

## OPERATIONAL TECHNIQUES AND TABLES FOR MAKING WEAK MSE TESTS FOR RESTRICTIONS IN REGRESSIONS

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Tables of critical points for the noncentral  $F$  are presented with noncentrality equal to  $\frac{1}{2}$  of numerator degrees of freedom for denominator degrees of freedom of 1-30, 40, 60, 120, 200, 400, and 1,000, and numerator degrees of freedom of 1-30, 40, 60, 120, and 200, and type one errors of 0.05, 0.10, 0.25, and 0.50. These critical points can be used to test the second weak MSE criterion discussed in the companion paper [2]. An approximation is suggested for noncentral  $F(\theta)$ , and accuracy checks are given. An appendix provides a Fortran function for the approximation. The approximation is intended for using the first weak MSE test discussed in the companion paper.

### 1.

Table I contains critical points for the  $F$  distribution with noncentrality of  $m/2$  where  $m$  is numerator degrees of freedom. Numerical integration was used to obtain the critical values. Those interested in the integration routine may obtain information from the senior author.

The reader will note that the critical values for testing the hypothesis that  $\lambda \leq m/2$  are larger for given degrees of freedom than corresponding critical values for testing the hypothesis  $\lambda \leq \frac{1}{2}$ .<sup>1</sup> This is in keeping both with the intuitive notion that the weak MSE criterion is less demanding and with the mathematical property that the noncentral  $F$  is monotone increasing in  $\lambda$ .

For a given set of restrictions,  $H'\beta = h$ , the test that  $\lambda \leq m/2$  can be carried out by computing the  $u$  statistic of equation (3) in [2] and comparing its value with the tabular  $F$  value (Table I) for whatever  $\alpha$  (type one error) the reader may choose.<sup>2</sup> If the reader wishes to calculate the probability of a larger  $F$  instead of testing for a pre-selected  $\alpha$ , he may use the approximation given in Section 2.

### 2.

The noncentral  $F$  approximation given here is one reported in a U.S. Department of Commerce handbook [1, p. 948]. The approximation is

$$(1) \quad \int_0^F g(u, m, q, \lambda) du \approx \int_{-\infty}^{x'} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

where

$$(2) \quad x' = \frac{\left(\frac{mF}{m+2\lambda}\right)^{\frac{1}{3}} \left[1 - \frac{2}{9q}\right] - \left[1 - \frac{2(m+4\lambda)}{9(m+2\lambda)^2}\right]}{\left[\frac{2(m+4\lambda)}{9(m+2\lambda)^2} + \frac{2}{9q} \left(\frac{mF}{m+2\lambda}\right)^{\frac{2}{3}}\right]^{\frac{1}{3}}}$$

<sup>1</sup> Except for  $m = 1$ , in which case, of course, equality holds. See reference [3] for the critical points for testing  $\lambda \leq \frac{1}{2}$ .

<sup>2</sup> Some examples of typical restrictions are given in reference [3] and will not be repeated here.

TABLE I

VALUES OF THE NONCENTRAL  $F$  WITH NONCENTRALITY PARAMETER EQUAL TO ONE-HALF OF THE NUMERATOR DEGREES OF FREEDOM

Denominator degrees of freedom	Probability of a larger $F$	Numerator degrees of freedom								
		1	2	3	4	5	6	7	8	9
1	.05	345.387	417.649	446.356	461.415	470.657	476.296	481.418	484.793	487.422
	.10	85.452	103.704	110.953	114.759	117.092	118.669	119.801	120.651	121.314
	.25	12.673	15.810	17.043	17.689	18.084	18.350	18.541	18.685	18.797
	.50	2.295	3.243	3.611	3.803	3.920	3.998	4.055	4.097	4.130
2	.05	37.502	38.251	38.496	38.616	38.690	38.741	38.777	38.803	38.824
	.10	17.505	18.240	18.486	18.609	18.683	18.733	18.769	18.795	18.816
	.25	5.517	6.218	6.459	6.581	6.654	6.704	6.739	6.766	6.786
	.50	1.561	2.175	2.403	2.520	2.591	2.640	2.674	2.700	2.721
3	.05	19.935	18.777	18.276	17.998	17.823	17.703	17.615	17.548	17.495
	.10	11.155	10.874	10.713	10.618	10.555	10.512	10.480	10.455	10.435
	.25	4.333	4.694	4.792	4.836	4.861	4.877	4.888	4.896	4.902
	.50	1.384	1.926	2.122	2.223	2.284	2.325	2.355	2.377	2.394
4	.05	14.921	13.443	12.815	12.468	12.246	12.094	11.982	11.897	11.829
	.10	9.060	8.515	8.247	8.092	7.991	7.920	7.868	7.828	7.796
	.25	3.869	4.104	4.148	4.161	4.166	4.168	4.168	4.169	4.169
	.50	1.305	1.818	2.000	2.093	2.150	2.188	2.215	2.236	2.252
5	.05	12.652	11.083	10.417	10.047	9.810	9.646	9.526	9.433	9.360
	.10	8.043	7.388	7.073	6.891	6.773	6.689	6.627	6.580	6.542
	.25	3.623	3.795	3.809	3.805	3.799	3.793	3.788	3.783	3.780
	.50	1.261	1.757	1.933	2.022	2.075	2.112	2.138	2.157	2.172
6	.05	11.378	9.776	9.095	8.715	8.471	8.302	8.176	8.080	8.004
	.10	7.448	6.734	6.394	6.197	6.068	5.977	5.909	5.856	5.815
	.25	3.472	3.604	3.601	3.586	3.573	3.561	3.552	3.545	3.538
	.50	1.232	1.718	1.889	1.976	2.028	2.063	2.088	2.107	2.122
7	.05	10.567	8.951	8.264	7.879	7.631	7.458	7.330	7.231	7.152
	.10	7.058	6.309	5.953	5.746	5.610	5.513	5.442	5.386	5.341
	.25	3.370	3.476	3.460	3.438	3.419	3.404	3.392	3.382	3.374
	.50	1.213	1.692	1.860	1.944	1.995	2.030	2.054	2.073	2.087
8	.05	10.007	8.386	7.696	7.307	7.057	6.881	6.751	6.650	6.570
	.10	6.783	6.011	5.645	5.430	5.289	5.189	5.114	5.056	5.009
	.25	3.296	3.383	3.358	3.331	3.308	3.290	3.276	3.264	3.255
	.50	1.198	1.672	1.838	1.921	1.972	2.005	2.029	2.048	2.062
9	.05	9.598	7.975	7.283	6.893	6.640	6.463	6.331	6.229	6.147
	.10	6.580	5.791	5.416	5.197	5.052	4.949	4.871	4.811	4.763
	.25	3.240	3.314	3.282	3.250	3.224	3.204	3.188	3.175	3.164
	.50	1.187	1.657	1.821	1.904	1.953	1.987	2.010	2.028	2.042
10	.05	9.286	7.664	6.971	6.579	6.325	6.146	6.013	5.923	5.827
	.10	6.423	5.622	5.241	5.018	4.870	4.764	4.685	4.629	4.573
	.25	3.197	3.259	3.222	3.187	3.159	3.137	3.119	3.107	3.093
	.50	1.178	1.646	1.808	1.890	1.939	1.972	1.995	2.014	2.027
11	.05	9.041	7.419	6.726	6.333	6.078	5.898	5.763	5.659	5.575
	.10	6.298	5.488	5.103	4.876	4.725	4.617	4.536	4.473	4.422
	.25	3.162	3.215	3.174	3.136	3.106	3.082	3.063	3.048	3.035
	.50	1.171	1.636	1.798	1.879	1.928	1.960	1.983	2.001	2.015
12	.05	8.843	7.223	6.529	6.135	5.879	5.698	5.562	5.457	5.373
	.10	6.197	5.379	4.990	4.760	4.608	4.498	4.416	4.351	4.299
	.25	3.133	3.180	3.135	3.094	3.062	3.037	3.017	3.001	2.987
	.50	1.165	1.628	1.789	1.870	1.918	1.950	1.973	1.991	2.004
13	.05	8.680	7.061	6.367	5.973	5.716	5.534	5.397	5.291	5.206
	.10	6.113	5.289	4.897	4.665	4.511	4.400	4.316	4.250	4.198
	.25	3.109	3.150	3.102	3.059	3.026	3.000	2.979	2.962	2.948
	.50	1.160	1.622	1.782	1.862	1.910	1.942	1.965	1.982	1.996
14	.05	8.543	6.926	6.232	5.837	5.579	5.396	5.260	5.153	5.067
	.10	6.042	5.213	4.818	4.585	4.429	4.317	4.232	4.165	4.112
	.25	3.089	3.124	3.074	3.029	2.995	2.968	2.946	2.928	2.914
	.50	1.156	1.616	1.776	1.855	1.903	1.935	1.958	1.975	1.989
15	.05	8.427	6.811	6.118	5.722	5.464	5.280	5.142	5.035	4.949
	.10	5.982	5.149	4.752	4.516	4.359	4.246	4.160	4.093	4.038
	.25	3.071	3.103	3.050	3.004	2.968	2.940	2.918	2.900	2.884
	.50	1.152	1.611	1.770	1.850	1.897	1.929	1.952	1.969	1.982
16	.05	8.327	6.713	6.019	5.623	5.364	5.180	5.042	4.934	4.847
	.10	5.930	5.093	4.694	4.457	4.299	4.184	4.098	4.030	3.975
	.25	3.056	3.084	3.029	2.982	2.945	2.917	2.894	2.875	2.859
	.50	1.149	1.607	1.766	1.845	1.892	1.924	1.947	1.964	1.977
17	.05	8.240	6.627	5.934	5.538	5.278	5.093	4.954	4.846	4.759
	.10	5.884	5.044	4.644	4.406	4.246	4.131	4.044	3.975	3.919
	.25	3.043	3.067	3.011	2.963	2.925	2.896	2.872	2.853	2.837
	.50	1.147	1.603	1.762	1.841	1.888	1.920	1.942	1.959	1.972
18	.05	8.164	6.552	5.859	5.462	5.202	5.017	4.878	4.769	4.681
	.10	5.844	5.002	4.599	4.360	4.200	4.084	3.996	3.927	3.871
	.25	3.031	3.053	2.995	2.945	2.907	2.877	2.853	2.833	2.817
	.50	1.144	1.600	1.758	1.837	1.884	1.916	1.938	1.955	1.968

