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Selling to consumers who cannot detect small differences ☆

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Abstract

This paper studies how sellers behave when their consumers have difficulty in detecting small differences. These consumers pose a problem because, even if a good deal exists, they cannot appreciate it if it is barely better than their outside options. This creates a role for a second deal, either marketed by the same seller or by another seller, even when consumers are homogeneous in their tastes. If the same seller markets the second deal, it will strategically position the first as a good deal, and the second as a bad deal, and use the bad deal to help consumers appreciate the good deal. If another seller markets the second deal, the two sellers will become specialized as, respectively, good-deal and bad-deal providers. Both sellers free-ride each other. The good-deal provider is happy because the bad deal helps consumers appreciate its good deal; while the bad-deal provider hides behind the presence of the good deal and manages to make a sale some of the time.

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1. Introduction

Adding a slightly worse option can sometimes help decision makers compare the top two contenders, while adding a much worse one cannot. Ketcham et al. (2015) use Medicare Part D data to study what nudges people to switch their Medicare prescription drug plans. They find that adding plans that are significantly worse does not help the switch, while adding plans that are slightly worse does.[1]

To understand why this may happen, consider the following illustrative example. Suppose you are to choose between two cups of coffee, A and B. You'd prefer the one with less sugar, but they taste more or less the same, so without further information you can only randomly pick one. You may or may not end up having the one with more sugar. Now comes a third cup of coffee, C, which is discernably sweeter than both A and B. This third cup of coffee apparently does not provide any help for your decision between A and B. Then comes a fourth cup, D, which is sweeter, but not much so. Specifically, it is discernably sweeter than A, but tastes more or less the same as B. With the help of D, you can now infer that B must be somewhere between A and D in terms of sugar content, and that A is the one with the least amount of sugar. Happily, you choose A over B.

Adding a slightly worse option helps because, when decision makers have difficulty in detecting small differences, they will struggle to compare similar alternatives. These difficulties arise more often when the values of the given options are themselves difficult to calculate,[2] when alternatives differ in multiple dimensions,[3] or when the decision maker is cognitively overloaded. The consequences of these difficulties loom larger when the decisions are irreversible, either because a product cannot be returned once purchased, or because the value cannot be known until (long) after the product is consumed.

In this paper, we explore how sellers behave when their consumers have difficulty detecting small differences between options. These consumers pose a problem because, even if a good deal exists, they cannot appreciate it if it is barely better than the alternatives. If there is a monopolist who can market only one deal, its profit necessarily decreases the more consumers suffer from this difficulty (Proposition 2). This creates a role for a second deal, either marketed by the same monopolist or by another seller, even when consumers are homogeneous in their tastes.

If the same monopolist markets the second deal, it will strategically position the first as a good deal and the second as a bad deal, and use the bad deal to help consumers appreciate the good deal (Proposition 6). If this monopolist can market many deals, all of them but one will be positioned as bad deals, and mainly serve the purpose of helping consumers appreciate the (single) good deal (Proposition 7). If another seller markets the second deal, the two sellers will become specialized as, respectively, good-deal and bad-deal providers. Both sellers free-ride each other. The good-deal provider is happy that the bad deal helps consumers appreciate its

[1] Ketcham et al. (2015) estimate the total costs (premium plus out of pocket costs) for each plan and for each individual. People who switch away from default plans on average save \$100-\$256, depending on the year and on how total costs are estimated. Adding plans whose total costs are more than \$500 above the best plan does not help nudge more switching, whereas adding plans that are less than \$500 above the best plan does.

[2] For example, to estimate the total costs associated with a Medicare prescription drug plan, one has to solve a complicated optimization problem regarding the number of prescription drug fills.

[3] Two products that are the same except for their prices would have been easy to compare, no matter how little their prices differ from each other.

good deal; while the bad-deal provider hides behind the presence of the good deal and manages to make a sale some of the time (Proposition 8).

In the context of Medicare prescription drug plans, Abaluck and Gruber (2011) find that people sometimes choose inferior plans. However, Ketcham et al. (2016) find that "the odds of choosing an inferior plan decline between 2006 and 2010 despite increasing availability of inferior plans" (p. 3934). These findings are not be surprising in light of our results. Suppose consumers are heterogeneous in their difficulty in detecting small differences. Then a bad deal can help some consumers, but not all, appreciate a good deal. Some consumers still erroneously purchase bad deals. However, when there are more and more bad deals, each helping a different group of consumers appreciate the good deal, the odds of purchasing the good deal will increase.

In order to model consumers' difficulty in detecting small differences, we follow an approach inspired by the decision-theoretic literature on intransitive indifference.[4] Specifically, each consumer has a *just noticeable difference* (jnd). If two alternatives offer utilities that differ by less than this jnd, the consumer will not be able to tell them apart.[5] Another popular approach often followed in the literature is called *categorization*, where a decision maker partitions alternatives into a few categories and cannot tell apart different alternatives within the same category.[6] This approach differs from ours in that, when two alternatives fall on opposite sides of a partition line, the decision maker will be able to tell them apart no matter how close they are to each other.

The consumers' difficulty in detecting small differences is one reason why they may not be responsive to competing offers. Other reasons for this lack of responsiveness that have been studied in the behavioral industrial organization literature include inattention (de Clippel et al. (2014)) and framing (Piccione and Spiegler (2012)). A recurring theme in this literature is that, when multiple sellers compete, consumers' surplus decreases with their ability to make comparisons. This phenomenon arises in our setting as well (Proposition 9). It will be interesting to investigate whether some common economic forces are at work. We leave this inquiry for future research.

Consumers' difficulty in detecting small differences also partly explains why the introduction of a third option affects their preference between the first two—this is a phenomenon generally known as the *context effect*. Two prominent special cases of the context effect are the attraction effect (Huber et al. (1982)) and the compromise effect (Simonson (1989)). The attraction effect arises when the appeal of x vis-à-vis y increases in the presence of a third option z that is dominated in every dimension by x but not by y. Ok et al. (2015) axiomatizes a reference-dependent choice model that can explain the attraction effect. Our consumer, however, cannot be modeled as a reference-dependent decision maker, because his behavior violates Ok et al.'s (2015) No-Cycle Condition.[7] The compromise effect arises when an option that is mediocre in every dimension is chosen over any other option that is the best in one dimension but the worst in another. de Clippel and Eliaz (2012) axiomatizes a reason-based choice model that can explain both the attraction and the compromise effects. Our consumer, however, cannot be

[4] See, for example, Luce (1956), Fishburn (1970a, 1970b), Jamison and Lau (1973), Fishburn (1975), and Beja and Gilboa (1992).

[5] In the literature on intransitive indifference, jnd is often deterministic. In this paper, we follow Gilboa's (2009) suggestion and postulate a probabilistic jnd instead, so that when two alternatives differ more and more in their offered utilities, the probability that the consumer can tell them apart increases gradually.

[6] See, for example, Chen et al. (2010), and Gul et al. (2017).

[7] Specifically, he may find x choosable when he is faced with the pair $\{x, y\}$ (because their difference is too small for him to detect), and find y choosable when he is faced with the pair $\{y, z\}$ (because their difference is too small for him to detect), but does not find x choosable when faced with the pair $\{x, z\}$ (because z is discernably better than x).

modeled as a reason-based decision maker, because his behavior violates de Clippel and Eliaz's (2012) Existence-of-a-Compromise Condition.[8]

Another explanation of the context effect is that the very act of introducing the third alternative is endogenous, and hence carries extra information (Kamenica (2008)). Kamenica's (2008) explanation can be readily separated from ours by a laboratory experiment where the third alternative is introduced by a robot instead of a strategic player. If a subject's ranking between the first two alternatives is still affected, the experimental result would lend support to our explanation.

Our result that a monopolist would market different deals with different values resembles Salop's (1977) classical result that a monopolist may permit a non-degenerate distribution of price for the same product. However, the mechanisms are different. In Salop (1977), consumers differ in their price elasticities, and the monopolist would like to charge those who have lower price elasticities a higher price. If consumers who have lower price elasticities also have higher search costs, then the monopolist can screen for these consumers by permitting a non-degenerate price distribution, because consumers who search less will on average pay a higher price. In our model, consumers' difficulty in detecting small differences is not correlated with any other characteristic, and hence screening does not play a role in our result.

Our model is also similar to Rubinstein (1993) in that different types of consumers differ neither in their preferences nor in the information they possess, but rather in their ability to process information. Like Salop (1977), Rubinstein's (1993) monopolist also permits a non-degenerate price distribution. However, in terms of the underlying mechanism, Rubinstein (1993) is closer to Salop (1977) than to us. In particular, Rubinstein (1993) shows that, when consumers differ in their ability to process information, with less able consumers also being more costly to serve, the monopolist can screen out these costly-to-serve consumers by offering a non-degenerate price distribution.[9]

We found that, in the case of multiple sellers, different sellers offer different deals with different values. This result resembles that of Salop and Stiglitz (1977), Varian (1980), and Rob (1985). However, our finding that both the good-deal and the bad-deal providers free-ride each other does not have a counterpart in these studies.

Finally, our paper is also related to Natenzon (2019), who looks at an environment where the choice between two deals may be affected by the presence of a third one. We differ in two aspects. First, while we focus on sellers' strategic choices over what alternatives to offer, Natenzon (2019) assume an exogenously fixed distribution of these alternatives. Second, in Natenzon (2019), the consumer finds some pair of alternatives easier to compare than other pairs. Such asymmetry is absent in our paper, where every pair is *a priori* similar, and contextual inference comes solely from the utility gap.

2. The model

There is a single consumer and up to two sellers. Sellers "compete in utility space" in the fashion of Armstrong and Vickers (2001). That is, they compete by offering various indivisible

[8] Specifically, he may find that the difference between any pair in $\{x, y, z\}$ too small to detect, and hence find every option choosable when he is faced with any pair as well as when his faced with the triplet.

[9] See also Piccione and Rubinstein (2003), where consumers differ *both* in their preferences and in their ability to process information. Again, when these two characteristics are correlated in a certain way, the monopolist can use a non-degenerate price distribution (or, more precisely, a deterministic price sequence that looks random to some consumers) as a screening device to separate consumers with different willingness to pay.

"deals", and the consumer demands at most one of them. Each deal is identified with the utility offered to the consumer. When a seller provides utility u to the consumer, the maximum profit it can extract from the consumer is $\pi(u)$, which decreases in u. We normalize the consumer's reservation utility to 0, and assume that $\pi(u)$ takes the simple linear form of $\pi(u) = \overline{\pi} - u$. We assume that $\overline{\pi} > 0$, and hence there is gain of trade.

At the moment of purchase, the consumer cannot observe the offered utility of any deal. All he can observe are *ordinal* rankings among the offered utilities and his reservation utility. We call these rankings his *primary ordering*.

The consumer comes with a random type $d \in [0, \infty)$. When the consumer is of type-d, he is able to *discern* two utilities, u_1 and u_2, if and only if $|u_1 - u_2| > d$; in which case his primary ordering is either $u_1 \widehat{\succsim} u_2$ or $u_2 \widehat{\succsim} u_1$, depending on which one is bigger. The type d is hence an inverse measure of the consumer's ability to discern the two utilities. If he cannot discern the two utilities, his primary ordering is $u_1 \widehat{\sim} u_2$.

When the consumer cannot discern between two utilities, he may nevertheless still be able to *a-discern* (where "a" stands for "assisted") them if he has a chance to examine another deal with offered utility u_3. For example, if he cannot discern between u_1 and u_2 (i.e., $u_1 \widehat{\sim} u_2$) and cannot discern between u_2 and u_3 (i.e., $u_2 \widehat{\sim} u_3$), but nevertheless feels that u_1 is better than u_3 (i.e., $u_1 \widehat{\succsim} u_3$), then, by simple logical deduction, he should be able to infer that u_1 is actually higher than u_2. We shall assume that the consumer is always able to make such an inference. In other words, while the consumer may have difficulty in detecting small differences, his rationality is undamaged.

Formally, we follow Kamada (2016) and construct *inferred ordering*, $\succ$, from the more primitive $\widehat{\succsim}$ as follows. Fix the set of deals that the consumer has a chance to examine (including his outside option), and let U be the set of utilities offered by these deals. Consider $u_1, u_2 \in U$. Then $u_1 \succ u_2$ if and only if at least one of the following holds:[10]

1. $u_1 \widehat{\succsim} u_2$;
2. $\exists u_3 \in U$ such that $u_1 \widehat{\succsim} u_3$ but $u_2 \widehat{\not\succsim} u_3$;
3. $\exists u_3 \in U$ such that $u_3 \widehat{\succsim} u_2$ but $u_3 \widehat{\not\succsim} u_1$.

We write $u_1 \sim u_2$ if and only if $u_1 \not\succ u_2$ and $u_2 \not\succ u_1$. We say that the consumer can *a-discern* u_1 and u_2 if $u_1 \not\sim u_2$.

An important assumption here is that u_3, which helps the consumer a-discern u_1 and u_2, has to come with a deal that the consumer has a chance to examine (as opposed to an imaginary deal). Without this assumption, our model would readily collapse into a traditional one. This is because, for example, a consumer with $d = 5$, though not being able to discern $u_1 = 10$ and $u_2 = 7$, would (had the implicit assumption relaxed) be able to a-discern them if he manages to imagine a fictitious deal with utility $u_3 = 4$.[11] One can think of the consumer's ability to discern between two utilities as already including his limited ability to imagine fictitious deals.

We assume that both the consumer and the sellers are risk neutral. Therefore, the consumer makes his purchase decision based on expected utilities, where expectation is taken conditional

[10] Kamada (2016) shows that further iterations of this logic would not result in new inferences. More precisely, if we define $\succ^*$ using the same method (with $\succ$ in place of $\widehat{\succsim}$), the new binary relation $\succ^*$ will remain the same as $\succ$.

[11] With $d = 5$, the consumer's primary ordering is $u_1 \widehat{\not\succsim} u_2$ and $u_2 \widehat{\not\succsim} u_3$ but $u_1 \widehat{\succsim} u_3$. From this primary ordering the consumer can derive $u_1 \succ u_2$, inferring that u_1 is higher than u_2.

on his primary ordering, and his knowledge of the sellers' strategies. Similarly, each seller maximizes expected profit, where the profit from offering utility u is $\overline{\pi} - u$.

We assume that *the consumer does not know his own type*. As a result, the consumer's purchasing strategy is independent of his type. It is commonly known that the distribution of his type, F, has support $\mathbb{R}_+$, and admits a differentiable density function, f, which, in turn, satisfies the following assumption:

Assumption 1. *The density function f is weakly decreasing and satisfies the monotone hazard rate property; i.e., $f/(1-F)$ is weakly increasing.*

Examples of a density function satisfying Assumption 1 include that of an exponential distribution.

If F is a point mass at 0, then our model collapses to a traditional one, where, in equilibrium, a monopolist would offer utility $u = 0$, and the consumer would purchase for sure. Gain of trade would be realized for sure, and the monopolist would capture all the surplus. We shall refer to this outcome as the *first best* for short.

It turns out that the first best remains an equilibrium outcome when the probability mass nearby 0 is sufficiently close to 1.[12] The intuition is that types with d close to 0 impose a discipline on the monopolist, discouraging it from lowering the offered utility below 0 and thus exploiting types with a larger d. In order for our model to generate results that are qualitatively different from a traditional one, we need F to be sufficiently different from a point mass at 0. The dividing line turns out to be the following condition.

Assumption 2. *The density at $d = 0$ is sufficiently small; specifically, $f(0)\overline{\pi} < 1$.*

Throughout this paper, our solution concept is the standard perfect Bayesian equilibrium, which we shall simply refer to as the *equilibrium.*

2.1. Discussion of the model

Before proceeding to the analysis, we will provide a discussion of our modeling assumptions.

We assume that the consumer's reservation utility is common knowledge. In Online Appendix A, we study an alternative model where the consumer's reservation utility is his private information.

We assume that the consumer cannot observe the offered utility of any deal at the moment of purchase, and possibly not until after consumption, and he is unable to return the deal when he learns that it is a bad one. The kind of applications we have in mind for our model are closer to credence goods and experience goods rather than search goods (Nelson (1970)).

We assume that, while the consumer may have difficulty in detecting small differences, his rationality is undamaged. In Online Appendix B, we briefly discuss the likely consequences of relaxing this rationality assumption.

We assume that the consumer does not know his own type. We can think of the consumer's type (i.e., his ability to discern between two utilities) as being dependent on how sharp his receptors are—for example, his ability to tell apart two cups of coffee depends on how sharp his taste

[12] See Appendix D for details.

buds are—which in turn depends on the quality of his previous night's sleep. The consumer does not know his own type because the quality of his previous-night's sleep contains much randomness beyond his grasp. This assumption is made mainly for tractability. In Online Appendix C, we study an alternative model where the consumer can observe a noisy signal about his type. All the key takeaways in our analysis carry over to this alternative model.

