\theta;\theta_G) \cap \Theta_C^{(k)}$. Retrieve the round j in which $\pi(\hat{s},\hat{\theta})$ increases to 1, and the corresponding structure $\eta^{*(j)}(\theta)$. Then

$$\xi^{(k)}(s,\theta;\theta_G) = \max_{\hat{\Theta}(s,\theta;\theta_G)\subset\Theta'} \left[\sum_{\theta\in\hat{\Theta}} \mathbb{P}(\theta)\pi(s,\theta)v(s,\theta) - V_0\right] - \underbrace{\xi^{(j)}(\hat{s},\hat{\theta};\theta_G^{*(j)})\pi(s,\hat{\theta})}_{\text{rebate}}$$

$$\text{(OB)} \quad s.t. \quad \frac{\sum_{\theta\in\hat{\Theta}} \mathbb{P}(\theta)\pi(s,\theta)L_i(s_{-i}, s_i = l, \theta)}{\sum_{\theta\in\hat{\Theta}} \mathbb{P}(\theta)\pi(s,\theta)} = \sum_{j\neq i} R_{ji}, \ (\forall i \in I_h(s))$$

$$\text{(S)} \qquad \pi(s,\hat{\theta}) = \min\{\underbrace{[1 - \sum_{i}^{k-1} \pi^{*(i)}(s^{*(i)},\theta_G)]\eta(s,\theta;\theta_G)}_{\text{residual space}} \underbrace{\frac{\eta^{*(j)}(\theta)}{\eta^{*(j)}(\theta) - \eta(s,\theta;\theta_G)}}_{\text{scale back}}, 1\},$$

$$\text{(P)} \qquad \pi(s,\theta) \in [0,1], \ \text{for } \theta \in \Theta'.$$

The above definition says, the index for rounds $k > 1$ rescales the allocation problem in Definition 2.3.1 of $k = 1$ such that the next corner solution is reached, and when signal switching is involved, additionally rebates the payoff of the part that is switched.

Notice that all the obedience constraints are binding. Intuitively, asset shocks are severe and independent across banks, so in the optimal solution the cross-state borrowing price of any bank should not be free. One might argue that some bank has a slack constraint under s', because it cannot borrow to some state θ' where the regulator reports other signals there with probability 1. However, the regulator has incentive to report s' at θ' exactly when there is positive payoff to switch to s', and the residual space from other signals is in effect not restrictive.

We illustrate the two cases in Definition 2.3.2 by a two-bank example. Suppose at round 1, hl is reported at gg and gb, and corner solution is reached,

$$gg : 0 < \pi(hl, gg) < 1, \quad gb : \pi(hl, gb) = 1.$$

In the second round, (lh, bg) does not involve signal switching. Specifically, the regulator reports lh at both gg for lending excess liquidity and bg for saving bank 2, such that

$$\pi(lh, gg) + \pi(hl, gg) = 1, \text{ or } \pi(lh, bg)' = 1,$$

whichever is reached first. In contrast, (hh, bg) involves signal switching of the assigned hl, as hh is reported at gg for lending liquidity, and at gb, bg symmetrically. The switched part of hl at gg and gb results in an enlarged residual space for hh, and requires rebating the associated payoff of hl. For general networks, these are reflected respectively in the scaling equation and the objective function.

The following Lemma 2.3.5 presents the result of $\xi^{(k)}(s, \theta; \theta_G)$ for $k > 1$.

Lemma 2.3.5 (**Benefit Cost Index** $k > 1$)**.** $\Delta\boldsymbol{v}$*,* $\Delta\boldsymbol{L}(s;\theta_G)$*, and* $-\Delta\boldsymbol{\mathcal{L}}(\boldsymbol{s})$ *are respectively the vector and matrix presentations of incremental payoff, excess-liquidity and shortage of payments. Let* $\phi^{(k)}(s, \theta; \theta_G)$ *be the scaling factor to the next corner solution absent signal switching. Under Assumption 3.3.1 and 2.3.2,*

1) No signal switching,

$$\xi^{(k)}(s, \theta; \theta_G) = \underbrace{\phi^{(k)}}_{\textit{scaling}} \cdot \underbrace{\mathbb{P}(\theta_G)\Delta\boldsymbol{L}(s;\theta_G)}_{\textit{excess liquidity}} \cdot \underbrace{\left[(\Delta\boldsymbol{\mathcal{L}}(\boldsymbol{s}))'\right]^{-1}\Delta\boldsymbol{v}}_{\textit{benefit-to-cost}}.$$

2) There is signal switching, let j tracks the previous round where the switched part is allocated,

$$\xi^{(k)}(s, \theta; \theta_G) = \underbrace{\phi^{(k)}\frac{\eta^{*(j)}}{\eta^{*(j)} - \eta}}_{\textit{scaling}} \cdot \underbrace{\mathbb{P}(\theta_G)\Delta\boldsymbol{L}(s;\theta_G)}_{\textit{excess liquidity}}$$
$$\cdot \underbrace{\left[(\Delta\boldsymbol{\mathcal{L}}(\boldsymbol{s}))'\right]^{-1}\Delta\boldsymbol{v}}_{\textit{benefit-to-cost}} - \underbrace{\left(\frac{\eta^{*(j)}}{\eta^{*(j)} - \eta} - 1\right)\xi^{*(j)}(s, \theta; \theta_G)}_{\textit{rebate}},$$

Suppose the optimal solution corresponds to round k. Then the price-approach presentation is

$$\xi \propto V_0 + \underbrace{Const \cdot \sum_{i \in I_h(s)} \lambda_i(s)\mu_i}_{candidate\ (s,\theta)}$$
$$+ \underbrace{\sum_{\tilde{\theta}} \left[q(\tilde{\theta}) - v(s_l, \tilde{\theta})\right] - \sum_{s' \neq s : \chi(s', \theta_G) > 0} \chi(s', \theta_G) \boldsymbol{v(s')}' [\Delta \boldsymbol{\mathcal{L}(s)}]^{-1} \boldsymbol{L}(s; \theta_G)}_{effect\ on\ assigned\ signals} \tag{2.25}$$

The above lemma says, the index of later rounds adjusts the gain-to-cost ratio that allows for multiple-h signals to account for the amount of excess liquidity and proper scaling to be comparable with other (s, θ) that involve signal switching. When the optimal solution ends at k such that the last unit of excess liquidity is exhausted, the allocation of (s, θ) reflects the shadow values in the original game. Intuitively, entering later rounds implies higher bank profitability and lower cross-state borrowing price. Hence, in the shadow value representation, $\xi^{(k)}(s, \theta; \theta_G)$ has an extra part that captures the pricing effects of (s, θ) on the assigned signals, in addition to the total cross-state borrowing price of h banks. This adjustment is similar to the virtual valuation in the bargaining literature.

The following Proposition 2.3.3 proposes an algorithm for the optimal solution from the indices that we constructed. Specifically, the regulator allocates the (s, θ) with the highest index at each round, given the residual excess liquidity from the last round. In the current round either excess liquidity is exhausted (optimal solution), or a next round is entered.

Proposition 2.3.3. *Under Assumption 3.3.1 and 2.3.2, the following algorithm results in* $\boldsymbol{\pi}^*$*:*

1). Get $(s^{*(1)}, \theta^{*(1)}, \theta_G^{*(1)} = \theta_1) \equiv \arg\max \xi^{(1)}(s, \theta; \theta_G)$ *and the cross-state structure*

$\Theta^{*(1)}$, $\{\eta^{*(1)}\}$. *If* $\max_{\theta \in \Theta^{*(1)}} \eta^{*(1)}(\theta) \leq 1$,

$$\pi^*(s^{*(1)}, \theta) = \begin{cases} 1, & \theta = \theta_1, \\ \eta^{*(1)}(\theta), & \theta \neq \theta_1, \theta \in \Theta^*(s^{*(1)}, \theta^{*(1)}; \theta_G^{*(1)}), \\ 0, & \theta \notin \Theta^*(s^{*(1)}, \theta^{*(1)}; \theta_G^{*(1)}). \end{cases}$$

$$\pi^*(s_l, \theta) = 1 - \pi^*(s^{*(1)}, \theta).$$

Otherwise, denote $\tilde{\theta} = \arg\max \eta^{*(1)}(\theta)$. *For* $\theta \in \Theta^*(s^{*(1)}, \theta^{*(1)}; \theta_G^{*(1)})$, *set*

$$\pi^{*(1)}(s^{*(1)}, \tilde{\theta}) = 1, \text{ and } \pi^{*(1)}(s^{*(1)}, \theta) = \frac{\eta^{*(1)}(\theta)}{\eta^{*(1)}(\tilde{\theta})},$$

and move to the next round.

2). At round $k > 1$, *get* $(s^{*(k)}, \theta^{*(k)}, \theta_G^{*(k)}) \equiv \arg\max \xi^{(k)}(s, \theta; \theta_G)$, $\Theta^{*(k)}$ *and* $\{\eta^{*(k)}\}$. *Set* $\pi^{*(k)}(s^{*(k)}, \tilde{\theta})$ *(no signal switching) or* $\pi^{*(k)}(s^{*(k)}, \hat{\theta})$ *(signal switching) as specified in Definition 2.3.2. Let* $\theta' \equiv \tilde{\theta}$ *or* $\hat{\theta}$,

$$\pi^{*(k)}(s^{*(k)}, \theta) = \begin{cases} \pi^{*(k)}(s^{*(k)}, \theta') \frac{\eta^{*(k)}(\theta)}{\eta^{*(k)}(\theta')}, & \theta \neq \theta' \equiv \tilde{\theta} \text{ or } \hat{\theta}, \theta \in \Theta^*(s^{*(k)}, \theta^{*(k)}; \theta_G^{*(k)}), \\ 0, & \theta \notin \Theta^*(s^{*(k)}, \theta^{*(k)}; \theta_G^{*(k)}). \end{cases}$$

With signal switching, adjustment on the switched signals,

$$\pi^{*(k)}(s^{*(j)}, \theta) = \frac{\eta^{*(j)}(\theta)}{\eta^{*(j)}(\hat{\theta})}(1 - \pi(s^{*(k)}, \hat{\theta})),$$

3) If all excess liquidity is consumed, we have reached $\boldsymbol{\pi}^*$. *Otherwise, round* $k+1$ *is entered.*

Under Assumption 3.3.1 and 2.3.2, we have the following corollaries on the optimal information structure:

Corollary 2.3.1. *In the optimal information structure* $\boldsymbol{\pi}^*$*, all obedience constraints are binding.*

Corollary 2.3.2. *For a multiple-h signal* s*, its cross-state structure is different from that of any single-h signals each reporting only one of the involved banks is* h*.*

Definition 2.3.3 introduces the reservation signals, under which all h banks do not consume liquidity.

Definition 2.3.3 (Reservation Signals)**.** For $\theta \neq \theta_1$, the reservation signals at θ are

$$s_r(\theta) \in \{s \in \mathbf{S} | s = s_l, \text{ or } s : \theta \in \Theta_G(s)\}$$

Corollary 2.3.3. *Reservation signals are reported only when signals with more* $s_i = h$ *are infeasible.*

Corollary 2.3.3 says, although under a reservation signal, the banks that are reported with h have excess liquidity, the regulator always more banks are h when possible, to exploit the cross-bank risk sharing. Otherwise, a reservation signal is reported for cross-state risk sharing only. An implication is the sequence in which the regulator exhausts the excess liquidities in the good states $\theta_G \in \Theta_G$.

The following Proposition 2.3.4 characterizes how the regulator discriminates bank test results.

Proposition 2.3.4. *Under Assumptions 1, 2, for any connected network, there exists a set of thresholds* $(\bar{A}_i)_{i=1,2,\cdots,n}$ *and* $(\bar{R}_i)_{i=1,2,\cdots,n}$ *such that*
1). when all $A_i \geq \bar{A}_i$*, the regulator does not separate banks and either reports* $s_h \equiv (h, \cdots, h)$ *or* $s_l \equiv (l, \cdots, l)$*.*

2). when all $R_i \geq \bar{R}_i$, except for the reservation signals, the regulator either reports s_h or s_l.

If A_i for all banks are high, there is abundant liquidity to borrow from the good states, and payoffs of reporting signals approach their upper bounds. Hence, the regulator does not discriminate banks, and either reports all banks are h or all banks are l. If the interbank exposure is high, the contagion effect and the interbank complementarity is large, the regulator also reports the same signal on all banks.

In the following sections to display $\boldsymbol{\pi}^*$ of specific networks, we abstract from later-rounds adjustments and focus on the key trade-offs in the first-round. The following are the assumptions that we use:

Assumption 2.3.4 (Interior Solution)**.** *Bank profitability is small, such that $\boldsymbol{\pi}^*$ is an interior solution.*

Another is to assume that s_h dominates in states with more asset shocks, which have very small probability and thus the excess liquidity at good states are relatively abundant. An example is $p \in (\underline{p}, \bar{p})$.

Assumption 2.3.5. *$p_i \in (\underline{p_i}, \bar{p_i})$, such that for states with $\sum_i \mathbf{1}_{\{\tilde{A}_i(\theta)=0\}} > \bar{m}$, $\pi^*(s_h, \theta) + \pi^*(s_l, \theta) = 1$.*

The above assumptions are used only when characterizing the analytical $\boldsymbol{\pi}^*$ in given networks for convenience. For solution properties and numerical solutions, they are relieved.

2.4 Symmetric Networks

This section discusses symmetric networks and the effect of connectivity. Specifically, we study the complete network and the ring network: first we illustrate with a three-bank example, then extends to n banks, and last compare the policy gains. Naturally, in this section $w_i = 1$.

2.4.1 Complete Network

In a complete network, each bank borrows from all others equally:

$$A_i = A,\ D_i = D,\ R_i = R, \quad \text{for all } i;\ R_{ij} = \frac{R}{n-1} \text{ for all } i \neq j.$$

In autarky, banks are ex post different only in loan asset realization $\tilde{A}_i$.

Three-Bank Complete Network

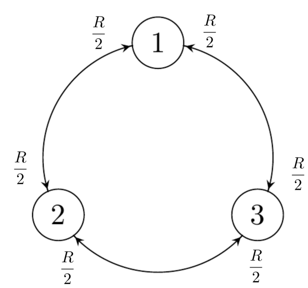

Figure 2.5: Three-Bank Complete Network Illustration

As illustrated by the graph, each bank borrows from the other two banks $\frac{R}{2}$. The parameter range follows from Assumption 3.3.1 and 2.3.2. Hence, $\mathbb{P}(s_h) = 1$ is infeasible, there is interbank contagion, and bad asset realization leads to senior default.

We study two sub-cases: (1) (fragile) $R > 2(A - D)$, and (2) (stable) $2(A - D) \geq R > A - D$. In Case (1), contagion is serious, and whenever there is asset shock, i.e. $\tilde{A}_1\tilde{A}_2\tilde{A}_3 = 0$, all banks default in autarky. In Case (2), the whole system default in autarky only under more than one asset shock.

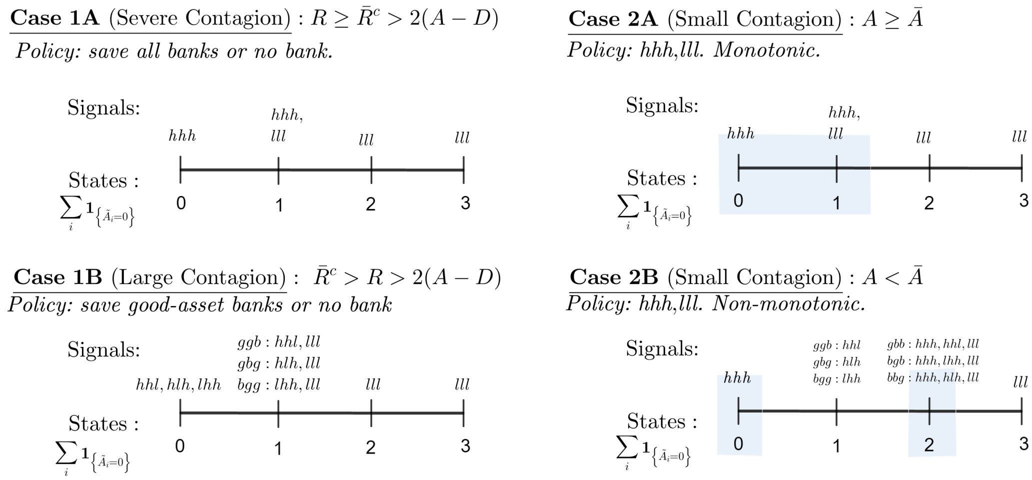

Figure 2.6: Optimal Information Structure of three-bank Complete Network

Banks are ex post separated into two groups by asset realizations. Hence, for any $s \neq s_l$, $s_i = h$ for banks with $\tilde{A}_i = A$; $s_j = h$ for bad-asset bank ($\tilde{A}_j = 0$) deteriorates the posterior of s, and the regulator trades off this cost against saving more banks and interbank spillover.

We show that at bad states, three types of signaling could show up: (1) s_h, reporting all banks are h; (2) truth-telling $\hat{s}(\theta)$, $s_i = h$ for $\tilde{A}_i > 0$ and $s_i = l$ for $\tilde{A}_i = 0$; (3) s_l, reporting all banks are l. Figure 2.6 summarizes the optimal disclosure policy when they are interior solutions.

Suppose that the following representation of Assumption 2.3.5 holds,

$$A - D \geq (\frac{1-p}{p})^2(3D - A), \tag{2.26}$$

i.e., good asset return is large and the shock probability is small. Under (2.26), s_h dominates at states with $m > 1$, where m is the number of asset shocks. Although we abstract from later-round index, and cross-bank policy is exemplified in the first round, and the cross-state policy is kept.

Denote the states with m asset shocks as

$$\theta^{(m)} \in \left\{ \theta | \sum_i \mathbf{1}_{\{\tilde{A}_i(\theta)=0\}} = m \right\}.$$

Case 1: $R > 2(A - D)$.

Information Structure

Suppose that Assumption 3.3.1, 2.3.2 and 2.3.5 (Eq (2.26)) hold, then

Case 1A). When $R > \bar{R}_1^c = \frac{10}{9}A - \frac{2}{3}D$, the regulator only reports s_h and s_l, monotonically across states; i.e., $\mathbb{P}(s_h) + \mathbb{P}(s_l) = 1$ and there exists a threshold $\hat{m} > 0$ such that

$$\pi^*(s_h, \theta) = \begin{cases} 1 & \text{if } m < \hat{m}, \\ \pi \in [0, 1] & \text{if } m = \hat{m}, \\ 0 & \text{if } m > \hat{m}. \end{cases}$$

Case 1B). When $2(A - D) < R \leq \bar{R}_1^c = \frac{10}{9}A - \frac{2}{3}D$, we summarize the solution depending on the liquidity level in the good state $\theta_1 = \theta^{(0)}$.

With small system liquidity (small p, A), at $\theta_1 = \theta^{(0)}$ the regulator reports $\hat{s}(\theta^{(1)})$, at the one-shock states ($m = 1$) randomizes between the truth-telling signals $\hat{s}(\theta)$ and s_l, and at worse states ($m > 1$) reports s_l. With larger system liquidity, the regulator additionally report s_h monotonically across states.

Case 2: $A - D < R \leq 2(A - D)$.

At states with one asset shock, the truth-telling signal $\hat{s}(\theta)$ is a reservation signal

(Definition 2.3.3) because banks with good asset realizations are solvent in autarky. According to Corollary 2.3.3 and Assumption 2.3.5, we can focus on the monotonicity of reporting s_h across states. Although the states with $m = 1$ are less costly to save, the regulator cares more about the states with $m > 1$ where the whole system fail in autarky. When bank profitability is smaller, the first effect is dominated, optimal policy is non-monotone across states.

Information Structure

Suppose that Assumption 3.3.1, 2.3.2 and 2.3.5(Eq (2.26)) hold, then

$$\theta : \begin{cases} m = 0, & \pi^*(s_h, ggg) = 1, \\ m = 1, & \pi^*(s_h, \theta^{(1)}) + \pi^*(\hat{s}(\theta^{(1)}), \theta^{(1)}) = 1, \\ m > 1, & \pi^*(s_h, \theta^{(m)}) + \pi^*(\hat{s}(\theta^{(1)}), \theta^{(m)}) + \pi^*(s_l, \theta^{(m)}) = 1. \end{cases} \tag{2.27}$$

Case 2A). When $A > \frac{6}{5}D$, the regulator reports s_h, $\hat{s}(\theta^{(1)})$ and s_l monotonically across states. i.e., there exists a threshold $\hat{m} > 0$ such that

$$\pi^*(s_h, \theta) = \begin{cases} 1 & \text{if } m < \hat{m}, \\ \pi \in [0, 1] & \text{if } m = \hat{m}, \\ 0 & \text{if } m > \hat{m}. \end{cases}$$

The monotonicity of $\hat{s}(\theta^{(1)})$ and s_l across states could be characterized similarly, in the residual space of respectively s_h, and s_h, $\hat{s}(\theta^{(1)})$, according to (2.27).

Case 2B). When $A \leq \frac{6}{5}D$, the regulator reports s_h non-monotonically across states. His priority is to report s_h at states with $m = 2$, before the states with $m = 1$.

General Complete Network

We extend the previous results to an n-bank complete network. In the benchmark case absent information design, there are only two groups of banks ex post by asset realizations. So effectively, the regulator chooses the number of bad-asset banks[29] to allow risk sharing, and trades-off the benefits of saving more banks plus the spillover effects against the tighter constraint that reduces the frequency of an average bank's risk sharing. Accordingly, at bad states the regulator reports (1) $\tilde{s}(\theta, b_h)$ where $b_h = 1, \cdots, m$,

$$\tilde{s}(\theta, b_h) \in \left\{ s | s_i = h \text{ for all } \tilde{A}_i > 0, \sum_i \mathbf{1}_{\{\tilde{A}_i = 0, s_i = h\}} = b_h \right\},$$

that only allows banks with good loans and h_b of the m banks with bad loans to raise funds; (2) s_l.

Notice that at the states with a small number of bad loans,

$$m(\theta) \leq \frac{n(A - D)}{A} \equiv \bar{m}_1,$$

s_h is a reservation signal because the cross-bank risk sharing alone is enough to convince the public that the average bank is solvent, and thus $\pi(s_h, \theta) = 1$. Hence, non-monotonic disclosure across states could arise only if at the states with more shocks $\theta :\ m(\theta) > \bar{m}_1$, in the benchmark case without disclosure, there is transition in system stability such that the regulator cares more about the states with more shocks that cause system failure. Specifically, introduce

$$\bar{m}_2 \equiv \frac{(n-1)(A - D)}{R},$$

[29] In the three-bank network, under the parameters associated with interior solutions, this number is either 0 or m in the optimal information structure.

and then in the benchmark case, at states where $m(\theta) \leq \bar{m}_2$, banks with $\tilde{A}_i > 0$ are solvent, and those with $\tilde{A}_i = 0$ default; at states with more shocks, all banks default.

When $\bar{m}_1 \geq \bar{m}_2$, all banks default at states with $m > \bar{m}_1$, and bank cash flows decrease with the number of shocks m, which lowers the frequency of risk sharing available at worse states results in monotone disclosure across states. On the other hand, when $\bar{m}_1 < \bar{m}_2$, non-monotonic disclosure arises when good loan delivers a smaller payoff, under which the cash flow disadvantage at worse states is dampened and dominated by the regulator's incentive to save system failure. Intuitively, to the extreme that the payoff of a good loan is very small, across different states system cash flow levels and thus the frequency of risk sharing are similar; to the other extreme that loan payoff is very large and bank cash flow approaches liabilities, risk sharing at the states with less shocks is almost free.

Case 1 $\bar{m}_2 \leq \bar{m}_1$: The parameter range consistent with this case and Assumption 2.3.2 is

$$A > D > R \geq \frac{n-1}{n} A > 0,\ A - D < R.$$

Under this parameter range, the contagion effect is serious and all banks default whenever there is bad asset realization. We show that, there exists a threshold state $\hat{\theta}$, such that risk sharing between the states with less shocks and the good state $\theta^{(0)}$ is most efficient when allowing all good asset banks and only one bad-asset bank to refinance; at states with more shocks, risk sharing is most efficient under s_h:

$$\arg\max_s \xi^{(1)}(s, \theta^{(m)}; \theta_1) = \begin{cases} \tilde{s}(\theta^{(m)}, b_h^* = 1) = s_h, & m(\theta) = 1, \\ \tilde{s}(\theta^{(m)}, b_h^* = 1), & 1 < m(\theta) \leq m(\hat{\theta}), \\ s_h, & m(\theta) > m(\hat{\theta}). \end{cases}$$

Case 2 $\bar{m}_2 > \bar{m}_1$: The parameter range consistent with this case and Assumption 2.3.2 is

$$A > D > R > 0,\ A - D < R < \frac{n-1}{n}A.$$

As we have discussed, in this case, the regulator needs to additionally choose whether to save the banks at states in $\theta : \bar{m}_1 < m(\theta) \leq \bar{m}_2$ or $\theta : m(\theta) > \bar{m}_2$. Within each of the two state groups, optimal disclosure is monotone across states. The discussion of the most efficient signal to implement risk sharing at states $\theta : m(\theta) > \bar{m}_2$ is identical to that in Case 1. For states $\theta : \bar{m}_1 \leq m(\theta) \leq \bar{m}_2$, s_h is preferred under large R, $\bar{m}_1$, $\bar{m}_2$, or small A, all of which increase the complementarity among banks.

Information Structure

Suppose that Assumption 3.3.1, 2.3.2 and 2.3.5 hold. In Case 1 where $\bar{m}_2 \leq \bar{m}_1$, the regulator only reports s_h and s_l, monotonically across states; i.e., $\mathbb{P}(s_h) + \mathbb{P}(s_l) = 1$ and there exists a threshold $\hat{m} > 0$ such that

$$\pi^*(s_h, \theta) = \begin{cases} 1 & \text{if } m < \hat{m}, \\ \pi \in [0,1] & \text{if } m = \hat{m}, \\ 0 & \text{if } m > \hat{m}. \end{cases}$$

In Case 2 where $\bar{m}_2 > \bar{m}_1$, there exists thresholds $\bar{R}$, $\bar{A}$ such that when $R \geq \bar{R}$ and $A < \bar{A}$, the regulator reports s_h to save default banks and non-monotonically across states:

$$\pi^*(s_h,\theta) = \begin{cases} s_h & \theta: \ 1 < m(\theta) \leq \bar{m}_1, \\ s_r(\theta'), \ s_r(\theta) & \theta: \ \bar{m}_1 < m(\theta) \leq \bar{m}_2, \\ s_h, \ s_r(\theta''), \ s_l & \theta: \ m(\theta) > \bar{m}_2, \end{cases}$$

where $s_r(\theta)$ is a reservation signal introduced in Definition 2.3.3, and $s_r(\theta) = s_l$ for states $m(\theta) > \bar{m}_2$. $s_r(\theta')$, $s_r(\theta'')$ represent the reservation signals from other states and are reported at θ for cross-state risk sharing. When $R \geq \bar{R}$ and $A \geq \bar{A}$, s_h is reported monotonically across states:

$$\pi^*(s_h,\theta) = \begin{cases} s_h & \theta: \ 1 < m(\theta) \leq \bar{m}_1, \\ s_h, \ s_r(\theta'), \ s_r(\theta) & \theta: \ \bar{m}_1 < m(\theta) \leq \bar{m}_2, \\ s_r(\theta''), \ s_l, & \theta: \ m(\theta) \leq \bar{m}_2. \end{cases}$$

2.4.2 Ring Network

In a symmetric ring network, bank $i+1$ (1) is the sole lender bank of i (n):

$$A_i = A, \ D_i = D, \ R_i = R, \quad \text{for all } i; \ R_{i+1,i} = R_{1n} = R \text{ for all } i \geq 1.$$

Thus banks are additionally asymmetric in the distance to the nearest borrower bank with bad asset.

Three-Bank Ring Network

In this example, $R_{21} = R_{32} = R_{13} = R$ and otherwise $R_{ij} = 0$, so bank 1 owes to bank 2, 2 to 3 and 3 to 1.

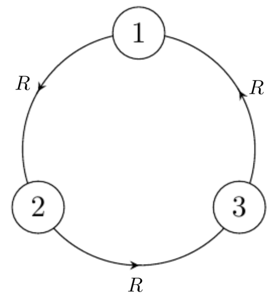

Figure 2.7: Three-Bank Ring Network Illustration

Unlike the complete network, cross-bank spillover effects in the ring network are directed from a borrower bank to its lender bank by repayments. Hence, when reporting two banks with h, the cross-bank risk sharing effect between them is asymmetric. For example, when the regulator reports hhl, the lender bank 2 cannot subsidize its borrower bank 1 the same way it is subsidized by bank 1, but instead the spillover goes via its lender bank 3 and then to bank 1. As a result, cross-bank risk sharing benefit is smaller than in complete network.

Besides Assumption 3.3.1 and 2.3.2, we focus on the "fragile" case of $R > 2(A - D)$, under which all banks default whenever there is asset shock in benchmark, and disclosure policy is monotone in m. If the regulator reports $s_i = h$, h should be reported on bank i's direct lender bank $i + 1$ whenever $\tilde{A}_{i+1} = A$. Hence, the regulator reports the following four types of signals at $\theta \neq \theta_1$: (1) s_h; (2) truth telling signal $\hat{s}(\theta)$; (3) $\tilde{s}(\theta)$: saving the more distant lender bank from asset shock, e.g., llh at bgg; (4) s_l. Note that a type (2) signal $\hat{s}(\theta)$ that reports two banks with h is also reported at another $\theta' \neq \theta_1$ for cross-bank risk sharing between the two banks. For example, $\hat{s}(ggb) = hhl$ is the truth-telling signal under which bank 1 subsidizes bank 2, and hhl is also reported at bgg because the lender bank 2 could only subsidize its borrower bank 1 via bank 3.

Information Structure

Under Assumption 3.3.1, 2.3.2, 2.3.4 (interior solution), and $R > 2(A - D)$, there exist thresholds $\bar{R}_1^r$ and $\bar{R}_2^r < \bar{R}^c$ (complete network threshold) that separate the cases of different cross-bank disclosures.

1). When $R \geq \max\{\bar{R}_1^r, \bar{R}_2^r\}$, the regulator allows all banks for cross-state risk sharing at one-shock states $\theta^{(1)}$: at ggg the regulator reports hhh, at $\theta^{(1)}$ mixes between hhh and lll, and otherwise reports lll.

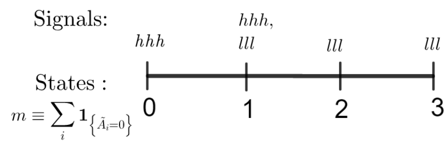

Figure 2.8: Ring Network Policy Case 1

2). When $\bar{R}_1^r \leq R < \bar{R}_2^r$, the regulator allows the two banks with good assets for cross-state risk sharing at $\theta^{(1)}$: at ggg the regulator reports hhl, hlh, lhh with equal probability $\frac{1}{3}$, at $\theta^{(1)}$ mixes between the truth-telling signal $\hat{s}(\theta)$, the truth-telling signal at another state for cross-bank risk sharing, and lll.

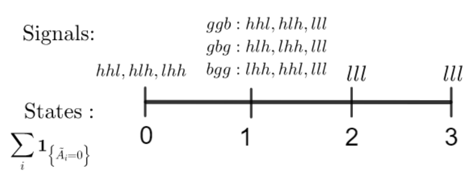

Figure 2.9: Ring Network Policy Case 2

3). When $R < \min\{\bar{R}_1^r, \bar{R}_2^r\}$, the regulator allows only the indirect lender from asset shock for cross-state risk sharing at $\theta^{(1)}$: at ggg he reports hll, lhl, llh with equal probability $\frac{1}{3}$, at $\theta^{(1)}$ mixes between $\tilde{s}(\theta)$ (lhl at ggb, hll at gbg and llh at bgg) and s_l, and at other states reports lll.

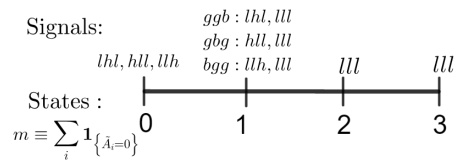

Figure 2.10: Ring Network Policy Case 3

When R increases, at the states with one asset shock, the relative efficiency of the truth-telling signals versus s_h decreases. Although cross-bank risk sharing is more efficient under $\hat{s}(\theta)$, the effect of contagion shock and low payments dominates. Hence, there exists a threshold $\bar{R}_2$ above which s_h is preferred, which is smaller than the threshold in the complete network due to asymmetric cross-bank risk sharing. The truth-telling is preferred to type (3) signals that save the indirect lender bank under large R. The cash flows advantage at the indirect lender bank is dampened under a larger contagion shock, and only the truth-telling signals that allow two banks for risk sharing enjoys the benefit of increased interbank complementarity. The comparison between s_h and type (3) signals depends on the relative importance of asset shock and contagion shock, and s_h is more favorable under large R.

Seemingly, the comparison of policy gains in the ring vs. complete network would be ambiguous, because on the one hand risk sharing is more efficient in the complete network, and on the other hand more discriminative signals are available in the ring network to separate banks when the system is fragile. However, when the networks are comparable in benchmark system stability, policy gain is always higher in the complete network in the three-bank example. We argue that the additional discriminative signals that separate indirect lender arise as the result of asymmetric cross-bank risk sharing, and would be dominated by reporting h on both banks with good assets if the cross-bank risk sharing were symmetric.

General Ring Network

For notation, we assume that any bank subscript that appear is no greater than n to avoid renumbering.

The extension to an $n-$bank ring network is complex because of the asymmetry in the distance from the nearest asset shock. One thing in particular is the need to track the relative positions of asset shocks. For example, asset shocks that are all connected affect other banks in similar way as only one asset shock, whereas disjoint asset shocks cut the ring network into sub-chains. The endogenous cross-state structure of signals that report multiple $s_i = h$ further complicates our analysis, because we cannot locally adjust the information structure at specific (s, θ) for replication arguments.