TABLE I (continued)

Numerator degrees of freedom									Denominator degrees of freedom	Probability of a larger F
10	11	12	13	14	15	16	17	18		
489.528	491.258	492.700	493.918	494.965	495.876	496.668	497.372	497.973	1	.05
121.846	122.281	122.645	122.952	123.216	123.444	123.645	123.821	123.976		.10
18.887	18.960	19.022	19.074	19.118	19.157	19.191	19.220	19.247		.25
4.157	4.179	4.197	4.212	4.225	4.236	4.246	4.255	4.263		.50
38.841	38.855	38.866	38.875	38.883	38.891	38.897	38.903	38.908	2	.05
18.833	18.846	18.857	18.867	18.875	18.882	18.889	18.894	18.899		.10
6.803	6.816	6.828	6.837	6.845	6.852	6.859	6.864	6.869		.25
2.737	2.750	2.761	2.771	2.779	2.786	2.792	2.798	2.803		.50
17.452	17.417	17.388	17.363	17.341	17.322	17.306	17.291	17.278	3	.05
10.419	10.406	10.395	10.386	10.378	10.371	10.364	10.359	10.354		.10
4.907	4.911	4.914	4.917	4.919	4.921	4.923	4.924	4.926		.25
2.408	2.420	2.429	2.437	2.444	2.450	2.456	2.460	2.464		.50
11.775	11.729	11.692	11.659	11.632	11.607	11.586	11.567	11.550	4	.05
7.770	7.749	7.731	7.715	7.702	7.691	7.680	7.671	7.663		.10
4.168	4.168	4.168	4.168	4.167	4.167	4.167	4.167	4.166		.25
2.265	2.275	2.284	2.292	2.298	2.304	2.309	2.313	2.317		.50
9.301	9.252	9.210	9.175	9.145	9.118	9.095	9.074	9.056	5	.05
6.511	6.485	6.464	6.445	6.429	6.415	6.403	6.392	6.382		.10
3.776	3.774	3.771	3.769	3.767	3.766	3.764	3.763	3.762		.25
2.185	2.195	2.203	2.210	2.217	2.222	2.227	2.231	2.234		.50
7.942	7.890	7.847	7.810	7.778	7.750	7.725	7.703	7.684	6	.05
5.781	5.752	5.728	5.708	5.690	5.675	5.661	5.649	5.638		.10
3.533	3.529	3.525	3.521	3.518	3.516	3.513	3.511	3.509		.25
2.134	2.144	2.152	2.159	2.165	2.170	2.174	2.178	2.182		.50
7.088	7.035	6.990	6.952	6.919	6.890	6.865	6.842	6.822	7	.05
5.305	5.275	5.249	5.227	5.208	5.191	5.177	5.164	5.152		.10
3.367	3.361	3.356	3.352	3.348	3.345	3.342	3.339	3.337		.25
2.099	2.108	2.116	2.123	2.129	2.134	2.138	2.142	2.146		.50
6.505	6.450	6.404	6.365	6.331	6.301	6.275	6.252	6.231	8	.05
4.971	4.939	4.912	4.889	4.869	4.851	4.836	4.822	4.810		.10
3.247	3.240	3.234	3.229	3.224	3.220	3.216	3.213	3.210		.25
2.073	2.082	2.090	2.097	2.103	2.108	2.112	2.116	2.119		.50
6.081	6.025	5.979	5.939	5.904	5.873	5.846	5.823	5.801	9	.05
4.723	4.690	4.662	4.638	4.617	4.599	4.582	4.568	4.555		.10
3.155	3.147	3.140	3.134	3.129	3.125	3.121	3.117	3.114		.25
2.053	2.063	2.070	2.077	2.083	2.087	2.092	2.096	2.099		.50
5.759	5.703	5.655	5.615	5.579	5.548	5.521	5.496	5.475	10	.05
4.532	4.498	4.469	4.444	4.422	4.403	4.387	4.372	4.358		.10
3.083	3.074	3.067	3.060	3.055	3.050	3.045	3.041	3.038		.25
2.038	2.047	2.055	2.061	2.067	2.072	2.076	2.080	2.083		.50
5.507	5.450	5.402	5.360	5.324	5.293	5.265	5.240	5.218	11	.05
4.380	4.345	4.315	4.290	4.268	4.248	4.231	4.215	4.201		.10
3.024	3.015	3.007	3.000	2.994	2.989	2.984	2.980	2.976		.25
2.026	2.035	2.042	2.049	2.054	2.059	2.063	2.067	2.070		.50
5.304	5.246	5.197	5.155	5.119	5.087	5.058	5.033	5.011	12	.05
4.257	4.221	4.190	4.164	4.141	4.121	4.103	4.088	4.073		.10
2.976	2.966	2.958	2.951	2.944	2.939	2.934	2.929	2.925		.25
2.015	2.024	2.032	2.038	2.044	2.048	2.052	2.056	2.059		.50
5.137	5.079	5.029	4.987	4.950	4.917	4.888	4.863	4.840	13	.05
4.154	4.117	4.086	4.060	4.036	4.016	3.998	3.981	3.967		.10
2.936	2.926	2.917	2.909	2.902	2.896	2.891	2.886	2.882		.25
2.007	2.016	2.023	2.029	2.035	2.039	2.044	2.047	2.050		.50
4.997	4.938	4.888	4.845	4.808	4.775	4.746	4.720	4.697	14	.05
4.067	4.030	3.999	3.971	3.948	3.927	3.908	3.891	3.877		.10
2.901	2.891	2.881	2.873	2.866	2.860	2.855	2.849	2.845		.25
1.999	2.008	2.016	2.022	2.027	2.032	2.036	2.040	2.043		.50
4.878	4.819	4.768	4.725	4.687	4.654	4.625	4.598	4.575	15	.05
3.993	3.956	3.924	3.896	3.872	3.850	3.831	3.814	3.799		.10
2.872	2.861	2.851	2.843	2.835	2.829	2.823	2.818	2.813		.25
1.993	2.002	2.009	2.015	2.021	2.025	2.029	2.033	2.036		.50
4.776	4.716	4.665	4.621	4.583	4.550	4.520	4.494	4.470	16	.05
3.929	3.891	3.859	3.830	3.806	3.784	3.765	3.748	3.732		.10
2.846	2.834	2.825	2.816	2.808	2.802	2.796	2.790	2.785		.25
1.988	1.996	2.004	2.010	2.015	2.020	2.024	2.027	2.031		.50
4.687	4.627	4.576	4.532	4.493	4.459	4.429	4.402	4.378	17	.05
3.873	3.835	3.802	3.773	3.748	3.726	3.707	3.689	3.674		.10
2.823	2.811	2.801	2.792	2.784	2.778	2.771	2.766	2.761		.25
1.983	1.992	1.999	2.005	2.010	2.015	2.019	2.022	2.026		.50
4.609	4.549	4.497	4.452	4.414	4.380	4.349	4.322	4.298	18	.05
3.824	3.785	3.752	3.723	3.697	3.675	3.655	3.638	3.622		.10
2.803	2.791	2.780	2.771	2.763	2.756	2.750	2.744	2.739		.25
1.979	1.987	1.995	2.001	2.006	2.010	2.014	2.018	2.021		.50