We assume that the distribution, F, of the consumer's type has full support on $\mathbb{R}_+$. This assumption is not necessary for any of our results. We can easily handle distributions with finite supports of the form [0, D], where $D < \infty$, at the expense of slightly messier proofs. Examples of a finite-support density function satisfying Assumption 1 include that of a uniform distribution. For any finite-support F that satisfies Assumptions 1 and 2, the upper limit of its support, D, is larger than $\overline{\pi}$. This is because, since f is weakly decreasing, we have $F(\overline{\pi}) \leq f(0)\overline{\pi} < 1$, and hence $D > \overline{\pi}$. An implication of this observation is that, even when the offered utility is as high as $\overline{\pi}$, thus erasing any profit for the seller, some types of the consumer will still be unable to discern between this utility and the reservation utility 0.

We have adopted the solution concept of a perfect Bayesian equilibrium. Technically, since all the games in this paper are infinite games, the solution concept of a sequential equilibrium is ill-defined. However, none of the equilibria we are looking at relies on the kind of "inconsistent" beliefs that a sequential equilibrium intends to preclude. See Online Appendix D for details.

3. A monopolist marketing a single deal

In this section, we start with the simplest possible case: a monopolist marketing a single deal. We can think of the costs of marketing a second deal as being prohibitively high, which is an assumption that we shall relax in the next two sections. The monopolist's strategy is represented by a distribution of the offered utility u. The consumer's strategy is represented by his probability of purchasing the deal conditional on his primary ordering (recall that he does not know his own type and hence cannot contingent his purchase probability on it). Since he is faced with only two utilities (u, and his reservation utility, 0) at the time of purchase, $\succ$ is the same as $\widehat{\succ}$. Utility maximization dictates that the consumer purchases with probability 1 (respectively, with probability 0) when his primary ordering is $u \widehat{\succ} 0$ (respectively, $0 \widehat{\succ} u$). Therefore, the consumer's strategy can be reduced to his purchase probability when his primary ordering is $u \widehat{\sim} 0$, which we shall denote by q.

Given the consumer's strategy q, the monopolist's profit as a function of its offered utility, u, is

$$\Pi(u; q) = \begin{cases} (\overline{\pi} - u)[1 - F(-u)]q & \text{if } u \leq 0 \\ (\overline{\pi} - u)\,(F(u) + [1 - F(u)]q) & \text{if } u \geq 0 \end{cases}.$$

In the case of $u \geq 0$, for example, $F(u)$ is the probability that the consumer can discern between u and 0 and hence would purchase with probability 1. With the complementary probability, $1 - F(u)$, he cannot discern between the two and hence would purchase with probability q.

Since the profit function has a kink at $u = 0$, we maximize it over the lower sub-range $u \in (-\infty, 0]$ and the upper sub-range $u \in [0, \overline{\pi}]$ separately, and then compare the maximized profit over each sub-range. In the proof of Proposition 1 below, we show that a unique maximizer exists in each sub-range, which we denote by $\underline{u}$ and $\overline{u}(q)$, respectively. Note that the maximizer in the lower sub-range does not depend on q, which can be readily verified by inspecting the profit function in that sub-range. The first best will be an equilibrium outcome iff $\underline{u} = 0 = \overline{u}(1)$. In the Appendix, we show that this is indeed the case if Assumption 2 is violated.

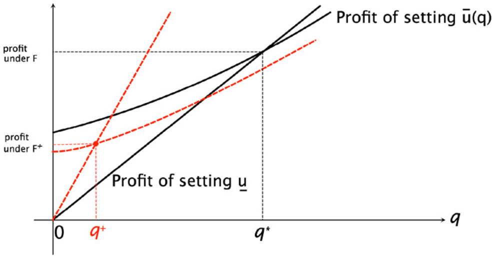

Fig. 1. The monopolist's profit in the one-deal case

In this sense, a distribution F that violates Assumption 2 is not different enough from a point mass at 0.

To understand why Assumption 2 guarantees that the first best cannot be an equilibrium outcome, it suffices to understand why $\underline{u} < 0$ under this assumption. Suppose the consumer purchases for sure even when he cannot discern between u and 0; i.e., suppose $q = 1$. Suppose the monopolist decreases the offered utility from $u = 0$ to $u = -\epsilon < 0$. For ϵ small, almost no type of the consumer could detect the decrease, and hence the monopolist earns ϵ more from almost every consumer type. There are approximately $\epsilon f(0)$ types whose receptors are very sharp (i.e., with $d < \epsilon$) and who would be able to detect the decrease and thus refuse to purchase. The lost profit from this small subset of types amounts to $\epsilon f(0)\overline{\pi}$. Under Assumption 2, the lost profit would be smaller than the gain from exploiting the rest of the types (i.e., $\epsilon f(0)\overline{\pi} < \epsilon$), and hence the monopolist could not resist the temptation to secretly decrease the offered utility below 0.

Indeed, for q sufficiently close to 1, low types (i.e., types with poor receptors) are so trusting and so willing to purchase that it is better for the monopolist to offer the low utility $\underline{u}$ in order to exploit these types. On the contrary, when q is sufficiently close to 0, low types are so untrusting and so unwilling to purchase that the only way to do business with them is to offer the high utility $\overline{u}(q)$ in the hope of convincing them that the deal is good. Neither case could result in an equilibrium, because the consumer's best response against $\underline{u}$ is $q = 0$ and that against $\overline{u}(q)$ is $q = 1$. In equilibrium, q must take some intermediate value q^* so that the monopolist is willing to randomize between $\underline{u}$ and $\overline{u}(q^*)$, and the monopolist must randomize in a way that makes the consumer willing to randomize between purchasing or not.

Fig. 1 illustrates how the equilibrium is uniquely determined. Here, the solid line passing through the origin represents the monopolist's profit if it offers the low utility $\underline{u}$, which in turn increases linearly in q. The solid curve represents the monopolist's profit if it offers the high utility $\overline{u}(q)$, which is convex in q because the monopolist re-optimizes the offered utility when it faces a different q. The convex curve is strictly above the linear line at $q = 0$, and is strictly below at $q = 1$. The shapes of the two profit functions dictate that they cross once and only once at q^*, at which point the monopolist will be willing to randomize between the high and the low utilities.

Proposition 1. *In the case of a monopolist marketing up to only one deal, the unique equilibrium is a mixed-strategy equilibrium, where*

- *the consumer purchases with a probability q^* that is strictly between 0 and 1/2 when he cannot discern the offered utility and his reservation utility u, and*

- *the monopolist randomizes between a low utility $\underline{u}^*$ that is strictly negative, and a high utility $\overline{u}^*$ that is strictly between 0 and $\overline{\pi}$.*

In equilibrium, gain of trade is not always realized. Some consumer types (those with $d \geq -\underline{u}^*$) sometimes obtain a strictly negative utility (when the monopolist offers the low utility $\underline{u}^*$). On average, however, the consumer obtains a strictly positive utility, meaning that the monopolist does not extract the full surplus, even when conditional on trade.

From Proposition 1, we can also obtain a very rough estimate of how much gain of trade is lost in equilibrium due to the consumer's difficulty in detecting small differences. Notice that every time the monopolist offers the low utility, the consumer either can tell that it is a bad deal (in which case there will be no sale), or cannot tell (in which case he purchases with probability $q^* < 1/2$). Therefore, conditional on a low offered utility, gain of trade is realized with probability at most $1/2$. Suppose the monopolist randomized between the high and low utilities with roughly equal probabilities. Then at least about $1/4$ of the gain of trade is lost in equilibrium due to the consumer's difficulty in detecting small differences.

It will be interesting to see how the monopolist's equilibrium profit changes when the consumer becomes less able to discern between two utilities. To answer that question, we compare the monopolist's equilibrium profit under two different distributions of the consumer's types, F and $F^\dagger$, where both satisfy Assumptions 1 and 2, but $F^\dagger$ first-order stochastically dominates (FOSD) F. In a market featuring $F^\dagger$, the consumer has a higher type (probabilistically) and hence is less able to discern between two utilities. Our next proposition says that the consumer's inability to discern between utilities actually hurts the monopolist.

Proposition 2. *In the case of a monopolist marketing up to only one deal, the monopolist's equilibrium profit decreases with an FOSD shift in F.*

Fig. 1 provides a pictorial proof of Proposition 2. When the distribution of the consumer's types undergoes an FOSD shift from F to $F^\dagger$, the consumer is less able to discern between utilities. This raises the monopolist's profit from offering a low utility, because the consumer is less able to tell if he is getting a bad deal. This results in an anti-clockwise tilt of the linear line. On the contrary, the monopolist's profit from offering a high utility is now lower, because the consumer is also less able to tell if he is getting a good deal. This results in a downward shift of the convex curve. In the new equilibrium, the consumer is less trusting ($q^\dagger < q^*$), and the monopolist's profit is lower.

This may seem a bit surprising. After all, a consumer who is less able to discern between two utilities seems vulnerable to exploitation, and hence should be welcomed by the monopolist. Such an intuition is incomplete, however. Recall that a consumer with difficulty discerning between two utilities is a person equipped with poor receptors. Although his instruments are poor, he is by no means irrational. He is aware that his instruments are poor, and is rationally untrusting when he finds the offered utility indiscernible from his reservation utility. A monopolist fares worse when the consumer is untrusting, because he cannot be easily convinced, even when the monopolist is indeed offering him a good deal.

While the consumer's inability to discern between utilities hurts the monopolist, it does not always benefit the consumer. Indeed, it is easy to see that the effect of an FOSD shift in F on the consumer's surplus is non-monotonic. Consider again the distributions F and $F^\dagger$, where the latter dominates the former in FOSD sense. In particular, this implies $F(\overline{\pi}) \geq F^\dagger(\overline{\pi})$. At the limit when $F^{\dagger}(\overline{\pi}) \searrow 0$, consumers almost surely cannot identify a good deal even when one exists (because the monopolist will never offer a utility beyond $\overline{\pi}$, and hence $\overline{u}$ cannot be larger than $\overline{\pi}$). Recall that whenever the consumer cannot identify a good deal he walks home with 0 surplus. Therefore, the consumer's surplus decreases with an FOSD shift from F to $F^{\dagger}$.

On the other hand, consider yet another distribution $F^{\dagger\dagger}$, which also satisfies Assumptions 1 and 2, and is dominated by F in FOSD sense. In particular, this implies that $f(0) \leq f^{\dagger\dagger}(0)$. At the limit when $f^{\dagger\dagger}(0) \nearrow 1/\overline{\pi}$, the first best is an equilibrium outcome, where the consumer walks home with 0 surplus.[13] Therefore, the consumer's surplus increases with an FOSD shift from $F^{\dagger\dagger}$ to F.

Proposition 3. *In the case of a monopolist marketing up to only one deal, the consumer's inability to discern utilities does not always benefit him. Indeed, the effect of an FOSD shift in F on the consumer's surplus is positive in some cases and negative in others.*

Intuitively, when the consumer's receptors are more likely to be poor (i.e., when d is larger in FOSD sense), there are two opposite forces at work. The first is that the monopolist now has to further sweeten its good deal (i.e., $\overline{u}^*$ has to increase)[14] in order for the consumer to be able to appreciate it (i.e., to be able to discern between $\overline{u}^*$ and 0). This tends to raise the consumer's welfare, specifically in the event that he has a small enough d to discern between the offered utility u and 0. The second is that the consumer is now less likely to have such a small d. Since the consumer receives 0 surplus whenever he is not able to discern between the offered utility u and 0 (otherwise he would be unwilling to randomize in equilibrium), this second force lowers the consumer's welfare by increasing the probability that he receives 0 surplus. The overall effect on the consumer's welfare delicately depends on which of these two opposing forces dominate.

4. A monopolist marketing two different deals

In this section, we will continue to study the case where there is only one seller, the monopolist. However, we now assume that the costs of marketing a second deal is negligible, while those of marketing more than two deals remain prohibitively high. We shall show that this improvement in the monopolist's marketing ability may paradoxically hurt its profit.

Formally, we assume that the costs of marketing a second deal are commonly known to be 0. The monopolist can market either a single deal or two different deals. In the case the monopolist markets two different deals, it can potentially offer a different utility for each deal. We shall name those two deals "deal 1" and "deal 2", with associated utilities u_1 and u_2, respectively. The consumer continues to demand at most one deal.

By allowing utilities to take the value of $-\infty$, we can wlog proceed as if both deals are actively being marketed. Marketing a single deal would then correspond to the case where $u_1 > -\infty = u_2$, whereas marketing two different deals would correspond to the case where both utilities are finite.

Some care should be taken in describing a game with two deals. It makes more sense to think of the deals as "anonymous", in the sense that the consumer's purchase decision depends on

[13] Continuity holds at the limit, meaning that when $f^{\dagger\dagger}(0) \approx 1/\overline{\pi}$ while still satisfying Assumption 2, the consumer still walks home with approximately 0 surplus. See the proof of Proposition 3 for details.

[14] That $\overline{u}^*$ has to increase can be proved formally for the families of exponential and uniform distributions. We conjecture that it holds more generally for other families of distributions as well.

the inferred ordering, $\succ$, of 0, u_1, and u_2, but otherwise does not depend on the names of the deals. This precludes, for example, the strategy of purchasing deal 1 with probability 4/5 when $u_1 \sim 0 \succ u_2$, while purchasing deal 2 with only probability 1/3 when $u_2 \sim 0 \succ u_1$. Similarly, it precludes the strategy of purchasing deal 1 and deal 2 with probabilities 1/10 and 9/10, respectively, when $u_1 \succ 0$, $u_2 \succ 0$, and $u_1 \sim u_2$. In other words, the names of the deals are artificial constructs that are for the convenience of we analysts only, but are otherwise meaningless to the consumer. All the consumers can learn about a specific deal is already summarized by the inferred ordering $\succ$.

Since the names of the deals are just artificial constructs that are meaningless to the consumer, we shall follow the convention that "deal 2" is the deal with a lower utility; i.e., $u_1 \geq u_2$ by our convention. This convention does not preclude positive sales for deal 2 in equilibrium. This is because, even though the consumer knows that deal 2 is less desirable than deal 1, a type who cannot a-discern between u_1 and u_2 cannot tell which deal is deal 2 (recall the assumption of anonymity above), and hence may end up purchasing deal 2 by chance.[15]

Under the convention of $u_1 \geq u_2$, there can only be 11 different primary orderings that the consumer could encounter.[16] For 8 out of these 11 primary orderings, there is an unambiguous highest-utility option in the resulting inferred ordering $\succ$, and utility maximization dictates that the consumer will choose this highest-utility option.[17] Among the remaining 3 primary orderings, the consumer's best response is also straightforward when the primary ordering is $u_1 \hat{\succ} 0$, $u_2 \hat{\succ} 0$, and $u_1 \hat{\sim} u_2$ (in which case the inferred ordering $\succ$ is the same as $\hat{\succ}$): in this case, anonymity dictates that the best the consumer can do is to purchase each deal with probability 1/2.

Therefore, the only two non-trivial cases are

- **s:** the **s**ingle-contender case, where the primary ordering is $u_1 \hat{\sim} 0$, $u_1 \hat{\succ} u_2$, and $0 \hat{\succ} u_2$, and hence deal 1 is the only possible good deal for the consumer; and
- **d:** the all-tied, **d**ouble-contender case, where the primary ordering is $u_1 \hat{\sim} 0$, $u_2 \hat{\sim} 0$, and $u_1 \hat{\sim} u_2$, and hence both deals 1 and 2 are possibly good deals for the consumer.

How the consumer behaves in these two cases will be determined in equilibrium. Let's denote by $q_s \in [0, 1]$ the probability that the consumer purchases deal 1 (i.e., the single contender) in the **s**ingle-contender case, and by $q_d/2 \in [0, 1/2]$ the probability that he purchases either contender in the all-tied, **d**ouble-contender case.

The reader may wonder how the monopolist could ever be hurt by its ability to market a second deal at negligible costs. Couldn't it guarantee at least its equilibrium profit in the one-deal case by simply marketing a single deal? The answer is no. If the consumer anticipates that the monopolist

[15] The anonymity assumption assumes that the consumer does not know how the monopolist names its two deals. Had he known which deal the monopolist names as "deal 2", he of course would never purchase that deal.

[16] There are 3 primary orderings that already form a linear order; for example, $u_1 \hat{\succ} 0 \hat{\succ} u_2$. For each of these 3 primary orderings, the inferred ordering $\succ$ is the same as $\hat{\succ}$. There are 4 primary orderings where exactly one pair of utilities are indiscernible; for example, $u_1 \hat{\sim} 0$, $u_1 \hat{\succ} u_2$, and $0 \hat{\succ} u_2$. For each of these 4 primary orderings, the inferred ordering $\succ$ is still the same as $\hat{\succ}$. There are 3 primary orderings where exactly two pairs of utilities are indiscernible; for example, $u_1 \hat{\succ} u_2$, $u_1 \hat{\sim} 0$, and $u_2 \hat{\sim} 0$. For each of these 3 primary orderings, the inferred ordering $\succ$ becomes a linear order; for example, from $u_1 \hat{\succ} u_2$, $u_1 \hat{\sim} 0$, and $u_2 \hat{\sim} 0$, the consumer obtains $u_1 \succ 0 \succ u_2$. Finally, there is 1 primary ordering where all three pairs of utilities are indiscernible. For this primary ordering, the inferred ordering $\succ$ is also the same as $\hat{\succ}$.