To simplify the characterization, first we suppose Assumption 2.3.4 holds such that the optimal policy is an interior solution. This does not mean we only consider the states with only one asset shock, because when expected bank profitability is high enough, at states with a smaller number of asset shocks an average bank is perceived as solvent by the cross-bank risk sharing effect alone, and the regulator reports s_h. Second, suppose all banks default in the benchmark when there are more asset shocks, so disclosure is monotone across states and we could focus on the choice of cross-bank disclosure. Specifically,

$$R > (n - \bar{m}_1 - 1)(A - D), \text{ where } \bar{m}_1 = \lfloor \frac{n(A-D)}{A} \rfloor. \tag{2.28}$$

[30] For states $\theta: \ 1 \leq m < \bar{m}_1$ the regulator always reports s_h, and $\theta^{(\bar{m}_1+1)}$ are the critical states in interior solutions for cross-state risk sharing with the state with no asset shock.

[30]To relieve the restrictiveness of all parameter assumptions, we could introduce a negative payoff $\tilde{A} < 0$ of the asset shocks to guarantee that banks with bad loans default on deposits.

To solve the problem, we compare the (s, θ) pairs with the same number of asset shocks m, and introduce the properties of the most efficient (s, θ) pairs in each sub-group of m.[31]

Definition 2.4.1 (Sub-chain). A sub-chain of the ring network is the collection of banks from a bad-asset bank (included) with good-asset direct lender to the indirect lender bank with the next asset shock (excluded):

$$j \in \left\{ j | i \leq j \leq i + d \text{ where } \tilde{A}_i = \tilde{A}_{i+d+1} = 0,\ \tilde{A}_{i+1} = \tilde{A}_{i+2} = \cdots = \tilde{A}_{i+d} > 0 \right\}.$$

The following lemma characterizes the subset of $(s, \theta^{(m)})$ that are not dominated, given the number and the positions of asset shocks.

Lemma 2.4.1 (Distance Strategy). *Suppose that Assumption 3.3.1, 2.3.2, 2.3.4 and Eq (2.28) hold. Given m, we can focus on the $(s, \theta^{(m)})$ pairs of the following type: in each sub-chain, the regulator reports h from d-th banks to the last bank, and l on the rest of the banks.*

Note that s_h and s_l are the special cases of the above structure. For interior solutions, we can decompose the analysis into sub-group comparisons. Intuitively, to exploit the directed interbank spillover in the ring network, once a regulator reports h on a borrower bank, he should also report h on its connected lender banks as long as they have good asset realizations. As the states with the same number of asset shocks are realized with equal probabilities, for any violation of Lemma 2.4.1, the regulator could move h reports on banks backward and the asset shocks forward to increase payoff.

[31] Cross group comparison is irrelevant for the interior solutions that we consider. Our analysis is robust to corner solutions if Assumption 2.3.5 holds as the cross-bank signal choice is degenerate. Otherwise, cross-group comparison is relevant, because in later rounds an (s, θ) which is dominated in its group of a smaller m, could be better than the best (s', θ') in another group of larger m'.

The next lemma captures how the regulator utilizes another benefit of the ring network that connected asset shocks do not accumulate due to limited liability.

Lemma 2.4.2 (Connected Asset Shocks)**.** *Suppose that Assumption 3.3.1, 2.3.2, 2.3.4 and Eq (2.28) hold. We can focus on the states where all asset shocks are connected.*

Intuitively, for the threshold-distance candidates (s, θ) in Lemma 2.4.1, disconnected asset shocks cuts the cross-bank risk sharing and recounts distance. This weakly decreases the occurrence and the number of banks allowed for risk sharing. This argument is robust to the endogenous cross-state structure of multiple-h signals. To see why, suppose a signal s of the distance strategy in Lemma 2.4.1 is reported at a state θ with scattered asset shocks. At another θ' where s is reported due to multiple obedience constraints, the disconnected $s_i = h$ imply asset shocks are in between or at the banks reported with h (Lemma 2.4.1), so multiple sub-chains still arise and cross-bank risk sharing effect is smaller.

To sum up, given the number of shocks m and equivalence of symmetric strategy, we can focus on the candidates $(s(\bar{d}, m), \theta(m))$ of the following type:

$$s(\bar{d}, m) : s(\bar{d}, m)_i = \begin{cases} h, & \text{if } m + \bar{d} \leq i \leq n, \\ l, & \text{otherwise}, \end{cases} \qquad \theta(m) : \tilde{A}_i(\theta) = \begin{cases} 0, & 1 \leq i \leq m, \\ A, & m + 1 \leq i \leq n. \end{cases}$$

From multiple obedience constraints, there is an endogenous collection of states $\Theta^*(s(\bar{d}, m), \theta(m); \theta_1)$ (defined in Definition 2.3.1 and Lemma 2.3.4) where $s(\bar{d}, m)$ is reported. The following

$$h_b = \sum_{i > \bar{d} + m}^{n} \mathbf{1}_{\{\tilde{A}_i = 0\}}$$

is a sufficient statistic to characterize the other states where $s(\bar{d}, m)$ is reported:

$$\theta' \in \Theta^*,\ \theta' \neq \theta(m) : \tilde{A}_i(\theta') = 0 \text{ for } 1 \leq i \leq m - h_b;$$
$$\sum_{i>\bar{d}+m}^{n} \mathbf{1}_{\{\tilde{A}_i=0\}} = h_b;\ \tilde{A}_i(\theta') = A,\ \text{otherwise.} \tag{2.29}$$

For example, suppose $n = 5$, $m = 2$, $\bar{d} = 1$ and $h_b = 1$. This means, in a five-bank ring network and given that there are two asset shocks, $s(\bar{d} = 1, m = 1) = llhhh$ is reported at states $\theta_1 = ggggg$ to enhance posterior belief of $llhhh$, at $\theta(m) : bbggg$ to allow only banks at least one distance away from the nearest asset shock for cross-state risk sharing, and at $\theta' : bggbg,\ bgggb$ as a result of the obedience constraints of bank 3, 4 and 5. Hence, when the public observes $s = llhhh$, they expect the true state to be either $ggggg$, or there are two asset shocks. In the latter case, either banks reported with h have good assets, or bank 1 has bad asset and bank 4 or 5 (the latter two banks reported with $s_i = h$) has bad asset. We rewrite the collection of bad states where $s(\bar{d}, m)$ is reported as a function of h_b:

$$\hat{\Theta}^*(s(\bar{d}, m), h_b)\Theta^*(s(\bar{d}, m), \theta(m); \theta_1) = \{\theta(m)\} \cup \{\theta' | \text{Eq. (2.29) holds.}\} \tag{2.30}$$

Proposition 2.4.1. *Suppose that Assumption 3.3.1, 2.3.2, 2.3.4 and Eq (2.28) hold. The optimal information structure is characterized as*

$$s = \begin{cases} s(\bar{d}^*, \bar{m}_1 + 1), & \theta = \theta_1, \\ s_h, & \theta : 1 < m(\theta) \leq \bar{m}_1, \\ s(\bar{d}^*, \bar{m}_1 + 1), s_h, s_l & \theta \in \hat{\Theta}^*(s(\bar{d}^*, \bar{m}_1 + 1), h_b^*), \\ s_l, & \textit{Otherwise,} \end{cases}$$

where $\bar{m}_1$ *is defined in Eq (2.28). Specifically,*

$$h_b^* \in \left\{ \lceil \frac{R}{A-D} \rceil - \bar{d}^*, \lceil \frac{R}{A-D} \rceil - \bar{d}^* + 1, \bar{m}_1 + 1 \right\}.$$

$h_b^* = \lceil \frac{R}{A-D} \rceil - \bar{d}^*$ *when* $R < \bar{R}_1(\bar{d}^*, \bar{m}_1 + 1)$*, and* $h_b^* = \bar{m}_1 + 1$ *may arise for* $R > \bar{R}_2(\bar{d}^*, \bar{m}_1 + 1)$.

The choice of the threshold distance $\bar{d}^*$ *decreases in counterparty exposure* R.

Proposition 2.4.1 says, at states where an average bank is perceived to be solvent when assuming full payments from other banks, i.e. $m(\theta) \in (1, \bar{m}_1]$, the regulator reports all banks are h. At $\theta_1 : m = 0$, for cross-state risk sharing with $\theta(\bar{m}_1 + 1)$, the regulator always reports $s(\bar{d}^*, \bar{m}_1 + 1)$, which assigns $s_i = h$ on banks from $\bar{m}_1 + 1 + \bar{d}^*$ to n and $s_i = l$ on the other banks. At state $\theta \in \hat{\Theta}^*(s(\bar{d}^*, \bar{m}_1 + 1), h_b^*)$, there are $\bar{m}_1 + 1$ asset shocks. They are either connected from bank 1 to $\bar{m}_1 + 1$, or connected from bank 1 to $\bar{m}_1 + 1 - h_b^*$ and the rest are in banks from $\bar{m}_1 + 2 + \bar{d}^*$ to n. At these states, the regulator mixes among $s(\bar{d}^*, \bar{m}_1 + 1)$, s_h for cross-state risk sharing with states $\theta : m \in [0, \bar{m}_1]$, and s_l. At all other states the regulator reports s_l. The distance strategy is reflected at $\theta(m)$, where the regulator either reports $s_i = h$ on all banks no less than $\bar{d}^*$ away from the nearest asset shock and $s_i = l$ on other banks. Other signals that are reported either involves reservation signals (s_h, s_l), or for cross-bank risk sharing as the result of multiple obedience constraints.

We show that, as counterparty exposure R increases, the optimal policy becomes less discriminative. Specifically, the number of bad-asset banks saved under $s(\bar{d}, m)$, h_b^* increases, and the threshold distance $\bar{d}^*$ decreases. Intuitively, disclosure is more discriminative when the level of net assets L_i vary significantly across banks and the regulator saves more distant lender banks. As R increases, the difference of accumulated cash flows across banks is overshadowed by the large contagion effect (efficiency measure ξ is concave in shortage of payments), and cross-bank risk sharing becomes more efficient among banks with more aligned levels of net assets, both of which favor less discriminative signals.

2.4.3 Complete and Ring Comparison

The following lemma compares the policy gains in the three-bank examples.

Lemma 2.4.3. *Suppose that Assumption 3.3.1, 2.3.2 hold, $n = 3$ and $R > 2(A - D)$, such that the ring and complete networks share the same system stability in the benchmark case V_0. Under the optimal disclosure, the complete network has a weakly higher expected solvency rate, $V(\boldsymbol{\pi}^*)$.*

This result comes from better risk sharing: first, any positive cash flows are shared equally among banks, which reduces the shortage of payments of an average bank; second, the shared cash flows make the banks' liquidity levels L_i more aligned, and thus cross-bank risk sharing is more efficient.

However, when $n > 3$, there is a counter force against the complete network that never shows up in three-bank networks. Specifically, in the ring network, multiple connected asset shocks do not accumulate to hurt the rest of the network due to limited liability, but instead act as if one single asset shock. In contrast, in the complete network, multiple asset shocks are equally shared among the rest of banks. This relates to the "robust yet fragile" result of interconnectedness in the literature: a connected network enjoys better risk sharing sharing in good times yet is subject to worse contagion in bad times.

Proposition 2.4.2. *Suppose that Assumption 3.3.1, 2.3.2 hold, and $R > \max\{(n - \bar{m}_1 - 1)(A - D), \frac{(n-\bar{m}_1)(A-D)}{\bar{m}_1+1}\}$, where $\hat{m} = \lfloor \frac{n(A-D)}{A} \rfloor$. When $n = 3$, the expected* *solvency rate under the optimal information structure, $V(\boldsymbol{\pi}^*)$, is weakly higher in the complete network. When $n \geq 4$, the comparison of $V(\boldsymbol{\pi}^*)$ is ambiguous.*

$R > \max\{(n - \bar{m}_1 - 1)(A - D), \frac{(n-\bar{m}_1)(A-D)}{\bar{m}_1+1}\}$ corresponds to the parameter range under which the two networks are comparable. For states with asset shocks $1 < m \leq \lfloor \frac{n(A-D)}{A} \rfloor$, the regulator always reports s_h in both networks. At states with more asset shocks, all banks default in the benchmark in both networks. In the complete network, the more efficient liquidity sharing among banks increases both an average bank's cash flows and the interbank complementarity that leads to better cross-bank risk sharing under multiple-h signals. However, in the complete network asset, the asset shocks are also shared among the rest of banks, whereas connected asset shocks do not accumulate in ring networks. This effect shows up only when $n \geq 4$. When $n = 3$ and at states with two shocks, the shortage from liabilities at the good-asset bank is the same in both networks. Now we provide an example where the ring network has a higher expected solvency rate under the optimal policy due to this effect.

Example. $n = 4,\ D = \frac{3}{4}A,\ R = 0.54,\ p = 0.2$. At states with $m = 1$, the regulator reports s_h at both networks, under which a typical bank is perceived to be just solvent. At states with $m = 2$, one can verify that reporting $s(\bar{d} = 2, \theta(2))$ in the ring network, i.e. *lllh* at *bbgg*, is more efficient than reporting any signal sent at states with $m = 2$ in the complete network. $p = 0.2$ ensures that the optimal disclosure is interior solution. Therefore, in this example $V(\boldsymbol{\pi}^*)$ is larger in the ring network.

2.5 Asymmetric Networks

In this part, I characterize the optimal information structures on networks that feature asymmetry among banks, and discuss which kind of banks receive preferential treatment. First I present a three-bank star network where banks differ in connectivity, and then an $n-$bank core-periphery network.

2.5.1 Three-Bank Star

Network Structure. Bank 1 is the center bank that has double-sided liability contracts with both bank 2 and bank 3. Bank 2, 3 each only interacts with bank 1. Specifically,

$$R_{12} = R_{13} = \frac{R}{2},\ R_{21} = R_{23} = \frac{R}{2},\ \text{and otherwise } R_{ij} = 0.$$

Under Assumption 2.3.3, we scale the size of good loan payoff A_i and deposits D_i accordingly:

$$D_1 = D,\ D_2 = D_3 = \frac{D}{2},\ A_1 = A,\ A_2 = A_3 = \frac{A}{2},$$

So the higher connectivity of the center bank in this example corresponds to better risk sharing. One could also assume that all banks share the same size of assets and deposits, and then connectivity corresponds to more risk taking. Accordingly, we assume $w_1 = 2w_2$. Figure 2.11 illustrates the network structure.

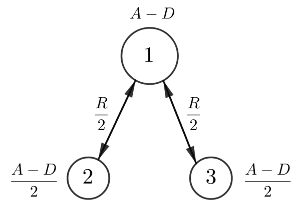

Illustration when $\tilde{A}_i = A_i, \ \forall i$.

Figure 2.11: Three-Bank Star Network Illustration

Parameter range. Assumption 3.3.1, 2.3.2, 2.3.3 hold and focus on $R > 2(A - D)$.

Under $R > 2(A-D)$, absent information design if bank 1 has bad asset, all banks default. If bank 1 and bank 2 (3) has good asset, bank 3 (2) has bad asset, bank 1 defaults but bank 2 (3) is solvent due to longer distance to asset shock. For other contingent states with bad asset(s), all banks default in the benchmark.

For notational convenience, we label the 8 states as ggg, ggb, gbg, gbb, bgg, bgb, bbg bbb and the 8 signals hhh, hhl, hlh, lhh, hll, lhl, llh, lll, in the order of bank 1, bank 2 and bank 3. E.g., $hhl: \ s_1 = h, \ s_2 = h, \ s_3 = l$.

In this example that 4 types of signals arise in equilibrium: (1) $s_h = hhh$; (2) Truth telling, e.g., reporting hhl at ggb; (3) Saving the bank more distant from shock, i.e., (lhl, ggb) and (llh, gbg); (4) $s_l = lll$. There are a couple of things worth emphasizing. First, a s that reports multiple h banks is sent at an endogenous collection of bad states such that any bank reported h is perceived as just solvent upon s. For example, truth-telling signal hhl sent at ggb is not truth-revealing, because the regulator also reports hhl at the good state ggg for cross-state lending and $\theta' \neq ggg, ggb$, such that upon hhl, investors expect bank 1 and bank 2 to have the same liquidity level on average. Second, when prior liquidity is low and the optimal information structure is an interior solution, truth-telling signals exclude reporting lhh at bgg or hll at gbb due to the strong complementarity between the each periphery bank and the center bank.

Information Structure.

1). If $(2-p)A \geq (3-2p)D$, liquidity is abundant and the regulator only reports hhh or lll.

2). If $(2-p)A < (3-2p)D$, the choice of $s \neq lll$ depends on R. When $R \geq \bar{R}$, hhh and lll show up in equilibrium, and possibly also lhl, llh. Specifically,

$$\pi^*(hhh, \theta) = \begin{cases} 1 & \text{if } m < \hat{m} \in \{1, 2\}, \\ \pi \in [0, 1] & \text{if } m = \hat{m}, \\ 0 & \text{if } m > \hat{m}, \end{cases}$$

$$\pi^*(lhl, ggb) = 1 - \pi^*(hhh, ggb), \ \pi^*(llh, gbg) = 1 - \pi^*(hhh, gbg).$$

If instead $R < \bar{R}$, we assume that hhh dominates at $\theta : m > 1$ to simplify the characterization. Then hhh, truth-telling $(hhl, ggb), (hlh, gbg)$, and lll could show up in equilibrium, depending on the parameter range.

The optimal policy is an application of Proposition 2.3.4. When prior liquidity is high (case (1)) or contagion is large, the regulator does not separate benchmark default banks. Possible reports of lhl (llh) arise in case (2) $R > \bar{R}$ because bank 2 (3) is solvent at bgb (bbg) in the benchmark, sending lhl (llh) at bgb (bbg) and another bad states allows cross-state risk sharing for bank 2 (3). When prior liquidity is lower and $R < \bar{R}$, truth-telling policies $(hhl, ggb), (hlh, gbg)$ are the most efficient marginally.

Lowering R in this region has complex effects: first the average distance to default at bad states weakly decreases, second the complementarity and thus the cross-bank risk sharing decreases, and third prior liquidity increases. Whether a separating signal that reports h on a subset of less harmed banks becomes more favorable by marginal comparison depends on the aggregate of the first two effects, and in this example $(hhl, ggb), (hlh, gbg)$ dominates due to the complementarity between each periphery bank and the center bank. The third effect pushes $(hhl, ggb), (hlh, gbg)$ to corner solution, and here hhh dominates in later rounds.

As banks could have different probabilities of being solvent in the benchmark, the following definition makes banks comparable by focusing on the cases where the regulator saves banks by borrowing from states where all banks are solvent under s_h.

Definition 2.5.1 (Preferred Treatment)**.** Let $\mathbb{P}(s|\theta;\theta_G)$ denote the conditional probability of s at a bad state $\theta \notin \Theta_G(s)$ (defined in Eq (2.17)) supported by reporting s at good state $\theta_G \in \Theta_G(s)$. Specifically, we can get $\mathbb{P}(s|\theta;\theta_G)$ by first setting $\pi(s_l, \theta_G) = 1$ and substract the resulting information structure $\boldsymbol{\pi}'$ from the optimal information structure π^*:

$$\mathbb{P}(s|\theta;\theta_G) = \pi^*(s,\theta) - \pi'(s,\theta).$$

A bank i receives preferred treatment than bank j if

$$\sum_{\theta_G \in \Theta_G(s_h)} \sum_{\theta \notin \Theta_G(s_h)} \mathbb{P}(\theta)\mathbb{P}(s|\theta;\theta_G)\mathbf{1}_{\{s_i=h\}} \geq \sum_{\theta_G \in \Theta_G(s_h)} \sum_{\theta \notin \Theta_G(s_h)} \mathbb{P}(\theta)\mathbb{P}(s|\theta;\theta_G)\mathbf{1}_{\{s_j=h\}} \tag{2.31}$$

The above definition says a bank receives preferred treatment if the regulator utilize the public excess liquidity ($\Theta_G(s_h)$) to report $s_i = h$ for bank i in the bad states with a higher probability. This rules out welfare improvement from the cross-state risk sharing of a reserve signal ($s \neq s_l, \theta$), where a subset of banks are solvent to start with. This adjustment makes banks with heterogeneous benchmark stability comparable. When all banks share the same benchmark outcome per contingent state, Eq (2.31) has a clean exposition:

$$\mathbb{P}(s_i = h) \geq \mathbb{P}(s_j = h).$$

Proposition 2.5.1. *Under Assumption 3.3.1,2.3.2,2.3.3 and $w_1 = 2w_2$, bank 1 typically receives preferred treatment except when*

$$A < \bar{A},\ \frac{D}{4} - \frac{A}{6} + 4(A - D) \geq \bar{R} > R > 4(A - D), \tag{2.32}$$

under which periphery banks could receive preferred treatment. $\bar{A}$ and $\bar{R}$ are functions of (p, D) and (A, D) respectively.

As the periphery banks are completely exposed to the center bank whereas the center bank's liability is spread-out between periphery banks, investors think it will be a rare occurrence where the center bank is subject to a large contagion (which requires both periphery banks to have bad assets). As an outcome of bank 1's better risk sharing and strong complementarity between each periphery bank, typically the regulator reports $s_1 = h$ whenever $s_2 = h$ and/or $s_3 = h$ is reported.

An exception is when (2.32) holds. When $A < \bar{A}$, prior liquidity is scarce and the regulator could separate a subset of less harmed banks with h and report l on other banks. When $R > 4(A-D)$, all banks default in the benchmark at states with asset shock(s). Then the regulator could save only bank 2 (3) by sending lhl (llh) at both ggg and ggb (gbg) if the prior liquidity is too scarce to report multiple $s_i = h$ under large R. When R is larger ($R > \bar{R}$), periphery banks lose preferred treatment because complementarity among banks increase and hhl (hlh) dominates.

2.5.2 Core-Periphery Network

We extend our result that a center bank receives preferred treatment to a general core-periphery network. We impose symmetry on banks within the core/ periphery group and focus on the difference in connectivity between groups.

Specifically, suppose there are n_c core banks and n_p periphery banks. Each core bank is connected to the same number of periphery banks, which is $\frac{n_p}{n_c}$, and each periphery bank is connected to one core. We also assume that the core banks are connected and banks are ex ante symmetric within the core/periphery part. One example is the star network. Another example is a symmetric complete network as the core part, plus each bank connected to $\frac{n_p}{n_c}$ extra banks (peripheries) via double sided interbank debt contracts, and these extra banks each has no other interbank links otherwise.

We denote a typical core bank by i^c, and a typical periphery bank by i^p. To illustrate connection between core and periphery, $c(i^p)$ is the core bank that periphery bank i^p is connected to, and $p(i^c)$ is a typical connected periphery bank for the core bank i^c. We introduce a core bank's periphery exposure as

$$\kappa(i^c) \equiv \frac{\sum_{j \in p(i)} R_{ij}}{\sum_{k \neq i} R_{ik}} = \kappa,$$

the proportion of liability borrowed from/lent to connected periphery banks.

For the parameter range, suppose that Assumption 3.3.1, 2.3.2, 2.3.3 hold. To focus on asymmetric connectivity, we assume that a bank's weight is proportional to its size

$$\frac{w_i}{w_j} = \frac{A_i}{A_j}, \ \forall i, \forall j \neq i.$$

The following proposition extends the result of preferred treatment to a general symmetric core-periphery network.

Proposition 2.5.2. *Given D and p, there exists $\hat{R}_1, \hat{R}_2$, $\hat{A}$, such that for $R \leq \hat{R}_1$ or $R \geq \hat{R}_2$ or $A \geq \hat{A}$, i^c receives preferred treatment than i^p. If each core is connected to only one periphery, $n_p = n_c$, outside of the above range ($\hat{R}_1 < R < \hat{R}_2$ and $D > \hat{D}$), periphery banks may receive preferred treatment.*

Typically core banks receive preferred treatment as the public believe it to be a rare occurrence that a core bank with spread-out interbank liabilities is subject to large contagion. Although under some circumstances periphery banks are less harmed than connected core bank due to distance, the disparity in liquidity levels is small and does not necessarily makes the regulator to single out the periphery banks with high signals.

2.6 Conclusion

We study the optimal stress test disclosure that maximizes the total solvency rate in financial networks. The network structure is exogenous, and characterizes how banks are connected via interbank liabilities. A passing stress test result enables the bank to share risk via the financial market. Disclosure has spillover effects among banks in our model: a passing result on one bank ensures repayments to its lender banks, which improves the perception of their financial health. We hence highlight that the stress test design on a financial network involves novel cross-bank risk sharing, in addition to cross-state risk sharing studied in single banks.

The optimal disclosure policy could be non-monotone in the physical states of asset realizations, and allows banks for risk sharing at worse states. This arises when

1) there is transition in system stability as the physical state deteriorates and 2) payoff of good project is small. The regulator cares more about worse states that lead to system failure, and the difference of belief deterioration between pooling different states with the good state narrows when the liquidity accumulated per extra good project is small. We highlight the value of coordinating cross-bank disclosures when designing the stress test at the system level. The regulator does not discriminate between banks, and either reports all banks are h or all banks are l, unless he is constrained with limited system liquidity and interbank exposure is relatively small, which forces him to separate the less harmed banks with h.

Optimal disclosure policy varies with the network structures. I show that stress test undoes the contagion effect shaped by the network structure, and favors interconnectedness. Specifically, in the complete network, the regulator passes the banks that would fail due to contagion; in the ring network, the regulator passes banks at least a specific distance away from the nearest asset impairment. Except for the advantage of less connected networks that connected asset shocks do not accumulate due to limited liability (depositors absorb shocks), the optimal policy brings higher value in a more connected network due to more efficient cash flow sharing. In a core-periphery network where banks are asymmetric in connectivity, I show that typically the core banks receive preferred treatment because of the more efficient risk sharing with connected counterparties. With limited system liquidity and intermediate contagion effect, periphery banks may receive preferred treatment as they are more distant from asset shocks.

Chapter 3

Selective Stress Testing in Financial Networks

3.1 Introduction

Bank stress tests have become a centerpiece of the post 2008 crisis banking supervision. Bank supervisors around the world conduct periodic bank stress tests and publicly disclose the test results to restore market confidence regarding the banking system.[1] Conducting stress testing is costly.[2] In practice, central banks in different regions have applied heterogeneous prudential rules to select the banks for stress tests, instead of unrealistically exhausting every bank. In the United States, banks above a size threshold are subject to stress tests and the Federal Reserve relaxed the threshold over the years.[3] In the European wide supervisory stress tests conducted by European Banking Authority, banks subject to stress tests are mainly selected by each country and standards are heterogeneous.[4] Choosing the optimal subset of banks for stress test is an important but under studied question. The literature thus

[1] For a detailed review of the history and goals of US stress testing, see [HL15].

[2] Financial data is retrieved from banks for the regulators to perform stress tests

[3] The first bank stress test in 2009 (Supervisory Capital Assessment Program, SCAP) involved the 19 largest US-owned BHCs; In 2014, 30 BHCs with assets of at least $50 billion participated in the two annual stress tests CCAR and DFAST; On October 10, 2019, Federal Reserve revised the prudential standards to exempt banks with total assets of less than $100 billion from the supervisory stress test, and to test banks with total assets between $100 billion and $250 billion on a two-year cycle.

[4] For example, Spain put up 95% of its banking sector against an average of about 60% in other countries.

far takes the set of stress test banks as given.

There is a growing literature that models stress test disclosure as an application of information design, and asks the optimal informativeness of stress testing to persuade market participants to act in a way that best enhances financial stability.[5] but they are mainly about stand-alone single banks. This highlights the unique feature of public disclosure to influencing market beliefs, as opposed to other regulatory tools that provide direct financial support to troubled financial institutions. However, most previous studies are stress test design of stand-alone single banks, and are thus less suitable frameworks to examine what banks should be required for stress tests in the financial system. The financial network framework became popular to characterize bank interdependence and implications on system stability. Specifically, network theories take an ex post view to see how local shocks to specific banks transmit across the system via contractual linkages among banks.

In this paper, we ask what subset of banks should be required for stress tests, taken as given the financial architecture in which banks are connected. A cash-constrained regulator commits to test a fixed number of banks and a disclosure scheme on these banks that specifies how likely each bank passes given the underlying fundamentals. In this way, the regulator is able to restore market confidence in the passing banks but needs to fail banks sometimes to make the passing results convincing to the market with low prior. Network structure is common knowledge, so disclosure about one bank influences the market beliefs of the counterparty risks on other banks, which is the vital difference from the stress test design on stand alone banks.

In our model, there are three types of risk neutral agents: a regulator minimizes the expected number of default banks by choosing the banks for stress tests and

[5]For stress test design in an information design approach to influence market beliefs, see [GL18]; [FeCMP16]; [LW17]; [OZS18]; [IP18]; [Ino19]; and others. [PP18], on the other hand, study how the regulator should design stress scenarios to learn about banks' exposures to risk factors.

disclosure policy;[6] n banks are connected via interbank liabilities which corresponds to the network structure; a financial market that provides banks with refinancing opportunities given public information. There are three dates $0, 1, 2$. At the first date, given the network structure, the regulator chooses a fixed number $m < n$ of banks for stress test, and commits to a disclosure policy which specifies how likely a bank passes or fails the test at each future contingency. Our model is an application of Bayesian persuasion ([KG11]) and information design ([BM16a, BM16b]), and is built in the framework of the latter. The key feature here is the disclosure spillovers via the interbank payment problem introduced in [EN01].

We introduce asset uncertainty to the bank balance sheets borrowed from [EN01] and [AOTS15]. Balance sheets are exogenously given at the first date $t = 0$ and are common knowledge. On the asset side, each bank has a risky loan project which is the source of uncertainty, and interbank claims on other banks. On the liability side, interbank liabilities are junior and shocks may transmit via default on the payments; there is also senior liabilities that should be paid before any payments to counterparty banks.

At the interim date $t = 1$, each bank's risky loan is realized independently. The state of nature is the collection of all banks' loan realizations, and not observed by anyone at this point.[7] Then a public signal of the m banks test results is released according to the committed disclosure policy. After that a bank chooses either to raise funds in the financial market, or stay put and wait till $t = 2$, in order to avoid default. The market value of a bank's total assets – loan plus interbank claims, which is the amount that could be raised, is determined by the Bayes rule given the public

[6]Exogenous bank weights are allowed.

[7]Our analysis is robust to the case where each bank observes its own loan project realization. Whether banks know their own types, there is remaining uncertainty regarding the types of their counter-party banks. Absent public signals, banks prefer to wait for the state to reveal in hope for the best. Hence, bank actions have no signaling effects.

signal and bank actions. If a bank's market value exceeds liabilities, it would raise enough funds and hence avoid default later. Otherwise, the bank waits in hope for the best of the underlying state, and may still be solvent if the state turns out to be favorable.

At the last date $t = 2$, the state is revealed and liabilities are due. After full payments to senior liabilities, a bank repays its interbank liabilities in proportion to the face values (pari passu). As the outgoing payments depends on what the banks receive from other banks, interbank payments are a set of fixed points such that the above rule is simultaneously satisfied for every interbank liability. We assume that a bank with bad project for sure defaults, yet a bank with good project may still default due to contagion via interbank payments.

After the set of stress test banks are chosen, we can apply the [BM16a, BM16b] framework to solve for the corresponding optimal disclosure policy. This representation connects with the idea of revelation principle, and recasts the complex multiple-agent problem of the regulator and banks into the regulator's problem: we impose obedience constraints on the way that signals are communicated, such that signals recommend actions to banks and banks would take the recommended actions under the resulting posteriors. Specifically, the regulator reports a joint signal that specifies for each stress test bank whether its value is h (high) or l (low) under the obedience constraints that results in high enough market values for banks reported with h to raise funds and survive at $t = 2$, and low market values for banks reported with l to wait in hope for the best. Note that although stress test only directly disclose on the chosen set of banks, the public signals influence the market values of the remaining banks as well, whose refinancing decisions are taken into account for the interbank payment at $t = 2$. It is also worth clarifying that l does not necessarily lead to bank failure but just denies the opportunity of risk sharing via market, nor does it imply giving up the bank in practice because here the regulator does not provide fiscal backstops but exclusively influences beliefs of the market. The optimal choice of banks for stress tests are then the set that best enhances system stability under the corresponding optimal disclosure rule.

The problem is solved in two steps. First, for each possible set of selected banks, we solve for the optimal disclosure scheme. Second, we choose the set of stress test banks that results in the highest expected numbers of solvent banks across under the corresponding optimal disclosure.

The first step of designing information structure is solved by comparing an efficiency index similar to that in [Hua19], which is a significant enrichment of the gain-to-cost ratio in [GL18] of single bank models. A bank reported with h will be solvent, on the condition that the market thinks an h bank's value is on average enough to cover the liabilities. Hence, when choosing which state to report h of a given stress test bank, the regulator trades off the incremental system solvency against how much the posterior about this bank deteriorates when informing that the state is a potential probability. The index in [Hua19] additionally captures the trade-off on the second dimension across banks, which is the efficiency of reporting h on specific banks at the same time. This is entangled with the cross-state dimension, because a coordinated refinancing makes each h bank healthier on average but imposes a tighter constraint to convince the public that the weaker banks in the set are also healthy. The major difference here is that the regulator only tests a subset of banks and are thus able to directly design beliefs about these banks, while taking into account the information spillovers to the other banks whose project uncertainties remain.

One key insight is the emphasis on systemic importance. Compared with the testing-all-bank model where it is feasible to design beliefs about contingent state project shocks and contagion effects, here the regulator could only influence the beliefs about remaining risks via the spillover from selected banks. As a result, selecting peripheral banks become less attractive as it is no longer possible to inform a very small potential contagion risk when risks from unselected banks remain. Systemically important banks, on the other hand, have a larger influence on the other banks. However, it is not always optimal to select these banks, because their project failure leads to a worse financial system and selection depends on the prior beliefs.

The other difference from the testing-all-bank model is the ambiguous effects when the size of interbank exposures R varies. Intuitively, the public knows counterparty contagion is always present, regardless of the selected banks' project risks. So cross-state risk sharing depends on R in a complex way because payoff and bank liquidity decrease state wise. When all banks are tested, on the other hand, the relative effects of R across banks is also monotone in R, as banks have different susceptibility to contagion which only arises at bad states. Here, an increase in R not only creates an incentive of selecting important banks and/ or a non-discriminatory disclosure to contain contagion, but also decreases the state-wise payoff and bank liquidity.