TABLE I (continued)

Denominator degrees of freedom	Probability of a larger F	Numerator degrees of freedom								
		1	2	3	4	5	6	7	8	9
19	.05	8.097	6.486	5.793	5.396	5.136	4.950	4.810	4.701	4.613
	.10	5.809	4.964	4.560	4.320	4.159	4.042	3.954	3.884	3.827
	.25	3.021	3.040	2.980	2.930	2.891	2.861	2.836	2.816	2.799
	.50	1.142	1.597	1.755	1.833	1.881	1.912	1.934	1.951	1.964
20	.05	8.037	6.427	5.734	5.337	5.076	4.890	4.750	4.640	4.552
	.10	5.777	4.930	4.525	4.284	4.122	4.005	3.916	3.846	3.788
	.25	3.011	3.028	2.967	2.917	2.877	2.846	2.821	2.800	2.783
	.50	1.140	1.595	1.752	1.830	1.877	1.909	1.931	1.948	1.961
21	.05	7.983	6.374	5.681	5.284	5.023	4.836	4.696	4.586	4.497
	.10	5.749	4.900	4.494	4.252	4.089	3.972	3.882	3.811	3.754
	.25	3.003	3.017	2.956	2.904	2.864	2.833	2.807	2.786	2.769
	.50	1.138	1.592	1.749	1.828	1.875	1.906	1.928	1.945	1.958
22	.05	7.935	6.327	5.634	5.237	4.975	4.788	4.647	4.537	4.448
	.10	5.723	4.872	4.466	4.223	4.060	3.941	3.851	3.780	3.722
	.25	2.995	3.008	2.945	2.893	2.852	2.820	2.795	2.774	2.756
	.50	1.137	1.590	1.747	1.825	1.872	1.903	1.926	1.942	1.955
23	.05	7.891	6.284	5.591	5.193	4.932	4.744	4.603	4.492	4.403
	.10	5.700	4.847	4.440	4.197	4.033	3.914	3.823	3.752	3.693
	.25	2.988	2.999	2.936	2.883	2.842	2.809	2.783	2.762	2.744
	.50	1.135	1.588	1.745	1.823	1.870	1.901	1.923	1.940	1.953
24	.05	7.851	6.245	5.552	5.154	4.892	4.705	4.563	4.452	4.362
	.10	5.678	4.825	4.417	4.173	4.008	3.889	3.798	3.726	3.667
	.25	2.982	2.992	2.927	2.874	2.832	2.799	2.773	2.751	2.733
	.50	1.134	1.586	1.743	1.821	1.868	1.899	1.921	1.938	1.951
25	.05	7.814	6.209	5.516	5.118	4.856	4.668	4.526	4.415	4.325
	.10	5.659	4.804	4.395	4.151	3.986	3.866	3.775	3.702	3.643
	.25	2.976	2.984	2.919	2.865	2.823	2.790	2.763	2.742	2.723
	.50	1.133	1.585	1.741	1.819	1.866	1.897	1.919	1.935	1.948
26	.05	7.781	6.176	5.484	5.086	4.823	4.635	4.493	4.381	4.291
	.10	5.641	4.785	4.375	4.130	3.965	3.845	3.753	3.681	3.621
	.25	2.971	2.978	2.912	2.857	2.815	2.782	2.755	2.733	2.714
	.50	1.131	1.583	1.739	1.817	1.864	1.895	1.917	1.934	1.947
27	.05	7.750	6.146	5.454	5.055	4.792	4.604	4.462	4.350	4.260
	.10	5.625	4.768	4.357	4.112	3.946	3.826	3.733	3.660	3.601
	.25	2.966	2.972	2.905	2.850	2.808	2.774	2.747	2.724	2.705
	.50	1.130	1.582	1.738	1.816	1.862	1.893	1.915	1.932	1.945
28	.05	7.721	6.118	5.426	5.027	4.764	4.576	4.433	4.321	4.230
	.10	5.609	4.751	4.341	4.095	3.928	3.808	3.715	3.642	3.582
	.25	2.961	2.966	2.899	2.844	2.801	2.767	2.739	2.717	2.698
	.50	1.129	1.581	1.737	1.814	1.861	1.892	1.914	1.930	1.943
29	.05	7.695	6.092	5.400	5.002	4.738	4.550	4.407	4.294	4.204
	.10	5.595	4.736	4.325	4.079	3.912	3.791	3.698	3.625	3.565
	.25	2.957	2.961	2.893	2.837	2.794	2.760	2.732	2.709	2.690
	.50	1.128	1.579	1.735	1.813	1.859	1.890	1.912	1.929	1.942
30	.05	7.670	6.068	5.376	4.978	4.714	4.525	4.382	4.270	4.179
	.10	5.582	4.722	4.311	4.064	3.897	3.775	3.682	3.609	3.549
	.25	2.953	2.956	2.888	2.832	2.788	2.753	2.726	2.703	2.683
	.50	1.128	1.578	1.734	1.812	1.858	1.889	1.911	1.927	1.940
40	.05	7.495	5.898	5.206	4.807	4.542	4.351	4.207	4.093	4.000
	.10	5.488	4.623	4.207	3.958	3.788	3.664	3.569	3.494	3.432
	.25	2.925	2.920	2.849	2.790	2.744	2.708	2.679	2.654	2.634
	.50	1.122	1.570	1.725	1.802	1.848	1.879	1.901	1.917	1.930
60	.05	7.326	5.733	5.042	4.642	4.375	4.183	4.037	3.921	3.826
	.10	5.396	4.525	4.107	3.854	3.682	3.556	3.458	3.381	3.317
	.25	2.897	2.886	2.810	2.749	2.701	2.663	2.632	2.606	2.585
	.50	1.116	1.562	1.717	1.793	1.839	1.869	1.891	1.907	1.920
120	.05	7.162	5.574	4.883	4.482	4.214	4.020	3.871	3.753	3.657
	.10	5.306	4.430	4.008	3.753	3.578	3.449	3.350	3.270	3.204
	.25	2.869	2.852	2.773	2.708	2.658	2.618	2.586	2.559	2.535
	.50	1.110	1.555	1.708	1.784	1.830	1.860	1.882	1.898	1.910
200	.05	7.097	5.511	4.821	4.420	4.151	3.956	3.807	3.688	3.590
	.10	5.271	4.393	3.970	3.713	3.537	3.407	3.307	3.226	3.159
	.25	2.859	2.838	2.758	2.692	2.641	2.601	2.567	2.539	2.516
	.50	1.107	1.551	1.705	1.781	1.826	1.856	1.878	1.894	1.906
400	.05	7.049	5.465	4.775	4.373	4.104	3.908	3.759	3.639	3.541
	.10	5.245	4.365	3.941	3.683	3.507	3.376	3.275	3.193	3.126
	.25	2.850	2.828	2.746	2.680	2.629	2.587	2.554	2.525	2.501
	.50	1.105	1.549	1.702	1.778	1.823	1.854	1.875	1.891	1.904
1000	.05	7.021	5.438	4.748	4.346	4.076	3.880	3.730	3.610	3.511
	.10	5.229	4.349	3.924	3.666	3.488	3.357	3.256	3.174	3.106
	.25	2.846	2.822	2.740	2.673	2.621	2.579	2.545	2.517	2.492
	.50	1.104	1.548	1.701	1.776	1.822	1.852	1.873	1.889	1.902