[17] There are 6 out of 11 primary orderings where the inferred ordering $\succ$ is a linear order (see Footnote 16), and hence an unambiguous highest-utility option exists. The other two primary orderings where an unambiguous highest-utility option exists are ($0 \hat{\succ} u_1$, $0 \hat{\succ} u_2$, and $u_1 \hat{\sim} u_2$) and ($u_1 \hat{\succ} u_2$, $u_1 \hat{\succ} 0$, and $u_2 \hat{\sim} 0$).

markets two different deals, then marketing a single deal will be an off-equilibrium event, and the consumer's off-equilibrium belief in such an event might be quite different from his equilibrium belief in the one-deal case. We present an equilibrium with such a flavor below in Proposition 4. We then provide an example of F such that the monopolist's profit in the equilibrium described in Proposition 4 is lower than its equilibrium profit in the one-deal case.

Proposition 4. *In the case of a monopolist marketing up to two different deals, there exists an equilibrium where*

- *the consumer refuses to purchase whenever there is no apparent highest-utility option (i.e., $q_s^* = q_d^* = 0$),*
- *the monopolist markets two different deals with offered utilities being mirror images of each other around 0 (i.e., $u_1^* > 0 > u_2^*$ and $(u_1^* + u_2^*)/2 = 0$), and*
- *given the consumer's behavior (i.e., $q_s^* = q_d^* = 0$), (u_1^*, u_2^*) is the unique maximizer of the monopolist's profit over all pairs of utilities that are mirror images of each other around 0.*

In this equilibrium, the monopolist's offered utilities are deterministic. Gain of trade, however, is still not always realized. The consumer purchases only if he can a-discern between u_1^* and 0, and hence he always obtains strictly positive surplus conditional on a purchase.

To further elaborate on the point we made in the paragraph immediately before Proposition 4, let's consider what would happen if the monopolist deviated from its equilibrium behavior by marketing a single deal. Specifically, suppose that, instead of offering utilities (u_1^*, u_2^*) as described in Proposition 4, the monopolist deviates and offers $u_2 = -\infty$ and randomizes between $u_1 = \overline{u}^*$ and $u_1 = \underline{u}^*$ as in Proposition 1. Given the consumer's equilibrium strategy $q_s^* = q_d^* = 0$, the monopolist cannot make any sales when the random utility u_1 takes the value of $\underline{u}^*$. Even when the random utility u_1 takes the value of $\overline{u}^*$, the monopolist makes a sale only if the consumer can discern between u_1 and 0. Its profit from such a deviation is hence much lower than its equilibrium profit in the one-deal case.

The reason behind this dismal deviation profit is that the consumer is very untrusting in the single-contender case ($q_s^* = 0$). In the equilibrium described in Proposition 4, the single-contender case is an off-equilibrium event, and the consumer's off-equilibrium belief can be quite pessimistic.

Note that, in the equilibrium described in Proposition 4, the monopolist never makes any sales from deal 2. The role of deal 2 is to convince the consumer that deal 1 is a good deal (which it is). Specifically, types d that satisfy $u_1^* - u_2^* > d \geq u_1^* - 0$, although unable to *discern* between u_1^* and 0, are nevertheless able to *a-discern* the two. The primary ordering received by these types is $u_1^* \widehat{\sim} 0$, $0 \widehat{\sim} u_2^*$ and $u_1^* \widehat{\succ} u_2^*$, which induces the inferred ordering of $u_1^* \succ 0 \succ u_2^*$, convincing the consumer to purchase deal 1.

Paradoxically, this helping hand from the second deal can backfire. In the one-deal case, it cannot be an equilibrium for the consumer to be totally untrusting. If the consumer were totally untrusting (i.e., if $q = 0$), the monopolist would not be able to make any sales unless it offered a strictly positive utility, but then the consumer should be totally trusting (i.e., $q = 1$) instead. This is no longer the case when there exists a bad deal purely to help the consumer to appreciate a good deal. Now the consumer who still cannot a-discern between utilities *can* justifiably remain totally untrusting (i.e., $q_s^* = q_d^* = 0$), worrying that he may inadvertently purchase a bad deal.

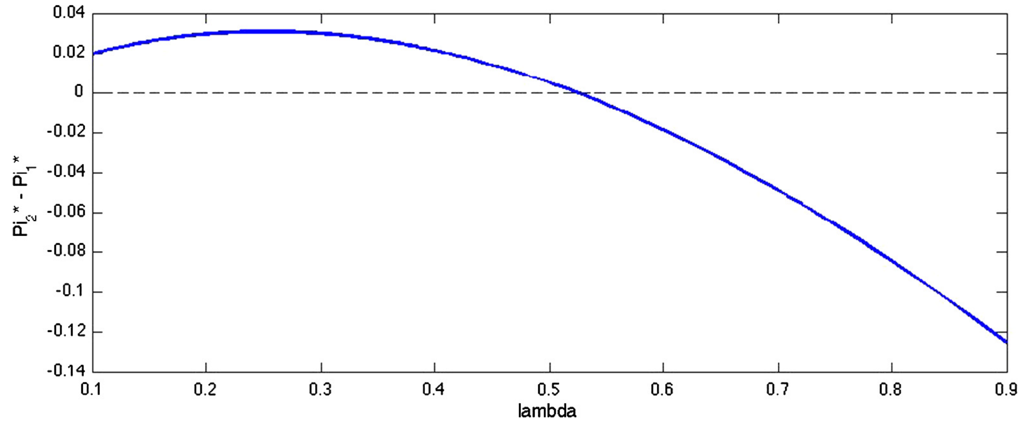

Fig. 2. Equilibrium profits in the one-deal and two-deal cases

That the consumer is more untrusting in the equilibrium described in Proposition 4 than in the unique equilibrium described in Proposition 1 is the main reason why the monopolist's profit can be lower in the former than in the latter. As an illustration, we compute the monopolist's profit in each of these two equilibria by letting $\overline{\pi} = 1$ and letting F be a member of the exponential class; i.e., $F(d) = 1 - e^{-\lambda d}$. Let Π_1^* denote the monopolist's profit in the equilibrium described in Proposition 1, and Π_2^* that in the equilibrium described in Proposition 4. In Fig. 2, we plot $\Pi_2^* - \Pi_1^*$ against λ, the parameter of the exponential distribution. Note that F satisfies Assumption 2 only if $\lambda < 1$. As λ increases towards 1, the consumer's ability to discern between utilities improves.[18] In the one-deal case, this generates more discipline on the monopolist's part, decreasing its incentive to offer a bad deal, and increasing its incentive to offer a good deal. Consumer types who cannot discern between utilities, by free-riding those who can, can hence afford to be more trusting, resulting in a higher q^* (see the proof of Proposition 2). Meanwhile, in the equilibrium described in Proposition 4, the consumer remains totally untrusting. As a result, $\Pi_2^* - \Pi_1^*$ becomes negative as λ increases towards 1.[19]

Proposition 5. *A monopolist's ability to market a second deal at negligible costs may paradoxically hurt its profit. Specifically, there exists an equilibrium (as described in Proposition 4) where the monopolist's expected profit, under certain distributions of types, is lower than its (unique) equilibrium expected profit when marketing a second deal is prohibitively costly.*

While there is a unique equilibrium in the one-deal case, there are multiple equilibria in the two-deal case, with the one described in Proposition 4 being merely one of them. For the sake of completeness, we fully characterize an important sub-class of equilibria, namely the seller-pure-strategy equilibria, in Proposition 6 below. These are equilibria where the monop-

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18 Formally, an exponential distribution with a smaller λ dominates one with a larger λ in the FOSD sense.

19 Fig. 2 actually illustrates a result that is stronger than what is stated in Proposition 5 below. Specifically, Fig. 2 illustrates that there exists a threshold $\hat{\lambda}$ such that $\Pi_1^* \gtreqless \Pi_2^*$ iff $\lambda \gtreqless \hat{\lambda}$. In Online Appendix E, we show that this result is in general true. Specifically, we provide a mild condition, which is satisfied by both the families of exponential and uniform distributions, such that, if $\Pi_1^* > \Pi_2^*$ (respectively, $\Pi_1^* < \Pi_2^*$) under a given distribution of d, then the same continues to be true under a second distribution that dominates (respectively, is dominated by) the first distribution in FOSD sense.

olist plays a pure strategy, in contrast to the mixed strategy played in the unique equilibrium in the one-deal case. Note that the equilibrium described in Proposition 4 is an example of a seller-pure-strategy equilibrium, where the monopolist offers utilities $(u_1, u_2) = (u_1^*, u_2^*)$ with probability 1.

Proposition 6. *In the case of a monopolist marketing up to two different deals, there is a* $q_d^{max} \in (0, 1)$ *such that*

- *every seller-pure-strategy equilibrium features a* $q_d^* \leq q_d^{max}$*;*
- *there exists a strictly decreasing function* $u_1(\cdot)$ *that maps* $[0, q_d^{max}]$ *into* $(0, \overline{\pi})$ *such that, in the seller-pure-strategy equilibrium featuring* $q_d^* \in [0, q_d^{max}]$*, the monopolist offers deterministic utilities* $u_1^* = u_1(q_d^*)$ *and* $u_2^* = -u_1^*$*;*
- *comparing any two seller-pure-strategy equilibria, the monopolist's expected profit is higher and the consumer's expected surplus is lower in the equilibrium with a higher* q_d^**; and*
- *if, in addition to Assumption 2,* f *further satisfies* $f(0)\overline{\pi} > 1/2$*, then*
  - *for every* $q_d^* \in [0, q_d^{max}]$*, there exists a seller-pure-strategy equilibrium featuring that specific* q_d^**; and*
  - *in the seller-pure-strategy equilibrium featuring* $q_d^* = q_d^{max}$*, the monopolist's expected profit is higher than its (unique) equilibrium expected profit in the one-deal case.*

In other words, every seller-pure-strategy equilibrium resembles the one described in Proposition 4, in the sense that the monopolist markets two different deals, with offered utilities being mirror images of each other (i.e., $u_1^* > 0 > u_2^*$ and $(u_1^* + u_2^*)/2 = 0$). As a result, the single-contender case is always an off-equilibrium event, rendering the exact description of q_s^* payoff-irrelevant. Each seller-pure-strategy equilibrium described in Proposition 6 hence is more precisely an equivalent class of equilibria featuring the same q_d^* but different q_s^*'s.

Comparing different seller-pure-strategy equilibria, the consumer is more trusting in those featuring higher q_d^*. Specifically, even when he has a low type (i.e., has poor receptors), and finds himself in the all-tied, double-contender case, he is still willing to purchase with probability q_d^*. To sustain such kind of equilibria, the consumer cannot have a low type too often, otherwise the monopolist would be too tempted to deviate and take advantage of his trust. This explains the extra condition of $f(0)\overline{\pi} > 1/2$, which roughly states that high types (i.e., types with d close to 0) appear sufficiently often, or, equivalently, that low types are sufficiently rare.

When the consumer is more trusting (i.e., when q_d^* is higher), the monopolist's expected profit is higher at the expense of the consumer's expected surplus. The equilibrium described in Proposition 4 is hence the worst for the monopolist and the best for the consumer among all seller-pure-strategy equilibria.

The consumer, however, will never be totally trusting in any seller-pure-strategy equilibrium. This is shown by the fact that q_d^* is capped from above by an upper bound q_d^{max} that is strictly smaller than 1. Therefore, once again, the first best cannot be achieved.

Finally, one may wonder whether there exists any (mixed-seller-strategy) equilibrium where the monopolist does not market deal 2 at all; i.e., $u_2 = -\infty$ with probability 1. Such an equilibrium, if it exists, must look exactly like the unique equilibrium in the one-deal case as described in Proposition 1; i.e., the monopolist plays a mixed strategy and randomizes between $u_1 = \overline{u}^*$ and $u_1 = \underline{u}^*$, and the consumer purchases with probability $q_s = q^*$ when he finds himself in the single-contender case. Let's call such an equilibrium, if it exists, the single-deal equilibrium.[20]

When the monopolist can market up to two different deals at negligible costs, the single-deal equilibrium may not exist. The reason is that, whenever the monopolist is to offer $u_1 = \overline{u}^*$—which is a good deal for the consumer because $\overline{u}^* > 0$—it would lament the fact that too few consumer types can *discern* between $\overline{u}^*$ and 0 and hence appreciate this good deal, and would have incentives to bring in the second deal in order to help the consumer to *a-discern* between $\overline{u}^*$ and 0.

Characterizing exactly when the single-deal equilibrium fails to exist turns out to be both tedious and non-illuminating. This is because there are many possible deviations involving "bringing in the second deal", and the single-deal equilibrium will fail to exist as long as one of these deviations is profitable. We shall therefore only provide one sufficient condition for the non-existence of the single-deal equilibrium, focusing on only one particular deviation, namely the deviation to offering $(u_1, u_2) = (u_1^*, u_2^*)$, where (u_1^*, u_2^*) is as defined in Proposition 4. In the following observation, Π_1^* and Π_2^* are as defined in the paragraph immediately before Proposition 5.

Observation 1. *In the case of a monopolist marketing up to two different deals, the single-deal equilibrium does not exist whenever* $\Pi_2^* > \Pi_1^*$.

Proof. This is because the monopolist's expected profit in the single-deal equilibrium, if it exists, must equal to Π_1^*, while its deviation expected profit is at least Π_2^* if it deviates to setting $(u_1, u_2) = (u_1^*, u_2^*)$, where (u_1^*, u_2^*) is as defined in Proposition 4.[21,22] □

For example, we can see from Fig. 2 that, when $\overline{\pi} = 1$, and F is an exponential distribution with parameter $\lambda < 1/2$, the single-deal equilibrium does not exist.

5. A monopolist marketing many deals

While a monopolist's ability to market a second deal at negligible costs may paradoxically hurt its profit (Proposition 5), we can, however, prove that its ability to market at negligible costs a sufficiently large number of deals, almost none of which meant to make any sales, will necessarily help its profit.

Formally, we assume that the costs of marketing the first n deals are commonly known to be 0, while those of marketing more than n deals remain prohibitively high. We shall show that, for a sufficiently large n, the monopolist's profit in *any* equilibrium is arbitrarily close to its first-best profit $\overline{\pi}$, and hence is higher than its equilibrium profit in the one-deal case. Since the proof is short and constructive, we include it here in the main text and let it help explain the underlying intuition.

[20] The single-deal equilibrium is more precisely an equivalent class of equilibria featuring different q_d^*. This is because $u_2 = -\infty$ with probability 1 implies that the all-tied case is an off-equilibrium event, and hence many different q_d^*'s can be supported by appropriately chosen off-equilibrium beliefs.

[21] Recall from Footnote 20 that the single-deal equilibrium can feature many different q_d^*.

[22] The deviation profit is exactly Π_2^* if $q_d^* = 0$, and is strictly higher than Π_2^* if $q_d^* > 0$. (See Proposition 6.) Note that q_s^* is irrelevant in calculating the deviation expected profit.

Proposition 7. *For any $\varepsilon > 0$, there exists $\overline{n}$ such that, for any $n \geq \overline{n}$, in the case of a monopolist marketing up to n different deals, the monopolist can achieve a profit higher than $\overline{\pi} - \varepsilon$ in any equilibrium.*

Proof. Pick any $\delta > 0$ small enough and $\overline{n}$ big enough so that $(\overline{\pi} - \delta)F(\overline{n}\delta) > \overline{\pi} - \varepsilon$. Suppose the monopolist is to market $n \geq \overline{n}$ deals, with offered utilities $u_1 = \delta$, $u_2 = -\delta$, $u_3 = -2\delta$, ..., and $u_n = -(n-1)\delta$, respectively. When the consumer has type $d < \delta$, he will be able to discern between u_1 and 0 and hence be able to tell that u_1 is strictly positive, and will purchase deal 1. When the consumer has type $d \in [\delta, 2\delta)$, he cannot discern between u_1 and 0, and cannot discern between 0 and u_2, but is able to discern between u_1 and u_2, and hence can infer that u_1 is strictly positive, and will also purchase deal 1. More generally, when the consumer has type $d \in [(k-1)\delta, k\delta)$, $k \in \{2, \ldots, n\}$, he cannot discern between u_1 and 0, and cannot discern between 0 and u_k, but is able to discern between u_1 and u_k, and hence can infer that u_1 is strictly positive, and will purchase deal 1. The monopolist's profit is hence at least $(\overline{\pi} - u_1)F(n\delta) \geq (\overline{\pi} - \delta)F(\overline{n}\delta) > \overline{\pi} - \varepsilon$. □

Intuitively, having many bad deals allows the monopolist to ensure that, no matter how blunt the consumer is, there is a sufficiently bad deal on the menu to help him appreciate the good deal. Therefore, when n is arbitrarily large, the consumer will almost certainly be able to appreciate the good deal and buy it, no matter how close its offered utility is to his outside option.