To highlight the influence of systemic importance, we examine the case of selecting only one bank for stress test and shut down the complementarity of multiple banks. We then examine more specific stress test policies. First, we discuss selecting only one bank for stress test to highlight the effect of the bank's systemic importance. The optimal policy alternates between selecting a more versus a less important bank as interbank exposure R increases. On the one hand, contagion becomes more important as additional banks fail but on the other hand there is a higher probability of reporting bad news. Second, we discuss how to select two banks for stress tests in a ring network to highlight the effect of interbank complementarity. In a star network example, this means that the center bank is selected for stress test when counterparty exposure is either sufficiently small or sufficiently large.

Last, we discuss how to select two banks for stress tests in a ring network to highlight the effect of interbank complementarity. We find that stress test is either "balanced" on banks positioned evenly and disclosure is non-discriminatory, or on "connected" banks and disclosure is truth-telling on potential shocks. The result is in stark contrast with that in the testing-all-bank model, where connected banks are passed together. Intuitively, connected positioning is most efficient for interbank spillover due to the direction of cross-bank risk sharing. Here, interbank complementarity becomes much smaller as only a very small subset of banks are selected and the remaining project risks dominate. Here balanced positioning arises mainly to best contain the remaining counterparty risks, whereas a truth telling disclosure that aims at sharing contagion risk is still most efficient with connected positioning.

Literature

Our paper is related to several strands of literature. The first strand is the literature on stress test. [GS14] and [Lei14] give overall discussions on the benefits and costs of regulatory disclosure on banks. Several papers study the optimal disclosures of test results, as applications of Bayesian persuasion ([KG11]) and information design ([BM16a]). In [GL18], the optimal disclosure stochastically reports bad banks to prevent market breakdown, and pools the rest of banks for cross-state risk sharing. [FeCMP16] argue that the optimal disclosure policy depends on fiscal capacity because of the costly backstops that ensue. [Wil17] examines the effects of disclosure on a bank's ex ante asset choice. [LW17] discuss whether to disclose stress test models to banks, and argue that model secrecy may lead to socially undesirable investments by banks. [OZS18] consider the design of macro-prudential stress tests with capital requirements on banks to avoid future default. A dynamic disclosure which forces weak banks to raise capital first leads to efficient recapitalization of stronger banks. [IP18] study the information design with multiple receivers in the global games framework of regime change.[8] The optimal policy removes strategic uncertainty, whereas the structural uncertainty of disagreement on fundamentals persists. [Ino19] incorporates rollover risks, and the optimal policy first discloses the banks with good assets, and additionally test the banks with bad assets on liquidity positions with contingent recapitalization requirements. All these papers examine stand-alone single banks, and to the best of our best knowledge, this is the first stress test paper to account for interbank contagion in a financial network. [FHK17] provide empirical evidence that stress testing provides information about both the tested banks and the banking industry in general, but there is no evidence on a reduction of private information production.

Our paper is also related to the more general literature of information disclosure on financial institutions. The classic concern about perfect disclosure is the Hirshleifer effect ([Hir71]) that reduces risk sharing opportunities. [DGHO17] explain why banks should be opaque, which is consistent with our implication of coordinating regulatory disclosures across banks. Other related contributions include [BCM15], [SS15], [HM16].

Our paper is an application of Bayesian persuasion with multiple receivers, and adopts the framework of information design.[9] This literature traces to [Mye86] who argues that the designer can restrict to action recommendations to the agents, in a general class of multi-stage games of incomplete information. [KG11] study the optimal persuasion between a sender and a single receiver. Other contributions on the

[8][GH16] also consider a global game model of regime change.

[9]See [Kam18], [BM19] for surveys of this literature.

single-sender, single receiver persuasion are [BC07], [RS10], [GK14]. [DM19] present a price-theoretic approach to Bayesian persuasion and characterize the conditions for monotone partitional signaling, when payoffs depend only on the mean of posterior. [EFK15], [Ely17], [Xan16] and [DE16] study persuasions in dynamic settings, and [GK16] allow for multiple senders. [BM16a, BM16b, BM19] present the information design framework with multiple receivers that unifies communication in games and Bayesian persuasion. We build on the notion of Bayes-correlated equilibrium (BCE) in [BM16a, BM16b], who argue that the set of Bayes-Nash equlibria that can arise correspond to the BCEs that are obedient – the agents take the recommended actions under the resulting posteriors. [Tan15], [MPT17], [AC16a, AC16b], [BHM15] and [BG18] also study the information design with multiple receivers. [GP18] formalize the dual problem of the [BM16a] information design framework. [GP19] study persuasion on a social network where a designer can communicate with only a limited number of agents, who then share the information with neighbors.[10]

The micro-foundation of the payoffs in our paper relates to the financial network literature.[11] In the seminal work of [AG00], interbank lending networks allow banks in different regions to share liquidity risk (a la [DD83]), and a complete network provides the most efficient risk sharing. [EN01] introduce a basic framework to study financial contagion in exogenous networks determined by interbank liabilities, and show that the set of fixed points of interbank repayments exists and is generically unique.[12] [AOTS15] study the extent of financial contagion under different network structures. We introduce uncertainty, and study the information design to maxi-

[10] [ES19] study how the optimal level of propaganda depends on the social network structure.

[11] For surveys see [AB09], [GY16] and [Sum13].

[12] This departure from the possible multiple equilibrium that arise from coordination on payments requires positive net cash flows in a subset of the network, or as in [AOTS15], the existence of senior liabilities.

mize system stability as determined by the interbank payments. Other contributions on financial contagion and system stability in exogenous network structures include [Das04], [CS13], [EGJ14], [GY15] and [GPY15]. Recent work on network interventions include [AB15], [GGG17], [Kan18], [Ram19], [BCS17] and [Ero18]. The closest to our paper is [AB15], who discuss whether mandatory disclosure can improve welfare as opposed to voluntary disclosures by banks who have strategic substitutability or complementarity in equity levels due to contagion. Other papers that study the role networks play in strategic behaviors include [BCAZ06], [GGJ+10], [BKD14], [CGG17], [BK18]. [BK18] micro-found a [Kyl89] model in connected intermediaries and analyze how decentralization affect information diffusion. There is also a growing literature that considers endogenous interbank linkages: [GVR05], [Far17], [EO17], [Wan16], [CM18] and others.

The rest of the paper is organized as follows. Model setting is presented in Section 2. Section 3 solves the problem and provides general properties of the optimal policy. Section 4 discusses the policy that tests only one bank to highlight systemic importance. Section 5 discusses the policy on a ring network to highlight interbank complementarity. Section 6 concludes.

3.2 Model Setup

The economy lasts for three dates $t = 0,\ 1,\ 2$ without discounting, and there are three types of risk-neutral agents: banks, a regulator and investors. The banking sector consists of n banks indexed by $i = 1,\ 2, \cdots,\ n$; the network structure corresponds to the collection of interbank liabilities that connects the banks, and is exogenous and common knowledge. Banks may raise cash from a continuum of Bayesian investors at their market values. The regulator maximizes the expected system stability by

Bank i's Balance Sheet (Book Value)

Assets:	**Liabilities:**
Risky loan $\tilde{A}_i$	Senior liabilities D_i
Interbank claims $\sum_{j\neq i} R_{ij}$	Interbank liabilities $\sum_{j\neq i} R_{ji}$
	Equity

Figure 3.1: A typical Bank's Balance Sheet

choosing a subset $m \leq n$ of banks for stress tests and committing to a disclosure policy at $t = 0$, before shocks realize at $t = 1$.

Banks and Financial Network. We introduce uncertainty to the financial network model of [EN01] and [AOTS15], who focus on interbank payments at contingent states. At the beginning of $t = 0$, bank balance sheet is exogenously given as follows and is common knowledge:

On the asset side, each bank i has a risky loan asset that delivers binary random payoff $\tilde{A}_i \in \{A_i > 0,\ 0\}$ independently across banks at the beginning of $t = 1$.

$$\mathbb{P}(\tilde{A}_i = A_i) = p_i,\ \mathbb{P}(\tilde{A}_i = 0) = 1 - p_i$$

is the common prior. State of nature $\theta \in \Theta$ is the collection of loan asset realizations:

$$\Theta \equiv \left\{\theta = (\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n) | \tilde{A}_i(\theta) \in \{A_i, 0\},\ \forall i = 1, \cdots, n\right\}.$$

Note that only a subset of banks are required for stress tests, so the states about which the regulator chooses the extent of information sharing are a partition of Θ related to the stress test banks. Bank i also has claims on its borrower banks, $\sum_{j\neq i} R_{ij}$, where R_{ij} is the face value lent to bank j. Throughout this paper, we use the first subscript to indicate the lender and the second subscript to indicate the borrower for notations regarding directed lending.

On the liability side, each bank i has deposits of face value D_i as senior liabilities, and junior interbank liabilities of face value $\sum_{j \neq i} R_{ji}$, in which lender banks are pari passu. All liabilities are due at the last date $t = 2$, when bank i uses the available cash flows, which is either its project return and payments from other banks or cash raised at $t = 1$ which we will characterize later, to first repay deposits, and then to lender banks. In the case of default, after deposits payments the bank pays lender banks in proportion to the face values. Banks are also protected by limited liability. We will later characterize the payment equilibrium of banks, which is a set of fixed point such that the above payment rule is satisfied for every interbank liability $R_i j$. Last, bank equity is the residual value.

At $t = 1$, banks can either raise funds against total assets or not in order to avoid default at $t = 2$, because we assume that there is a sufficiently large default penalty and bank i's utility at $t = 2$ is:

$$u_i = \begin{cases} 0, & \text{solvent}, \\ -K < 0, & \text{default}. \end{cases} \tag{3.1}$$

We emphasize that the network structure corresponds to the collection of interbank liabilities $\{R_{ij}\}$.

Information Design. At $t = 0$, the regulator chooses $m \leq n$ out of the total n banks for stress tests. We denote the collection of all banks as $\mathcal{N}$, and the subset subject to stress tests as $\mathcal{M}$. In addition, the regulator commits to an information structure $\{\boldsymbol{S}_{\mathcal{M}}, \boldsymbol{\pi}_{\mathcal{M}}\}$ which characterizes how much information about the projects of $\mathcal{M}$ to share publicly, in order to optimally influence the banks' refinancing opportunities at the interim date $t = 1$.

The information structure consists of the signal space $\hat{\mathbf{S}}_{\mathcal{M}}$, which characterizes all the available signals to report about $\mathcal{M}$, and distribution of signals $\boldsymbol{\pi}_{\mathcal{M}}$, which is the conditional probability of reporting any signal given project realizations of $\mathcal{M}$. Our model assumes a binary action set for each bank, and it is without loss of generality to assume binary signals for each bank in $\mathcal{M}$.[13] Hence,

$$\boldsymbol{S}_{\mathcal{M}} \equiv \{s = (\cdots, s_i, \cdots)|i \in \mathcal{M},\ s_i = h \text{ or } l\},$$

where a typical signal $s \in \hat{\mathbf{S}}_{\mathcal{M}}$ is an m-element vector. Let

$$\Theta_{\mathcal{M}} \equiv \{\theta_{\mathcal{M}}(\cdots, \tilde{A}_i, \cdots)|i \in \mathcal{M}\}$$

represent the project realizations of banks subject to stress tests $\mathcal{M}$. A typical element $\theta_{\mathcal{M}} \in \Theta_{\mathcal{M}}$ corresponds to a collection of elements $\theta \in \Theta$ for given project realizations of banks in $\mathcal{M}$. Distribution of signals $\boldsymbol{\pi}_{\mathcal{M}}$: $\Theta_{\mathcal{M}} \to \Delta(\boldsymbol{S}_{\mathcal{M}})$ specifies the conditional probability of receiving any stress test results $s \in \boldsymbol{S}_{\mathcal{M}}$ given any collection of the tested banks' asset realizations, $\theta_{\mathcal{M}} \in \Theta_{\mathcal{M}}$. $\boldsymbol{\pi}_{\mathcal{M}}$ is an $|\boldsymbol{S}_{\mathcal{M}}| \times |\Theta_{\mathcal{M}}|$ matrix, and a typical element $\boldsymbol{\pi}_{\mathcal{M}}$ is

$$\pi_{\mathcal{M}}(s, \theta_{\mathcal{M}}) \equiv \mathbb{P}(s|\theta_{\mathcal{M}}) \in [0, 1],\ \text{and} \sum_{s \in \boldsymbol{S}_{\mathcal{M}}} \pi_{\mathcal{M}}(s, \theta_{\mathcal{M}}) = 1,\ \forall \theta_{\mathcal{M}} \in \Theta_{\mathcal{M}}.$$

Therefore, the regulator commits to a stress test policy $\{\mathcal{M}, \pi_{\mathcal{M}}\}$ at $t = 0$, which includes the subset of banks required for stress tests and a disclosure policy on these banks. Hence, the regulator directly influences the beliefs about only the banks in

[13] We will see later that the actions of non-stress test banks might be a source of multiple equilibrium. Under the parameter assumption of low priors multiplicity does not arise. Hence, extra signals are not needed for coordination purpose.

$\mathcal{M}$, but takes into account the information spillover to the remaining banks.[14]

Now we characterize how the policy influences banks' refinancing opportunities at $t = 1$. At the beginning of $t = 1$, state $\theta = (\tilde{A}_1, \cdots, \tilde{A}_n)$ realizes, which is not observed by anyone.[15] Then a public signal s about the selected banks $\mathcal{M}$ is released according to $\boldsymbol{\pi}_{\mathcal{M}}$. Each bank i, including those not selected for stress tests, chooses either to "raise funds" in the financial market, or or "wait" till $t = 2$. We use x_i to represent bank i's action, and $x \in X \equiv X_1 \times \cdots \times X_n$ the collection of all banks actions. We assume that a bank could only refinance against its total assets– loan plus interbank claims receivable, and the amount raised is financial market's Bayes' posterior $m: \mathbf{S}_{\mathcal{M}} \times \boldsymbol{\pi}_{\mathcal{M}} \times X \to \mathbb{R}^n$ given the public signal s and bank actions x.[16]

Default penalty (see Eq (3.1)) at $t = 2$ creates the incentive for risk sharing at $t = 1$. If a bank's market value m_i is enough to repay total liabilities, the bank raises funds and hence avoid default, and otherwise waits till $t = 2$ but may not necessarily default depending on the underlying state:

$$x_i = \begin{cases} \text{raise funds}, & \text{if } m_i \geq D_i + \sum_{j \neq i} R_{ji}, \\ \text{wait}, & \text{if } m_i < D_i + \sum_{j \neq i} R_{ji}, \end{cases} . \tag{3.2}$$

[14] Note that the above information structure is different from the mapping $\Theta \to \Delta(\boldsymbol{S}_{\mathcal{M}})$, which corresponds to testing all the banks but commits to directly disclose signals on a subset of $\mathcal{M}$ banks.

[15] Our model is robust to the case where each bank i privately observes its own asset realization $\tilde{A}_i$. Whether banks know their types, there is remaining uncertainty about the actual interbank payments they receive, which depend on the other banks' types. Absent public signals, they prefer to wait in hope for the best of the underlying state to be revealed. In contrast, there is signaling on private information in [GL18], where the reservation values of trading in the financial market are heterogeneous across bank types. In our application [BM16a]'s framework, the obedience constraints are thus degenerate in bank types. Hence, without loss of generality we assume that the state is latent at $t = 1$.

[16] This could be interpreted as derivative contracts that creates super-seniority for the investors, or directly selling assets for cash as in [GL18]. In contrast, the financial market in [OZS18] serves as the means of costly liquidation, where the regulator requires banks to sell part of the risky asset for cash buffers against future loss.

Without loss of generality we can represent the information design problem by the [BM16a] framework in which signals recommend actions to banks, h for "raise funds" and l for "wait", and accordingly restricts how signals are sent ($\boldsymbol{\pi}_{\mathcal{M}}$) with obedience constraints. These constraints ensures that under the resulting posterior of the signal, banks in $\mathcal{M}$ would indeed take the recommended action. Let $\psi : \mathbf{S}_{\mathcal{M}} \to X_{\mathcal{M}}$ represent decision rules of banks subject to stress tests. Then signal recommendations say

$$x_i = \phi(x) = \begin{cases} \text{raise funds}, & \text{if } s_i = h, \\ \text{wait}, & \text{if } s_i = l, \end{cases} \tag{3.3}$$

and obedience constraints ensures that the resulting bank i's interim market value m_i is in the consistent range for the bank to take the recommended actions,

$$m_i(s, \boldsymbol{\pi}, \psi(s)) \geq D_i + \sum_{j \neq i} R_{ji} \text{ if } s_i = h,$$
$$m_i(s, \boldsymbol{\pi}, \psi(s)) < D_i + \sum_{j \neq i} R_{ji} \text{ if } s_i = l.$$

If prior about banks is low, $pA_i + \sum_{j \neq i} R_{ij} < D_i + \sum_{j \neq i} R_{ji}$ for some i, an example of violation is to always report all banks are h,

$$s_h : \; s_i = h, \; \forall i \in \mathcal{M}; \; \pi(s_h, \theta)_{\mathcal{M}} = 1, \; \forall \theta. \tag{3.4}$$

Then banks are valued at the prior, under which no bank is willing to raise funds.

Note that the [BM16a] framework assumes coordination among banks given the public signal. Here the actions of non-test banks $i \notin \mathcal{M}$ might be a source of multiple equilibrium. Later we impose a parameter assumption that ensures waiting is a dominant strategy for the banks that are not tested.[17]

Interbank Payments. At $t = 2$, the state is revealed and liabilities are due. Let $y_{ij}(s, \theta)$ denote the actual payment from bank j to bank i given the signal s and the revealed state θ,

$$y_{ij}(s,\theta) = \begin{cases} R_{ij}, & s_j = h, \\ \left\{\min\left[R_{ij}, \frac{R_{ij}}{\sum_{k\neq j} R_{kj}}\left(\tilde{A}_j(\theta) + \sum_{k\neq j} y_{jk}(s,\theta) - D_j\right)\right]\right\}^+, & s_j = l. \end{cases} \tag{3.5}$$

Bank j's available cash flows at $t = 2$ is either cash raised at $t = 1$, or project returns plus the payments from borrower banks. If $s_j = h$, bank j raised enough cash and repays in full. If instead $s_j = l$, whether it defaults is determined by the payment rule (3.5): after full deposit payments, it repays junior creditor bank i up to the face value R_{ij}, and when default pays out all cash flows $\tilde{A}_j(\theta) + \sum_{k\neq j} y_{jk}(s, \theta) - D_j$ in proportion to the lender's share of total liabilities $\frac{R_{ij}}{\sum_{k\neq j} R_{kj}}$ and is protected by limited liability. Interbank payments $\{y_{ij}\}$ are a set of fixed point in which the above payment rule (3.5) is simultaneously satisfied for every interbank liability.[18]

Regulator's Payoff. The regulator commits to a stress test policy $\{\mathcal{M}, \pi_{\mathcal{M}}\}$ at $t = 0$ to maximize the total weighted number of banks that survive. Let w_i be the exogenous weight of bank i, $v(s, \theta)$ the contingent state payoff at $t = 2$, and $V_{\mathcal{M}}(\boldsymbol{\pi_{\mathcal{M}}})$

[17] The assumption extends the idea of focusing on one specific equilibrium as is standard in the information design literature, and is embedded in the low prior assumption in the model where all banks are tested. If the condition is relaxed and multiplicity arises, we suggest that banks $i \notin \mathcal{M}$ coordinate to act on the worst outcome. This is a reasonable assumption because stress test is costly and provides coordination in addition to share information about fundamental.

[18] The payment rule here is consistent with [EN01] and [AOTS15], where the $s_j = h$ case is absent. $s_i = h$ is as if assets were sold with enough cash in bank i, so that it always make full payments under their payment rule.

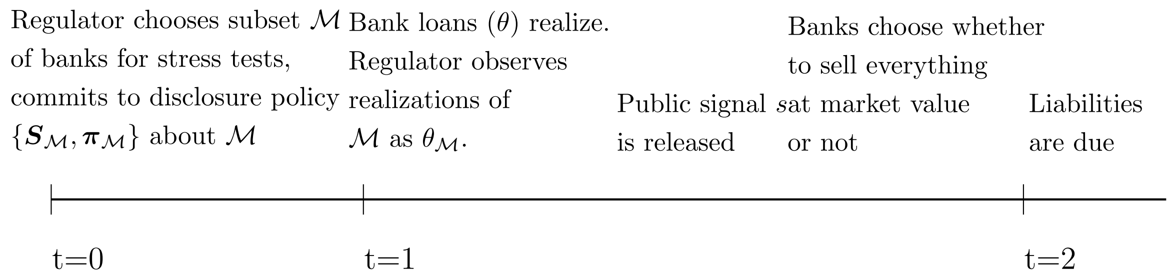

Figure 3.2: Model Timeline

the regulator's payoff at $t = 0$:

$$v(s,\theta) = \sum_i w_i \mathbf{1}_{\{\sum_{j\neq i} y_{ji}(s,\theta) = \sum_{j\neq i} R_{ji}\}}, \tag{3.6}$$

$$V_{\mathcal{M}}(\boldsymbol{\pi_{\mathcal{M}}}) = \sum_{\theta_{\mathcal{M}}\in\Theta_{\mathcal{M}}} \mathbb{P}(\theta_{\mathcal{M}}) \left[\sum_{s\in\boldsymbol{S}_{\mathcal{M}}} \pi_{\mathcal{M}}(s,\theta_{\mathcal{M}}) \mathbb{E}_{\theta}\left[v(s,\theta)|\theta_{\mathcal{M}}\right] \right]. \tag{3.7}$$

Timeline and Model Summary. The model timeline is summarized as follows:

Regulator's Problem. The regulator's problem $\mathcal{P}_{\mathcal{M}}$ is to choose the optimal stress test policy $\{\mathcal{M}, \pi_{\mathcal{M}}\}$ that includes the subset of banks for stress testing $\mathcal{M}$ and a corresponding disclosure scheme $\boldsymbol{\pi_{\mathcal{M}}}$ to maximize the expected system stability, while taking into account the uncertainty of the untested banks' projects and the counterparty risks across the network. Specifically,

$$\max_{\mathcal{M},\boldsymbol{\pi_{\mathcal{M}}}} V_{\mathcal{M}}(\boldsymbol{\pi_{\mathcal{M}}}) \equiv \sum_{\theta_{\mathcal{M}}\in\Theta_{\mathcal{M}}} \mathbb{P}(\theta_{\mathcal{M}}) \left[\sum_{s\in\boldsymbol{S}_{\mathcal{M}}} \pi_{\mathcal{M}}(s,\theta_{\mathcal{M}}) \mathbb{E}_{\theta}\left[v(s,\theta)|\theta_{\mathcal{M}}\right] \right]$$

$$\text{(Obedience: } h) \qquad s.t. \sum_{\theta_{\mathcal{M}}\in\Theta_{\mathcal{M}}} \left\{ \frac{\mathbb{P}(\theta_{\mathcal{M}})\pi_{\mathcal{M}}(s,\theta_{\mathcal{M}})}{\sum_{\theta_{\mathcal{M}}\in\Theta_{\mathcal{M}}} \mathbb{P}(\theta)\pi_{\mathcal{M}}(s,\theta_{\mathcal{M}})} \cdot \mathbb{E}_{\theta}\left[L_i(s_{-i}, s_i = l, \theta)|\theta_{\mathcal{M}}\right] \right\}$$

$$\geq \sum_{j\neq i} R_{ji}, \ (\forall s_i = h) \tag{3.8}$$

$$\text{(Obedience: } l\text{)} \qquad \sum_{\theta_{\mathcal{M}}\in\Theta_{\mathcal{M}}} \left\{ \frac{\mathbb{P}(\theta_{\mathcal{M}})\pi_{\mathcal{M}}(s,\theta_{\mathcal{M}})}{\sum_{\theta_{\mathcal{M}}\in\Theta_{\mathcal{M}}} \mathbb{P}(\theta)\pi_{\mathcal{M}}(s,\theta_{\mathcal{M}})} \cdot \mathbb{E}_{\theta}\left[L_i(s,\theta)|\theta_{\mathcal{M}}\right] \right\}$$

$$< \sum_{j\neq i} R_{ji}, \ (\forall s_i = l) \tag{3.9}$$

$$\text{(Prior Consistency)} \qquad \sum_{s\in \boldsymbol{S}_{\mathcal{M}}} \pi_{\mathcal{M}}(s,\theta_{\mathcal{M}}) = 1, \ (\forall \theta_{\mathcal{M}}) \tag{3.10}$$

$$\text{(Probability)} \qquad 0 \leq \pi_{\mathcal{M}}(s,\theta_{\mathcal{M}}) \leq 1. \ (\forall \theta, \mathbf{s}) \tag{3.11}$$

The constraints on the information structure $\boldsymbol{\pi_{\mathcal{M}}}$ are two sets of obedience constraints which ensure banks reported with h refinance and banks reported with l wait, prior consistency constraints that for each possible project realizations of tested banks the conditional probability of reporting all signals add up to 1, and probability constraints of each $\pi_{\mathcal{M}}(s,\theta_{\mathcal{M}})$.

3.3 Existence, Uniqueness and Methodology

In this part, we argue that the optimal policy $\{\mathcal{M}, \pi_{\mathcal{M}}\}$ of selecting banks for stress tests and the corresponding information structure exists and is generically unique. Then we characterize how to solve for the optimal policy.

Assumption 3.3.1. *(1) (Low Prior)*

$$pA_i + \sum_{j\neq i} R_{ij} < D_i + \sum_{j\neq i} R_{ji}, \ \forall i = 1, \cdots, n. \tag{3.12}$$

(2) There exists a bank i, such that

$$A_i + \mathbb{E}_{\theta}[\sum_{j\neq i} R_{ij}|\tilde{A}_i = A_i] \geq D_i + \mathbb{E}_{\theta}[\sum_{j\neq i} R_{ji}|\tilde{A}_i = A_i] \tag{3.13}$$

Assumption 3.3.1 adds a second part compared with the model that tests all banks. The first part guarantees that without any disclosure to signal a bank's health, the bank is expected to default regardless of counterparty payments. This part rules out the uninteresting case of always reporting h for any bank.[19] The second part says there exists a bank i, such that if it has a good project, it is solvent in expectation; otherwise the only feasible information structure is to always report l for all banks.

Lemma 3.3.1. *We can solve the relaxed problem $\mathcal{P}'_{\mathcal{M}}$ without the obedience constraint of l (3.9).*

Proof: See Appendix. ■

Proposition 3.3.1 (Existence)**.** *For any (s, θ), the interbank payment equilibrium $\{y_{ij}(s, \theta)\}$ exists and is generically unique.*[20] *The optimal subset of banks for stress test $\mathcal{M}^*$ and the corresponding information structure $\boldsymbol{\pi}^*_{\mathcal{M}^*}$ exist.*

Proof: See Appendix. ■

Proposition 3.3.2 (Uniqueness)**.** *The optimal policy $\{\mathcal{M}^*, \boldsymbol{\pi}_{\mathcal{M}}{}^*\}$ is generically unique.*

Conditional on selected banks for stress tests $\mathcal{M}$, there exists an index that summarizes the efficiency of disclosing any potential shocks in $\mathcal{M}$, and an algorithm to derive the optimal information structure $\boldsymbol{\pi}^*_{\mathcal{M}}$ based on the index.

Proposition 3.3.3 (Algorithm)**.** *All obedience constraints in $\mathcal{P}'_{\mathcal{M}}$ are binding. Efficiency index $\xi^{(k)}$ are similarly defined as in the model that tests all banks, except for*

[19] (3.12) is a sufficient condition and could be relaxed with $m < n$. If bank i is not tested, in expectation the maximum repayment from bank i is pR_{ji} to bank j. Nevertheless we use (3.12) for unified representation regardless of $\mathcal{M}$, and the implication that banks not tested will wait.

[20] In the non-generic case, there may exist a continuum of payment equilibria. For example, in a system of two symmetric banks, $R_{12} = R_{21} = R$, $D_1 = D_2 = 0$, $\theta : \tilde{A}_1 = \tilde{A}_2 = 0$, then any set of payments with $y_{12} = y_{21} \in [0, R]$ is an equilibrium. The key for generic uniqueness is the outside-network flows. Eg., $y_{12} = y_{21} > 0$ cannot be an equilibrium if $D_1 > 0$, which should be paid first from the y_{12} received.

switching to conditional information $\mathbb{E}_\theta[\cdot|\theta_{\mathcal{M}}]$ *in the representation. In addition, let*

$$s_h : s_i = h, \text{ for all } i \in \mathcal{M},$$

we have

$$\xi^{(1)}(s_h(\mathcal{M}), \theta_{\mathcal{M}}; \theta_0) = \underbrace{\mathbb{E}_\theta\left[v(s_h(\mathcal{M}), \theta) - v_0(\theta)|s_h(\mathcal{M})\right]}_{\text{Incremental System Stability}} \cdot \frac{\mathbb{P}(\theta_0)\left(\sum_{k\in\mathcal{M}} \frac{1}{\frac{a_k - a'_k}{a_k}}\right)}{1 - \left(\sum_{k\in\mathcal{M}} \frac{1}{\frac{a_k - a'_k}{a_k}}\right)}, \tag{3.14}$$

which is increasing in incremental system stability and decreasing in the effect of bank i*'s project shock* $\frac{a_k - a'_k}{a_k}$ *as defined in (A.3).*

Proof: See Appendix. ■

The first part of Proposition 3.3.3 characterizes how to solve for the optimal disclosure about $\mathcal{M}$ based on an index that summarizes the efficiency of informing any possible potential shocks when some banks are reported with h. The methodology is very similar to that in the testing-all-bank model except for changing contingent state representations to conditional representations. All obedience constraints regarding h signals are binding, which means any bank reported with h exhausts the excess credibility at good states and is expected to be just solvent on average. This is because project shock is likely for any bank, and is more serious than any counterparty contagion.

The second part of Proposition 3.3.3 says index representation could be further simplified under certain circumstances. This is essentially because the regulator inspects only a subset of banks, he shares information about coarser states. As a result, first the dimension of $\boldsymbol{\pi}_{\mathcal{M}}$ shrinks. Second, signals are more informative about the expected system stability: banks reported with h survive and repay full amounts to other banks, and payments from other banks are largely shaped by the remaining project risks in the system.

Equation (3.14) is the efficiency index of reporting all tested banks pass. It consists of two parts: how would reporting these banks pass increase the expected system stability, and the probability to report h on these banks for the market to believe h means solvent on average.[21] The key takeaway in the representation is that the regulator cares more about systemic risk. systemically important banks have a larger influence on both the survival probability and the payments of the other banks. As we will see later, the latter part could work against selecting systemically important banks, because they are worse in health when projects fail as they receive less interbank payments from network feedback. Hence, prior is low, the regulator might prefer selecting peripheral banks to systemically important banks with high probability of shocks.

This emphasis on systemic importance extends to general disclosure policies in selective stress testing. Compared with the testing-all-bank model where it is feasible to design beliefs about any project risks and counterparty contagion effects, here he could only influence the remaining risks via the spillover from selected banks. First,

[21] $a_k - a'_k$ has two parts: 1) a direct effect of losing project return $A_{(k)}$ of the k-th bank in $\mathcal{M}$; 2) the indirect effect of lost in expected interbank payments $\Delta^{\tilde{A}_{(k)}=A_{(k)}}_{\tilde{A}_{(k)}=0}\mathbb{E}_\theta\left[\sum_{j\neq(k)} y_{(k)j}|s_{-(k)},\tilde{A}_{(k)}\right]$, conditional on other banks in $\mathcal{M}$ paying in full. Hence, the second part is the remaining counterparty uncertainty

$$\Delta^{\tilde{A}_{(k)}=A_{(k)}}_{\tilde{A}_{(k)}=0}\mathbb{E}_\theta\left[\sum_{j\neq(k)} y_{(k)j}|s_{-(k)},\tilde{A}_{(k)}\right] = \Delta^{\tilde{A}_{(k)}=A_{(k)}}_{\tilde{A}_{(k)}=0}\mathbb{E}_\theta\left[\sum_{j\notin\mathcal{M}} y_{(k)j}|s_{-(k)},\tilde{A}_{(k)}\right].$$

Choosing $\mathcal{M}$ to decrease remaining counterparty uncertainty is consistent with increasing the other part in the index, incremental system stability

$$\mathbb{E}_\theta\left[v(s_h(\mathcal{M}),\theta) - v_0(\theta)|s_h(\mathcal{M})\right].$$

in a testing-all-bank model, peripheral banks may be favorable as the regulator has the option to inform a very small potential counterparty risk corresponding to a contingent state, whereas here there are always potential counterparty risks from unselected banks. Second, this introduces a wedge on selecting systemically important banks who have a larger influence on both the survival probability and the payments of the other banks.

The following lemma characterizes the properties of signals that report l on some banks if they arise in equilibrium.

Lemma 3.3.2. *For a given set of stress test banks $\mathcal{M}$, if the corresponding optimal information structure reports $s \neq s_h(\mathcal{M}) \equiv (h, \cdots, h), s_l(\mathcal{M}) \equiv (l, \cdots, l)$, i.e.,*

$$s : \exists i, j \in \mathcal{M}, s_i = h, s_j = l; \text{ and } (s, \Theta_{\mathcal{M}}^{(1)*}(s, \theta_{\mathcal{M}})) = \arg\max \xi^{(1)}(s, \theta_{\mathcal{M}}),$$

then there exists a bank $\hat{i} \in \mathcal{M}$ and $s_{\hat{i}} = h$ who is subject to only contagion risk when s is reported:

$$\exists \hat{i} \text{ with } s_{\hat{i}} = h : \tilde{A}_{\hat{i}} = A_{\hat{i}} \text{ for all } \theta_{\mathcal{M}} \in \Theta_{\mathcal{M}}^{(k)*} \equiv \{\theta_{\mathcal{M}} | \boldsymbol{\pi}^*{}_{\mathcal{M}}(s, \theta_{\mathcal{M}}) > 0\}.$$

Similar to the model that tests all banks, under some parameter range the optimal policy reports l on some banks. This arises when the potential shocks for some h banks come from their counterparties and not themselves. Then a smaller number of banks are able to refinance with a higher probability compared with all banks in $\mathcal{M}$ refinancing with a smaller probability because project shocks are more serious than counterparty contagion.

