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Structurally Rigid Elastic Composites for Acoustic Imaging Countermeasures

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Abstract: We explore the possibilities coming from transformation acoustics and beyond for creating rigid elastic composite shells capable of suppressing the total scattering cross-section of acoustically large objects. The reported design methodology is based on generalized shape and topology optimization, and the outcomes are suitable for rapid prototyping techniques.

Acoustic cloaking has been a subject of interest to the phononics community since the advent of transformation acoustics (TA). TA methodology is based on the form-invariance of the scalar Helmholtz equation describing acoustic pressure waves in fluids. Much of the past effort in TA was focused on the so-called *acoustic metafluids* [1],[2],[3], which are materials with vanishingly small shear modulus. The acoustics of metafluids is mathematically almost as simple as acoustics of regular fluids, with the only complication that the density is anisotropic and described by a rank-2 tensor. This explains why virtually all recent effort (with a few exceptions) in transformation acoustics was devoted to metafluids[1],[2],[3].

Unfortunately, the structural properties of metafluids are not particularly suitable for deployable acoustic devices. Fluids (and metafluids) are unable to support any amount of shear stress: an infinitesimal amount of shear stress leads to a finite deformation. This presents a fundamental difficulty in integrating them with any on-board devices mounted on air vehicles, or even on stationary platforms that are subject to wind or streaming water. This lack of structural robustness was overcome in the proof-of-concept demonstrations involving acoustic metafluids by restricting device geometries to two-dimensional (in-plane) propagation. These geometries allowed the use of metafluids formed by arrays of rigid inclusions (rods, bars) which are mechanically disconnected from each other and embedded in a homogeneous fluid (air, water, etc.) By construction, such a metamaterial geometry does not support in-plane shear wave propagation and therefore acts as a two-dimensional metafluid.

The difficulty with generalizing this strategy into three dimensions is so fundamental that no demonstration of a non-trivially three-dimensional transformation acoustical device has been offered to date, even as merely a proof-of-concept experiment. The need for structural integrity and rigidity calls for the development of elastic metamaterials with a suitably large shear modulus. Such media would be capable of tolerating finite stress in arbitrary directions, whose amount is limited only by their yield or fracture properties.

Pentamode elastic media. Unlike the scalar Helmholtz equation, the elastic wave equation, at least in its conventional form, is generally not form-invariant[4],[5], which makes coordinate transformation techniques like TA difficult to apply. However, if one restricts the class of elastic materials to a very small subset, known as pentamode media, the elastic wave equation reduces to a single, scalar Helmholtz wave equation. A pentamode medium is defined as an elastic medium whose elasticity tensor **C**, expressed as a 6-dimensional matrix, can be factorized as follows:

$$C_{ij} = KQ_iQ_j, \tag{1}$$

where K is scalar elastic coefficient, and Q is a constant 6-dimensional vector. Substitution of this ansatz into the elastic wave equation,

$$\rho \ddot{u}_i = \nabla \cdot \sigma, \tag{2}$$

where $\sigma = C\,\varepsilon$ is the stress tensor and $\varepsilon = \nabla^T u$ is the strain tensor, u being the displacement vector, gives a scalar equation

$$\rho K^{-1} \ddot{p} = \nabla^2 p \tag{3}$$

for the effective scalar pressure $p = -KQ^T\varepsilon$. In the frequency domain, $\ddot{p} = -\omega^2 p$, and the usual scalar Helmholtz equation is retrieved. Allowing the scalar modulus K as well as the density tensor ρ to be functions of coordinates (but not the pentamode vector Q), one obtains an equation with inhomogeneous coefficients that is form-invariant with respect to arbitrary three-dimensional coordinate transformations. The recipes developed for acoustic metafluid-based TA devices should therefore be implementable with pentamode elastic media as well.

Non-pentamode approaches to shear wave management. From the less mathematical angle, the physical reason for the difficulties with general elastic media is co-existence of vastly different types of waves: longitudinal pressure waves, or *p*-waves, and transverse shear waves (*s*-waves), which co-propagate and generally couple with each other. This issue associated with acoustics of elastic media can be referred to as the multimode propagation problem. The coupling occurs at the sharp interfaces between different media, including the fluid/elastic medium boundary. In gradient-index structures, the coupling may also occur due to both gradients of effective density and its anisotropy, as was shown in Ref.[6]. Density gradients and anisotropy arise naturally in most Transformation Acoustics scenarios. Although gradients and anisotropy can both be mitigated by optimizing the choice of coordinate transformations, complete elimination of anisotropy and/or density gradients is usually impossible without degrading the performance of resulting devices, or losing versatility in the choices of device shapes.

The strategies described above – metafluids and pentamode media – can be all seen as approaches to manage multimode coupling on the microscopic level, by means of adjusting the material properties and/or the metamaterial unit cell architectures. Though such strategies, if proven successful, would be widely applicable to virtually all conceivable TA devices, it is worthwhile to consider more narrowly focused strategies based on macroscopic management of the multimode issues. Recently, Urzhumov et al. demonstrated in Ref.[9] that, under certain resonant conditions, one of the co-propagating modes (shear waves) can be effectively eliminated from the picture at discrete wavelengths, a concept that opens the door to TA devices composed of non-pentamode elastic media with substantial shear modulus.

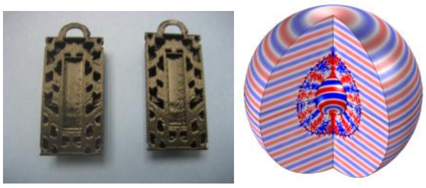

Figure 1. Left: Structurally rigid directional acoustic cloak fabricated from a hard plastic by Fused Deposition Modeling. Right: Full-wave acousto-elastodynamic simulation of a spherical-shape cloak in water. Color shows pressure amplitude.

In this work, we demonstrate macroscopic management of shear wave-related scattering in a directional acoustic cloak composed of an ABS plastic or another elastic material with a Poisson ratio about 0.4 (Figure 1). It is shown that, by means of shape optimization of the air voids, scattering cancellation can be achieved for virtually any desirable Poisson ratio and in a wide range of wavelength-to-diameter ratios. Composite structures with strongly reduced scattering cross-section can be designed in a variety of shapes; cylindrical and spherical ones have the advantage of reduced design time due to the availability of efficient full-wave axisymmetric Helmholtz solvers.[7]

References

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