6. Two sellers marketing one deal each

In Section 4, we study the case of a single seller marketing up to two different deals. In this section, we study the case where the ability to market the second deal comes from a second seller instead of from the original seller. Specifically, we study the case where there are two identical sellers, each marketing up to only one deal.

We shall name the two sellers "seller 1" and "seller 2", with associated offered utilities u_1 and u_2 for their respective deals. The consumer continues to demand one and only one deal. Following the second half of Section 4, we focus on seller-pure-strategy equilibria, meaning those equilibria where each seller plays a pure strategy.

As in Section 4, we assume that deals are "anonymous", in the sense that the names of the deals are artificial constructs that are for the convenience of we analysts only, but are otherwise meaningless to the consumer. All the consumers can learn about a specific deal is already summarized by the inferred ordering $\succ$. As such, and since sellers play pure strategies, we can follow the convention in Section 4 that "deal 2" is the deal with a lower equilibrium offered utility; i.e., $u_1^* \geq u_2^*$ by our convention. As before, this convention does not preclude an asymmetric equilibrium where the two sellers offer different utilities. This is because, even though the consumer knows that deal 2 is worse than deal 1, if he cannot a-discern between u_1 and u_2, he cannot tell which deal is deal 2, and hence may end up purchasing deal 2 by chance.

As in Section 4, we describe the consumer's strategy by describing his behavior in the single-contender case and the all-tied, double-contender case, which are defined in exactly the same way as in Section 4. Let's continue to denote by $q_s \in [0, 1]$ the probability that the consumer purchases deal 1 (i.e., the single contender) in the single-contender case, and by $q_d/2 \in [0, 1/2]$ the (necessarily common) probability that he purchases either contender in the all-tied, double-contender case.

The reader may wonder why more competition does not always benefit the consumer. Wouldn't compete sellers undercut each other and lead to higher offered utilities as in the traditional Bertrand model? The answer is no. For starters, when the consumer has difficulty in detecting small differences, undercutting one's opponent does not enable a seller to capture the whole market, because the consumer often is not able to tell that its offered utility is higher than its opponent's. This reduces one's incentive to undercut its opponent. Indeed, if sellers are anticipated to undercut each other aggressively, the consumer will become fairly trusting, and will be fairly willing to purchase even when he cannot a-discern between utilities. Sellers will then have incentives to lower their offered utilities in order to exploit this trusting consumer, thus invalidating the original anticipation.

When the consumer has difficulty in detecting small differences, sellers actually free-ride instead of undercut each other. There are two different kinds of free-riding behavior, and are respectively adopted by the two sellers. In a seller-pure-strategy equilibrium, one seller (seller 1) will specialize in offering a good deal, while the other (seller 2) specializes in offering a bad deal. Seller 2 free-rides seller 1's good deal, which keeps the consumer trusting, and offers a bad deal to exploit this trusting consumer.[23] Seller 1, on the other hand, free-rides seller 2's bad deal, which enables the consumer to sometimes recognize the good deal offered by seller 1—when the consumer cannot *discern* between u_1 and 0, but can *a-discern* between them with the help of u_2—and avoids the need to offer an even higher utility to win over the consumer.[24] As a result, both sellers manage to alleviate some upward pressure on their offered utilities by free-riding each other, albeit free-riding in very different ways.

A seller-pure-strategy equilibrium in this two-seller case ends up being very similar to the one in the two-deal case studied in Section 4 in the sense that the offered utilities of the two deals are mirror images of each other around 0. The consumer purchases either when he can a-discern between utilities—in which case he purchases the better deal and obtains a strictly

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[23] More formally, $u_2 < 0$ can be a best response for seller 2 only when $q_d > 0$ (because seller 2 can sell a bad deal only when the consumer finds himself in the all-tied case), and $q_d > 0$ can be part of the consumer's best response only when $u_1 \geq 0$ (otherwise $(u_1 + u_2)/2 \leq u_1 < 0$ and hence the consumer's best response must feature $q_d = 0$).

[24] More formally, consider the case where $q_s = 0 < q_d$ (which, as we shall argue soon, can be assumed wlog in our search for seller-pure-strategy equilibria). Suppose seller 2 offers $u_2 = -\infty$, effectively dropping out from the market. Then seller 1's best response is to set $u_1 = \overline{u}(0)$, where $\overline{u}(\cdot)$ is as defined in Section 3. Suppose seller 2 now raises its offered utility, but not so high as to offer the consumer a genuine good deal. Specifically, suppose seller 2 raises its offered utility from $-\infty$ to some finite $u_2 \in (-u_1, 0)$. Instead of imposing competitive pressure on seller 1, this move actually *raises* seller 1's profit from $(\overline{\pi} - u_1)F(u_1)$ to

$$\Pi_1 := (\overline{\pi} - u_1)\{F(u_1 - u_2) + q_d[1 - F(u_1 - u_2)]\}.$$

More importantly for the consumer, seller 1 would now have incentives to further *lower* its offered utility. Intuitively, with the help of a finite u_2, the consumer can more often appreciate the good deal that seller 1 is offering him. With a bigger consumer base, seller 1 now has an incentive to lower its offered utility. Formally,

$$\begin{aligned}\left.\frac{\partial \Pi_1}{\partial u_1}\right|_{u_1=\overline{u}(0)} &= -\{F(u_1 - u_2) + q_d[1 - F(u_1 - u_2)]\} + (\overline{\pi} - u_1)(1 - q_d) f(u_1 - u_2) \\ &= -q_d - (1 - q_d) f(u_1 - u_2)\left[\frac{F(u_1 - u_2)}{f(u_1 - u_2)} - \frac{F(u_1)}{f(u_1)}\right] \\ &< 0,\end{aligned}$$

where the second equality makes use of the first-order condition $\overline{\pi} - u_1 = F(u_1)/f(u_1)$ that characterizes $u_1 = \overline{u}(0)$, and the inequality makes use of Assumption 1.

positive surplus—or when he finds himself in the all-tied, double-contender case—where he randomizes between purchasing or not, and randomizes between the two deals when he does purchase, and obtains zero surplus on average. The consumer's surplus hence depends solely on how high deal 1's offered utility is, which is the same as in a seller-pure-strategy equilibrium in the two-deal case. It turns out that the free-riding logic mentioned in the previous paragraph and in Footnote 24 will depress u_1 so low that the consumer's surplus in this two-seller case is even lower than that in the seller-pure-strategy equilibrium described in Proposition 4.

Proposition 8. *In the case of two sellers marketing up to only one deal each, in any seller-pure-strategy equilibrium,*

- *the two sellers offer utilities that are mirror images of each other around* 0*; specifically,* $(u_1^*, u_2^*) = (x^*, -x^*)$*, where* $x^* > 0$ *is the unique solution to*

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + \overline{\pi};$$

- *the consumer's expected surplus is strictly lower than in the equilibrium described in Proposition 4 for the case of a monopolist marketing up to two different deals.*

In the special case where F belongs to the exponential class, the consumer's expected surplus in any seller-pure-strategy equilibrium is especially easy to calculate. First, using the equation in Proposition 8, we have

$$x^* + \overline{\pi} = \frac{1 - F(2x^*)}{f(2x^*)} = \frac{\exp(-\lambda(2x^*))}{\lambda \exp(-\lambda(2x^*))} = \frac{1}{\lambda},$$

which gives us $x^* = 1/\lambda - \overline{\pi}$. When the consumer has type $d < 2x^*$, he is able to a-discern between u_1^* and 0 (either by directly *discerning* them, or by *comparing* them with the help of u_2^*) and hence will walk home with surplus $u_1^* = x^*$. When the consumer has type $d \geq 2x^*$, on the other hand, he cannot a-discern between u_1^* and 0, and hence will purchase both deals 1 and 2 with the same probability if he ever does any purchase. The consumer then walks home with zero surplus, given that u_1^* and u_2^* are mirror images of each other around 0. On average, the consumer's surplus is

$$x^* F(2x^*) = x^*[1 - \exp(-\lambda(2x^*))] = \left[\frac{1}{\lambda} - \overline{\pi}\right]\left[1 - \exp(-2 + 2\lambda\overline{\pi})\right],$$

which is strictly decreasing in λ. Recall that, in the special case of the exponential class, a smaller λ represents an FOSD shift in F, meaning that the consumer is less able to discern between utilities. We conclude that the consumer's expected surplus is increasing in his *in*ability to discern utilities, echoing similar results in Piccione and Spiegler (2012) and de Clippel et al. (2014). Intuitively, when the consumer has more difficulty in discerning between utilities, there are two opposite effects on the consumer's expected surplus. On the one hand, the consumer is less able to recognize the good deal offered by seller 1, which reduces his expected surplus. On the other hand, seller 1 is pressured to further raise its offered utility in order to convince the consumer that its deal is good, which increases his expected surplus. Depending on the distribution of the consumer's types, it is possible that the second effect dominates the first effect.

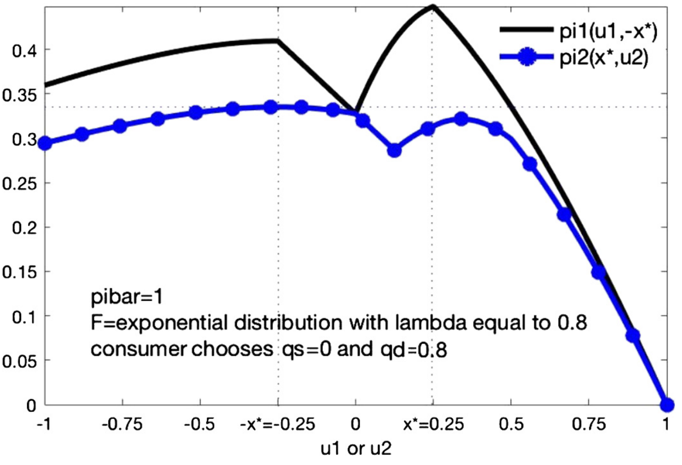

Fig. 3. The solid line is seller 1's profit as a function of u_1, given the opponent's offered utility $u_2 = -x^*$. The dotted line is seller 2's profit as a function of u_2, given the opponent's offered utility $u_1 = x^*$. Both are drawn with $\overline{\pi} = 1$, F being the exponential distribution with parameter $\lambda = 0.8$ (which implies $x^* = 0.25$), and $(q_s, q_d) = (0, 0.8)$.

Proposition 9. *Suppose F belongs to the exponential class. In the case of two sellers marketing up to only one deal each, the consumer's seller-pure-strategy-equilibrium expected surplus increases with an FOSD shift in F.*[25]

A seller-pure-strategy equilibrium, however, may not exist. Indeed, one can prove that it does not exist if $f(0)\overline{\pi} < 1/3$. In order to make sure that Proposition 8 above is not a characterization of the empty set, we numerically demonstrate the existence of a seller-pure-strategy equilibrium given some value of $\overline{\pi}$, and some distribution F that satisfies Assumptions 1 and 2.

Specifically, let $\overline{\pi} = 1$, and F be the exponential distribution with parameter $\lambda = 0.8$. Plugging these into the equation in Proposition 8, one readily computes that $x^* = 0.25$. According to Proposition 8, a seller-pure-strategy equilibrium, if one exists, must feature $(u_1^*, u_2^*) = (x^*, -x^*) = (0.25, -0.25)$. We can plot seller 1's profit $\Pi_1(u_1, u_2)$ (respectively, seller 2's profit $\Pi_2(u_1, u_2)$) as a function of its offered utility u_1 (respectively, u_2), given the opponent's offered utility $u_2 = -0.25$ (respectively, $u_1 = 0.25$), and given any (q_s, q_d). A seller-pure-strategy equilibrium exists iff there exists some (q_s, q_d) such that $u_1 = 0.25$ and $u_2 = -0.25$ are global optima of $\Pi_1(\cdot, -0.25)$ and $\Pi_2(0.25, \cdot)$, respectively. An argument we used in the proof of Proposition 6

[25] As explained in Section 2.1, we can easily handle distributions with finite supports (such as uniform distributions) at the expense of messier proofs. In the special case where F belongs to the uniform class $U[0, D]$, the equation in Proposition 8 becomes

$$x^* + \overline{\pi} = \frac{1 - F(2x^*)}{f(2x^*)} = D - 2x^*,$$

which gives us $x^* = (D - \overline{\pi})/3$. Therefore, consumers' surplus is

$$x^* F(2x^*) = x^* \frac{2x^*}{D} = \frac{2(D - \overline{\pi})^2}{9D},$$

which is strictly increasing in D if $D > \overline{\pi}$, which in turned is guaranteed by Assumption 2. Recall that, in the special case of the uniform class, a bigger D represents an FOSD shift in F. We once again arrive at the same conclusion that the consumer's expected surplus is increasing in his *in*ability to discern between utilities.

suggests that, in our search for (q_s, q_d) with such a property, it is wlog to let $q_s = 0$, because this does not affect either seller's profit at its candidate-equilibrium offered utility, while weakly lowering its profit at other offered utilities.

This leaves us only one variable to tune with. As we tune q_d, the shapes of $\Pi_1(\cdot, -0.25)$ and $\Pi_2(0.25, \cdot)$ change. In the neighborhood of $q_d = 0.8$, $u_1 = 0.25$ and $u_2 = -0.25$ indeed become the global optima of $\Pi_1(\cdot, -0.25)$ and $\Pi_2(0.25, \cdot)$, respectively, as shown in Fig. 3, verifying the existence of a seller-pure-strategy equilibrium for such $\overline{\pi}$ and F.

Fig. 3 is robust to perturbation in q_d, meaning that $(u_1^*, u_2^*, q_s^*, q_d^*) = (0.25, -0.25, 0, q_d^*)$ remains a seller-pure-strategy equilibrium for an open set of q_d^* containing 0.8. Indeed, $\Pi_1(\cdot, -0.25)$ has a kink at $u_1 = 0.25$. As we perturb q_d, the left and right derivatives of $\Pi_1(\cdot, -0.25)$ at $u_1 = 0.25$ will be perturbed, and so will the values of $\Pi_1(\cdot, -0.25)$ at the two local optima, but $u_1 = 0.25$ will remain the unique global optimum. The case for $\Pi_2(0.25, \cdot)$ is slightly different: its slope at $u_2 = -0.25$ remains flat regardless of the value of q_d. Indeed, this property is the geometric meaning of the equation in Proposition 8, which we used earlier to compute x^*. Therefore, as we perturb q_d, $u_2 = -0.25$ remains a local optimum. Since the value of $\Pi_2(0.25, \cdot)$ at $u_2 = -0.25$ is strictly higher than that at the other local optimum, $u_2 = -0.25$ will remain the unique global optimum upon perturbating q_d.

It is also worth highlighting some interesting features of Fig. 3 that are not mentioned in Proposition 8. First, the two sellers earn different profits, with the one offering a bad deal earning strictly less than the one offering a good deal. While seller 2 may envy seller 1, it cannot mimic the latter by also offering a good deal. If it were to do so, the best way to do it is to undercut seller 1, which is shown by the fact that the right local optimum of $\Pi_2(0.25, \cdot)$ is located on the right of 0.25. However, by doing so, seller 2 would actually earn even less than free-riding seller 1 and exploiting the trusting consumer.

Second, sellers' profits as functions of their own offered utilities are not quasi-concave. This explains why it is difficult to provide interesting sufficient conditions for the existence of a seller-pure-strategy equilibrium beyond numerical examples such as the one depicted in Fig. 3.

7. Conclusion

In this paper, we have explored how sellers behave when their consumers have difficulty detecting small differences. These consumers pose a problem because they cannot appreciate a good deal, even when one exists, if it is barely better than their outside options. This creates a role for a second deal, either marketed by the same seller, or by another seller, even when consumers are homogeneous in their tastes. If it is the same seller who markets the second deal, it will strategically position the first as a good deal, and the second as a bad deal, and use the bad deal to help consumers appreciate the good deal. If it is another seller who markets the second deal, the two sellers will specialize into a good-deal and a bad-deal providers. Both sellers free-ride each other. The good-deal provider is happy that the bad deal helps consumers appreciate its good deal; while the bad-deal provider hides behind the presence of the good deal and manages to make a sale some of the time.

Appendix A. Omitted proofs in Section 3

The following lemma will be used in the proof of Proposition 1.