TABLE I (continued)

Numerator degrees of freedom									Denominator degrees of freedom	Probability of a larger <i>F</i>
10	11	12	13	14	15	16	17	18		
4.540	4.479	4.427	4.383	4.344	4.309	4.279	4.251	4.227	19	.05
3.780	3.741	3.707	3.678	3.652	3.630	3.610	3.592	3.576		.10
2.785	2.773	2.762	2.753	2.744	2.737	2.731	2.725	2.719		.25
1.975	1.983	1.991	1.997	2.002	2.007	2.011	2.014	2.017		.50
4.479	4.418	4.365	4.320	4.281	4.246	4.216	4.188	4.163	20	.05
3.741	3.701	3.667	3.638	3.612	3.589	3.569	3.551	3.534		.10
2.769	2.756	2.745	2.736	2.727	2.720	2.713	2.707	2.702		.25
1.971	1.980	1.987	1.993	1.999	2.003	2.007	2.011	2.014		.50
4.424	4.362	4.310	4.264	4.225	4.190	4.159	4.131	4.106	21	.05
3.706	3.666	3.631	3.602	3.575	3.552	3.532	3.514	3.497		.10
2.754	2.741	2.730	2.721	2.712	2.704	2.698	2.691	2.686		.25
1.968	1.977	1.984	1.990	1.995	2.000	2.004	2.007	2.011		.50
4.374	4.312	4.259	4.214	4.174	4.139	4.108	4.080	4.055	22	.05
3.674	3.634	3.599	3.569	3.543	3.519	3.499	3.480	3.463		.10
2.741	2.728	2.717	2.707	2.698	2.690	2.683	2.677	2.671		.25
1.966	1.974	1.981	1.987	1.993	1.997	2.001	2.005	2.008		.50
4.329	4.267	4.214	4.168	4.128	4.093	4.061	4.033	4.008	23	.05
3.645	3.604	3.569	3.539	3.513	3.489	3.468	3.450	3.433		.10
2.729	2.716	2.704	2.694	2.685	2.677	2.670	2.664	2.658		.25
1.963	1.972	1.979	1.985	1.990	1.995	1.998	2.002	2.005		.50
4.288	4.226	4.173	4.126	4.086	4.051	4.019	3.991	3.965	24	.05
3.619	3.578	3.542	3.512	3.485	3.462	3.440	3.422	3.405		.10
2.718	2.704	2.693	2.683	2.674	2.666	2.658	2.652	2.646		.25
1.961	1.969	1.977	1.983	1.988	1.992	1.996	2.000	2.003		.50
4.251	4.188	4.135	4.088	4.048	4.012	3.980	3.952	3.926	25	.05
3.595	3.553	3.518	3.487	3.460	3.436	3.415	3.396	3.379		.10
2.707	2.694	2.682	2.672	2.663	2.655	2.647	2.641	2.635		.25
1.959	1.967	1.974	1.980	1.986	1.990	1.994	1.997	1.999		.50
4.216	4.153	4.100	4.053	4.013	3.977	3.945	3.916	3.891	26	.05
3.572	3.531	3.495	3.464	3.437	3.413	3.392	3.372	3.355		.10
2.698	2.684	2.673	2.662	2.653	2.645	2.637	2.631	2.624		.25
1.957	1.965	1.972	1.978	1.984	1.988	1.992	1.995	1.998		.50
4.185	4.122	4.068	4.021	3.980	3.944	3.912	3.883	3.857	27	.05
3.552	3.510	3.474	3.443	3.416	3.391	3.370	3.351	3.333		.10
2.689	2.676	2.664	2.653	2.644	2.635	2.628	2.621	2.615		.25
1.955	1.964	1.971	1.977	1.982	1.986	1.990	1.994	1.997		.50
4.155	4.092	4.038	3.991	3.950	3.914	3.882	3.853	3.827	28	.05
3.533	3.491	3.455	3.423	3.396	3.371	3.350	3.330	3.313		.10
2.681	2.667	2.655	2.645	2.635	2.627	2.619	2.612	2.606		.25
1.953	1.962	1.969	1.975	1.980	1.984	1.988	1.992	1.995		.50
4.128	4.065	4.011	3.963	3.922	3.886	3.854	3.825	3.798	29	.05
3.515	3.473	3.437	3.405	3.377	3.353	3.331	3.312	3.294		.10
2.674	2.660	2.648	2.637	2.627	2.619	2.611	2.604	2.598		.25
1.952	1.960	1.967	1.973	1.978	1.983	1.987	1.990	1.993		.50
4.103	4.039	3.985	3.938	3.896	3.860	3.827	3.798	3.772	30	.05
3.498	3.456	3.420	3.388	3.360	3.336	3.314	3.294	3.276		.10
2.667	2.653	2.640	2.629	2.620	2.611	2.603	2.596	2.590		.25
1.950	1.959	1.966	1.972	1.977	1.981	1.985	1.989	1.992		.50
3.923	3.858	3.802	3.754	3.711	3.673	3.640	3.610	3.582	40	.05
3.380	3.336	3.299	3.266	3.237	3.211	3.188	3.167	3.149		.10
2.616	2.601	2.588	2.576	2.566	2.557	2.548	2.540	2.533		.25
1.940	1.949	1.956	1.961	1.966	1.971	1.975	1.978	1.981		.50
3.748	3.681	3.624	3.574	3.530	3.491	3.456	3.424	3.396	60	.05
3.264	3.218	3.179	3.145	3.114	3.087	3.063	3.041	3.022		.10
2.566	2.550	2.535	2.523	2.512	2.502	2.492	2.484	2.475		.25
1.930	1.938	1.945	1.951	1.956	1.960	1.964	1.968	1.971		.50
3.576	3.508	3.448	3.397	3.351	3.310	3.274	3.241	3.211	120	.05
3.149	3.101	3.060	3.024	2.993	2.964	2.939	2.915	2.894		.10
2.516	2.498	2.483	2.469	2.457	2.446	2.436	2.427	2.418		.25
1.920	1.929	1.935	1.941	1.946	1.950	1.954	1.957	1.960		.50
3.509	3.440	3.379	3.327	3.280	3.239	3.202	3.168	3.137	200	.05
3.103	3.055	3.013	2.977	2.944	2.915	2.889	2.865	2.843		.10
2.495	2.477	2.462	2.448	2.435	2.423	2.413	2.403	2.395		.25
1.916	1.925	1.931	1.937	1.942	1.946	1.950	1.953	1.956		.50
3.459	3.389	3.328	3.275	3.228	3.186	3.148	3.113	3.082	400	.05
3.069	3.020	2.978	2.941	2.907	2.878	2.851	2.827	2.804		.10
2.480	2.462	2.446	2.431	2.418	2.406	2.396	2.386	2.377		.25
1.914	1.922	1.929	1.934	1.939	1.943	1.947	1.950	1.953		.50
3.429	3.358	3.297	3.243	3.196	3.154	3.115	3.081	3.049	1000	.05
3.049	2.999	2.957	2.919	2.885	2.855	2.828	2.804	2.781		.10
2.471	2.453	2.436	2.421	2.408	2.396	2.385	2.375	2.366		.25
1.912	1.920	1.927	1.933	1.937	1.942	1.945	1.949	1.952		.50