3.4 Selecting One Bank

In this section, we examine the case of selecting only one bank for stress test $m = 1$ to highlight the influence of systemic importance, while shutting down the complementarity of designing the beliefs of multiple banks. First we derive the efficiency index for a typical bank, and discuss how the size of interbank exposure R influences bank choice. Then we exemplify the trade-offs using a three-bank star network example.

When $m = 1$, both $\Theta_{\mathcal{M}}$ and $\boldsymbol{S}_{\mathcal{M}}$ are binary, which greatly simplifies the information structure. From the second part of Assumption 3.3.1, some banks are solvent when $\tilde{A}_i > 0$. The regulator only selects from these banks, and when selected, the bank is always reported with h when it has good project, $\pi_{\mathcal{M}}(s_i = h, \tilde{A}_i = A_i) = 1$. Then the key considerations for selecting the bank is, i) the expected system stability when the bank is solvent (reported with h) and when the bank has bad project and defaults (reported with l); ii) the probability of reporting h or l.

For notational convenience, let

$$\chi_i(s_i = h, \tilde{A}_i = 0) \equiv \mathbb{P}(\tilde{A}_i = 0)\pi_{\mathcal{M}}(s_i = h, \tilde{A}_i = 0). \tag{3.15}$$

Then

$$p_i + \chi_i = \mathbb{P}(s_i = h).$$

Lemma 3.4.1. *The efficiency index associated with selecting bank i for stress test is,*

$$\xi_i = p_i \left\{ \left(1 + \frac{\chi_i}{p_i}\right) \cdot \mathbb{E}_\theta \left[v(s,\theta)|s_i = h\right] + \left[\frac{1}{p_i} - \left(1 + \frac{\chi_i}{p_i}\right)\right] \cdot \mathbb{E}_\theta \left[v_0(\theta)|\tilde{A}_i = 0\right] \right\}. \tag{3.16}$$

If $\sum_{j \neq i} p_j R_{ij} \leq R_i \equiv \sum_{j \neq i} R_{ji}$, ξ_i *is decreasing in* R.

Proof: See Appendix. ■

Lemma 3.4.1 characterizes the efficiency index of selecting a typical bank i. It is a weighted average of the expected system stability when the bank is solvent and when the bank has bad project and defaults, and the weight is the probability of reporting h or l. The second part says, as long as a bank's total interbank claims is not much higher than interbank liabilities, the expected payoff is decreasing in the size of interbank exposures regardless of what bank is selected for stress tests. Intuitively, any bank's health is expected to worsen with higher level of interbank dependence, which allows a larger proportion of project shocks from counterparties to go through. If a bank's exposure is unbalancedly higher in interbank claims, its healthiness may be predominantly determined by payments from other banks. In that case, the effect of the scale of exposure is ambiguous.

Now we discuss how the size of interbank exposures R influences the relative payoffs of selecting a more systemically important bank versus a less systemically important bank. We introduce the following definition of systemic importance in the context of this section.

Definition 3.4.1 (systemic importance)**.** For bank i and j, if

$$\mathbb{E}_\theta\left[v(s,\theta)|s_i = h\right] \geq \mathbb{E}_\theta\left[v(s,\theta)|s_j = h\right] \text{ and } \mathbb{E}_\theta\left[v_0(\theta)|\tilde{A}_i = 0\right] < \mathbb{E}_\theta\left[v_0(\theta)|\tilde{A}_j = 0\right], \tag{3.17}$$

bank i is more systemically important than bank j.

Inequalities (3.17) means bank i and largely determines system stability due to potential contagion effects on other banks and is thus systemically important. For example, an extreme case is that a center of the star network and parameters are such that all peripheral banks with good assets will default or survive depending on whether the center bank delivers payments. On the other hand, bank j is less systemically important as system stability is largely determined by other banks. An extreme example is when j is an isolated bank, and therefore influencing the refinancing opportunity of j only makes the difference of bank j's own survival.

The following proposition discusses how R influences selecting a systemically more important bank for stress test.

Proposition 3.4.1. *Suppose all banks have equal weights $w_i = 1$ and the asset shock probability $p_i = p$. For a systemically more important bank i and less important bank j as characterized in Definition 3.4.1, there exists a set of cutoffs $\{\hat{R}\}$ such that in the intervals determined by $\hat{R}$, the relative magnitude of the efficiency index ξ_i and ξ_j, or selection between bank i and j, alternates.*

Proof: See Appendix. ■

As the health of a systemically important bank i decisively determines the system stability, a signal about i is very informative and the regulator's payoff depends on the probability to report h. On the other hand, system stability changes less whether a less important bank j survives, but rather depends on the prior belief or the size of contagion cascades on average.

The benefit of selecting a systemically important bank is not monotone in R because there are two effects when R increases. First, certain thresholds of R trigger phase transition of system stability, as additional banks fail due to the same set of project shocks and the size of contagion cascades increases. This makes it favorable to test a systemically important bank because otherwise disclosure about other banks is less informative of system stability, whose prior is low for large R. Second, as argued in Lemma 3.4.1, the probability of reporting h decreases regardless which bank is selected. Hence, for a range of R in which the prior of contagion cascade is small, but R is relatively large such that disclosure about a systemically important bank involves a high probability of l, the regulator prefers an uninformative disclosure by selecting an unimportant bank.

3.4.1 Example: Three-bank Star Network

This subsection illustrates the tradeoffs in Proposition 3.4.1 using a star network example. Bank 1 is the center bank, and bank 2 and 3 are two symmetric smaller periphery banks. Specifically,

$$w_i = 1; \ \mathbb{P}(\tilde{A}_i = A_i) = p;$$
$$A_1 = 2A_2 = 2A_3 = A, \ D_1 = 2D_2 = 2D_3 = D,$$
$$R_{12} = R_{13} = R_{21} = R_{31} = \frac{R}{2}, R_{ij} = 0 \text{ otherwise.}$$

Figure 3.3 illustrates the efficiency index ξ_i as a function counterparty exposure R, and thus summarizes how the regulator selects the bank for stress test as R increases. ξ_1 is illustrated in red color. $\xi_2 = \xi_3$ is illustrated in grey and is discontinuous at $R = A - D, 2(A - D)$ and $4(A - D)$. These discontinuity represents the increasing size of contagion cascades at critical values, as illustrated by different shades of the rectangular areas.

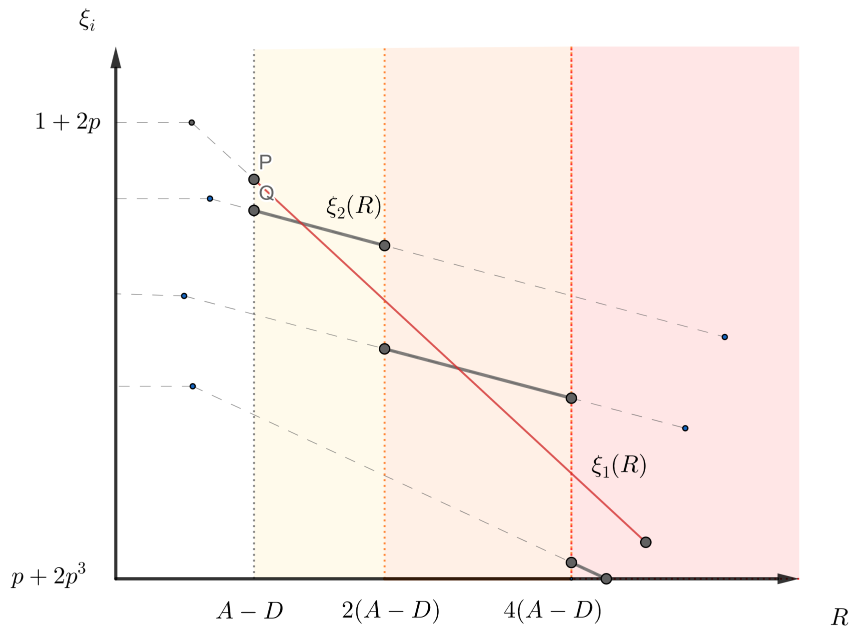

Figure 3.3: Efficiency Index as Function of R

The efficiency index ξ_i is linear because it is a weighted average of payoffs when $s_i = h$ the bank is solvent and when $s_i = l$ which reveals that the bank has bad asset. R determines the probability of $s_i = l$. From Lemma 3.4.1 we know ξ_i is downward sloping. Note that the weight $1 + \frac{\chi_i}{p}$ is generically different across banks.

Bank 1 is the systemically important bank, as illustrated by the steeper slope of ξ_1 and a continuous function when $R > A-D$. Intuitively, the expected system stability changes drastically depending on whether bank 1 has good asset. In addition, how the center bank determines system stability does not change with the shock size. On the other hand, the peripheral banks 2 and 3 only influences the system stability to a smaller extent and hence the flatter slope of ξ_2. The overall system stability depends on the size of contagion cascade, as illustrated by the downward jump of ξ_2 as R exceeds critical values where additional banks default due to contagion.

Therefore, the optimal policy depends on the expected profitability of banks (A, p)

and counterparty exposure R. Intuitively, stress testing the center bank 1 is more informative and the regulator selects bank 1 when the prior about system stability is low, or the prior about the bank is not too low. Otherwise $s_1 = l$ is reported with large probability, and the regulator prefers selecting a peripheral bank which reveals little information and the prior about system stability is relatively high.

A larger R benefits the center bank when it leads to a larger size of contagion failure, but on the other hand the regulator is able to design the beliefs to a smaller extent. The following proposition summarizes the optimal policy depending on the parameters.

Proposition 3.4.2. *Within each interval of* $R \in (A-D, 2(A-D)], (2(A-D), 4(A-D)], (4(A-D), D)$*, there exists a threshold* $\underline{A}(D,p)$*,* $\bar{A}(D,p)$ *and* $\bar{R}$ *such that*

$$\mathcal{M}^* = \begin{cases} 1 & A \geq \bar{A}(D,p);\ \ \textit{or}\ A \in (\bar{A}(D,p), \underline{A}(D,p))\ \textit{and}\ R \leq \bar{R} \\ 2 & A < \bar{A}(D,p);\ \ \textit{or}\ A \in (\bar{A}(D,p), \underline{A}(D,p))\ \textit{and}\ R > \bar{R} \end{cases}$$

Proof: See Appendix. ■

3.5 Selecting Multiple Banks in Ring Network

This section highlights the complementarity of selecting multiple banks for stress tests by focusing on a ring network, where banks are ex ante symmetric and effect of systemic importance of an individual bank is held fixed. We characterize the optimal policy of selecting two banks for stress test, $m = 2$. This is because direct spillover of disclosures only arise locally between two nearest selected banks. The problem with a larger m is similar in a qualitative way, but introduces complexity in information structures.

First we characterize the optimal policy, and this consists of the choice of stress test banks $\mathcal{M}$ and the optimal disclosure policy on $\mathcal{M}$. Then we compare it with 1) the optimal disclosure policy in the baseline model where all banks are stress tested; 2) the optimal policy when the regulator inspects all banks but is constrained to disclose on a subset of banks.

Contagion Cascade. Suppose $R > (n-1)(A-D)$. Under this set of parameters contagion cascade is the largest. Specifically all banks default whenever there is bad asset realization in the system. This rules out cases where distance between two selected banks becomes irrelevant locally, as banks with good assets will be solvent when they are distant enough from potential shocks. When we relax this parameter restriction, key trade offs are robust qualitatively.

For notation, suppose the regulator selects bank 1 and bank $1+\Delta$ without loss of generality. We use

$$\Theta_{\mathcal{M}} = \{gg, gb, bg, bb\}$$

to denote the project realizations of the tested banks. For example,

$$\theta_{\mathcal{M}} = gb: \ \tilde{A}_1 = A, \ \tilde{A}_{1+\Delta} = 0.$$

Correspondingly, we use

$$\boldsymbol{S}_{\mathcal{M}} = \{hh, hl, lh, ll\}$$

for available signals. The relevant disclosure policies are:

Case 1 (non-discriminatory)**.** *Report hh at gg, randomize between hh and ll at gb* *and bg, and report ll otherwise.*[22] *i.e.,*

$$\pi_{\mathcal{M}}(gg, hh) = 1,\ \pi_{\mathcal{M}}(gb, hh) = \pi_{\mathcal{M}}(bg, hh) \leq 1,\ \pi_{\mathcal{M}}(bb, ll) = 1.$$

Case 2 (truth telling). *Report hl at gg, randomize between hl and ll at gb, and report ll otherwise. i.e.,*

$$\pi_{\mathcal{M}}(gg, hl) = 1,\ \pi_{\mathcal{M}}(gb, hl) \leq 1,\ \pi_{\mathcal{M}}(bg, ll) = \pi_{\mathcal{M}}(bb, ll) = 1.$$

The above cases are without loss of generality, as other types of information structures are dominated by Case 1 (Lemma 3.3.2). Now we introduce two lemmas that summarizes the optimal Δ associated with each of the above disclosure policy.

Lemma 3.5.1. *Suppose the regulator either reports both stress test banks are h or both are l as in Case 1. There exists a $\bar{R}$, and when $R < \bar{R}$, we have "balanced" stress testing:* $\mathcal{M}^* = \{1, \lfloor \frac{n}{2} \rfloor\}$.

Proof: See Appendix. ■

The above lemma says, if the regulator is required to report either hh or ll and interbank liability R is not too large, stress testing is "balanced" as the two positions cut the ring network in halfs. This result is in stark contrast with that of passing connected banks together in a ring network when all banks are conducted with stress test. In that setting, the regulator passes the bank as long as it is a specific distance away from the nearest bank with asset impairment when the potential asset shocks are on connected banks. The result is driven by the direction of cross-bank risk sharing – when a borrower bank is reported with h, its lender banks, direct or indirect, are

[22] When the prior is high enough, the regulator is able to always report hh at gb and bg, and randomly report hh at bb. This information structure also corresponds to Case 1.

solvent if they have good assets; when potential shocks are on connected banks, this creates a quarantine effect and the rest of the network is perceived to be healthier.

The key difference here is that the regulator only tests a very small subset of banks. In contrast to testing all banks where it is feasible to inform location of potential counterparty risk, here manipulating the belief about contagion is less relevant: the remaining asset realization uncertainties dominate counterparty payment uncertainty.

Still the cross-bank spillover is large between connected banks, say bank 1 and 2; but we show that when $R < \bar{R}$, the optimal policy is a "balanced" stress test, which minimizes remaining project uncertainties. Intuitively, a bank is less likely to be solvent if it is more distant away from a bank reported with h, as asset shock uncertainties accumulate in between. Hence, the expected number of solvent banks increases concavely in between two selected banks as the distance increases. Passing bank 1 and $\lfloor \frac{n}{2} \rfloor$ together therefore maximizes the conditional expected system stability. In addition, when $R < \bar{R}$, the probability of reporting hh under a balanced stress test is also relatively higher. This depends on the expected payments that bank 1 and $\Delta + 1$ receive at gb and bg, or their expected healthiness. Cash flow generated in a more distant bank is less likely to be collected as uncertainty accumulates; but there is a small chance that the bank receive full payment similar to a "prize", and this may change the curvature of the expected payments received as a function of Δ. When $R > \bar{R}$, expected payment is decreasing and convex in Δ, and selecting bank 1 and 2 may dominate with its high probability of reporting hh.

In sum, a sufficient condition for balanced stress test is $R \leq \bar{R}$ when the regulator is restricted to a non-discriminatory disclosure of hh and ll. We will later argue that the sufficient condition is not restrictive when n is large.

The following lemma says stress testing is "connected" when the regulator is required with a "truth-telling" type of disclosure as in Case 2.

Lemma 3.5.2. *Suppose the regulator truthfully reports the potential shock in the two banks as in Case 2. When* $R > \frac{1-p^{n-1}}{1-p}\cdot$*, we have "connected" stress testing:* $\mathcal{M}^* = \{1, 2\}$ *or* $\mathcal{M}^* = \{1, n\}$*.*

Proof: See Appendix. ■

In the Case 2 type of disclosure, we show that Δ takes corner solution as the local extremal points, if there are any, are local minimums. The trade off is more ambiguous compared with that in choosing the distance in the stress testing all banks model. There, when the optimal disclosure informs that the potential shock is more distant, less banks are solvent conditional on reporting h, but h signals are reported with a higher probability as the banks are perceived to be healthier when the shocks are present. The complexity here comes from the remaining uncertainty of $n-2$ banks, so varying the stress test distance Δ also affects the expected system stability and bank healthiness when both banks have good assets. Specifically, postponing the potential shock $\tilde{A}_{\Delta+1}$ (increasing Δ) decreases the expected system stability at gg but increases bank 1's expected healthiness at gg. On the other hand gb, a higher Δ increases the expected system stability at gb but decreases bank 1's expected healthiness. We show that one direction of effect in the aggregate dominates the other and Δ takes corner solutions. Note that the sufficient condition is easily satisfied when n is large.

The following proposition characterizes the optimal policy when conducting stress tests on two banks in a ring network.

Proposition 3.5.1. *When* n *is relatively large, the optimal policy is either i) a "balanced" test* $\mathcal{M}^* = \{1, \lfloor \frac{n}{2} \rfloor\}$ *with nondiscriminatory signals hh and ll, or ii) a "connected" test* $\mathcal{M}^* = \{1, 2\}$ *with truth-telling signals. There exist threshold* $\hat{R}_1 < \hat{R}_2$ *such that when* R *is either very small* $R \leq \hat{R}_1$ *or very large* $R \geq \hat{R}_2$*, the optimal policy is the "balanced" stress testing with nondiscriminatory signals.*

Proof: See Appendix. ■

Proposition 3.5.1 characterizes the optimal subset of banks for stress test and the corresponding disclosure policy. In contrast to the model where all banks are tested, now the relative benefit of passing banks together (non-discriminatory disclosure) is ambiguous in counterparty exposure R. The complexity arises because even at the good state gg, there is still contagion exposure to remaining counterparties who are not tested. So for implications on the extent of cross state risk sharing, the contagion effect at the good states also matters. When passing banks together, the contagion effect across states is overall smaller, but the relative efficiency is ambiguous in contagion size.

The following Figure (3.4) illustrates the two types of policies in Proposition 3.5.1, alongside with disclosures in the model of testing all banks. The horizontal axis characterizes the probability of passing some banks. Area to the left of the dashed vertical line corresponds to the good state where the signal is always reported; area to the right corresponds to states where some stress test banks have bad asset realizations. The vertical axis characterizes the system stability. Hence, the area of the shaded polygons corresponds to the associated efficiency index. On both subfigures, a less discriminatory disclosure policy is shaded with red area, and a more discriminatory disclosure is shaded with blue.

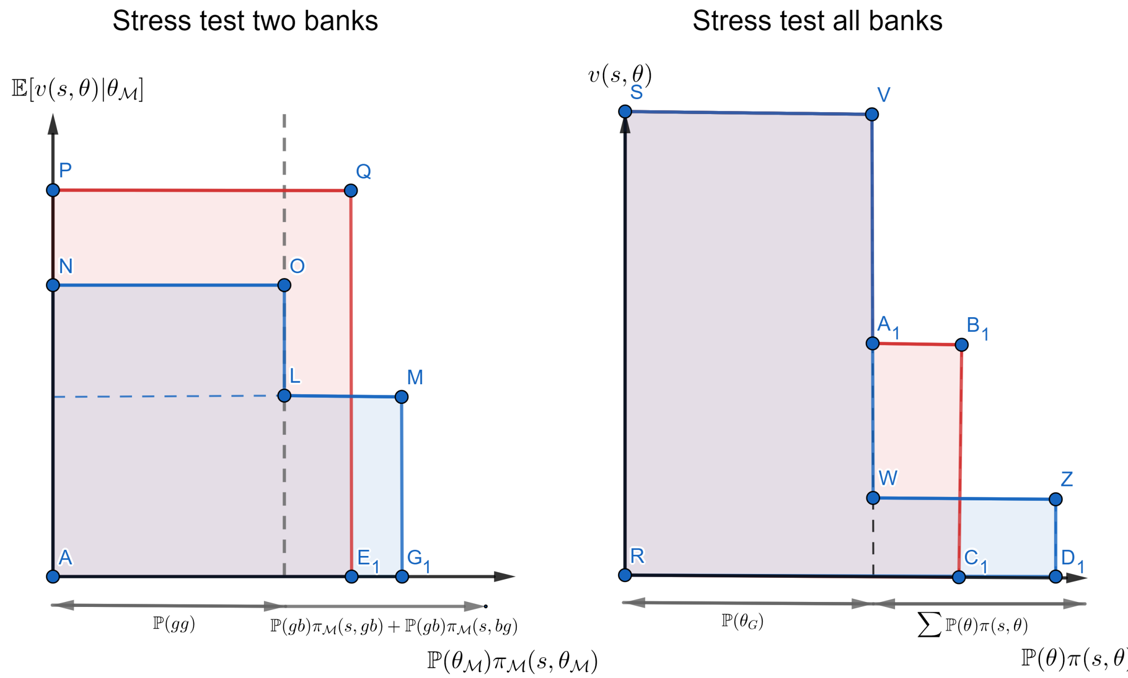

Figure 3.4: Disclosure policies in the ring network

When all banks are tested, the regulator trades off a higher system stability versus tighter obedience constraint when passing more banks together. The cost is relatively small when contagion R is larger, because a more discriminatory signal that tells banks of different healthiness apart is also reported with a small probability. Note that at the good state, the system stability and bank's healthiness is constant regardless of the disclosure policy.

On the other hand, when only two banks are tested and both have good assets, contagion is still present. Expected system stability is overall higher when passing both banks, because passing an additional bank reduces the counterparty risks on the remaining $n-2$ banks. The relative areas to the right of the dashed line is the incremental system stability at bad states. The relative areas is not necessarily monotone in R. The blue area decreases relatively slower with R, as banks recover a relatively higher payment at gg where contagion effect is smaller. In the model where all banks are tested, the blue area instead decreases relatively faster with R, as contagion effect is larger under a less discriminatory disclosure.

In sum, as the regulator is not able to completely influence the belief about contagion when only a subset of banks are tested, the choice of stress test and disclosure does not directly depend on contagion size. Remaining counterparty uncertainty becomes important. A less discriminatory disclosure that passes both banks together also serves to stabilize expected system stability to a larger extent, and therefore it is optimal to "balance" the tests on the ring.

Ring: Testing All Banks and Constrained Disclosure

Now we compare with another setting where the regulator inspects all banks,

$$\mathcal{M} = \mathcal{N},$$

but commits to disclose information on two banks,

$$\mathcal{M}^D = \{1, 1+\Delta\},\ \boldsymbol{S} = \{s | i \notin \mathcal{M}^D : s_i = l\},$$

the implications immediately change. The regulator is now able to choose the size of bank $i \in \mathcal{M}^D$'s counterparty risk at contingent state to shock cover under h. This is summarized by the distance to the nearest project shock (d', d'') of two banks. Now the choice of Δ matters via the feasible choice set of (d', d''). The following proposition summarizes the optimal policy.

Proposition 3.5.2. *In a ring network, suppose the regulator inspects all banks but commits to disclose information on two banks $\mathcal{M}^D = \{1, 1+\Delta\}$.*

(1). When R is relatively small, the regulator inspects adjacent banks and reports h

when contagion risk is smallest,

$$\mathcal{M}^{D*} = \{1, 2\}, \text{ and } d'^{*} + d''^{*} = n - 2,$$

(2). Otherwise when R is relatively large, Δ is irrelevant and reports h to cover project risk.

$$d'^{*} = d''^{*} = 0.$$

Proof: See Appendix. ■

3.6 Conclusion

Bank stress tests are a centerpiece of post crisis banking supervision. Existing literature has thus far mostly examined stand-alone single banks, and taken the banks subject to stress tests as given. However, stress testing is costly and we observe selective stress tests in practice. To the best of my knowledge, this is the first paper to study selecting the optimal subset of banks for stress test in a financial network, which is a natural environment to examine interbank spillovers with selective stress testing. A cash-constrained regulator commits to test a fixed number of banks and a disclosure scheme on these banks that specifies how likely each bank passes given the underlying fundamentals. The network structure is common knowledge, so disclosure about one bank influences the market beliefs of the counterparty risks on other banks.

The key insights are the emphasis on systemic importance and the ambiguity in how interbank exposures influence the selection of banks. Compared with the testing-all-bank model where it is feasible to design beliefs about contingent state project shocks and contagion effects, here the regulator could only influence the beliefs about remaining risks via the spillover from selected banks. As a result, selecting peripheral banks become less attractive as it is no longer possible to inform a very small potential contagion risk when risks from unselected banks remain. Systemically important banks, on the other hand, better contain the risks from unselected banks if they are solvent. However, the selection still depends on the prior beliefs, because an important bank's project failure leads to a worse financial system and it is selected only when the bank is passed with a high probability or the system prior is already low. This discussion relates to our second insight. Counterparty contagion is always present regardless of the selected banks' project risks, which makes the effects on the prior beliefs and cross-state risk sharing much more complex.

We then examine more specific stress test policies. First, we discuss selecting only one bank for stress test to highlight the effect of the bank's systemic importance. The optimal policy alternates between selecting a more versus a less important bank as interbank exposure R increases. On the one hand, contagion becomes more important as additional banks fail but on the other hand there is a higher probability of reporting bad news. Second, we discuss how to select two banks for stress tests in a ring network to highlight the effect of interbank complementarity. We find that stress test is either "balanced" on banks positioned evenly and disclosure is non-discriminatory, or on "connected" banks and disclosure is truth-telling on potential shocks. Intuitively, connected positioning is most efficient for interbank spillover, but here balanced positioning arises mainly to best contain the remaining counterparty risks.

Chapter 4

Conclusion

This dissertation extend the stress test literature in a very relevant dimension. Financial network is a well accepted framework to think about systemic risk. By putting stress test design in financial network, this dissertation studies the macro-prudential regulatory disclosure. The network structure is exogenous, and characterizes how banks are connected via interbank liabilities. A passing stress test result enables the bank to share risk via the financial market. Disclosure has spillover effects among banks in our model: a passing result on one bank ensures repayments to its lender banks, which improves the perception of their financial health. We hence highlight that the stress test design on a financial network involves novel cross-bank risk sharing, in addition to cross-state risk sharing studied in single banks.

For findings in general networks, the optimal disclosure policy could be non-monotone in the physical states of asset realizations. We highlight the value of coordinating cross-bank disclosures when designing the stress test at the system level. However, when stress test is costly and selective, the benefit of coordinate banks via non-discriminatory signals is more about containing remaining shocks from unselected banks, rather than designing beliefs about contagion between these banks in the most favorable way. The key takeaways with selective stress testing is the emphasis on systemic risk and the uncertain effect of the size of counterparty exposures.

The optimal disclosure policy varies with the network structures. When all banks are tested, I show that stress test undoes the contagion effect shaped by the network structure, and favors interconnectedness. In a core-periphery network where banks are asymmetric in connectivity, I show that typically the core banks receive preferred treatment because of the more efficient risk sharing with connected counterparties. With selective stress testing, I show that (i) in a ring network, stress test is either "balanced" on banks positioned evenly and disclosure is non-discriminatory, or on "connected" banks and disclosure is truth-telling on potential shocks; (ii) in a star network, stress test is conducted on the center bank when counterparty exposure is either sufficiently small or sufficiently large.

Appendix A

Appendix to Chapter 2

A.1 Notation Summary

Notation	*Definition and Meaning*	*Characterization*
n	Number of banks	
$\tilde{A}_i$	Loan asset return of bank i	$\tilde{A}_i \in \{A_i > 0, 0\}$
p_i	Probability of good loan asset realization for bank i	$p_i \equiv \mathbb{P}(\tilde{A}_i = A_i)$
θ	State of nature	$\theta \equiv \tilde{A}_1 \times \cdots \times \tilde{A}_n \in \Theta$
D_i	Face value of senior debts owed by bank i	
R_{ij}	Face value of interbank debts borrowed by bank j from bank i	
$\hat{\mathbf{S}}$	Signal space	$\hat{\mathbf{S}} = \hat{\mathbf{S}}_\mathbf{1} \times \cdots \times \hat{\mathbf{S}}_\mathbf{n},\ \hat{\mathbf{S}}_i$ finite
$\mathbf{S}$	Signal space of binary signal for each bank	$\mathbf{S} = \mathbf{S}_\mathbf{1} \times \cdots \times \mathbf{S}_\mathbf{n},\ \mathbf{S}_i = \{h, l\}$
s	A typical signal	$s \in \mathbf{S}$
$\boldsymbol{\pi}$	Signal distribution	$\boldsymbol{\pi} : \Theta \to \triangle\mathbf{S},\ \pi(s,\theta) \equiv \mathbb{P}(s\vert\theta)$
x_i	Bank i's action given signal realization	$x_i \in X_i \equiv \{\text{raise funds},\ \text{wait}\}$
u_i	Bank i's payoff of its stability outcome at $t = 2$.	Default punishment $-K < 0$
m_i	Investors' valuation of bank i's total assets	$m_i : \mathbf{S} \times \boldsymbol{\pi} \times X \to \mathbb{R}$
$y_{ij}(s,\theta)$	Actual interbank payment from j to i	See (3.5)
w_i	Weight assigned to bank i	
$v(s,\theta)$	Total weighted solvency rate given the signal and contingent state	See (2.8)
$V(\boldsymbol{\pi})$	Expected total solvency rate, or the regulator's payoff at $t = 0$	See (3.7)
$y^0_{ij}(\theta)$	Actual interbank payment from j to i in autarky	
$v_0(\theta)$	Total weighted solvency rate in autarky	
V_0	Expected total weighted solvency rate	
L_i	Bank i's liquidity after senior debt payment	See (2.11)
$I_h(s)$	Banks that receive $s_i = h$ under s	See (2.16)
$\theta_G(s)$	State with excess liquidity given s: all banks with h have excess liquidity	See (2.17)
$\Theta_G(s)$	Collection of states with excess liquidity given s	See (2.17)
$\Theta'(s,\theta_G)$	Collection of states that consume liquidity under s, plus $\theta_G(s)$	See (2.18)
$\Theta'^{*(k)}(s,\theta;\theta_G)$	Support of the cross-state distribution of s given θ, θ_G, excluding θ_G	See Definition 2.3.1, 2.3.2, (2.21)
$\xi^{(k)}(s,\theta;\theta_G)$	Benefit cost index of (s,θ) at the k-th round	See Definition 2.3.1, 2.3.2
η	Cross-state distribution of s	$\eta(s,\theta;\theta_G) \equiv \dfrac{\pi(s,\theta)}{\pi(s,\theta_G)}$
$\chi(s,\theta)$	Joint signal-state probability	$\chi(s,\theta) = \mathbb{P}(\theta)\pi(s,\theta)$
$\lambda_i(s)$	Multiplier of bank i's obedience constraint under s	
$q(\theta)$	Multiplier of the prior consistency constraint at θ	

Table A.1: Notation Summary

A.2 Mathematical Appendix

Proof of Lemma 2.3.1.

Proof: Suppose all banks default on interbank liabilities and no bank defaults on deposits,

$$0 < y_{ij} < R_{ij}, \ \forall i, \ j \neq i$$

For bank i,

$$\sum_{j \neq i} y_{ji} = \tilde{A}_i - D_i + \sum_{j \neq i} y_{ij}.$$

Sum over all banks,

$$\sum_i \sum_{j \neq i} y_{ji} = \sum_i \tilde{A}_i - nD_i + \sum_i \sum_{j \neq i} y_{ij} \Rightarrow \sum_i \tilde{A}_i - nD_i = 0,$$

which is not true for a general set of parameters. ∎

Proof of Lemma 2.3.2.

Proof: For the relaxed problem $\mathcal{P}$, suppose in the optimal $\boldsymbol{\pi}^*$ there exists s' with $\mathbb{P}(s') > 0$ and $s'_i = l$ such that

$$\frac{\sum_\theta \mathbb{P}(\theta)\pi^*(s',\theta)L_i(s'_{-i},\theta|s_i = l)}{\sum_\theta \mathbb{P}(\theta)\pi^*(s',\theta)} \geq \sum_{j \neq i} R_{ji}. \tag{A.1}$$

Let $s'' \equiv (s'_{-i}, s_i = h)$. We can make the following adjustment in signal probability

$$\tilde{\pi}(s,\theta) = \begin{cases} 0, & if \ s = s' \\ \pi^*(s,\theta) + \pi^*(s',\theta), & if \ s = s'' \\ \pi^*(s,\theta), & if \ s \neq s', s''. \end{cases}$$

and verify that all constraints in $\mathcal{P}$ still hold but $V(\tilde{\pi}) \geq V(\pi^*)$.

We only need to verify for $s = s''$ that

$$\sum_{\theta} \mathbb{P}(\theta)[\pi^*(s'', \theta) + \pi^*(s', \theta)][L_i(s''_{-i}, \theta | s_i = l) - \sum_{j \neq i} R_{ji}] \geq 0. \qquad \text{(A.2)}$$

As π^* is the solution of $\mathcal{P}$,

$$\sum_{\theta} \mathbb{P}(\theta)\pi^*(s'', \theta)[L_i(s''_{-i}, \theta | s_i = l) - \sum_{j \neq i} R_{ji}] \geq 0.$$
$$\sum_{\theta} \mathbb{P}(\theta)\pi^*(s', \theta)[L_i(s'_{-i}, \theta | s_i = l) - \sum_{j \neq i} R_{ji}] \geq 0.$$

For bank $j \neq i$, we have $L_j(s''_{-j}, \theta | s_j = l) \geq L_j(s'_{-j}, \theta | s_j = l)$, and plus (3.1) we have (3.2). ■

Proof of Proposition 2.3.1.