Lemma 1. *Consider the function* $\Pi(u) := (\overline{\pi} - u)[1 - F(-u)]$. *In the sub-range* $u \in (-\infty, 0]$, *the function* $\Pi(u)$ *is strictly quasi-concave, and has a unique maximizer, which is interior in this sub-range.*

Proof. At any $u \in (-\infty, 0)$, we have $\Pi(u) > 0$ and

$$\frac{d\Pi(u)}{du} = f(-u)\left[(\overline{\pi} - u) - \frac{1 - F(-u)}{f(-u)}\right].$$

By Assumption 1, the term in the square parentheses is strictly decreasing in u. Therefore, the function $\Pi(u)$ is strict quasi-concave, and has at most one maximizer. For u sufficiently negative, the term in the square parentheses is strictly positive. For u sufficiently close to 0, by Assumption 2, the term in the square parentheses is strictly negative. Therefore, a unique maximizer exists and is interior in the sub-range $u \in (-\infty, 0]$. □

Proof of Proposition 1. By Lemma 1, for any $q > 0$, the function $\Pi(u; q)$ is strictly quasi-concave and has a unique maximizer in the sub-range $u \in (-\infty, 0]$. Let $\underline{u} \leq 0$ denote this unique maximizer. Apparently $\underline{u}$ does not depend on q.[26] Moreover, we have

$$\underline{u} < 0 \quad \text{iff} \quad \left.\frac{\partial\Pi(u; q)}{\partial u}\right|_{u=0^-} < 0 \quad \text{iff} \quad f(0)\overline{\pi} < 1,$$

which is guaranteed by Assumption 1.

In the sub-range $u \in (0, \overline{\pi})$,

$$\frac{\partial\Pi(u; q)}{\partial u} = -(q + F(u)(1 - q)) + (\overline{\pi} - u)f(u)(1 - q),$$
$$\frac{\partial^2\Pi}{\partial u^2} = -2f(u)(1 - q) + (\overline{\pi} - u)f'(u)(1 - q) < 0,$$

where the last inequality follows from Assumption 1. Therefore, $\Pi(u; q)$ is strictly concave in u, and has a unique maximizer in the sub-range $u \in [0, \overline{\pi}]$. Let $\overline{u}(q) \geq 0$ denote this unique maximizer. Since

$$\frac{\partial^2\Pi(u; q)}{\partial u \partial q} = -(1 - F(u)) - (\overline{\pi} - u)f(u) < 0,$$

$\overline{u}(q)$ is decreasing in q (strictly so if $\overline{u} \in (0, \overline{\pi})$). It attains its lower bound 0 iff

$$\left.\frac{\partial\Pi(u; q)}{\partial u}\right|_{u=0^+} = -q + \overline{\pi}f(0)(1 - q) \leq 0 \quad \text{iff} \quad f(0)\overline{\pi} \leq \frac{q}{1 - q}.$$

Let $\overline{q}$ be the unique solution to $f(0)\overline{\pi} = \overline{q}/(1 - \overline{q})$. Note that $0 < \overline{q} < 1/2$ by Assumption 2.

At $q = 0$, $\Pi(u; q) = 0$ for any $u \leq 0$, and hence $\Pi(\underline{u}; q) = 0 < \Pi(\overline{u}(q); q)$. At any $q \in [\overline{q}, 1]$, we have $\overline{u}(q) = 0$, and hence $\Pi(\underline{u}; q) > \Pi(0; q) = \Pi(\overline{u}(q); q)$. At any $q \in (0, \overline{q})$, $\Pi(\underline{u}; q)$ is linear in q, while $\Pi(\overline{u}(q); q)$ is convex in q:

$$\frac{d\Pi(\overline{u}(q); q)}{dq} = \frac{\partial\Pi(\overline{u}(q); q)}{\partial q} = (\overline{\pi} - \overline{u}(q))[1 - F(\overline{u}(q))],$$

[26] If $q = 0$, $\Pi(u; q) \equiv 0$ for any $u \leq 0$, and hence $\underline{u}$ remains a maximizer.

which is increasing in q. Therefore, $\Pi(\underline{u};\cdot)$ crosses $\Pi(\overline{u}(\cdot);\cdot)$ once and only once, and crosses from below. Let $q^* \in (0, \overline{q})$ denote the unique solution of $\Pi(\underline{u};q) = \Pi(\overline{u}(q);q)$.

Any equilibrium must have $q = q^*$. Indeed, if $q < q^*$ in equilibrium, $\overline{u}(q) > 0$ will be the unique maximizer of $\Pi(u;q)$, leading to $q = 1$ as the consumer's best response, a contradiction. Similarly, if $q > q^*$ in equilibrium, $\underline{u} < 0$ will be the unique maximizer of $\Pi(u;q)$, leading to $q = 0$ as the consumer's best response, a contradiction again. At $q = q^*$, the monopolist is indifferent between offering utility $\underline{u}$ and $\overline{u}(q^*)$. In equilibrium it must randomize between these two in a way that makes the consumer willing to randomize between purchasing and not purchasing when he cannot discern the offered utility and his reservation utility 0.

Let α be the probability that the monopolist offers the bad deal $\underline{u}$ in equilibrium. Conditional on the event that the consumer cannot discern the offered utility and his reservation utility 0, the conditional expected utility of the offered deal is

$$\frac{\alpha\left[1-F\left(-\underline{u}\right)\right]\underline{u}+(1-\alpha)\left[1-F\left(\overline{u}(q^*)\right)\right]\overline{u}(q^*)}{\alpha\left[1-F\left(-\underline{u}\right)\right]+(1-\alpha)\left[1-F\left(\overline{u}(q^*)\right)\right]}.$$

In order for the consumer to be indifferent between purchasing and not purchasing, this conditional expected utility must be the same as his reservation utility 0, or equivalently,

$$\alpha=\frac{\overline{u}(q^*)\left[1-F\left(\overline{u}(q^*)\right)\right]}{\left(-\underline{u}\right)\left[1-F\left(-\underline{u}\right)\right]+\overline{u}(q^*)\left[1-F\left(\overline{u}(q^*)\right)\right]}.$$

The consumer obtains strictly positive surplus in expectation because he obtains his reservation utility 0 either when he feels that the offered utility is discernibly lower than 0, or when he cannot discern the two, yet with strictly positive probability (more precisely, with probability $(1-\alpha)F\left(\overline{u}(q^*)\right) > 0$) he obtains a strictly positive surplus of $\overline{u}(q^*) > 0$. □

Proof of Proposition 2. Suppose $F^\dagger$ is a distribution that also satisfies Assumptions 1 and 2, and dominates F in the FOSD sense. Let's write the distribution explicitly as an argument of the profit function. Then $\Pi(u;q,F^\dagger) > \Pi(u;q,F)$ for any $q > 0$ and any $u < 0$ (this is because when the monopolist offers a utility lower than the consumer's reservation utility 0, it will fare better if the consumer is less able to discern these two utilities and hence cannot tell for sure that this is a bad deal), and hence $\Pi(\underline{u}^\dagger;q,F^\dagger) > \Pi(\underline{u};q,F)$ for any $q > 0$, where $\underline{u}^\dagger$ is the unique maximizer of $\Pi(u;q,F^\dagger)$ in the sub-range $u \in (-\infty, 0]$.

On the other hand, $\Pi(u;q,F^\dagger) < \Pi(u;q,F)$ for any q and any $u > 0$ (this is because when the monopolist offers a utility higher than the consumer's reservation utility 0, it will fare worse if the consumer is less able to discern these two utilities and hence cannot appreciate this good deal), and hence $\Pi(\overline{u}^\dagger(q);q,F^\dagger) < \Pi(\overline{u}(q);q,F)$ for any q, where $\overline{u}^\dagger(q)$ is the unique maximizer of $\Pi(u;q,F^\dagger)$ in the sub-range $u \in [0, \overline{\pi}]$.

Recall that $\Pi\left(\underline{u};\cdot,F\right)$ and $\Pi\left(\overline{u}(\cdot);\cdot,F\right)$ are both increasing functions of q, with the former crossing the latter once and only once and from below at some $q^* \in (0, 1/2)$. Similarly $\Pi\left(\underline{u};\cdot,F^\dagger\right)$ and $\Pi\left(\overline{u}(\cdot);\cdot,F^\dagger\right)$ are both increasing functions of q, with the former crossing the latter once and only once and from below at some $q^\dagger \in (0, 1/2)$. The facts that $\Pi\left(\underline{u};\cdot,F\right)$ lies pointwise below $\Pi\left(\underline{u};\cdot,F^\dagger\right)$ and that $\Pi\left(\overline{u}(\cdot);\cdot,F\right)$ lies pointwise above $\Pi\left(\overline{u}(\cdot);\cdot,F^\dagger\right)$ hence imply $q^* \geq q^\dagger$. The monopolist's equilibrium profit under distribution $F^\dagger$ is hence

$$\begin{aligned}\Pi\left(\overline{u}^\dagger(q^\dagger);q^\dagger,F^\dagger\right) &\leq \Pi\left(\overline{u}^\dagger(q^\dagger);q^\dagger,F\right)\\ &\leq \Pi\left(\overline{u}^\dagger(q^\dagger);q^*,F\right)\end{aligned}$$

$$\leq \Pi\left(\overline{u}(q^*); q^*, F\right),$$

where the first inequality follows from the fact that the monopolist benefits from the higher probability that the consumer can appreciate its good deal, the second inequality from the fact that $\Pi(u; q, F)$ is strictly increasing in q for any $u < \overline{\pi}$, and the third inequality from the optimality of $\overline{u}(q^*)$ given q^* and F. This proves that the monopolist's equilibrium profit decreases with an FOSD shift in F. □

Proof of Proposition 3. Start with any arbitrary distribution F that satisfies Assumptions 1 and 2. By Proposition 1 the consumer's expected surplus is strictly positive. Construct an *increasing* sequence of distributions $\{F_n\}_{n\geq 0}$ satisfying Assumptions 1 and 2 such that $F_0 = F$ and $f_n(0) \nearrow 1/\overline{\pi}$. Note that, along this sequence, we have F_n dominates F_{n+1} in the FOSD sense for all n. We shall prove that, for n sufficiently large, the consumer's expected surplus in an economy featuring F_n is lower than that in an economy featuring F. To this end, it suffices to prove that the consumer's expected surplus converges to 0 as $n \to \infty$.

Let $\overline{u}_n^*$, $\underline{u}_n^*$, and q_n^* be the corresponding equilibrium variables in an economy featuring F_n. Recall that $\underline{u}_n^*$ maximizes $\Pi_n\left(u; q_n^*\right) = (\overline{\pi} - u)[1 - F_n(-u)]q_n^*$ over the sub-range $u \in (-\infty, 0]$, and that $q_n^* > 0$. Therefore, $\underline{u}_n^*$ solves the following first-order condition:

$$\overline{\pi} - \underline{u}_n^* = \frac{1 - F_n\left(-\underline{u}_n^*\right)}{f_n\left(-\underline{u}_n^*\right)} \leq \frac{1 - F_n(0)}{f_n(0)} = \frac{1}{f_n(0)} \searrow \overline{\pi},$$

where the inequality follows from Assumption 1. We hence have $\underline{u}_n^* \nearrow 0$, which implies

$$\lim_{n\to\infty} \Pi_n\left(\underline{u}_n^*; q\right) = q\overline{\pi}.$$

We next prove that $q_n^* \nearrow 1/2$. Recall from the proof of Proposition 1 that $\Pi_n\left(\underline{u}_n^*; \cdot\right)$ crosses $\Pi_n\left(\overline{u}_n(\cdot); \cdot\right)$ from below at $q_n^* < 1/2$. Therefore, it suffices to prove that, for any $q < 1/2$, $\Pi_n\left(\underline{u}_n^*; q\right) < \Pi_n\left(\overline{u}_n(q); q\right)$ for n sufficiently large.

Recall from the proof of Proposition 1 that $\overline{u}_n(q) > 0$ for all $q < \overline{q}_n$, where $\overline{q}_n$ is the unique solution of $f_n(0)\overline{\pi} = \overline{q}_n / \left(1 - \overline{q}_n\right)$. Apparently $\overline{q}_n \nearrow 1/2$. Therefore, for any $q < 1/2$, we have $\overline{u}_n\left(q\right) > 0$ for n sufficiently large. Moreover, for such q and n, we have

$$\Pi_n\left(\overline{u}_n(q); q\right) > \Pi_n(0; q) = q\overline{\pi} = \lim_{m\to\infty} \Pi_m(\underline{u}_m^*; q),$$

where the inequality follows from the fact that $\overline{u}_n(q)$ is the unique maximizer of $\Pi_n(\cdot; q)$ in the sub-range $u \in [0, \overline{\pi}]$. For any $m > n$, $\Pi_m\left(\overline{u}_m(q); q\right) \geq \Pi_n\left(\overline{u}_n(q); q\right)$ (recall the proof of Proposition 2). Therefore, we have

$$\lim_{m\to\infty} \Pi_m\left(\overline{u}_m(q); q\right) \geq \Pi_n\left(\overline{u}_n(q); q\right) > \lim_{m\to\infty} \Pi_m(\underline{u}_m^*; q),$$

and hence we have $\Pi_m\left(\overline{u}_m(q); q\right) > \Pi_m\left(\underline{u}_m^*; q\right)$ for m sufficiently large. Since $q < 1/2$ is arbitrary, we hence have $q_n^* \nearrow 1/2$ as claimed.

Finally, we prove that $\overline{u}_n^* \searrow 0$. Since $\overline{u}_n^* = \overline{u}_n\left(q_n^*\right)$, it maximizes $\Pi_n\left(u; q_n^*\right) = (\overline{\pi} - u)\left(F_n(u) + [1 - F_n(u)]q_n^*\right)$ over the sub-range $u \in [0, \overline{\pi}]$, and hence solves the following first-order condition:

$$\overline{\pi} - \overline{u}_n^* = \frac{F_n\left(\overline{u}_n^*\right)}{f_n\left(\overline{u}_n^*\right)} + \frac{q_n^*}{\left(1 - q_n^*\right) f_n\left(\overline{u}_n^*\right)} > \frac{q_n^*}{\left(1 - q_n^*\right) f_n\left(\overline{u}_n^*\right)} \geq \frac{q_n^*}{\left(1 - q_n^*\right) f_n(0)}.$$

Taking limit on both sides, we have

$$\lim_{n\to\infty} \overline{\pi} - \overline{u}_n^* \geq \lim_{n\to\infty} \frac{q_n^*}{\left(1-q_n^*\right) f_n(0)} = \overline{\pi},$$

and hence $\overline{u}_n^* \searrow 0$ as claimed.

That the consumer's expected surplus converges to 0 as $n \to \infty$ now follows from $\overline{u}_n^* \searrow 0$.

Similarly, we can construct a *decreasing* sequence of distributions $\{F_n\}_{n\geq 0}$ satisfying Assumptions 1 and 2 such that $F_0 = F$ and $F_n(\overline{\pi}) \searrow 0$. Note that, along this sequence, we have F_n dominates F_{n-1} in the FOSD sense for all n. As explained in the main text, the consumer's expected surplus converges to 0 as $n \to \infty$, and hence the consumer's expected surplus in an economy featuring F_n is lower than that in an economy featuring F for n large enough. □

Appendix B. Omitted proofs in Section 4

We first prove three lemmas that will be used in the proofs of both Proposition 4 and Proposition 6. Define

$$\mathbf{U} := \{(u_1, u_2) \mid u_1 \geq 0, u_2 = -u_1\}.$$

Lemma 2. *Consider the case of a monopolist marketing up to two different deals. Suppose the consumer's strategy (q_s, q_d) is such that $q_s = 0$. Then, for every (u_1, u_2) such that $u_1 > 0$ and $u_1 \geq u_2 > -u_1$, there exists $(u_1', u_2') \in \mathbf{U}$ such that the monopolist makes strictly higher profit (i.e., $\Pi(u_1, u_2) < \Pi(u_1', u_2')$).*

Proof. Fix any $u_1 > 0$. If the monopolist is to offer $u_2 \in (-u_1, 0]$, its profit will be

$$\begin{aligned}\Pi\left(u_1, u_2\right) &= \left(\overline{\pi} - u_1\right) F\left(u_1 - u_2\right) + q_d \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)\left[1 - F\left(u_1 - u_2\right)\right] \\ &< \left(\overline{\pi} - \frac{u_1 - u_2}{2}\right) F(u_1 - u_2) + q_d\left(\overline{\pi} - 0\right)\left[1 - F(u_1 - u_2)\right] \\ &= \Pi\left(\frac{u_1 - u_2}{2}, -\frac{u_1 - u_2}{2}\right),\end{aligned}$$

where the strict inequality follows from $u_1 > \left(u_1 - u_2\right)/2 > 0$.