TABLE I (continued)

Denominator degrees of freedom	Probability of a larger F	Numerator degrees of freedom									
		19	20	21	22	23	24	25	26	27	
1	.05	498.536	499.040	499.501	499.915	500.301	500.649	500.965	501.258	501.532	
	.10	124.117	124.244	124.359	124.463	124.560	124.647	124.727	124.801	124.870	
	.25	19.270	19.292	19.311	19.329	19.345	19.360	19.373	19.386	19.397	
	.50	4.270	4.276	4.282	4.287	4.292	4.296	4.300	4.304	4.307	
2	.05	38.912	38.916	38.919	38.923	38.926	38.928	38.931	38.933	38.935	
	.10	18.903	18.907	18.911	18.914	18.917	18.920	18.922	18.925	18.927	
	.25	6.873	6.877	6.881	6.884	6.887	6.890	6.892	6.894	6.897	
	.50	2.807	2.811	2.814	2.818	2.820	2.823	2.826	2.828	2.830	
3	.05	17.267	17.256	17.247	17.238	17.230	17.223	17.216	17.210	17.204	
	.10	10.349	10.345	10.342	10.338	10.335	10.333	10.330	10.328	10.326	
	.25	4.927	4.928	4.929	4.930	4.931	4.932	4.932	4.933	4.933	
	.50	2.468	2.472	2.475	2.477	2.480	2.482	2.484	2.486	2.488	
4	.05	11.535	11.522	11.509	11.498	11.488	11.478	11.470	11.462	11.454	
	.10	7.656	7.650	7.644	7.638	7.633	7.629	7.625	7.621	7.617	
	.25	4.166	4.166	4.166	4.166	4.165	4.165	4.165	4.165	4.165	
	.50	2.320	2.323	2.326	2.329	2.331	2.333	2.335	2.337	2.339	
5	.05	9.039	9.024	9.010	8.998	8.987	8.976	8.967	8.958	8.950	
	.10	6.374	6.366	6.359	6.352	6.346	6.341	6.336	6.331	6.327	
	.25	3.761	3.760	3.759	3.758	3.757	3.756	3.756	3.755	3.755	
	.50	2.238	2.241	2.243	2.246	2.248	2.250	2.252	2.254	2.255	
6	.05	7.666	7.651	7.636	7.623	7.611	7.600	7.590	7.580	7.572	
	.10	5.628	5.620	5.612	5.604	5.598	5.591	5.586	5.581	5.576	
	.25	3.508	3.506	3.505	3.503	3.502	3.501	3.500	3.499	3.498	
	.50	2.185	2.188	2.191	2.193	2.195	2.197	2.199	2.201	2.202	
7	.05	6.804	6.787	6.772	6.758	6.746	6.734	6.724	6.714	6.705	
	.10	5.142	5.132	5.123	5.116	5.108	5.102	5.096	5.090	5.085	
	.25	3.334	3.332	3.331	3.329	3.327	3.326	3.325	3.323	3.322	
	.50	2.149	2.152	2.154	2.157	2.159	2.162	2.162	2.164	2.166	
8	.05	6.212	6.195	6.180	6.166	6.153	6.141	6.130	6.120	6.110	
	.10	4.798	4.788	4.779	4.771	4.763	4.756	4.750	4.744	4.738	
	.25	3.208	3.205	3.203	3.201	3.199	3.198	3.196	3.195	3.193	
	.50	2.122	2.125	2.128	2.130	2.132	2.134	2.136	2.137	2.139	
9	.05	5.782	5.765	5.749	5.734	5.721	5.709	5.697	5.687	5.677	
	.10	4.543	4.533	4.523	4.514	4.506	4.499	4.492	4.486	4.480	
	.25	3.111	3.108	3.106	3.104	3.102	3.100	3.098	3.096	3.095	
	.50	2.102	2.105	2.107	2.109	2.111	2.113	2.115	2.117	2.118	
10	.05	5.455	5.437	5.421	5.406	5.392	5.380	5.368	5.358	5.348	
	.10	4.346	4.335	4.325	4.316	4.308	4.300	4.293	4.286	4.280	
	.25	3.035	3.032	3.029	3.026	3.024	3.022	3.020	3.018	3.017	
	.50	2.086	2.089	2.091	2.093	2.095	2.097	2.099	2.100	2.102	
11	.05	5.198	5.180	5.163	5.148	5.134	5.121	5.109	5.099	5.088	
	.10	4.189	4.178	4.167	4.158	4.149	4.141	4.134	4.127	4.120	
	.25	2.973	2.969	2.966	2.964	2.961	2.959	2.957	2.955	2.953	
	.50	2.073	2.076	2.078	2.080	2.082	2.084	2.086	2.087	2.089	
12	.05	4.990	4.972	4.955	4.940	4.925	4.912	4.900	4.889	4.879	
	.10	4.060	4.049	4.038	4.028	4.019	4.011	4.004	3.997	3.990	
	.25	2.921	2.918	2.915	2.912	2.909	2.907	2.904	2.902	2.900	
	.50	2.062	2.065	2.067	2.069	2.071	2.073	2.075	2.077	2.078	
13	.05	4.819	4.800	4.783	4.768	4.753	4.740	4.728	4.716	4.706	
	.10	3.954	3.942	3.931	3.921	3.911	3.903	3.895	3.888	3.881	
	.25	2.878	2.874	2.871	2.868	2.865	2.863	2.860	2.858	2.856	
	.50	2.053	2.056	2.058	2.060	2.062	2.064	2.066	2.067	2.069	
14	.05	4.676	4.657	4.639	4.623	4.609	4.595	4.583	4.571	4.561	
	.10	3.863	3.851	3.840	3.829	3.820	3.811	3.803	3.796	3.789	
	.25	2.841	2.837	2.834	2.830	2.827	2.825	2.822	2.820	2.818	
	.50	2.046	2.048	2.051	2.053	2.055	2.057	2.058	2.060	2.061	
15	.05	4.553	4.534	4.517	4.500	4.486	4.472	4.459	4.448	4.437	
	.10	3.786	3.773	3.762	3.751	3.742	3.733	3.725	3.717	3.710	
	.25	2.809	2.805	2.801	2.798	2.795	2.792	2.789	2.787	2.785	
	.50	2.039	2.042	2.044	2.046	2.048	2.050	2.052	2.053	2.055	
16	.05	4.448	4.429	4.411	4.395	4.380	4.366	4.353	4.341	4.330	
	.10	3.718	3.706	3.694	3.683	3.674	3.664	3.656	3.648	3.641	
	.25	2.781	2.777	2.773	2.769	2.766	2.763	2.761	2.758	2.756	
	.50	2.033	2.036	2.038	2.040	2.042	2.044	2.046	2.047	2.049	
17	.05	4.357	4.337	4.319	4.302	4.287	4.273	4.260	4.248	4.237	
	.10	3.659	3.646	3.635	3.624	3.614	3.605	3.596	3.588	3.581	
	.25	2.756	2.752	2.748	2.744	2.741	2.738	2.735	2.733	2.730	
	.50	2.028	2.031	2.033	2.035	2.037	2.039	2.041	2.042	2.044	
18	.05	4.276	4.256	4.238	4.221	4.206	4.192	4.178	4.166	4.155	
	.10	3.607	3.594	3.582	3.571	3.561	3.552	3.543	3.535	3.528	
	.25	2.734	2.730	2.726	2.722	2.719	2.716	2.713	2.710	2.707	
	.50	2.024	2.027	2.029	2.031	2.033	2.035	2.036	2.038	2.039	