Proof: We introduce notations of vector and matrix representation. Introduce $\mathbf{e}$ as the cash flows irrelevant of interbank payments and liabilities, $\mathbf{R}$ the vector for the total face value of interbank liabilities, and $\mathbf{y}$ the vector of total actual payments to interbank creditors,

$$\mathbf{e} = \vec{A}(\theta) - D\mathbf{1},$$
$$\mathbf{R} = [R_1, \cdots, R_n] \equiv [\sum_{k \neq 1} R_{k1}, \cdots, \sum_{k \neq n} R_{kn}],$$
$$\mathbf{y} = [y_1, \cdots, y_n] \equiv [\sum_{k \neq 1} y_{k1}, \cdots, \sum_{k \neq n} y_{kn}].$$

We also introduce $\mathbf{Q}$ as the interbank liability weight matrix. Its element $\mathbf{Q}_{ij}$ is the fraction of bank i's claim on bank j in bank j's total interbank liabilities.

Specifically, $\mathbf{Q}$ is defined as

$$\mathbf{Q}_{ij} = \begin{cases} \dfrac{R_{ij}}{R_j}, & \text{if } R_j > 0 \text{ and } i \neq j, \\ 0, & \text{if } R_j > 0 \text{ and } i = j, \\ \dfrac{1}{n}, & \text{if } R_j = 0. \end{cases}$$

If bank j owes nothing to other banks, $R_j = 0$, we set $\mathbf{Q}_{ij} = \frac{1}{n}$ as normalization. Any $\mathbf{Q}_{ij} \in [0,1]$ with $\sum_{k \neq j} \mathbf{Q}_{kj} = 1$ will do. Under this construction, the column sums $\sum_i \mathbf{Q}_{ij} = 1$. Then, absent information design, the vector of total actual interbank payments $\mathbf{y}$ satisfy

$$\mathbf{y} = [\min\{\mathbf{R}, \mathbf{e} + \mathbf{Q}\mathbf{y}\}]^+.$$

First, we prove that for any information structure $\boldsymbol{\pi}$ in the feasible set

$$\begin{aligned} \{\boldsymbol{\pi} | & \frac{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)L_i(\theta, s_{-i}|s_i = l)}{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)} \geq \sum_{j \neq i} R_{ji}, \ (\forall s, \forall i \text{ with } s_i = h \text{ in } s) \\ & \sum_s \pi(s,\theta) = 1, \ (\forall \theta) \\ & 0 \leq \pi(s,\theta) \leq 1. \ (\forall \theta, \mathbf{s}), \} \end{aligned}$$

under any possible realization of state and signal (θ, s), the actual interbank payment vector $\mathbf{y}(\theta, s)$ exists and is generically unique.

Given that $\boldsymbol{\pi}$ is feasible, $y_i = R_i$ if $s_i = h$. Then we can construct a new network with modifications on cash flows and interbank liability face values to apply [AOTS15]'s proof. For any bank i with $s_i = h$ and bank j with $s_j = l$, set

$$\hat{A}_j(\theta) = \tilde{A}_j(\theta) + \sum_{s_i = h} R_{ji},$$

$$\hat{R}_i = 0,$$

In this network, the interbank payment equilibrium exists and is generically unique, according to Proposition 1 in [AOTS15].

Then we need to prove that the feasible set is non-empty. Consider the following information structure that corresponds to truth-telling: at each state θ, report h on banks that are solvent and l on banks that default in autarky. i.e.,

$$\mathbb{P}(s_i = h|\theta) = 1, \text{ if } \tilde{A}_i + \sum_{j\neq i} y^0_{ij}(\theta) - D_i \geq \sum_j R_{ji},$$
$$\mathbb{P}(s_i = l|\theta) = 1, \text{ if } \tilde{A}_i + \sum_{j\neq i} y^0_{ij}(\theta) - D_i < \sum_j R_{ji},$$

where $y^0_{ij}(\theta)$ is the interbank payment from bank j to i in autarky at state θ. Hence, the feasible set is a nonempty compact set and the object function is continuous. By the Weierstrass extreme value theorem there exists the optimal $\boldsymbol{\pi}$. ■

Proof of Proposition 2.3.2.

Proof: Our problem is linear programming with bounded convex feasible set. With $2^n \cdot 2^n$ variables and $n2^{n-1} - 1 + 2^n + 2 \cdot 2^n \cdot 2^n$ constraints, there are finite many vertices of the feasible set polyhedron. The optimal vertex is an optimal solution of the problem.

If there exists another optimal solution, it must be that the projection of the object function is parallel to one of the constraints, or some constraints are identical, which does not hold generically. ■

Auxiliary Game. We define the auxilary game $\mathcal{G}(s, \theta_G, T)$ for each combination of $s \neq s_l$ and $\theta_G \in \Theta_G(s)$. The game is to solve for the optimal signal distribution

$\boldsymbol{\pi}_{\mathcal{G}(s,\theta_G,T)}$ with restricted signaling space $\mathbf{S}'$, states Θ' and conditional probability space T:

$$\mathbf{S}' = \{s, s_l\}$$
$$\Theta' = \{\theta_G\} \cup \{\theta | \theta \notin \Theta_G(s)\}$$
$$T = T_{\theta_1} \times \cdots \times T_{\theta_{2^n}},\ T\theta^i \subset [0,1].$$

To be more specific, the regulator's problem for $\mathcal{G}(s,\theta_G,T)$ is $\mathcal{P}_{\mathcal{G}}(s,\theta_G,T)$:

$$\begin{aligned} V = V_0 + &\max_{\boldsymbol{\pi}_{\mathcal{G}}} \sum_{\theta\in\Theta'} \mathbb{P}(\theta)\pi(s,\theta) \sum_i w_i \mathbf{1}_{\{s_i = h \cap i \in D(\theta)\}} \\ s.t.\ &\frac{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta) L_i(s_{-i},\theta | s_i = l)}{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)} \geq \sum_{j\neq i} R_{ji},\ (\forall i \in I_h(s)) \\ &\pi(s,\theta) \in T_\theta,\ (\forall \theta \in \Theta'). \end{aligned}$$

The auxiliary game corresponds to the subgame of a specific signal, given the good state $\theta_G(s)$ from which to borrow liquidity. According to Lemma 2.3.2, we remove the obedience constraints associated with $s_i = l$.

For any given conditional probability space T, the optimal solution to the auxilary game problem $\boldsymbol{\pi}_{\mathcal{G}}$ exists and is generically unique, by the argument in Proposition 2.3.1 and 2.3.2. We show that $\boldsymbol{\pi}_{\mathcal{G}}$ saves the states in the sequence of their associated efficiencies: per extra unit of liquidity available $\theta_G(s)$, the regulator would report s at $\boldsymbol{\pi}_{\mathcal{G}}$ and the $\theta \notin \Theta_G(s)$ with the highest resulting incremental system solvency. We formalize this in the following proposition:

Proposition A.2.1. *In the auxilary game $\mathcal{G}(s,\theta_G,T)$, there is a partition of states $(\Theta'_m)_{m=0,1,\cdots,M}$ with $\Theta'_0 = \{\theta_G\}$ such that whenever $m' < m''$, for any $\theta' \in \Theta'_{m'}$ and* *any* $\theta'' \in \Theta'_{m''}$,

$$\pi(s, \theta'') = 0 \textit{ if } \pi(s, \theta') < T_{\theta'}.$$

1). If $I_h(s)$ *is a singleton, generically the partition sets are singletons and regulator ranks the states according to*

$$\mathbb{P}(\theta_G)[v(s,\theta) - v_0(\theta)] \cdot \frac{L_i(s_{-i}, \theta_G | s_i = l) - \sum_{j \neq i} R_{ji}}{\sum_{j \neq i} R_{ji} - L_i(s_{-i}, \theta | s_i = l)} \tag{A.3}$$

2). If $I_h(s)$ *is not a singleton, the regulator never partition the states according to (A.3) of any individual bank* $\imath \in I_h(s)$.

Proof: We provide a short proof of part (2) here. To do this, we show that the multiple solvency constraints impose extra structure on π so that we cannot rank the states by (A.3).

Still let λ_s^i denote the Lagrangian multiplier of bank i's obedience constraint under s in the auxiliary game, and we show that

$$\lambda_s^i > 0, \text{ or } \lambda_s^i = 0, \ \pi(s, \theta_i) = T_{\theta_i},$$

where $\theta_i : \tilde{A}_i = 0, \tilde{A}_k > 0, \forall k \neq i$. Due to the scarce liquidity assumption, there exists a bank j such that its solvency constraint binds

$$\sum_{\theta} \mathbb{P}(\theta)\pi(s,\theta)[L_j(s_{-j}, \theta | s_j = l) - \sum_{k \neq j} R_{kj}] = 0,$$

Suppose another bank i with $s_i = h$ has loose solvency constraint,

$$\sum_{\theta} \mathbb{P}(\theta)\pi(s,\theta)[L_i(s_{-i}, \theta | s_i = l) - \sum_{k \neq i} R_{ki}] > 0.$$

We focus on two states $\theta_j, \theta_i \in \Theta'$ with switching bad fundamental asset realizations of bank j and i,

$$\theta_j : A_i > 0, A_j = 0, A_k > 0, \forall k \neq i, j,$$

$$\theta_i : A_j > 0, A_i = 0, A_k > 0, \forall k \neq i, j.$$

and then

$$L_i(s_{-i}, \theta_j | s_i = l) - \sum_{k \neq i} R_{ki} > 0,\ L_j(s_{-j}, \theta_j | s_j = l) - \sum_{k \neq i} R_{kj} < 0$$

$$L_i(s_{-i}, \theta_i | s_i = l) - \sum_{k \neq i} R_{ki} < 0,\ L_j(s_{-j}, \theta_i | s_j = l) - \sum_{k \neq i} R_{kj} > 0.$$

If $\lambda_s^i = 0$ we have $\pi(s, \theta_i) = T_{\theta_i}$ or else we can increase $\pi(s, \theta_i)$ marginally, and all the constraints still hold and the objective function value is higher.

If $\lambda_s^i > 0$, by subtracting the constraints of i and j we have

$$\sum_{\theta} \mathbb{P}(\theta) \pi_s(\theta) \left\{ [L_i(s_{-i}, \theta | s_i = l) - \sum_{k \neq i} R_{ki}] - [L_j(s_{-j}, \theta | s_j = l) - \sum_{k \neq i} R_{kj}] \right\} = 0.$$

The above equation shows smoothing across the individual bank's rankings. ■

The following lemma says, in the original relaxed problem $\mathcal{P}$, the across-state distribution of any signal sent with positive probability is consistent with that in its auxiliary games.

Lemma A.2.1. *Suppose* $\boldsymbol{\pi}^*$ *is the solution of* $\mathcal{P}$ *and* $\mathbb{P}(s) > 0$. *Set*

$$T_\theta = 1 - \sum_{s' \neq s, s_l} \pi(s, \theta),\ T = T_{\theta_1} \times \cdots \times T_{\theta_{2^n}}.$$

If $\Theta_G(s)$ *has* $M \geq 1$ *elements, there exists a set of* (T^m) *with* $\sum_m T^m_\theta = T$ *such that*

$$\pi^*(s,\theta) = \sum_m \pi_{\mathcal{G}(s,\theta^m_G,T^m)}(s,\theta).$$

The cross-state distribution of any signal $s \neq s_l$ that shows up in the optimal information structure is optimal in its subgames. Then we show that the planner assigns the excess liquidities in the sequence of the efficiencies associated with each (s,θ). To do this, we first propose the efficiency measures, and then argue that an algorithm based on these measures leads to the optimal information structure.

Dual Approach.

Following is the dual for the planer's original problem:

Dual

$$\mathcal{P}^* = \min_{q\in\mathbb{R}^\Theta} \sum_\theta \mathbb{P}(\theta)q(\theta)$$

$$q(\theta) \geq v(s,\theta) + \sum_{i\in I_h(s)} \lambda_i(s_i,s_{-i})\left[L_i(s_{-i},\theta|s_i=h) - R_i\right], (\forall s,\theta) \quad (\chi(s,\theta)) \quad \text{(A.4)}$$

$$\lambda_i(s) \geq 0,\ q(\theta) \geq 0. \quad \text{(A.5)}$$

From the slack obedience constraints associated with $s_i = l$ in the primal of the planer's original problem and the complementary slackness condition, we know

$$\lambda_i(s_i = l, s_{-i}) = 0.$$

(A.4) says any signal $s \in \mathbf{S}$ puts a lower bound for the state contingent price of sending signals, $q(\theta)$. In particular, $s = s_l = (l,\cdots,l)$ puts a constant bound on

$q(\theta)$:

$$q(\theta) \geq v(s_l, \theta).$$

From the complementary slackness condition, the price of sending signal at a particular state $q(\theta)$ is determined by the value of the equilibrium signal(s) sent at this state θ,

$$q(\theta) = v(s,\theta) + \sum_{i \in I_h(s)} \lambda_i(s_i, s_{-i}) \left[L_i(s_{-i}, \theta | s_i = h) - R_i\right], \ \forall s \text{ with } \chi(s,\theta) > 0.$$

Let $\mu_i \equiv \frac{A_i}{A} = \frac{D_i}{D} = \frac{R_i}{R}$ be the adjustment for size under Assumption 2.3.3.

Proof of Lemma 2.3.4.

Proof: Suppose excess liquidity is scarce such that the regulator is afford to send only one signal $s^* \neq s_l$ to save one set of bad states.[1] The following conditions in the dual problem hold:

$$\theta_1 \ (\tilde{A}_i > 0, \forall i): \qquad q(\theta_1) = \sum_i w_i + (A_i + \sum_{j \neq i} R_{ij} - D_i - \sum_{j \neq i} R_{ji}) \sum_{i \in I_h(s^*)} \lambda_i(s_i^*, s_{-i}^*), \tag{A.6}$$

$$\theta \neq \theta_1, \ \chi(s^*, \theta) > 0: \quad q(\theta) = v(s^*, \theta) + \sum_{i \in I_h(s^*)} \lambda_i(s_i^*, s_{-i}^*) \left[L_i(s_{-i}^*, \theta | s_i = h) - R_i\right] = v(s_l, \theta). \tag{A.7}$$

Let μ_i be a size scaler for the amount of bank i's excess liquidity at θ_1,

$$\mu_i \propto A_i + \sum_{j \neq i} R_{ij} - D_i - \sum_{j \neq i} R_{ji}.$$

[1] If s reports multiple $s_i = h$ then $\chi(s, \theta) > 0$ for multiple bad states $\theta \notin \Theta_G(s)$.

We rewrite (A.7) in vector matrix form to draw the connection between the benefit-cost index and shadow values. Introduce the following vector and matrix notations,

$$\Delta \boldsymbol{v}(s)_{|\Theta|\times 1} = [\cdots, v(s,\theta) - v_0(\theta), \cdots]'_{\theta\in\Theta},$$
$$\boldsymbol{\mu}_{|I_h(s)|\times 1} = [\cdots, \mu_i, \cdots]'_{i\in I_h(s)},$$
$$\boldsymbol{\chi}(s)_{|\Theta|\times 1} = [\cdots, \chi(s,\theta), \cdots]'_{\theta\in\Theta},$$
$$\boldsymbol{\lambda}(s)_{|I_h(s)|\times 1} = [\cdots, \lambda_i(s_i, s_{-i}), \cdots]'_{i\in I_h(s)},$$
$$\Delta \boldsymbol{\mathcal{L}}(s)_{|I_h(s)|\times|\Theta|} = [L(s)_{i\theta}]_{i\in I_h(s),\theta\in\Theta} = [L_i(s_{-i},\theta|s=h) - R_i],$$

where $\Theta(s) \equiv \{\theta \neq \theta_1 | \chi(s,\theta) > 0\}$ is the set of bad signals on which s is sent. By optimality, $\Theta(s) = \Theta^*(s,\theta;\theta_G) \setminus \{\theta_G\}$ in the benefit-cost index approach. By binding obedience constraints,[2] $|I_h(s)| = |\Theta|$. Hence,

$$\left(-\Delta \boldsymbol{\mathcal{L}}(s^*)_{|I_h(s^*)|\times|\Theta|}\right)' \boldsymbol{\lambda}(s^*)_{|I_h(s^*)|\times 1} = \Delta \boldsymbol{v}_{|\Theta|\times 1}(s^*) \Rightarrow \boldsymbol{\lambda}(s^*) = \left[(\Delta - \boldsymbol{\mathcal{L}}\,(s^*))'\right]^{-1} \Delta \boldsymbol{v}(s^*). \tag{A.8}$$

Remember the first-round benefit-cost index is defined as

$$\xi^{(1)}(s,\theta;\theta_1) = \sum_{\theta\in\Theta^*(s,\theta;\theta_G)} \chi(s,\theta)[v(s,\theta) - v(s_l,\theta)],$$

where the quantities $\chi(s,\theta)$ are derived from the binding obedience constraints,

$$\Delta \boldsymbol{\mathcal{L}}(s)_{|I_h(s)|\times|\Theta|} \boldsymbol{\chi}_{|\Theta|\times 1} = -\mathbb{P}(\theta_1)\cdot Const \cdot \boldsymbol{\mu}_{|I_h(s)|\times 1} \Rightarrow \boldsymbol{\chi} = \mathbb{P}(\theta_1)\cdot Const \cdot [-\Delta \boldsymbol{\mathcal{L}}(s)]^{-1} \boldsymbol{\mu}.$$

[2]Obedience constraints are binding in the first round interior solution. For corner solutions, we argue that the obedience constraints are binding in the proof of Proposition 2.3.3. For this part, we construct the indices and show the price-approach representations based on Definition 2.3.1, 2.3.2, assuming binding obedience constraints.

Then for the equilibrium signal $s^* = s^{*(1)}$ on the set of bad states $\Theta(s^*) = \Theta^*(s^*, \theta^{*(1)}; \theta_1)$,

$$\begin{aligned}\max_{(s,\theta)} \xi^{(1)}(s, \theta; \theta_1) &= \left\{\mathbb{P}(\theta_1) \cdot Const \cdot [-\Delta \boldsymbol{\mathcal{L}}(s^*)]^{-1} \boldsymbol{\mu}\right\}^{'} \Delta \boldsymbol{v} \\ &= \mathbb{P}(\theta_1) \cdot Const \cdot \boldsymbol{\mu}' \boldsymbol{\lambda}(s^*) \\ &\propto \sum_{i \in I_h(s^*)} \lambda_i(s^*, s^*_{-i}) \mu_i.\end{aligned}$$

For signal $s \neq s^*$,

$$\begin{aligned}\theta_1\ (\tilde{A}_i > 0, \forall i): \qquad & q(\theta_1) = \sum_i w_i + Const \cdot \sum_{i \in I_h(s^*)} \lambda_i(s^*_i, s^*_{-i}) \mu_i \\ & > \sum_i w_i + Const \cdot \sum_{i \in I_h(s)} \lambda_i(s_i, s_{-i}) \mu_i \qquad \text{(A.9)} \\ \theta \neq \theta_1: \qquad & q(\theta) = v(s_l, \theta) \geq v(s, \theta) + \sum_{i \in I_h(s)} \lambda_i(s_i, s_{-i}) \left[L_i(s_{-i}, \theta | s_i = h) - R_i\right]. \qquad \text{(A.10)}\end{aligned}$$

As $\lambda_i(s)$ (the cross-state borrowing prices of $s \neq s^*$) don't show up in the objective function,

$$\sum_\theta q(\theta) = \underbrace{\sum_i w_i + Const \cdot \sum_{i \in I_h(s^*)} \lambda_i(s^*_i, s^*_{-i}) \mu_i}_{q(\theta_1)} + \sum_{\theta \neq \theta_1} v(s_l, \theta),$$

we can pick $\lambda_i(s)$ as long as (A.9) and (A.10) are satisfied. Consider the following problem with restricted signal space $\{s, s_l\}$:

$$\begin{aligned}&\min_\lambda \sum_{i \in I_h(s)} \lambda_i(s_i, s_{-i}) \mu_i \\ &s.t.\ q(\theta) = v(s_l, \theta) \geq v(s, \theta) + \sum_{i \in I_h(s)} \lambda_i(s_i, s_{-i}) \left[L_i(s_{-i}, \theta | s_i = h) - R_i\right],\end{aligned}$$

$$\lambda_i(s_i, s_{-i}) \geq 0.$$

The optimality of s^* ensures (A.9). By the above argument that $\sum_{i \in I_h(s^*)} \lambda_i(s^*, s^*_{-i})\mu_i \propto \max_{(s,\theta)} \xi^{(1)}(s, \theta; \theta_1)$, we have

$$\sum_{i \in I_h(s)} \lambda_i(s_i, s_{-i})\mu_i \propto \max_{\theta} \xi^{(1)}(s, \theta; \theta_1).$$

■

Proof of Lemma 2.3.5.

Proof: Suppose there is excess liquidity in the good state(s) even if the assignment of the most efficient (s, θ) hits corner solution $\chi(s, \theta') = 1$ for $\theta' \in \Theta^*(s, \theta; \theta_1)$. Then the optimal information structure does not take the form in the previous case, and multiple (s, θ) pairs determine the prices of sending signals $q(\theta)$.

WLOG suppose $\theta_G = \theta_1$, as the regulator consumes the excess liquidity at $\theta_G \neq \theta_1, \theta_G \in \Theta_G(s)$ for $s \neq s_h \equiv (h, \cdots, h)$ under the residual conditional probability space after θ_1-liquidity consumption.

Let $\hat{s} \neq s_l, \hat{\Theta}(\hat{s})$ be the marginal signal and the set of bad states where the last unit of excess liquidity at θ_1 is exhausted, $s' \neq s_l, \hat{s}$ be the other equilibrium signals $(\chi(s', \theta_1) > 0)$, and $\hat{\theta}$any states with $\sum_{s \neq s_l} \chi(s, \hat{\theta}) = 1$. Then in the dual,

$$V = \sum_{\theta} q(\theta) = \underbrace{\sum_i w_i + Const \cdot \sum_{i \in I_h(\hat{s})} \lambda_i(\hat{s})\mu_i}_{q(\theta_1)} + \sum_{\hat{\theta}: \sum_{s \neq s_l} \chi(s, \hat{\theta}) = 1} q(\hat{\theta}) + \sum_{\theta: \chi(s_l, \theta) > 0} v(s_l, \theta);$$

In the Primal,

$$V = \mathbb{P}(\theta_1) Const \cdot \sum_{s' \neq s_l, \hat{s}} \chi(s', \theta_1) \sum_{\Theta(s')} \Delta \boldsymbol{v}(s')' [-\Delta \boldsymbol{L}(s')_{|I_h(s')| \times |\Theta(s')|}]^{-1} \boldsymbol{\mu}(s')$$

$$+\underbrace{\mathbb{P}(\theta_1)Const\cdot\chi(\hat{s},\theta_1)\cdot\Delta\boldsymbol{v}(\hat{s})'[-\Delta\boldsymbol{L}(\hat{s})_{|I_h(\hat{s})|\times|\Theta(\hat{s})|}]^{-1}\boldsymbol{\mu}(\hat{s})}_{\text{Benefit-Cost Index}\xi^{(k>1)}}.$$

Therefore,

$$\begin{aligned}\max_{(s,\theta)}\xi^{(k>1)}(s,\theta;\theta_1)=&\sum_i w_i+Const\cdot\sum_{i\in I_h(\hat{s})}\lambda_i(\hat{s})\mu_i(\hat{s})+\sum_{\hat{\theta}:\sum_{s\neq s_l}\chi(s,\hat{\theta})=1}q(\hat{\theta})\\&+\sum_{\theta:\chi(s_l,\theta)>0}v(s_l,\theta)-\mathbb{P}(\theta_1)Const\cdot\\&\sum_{s'\neq s_l,\hat{s}}\chi(s',\theta_1)\sum_{\Theta(s')}\Delta\boldsymbol{v}(s')'[-\Delta\boldsymbol{L}(s')]^{-1}\boldsymbol{\mu}(s')\\=&\sum_\theta v(s_l,\theta)+\underbrace{Const\cdot\sum_{i\in I_h(\hat{s})}\lambda_i(\hat{s})\mu_i}_{\text{marginal}(\hat{s},\theta)}+\underbrace{\sum_{\hat{\theta}}\left[q(\theta)-v(s_l,\hat{\theta})\right]}_{\text{effect on assigned signals}}\\&\underbrace{-\mathbb{P}(\theta_1)Const\cdot\sum_{s'\neq s_l,\hat{s}}\chi(s',\theta_1)\sum_{\Theta(s')}\Delta\boldsymbol{v}(s')'[-\Delta\boldsymbol{L}(s')]^{-1}\boldsymbol{\mu}(s')}_{\text{payoffs of assigned signals}}\end{aligned}$$

By the same argument in (1), the cross-state borrowing prices $\{\lambda_i(s)\}_{i\in I_h(s)}$ of signals with $\mathbb{P}(s)=0$ don't affect the price of sending signals $q(\theta)$ and thus the total value V. Thus we can extend the above connection to each candidate signal s paired with the accordingly most effecient state θ:

$$\begin{aligned}\max_{\theta}\xi^{(k>1)}(s,\theta;\theta_1)=&\sum_\theta v(s_l,\theta)+\underbrace{Const\cdot\sum_{i\in I_h(s)}\lambda_i(s)\mu_i}_{\text{candidate}(s,\theta)}+\underbrace{\sum_{\hat{\theta}}\left[q(\theta)-v(s_l,\hat{\theta})\right]}_{\text{new price of assigned signals}}\\&\underbrace{-\mathbb{P}(\theta_1)Const\cdot\sum_{s'\neq s_l,\hat{s}}\chi(s',\theta_1)\sum_{\Theta(s')}\Delta\boldsymbol{v}(s')'[-\Delta\boldsymbol{L}(s')]^{-1}\boldsymbol{\mu}(s')}_{\text{payoffs of assigned signals}},\end{aligned}$$

where $\xi^{(k>1)}(s,\theta;\theta_1)$ is the proposed benefit-cost index and $\{\lambda_i(s)\}_{i\in I_h(s)}$ are the solutions to the marginal assignment problem with restricted signal space$\{s, s_l\}$, given the previous round assignments of all $(s', \Theta(s'))$. The constraints regarding $\lambda_i(s)_{i\in I_h(s)}$ are the same in the two problems except $\sum_{i\in I_h(s)} \lambda_i(s)\mu_i < \sum_{i\in I_h(\hat{s})} \lambda_i(\hat{s})\mu_i$ in the original problem, which is satisfied by the optimality of the dual. ■

Proof of Proposition 2.3.3.

Proof: First, we introduce auxiliary games and state that the cross-state distribution of any signals sent with positive probability in the optimal information structure is consistent with that in its auxiliary games.

Remember

$$\begin{aligned} I_h(s) &\equiv \{i|s_i = h \text{ in } s\}, \\ \Theta_G(s) &\equiv \{\theta : L_i(s_{-i}, \theta|s_i = l) > \sum_{k\neq i} R_{ki}, \forall i \in I_h(s)\}. \end{aligned}$$

An auxilary game $\mathcal{G}(s, \theta_G, T)$ is defined for each combination of $s \neq s_l$ and $\theta_G \in \Theta_G(s)$. The game is to solve for the optimal signal distribution $\boldsymbol{\pi}_{\mathcal{G}(s,\theta_G,T)}$ with restricted signaling space $\mathbf{S}'$, states Θ' and conditional probability space T:

$$\begin{aligned} &\mathbf{S}' = \{s, s_l\} \\ &\Theta' = \{\theta_G\} \cap \{\theta|\theta \notin \Theta_G(s)\} \\ &T = T_{\theta_1} \times \cdots \times T_{\theta_{2^n}},\ T_i \subset [0, 1]. \end{aligned}$$

To be more specific, the regulator's problem for $\mathcal{G}(s, \theta_G, T)$ is $\mathcal{P}_{\mathcal{G}}(s, \theta_G, T)$:

$$V_{\mathcal{G}} = V_{\mathcal{G}0} + \max_{\boldsymbol{\pi}_{\mathcal{G}}} \sum_{\theta\in\Theta'} \mathbb{P}(\theta)\pi(s, \theta) \sum_i w_i \mathbf{1}_{\{s_i=h\cap i\in D(\theta)\}}$$

$$s.t. \ \frac{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)L_i(s_{-i},\theta|s_i=l)}{\sum_\theta \mathbb{P}(\theta)\pi(s,\theta)} \geq \sum_{j\neq i} R_{ji}, \ (\forall i \in I_h(s))$$

$$\pi(s,\theta) \in T_\theta, \ (\forall \theta \in \Theta').$$

We get existence and generic uniqueness of $\mathcal{P}_\mathcal{G}(s,\theta_G)$ by an argument similar to the proof of Proposition 1 and 2. Then we claim that in the solution $\boldsymbol{\pi}^*$ to the original problem $\hat{\mathcal{P}}$, if $\mathbb{P}(s) > 0$ and $|\Theta_G(s)| = M \geq 1$, there exists a set of (T^m) with $\sum_m T^m_\theta = 1 - \sum_{s'\neq s, s_l} \pi(s,\theta)$ such that

$$\pi^*(s,\theta) = \sum_m \pi_{\mathcal{G}(s,\theta^m_G,T^m)}(s,\theta).$$

If not, the regulator is not optimizing at least one of the auxiliary game of s, taking the conditional probability distributions of other signals as given. Then the planner can increase value by reseting this $\pi_\mathcal{G}$, which contradicts π^* being optimal. From this result and the linearity nature of the problem, we can break the optimal information structure into several rounds consuming excess liquidity.

Second, we prove the first part of Proposition 3. All obedience constraints must be binding due to limited scarcity, switching liquidity levels of banks across-states, and the parameter range assumption in 1). Then part 1) is due to $(s^{*(1)}, \theta^{*(1)}, \theta_G^{*(1)}) \equiv \arg\max \xi^{*(1)}(s,\theta;\theta_G)$ and linearity. As $L_i(s_{-i},\theta_1|s_i=l) \geq L_i(s_{-i},\theta|s_i=l)$ for any state $\theta \neq \theta_1$ and bank i, $\theta_G^{*(1)} = \theta_1$.

We argue that the method in part 2) that compares the index of single $(s,\theta;\theta_G)$ is WLOG, if all obedience constraints are binding. Otherwise, convex combinations of $(s,\theta;\theta_G)$ may affect the cross-state structures of the involved (s,θ). With binding obedience constraints, convex combinations of $(s,\theta;\theta_G)$ are nested, as linearity puts weight 1 on the element that delivers the highest value.

Obedience constraints of h (3.8) are binding when the optimal information structure $\boldsymbol{\pi}^*$ corresponds to $k = 1$. We show that, if $\boldsymbol{\pi}^*$ corresponds to $k \geq 1$ and all obedience constraints are binding, then obedience constraints are binding when $\boldsymbol{\pi}^*$ corresponds to $k + 1$. The parameters under which the round jumps from k to $k + 1$ could be approximated by a sequence of parameters in the $k-$round case.[3]

For expositional convenience, we make the argument for $k = 1$. The same reasoning applies for $k > 1$. In Round 2, the regulator moves to (s, θ) with

$$s \neq s^{*(1)} \text{ or } \theta \notin \Theta^*(s^{*(1)}, \theta^{*(1)}; \theta_G^{*(1)}),$$

where Θ^* is the set of states that consume liquidity. Otherwise, $\exists \hat{\theta} \in \Theta^*(s^{*(1)}, \theta^{*(1)}; \theta_G^{*(1)})$ and another $\theta' \in \Theta^*(\theta_G^{*(1)}, s^{*(1)}, \theta^{*(1)})$ such that at Round 2,

$$\pi(s^{*(1)}, \hat{\theta}) = 1, \quad \pi(s^{*(1)}, \theta') > \eta^{(1)}(\theta')\pi(s^{*(1)}, \theta^{*(1)}).$$

From binding obedience constraints in Round 1, $\exists i \in I_h(s)$ with failing obedience constraint.

Thus, the candidate (s, θ) for Round 2 either involves a new signal $s' \neq s^{*(1)}$, or keeps $s^{*(1)}$ and involves a new state $\theta \notin \Theta^{*(1)}$. If $s^{*(1)}$ is kept, or s' does not involve signal switching,[4] obedience constraints are binding for Round 2, because in the signal's subgame the allocation resembles that in Round 1. Whenever signal

[3]For any such threshold parameters, there exists an approximating sequence of parameters that correspond to k rounds and does not results in jumps in the optimal assignments of previous rounds. Hence, binding obedience constraints is robust to possible jumps in policy (information structure) when we vary the parameters.

[4]For the bad states where s' is reported at Round 2, i.e. $\Theta^{*(2)}(s', \theta; \theta_G)$, the residual conditional probability space from the allocation of Round 1 is positive.

switching is involved, it must be that

$$I_h(s^{*(1)}) \in I_h(s'),$$

otherwise $s' = s^{*(1)}$. Because the state where corner solution is reached involves signal switching, in the subgame of s' the allocation resembles that in Round 1.

Next, we argue Corollary 3 holds – the regulator exhausts the excess liquidities in $\theta_G \in \Theta_G$ in sequence. Otherwise, the information is not optimal due to residual excess liquidity. In addition, as we start with $\theta_G^{*(1)} = \theta_1$, allocation that corresponds to $\theta_G \neq \theta_1$ can be replicated by $\theta_G = \theta_1$.

Finally, we argue that the index formulas in part 2) is correct. Case 1) formula is derived by the optimality of the auxiliary game. For case 2), we need to prove at the state that involves signal switching, generically only one signal is reported after the previous round.[5] Otherwise, by linearity and binding obedience constraints, the regulator decreases the weight of the worst signal. ■

Proof of Proposition 2.3.4.

Proof: Remember we use R_i to represent the total face value of bank i's interbank liabilities, and R_i^c to represent the total face value of its interbank claims,

$$R_i = \sum_{j \neq i} R_{ji}, \; R_i^c = \sum_{j \neq i} R_{ij}.$$

For the first part, we prove that any optimal information structure with $\pi(s', \theta) > 0$ for $s' \neq s_h, s_l$ is only consistent with $A_i < \bar{A}_i \leq \tilde{A}_i$.

[5] First, the exceptions include, for example networks that involve symmetry. Second, by signal switching, we refer to the cases where the previously assigned signal s' and the new signal s'' borrow excess liquidity from the same θ_G.