If the monopolist is to offer $u_2 \in (0, u_1/2]$, its profit will be

$$\Pi(u_1, u_2) = \left(\overline{\pi} - u_1\right) F\left(u_1\right) + q_d \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)\left[1 - F\left(u_1\right)\right],$$

which is weakly decreasing in u_2 (strictly so if $q_d > 0$), and hence according to the last paragraph is also strictly worse than $(u_1', u_2') = (u_1/2, -u_1/2) \in \mathbf{U}$.

If the monopolist is to offer $u_2 \in (u_1/2, u_1]$, its profit will be

$$\begin{aligned}\Pi(u_1, u_2) = &\left(\overline{\pi} - u_1\right) F(u_1 - u_2) \\ &+ \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)\left[F(u_2) - F(u_1 - u_2)\right] \\ &+ \left(\overline{\pi} - u_1\right)\left[F(u_1) - F(u_2)\right]\end{aligned}$$

$$
\begin{aligned}
&+ q_d \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(u_1)] \\
&\leq \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)(F(u_1) + q_d [1 - F(u_1)]).
\end{aligned}
$$

However, if the monopolist is to offer $\left(u_1', u_2'\right) = (u_2, -u_2) \in \mathbf{U}$, its profit will be

$$
\begin{aligned}
\Pi(u_1', u_2') &= \Pi(u_2, -u_2) \\
&= (\overline{\pi} - u_2) F(2u_2) + q_d \overline{\pi}[1 - F(2u_2)] \\
&> \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right) F(2u_2) + q_d \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(2u_2)] \\
&> \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)(F(u_1) + q_d [1 - F(u_1)]) \\
&\geq \Pi(u_1, u_2),
\end{aligned}
$$

where the second inequality follows from $2u_2 > u_1$. □

Lemma 3. *Consider the case of a monopolist marketing up to two different deals. Suppose the consumer's strategy (q_s, q_d) is such that $q_s = 0$. Suppose, furthermore, either $q_d = 0$ or $f(0)\overline{\pi} \geq 1/2$. Then, for every (u_1, u_2) such that $u_1 > 0$ and $u_2 < -u_1$, there exists $(u_1', u_2') \in \mathbf{U}$ such that the monopolist makes weakly higher profit (i.e., $\Pi(u_1, u_2) \leq \Pi(u_1', u_2')$).*

Proof. Fix any $u_1 > 0$. If the monopolist is to offer $u_2 < -u_1$, its profit will be

$$
\begin{aligned}
\Pi(u_1, u_2) = &(\overline{\pi} - u_1) F(u_1) \\
&+ (\overline{\pi} - u_1)[F(u_1 - u_2) - F(-u_2)] \\
&+ q_d \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(u_1 - u_2)].
\end{aligned}
$$

Partial-differentiating $\Pi(u_1, u_2)$ wrt u_2, we have

$$
\begin{aligned}
\frac{\partial \Pi(u_1, u_2)}{\partial u_2} &= (\overline{\pi} - u_1)[-f(u_1 - u_2) + f(-u_2)] \\
&\quad + q_d \left[-\frac{1 - F(u_1 - u_2)}{2} + \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right) f(u_1 - u_2)\right] \\
&\geq \frac{q_d f(u_1 - u_2)}{2}\left[-\frac{1 - F(u_1 - u_2)}{f(u_1 - u_2)} + 2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)\right] \\
&\geq \frac{q_d f(u_1 - u_2)}{2}\left[-\frac{1}{f(0)} + 2\overline{\pi}\right] \\
&\geq 0,
\end{aligned}
$$

where the first and the second inequalities follow from Assumption 1, and the third inequality follows from the supposition that either $q_d = 0$ or $f(0)\overline{\pi} \geq 1/2$. Therefore, (u_1, u_2) is weakly worse than $(u_1', u_2') = (u_1, -u_1) \in \mathbf{U}$. □

Lemma 4. *Consider the case of a monopolist marketing up to two different deals. Consider the following constrained maximization problem: the monopolist is to maximize its profit by setting*

(u_1, u_2), subject to the constraints that $(u_1, u_2) \in \mathbf{U}$, and given the consumer's strategy (q_s, q_d). The monopolist's problem has a unique solution that depends only on q_d but not on q_s. There exists a strictly decreasing function $u_1(\cdot)$, with $u_1(0) < \overline{\pi}$ and $u_1(1) = 0$, such that, for any $q_d \in [0, 1]$, the monopolist's unique solution is $u_1 = u_1(q_d)$ and $u_2 = -u_1(q_d)$.

Proof. Fix any (q_s, q_d). For any $(u_1, u_2) \in \mathbf{U}$, the monopolist's profit is

$$\Pi(u_1, -u_1) = (\overline{\pi} - u_1)F(2u_1) + q_d\overline{\pi}[1 - F(2u_1)],$$

which apparently depends only on q_d but not on q_s.

Total-differentiating $\Pi(u_1, -u_1)$ wrt u_1, we have

$$\begin{aligned}\frac{d\Pi(u_1, -u_1)}{du_1} &= -F(2u_1) + 2(\overline{\pi} - u_1)f(2u_1) - 2q_d\overline{\pi}f(2u_1)\\ &= f(2u_1)\left[-\frac{F(2u_1)}{f(2u_1)} + 2(\overline{\pi} - u_1) - 2q_d\overline{\pi}\right].\end{aligned} \tag{1}$$

By Assumption 1, F/f is weakly increasing, and hence $-F(2u_1)/f(2u_1)$ is weakly decreasing in u_1. Therefore, the term inside the square brackets is strictly decreasing in u_1. This shows that $\Pi(u_1, -u_1)$ is strictly quasi-concave in u_1, and has at most one maximizer, denoted by $u_1(q_d)$.

Since the term inside the square brackets is strictly positive (unless $q_d = 1$) at $u_1 = 0$ and strictly negative at $u_1 = \overline{\pi}$, we have $u_1(q_d) \in (0, \overline{\pi})$ for all $q_d \in [0, 1)$. As for $q_d = 1$, the term inside the square brackets is strictly negative at any $u_1 > 0$ and is 0 at $u_1 = 0$. Therefore, we have $u_1(1) = 0$.

Finally, since an increase in q_d strictly decreases the term inside the square brackets, $u_1(\cdot)$ is strictly decreasing in q_d. □

Proof of Proposition 4. If the monopolist always offers utilities that are mirror images of each other around 0, then $q_d = 0$ is apparently a best response of the consumer contingent on the all-tied case. Moreover, the single-contender case will never arise, and hence we are free to specify the consumer's (off-equilibrium) belief contingent on such an event. In particular, one possible (off-equilibrium) belief contingent on the single-contender case is that u_1 is strictly negative, while at the same time the consumer's type d is strictly larger than $-u_1$. Against such a belief, $q_s = 0$ is the consumer's best response contingent on the single-contender case.

On the other hand, if $q_s = q_d = 0$, the consumer will never make a purchase unless he finds at least one deal offering strictly positive utility. The monopolist will then make 0 sales if it offers $u_1 \le 0$ (recall that $u_2 \le u_1$ by our convention). Therefore, the monopolist's optimal strategy must have $u_1 > 0$. By Lemmas 2 and 3, for every (u_1, u_2) such that $u_1 > 0$ and $u_1 \ge u_2 \ne -u_1$, there exists $(u_1', u_2') \in \mathbf{U}$ such that the monopolist makes weakly higher profit. Therefore, solutions of the constrained maximization problem described in Lemma 4 are also the monopolist's *un*constrained best responses. By Lemma 4, the constrained maximization problem admits a unique solution, namely $u_1^* = u_1(0)$ and $u_2^* = -u_1(0)$, where $u_1(0) \in (0, \overline{\pi})$. □

We prove Proposition 6 through a series of lemmas.

Lemma 5. *In the case of a monopolist marketing up to two different deals, in any seller-pure-strategy equilibrium, $(u_1^* + u_2^*)/2 \le 0$.*

Proof. Suppose not. Then the monopolist's equilibrium profit is bounded from above by $\overline{\pi} - (u_1^* + u_2^*)/2 < \overline{\pi}$, because if the consumer purchases deal 2 with positive probability he must also purchase deal 1 with the same probability.

Note that in equilibrium the consumer knows the monopolist's strategy. Therefore, $q_d^* = 1$, as the consumer's unique best response is to purchase either deal randomly when he finds himself in the all-tied case. Given $q_d^* = 1$, however, the monopolist can profit from deviating to $u_1 = u_2 = 0$, because its profit will increase to $\overline{\pi}$, a contradiction. □

Lemma 6. *In the case of a monopolist marketing up to two different deals, in any seller-pure-strategy equilibrium, if $(u_1^* + u_2^*)/2 < 0$, then $u_1^* = 0$, $u_2^* = -\infty$, and $q_s^* > 0$.*

Proof. Suppose there is a seller-pure-strategy equilibrium where $(u_1^* + u_2^*)/2 < 0$. Then $q_d^* = 0$ (when the consumer finds himself in the all-tied case, he rationally refrains from purchasing any deal). Apparently $u_1^* \geq 0$, otherwise we would have had $q_s^* = 0$ as well, and the monopolist's equilibrium profit would have been 0, and would have profited strictly from deviating to, say, $u_1 = u_2 = \overline{\pi}/2$.

We claim that, if $q_s^* = 1$, then $u_2^* = -\infty$. The presumption of $(u_1^* + u_2^*)/2 < 0$ implies $u_2^* < -u_1^*$. For any $u_2 < -u_1^*$, given $q_s^* = 1$ and $q_d^* = 0$, the monopolist's profit is

$$\Pi(u_1^*, u_2) = (\overline{\pi} - u_1^*) F(u_1^* - u_2),$$

which is maximized by setting $u_2 = -\infty$.

Suppose $u_1^* > 0$. Then $q_s^* = 1$ indeed (when the consumer finds himself in the single-contender case, he would know for sure that the only contender is a good deal). Then, according to the claim in the above paragraph, we have $u_2^* = -\infty$. However, by setting $u_1 > 0$ and $u_2 = -\infty$, given $q_s^* = 1$, the monopolist's profit is $\overline{\pi} - u_1$ (the consumer will purchase deal 1 for sure regardless whether he can discern u_1 and 0), which is strictly decreasing in u_1. Hence the only possible candidate for u_1^* is 0.

It remains to prove that $u_2^* = -\infty$ given that $u_1^* = 0$. Note that $q_s^* = 0$ is not an equilibrium, otherwise the monopolist"s equilibrium profit would have been 0, and would have profited strictly from deviating to, say, $u_1 = u_2 = \overline{\pi}/2$. For any $q_s^* > 0$, the monopolist's profit is

$$\Pi(u_1^*, u_2) = \Pi(0, u_2) = q_s^* \overline{\pi} F(-u_2),$$

which is maximized by setting $u_2 = -\infty$. □

Corollary 1. *In the case of a monopolist marketing up to two different deals, in any seller-pure-strategy equilibrium, $(u_1^* + u_2^*)/2 = 0$.*

Proof. By Lemma 5, it suffices to show that $(u_1^* + u_2^*)/2 < 0$ is impossible. By Lemma 6, if $(u_1^* + u_2^*)/2 < 0$, then $u_1^* = 0$, $u_2^* = -\infty$, and $q_s^* > 0$. Given $u_2^* = -\infty$ and $q_s^* > 0$, the monopolist's profit from setting any $u_1 \leq 0$ is

$$\Pi(u_1, u_2^*) = \Pi(u_1, -\infty) = q_s^* (\overline{\pi} - u_1) [1 - F(-u_1)].$$

By the same argument as in the proof of Proposition 1, there exists some $\underline{u} < 0$ such that $\Pi\left(\underline{u}, -\infty\right) > \Pi(0, -\infty) = \Pi(u_1^*, u_2^*)$, a contradiction. □

Lemma 7. *Consider the case of a monopolist marketing up to two different deals. Consider the following constrained maximization problem: the monopolist is to maximize its profit by* *offering* (u_1, u_2)*, subject to* $u_2 \le u_1 \le 0$*, and given the consumer's strategy* (q_s, q_d) *where* $q_s = 0$*. A solution of the monopolist's problem is* $(u_1, u_2) = (\underline{u}, \underline{u})$*, where* $\underline{u} < 0$ *is as defined in Proposition 1.*

Proof. For any (u_1, u_2) such that $u_2 \le u_1 \le 0$, the monopolist's profit is

$$\begin{aligned}\Pi(u_1, u_2) &= q_d\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(-u_2)] \\ &\le q_d(\overline{\pi} - u_2)[1 - F(-u_2)] \\ &\le q_d\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right] \\ &= \Pi\left(\underline{u}, \underline{u}\right),\end{aligned}$$

where the second inequality follows from the definition of $\underline{u}$ in the proof of Proposition 1. □

Proof of Proposition 6. By Corollary 1, all seller-pure-strategy equilibria resemble the one described in Proposition 4, in the sense that $(u_1^* + u_2^*)/2 = 0$. Therefore, if (q_s^*, q_d^*) is the consumers' equilibrium strategy, the monopolist's equilibrium strategy must also solve the constrained maximization problem described in Lemma 4. That is, any seller-pure-strategy equilibrium must take the form of $(u_1^*, u_2^*, q_s^*, q_d^*) = \left(u_1(q_d^*), -u_1(q_d^*), q_s^*, q_d^*\right)$.

Apparently, if the monopolist is to set (u_1, u_2) such that $(u_1 + u_2)/2 = 0$, any (q_s, q_d) would be the consumer's best response (see the proof of Proposition 4). Therefore, a candidate equilibrium $(u_1^*, u_2^*, q_s^*, q_d^*) = \left(u_1(q_d^*), -u_1(q_d^*), q_s^*, q_d^*\right)$ is a valid equilibrium as long as (u_1^*, u_2^*) is also the monopolist's *un*constrained optimal choice given (q_s^*, q_d^*).

Note that, if $\left(u_1(q_d^*), -u_1(q_d^*), q_s^*, q_d^*\right)$ is a seller-pure-strategy equilibrium, then $\left(u_1(q_d^*), -u_1(q_d^*), 0, q_d^*\right)$ will also be a seller-pure-strategy equilibrium. This is because (i) the single-contender case is an off-equilibrium event, and any q_s can be supported by some pessimistic enough off-equilibrium beliefs (see the proof of Proposition 4), and (ii) lowering q_s to 0 does not affect the monopolist's equilibrium profit, but weakly lowers its profit if it were to deviate. In what follows we shall hence focus on equilibria of the form $(u_1^*, u_2^*, q_s^*, q_d^*) = \left(u_1(q_d^*), -u_1(q_d^*), 0, q_d^*\right)$.

Let $\mathbf{Q_d}$ denote the compact[27] set of q_d's such that $(u_1(q_d), -u_1(q_d), 0, q_d)$ is a seller-pure-strategy equilibrium. From Proposition 4 we know that $\mathbf{Q_d}$ is not empty, and contains the point 0.

For any $q_d, q_d' \in \mathbf{Q_d}$ such that $q_d < q_d'$, the monopolist's equilibrium profit is

$$\begin{aligned}\Pi^*(q_d) &= (\overline{\pi} - u_1(q_d))\, F\,(2u_1(q_d)) + q_d\overline{\pi}\,[1 - F\,(2u_1(q_d))] \\ &< (\overline{\pi} - u_1(q_d))\, F\,(2u_1(q_d)) + q_d'\overline{\pi}\,[1 - F\,(2u_1(q_d))] \\ &\le \left(\overline{\pi} - u_1(q_d')\right) F\left(2u_1(q_d')\right) + q_d'\overline{\pi}\left[1 - F\left(2u_1(q_d')\right)\right] \\ &= \Pi^*(q_d'),\end{aligned}$$

where the second inequality follows from the definition of $u_1(\cdot)$. Therefore, different equilibria are rankable in terms of the monopolist's equilibrium profit, with higher q_d^* implying higher profit.

[27] Compactness follows from the usual continuity argument.

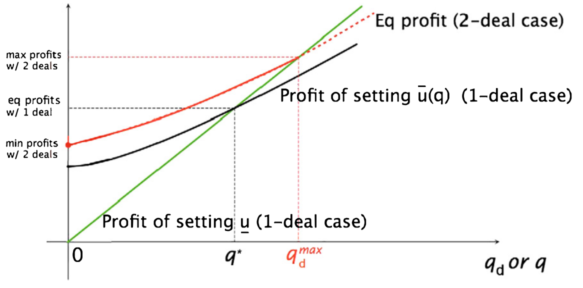

Fig. 4. Equilibrium profits in the one-deal and two-deal cases

On the other hand, in any seller-pure-strategy equilibrium, the consumer gets a positive surplus only when he has a type $d < 2u_1(q_d^*)$, which enables him to a-discern $u_1(q_d^*)$ and 0 and hence identify deal 1 out of the two deals. His expected surplus is hence

$$\overline{CS} = u_1(q_d^*)F\left(2u_1(q_d^*)\right), \tag{2}$$

which is strictly decreasing in q_d^*. Therefore, different equilibria are also rankable in terms of the consumer's expected surplus, with higher q_d^* implying lower expected surplus for the consumer.