TABLE I (continued)

Numerator degrees of freedom							Denominator degrees of freedom	Probability of a larger F
28	29	30	40	60	120	200		
501.786	502.020	502.239	503.836	505.430	507.032	507.672	1	.05
124.934	124.993	125.048	125.450	125.852	126.253	126.614		.10
19.408	19.418	19.427	19.495	19.563	19.631	19.658		.25
4.311	4.314	4.316	4.336	4.356	4.376	4.384		.50
38.937	38.939	38.941	38.953	38.966	38.978	38.983	2	.05
18.929	18.931	18.932	18.945	18.957	18.970	18.975		.10
6.899	6.900	6.902	6.915	6.927	6.940	6.945		.25
2.832	2.834	2.836	2.848	2.860	2.873	2.878		.50
17.199	17.194	17.189	17.155	17.121	17.087	17.073	3	.05
10.323	10.322	10.320	10.307	10.294	10.281	10.275		.10
4.934	4.934	4.935	4.938	4.942	4.945	4.946		.25
2.490	2.491	2.493	2.504	2.514	2.525	2.529		.50
11.447	11.441	11.435	11.391	11.346	11.301	11.283	4	.05
7.614	7.611	7.608	7.587	7.565	7.543	7.535		.10
4.165	4.165	4.165	4.164	4.163	4.162	4.162		.25
2.340	2.342	2.343	2.353	2.363	2.373	2.377		.50
8.942	8.935	8.928	8.879	8.830	8.780	8.760	5	.05
6.323	6.319	6.315	6.289	6.263	6.237	6.226		.10
3.754	3.754	3.753	3.750	3.746	3.743	3.741		.25
2.257	2.258	2.260	2.269	2.279	2.288	2.292		.50
7.564	7.556	7.549	7.497	7.445	7.392	7.370	6	.05
5.571	5.567	5.563	5.534	5.504	5.475	5.463		.10
3.497	3.497	3.496	3.491	3.485	3.479	3.477		.25
2.204	2.205	2.206	2.216	2.225	2.234	2.238		.50
6.696	6.689	6.681	6.627	6.572	6.516	6.494	7	.05
5.080	5.075	5.071	5.039	5.007	4.975	4.962		.10
3.321	3.320	3.319	3.312	3.305	3.298	3.295		.25
2.167	2.168	2.170	2.179	2.188	2.197	2.201		.50
6.102	6.093	6.086	6.030	5.972	5.914	5.891	8	.05
4.733	4.728	4.723	4.690	4.656	4.621	4.607		.10
3.192	3.191	3.190	3.182	3.173	3.164	3.161		.25
2.140	2.141	2.143	2.152	2.160	2.170	2.173		.50
5.668	5.660	5.652	5.594	5.535	5.475	5.450	9	.05
4.474	4.469	4.464	4.429	4.393	4.356	4.341		.10
3.093	3.092	3.091	3.081	3.072	3.062	3.058		.25
2.120	2.121	2.122	2.131	2.140	2.149	2.152		.50
5.338	5.330	5.322	5.262	5.201	5.139	5.114	10	.05
4.274	4.269	4.264	4.227	4.189	4.151	4.135		.10
3.015	3.014	3.012	3.002	2.991	2.980	2.976		.25
2.103	2.105	2.106	2.114	2.123	2.132	2.136		.50
5.079	5.070	5.062	5.001	4.938	4.875	4.849	11	.05
4.115	4.109	4.104	4.065	4.026	3.986	3.969		.10
2.951	2.950	2.948	2.937	2.925	2.913	2.908		.25
2.090	2.091	2.093	2.101	2.110	2.119	2.122		.50
4.869	4.860	4.852	4.789	4.725	4.660	4.633	12	.05
3.984	3.978	3.973	3.933	3.892	3.850	3.833		.10
2.899	2.897	2.895	2.883	2.871	2.858	2.852		.25
2.079	2.081	2.082	2.090	2.099	2.108	2.111		.50
4.696	4.687	4.678	4.615	4.549	4.482	4.455	13	.05
3.875	3.869	3.863	3.823	3.781	3.737	3.720		.10
2.854	2.852	2.850	2.838	2.824	2.811	2.805		.25
2.070	2.071	2.073	2.081	2.090	2.098	2.102		.50
4.550	4.541	4.532	4.468	4.401	4.332	4.304	14	.05
3.782	3.776	3.771	3.729	3.686	3.641	3.623		.10
2.816	2.814	2.812	2.799	2.785	2.770	2.764		.25
2.063	2.064	2.065	2.073	2.082	2.090	2.094		.50
4.427	4.417	4.408	4.342	4.275	4.204	4.176	15	.05
3.703	3.697	3.691	3.648	3.604	3.558	3.539		.10
2.783	2.781	2.779	2.765	2.750	2.734	2.728		.25
2.056	2.057	2.058	2.067	2.075	2.084	2.087		.50
4.320	4.310	4.301	4.234	4.165	4.094	4.064	16	.05
3.634	3.628	3.622	3.578	3.533	3.486	3.466		.10
2.753	2.751	2.749	2.735	2.720	2.703	2.697		.25
2.050	2.051	2.052	2.061	2.069	2.078	2.081		.50
4.226	4.217	4.207	4.140	4.069	3.997	3.967	17	.05
3.574	3.568	3.561	3.517	3.470	3.422	3.402		.10
2.728	2.726	2.724	2.709	2.693	2.676	2.669		.25
2.045	2.046	2.047	2.056	2.064	2.072	2.076		.50
4.144	4.134	4.125	4.056	3.985	3.911	3.881	18	.05
3.521	3.514	3.508	3.462	3.415	3.365	3.345		.10
2.705	2.703	2.701	2.685	2.669	2.651	2.644		.25
2.040	2.042	2.043	2.051	2.059	2.068	2.071		.50

TABLE I (continued)