Suppose $\Theta(s') \equiv \{\theta | \pi(s', \theta) > 0\}$ is the collection of states where s' is sent. Then the corresponding payoff of s' on $\Theta(s')$

$$\sum_{\theta \in \Theta(s')} \mathbb{P}(\theta)\pi(s', \theta)\left[v(s', \theta) - v(s_l, \theta)\right] \leq |I_h(s')| \sum_{\theta \in \Theta(s')} \mathbb{P}(\theta)$$

Now suppose $A_i \geq \tilde{A}_i$ for all i such that when the regulator replaces $\pi(s', \theta) > 0$ by $\pi(s_h, \theta) > 0$ we have

$$\pi(s_h, \theta) \geq \frac{|I_h(s')|}{n}.$$

As s_h is be sent on a larger set of states than s', ($\Theta(s') \subsetneq \Theta(s_h)$), payoff of s_h on $\Theta(s_h)$ must exceed that of s' on $\Theta(s')$,

$$\sum_{\theta \in \Theta(s')} \mathbb{P}(\theta)\pi(s', \theta)\left[v(s', \theta) - v(s_l, \theta)\right] < \sum_{\theta \in \Theta(s_h)} \mathbb{P}(\theta)\pi(s_h, \theta)\left[v(s_h, \theta) - v(s_l, \theta)\right],$$

which contradicts the optimality of $\pi(s', \theta) > 0$.

For the second part, we use the dual approach. We first suppose that the cross-state borrowing prices $\{\lambda_i(s)\}_{i \in I_h(s)}$ for equilibrium signals ($s : \mathbb{P}(s) > 0$) are pinned down by s_l (as the opportunity cost of sending s) and later verify that these conditions hold in the solution structure. To be more specific, for any $s : \pi(s, \theta_1) > 0$ ($\theta_1 : \tilde{A}_i > 0\ \forall i$),$\exists \Theta(s)$ with $|\Theta(s)| = |I_h(s)|$

$$\theta \in \Theta(s), \theta \neq \theta_1 : \ q(\theta) = v(s, \theta) + \sum_i \lambda_i(s)\left[L_i(s_{-i}, \theta | s_i = l) - R_i\right] = v(s_l, \theta). \quad \text{(A.11)}$$

For fixed $p, \{A_i\}_i, \{D_i\}_i$, notice that $\{\lambda_i(s_h)\}_{i \in I_h(s_h)}$ are a set of constants while

$\{\lambda_i(s)\}_{i\in I_h(s)}$ for $s \neq s_h$ decrease in $\{R_i\}_i$:

$$L_i(s_{-i},\theta|s_i = l) - R_i = \begin{cases} \tilde{A}_i + R_i^c - D_i - R_i & s = s_h \\ \tilde{A}_i - D_i + \sum_j \mathbf{1}_{\{y_{ij}<R_{ij}\}} y_{ij} - \frac{\sum_j \mathbf{1}_{\{y_{ij}<R_{ij}\}}}{R_i} \cdot R_i & s \neq s_h \end{cases}.$$

Then there exists $\left\{\tilde{R}_i\right\}_i$ such that for $R_i \geq \tilde{R}_i \geq \bar{R}_i$ and $\forall s \neq s_h$

$$q(\theta_1) = \sum_i w_i + \sum_i \lambda_i(s_h)(\tilde{A}_i + R_i^c - D_i - R_i) > \sum_i w_i + \sum_{i\in I_h(s)} \lambda_i(s)(\tilde{A}_i + R_i^c - D_i - R_i),$$

$\pi(s_h, \theta_1) = 1$.

$\{\lambda_i(s)\}_{i\in I_h(s)}$ that are pinned down this way relate to the marginal benefit-cost indices $\xi^{(1)}(s,\theta;\theta_1)$, which measures the tradeoff of assigning the first unit of excess liquidity. Conditional on the same set of states, the upper bound of s_h's payoff must exceed that of $s \neq s_h$. Therefore, the regulator never reallocates the excess liquidity consumed by (s_h,θ) to (s,θ) for $s \neq s_h$ in later rounds, verifying the structure we imposed in (A.11). ■

Three-Bank Symmetric Complete Network

Proof: For notational convenience, let $\theta^{(m)}$ refer a state where the number of bad assets is m:

$$\theta^{(m)} \in \{\theta | \sum_i \mathbf{1}_{\{\tilde{A}_i=0\}} = m\}.$$

Case 1: $R > 2(A - D)$.

In Case 1, all banks default whenever there is bad asset realization, so $\Theta_G(s) = \{\theta_1\}$ for all s. Following Proposition 3, we calculate the first round indices $\xi^{(1)}(s,\theta;\theta_1)$:

$$\max_\theta \xi^{(1)}(s_h,\theta;\theta_1) = \xi^{(1)}(s_h,\theta^{(1)};\theta_1) = \mathbb{P}(\theta_1)\cdot\frac{9(A-D)}{3D-2A},$$
$$\max_\theta \xi^{(1)}(hhl,\theta;\theta_1) = \xi^{(1)}(hhl,ggb;\theta_1) = \mathbb{P}(\theta_1)\cdot\frac{2(A-D)}{\frac{R}{2}-(A-D)} > \max_\theta \xi^{(1)}(hll,\theta;\theta_1).$$

Hence,

$$\arg\max_{(s,\theta)} \xi^{(1)}(s,\theta;\theta_1) = \begin{cases}(s_h,\theta^{(1)}), & R \geq \bar{R} = -\frac{6}{9}D + \frac{10}{9}A,\\ (hhl,ggb), & R < \bar{R} = -\frac{6}{9}D + \frac{10}{9}A.\end{cases}$$

The scope of banks that the regulator saves at the one-shock states (ggb, gbg, bgg) by consuming the first unit of liquidity at ggg depends on the size of interbank exposure R. When contagion is severe, he saves all banks (s_h); Otherwise he saves only the less harmed banks – those default due to contagion but have good assets – by a truth-telling policy that sends hhl at ggb, hlh at gbg and lhh at bgg (with equal probability) and also sends hhl, hlh, lhh at ggg for cross-state borrowing.

We need to further compare $\xi^{(2)}(s,\theta;\theta_1)$ if there is extra liquidity ($\pi(s_l, ggg) > 0$) even when $\pi(s_h,\theta^{(1)}) = 1$ or $\pi(hhl,ggb) = \pi(hlh,gbg) = \pi(lhh,bgg) = 1$. Notice that the marginal comparative advantage of s_h over other signals

$$\frac{\xi^{(1)}(s_h,\theta;\theta_1)}{\xi^{(1)}(s,\theta;\theta_1)}$$

increases with the number of shocks. This is because in this example the liquidity gap between sending signals

$$\sum_{\theta \in \Theta(s_h)} \pi(s_h, \theta) L_i(s_{h(-i)}, \theta | s_i = h) - \sum_{\theta \in \Theta(s)} \pi(s, \theta) L_i(s_{-i}, \theta | s_i = h)$$

is fixed across states while the distance to default

$$\sum_{\theta \in \Theta(s)} \pi(s, \theta) \left[R_i - L_i(s_{-i}, \theta | s_i = h) \right]$$

increases as the state gets worse, dampening the liquidity advantage of $s \neq s_h$ from separating less harmed banks as the index function is increasing and convex in liquidity levels.

Therefore, if $R \geq \bar{R} = -\frac{6}{9}D + \frac{10}{9}A$ the regulator sends s_h monotonically across states as liquidity increases (part (1) in the information structure).

However, if $2(A - D) < R < \bar{R}$ and with larger liquidity, $(2(A - D), \bar{R})$ could be further separated into $(2(A - D), \hat{R}(s)$ where $s \neq s_h$ is more efficient at worse states $(m > 1)$ and $[\hat{R}(s), \bar{R})$ where s_h is more efficient at states $\theta : m > 1$ and even $\theta : m = 1$ as the abundant liquidity effect (Proposition 4 part (1)) dominates.

To concentrate on the interesting part of the result, we make the following assumption such that the abundant liquidity effect dominates at worse-off states $(\theta : m > 1)$ and there is no need to further separate $(2(A - D), \bar{R})$:

$$A - D \geq (\frac{1 - p}{p})^2 (3D - A),$$

which is easily satisfied if there is reasonable amount of asset upside and the shock probability is not large. Therefore, we have proved the second part of the optimal information structure.

Case 2: $A - D < R \leq 2(A - D)$.

Proof of the optimal information structure is similar as that of Case 1.

As the banks with good assets in the one-shock states (ggb, gbg, bgg) are solvent, the regulator either reports all banks are h or truth-tells at theses states:

$$\pi(s_h, \theta^{(1)}) + \pi(\hat{s}(\theta^{(1)}), \theta^{(1)}) = 1.$$

For the latter case $\theta^{(1)} \in \Theta_G(\hat{s}(\theta^{(1)}))$, the regulator lends liquidity to other bad states. Then the only relevant first-round index at $\hat{s}(\theta^{(1)})$ is

$$\xi^{(1)}(s_h, \theta^{(1)}; \theta_1) = \frac{3\mathbb{P}(\theta_1)(A - D)}{3D - 2A}.$$

The other candidates for signals sent at bad states $\theta \neq \hat{s}(\theta^{(1)})$ are

$$\xi^{(1)}(s_h, \theta^{(2)}; \theta_1) = \frac{9\mathbb{P}(\theta_1)(A - D)}{3D - A},$$
$$\xi^{(1)}(hhl, gbb; \theta_1) = \xi^{(1)}(hhl, bgb; \theta_1) = \frac{4\mathbb{P}(\theta_1)(A - D)}{2D + R - A},$$
$$\xi^{(1)}(hll, gbb; \theta_1) = \frac{\mathbb{P}(\theta_1)(A - D)}{R - (A - D)}, \ \xi^{(1)}(hll, bbg; \theta_1) = \frac{3\mathbb{P}(\theta_1)(A - D)}{D + R - \frac{A-D}{2}} < \xi^{(1)}(s_h, \theta^{(2)}; \theta_1).$$

The comparative advantage of s_h over other signals depends on the size of R as Case 2 corresponds to a smaller exposure range. We follow the assumption that the abundant liquidity effect dominates at $\theta : m > 1$:

$$A - D \geq (\frac{1 - p}{p})^2 (3D - A),$$

and thus do no need to further separate $(A - D, 2(A - D)]$ into $(A - D, \hat{R}(s))$ where

$s \neq s_h$ that separates less harmed banks is chosen and $[\hat{R}(s), 2(A-D)]$ where s_h is chosen.

What remains is to compare sending s_h at different states. If $A \leq \frac{6}{5}D$, the effect of asset shock is dominated by the importance of system failure, and the regulator borrows the first unit of liquidity at θ_1 by sending s_h at ggg and two-shock states gbb, bgb, bbg. If liquidity is abundant, the regulator extends s_h at one-shock states. Hence, if $A \leq \frac{6}{5}D$, s_h is sent non-monotonically across states. Otherwise, if $A > \frac{6}{5}D$, asset shock dominates and s_h is sent non-monotonically across states. ■

Lemma A In the optimal information structure, if $\pi^*(s,\theta) > 0$ and s reports $s_i = h$ for i and a set of banks symmetric to i at θ and $s_k = l$ otherwise, then any bank j with higher adjusted liquidity

$$\frac{L_j(s_{-j},\theta|s_j = h) - R_j}{A_j - D_j} \geq \frac{L_i(s_{-i},\theta|s_i = h) - R_i}{A_i - D_i} \tag{A.12}$$

must be reported with $s_j = h$.

Proof: If not, propose a new information structure π' that replaces $\pi^*(s,\theta) > 0$ with $\pi(s',\theta)$ where $s'_j = h$ and $s'_{-j} = s_{-j}$. The obedience constraint of bank j holds due to (A.12). As

$$y_{ij}(s'_{-i},\theta|s'_i = h) = R_{ij} \geq y_{ij}(s_{-i},\theta|s_i = h),$$

and payment fixpoint is monotone increasing, we have

$$L_i(s'_{-i},\theta|s'_i = h) \geq L_i(s_{-i},\theta|s_i = h),$$

and thus the obedience constraints of banks reported with h in s hold. As $v(s', \theta) \geq v(s, \theta)$ by including $s_j = h$, $V(\pi') \geq V(\pi^*)$ which contradicts optimality. ■

General Symmetric Complete Network

Proof: This part is similar to that of the three-bank example, except that the stability outcome is a more general step function of R, which further complicates our discussion.

By Lemma A we argue that the following 4 types of signals sent at the corresponding bad states ($\theta \notin \Theta_G(s)$) summarize the candidate policies: (1) s_h, (2) saving a subset of bad-asset banks $\tilde{s}(\theta)$, (3) truth-telling $\hat{s}(\theta)$, (4) s_l, where

$$\tilde{s}(\theta) : \begin{cases} \forall i, \ \tilde{A}_i > 0, & s_i = h, \\ \exists i, \ \tilde{A}_i = 0, & s_i = h, \\ \exists i, \ \tilde{A}_i = 0, & s_i = l. \end{cases} \qquad \hat{s}(\theta) : \begin{cases} \forall i, \ \tilde{A}_i > 0, & s_i = h, \\ \forall i, \ \tilde{A}_i = 0, & s_i = l. \end{cases}$$

Let $m \equiv \sum_i \mathbf{1}_{\{\tilde{A}_i = 0\}}$, $\theta^{(m)} \in \left\{\theta | \sum_i \mathbf{1}_{\{\tilde{A}_i(\theta) = 0\}} = m\right\}$ and $b_h \equiv \sum_i \mathbf{1}_{\{\tilde{A}_i = 0, s_i = h\}}$ respectively denote the number of asset shocks, any state with a specific number of asset shocks and the number of bad-asset banks reported with h.

First, notice that if the number of shocks

$$m(\theta) \leq \frac{n(A - D)}{A} \equiv \bar{m}_1,$$

$\pi(s_h, \theta) = 1$ is a dominant strategy as the cross-bank subsidy alone is enough to convince the public that the average bank is solvent.

Second, we introduce $\bar{m}_2$, another threshold for the number of asset shocks, to help characterize the benchmark stability outcome:

$$m \equiv \sum_i \mathbf{1}_{\{\tilde{A}_i = 0\}} \leq \frac{(n-1)(A-D)}{R} \equiv \bar{m}_2.$$

Under the null information structure, for states where $m(\theta) \leq \bar{m}_2$ banks with good assets ($\tilde{A}_i > 0$) are solvent, and those with bad assets ($\tilde{A}_i = 0$) default; For states with more asset shocks $m(\theta) > \bar{m}_2$, all banks default.

When $\bar{m}_1 \geq \bar{m}_2$, the optimal disclosure policy is monotone in states number of asset shocks. However, when $\bar{m}_1 < \bar{m}_2$, optimal disclosure is not necessarily monotone. One counter example is the three-bank complete network with small R and A we discussed earlier. In that example, to boost the unconditional system stability, saving the 2-shock states is preferred as the cascade of failure is larger there and the distance to default is relatively smaller with small A.

Case 1 $\bar{m}_2 \leq \bar{m}_1 : A > D > R \geq \frac{n-1}{n} A > 0$, $A - D < R$.

We show that the truth-telling signal $\hat{s}(\theta)$ is dominated in case 1 where contagion shock is large:

$$\begin{aligned}
\xi^{(1)}(s_h, \theta^{(m)}; \theta_1) &= \frac{n^2 \mathbb{P}(\theta^{(0)})(A-D)}{-\left[n(A-D) - mA\right]}, \\
\xi^{(1)}(\hat{s}(\theta^{(m)}), \theta^{(m)}; \theta_1) &= \frac{\mathbb{P}(\theta^{(0)})(A-D)\,(n-m)}{\left[\frac{mR}{n-1} - (A-D)\right]}, \\
\frac{\xi^{(1)}(s_h, \theta^{(m)}; \theta_1)}{\xi^{(1)}(\hat{s}(\theta^{(m)}), \theta^{(m)}; \theta_1)} &= \frac{n^2\left[\frac{mR}{n-1} - (A-D)\right]}{(n-m)\left[mA - n(A-D)\right]} > \frac{\frac{mR}{n-1} - (A-D)}{\frac{mA}{n} - (A-D)} \\
&\geq \frac{\frac{m}{n-1}\frac{n-1}{n}A - (A-D)}{\frac{mA}{n} - (A-D)} = 1.
\end{aligned}$$

The relative efficiencies of the other two candidate signal types, s_h and saving a subset of bad-asset banks $\tilde{s}(\theta)$, is ambiguous. For a given $m(\theta)$, the optimal $b_h^* \equiv \sum_i \mathbf{1}_{\{\tilde{A}_i=0, s_i=h\}}$ trades-off saving more banks and the average distance to default in these banks. As

$$\xi^{(1)}(\tilde{s}(\theta^{(m)}|b_h), \theta^{(m)}; \theta_1) = \frac{(n-m+b_h)^2 \mathbb{P}(\theta^{(0)})(A-D)}{\left[\frac{(m-b_h)R}{n-1} + b_h D - (n-m)(A-D)\right]}, \tag{A.13}$$

$$\frac{\partial \xi^{(1)}(\tilde{s}(\theta^{(m)}|b_h), \theta^{(m)}; \theta_1)}{\partial b^h} = \underbrace{F(b_h)}_{+} \cdot \left[b_h(D - \frac{R}{n-1}) - 2(n-m)A + (n-m)D + \frac{(n+m)R}{n-1} \right], \tag{A.14}$$

we argue that there exists a threshold state $\bar{\theta}$, such that

$$b_h^* = \begin{cases} 1, & m(\theta) \leq \sum_i \mathbf{1}_{\{\tilde{A}_i(\bar{\theta})=0\}}, \\ m(\theta), & m(\theta) > \sum_i \mathbf{1}_{\{\tilde{A}_i(\bar{\theta})=0\}}. \end{cases}$$

Then for states with relatively more asset shocks, saving a subset of bad-asset banks $\tilde{s}(\theta)$, is dominated by reporting all banks are h, s_h.

For $\theta: \; m(\theta) \leq \sum_i \mathbf{1}_{\{\tilde{A}_i(\theta)=0\}}$, plug in the optimal $b_h^* = 1$:

$$\frac{\xi^{(1)}(s_h, \theta^{(m)}; \theta_1)}{\xi^{(1)}(\tilde{s}(\theta^{(m)}|b_h^*=1), \theta^{(m)}; \theta_1)} = \underbrace{\left(\frac{n}{n-m+1}\right)^2}_{\text{ratio: stability gain}} \underbrace{\frac{\frac{(m-1)R}{n-1} + D - (n-m)(A-D)}{mA - n(A-D)}}_{\text{ratio: distance to default}}, \tag{A.15}$$

where the first part increases in m and one can verify that the second part decreases in m under the Case 1 parameters. For small m, the second effect dominates whereas the first effect dominates for larger m. As A.15 equals 1 when $m = 1$, there exists a threshold state $\hat{\theta}$ such that

$$\arg\max \xi^{(1)}(s, \theta^{(m)}; \theta_1) = \begin{cases} s_h, & m(\theta) = 1, \\ \tilde{s}(\theta^{(m)} | b_h^* = 1), & 1 < m(\theta) \leq \sum_i \mathbf{1}_{\{\tilde{A}_i(\hat{\theta})=0\}}, \\ s_h, & m(\theta) > \sum_i \mathbf{1}_{\{\tilde{A}_i(\hat{\theta})=0\}}. \end{cases}$$

We focus on the first-round marginal comparison, and extend the assumption that with abundant liquidity the regulator sends s_h in later rounds at the inferior states as the liquidity effect dominates. One can verify that under the Case 1 parameter range, $\bar{m}_2 \leq \bar{m}_1$. Therefore, the optimal disclosure policy is sending s_h monotonically across states.

Case 2 $\bar{m}_2 > \bar{m}_1 : A > D > R > 0$, $A - D < R < \frac{n-1}{n} A$.

The discussion is similar to that in Case 1, except that the states $\theta : \bar{m}_1 < m(\theta) \leq \bar{m}_2$ complicate things in two ways. First, the regulator needs to choose from the candidate signals $s_h, \tilde{s}(\theta)$ (that consume liquidity) at $\theta : \bar{m}_1 < m(\theta) \leq \bar{m}_2$; Second, the regulator needs to choose whether to save the states in $\theta : \bar{m}_1 < m(\theta) \leq \bar{m}_2$ or $\theta : m(\theta) > \bar{m}_2$. The index ξ at $\theta : m(\theta) > \bar{m}_2$ are identical to those in Case 1, except under different parameter range for A, D, R, which makes the pooling signal s_h less favorable.

At states $\theta : \bar{m}_1 \leq m(\theta) < \bar{m}_2$, for a small number of asset shocks a subset of bad-asset banks could be saved by the cross-bank subsidy only, which corresponds to the first case of b_h^* new from that in Case 1 and $\theta : m(\theta) > \bar{m}_2$ in Case 2:

$$b_h^* = \begin{cases} \lfloor \frac{(n-m)(A-D)-\frac{mR}{n-1}}{D-\frac{R}{n-1}} \rfloor + 1, & m(\theta) \leq \bar{m}', \\ 1, & \bar{m}' < m(\theta) \leq \bar{m}'', \\ m(\theta), & m(\theta) > \bar{m}''. \end{cases}$$

Monotonicity across states holds within $\theta : \bar{m}_1 \leq m(\theta) \leq \bar{m}_2$ and within $\theta : m(\theta) > \bar{m}_2$. For the signal choice,

$$\arg\max \xi^{(1)}(s, \theta^{(m)}; \theta_1) = \begin{cases} s_h, & \text{large } R\text{, or } \bar{m}_1 \ (\bar{m}_2)\text{, or small } A, \\ \tilde{s}(\theta) \text{ or } \hat{s} & \text{otherwise,} \end{cases}$$

where b_h^* was shown earlier.

For Case (2) we emphasize the policy monotonicity across states, which does not necessarily hold when A is relatively small, as shown by our three-bank example. It's easily verified that the relative cost of saving worse-off states against better-off states is increasing in A.

In sum, with larger R and smaller A, s_h is randomly reported and non-monotonically across states to borrow from θ_1 liquidity; Otherwise $h-$report is monotone across states: $\pi(s_h, \theta) = 1$ for $\theta :\ 1 < m(\theta) \leq \bar{m}_1$, and then s_h, $\hat{s}$ or $\tilde{s}$ is used at inferior states $\theta :\ \bar{m}_1 < m(\theta) \leq \bar{m}_2$. ■

Three-Bank Ring Network

Proof: From Lemma A, we know that if the regulator report $s_i = h$, he would report h on bank i's lender banks with $\tilde{A} > 0$. Then there are 4 types of candidate

(s,θ): (1) s_h; (2) truth telling signal $\hat{s}(\theta)$, (3) $\tilde{s}(\theta)$: saving the lender further from asset shock, i.e., $s_i = s_{i+1} = l,\ s_{i+2} = h$ for $\tilde{A}_i = 0,\ \tilde{A}_{i+1} = \tilde{A}_{i+2} = A$; (4) s_l. The corresponding indices are

$$\xi^{(1)}(s_h, \theta^{(1)}; \theta_1) = \frac{9\mathbb{P}(\theta_1)(A-D)}{3D-2A},$$
$$\xi^{(1)}(hhl, ggb; \theta_1) = \frac{2\mathbb{P}(\theta_1)(A-D)}{\frac{R}{2}\frac{3A-D}{(A+\frac{3}{2}R)} - (A-D)},$$
$$\xi^{(1)}(hll, gbg; \theta_1) = \frac{\mathbb{P}(\theta_1)(A-D)}{R-2(A-D)}.$$

We argue that the comparative efficiencies of s_h over the other signals $\hat{s}$, $\tilde{s}$

$$\frac{\xi^{(1)}(s_h, \theta^{(1)}; \theta_1)}{\xi^{(1)}(hhl, ggb; \theta_1)},\quad \frac{\xi^{(1)}(s_h, \theta^{(1)}; \theta_1)}{\xi^{(1)}(hll, gbg; \theta_1)}$$

increases in R. The result of s_h vs. $\tilde{s}$ (e.g., hll at gbg) comes from the fact that a larger counterparty exposure or contagion effect increases complementarity in liquidity levels, which makes s_h more favorable. For s_h vs. $\hat{s}$ (e.g., hhl at ggb),

$$\frac{\partial \xi^{(1)}(hhl, ggb; \theta_1)}{\partial R} < 0,$$

there is an additional effect as a larger R improves the cross-bank risk sharing under $\hat{s}$, which is dominated by the first effect.

For the comparison between $\hat{s}$ and $\tilde{s}$, $\hat{s}$ is relatively more favorable as we increase R, because the average distance to default, $\frac{R}{2}\frac{3A-D}{(A+\frac{3}{2}R)} - (A-D)$,

$$\frac{\partial\left[\frac{R}{2}\frac{3A-D}{(A+\frac{3}{2}R)} - (A-D)\right]}{\partial R} > 0,\quad \frac{\partial^2\left[\frac{R}{2}\frac{3A-D}{(A+\frac{3}{2}R)} - (A-D)\right]}{\partial R^2} < 0.$$

Intuitively, distance to default increases under both signals which dampens the liquidity advantage under $\tilde{s}$. Additionally, R enhances the cross-bank risk sharing under $\hat{s}$ as $\frac{A}{R}$ is closer to 1.

When there is abundant liquidity and the first round assignment reaches corner solution $\pi(s,\theta)=1$, we assume that the liquidity effect dominates and s_h is assigned.

In sum, we have proved that the stated optimal information structure that disclosure is monotone is state and the regulator saves the state with one-shock. As R decreases, the regulator saves fewer banks: he changes from reporting all banks are h, to truth-telling of the state, to reporting only the bank further away from shock is h. ■

General Symmetric Ring Network.

By symmetry we can assume that any bank subscript that follows, e.g. $i+d$, is less or equal than n to avoid renumbering. We characterize the pattern of any optimal information structures in interior solutions. For marginal comparison, we can first compare the (s,θ) pairs conditional on the number of asset shocks m and argue that those who contradict the above patterns are weakly dominated in their $m-$shock group.

Proof of 2.4.1

Proof: First, we argue that any candidate (s,θ) with the following structure: $\pi(s,\theta')>0$ for

$$\theta' : \tilde{A}_j > 0, \text{ for } i \leq j \leq i+d, \; d > 1, \; \tilde{A}_{i+d+1} = 0$$

$$s : s_j = \begin{cases} h, & i \leq j \leq i + d_1 < i + d, \\ l, & i + d_1 < j \leq i + d, \\ h \text{ or } l, & \text{otherwise.} \end{cases}$$

is dominated. We can slightly change the cross-state distribution of s from the original one, i.e., $\pi(s, \theta)$ for $\theta \in \Theta'(s, \theta; \theta_1)$, and keep everything else unchanged:

$$\pi'(s, \theta) = \begin{cases} \pi(s, \theta') + \pi(s, \theta), & \theta = \theta'' : \tilde{A}_j(\theta'') = \begin{cases} \tilde{A}_{j+1}(\theta'), & j \in \{\} \\ \tilde{A}_j(\theta'), & \text{otherwise} \end{cases} \\ 0, & \theta = \theta', \\ \pi(s, \theta), & \theta \neq \theta', \theta''. \end{cases}$$

Note that we are calculating the marginal index $\xi^{(1)}$, so the distributions for other signals could be neglected. One can verify that the new information structure is feasible (the obedience constraint for bank i could be slack while those for other banks are equivalent) and delivers the same payoff to the regulator.

Second, we argue that any candidate (s, θ) with the following structure: $\pi(s', \theta') > 0$ for

$$\theta' : \tilde{A}_j > 0, \text{for } i < j \leq i + d, d > 1, \tilde{A}_{i+d+1} = 0$$

$$s' : s_j = \begin{cases} h, & j = i, i + d_1 + 1, \cdots, i + d, \\ l, & j = i + 1, \cdots, i + d_1, \\ h \text{ or } l, & \text{otherwise.} \end{cases}$$

is dominated. The above structure implies that for any bank j in $i < j \leq i + d_1$, there exists a state θ with $\pi(s',\theta) > 0$ and $\tilde{A}_j = 0$; Otherwise, there exists $\hat{j}$ from $i < \hat{j} \leq i + d_1$,

$$\hat{j}: \; s'_j = l, \; L_{\hat{j}}(s',\theta) = A > R, \; \forall \theta : \pi(s',\theta) > 0,$$

which is dominated.

Then we argue any candidate (s',θ) with the above described feature could be replaced by s'', where

$$s'' : s''_j = \begin{cases} h, & j = i+1, i+d_1+1, \cdots, i+d, \\ l, & j = i, i+2, \cdots, i+d_1, \\ h \text{ or } l, & \text{otherwise,} \end{cases} \quad \text{or } s''_j = \begin{cases} s'_j, & j \neq i, i+1, \\ h, & j = i+1, \\ l, & j = i, \end{cases}$$

and the corresponding cross-state distribution:

$$\pi'(s'',\theta) = \begin{cases} \pi(s',\theta), & \theta \in \left\{\theta | \tilde{A}_{i+1} > 0\right\} \cup \left\{\theta | \tilde{A}_{i-1} = \tilde{A}_i = \tilde{A}_{i+1} = 0\right\} \\ 0, & \theta : \tilde{A}_{i-1} > 0, \tilde{A}_{i+1} = 0 \\ \pi(s',\theta) + \pi(s',\theta'), & \theta' : \tilde{A}_{i-1} > 0, \tilde{A}_i = \tilde{A}_{i+1} = 0, \\ & \theta : \tilde{A}_i > 0, \tilde{A}_{\hat{i}} = 0, \tilde{A}_j(\theta) = \tilde{A}_j(\theta'), j \neq i, \hat{i}, \\ \pi(s',\theta) + \pi(s',\theta''), & \theta' : \tilde{A}_{i-1} > 0, \tilde{A}_i > 0, \tilde{A}_{i+1} = 0, \\ & \theta : \tilde{A}_{i+1} > 0, \tilde{A}_{\hat{i}} = 0, \tilde{A}_j(\theta) = \tilde{A}_j(\theta'), j \neq i, \hat{i}, \end{cases}$$

where

$$\hat{i} \equiv \sup_k \left\{ k < i | \tilde{A}_{k+1}(\theta') = 0, s'_{k+1} = l \right\}.$$

The idea is to switch the h report backward on banks and the asset shocks forward. One can verify that the above structure is feasible and delivers the same payoff to the regulator, and banks reported with h have weakly higher expected liquidity levels or weakly smaller distance to default. The adjustment is free of the distribution of other signals as we are discussing about the marginal efficiencies. By an iterated argument, we know the discussed candidate is dominated for any $d_1 > 1$.

As multiple disconnected asset shocks separate the ring network to sub-chain networks with different length, by symmetry we know the above property extends to all sub-chains. The concerns of conditioning on the same asset shock number are irrelevant here. The dominated structures could be better than the candidates in an inferior group, but we do not need the whole ranking of (s, θ) for interior solutions and for the welfare comparison between the ring and the complete network. Also, a signal could be sent on states across different groups, but dominated by the same signal sent on the best one (the minimum shock number) across the previous groups.

■

Proof of 2.4.2

Proof: We assume that there is at least one h report in each sub-chain, otherwise θ could be replaced by θ' where the all-l sub-chain is combined with the next following sub-chain, by switching the asset realizations between the starting bank (bad asset) of the latter chain with the second bank (good asset) in the previous chain. For notational convenience we use a common threshold distance $\bar{d}$ for every sub-chain, and the proof extends to different distance thresholds by replacing $\bar{d}$ with $\sup d$ in the index upper bound.

There are two sub cases:

(1). Within each sub-chain, at most one bank is reported with h. i.e., θ is such that for any $\tilde{A}_i = 0$, if $\tilde{A}_j > 0$ for $i < j \leq i + \bar{d}$ we have $\tilde{A}_{i+\bar{d}+1} = 0$. Then there is no cross-bank subsidy and the signal s is sent on only $\theta_1 : \tilde{A}_k > 0, \forall k$ and θ. The

$\bar{d}$ strategy at connected asset shock states is more efficient because the number of $l-$banks weakly decreases and there is cross-bank subsidy.

(2). There exists a sub-chain where more one bank is reported with h. Note that the sufficient statistics for any $\xi^{(1)}(s,\theta;\theta_1)$ are $\bar{d}$ and the structures of the sub-chains: number of sub-chains, $q(\theta)$, and the sequence of their lengths. Hence, WLOG we focus on the states where there is only one area of connected asset shocks and the other asset shocks are disjoint.

To be more specific, for any candidate $(s,\theta^{(m)})$ and threshold distance $\bar{d}>0$, we can find an equivalent $(s,\theta'^{(m)})$ such that in between disjoint h reports, there is only one asset shock. i.e., under s for all bank i with $s_i=h$ ($\tilde{A}_i=A$ by $\bar{d}>0$),

$$\exists! j,\ i'<j<i:\tilde{A}_j=0, s_j=l, j=\sup_k\left\{k<i|\tilde{A}_k=0\right\}, i'\equiv\sup_k\{k<j|s_{i'}=h\}.j=i'+1$$

Therefore, for any $\theta\in\Theta'(s,\theta;\theta_1)$ where $\pi(s,\theta)>0$,

$$L_i(s_{-i},\theta|s_i=l)\leq\begin{cases}\bar{d}(A-D)-R, & \theta:\tilde{A}_i>0,\\ x\leq -D, & \theta:\tilde{A}_i=0,\end{cases}$$

from the observation that $\forall\theta\in\Theta'(s,\theta;\theta_1)$, with $\tilde{A}_i>0$, $\exists j, i'<j<i, \tilde{A}_j=0$. We argue that

$$\xi^{(1)}(s,\theta;\theta_1)\leq X\cdot\max\left\{\frac{|I_h(s)|+[q(s,\theta)-1](\bar{d}-1)}{R-\bar{d}(A-D)},\ \frac{|I_h(s)|+q(s,\theta)(\bar{d}-1)}{D}\right\}$$

where X is a constant that captures the excess liquidity at good state(s), and q is the number of sub-chains separated by the asset shocks at the original state. If $\tilde{A}_i=0$, then any banks between j and $i+1$ are reported with l but turn out to be solvent. The maximum total number of these banks is $|I_h(s)|+(q-1)(\bar{d}-1)$ when

$\tilde{A}_i > 0$, and $|I_h(s)| + q(\bar{d} - 1)$ when $\tilde{A}_i = 0$.