Let $q_d^{max} := \sup \mathbf{Q_d}$. We shall now prove that $q_d^{max} < 1$.

Recall from the proof of Proposition 1 that $\underline{u}$ is the unique solution of $\max_{u \le 0}(\overline{\pi} - u)[1 - F(-u)]$. Since $\underline{u} < 0$, we have

$$\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right] > \overline{\pi}.$$

Suppose $q_d^{max} = 1$, then by compactness of $\mathbf{Q_d}$ there is a seller-pure-strategy equilibrium featuring $q_d^* = 1$. In such an equilibrium, by Lemma 4, we must have $u_1^* = u_2^* = 0$. The monopolist's equilibrium profit is hence $\overline{\pi}$.

If the monopolist deviates to $u_1 = u_2 = \underline{u}$, its profit will increase to $\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right]$ thanks to $q_d^* = 1$, a contradiction.

In the remainder of this proof, suppose $f(0)\overline{\pi} \ge 1/2$. Then, by Lemmas 2, 3, 4, and 7, when the consumer's strategy (q_s, q_d) is such that $q_s = 0$, the monopolist's profit is maximized either at $(u_1, u_2) = (u_1(q_d), -u_1(q_d))$, or at $(u_1, u_2) = \left(\underline{u}, \underline{u}\right)$.

In Fig. 4, we plot the monopolist's profits at each of these two candidate maximizers as functions of q_d. The profit at $(u_1, u_2) = \left(\underline{u}, \underline{u}\right)$ is depicted by the straight line passing through the origin. It is the same straight line in Fig. 1, with q_d replacing q in the expression of $\Pi = q\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right]$.

The profit at $(u_1, u_2) = (u_1(q_d), -u_1(q_d))$ is depicted by the upper convex curve, and has the expression of $\Pi = (\overline{\pi} - u_1(q_d))\, F\,(2u_1(q_d)) + q_d\overline{\pi}\,[1 - F\,(2u_1(q_d))]$. That it is strictly increasing and convex can be seen by totally differentiating it wrt q_d using the Envelope Theorem, yielding

$$\frac{d\Pi}{dq_d} = \overline{\pi}\,[1 - F\,(2u_1(q_d))]\,,$$

which is strictly positive and strictly increasing in q_d.

The convex curve is strictly above the straight line at $q_d = 0$, and is strictly below at $q_d = 1$ (recall that $u_1(1) = 0$). The shapes of the two profit functions dictate that the convex curve crosses the straight line once and only once, and crosses from above, at some $q_d^{max} \in (0, 1)$. Therefore, the candidate equilibrium $(u_1^*, u_2^*, q_s^*, q_d^*) = (u_1(q_d), -u_1(q_d), 0, q_d)$ is a valid equilibrium if and only if $q_d \in \left[0, q_d^{max}\right]$.

We also superimpose onto Fig. 4 the monopolist's profit if it offers the good deal $\overline{u}(q)$ in the one-deal case. It is depicted by the lower convex curve. Indeed, it is strictly below the upper convex curve at every $q < 1$, because it has the expression of

$$\begin{aligned}\Pi &= (\overline{\pi} - \overline{u}(q))\,(F\,(\overline{u}(q)) + q\,[1 - F\,(\overline{u}(q))]) \\ &\leq (\overline{\pi} - \overline{u}(q))\,(F\,(2\overline{u}(q)) + q\,[1 - F\,(2\overline{u}(q))]) \\ &\leq (\overline{\pi} - \overline{u}(q))\,F\,(2\overline{u}(q)) + q\overline{\pi}\,[1 - F\,(2\overline{u}(q))] \\ &\leq (\overline{\pi} - u_1(q))\,F\,(2u_1(q)) + q\overline{\pi}\,[1 - F\,(2u_1(q))]\end{aligned}$$

where the first two inequalities are strict if $\overline{u}(q) > 0$, and the third inequality is strict if $\overline{u}(q) = 0 \neq u_1(q)$, and hence at least one of these inequalities is strict for every $q < 1$. Note that the last expression is exactly the same as the monopolist's profit at $(u_1, u_2) = (u_1(q_d), -u_1(q_d))$ if we replace q with q_d, which proves that the upper and the lower convex curves touch only at $q = 1$.

It becomes apparent from Fig. 4 that $q^* < q_d^{max}$, and that in the seller-pure-strategy equilibrium featuring $q_d^* = q_d^{max}$, the monopolist's profit is higher than its (unique) equilibrium profit in the one-deal case. □

Appendix C. Omitted proofs in Section 6

We prove Proposition 8 through a series of lemmas.

Lemma 8. *In the case of two sellers marketing up to only one deal each, in any seller-pure-strategy equilibrium, if $u_1^* = u_2^* = u^*$, then $u^* > 0$.*

Proof. Suppose $u^* < 0$. Then we must have $q_d^* = 0$ (when the consumer finds himself in the all-tied case, he would rationally refuse to purchase either brand), resulting in 0 equilibrium profit for both sellers. Seller 1, for example, can profit from deviating to $u_1 = \overline{\pi}/2 > 0$. After such deviation, seller 1 can make sales with probability at least $F(\overline{\pi}/2)$, a contradiction.

Suppose $u^* = 0$. Then $u_1 = 0$ must be a best response against $u_2^* = 0$. When $u_2^* = 0$, seller 1's profit as a function of u_1 is

$$\Pi_1(u_1, 0) = \begin{cases} (\overline{\pi} - u_1)[1 - F(-u_1)]q_d/2 & \text{if } u_1 \leq 0 \\ (\overline{\pi} - u_1)\,(F(u_1) + [1 - F(u_1)]q_d/2) & \text{if } u_1 \geq 0 \end{cases},$$

which is exactly the same profit function as that in the one-deal case (with q replaced by $q_d/2$, and u replaced by u_1). However, from the proof of Proposition 1, we know that $u_1 = 0$ is never a best response regardless of $q_d/2$, a contradiction. □

Lemma 9. *In the case of two sellers marketing up to only one deal each, in any seller-pure-strategy equilibrium, we have $u_2^* < u_1^* < \overline{\pi}$.*

Proof. By Lemma 8, it suffices to prove that there is no seller-pure-strategy equilibrium with $u_1^* = u_2^* = u^* > 0$. Suppose, on the contrary, such an equilibrium exists. Then we must have $q_d^* = 1$ (when the consumer finds himself in the all-tied case, he can guarantee a strictly positive surplus of u^* by purchasing randomly from one of the two sellers). Note that, since the single-contender case is an off-equilibrium event, any q_s^* can be supported by some off-equilibrium belief. It is wlog to set $q_s^* = 0$, because that leaves the sellers' equilibrium profits intact, while making their deviation profits weakly lower.

Given $(q_s^*, q_d^*) = (0, 1)$ and $u_1^* = u_2^* = u^*$, seller 1's equilibrium profit is

$$\Pi_1^* = (\overline{\pi} - u^*)/2.$$

If seller 1 deviates to $u_1 = 0$, its profit will become

$$\Pi_1 = \overline{\pi}[1 - F(u^*)]/2.$$

We shall show that $\Pi_1 > \Pi_1^*$, which will be a contradiction. Let $H(u^*) := 2(\Pi_1 - \Pi_1^*)$, and differentiate it wrt u^*, we have

$$H'\left(u^*\right) = 1 - \overline{\pi} f(u^*) \geq 1 - \overline{\pi} f(0) > 0,$$

where the first and the second inequalities follow from Assumptions 1 and 2, respectively. Therefore, $H(u^*) > H(0) = 0$ for any $u^* > 0$, and hence $\Pi_1 > \Pi_1^*$ as claimed.

If $u_1^* \geq \overline{\pi}$, seller 1 makes non-positive profit, and can strictly profit from deviating to $u_1 = (\overline{\pi} + u_2^*)/2$, a contradiction. □

Lemma 10. *In the case of two sellers marketing up to only one deal each, in any seller-pure-strategy equilibrium, we have* $(u_1^* + u_2^*)/2 \geq 0$.

Proof. By Lemma 9, we have $u_2^* < u_1^* < \overline{\pi}$. Suppose $(u_1^* + u_2^*)/2 < 0$. Then $u_2^* < 0$, and hence the only chance that seller 2 can make any sales is when the consumer finds himself in the all-tied case. However, $(u_1^* + u_2^*)/2 < 0$ also implies that when the consumer finds himself in the all-tied case, he would rationally refuse to purchase; i.e., $q_d^* = 0$. Therefore, seller 2 makes 0 equilibrium profit, and can strictly profit from deviating to $u_2 = (u_1^* + \overline{\pi})/2$, a contradiction. □

Lemma 11. *In the case of two sellers marketing up to only one deal each, in any seller-pure-strategy equilibrium, we have* $u_2^* < 0$.

Proof. By Lemma 9, we have $u_2^* < u_1^* < \overline{\pi}$. Suppose $u_2^* \geq 0$. Then $(u_1^* + u_2^*)/2 > 0$, and hence when the consumer finds himself in the all-tied case, he would rationally purchase for sure; i.e., $q_d^* = 1$.

Divide the consumer's type space into two (disjoint and exhaustive) subsets. The first subset are types who purchase from seller 1 for sure; i.e., those types in the set

$$D_1 := \left\{d \;\middle|\; d < u_1^* - u_2^*\right\} \cup \left\{d \;\middle|\; \max\{u_1^* - u_2^*, u_2^*\} \leq d < u_1^*\right\}.$$

The second subset are types who purchase from each seller with probability 1/2; i.e., those types in the set

$$D_2 := \left\{d \;\middle|\; u_1^* - u_2^* \leq d < u_2^*\right\} \cup \left\{d \;\middle|\; d \geq u_1^*\right\}.$$

Seller 1's equilibrium profit is

$$\Pi_1^* = (\overline{\pi} - u_1^*)[Pr(D_1) + Pr(D_2)/2].$$

If seller 1 deviates to $u_1 = u_2^*$, it will share the market with seller 2, resulting in profit

$$\Pi_1 = (\overline{\pi} - u_2^*)/2 = (\overline{\pi} - u_2^*)[Pr(D_1)/2 + Pr(D_2)/2].$$

Equilibrium requires that such deviation is not profitable; i.e.,

$$(\bar{\pi} - u_1^*)[Pr(D_1) + Pr(D_2)/2] \geq (\bar{\pi} - u_2^*)[Pr(D_1)/2 + Pr(D_2)/2]. \quad (3)$$

On the other hand, seller 2's equilibrium profit is

$$\Pi_2^* = (\bar{\pi} - u_2^*)Pr(D_2)/2.$$

If seller 2 deviates to $u_2 = u_1^*$, it will share the market with seller 1, resulting in profit

$$\Pi_2 = (\bar{\pi} - u_1^*)/2 = (\bar{\pi} - u_1^*)[Pr(D_1)/2 + Pr(D_2)/2].$$

Equilibrium requires that such deviation is not profitable; i.e.,

$$(\bar{\pi} - u_2^*)Pr(D_2)/2 \geq (\bar{\pi} - u_1^*)[Pr(D_1)/2 + Pr(D_2)/2]. \quad (4)$$

Adding (3) and (4), we have $u_1^* \leq u_2^*$, a contradiction. □

Lemma 12. *In the case of two sellers marketing up to only one deal each, in any seller-pure-strategy equilibrium, we have* $(u_1^*, u_2^*) = (x^*, -x^*)$*, where* $x^* > 0$ *is the unique solution to*

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + \bar{\pi}.$$

Proof. By Lemmas 10 and 11, we have $(u_1^* + u_2^*)/2 \geq 0$ and $u_2^* < 0$, which together imply $u_1^* > 0 > u_2^*$. Suppose $(u_1^* + u_2^*)/2 > 0$. Then, when the consumer finds himself in the all-tied case, he would rationally purchase for sure; i.e., $q_d^* = 1$. In addition, $(u_1^* + u_2^*)/2 > 0$ implies $u_1^* > -u_2^*$, and hence the single-contender case is off the equilibrium path. Fix any $q_s^* \in [0, 1]$. Against $u_1^* > 0$ and $(q_s^*, q_d^*) = (q_s^*, 1)$, seller 2's profit for any $u_2 < 0$ is

$$\Pi_2 = (\bar{\pi} - u_2)[1 - F(u_1^* - u_2)]/2,$$

which implies that u_2^* satisfies the FOC of

$$1 - F(u_1^* - u_2^*) = (\bar{\pi} - u_2^*)f(u_1^* - u_2^*). \quad (5)$$

Similarly, against $u_2^* < 0$ and $(q_s^*, q_d^*) = (q_s^*, 1)$, seller 1's profit for any $u_1 > -u_2^*$ is

$$\Pi_1 = (\bar{\pi} - u_1) - (\bar{\pi} - u_1)[1 - F(u_1 - u_2^*)]/2,$$

which implies that u_1^* satisfies the FOC of

$$1 - [1 - F(u_1^* - u_2^*)]/2 = (\bar{\pi} - u_1^*)f(u_1^* - u_2^*)/2. \quad (6)$$

Multiply (6) by 2, and subtract (5) from it, we have $2F(u_1^* - u_2^*) = (u_2^* - u_1^*)f(u_1^* - u_2^*) < 0$, a contradiction. This proves that $(u_1^* + u_2^*)/2 = 0$.

Recall that seller 2 must make strictly positive equilibrium profit (otherwise it can strictly profit from deviating to $u_2 = (\bar{\pi} + u_1^*)/2$), hence we must have $q_d^* > 0$ (because the only chance that seller 2 makes any sales is when the consumer finds himself in the all-tied case). Against $u_1^* > 0$ and $q_d^* > 0$, for any $u_2 < 0$, seller 2's profit is

$$\Pi_2 = (\bar{\pi} - u_2)[1 - F(u_1^* - u_2)]q_d^*/2,$$

which implies that u_2^* satisfies the same FOC as (5). Rewrite (5) as

$$\frac{1-F(2x^*)}{f(2x^*)} = x^* + \overline{\pi}, \tag{7}$$

where $x^* := u_1^* = -u_2^* > 0$. Since the LHS of (7) is weakly decreasing in x^* by Assumption 1, while the RHS is strictly increasing in x^* without bound, a solution to (7) is unique if it exists. Existence of a strictly positive solution follows from the fact that, by Assumption 2, we have the LHS strictly bigger than the RHS at $x^* = 0$. □

Proof of Proposition 8. The first half of Proposition 8 follows from Lemma 12. It remains to prove the second half.

By the first half of Proposition 8, in any seller-pure-strategy equilibrium, the consumer gets a positive surplus iff he has a type $d < 2x^*$ (in which case he can a-discern utilities and is able to identify the positive-utility deal 1). His expected surplus is hence

$$CS = x^* F(2x^*).$$

In the equilibrium described in Proposition 4 for the case of a monopolist marketing up to two different deals, by (2) in the proof of Proposition 6, the consumer's expected surplus is

$$\overline{CS} = u_1(0)F(2u_1(0)) = \overline{x}F(2\overline{x}),$$

where $\overline{x} := u_1(0)$ is the unique solution to the first-order condition

$$\frac{F(2\overline{x})}{f(2\overline{x})} = 2(\overline{\pi} - \overline{x}) \tag{8}$$

by (1) in the proof of Lemma 4 (where we have simplified using $q_d = 0$). To prove the second half of Proposition 8, it suffices to prove that $x^* < \overline{x}$.

Note that the LHS of (8) is strictly increasing and the RHS is strictly decreasing, and they are equal to each other when evaluated at $\overline{x}$. Therefore, to prove that $x^* < \overline{x}$, it suffices to prove that the LHS of (8) is strictly smaller than the RHS when they are evaluated at x^*. That is, it suffices to prove that

$$\frac{F(2x^*)}{f(2x^*)} < 2(\overline{\pi} - x^*). \tag{9}$$

Step 1: We first give a lower bound for $(\overline{\pi} - x^*)$.

Let $\Pi_1(u_1, u_2^*)$ and $\Pi_2(u_1^*, u_2)$ be sellers 1's and 2's profits as functions of their own offered utilities, where we have suppressed their dependence on the consumer's equilibrium strategy (q_s^*, q_d^*). Recall from Fig. 3 that $\Pi_1(\cdot, -x^*)$ has a kink at $u_1 = x^*$. In order for $u_1 = x^*$ to be a local optimum for $\Pi_1(u_1, -x^*)$, the left derivative at $u_1 = x^*$ must be non-negative.