Denominator degrees of freedom	Probability of a larger F	Numerator degrees of freedom									
		19	20	21	22	23	24	25	26	27	
19	.05	4.205	4.184	4.166	4.149	4.134	4.119	4.106	4.094	4.082	
	.10	3.561	3.548	3.536	3.524	3.514	3.505	3.496	3.488	3.480	
	.25	2.714	2.710	2.706	2.702	2.699	2.695	2.692	2.690	2.687	
	.50	2.020	2.023	2.025	2.027	2.029	2.031	2.032	2.034	2.035	
20	.05	4.141	4.121	4.102	4.085	4.069	4.055	4.041	4.029	4.017	
	.10	3.520	3.506	3.494	3.482	3.472	3.462	3.453	3.445	3.437	
	.25	2.697	2.692	2.688	2.684	2.681	2.677	2.674	2.671	2.669	
	.50	2.017	2.019	2.021	2.023	2.025	2.027	2.029	2.030	2.032	
21	.05	4.084	4.063	4.044	4.027	4.011	3.997	3.983	3.971	3.959	
	.10	3.482	3.468	3.456	3.445	3.434	3.424	3.415	3.407	3.399	
	.25	2.681	2.676	2.672	2.668	2.664	2.661	2.658	2.655	2.652	
	.50	2.013	2.016	2.018	2.020	2.022	2.024	2.025	2.027	2.028	
22	.05	4.032	4.011	3.992	3.975	3.959	3.944	3.931	3.918	3.906	
	.10	3.448	3.434	3.422	3.410	3.400	3.390	3.380	3.372	3.364	
	.25	2.666	2.661	2.657	2.653	2.649	2.646	2.643	2.640	2.637	
	.50	2.010	2.013	2.015	2.017	2.019	2.021	2.023	2.024	2.025	
23	.05	3.985	3.964	3.945	3.928	3.912	3.897	3.883	3.870	3.858	
	.10	3.417	3.403	3.391	3.379	3.368	3.358	3.349	3.340	3.332	
	.25	2.653	2.648	2.643	2.639	2.636	2.632	2.629	2.626	2.623	
	.50	2.008	2.010	2.013	2.015	2.017	2.018	2.020	2.021	2.023	
24	.05	3.942	3.921	3.902	3.885	3.868	3.853	3.840	3.827	3.815	
	.10	3.389	3.375	3.362	3.350	3.339	3.329	3.320	3.311	3.303	
	.25	2.641	2.636	2.631	2.627	2.623	2.619	2.616	2.613	2.610	
	.50	2.005	2.008	2.010	2.012	2.014	2.016	2.017	2.019	2.020	
25	.05	3.903	3.882	3.863	3.845	3.829	3.814	3.800	3.787	3.774	
	.10	3.363	3.349	3.336	3.324	3.313	3.303	3.293	3.285	3.276	
	.25	2.629	2.624	2.620	2.615	2.611	2.608	2.604	2.601	2.598	
	.50	2.003	2.006	2.008	2.010	2.012	2.014	2.015	2.017	2.018	
26	.05	3.867	3.846	3.827	3.809	3.792	3.777	3.763	3.750	3.738	
	.10	3.339	3.325	3.312	3.300	3.289	3.278	3.269	3.260	3.252	
	.25	2.619	2.614	2.609	2.605	2.601	2.597	2.594	2.590	2.587	
	.50	2.001	2.004	2.006	2.008	2.010	2.012	2.013	2.015	2.016	
27	.05	3.834	3.813	3.793	3.775	3.759	3.743	3.729	3.716	3.704	
	.10	3.317	3.303	3.290	3.277	3.266	3.256	3.246	3.237	3.229	
	.25	2.609	2.604	2.599	2.595	2.591	2.587	2.584	2.580	2.577	
	.50	1.999	2.002	2.004	2.006	2.008	2.010	2.011	2.013	2.014	
28	.05	3.803	3.782	3.762	3.744	3.727	3.712	3.698	3.684	3.672	
	.10	3.297	3.282	3.269	3.257	3.245	3.235	3.225	3.216	3.208	
	.25	2.600	2.595	2.590	2.586	2.582	2.578	2.574	2.571	2.568	
	.50	1.998	2.000	2.002	2.004	2.006	2.008	2.010	2.011	2.012	
29	.05	3.775	3.753	3.733	3.715	3.698	3.683	3.669	3.655	3.643	
	.10	3.278	3.263	3.250	3.237	3.226	3.215	3.205	3.196	3.188	
	.25	2.592	2.587	2.582	2.577	2.573	2.569	2.566	2.562	2.559	
	.50	1.996	1.998	2.001	2.003	2.005	2.006	2.008	2.009	2.011	
30	.05	3.748	3.726	3.707	3.688	3.671	3.656	3.641	3.628	3.615	
	.10	3.260	3.245	3.232	3.219	3.208	3.197	3.187	3.178	3.169	
	.25	2.584	2.579	2.574	2.569	2.565	2.561	2.557	2.554	2.551	
	.50	1.995	1.997	1.999	2.001	2.003	2.005	2.006	2.008	2.009	
40	.05	3.558	3.535	3.514	3.495	3.478	3.461	3.446	3.432	3.419	
	.10	3.132	3.116	3.102	3.089	3.076	3.065	3.055	3.045	3.036	
	.25	2.527	2.521	2.516	2.511	2.506	2.502	2.498	2.494	2.490	
	.50	1.984	1.986	1.988	1.990	1.992	1.994	1.995	1.997	1.998	
60	.05	3.370	3.346	3.324	3.304	3.286	3.268	3.252	3.237	3.223	
	.10	3.004	2.982	2.972	2.958	2.945	2.933	2.921	2.911	2.901	
	.25	2.469	2.463	2.457	2.451	2.446	2.441	2.437	2.433	2.429	
	.50	1.973	1.975	1.978	1.980	1.982	1.983	1.985	1.986	1.988	
120	.05	3.184	3.159	3.136	3.114	3.094	3.076	3.059	3.042	3.027	
	.10	2.875	2.857	2.841	2.826	2.811	2.798	2.786	2.775	2.764	
	.25	2.411	2.403	2.397	2.391	2.385	2.379	2.374	2.369	2.365	
	.50	1.963	1.965	1.968	1.969	1.971	1.973	1.974	1.976	1.977	
200	.05	3.109	3.084	3.060	3.038	3.017	2.998	2.981	2.964	2.948	
	.10	2.823	2.805	2.788	2.772	2.757	2.744	2.731	2.719	2.708	
	.25	2.387	2.379	2.372	2.366	2.360	2.354	2.348	2.343	2.339	
	.50	1.959	1.961	1.963	1.965	1.967	1.969	1.970	1.972	1.973	
400	.05	3.054	3.027	3.003	2.980	2.959	2.940	2.922	2.904	2.888	
	.10	2.784	2.765	2.748	2.732	2.716	2.702	2.689	2.677	2.665	
	.25	2.368	2.361	2.353	2.347	2.340	2.334	2.329	2.324	2.319	
	.50	1.956	1.958	1.961	1.962	1.964	1.966	1.967	1.969	1.970	
1000	.05	3.020	2.993	2.969	2.946	2.925	2.905	2.886	2.868	2.852	
	.10	2.760	2.741	2.724	2.707	2.692	2.677	2.664	2.651	2.639	
	.25	2.357	2.350	2.342	2.335	2.329	2.323	2.317	2.311	2.306	
	.50	1.954	1.957	1.959	1.961	1.962	1.964	1.966	1.967	1.968	



TABLE I (continued)