Then we can find a corresponding candidate (s', θ') with connected asset shocks in the $\theta^{(m)}$ group that dominates the previous (s, θ). By symmetry, we pick

$$\theta' : \tilde{A}_1 = \tilde{A}_2 = \cdots = \tilde{A}_m = 0, \tilde{A}_{m+1} = \tilde{A}_{m+2} = \cdots = \tilde{A}_n = A.$$

and

$$\xi^{(1)}(s', \theta'; \theta_1) > \begin{cases} \frac{n-m-\bar{d}+1}{R-\bar{d}(A-D)} > \frac{|I_h(s)|+(q(s,\theta)-1)(\bar{d}-1)}{R-\bar{d}(A-D)}, & s' : s'_i = h \\ & \text{if } m + \bar{d} - 1 \leq i \leq n, s'_i = l \\ & \text{otherwise;} \\ \frac{n-m+q(s,\theta)}{D} > \frac{|I_h(s)|+q(s,\theta)(\bar{d}-1)}{D}, & s' : s'_1 = s'_2 = \cdots \\ & s'_{q-1} = s'_m = h, s'_i = l \text{ otherwise.} \end{cases}$$

■

Proof of Proposition 2.4.1

Proof: We focus on the cases $\bar{d} \geq 1$ as an example to characterize the choice of $\bar{d}$, and $\bar{d} = 0$ corresponds to a reduced case and the discussion is similar.

From binding obedience constraints and symmetry of the banks, $i = m + \bar{d} + 1, \cdots, n$, we know at the other states where $s(\bar{d}, m)$ is sent, at least one of them has bad asset and h_b is the sufficient statistic to characterize the cross-state distribution and payoff of $(s(\bar{d}, m))$.

Now we calculate the ratios of payoff over average distance to default for various h_b, and the one associated with the highest ratio, h_b^*, corresponds to $\xi^{(1)}(s(\bar{d}, m), \theta(m); \theta_1)$.

With a little abuse of notation, let

$$\theta^{(m,h_b)} \in \left\{ \theta | \sum_i \mathbf{1}_{\left\{\tilde{A}_i=0\right\}} = m, \sum_{i>\bar{d}+m}^{n} \mathbf{1}_{\left\{\tilde{A}_i=0\right\}} = h_b, \sum_{i=1}^{m-h_b} \mathbf{1}_{\left\{\tilde{A}_i=0\right\}} = m - h_b, \right\}$$

and then $\theta(m) = \theta^{(m,h_b=0)}$. To simplify the notation for calculating the cross-state distribution of s, let

$$X \equiv \mathbb{P}(\theta_1)(A - D)$$

denote the total liquidity borrowed from state $\theta_1 : \tilde{A}_i > 0, \ \forall i$, and the followings for the unconditional probability of (s, θ) pairs

$$x \equiv \mathbb{P}(\theta(m))\pi(s(\bar{d}, m), \theta(m)), \quad y \equiv \mathbb{P}(\theta^{(m)})\pi(s(\bar{d}, m), \theta^{(m,h_b)}), \quad z \equiv \binom{|I_h(s)| - 1}{h_b} y.$$

From symmetry, the binding obedience constraints for $i = m + \bar{d}$ and $i = m + \bar{d} + 1, \cdots, n - 1$ are respectively,

$$\begin{aligned} X + \left[\bar{d}(A - D) - R\right] x + \max\left\{(\bar{d} + h_b)(A - D) - R, (A - D)\right\} z = 0, \\ X + (A - D)x + \left[(A - D) - \frac{h_b}{|I_h(s)| - 1} A\right] z = 0. \end{aligned}$$

Similar to the construction of $\xi^{(1)}$, the marginal efficiency of $s(\bar{d}, m)$ sent across the collection of states $\{\theta(m), \theta^{(m,h_b)}\}$ is

$$X \cdot \left[|I_h(s)| x + v(s, \theta^{(m,h_b)}) z\right]. \tag{A.16}$$

Depending on the value of $\left[R - (\bar{d} - 1 + h_b)(A - D)\right]^+$ and $v(s, z)$, (A.16) has 3 types of representations.

(1). When $R-(\bar{d}-1+h_b)(A-D)>0$, we have $v(s,\theta^{(m,h_b)})=|I_h(s)|$.

$$X|I_h(s)|\cdot(x+z)=\frac{|I_h(s)|X}{\left[R-(\bar{d}-1)(A-D)\right]\frac{\frac{A}{|I_h(s)|-1}}{(A-D)+\frac{A}{|I_h(s)|-1}}-(A-D)}, \tag{A.17}$$

which does not depend on h_b because the asset shocks absorbed by $i=m+\bar{d}+1,\cdots,n$ are exactly canceled by the increased liquidity level of bank $m+\bar{d}$.

(2). When $R-(\bar{d}-1+h_b)(A-D)\leq 0$ and $v(s,\theta^{(m,h_b)})=|I_h(s)|$,

$$X|I_h(s)|\cdot(x+z)=\frac{|I_h(s)|X}{\left[R-(\bar{d}-1)(A-D)\right]\frac{\frac{h_bA}{|I_h(s)|-1}}{R-(\bar{d}-1)(A-D)+\frac{h_bA}{|I_h(s)|-1}}-(A-D)}, \tag{A.18}$$

which is weakly smaller than (A.17) as $R-(\bar{d}-1+h_b)(A-D)\leq 0$.

(3). When $v(s,\theta^{(m,h_b)})>I_h(s)$, we have $R-(\bar{d}-1+h_b)(A-D)\leq 0$.

$$\begin{aligned}&X\left[|I_h(s)|x+v(s,\theta^{(m,h_b)})z\right]\\&=\frac{|I_h(s)|X}{\left[R-(\bar{d}-1)(A-D)\right]\frac{\frac{h_bA}{|I_h(s)|-1}+\frac{\Delta v(s,\theta^{(m,h_b)})}{|I_h(s)|}(A-D)}{\frac{h_bA}{|I_h(s)|-1}+\frac{v(s,\theta^{(m,h_b)})}{|I_h(s)|}\left[R-(\bar{d}-1)(A-D)\right]}-(A-D)}.\end{aligned} \tag{A.19}$$

Notice that

$$v(s,\theta^{(m,h_b)})=\begin{cases}\left[h_b-\lfloor\frac{R}{A-D}-(\bar{d}-1)\rfloor\right]^++|I_h(s)|, & 1\leq h_b<m,\\ n, & h_b=m,\end{cases}$$

so conditional on $s(\bar{d},m)$, (A.19) is monotone in h_b. Therefore,

$$h_b^*=\lceil\frac{R}{A-D}-(\bar{d}-1)\rceil \text{ or } m. \tag{A.20}$$

Now we plug (A.20) into (A.19) and compare its value with (A.17). One can verify that

$$\frac{\frac{A}{|I_h(s)|-1}}{\frac{A}{|I_h(s)|-1}+(A-D)} - \frac{\frac{hb}{|I_h(s)|-1}A+\frac{\Delta v(s,\theta^{(m,h_b)})}{|I_h(s)|}(A-D)}{\frac{hb}{|I_h(s)|-1}A+\frac{v(s,\theta^{(m,h_b)})}{|I_h(s)|}\left[R-(\bar{d}-1)(A-D)\right]}$$
$$=\text{Positive Const}\cdot\left\{\frac{Av(s,\theta^{(m,h_b)})}{|I_h(s)|\left(|I_h(s)|-1\right)}\left[R-\bar{d}(A-D)\right]-\text{Positive Const}\right\},$$

so there exists threshold $\bar{R}(h_b^*)$ above which (A.20) is the optimal choice, and below $\min\bar{R}(h_b^*)$ (A.17) is optimal, in which case we pick

$$h_b = \lceil\frac{R}{A-D}-\bar{d}\rceil$$

as all h_b satisfying the conditions of (A.17) are equivalent.

Within Case (3), one can verify

$$\frac{\frac{m}{|I_h(s)|-1}A+\frac{n-|I_h(s)|}{|I_h(s)|}(A-D)}{\frac{m}{|I_h(s)|-1}A+\frac{n}{|I_h(s)|}\left[R-(\bar{d}-1)(A-D)\right]} - \frac{\frac{h_b}{|I_h(s)|-1}A+\frac{1}{|I_h(s)|}(A-D)}{\frac{h_b}{|I_h(s)|-1}A+\frac{|I_h(s)|+1}{|I_h(s)|}\left[R-(\bar{d}-1)(A-D)\right]}$$
$$=\text{Positive Const}$$
$$\cdot\left\{\left[\frac{m(|I_h(s)|+1)-h_bn}{(|I_h(s)|-1)|I_h(s)|}A+\frac{n-(|I_h(s)|+1)}{|I_h(s)|}(A-D)\right]\left[R-\bar{d}(A-D)\right]+\text{Positive Const}\right\}$$

which is either positive, or decreasing in R. Hence, $h_b^*=m$ arises only with large R.

Then we argue that conditional on the choice of h_b, the relevant efficiencies (A.17) and (A.19) are decreasing in R, which is similar to the proof in the three-bank example. Therefore, the choice of $\bar{d}$ is decreasing in R. ■

Proof of Lemma 2.4.3.

Proof:

We prove that under the parameter range that both networks share the same benchmark stability outcomes, policy gain is higher in the complete network.

We argue that

$$\max_{s,\theta} \xi_C^{(1)}(s,\theta;\theta_1) \geq \max_{s,\theta} \xi_R^{(1)}(s,\theta;\theta_1). \tag{A.21}$$

As the optimal (s,θ) is endogenous, we prove (A.21) conditional on each optimal s.

As the threshold $\bar{R}$ below which the optimal policy is switched to truth-telling at one shock states is larger in the complete network,

$$\bar{R}_C = \frac{10A - 6D}{9} > \bar{R}_R = \frac{5}{6}A - \frac{D}{2},$$

(A.21) holds when $\arg\max_{s,\theta} \xi_R^{(1)}(s,\theta;\theta_1) = (s_h, \theta^{(1)})$ and takes equality when

$$\arg\max_{s,\theta} \xi_C^{(1)}(s,\theta;\theta_1) = (s_h, \theta^{(1)}).$$

When the optimal policy in both networks are truth-telling at the one shock states, we show that the distance to default for the average bank reported with h is larger in the ring network,

$$\frac{R}{2}\frac{3A-D}{(A+\frac{3}{2}R)} - (A-D) > \frac{R}{2} - (A-D). \tag{A.22}$$

(A.22) holds at $R = 2(A-D)$, the LHS is strictly increasing and concave in R while the RHS is linearly increasing in R, and the relevant parameters are $2(A-D) < R < \bar{R}$ with $2(A-D) < \bar{R}_R < \bar{R}_C$. Hence, (A.22) holds.

The payoff of the additional type of signal in the ring network, $\tilde{s}$ which reports h on the further away lender bank, is dominated by the truth-telling signal in the complete network. ■

Proof of Proposition 2.4.2

Proof: See Proof of Lemma 2.4.3 and the counter example. ■

Proof of Proposition 2.5.1

Proof: First, notice that for any states that $\tilde{A}_i(\theta) = 0$, the regulator never sends the signal that reports only bank i has high asset value:

$$\pi(s', \theta) = 0 \text{ where } s' : s_i = h,\ s_j = l,\ \forall j \neq i,\ \text{and } \theta \in \{\theta | \tilde{A}_i(\theta) = 0\}.$$

This is because for any of the above θ, there exists a bank $j \neq i$ with $\tilde{A}_j > 0$, and then

$$\frac{L_i(s_l, \theta)}{R_i} \leq \frac{-D_i}{R_i} < \frac{A_j - D_j - R_j}{R_j} \leq \frac{L_j(s_l, \theta)}{R_j},$$

as bad asset is more serious than the worst case contagion. So if $\pi(s', \theta) > 0$ by consuming $\theta_1(\tilde{A}_i > 0, \forall i)$ liquidity, the planner is better off by setting

$$\pi(s'', \theta) = \pi(s', \theta), \pi(s'', \theta_1) = \pi(s', \theta)\frac{L_i(s_l, \theta)}{L_i(s_1, \theta),} \text{ where } s'' : s_i = s_j = h, s_k = l, \forall k \neq i, j.$$

as bank j has a better posterior under the original probability distribution. If $\pi(s', \theta_1) = 0$ and $\pi(s', \theta_G) > 0$ for some $\theta_G \neq \theta_1$, by Assumption 2 that banks facing the worst case contagion default $(A - D < R)$,

$$\theta_G \in \{\theta_2, \theta_3\}.$$

However, it's easy to verify that for any first round allocation $\theta_2, \theta_3 \in \Theta^{*(1)}$ and $\theta_6, \theta_7 \notin \Theta^{*(1)}$. Hence, if $\pi(s', \theta_G) > 0, \pi(s', \theta) > 0$ for $\tilde{A}_(\theta) = 0$, we have $\pi(s', \theta_6) = 1$

or $\pi(s', \theta_7) = 1$. WLOG say $\pi(s', \theta_6) = 1$,

$$\mathbb{P}(\theta_G)\pi(s', \theta_G)L_i(s_l, \theta) + \mathbb{P}(\theta_6)L_i(s_l, \theta_6) > 0$$
$$\Rightarrow (1-p)[R-(A-D)] < p\pi(s', \theta_G)(A-D) < p(A-D),$$

and then the planner is better off switching to

$$\pi(hhl, \theta_1) = \pi(hlh, \theta_1) = 0.5, \pi(hhl, \theta_2) = \pi(hlh, \theta_3) = 1,$$

which is feasible by verifying the obedience constraints under $(1-p)[R-(A-D)] < p(A-D)$.

Therefore, $\pi(s', \theta) = 0$ and we only needs to consider the states where an individual bank is exposed to contagion shock only. The fact that the center bank in the aggregate is facing a smaller contagion compared to the periphery banks is the key to Proposition 5.

Second, we argue that if $4(A-D) \geq R$, conditional on any $s \in \{hll, lhl, llh, lhh\}$ the maximum first-round benefit-cost index among possible candidate state is

$$\max_{\theta} \xi^{(1)}(s, \theta; \theta_1) = \left(\sum_i w_i \mathbf{1}_{\{s_i = h \cap i \in D(\theta)\}}\right) \cdot \frac{\mathbb{P}(\theta_1)(A-D)}{R-(A-D)},$$

If $4(A-D) \geq R$, a periphery bank is solvent if only the other periphery bank has bad asset realization. For states where periphery banks default due to contagion, $L_i - R_{i1} = \frac{A-D-R}{2} (< 0)$ for $s = lhl, llh$. The center bank could be exposed to partial contagion when some periphery bank has good asset, and $L_1 - \sum_{i \neq 1} R_{i1} > A - D - R$ for $s = hll$. However such states could be ruled out for $s = hll$ as the regulator should send hhl or hlh instead informing the healthy periphery also has good asset. It's also easy to verify that $\pi(lhh, \theta) = 0$ for $\theta \in \{\theta_2, \theta_3, \theta_4, \theta_6, \theta_7\}$ as

$\xi^{(1)}(lhh, \theta; \theta_1) < \xi^{(1)}(hhh, \theta; \theta_1)$ and the weights multiplier is larger at hhh. So for $s = lhh$ the above result holds. So far we have also proved that the non-dominated (s, θ) combination for $s \in \{hll, lhl, llh, lhh\}$ are

$$(hll, \theta_4),\ (lhl, \theta_6),\ (llh, \theta_7),\ (lhh, \theta_5).$$

Next, we state that

$$\xi^{(1)}(hhl, \theta_2; \theta_1) = \xi^{(1)}(hlh, \theta_3; \theta_1) > \max_{\theta} \xi^{(1)}(s, \theta; \theta_1) \text{ for } s \in \{hll, lhl, llh, lhh\},$$

so these signals will not be considered for the first round allocation. This is because at (hhl, θ_2) or (hlh, θ_3), bank 1 is exposed to only partial contagion, gets externality from the healthy periphery, and cross-bank smoothing requires to send $hhl(hlh)$ at bank 1's more favorable states.

If the parameter range is such that first round allocation exhausts θ_1 excess liquidity, we have proved Proposition 5. If not, after the first round,

$$\begin{cases} s^{*(1)} = hhh: & \pi(hhh, \theta_2) = \pi(hhh, \theta_3) = \pi(hhh, \theta_5) = 1; \\ s^{*(1)} = hhl, hlh: & \pi(hhh, \theta_2) = \pi(hhh, \theta_3) = 1. \end{cases}$$

If in equilibrium $\mathbb{P}(hhh) = 0$, $s^{*(1)} = hhl, hlh$. In the first round $\mathbb{P}(s_1 = h) = 2\mathbb{P}(s_2 = h) = 2\mathbb{P}(s_3 = h)$, and bank 1 is treated nicer in equilibrium as

$$\begin{aligned} \mathbb{P}(\theta_6)\pi(lhl, \theta_6) + \mathbb{P}(\theta_5)\pi(lhh)\pi(lhh, \theta_5) &\leq \mathbb{P}(\theta_6) + \mathbb{P}(\theta_5)\pi(lhh)\pi(lhh, \theta_5) \\ &\leq \mathbb{P}(\theta_2)\pi(hhl, \theta_2) + \mathbb{P}(\theta_4)[\pi(hhl, \theta_4) + \pi(hll, \theta_4)]. \end{aligned}$$

If $s^{*(1)} = hhh$, $\mathbb{P}(lhh) = 0$. If $\mathbb{P}(hhl) > 0$,

$$\mathbb{P}(\theta_6)\pi(lhl, \theta_6) \leq \mathbb{P}(\theta_4)[\pi(hhl, \theta_4) + \pi(hll, \theta_4)].$$

If $\mathbb{P}(hhl) = 0$, the residual space for hll, lhl, llh is symmetric, but hll has a larger weight.

$$\mathbb{P}(\theta_6)\pi(lhl, \theta_6) \geq \mathbb{P}(\theta_4)\pi(hll, \theta_4).$$

When $R > 4(A - D)$, all banks default whenever there is bad asset realization. Then a periphery bank is faced with a smaller contagion exposure (as a proportion to total interbank debts) than the center bank when only the other periphery bank has bad asset valuation. Note that under this range

$$\begin{aligned}
\xi^{(1)}(lhl, \theta_2; \theta_1) =& \frac{w_2}{\frac{R}{2} - 2(A - D)} \cdot \mathbb{P}(\theta_1)\frac{A - D}{2}, \\
\xi^{(1)}(hhl, \theta_2; \theta_1) =& \left[\frac{(w_1 + w_2)(1 + \frac{R}{2A})}{\frac{D}{2}\frac{R}{2A} - \frac{A-D}{2}}\right] \cdot \mathbb{P}(\theta_1)\frac{A - D}{2}, \\
\xi^{(1)}(hhh, \theta_2; \theta_1) =& \left[\frac{3(w_1 + 2w_2)}{\frac{3D-2A}{2}}\right] \cdot \mathbb{P}(\theta_1)\frac{A - D}{2}.
\end{aligned}$$

So there exists a $\bar{R}$ for $R > \max\{\bar{R}, 4(A - D)\}$,

$$\xi^{(1)}(lhl, \theta_2; \theta_1) < \min\{\xi^{(1)}(hhl, \theta_2; \theta_1), \xi^{(1)}(hhh, \theta_2; \theta_1)\}$$

and then the center bank is treated nicer. When $4(A-D) < R < \max\{\bar{R}, 4(A - D)\}$, in the first round the regulator sends lhl and llh and it could be that in equilibrium the periphery is treated nicer.

■

Proof of Proposition 2.5.2

Proof: The idea of the proof is to verify that whenever the regulator saves a default periphery bank by reporting $s_{i^p} = h$, he would report $s_{c(i^p)} = h$ due to the strong complementarity between each periphery bank and its connected core. In addition, as the core banks have better risk sharing with more counter-parties, the cross-state distribution of size-adjusted distance to default is flatter for the core banks, whereas the periphery banks have fewer mass points and a left tail more concentrated on lower liquidities. Specifically, a periphery bank is subject to the largest contagion shock whenever its connected core bank has bad asset, but it's a rare occurrence for the core bank to have the same size adjusted liquidity level where all its counter-parties have bad assets.

For the proof of this part, we refer to the information structure generated by consuming the public excess liquidity only, which is introduced in Definition 2.5.1. Specifically,

$$\pi(s,\theta) = \begin{cases} \pi^*(s,\theta), & \theta \in \Theta_G(s_h), \\ \mathbb{P}(s|\theta,\theta_G), & \theta \notin \Theta_G(s_h). \end{cases}$$

This adjustment makes comparable banks with different stabilities in the benchmark.

According to Proposition 2.3.4, for smaller A and smaller R, the regulator reports $s_i = h$ on only a subset of default banks. Here we argue that there exists thresholds $\hat{A}$, $\hat{R}_1$, $\hat{R}_2$ such that when $A < \hat{A}$ and $\hat{R}_1 < R < \hat{R}_2$, the regulator separates a periphery bank to allow for refinance but not its connected core bank,

$$s_{i^p} = h, \; s_{c(i^p)} = l,$$

at some states with

$$\tilde{A}_{i^p} > 0,\ \tilde{A}_{c(i^p)} > 0.$$

First, we introduce size adjusted liquidity level,

$$\eta_i(s,\theta) \equiv \frac{L_i(s_{-i}, s_i = l, \theta)}{A_i - D_i}.$$

By Assumption 2.3.3,

$$\eta_i(s_l,\theta) = \frac{\tilde{A}_i - D_i}{A_i - D_i} + \sum_{j\neq i} \frac{R_{ij}}{\sum_{s\neq i} R_{is}} \left\{ \min\left[\eta_j(s_l,\theta), \frac{R}{A-D} \right] \right\}^+,$$

Without loss of generality, we can focus on candidates (s,θ) such that if $s_i = h$ then for all $j \neq i$ with $\eta_j(s,\theta) \geq \eta_i(s,\theta)$ we have $s_j = h$. In core-periphery networks, under $s : s_{i^c} = h, s_{p(i^c)} = l$

$$\hat{\eta}_{p(i^c)} = \begin{cases} 1 + \left\{\min\left[\eta_{i^c}, \frac{R}{A-D}\right]\right\}^+, & \text{if } \tilde{A}_{p(i^c)} > 0 \\ -\frac{D}{A-D} + \left\{\min\left[\eta_{i^c}, \frac{R}{A-D}\right]\right\}^+, & \text{if } \tilde{A}_{p(i^c)} = 0 \end{cases},$$

where $\hat{\eta}$ corresponds to evaluation under $s_{-j}, s_j = l, \theta$. So whenever $s_{i^c} = h$, for any $\tilde{A}_{p(i^c)} > 0$, $s_{p(i^c)} = h$.

If bank profitability is higher, the marginal (s,θ) has smaller η.

For candidates (s,θ) where $\tilde{A}_{i^c}(\theta) > 0$ for a core bank and $\tilde{A}_{i^p}(\theta) > 0,\ \tilde{A}_{c(i^p)}(\theta) > 0$ for a periphery bank, we discuss the trade-off of $s_{c(i^p)}$ conditional on $s_{i^p} = h$. On the one hand, there is strong externality of $c(i^p)$ on i^p:

$$s_{c(i^p)} = h, \Rightarrow\ \eta_{i^p} = \sup \eta = 1 + \frac{R}{A-D}.$$

On the other hand,

$$\eta_{i^p}(s_l,\theta) = 1 + \eta_{c(i^p)}(s_l,\theta) > \eta_{c(i^p)}(s_l,\theta),$$

the periphery bank is one-bank (connected core) away from any shock when $\tilde{A}_{i^p} > 0,\ \tilde{A}_{c(i^p)} > 0$. As payoff is strictly decreasing and convex in $\frac{R}{A-D} - \eta(s,\theta)$ and the gap $\eta_{i^p}(s_l,\theta) - \eta_{c(i^p)}(s_l,\theta) = 1$ is a constant, and cross-bank risk sharing is more efficient under large R, there exists threshold parameters out of which the regulator reports h on both $i^p, c(i^p)$.

Complementarity weakly increases in partial ranked states (added asset shocks $\sum_i \mathbf{1}_{\{\tilde{A}_i=0\}}$) and contagion exposure. We further argue that there exists a uniform set of thresholds $\hat{A},\ \hat{R}_1,\ \hat{R}_2$ such that when $A < \hat{A}$ and $\hat{R}_1 \geq R < \hat{R}_2$, and a state $\theta : \tilde{A}_{i^p} > 0,\ \tilde{A}_{c(i^p)} > 0$ where the regulator reports s with positive probability and

$$s : s_{i^p} = h,\ s_{c(i^p)} = l.$$

The lower threshold $\hat{R}_1$ arises because a smaller R increases the prior liquidity level such that $s : s_{i^p} = s_{c(i^p)} = h$ dominates as the regulator reports it with a large probability that approaches its upper-bound payoff.

This does not translate into $\mathbb{P}(s_{i^p} = h) \geq \mathbb{P}(s_{c(i^p)} = h)$ under the parameter range. For example, when $n_P > n_C$, $\pi(s,\theta) > 0,\ s :\ s_{i^p} = s_{c(i^p)} = h,\ \theta :$ with $\tilde{A}_{i^p}, \tilde{A}_{c(i^p)} > 0$, but there exists a symmetric θ',

$$\begin{aligned} \theta' :\ & \tilde{A}_{i^p}(\theta') = 0,\ \tilde{A}_{c(i^p)}(\theta') > 0,\ \sum_{p(c(i^p))} \mathbf{1}_{\{\tilde{A}_{p(c(i^p))}(\theta')>0\}} = \sum_{p(c(i^p))} \mathbf{1}_{\{\tilde{A}_{p(c(i^p))}(\theta)>0\}}, \\ & \tilde{A}_i(\theta') = \tilde{A}_i(\theta) \text{ otherwise,} \end{aligned}$$

and $\tilde{A}_{i^p} = 0,\ s_{i^p} = l$ by symmetry. Therefore, if $n_p = n_c$ and $A < \hat{A},\ \hat{R}_1 \geq R < \hat{R}_2$,

$$\mathbb{P}(s_{i^p} = h) \geq \mathbb{P}(s_{c(i^p)} = h).$$

If $A \geq \hat{A}$, the abundant liquidity effect dominates and $\forall \pi(s,\theta) > 0,\ s:\ s_{i^p} = h$, we have $s_{c(i^p)} = h$. If For $A < \hat{A}$ and $\hat{R}_1 \geq R < \hat{R}_2$, for all states $\theta : \tilde{A}_{i^p} > 0,\ \tilde{A}_{c(i^p)} > 0$, $\pi(s,\theta) > 0$ and $s_{i^p} = h$, we have $s_{c(i^p)} = h$.

Now we discuss the case where in the marginal (s,θ) (at later rounds) $\theta : \tilde{A}_{i^p} > 0,\ \tilde{A}_{c(i^p)} > 0$ fails. A typical core bank is more efficient in risk sharing as it has more counter-parties and asset shocks are i.i.d. across banks. Specifically,

$$\mathbb{P}\left(\eta_{i^p}(s_l,\theta) \geq 1 - \frac{R}{A-D}\right) = p^2,$$
$$\mathbb{P}\left(\eta_{i^c}(s_l,\theta) \geq 1 - \frac{R}{A-D}\right) = p[1-(1-p)^{t(i^c)}],$$

where $t(i^c) = \sum_{j \neq i^c} \mathbf{1}_{\{R_{i^c,j}>0\}}$ is the number of bank i^c's counter-parties. Moreover, For any $\eta' < \sup \eta(s_l,\theta) = 1 + \frac{R}{A-D}$,

$$\mathbb{P}(\eta_{i^p} \geq \eta') \leq \mathbb{P}(\eta_{c(i^p)} \geq \eta' - 1).$$

Hence, when the marginal (s,θ) moves to $\eta_i(s,\theta) = 1 - \frac{R}{A-D}$, we have $\mathbb{P}(s_{i^p} = h) \leq \mathbb{P}(s_{c(i^p)} = h)$. By the similar arguement of spread-out interbank liability, $\mathbb{P}(s_{c(j^p)} = h | \tilde{A}_{j^p} = 0) \geq \mathbb{P}(s_{j^p} = h | \tilde{A}_{j^p} = 0)$.

In sum, $\mathbb{P}(s_{i^p} = h) \leq \mathbb{P}(s_{c(i^p)} = h)$.

Now we change Assumption 2.3.3 and instead assume that the out-of-network cash flows are the same across banks,

$$\tilde{A}_i = \tilde{A},\ A_i = A,\ D_i = D,\ \forall i.$$

Under this specification, more counter parties correspond to purely added risk, instead of purely better risk sharing under Assumption 2.3.3. We have

$$L_i(s_l,\theta) - R_i = \tilde{A}_i - D + \sum_{j\neq i} \frac{R_{ij}}{R_j} \left\{\min\left[L_j(s_l,\theta), R_j\right]\right\}^+ - R_i.$$

Still, whenever $s_{i^c} = h$, for any $\tilde{A}_{p(i^c)} > 0$, $s_{p(i^c)} = h$. Peripheries with good loan asset has higher relative liquidity compared to connected core bank,

$$\begin{aligned}\frac{L_{p(i^c)}(s_l,\theta) - R_{p(i^c)}}{A-D} &= 1 + \frac{R_{p(i^c)}}{R_{i^c}} \min\left\{\frac{\left[L_{i^c}(s_l,\theta) - R_{i^c}\right]^-}{A-D}, -\frac{R_{i^c}}{A-D}\right\} \\ &> \frac{L_{i^c}(s_l,\theta) - R_{i^c}}{A-D}.\end{aligned}$$

Note that the cost of cross-bank risk sharing by allowing both the above periphery bank and its connected core bank increases due to the larger relative liquidity gap. Hence, the set of parameters under which the regulator pools good-asset periphery and connected core shrinks in this specification.

When the threshold relative liquidity decreases as system liquidity increases, still the regulator treats periphery banks preferably, because Now we discuss the case where in the marginal (s,θ) (at later rounds) $\theta : \tilde{A}_{i^p} > 0,\ \tilde{A}_{c(i^p)} > 0$ fails. A typical core bank is more efficient in risk sharing as it has more counter-parties and asset shocks are i.i.d. across banks. Specifically,

$$\begin{aligned}\mathbb{P}\left(\eta_{i^p}(s_l,\theta) \geq 1 - \frac{R}{A-D}\right) &= p^2, \\ \mathbb{P}\left(\eta_{i^c}(s_l,\theta) \geq 1 - \frac{R}{A-D}\right) &\leq p \cdot p^{t(i^c)},\end{aligned}$$

where $t(i^c) = \sum_{j\neq i^c} \mathbf{1}_{\{R_{i^c,j}>0\}}$ is the number of bank i^c's counter-parties.

Therefore, if each core bank is connected with only one periphery, $n_c = n_p$, periphery banks receive preferred treatment. ■

Appendix B

Appendix to Chapter 3

B.1 Mathematical Appendix

Proof of Lemma 3.3.1.

Proof: [Sketch] Whenever (3.9) fails, the regulator prefers to switch the associated l to h. To see this, as additional banks are reported with h, at each physical state θ interbank payments are weakly higher. So conditional on $\theta_{\mathcal{M}}$, both the expected counterparty payments $\mathbb{E}_\theta\left[L_i(s_{-i}, s_i = l, \theta_{\mathcal{M}})\right]$ and expected system stability $\mathbb{E}_\theta\left[v(s,\theta)|s, \theta_{\mathcal{M}}\right]$ weakly increase. ■

Proof of Proposition 3.3.1.

Proof: [Sketch] We have proved in the baseline model that $L_i(s_{-i}, s_i = l, \theta)$ exists and is generically unique. From (2) of Assumption 3.3.1, there exists a bank i and a feasible policy is

$$\mathcal{M} = \{i\},\ \pi_{\mathcal{M}}(h, \tilde{A}_i = A_i) = 1,\ \pi_{\mathcal{M}}(l, \tilde{A}_i = 0) = 1.$$

Then we can apply the extreme value theorem. ■

Proof of Proposition 3.3.3.

Proof: [First Part Sketch] Conditional on $\mathcal{M}$, the difference in the obedience constraints from the testing-all-bank model is conditional expectation of the bank's net assets (from $L_i(s_{-i}, s_i = l, \theta)|\theta_{\mathcal{M}}$ to $\mathbb{E}_\theta\left[L_i(s_{-i}, s_i = l, \theta)|\theta_{\mathcal{M}}\right]$). Subgame optimality for given $\mathcal{M}$ and s applies.

Therefore, the first marginal unit of excess liquidity is borrowed to the $(s, \theta_{\mathcal{M}})$ with highest first-round index $\xi^{(1)}$. As for every $i \in \mathcal{M}$, there is a

$$\theta_i \in \Theta_{\mathcal{M}} : \ \tilde{A}_i = 0, \ \tilde{A}_j = A_j \text{ for all } j \neq i, j \in \mathcal{M}, \tag{A.1}$$

all obedience constraints are binding in the subgame for calculating $\xi^{(1)}$.

As the regulators only inspect a subset of banks, the signals are relatively more informative regarding $\mathcal{M}$, and the representation of the index $\xi^{(1)}$ could be further simplified.

For notation, let $I_{h,(-i)}(s)$ be the collection of banks with h under s excluding bank i, $I_l(s)$ the set of banks with l under s, and $\theta_0 \in \Theta_{\mathcal{M}}$ be the state with $\tilde{A}_i = A_i, \ \forall i \in \mathcal{M}$.