Whenever $u_2^* < 0$, for any $u_1 \in [0, -u_2^*]$,

$$\begin{aligned}\Pi_1(u_1, u_2^*) &= (\overline{\pi} - u_1)\Big(F(u_1) + q_s^*\left[F(-u_2^*) - F(u_1)\right] + \left[F(u_1 - u_2^*) - F(-u_2^*)\right] \\ &\qquad + \frac{q_d^*}{2}\left[1 - F(u_1 - u_2^*)\right]\Big) \\ &= (\overline{\pi} - u_1)\Big(F(u_1 - u_2^*) + \frac{q_d^*}{2}\left[1 - F(u_1 - u_2^*)\right] \\ &\qquad - (1 - q_s^*)\left[F(-u_2^*) - F(u_1)\right]\Big).\end{aligned}$$

Therefore, for any $u_1 \in (0, -u_2^*)$,

$$\begin{aligned}\frac{\partial \Pi_1(u_1, u_2^*)}{\partial u_1} &= -\left(F(u_1 - u_2^*) + \frac{q_d^*}{2}\left[1 - F(u_1 - u_2^*)\right] - (1 - q_s^*)\left[F(-u_2^*) - F(u_1)\right]\right) \\ &\quad + (\overline{\pi} - u_1)\left[\left(1 - \frac{q_d^*}{2}\right) f(u_1 - u_2^*) + (1 - q_s^*) f(u_1)\right] \\ &\leq -\left(F(u_1 - u_2^*) + \frac{q_d^*}{2}[1 - F(u_1 - u_2^*)] - [F(-u_2^*) - F(u_1)]\right) \\ &\quad + (\overline{\pi} - u_1)\left[\left(1 - \frac{q_d^*}{2}\right) f(u_1 - u_2^*) + f(u_1)\right].\end{aligned}$$

Plugging in $u_2^* = -x^*$, local optimality of $u_1 = x^*$ hence requires that

$$\begin{aligned}0 &\leq \left.\frac{\partial \Pi_1(u_1, -x^*)}{\partial u_1}\right|_{u_1 = x^*-} \\ &\leq -\left(F(2x^*) + \frac{q_d^*}{2}\left[1 - F(2x^*)\right]\right) + \left[\left(1 - \frac{q_d^*}{2}\right) f(2x^*) + f(x^*)\right](\overline{\pi} - x^*) \\ &= -\left[F(2x^*) + \frac{q_d^*}{2} f(2x^*)(\overline{\pi} + x^*)\right] + \left[\left(1 - \frac{q_d^*}{2}\right) f(2x^*) + f(x^*)\right](\overline{\pi} - x^*) \\ &= -F(2x^*) - q_d^* f(2x^*)\overline{\pi} + \left[f(2x^*) + f(x^*)\right](\overline{\pi} - x^*),\end{aligned}$$

where the first equality follows from (7). This implies

$$q_d^* \leq \frac{[f(2x^*) + f(x^*)](\overline{\pi} - x^*) - F(2x^*)}{f(2x^*)\overline{\pi}}. \tag{10}$$

Rearranging terms, we can express (10) as a lower bound for $(\overline{\pi} - x^*)$:

$$\overline{\pi} - x^* \geq \frac{F(2x^*) + q_d^* f(2x^*)\overline{\pi}}{f(2x^*) + f(x^*)} \geq \frac{F(2x^*)}{f(2x^*) + f(x^*)}. \tag{11}$$

STEP 2: We next give an upper bound for $F(2x^*)$.
In order for $u_2 = -x^*$ to be a global optimum for $\Pi_2(x^*, u_2)$, we must have

$$\begin{aligned}&\Pi_2(x^*, -x^*) \geq \Pi_2(x^*, x^*) \\ \iff \quad &\frac{q_d^*}{2}\left[1 - F(2x^*)\right](\overline{\pi} + x^*) \geq \left(\frac{1}{2} F(x^*) + \frac{q_d^*}{2}\left[1 - F(x^*)\right]\right)(\overline{\pi} - x^*) \\ \iff &\left\{\left[1 - F(2x^*)\right](\overline{\pi} + x^*) - \left[1 - F(x^*)\right](\overline{\pi} - x^*)\right\} q_d^* \geq F(x^*)(\overline{\pi} - x^*).\end{aligned}$$

Since the RHS is strictly positive, the term in the curly brackets is strictly positive. Dividing both sides by the term in the curly brackets, we have

$$q_d^* \geq \frac{F(x^*)(\overline{\pi} - x^*)}{[1 - F(2x^*)](\overline{\pi} + x^*) - [1 - F(x^*)](\overline{\pi} - x^*)}. \tag{12}$$

Combining (10) and (12), we have

$$\frac{F(x^*)(\overline{\pi} - x^*)}{[1 - F(2x^*)](\overline{\pi} + x^*) - [1 - F(x^*)](\overline{\pi} - x^*)} \leq \frac{[f(2x^*) + f(x^*)](\overline{\pi} - x^*) - F(2x^*)}{f(2x^*)\overline{\pi}}.$$

Rearranging terms, we have

$$\begin{aligned}
F(2x^*) &\le [f(2x^*)+f(x^*)](\overline{\pi}-x^*) - \frac{F(x^*)(\overline{\pi}-x^*)f(2x^*)\overline{\pi}}{[1-F(2x^*)](\overline{\pi}+x^*)-[1-F(x^*)](\overline{\pi}-x^*)} \\
&= f(2x^*)(\overline{\pi}-x^*)\Bigg(1+\frac{f(x^*)}{f(2x^*)} \\
&\quad - \frac{F(x^*)\overline{\pi}}{[1-F(2x^*)](\overline{\pi}+x^*)-[1-F(x^*)](\overline{\pi}-x^*)}\Bigg) \\
&\le f(2x^*)(\overline{\pi}-x^*) \\
&\quad \times \Bigg(1+\frac{f(x^*)}{f(2x^*)} \\
&\quad - \frac{x^*f(x^*)\overline{\pi}}{[1-F(2x^*)](\overline{\pi}+x^*)-[1-F(2x^*)+x^*f(2x^*)](\overline{\pi}-x^*)}\Bigg) \\
&= f(2x^*)(\overline{\pi}-x^*)\left(1+\frac{f(x^*)}{f(2x^*)}-\frac{x^*f(x^*)\overline{\pi}}{2x^*[1-F(2x^*)]-x^*f(2x^*)(\overline{\pi}-x^*)}\right) \\
&= f(2x^*)(\overline{\pi}-x^*)\left(1+\frac{f(x^*)}{f(2x^*)}-\frac{f(x^*)\overline{\pi}}{2f(2x^*)(\overline{\pi}+x^*)-f(2x^*)(\overline{\pi}-x^*)}\right) \\
&= f(2x^*)(\overline{\pi}-x^*)\left[1+\frac{f(x^*)}{f(2x^*)}\left(1-\frac{\overline{\pi}}{2(\overline{\pi}+x^*)-(\overline{\pi}-x^*)}\right)\right] \\
&= f(2x^*)(\overline{\pi}-x^*)\left[1+\frac{f(x^*)}{f(2x^*)}\left(1-\frac{\overline{\pi}}{\overline{\pi}+3x^*}\right)\right],
\end{aligned}$$

where the second inequality follows from Assumption 1,[28] and the third equality follows from (7). Therefore, to prove (9), it suffices to prove that

$$\frac{f(x^*)}{f(2x^*)}\left(1-\frac{\overline{\pi}}{\overline{\pi}+3x^*}\right)<1. \tag{13}$$

STEP 3: We shall now prove (13).

By (11), we have

$$\begin{aligned}
\left[f(2x^*)+f(x^*)\right](\overline{\pi}-x^*) &\ge F(2x^*) \\
&= 1-f(2x^*)(\overline{\pi}+x^*),
\end{aligned}$$

where the equality follows from (7). Rearranging, we have

$$\begin{aligned}
1 &\le 2f(2x^*)\overline{\pi}+f(x^*)(\overline{\pi}-x^*) \\
&\le 2f(0)\overline{\pi}+f(0)\overline{\pi} \\
&= 3f(0)\overline{\pi},
\end{aligned} \tag{14}$$

where the second inequality follows from Assumption 1.

By (7), we have

$$x^* = \frac{1-F(2x^*)}{f(2x^*)}-\overline{\pi} \le \frac{1}{f(0)}-\overline{\pi},$$

[28] By Assumption 1, f is weakly decreasing. Hence $F(x^*) \ge x^* f(x^*)$ and $F(2x^*) - F(x^*) \ge x^* f(2x^*)$.

where the inequality follows from Assumption 1. Therefore,

$$\frac{x^*}{\overline{\pi}+x^*} \le \frac{1/f(0)-\overline{\pi}}{\overline{\pi}+1/f(0)-\overline{\pi}} = 1-f(0)\overline{\pi} \le \frac{2}{3}, \tag{15}$$

where the last inequality follows from (14).

By (11) again, we have

$$\overline{\pi}-x^* \ge \frac{F(2x^*)}{f(2x^*)+f(x^*)} \ge \frac{2x^* f(2x^*)}{f(2x^*)+f(x^*)},$$

where the second inequality follows from Assumption 1. Therefore,

$$\overline{\pi} \ge x^*\left(1+\frac{2f(2x^*)}{f(2x^*)+f(x^*)}\right)$$

$$\frac{x^*}{\overline{\pi}} \le \frac{f(2x^*)+f(x^*)}{3f(2x^*)+f(x^*)}. \tag{16}$$

By Assumption 1,

$$0 \le \left(\frac{f}{1-F}\right)' = \frac{f'}{1-F}+\left(\frac{f}{1-F}\right)^2 = \frac{f}{1-F}\left(\frac{f'}{f}+\frac{f}{1-F}\right)$$
$$= \frac{f}{1-F}\left((\ln f)'+\frac{f}{1-F}\right).$$

Therefore, for any $x \in [x^*, 2x^*]$,

$$\frac{d\ln f(x)}{dx} \ge -\frac{f(x)}{1-F(x)} \ge -\frac{f(2x^*)}{1-F(2x^*)},$$

where the second inequality follows from Assumption 1. Therefore,

$$\ln f(2x^*)-\ln f(x^*) = \int_{x=x^*}^{2x^*} \frac{d\ln f(x)}{dx}dx \ge -\frac{x^* f(2x^*)}{1-F(2x^*)} = -\frac{x^*}{\overline{\pi}+x^*} \ge -\frac{2}{3},$$

where the second equality follows from (7), and the last inequality follows from (15). We hence have

$$\frac{f(x^*)}{f(2x^*)} \le \exp\left(\frac{x^*}{\overline{\pi}+x^*}\right) \le \exp(2/3), \tag{17}$$

and, by (16),

$$\frac{x^*}{\overline{\pi}} \le \frac{f(2x^*)+f(x^*)}{3f(2x^*)+f(x^*)} = \frac{1+f(x^*)/f(2x^*)}{3+f(x^*)/f(2x^*)} \le \frac{1+\exp(2/3)}{3+\exp(2/3)}. \tag{18}$$

The upper bound (18) allows us to give an even tighter upper bound for $f(x^*)/f(2x^*)$ than (17):

$$\frac{f(x^*)}{f(2x^*)} \le \exp\left(\frac{x^*}{\overline{\pi}+x^*}\right) = \exp\left(\frac{x^*/\overline{\pi}}{1+x^*/\overline{\pi}}\right) \le \exp\left(\frac{1+\exp(2/3)}{4+2\exp(2/3)}\right). \tag{19}$$

Plugging (16) and (19) into the LHS of (13), we then have

$$\begin{aligned}\frac{f(x^*)}{f(2x^*)}\left(1-\frac{\overline{\pi}}{\overline{\pi}+3x^*}\right) &= \frac{f(x^*)}{f(2x^*)}\left(\frac{3x^*/\overline{\pi}}{1+3x^*/\overline{\pi}}\right)\\ &\leq \exp\left(\frac{1+\exp(2/3)}{4+2\exp(2/3)}\right)\times\left(\frac{3+3\exp(2/3)}{6+4\exp(2/3)}\right)\\ &= 0.9314\\ &< 1,\end{aligned}$$

as claimed. □

Appendix D. When Assumption 2 is violated

This appendix contains the complementary analysis for the case when Assumption 2 is violated. We start with a monopolist marketing a single deal.

Proposition 10. *Consider a monopolist marketing up to only one deal. In the case of $f(0)\overline{\pi} \geq 1$, there exists a continuum of equilibria, indexed by different $q^* \in [\overline{q}, 1]$ with some $\overline{q} \geq 1/2$, where the monopolist offers the utility $u^* = 0$, and the consumer purchases with probability q^* whenever he cannot discern the utility from the deal and his reservation utility. Among these equilibria, the most efficient one is the one with $q^* = 1$, which also achieves the first best.*

Proof. Define $\underline{u}$ and $\overline{u}(q)$ as in the proof of Proposition 1; i.e., $\underline{u}$ is the unique maximizer of $\Pi(u; q)$ in the sub-range $u \in (-\infty, 0]$, while $\overline{u}(q)$ is that in the sub-range $u \in [0, \overline{\pi}]$. Since $\left.\frac{\partial \Pi}{\partial u}\right|_{u=0-} = f(0)\overline{\pi} - 1 \geq 0$, we have $\underline{u} = 0$, and hence $\overline{u}(q)$ is also the unique maximizer of Π in the whole range $u \in (-\infty, \overline{\pi}]$.

Define $\overline{q}$ as in the proof of Proposition 1; i.e., $\overline{q}$ is the point at which $\overline{u}(q)$ as a decreasing function of q first reaches 0 (in other words, $\overline{u}(q) = 0$ iff $q \geq \overline{q}$). Suppose $q^* < \overline{q} < 1$ in equilibrium. Then we must have $u^* = \overline{u}(q^*) > 0$ in equilibrium as well. But then when the consumer is unable to discern u^* and 0, he can still infer from the monopolist's equilibrium strategy that $u^* = \overline{u}(q^*) > 0$. His best response is hence to purchase the deal for sure (i.e., $q^* = 1$), contradicting the presumption that $q^* < \overline{q} < 1$.

On the other hand, any $q^* \in [\overline{q}, 1]$ can be part of an equilibrium, with the monopolist's best response being $u^* = 0$. □

We then move on to the case of a monopolist marketing up to two different deals. When Assumption 2 is violated, there are more seller-pure-strategy equilibria. Not only that there are more seller-pure-strategy equilibria where the monopolist markets two different deals (and offers utilities that are mirror images of each other around 0), there is also a seller-pure-strategy equilibrium where the monopolist markets only one deal.

Proposition 11. *Consider a monopolist marketing up to two different deals. In the case of $f(0)\overline{\pi} \geq 1$,*

- *there exists a strictly decreasing function $u_1(\cdot)$ that maps $[0, 1]$ into $[0, \overline{\pi})$, with $u_1(1) = 0$, such that, for every $q_d^* \in [0, 1]$, there exists a seller-pure-strategy equilibrium featuring that specific q_d^*, in which the monopolist offers deterministic utilities $u_1^* = u_1(q_d^*)$ and $u_2^* = -u_1^*$;*

- *there also exists a seller-pure-strategy equilibrium featuring* $(u_1^*, u_2^*) = (0, -\infty)$ *and* $(q_s^*, q_d^*) = (1, 0)$.

Proof. Note that Lemmas 2, 3, 4, 5, 6, 7 (with "$\underline{u} < 0$" replaced by "$\underline{u} = 0$"), and Proposition 4 remain valid, because their proofs do not rely on Assumption 2. However, the proof of Corollary 1 falls apart, because it relies on an argument made in the proof of Proposition 1, which in turn relies on Assumption 2. Therefore, we continue to have seller-pure-strategy equilibria of the form described in Proposition 6, but we can no longer use Corollary 1 to rule out seller-pure-strategy equilibria featuring $(u_1^*, u_2^*) = (0, -\infty)$.

Let q_d^{max} be defined as in the proof of Proposition 6. Since $f(0)\overline{\pi} > 1/2$ when Assumption 2 is violated, the same argument as in the last part of the proof of Proposition 6 suggests that (i) for every $q_d^* \in [0, q_d^{max}]$, there exists a seller-pure-strategy equilibrium featuring that specific q_d^*, in which the monopolist offers deterministic utilities $u_1^* = u_1(q_d^*)$ and $u_2^* = -u_1^*$ (where $u_1(\cdot)$ is defined in Lemma 4), and (ii), q_d^{max} is the maximum q_d such that profit at $(u_1, u_2) = (u_1(q_d), -u_1(q_d))$ is weakly higher than profit at $(u_1, u_2) = (\underline{u}, \underline{u})$. However, when Assumption 2 is violated, $\underline{u} = 0$, and hence the former profit is weakly higher than the latter profit for every q_d by the definition of $u_1(\cdot)$. Therefore, we have $q_d^{max} = 1$, which implies the first half of the proposition.

To prove the second half of the proposition, note that if $(u_1^*, u_2^*) = (0, -\infty)$, then any q_s is a best response, and any q_d is a best response against some off-equilibrium belief. So it suffices to prove that $(u_1, u_2) = (0, -\infty)$ is a best response against $(q_s^*, q_d^*) = (1, 0)$. The proof, however, is almost the same as that for $u = 0$ to be a best response against $q = 1$ in the one-deal case when Assumption 2 is violated (see the proof of Proposition 10), and hence is omitted. □

Appendix E. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2021.105186.

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