Numerator degrees of freedom						Denominator degrees of freedom	Probability of a larger F	
28	29	30	40	60	120			200
4.072	4.062	4.052	3.983	3.910	3.835	3.804	19	.05
3.473	3.466	3.460	3.414	3.365	3.315	3.294		.10
2.685	2.682	2.680	2.664	2.647	2.629	2.621		.25
2.036	2.038	2.039	2.047	2.055	2.064	2.067		.50
4.007	3.996	3.987	3.916	3.843	3.767	3.735	20	.05
3.430	3.423	3.417	3.370	3.321	3.269	3.248		.10
2.666	2.664	2.661	2.645	2.627	2.609	2.601		.25
2.033	2.034	2.035	2.043	2.052	2.060	2.063		.50
3.948	3.938	3.928	3.857	3.783	3.705	3.673	21	.05
3.392	3.385	3.378	3.330	3.280	3.228	3.206		.10
2.649	2.647	2.645	2.628	2.610	2.590	2.582		.25
2.030	2.031	2.032	2.040	2.048	2.057	2.060		.50
3.895	3.885	3.875	3.803	3.728	3.649	3.617	22	.05
3.357	3.350	3.343	3.295	3.244	3.190	3.168		.10
2.634	2.632	2.629	2.612	2.593	2.573	2.565		.25
2.027	2.028	2.029	2.037	2.045	2.054	2.057		.50
3.847	3.837	3.827	3.754	3.678	3.598	3.565	23	.05
3.325	3.318	3.311	3.262	3.210	3.156	3.133		.10
2.620	2.618	2.615	2.597	2.578	2.558	2.550		.25
2.024	2.025	2.026	2.034	2.043	2.051	2.054		.50
3.803	3.793	3.783	3.709	3.633	3.552	3.518	24	.05
3.295	3.288	3.282	3.232	3.179	3.124	3.101		.10
2.607	2.605	2.602	2.584	2.565	2.544	2.535		.25
2.022	2.023	2.024	2.032	2.040	2.048	2.052		.50
3.763	3.752	3.743	3.668	3.591	3.509	3.475	25	.05
3.269	3.261	3.255	3.204	3.151	3.095	3.072		.10
2.595	2.593	2.590	2.572	2.552	2.531	2.522		.25
2.019	2.021	2.022	2.030	2.038	2.046	2.049		.50
3.726	3.715	3.705	3.631	3.552	3.469	3.435	26	.05
3.244	3.237	3.230	3.179	3.125	3.068	3.044		.10
2.584	2.582	2.579	2.560	2.540	2.518	2.509		.25
2.017	2.018	2.020	2.028	2.036	2.044	2.047		.50
3.692	3.681	3.671	3.596	3.516	3.433	3.398	27	.05
3.221	3.214	3.207	3.155	3.101	3.043	3.019		.10
2.574	2.572	2.569	2.550	2.529	2.507	2.498		.25
2.015	2.017	2.018	2.026	2.034	2.042	2.045		.50
3.660	3.649	3.639	3.563	3.483	3.399	3.363	28	.05
3.200	3.192	3.185	3.133	3.078	3.020	2.995		.10
2.565	2.562	2.559	2.540	2.519	2.496	2.487		.25
2.014	2.015	2.016	2.024	2.032	2.040	2.043		.50
3.631	3.620	3.610	3.533	3.452	3.367	3.331	29	.05
3.180	3.172	3.165	3.113	3.057	2.998	2.973		.10
2.556	2.553	2.551	2.531	2.509	2.486	2.477		.25
2.012	2.013	2.014	2.022	2.030	2.038	2.042		.50
3.604	3.593	3.582	3.505	3.424	3.337	3.301	30	.05
3.161	3.154	3.147	3.094	3.037	2.977	2.952		.10
2.548	2.545	2.542	2.522	2.500	2.477	2.467		.25
2.010	2.012	2.013	2.021	2.029	2.037	2.040		.50
3.406	3.395	3.384	3.302	3.215	3.121	3.081	40	.05
3.027	3.019	3.011	2.954	2.893	2.827	2.799		.10
2.487	2.484	2.481	2.459	2.434	2.407	2.395		.25
1.999	2.001	2.002	2.010	2.017	2.025	2.029		.50
3.210	3.198	3.186	3.098	3.003	2.898	2.852	60	.05
2.892	2.883	2.875	2.812	2.744	2.669	2.636		.10
2.425	2.422	2.418	2.393	2.364	2.332	2.318		.25
1.989	1.990	1.991	1.999	2.006	2.014	2.017		.50
3.013	3.000	2.987	2.890	2.783	2.660	2.603	120	.05
2.753	2.744	2.735	2.665	2.587	2.497	2.456		.10
2.361	2.357	2.353	2.323	2.289	2.249	2.230		.25
1.978	1.979	1.980	1.988	1.996	2.003	2.006		.50
2.933	2.919	2.906	2.805	2.691	2.556	2.492	200	.05
2.697	2.687	2.677	2.604	2.521	2.421	2.374		.10
2.334	2.330	2.326	2.291	2.257	2.211	2.189		.25
1.974	1.975	1.976	1.984	1.991	1.999	2.002		.50
2.873	2.859	2.845	2.740	2.619	2.472	2.399	400	.05
2.654	2.644	2.634	2.557	2.469	2.359	2.305		.10
2.314	2.309	2.305	2.271	2.231	2.181	2.155		.25
1.971	1.972	1.973	1.981	1.988	1.996	1.999		.50
2.836	2.822	2.808	2.700	2.575	2.418	2.338	1000	.05
2.628	2.617	2.607	2.528	2.436	2.319	2.259		.10
2.301	2.297	2.292	2.257	2.216	2.161	2.132		.25
1.969	1.970	1.972	1.979	1.986	1.994	1.997		.50

The approximation was tested with the 5184 points given in reference [3]. In Table I of the present paper 4896 points are given and 2117 additional points were generated with various values of  $F$  ranging from 1 to 10,  $m$  and  $q$  ranging from 3 to 30, and  $\lambda$  ranging from 0 to 10.<sup>3</sup>

Checking the approximation against values obtained by numerical integration indicated good accuracy. Over all values of  $F$ ,  $m$ ,  $q$ , and  $\lambda$ , only about two per cent of the errors were greater than .015, and those errors occurred mostly for very small  $F$ 's and  $q$ 's. For  $F > 2$  and  $q > 1$ , there were no errors in excess of .015, and only about one per cent of the errors were greater than .01. A Fortran function for the approximation is given in the Appendix.

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#### APPENDIX

##### THE APPROXIMATION TO THE NONCENTRAL $F$

We make use of the standard error function supplied in most Fortran libraries, defined to be

$$\text{ERF}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-u^2} du$$

and the relationship

$$\int_{-\infty}^X \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = \frac{1 + \text{ERF}(x/\sqrt{2})}{2}.$$

The following Fortran function written in double precision for the IBM 360 in Fortran IV can be used to approximate noncentral  $F$  integrals of the form:

$$\int_{F'}^{\infty} g(F, m, q, \lambda) dF.$$

```

REAL FUNCTION PGNCF*8 (F, NUMDF, DENDF, LAMBDA)
IMPLICIT REAL*8 (A-Z)
c  INTEGER NUMDF, DENDF
c  EVALUATES PROBABILITY OF A GREATER NON-CENTRAL F
c  IF(F.GT.φ.Dφ. AND.NUMDF.GT.φ.AND.DENDF.GT.φ.AND.LAMBDA.GE.φ.Dφ)GO
TO 5
PGNCF = 1.Dφ
RETURN
5  N2L = NUMDF + 2.Dφ*LAMBDA
N4L = N2L + 2.Dφ*LAMBDA
T9 = 2.Dφ/9.Dφ
X1 = (NUMDF*F/N2L)**(1.Dφ/3.Dφ)
X2 = X1*(1.Dφ - T9/DENDF) - 1.Dφ + T9*N4L/(N2L*N2L)
X2 = X2/DSQRT(2.Dφ*(T9*N4L/(N2L*N2L) + T9/DENDF*X1*X1))
PGNCF = .5Dφ *DERFC (X2)
RETURN
END

```

DERFC is the double precision complemented error function defined to be  $1 - \text{ERF}(X)$ .

<sup>3</sup> All symbols are as previously defined with  $q = t - K$ . The approximation given in equation (1) differs slightly from that given in reference [1] in that  $g(u, m, q, \lambda)$  uses  $\lambda$  rather than  $\lambda/2$  in its definition.

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