Suppose $s_h(\mathcal{M})$ reports h on every $i \in \mathcal{M}$, and then bank i's expected net asset only depends on its own loan project realization, as other stress testing banks repay in full:

$$\mathbb{E}_\theta \left[L_i(s_{-i}, s_i = l, \theta) | \theta_{\mathcal{M}} \right] = \mathbb{E}_\theta \left[L_i(s_{-i}, s_i = l, \theta) | I_{h,(-i)}(s), \tilde{A}_i = A_i \right]$$

From feasibility of obedience constraint (3.8), it must be

$$\mathbb{E}_\theta \left[\tilde{A}_i + \sum_{j \neq i} y_{ij}(s_{-i}, s_i = l, \theta) | I_{h,(-i)}(s), \tilde{A}_i = A_i \right] - v_i - \sum_{j \neq i} R_{ji} > 0, \text{ for all } i \in \mathcal{M}.$$

From binding obedience constraints and maximizing the probability of reporting $s_h(\mathcal{M})$, $s_h(\mathcal{M})$ is reported at θ_0 and θ_i for all $i \in \Theta_{\mathcal{M}}$ (see (A.1)), or

$$\begin{aligned} &\Theta_{\mathcal{M}}^{(1)*}(s_h(\mathcal{M}), \theta_{\mathcal{M}}; \theta_0) \\ &= \left\{ \theta_{\mathcal{M}} \in \Theta_{\mathcal{M}} | \tilde{A}_i(\theta_{\mathcal{M}}) = 0, \ \tilde{A}_j(\theta_{\mathcal{M}}) = A_j \text{ for all } i \in \mathcal{M} \text{ and } j \neq i, j \in \mathcal{M} \right\} \end{aligned} \tag{A.2}$$

For notation, numerate the banks in $I_h(s)$ as $(1),(2),\cdots,|(I_h(s))|$ and let

$$\theta_{(1)},\theta_{(2)},\cdots,\theta_{(I_h(s))}$$

correspondingly denote the state $\theta_{\mathcal{M}}$ where only bank (k) has bad loan realization as in (A.1):

$$\theta_{(k)}\in\Theta_{\mathcal{M}}:\ \tilde{A}_{(k)}=0,\tilde{A}_j=A_j \text{ for all } j\in\mathcal{M} \text{ and } j\neq(k).$$

Hence,

$$\begin{aligned}
&\xi^{(1)}(s_h(\mathcal{M}),\theta_{\mathcal{M}};\theta_0)=\mathbb{E}_\theta\left[v(s,\theta)|s\right]\cdot\left[\sum_{\theta_{\mathcal{M}}\in\Theta^{(1)*}_{\mathcal{M}}}\mathbb{P}(\theta_{\mathcal{M}})\pi_{\mathcal{M}}(s,\theta_{\mathcal{M}})\right]\\
&s.t. \text{ for } k=1,\cdots,|I_h(s)|,\quad \mathbb{P}(\theta_0)\\
&\left\{\mathbb{E}_\theta\left[A_{(k)}+\sum_{j\neq(k)}y_{(k)j}(s_{-(k)},s_{(k)}=l,\theta)|I_{h,(-(k))}(s),\tilde{A}_{(k)}=A_{(k)}\right]-v_{(k)}-R_{(k)}\right\}\\
&+\sum_{\theta_{(m)}\neq\theta_{(k)}}\mathbb{P}(\theta_{(m)})\pi_{\mathcal{M}}(s,\theta_{(m)})\cdot\\
&\left\{\mathbb{E}_\theta\left[A_{(k)}+\sum_{j\neq(k)}y_{(k)j}(s_{-(k)},s_{(k)}=l,\theta)|I_{h,(-(k))}(s),\tilde{A}_{(k)}=A_{(k)}\right]-v_{(k)}-R_{(k)}\right\}\\
&+\mathbb{P}(\theta_{(k)})\pi_{\mathcal{M}}(s,\theta_{(k)})\\
&\cdot\left\{\mathbb{E}_\theta\left[\sum_{j\neq(k)}y_{(k)j}(s_{-(k)},s_{(k)}=l,\theta)|I_{h,(-(k))}(s),\tilde{A}_{(k)}=0\right]-v_{(k)}-R_{(k)}\right\}=0.
\end{aligned}$$

From the above problem we have

$$\xi^{(1)}(s_h(\mathcal{M}),\theta_{\mathcal{M}};\theta_0)=\mathbb{E}_\theta\left[v(s_h(\mathcal{M}),\theta)-v_0(\theta)|s_h(\mathcal{M})\right]\cdot\frac{\mathbb{P}(\theta_0)\left(\sum_k\frac{1}{\frac{a_k-a'_k}{a_k}}\right)}{1-\left(\sum_k\frac{1}{\frac{a_k-a'_k}{a_k}}\right)},$$

where

$$\frac{a_k - a'_k}{a_k} \equiv \frac{A_{(k)} + \Delta^{\tilde{A}_{(k)}=A_{(k)}}_{\tilde{A}_{(k)}=0} \mathbb{E}_\theta \left[\sum_{j \neq (k)} y_{(k)j}(s_{-(k)}, s_{(k)} = l, \theta) | I_{h,(-(k))}(s), \tilde{A}_{(k)} \right]}{A_{(k)} + \mathbb{E}_\theta \left[\sum_{j \neq (k)} y_{(k)j}(s_{-(k)}, s_{(k)} = l, \theta) | I_{h,(-(k))}(s), \tilde{A}_{(k)} = A_{(k)} \right] - v_{(k)} - R_{(k)}}, \tag{A.3}$$

and define

$$\begin{aligned} &\Delta^{\tilde{A}_{(k)}=A_{(k)}}_{\tilde{A}_{(k)}=0} \mathbb{E}_\theta \left[\sum_{j \neq (k)} y_{(k)j}(s_{-(k)}, s_{(k)} = l, \theta) | I_{h,(-(k))}(s), \tilde{A}_{(k)} \right] \equiv \\ &\mathbb{E}_\theta \left[\sum_{j \neq (k)} y_{(k)j}(s_{-(k)}, s_{(k)} = l, \theta) | I_{h,(-(k))}(s), \tilde{A}_{(k)} = A_{(k)} \right] \\ &- \mathbb{E}_\theta \left[\sum_{j \neq (k)} y_{(k)j}(s_{-(k)}, s_{(k)} = l, \theta) | I_{h,(-(k))}(s), \tilde{A}_{(k)} = 0 \right]. \end{aligned}$$

The index is increasing in incremental system stability

$$\mathbb{E}_\theta \left[v(s_h(\mathcal{M}), \theta) - v_0(\theta) | s_h(\mathcal{M}) \right],$$

and decreasing in size of feedback effect of a stress testing bank's own project shock to itself

$$\frac{a_k - a'_k}{a_k}.$$

It also implicitly depends on the complementarity among the stress test banks, which determines the size of feedback effect. ■

Proof of Lemma 3.4.1

Proof: When $m = 1$, the states that could be inspected and the available signals to communicate are simple:

$$\Theta_{\mathcal{M}} = \{\tilde{A}_i = A_i,\ \tilde{A}_i = 0\},\ \boldsymbol{S_{\mathcal{M}}} = \{s_i = h,\ s_i = l\}.$$

Then a little abuse of notation, and let ξ_i represent the efficiency index associated with choosing bank i for stress test. Accordingly,

$$\begin{aligned}\xi_i = \mathbb{E}_\theta\left[v(s,\theta)|s_i=h\right]\cdot(p_i+\chi_i) + \mathbb{E}_\theta\left[v_0(\theta)|\tilde{A}_i=0\right]\cdot\left[1-(p_i+\chi_i)\right]\\ s.t.\ p_i\left\{A_i - D_i + \mathbb{E}_\theta\left[\sum_{j\neq i} y_{ij}(s_i=l,\theta)|\tilde{A}_i=A_i\right] - R_i\right\}\\ +\chi_i\left\{-D_i + \mathbb{E}_\theta\left[\sum_{j\neq i} y_{ij}(s_i=l,\theta)|\tilde{A}_i=0\right] - R_i\right\} = 0,\\ 0\leq\chi_i\leq 1-p_i.\end{aligned}$$

We have

$$\xi_i = p_i\left\{\left(1+\frac{\chi_i}{p_i}\right)\cdot\mathbb{E}_\theta\left[v(s,\theta)|s_i=h\right] + \left[\frac{1}{p_i} - \left(1+\frac{\chi_i}{p_i}\right)\right]\cdot\mathbb{E}_\theta\left[v_0(\theta)|\tilde{A}_i=0\right]\right\},$$

where

$$\frac{\chi_i}{p_i}+1 = \max\left\{\frac{A_i + \Delta_{\tilde{A}_i}\mathbb{E}_\theta\left[\sum_{j\neq i} y_{ij}(s_i=l,\theta)|\tilde{A}_i\right]}{D_i + R_i - \mathbb{E}_\theta\left[\sum_{j\neq i} y_{ij}(s_i=l,\theta)|\tilde{A}_i=0\right]}, \frac{1}{p_i}\right\}. \tag{A.4}$$

Suppose R characterizes the scale of counterparty exposure,

$$R_{ij} = r_{ij}\cdot R,\ \ r_{ij}\text{ is constant.}$$

We claim that the efficiency index ξ_i (3.16) is decreasing in R. To see this, note that $y_{ij}(s_i=l,\theta)$ for all θ, i and j are continuous in R, whereas $\mathbb{E}_\theta\left[v(s,\theta)|s_i=h\right]$ and $\mathbb{E}_\theta\left[v_0(\theta)|\tilde{A}_i=0\right]$ are step functions of R and may jump downwards as R increases. In the neighborhood of R where v, v_0 are constants, we need to prove that $\frac{\chi_i}{p_i}+1$

(A.4) decreases in R. A sufficient condition is

$$\sum_{j \neq i} p_j R_{ij} \leq R_i \equiv \sum_{j \neq i} R_{ji}. \tag{A.5}$$

To see this, the expected interbank payments that bank i receives

$$\mathbb{E}_\theta \left[\sum_{j \neq i} y_{ij}(s_i = l, \theta) | \tilde{A}_i = 0 \right] = \sum_{j \neq i} p_j (c_{ij}^R R_{ij} + \sum_{k \neq j} c_k^A A_k - \sum_{k \neq j} c_k^D D_k)$$

is a linear function, where all c are constants. As bank i with $\tilde{A}_i = 0$ repays 0, $\sum_{k \neq j} c_k^A A_k - \sum_{k \neq j} c_k^D D_k > 0$ for all j and thus $c_{ij}^R \leq 1$. The inequality may bind as all other banks $j \neq i$ are solvent and repay in full when $\tilde{A}_i > 0$.

Hence, (A.5) guarantees that $\frac{\partial \left(\frac{\chi_i}{p_i} + 1 \right)}{\partial R} \leq 0$ considering $\frac{\chi_i}{p_i} + 1 \in [1, \frac{1}{p_i}]$. This means as long as a bank's total interbank claims and liabilities are not too unbalanced, in which case the bank's healthiness may be predominantly determined by payments from other banks, the efficiency index ξ_i (3.16) is decreasing in R. We will directly discuss the dynamics of the efficiency index outside (A.5) and choice of banks. ■

Proof of Proposition 3.4.1

Proof: For bank i and j, if

$$\mathbb{E}_\theta \left[v(s, \theta) | s_i = h \right] \geq \mathbb{E}_\theta \left[v(s, \theta) | s_j = h \right] \text{ and } \mathbb{E}_\theta \left[v_0(\theta) | \tilde{A}_i = 0 \right] \geq \mathbb{E}_\theta \left[v_0(\theta) | \tilde{A}_j = 0 \right],$$

clearly $\xi_i \geq \xi_j$ and the regulator never stress tests bank j. The interesting case arises when

$$\mathbb{E}_\theta \left[v(s, \theta) | s_i = h \right] \geq \mathbb{E}_\theta \left[v(s, \theta) | s_j = h \right] \text{ and } \mathbb{E}_\theta \left[v_0(\theta) | \tilde{A}_i = 0 \right] < \mathbb{E}_\theta \left[v_0(\theta) | \tilde{A}_j = 0 \right]. \tag{A.6}$$

The solvency of bank i influences other banks' solvency via contagion, as banks with bad asset would for sure default:

$$\mathbb{E}_\theta \left[v(s,\theta)|s_i = h\right] = w_i + \sum_{j\neq i} w_j p_j \left[1 - \mathbb{P}(j \text{ defaults}|\tilde{A}_j > 0, s_i = h)\right],$$
$$\mathbb{E}_\theta \left[v_0(\tilde{A}_i = 0)\right] = \sum_{j\neq i} w_j p_j \left[1 - \mathbb{P}(j \text{ defaults}|\tilde{A}_j > 0, \tilde{A}_i = 0)\right],$$

Suppose the regulator attaches equal weights $w_i = 1$ to all banks, and the asset shock probability $p_i = p$ is the same for all banks. Then (3.17) means bank i is systemically important and largely determines system stability via contagion. On the other hand, bank j is less systemically important; for example, one can think of an extreme where bank j is an isolated bank, and then

$$\mathbb{E}_\theta \left[v(s,\theta)|s_i = h\right] - \mathbb{E}_\theta \left[v_0(\tilde{A}_i = 0)\right] = w_i$$

takes the minimum value.

When R is very small and contagion never arises (violating Assumption 2), each bank is in effect an isolated bank and choosing any bank for stress test is equivalent.

Sketch: (sketch proof, see the star example for cleaner results and intuition)

Let $(\hat{\underline{R}}, \bar{\hat{R}})$ be an interval in which $\mathbb{E}_\theta \left[v(s,\theta)|s_i = h\right]$ and $\mathbb{E}_\theta \left[v_0(\tilde{A}_i = 0)\right]$ are constants. ξ_i is linear and decreasing in R when $R \in (\hat{\underline{R}}, \bar{\hat{R}})$, and the slope of ξ_i is steeper if i is systemically more important. There exists $\tilde{R}$ where $\xi_i(R) = \xi_j(R)$, and $\tilde{R}$ is linear functions of A_i and D_i. So there exists a $\bar{A}$ and when $A \leq \bar{A}$ the optimal policy tests the most systemically important bank (the steepest slope) when $R \in (\hat{\underline{R}}, \tilde{R})$.

■

Proof of Proposition 3.4.2

Proof: (1) When $R \leq A - D$ (ruled out by Assumption 2), there is no contagion effect in the system, and thus it is equivalent to choose any bank for stress testing;

$$\xi_i = \left(w_i + p\sum_{j\neq i} w_j\right) \cdot \frac{A_i}{D_i + (1-p)R_i} + \left(p\sum_{j\neq i} w_j\right)\left[\frac{1}{p} - \frac{A_i}{D_i + (1-p)R_i}\right]$$

(2) When $A - D < R \leq 2(A - D)$, bank failure from contagion arises only when the center bank 1 has bad asset and/or both small banks 2 and 3 have bad asset, in which case all banks default; otherwise, if bank 1 has good asset and/or there exists a small bank with good asset, there is no failure due to contagion. We show the algebra in this case and the proof is similar for (3) and (4).

$$\begin{aligned}
\xi_1 &= \frac{A + p[R - (A-D)]}{D + R - p(A-D)} \cdot (1+2p)\,, \\
\xi_2 &= \frac{A + \{pR - p\left[pR + (1-p)(A-D)\right]\}}{D + R - p\left[pR + (1-p)(A-D)\right]} \cdot (1+p+p^2) \\
&+ \left\{\frac{1}{p} - \frac{A + p(1-p)\cdot[R - (A-D)]}{D + R - p\left[pR + (1-p)(A-D)\right]}\right\} \cdot 2p^2. \\
\xi_1|_{R\to A-D} &= \frac{A}{D + (1-p)(A-D)} \cdot (1+2p)\,, \\
\xi_1|_{R\to A-D} &= \frac{A}{D + (1-p)(A-D)} \cdot (1+p+p^2) + \left\{\frac{1}{p} - \frac{A}{D + (1-p)(A-D)}\right\} \cdot 2p^2 \\
\xi_1|_{R=2(A-D)} &= \frac{A + p(A-D)}{D + (2-p)(A-D)} \cdot (1+2p)\,, \\
\xi_2|_{R=2(A-D)} &= \frac{A + p(A-D) - p^2(A-D)}{D + (2-p)(A-D) - p^2(A-D)} \cdot (1+p+p^2) \\
&+ \left\{\frac{1}{p} - \frac{A + p(A-D) - p^2(A-D)}{D + (2-p)(A-D) - p^2(A-D)}\right\} \cdot 2p^2
\end{aligned}$$

So

$$\text{When} \begin{cases} \frac{A+p(A-D)}{D+(2-p)(A-D)} \geq p' & \xi_1 \geq \xi_2; \\ \frac{A}{D+(1-p)(A-D)} \leq \frac{2}{1+p}, & \xi_1 \leq \xi_2; \cdot \\ \text{otherwise depends on } R: R \leq \bar{R}, & \xi_1 \geq \xi_2; \end{cases}$$

(3). When $2(A-D) < R \leq 4(A-D)$, the size of contagion cascade increases: still system fails if the center bank and/or both small banks have bad asset; now additionally, if one small bank has bad asset, the center bank 1 fails due to contagion and the other small bank is solvent if it has good asset.

(4). When $R > 4(A-D)$, contagion cascade is the largest possible, as any asset shock to any bank would cause the whole system default. ■

Proof of Lemma 3.5.1

Proof: Suppose the regulator tests bank 1 and bank $1+\Delta$. Without loss of generality, assume $\Delta \geq n-1-\Delta$. For notational convenience, we use

$$\Theta_{\mathcal{M}} = \{gg, gb, bg, bb\}$$

to denote the project realizations of tested banks, and

$$\boldsymbol{S}_{\mathcal{M}} = \{hh, hl, lh, ll\}$$

for available signals. Following the algorithm in Proposition 3.3.3,

$$\xi^{(1)}(hh, \theta_{\mathcal{M}}; gg) = \mathbb{E}_{\theta}\left[v(hh,\theta)|\theta_{\mathcal{M}}\right] \sum_{\theta_{\mathcal{M}}} \mathbb{P}(\theta_{\mathcal{M}})\pi(hh,\theta_{\mathcal{M}}) - \mathbb{P}(gg)\pi(hh,gg)p^{n-2} \cdot n,$$

$$\begin{aligned}
s.t.\ & [\mathbb{P}(gg) + \mathbb{P}(gb)\pi(hh,gb)][A + \mathbb{E}(y_{1,n}|y_{\Delta+1} = R) - D - R] \\
& + \mathbb{P}(bg)\pi(hh,bg)[\mathbb{E}(y_{1,n}|y_{\Delta+1} = R) - D - R] = 0, \\
& [\mathbb{P}(gg) + \mathbb{P}(bg)\pi(hh,bg)][A + \mathbb{E}(y_{\Delta+1,\Delta}|y_1 = R) - D - R] \\
& + \mathbb{P}(gb)\pi(hh,gb)[\mathbb{E}(y_{\Delta+1,\Delta}|y_1 = R) - D - R] = 0.
\end{aligned}$$

Hence,

$$\begin{aligned}\xi^{(1)}(hh,\theta_{\mathcal{M}};gg) \propto & \mathbb{E}_\theta\left[v(hh,\theta)|\theta_{\mathcal{M}}\right]\left[\mathbb{P}(gb)\pi(hh,gb)+\mathbb{P}(bg)\pi(hh,bg)\right], \\ = & \mathbb{E}_\theta\left[v(hh,\theta)|\theta_{\mathcal{M}}\right]\cdot\mathbb{P}(gg) \\ & \cdot\left[\frac{A}{2D+2R-A-\mathbb{E}(y_{1,n}|y_{\Delta+1}=R)-\mathbb{E}(y_{\Delta+1,\Delta}|y_1=R)}-1\right].\end{aligned}$$

We argue that to enhance the expected system stability $\mathbb{E}_\theta\left[v(hh,\theta)|\theta_{\mathcal{M}}\right]$, the regulator should increase Δ. This is because the expected number of solvent banks between bank 1 and Δ

$$\mathbb{E}_\theta\left[\sum_{i=1}^{\Delta}\mathbf{1}_{\{\sum_{j\neq i}y_{ji}(s,\theta)=\sum_{j\neq i}R_{ji}\}}|\theta_{\mathcal{M}}\right]=\frac{1-p^\Delta}{1-p}$$

is strictly concave in the length Δ.

The second part, probability of reporting hh, $\mathbb{P}(gb)\pi(hh,gb)+\mathbb{P}(bg)\pi(hh,bg)$ increases in $\mathbb{E}(y_{1,n}|y_{\Delta+1}=R)+\mathbb{E}(y_{\Delta+1,\Delta}|y_1=R)$, or the total remaining counterparty risks. We examine the curvature of

$$\mathbb{E}(y_{\Delta+1,\Delta}|y_1=R)=\frac{p}{1-p}(A-D)+p^{\Delta-1}\left[R-(\Delta-2+\frac{1}{1-p})(A-D)\right],$$

which then depends on R.

$$\frac{\partial\mathbb{E}(y_{1,n}|y_{\Delta+1}=R)}{\partial\Delta}=p^{\Delta-1}\ln p\left\{R-\left[(\Delta-2+\frac{1}{1-p})+\frac{1}{\ln p}\right](A-D)\right\}, \quad \text{(A.7)}$$

$$\frac{\partial^2\mathbb{E}(y_{1,n}|y_{\Delta+1}=R)}{\partial\Delta^2}=(\ln p)^2\cdot p^{\Delta-1}\left\{R-\left[(\Delta-2+\frac{1}{1-p})+\frac{2}{\ln p}\right](A-D)\right\}. \quad \text{(A.8)}$$

If R is relatively smaller, $\mathbb{E}(y_{1,n}|y_{\Delta+1}=R)$ is concave in Δ, and to economize on the martingale belief budget, the regulator would decrease Δ provided that $\Delta\geq n-2-\Delta$.

∎

Proof of Lemma 2

Proof: Another type of information structure is to report only one of the stress test banks is h. Without loss of generality, we consider reporting hl at gg, randomizes between hl and ll at gb, and ll otherwise. Other candidates of this class of information structure are dominated by randomly reporting hh when one of the stress test bank has bad asset ($\theta_{\mathcal{M}} = gb, bg$). The corresponding index is,

$$\begin{aligned}\xi^{(1)}(hl, gb; gg) =& \mathbb{P}(gg)\mathbb{E}_\theta\left[v(hl,\theta)|gg\right] + \mathbb{P}(gb)\pi(hl,gb)\mathbb{E}_\theta\left[v(hl,\theta)|gb\right] \\ &+ \mathbb{P}(gb)\left[1-\pi(hl,gb)\right]\mathbb{E}_\theta\left[v_0(\theta)|gb\right] \qquad \text{(A.9)} \\ \propto& \mathbb{E}_\theta\left[v(hl,\theta)|gg\right] - \mathbb{E}_\theta\left[v(hl,\theta)|gb\right] + \mathbb{E}_\theta\left[v(hl,\theta)|gb\right] \\ &\cdot \underbrace{\frac{\mathbb{E}(y_{1,n}|\tilde{A}_1 = A,\ \tilde{A}_{1+\Delta} = A) - \mathbb{E}(y_{1,n}|\tilde{A}_1 = A,\ \tilde{A}_{1+\Delta} = 0)}{D + R - A - \mathbb{E}(y_{1,n}|\tilde{A}_1 = A,\ \tilde{A}_{1+\Delta} = 0)}}_{\in[1,\frac{1}{p}]} \qquad \text{(A.10)} \\ =& \frac{p^{\Delta-1} - p^{n-1}}{1-p} \\ &+ \frac{1-p^\Delta}{1-p} \cdot \frac{p^{n-2}R - \left[(n-2+\frac{1}{1-p})p^{n-2} - (1+\frac{1}{1-p})p^{n-(\Delta+1)}\right](A-D)}{R - \frac{1-p^{n-\Delta-1}}{1-p}\cdot(A-D)} \\ & \qquad \text{(A.11)}\end{aligned}$$

As bank 1 repays R under $s_1 = h$, the gap in the expected due to $\tilde{A}_{1+\Delta}$ realization is determined by bank $\Delta + 1$ to n, so $\mathbb{E}_\theta\left[v(hl,\theta)|gg\right] - \mathbb{E}_\theta\left[v(hl,\theta)|gb\right] = \frac{p^{\Delta-1}(1-p^{n-\Delta})}{1-p}$. This part decreases with Δ as the potential shock becomes more distant. The expected system stability conditional on $s_1 = h, \tilde{A}_{1+\Delta} = 0$, $\mathbb{E}_\theta\left[v(hl,\theta)|gb\right] = \frac{1-p^\Delta}{1-p}$, is the expected total survival from banks 1 to Δ, and increases with Δ as the shock is postponed to a more distant bank. The third term captures the probability of reporting hl, such that when $s_1 = h$ the market thinks bank 1 is solvent on average. This part changes ambiguously when the regulator varies Δ as both the numerator and denominator increases with Δ: expected actual payment received by bank 1 varies more when the potential shock is nearer, but bank 1 is also less healthy when $\tilde{A}_{\Delta+1} = 0$. Therefore, how $\xi^{(1)}(hl, gb; gg)$ varies with Δ is ambiguous at first glance.

Now we show that $\xi^{(1)}(hl, gb; gg)$ is convex with respect to Δ. Rearranging

$$\begin{aligned}
& \frac{p^{\Delta-1} - p^{n-1}}{1-p} + \frac{1-p^{\Delta}}{1-p} \frac{p^{n-2}R - \left[\left(n-2+\frac{1}{1-p}\right)p^{n-2} - \left(1+\frac{1}{1-p}\right)p^{n-\Delta-1}\right](A-D)}{R - \frac{1-p^{n-\Delta-1}}{1-p}(A-D)} \\
= \; & \frac{p^{\Delta-1} - p^{n-1}}{1-p} + \frac{p^{n-2}}{1-p} \cdot \frac{-\alpha x^2 + (\alpha-\gamma)x + \gamma}{\phi x + \psi},
\end{aligned}$$

where

$$\begin{aligned}
x &= p^{\Delta}, \\
\alpha &= (1-p)R - \left[(n-2)(1-p)+1\right](A-D), \\
\gamma &= p(2-p)(A-D) > 0, \\
\phi &= (1-p)R - (A-D), \\
\psi &= p^{n-1}(A-D) > 0.
\end{aligned}$$

Then (A.11) becomes

$$\begin{aligned}
& \frac{p^{\Delta-1} - p^{n-1}}{1-p} + \frac{p^{n-2}}{1-p} \cdot \frac{-\alpha x^2 + (\alpha-\gamma)x + \gamma}{\phi x + \psi} \\
= \; & \frac{x}{p(1-p)} - \frac{p^{n-1}}{1-p} + \frac{p^{n-2}}{1-p} \cdot \left[-\frac{\alpha}{\phi}x - \frac{1}{\phi^2} \cdot \frac{(\alpha-\gamma)\psi\phi + \alpha\psi^2 - \gamma\phi^2}{\phi x + \psi} + \frac{\alpha-\gamma}{\phi} + \frac{\alpha\psi}{\phi^2}\right] \\
= \; & \left[\frac{1}{p(1-p)} - \frac{\alpha}{\phi} \cdot \frac{p^{n-2}}{1-p}\right] x - \frac{p^{n-2}}{1-p} \cdot \frac{1}{\phi^2} \cdot \frac{(\alpha-\gamma)\psi\phi + \alpha\psi^2 - \gamma\phi^2}{\phi x + \psi} + Constant.
\end{aligned}$$

Suppose

$$\phi = (1-p)R - (A-D) > 0. \tag{A.12}$$

Note that this condition is easy to satisfy when n is relatively large. To see this,

$$\phi x + \psi = (1-p)p^{\Delta} \cdot \left\{ D + R - A - \mathbb{E}(y_{1,n} | \tilde{A}_1 = A,\ \tilde{A}_{1+\Delta} = 0) \right\} > 0.$$

When $\Delta = 1$, this becomes

$$(1-p)R - (A-D) + p^{n-1}(A-D) > 0.$$

When (A.12) $\phi > 0$, we have $\frac{\psi}{\phi} > 0$ and

$$\begin{aligned} \frac{1}{p(1-p)} - \frac{\alpha}{\phi} \cdot \frac{p^{n-2}}{1-p} &= \frac{1}{p(1-p)} - \frac{p^{n-2}}{1-p} \cdot \frac{(1-p)R - \left[(n-2)(1-p)+1\right](A-D)}{(1-p)R-(A-D)} \\ &= \frac{1}{p(1-p)} - \frac{p^{n-2}}{1-p} + \frac{p^{n-2}}{1-p} \cdot \frac{(n-2)(1-p)(A-D)}{(1-p)R-(A-D)} > 0. \end{aligned}$$

Then (A.11) is proportional to the following function

$$G(\Delta) = F\left(p^{\Delta}\right) = F(x) = x + \frac{\eta}{x + \frac{\psi}{\phi}} + const,$$

and we need to show that $G(\Delta)$ has no interior maximal points. Note that p^{Δ} is a monotone transformation of Δ, so it is sufficient to check the extremal points of $F(x)$. The second order derivatives

$$\begin{aligned} G'(\Delta) &= F'\left(p^{\Delta}\right) p^{\Delta} (\ln p) \\ G''(\Delta) &= F''\left(p^{\Delta}\right) p^{2\Delta} (\ln p)^2 + F'\left(p^{\Delta}\right) p^{\Delta} (\ln p)^2 \end{aligned}$$

At the interior optimum point $G'\left(\widehat{\Delta}\right) = 0$ so

$$G''\left(\widehat{\Delta}\right) = F''\left(p^{\widehat{\Delta}}\right) p^{2\widehat{\Delta}} (\ln p)^2$$

There are two subcases

Case 1. *If $\eta < 0$, then $F(x)$ is monotone in x. Done.*

Case 2. *If $\eta > 0$, then $F(x)$ might take some interior extremal points. Denote it as $\widehat{x}$. The FOC*

$$G'\left(\widehat{\Delta}\right) = \left(1 - \frac{\eta}{\left(p^{\widehat{\Delta}} + \frac{\psi}{\phi}\right)^2}\right) p^{\widehat{\Delta}} (\ln p) = 0$$

and the SOC at this point

$$G''\left(\widehat{\Delta}\right) = \frac{2\eta}{\left(p^{\widehat{\Delta}} + \frac{\psi}{\phi}\right)^3} p^{2\widehat{\Delta}} (\ln p)^2$$

But since

$$p^{\widehat{\Delta}} + \frac{\psi}{\phi} > 0$$

always, the interior optimal point has to be a local minimum.

■

Proof of Proposition 3.5.1

Proof: When n is relatively large, the sufficient conditions in Lemma 3.5.1, 3.5.2 hold. Following Proposition 3.3.3, the index associated with the two cases of disclosure are:

$$\xi^{(1)}(hh, gb; gg) = \mathbb{E}_\theta \left[v(hh, \theta)|\theta_{\mathcal{M}}\right] \left[\mathbb{P}(gg) + \mathbb{P}(gb)\pi_{\mathcal{M}}(hh, gb) + \mathbb{P}(bg)\pi_{\mathcal{M}}(hh, bg)\right]$$

$$\xi^{(1)}(hl, gb; gg) = \mathbb{P}(gg)\mathbb{E}_\theta \left[v(hl, \theta)|gg\right] + \mathbb{P}(gb)\pi_{\mathcal{M}}(hl, gb)\mathbb{E}_\theta \left[v(hl, \theta)|gb\right]$$

As

$$\mathbb{E}_\theta \left[v(hh, \theta)|\theta_{\mathcal{M}}\right] > \mathbb{E}_\theta \left[v(hl, \theta)|gg\right] > \mathbb{E}_\theta \left[v(hl, \theta)|gb\right],$$

$\xi^{(1)}(hh, gb; gg) \geq \xi^{(1)}(hl, gb; gg)$ when $\mathbb{P}(gb)\pi_{\mathcal{M}}(s, gb) + \mathbb{P}(bg)\pi_{\mathcal{M}}(s, bg)$ is either big enough for $s = hh$ or small enough for $s = hl$. Note that $\mathbb{P}(gb)\pi_{\mathcal{M}}(s, gb)+\mathbb{P}(bg)\pi_{\mathcal{M}}(s, bg)$ is decreasing in R, and the observation translates into "balanced" tests with nondiscriminatory disclosure in the range of small $R \leq \hat{R}_1$ and large $R \geq \hat{R}_2$. To see this,

$$\frac{\mathbb{P}(gg) + \mathbb{P}(gb)\pi_{\mathcal{M}}(hh, gb) + \mathbb{P}(bg)\pi_{\mathcal{M}}(hh, bg)}{\mathbb{P}(gg)}$$
$$= \frac{A}{2D + 2R - A - \mathbb{E}(y_{1,n}|y_{\Delta+1} = R) - \mathbb{E}(y_{\Delta+1,\Delta}|y_1 = R)},$$
$$\frac{\mathbb{P}(gg) + \mathbb{P}(gb)\pi_{\mathcal{M}}(hl, gb)}{\mathbb{P}(gg)}$$
$$= \frac{p^{n-2}R - \left[(n - 2 + \frac{1}{1-p})p^{n-2} - (1 + \frac{1}{1-p})p^{n-(\Delta+1)}\right](A - D)}{R - \frac{1-p^{n-\Delta-1}}{1-p} \cdot (A - D)},$$

where $\mathbb{E}(y_{1,n}|y_{\Delta+1} = R)$ and $\mathbb{E}(y_{\Delta+1,\Delta}|y_1 = R)$ are linear functions of R with slopes smaller than 1.

Next we argue that in the middle range of R,

$$\frac{\mathbb{P}(gg) + \mathbb{P}(gb)\pi_{\mathcal{M}}(hh, gb) + \mathbb{P}(bg)\pi_{\mathcal{M}}(hh, bg)}{\mathbb{P}(gg) + \mathbb{P}(gb)\pi_{\mathcal{M}}(hl, gb)}$$
$$= \frac{A}{\left(c1\left[R - \frac{1-p^{n-\Delta-1}}{1-p} \cdot (A - D)\right] + \frac{c2}{R - \frac{1-p^{n-\Delta-1}}{1-p} \cdot (A-D)} + const\right)},$$

where $c1 > 0$, is not necessarily monotone in R. Hence the optimal policy is ambiguous. ■

Proof of Proposition 3.5.2

Proof: [Sketch] The regulator chooses (d', d'') to trade off between system stability and the cost efficiency of manipulating beliefs. Intuitively, when R is relatively small but still serious to cause system failure, the cost of manipulating belief at a larger contagion shock state dominates, and the regulator wants to maximize d.

After some algebra, we show that if $(n+1)(A-v) \geq R$,

$$\frac{\partial \xi^{(1)}(s_{hh,\Delta}, \theta_{n+1-d'}; \theta_0)}{\partial (d' + d'')} > 0, \ \frac{\partial \xi^{(1)}(s_{hh,\Delta}, \theta_{n+1-d'}; \theta_0)}{\partial (d')} > 0.$$

but

$$\frac{\partial \xi^{(1)}(s_{hh,\Delta}, \theta_{n+1-d'}; \theta_0)}{\partial (d'd'')} < 0.$$

The signs switch when $R > (n+1)(A-v)$.

$\blacksquare$

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